Ph.D. Thesis
Ph.D. Program in Economics

Life Expectancy and Economic Growth

Tamara Fioroni

Department of Economics
University of Pisa
Via Ridolfi 10, Pisa, Italy
zanelli@ec.unipi.it

Supervisor
Prof. Davide Fiaschi

Department of Economics
University of Pisa
Via Ridolfi 10, Pisa, Italy
dfiaschi@ec.unipi.it

Contents

Preface

The growing interest in economics on the relationship between health and economic growth arises from the persistence of a strong gap in living standards between rich and poor countries. Despite the remarkable progress in health improvement in the last half century, mortality rates remain much higher in poor countries, with a difference in life expectancy between rich and poor countries of about 30 years (Cutler et al., 2006). This thesis focuses on this topic exploring the mechanisms by which health status affects economic growth including the relevant interconnections with investment in education, saving decisions and the intergenerational transmission of wealth.

The first chapter reviews the literature on the effects of mortality reductions on economic growth. The effect of longevity on economic growth has been analyzed in two strands of literature. The first strand assumes exogenous longevity and shows that increases in life expectancy improve economic growth in poor countries while have a null or negative effect in rich countries. The second strand identifies human capital as the principal factor affecting longevity. These contributions show that poor countries can be trapped in an equilibrium where life expectancy is low, education is low and fertility is high whereas rich countries grow in the long-run.
A recent strand of literature focuses on the explicit effect of health spending on life expectancy stressing the central role of both quantity and quality of life on economic welfare. This studies emphasize the value of life and the willingness to pay criterion to reduce the mortality risk.

The second chapter analyzes the relationship between human capital accumulation and life expectancy in a three periods overlapping generations economy. Agents are altruistic and differ in their probability of surviving to the second and third period. The model shows the existence of multiple steady states depending on the initial distribution of human capital. Poor economies converge to a stable steady state where mortality is high and education is low. On the other hand, rich economies show high levels of education and high life expectancy.

The third chapter focuses on the direct effect of health spending on life expectancy. In particular, we investigate the relationship between saving and health expenditure in a two-periods overlapping generations economy. Individuals work in the first period and live in retirement in the old age. Health investment is an activity that increases the quality of life and the probability of surviving from the first period to the next. Empirical evidence shows that both health spending and saving, i.e. the consumption when old, appear to be luxury goods but their behavior is strongly different according to the level of per capita GDP. The share of saving on GDP appears to be concave with respect to per capita GDP. On the opposite, the share of health expenditure on GDP increases more than proportionally with respect to per capita GDP. The ratio of saving to health investment is nonlinear with respect to per capita GDP, i.e. first increasing and then decreasing. In the proposed model, the ratio of health spending to saving is equal to the ratio between the elasticity of the survival function and the elasticity of the utility function. We prove that the model can replicate empirical results if the elasticity of the utility function vary with the consumption, e.g. agents have H.A.R.A. (hyperbolic absolute risk aversion) preferences, and the survival function presents a positive and increasing elasticity with respect to health investment. Moreover we show that C.E.S. (constant elasticity of substitution) preferences are not consistent with empirical evidence.

Acknowledgments

I want to thank all people who help me to complete this thesis. I am deeply grateful to my supervisor Prof. D. Fiaschi for detailed and constructive comments and suggestions and the encouragement that helped me in all the time writing of this thesis. I have furthermore to thank the Director of the Ph.D. Program in Economics, Prof. C. Bianchi. I want to thank my colleagues from the Department of Economics who supported me in this thesis. Especially I am obliged to Simone D'Alessandro for many helpful comments, to Lucia Zanelli, Lorenzo Corsini and Marco Guerrazzi. I want to thank the participants at the 4th Phd Conference research in economics in Volterra (September 2006), and the participants at the 11 Convegno Annuale dell' Associazione Italiana Economia Sanitaria in Venezia (November 2006). Finally I would like to give my special thanks to my family and my friends for their indispensable and fundamental support. Usual disclaimer applies.

Chapter 1

Mortality Decline and Economic Growth

1.1 Introduction

For a long time, the theory of economic growth has given few relevance to health as an important determinant of economic growth (Zon and Muysken, 2005). The pioneering contributions on health, mainly, focused on the characterization of a demand's function for health services (Grossman, 1972; Ehrlich and Chuma, 1990) and, on the link which runs from better economic conditions to mortality reductions (Preston, 1975), ignoring the relationship which runs in the opposite direction, that is, from good health to economic growth.

The endogenous growth theory (Romer, 1986; Lucas, 1988), identifying in human capital the critical factor for economic growth, implicitly recognizes the importance of health on economic growth. The paper by Fogel (1994) is the first contribution on the effects of good health on economic growth:

> Changes in health, in the composition of diet, and in clothing and shelter can significantly affect the efficiency with which ingested energy is converted into work output. Reductions in the incidence of infectious diseases increase the proportion of ingested energy.....(Fogel (1994), p. 386)
>
> ....I believe, on the huge social investments made between 1870 and 1930, whose payoffs were not counted as a part of national

> income during the 1920's and 1930's even though they produced a large stream of benefits during these decades.... (Fogel (1994), p.388)

Good health and longevity affect economic growth through different channels such as the saving rate, human capital accumulation and agent's productivity. In accordance with this point, the Word Health Organization report, published in 2001, defines good health as the basis for job productivity, a critical input for poverty reduction, economic growth and long-term development (World Health Organization, 2001).

Bloom and Canning (2000) distinguish four ways in which health improvements can lead to economic growth: 1) productivity, i.e. healthier populations tend to be physically more energetic and mentally more robust; 2) education, that is people who live longer have stronger incentives to invest in their human capital because they can enjoy the benefits of such investments over longer periods. In addition, increased schooling promotes greater productivity and, in turn, higher income; 3) investment in physical capital since improvements in longevity create a greater need for people to save for their retirement; 4) demographic transition from high to low rates of mortality and fertility.

The interest for the effects of good health and longevity on economic growth comes from the extraordinary gains in life expectancy in second half of 20th century (see Figure 1.1). For example, between 1820 and 1870, in England, life expectancy was about 41 years (Cutler et al., 2006), in the first decade of the twentieth century was 50 years whereas in 2004 climbed to about 78 years (W.D.I. 2006). Mortality reduction in France was broadly similar to that in England. In the United States, life expectancy at birth rose from 47 years in 1900 to 77 years in 2004 (W.D.I. 2006). In general, a similar transition, with some moderate differences, took place in all developed countries (Cutler et al., 2006). Developing countries, also, show a rapid increase in life expectancy which, however, stop to increase from 1980 (see Figure 1.1 and 1.2). From 1980, indeed, in several poor countries the HIV/AIDS epidemic reverse the positive trend in life expectancy (Becker et al., 2005; Cutler et al., 2006). In 1980, many developing countries showed a life expectancy at birth lower than in 1960 (Cutler et al., 2006) as we can see in Figure 1.1.

Figure 1.1 shows that income growth improves life expectancy and it has a larger effect on mortality reduction among the poor than among the rich countries. In addition, we can see a dimension of change in longevity which is not associated with income, that is, for constant levels of income, life expectancy has been rising from 1960 to 2002 (see also Soares (2005)). This phenomenon was first noticed in the Preston (1975)'s seminal paper which analyzed the data between 1930 and 1960. In particular, he argues that the raise in income can explain only a part of the increase in life expectancy from 1930 to 1960 and attributes the shift upward of the relationship between life expectancy and income to the improved public health measures, particularly in middle income countries (Deaton, 2003). According to this approach, Amartya Sen (1999) argues that the increase in life expectancy can not be imputable only to the increase in the per capita income. For example, he argues that the increase in life expectancy in England during the two wars was imputable mostly to the strong and direct nutritional and public health interventions that took place during and immediately after the World War II.

Figure 1.2 shows the international relationship between life expectancy at birth and per capita income in 2002. The size of the circles is proportional to population, the black ones indicate Sub-Saharan countries and the gray ones indicate east Asian and Pacific countries (source W.D.I. 2006). In 2002, we can observe, a strong "divergence" in life expectancy at birth between rich and poor countries: in many poor countries life expectancy at birth was lower than 60 year whereas in rich countries life expectancy was about 80 years.

A critical point is the strong difference in the causes of mortality between developed and developing countries. In poor countries there are a high infant mortality rate and most deaths are caused by infectious disease. Health delivery is often of low quality in both public and private sector. At the same time many countries spend so little on health care that, no matter how organized, it is unlikely to be effective (Cutler et al., 2006). In rich countries the scenario is different, i.e. infant mortality rate is very low and most deaths are caused by cancers and cardiovascular diseases (Cutler et al., 2006).

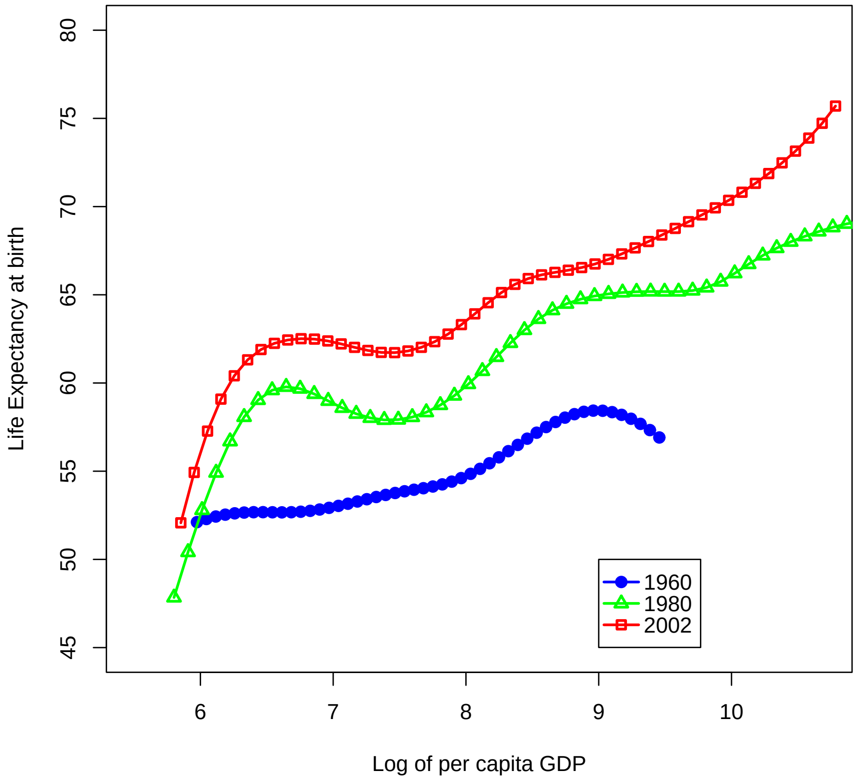

Figure 1.1: Life Expectancy versus Log of GDP per capita (1960, 1980, 2002). Source: World Development Indicators CD-ROM, World Bank (2006).

This chapter is substantially devoted to review the literature which analyzes the effects of good health and longevity on country's per capita GDP over time. We begin with the study of economic growth models in which life expectancy is assumed exogenous. We discuss, then, theoretical and empirical contributions which assume human capital as the principal determinant of mortality reduction. Finally we review the recent literature which emphasizes the direct effect of health spending on life expectancy.

In particular, the structure of the chapter is outlined as follows. Section 2 discusses the strict relation between human capital and health. Section 3 reviews the literature which analyzes the effect of life expectancy on economic

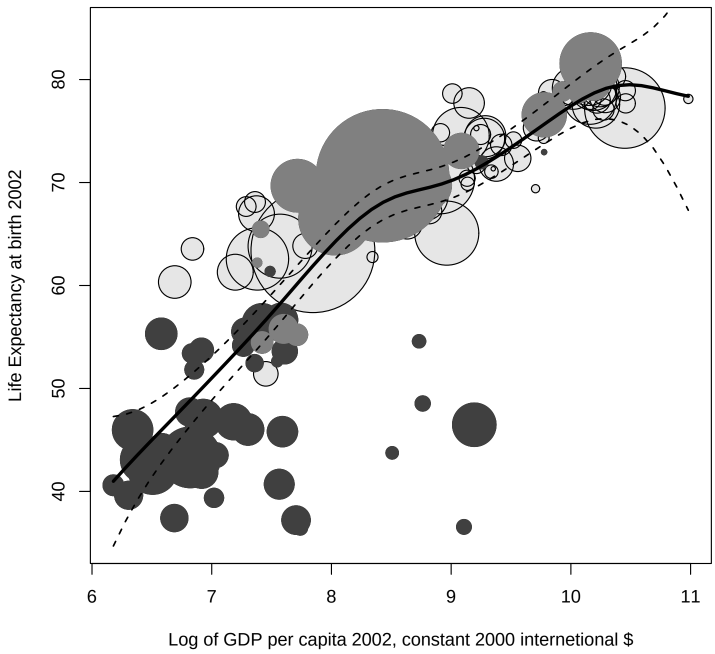

Figure 1.2: Life Expectancy versus Log of GDP per capita. Source: World Development Indicators CD-ROM, World Bank (2006)

growth. In section 4 we analyze the recent health literature on the value of life and the willingness to pay criterion to reduce the mortality risk. Finally, section 5 contains some concluding remarks.

1.2 Health and Human capital

A large body of the literature emphasizes the strict relation and the similarities between health investment and education (see for example Mushkin (1962) and Becker (1962)).

Both health and education are the basis of human capital accumulation, satisfy human wants and lead to higher standard of living. The paper of Mushkin (1962) presents an extensive analysis of the relationship between education and health:

> Health and education are joint investment is the same individual. The individual is more effective in society as producers and as consumers because of these investments (Mushkin (1962) p.130).

Lower levels of health and education reflect lower levels of economic development (Shultz, 1999) and agent's spending in health and education capital presents positive externality for the entire community. Purchase of health services for the prevention of contagious and infectious disease benefits the community as a whole (Cutler et al., 2006).

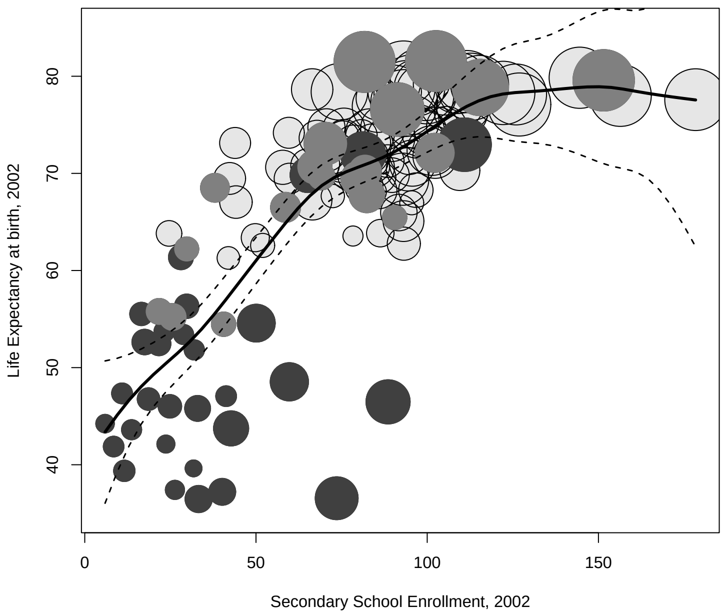

Figure 1.3: Life Expectancy versus Education. Source: World Development Indicators CD-ROM, World Bank (2006)

In Figure 1.3 we propose an estimation of the relationship between life expectancy and secondary education[1]. The circles are proportional to the

[1]We use the secondary gross enrollment ratio as a proxy of education (Source WDI 2006). The gross enrollment ratio is given by the ratio of total enrollment, regardless of age, to the population of the age group that officially corresponds to the level of education shown. Secondary education completes the provision of basic education that began at the primary level, and aims at laying the foundations for lifelong learning and human

country's per capita income, the black ones indicate Sub-Saharan countries and the gray ones indicate east Asian and Pacific countries (W.D.I, 2006). Figure 1.3 shows a positive relationship between health and education and low levels of education and health in poor countries.

In the literature the relationship between education and health has been explained in three ways (Grossman, 2004). Firstly a higher level of education leads to higher level of health: higher agent's education implies a higher willingness to invest in health care either because education makes people better decision makers or because more educated people have better informations about health. Education can improve health through a better choice of health inputs, that is education reduces smoking, improves eating habits and increases exercise (Adams, 2002). In addition, home environment in general and mother's schooling in particular play an extremely important role in the determination of child and adolescent health (Grossman, 1982; Shultz, 1999).

Secondly, the direction of causality runs from better health to more schooling, i.e. good health is fundamental to acquire education. If children's health is not good his/her formal schooling is impossible. Loss in days of schooling due to ill health reduces the effectiveness of investment in education (Mushkin, 1962; Cutler et al., 2006). In addition, an increase in life expectancy raises the investment in education since it reduces the rate of depreciation of investment in education and increases the return to it (Mushkin, 1962; De la Croix and Licandro, 1999).

Thirdly there are some unobserved variables such as physical and mental abilities, genetic characteristics and parental background that affects both health and schooling. For example, both education and health are largely affected by parental input. The decisions made by parents concerning where and how to raise a family depends on all family background characteristics that affect both health and education investments. In addition intergenerational transmissions of health have a strong effect on health.

Grossman (1972, 1982)'s papers are closely related to the first explanation, i.e. he argues that years of schooling have a significant impact on adult

development, by offering more subject or skill oriented instruction using more specialized teachers.

population's health even when income is held constant (Grossman, 1972). In particular, he assumes the following specification for the stock of health in the period $t+1$:

$$H_{t+1} = I_t(M_t, TH_t; E_t) + H_t\,(1-\delta)\,, \tag{1.1}$$

where I_t is the gross investment production function which depends on medical care (M_t), the stock of human capital (E_t) and the time input (TH_t) , δ is the rate of depreciation. Shifts in human capital, measured by education, i.e. E_t , change the productivity in health investment. In particular, the effect on investment in health of one-unit change in education, that is the marginal product of E_t, is given as follows:

$$\frac{\partial I_t}{\partial E_t} = M_t \frac{\partial\,(I_M)}{\partial E_t} + TH \frac{\partial\,(I_{TH})}{\partial E_t}, \tag{1.2}$$

where $I_M = \partial I_t/\partial M_t$ is the marginal product of medical care and $I_{TH} = \partial I_t/\partial TH_t$ is the marginal product of time. Education raises the marginal product of medical care and the time inputs and consequently it reduces the quantity of these inputs required to produce a given amount of health investment (I_t). Hence, with no changes in input prices, an increase in education lowers the marginal cost of health investment. For example, if education increases the marginal products of medical care and time by 3 percent, it would reduces the price of health investment by 3 percent (Grossman (1972) p.243-246).

Closely related to Grossman's paper, Adams (2002)'s work provides evidence of a pronounced relationship between education and health among older people in the U.S. in 1992. He uses the data from the Health and Retirement Study (HRS) and restricts the sample to US individuals between the ages of 51 and 61. In particular he estimates the following relationship:

$$H_i = \beta X_i + \gamma E_i + \varepsilon_i, \tag{1.3}$$

where H_i is the health level of individual i, E_i is agent's educational attainment measured as years of schooling completed and X_i is a set of variables that affect education and health. These include basic individual characteristics, such as race and region of birth[2]. Both the OLS and the 2SLS results show that higher levels of education lead higher level of health for older people (at least at the 0.10 level of significance).

Lleras-Muney (2005) estimates the effect of compulsory education laws on mortality. He uses the 1960, 1970 and 1980 censuses of the U.S. and analyzes the effect of change in compulsory schooling laws happened between 1915 and 1939. He estimates the following aggregate model:

$$D_{tcs} = b + \lambda E_{tcs} + \beta X_{tcs} + \delta W_{tcs} + \gamma_c + \alpha_s + \varepsilon_{cs}, \tag{1.4}$$

where D_{tcs} is the proportion of individuals, belonging to cohort c and born in state s, which died at time t, E is the education of that group (measured by completed years of education), X_{tcs} are the average time invariant characteristics such as gender, W_{tcs} is a set of characteristics of individual i's state of birth at age 14, γ_c is a set of cohort dummies and α_s is a set of state of birth dummies (Lleras-Muney, 2005). He estimates that an additional year of schooling lowers the probability of dying in the next 10 years by 3.6 percentage points. In addition, he shows a direct effect of compulsory schooling laws on mortality during adulthood: one more year of compulsory schooling decreased mortality after age 35 by about 3%.

In accordance with this approach, many contributions focus on the positive relationship between parent's human capital and child's health status. Increases in parent's education lead to a higher demand for children's schooling since more educated parents may value it more highly for their children and can better help their children to learn (Shultz, 1993). According this point, Shultz (1993) shows that higher level of parents education are correlated with lower child mortality, even after holding per capita income constant. In addition, the relationship from parent's education to child health is almost always more strong for the mother than for the father. In particular, a year of additional schooling for the mother is often associated, in a low-income country, with 5-10 percent reduction in her child's probability of dying in the first five years of life (Shultz, 1993).

[2] The average female in the sample is 56.08 years of age and has 12.37 years of completed schooling. The average male is 56.05 years old and has completed 12.65 years of schooling. There are 4577 men in the sample and 4059 women (Adams, 2002).

With respect to the second direction of causality which runs from good health to better education, Rivera and Currais (1999) emphasize the effect of health investment on labor productivity and human capital accumulation. They develop an extension of the augmented Solow model which introduces health investment. In particular, health enters in the production function as follows:

$$Y(t) = K(t)^{\alpha} E(t)^{\beta} H(t)^{\eta} \left(A(t)L(t)\right)^{\mu}, \tag{1.5}$$

where Y is the aggregate output, K is the stock of physical capital, E is the stock of education, H is the stock of health, A is the level of technology and L is the labor. Some calculations lead to the following expression for the log of the steady state level of income in units of effective labor:

$$\ln y^{*} = \frac{\alpha}{\mu} \ln s_k + \frac{\beta}{\mu} \ln s_e + \frac{\eta}{\mu} \ln s_h - \frac{1-\mu}{\mu} \ln\left(n+g+\delta\right) \tag{1.6}$$

where n is the rate of population growth, g is the technological progress and δ is the rate of depreciation. The evolution of income in the long run is given as follows:

$$\begin{aligned} \ln\left(\frac{y(t)}{y(0)}\right) = & \frac{\left(1-e^{-\lambda t}\right)}{\mu+\beta} \left[\alpha \ln s_k + \beta \ln e^{*} + \eta \ln s_h - \left(1-\mu-\beta\right) \ln\left(n+g+\delta\right)\right] \\ & - \left(1-e^{-\lambda t}\right) \ln y(0), \end{aligned} \tag{1.7}$$

where $\lambda = (n+g+\delta)\,\mu$ is the rate of convergence. Equation (1.7) is estimated by ordinary least square for the 24 OECD countries in the period 1960-1990. The total health expenditure on GDP is used as a proxy of investment in health. The results show that adding health investment to the model reduces the size of coefficient for physical capital (from 0.37 to 0.33) and education (from 0.25 to 0.20) whereas the fit of the regression improves little (R^2 increases from 0.87 to 0.88) (the coefficient associated to health is 0.22, for more details see (Rivera and Currais, 1999)).

De la Croix and Licandro (1999)'s paper shows that longevity affects human capital accumulation through agent's decisions on the time to spend in education. In particular, they assume that agent's human capital accumulates according to the following expression:

$$h(t) = A\overline{H}(t)\,T(t), \tag{1.8}$$

where A is a productivity parameter, $\overline{H}(t)$ is the average human capital in the economy and $T(t)$ is the time spent at school. Given a linear utility function the first order condition for $T(t)$ yields the following expression for the optimal time spent on education (for more details see Section 3.1):

$$T(t) = \frac{1}{\theta + \beta}, \tag{1.9}$$

where θ is the subjective discount rate and β is the rate at which the members of a given generation die. Thus an increase in the death rate β implies a reduction of the optimal time spent at school. Hence, when life expectancy is short people choose to start working early in their life and not to stay at school to long. In the other hand, when agents die later on average, they prefer to devote more time to schooling in order to obtain higher future wages.

Contrary to the approach showed up to now there are some studies that argue the existence of a trade-off between health and human capital. For example, the model of Van Zon and Muysken (2001) shows that an expansion of health sector can promote economic growth through increased health of population, while a contraction of the health sector could also free resources necessary to promote growth by means of an increase in human capital accumulation activities. This approach provides an interesting contrast to the analysis that suggests the existence of complementarity between health and education, and it lead to analyze empirically the relationship between health spending and education.

1.3 Life expectancy and economic growth

The literature which studies the economic consequences of mortality decline in growth models, can be divided in two groups, on the basis of the main assumptions and the results obtained: the first group assumes *exogenous* life expectancy and the second group develops growth model in which life expectancy depends on the level of human capital.

