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## Asymptotically Safe Standard Model via Vectorlike Fermions

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We construct asymptotically safe extensions of the standard model by adding gauged vectorlike fermions. Using large number-of-flavor techniques we argue that all gauge couplings, including the hypercharge and, under certain conditions, the Higgs coupling, can achieve an interacting ultraviolet fixed point.

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Although the standard model (SM) of particle interactions is an extremely successful theory of nature, it is an effective theory but not a fundamental one. Following Wilson [1,2], a theory is fundamental if it features an ultraviolet fixed point. The latter can be either noninteracting (asymptotic freedom) [3–12] or interacting (asymptotically safe) [13–15] or mixed [9,15–17]. Except for the non-Abelian gauge couplings none of the remaining SM couplings features an ultraviolet fixed point.

Here we extend the idea of a safe QCD scenario in Ref. [18] to the entire SM. We argue that an asymptotically safe completion of the SM can be realized via new vectorlike fermions. Recently an interesting complementary approach appeared [20] in which new fermions in higher dimensional representations of the SM gauge groups were added, hoping for a (quasi) perturbative UV fixed point. However these models were unable to lead to a safe hypercharge and Higgs self-coupling. Our work relies on the limit of a large number of fermion matter fields, which allows us to perform a  $1/N_F$  expansion [21,22]. Here the relevant class of diagrams can be summed up to arbitrary loop order, leading to an UV interacting fixed point for the (non) Abelian interactions of the SM. Thus, we go beyond the cornerstone work of Ref. [13] where UV safety is realized in the Veneziano-Witten limit by requiring both  $N_c$  and  $N_F$  to go to infinity with their ratio fixed, and adjusting it close to the value for which asymptotic freedom is lost.

Depending on how these new vectorlike fermions obtain their masses, we can either introduce new scalars that generate fermion masses through new Yukawa operators or simply introduce explicit vectorlike mass operators.

In the following, we focus on the latter most economical case and explore the following three distinct SM  $SU(3) \times SU_L(2) \times U(1)$  charge assignments and multiplicity: (i)  $N_F(3, 2, 1/6)$ ; (ii)  $N_{F_3}(3, 1, 0) \oplus N_{F_2}(1, 2, 1/2)$ ; (iii)  $N_{F_3}(3, 1, 0) \oplus N_{F_2}(1, 3, 0) \oplus N_{F_1}(1, 1, 1)$ . To the above, one needs to add, for each model, the associated right charge-conjugated fermions. The above models are to be viewed as templates that allow us to exemplify our novel approach in the search of an asymptotically safe extension of the SM. The basic criterion is that different fermions should have the same charge if it is nonzero; otherwise the summation technique fails [see Eq. (4) and the corresponding discussion]. In fact, we have checked that other models [e.g.,  $N_{F_3}(3, 1, 2/3) \oplus N_{F_2}(1, 3, 0)$  featuring new top primes] lead to similar results as the ones used here. We note that, following our innovative approach, a recent follow-up paper appeared [33], in which the set  $N_{F_3}$  is abandoned. Here QCD remains asymptotically free while the rest of the SM gauge couplings are still safe. Finally, we neglect [in model (i)] possible mixing among the new vectorlike fermions and SM quarks.

We start by considering the RG equations describing the gauge-Yukawa-quartic to two loop order including vectorlike fermions. We have checked that our results agree for the SM case with the ones in Refs. [23,24]. We used Refs. [25,26] for the vectorlike fermion contributions to gauge couplings and [27,28] for the contributions to the Higgs quartic. The associated beta functions read

