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VECTOR-LIKE QUARKS IN A "COMPOSITE" HIGGS MODEL

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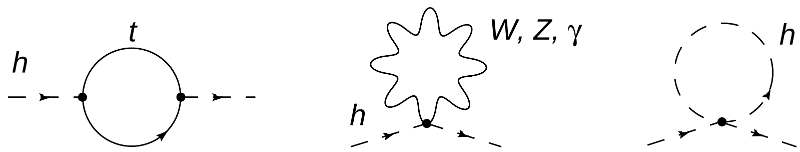
Contents

Chapter 1

Introduction and motivations

The great success of the Standard Model (SM) in predicting the electroweak observables leaves many theoretical open questions. One of them is about the Higgs boson mass. More precisely, the fit of the SM prediction to electroweak precision data strongly suggests that electroweak symmetry is (spontaneously) broken by one or more scalar $SU(2)_{weak}$ doublets. This leads to the prediction of the existence of a Higgs boson whose mass is maybe the most important unknow parameter left in the SM.

The problem is that scalar particle masses suffer of radiative instability. For example diagrams:



give the following quadratically divergent contributions to m_h^2:

Top loop	$-\frac{3}{4\pi^2}\lambda_t^2\Lambda^2$
Gauge boson loops	$\frac{9}{32\pi^2}g^2\Lambda^2$
Higgs boson loop	$\frac{1}{8\pi^2}\lambda^2\Lambda^2$

Ununderstood cancellations between parameters are not liked, so to keep the Higgs boson mass near the weak-scale expectation value v with no more than 10% finetuning these loops (expecially the top one) must be cut-off at a scale $\Lambda_{nat} \lesssim 1-2$ TeV. This is the famous "naturalness problem" of the Fermi scale: one looks for a non-accidental reason that explains why the Higgs boson is so light relatively to any other short distance scale in Physics.

This argument tells us that the SM is not natural at the energy of the Large Hadron Collider (LHC), and more specifically new physics that cuts-off the divergent loops has to be expected at or below 2 TeV. In a weakly coupled theory this means new particles with masses below 2 TeV and related to the SM particles by some symmetry. For concreteness, the dominant contribution comes from the top loop. Thus naturalness arguments predict new multiplet(s) of top-symmetry-related particles that should be easily produced at the LHC, which has a maximum available energy of 14 TeV.

History suggests exactly the same thing. As noted for example in [1] it would not be the first time in Physics that unespected and precise cancellations among parameters are actually signals of the existence of new particles. In classical electrodynamics the electron self energy has a power divergence which leads to a naturalness problem cured only by the introduction of the positron, the electromagnetic contribution to the $\pi^+ - \pi^0$ mass difference needs to be cut-off at the scale of the ρ meson, and so on.

Many possibilities can be considered, for example in Supersymmetry these cutoffs come from supersymmetrical partners. Here a model is presented in which the Higgs particle is realized as a pseudo-Goldstone boson (in this case associated to the breaking $SO(5) \to SO(4)$ at a scale $f > v$) in order to protect his mass from self-coupling corrections and raise the cutoff scale at least up to 2.5 TeV. The $SO(5)$ symmetry has then to be extended to the top sector by adding new vector-like quarks in order to reduce the UV sensitivity of m_h to the top loop. In doing this it is necessary to fulfill the requirements of the Electro Weak Precision Tests (EWPT), as it will be discussed. In principle new heavy vectors should also be included in order to cut-off the gauge boson loops, however here only the quark sector will be studied for two reasons: first of all from a logical point of view because the dominant contribution comes from the top; second, from a phenomenological point of view since heavy quark searches at the LHC may be easier than heavy vector searches.

The experimental constraints on this model come from the EWPT fit of the Peskin-Tekeuchi parameters and from the modified bottom coupling to the Z boson. It will be shown that in principle compatibility with experimental data can be fulfilled in a small slice of parameter space of a suitably extended model. The minimal allowed values for their masses will be reported.

The content of this thesis is as follows:

- In chapter 2 the model is introduced and described in the Higgs and gauge sector. The consequences on EWPT are discussed.
- In chapter 3 the top sector is enlarged in a minimal way. The difficulty in satisfying experimental requirements is then discussed.
- Chapter 4 contains the explicit description of a non-minimal model which can be compatible with data.
- In chapter 5 I briefly describe two other models which are ruled out by electroweak precision data.

The results of chapters 2 and 3 can be found in [2], while chapters 4 and 5 contain my original contribution.

Chapter 2

The $SO(5)/SO(4)$ model

In the following I consider a model based on the $SO(5)$ symmetry as a minimal extension of the $SO(4)$ symmetry of the Higgs boson potential.

As discussed in the Introduction, the idea is to realize the Higgs boson as a pseudo Nambu-Goldstone boson in order to protect his mass from radiative corrections due to the Higgs boson itself. In the next chapter the top sector will also be enlarged to protect the Higgs boson mass from the top loop contribution. Additional heavy vectors should be included for the same reason, however I will concentrate on quarks because the main correction to the Higgs boson mass comes from the top loop, and moreover new heavy quarks may be more easily detectable than new vectors (see [3]).

2.1 Higgs and Gauge sector

2.1.1 Custodial symmetry in the SM

I recall that in the Standard Model the Higgs sector before Electro Weak Symmetry Breaking (EWSB) can be represented by a scalar fuor-plet or by a complex doublet:

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} \quad \leftrightarrow \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_4 \\ \phi_1 + i\phi_3 \end{pmatrix}, \quad H^c = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 - i\phi_3 \\ -\phi_2 + i\phi_4 \end{pmatrix} \tag{2.1}$$

which can be usefully rewritten in the matrix form:

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 - i\phi_3 & \phi_2 + i\phi_4 \\ -\phi_2 + i\phi_4 & \phi_1 + i\phi_3 \end{pmatrix} = (H^c, H)$$

With such parametrization and the definitions:

$$A_1 : \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad A_2 : \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \quad A_3 : \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$B_1 : \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad B_2 : \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad B_3 : \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

the isomorphism between the $SO(4)$ and $SU(2)_L \times SU(2)_R$ (custodial) algebras can be made explicit in the following form:

$$e^{\alpha_j A_j + \beta_k B_k} \phi \quad \leftrightarrow \quad e^{i\alpha_j \sigma_j^L} \hat{H} (e^{i\beta_k \sigma_k^R})^+ = e^{i\alpha_j \sigma_j^L} \hat{H} e^{-i\beta_k \sigma_k^R}. \tag{2.2}$$

The SM Lagrangian (without Yukawa terms and with $g' = 0$) can be written in terms of $\hat{H}$, explicitly:

$$\phi^+ \phi = \frac{1}{2} tr(\hat{H}^+ \hat{H}) \qquad (D_\mu \phi)^+ D^\mu \phi = \frac{1}{2} tr((D_\mu \hat{H})^+ D^\mu \hat{H}).$$

After EWSB only the ϕ_1 component acquires a vacuum expectation value $\langle \phi_1 \rangle = v\sqrt{2}$, so that $\hat{H}$ becomes proportional to the identity and the residual (approximate) symmetry is the diagonal $SU(2)_V$ with $\alpha_i = \beta_i$.

This symmetry is called "custodial" because it is responsible of the fact:

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} \approx 1$$

and it would be $\rho = 1$ if the symmetry were exact (in fact the hypercharge gauge coupling and the Yukawa terms break this symmetry).

2.1.2 $SO(5)$ model

Following [2], the dynamics in the Higgs sector is now described by the sigma-model with an $SO(5)$ global symmetry broken spontaneously to $SO(4)$. After the symmetry breaking the scalar five-plet will be subject to the constraint[1]:

$$\phi^2 \approxeq f^2$$

[1] This constraint is only approximate in the perturbative model, for the strong coupling case ($\lambda \to \infty$ in 2.3) see [2].

where f is the scale of the $SO(5) \to SO(4)$ breaking, which is assumed to be higher than the EWSB scale $v = 175$ GeV.

The SM group $G_{SM} = SU(2)_L \times U(1)$ is generated by the $SU(2)_L$ and the T_3 of the $SU(2)_R$ of a fixed subgroup $SO(4) = SU(2)_L \times SU(2)_R \subset SO(5)$ acting on $\overrightarrow{\phi} \stackrel{\text{def}}{=}$ (first four components of ϕ), so that the covariant derivative is given by:

$$D_\mu \phi = \partial_\mu \phi - i(W_\mu^a T_L^a + B_\mu T_R^3)\phi.$$

The important point is that there be a misalignement between the $SO(4)$ acting on the first four components of ϕ and the $SO(4)$ of the $SO(5) \to SO(4)$ breaking. If there is perfect alignement, that is $\langle\phi\rangle = (0,0,0,0,f)$, there is no EWSB and the W and Z bosons are massless. In the general case the standard relation holds:

$$m_W^2 = \frac{g^2 v^2}{2} \quad , \quad v^2 = \frac{1}{2}\left\langle \overrightarrow{\phi}^2 \right\rangle.$$

The collective symmetry breaking is described by a potential given by an $SO(5)$ symmetric term (which for big λ means $\phi^2 \approx f^2$) and the most general soft-breaking terms up to dimension 2 and consistent with the gauge symmetry:

$$V = \lambda(\phi^2 - f^2)^2 - A f^2 \overrightarrow{\phi}^2 + B f^3 \phi_5 \tag{2.3}$$

Minimization gives a VEV to $\overrightarrow{\phi}$ provided that $A > 0$, and it comes out:

$$\begin{cases} v^2 = \frac{1}{2}\left\langle \overrightarrow{\phi}^2 \right\rangle = \frac{f^2}{2}\left[1 + \frac{A}{2\lambda} - \left(\frac{B}{2A}\right)^2\right] \\ \langle \phi_5 \rangle = -\frac{B}{2A} f. \end{cases}$$

The dimensionless coefficients A, B, λ are treated as free parameters. To have $v \ll f$ requires a finetuning which can be quantified by the logarithmic derivative:

$$\Delta = \frac{A}{v^2}\frac{\partial v^2}{\partial A} \approx \frac{f^2}{v^2}(1 + \frac{3A}{4\lambda}).$$

This finetuning will be taken as acceptable for $\frac{1}{\Delta} \gtrsim 10\%$.

The cutoff scale can be obtained by requiring that the one loop correction to the squared mass of ϕ from the symmetric coupling does not exceed the tree level term (a "naturalness" requirement):

$$\Lambda_{\text{nat}} \stackrel{\text{def}}{=} \frac{4\pi f}{\sqrt{N+2}} \approxeq 4.7 f \tag{2.4}$$

where $N = 5$. For example for $f = 500$ GeV it is $\Lambda_{\text{nat}} \backsim 2.4$ Tev.

For moderate λ there are now two scalar particles below the cutoff:

$$h = \cos\alpha\,\phi_3 + \sin\alpha\,\phi_5 \quad , \quad \sigma = -\sin\alpha\,\phi_3 + \cos\alpha\,\phi_5 \tag{2.5}$$

where:

$$\cos\alpha = \left(1 - \frac{2v^2}{f^2}\right)^{1/2} \tag{2.6}$$

with masses:

$$m^2 = 4\lambda f^2\left(1 + \frac{3A}{4\lambda} \pm [1 + \frac{3A}{2\lambda}(1 - 4v^2/3f^2) + \left(\frac{3A}{4\lambda}\right)^2]^{1/2}\right).$$

In the strongly coupled case ($\lambda \to \infty$) the lighter h has a mass:

$$m_h = 2v\sqrt{A}, \tag{2.7}$$

whereas the heavier σ goes above the cutoff. Notice that the Higgs mass is controlled by the A parameter, that is by the "asymmetric" mass term in 2.3. This reflects the pseudo-Goldstone nature of the Higgs boson and will be relevant in section 3.2.

In the region of interest ($f \approx 500$ GeV, m_h between 100 GeV and 300 GeV) the σ particle can have a mass from 1 TeV to over 2.5 TeV, depending on the value of λ.