Generally the literature which studies the effect of mortality reductions on economic growth uses as basic framework the overlapping generation model à la Diamond (1965) in which agents have a probability of surviving to the second period (or third period). In particular, the basic model is as follows:

$$U = u(c_t) + p\beta u(c_{t+1}), \tag{1.10}$$

subject to the following budget constraints:

$$\begin{cases} c_t = y_t - s_t \quad \text{and} \\ c_{t+1} = (1 + r_{t+1})\, s_t, \end{cases} \tag{1.11}$$

where c_t is the consumption in the first period, y_t is the income, s_t is the saving, c_{t+1} is the consumption in the old age which is given, in an economy without the state, by the saving augmented for the interest rate in the second period (r_{t+1}), p is the probability of surviving to the second period and β is the intertemporal discount rate.

1.3.1 Exogenous Life Expectancy

The literature which analyzes the effect of exogenous longevity on economic growth yields homogeneous results: increasing longevity exerts, through different channels, opposing forces on economic growth. The net effect depends on the initial mortality's level of the country. In the poor ones, where life expectancy is low, a reduction in the mortality rate has a positive effect on economic growth, in the rich ones, where instead life expectancy is high, the effect of a further increase in the average life on economic growth can be null or negative.

In particular, these studies evidences two important effects produced by increase in life expectancy. The first is that a higher life expectancy raises the saving rate leading to physical capital accumulation. The papers of Zhang et al. (2003) and Kageyama (2003) are two examples that generates this effect. The second effect is that gains in life expectancy raise the investment in education promoting human capital accumulation. One of the earlier contribution that shows this result is the paper of De la Croix and Licandro (1999).

The theoretical contribution of Zhang et al. (2003) shows that a higher longevity produces two opposite effect: on the one hand increases the propensity to save and on the other hand lowers the accidental bequest. These results are obtained in an overlapping generations model in which agents live for three periods and have an exogenous probability of surviving to old age. In particular, they add to the general utility function in equation (1.10) the quality of school for children, i.e. q_t :

$$U = u(c_t) + p\beta u(c_{t+1}) + \phi u(q_t), \tag{1.12}$$

where $p \in (0, 1)$ is the probability of surviving to the second period, c_t and c_{t+1} are defined in equation (1.11). A key assumption of this model is that there is no annuity market, so that the saving of a deceased person becomes an accidental bequest to his child, that is:

$$b_t = \begin{cases} 0 & \text{if } p = 1 \\ s_{t-1} & \text{if } p = 0. \end{cases} \tag{1.13}$$

Hence, this assumption introduces an inequality between agents who receive a positive bequest and agents who receive no bequest. In addition, given the accumulation rule for capital states which states that the savings are transformed into productive capital for the next period, that is $k_{t+1} = s_t$, we get:

$$b_t = ps_{t-1} = pk_t. \tag{1.14}$$

Equation (1.14) shows the first effect of an increase in life expectancy, that is it reduces the accidental bequest and therefore the rate of physical capital accumulation.

From the first order conditions, the optimal saving increases with the probability of surviving to the second period, that is:

$$s_t = \frac{\beta p}{1 + \beta p} y_t. \tag{1.15}$$

Therefore an increase in life expectancy leads two opposite effects: firstly, it reduces the accidental bequest (see equations (1.13) and (1.14)), secondly it raises the propensity to save and therefore increases the rate of physical capital accumulation (see equation (1.15)). The net effect of increases in life expectancy on economic growth depends on the life expectancy in the country. If initial mortality is low, rising longevity may have a positive effect on economic growth. Starting with low mortality, the effect of rising longevity on growth can be negative (Zhang et al., 2003).

The positive effect of longevity on the saving emphasized in the Zhang et al. (2003)'s paper is accepted by many theoretical and empirical contributions. Yaari (1965) is the first one which introduces the effect of uncertain lifetime on the saving. He shows that when the survival function is introduced in the intertemporal utility function the future is discounted more heavily because of the uncertainty of survival. Levhary and Miriam (1977), studying the effect of lifetime uncertainty on optimal consumption decisions, show that when the utility function is CES (constant relative risk aversion) and the interest rate is constant, an increase in the relative risk aversion implies a lower consumption in the initial period. In other words, the individual being more sensitive to the possibility of lower consumption in the future, saves more in the initial period. In accordance to this view, the Modigliani's (1988) life cycle hypothesis shows that, even in absence of bequest, the fact that income decreases with retirement, could generate an amount of wealth quite large relative to income.

Strictly related to the paper of Zhang et al. (2003) are the papers of Kageyama (2003) and Bloom and Canning (2000) which focus on the relationship between increases in life expectancy and the allocation of wealth between consumption and saving.

Kageyama (2003) shows both theoretically and empirically that increases in lifetime raise the aggregate saving. This is because higher longevity implies that the younger cohort, in order to secure consumption for a longer retirement, saves more than the older cohort consume (Kageyama, 2003). In particular, he supposes that agents, born in the period t, maximize the intertemporal utility function given by equation (1.10) subject to the following constraints:

$$w_t = c_t + s_t, \tag{1.16}$$

$$c_{t+1} = \frac{(1+r)}{p} s_t. \tag{1.17}$$

Equation (1.17) embody the restrictive assumption that the total returns from the savings of those who are deceased before reaching their old age is equally redistributed, in the form of hump-shaped transfer, to the agents who survive to the third period[3].

Given a constant fertility rate, the optimal aggregate saving, in the period t, is the difference between the saving of adults s_t and the consumption of elderly, i.e. c_{t-1}, that is:

$$S_t = \frac{p}{(1+r)} \left(\frac{1+r}{\beta} \right)^{\frac{1}{\gamma}} c_t - p c_{t-1}, \tag{1.18}$$

where c_{t-1} is the consumption in the period t of agents born in $t-1$. Given equation (1.18) the length of lifetime does not necessarily affect the aggregate saving. The central point is not the level of lifetime, but the *size* of the increase in lifetime. This indicates that the rate at which lifetime is increasing is positively correlated with aggregate saving. This result is tested and supported by the empirical analysis using the household data in the period 1960-1989.

De la Croix and Licandro (1999) focus on the effect of increases in life expectancy on human capital accumulation. The basic idea is that longevity, influencing agent's investment in education, affects human capital accumulation. They analyze an overlapping generations economy where agents decide the time to devote to education before starting to work. In particular, the expected utility of agents born in t and living in z is:

$$\int_t^{\infty} u(c\,(z,t)) e^{-(\beta+\theta)(z-t)} dz,$$

where θ is the subjective discount rate, β is the death rate and $u(c\,(z,t))$ is the instantaneous utility function of consumption. In particular, $u(c\,(z,t))$

[3] In particular, the assumption is that c_{t+1} is given as follows:

$$c_{t+1} = s_t\,(1+r) + \tau_t,$$

where:

$$\tau_t = \frac{(1-p)\,s_t\,(1+r)}{p}.$$

is assumed linear. The agent's budget constraint is:

$$\int_{t}^{\infty} c(z,t)\, R(z,t)\, dz = \int_{t+T(t)}^{\infty} h(t)w(z)R(z,t)\, dz,$$

where $T(t)$ is the time devoted to schooling, $R(z,t)$ is the discount factor and $h(t)$ is the individual's human capital defined in equation (1.8). Since total output is assumed a linear function of the aggregate stock of human capital, the wage per unit of human capital is equal to one, that is $w(z) = 1$ for each z.

The effect of increases in life expectancy on the growth rate results from combining three factors. First, given the optimal time spent in education, i.e. $T(t)$, as follows:

$$T(t) = \frac{1}{\theta + \beta}$$

a reduction in the death rate, that is β, leads to an increase in the time devoted to education. Second, increases in life expectancy implies reductions in the depreciation rate of aggregate human capital. Indeed, the aggregate stock of human capital given by the following expression:

$$H(t) = H(0)\, e^{-\beta t} + \int_{-T}^{t-T} \beta e^{-\beta(t-z)} \left[ATH(z)\right] dz$$

decreases at a rate β as time passes and people die.

Third, the economy consists of more of old agents who did their schooling a long time ago.

The two first effects have a positive influence on the growth rate but the third effect has a negative influence. Numerical computations show that, when life expectancy is below a certain threshold, a higher longevity affects positively economic growth. In rich countries where life expectancy is high the net effect of a higher life expectancy could be negative. This is because the positive effect of a longer life on economic growth could be offset by an increase in the average age of the workers.

The literature discussed in this section obtains important insights on the relationship between life expectancy and economic growth. Yet, the mechanism through which mortality declines is not specified, so any link between health spending and saving or investment in education and saving is not analyzed. Life expectancy, as empirical evidence shows (see Figures 1.2 and 1.3), is not exogenous, but it depends on individual's income, human capital and, in general, on the economic development of the country. Thus in poor countries, without exogenous shock or external interventions, life expectancy can not increase. Empirical analysis, indeed, shows that several poor countries are in a low equilibrium characterized by a low per capita income, a low life expectancy, low health investment and education spending (Cutler et al., 2006). The literature which assumes endogenous longevity investigates some of these central issues.

1.3.2 Endogenous Life Expectancy

Many theoretical and empirical contributions which study the effect of gains in health on economic growth suppose that human capital is the main variable affecting life expectancy. In particular, this literature develops growth models that can explain the demographic transition from high to low rates of mortality and fertility. This approach, described by Galor and Weil (1999), characterizes the process of economic development passing through three distinct regimes (Galor and Weil, 1999). The first is called the *Malthusian Regime*. Here the relationship between income per capita and population growth is positive: small increases in income lead to population growth (Lagerlof, 2003). In the second regime, called the post-Malthusian regime, both per capita income and population present a positive growth rate and their relationship remains positive as in the Malthusian regime. The final stage of development is the *Modern Growth Regime*. In this latter, both income per capita and the level of technology present a positive growth rate whereas population growth declines. Galor and Weil (1999) focus on the technology, the evolution of population and the output growth as the key elements which can explain the transition process through to this three stages. They argue that the technological progress raises the rate of return to human capital inducing parents to invest in children education. In particular, the technological progress has two effects on population growth. On the one hand, improved technology increases households' budget constraints, allowing them to spend more resources on raising children. On the other hand, it induces a reallocation of these increased resources toward child quality (Galor and Weil, 1999). In the Post-Malthusian Regime, the former effect dominates, and so population grows. However, since the return to child quality continues to rise, the shift away from child quantity becomes more significant causing a reduction in the population growth rate and an increase in the output growth rate (Galor and Weil, 1999).

According to this approach, (Lagerlof, 2003)'s paper models demographic and economic long-run development in a setting where mortality depends on agent's human capital and subjects to epidemic shocks. He considers an overlapping generations economy where agents potentially live for two periods. In particular, the probability of surviving to the second period is specified as follows:

$$T_t = \frac{H_t/P_t}{\omega_t + H_t/P_t}, \tag{1.19}$$

where P_t denotes the adult population size in the period t, H_t is the human capital inherited from parents and ω_t the epidemic shocks. Thus, the survival rate T_t lies between 0 and 1, it decreases with epidemic shocks and population density, and it increases with human capital. Advances in medical skills reduce the impact on mortality of epidemics. The production function of human capital takes the following form:

$$H_{t+1} = A\left(P_t\right)\left[L + H_t\right]\left(p\upsilon + h_t\right), \tag{1.20}$$

where $A\left(P_t\right)$ is a productivity parameter which denotes the effectiveness with which one generation transmits human capital to the next, L is the agent's innate human capital, υ is the time spent nursing each child and $p\upsilon$ measures the direct inheritance of human capital from one generation to the next, and h_t is the time spent educating the child.

Equation (1.19) and (1.20) show the two opposite effects of population density: on the one hand it increases the impact on mortality of epidemic shocks (see equation (1.19)), on the other hand it have a positive effect on productivity in the human capital sector (see equation (1.20)).

Agent maximizes the following utility function:

$$U = \ln\left(C_t\right) + \alpha \ln\left(B_t T_t\right) + \alpha\delta \ln\left(L + H_{t+1}\right), \tag{1.21}$$

where B_tT_t is the number of children who survive to the second period and $L+H_{t+1}$ captures the parent's utility get from the quality of children. Optimal conditions imply that the number of children decreases with the time spent on children $(\upsilon + h_t)$, that is:

$$B_t = \left(\frac{\alpha}{1+\alpha}\right)\frac{1}{(\upsilon + h_t)}.$$

The optimal time spent in education increases with the rate of return to human capital investment $A(P_t)[L+H_t]$,that is:

$$h_t \geq \frac{1}{1-\delta}\left[\upsilon(\delta-\rho) - \frac{L}{A(P_t)[L+H_t]}\right], \tag{1.22}$$

where $(\delta - \rho) > 0$ since $\rho < \delta < 1$. The inequality in equation (1.22) follows from the condition that $h_t \geq 0$. Thus if $\upsilon(\delta-\rho) < L/A(P_t)[L+H_t]$ then $h_t = 0$ and the inequality is strict. Otherwise equation (1.22) holds with equality.

Lagerlof (2003) distinguish two cases: a high-ω world and low-ω world. In the first case the economy shows three steady state. The first steady state is a Malthusian equilibrium where the time spent in education is zero. The stability originates from the increased mortality effect of a larger population. The second steady state is unstable and it is a threshold that separates the two growth regions. Above the threshold the economy converges to a sustained growth in human capital and population, below it converges to the Malthusian equilibrium. In the second case, that is low-ω world, there is no Malthusian equilibrium and the economy exhibits sustained growth regardless of where it starts off.

The transition from the Malthusian trap to the sustained growth is generated from a series of mild epidemic shocks. When the economy experiences a phase of relatively mild epidemic shocks, mortality rates fall leading to a positive population growth rate. However, birth rates remain unchanged and parents do not invest in children's education. Thus, the economy is in the post Malthusian regime. When the education time becomes positive the economy transits into the modern growth regime. In this regime, the economy experience a quality-quantity substitution in children, i.e. birth rates fall since education time make children more expensive. Once the growth rate of human capital is high the impact of further epidemics becomes negligible and the economy remains in the modern growth regime (Lagerlof, 2003).

Blackburn and Cipriani (2002) show a framework where changes in human capital characterize the demographic transition. Agents live potentially three periods and have an endogenous probability of surviving to the second period. In the first period agent is raised by her parent, in the second period she/he works, invests in education and bears their children and in the third period she/he works and bears their children. Thus, since agents work in both the second and third period they do not save to finance the consumption in the old age. This assumption rules out the intergenerational transmission of wealth through the saving of parents which do not survive to the old age.

People are non altruistic and children are treated as consumption goods, i.e. parents derive utility from the production of offspring. In particular, the utility from the number of children n is add to the expected utility function in equation (1.10). Longevity π_{t+1}, is increasing in agent's human capital. This latter depends on agent's innate potential $\underline{h}$, and the human capital inherited from her/his parents h_{t+1}. In particular, human capital accumulates according to:

$$h_{t+2} = B\left(h_{t+1} + \underline{h}\right)\left(1 - l_{t+1} - qn_{t+1}\right), \tag{1.23}$$

where l_{t+1} is the time spent on work, q is the time spent to raise each child and n_{t+1} is the number of children. Hence $(1 - l_{t+1} - qn_{t+1})$ is the total time spent in education. Optimal conditions imply that the number of children and the supply of labor decreases with life expectancy, that is:

$$n_{t+1} = \frac{\gamma}{q\left(1 + \gamma + \pi_{t+1}\theta\right)}, \tag{1.24}$$

$$l_{t+1} = \frac{1}{1 + \gamma + \pi_{t+1}\theta}, \tag{1.25}$$

where θ is the discount factor and γ is the the utility weight on offspring, i.e. $\gamma u(n_{t+1})$. Combining equation (1.24) and (1.25) it follows that total time spent in education increases with life expectancy:

$$1 - l_{t+1} - qn_{t+1} = \frac{\pi_{t+1}\theta}{1 + \gamma + \pi_{t+1}\theta}, \tag{1.26}$$

Substituting equation (1.26) in equation (1.23) we obtain that increases in π_{t+1} has a positive effect on human capital accumulation by increasing the amount of time devoted to education, that is:

$$h_{t+2} = \frac{B\left(h_{t+1} + \underline{h}\right)\pi_{t+1}\left(h_{t+1}\right)\theta}{1 + \gamma + \pi_{t+1}\left(h_{t+1}\right)\theta}. \tag{1.27}$$

This framework produces multiple development regimes such that the growth of economy depends on initial conditions (see Figure 1.4).

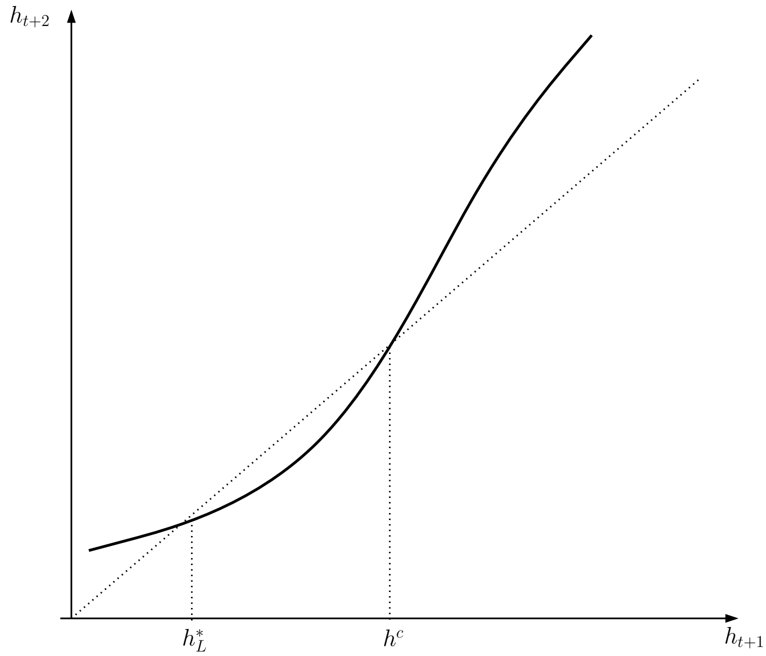

Figure 1.4: Multiple development regimes

There is a threshold level of human capital h_c, below which the economy is on a low development path and above which the economy is on high development path. An economy with an initial human capital below h_c converge to a low steady state h_L^* where life expectancy is low, education is low and fertility is high. An economy with an initial human capital above h_c is on a high development path, where life expectancy is high, education is high and fertility is low (see Figure 1.4). Thus, the initial stock of human capital determines the initial probability of survival and the initial allocation between education, working and child rearing. The demographic transition from the low to high equilibrium is determined by exogenous shifts in the stock of human capital that can push the existing level of human capital above the threshold (Blackburn and Cipriani, 2002).

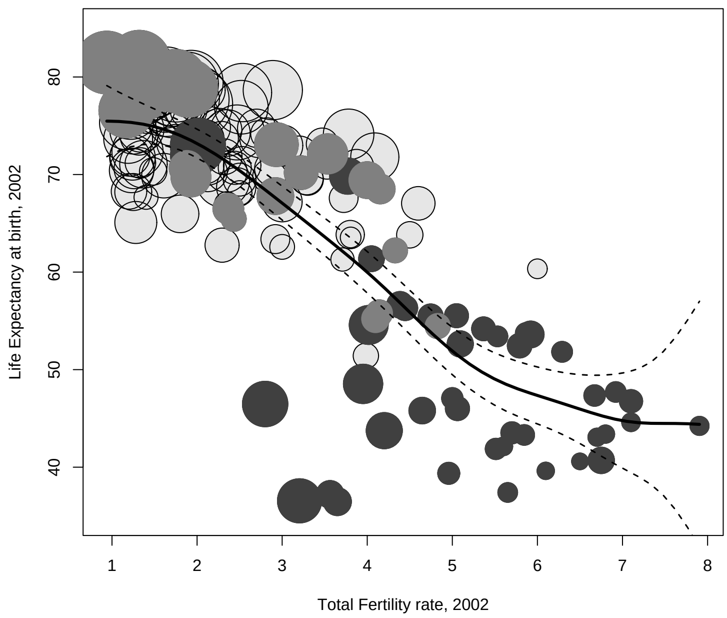

Figure 1.5: Life Expectancy versus Fertility. Source: World Development Indicators CD-ROM, World Bank (2006)

Looking at Figure 1.5 we may have an insight of the relationship between life expectancy and fertility (2002). The circles are proportional to the country's per capita income, the black ones indicate Sub-Saharan countries and the gray ones indicate east Asian and Pacific countries. Figure 1.5 supports the inverse relationship between life expectancy and fertility. In addition, in poor countries (black circles) low per capita incomes are associated to high fertility rate.

The recent contribution by Cervellati and Sunde (2005) analyzes the interactions between life expectancy and agent's decisions to acquire human capital, that is to be skilled or unskilled workers. The demographic transition is based on a positive relationship between life expectancy, human capital formation, and endogenous technological progress. Human capital is a central factor of production, affects positively longevity and the productivity of future generations. Individuals themselves, rather than their parents, decide about their education. Thus, the model exclude parent's investment in education, public provision of education and any link between generations, through savings and bequest.

Agents are endowed with a life expectancy T_t and can decide to invest part of their lifetime in education and supply skilled labor or supply, for all their lifetime T_t, unskilled labor which requires no education. In the first case they spent a fixed cost in terms of time (e), earning a wage $\left(w^H\right)$ for the time $T_t - e$. In particular, the total lifetime income for skilled individuals is:

$$V^H = a(T_t - e)w^H, \tag{1.28}$$

where a is agent's ability. Alternatively agents can supply unskilled labor earning the following income for the whole lifetime:

$$V^L = T_t w^L. \tag{1.29}$$

If $V^H = V^L$ then agent is indifferent whether or not to acquire human capital. The equality $V^H = V^L$ yields the following ability threshold:

$$\tilde{a}_t = \frac{T_t}{(T_t - e)} \frac{w^L}{w^H}. \tag{1.30}$$

Hence, agents with $a > \tilde{a}$ find it profitable to incur the cost and acquire human capital, while those with $a < \tilde{a}$ prefer to remain unskilled.

From the first order conditions, the equilibrium fraction of the population acquiring human capital λ_t, is given as follows:

$$\lambda_t = \Lambda^{-1}\left(T_t, A_t\right), \tag{1.31}$$

which comes from the following relationship between life expectancy and λ_t :

$$T_t = \Lambda\left(\lambda_t, A_t\right) = \frac{e}{1 - \Omega\left(A_t\right) g\left(\lambda_t\right)}, \tag{1.32}$$

where $\Omega\left(A_t\right) = 1/\left(2^{1-\alpha} A_t\right), A_t = A_t^H / A_t^L$, and $g(\lambda_t) = [(1-(1-\lambda_t)^2)^{1-\alpha}/(1-\lambda_t)^{2-\alpha}]$.

From equation (1.31) λ_t is increasing in life expectancy T_t and in the relative productivity of the human-capital intensive sector, that is A_t. Thus a higher life expectancy implies more people that invest a part of their lifetime to acquire education.

The relationship between longevity and human capital acquisition in equation (1.32) is nonlinear and S-shaped, it is more pronounced for intermediate values of T and λ. When life expectancy is low people prefer to remain unskilled since the fixed time cost (e) which the acquisition of education implies. On the other hand when λ_t is high, are necessary large increases in life expectancy to lead individuals to acquire human capital. The decreasing returns in both sector, indeed, imply that when there are few unskilled workers their marginal productivity is relatively large so that they do not find profitable to invest in human capital even if life expectancy is very high.

Cervellati and Sunde (2005) assume that life expectancy of generation t increases in the fraction of the population of the previous generation that acquired human capital (λ_{t-1}), that is:

$$T_t = \Upsilon(\lambda_{t-1}) = \underline{T} + \rho\lambda_{t-1}, \tag{1.33}$$

where $\rho > 0$ reflects the extent of the externality and $\underline{T} > e$ is the minimum life expectancy when $\lambda_{t-1} = 0$.

This framework captures two crucial features of the human capital formation process. First, as in Blackburn and Cipriani (2002), longer life expectancy motivates agents to accumulate human capital. Second higher innate ability, which is less important for providing manual labor, facilitates the acquisition of human capital.

Figure (1.6) illustrates the system given by equations (1.31) and (1.33) for a given level of technology $A > 0$ and with equation (1.31) defined for $T \in [e, \infty)$.