$$\begin{aligned}\beta_1 &= \frac{d\alpha_1}{dt} = \left( b_1 + c_1\alpha_1 + d_1\alpha_3 + e_1\alpha_2 - \frac{17}{3}\alpha_{y_i} \right) \alpha_1^2, \\ \beta_2 &= \frac{d\alpha_2}{dt} = \left( -b_2 + c_2\alpha_2 + d_2\alpha_3 + e_2\alpha_1 - 3\alpha_{y_i} \right) \alpha_2^2, \\ \beta_3 &= \frac{d\alpha_3}{dt} = \left( -b_3 + c_3\alpha_3 + d_3\alpha_2 + e_3\alpha_1 - 4\alpha_{y_i} \right) \alpha_3^2, \\ \beta_{y_i} &= \frac{d\alpha_{y_i}}{dt} = \left( 9\alpha_{y_i} - \frac{9}{2}\alpha_2 - 16\alpha_3 - \frac{17}{6}\alpha_1 \right) \alpha_{y_i} + \beta_{y_i}^{2\text{loop}},\end{aligned}$$

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$$\begin{aligned}
\beta_{\alpha_h}^{\text{1loop}} &= \frac{d\alpha_h}{dt} = \frac{3}{8}(\alpha_1^2 + 3\alpha_2^2 + 2\alpha_1(\alpha_2 - 4\alpha_h) \\
&\quad + 64\alpha_h^2 - 24\alpha_2\alpha_h + 32\alpha_h\alpha_{y_i} - 16\alpha_{y_i}^2), \\
\beta_{\alpha_h}^{\text{2loop}} &= \frac{1}{6}[-4D_{R_3}S_2(R_2)\alpha_2^2N_{F_2}(2\alpha_1 + 6\alpha_2 - 15\alpha_h) \\
&\quad - 4D_{R_3}D_{R_2}\alpha_1^2Y^2N_{F_1}(2\alpha_1 + 2\alpha_2 - 5\alpha_h)] + \beta_{\alpha_h}^{\text{SM}}, \\
\beta_{\alpha_h}^{\text{SM}} &= \frac{1}{48}(-379\alpha_1^3 - 559\alpha_2\alpha_1^2 - 289\alpha_2^2\alpha_1 + 915\alpha_2^3) \\
&\quad + \frac{1}{48}(1258\alpha_1^2 + 468\alpha_2\alpha_1 - 438\alpha_2^2)\alpha_h - 312\alpha_h^3 \\
&\quad + \frac{1}{48}(1728\alpha_1 + 5184\alpha_2)\alpha_h^2, \quad (1)
\end{aligned}$$

where  $t = \ln(\mu/M_Z)$  and  $\alpha_1, \alpha_2, \alpha_3, \alpha_{y_i}, \alpha_h$  are the U(1), SU(2), SU(3), top-Yukawa, and Higgs self-couplings, respectively, and we have used the normalization

$$\alpha_i = \frac{g_i^2}{(4\pi)^2}, \quad \alpha_{y_i} = \frac{y_i^2}{(4\pi)^2}, \quad \alpha_h = \frac{\lambda_h}{(4\pi)^2}. \quad (2)$$

$\beta_{\alpha_h}^{\text{SM}}$  and  $\beta_{\alpha_h}^{\text{2loop}}$  represent two loop SM contributions to the RG functions of  $\alpha_h$  and  $\alpha_{y_i}$ , which are not shown explicitly.  $D_{R_2}, D_{R_3}$  represent the dimensions of the representations ( $R_2, R_3$ ) under SU(2) and SU(3), while  $S_2(R_2)$  represents the Dynkin index of the representation  $R_2$ . The contributions of the SM chiral fermions are encoded in  $b_1, b_2, b_3, c_1, c_2, c_3, d_1, d_2, e_2, e_3$  in Eq. (3) and can be distinguished from the new vectorlike contributions that are all proportional to a “ $D_R$ ” coefficient