2.2 Reduced coupling and EWPT

As explained at the beginning of the previous section, the gauge group acts on $\overrightarrow{\phi} \stackrel{\text{def}}{=}$ (first four components of ϕ), so the mass eigenstates 2.5 are not "eigenstates" of gauge interaction. More precisely, the Higgs particle h has its couplings to the gauge bosons (as well as to any other SM particle) reduced by the factor $\cos\alpha$ (equation 2.6), and this fact has important consequences on the Electro Weak Precision Tests (EWPT).

2.2.1 Peskin-Takeuchi parameters

I recall the definition of Peskin-Takeuchi parameters, which can be given in terms of the vacuum polarization amplitudes of the gauge bosons:

$$\hat{S} = \frac{g}{g'}\Pi'_{30}|_0 = \cos^2\theta\left(\Pi'_{ZZ} - \Pi'_{\gamma\gamma} + \frac{\cos^2\theta - \sin^2\theta}{\cos\theta\sin\theta}\Pi'_{Z\gamma}\right)|_0 \tag{2.8}$$

$$\hat{T} = \frac{\Pi_{33} - \Pi_{11}}{m_W^2}|_0 = \frac{\cos^2\theta\Pi_{ZZ} - \Pi_{WW}}{m_W^2}|_0 \tag{2.9}$$

where the amplitudes are:

$$\Pi_{VV}^{\mu\nu} = \Pi_{VV}(q^2)g^{\mu\nu} + F(q^2)q^\mu q^\nu.$$

The original definition given by Peskin and Takeuchi will also be used:

$$S = \tfrac{16\pi}{g^2}\hat{S} \qquad\qquad T = \tfrac{1}{\alpha_{EM}}\hat{T}$$

while the U parameter is not useful here. The T parameter measures custodial symmetry violation, since it is mainly sensible to the top-bottom mass splitting which breaks the custodial symmetry. The S parameter can be nonzero also without custodial-symmetry violation.

More precisely let us consider the one loop correction to the ρ parameter. To sufficient accuracy, gauge boson masses are given by:

$$\begin{cases} m_Z^2 = (g^2+g'^2)\frac{v^2}{2} - \Pi_{ZZ}(0) \\ m_W^2 = g^2\frac{v^2}{2} - \Pi_{WW}(0) \end{cases}$$

so that:

$$\rho \overset{\text{def}}{=} \frac{m_W^2}{m_Z^2\cos^2\theta} \approx \frac{1-\Pi_{WW}/m_W^2}{1-\Pi_{ZZ}/m_Z^2}|_0 \approx 1 + \frac{\Pi_{ZZ}\cos^2\theta - \Pi_{WW}}{m_W^2}|_0 \overset{\text{def}}{=} 1+\hat{T} \quad .$$

The main SM effect, due to the top-bottom splitting, is:

$$\hat{T}_{SM} = \frac{3g^2m_t^2}{64\pi^2 m_W^2} = \frac{3G_F m_t^2}{8\sqrt{2}\pi^2} \approx 0.01 \tag{2.10}$$

as we shall see.

2.2.2 Effects of the reduced coupling

In the heavy Higgs approximation ($m_h \gg m_W, m_Z$) the electroweak parameters at one loop are given by:

$$S, T = a_{S,T}\log m_h + b_{S,T}$$

where $a_{S,T}, b_{S,T}$ are convenient constants.

In this $SO(5)$ model, in the same approximation, due to the reduced coupling 2.6 and to the presence of two interacting bosons 2.5 we have[2]:

$$S, T = a_{S,T}[(\cos\alpha)^2\log m_h + (\sin\alpha)^2\log m_\sigma] + b_{S,T}$$

[2] In the strongly coupled case the reduced coupling is seen as a wavefunction renormalization effect, and m_σ is replaced by the cutoff, see [2].

which amounts to replace m_h in the SM with an (increased) effective mass:

$$m_{\text{eff}} = m_h \left(\frac{m_\sigma}{m_h}\right)^{\sin^2\alpha} . \qquad (2.11)$$

Note that this contribution is important for low f, which is the more interesting (less finetuned) case.

Contribution from physics at the cutoff should also be expected, which can be estimated by means of proper higher dimensional operators. The $\hat{T}$ parameter is protected by the custodial $SO(4)$ contained in the $SO(5)$, but there will be in general a contribution to $\hat{S}$ which can be estimated as[3]:

$$\delta\hat{S}|_\Lambda \approx +\frac{g^2 v^2}{\Lambda^2} \approxeq 1.4\times 10^{-3}\left(\frac{3\text{TeV}}{\Lambda}\right)^2 . \qquad (2.12)$$

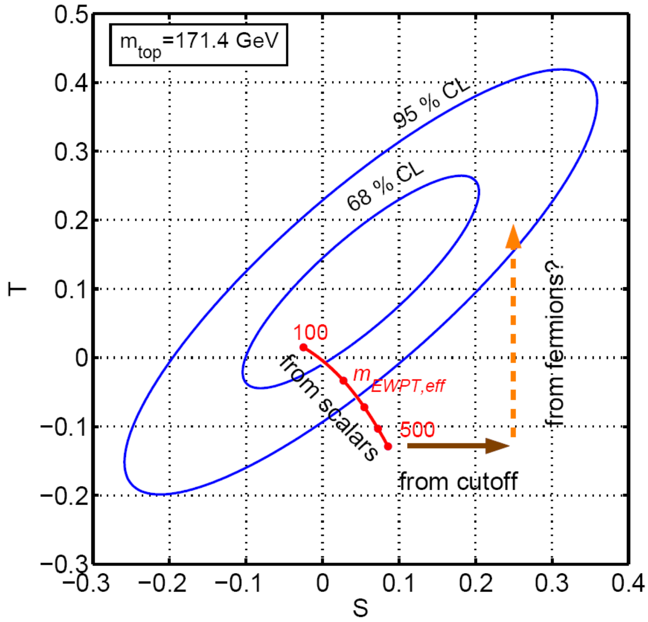


Figure 2.1: The experimentally allowed region in the ST plane, including contributions 2.11 "from scalars" and 2.12 "from cutoff". In the next chapter another contribution "from fermions" will be considered. This figure is taken from [2].

These two contributions ("from scalars" and "from cutoff" in Figure 2.1) make the model as it stands so far inconsistent with the EWPT. One obvious

[3]See [2], sections 2 and 6.

way out is to increase the f scale: for example in principle $f = 1$ TeV would be at the border of the 2σ ellipse, but the finetuning price would be $\approx 3\%$ which is unconfortably large.

A better strategy is to take advantage of the fermion sector enlargement. As outlined in the Introduction the quark sector needs to be extended in a $SO(5)$-symmetrical way, since the Higgs boson mass is quadratically sensitive to the top loop. So to avoid finetuning we set $f = 500$ GeV and try to enlarge the fermion sector in such a way that an extra positive contribution is provided to the T parameter.

Another important consequence of this reduced coupling of the Higgs particle to gauge bosons is that the divergence of longitudinal WW scattering amplitude is no longer cancelled. Instead it grows now as:

$$\mathcal{A}(W_L W_L \to W_L W_L) = -\frac{G}{\sqrt{2}}(1+\cos\theta)(\sin^2\alpha)s$$

where s is the square center of mass energy and θ is the scattering angle. This growth can be used to give an alternative estimate of the cutoff of the model. Indeed the unitarity bound is now saturated at:

$$s_c = \frac{s_c^{SM}}{\sin^2\alpha}$$

where $s_c^{SM} = (1.2\text{ TeV})^2$ is the analogous bound in the Higgsless SM. For example for $f = 500$ GeV we would have $\sqrt{s_c} \approx 2.4$ TeV, which is nearly the same as 2.4.

As noted for example in [1] this fact can be used in general as a genuine test of the Higgs particle compositeness, so analyzing longitudinal gauge-boson scattering amplitude is a very important process which can provide useful informations on the nature of the Higgs boson.

Chapter 3

Top mass and $SO(5)$: minimal model

As outlined in the previous chapters, there are two importat motivations for extending the $SO(5)$ symmetry to the top sector.

First of all the very logic of the extended symmetry is based on the enlargement of the (symmetry of the) scalar, quark, and in principle also the gauge boson sectors in order to keep the Higgs boson light by reducing the one loop contributions to m_h.

A second motivation is to see if one can provide the additional positive contribution to T needed to be consistent with the EWPT constraints, as shown in Figure 2.1.

3.1 Minimally extended top sector

3.1.1 Fermionic content

The fermion sector has to be enlarged in such a way that the top quark is ($SO(5)$ symmetrically) given the right mass $m_t = 171$ GeV, and new haevy quarks are vector like in the $v/f \to 0$ limit. With this I mean that ψ_L and ψ_R transform in the same way under $SU(2)_L \times U(1)_Y$ so that the mass term: $m\overline{\psi}_L\psi_R$ is consistent with the gauge symmetry. The bottom quark can be considered massless at this level of approximation while lighter quarks will be completely neglected.

Following [2], the minimal way to do this is to enlarge the left-handed top-bottom doublet q_L to a vector (one for each colour) Ψ_L of $SO(5)$, which under $SU(2)_L \times SU(2)_R$ breaks up as $(2, 2) + 1$. The full fermionic content of the third quark generation is now:

$$\Psi_L = \left(q = \begin{pmatrix} t \\ b \end{pmatrix}, X = \begin{pmatrix} X^{5/3} \\ X \end{pmatrix}, T \right)_L, t_R, b_R, X_R = \begin{pmatrix} X^{5/3} \\ X \end{pmatrix}_R, T_R,$$

where the needed right handed states have been introduced in order to preserve parity of strong and electromagnetic interactions and to give mass to the new fermions. Defining:

$$\Psi_L = \left(\hat{\Psi}_L = \begin{pmatrix} t & X^{5/3} \\ b & X \end{pmatrix}_L \right) \oplus T_L \,, \tag{3.1}$$

we have that $SU(2)_L \times SU(2)_R$ acts on $\hat{\Psi}_L$ in the same way as it acted on $\hat{H}$ in 2.2.

Equivalently, we can rewrite 3.1 in a five-dimensional notation with $SU(2)_L \times SU(2)_R \approx SO(4)$ acting on the first four components in the same way as it acted on ϕ in 2.2:

$$\Psi_L = \frac{1}{\sqrt{2}} \begin{pmatrix} t + X \\ X^{5/3} - b \\ i(t - X) \\ -i(X^{5/3} + b) \\ \sqrt{2}\,T \end{pmatrix}_L . \tag{3.2}$$

According to 2.2 and 3.1 it is: $T^3_R(q_L) = \frac{1}{2}$, $T^3_R(X_L) = -\frac{1}{2}$, $T^3_R(T_L) = 0$, so in order to obtain the correct value of the electric charges we must set:

$$Y(X_{L,R}) = \frac{7}{6} \quad , \quad Y(t_L, b_L) = \frac{1}{6} \quad , \quad Y(t_R, b_R, T_{L,R}) = \frac{2}{3}, \tag{3.3}$$

Note that the upper component of the "exotic" X has electric charge 5/3.