The system given by equations (1.31) and (1.33) displays at least one stable steady state and at most three steady state equilibria (Cervellati and Sunde, 2005). Figure (1.6) illustrates the system in the case of three equilibria. The low equilibrium is locally stable and it is characterized by low life expectancy and a small share of the population acquiring human capital, the high equilibrium is locally stable and is characterized by a relatively large fraction of skilled individuals and high life expectancy. The third equilibrium is unstable and shows a positive supply of skilled and unskilled labor. The

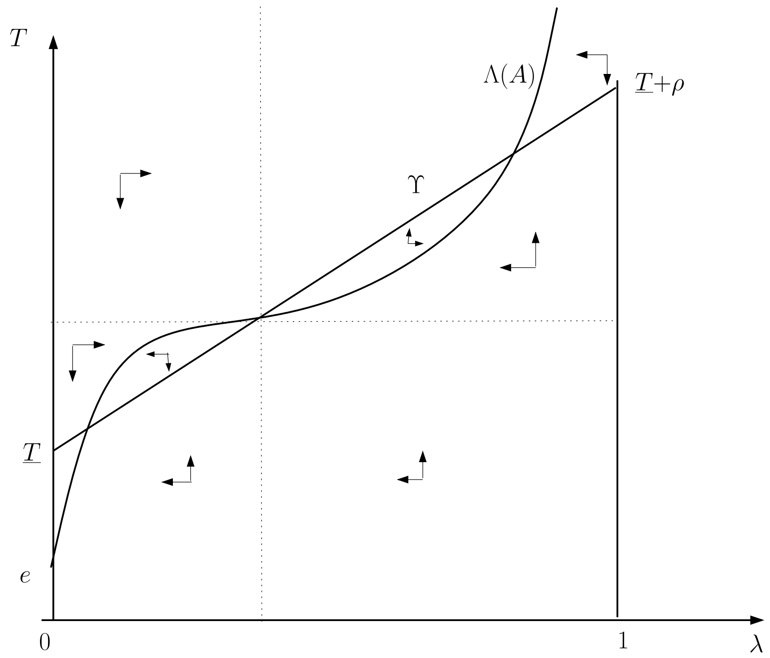

Figure 1.6: Phase Diagram

transition from the low steady state to the high equilibrium occurs generations by generations and requires an exogenous technological change, since during the phase of stagnancy people not find optimal to invest in education. In particular, an economy with sufficiently low A and sufficiently large e passes through the following phases of development: i) a period, that could be very long, of low-type equilibria; ii) a rapid transition toward a sequence of high-type equilibria; (iii) a phase of sustained growth (Cervellati and Sunde, 2005) .

Kalemli-Ozcan (2002) focuses on the effect of reductions in children's mortality on parent's choice about the quantity and quality of children. With respect to the work of Blackburn and Cipriani (2002) this model assumes altruistic agents which derive utility from consumption and the human capital of their surviving children, that is:

$$U_t = \sum_{N_t=0}^{n_t} \{\gamma \ln (c_t) + (1-\gamma) E_t [N_t w_{t+1} h_{t+1}]\} f(N_t; n_t, q_t),$$

where $N_t w_{t+1} h_{t+1}$ is the future income of surviving children. In particular

N_t is the number of survivors, w_{t+1} is the future wage of surviving children per unit of human capital and h_{t+1} is children's human capital. The function $f(N_t; n_t, q_t)$ is the probability that N_t out of n_t children will survive and q_t is the survival probability of each child.

Parents choose the number of children and the optimal amount of education to give each child. Agent's prudence on the uncertainty about child survival causes a precautionary demand for children. An exogenous decline in the mortality rate (q_t) implies a reduction in the uncertainty which in turn causes a lower precautionary demand of children and a rise in educational investment.

When the survival probability of each child is endogenous, that it is a concave function of the per capita income, $q_t = q(y_t)$, the income growth rate shows multiple development path. At low levels of income per capita, population increases with income causing a reduction of income per capita. Thus the economy is in a stable Malthusian steady state where fertility is high and human capital investment is low. At high levels of income per capita population growth falls as income per capita increases. This leads to a unstable growth steady state with low fertility and high human capital investment, above which sustained growth is achieved.

Soares (2005) develops a model where exogenous increases in life expectancy together with a stable relationship between life expectancy, investment in education and fertility, generate the demographic transition. In particular, the model incorporates the effect of both reductions in child and adult mortality on human capital accumulation.

This framework differs from the others for two main assumptions. First, the utility that parents derive from each child is affected by child mortality and adult longevity. This assumption implies that parents care about the number of children surviving into adulthood and the lifetime that each child will enjoy as an adult. In particular, the expected utility takes the following form:

$$U_t = T\frac{c^{\sigma}}{\sigma}dt + \rho(n, T, \beta)\frac{{h_c}^{\alpha}}{\alpha}, \tag{1.34}$$

where the subjective discount rate and the interest rate are equal zero, h_c is the human capital of each children and $\rho(n, T, \beta)$ is the discount factor which is a function of the number of children n, child mortality β and adult longevity T.

Second, the model distinguishes between investment of parents in their own human capital e and in that of their children b. In particular, adult human capital H_p depends on the time invested in education e and on the basic human capital h_p that individuals have when enter in adulthood:

$$H_p = Aeh_p + H_o, \tag{1.35}$$

where H_o is the level of adult's human capital when they do not invest in education. The basic idea is that adults, deciding the time to devote to education, can improve their basic level of human capital h_p. Adult human capital and the time invested in children b, determines the basic human capital of each child:

$$h_c = DbH_p + h_o, \tag{1.36}$$

where h_o is the children's human capital in the absence of investment in education.

The first order conditions show that increases in longevity affect the economy through two channels. First, the optimal time devoted to adult education increases with adult longevity since gains in adult lifetime increase the period over which returns from investments in education can be enjoyed, that is:

$$e = \frac{T}{2}. \tag{1.37}$$

This result leads to a higher productivity in the labor market and in raising children (see equations 1.35 and 1.36). Indeed, a higher investment in adult education raises the basic human capital of each child.

Second, since the optimal number of children n decreases with adult longevity, that is $dn/dT < 0$, the shadow price of h_c is reduced. Thus, a lower fertility and a higher household productivity makes relatively cheaper for parents to give more human capital to each child, that is $dh_c/dT > 0$.

Given equations (1.35) and (1.36), and assuming $Ho = 0$ and $ho = 0$, the growth rate of basic human capital is $h_c/h_p = DAbe > 1$. Hence from equation (1.37) longevity gains increase the steady state rate of growth of the economy.

When $h_p = h_o$, the economy is in a Malthusian equilibrium where there is no investment in education, that is the optimal individual choice implies $b = e = 0$. In this situation, changes in longevity are positively related to changes in both consumption and fertility, i.e. $dn/dT > 0$ and $dc/dT > 0$.

The demographic transition from the Malthusian equilibrium to the sustained growth is characterized as a consequence of successive reductions in mortality. Increases in adult life expectancy lead to higher returns from investment in adult education because of the longer period over which education is productive. When these gains are large enough, parents decide to start investing in their own education ($e > 0$). In the other hand, gains in adult longevity increase the amount of resources available implying an income effect that tends to raise fertility. However, the assumptions in relation to $\partial\rho/\partial n$ (that is $\partial\rho/\partial n = 0$ for n large enough) imply that increases in fertility tend to reduce its marginal utility to a point where parents start to reduce the number of children and to invest in their children ($b > 0$). After both these thresholds are reached the economy moves to the sustained growth (Soares, 2005).

The literature discussed so far identifies in the human capital the principal determinant of reductions in mortality. However, the explicit effect of health spending on life expectancy and, through this channel, the effect of health investment on economic growth is not analyzed. The recent health economics literature, founding on the earlier theoretical contributions on the demand of health, focus on the direct effect of health investment on life expectancy and the effect of both quantity and quality of life on economic growth.

1.4 Mortality and Health Spending

The recent health literature focuses on the explicit role of health spending on life expectancy, on the concept of the value of life and the willingness to pay criterion to reduce mortality risk. This studies emphasizes the central importance of both quality and quantity of life for the overall economic welfare. Welfare depends not only on income but also on the numbers of years over which this income is enjoyed (Becker et al., 2005). The income per capita, indeed, measures material gains that are only one of many aspects of life that enhance economic welfare. In accordance with this point, the paper of Becker et al. (2005) develops a growth model in which the monetary value of longevity is add to the observed gains in income per capita.

They estimate that when longevity changes are included in the income's growth rate, countries became significantly more equal between 1960 and 2000 (Becker et al., 2005). there is an average yearly growth of 4.1 percent, of which 1.7 percentage points are due to health. For the richest 50 percent of countries the average yearly growth is 2.6 percent and only 0.4 percentage points are due to health (Becker et al., 2005).

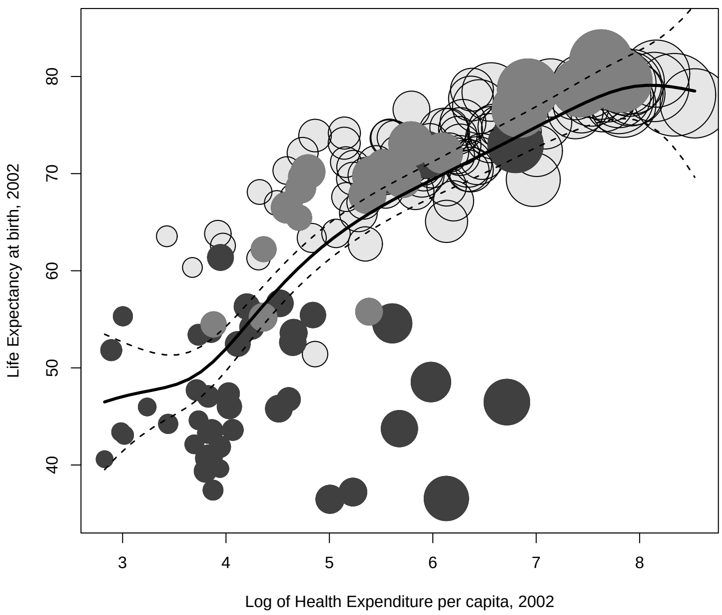

Figure 1.7: Life Expectancy versus Health Spending. Source: World Development Indicators CD-ROM, World Bank (2006)

They impute the strong decline in life expectancy inequality between 1960 and 1990 to the fact that countries starting with low longevity tended to gain more in life expectancy than countries starting with high longevity. Countries with higher initial mortality would have larger mortality reductions because they have much higher returns on investment in health than do countries with lower mortality (Becker et al., 2005).

Figure 1.7 shows the relationship between life expectancy at birth and the per capita health expenditure in 2002. In many poor countries low level of health spending are associated to low level of life expectancy.

We start the section with the demand of health developed in the Grossman (1972)'s seminal paper and then we present the recent results on the value of life and the willingness to pay criterion to reduce the mortality risk.

1.4.1 Demand for health

The seminal work of Grossman (1972) was the first one which develops a model for the demand of "health". Health, as other commodities, directly enters in the utility function and people demand health since it increases the quality of life and time available for market and nonmarket activities (Grossman, 1972). Thus, the intertemporal utility function takes the following form:

$$U = U\left(\phi_0 H_0, ...\phi_n H_n, Z_0 .....Z_n\right), \tag{1.38}$$

where H_0 is the inherited stock of health, H_n is the stock of health in the period n, ϕ_n is the service flow per unit of stock, $h_n = \phi_n H_n$ is the total consumption of health services and Z_n may be viewed as an aggregate of other commodities besides health (Grossman, 1972).

Health is treated as a durable commodity and agents inherit an initial stock of health that depreciates over time and can be augmented by investment. Death occurs when the stock of health falls below a minimum level i.e. H_m. In particular, the stock of health in the period $t+1$ is:

$$H_{t+1} = I_t + H_t\left(1 - \delta_t\right), \tag{1.39}$$

where I_t is the investment in health care and δ_t is the rate of depreciation during the period t. Health investment I_t depends on medical care, the stock of human capital and the time input. In particular, the household's production function of gross investments in health (Becker, 1965) is defined as follows:

$$I_t = I_t(M_t, TH_t; E_t), \tag{1.40}$$

where M_t are the medical care, TH_t is the time input and E_t is the stock of human capital.

The optimal investment in health is determined when the value of the marginal cost of gross investment in health is equal to the value of marginal benefits, that is:

$$\pi_{t-1}\left(r-\tilde{\pi}_{t-1}+\delta_t\right)=G_t\left[W_t+\frac{U_{h_t}}{\lambda}\left(1+r\right)^t\right], \tag{1.41}$$

where the left side is the supply price of health capital and contains the terms $\pi_{t-1} = W_{t-1}/(\partial I/\partial TH)$ which is the marginal cost of gross investment in health in period $t-1$, $\tilde{\pi}_{t-1}$ is the percentage rate of change in marginal cost between period $t-1$ and period t, r is the interest rate and δ_t is the depreciation rate. The right side is the value of the marginal product of the optimal stock of health capital. In particular, G_t is the marginal product of the stock of health in the production of healthy days, that is the increase in the number of healthy days caused by a one-unit increase in the stock of health. U_{h_t} is the marginal utility of healthy days, λ is the marginal utility of wealth. The wage rate W_t and the term U_{h_t}/λ convert the marginal product G_t into value terms. In particular, the discounted wage rate measures the monetary value of a one-unit increase in the total amount of time available for market and nonmarket activities. The term U_{h_t}/λ measures the discounted monetary equivalent of the increase in utility due to a one-unit increase in healthy time. Thus the sum of these two terms measures the discounted marginal value to consumers of the output produced by healthy time.

When health is assumed only a form of investment, i.e. it does enter in the utility function, the optimal health investment is given by the equilibrium between the demand and supply curve of health, that is:

$$r-\tilde{\pi}_{t-1}+\delta_t=\frac{G_tW_t}{\pi_{t-1}}. \tag{1.42}$$

The demand curve of health (right side of equation 1.42) is negatively inclined and shows the relationship between the stock of health and the marginal return of health. The supply curve (left side of equation 1.42), given by the cost of health capital, is infinitely elastic since it is independent on health. The demand curve is negatively related with health's shadow price which depends on the price of medical care, the agent's age, education, wealth and wage rates. The optimal amount of health capital is obtained when the cost of health investment is equal to the future benefit of the health capital (Grossman, 1972).

When the rate of depreciation increases with age the supply curve of health capital shifts upward. This determines a reduction of the quantity of health capital demanded over the life cycle. In particular, the greater is the elasticity of the health curve demand, the greater is the decrease in the optimal stock with age. An increase in agent's wage rate, implying a higher value of healthy time, shifts upward the demand curve. This lead to a higher health capital demanded.

A central result is the effect of variations in the level of human capital on the demand of health. The basic assumption is that more educated people are more efficient producers of money earnings. Consequently, shifts in human capital, measured by education, change the agent's productivity. The effect on gross health investment of one-unit change in education, i.e. $\partial I/\partial E$, is a weighted average of the percentage change in the marginal products of medical care M and time inputs TH (see equation 1.40). Because education raises the marginal efficiency of the inputs M and TH then it reduces the quantity of these inputs required to produce a given amount of gross investment. Hence, with no changes in input prices, an increase in the human capital lowers the marginal cost. In addition, if the wage rate and the marginal product of a given stock of health are held constant, an increase in the education would raise the marginal efficiency of health capital and shifts the demand curve to the right. This implies that the more educated people would demand a larger optimal stock of health (Grossman, 1972).

Ehrlich and Chuma (1990)'s paper is strictly related to the Grossman's seminal work on the demand for health. As the Grossman's model, they specify a demand function for longevity, or "quantity of life"to which corresponds demand functions for indicators of "quality of life"and a value of life extension function (Ehrlich and Chuma, 1990).

They start with Grossman's basic formulation, in continuous time, where health increases the lifetime utility and earnings capacity (see equation 1.38, 1.39 and 1.40). Health capital has two effects: firstly it augments the amount of healthy time available in any instant of life, secondly it delays the approach of death since the latter is assumed to occur when health deteriorates to its minimum "subsistence"level. Contrary to Grossman's model where health investment is produced through constant returns to scale they assume that health production, that is $I(t) = I(M, TH; E)$ is subject to decreasing returns to scale. In particular, the cost of health investment is:

$$C(I) = \pi I(t)^{\alpha} \qquad \alpha > 1, \tag{1.43}$$

where $\pi = \pi(M, TH; E)$ denotes the one-unit cost of producing $I(t)$ and it supposed constant over time. Optimal health investment is determined when the marginal cost of health investment intersect the shadow price of health capital, that is:

$$\alpha\pi I(t)^{\alpha-1} = \frac{\lambda_H(t)}{\lambda_A(t)}, \tag{1.44}$$

where the left hand side represents the marginal cost of producing health and the right hand side the shadow price which is given by the ratio between the marginal utility of health λ_H and the marginal utility of wealth λ_A. Given the optimal health investment in equation (1.44), they emphasize the central role of initial wealth $A(0)$ rather than current income in determining longevity. Their analysis shows that an increase in $A(0)$ leads to an increase in lifetime wealth and raises the entire paths of health stocks and consumption over the life cycle. The main result is that health behaves as a superior good in response to a higher initial wealth level, i.e. health would rise with initial wealth in order to obtain a higher longevity.

In addition, Ehrlich and Chuma (1990) stress the effect of the time preference rate for consumption on the health demand. Even in the case in which current health makes no contribution to current utility, a higher rate of time preference reduces the demand for longevity and thus optimal health investment (Ehrlich and Chuma, 1990). The basic idea is that there exists an implicit trade-off between "quality of life", obtained through an initial consumption level, and longevity. Thus a higher rate of time preferences increases current consumption, i.e. the demand for the quality of life and reduces the propensity to save, i.e. the demand for longevity (Ehrlich and Chuma, 1990).

The current variables, such as education, the rate of health depreciation, the wage and the price of medical cares affect the demand of longevity through the wealth effects that they generate. Agent's level of education, for example, affects positively the demand of health, in a given wage level, both since more educated people invest better in health and since education generates a wealth effect. Finally, the uncertainty of the incidence of illness implies that agents purchase extra medical care and increase their saving as a precaution against future periods of illness.

1.4.2 Value of life and willingness to pay to reduce mortality risk

Ehrlich and Yin (2005) develop a model to derive a theoretical measure of the "value of life saving". The basic idea is to quantify the importance of individual efforts at health and life protection in explaining the trend and persistent disparities in age specific life expectancies across population groups (Ehrlich and Yin, 2005). A central aspect of their approach is the correspondence between efforts to reduce mortality risk, that is "life protection"measures s, and the definition of "value of life saving".

There are two states of the world: "life" with probability $1 - p(s)$, and "death" with probability $p(s)$. Agents enjoy a higher utility in the state life, that is:

$$U = \begin{cases} U(W-s) & \text{with probability } 1 - p(s) \\ V(W-s) & \text{with probability } p(s), \end{cases}$$

where W is the wealth constraint and $U(W - s) > V(W - s)$.

The "willingness to pay"for a marginal reduction in the probability of mortality or the "value of life saving"is given by the equality between the marginal cost of life protection and its marginal benefit, that is:

$$-1/p'(s) = \frac{U(W-s) - V(W-s)}{(1-p(s))U'(W-s) + p(s)V'(W-s)}, \tag{1.45}$$

where the numerator defines the difference in utility between being alive or dead and the denominator is the marginal expected utility of wealth.

Ehrlich and Yin (2005) estimate for the US population in 1996 that the value of life saving exhibits an inverted U shape; it starts a \$1.236 at age 18 (in 1996 dollars), peaks at \$1.440 million at age 38 and falls monotonically at the early 60s. This inverted U shape reflects the influence of the life-cycle profiles of human wealth. The human wealth represents the capitalized sums of remaining net earning flows, that is the difference between employment earning and expenditure on self protection, discounted for both the cost of future funds and mortality risk. Based on markets wage, human wealth peaks at age 29, then decreases continuously and becomes negative after age 78 which is the last age with projected positive earnings. This nonlinear path of the human wealth reflects both increasing expenses for life protection and increasing risks of mortality with age.

A higher degree of relative risk aversion increases the value of life saving and the demand for life protection. A higher bequest preferences lowers the demand for life protection, that is if agents give a high value to the bequest for their descendants then the marginal value of longevity decreases. An increase in the wage implies higher earnings which raise the human wealth and thus the demand for life protection, since the only way to secure future earnings is through survival. Finally, education raises the efficiency of life protection, the opportunity cost of time devoted to it and, the incentive to protect the wealth through survival. This latter effect is based on the assumption that more educated people posses relatively higher wealth.

According to the concept of "value of life saving", the Murphy and Topel (2006)'s paper emphasizes that the valuation of improvements in health is important since they are a form of economic progress. Health improvements presents a value which increases with the population, the lifetime income, the existing level of health and the closer is the age of population to the age of disease. They distinguish two types of health improvements: those that increase life expectancy and those which raise the quality of life. A higher life expectancy allows agents to enjoy utility from goods and leisure for a longer period of life, improvements in the quality of life raise utility from a given amount of goods and leisure. Improvements in agent's health level tends to be complementary, that is an increase in life expectancy tend to increase the willingness to pay for further health improvements by increasing the value of remaining life.

They distinguishes two types of technologies. The first $H(t)$ raises the quality of life without affecting mortality. For example technologies that improve mental health may increase instantaneous utility without affecting mortality. $H(t)$ enters in the utility function and affect the quality of life by increasing the utility from consumption $c(t)$ and nonmarket time $l(t)$. The second $G(t)$ affects mortality without affecting the quality of life. Thus $G(t)$ affects the survival function which is specified as follows:

$$\tilde{S}(t,a) = e^{-\int_a^t \lambda(\tau,G(\tau))d\tau},$$

where $\tilde{S}(t,a)$ is the probability of surviving from age a to t and $\lambda(\tau, G(\tau))$ is the instantaneous mortality rate at the period τ. They assume a perfect annuity market, that is the expected discounted value of future consumption equals expected wealth.

From the first order conditions, the value of life at age a is:

$$V_\lambda(a) = \int_a^\infty v(t)\, e^{-r(t-a)} \tilde{S}(t,a)\, dt, \tag{1.46}$$

where $v(t)$ is the value of a life year, that is:

$$v(t) = \frac{u(c(t), l(t))}{u'(c(t), l(t))} + y(t) - c(t), \tag{1.47}$$

which is given by the value of utility and net savings at age t. Savings affect $v(t)$ because they finance consumption in other periods. The term H does not appear explicitly in equation(1.46) since the model assumes that it raises the total utility and the marginal utility of consumption by the same proportional amount. However, H affects the rate of change in the value of a life year $\dot{v}(t) = f(\dot{H})$. That is, life years become less valuable as health deteriorates and persons in declining health are more impatient.

The willingness to pay for any factor α which affects health is given as follows:

$$V_\alpha(a) = \int_a^\infty v(t)\, S_\alpha(t,a) + \int_a^\infty \frac{H'_\alpha(t)}{H(t)} \frac{u(c(t), l(t))}{u_c} S(t,a)\, dt, \tag{1.48}$$

where the first term is the value of additional lifetime utility from changes in mortality and the second term is the value of changes in H. From equation

(1.48) and (1.47) we can see that the willingness to pay for health includes the value of non-market time and rises with wealth, that is richer societies invest proportionally more in health.

Strictly related to the willingness to pay criterion and the value of life saving is the recent contribution of Jones and Hall (2006). The principal aim of the paper is to explain, through agent's preferences, the causes of the rise in health spending in the United States in the period 1950-2000. They present a framework in which agents have to allocate their total resources between consumption c and health investment h. Agents do not save so there is not any reference to the intergenerational transmission of wealth.

As in the Murphy and Topel (2006)'s model, utility depends on the length of life and the quality of life. Spending on health therefore affects utility in two ways: i) by increasing the quantity of life through a mortality reduction and ii) by increasing the quality of life. However, in this model the quality of life is given by the consumption whereas in the model of Murphy and Topel there is some medical services which affects only the quality of life.

Optimal conditions imply that the ratio of health spending to consumption is equal to the ratio of the elasticities of health production function and the flow utility function, that is:

$$\frac{h}{c} = \frac{\eta_h}{\eta_c}, \tag{1.49}$$

where $\eta_h = f'(h)h/f(h)$, $f(h)$ is the health production function which governs the individual's state of health and $\eta_c = u'(c)c/u(c)$.

Adding a constant to the standard utility function they can explain the rising path of health share. In particular, this behavior depends on increasing returns of health spending and the decreasing marginal utility of consumption. As income rises people prefer to devote more resources to health care, which allows to live additional years of life rather than to the consumption.

The optimal health spending is determined by the equality between the marginal benefit of saving a life and its marginal cost, that is:

$$\frac{\beta v_{a+1,t+1}}{u_c} + \frac{u_{f(h)} f(h_{a,t})^2}{u_c} = \frac{f(h_{a,t})^2}{f'(h_{a,t})}, \tag{1.50}$$

where the left hand side is the marginal benefit of saving a life which is given by the sum of the social value of life $\beta v_{a+1,t+1}/u_c$ and the additional utility which agents enjoy as a result of the increase in health status $\left[u_{f(h)}f(h_{a,t})^2\right]/u_c$. In particular, $v_{a+1,t+1} = \partial V_t/\partial N_{a,t}$ denotes the change in social welfare V_t associated with having an additional person of age a alive ($N_{a,t}$ is the number of people of age a alive at time t). The right hand side is the marginal cost of saving a life, that is the ratio between the increase in resources devoted to health care and the reduction in mortality rate. In particular, $1/f\prime(h)$ is the cost of increasing the survival rate $f(h)$.