$$\begin{aligned}
b_1 &= \frac{41}{3} + \frac{8}{3}Y^2N_F D_{R_2}D_{R_3}, & c_1 &= \frac{199}{9} + \frac{8}{3}Y^4N_F D_{R_2}D_{R_3}, \\
b_2 &= \frac{19}{3} - \frac{4N_F}{3}D_{R_3}, & c_2 &= \frac{35}{3} + \frac{49N_F}{3}D_{R_3}, \\
b_3 &= 14 - \frac{4N_F}{3}D_{R_2}, & c_3 &= -52 + \frac{76N_F}{3}D_{R_2}, \\
d_1 &= \frac{88}{3} + \frac{32}{3}Y^2N_F D_{R_2}D_{R_3}, & e_1 &= 9 + 6Y^2N_F D_{R_2}D_{R_3}, \\
d_2 &= 24 + \frac{16}{3}N_F D_{R_3}, & e_2 &= 3 + 4Y^2N_F D_{R_3}, \\
d_3 &= 9 + 3N_F D_{R_2}, & e_3 &= \frac{11}{3} + 4N_F Y^2 D_{R_2}, \quad (3)
\end{aligned}$$

where for simplicity, the above explicit coefficients only apply to fundamental representations [models (i) and (ii)]; for higher dimension representations the corresponding Casimir invariants and the Dynkin index should be incorporated.

The following diagrams (see Fig. 1) encode the infinite tower of higher order contributions to the self-energies related to the gauge couplings. These diagrams can be

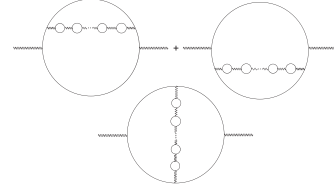


FIG. 1. Higher order self-energy diagram.

summed up analytically (the Abelian and non-Abelian cases were first computed, respectively, in Refs. [29,30]).

To the leading  $1/N_F$  order, the higher order (ho) contributions to the RG functions of  $\beta_2$  and  $\beta_3$  are given by Ref. [21] and have been generalized to the case with any hypercharge  $Y$  and semi-simple group ( $F_1$  first appeared in Ref. [29]):

$$\beta_{\text{ho}i} = \frac{2A_1\alpha_1 F_1(A_1)}{3 N_F}; \quad \beta_{\text{ho}i} = \frac{2A_i\alpha_i H_{1i}(A_i)}{3 N_F} \quad (i = 2, 3), \quad (4)$$

where

$$\begin{aligned}
A_1 &\equiv 4\alpha_1 N_F Y^2 D_{R_2} D_{R_3}; & A_2 &\equiv 2\alpha_2 N_F D_{R_3}, \\
A_3 &\equiv 2\alpha_3 N_F D_{R_2}, & F_1 &= \int_0^{A/3} I_1(x) dx, \\
H_{1i} &= \frac{-11}{2} N_{ci} + \int_0^{A/3} I_1(x) I_2(x) dx \quad (N_{ci} = 2, 3), \\
I_1(x) &= \frac{(1+x)(2x-1)^2(2x-3)^2 \sin(\pi x)^3}{(x-2)\pi^3} \\
&\quad \times (\Gamma(x-1)^2 \Gamma(-2x)), \\
I_2(x) &= \frac{N_{ci}^2 - 1}{N_{ci}} + \frac{(20 - 43x + 32x^2 - 14x^3 + 4x^4)}{2(2x-1)(2x-3)(1-x^2)} N_{ci}.
\end{aligned}$$

We recall that the validity of the summation depends on our first criterion which implies that for each gauge group we have only a single  $A_i$ , constraining the possible vectorlike models.  $F_1$  has poles at  $A = 15/2 + 3n$  while  $H_{1i}$  has poles at  $A = 3, 15/2, \dots, 3n + 9/2$ . In this Letter we concentrate on the first UV pole branch ( $A = 15/2$  for  $F_1$  and  $A = 3$  for  $H_{1i}$ ). Note that the pole structure of  $H_{1i}$  is the same for all the non-Abelian groups, implying that when  $N_F$  is fixed, the non-Abelian gauge coupling values will be very close to each other if  $D_{R_2} = D_{R_3}$ . The presence of the UV poles at  $F_1$  and  $H_{1i}$  guarantees the existence of an UV safe fixed point for the gauge couplings. Note that the functions  $F_1$  and  $H_{1i}$  are scheme independent according to [31]. We therefore expect the pole structure and the related UV fixed points to be scheme independent. Physical quantities, such as scaling exponents, were computed in [13]. The  $1/N_F^2$  terms are negligible for  $N_F$  sufficiently large. Specifically, as pointed out in Ref. [21], for SU(3) one finds that  $N_F$  needs to be larger than 32 while for U(1) one finds  $N_F \geq 16$ .