The Yukawa Lagrangian of this sector consists of an $SO(5)$ symmetric mass term for the top (this guarantees the absence of quadratic divergences in the contribution to m_h, as shown in section 3.2) and the most general (up to redefinitions) gauge invariant mass terms for the heavy X and T:

$$\mathcal{L}_{top} = \lambda_1 \overline{\Psi}_L \phi t_R + \lambda_2 f \overline{T}_L T_R + \lambda_3 f \overline{T}_L t_R + M_X \overline{X}_L X_R + h.c. \tag{3.4}$$

Since explicitly, using 2.1 with 3.2 or 3.1:

$$\overline{\Psi}_L \phi = tr\left(\hat{\Psi}^+_L \hat{H}\right) + \overline{T}_L \phi_5 = \overline{q}_L H^c + \overline{X}_L H + \overline{T}_L \phi_5,$$

the full mass and Yukawa Lagrangian 3.4 is rewritten as:

$$\mathcal{L}_{top} = \lambda_1 \overline{q}_L H^c t_R + \lambda_1 \overline{X}_L H t_R + \lambda_2 f \overline{T}_L T_R + (\lambda_1 \phi_5 + \lambda_3 f) \overline{T}_L t_R + M_X \overline{X}_L X_R + h.c.$$

which up to a field rotation:

$$T_R^{new} = \frac{\lambda_2 T_R + (\lambda_1+\lambda_3) t_R}{\sqrt{(\lambda_1+\lambda_3)^2+\lambda_2^2}} \quad , \quad t_R^{new} = \frac{-(\lambda_1+\lambda_3) T_R + \lambda_2 t_R}{\sqrt{(\lambda_1+\lambda_3)^2+\lambda_2^2}}$$

and parameter redefinition:

$$\lambda_t = \frac{\lambda_1\lambda_2}{\sqrt{(\lambda_1+\lambda_3)^2+\lambda_2^2}} \, , \, \lambda_T = \frac{\lambda_1(\lambda_1+\lambda_3)}{\sqrt{(\lambda_1+\lambda_3)^2+\lambda_2^2}} \, , \, M_T = f\sqrt{(\lambda_1+\lambda_3)^2+\lambda_2^2}$$

can be written as:

$$\mathcal{L}_{top} = \overline{q}_L H^c(\lambda_t t_R + \lambda_T T_R) + \overline{X}_L H(\lambda_t t_R + \lambda_T T_R) + M_T \overline{T}_L T_R + M_X \overline{X}_L X_R + h.c. \tag{3.5}$$

Note that the T_R and t_R fields have exactly the same quantum numbers, so this rotation doesn't change any physical observable.

3.1.2 Mass matrix

Keeping only mass terms and adding a superscript "0" to the fields, the Lagrangian 3.5 becomes:

$$\mathcal{L}_{top} = \lambda_t v \overline{t}_L^0 t_R^0 + \lambda_t v \overline{X}_L^0 t_R^0 + \lambda_T v \overline{t}_L^0 T_R^0 + \lambda_T v \overline{X}_L^0 T_R^0 + M_T \overline{T}_L^0 T_R^0 + M_X \overline{X}_L^0 X_R^0 + h.c.$$

which can be written in the matrix form:

$$\mathcal{L} = \begin{pmatrix} \overline{t}_L^0 & \overline{T}_L^0 & \overline{X}_L^0 \end{pmatrix} \begin{pmatrix} \lambda_t v & \lambda_T v & 0 \\ 0 & M_T & 0 \\ \lambda_t v & \lambda_T v & M_X \end{pmatrix} \begin{pmatrix} t_R^0 \\ T_R^0 \\ X_R^0 \end{pmatrix} + h.c. \tag{3.6}$$

From now on I will define:

$$\epsilon_R = \frac{\lambda_t v}{M_X} \qquad \epsilon_L = \frac{\lambda_T v}{M_T}. \tag{3.7}$$

To obtain the physical states and their respective masses it is necessary to diagonalize this matrix, and up to second order the result is:

$$\begin{pmatrix} t_R^0 \\ T_R^0 \\ X_R^0 \end{pmatrix} = \begin{pmatrix} 1-\frac{\epsilon_R^2}{2} & (\frac{M_X}{M_T} - \frac{M_T M_X}{M_X^2 - M_T^2})\epsilon_R\epsilon_L & \epsilon_R \\ -\frac{M_X}{M_T}\epsilon_R\epsilon_L & 1 - \frac{M_T^2 M_X^2}{2(M_X^2-M_T^2)^2}\epsilon_L^2 & \frac{M_T M_X}{M_X^2-M_T^2}\epsilon_L \\ -\epsilon_R & -\frac{M_T M_X}{M_X^2-M_T^2}\epsilon_L & 1 - \frac{M_T^2 M_X^2}{2(M_X^2-M_T^2)^2}\epsilon_L^2 - \frac{\epsilon_R^2}{2} \end{pmatrix} \begin{pmatrix} t_R \\ T_R \\ X_R \end{pmatrix}$$

$$\begin{pmatrix} t_L^0 \\ T_L^0 \\ X_L^0 \end{pmatrix} = \begin{pmatrix} 1-\frac{\epsilon_L^2}{2} & \epsilon_L & \epsilon_R^2 + \frac{M_T^2}{M_X^2-M_T^2}\epsilon_L^2 \\ -\epsilon_L & 1 - \frac{\epsilon_L^2}{2} - \frac{M_T^4}{2(M_X^2-M_T^2)^2}\epsilon_L^2 & \frac{M_T^2}{M_X^2-M_T^2}\epsilon_L \\ -\epsilon_R^2 & -\frac{M_T^2}{M_X^2-M_T^2}\epsilon_L & 1 - \frac{M_T^4}{2(M_X^2-M_T^2)^2}\epsilon_L^2 \end{pmatrix} \begin{pmatrix} t_L \\ T_L \\ X_L \end{pmatrix}.$$

Actually to evaluate the physical quantities of paragraph 3.3.1 and 3.3.2 it has been necessary to diagonalize this mass matrix up to the fourth order in $\epsilon_{L,R}$, but this result is not useful here.

Note that the composition of the bottom quark is completely standard, and this is a very important point if the whole model has to be phenomenologically interesting. In fact from LEP data ($Z \to b\bar{b}$) it is known that no such botton mixing can have any hope to be consistent with experiments.

According to these rotations we obtain the physical masses in terms of the parameters of Lagrangian 3.6:

$$\begin{aligned} m_t^{phys} &= \lambda_t v(1 - \frac{\epsilon_R^2}{2} - \frac{\epsilon_L^2}{2}) \\ M_T^{phys} &= M_T(1 + \frac{\epsilon_L^2}{2} - \frac{M_T^2 \epsilon_L^2}{2(M_X^2 - M_T^2)}) \\ M_X^{phys} &= M_X(1 + \frac{\epsilon_R^2}{2} + \frac{M_T^2 \epsilon_L^2}{2(M_X^2 - M_T^2)}) \end{aligned} \tag{3.8}$$

while $M_{X^{5/3}}^{phys}$ remains M_X and the bottom mass is neglected.

It is now mandatory to evaluate the relevant physical observables in order to check whether this construction can be consistent with experimental data or not. First of all let us check the cancellation of the quadratically divergent contribution to m_h due to the top loop.

3.2 One loop corrections to m_h

Let us start from potential 2.3:

$$V = \lambda(\phi^2 - f^2)^2 - Af^2 \vec{\phi}^2 + Bf^3 \phi_5$$

As shown by equation 2.7 the Higgs mass is controlled by the A parameter, that is by the $SO(5)$-breaking term. This is reasonable since if everything were $SO(5)$-symmetric the Higgs particle would be a massless Goldstone boson.

To compute the one loop correction to A due to fermions we can use Lagrangian 3.4 and apply a very general formula[1] for the effective potential which takes into account of all the one-fermion-loop contributions. From:

$$\mathcal{L}_{ferm} = \sum_{i,j=1}^{N} \overline{\psi}_L^i \left(i\delta_{ij}\partial_\mu - \frac{1}{2} M(\phi)_{ij} \right) C^{-1} \psi_L^j$$

[1] This formula can be found for example in [5], and is carefully derived in the Coleman's lecture about "secret symmetry" ([6]).

where ψ_L^i are a general set of N left handed Weyl spinors, one obtains the divergent terms:

$$V_{ferm}^{eff}(\phi) = \frac{[-8\Lambda^2 tr(M(\phi)^+ M(\phi)) + 2\log\Lambda^2\, tr((M(\phi)^+ M(\phi))^2)]}{64\pi^2}. \quad (3.9)$$

Using the basis of equation 3.6 we can write:

$$M(h,\sigma)_{ij} = \begin{pmatrix} \lambda_1(v+\frac{h}{\sqrt{2}}) & 0 & 0 \\ \lambda_3 f + \lambda_1(\sqrt{f^2-2v^2}+\sigma) & f\lambda_2 & 0 \\ \lambda_1(v+\frac{h}{\sqrt{2}}) & 0 & M_X \end{pmatrix}$$

where a 2 (Dirac spinors) $\times 3$ (QCD colors) multiplicity is understood.

The one loop correction to A comes from the terms of equation 3.9 which are quadratic in the fields h and σ. Setting $v=0$ for simplicity we have:

$$\begin{aligned} \delta V^{(2)} &= f^2\left(C_\sigma \sigma^2 + C_h h^2\right) \\ &= \frac{12}{64\pi^2}[(-4\lambda_1^2\Lambda^2 + 2\lambda_1^2 f^2(3(\lambda_1+\lambda_3)^2+\lambda_2^2)\log\Lambda^2)\sigma^2 \\ &\quad +(-4\lambda_1^2\Lambda^2 + \lambda_1^2(2f^2(\lambda_1^2+\lambda_3)^2 + M_X^2)\log\Lambda^2)h^2] \end{aligned}$$

The A parameter correction comes from the difference between these two coefficents $C_h - C_\sigma$, so there is no quadratic divergence and the coefficent of the logarithm is[2]:

$$\begin{aligned} -\delta A &= \frac{12}{64\pi^2}\lambda_1^2\left(\frac{M_X^2}{f^2} - 4\left(\lambda_1+\lambda_3\right)^2 - 2\lambda_2^2\right)\log\Lambda^2 \\ &= \frac{3}{16\pi^2}\left(\lambda_t^2+\lambda_T^2\right)\left(M_X^2 + M_T^2\left(\frac{2}{1+\lambda_T^2/\lambda_t^2} - 4\right)\right)\log\Lambda^2. \quad (3.10) \end{aligned}$$

Notice that M_X and M_T take the role of the cutoff Λ in the original top-loop quadratic divergence. For this reason we cannot allow them to be above 2 TeV.

[2]Obviously the finite terms in equation 3.9 give an infrared scale of order of the heavy quark masses.

3.3 Phenomenology

There are two important experimental constraints on this construction: first of all as shown in Figure 2.1 the model begs for an extra positive contribution to $\hat{T} = \rho - 1$, and this is discussed in paragraph 3.3.1. A second constraint comes from the modified coupling of the bottom to the Z gauge boson due to the "vertex" correction, as discussed in paragraph 3.3.2.

3.3.1 The ρ parameter

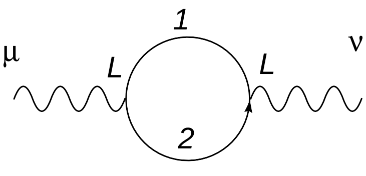
To compute the relative deviation of the T parameter with respect to the usual SM result 2.10, we write down the "new" currents:

$$J_1^\mu = \frac{g}{2}(\bar{t}^0_L \gamma^\mu b_L + \overline{X}^0_L \gamma^\mu X^{5/3}{}_L + \overline{X}^0_R \gamma^\mu X^{5/3}{}_R + h.c.)$$

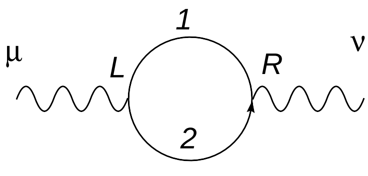
$$J_3^\mu = \frac{g}{2}(\bar{t}^0_L \gamma^\mu t^0_L - \bar{b}_L \gamma^\mu b_L + \overline{X^{5/3}}_L \gamma^\mu X^{5/3}{}_L - \overline{X}^0_L \gamma^\mu X^0_L + \overline{X^{5/3}}_R \gamma^\mu X^{5/3}{}_R - \overline{X}^0_R \gamma^\mu X^0_R)$$

and we evaluate the various fermionic one loop diagrams contributing to the vacuum polarization amplitudes of equation 2.9 in terms of mass eigenstates instead of the zero-superscript fields.