Many other contributions stress the luxury good behavior of health spending. For example, Chakraboty and Das (2005) investigates the relationship between health and wealth inequality. They analyze a two-period overlapping generations model with altruistic parents who leave a part of their earnings to their children as bequest at the end of the second period (b_{t+1}). The probability of survival from the first period of life to the next depends on private health spending, i.e. $p = \phi(h)$ and it is specified as follows:

$$\phi(h) = \begin{cases} ah^{\varepsilon}, & \text{if } h \in \left[0, \hat{h}\right] \\ \phi, & h > \hat{h}. \end{cases} \tag{1.51}$$

When parents do not survive to the second period, their savings are passed on to their offspring as unintended bequest. From the first order conditions optimal health investment is a function of wealth W_t, that is:

$$h_t = \eta(W_t). \tag{1.52}$$

In particular health behaves as a luxury good if the elasticity of intertemporal substitution of the CES instantaneous utility function σ is greater than ε (see equation (1.51)).

Parent's low income status transmits to her descendants since endogenous mortality implies that poorer people are more likely die prematurely so they leave their offspring lower assets. In particular, intergenerational wealth dynamics follows a nonlinear Markov process, that is:

$$W_{t+1} = \begin{cases} b^*_{t+1}(W_t) & \text{with probability } \phi(h(W_t)) \\ s^*_t(W_t) & \text{with probability } 1-\phi(h(W_t)). \end{cases} \tag{1.53}$$

They assume that parents altruism is very high, that is:

$$\theta > (1/\bar{r})^{\sigma}, \tag{1.54}$$

where θ is the parameter in the expected utility function which measures agent's altruism and $\bar{r}$ is the constant interest rate. The assumption in equation (1.54) ensures that:

$$b^*_{t+1}(W_t) > s^*_t(W_t), \tag{1.55}$$

which implies that agents receive a larger amount of resources when parents survive to the second period.

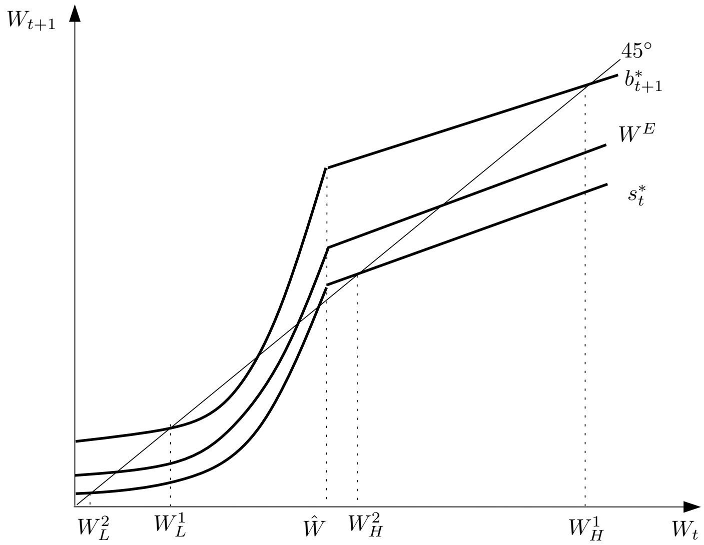

Figure 1.8: Wealth Dynamics

From equation (1.53) the expected bequest is equals to:

$$W^E = \phi(h(W_t))b^*_{t+1}(W_t) + [1 - \phi(h(W_t))]\, s^*_t(W_t). \tag{1.56}$$

Figure 1.8 shows the wealth dynamics in equation (1.53) and the expected bequest in equation (1.56). Poor families with a wealth level lower to the threshold level, that is $W < \bar{W}$ converge on the support $[W_L^2, W_L^1]$, and rich families with a wealth level $W > \bar{W}$ converge to a high income equilibrium. The long run persistence of wealth and health inequality is due to the dependence of health and mortality to economic status.

1.5 Conclusions

This chapter presents an overview of the main contributions which focus on the effects of health and longevity on economic growth. A part of this literature assumes exogenous life expectancy and shows that the mortality reductions can promote economic growth in poor countries whereas can lead to an aging of population in the rich ones. Although, this literature shows important effects of mortality reductions on economic growth it does not explain the mechanism through which life expectancy increases.

A second strand of literature analyzes how mortality declines affect fertility decisions and investments in human capital. This contributions show that poor countries can be trapped in a Malthusian equilibrium with a low life expectancy, a low investment in human capital and a high fertility rate. The opposite occurs in rich countries where increases in human capital are associated to increases in life expectancy which in turn stimulate investments in education. However in this literature longevity depends human capital and does not analyze the explicit effect of health investment on life expectancy.

Finally, we review the recent health literature which focuses, principally, on the agent's demand for health and the value of life saving. In this literature health is treated as a commodity that agents demand to increase their utility. One of the main results of these contributions is that health presents a luxury good behavior. That is, when income is low, people do not spend in health care, whereas for high level of income agents prefer to devote a higher proportion of their income to additional years of life rather than to the consumption.

Chapter 2

Human Capital Accumulation and Longevity

2.1 Introduction

This chapter focus on the dynamic interaction between life expectancy and human capital accumulation. In particular, we explore the intergenerational transmission of inequality and its persistence through life expectancy.

The literature which analyzes the relationship between economic growth and longevity can be divided in two groups. The first group, assumes exogenous life expectancy and shows that increases in life expectancy tend to have a positive effect in poor countries and a null or negative effect in rich countries. This is because starting from a high mortality rate an increase in life expectancy is imputable principally to a decline in infant's death rates. Population growth rates rise rapidly, making the population younger, not older. Rates of return to investment in human capital rise. However, starting from a low mortality rate such as is found in most industrial populations, the net effect of a further decline in mortality is to reduce the growth rate.

The second group assuming life expectancy affected by the level of development in the country, shows the existence of multiple development regimes. There is a threshold level of human capital: below the threshold the economy is on a low development path, while above the threshold the economy is on a high development path. Correspondingly, there is a low steady state in which life expectancy is low, education is low, and an equilibrium with positive long-run growth in which life expectancy and education are high.

Dealing with the first group De la Croix and Licandro (1999) and Boucekkine et al. (2002) show that higher longevity produces two opposite effects: (i) it leads to human capital accumulation since people devote more time of their life to education and (ii) an aging of the population which implies a reduction in the saving rate. The net effect of increases in life expectancy on growth is positive for economies with a relatively low life expectancy, and null or negative in more advanced economies. In rich countries the positive effect of longevity on human capital accumulation could indeed be offset by an increase in the average age of the workers.

Zhang et al. (2003) develop an overlapping generation model with non altruistic parents. They show that a decline in mortality on one side raises the saving rate and thereby increases the rate of physical capital accumulation, on the other side it reduces the endowment of the next generations. In poor countries the net effect of the increasing longevity is positive. On the contrary in rich countries this effect could be negative.

The second group shows that at low levels of income population growth rises as income per capita rises leading to a Malthusian steady-state equilibrium, whereas at high levels of income population growth declines leading to a sustained growth steady state.

On this respect Kalemli-Ozcan (2002), Blackburn and Cipriani (2002) and Lagerlof (2003) argue that in rich countries an increase in life expectancy raises the opportunity cost of current work and reproduction by raising the future returns to human capital accumulation. Under such circumstances, agents devote more of their time to education and have fewer numbers of children when young, implying a higher growth rate of output and a lower growth rate of population. At low levels of income per capita population growth rises as income per capita and life expectancy rise. This in turn leads to dilution of resources and results in lower income per capita and hence in a stable Malthusian steady state.

This chapter departs from this literature by stressing the effect of longevity on the intergenerational transmission of inequality. We start to analyze the effect of exogenous longevity on economic growth and then we suppose life

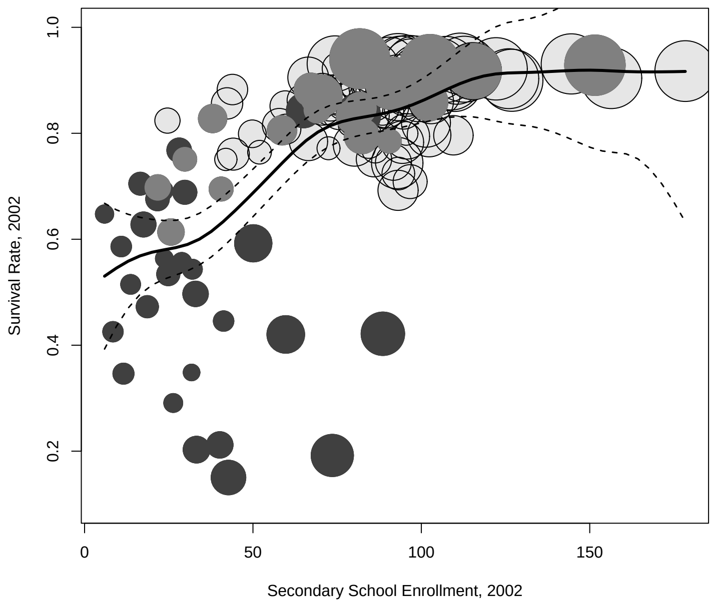

Figure 2.1: Longevity versus Education. Source: World Development Indicators CD-ROM, World Bank (2006)

expectancy depending on human capital. In particular we consider altruistic parents which leave a bequest to their children and when they do not survive to the old age leave also the saving accumulated in the working age as unintended bequest. We analyze an overlapping generations economy where agent's life expectancy extends probabilistically to three periods. In the first period agents acquire formal education, in the second period individuals work and receive a wage proportional to the amount of human capital acquired in the first period. This income is allocated between current consumption, saving for old-age consumption and the bequest for the next generation. When old, individuals live in retirement and consume entirely the saving. The main result of this model is that the economy displays a multiple development regimes as in, for example, Blackburn and Cipriani (2002); Kalemli-Ozcan (2002). According to the literature we suppose that longevity depends on agent's human capital. On this respect, Figure 2.1 shows the cross-countries relationship between life expectancy at birth and education [1].

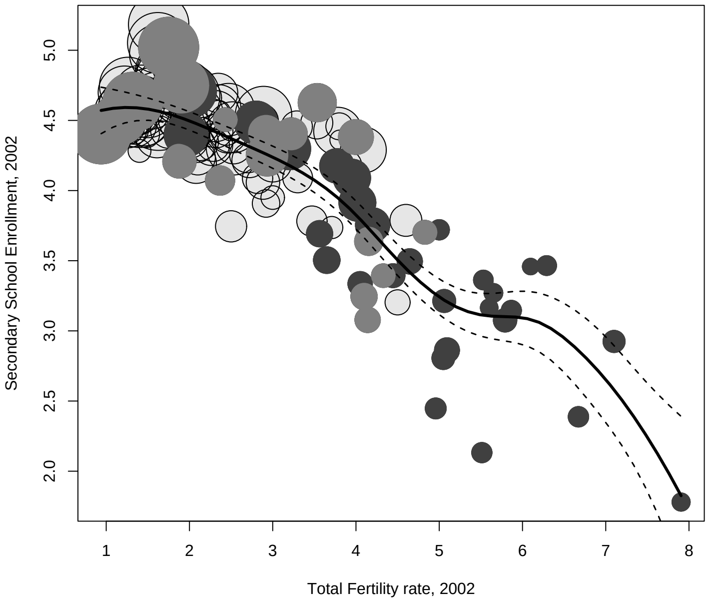

Figure 2.2: Education versus Fertility. Source: World Development Indicators CD-ROM, World Bank (2006)

The circles are proportional to the country's income. In particular the black circles indicate the sub-saharian countries whereas and the gray circles show the east Asian countries. We can see that high levels of education are associated to high longevity. Finally Figure 2.2 shows the inverse relationship between education and fertility[2]. Poorest countries show a high fertility rate

[1]We use the secondary gross enrollment ratio as a proxy of education (Source WDI 2006). The gross enrollment ratio is given by the ratio of total enrollment, regardless of age, to the population of the age group that officially corresponds to the level of education shown. Secondary education completes the provision of basic education that began at the primary level, and aims at laying the foundations for lifelong learning and human development, by offering more subject or skill oriented instruction using more specialized teachers (World Bank, 2006).

[2]Total fertility rate represents the number of children that would be born to a woman if she were to live to the end of her childbearing years and bear children in accordance with prevailing age-specific fertility rates (World Bank, 2006).

and a low education whereas in rich countries education is high and fertility is low.

The chapter is organized as follows. The model is set out in Section 2. In section 3 we analyze the growth rate of the economy when life expectancy is exogenous. Section 4 extends the analysis to consider an endogenous survival probability in an economy with representative agent.

2.2 The model

We suppose an economy where altruistic agents potentially live three periods: childhood, youth and old age. Each adult has $n \geq 1$ children which have a probability $\pi \in (0, 1]$ of surviving to the second period. In particular, as soon as individuals are born they face the child mortality rate and, if they survive, become adults (Soares, 2005). Adult agents have a probability $p \in (0, 1)$ of surviving to the old age.

In the first period agents learn in school , in the second period work, and in the third period live in retirement. In the working age, people receive an income (y_{t+1}) which is composed by a constant share (w) and an amount proportional to the human capital accumulated in the first period (h_{t+1}), that is:

$$y_{t+1} = w(1 + h_{t+1}). \tag{2.1}$$

This income is allocated between current consumption, saving for old-age consumption and the investment in children's education. Hence, agent's budget constraint, in the second period is given:

$$c_{t+1} = w(1 + h_{t+1}) - s_{t+1} - b_{t+1}, \tag{2.2}$$

where s_{t+1} is the saving and b_{t+1} is the amount devoted to finance children's education.

In the third period agents do not work and consume entirely the saving accumulated in the working age, that is:

$$c_{t+2} = Rs_{t+1}, \tag{2.3}$$

where $R = 1 + r_{t+1}$ is the interest rate which is assumed constant.

The total resources devoted to children's education depends on the mortality of their parents: when parents survive to the old age their children will receive the planned transfer b_{t+1}, otherwise when parents die before entering to the old age their children will receive the planned transfer b_{t+1}, and the saving accumulated to finance the consumption in the old age s_{t+1}[3]. Thus, parents leave to each child who survives to the second period the amount $b_{t+1}/\pi n$ with a probability p, and $(b_{t+1} + s_{t+1})/\pi n$ with a probability $(1-p)$. Hence the total resources devoted to finance education of each child, i.e. e_{t+1}, are given by[4]:

$$e_{t+1} = \begin{cases} b_{t+1}/\pi n & \text{with probability } p \\ (s_{t+1} + b_{t+1})/\pi n & \text{with probability } (1-p)\,. \end{cases} \tag{2.4}$$

Adult human capital together with the investment in children education in equation (2.4) determines the human capital of children (De la Croix and Michel, 2002):

$$h_{t+2} = h_{t+1}^{1-\alpha} e_{t+1}^{\alpha}, \qquad \text{with} \qquad 0 < \alpha < 1,$$

Assuming zero utility from death (Rosen, 1988), the intertemporal utility function of an adult agent is given by[5]:

$$\begin{aligned} U &= \log(c_{t+1}) + \beta p \log(c_{t+2}) + \\ &+ \phi \pi n \left[p \log\left(\frac{b_{t+1}}{\pi n}\right) + (1-p)\log\left(\frac{b_{t+1} + s_{t+1}}{\pi n}\right)\right]. \end{aligned} \tag{2.5}$$

[3]This saving could be considered as the whole of all activities that an individual accumulates in young adulthood to finance the consumption in retirement. These financial resources are available for the children when their parents do not survive to the old age. We suppose that the saving is devoted to finance education. However this saving could be devoted to finance children's consumption. In this case the results of our analysis could change.

[4]We do not accept the hypothesis that the total returns from the savings of those who are deceased before reaching their old age will be equally redistributed, in the form of a lump-sum transfer, to the remaining survivors within the same generation.

[5]Following Rosen (1988) the expected utility in the second period is given by the utility of consumption if agents survive to the second period and the utility of death if they do not survive, that is:

$$EU = p\log(c_{t+1}) + (1-p)\,M.$$

where M is the utility of death that we assume equal to zero.

where $0 < \beta < 1$ is the psychological discount factor [6] and $0 < \phi < 1$ the agent's sensibility to the resources left to his/her offspring. The first two terms denote the utility that parents derive from consumption, and the second two terms denote the utility that they derive from children who survive to the second period. In particular, the last two terms in equation (2.5) define the expected utility from the amount devoted to children's education which is equal to the planned transfer with probability p and to the parent's saving and the planned transfer with probability $(1-p)$ (see equation (2.4)). The parent's intensity of altruism, i.e.ϕ, is the same for both types of investment in education.

The first order conditions associated with s_{t+1} and b_{t+1} are as follows:

$$\frac{1}{c_{t+1}} = \frac{\beta p}{s_{t+1}} + \frac{\phi(1-p)\pi n}{b_{t+1}+s_{t+1}}, \tag{2.6}$$

$$\frac{1}{c_{t+1}} = \frac{\phi p \pi n}{b_{t+1}} + \frac{\phi(1-p)\pi n}{b_{t+1}+s_{t+1}}, \tag{2.7}$$

from which:

$$s_{t+1} = \frac{\beta}{\phi \pi n} b_{t+1}. \tag{2.8}$$

From equations (2.6) and (2.7) we get the optimal saving and the optimal bequest (see Appendix A.1) as follows:

$$s_{t+1} = \frac{w(1+h_{t+1})\beta\left(\beta p + \phi\pi n\right)}{\left(\phi\pi n + \beta\right)\left(1 + \beta p + \phi\pi n\right)}, \tag{2.9}$$

$$b_{t+1} = \frac{w(1+h_{t+1})\phi\pi n\left(\beta p + \phi\pi n\right)}{\left(\phi\pi n + \beta\right)\left(1 + \beta p + \phi\pi n\right)}. \tag{2.10}$$

Saving increases with longevity, i.e. $\partial s_{t+1}/\partial p > 0$, since gains in adult lifetime increase the consumption needs for a longer retirement period. In particular, from equations (2.4) and (2.9) we can see that increase in p affects the economy through two channels: first, higher longevity lead to higher saving;

[6] In particular we have that:

$$\beta = \frac{1}{(1+\sigma)}$$

where σ is the rate of time preference, which varies inversely with β.

second if parents survive to old age, they consume entirely the saving accumulated in the working age so that their children will receive b_{t+1} rather than $b_{t+1}+s_{t+1}$. These effects of life expectancy on the intergenerational transmission of wealth are in accordance with the literature on exogenous longevity (see for example Zhang et al. (2003)). Increases in adult longevity raises the planned transfer b_{t+1} devoted to children's education, that is $\partial b_{t+1}/\partial p > 0$. A higher probability of surviving to the second period (π) or a higher number of children (n) increase b_{t+1}, i.e. $\partial b_{t+1}/\partial \pi > 0$ and $\partial b_{t+1}/\partial n > 0$, since parents should invest in education of more children. Hence, a higher n or π, implies that parents have lower resources to devote to saving, i.e. $\partial s_{t+1}/\partial \pi < 0$ and $\partial s_{t+1}/\partial n < 0$ (see Appendix A.1).
Substituting equations (2.9) and (2.10) in equation (2.4) the optimal education spending for each child is given as follows:

$$e_{t+1} = \begin{cases} \frac{w(1+h_{t+1})\phi(\beta p+\phi\pi n)}{(\phi\pi n+\beta)(1+\beta p+\phi\pi n)} & \text{with probability } p \\ \\ \frac{w(1+h_{t+1})(\beta p+\phi\pi n)}{\pi n(1+\beta p+\phi\pi n)} & \text{with probability } (1-p)\,, \end{cases} \tag{2.11}$$

from which the expected education spending for each child is a weighted mean of the education spending in the state p and in the state $(1-p)$, that is:

$$E\left[e_{t+1}\right] = \frac{w(1+h_{t+1})\left(\beta p+\phi\pi n\right)}{(1+\beta p+\phi\pi n)}\left[\frac{\phi\pi n+\beta\left(1-p\right)}{(\phi\pi n+\beta)\pi n}\right]. \tag{2.12}$$

Thus if parents survive to the old age, i.e. $p=1$, then $E\left[e_{t+1}\right]_{p=1} = b_{t+1}|_{p=1}$. When parents do not survive to the old age $E\left[e_{t+1}\right]_{p=0} = s_{t+1} + b_{t+1}|_{p=0}$. The aim of the next section is to analyze the role played by exogenous changes in adult longevity, in the children probability of surviving to the second period and in the fertility rate on human capital accumulation.

2.3 Exogenous Adult Longevity, Fertility and Children Mortality

Given the optimal education spending from equation (2.11) above, human capital follows a nonlinear Markov process:

$$h_{t+2} = \begin{cases} h_{t+1}^{1-\alpha}\left[\frac{w(1+h_{t+1})\phi(\beta p+\phi\pi n)}{(\phi\pi n+\beta)(1+\beta p+\phi\pi n)}\right]^{\alpha} & \text{with probability } p \\ h_{t+1}^{1-\alpha}\left[\frac{w(1+h_{t+1})(\beta p+\phi\pi n)}{\pi n(1+\beta p+\phi\pi n)}\right]^{\alpha} & \text{with probability } (1-p) \end{cases} \tag{2.13}$$

where the expected human capital is a weighted mean (with weigh p) of human capital in the state p and the human capital in the state $(1-p)$, that is:

$$\begin{aligned} E(h_{t+2}) &= h_{t+1}^{1-\alpha}\left[\frac{(w(1+h_{t+1})\beta p+\phi\pi n)}{\pi n(1+\beta p+\phi\pi n)(\phi\pi n+\beta)}\right]^{\alpha}\cdot \\ &\cdot \quad [p(\phi\pi n)^{\alpha}+(1-p)(\phi\pi n+\beta)^{\alpha}]. \end{aligned} \tag{2.14}$$

Thus the growth factor of human capital depends on the wage and on the variation in the adult's mortality rate, children's mortality and the fertility rate.

Given the expected human capital in equation (2.14) we suppose that the economy grows in the long run, which means that the following condition holds:

$$w > \frac{\pi n\,(1+\beta p+\phi\pi n)\,(\phi\pi n+\beta)}{(\beta p+\phi\pi n)\,[p\,(n\phi\pi)^{\alpha}+(1-p)\,(\phi\pi n+\beta)^{\alpha}]^{1/\alpha}}. \tag{2.15}$$

Figure (2.3) shows h_{t+2} in both states p and $1-p$ and the corresponding expected human capital $E(h_{t+2})$ (in particular we choose the following calibration: $\phi = 0.3$, $\pi = 0.6$, $n = 2$, $w = 2.7$, $\alpha = 0.6$, $\beta = 0.4$ and $p = 0.5$). Human capital in the state $1-p$ is higher than human capital in the state p since when parents do not survive to old age a larger amount of resources is invested in education (see equation 2.11).

In Figure 2.3 h^S is the equilibrium in the state p and h^D is the equilibrium in the state $(1-p)$. Given the assumption in equation (2.15) are both unstable. In particular, the equilibria in the two states are given as follows:

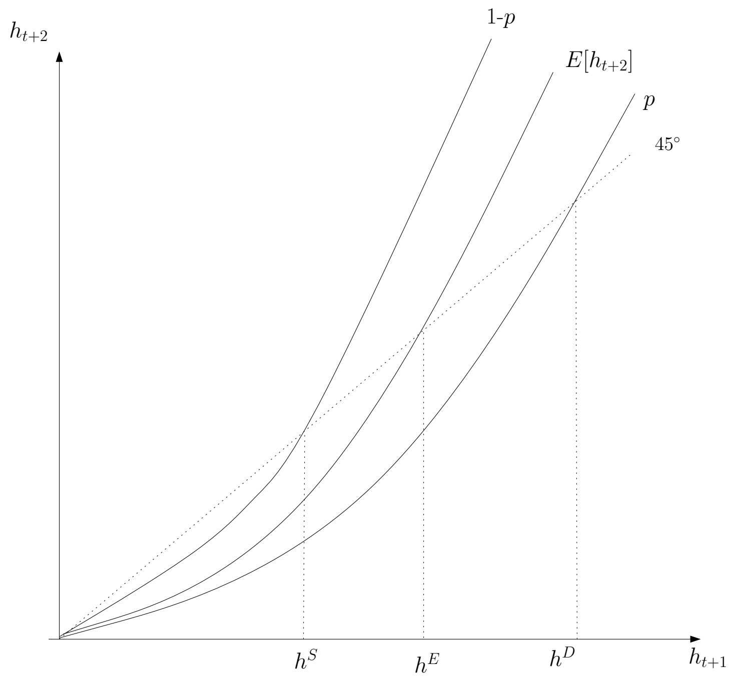

Figure 2.3: Human Capital accumulation and Exogenous Longevity

$$h^S = \frac{w\phi\pi\left(\beta p + \phi\pi n\right)}{\left(\beta p + \phi\pi n\right)\left(\phi\pi n + \beta - \phi w\right) + \phi\pi n + \beta},$$

$$h^D = \frac{w\left(\beta p + \phi\pi n\right)}{\left(\beta p + \phi\pi n\right)\left(\pi n - w\right) + \pi n}.$$

For a given values of other parameters, an increase in p raises the equilibrium in both states p and $1-p$, i.e. $\partial h^S/\partial p > 0$ and $\partial h^D/\partial p > 0$.