Thus the total RG functions for the gauge-Yukawa subsystem can be written as

$$\begin{aligned}\beta_{1\text{tot}} &= \beta_1(\alpha_{1\text{tot}}, \alpha_{2\text{tot}}, \alpha_{3\text{tot}}, \alpha_{y,\text{tot}}) + \beta_{\text{ho1}}(\alpha_{1\text{tot}}), \\ \beta_{3\text{tot}} &= \beta_3(\alpha_{1\text{tot}}, \alpha_{2\text{tot}}, \alpha_{3\text{tot}}, \alpha_{y,\text{tot}}) + \beta_{\text{ho3}}(\alpha_{3\text{tot}}), \\ \beta_{2\text{tot}} &= \beta_2(\alpha_{1\text{tot}}, \alpha_{2\text{tot}}, \alpha_{3\text{tot}}, \alpha_{y,\text{tot}}) + \beta_{\text{ho2}}(\alpha_{2\text{tot}}), \\ \beta_{y,\text{tot}} &= \left(9\alpha_{y,\text{tot}} - \frac{9}{2}\alpha_{2\text{tot}} - 16\alpha_{3\text{tot}}\right)\alpha_{y,\text{tot}} + \beta_{y,\text{tot}}^{2\text{loop}},\end{aligned}\quad (5)$$

where  $\alpha_{i\text{tot}}$  corresponds to the gauge couplings including the leading  $1/N_F$  contribution to the self-energy diagrams, and  $\alpha_{y,\text{tot}}$  is the accordingly modified Yukawa coupling. We also avoided the double counting problem due to the simultaneous presence of the  $c_i$  ( $i = 1, 2, 3$ ) terms in Eq. (1) and the leading terms of  $\beta_{\text{ho2}}$ ,  $\beta_{\text{ho3}}$  in Eq. (4). We employ the MS scheme, which is a mass independent RG scheme allowing us to investigate the running of the couplings independently of the running vectorlike masses, except for threshold corrections that can be shown to be controllably small.

Solving Eq. (5), we obtain the running coupling solutions depicted in Fig. 2 by the blue, green, red, and purple curves, corresponding, respectively, to the U(1), SU(2), SU(3) gauge couplings, and top Yukawa coupling; the orange curve corresponding to the Higgs coupling (discussed below) has not yet been included. It is clear that all the gauge couplings are UV asymptotically safe while the top Yukawa coupling is asymptotically free. Note that the sub-system encounters an interacting UV fixed point at  $3.2 \times 10^{13}$  GeV which is safely below the Planck scale and so gravity contributions can be safely ignored. For the UV