For reference, the following expressions have been used:



$$\Pi_{LL}(0) = \frac{1}{16\pi^2} \int_0^1 dx \left[\Lambda^2 - 2\Delta \log \frac{\Lambda^2}{\Delta}\right]$$



$$\Pi_{LR}(0) = \frac{2}{16\pi^2} \int_0^1 dx \; m_1 m_2 \left(\log \frac{\Lambda^2}{\Delta} - 1\right)$$

where 1,2 are two fermions of mass m_1, m_2 "L" and "R" stand for left-right projectors, and:

$$\Delta = (x\, m_2^2 + (1-x) m_1^2) \qquad \Pi_{RR} = \Pi_{LL}\,,\ \Pi_{RL} = \Pi_{LR}$$
$$\Pi^{\mu\nu}(q) = \Pi(q^2) g^{\mu\nu} + f(q^2) q^\mu q^\nu \qquad \Rightarrow \Pi^{\mu\nu}(0) = \Pi(0) g^{\mu\nu}.$$

The "technical" cutoff Λ must disappear from the final result, and this fact has been used as an overall check for the correctness of the calculation.

Up to second order in ϵ_L and ϵ_R (see 3.7), the final result is $T = T_{SM} + \delta T$ where:

$$\frac{\delta T}{T_{SM}} = \epsilon_R^2(-8\log\frac{M_X}{m_t} + \frac{22}{3}) + \epsilon_L^2\{4\log\frac{M_X}{m_t} - 4\log\frac{M_X}{M_T}[1 - \frac{M_T^2}{M_X^2} -3\frac{M_T^4}{M_X^4} + 3\frac{M_T^6}{M_X^6} + \frac{\lambda_T^2}{\lambda_t^2}(2\frac{M_T^2}{M_X^2} - 3\frac{M_T^4}{M_X^4} - 3\frac{M_T^6}{M_X^6})]/(1 - \frac{M_T^2}{M_X^2})^5 +[-2 + \frac{\lambda_T^2}{\lambda_t^2} + (\frac{50}{3} + \frac{7}{3}\frac{\lambda_T^2}{\lambda_t^2})\frac{M_T^2}{M_X^2} - (\frac{146}{3} + 15\frac{\lambda_T^2}{\lambda_t^2})\frac{M_T^4}{M_X^4} +(62 + 11\frac{\lambda_T^2}{\lambda_t^2})\frac{M_T^6}{M_X^6} + (-\frac{104}{3} + \frac{2}{3}\frac{\lambda_T^2}{\lambda_t^2})\frac{M_T^8}{M_X^8} + \frac{20}{3}\frac{M_T^{10}}{M_X^{10}}]/(1 - \frac{M_T^2}{M_X^2})^5\} \tag{3.11}$$

which in the $M_X \gg M_T$ limit reduces to:

$$\frac{\delta T}{T_{SM}} \approx 2\epsilon_L^2(\log\frac{M_T^2}{m_t^2} - 1 + \frac{\lambda_T^2}{2\lambda_t^2})$$

while for $M_T \gg M_X$ it is:

$$\frac{\delta T}{T_{SM}} \approx -4\epsilon_R^2(\log\frac{M_X^2}{m_t^2} + \frac{11}{6}) \quad .$$

These results can be found also in [2]. The issue of compatibility with experimental data will be discussed in paragraph 3.4, where this calculation will be done numerically without the approximation $\epsilon_{L,R} \ll 1$. The analitic expression 3.11 is then used as an important check of the correctness of the numerical calculation, as shown in Figure 3.5. The same procedure is followed in the evaluation of the $Z \to b\bar{b}$ correction of the next section.

3.3.2 The $A^{bb}(Z \to b\bar{b})$ parameter

In the SM one loop exchanges of the top modify the coupling of the bottom quark to the Z boson as:

$$\left(-\frac{1}{2} + \frac{\sin^2\theta_W}{3} + A^{bb}\right)\frac{g}{\cos\theta_W}Z_\mu\bar{b}_L\gamma^\mu b_L \qquad , \qquad A^{bs}\frac{g}{\cos\theta_W}Z_\mu\bar{b}_L\gamma^\mu s_L$$

and in the SM in the limit $m_t \gg m_W$ it is:

$$A^{bb}_{SM} = \frac{\lambda_t^2}{32\pi^2} \qquad , \qquad A^{bs}_{SM} = V_{ts}V^*_{tb}A^{bb}_{SM}. \tag{3.12}$$

In the $SO(5)$ model heavy quark exchange contributions have also to be included.

In order to simplify the calculations I note that in the approximation $m_t \gg m_W$ we can work in the "gaugeless limit". As formally explained in [8] or [9], this follows from appropriate Ward identities which relate gauge vectors to the derivative couplings of the eaten up Goldstone bosons. The point is that because of gauge invariance the quark couplings to vector bosons always appear in the combinations:

$$\left(Z_\mu + \frac{\cos\theta_W}{gv}\partial_\mu \pi^0\right)\bar{b}_L\gamma^\mu b_L \quad , \quad \left(W^\pm_\mu \pm \frac{1}{gv}\partial_\mu \pi^\pm\right)\bar{t}_L\gamma^\mu b_L \, .$$

This implies that the quark-vector boson vertex can be evaluated in terms of the quark-scalar derivative coupling. Note that this coupling is a non renormalizable interaction and the diagram under consideration is already finite, which makes a significant simplification already at the SM level and a fortiori here.

In this logic by rewriting 3.5 in terms of scalars we obtain the Yukawa Lagrangian (which again must be expressed in terms of physical quark states):

$$\mathcal{L} = \pi^-(\lambda_t \bar{b}_L t^0_R + \lambda_T \bar{b}_L T^0_R) + \frac{\pi^0}{\sqrt{2}}\left[\lambda_t \bar{t}^0_R t^0_L + \lambda_T \overline{T}^0_R t^0_L + \lambda_t \bar{t}^0_R X^0_L + \lambda_T \overline{T}^0_R X^0_L\right] + h.c.$$

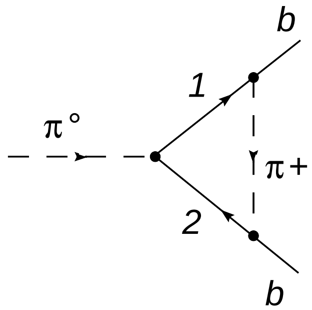


Figure 3.1: One loop contribution to A^{bb}. Here 1,2 are charge 2/3 quarks with mass m_1, m_2.

Denoting by λ^x_{ij} the coupling of two physical quarks q_i and q_j with the π^x boson, $x = \pm, 0$, the relevant amplitude comes from the vertex diagram in Figure 3.1:

$$A_{12} = \frac{\sqrt{2}}{32\pi^2}\lambda^+_{1b}\lambda^+_{2b}\lambda^0_{12}\frac{m_2}{m_1^2 - m_2^2}\log\frac{m_1^2}{m_2^2}.$$

For example in the usual SM the dominant contribution comes from the diagram with $1 = 2 = t$, and it is $\sqrt{2}\lambda^0_{tt} = \lambda^+_{tb} = \lambda_t$, $m_t = \lambda_t v$, so that 3.12 is recovered[3].

The final result obtained by rewriting the Yukawa Lagrangian above in terms of the physical fields and evaluating the various contributions is, up to second order in $\epsilon_{L,R}$:

$$\begin{aligned}\frac{A^{bb}}{A^{bb}_{SM}} &= 1+\epsilon^2_R(2\log\frac{M_X}{m_t}-1)+[-2+\frac{\lambda^2_T}{\lambda^2_t}\frac{1-2\frac{M^2_T}{M^2_X}}{1-\frac{M^2_T}{M^2_X}}+4\log\frac{M_X}{m_t} \qquad (3.13)\\ &\quad -4\log\frac{M_X}{M_T}\frac{1-(3+\frac{\lambda^2_T}{2\lambda^2_t})\frac{M^2_T}{M^2_X}-2\frac{M^4_T}{M^4_X}}{(1-\frac{M^2_T}{M^2_X})^2}]\epsilon^2_L\end{aligned}$$

which in the limit $M_X >> M_T$ becomes:

$$\frac{A^{bb}}{A^{bb}_{SM}} \approx 1+2\epsilon^2_L(\log\frac{M^2_T}{m^2_t}-1+\frac{\lambda^2_T}{2\lambda^2_t})$$

while for $M_T >> M_X$:

$$\frac{A^{bb}}{A^{bb}_{SM}} \approx 1+\epsilon^2_R(\log\frac{M^2_X}{m^2_t}-1) \quad .$$

3.4 Experimental constraints

The question is now whether corrections 3.11 and 3.13 can be consistent with the experimental constraints of Table 3.1. The former condition comes from Figure 2.1, the latter from LEP[4] precision measurements of $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{had})$.

In principle one could consider also the constraint coming from the b-factories data on $B \to X_s l^+ l^-$ decays:

$$\frac{A^{bs}}{A^{bs}_{SM}} = 0.95 \pm 0.20,$$

however using this constraint or the one in Table 3.1, the final conclusions do not change.

[3] In fact with our notation to obtain the contribution to A^{bb} it is necessary to divide A_{12} by λ_t/m_t.

[4] See [2], par. 3.2.2. Experimental data are from [10].

$0.25 \lesssim \delta T_{fermions} \lesssim 0.50$
$\frac{A^{bb}}{A^{bb}_{SM}} = 0.88 \pm 0.15$

Table 3.1: Experimental constraints on ρ and $Z \to b\bar{b}$.

In the following the results of numerical computations for the one-loop δT and A^{bb}/A^{bb}_{SM} are reported in terms of the parameters of Lagrangian 3.6 (however M_X and M_T can be considered roughly equal to the physical masses). The only effective free parameters are λ_t/λ_T (taken from 1/3 to 3, so that the theory is not strongly coupled), M_X and M_T. From Figures 3.2, 3.3, and 3.4 we can see that there are no allowed regions in parameter space for this minimal model with vector-like quark masses below[5] 2 TeV, in particular the δT parameter would "prefere" $\lambda_t/\lambda_T \geq 1$, while A^{bb} is better for lower values. This conclusion was already reached in [2].

This suggests to consider the non minimal models of the next chapters.

3.4.1 Analytic vs numerical results

As already mentioned, the results of the analytic calculation in equations 3.11 and 3.13 has been used as a check of this numerical result. In Figure 3.5 we show the relative difference between the results of the numerical exact calculation and the asymptotic analytic expressions for δT and A^{bb}. Note that the agreement is very good for masses above 1 TeV, while for small masses the analytic expressions are not reliable.

[5]New quark masses over 2 TeV would give an exceedingly large logarithmically divergent contribution to m_h^2, as shown in section 3.2

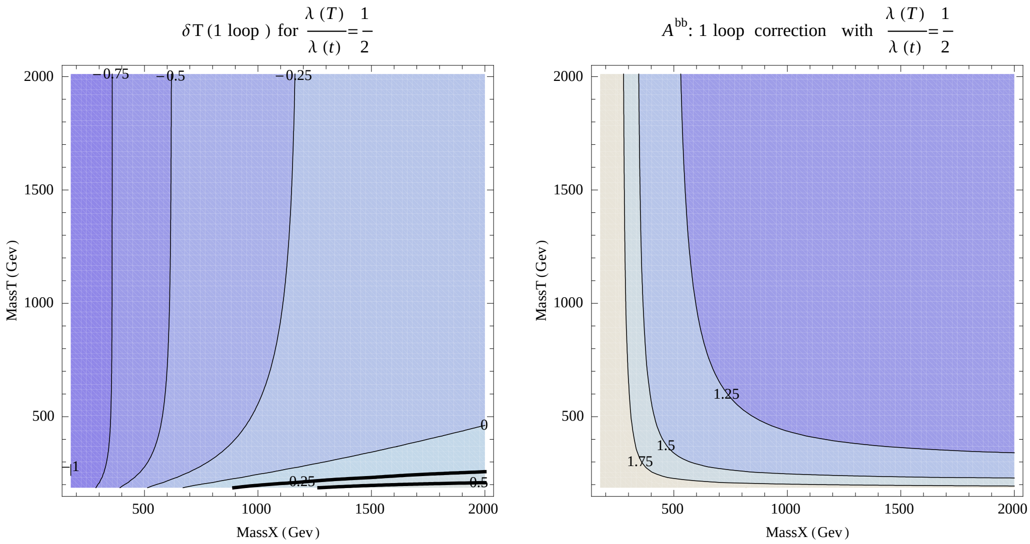


Figure 3.2: Numerical isoplot of the one loop corrections to δT and A^{bb}/A^{bb}_{SM} versus the Lagrangian parameters (M_X, M_T), for $\lambda_T/\lambda_t = 1/2$. Note that there is no experimentally allowed region.