The effect of exogenous changes in adult longevity on the expected human capital depends on the value of initial adult mortality, that is when p is lower than a certain threshold, i.e. p^* (see appendix A.2) increases in longevity have a positive effect on human capital, that is $\partial E\left[h_{t+2}\right]/\partial p > 0$. When $p > p^*$ increases in life expectancy have a negative effect on human capital accumulation, i.e. $\partial E\left[h_{t+2}\right]/\partial p < 0$ (see Appendix A.2).

Proposition 1 *An exogenous increase in adult's longevity has a positive effect on the expected human capital if the initial life expectancy is below a certain threshold p^*. When life expectancy is above this threshold p^* an increase* *in adult's life expectancy has a negative effect on human capital accumulation.*

Proof. For technical details see appendix A.2 ■

To understand the effect of exogenous changes in life expectancy on human capital accumulation, we analyze the effect of longevity on the expected investment in education given by equation (2.12).

Figure 2.4: Education Spending

When parents do not survive to the old age, i.e $p = 0$, children's education spending is higher than when parents survive to the old age, that is (see Figure 2.4):

$$E\left[e_{t+1}\right]_{p=0} - E\left[e_{t+1}\right]_{p=1} = \frac{\phi\beta}{(1+\phi\pi n)\,(1+\beta+\phi\pi n)} > 0.$$

The expected education spending is nonlinear with respect to longevity: for low levels of life expectancy it increases and for high levels of life expectancy it decreases (see Appendix A.3). In particular, the expected education shows the maximum value when the probability of surviving to the first period is equal to:

$$\hat{p} = \frac{(1+\beta+2n\pi\phi)^{1/2} - (1+n\pi\phi)}{\beta}, \tag{2.16}$$

which is positive if the following condition is satisfied:

$$\beta > (n\pi\phi)^2 .$$

Therefore, when $p < \hat{p}$ the expected education spending increases and when $p > \hat{p}$ it decreases (see Appendix A.3). Thus, when the initial adult mortality is high, exogenous increases in adults life expectancy improve the investment in children's education, whereas when initial adult mortality is low, rising longevity leads to a reduction in children's investment in education (De la Croix and Licandro, 1999; Zhang et al., 2003).

This path depends on the two opposite effects of increases in adult longevity on saving and bequest. First it affects positively both the saving and the planned transfer ($\partial s_{t+1}/\partial p > 0; \partial b_{t+1}/\partial p > 0$). Rising longevity, indeed, requiring higher consumption in old age, leads individuals to save more, in the working age, to finance increased consumption needs in the retirement. The second effect is that when parents do not survive to the old age their children will receive both the planned transfer b_{t+1} and s_{t+1} (see equation (2.11)) otherwise children will receive b_{t+1}.

A higher fertility rate or an increase in the probability of surviving to the second period imply lower resources for each child (indeed $\partial E\left[e_{t+1}\right]/\partial n < 0$ and $\partial E\left[e_{t+1}\right]/\partial \pi < 0$ see Appendix A.2.1 and A.2.2) which in turn has a negative effect on human capital accumulation, that is:

$$\frac{\partial E(h_{t+2})}{\partial n} < 0$$

$$\frac{\partial E(h_{t+2})}{\partial \pi} < 0$$

Proposition 2 *Exogenous increases in the fertility rate or in the children's survival probability have a negative effect on human capital accumulation*

Proof. The technical part of this proposition is proved in Appendix A.2.1 and A.2.2 ■

Finally an increase in the fertility rate or a reduction in children's mortality move to the left the turning point in Eq (2.16). Indeed:

$$\frac{\partial \hat{p}}{\partial n} = -\frac{\pi\phi}{\beta}\left[\frac{(1+\beta+2n\pi\phi)^{1/2}-1}{(1+\beta+2n\pi\phi)^{1/2}}\right] < 0$$

$$\frac{\partial \hat{p}}{\partial \pi} = -\frac{n\phi}{\beta}\left[\frac{(1+\beta+2n\pi\phi)^{1/2}-1}{(1+\beta+2n\pi\phi)^{1/2}}\right] < 0$$

In accordance with the literature (Kalemli-Ozcan, 2002; Blackburn and Cipriani, 2002) the model shows a trade-off between quantity and quality of children. An increase in the fertility rate or in the children's probability of surviving to the second period leads to a reduction in the expected level education. Indeed a higher number of children, implies lower resources for the next generations in both states.

2.4 Endogenous longevity

In this section we analyze the effect of longevity when it is endogenously specified. In particular, we focus on the probability of an adult agent of surviving to the second period whereas the probability of surviving to the second period is assumed exogenous. We suppose that life expectancy depends on human capital: a higher agent's education implies a higher willingness to invest in health care either because education makes people better decision makers or because more educated people have better informations about health. Hence we define the probability of surviving to the second period as a function of human capital:

$$p = p\left(h_{t+1}\right), \tag{2.17}$$

Following empirical evidence (Figure 2.1) longevity is assumed to satisfy the following properties:

$$\partial p/\partial h_{t+1} > 0; \tag{2.18}$$

$$\partial^2 p/\partial h_{t+1}^2 < 0; \tag{2.19}$$

$$p\left(0\right) = \underline{p} \geq 0; \tag{2.20}$$

$$\lim_{h\to\infty} p\left(h\right) = \bar{p} \leq 1. \tag{2.21}$$

Following Blackburn and Cipriani (2002) we specify the probability of surviving to the old age as:

$$p = \frac{\underline{p} + \overline{p}\gamma\left(h_{t+1}\right)^{\eta}}{1+\gamma\left(h_{t+1}\right)^{\eta}}, \tag{2.22}$$

where the parameters $0 < \eta \leq 1$ and $\gamma > 0$ jointly determine both the turning point in $p'(h)$ and the speed at which $p(\cdot)$ traverses the interval $(\underline{p}, \overline{p})$. For a given value of η, an increase (decrease) in γ reduces the turning point, while for a given value of such a point, an increase (decrease) in η raises the speed of transition (the limiting case of which is when $p(\cdot)$ changes value from $\underline{p}$ to $\overline{p}$ instantaneously, which corresponds to the case of a step function (Blackburn and Cipriani, 2002). For simplicity we assume that $\eta = 1$. This function satisfies the properties in equations (2.18), (2.19), (2.20) and (2.21), that is:

$$p' = \frac{\gamma(\overline{p} - \underline{p})}{(1+\gamma h)^2} > 0,$$

$$p'' = \frac{-2\gamma}{1+\gamma h} < 0$$

and it is bounded in $\underline{p}$ and $\overline{p}$, that is:

$$\lim_{h\to 0} p = \underline{p};$$

$$\lim_{h\to\infty} p = \overline{p}.$$

The dynamic of human capital accumulation is obtained by combining equations (2.22) and (2.13):

$$h_{t+2} = \begin{cases} h_{t+1}^{1-\alpha}\left[\frac{w(1+h_{t+1})\phi(\beta p(h_{t+1})+\phi\pi n)}{(\phi\pi n+\beta)(1+\beta p(h_{t+1})+\phi\pi n)}\right]^{\alpha} & \text{with probability } p \\ \\ h_{t+1}^{1-\alpha}\left[\frac{w(1+h_{t+1})\beta p(h_{t+1})+\phi\pi n}{n\pi(1+\beta p(h_{t+1})+\phi\pi n)}\right]^{\alpha} & \text{with probability } (1-p) \end{cases} \tag{2.23}$$

The growth rate , i.e. g, when h_{t+1} goes to zero is as follows:

$$\lim_{h\to 0} g = \begin{cases} \left[w\left(\frac{1+h_{t+1}}{h_{t+1}}\right)\frac{\phi\left(\beta\underline{p}+\phi\pi n\right)}{(\phi\pi n+\beta)\left(1+\beta\underline{p}+\phi\pi n\right)}\right]^{\alpha} - 1 = \infty & \text{with probability } p \\ \\ \left[w\left(\frac{1+h_{t+1}}{h_{t+1}}\right)\frac{\beta\underline{p}+\phi\pi n}{n\pi\left(1+\beta\underline{p}+\phi\pi n\right)}\right]^{\alpha} - 1 = \infty & \text{with probability } (1-p)\,. \end{cases} \tag{2.24}$$

We suppose that the economy growth in the long run, that is:

$$\lim_{h\to\infty} g^{LR} = \begin{cases} \left(\frac{w\phi}{\phi\pi n+\beta}\right)\left(\frac{\beta\bar{p}+\phi\pi n}{1+\beta\bar{p}+\phi\pi n}\right) > 1 & \text{with probability } p \\ \\ \left(\frac{w}{\pi n}\right)\left(\frac{\beta\bar{p}+\phi\pi n}{1+\beta\bar{p}+\phi\pi n}\right) > 1 & \text{with probability } (1-p)\,. \end{cases} \tag{2.25}$$

An increase in the probability of surviving to the second period or a high number of children have a negative effect on the growth rate in the long run in both states, that is[7]:

$$\frac{\partial g^{LR}}{\partial \pi} < 0; \frac{\partial g^{LR}}{\partial n} < 0.$$

This result confirms the existence of a trade off between quantity and quality of children.

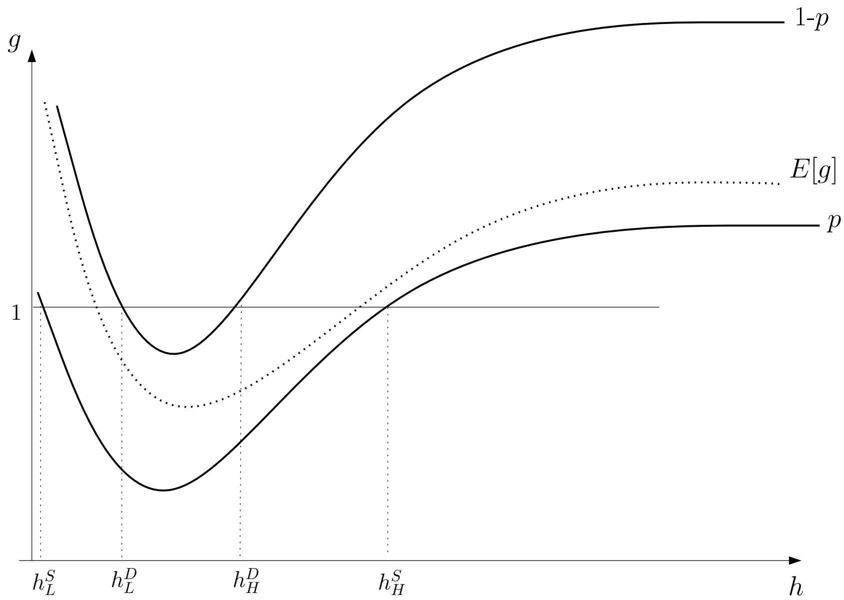

Figure 2.5: Human Capital Accumulation: Poverty Trap

The expected growth rate presents a stable steady state if both curves in equation (2.23) show a stable steady state (see Figure 2.5)[8]. In this case an economy that start with a level of human capital $h_0 < h_L^S$ or a $h_0 < h_H^D$ converges to the interval $\left[h_L^S, h_L^D\right]$. An economy with an initial level of human

[7]In particular in the state p we assume that:

$$\frac{\partial g^{LR}}{\partial \pi} = \frac{\partial g^{LR}}{\partial n} = \beta\left(1 - \overline{p}\right) - \left(\beta\overline{p} + \phi\pi n\right)^2 < 0$$

[8]Our reference calibration is $\pi = 1$, $f = 1$, $\underline{p} = 0.2$, $\overline{p} = 0.9$, $\eta = 1$, $\gamma = 0.01$, $\alpha = 0.6$, $w = 2.7$, $\beta = 0.4$, $\phi = 0.3$.

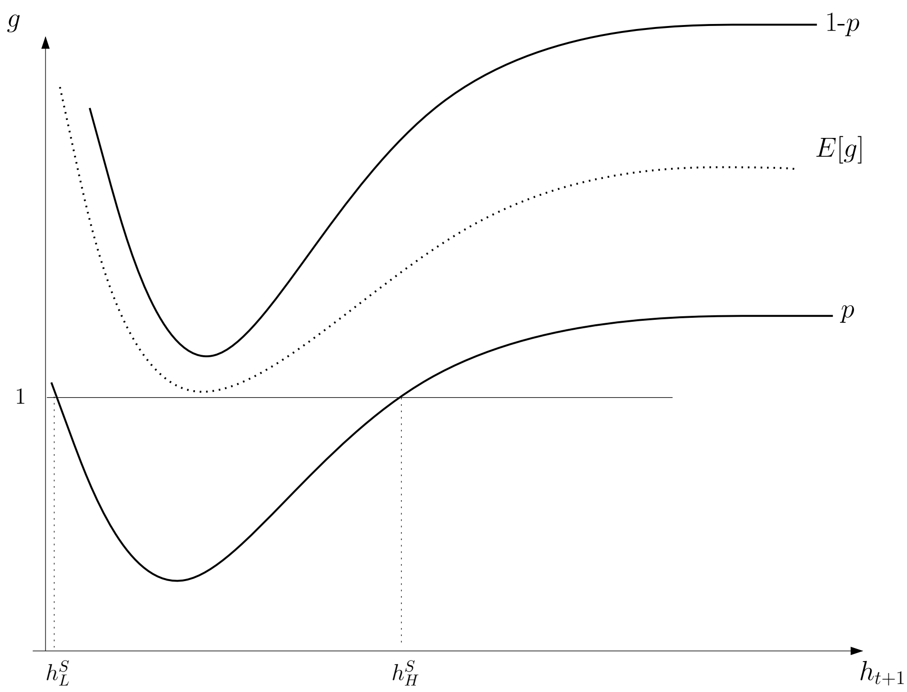

Figure 2.6: Human Capital Accumulation: No Poverty Trap

capital $h_0 > h_H^S$ shows a positive growth rate and an economy that start with $h_H^D < h_0 < h_H^S$ converges to the interval $\left[h_L^S, h_L^D\right]$ with probability p and grows in the long run with probability $1 - p$.

Proposition 3 *When the economy start with a human capital $h_0 < h_L^S$ and $h_0 < h_H^D$ it converges to the interval $\left[h_L^S, h_L^D\right]$. For high levels of initial human capital, i.e. $h_0 > h_H^S$ the expected human capital rises leading to a sustained growth. Finally if $h_H^D < h_0 < h_H^S$ the economy converges to the interval $\left[h_L^S, h_L^D\right]$ with probability p and it grows a the long-run growth rate with probability $1 - p$.*

Proof. See Appendix A.4. ■

Figure (2.6) shows the convergence of both poor and rich economies to the long-run expected growth rate.

Therefore since the distribution in the long run depends on the path of the growth rate in the state $(1 - p)$ we focus on this state. From equations (2.24)

and (2.25) we know that the growth rate goes to infinity when the human capital tends to zero and that the economy grows in the long run.
The growth rate in the state $(1-p)$ shows two equilibria, that is h_L and h_H, if the following condition is satisfied (see appendix A.4):

$$\gamma < \hat{\gamma}, \tag{2.26}$$

where $\hat{\gamma}$ is defined in appendix A.4. In particular the economy shows a minimum growth rate h_{min} if the two following conditions are satisfied (see appendix A.4):

$$\gamma > \frac{\underline{p}}{\overline{p}},$$

and:

$$\gamma > \overline{\gamma}.$$

Figure (2.7) illustrates the growth rate in the death state (our reference calibration is $\pi = 1$, $f = 1$, $\underline{p} = 0.2$, $\overline{p} = 0.9$, $\eta = 1$, $\gamma = 0.01$, $\alpha = 0.6$, $w = 2.7$, $\beta = 0.4$, $\phi = 0.3$.).

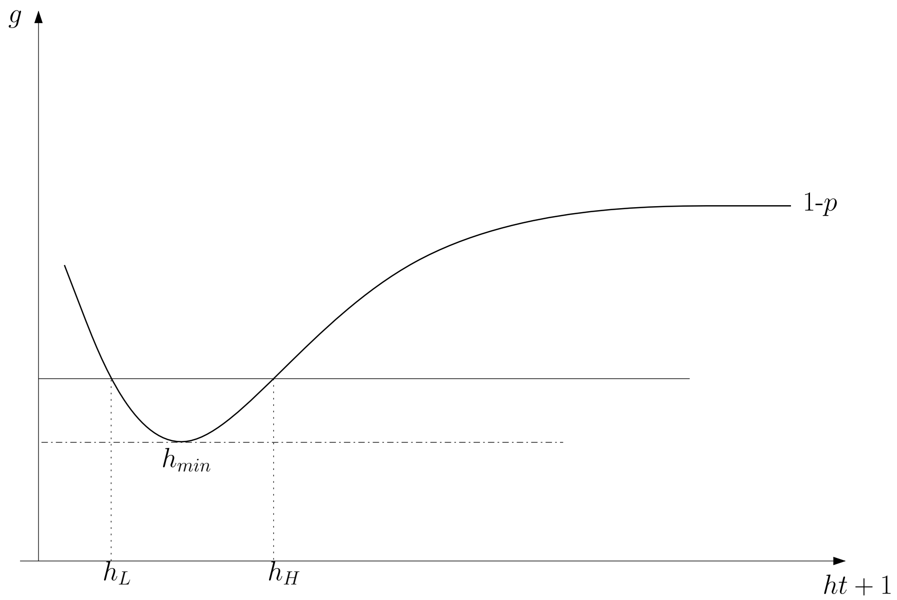

Figure 2.7: Human Capital Accumulation in the death state

It is nonlinear, it first decreases and then increases. An economy with an initial human capital $h_0 < h_L$ or $h_L < h_0 < h_H$ converges to the stable steady state h_L. In the opposite an economy with an initial human capital $h_0 > h_H$ shows a positive growth rate in the long run. Therefore to the left of h_L , the economy is on a low development path where life expectancy is low and education spending is low. The human capital level h_L acts as threshold determining the persistence of intergenerational human capital inequality. The following proposition describes the conditions under which the human capital growth rate presents two steady state (the technical aspects are proved in A.4).

Proposition 4 *Human capital growth rate shows two steady states if* $\gamma < \hat{\gamma}$. *The low steady state, i.e.* h_L, *is locally stable whereas the high steady state, i.e.* h_H, *is unstable.*

Proof. See Appendix A.4 ■

2.5 Concluding remarks

In this chapter we analyze the effect of increases in life expectancy on human capital accumulation. In particular, we focus on the effects of adult's mortality on the intergenerational transmission of wealth. When longevity is exogenous, it exerts opposite effects on economic growth. On one hand, an increase in longevity leads to higher savings because the increased consumption needs in later life. On the other hand, a high longevity reduces accidental bequests. This implies lower resources for the next generation. The net outcome depends on the initial level of mortality. When initial life expectancy is low, the positive effects dominate and hence rising longevity stimulates growth. When initial life expectancy is high, however, the negative effects tend to dominate and thus rising longevity tends to hinder growth.

When longevity depend on human capital the economy shows multiple development regimes. The economies with a low initial human capital converge to a stable steady state where human capital is low and mortality is high. The economies with a high human capital show a high life expectancy, a high level of human capital and show a positive growth rate in the long run.

Chapter 3

Life Expectancy, Health Spending and Saving

3.1 Introduction

Through the last two centuries, economic development gradually contributed to the increase in the human life span. In 1840 life expectancy at birth was 40 years in England, 44 years in Denmark and 45 years in Sweden (Livi-Bacci, 2001). According to recent life tables, in 2004 life expectancy at birth in England, Denmark and Sweden is 78, 77 and 80 years respectively. In particular, in most developed countries, life expectancy at birth is around 80 years (World Development Indicators 2006).

The increase in life expectancy has significant implications for various aspects of the society. In the literature, Bloom et al. (2003), Kageyama (2003), and Zhang et al. (2003), for example, show that increases in life expectancy lead to higher savings rates. This is because agents, in the working age, increase their saving to finance higher consumption needs in old age (Modigliani and Brumberg, 1980). Blackburn and Cipriani (2002) analyze the relationship between life expectancy, human capital and fertility. However, in this literature life expectancy is exogenous or depends on the level of human capital. Thus the explicit role of health investment on life expectancy is not analyzed.

Some theoretical contributions focus on the willingness of people to pay to reduce mortality risk. The willingness to pay criterion is based on the principle that living is a generally enjoyable activity for which consumers should be willing to sacrifice other pleasures (Murphy and Topel, 2006; Ehrlich and Yin, 2005).

Strictly related to the willingness to pay criterion is the Grossman's (1972) paper which analyzes the demand for the commodity "good health". In this model agents demand health since it increases the time available for market and non market activities. Indeed, a rise in the stock of health reduces the amount of time lost for these activities and the monetary value of this reduction is an index of the return to the investment in health (Grossman, 1972). A central result of the Grossman model is that the consumer's demand for health and medical care is positively correlated with his\her wage rate and his\her education level.

The recent paper of Jones and Hall (2006) considers the optimal choice between length of life and consumption. They show that health is a superior good, that is as income rises the marginal utility of consumption falls quickly more than the marginal utility of health spending.

The aim of our paper is to analyze the direct effect of health investment on life expectancy. This framework allows us to investigate the agent's decision on the allocation of total resources between saving and health investment, i.e the consumption in old age and the length of life. We analyze a two-period overlapping generations model in which agents work in the first period and live in retirement in the old age. Health investment is an activity that increases the quality of life and the probability of surviving from the first period of life to the next. Longevity depends on agent's specific health level which in turn offers an important contribution to agent's enjoyment of life (Ehrlich and Chuma, 1990). On the other hand, agents can ensure a good quality of life in the old age by increasing the saving in the working age.

Empirical evidence shows that both health spending and saving, i.e. the consumption when old, appear to be luxury goods but their behavior is strongly different according to the level of per capita GDP. The share of saving on GDP appears to be concave with respect to per capita GDP. On the opposite, the share of health expenditure on GDP increases more than proportionally with respect to per capita GDP. The ratio of saving to health investment is nonlinear with respect to per capita GDP, it is first increasing and then decreasing.

In the proposed model, the ratio of health spending to saving is equal to the ratio between the elasticity of the survival function and the elasticity of the utility function. We prove that the model can replicate empirical results if the utility function is HARA (hyperbolic absolute risk aversion) and the survival function presents a non-constant elasticity with respect to health investment. We show that CES (constant elasticity of substitution) preferences don't allows to understand the different path of saving and health spending.

The structure of the paper is outlined as follows. Section 1 presents empirical analysis. Section 2 introduces the general model. Section 3 discusses some possible specification of the instantaneous utility function and the survival function. Section 4 demonstrates that using HARA (hyperbolic absolute risk aversion) preferences we can replicate empirical results. Finally, section 5 draws some concluding remarks.

3.2 Empirical evidence

The data used in the analysis are taken from World Development Indicators (World Bank, 2006), they are for the period 1960-2005 and cover 208 countries. In Figure 3.1 we present a recent version of the Preston curve (1975), that is the international relationship between adult survival rate[1] and per capita GDP in purchasing power parity. Whereas Preston (1975) uses the data on life expectancy we use the data on the survival rate that is less sensitive to child mortality. This is because we are interested in adult's health investment decisions to improve his\her probability of surviving to old age[2]. We estimate the Preston curve using a cross-country nonparametric

[1] The survival rate is the difference between 1 and adult mortality rate. The adult mortality rate is defined from the World Bank as the probability of dying between the ages of 15 and 60, that is, the probability of a 15-year-old dying before reaching age 60, if subject to current age specific mortality rates between ages 15 and 60.

[2] However if we use the data on life expectancy the path of the life expectancy with respect to the per capita income is very similar to the path of the survival rate.

regression (year 2002, 158 countries).

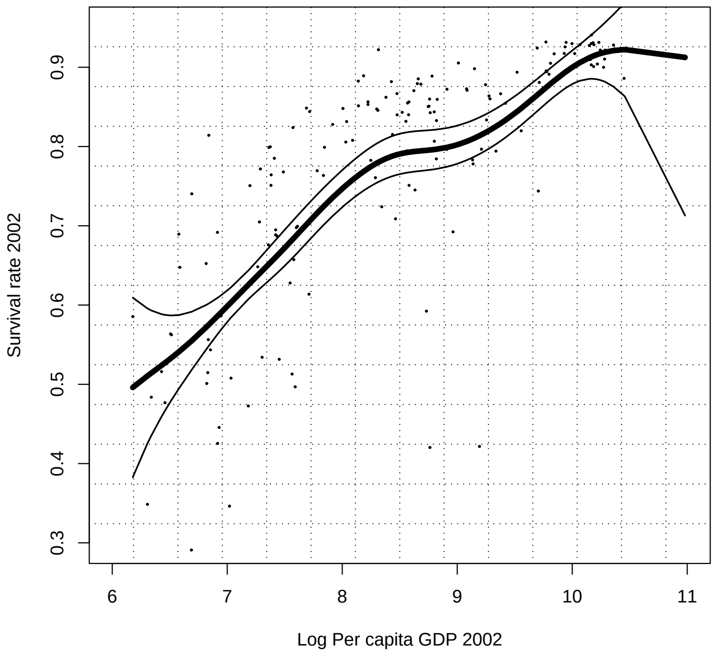

Figure 3.1: The Preston Curve: Survival Rate versus GDP Per Capita. Nonparametric kernel smoother (bandwidth = 0.45), year 2002, n = 158. Source: World Development Indicators CD-ROM, World Bank (2006)

We prefer to perform nonparametric regression since it allows us to investigate the relationship between dependent variable and one or more explanatory variables, without making any a priori explicit or implicit assumption about the shape of such relationship. The confidence interval in Figure 3.1 identifies clearly a positive relationship between survival rate and per capita GDP. In particular, the confidence interval is an indication of the degree of variability present in the estimate but it cannot be used to draw firm conclusions about the shape of the curve in particular regions [3] To asses the shape of the curve we carry out a test which compare nonparametric regression

[3]The confidence interval describes the level of variability present in the estimate without attempting to adjust for the inevitable presence of bias. The wideness of the confidence interval is determined by an estimate of the standard error (Bowman and Azzalini, 1997;

with a simple linear regression. This test indicates that the relationship between survival rate and the per capita GDP, can be represented by a linear model, i.e. the significance test for the nonparametric regression shows a $p - value = 0.119$. However, in Figure 3.1 we can see that the relationship is not clearly linear, indeed, in low income countries, increases in the per capita GDP are strongly associated with increases in life expectancy, as income per head rises the relationship flattens out. This path reflects the influence of a country's own level of income on mortality through such factor as nutrition, education, leisure and health expenditure. With respect the latter factor Figure 3.2 shows the direct relationship between survival rate and per capita health investment in 2002 for 155 countries. Per capita health investment includes both public and private expenditures on health. It covers the provision of health services (preventive and curative), family planning activities, nutrition activities, and emergency aid designated for health but does not include provision of water and sanitation (World Bank, 2006). The relationship between survival rate and per capita health is clearly positive and can be represented by a linear model ($p - value = 0.618$). However, like the Preston curve, figure 3.2 shows that countries with low health expenditure tend to gain more in life expectancy than countries starting with high level of health spending.

In figures 3.3 and 3.4 we examine the path of health expenditure and saving with respect to income. The aim is to analyze the behaviour of health spending with respect to different level of income and the relationship between health investment and saving; i.e. on one side agents, investing in health, can increase their length of life, on the other side a high saving imply more consumption in old age.

Figures 3.3 and 3.4 show nonparametric regressions for the saving on GDP, the health expenditure on GDP and the ratio between saving and health. In particular, we perform a pooling of all observation in the period 1997-2002 for 147 countries.

Figure 3.3 shows that both health expenditure on GDP and saving on

---

Hardle et al., 2004). However the confidence interval cannot be used to asses the shape of the curve in particular regions. This is not only because the presence of bias, but also because of the pointwise nature of the bands (Bowman and Azzalini, 1997).

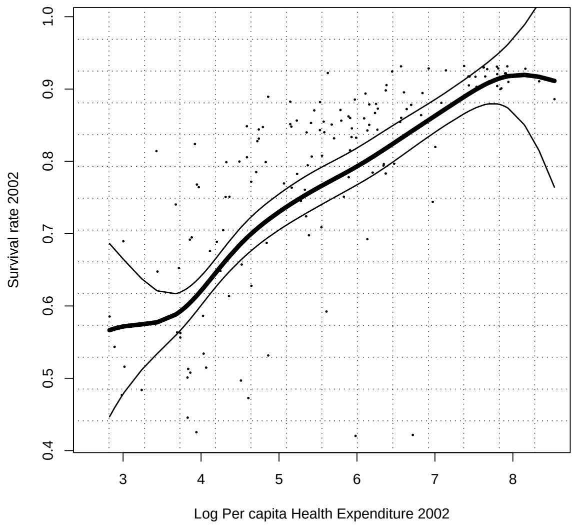

Figure 3.2: Survival rate versus per capita Health Expenditure. Nonparametric kernel smoother (bandwidth = 0.53), year 2002, $n = 155$. Source: World Development Indicators CD-ROM, World Bank (2006)

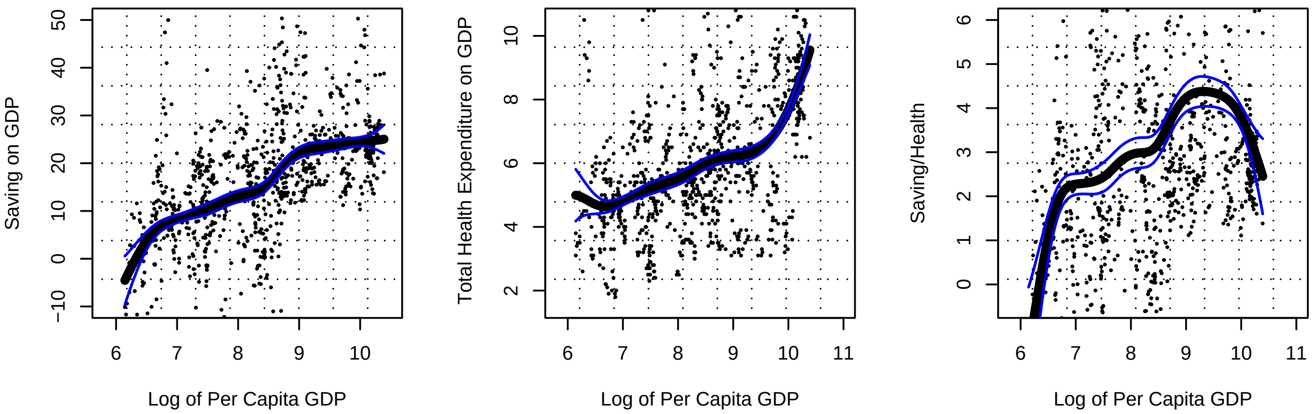

Figure 3.3: Saving and Health versus GDP Per Capita. Nonparametric kernel smoother (bandwidth = 0.31), years from 1997 to 2002, $n = 863$. Source: World Development Indicators CD-ROM, World Bank (2006)

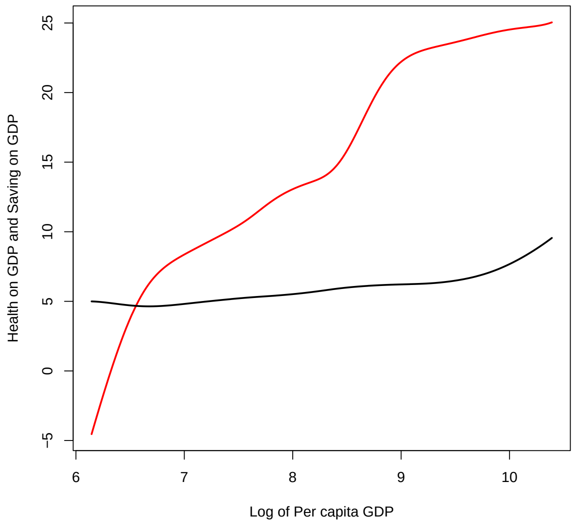

Figure 3.4: Saving Share and Health Share. Years from 1997 to 2002. Source: World Development Indicators CD-ROM, World Bank (2006)

GDP present a luxury goods behavior. However the path of health share and saving share is strongly different according to different levels of per capita GDP. The share of saving on GDP appears to be a concave function with respect to per capita GDP, i.e. the comparison between nonparametric regression and a simple linear model yields that the linear model can be refused ($p - value = 0$). In the opposite, health spending on GDP increases more than proportionally with respect to per capita GDP. The test for a linear model provides indication that a simple linear model is inappropriate ($p - value = 0$). The path of the ratio between saving and health expenditure is clearly nonlinear, it is first increasing and then decreasing ($p - value = 0$). This suggests that the investment in health increases faster than the saving when a country is sufficiently developed. The intuition is that as income increases, the saturation occurs faster in saving than in health spending.

Figure 3.4 compares the path of saving share and health share. We can see that when the log of per capita GDP is very low (6 and 7) the saving share is below the health investment. This result can be explained by the fact that health investment covers a part of public expenditure as emergency aid. When income increases health on GDP grows more quickly rather than the saving share.

It is possible to give different explanations for this luxury good behavior of health expenditure. One explanation can be the progressiveness of the tax schedule since the average tax rate increases with income. Others explanations are based on individuals preferences. The idea is that as income grows individual preferences extend not only on the amount of the good consumed but also on the length of life which allows to enjoy additional period of utility (Jones, 2004; Jones and Hall, 2006). In other words, when people became richer decide to increase the consumption of health services to extend their life expectancy. In the next section we propose a model based on the latter explanation.

3.3 A general model

In this section we present a general model to analyze agent's decision about the allocation of total resources between saving and health spending.

We consider an overlapping generations economy in which agents live for two periods "youth" and "old age". At the end of the first period agents give birth to a single child. Parents are non altruistic and when they do not survive to the old age, their saving is passed on their offspring as unintended bequest. Hence in the first period of life agents inherit a certain amount of wealth as unintended bequest[4], $b_t \geq 0$, and work receiving a constant wage equal to $\overline{w}$. The total resources of agents, i.e. $y_t = \bar{w} + b_t$, are allocated between current consumption, health expenditure and saving for the old age consumption . Thus, in the first period, the budget constraint of the representative agent is:

$$c_t = y_t - m_t - s_t, \tag{3.1}$$

[4]The unintended bequest b_t is given by the saving of the parents that did not survive to the old age, that is:

$$b_t = (1 - p_{t-1})s_{t-1}.$$

This implies that in period t agents whose parents die prematurely have higher endowment. In the proposed model we assume that the initial distribution of wealth is given.

where m_t is the health investment[5] and s_t is the saving.

In the second period agents live in retirement and consume entirely their savings, hence the budget constraint in the old age is:

$$c_{t+1} = s_t R, \tag{3.2}$$

where R is the constant interest rate in the period $t+1$.

Agents have a probability of surviving to the second period which depends on the health investment undertaken in the working age. Following empirical evidence (see Figure 3.2), we suppose that the probability of surviving increases with health investment:

$$p_t = p(m_t), \tag{3.3}$$

where $p_t \in (0, \bar{p}]$, $p'_t > 0$, $p''_t < 0$.

We assume that health spending, beyond the increase in the length of life, allows agents to enjoy better life. Thus agent's health level, h_{t+1}, is a

[5]We suppose a perfect substitutability between public health expenditure and private health spending. This implies that a higher proportion of government expenditure devoted to health services reduce private health spending. Indeed, health investment, m_t, in the consumer's budget constraint is the sum of private health investment, m_t^{PRI}, and public health investment, m_t^{PUB}. The latter is equal to a proportional tax on income that is $m_t^{PUB} = \tau y_t$. Thus the budget constraint in the first period is:

$$c_t = (1-\tau)\, y_t - m_t^{PRI} - s_t,$$

where substituting $m_t^{PUB} = \tau y_t$ we obtain:

$$c_t = y_t - s_t - \left(m_t^{PRI} + m_t^{PUB}\right),$$

where $m_t^{PRI} + m_t^{PUB} = m_t$ in equation (3.1).

The idea is that if agents pay high tax then receive high quality public health services and therefore decide to devote a low proportion of income to private health expenditure. Otherwise when public health sector is absent, health spending is private.

positive function of investment in health services[6] (Grossman, 1972):

$$h_{t+1} = h\left(m_t\right). \tag{3.4}$$

For simplicity we consider health level a linear function of health investment, that is:

$$h_{t+1} = m_t. \tag{3.5}$$

The lifetime utility of a representative agent is:

$$U_t = u(c_t) + \beta p(m_t)\hat{u}(c_{t+1}, h_{t+1}) + \left[1 - p(m_t)\right] M, \tag{3.6}$$

where $0 < \beta < 1$ is the psychological discount factor, M is the utility in the death state (Rosen, 1988), $u(c_t)$ is the utility in the first period, and $\hat{u}(c_{t+1}, h_{t+1})$ is the utility in the second period. In particular, if agents survive to the second period enjoys an utility which depends on consumption and health level.

Assuming zero utility from death, i.e. $M = 0$[7], and substituting equation (3.5) into equation (3.6) we get:

$$U_t = u(c_t) + \beta p(m_t)\hat{u}(c_{t+1}, m_t). \tag{3.7}$$

[6] In particular Grossman (1972) assumes that individuals inherit an initial amount of health that depreciates with age and can be increased by investment in health services:

$$h_{t+1} = m_t + (1 - \delta_t)h_t$$

where m_t is the investment in health, δ_t is the depreciation rate that depends on age, and h_t is the inherited health level.

[7] Following Rosen (1988) the expected utility in the second period is:

$$EU = p(m_t)u(c_{t+1}, h_{t+1}) + \left(1 - p\left(m_t\right)\right) M.$$

Subtracting M from utility in each state we normalize the utility of nonsurvival to zero:

$$EU = p(m_t)\left[u(c_{t+1}, h_{t+1}) - M\right] + \left(1 - p\left(m_t\right)\right)\left[M - M\right].$$

Therefore the expected utility in the second period is given by the differences in utility between life and death, that is:

$$EU = p(m_t)\left[u(c_{t+1}, h_{t+1}) - M\right].$$

3.3.1 Optimal saving and health spending

Proposition 5 characterizes the optimal condition for saving and health spending:

Proposition 5 *The optimal allocation of resources implies that the ratio of saving to health investment is:*

$$\frac{s_t}{m_t} = \frac{\varepsilon_{\hat{u}_c}}{\varepsilon_{\hat{u}_m} + \varepsilon_p}, \tag{3.8}$$

where $\varepsilon_{\hat{u}_c} = \hat{u}_c(c_{t+1}, m_t)c_{t+1}/\hat{u}(c_{t+1}, m_t)$ *is the elasticity of the instantaneous utility function with respect to consumption,* $\varepsilon_{\hat{u}_m} = \hat{u}_m(c_{t+1}, m_t)m_t/\hat{u}(c_{t+1}, m_t)$ *is the elasticity of the instantaneous utility function with respect to health investment*[8] *and* $\varepsilon_p = p'(m_t)m_t/p(m_t)$ *is the elasticity of the survival function with respect to health investment.*

Proof. Given the budget constraints in equations (3.1) and (3.2), the first order conditions with respect to s_t and m_t are:

$$\frac{u'(c_t)}{\hat{u}_c(c_{t+1}, m_t)} = \beta p_t(m_t)R, \tag{3.9}$$

and:

$$u'(c_t) = \beta p_t'(m_t)\hat{u}(c_{t+1}, m_t) + \beta p(m_t)\hat{u}_m(c_{t+1}, m_t). \tag{3.10}$$

The substitution of equation (3.10) in equation (3.9) yields the ratio between the saving and health investment. ■

Equation (3.9) is the usual condition that requires the marginal rate of substitution between current and future consumption should to be equal to the expected return on saving. Equation (3.10) captures the trade-off between the marginal cost and marginal benefit of health care spending. By investing in health care, agents renounce to the current consumption to

[8] We define:

$$\hat{u}_c = \frac{\partial \hat{u}(c_{t+1}, m_t)}{\partial c_{t+1}},$$

and:

$$\hat{u}_m = \frac{\partial \hat{u}(c_{t+1}, m_t)}{\partial m_t}.$$

increase their health level and the probability of surviving to the second period.

According to Proposition 5 the response of the ratio between saving and health spending to variations in the level of income depends on the behavior of the elasticities in equation (3.8). Empirical evidence (Figures 3.3 and 3.4) shows that both saving and health investment rise with income but, when income is high, health spending on GDP grows faster rather than the saving on GDP. The intuition is that when income becomes higher than a certain threshold, consumption elasticity falls relative to the health elasticity causing the ratio between saving and health to decrease.

3.4 Alternative specifications of the Utility function and the Survival function

In this section we analyze the effect of alternative specifications of instantaneous utility function and survival function on the ratio between saving and health investment in equation(3.8).

Constant elasticity of utility function and survival function

The intuition from figures 3.3 and 3.4 is that when income is low people prefer to devote more income to the consumption rather than health spending, but when income rises the marginal utility of consumption appears to decreases faster than the marginal utility of health spending. We cannot replicate this empirical evidence using an utility function with constant elasticity with respect consumption and health investment, e.g. $\hat{u} = \left[c^{\beta} m^{1-\beta}\right]^{1-\gamma} / (1-\gamma)$, and a survival function with constant elasticity with respect to health investment, i.e. $p = m_t^{\delta}$. Indeed using this specification the ratio s_t/m_t is constant. In particular, from equation (3.8) we obtain $s_t/m_t = \beta(1-\gamma) / ((1-\beta)(1-\gamma)+\delta)$.

Constant elasticity of utility function with respect to consumption

Using an utility function with constant $\varepsilon_{\hat{u}_c}$and non-constant $\varepsilon_{\hat{u}_m}$, and a survival function $p(m)$ with non-constant ε_p, we have that the ratio s_t/m_t is consistent with empirical evidence if the sum $\varepsilon_{\hat{u}_m} + \varepsilon_p$ is first decreasing and then increasing. This specification implies that the model is intractable with analytical tools.

Constant elasticity of utility with respect to investment in health

In a model with non-constant $\varepsilon_{\hat{u}_c}$, constant $\varepsilon_{\hat{u}_m}$and non-constant ε_p the path of the ratio s_t/m_t depends on the movements of $\varepsilon_{\hat{u}_c}$, ε_p and on the value of the constant elasticity $\varepsilon_{\hat{u}_m}$.

In the next section we present a model where the utility function presents a zero elasticity with respect to health investment. This specification allows us to replicate the empirical results.

3.5 A Model with zero elasticity of utility with respect to investment in health

In this section we present a simplified version of the general utility function displayed in equation (3.7). In particular, we suppose that health does not enter in the utility function and affects only the survival function. Thus, the lifetime utility takes the following form:

$$U_t = u(c_t) + \beta p(m_t) u(c_{t+1}), \tag{3.11}$$

subject to the budget constraints given by equations (3.1) and (3.2).

Given zero utility from health level, the ratio between saving and health investment is equal to the ratio between the elasticity of the utility with respect to consumption in old age and the elasticity of the probability function with respect to health investment. Thus equation (3.8) becomes:

$$\frac{s_t}{m_t} = \frac{\varepsilon_{u_c}}{\varepsilon_p}. \tag{3.12}$$

3.5.1 Survival function

Given equation (3.3) we specify the following probability of surviving to old age:

$$p(m_t) = \begin{cases} \underline{p} + \lambda m_t^{\delta}, & \text{if } m_t \in [0, \hat{m}] \\ \overline{p} & \text{if } m_t > \hat{m} \end{cases} \tag{3.13}$$

where $0 < \delta < 1$, $\lambda > 0$, $\underline{p}$ is the minimum agent's survival probability if they do not invest in health services and $\overline{p}$ is the highest probability of surviving to old age[9]. This means that an increase in the level of health investment beyond $\hat{m}$ cannot increase the probability of surviving[10]. In particular $\hat{m}$ is given by:

$$\hat{m}_t = \left(\frac{\overline{p} - \underline{p}}{\lambda}\right)^{1/\delta}. \tag{3.14}$$

The elasticity of the survival function is concave with respect to health investment, that is:

$$\varepsilon_p(m_t) = \frac{\delta \lambda m_t^{\delta}}{\underline{p} + \lambda m_t^{\delta}}, \tag{3.15}$$

where:

$$\varepsilon_p(0) = 0,$$

$$\lim_{m \to \infty} \varepsilon_p = \delta.$$

3.5.2 Preferences

Jones and Hall (2006) to explain the luxury good behavior of health spending choose to add a constant term to the standard utility function with constant elasticity of substitution (C.E.S.). Using this specification in our model we obtain intractable results. Thus, we choose to use H.A.R.A (hyperbolic absolute risk aversion function)[11] preferences which present a non-constant

[9] Assuming $\underline{p} = 0.1$, non linear least square estimates of the parameters λ and δ in equation(3.13) yields $\lambda = 0.2$ and $\delta = 0.6$.

[10] Empirical analysis (figure 3.3) shows that in rich countries health investment is still increasing. This stylized fact support the idea that health investment did not yet reach its maximum level $\hat{m}$.

[11] The HARA family is rich, in the sense that by suitable adjustment of the parameters we can have an utility function with absolute o relative risk aversion increasing, decreasing or constant. Thus, isolelastic (constant relative risk aversion for $\theta = 0$), exponential

elasticity with respect to the consumption. Hence the utility function is:

$$u(c) = \frac{(\theta + \sigma c)^{\frac{\sigma-1}{\sigma}}}{\sigma - 1}, \tag{3.16}$$

where[12] the constant $\theta > 0$ can be considered as the minimum required consumption at the end of the horizon. We assume that $\sigma > 1$, which implies that the function is D.A.R.A like the standard utility function C.E.S.

Given equations (3.13) and (3.16), equation (3.12) yields the following relationship between saving and health investment:

$$\frac{s_t}{m_t} = \frac{1}{\delta}\left(\frac{\sigma - 1}{\sigma}\right)\left(1 + \frac{\underline{p}}{\lambda m_t^{\delta}}\right) - \frac{\theta}{\sigma R m_t}, \tag{3.17}$$

which implies that the saving is concave in health investment, i.e. $\partial s_t/\partial m_t > 0$ and $\partial^2 s_t/\partial m_t^2 < 0$ (see Appendix B.3).

The first order conditions corresponding to equation (3.11) in the range $[0, \hat{m}]$ are given by:

$$c_t = \frac{\theta + \sigma c_{t+1}}{\sigma\left[\beta R\left(\underline{p} + \lambda m_t^{\delta}\right)\right]^{\sigma}} - \frac{\theta}{\sigma}, \tag{3.18}$$

$$c_{t+1} = R\left(\frac{\sigma - 1}{\sigma}\right)\frac{m_t}{\delta}\left(1 + \frac{\underline{p}}{\lambda m_t^{\delta}}\right) - \frac{\theta}{\sigma}. \tag{3.19}$$

From equations (3.1), (3.19) and (3.18) we obtain the following implicit relation between health investment and income, that is:

$$F(y_t, m_t) = 0,$$

(constant absolute risk aversion) and quadratic utility functions are subsets of HARA family (Merton, 1992). In particular:

$$\begin{array}{ll} if & \sigma > 0 \Rightarrow D.A.R.A \\ if & \sigma < 0 \Rightarrow I.A.R.A \\ if & \sigma = \infty \Rightarrow A.R.A = 0 \end{array}$$

In this paper we assume $\sigma > 0$.

[12]This utility function shows an elasticity which increases with the consumption, that is:

$$\varepsilon_{u_c} = \frac{c(\sigma - 1)}{\theta + \sigma c}$$

where:

$$F(y_t, m_t) \equiv \left(\frac{\sigma-1}{\sigma}\right)\frac{m_t}{\delta}\left(1+\frac{\underline{p}}{\lambda m_t^{\delta}}\right)\left[\frac{R^{1-\sigma}}{\left[\beta\left(\underline{p}+\lambda m_t^{\delta}\right)\right]^{\sigma}}+1\right]+ + m_t - y_t - \frac{\theta}{\sigma}\left[1+\frac{1}{R}\right]. \quad (3.20)$$

We are interested in analyzing the behavior of saving and health investment according to different levels of per capita income. The aim is to show that the elasticity of saving falls more rapidly than the elasticity of health investment, that is as people became richer, saving rises but they prefer to devote an increasing share of income to additional years of life. The following proposition define the properties of health share and saving share.

Proposition 6 *In the range* $[0, \hat{m}]$, *a sufficient condition to have health investment increasing and convex in income, i.e.* $\partial m_t/\partial y_t > 0$ *and* $\partial^2 m_t/\partial y_t^2 > 0$, *is* $\delta \leq \frac{1}{\sigma}$. *When this condition is satisfied optimal health share presents the following properties*[13] *(see figure 3.5):*

(1) $\lim_{m \to m_0} \frac{m_t}{y_t} = \infty$,

(2) $\lim_{m \to \hat{m}} \frac{m_t}{y_t} = \frac{\hat{m}}{\hat{y}} > 0$,

(3) $\frac{\partial(m_t/y_t)}{\partial y_t} = 0$ *for* $y_t = y_m$; $\frac{\partial(m_t/y_t)}{\partial y_t} < 0$ *for* $y_t < y_m$; $\frac{\partial(m_t/y_t)}{\partial y_t} > 0$ *for* $y_t > y_m$.