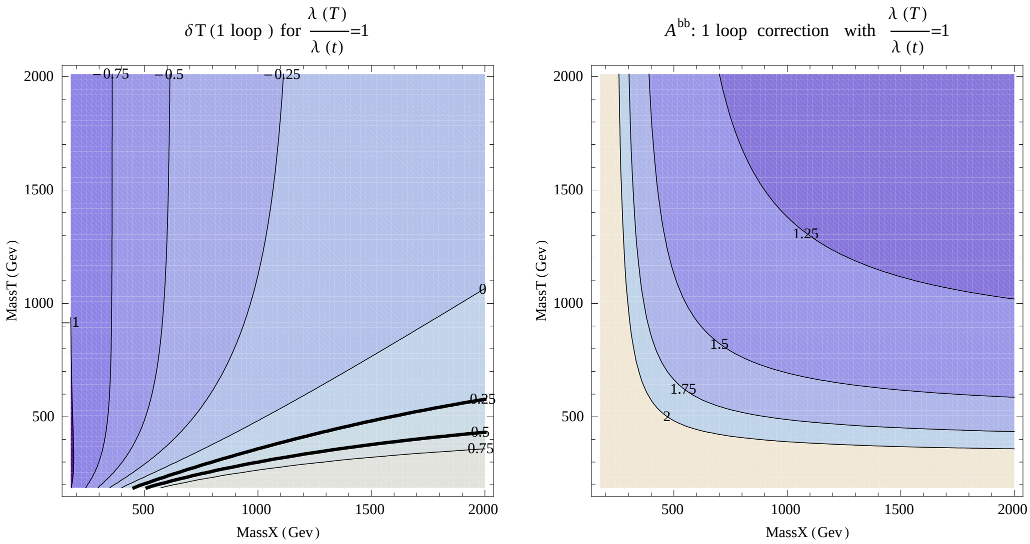


Figure 3.3: As in Figure 3.2, with $\lambda_T/\lambda_t = 1$.

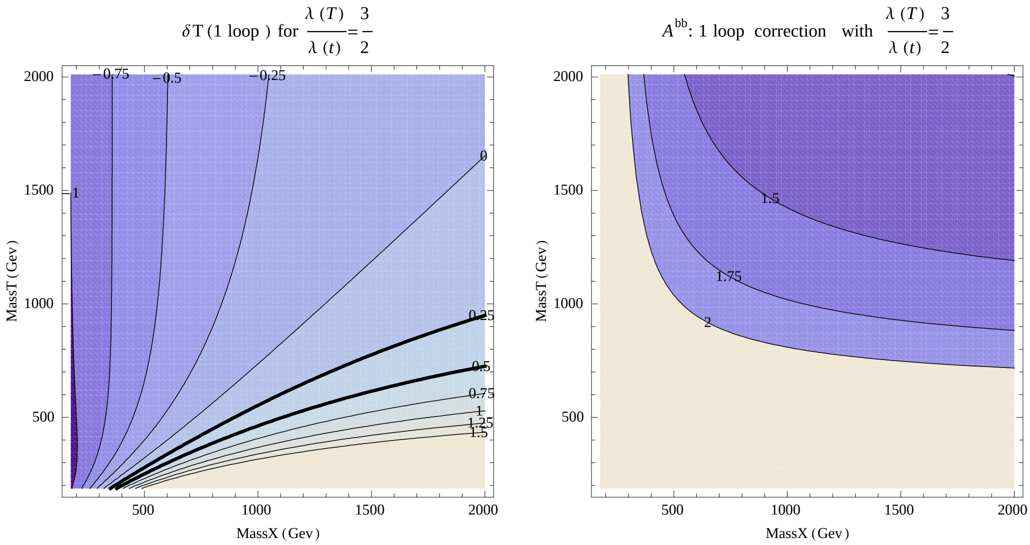


Figure 3.4: As in Figure 3.2, with $\lambda_T/\lambda_t = 3/2$.

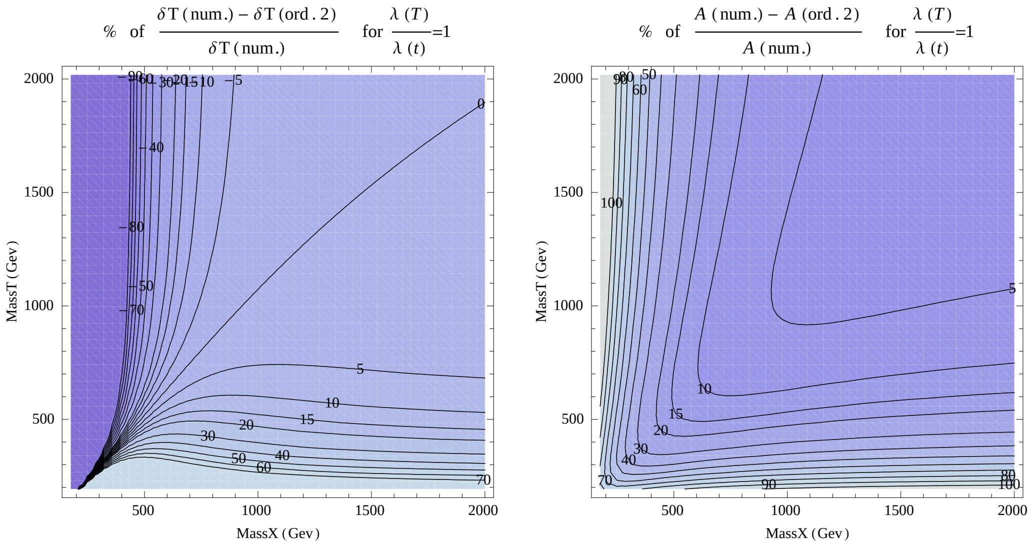


Figure 3.5: Isoplot of the % relative difference between the numerical exact results and the asymptotic analytic results 3.11 and 3.13, represented in the (M_X, M_T) plane. Note that for masses above 1 Tev the difference is below 5%; on the contrary for small masses the approximation $\epsilon_{L,R} \ll 1$ is not reliable.

Chapter 4

An extended model in the top sector

The minimal model described in chapter 3 failed to be consistent with the experimental constraints of Table 3.1. It remains to be seen whether a suitably extended model can come out of this problem. In this chapter we shall exhibit an explicit example where this is the case. This construction is motivated by suitable 5-dimensional extensions of the model under consideration (see [11] and [12]).

4.1 Non-minimal model

In the next-to-minimal interesting[1] model the $SO(5)$ symmetric quark sector is completely made up of new quarks and it generates a symmetric mass term for the top through the order v/f mass mixing. The fermionic content is now:

$$\Psi_{L,R} = \left(Q = \begin{pmatrix} Q^u \\ Q^d \end{pmatrix}, X = \begin{pmatrix} X^u \\ X^d \end{pmatrix}, T \right)_{L,R}, q_L = \begin{pmatrix} t \\ b \end{pmatrix}_L, t_R, b_R,$$

where Q is now a standard ($Y = 1/6$) $SU(2)_L$ doublet and the quantum numbers are the same as 3.3. The Yukawa Lagrangian is now $\mathcal{L} = \mathcal{L}_{int} + \mathcal{L}_{BSM}$, where $\mathcal{L}_{BSM}$ involves only "beyond the SM" fields with a non renormalizable Yukawa interaction, and $\mathcal{L}_{int}$ describes the mass mixing of the standard fields

[1]Actually a spinor representation could also be considered, but as shown in [2] there would be a mixing of the bottom quark at first order in v/f, which is not phenomenologically acceptable.

with the heavy fermions:

$$\begin{aligned}\mathcal{L}_{int} &= \lambda_1 f \overline{q}_L Q_R + \lambda_2 f \overline{T}_L t_R \\ \mathcal{L}_{BSM} &= \frac{y}{f}\overline{\Psi}_L \phi\phi^T \Psi_R + m_Q \overline{Q}_L Q_R + m_X \overline{X}_L X_R + m_T \overline{T}_L T_R + h.c.\end{aligned} \tag{4.1}$$

Defining:

$$\lambda_t = \frac{y\lambda_1\lambda_2 f^2}{\sqrt{(m_T+yf)^2+(\lambda_2 f)^2}\sqrt{m_Q^2+(\lambda_1 f)^2}}$$

$$\begin{array}{ll} M_T = \sqrt{(m_T+yf)^2+(\lambda_2 f)^2} & A = \frac{m_T+yf}{\lambda_2 f} \\ M_Q = \sqrt{m_Q^2+(\lambda_1 f)^2} & B = \frac{m_Q}{\lambda_1 f} \\ M_X = m_X & M_f = \lambda_t f \end{array},$$

up to rotations which preserve quantum numbers the charge 2/3 mass matrix becomes, with the same notation of 3.6 and "quark vectors" $(t, T, Q, X)_{L,R}$:

$$\begin{pmatrix} \lambda_t v & -A\lambda_t v & -\frac{\sqrt{1+A^2}(\lambda_t v)^2}{M_f} & -\frac{\sqrt{1+A^2}(\lambda_t v)^2}{M_f} \\ 0 & M_T & \sqrt{1+A^2}\sqrt{1+B^2}\lambda_t v & \sqrt{1+A^2}\sqrt{1+B^2}\lambda_t v \\ -B\lambda_t v & AB\lambda_t v & M_Q + \frac{B\sqrt{1+A^2}(\lambda_t v)^2}{M_f} & \frac{B\sqrt{1+A^2}(\lambda_t v)^2}{M_f} \\ -\sqrt{1+B^2}\lambda_t v & A\sqrt{1+B^2}\lambda_t v & \frac{\sqrt{1+A^2}\sqrt{1+B^2}(\lambda_t v)^2}{M_f} & M_X + \frac{\sqrt{1+A^2}\sqrt{1+B^2}(\lambda_t v)^2}{M_f} \end{pmatrix} \tag{4.2}$$

The $o(v^2)$ mixing terms have been neglected.

In full analogy with 3.8 the physical masses will be corrected by diagonalization:

$$\begin{cases} m_t^{phys} = \lambda_t v(1+O(\epsilon^2)) \\ M_T^{phys} = M_T(1+O(\epsilon^2)) \\ M_Q^{phys} = M_Q(1+O(\epsilon^2)) \\ M_X^{phys} = M_X(1+O(\epsilon^2)) \end{cases} \tag{4.3}$$

where $\epsilon = \lambda_t v/M_{heavy}$. The Q^d (charge $-\frac{1}{3}$) and X^u (charge $\frac{5}{3}$) masses remain exactly M_Q and M_X since there is no state to mix with. Note that actually M_f is not a free parameter since to avoid finetuning in the scalar potential we want to be $f = 500$ GeV, as explained in section 2.2.2.

The exact one loop results for δT and A^{bb} up to order ϵ^2 are now very long and complicated, so here is reported only the limit in which three masses are much bigger than the other one.