Proof. The technical part of this proposition is proved in Appendix B.2. ■

Proposition 7 *Given the condition* $\delta \leq \frac{1}{\sigma}$, *optimal saving share in income satisfies the following properties (see figure 3.5):*

(1) $\lim_{m_t \to m_0} \frac{s_t}{y_t} = -\infty$ *if* $s_0 < 0$

(2) $\lim_{m_t \to \hat{m}} \frac{s_t}{y_t} = \frac{\hat{s}}{\hat{y}} > 0$

(3) $\frac{\partial s_t/y_t}{\partial y_t} > 0$ *if* $R > \frac{\sigma\delta}{(\sigma-1)}$

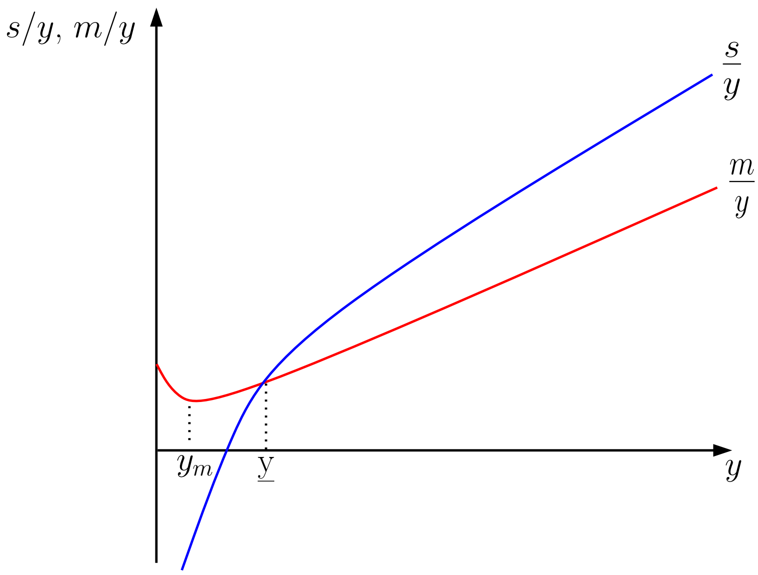

Figure 3.5: Saving share and Health share versus Income.

Proof. See Appendix B.3 ■

Propositions 6 and 7 imply that both saving and health investment behave like luxury goods. In particular, when income is low, i.e. $y_t < y_m$, health share is decreasing and presents an elasticity with respect to income $\varepsilon_m < 1$(see figure 3.5[14]). When income increases, i.e. $y_t > y_m$, the elasticity of health with respect to income rises, i.e. $\varepsilon_m > 1$.(see Appendix B.2). This results support some theoretical contributions which shows that the income elasticity of demand for health care is larger than one. In particular Blomqvist and Carter (1997) estimate that the income elasticity of health care spending, for OECD countries in the period 1960 to 1991, is significantly above one.

In Figure3.5 we can see that there exist a value of $\underline{y}$ so that the saving share is equal to the health share (for the technical part see appendix B.4). Thus when the income is equal to $\underline{y}$ the elasticity of utility function is equal to the elasticity of the survival function.

Figure 3.6 illustrates the results of our calibration for the ratio between

[13]The value m_0 define the value of m_t so that y_t is equal to zero (see appendix B.1).
[14]Our calibration is $\sigma = 2$, $\beta = 0.7$, $R = 3$, $\delta = 0.5$, $\theta = 1$, $\lambda = 0.2$, $\underline{p} = 0.3$.

optimal saving and optimal health investment with respect to different income levels (our baseline parameters values are $\sigma = 2$, $\beta = 0.7$, $R = 3$, $\delta = 0.5$, $\theta = 1$, $\lambda = 0.2$, $\underline{p} = 0.3$). The following proposition characterizes the properties of the ratio between the saving share and health share.

Proposition 8 *When $y_t < \tilde{y}$ the saving grows more quickly than health investment; hence the ratio s_t/m_t is increasing as income increases. For $y_t > \tilde{y}$ the ratio between saving and health investment decreases as income increases (see figure 3.6).*

Proof. See Appendix B.4 ■

Proposition 8 implies that when income is low people devote more resources to the consumption, when income becomes higher than a certain threshold agents spend more income to increase their probability of surviving to old age. Thus for $y_t > \tilde{y}$ while the marginal utility of consumption decreases the marginal utility of additional years of life does not decrease. This implies that as income grows the optimal composition of spending shifts toward health investment (see appendix B.4).

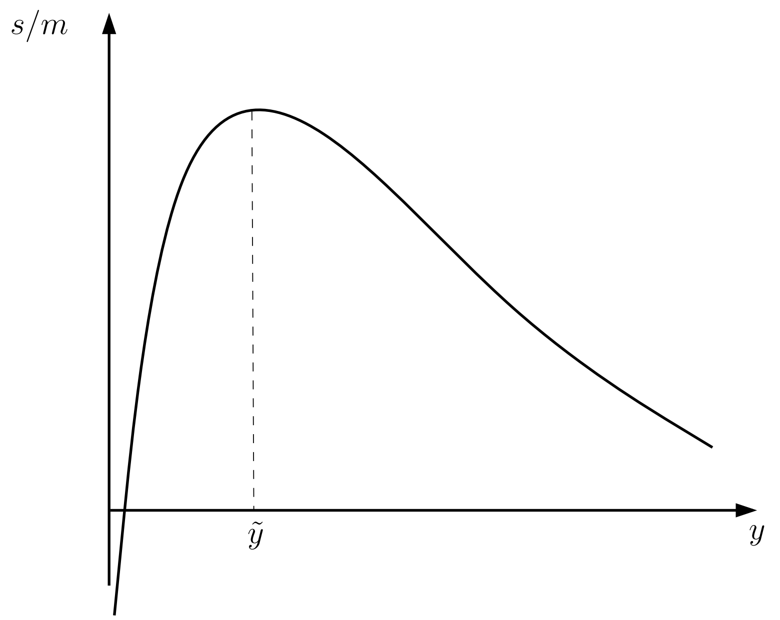

Figure 3.6: Ratio between Saving and Health Expenditure versus Income.

3.6 Conclusion

This paper analyzes agent's decision on the allocation of total resources between health investment and saving. Empirical evidence shows that when income is low agents devote more income to saving to assure consumption in the old age. As income rises the saving continues to rise but health spending increases more quickly. This indicates that for low levels of income, the elasticity of the utility function with respect to consumption is greater than the elasticity of the survival function with respect to health investment. When income rises the opposite occurs. The intuition for this results is that as income grows people become saturated in non-health consumption and choose to spend more income to purchase additional years of life. This mechanism is supported with a theoretical model in which agents present HARA preferences and the survival function shows a non-constant elasticity with respect to health investment.

In the future, we plain to specify a model in which health level directly enters in the utility function. We need to know health inequality within countries and the effect of public and private health investment on health inequality. This determines whether and by how much income redistribution can improve population health.

Appendix A

Appendix of Chapter 2

A.1 Proof of optimal conditions

Given the intertemporal utility function:

$$U = \log c_{t+1} + \beta p \log c_{t+2} + \phi\pi n \left[p \log\left(\frac{b_{t+1}}{\pi n}\right) + (1-p)\log\left(\frac{b_{t+1}+s_{t+1}}{\pi n}\right)\right], \quad \text{(A.1)}$$

the first order conditions associated with s_{t+1} and b_{t+1} are respectively:

$$\frac{1}{c_{t+1}} = \frac{\beta p}{s_{t+1}} + \frac{\phi\pi n(1-p)}{b_{t+1}+s_{t+1}}, \quad \text{(A.2)}$$

$$\frac{1}{c_{t+1}} = \phi\pi n\left[\frac{p}{b_{t+1}} + \frac{(1-p)}{b_{t+1}+s_{t+1}}\right],$$

from which:

$$b_{t+1} = \frac{\phi\pi n}{\beta} s_{t+1}. \quad \text{(A.3)}$$

Using equation (A.2) we obtain:

$$s_{t+1}(b_{t+1}+s_{t+1}) = [y_{t+1} - (b_{t+1}+s_{t+1})]\left[\beta p\,(b_{t+1}+s_{t+1}) + \phi\pi(1-p)ns_{t+1}\right],$$

where substituting equation (A.3) the optimal saving and the optimal bequest are given as follows:

$$s_{t+1} = y_{t+1}\frac{\beta}{(\phi\pi n+\beta)}\frac{(\beta p+\phi\pi n)}{(1+\beta p+\phi\pi n)}, \quad \text{(A.4)}$$

$$b_{t+1} = y_{t+1}\frac{\phi\pi n}{(\phi\pi n+\beta)}\frac{(\beta p+\phi\pi n)}{(1+\beta p+\phi\pi n)}. \quad \text{(A.5)}$$

An increase in the probability of surviving to the third period leads to an increase in both saving and bequest, that is:

$$\frac{\partial s_{t+1}}{\partial p} = y_{t+1}\frac{\beta}{(\phi\pi n+\beta)}\left[\frac{\beta}{(1+\beta p+\phi\pi n)^2}\right] > 0,$$

$$\frac{\partial b_{t+1}}{\partial p} = y_{t+1}\frac{\phi\pi n}{(\phi\pi n+\beta)}\left[\frac{\beta}{(1+\beta p+\phi\pi n)^2}\right] > 0.$$

An increase in child probability of surviving to the second period has a negative effect on the saving if $\pi > \hat{\pi}$, that is:

$$\frac{\partial s_{t+1}}{\partial \pi} = -\frac{y\beta\phi n\left[(\beta p+\phi\pi n)^2-\beta(1-p)\right]}{\left[(\phi\pi n+\beta)(1+\beta p+\phi\pi n)\right]^2} < 0,$$

if:

$$\pi > \frac{[\beta(1-p)]^{1/2}-\beta p}{\phi n} = \hat{\pi}. \tag{A.6}$$

An increase in child survival probability has a positive effect on the optimal bequest:

$$\frac{\partial b_{t+1}}{\partial \pi} = \frac{y\beta\phi n\left[\beta p+(\phi\pi n)^2+2\phi\pi n+(\beta p+\phi\pi n)^2\right]}{\left[(\phi\pi n+\beta)(1+\beta p+\phi\pi n)\right]^2} > 0.$$

Finally an increase in the fertility rate has the same effect of reductions in child survival probability.

A.2 Expected human capital and exogenous longevity

Given the expected human capital:

$$E(h_{t+2}) = h_{t+1}^{1-\alpha}\left[\frac{w(1+h_{t+1})(\beta p+\phi\pi n)}{\pi n(1+\beta p+\phi\pi n)(\phi\pi n+\beta)}\right]^{\alpha}\left[p(n\phi\pi)^{\alpha}+(1-p)(\phi\pi n+\beta)^{\alpha}\right]$$

it follows that:

$$\frac{\partial E(h_{t+2})}{\partial p} = G\left[-(p\beta)^2 - p\beta(1+\alpha+2\phi\pi n) + \frac{\alpha\beta(\beta+\phi\pi n)^{\alpha}}{(\beta+\phi\pi n)^{\alpha}-(\phi\pi n)^{\alpha}} - \phi\pi n(1+\phi\pi n)\right]$$

where:

$$G = \frac{h_{t+1}^{1-\alpha}}{(\beta p+\phi\pi n)(1+\beta p+\phi\pi n)}\left[\frac{w(1+h_{t+1})(\beta p+\phi\pi n)}{\pi n(1+\beta p+\phi\pi n)(\phi\pi n+\beta)}\right]^{\alpha} > 0$$

Given the following assumption:

$$\frac{\alpha\beta(\beta+\phi\pi n)^{\alpha}}{(\beta+\phi\pi n)^{\alpha}-(\phi\pi n)^{\alpha}} - \phi\pi n(1+\phi\pi n) > 0,$$

it follows that $\partial E(h_{t+2})/\partial p$ is parabola concave down which shows two solutions one negative and one positive. In particular, we obtain the two following real solutions p_1 and p_2, that is:

$$p_{1,2} = \frac{(1+\alpha+2\phi\pi n)}{2\beta} \pm \beta\sqrt{(1+\alpha+2\phi\pi n)^2 + 4\left[\frac{\alpha\beta(\beta+\phi\pi n)^\alpha}{(\beta+\phi\pi n)^\alpha - (\phi\pi n)^\alpha} - \phi\pi n(1+\phi\pi n)\right]}$$

where:

$$(1+\alpha+2\phi\pi n)^2 + 4\left[\frac{\alpha\beta(\beta+\phi\pi n)^\alpha}{(\beta+\phi\pi n)^\alpha - (\phi\pi n)^\alpha} - \phi\pi n(1+\phi\pi n)\right] > 0$$

and $p_1 > 0$ and $p_2 < 0$.
Since $0 < p < 1$ we choose the positive solution, that is $p_1 = p^*$. If $p < p^*$ then $\partial E(h_{t+2})/\partial p > 0$ and if $p > p^*$ then $\partial E(h_{t+2})/\partial p < 0$.

A.2.1 The effect of fertility on the expected human capital

Given the expected human capital we obtain that exogenous changes in the number of children reduce the expected human capital. In particular we have that:

$$\frac{\partial E(h_{t+2})}{\partial n} = -D\left[\left(\beta p(1+\beta p) + 2\beta\pi\phi p n + (\pi\phi n)^2\right)\left(p + (1-p)\left(\frac{\phi\pi n+\beta}{\phi\pi n}\right)^{1+\alpha}\right) - \beta p\right],$$

where:

$$D = \frac{\alpha h_{t+1}^{1-\alpha}}{\pi\left[n(\beta p+\phi\pi n)(1+\beta p+\phi\pi n)\right]^2}\left[\frac{w(1+h_{t+1})(\beta p+\phi\pi n)}{n(1+\beta p+\phi\pi n)(\phi\pi n+\beta)}\right]^{\alpha-1} > 0.$$

Thus:

$$\frac{\partial E(h_{t+2})}{\partial n} < 0$$

if:

$$\left(\beta p(1+\beta p) + 2\beta\pi\phi p n + (\pi\phi n)^2\right)\left(p + (1-p)\left(\frac{\phi\pi n+\beta}{\phi\pi n}\right)^{1+\alpha}\right) - \beta p > 0$$

which is can be written as follows:

$$\beta p\left[\beta p^2 - (1-p) + p + (1+\beta p)(1-p)\left(\frac{\phi\pi n+\beta}{\phi\pi n}\right)^{1+\alpha}\right]$$
$$+\left[2\beta\pi\phi p n + (\pi\phi n)^2\right]\left(p + (1-p)\left(\frac{\phi\pi n+\beta}{\phi\pi n}\right)^{1+\alpha}\right)$$

which is positive since both the second term and the first term are positive. In particular, the first term is positive since:

$$\left(\frac{\phi\pi n+\beta}{\phi\pi n}\right)^{1+\alpha} - 1 > 0,$$

because $\beta > 0$.
In addition education spending for each child decreases with the number of children, that is:

$$\frac{\partial E\left[e_{t+1}\right]}{\partial n} = -(1+h_{t+1})\left[\begin{array}{c} \beta^3\left(1-p\right)p\left(1+\beta p\right)+2\beta^2\pi n\left(1-p\right)p\left(1+\beta+\beta p\right)\phi+ \\ +\left(\beta\pi n\right)^3\left(1+3\left(1-p\right)p\right)\phi^2+\left(\pi n\phi\right)^3\left(2\beta+\pi n\phi\right) \end{array}\right] < 0$$

A.2.2 The effect of children's probability of surviving to the second period

The effect of exogenous changes in the probability of surviving to the second period is the same of changes in the number of children, that is:

$$\frac{\partial E(h_{t+2})}{\partial \pi} = -A\left[\left(\beta p\left(1+\beta p\right)+2\beta\pi\phi pn+\left(\pi\phi n\right)^2\right)\left(p+\left(1-p\right)\left(\frac{\phi\pi n+\beta}{\phi\pi n}\right)^{1+\alpha}\right)-\beta p\right] < 0$$

where:

$$A = \frac{\alpha h_{t+1}^{1-\alpha} n}{n\left[\pi\left(\beta p+\phi\pi n\right)\left(1+\beta p+\phi\pi n\right)\right]^2}\left[\frac{w(1+h_{t+1})\left(\beta p+\phi\pi n\right)}{n\left(1+\beta p+\phi\pi n\right)\left(\phi\pi n+\beta\right)}\right]^{\alpha-1} > 0$$

Thus a higher probability of surviving to the second period implies a reduction in the resources for each child and thus a lower expected human capital.

A.3 Expected Education Spending

Given the expected education spending:

$$E\left[e_{t+1}\right] = w(1+h_{t+1})\frac{\beta p+\phi\pi n}{(1+\beta p+\phi\pi n)}\left(\frac{\phi\pi n+\beta\left(1-p\right)}{n(\phi\pi n+\beta)}\right),$$

when $p = 0$, it follows that:

$$E\left[e_{t+1}\right]_{p=0} = w(1+h_{t+1})\frac{\phi\pi}{(1+\phi\pi n)},$$

and when $p = 1$ it follows that:

$$E\left[e_{t+1}\right]_{p=1} = w(1+h_{t+1})\frac{\phi\pi}{(1+\beta+\phi\pi n)}.$$

Therefore, education spending is higher when $p = 0$ than when parents survive to the old age, that is:

$$E\left[e_{t+1}\right]_{p=0} - E\left[e_{t+1}\right]_{p=1} = \frac{\phi\pi\beta}{(1+\phi\pi n)\left(1+\beta+\phi\pi n\right)} > 0.$$

Given the expected education spending as follows:

$$E\left[e_{t+1}\right] = w(1+h_{t+1})\frac{\beta p + \phi\pi n}{(1+\beta p+\phi\pi n)}\left\{\frac{\phi\pi n + (1-p)\beta}{(\phi\pi n+\beta)\,\pi n}\right\},$$

when $p < \hat{p}$ it increases and when $p > \hat{p}$ it decreases. In particular, the effect of exogenous changes of p in the expected education spending is given by:

$$\frac{\partial E\left[e_{t+1}\right]}{\partial p} = -\left(\beta p\right)^2 - p\left[2\beta\left(1+n\pi\phi\right)\right] + \beta - \left(n\pi\phi\right)^2,$$

which is a parabola concave downwards and when $p = 0$ it follows that:

$$\left.\frac{\partial E\left[e_{t+1}\right]}{\partial p}\right|_{p=0} = \beta - \left(n\pi\phi\right)^2,$$

which is positive if:

$$\beta > \left(n\pi\phi\right)^2$$

and when $p = 1$, $\partial E\left[e_{t+1}\right]/\partial p < 0$, that is:

$$\left.\frac{\partial E\left[e_{t+1}\right]}{\partial p}\right|_{p=1} = -\beta\left[\beta+1\right] - n\pi\phi\left[2\beta - n\pi\phi\right] < 0.$$

Moreover we have that:

$$\frac{\partial E\left[e_{t+1}\right]}{\partial p} = 0,$$

shows two solutions $p_{1,2}$:

$$p_{1,2} = \frac{-\left(1+n\pi\phi\right) \pm \left(1+\beta+2n\pi\phi\right)^{1/2}}{\beta},$$

where we chose $p_1 = \hat{p}$ which is positive if the following condition is satisfied:

$$\beta > \left(n\pi\phi\right)^2.$$

In addition we show that $\hat{p} < 1$:

$$1+\beta+2n\pi\phi < \left(\beta+1+n\pi\phi\right)^2$$

$$-\beta\left(2n\pi\phi+1\right) - \beta^2 - \left(n\pi\phi\right)^2 < 0$$

Therefore the expected education spending increases when $p < \hat{p}$ and it decreases when $p > \hat{p}$.

A.4 Proof Proposition 4

This appendix provides a formal description of the conditions for the existence of a poverty trap. The expected growth rate presents a stable steady state if both curves in equation (2.23) show a stable steady state. Therefore we analyze human capital accumulation in the state $(1-p)$ given by:

$$h_{t+2} = h_{t+1}\left[w\left(\frac{1+h_{t+1}}{h_{t+1}}\right)\frac{\beta p(h_{t+1})+\phi\pi n}{\pi n\left(1+\beta p(h_{t+1})+\phi\pi n\right)}\right]^{\alpha}. \tag{A.7}$$

There exists at least a steady state, i.e. $\bar{h}$ if:

$$\left[w\left(\frac{1+\bar{h}}{\bar{h}}\right)\frac{\beta p(\bar{h})+\phi\pi n}{\pi n\left(1+\beta p(\bar{h})+\phi\pi n\right)}\right] = 1. \tag{A.8}$$

Substituting equation (2.22) in equation (A.8):

$$w\left(\frac{1+\bar{h}}{\bar{h}}\right)\frac{\beta\left(\underline{p}+\overline{p}\gamma\bar{h}\right)+\phi\pi n\left(1+\gamma\bar{h}\right)}{\pi n\left(\left(1+\gamma\bar{h}\right)(1+\phi\pi n)+\beta\left(\underline{p}+\overline{p}\gamma\bar{h}\right)\right)} = 1,$$

some computations lead to:

$$\begin{aligned}&\bar{h}\gamma\left[w(\beta\overline{p}+\phi\pi n)-n\pi(1+\beta\overline{p}+\phi\pi n)\right]+\\&\bar{h}\left[w\gamma(\beta\overline{p}+\phi\pi n)+w\left(\beta\underline{p}+\phi\pi n\right)-\pi n(1+\beta\underline{p}+\phi\pi n)\right]+w(\beta\underline{p}+\phi\pi n)=0,\end{aligned} \tag{A.9}$$

where from the condition in equation (2.25) the first term is positive:

$$w(\beta\overline{p}+\phi\pi n) > \pi n(1+\beta\overline{p}+\phi\pi n).$$

We suppose that the second term is negative that is:

$$w\left[\gamma(\beta\overline{p}+\phi\pi n)+\beta\underline{p}+\phi\pi n\right]-\pi n(1+\beta\underline{p}+\phi\pi n) < 0,$$

which implies that:

$$\gamma < \frac{\pi n+\left(\beta\underline{p}+\phi\pi n\right)(\pi n-w)}{w(\beta\overline{p}+\phi\pi n)} = \hat{\gamma}. \tag{A.10}$$

The condition in equation (A.10) implies the existence of two real solutions, i.e. h_L and h_H.

If the growth rate shows a minimum value h_{min} so that when $h < h_{min}$ the growth rate decreases, i.e. $\partial g/\partial h < 0$, and when $h > h_{min}$ the growth rate increases, i.e. $\partial g/\partial h > 0$, then the low steady state $\bar{h}_L$ is stable and the steady state $\bar{h}_H$ is unstable.

The derivative of the growth rate is:

$$\begin{aligned}\frac{\partial g}{\partial h} &= \alpha\left[w\left(\frac{1+h}{h}\right)\frac{\beta p(h)+\phi\pi n}{\pi n\left(1+\beta p(h)+\phi\pi n\right)}\right]^{\alpha}\\&\quad\left[\frac{\beta p'}{(\beta p(h)+\phi\pi n)\left(1+\beta p(h)+\phi\pi n\right)}-\frac{1}{h\left(1+h\right)}\right]\end{aligned} \tag{A.11}$$

where:

$$\frac{\partial g}{\partial h}_{h=0} = -\infty$$

$$\lim_{h=\infty} \frac{\partial g}{\partial h} = 0.$$

Now we study the existence of a h_{min}. Given equation (A.11) there exist a h_{min} if:

$$\left[\frac{\beta p'}{(\beta p(h) + \phi \pi n)(1 + \beta p(h) + \phi \pi n)} - \frac{1}{h(1+h)} \right] = 0,$$

which is equal to:

$$\begin{aligned}\beta \left(\overline{p} \gamma h^2\right)(\gamma + \beta \gamma \overline{p} - 1) = (1+\gamma h)^2 (\phi \pi n)^2 + \beta \underline{p}\left(1 + \beta \underline{p}\right) + \gamma h \left(2 + h + 2\beta \overline{p}\right) \underline{p} + \\ + (1+\gamma h)(\phi \pi n)\left[1 + \gamma\left(h + 2\beta h \overline{p}\right) + 2\beta \underline{p}\right], \end{aligned} \tag{A.12}$$

that shows two solutions:

$$h_{1,2} = \frac{1}{D} \left\{ \begin{array}{c} -\gamma \left(1 + \beta \overline{p} + \phi \pi n\right)\left(\beta \underline{p} + \phi \pi n\right) \pm \\ \left[\beta \gamma \left(\overline{p} - \underline{p}\right)\left(\phi \pi n + \beta \underline{p}\right)\left[(1+\phi \pi n)(1-\gamma) + \beta\left(\gamma \overline{p} - \underline{p}\right)\right]\right]^{1/2} \end{array} \right\} \tag{A.13}$$

where:

$$D = \gamma \left(1 + \beta \overline{p} + \phi \pi n\right)\left(\beta \underline{p} + \phi \pi n\right) - \beta \left(\overline{p} - \underline{p}\right).$$

The two solutions in equation (A.13) are real if the following conditions are satisfied:

$$\gamma > \frac{\underline{p}}{\overline{p}}$$

and:

$$D > 0$$

which implies:

$$\gamma > \frac{\beta \left(\overline{p} - \underline{p}\right)}{\left(1 + \beta \overline{p} + \phi \pi n\right)\left(\beta \underline{p} + \phi \pi n\right)} = \tilde{\gamma}$$

Thus in our model $h_2 > 0$ is the minimum growth rate, i.e. $h_{\min}$.