δT correction:

$$
\begin{aligned}
&\text{For } M_Q, M_X, M_f >> M_T: \quad \rightarrow \quad \frac{\delta T}{T_{SM}} \approx 2A^2(\log\frac{M_T^2}{m_t^2} - 1 + \frac{A^2}{2})(\frac{m_t}{M_T})^2 \\
&\text{For } M_T, M_X, M_f >> M_Q: \quad \rightarrow \quad \frac{\delta T}{T_{SM}} \approx 4B^2(\log\frac{M_Q^2}{m_t^2} - \frac{3}{2} + \frac{1}{3}B^2)(\frac{m_t}{M_Q})^2 \qquad (4.4) \\
&\text{For } M_T, M_Q, M_f >> M_X: \quad \rightarrow \quad \frac{\delta T}{T_{SM}} \approx -4(1+B^2)(\log\frac{M_X^2}{m_t^2} - \frac{11}{6} - \frac{1}{3}B^2)(\frac{m_t}{M_X})^2
\end{aligned}
$$

while for $M_T, M_Q, M_X >> M_f$:

$$
\begin{aligned}
\frac{\delta T}{T_{SM}} \approx\ & \frac{2(1+A^2)}{3(M_Q^2-M_X^2)^2}\{12B\sqrt{1+B^2}(M_Q^3M_X + M_QM_X^3) - \\
& -(1+2B^2)(7(M_Q^4+M_X^4) - 26M_Q^2M_X^2) + \\
& +\frac{6\log\frac{M_Q^2}{M_X^2}}{M_Q^2-M_X^2}(-4B\sqrt{1+B^2}M_Q^3M_X^3 - 3M_Q^2M_X^4 + M_X^6 + \\
& +B^2(M_Q^6 - 3M_Q^4M_X^2 - 3M_Q^2M_X^4 + M_X^6))\}(\frac{\lambda_t v}{M_f})^2.
\end{aligned}
$$

$Z \rightarrow b\bar{b}$ correction:

$$
\begin{aligned}
&\text{For } M_Q, M_X, M_f >> M_T: \quad \rightarrow \quad \frac{\delta A^{bb}}{A^{bb}_{SM}} \approx 2A^2(\log\frac{M_T^2}{m_t^2} - 1 + \frac{A^2}{2})(\frac{m_t}{M_T})^2 \\
&\text{For } M_T, M_X, M_f >> M_Q: \quad \rightarrow \quad \frac{\delta A^{bb}}{A^{bb}_{SM}} \approx B^2(\log\frac{M_Q^2}{m_t^2} - 1)(\frac{m_t}{M_Q})^2 \qquad (4.5) \\
&\text{For } M_T, M_Q, M_f >> M_X: \quad \rightarrow \quad \frac{\delta A^{bb}}{A^{bb}_{SM}} \approx (1+B^2)(\log\frac{M_X^2}{m_t^2} - 1)(\frac{m_t}{M_X})^2 \\
&\text{For } M_T, M_Q, M_X >> M_f: \quad \rightarrow \quad \frac{\delta A^{bb}}{A^{bb}_{SM}} \approx \frac{\sqrt{1+A^2}(BM_X + M_Q\sqrt{1+B^2})}{M_QM_X}(\frac{\lambda_t v}{M_f})
\end{aligned}
$$

In the following, through numerical diagonalization of the mass matrix, it will be shown that compatibility with experimental constraints (Table 3.1) is now allowed in a thin slice of parameter space.

4.2 Minimal values for heavy masses

The parameter space has been studied for $\frac{1}{3} \leq A, B \leq 3$ with vector-like quark masses all below 2.5 TeV, looking for experimentally allowed configurations with relatively light vector-like quarks. Some results are summarized in section 4.2.1. A typical situation, for example $A = 1.8$, $B = 1.1$, $M_Q = 900$ GeV, is represented in Figure 4.1, where we give the isolines of δT and A^{bb} in the plane (M_X, M_T). The lower plots represent the (small) overall allowed region for the input parameters M_X, M_T and for the physical masses M_X^{phys}, M_T^{phys}.

For a better illustration of this case, consider for example an exact one loop calculation (masses in GeV) which corresponds to a "point" in Figure 4.1:

$$\boxed{\begin{array}{ll} M_T = 1000 & M_X = 1100 \\ M_Q = 900 & \\ A = 1.8 & B = 1.1 \end{array}} \quad \rightarrow \quad \boxed{\begin{array}{ll} M_T^{phys} = 1010 & \\ M_Q^{phys} = 550 & M_{Q^{-1/3}} = 920 \\ M_X^{phys} = 1940 & M_{X^{5/3}} = 1130 \\ \hline \delta T = 0.28 & A^{bb} = 0.97 \end{array}} \; .$$

First of all note that there is a significant difference between M_{heavy} and M_{heavy}^{phys}, and this is also apparent from the "allowed region" graphics. This fact however is not surprising since relations 4.3 are valid only in absence of quasi degeneracy between the input masses. In the quasi degenerate case $(\lambda_t v / M_{heavy})^2$ should be actually replaced by $(\lambda_t v)^2/(M_{heavy}^a - M_{heavy}^b)^2$, as it happens for example in 3.8.

A second observation is about the way these numbers are obtained. The point is that a mass-matrix rescaling is necessary in order to get the correct value for the top mass, which is modified by the diagonalization. For example at the beginning one can set $\lambda_t v = 170$ GeV; the physical top mass will be unavoidably modifyed by the mixing, say $m_{top}^{phys} = 200$ GeV. Then to obtain a physical result it is necessary to multiply the whole mass matrix by a factor 170/200, so that all its eigenvalues will be rescaled by the same factor. What is called M^{phys} in the following is the corrected value taking into account both these effects (diagonalization and rescaling).

Note also that with this procedure the correct value of $M_{X^{5/3}}$ (or $M_{Q^{-1/3}}$) is not exactly M_X (or M_Q), but it has to be rescaled in the same way (see also section 4.2.1).

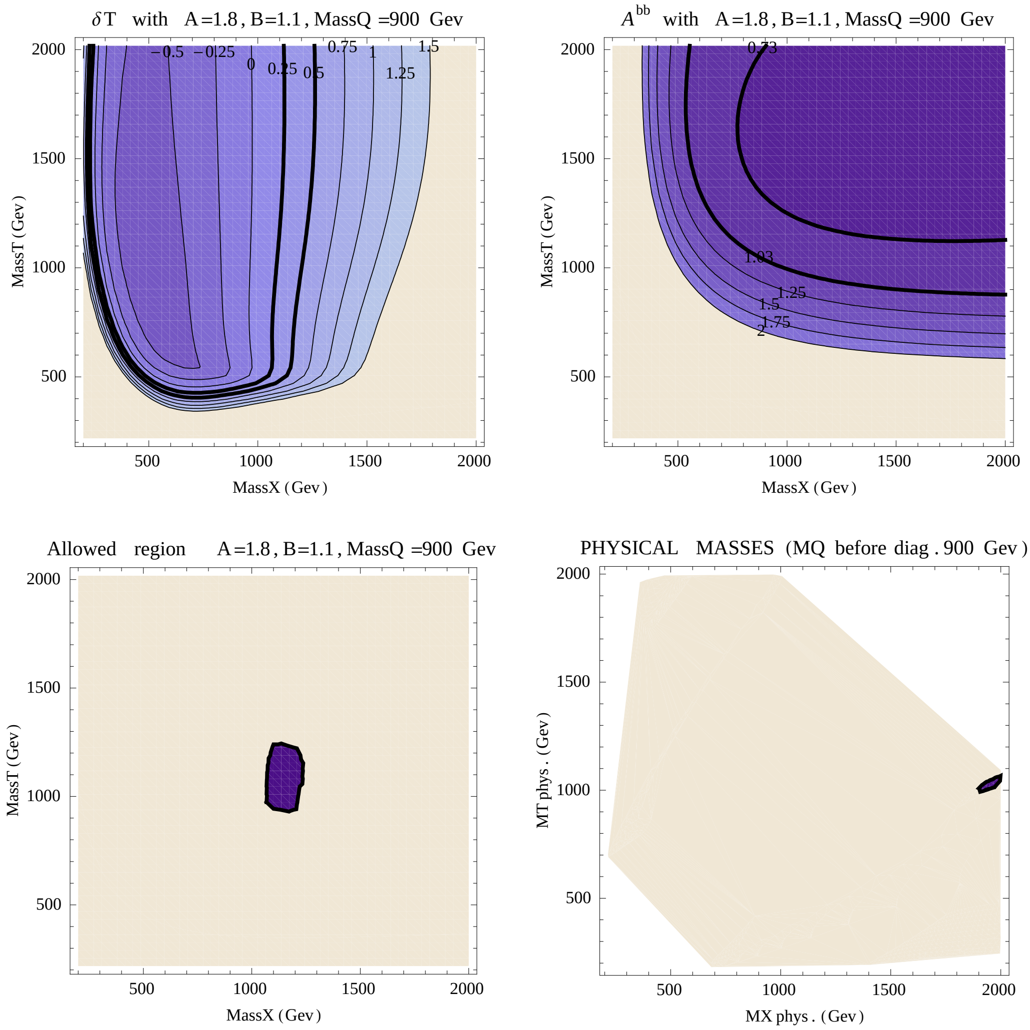


Figure 4.1: Isoplot of δT and A^{bb}/A^{bb}_{SM} in the (M_X, M_T) plane. In the lower graphics is reported the (M_X, M_T) plane and in the (M_X^{phys}, M_T^{phys}) plane. Plot for: ($A = 1.8$, $B = 1.1$, $M_Q = 900$ GeV). Physical masses take into account of diagonalization and of the scaling factor needed to get the correct top mass.

4.2.1 Numerical results

The main results of the numerical study of parameter space, looking for the lightest experimentally allowed configurations for new quarks, are here reported without contour plots, which are very similar to one another and to Figure 4.1. The following properties hold:

1. At least one of the new charge 2/3 quarks has to be heavy, that is around 1.9 TeV.

2. *Light Q:* The lightest possible new-quark state is the $Q^{2/3}$, which in principle can be as light as 290 GeV. In such configurations a heavy T or X is required, for example:

$M_T = 1225$	$M_X = 630$
$M_Q = 320$	
$A = 2.81$	$B = 0.33$

$\rightarrow$

$M_T^{phys} = 1890$	
$M_Q^{phys} = 290$	$M_{Q^{-1/3}} = 340$
$M_X^{phys} = 555$	$M_{X^{5/3}} = 670$
$\delta T = 0.40$	$A^{bb} = 1.02$

3. *Light T:* Allowing $M_X \approx 1.9$ TeV it is possible to obtain a T quark mass around 500 GeV for example:

$M_T = 940$	$M_X = 1200$
$M_Q = 960$	
$A = 1.86$	$B = 1.1$

$\rightarrow$

$M_T^{phys} = 510$	
$M_Q^{phys} = 1060$	$M_{Q^{-1/3}} = 955$
$M_X^{phys} = 1940$	$M_{X^{5/3}} = 1195$
$\delta T = 0.43$	$A^{bb} = 1.01$

4. *Light X:* $M_X^{2/3}$ can be as low as 450 GeV with $X^{5/3}$ at 950 GeV, and heavy T, for example:

$M_T = 1152$	$M_X = 969$
$M_Q = 971$	
$A = 2.99$	$B = 0.71$

$\rightarrow$

$M_T^{phys} = 2050$	
$M_Q^{phys} = 925$	$M_{Q^{-1/3}} = 1026$
$M_X^{phys} = 460$	$M_{X^{5/3}} = 1024$
$\delta T = 0.28$	$A^{bb} = 0.93$

Note however from point 2 that the $X^{5/3}$ can be also as light as 670 GeV.

4.2.2 Allowed volume in parameter space

Another question that could be asked about this model is how extended is the volume of parameter space which is allowed by the experimental data. To answer this question one can consider the fractional volume (making a linear sampling) of the experimentally allowed region in the relevant parameter space:

$$\left\{\frac{1}{3} \leq A, B \leq 3\right\} \cap \{200 \text{ GeV } \leq M_{T,X,Q} \leq \text{ some TeV}\}$$

For this purpose I take 10 equally spaced samplings for each of the 5 parameter intervals (with the "technological means" at my disposal I couldn't do much better). I call "probability" of the model this fractional volume. In Figure 4.2 the result of this calculation is given as a function of the maximum allowed $M_{X,T,Q} \neq M^{phys}_{X,T,Q}$.

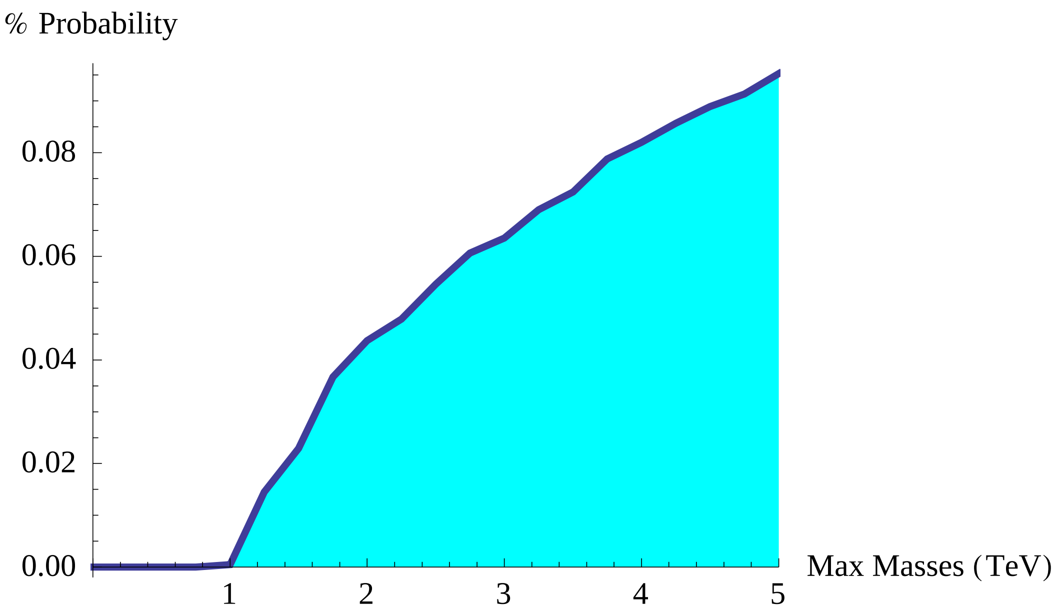


Figure 4.2: % Probability of being in the experimentally allowed region by randomly choosing a point in parameter space, as a function of the maximum allowed quark mass (before diagonalization).

For all fermion masses below the cutoff we obtain:

$$\frac{\text{Allowed volume}}{\text{Total volume } (M_{heavy} < 2.5 \text{ TeV})} \approx 0.05\% = \frac{1}{2000} \quad . \tag{4.6}$$

Moreover note that for the model to be consistent with data it is necessary to have at least one $M_{heavy} \gtrsim 1$ TeV (which actually leads to one $M^{phys}_{heavy} \gtrsim 1.8$ TeV because of the mass splitting and rescaling, see section 4.2).

Chapter 5

Alternative models

In the course of my investigation I have also considered other Lagrangian models for fermion masses, all involving more fields than the minimal model of chapter 3. In this chapter I briefly report about two of them, with different motivations. None gives an acceptable region of parameter space.

5.1 A different coupling

One can ask himself what happens considering Lagrangian 4.1 with a standard Yukawa fermion-scalar interaction instead of the non-renormalizable one studied in the previous chapter.

This can be done by taking exactly the same fermion sector extension described in section 4.1 with:

$$\begin{aligned} \mathcal{L} &= \lambda_1 f \overline{q}_L Q_R + \lambda_2 f \overline{T}_L t_R + y \overline{\Psi}_L \phi t_R \\ &\quad + m_Q \overline{Q}_L Q_R + m_X \overline{X}_L X_R + m_T \overline{T}_L T_R + h.c. \end{aligned} \tag{5.1}$$

Note that in this case there is no separation between $\mathcal{L}_{int}$ and $\mathcal{L}_{BSM}$ as in the model of chapter 4. Up to rotations which preserve quantum numbers the charge 2/3 mass matrix is now, with the same notation of 3.6 and 4.2:

$$\begin{pmatrix} \overline{t}^0_L \\ \overline{T}^0_L \\ \overline{Q}^0_L \\ \overline{X}^0_L \end{pmatrix} \begin{pmatrix} -\lambda_t v & -A\lambda_t v & 0 & 0 \\ 0 & M_T & 0 & 0 \\ B\lambda_t v & AB\lambda_t v & M_Q & 0 \\ -\sqrt{1+B^2}\lambda_t v & A\sqrt{1+B^2}\lambda_t v & 0 & M_X \end{pmatrix} \begin{pmatrix} t^0_R \\ T^0_R \\ Q^0_R \\ X^0_R \end{pmatrix} + h.c, \tag{5.2}$$

where it is:

$$\lambda_t = \frac{y\lambda_1 f^2}{\sqrt{1+\frac{f^2(\lambda_2+y)^2}{(m_T)^2}}\sqrt{(m_Q+(\lambda_1 f)^2}}$$

$$\begin{aligned} M_T &= \sqrt{m_T^2 + f^2(\lambda_2 + y)^2} \quad & A &= \tfrac{(\lambda_2+y)f}{m_T} \\ M_Q &= \sqrt{m_Q^2 + (\lambda_1 f)^2} \quad & B &= \tfrac{m_Q}{\lambda_1 f} \\ M_X &= m_X. \end{aligned}$$

Repeating the procedure to evaluate the δT and A^{bb} correction we obtain now the results reported in equations 5.3 and 5.4:

δT **correction:**

$$\begin{aligned} \text{For } M_Q, M_X >> M_T: \;\; \rightarrow \;\; & \frac{\delta T}{T_{SM}} \approx 2A^2(\log\frac{M_T^2}{m_t^2} - 1 + \frac{A^2}{2})(\frac{m_t}{M_T})^2 \\ \text{For } M_T, M_X >> M_Q: \;\; \rightarrow \;\; & \frac{\delta T}{T_{SM}} \approx 4B^2(\log\frac{M_Q^2}{m_t^2} - \frac{3}{2} + \frac{1}{3}B^2)(\frac{m_t}{M_Q})^2 \\ \text{For } M_T, M_Q >> M_X: \;\; \rightarrow \;\; & \frac{\delta T}{T_{SM}} \approx -4(1+B^2)(\log\frac{M_X^2}{m_t^2} - \frac{11}{6} - \\ & -\frac{1}{3}B^2)(\frac{m_t}{M_X})^2 \end{aligned} \tag{5.3}$$

$Z \rightarrow b\bar{b}$ **correction:**

$$\begin{aligned} \text{For } M_Q, M_X >> M_T: \;\; \rightarrow \;\; & \frac{\delta A^{bb}}{A^{bb}_{SM}} \approx 2A^2(\log\frac{M_T^2}{m_t^2} - 1 + \frac{A^2}{2})(\frac{m_t}{M_T})^2 \\ \text{For } M_T, M_X >> M_Q: \;\; \rightarrow \;\; & \frac{\delta A^{bb}}{A^{bb}_{SM}} \approx B^2(\log\frac{M_Q^2}{m_t^2} - 1)(\frac{m_t}{M_Q})^2 \\ \text{For } M_T, M_Q >> M_X: \;\; \rightarrow \;\; & \frac{\delta A^{bb}}{A^{bb}_{SM}} \approx (1+B^2)(\log\frac{M_X^2}{m_t^2} - 1)(\frac{m_t}{M_X})^2 \end{aligned} \tag{5.4}$$

Note that these corrections, because of the choice of A and B (which however have here a different meaning), are exactly of the same form 4.4 and 4.5, although with the non-renormalizable interaction there is an additional contribuition from f. In Figure 5.1 is reported the typical result of a numerical calculation, showing that there is no experimentally allowed region.

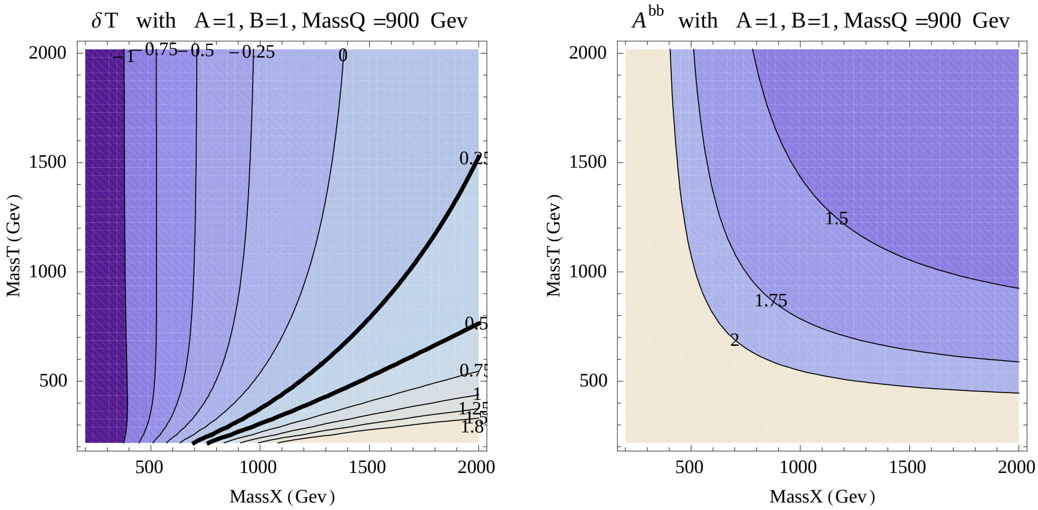


Figure 5.1: Isolines of δT and A^{bb}/A^{bb}_{SM} in the (M_X, M_T) plane for the the $SO(5)/SO(4)$ model with Lagrangian 5.1. Plot for: $A = 1$, $B = 1$, $M_Q = 900$ GeV. Note that there is no overlapping between the allowed regions.

5.2 A different model: $SU(4)/Sp(4)$

In this section the $SU(4)/Sp(4)$ Composite-Higgs theory described in [13] is briefly outlined and then analyzed with the same strategy used in the $SO(5)/SO(4)$ case. The number of fields which can mix with the top is exactly the same as in the $SO(5)/SO(4)$ extended model, with the same number of free parameters.

Scalar sector

Making reference to [13] for a detailed description of the model, here I just recall that the scalar sector is described by an antisymmetric 4×4 matrix Σ (12 real fields) trasforming under $SU(4)$ as:

$$\Sigma \to V \Sigma V^T \qquad V \in SU(4).$$

Following the general method described in [14], it can be shown that the most general invariant potential up to order four in the fields:

$$V(\Sigma) = -\frac{1}{2}\mu^2 tr(\Sigma\Sigma^*) + \frac{1}{4}\alpha(tr(\Sigma\Sigma^*))^2 + \frac{1}{4}\beta tr((\Sigma\Sigma^*)^2)$$

leads for $\beta < 0$, to the breaking of $SU(4)$ spontaneously down to $Sp(4)$.

Five Pseudo-Goldstone bosons are associated with the 5 broken generators: they transform as an $SU(2)$ doublet (like the standard Higgs doublet) plus an $SU(2)$ singlet. The others 7 scalars are heavier.

The $SU(2)_L \times U(1)_{T_3} \subset SU(2)_L \times SU(2)_R \subset Sp(4)$ is then gauged in the usual way. The detailed dynamics of the collective symmetry breaking of $SU(4)$ and $SU(2)_L \subset Sp(4) \subset SU(4)$ can be studied through minimization of a potential including soft-breaking terms. Also without doing so, one can safely assume that the unavoidable mixing of the Higgs particle with some heavier scalar will induce an "effective-mass" contribution to the S and T parameter similar to that of the $SO(5)$ model, so that one will need a: $\delta T_{needed} \backsim 0.25 - 0.5$. This is, however, not a crucial assumption since the main problem in this model is not to fulfill this requirement, but instead to satisfy the experimental condition:

$$A^{bb}(Z \to b\bar{b}) = 0.88 \pm 0.15.$$

Fermion sector

Enlarging the top sector with new fermions in the vectorial representation of $SU(4)$, as done in [13], is problematic because there is an $SU(2)_L$ singlet with electric charge $-1/3$ which will mix at tree level with the bottom, and this is phenomenologically not defendable. This problem is avoided (in a "minimal way") with new fermions in the antisymmetric representation:

$$A_L = \begin{pmatrix} 0 & Q_L & X_L^{5/3} & t_L \\ & 0 & X_L & b_L \\ & & 0 & T_L \\ & & & 0 \end{pmatrix}.$$

The quantum numbers of the fields are fixed by the natural[1] way $SU(2)_L \times SU(2)_R$ is embedded in $Sp(4)$. Introducing the needed right-handed states the third generation is therefore enlarged as:

$$\begin{pmatrix} X_{L,R}^{5/3} \\ X_{L,R} \end{pmatrix} = (2)_{7/6} \quad , \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix} = (2)_{1/6} \quad , \quad t_R, Q_{L,R}, T_{L,R} = (1)_{2/3}.$$

The most general Lagrangian respecting $SU(2) \times U(1)$ gauge invariance and the $SU(4)$ symmetry of the Yukawa interaction is:

$$\begin{aligned} \mathcal{L} &= \lambda_1 f \bar{t}_R Q_L + \lambda_2 f \bar{t}_R T_L \\ &+ \frac{1}{2} y_1 \overline{Q}_R tr(\Sigma^* A_L) + y_2 f \overline{Q}_R T_L \\ &+ m_Q \overline{Q}_R Q_L + m_T \overline{T}_R T_L + m_X \left(\overline{X}_R X_L + \overline{X}_R^{5/3} X_L^{5/3} \right) \end{aligned}$$

[1] For this and other details see [13].

where, forgetting about the fifth pseudo-Goldstone boson and the other scalars:

$$\frac{1}{2}tr(\Sigma^* A_L) = f(Q_L + T_L) + H \begin{pmatrix} t_L \\ b_L \end{pmatrix} + H^c \begin{pmatrix} X_L^{5/3} \\ X_L \end{pmatrix}.$$

Here f is the scale of the $SU(4)/Sp(4)$ breaking and H is the Higgs doublet.