Appendix B

Appendix of Chapter 3

B.1 Proof of the existence of m_0

When $y_t = 0$, from equation (3.1) we have that:

$$m_t = -(c_t + s_t),$$

which, from equations (3.18) and (3.19), yields :

$$m_t + \frac{m_t}{\delta}\left(\frac{\sigma-1}{\sigma}\right)\left(1+\frac{\underline{p}}{\lambda m_t^\delta}\right)\left[\frac{R}{\left[\beta R\left(\underline{p}+\lambda m_t^\delta\right)\right]^\sigma}+1\right]-\frac{\theta}{\sigma}\left[\frac{1}{R}+1\right]=0 \tag{B.1}$$

We show here the existence of a value of m_t, i.e. m_0, so that the income is equal to zero. The value m_0 can be considered as the activities that agents undertake to survive when they do not have resources. Moreover if we consider health spending as the sum of public health investment and private health investment, we can think that when income is equal zero agents receive a subsistence amount of resources to survive (see note 5).

From equation (B.1) we can define the two functions:

$$\Phi_1(m_t) = \frac{m_t}{\delta}\left(\frac{\sigma-1}{\sigma}\right)\left(1+\frac{\underline{p}}{\lambda m_t^\delta}\right)\left[\frac{R}{\left[\beta R\left(\underline{p}+\lambda m_t^\delta\right)\right]^\sigma}\right], \tag{B.2}$$

$$\Phi_2(m_t) = \frac{\theta}{\sigma}\left[1+\frac{1}{R}\right]-m_t\left[1+\frac{1}{\delta}\left(\frac{\sigma-1}{\sigma}\right)\left(1+\frac{\underline{p}}{\lambda m_t^\delta}\right)\right]. \tag{B.3}$$

The function in equation (B.2) increases with respect to health investment, that is:

$$\frac{\partial\Phi_1(m_t)}{\partial m_t} = \frac{1}{\delta}\left(\frac{\sigma-1}{\sigma}\right)\frac{R}{(\beta R)^\sigma}\left[\frac{\lambda m_t^\delta(1-\sigma\delta)+\underline{p}(1-\delta)}{\lambda m_t^\delta\left(\underline{p}+\lambda m_t^\delta\right)^\sigma}\right] > 0,$$

since $(1-\sigma\delta)$ is assumed positive from proposition 6, and $\Phi_1(0) = 0$.

The function $\Phi_2(m_t)$ in equation (B.3) is decreasing with respect to health investment, that is:

$$\frac{\partial \Phi_2(m_t)}{\partial m_t} = -\left[\frac{\lambda m_t^{\delta}(\sigma - 1 + \delta\sigma) + \underline{p}(\sigma-1)(1-\delta)}{\sigma\delta\lambda m_t^{\delta}}\right] < 0$$

and $\Phi_2(0) = \frac{\theta}{\sigma}\left[1 + \frac{1}{R}\right]$.

Thus since $\Phi_1(m_t)$ and $\Phi_2(m_t)$ have different intercepts, i.e. $\Phi_1(0) = 0$ and $\Phi_2(0) = \frac{\theta}{\sigma}\left[1 + \frac{1}{R}\right]$, and $\Phi_1(m_t)$ is increasing in health and $\Phi_2(m_t)$ is decreasing in health, we obtain that there exist a value of m_t, i.e m_0, such that the two functions intersect.

B.2 Proof of proposition 6

Equation (3.20) implicitly defines optimal health investment as a function of income. Applying the implicit function theorem to equation (3.20) we get:

$$\frac{\partial m_t}{\partial y_t} = \frac{\sigma\delta\lambda m_t^{\delta} G(m_t)}{(1-\delta)(\sigma-1)\underline{p}\left[G(m_t) + R\right] + \lambda m_t^{\delta}\left[R(1-\sigma\delta)(\sigma-1) + (\sigma - 1 + \sigma\delta)G(m_t)\right]},$$

where:

$$G(m_t) = \left[\beta R(\underline{p} + \lambda m_t^{\delta})\right]^{\sigma}$$

A sufficient condition to have health increasing in income is that:

$$\delta \leq \frac{1}{\sigma}. \tag{B.4}$$

We have that $\partial^2 m_t/\partial y_t^2 > 0$ if:

$$R\sigma\lambda m_t^{\delta}\left[\lambda m_t^{\delta}(1-\sigma\delta) + \underline{p}(1-\delta)\right] + \underline{p}(1-\delta)\left(\lambda m_t^{\delta} + \underline{p}\right)\left[G(m_t) + R\right] > 0.$$

which is satisfied when inequality (B.4) holds.

Analysis of Health Share

From equation (3.20) the following expression defines the health share:

$$\begin{aligned}\frac{m_t}{y_t} &= m_t\left\{\left(\frac{\sigma-1}{\sigma}\right)\frac{m_t}{\delta}\left(1 + \frac{\underline{p}}{\lambda m_t^{\delta}}\right)\left[\frac{R^{1-\sigma}}{\left[\beta\left(\underline{p} + \lambda m_t^{\delta}\right)\right]^{\sigma}} + 1\right] + \right. \\ &\left. + m_t - \frac{\theta}{\sigma}\left[1 + \frac{1}{R}\right]\right\}^{-1}.\end{aligned} \tag{B.5}$$

When income tends to zero, i.e. $m_t \to m_0$, we get:

$$\lim_{m \to m_0}\frac{m_t}{y_t} = \frac{m_0}{0} = \infty. \tag{B.6}$$

When $y_t \to \infty$ which, from equation (3.13), implies that $m_t \to \hat{m}$, health share is equal to a positive constant:

$$\lim_{m \to \hat{m}}\frac{m_t}{y_t} = \frac{\hat{m}}{\hat{y}} > 0.$$

Deriving equation (B.5) with respect to income we obtain:

$$\frac{\partial\left(m_t/y_t\right)}{\partial y_t}=\frac{\left(\partial m_t/\partial y_t\right)y_t-m_t}{y_t^2}, \tag{B.7}$$

where $\partial\left(m_t/y_t\right)/\partial y_t>0$ if:

$$\varepsilon_m=\frac{\left(\partial m_t/\partial y_t\right)y_t}{m_t}>1, \tag{B.8}$$

where ε_m is the elasticity of health spending with respect to income. Thus health share behaves like a luxury good if presents an elasticity with respect to income larger than one.

Since the denominator of equation (B.7) is always positive we study the numerator that is given by the following expression:

$$(\sigma-1)\,\delta m_t\left(\sigma\lambda m_t^{\delta}+\underline{p}\right)R^2+\delta G\left(m_t\right)\left[\left(\sigma-1\right)Rm_t\underline{p}-\lambda m_t^{\delta}\theta\left(1+R\right)\right],$$

from which $\varepsilon_m=1$ if:

$$\frac{(\sigma-1)\,m_t^{1-\delta}\left(\sigma\lambda m_t^{\delta}+\underline{p}\right)R^2}{G\left(m_t\right)}=-m_t^{1-\delta}\left(\sigma-1\right)R\underline{p}+\lambda\theta\left(1+R\right). \tag{B.9}$$

Thus we can analyze the two functions:

$$\psi_1\left(m_t\right)=\frac{(\sigma-1)\,m_t^{1-\delta}\left(\sigma\lambda m_t^{\delta}+\underline{p}\right)R^2}{G\left(m_t\right)},$$

$$\psi_2\left(m_t\right)=-m_t^{1-\delta}\left(\sigma-1\right)R\underline{p}+\lambda\theta\left(1+R\right).$$

From condition in equation (B.4) we have that the function $\psi_1\left(m_t\right)$ is increasing in health investment, that is:

$$\frac{\partial\psi_1}{\partial m_t}=\frac{(\sigma-1)}{G\left(m_t\right)}\left\{\sigma\lambda\left[\lambda m_t^{\delta}\left(1-\sigma\delta\right)+\underline{p}\left(1-\delta\right)\right]+\frac{\left(1-\delta\right)\underline{p}}{m_t^{\delta}}\right\}>0,$$

and:

$$\psi_1\left(0\right)=0,$$

$$\lim_{m\to\infty}\psi_1\left(m_t\right)=\infty.$$

The function ψ_2 decreases in health investment, that is:

$$\frac{\partial\psi_2}{\partial m_t}=-\frac{(\sigma-1)\left(1-\delta\right)R\underline{p}}{m_t^{\delta}}<0,$$

and:

$$\psi_2\left(0\right)=\lambda\theta\left(1+R\right),$$

$$\lim_{m\to\infty}\psi_2\left(m_t\right)=-\infty.$$

Thus there exist a value $\bar{m}$ so that equation (B.9) is satisfied, that is $\varepsilon_m=1$. Substituting this value $\bar{m}$ to the equation (3.20) we obtain the value y_m so that $\varepsilon_m=1$. When $y_t<y_m$ then $\psi_2\left(m_t\right)>\psi_1\left(m_t\right)$, that is $\varepsilon_m<1$ and the health share is decreasing in income. When $y_t>y_m$ then $\psi_2(m_t)>\psi_1(m_t)$ and $\varepsilon_m>1$, that is the health share increases.

B.3 Proof of proposition 7

The relationship between saving and health is positive and concave. That is, differentiation of equation (3.19) with respect to health investment give us:

$$\frac{\partial s_t}{\partial m_t} = \frac{1}{\delta}\left(\frac{\sigma - 1}{\sigma}\right)\left[\frac{\underline{p}\,(1-\delta) + \lambda m_t^{\delta}}{\lambda m_t^{\delta}}\right], \tag{B.10}$$

and :

$$\frac{\partial^2 s_t}{\partial m_t^2} = -\left(\frac{\sigma - 1}{\sigma}\right)\frac{\underline{p}\,(1-\delta)}{m_t^{(\delta+1)}}.$$

Thus $\partial s_t/\partial m_t > 0$ and $\partial^2 s_t/\partial m_t^2 < 0$ since $0 < \delta < 1$ and $\sigma > 1$.

When $m_t = 0$ we have that the saving is negative, that is:

$$s_t = -\frac{\theta}{\sigma R}$$

We suppose that when $m_t = m_0$ the saving is negative, that is:

$$s_0 = m_0^{1-\delta}\left(\lambda m_0^{\delta} + \underline{p}\right) - \frac{\theta\delta}{R\,(\sigma - 1)} < 0 \tag{B.11}$$

From condition in equation (B.4) we obtain that the saving behaves like a luxury good, that is:

$$\frac{\partial s_t}{\partial y_t} = \frac{\partial s_t}{\partial m_t}\frac{\partial m_t}{\partial y_t} > 0$$

Analysis of Saving Share

Equations (3.19) and (3.20) yield the following expression for the saving share on income:

$$\frac{s_t}{y_t} = \frac{1}{y_t}\left[\frac{m_t}{\delta}\left(\frac{\sigma - 1}{\sigma}\right)\left(1 + \frac{\underline{p}}{\lambda m_t^{\delta}}\right) - \frac{\theta}{\sigma R}\right]. \tag{B.12}$$

From equation (B.1)and given the condition in equation (B.11), when $y_t = 0$, i.e $m_t = m_0$ it follows that:

$$\lim_{m \to m_0} \frac{s_t}{y_t} = -\infty. \tag{B.13}$$

When $m_t \to \hat{m}$ we have that:

$$\lim_{m \to \hat{m}} \frac{s_t}{y_t} = \frac{\hat{s}}{\hat{y}} > 0$$

Deriving the saving share with respect to income we get:

$$\frac{\partial(s_t/y_t)}{\partial y_t} = \frac{1}{y_t^2}\left[\frac{\partial s_t}{\partial y_t} y_t - s_t\right], \tag{B.14}$$

Equation (B.14) is given by the following expression:

$$\begin{aligned}&(\sigma - 1)\,R\left\{\theta\left[(1-\sigma\delta)\,\lambda m_t^{\delta} + \underline{p}\,(1-\delta)\right] + (\sigma-1)\,\sigma m_t\left(\underline{p} + \lambda m_t^{\delta}\right)R\right\} + \\ &G(m_t)\left[(\sigma-1)\left(\theta\delta - \theta - \sigma m_t\delta\right)\underline{p}R + \theta\lambda m_t^{\delta}\left(\sigma\left(\delta - R\right) + R\right)\right],\end{aligned}$$

from which $\partial(s_t/y_t)/\partial y_t > 0$ if :

$$(\sigma-1)\,R\left\{\theta\left[(1-\sigma\delta)\,\lambda m_t^{\delta}+\underline{p}\,(1-\delta)\right]+(\sigma-1)\,\sigma m_t\left(\underline{p}+\lambda m_t^{\delta}\right)R\right\}+$$

$$+G(m_t)\left[\frac{(\sigma-1)\,(\theta\delta-\theta-\sigma m_t\delta)\,\underline{p}R}{(\sigma\,(\delta-R)+R)}\right]>-G(m_t)\theta\lambda m_t^{\delta}.$$

We define the function in left side $\Upsilon_1(m_t)$ and the function in the right side $\Upsilon_2(m_t)$. The function $\Upsilon_1(m_t)$ at $m=0$ is positive:

$$\Upsilon_1(0)=(\sigma-1)\,R\left\{\theta\left[\underline{p}\,(1-\delta)\right]\right\}+\left[\frac{(\sigma-1)\,\theta\,(1-\delta)\,\underline{p}R}{R\,(\sigma-1)-\sigma\delta}\right]>0,$$

if:

$$R>\frac{\sigma\delta}{(\sigma-1)}. \tag{B.15}$$

If the condition in equation (B.4) and the condition in equation (B.15) are satisfied, it follows that $\Upsilon_1(m_t)$ is increasing in m_t, that is:

$$\frac{\partial\Upsilon_1}{\partial m_t}=(\sigma-1)\,\sigma\left[(1+\delta)\,\lambda m_t^{\delta}+\underline{p}\right]+\frac{\sigma\beta\delta\underline{p}\left[(1+\sigma\delta)\,\lambda m_t^{\delta}+\underline{p}\right]RG(m_t)}{(R\,(\sigma-1)-\sigma\delta)\,G(m_t)^{1/\sigma}}+$$

$$+\theta\delta\lambda m_t^{\delta-1}\left[1-\sigma\delta+\frac{\sigma\beta\,(1-\delta)\,\underline{p}RG(m_t)}{(R\,(\sigma-1)-\sigma\delta)\,G(m_t)^{1/\sigma}}\right]>0,$$

The function $\Upsilon_2(m_t)$ is decreasing in m_t and when $m_t=0$ is equal to zero, that is:

$$\Upsilon_2(0)=0,$$

and:

$$\frac{\partial\Upsilon_2}{\partial m_t}=-G(m_t)\theta\lambda m_t^{\delta}<0$$

Thus since the function $\Upsilon_1(m_t)>\Upsilon_2(m_t)$ when the condition in equation (B.4) and the condition in equation (B.15) are satisfied, it follows that the saving share increases with income.

B.4 Proof of proposition 8

Given equation (3.19) we get that:

$$\frac{\partial(s_t/m_t)}{\partial y_t}=\frac{1}{m_t^2}\frac{\partial m_t}{\partial y_t}\left[\frac{\partial s_t}{\partial m_t}m_t-s_t\right]$$

where from equation (B.4) $\partial m_t/\partial y_t>0$. From equations (B.10) and (3.19) we obtain:

$$\frac{\partial s_t}{\partial m_t}m_t-s_t=\frac{\theta}{\sigma R}-\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\underline{p}}{\lambda}m_t^{1-\delta}\right)=0,$$

when:

$$\tilde{m} = \left[\frac{\lambda\theta}{\underline{p}R\left(\sigma-1\right)}\right]^{\frac{1}{1-\delta}}. \tag{B.16}$$

Substituting equation (B.16) in equation (3.20) we get:

$$\tilde{y} = y\left(\tilde{m}\right).$$

If $y < \tilde{y}$ then $\partial(s_t/m_t)/\partial y_t > 0$, that is:

$$\frac{\theta}{\sigma R} - \left(\frac{\sigma-1}{\sigma}\right)\left[\frac{\underline{p}}{\lambda} m_t^{1-\delta}\right] > 0,$$

when:

$$m_t < \tilde{m}.$$

When $y_t > \tilde{y}$ the ratio s_t/m_t is decreasing, that is:

$$\frac{\theta}{\sigma R} - \left(\frac{\sigma-1}{\sigma}\right)\left[\frac{\underline{p}}{\lambda} m_t^{1-\delta}\right] < 0$$

if:

$$m_t > \tilde{m}$$

Thus the ratio s_t/m_t, for $y < \tilde{y}$ is increasing and for $y > \tilde{y}$ is decreasing.
Given equation (3.17) we show that there exist a value of y such that health share is equal to the saving share:

$$\frac{s_t}{m_t} = 1, \tag{B.17}$$

thus:

$$\frac{1}{\delta}\left(\frac{\sigma-1}{\sigma}\right)\left(1 + \frac{\underline{p}}{\lambda m_t^{\delta}}\right) - \frac{\theta}{\sigma R m_t} - 1 = 0,$$

from which we study the two function:

$$m_t\left[(\sigma-1) - \delta\sigma\right] - \theta\delta = -\frac{R\left(\sigma-1\right)}{\lambda} m_t^{1-\delta}$$

where the function in the left side for $m_t = 0$ is equal $-\theta\delta$, for $m_t \to \infty$ it goes to infinity and finally it increases with m_t if the following condition is satisfied:

$$\sigma > \frac{1}{1-\delta}$$

The function in the right is decreasing and for $m_t = 0$ it is equal to zero and $m_t \to \infty$ it is equal to $-\infty$. Thus since the function in the left increases and the function in the right decreases, the two functions cross at $\underline{m}$. Substituting $\underline{m}$ in equation (3.20) we obtain the value $\underline{y}$ so that the saving share is equal to the health share, that is $\varepsilon_{u_c} = \varepsilon_p$.

Bibliography

Adams, S. (2002). "Educational Attainment and Health Evidence from a Sample of Older Adult". *Education Economics 10*, 97–109.

Becker, G. S. (1962). "Investment in Human Capital: A Theoretical Analysis". *Journal of Political Economy 70*, 9–49.

Becker, G. S. (1965). "A Theory of The Allocation of Time". *Economics Journal 75*, 493–517.

Becker, G. S., T. Philipson, and R. R. Soares (2005). "The Quantity and Quality of Life and the Evolution of World Inequality". *American Economic Review 95*, 277–291.

Blackburn, K. and G. P. Cipriani (2002). "A model of longevity fertility and Growth". *Journal of Economic Dynamics and Control 26*, 187–204.

Blomqvist, A. and R. Carter (1997). "Is Health Care Really a Luxury?". *Journal of Health Economics 16*, 207–229.

Bloom, D. and D. Canning (2000). "The health and wealth of nations". *Science 287*, 1207–1209.

Bloom, D., D. Canning, and B. Graham (2003). "Longevity and Life-cycle Saving". *Scandinavian Journal of Economics 105*, 319–338.

Boucekkine, R., D. D. Croix, and O. Licandro (2002). "Vintage Human Capital, Demographic Trends, and Endogenous Growth". *Journal of Economic Theory 104*, 340–375.

Bowman, A. W. and A. Azzalini (1997). *Applied Smoothing Techniques for Data Analysis*. Oxford: Clarendon Press.

Cervellati, M. and U. Sunde (2005). "Human Capital Formation, Life Expectancy and the Process of Development". *American Economic Review 95*, 1653–1672.

Chakraboty, S. and M. Das (2005). "Mortality Human Capital and Persistent Inequality". *Journal of Economic Growth 10*, 159–192.

Cutler, D., A. Deaton, and A. Lleras-Muney (2006). "The Determinants of Mortality". *Journal of Economic Perspectives 20*, 97–120.

De la Croix, D. and O. Licandro (1999). "Life Expectancy and Endogenous Growth". *Economic Letters* (65), 225–263.

De la Croix, D. and P. Michel (2002). *A Theory of Economic Growth*. Cambridge University Press.

Deaton, A. (2003). "Health, Inequality and Economic Development". *Journal of Economic Literature 41*, 113–158.

Diamond, P. A. (1965). "National Debt in a Neoclassical Growth Model". *American Economic Review 55*, 1126–1150.

Ehrlich, I. and H. Chuma (1990). "A Model of the Demand for Longevity and the value of Life Extension". *Journal of Political Economy 98*, 761–782.

Ehrlich, I. and Y. Yin (2005). "Explaining Diversities in Age-Specific Life Expectancies and Values of Life Saving: A Numerical Analysis". *Journal of Risk and Uncertainty 31*, 129–162.

Fogel, R. W. (1994). "Economic Growth, Population Theory, and Physiology: The Bearing of Long-Term Processes on the Making of Economic Policy". *American Economic Review 84*, 369–395.

Galor, O. and D. N. Weil (1999). "From Malthusian Stagnation to Modern Growth". *American Economic Review 89*, 150–154.

Grossman, M. (1972). "On the Concept of Health Capital and the Demand for Health". *Journal of Political Economy 80*, 223–255.

Grossman, M. (1982). "Government and Health Outcomes". *American Economic Review 72*, 191–195.

Grossman, M. (2004). "TheDemand for health, 30 years later: a very personal retrospective and prospective reflection". *Journal of Health Economics 23*, 629–636.

Hardle, W., M. Muller, S. Sperlich, and A. Werwatz (2004). "Nonparametric and Semiparametric Models. ". *Springer,http://www.xplore-stat.de/ebooks/ebooks.html.*.

Jones, C. I. (2004). "Why Have Health Expenditure as a Share of GDP Risen so Much?". *mimeo University of Berkeley.*

Jones, C. I. and R. E. Hall (2006). "The Value of Life and the Rise in Health Spending". *Quarterly Journal of Economics 122*, 39–72.

Kageyama, J. (2003). "Theeffects of a continuos increase in lifetime on saving". *Review of Income and Wealth 49*, 163–183.

Kalemli-Ozcan, S. (2002). "Does the mortality decline Promote Economic Growth?". *Journal of Economic Growth 7*, 411–439.

Lagerlof, N. P. (2003). "From Malthus to Modern Growth: Can Epidemics Explain the three Regimes". *International Economic Review 44*, 755–777.

Livi-Bacci, M. (2001). *A concise history of world population.* Oxford: Blackwell publishers.

Lleras-Muney, A. (2005). "The Relationship Between Education and Adult Mortality in the U.S. ". *Review of Economic Studies 72*, 189–221.

Lucas, R. E. (1988). "On the mechanics of Economic Development". *Journal of Monetary Economics 22*, 3–42.

Merton, R. C. (1992). *Continuous-Time Finance, revised edition.* Cambridge, MA: Blackwell Publishers.

Modigliani, F. and R. Brumberg (1980). "Utility Analysis and Aggregate Consumption Functions: An Attempt at Integration". *in The Collected Papers of Franco Modigliani Edited by Abel A.*(MA: The MIT Press), 128–197.

Murphy, K. M. and R. Topel (2006). "The Value of Health and Longevity". *Journal of Political Economy 114*, 871–904.

Mushkin, S. J. (1962). "Health as an Investment". *Journal of Political Economy 70*(5), 129–157.

Preston, S. (1975). "The Changing Relation between Mortality and Level of Economic Development". *Population Studies 29*(2), 231–248.

Rivera, B. and L. Currais (1999). "Income Variation and Health Expenditure: Evidence from OECD Countries". *Review of Development Economics 3*, 258–267.

Romer, P. M. (1986). "Increasing Returns and Long-run Growth". *Journal of Political Economy 94*(5), 1002–1037.

Rosen, S. (1988). "The Value of Changes in Life Expectancy". *Journal of Risk and Uncertainty*, 285–304.

Sen, K. A. (1999). *Development as Freedom.* London: Oxford University Press.

Shultz, T. (1993). "Mortality Decline in the Low-Income World: Causes and Consequences". *American Economic Review 83*, 337–342.

Shultz, T. (1999). "Health and Schooling Investments in Africa". *Journal of Economic Perspectives 13*, 67–88.

Soares, R. (2005). "Mortality Reductions, Educational Attainment and Fertility Choice". *American Economic Review 95*(3), 580–601.

World Health Organization (2001). "Macroeconomics and Health: Investing in Health for Economic Development". *Report of the Commission on Macroeconomics and Health* (Geneva: World Health Organization).

Zhang, J., J. Zhang, and R. Lee (2003). "Rising Longevity, Education, Savings, and Growth". *Journal of Development Economics 70*, 83–101.

Zon, A. V. and J. Muysken (2001). "Health and EndogenousGrowth". *Journal of Health Economics 20*, 169–185.

Zon, A. V. and J. Muysken (2005). "Health as a principal determinant of Economic Growth". *in Health and Economic Growth: Findings and Policy Implications Cambridge, MA: MIT Press*(G.L. Casasnovas and B. Rivera and L. Currais), 41–66.