This Lagrangian can be analyzed in a totally analogous way as we have done in the previous chapters. The mass matrix, concentrating on charge 2/3 quark mass terms and up to quantum-number preserving rotations, is:

$$\mathcal{L} = \begin{pmatrix} \bar{t}_L \\ \overline{T}_L \\ \overline{Q}_L \\ \overline{X}_L^d \end{pmatrix}^T \begin{pmatrix} \lambda_t v & A\lambda_t v & B\lambda_t v & 0 \\ 0 & M_T & 0 & 0 \\ 0 & 0 & M_Q & 0 \\ \lambda_t v & A\lambda_t v & B\lambda_t v & M_X \end{pmatrix} \begin{pmatrix} t_R \\ T_R \\ Q_R \\ X_R^d \end{pmatrix} + h.c. \tag{5.5}$$

where for example:

$$\lambda_t = \frac{\lambda_1 y_1 m_T f}{\sqrt{2}\sqrt{f^2\lambda_1^2 + (m_Q + f y_1)^2}\sqrt{m_T^2 + \frac{f^2(\lambda_2(m_Q+fy_1) - f\lambda_1(y_1+y_2))^2}{f^2\lambda_1^2 + (m_Q + f\lambda_1)^2}}}$$

while the other new parameters are much more complicated combinations of the original ones. Note that now Q is a singlet like T, while X is a component of an $Y = 7/6$-doublet.

Computing the one loop correction to the T parameter and to $Z \to b\bar{b}$ up to second order in $\lambda_t v/M_{heavy}$, like in sections 3.3 and 4.1, we now obtain:

δT correction:

$$M_X, M_Q >> M_T: \quad \rightarrow \quad \frac{\delta T}{T_{SM}} \approx 2A^2(\log\frac{M_T^2}{m_t^2} - 1 + \frac{A^2}{2})(\frac{m_t}{M_T})^2$$

$$M_X, M_T >> M_Q: \quad \rightarrow \quad \frac{\delta T}{T_{SM}} \approx 2B^2(\log\frac{M_Q^2}{m_t^2} - 1 + \frac{B^2}{2})(\frac{m_t}{M_Q})^2$$

$$M_Q, M_T >> M_X: \quad \rightarrow \quad \frac{\delta T}{T_{SM}} \approx -4(\log\frac{M_X^2}{m_t^2} - \frac{11}{6})(\frac{m_t}{M_X})^2$$

$Z \to b\bar{b}$ correction:

$$M_Q, M_X >> M_T: \quad \rightarrow \quad \frac{\delta A^{bb}}{A^{bb}_{SM}} \approx 2A^2(\log\frac{M_T^2}{m_t^2} - 1 + \frac{A^2}{2})(\frac{m_t}{M_T})^2$$

$$M_T, M_X >> M_Q : \quad \rightarrow \quad \frac{\delta A^{bb}}{A^{bb}_{SM}} \approx 2B^2(\log \frac{M_Q^2}{m_t^2} - 1 + \frac{B^2}{2})(\frac{m_t}{M_Q})^2$$

$$M_Q, M_T >> M_X : \quad \rightarrow \quad \frac{\delta A^{bb}}{A^{bb}_{SM}} \approx (\log \frac{M_X^2}{m_t^2} - \frac{1}{2})(\frac{m_t}{M_X})^2$$

Note the strict similarity with respect to the results of the $SO(5)$ models. In particular it is evident that the form of the δT and of the A^{bb} contributions depend very much on the $SU(2)_L$ properties of the new cherge 2/3 particle (as could be expected).

Numerical results and conclusions

The experimental consistency of the model has been checked via numerical diagonalization of the mass matrix (5.5) in the relevant region in parameter space:

$$\frac{1}{3} \leq A, B \leq 3 \quad , \quad 200 \text{ GeV } \leq M_T, M_Q, M_X \leq 2 \text{ TeV}.$$

The final result is that, because of the $Z \to b\bar{b}$ contribution, this model can not be consistent with experimental data. In Figure 5.2 is illustrated an example of the tipical situation.

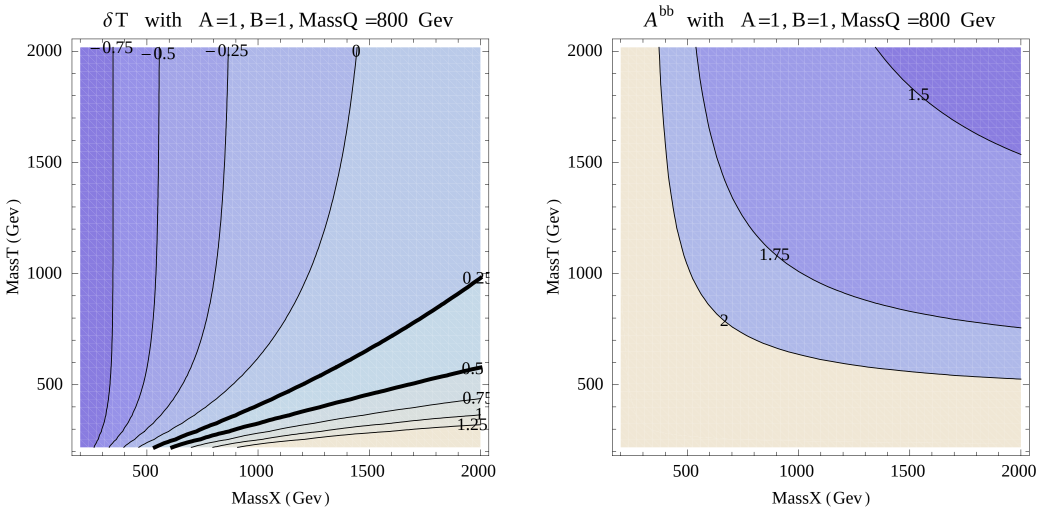


Figure 5.2: Isolines of δT and A^{bb}/A^{bb}_{SM} in the (M_X, M_T) plane for the the $SU(4)/Sp(4)$ model. Plot for $A = B = 1$, $M_Q = 800$ GeV. Note that there is no overlapping between the allowed regions.

Chapter 6

Conclusions

Naturalness arguments strongly suggest that new physics should manifest itself at LHC energies. This motivates many attempts to increase the SM symmetry and, most of the time, also its particle content.

In this thesis I discuss a composite-Higgs model in which the Higgs-top sector comes from an approximate $SO(5)$ symmetry as a minimal extension of the $SO(4)$ of the SM. The Higgs sector is represented by a real 5-plet of $SO(5)$. After symmetry breaking at a scale f of $SO(5)$ down to $SO(4)$, the standard Higgs doublet emerges as a pseudo Goldstone boson. Similarly the top-bottom $SU(2)_L$ doublet is extended to a 5-plet of Weyl spinors adding heavy vector-like quarks. In this way the one loop quadratic divergence of m_h due to the top loop is cancelled by loops involving heavy vector-like quarks.

The one-loop impact of the model on Electro Weak Precision Tests and B-physics has been studied and used as a constraint on the free parameters of the fermion sector. Confirming the results of [2], we have found that the minimal extension of the top sector has problems in fulfilling the experimental requirements. For this reason we have considered other possible extensions of the fermion sector, as well as another model based on a different symmetry. These models have received attention in the literature and have different motivations.

Our main result is that one such extension is consistent with the constraints coming from the EWPT, including B-physics, in a thin region of its parameter space. To the third generation quarks of the Standard Model one has to add a full vector-like 5-plet of SO(5), i.e. in particular three new quarks of charge 2/3 which mix with the top: T, Q, X.

The effects of this mixings on the ρ-parameter and on the $Z-$coupling to the b quark are shown to be compatible with the experimental data in a small region of the parameter space. In this region the new quarks can be as light as a few hundreds GeV and might therefore be accessible at the LHC. The range of possible masses is summarized in the following Table (see section 4.2.1):

Quark	$SU(2)_L \times U(1)_Y$	Constraints on mass
Q	$(2)_{1/6}$	$M_{Q^{2/3}} \gtrsim 300$ GeV, $M_{Q^{-1/3}} \gtrsim 350$ GeV
T	$(1)_{2/3}$	$M_{T^{2/3}} \gtrsim 500$ GeV
X	$(2)_{7/6}$	$M_{X^{2/3}} \geq 450$ GeV, $M_{X^{5/3}} \gtrsim 650$ GeV

It is of interest that, randomly "picking up a point" in the relevant parameter space with all fermion masses below 2.5 TeV, the probability of being consistent with data is very small, roughly 1/2000.

None of the other models that we have examined have regions of the corresponding parameter space which are compatible with the experimental data.

Bibliography

[1] G. F. Giudice, *Theories of the Fermi Scale* [arXiv:hep-ph/07103294v2]

[2] R. Barbieri, B. Bellazzini, V. S. Rychkov, A. Varagnolo, *The Higgs boson from an extended symmetry*, Phys. Rev. D 76, (2007) 115008.

[3] R. Barbieri, *Signatures of new physics at 14 Tev* [arXiv:hep-ph/08023988v1]

[4] M. Schmaltz, D. Tucker-Smith, *Little Higgs Review* [arXiv:hep-ph/0502182v1]

[5] R. Barbieri, *Supersymmetric gauge models of the fundamental interactions*, Acta Physica Austriaca, Suppl. XXIV, 363-392 (1982)

[6] Sidney Coleman, *Aspects of symmetry*, Cambridge University Press

[7] Michael E. Peskin, Daniel V. Schroeder, *An introduction to Quantum Field Theory*, Perseus Books, 1995.

[8] R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicerè, *Radiative correction effects of a very heavy top*, Phys. Lett. B288 (1992) 95.

[9] R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicerè, *Two-loop heavy-top effects in the Standard Model*, Nucl. Phys. B409 (1993) 105-127.

[10] ALEPH, DELPHI, L3, OPAL, SLD Collaborations, LEP and SLD Electroweak Working Groups, SLD Heavy Flavour Group, *Precision electroweak measurments on the Z resonance*, Phys. Rept. 427, 257 (2006) [arXiv:hep-ex/0509008]

[11] R.Contino, L. Da Rold, A. Pomarol, *Light custodians in a natural composite Higgs model*, Phys Rev. D 75, 055014 (2007) [arXiv:hep-ph/0612048]

[12] R.Contino, A. Pomarol, *The minimal composite Higgs model*, Nucl. Phys. B 719, 165 (2005) [arXiv:hep-ph/0412089]

[13] E. Katz, A. Nelson, D. G. E. Walker, *The Intermediate Higgs* [arXiv:hep-ph/ 0404252v1]

[14] Ling-Fong Li, *Group theory of spontaneously broken gauge symmetries*, Phys. Rev. D 9,6,1723 (1974)

[15] H. Georgi, *Lie Algebras in Particle Physics*, Westview Press, 1999.

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