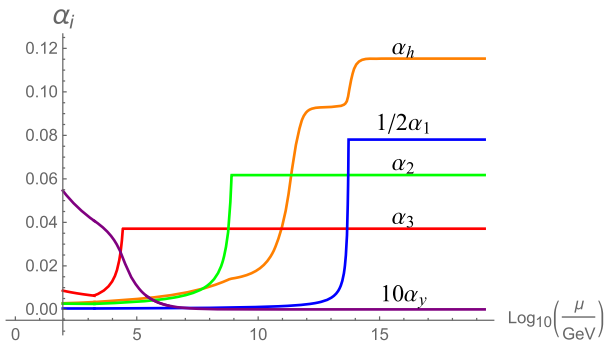


FIG. 2. Running of the gauge-Yukawa couplings as a function of the RG time with  $\log_{10}$  base using model (ii)  $[N_{F3}(3, 1, 0) \oplus N_{F2}(1, 2, 1/2)]$ . The blue, green, red, and purple curves correspond, respectively, to the U(1), SU(2), SU(3) gauge, and top Yukawa couplings. The top Yukawa coupling  $\alpha_y$  and U(1) gauge coupling  $\alpha_1$  have been rescaled by a factor of 10 and  $1/2$ , respectively, to fit all couplings on one figure. The orange curve depicts the two loop level Higgs quartic coupling  $\alpha_h$  in the same model. Here  $N_{F3} = 40$ ,  $N_{F2} = 24$ , and the initial values of the gauge and Yukawa couplings are chosen to be the SM coupling values at 2 TeV, while the Higgs quartic coupling is chosen to be 0.0034.

fixed point to exist, the choice of the initial value of the gauge coupling is not crucial since the only requirement is  $\alpha_i(t_0) < \alpha_i(t_*)$ , ( $i = 1, 2, 3$ ) where  $t_0 = \ln(\mu_0/M_Z)$  is an arbitrary initial scale and  $t_*$  is the scale for the UV fixed point. For simplicity, instead of sequentially introducing new vectorlike fermions, we assume they are introduced all at once at a particular scale near their MS-scheme mass  $m(m) = m$  ( $m \approx \mu = 2$  TeV (or  $t = 3$ ) in Fig. 2). We have checked that our results change very little if we employ different vector-like fermion masses corresponding to a larger matching scale, e.g.,  $m \approx \mu = 100$  TeV; the UV fixed point transition scale increases accordingly to around  $10^{16}$  GeV. Note that a too small  $N_F$  will fail the  $1/N_F$  expansion. To produce Fig. 2, we have used model (ii) with  $N_{F3} = 40$ ,  $N_{F2} = 24$  with the initial values of the gauge and Yukawa couplings chosen to be the SM coupling values at 2 TeV corresponding to  $t_0 = 3$ :

$$\begin{aligned}\alpha_3(t_0) &= 0.00661, & \alpha_2(t_0) &= 0.00256, \\ \alpha_1(t_0) &= 0.00084, & \alpha_y(t_0) &= 0.00403.\end{aligned}\quad (6)$$

We emphasize that the basic features of the gauge and Yukawa curves in Fig. 2 are generic and not limited only to model (ii). Figures similar to Fig. 2 result for all three vectorlike fermion models (i), (ii), and (iii).

We next consider the Higgs quartic coupling whose beta function to two loop order is given in Eq. (1). We first plot  $\beta_{\alpha_h}$  as a function of  $\alpha_h$  for model (ii) with the values of the gauge and Yukawa couplings at the fixed point and  $N_{F3} = 40$ ,  $N_{F2} = 24$ . Figure 3 shows that there exist four different regions denoted as I, II, III, IV. Depending on the choice of the initial value of  $\alpha_h$ , the Higgs self-coupling can be in any of these distinct phases. Because we are searching for asymptotic safety we are only interested in phase III. To guide the reader we mark with a red dot in Fig. 3 the

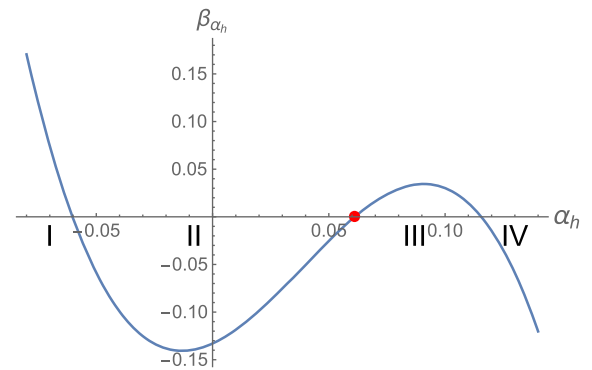


FIG. 3. This figure shows  $\beta_{\alpha_h}$  with  $\alpha_h$  with the values of the gauge and Yukawa couplings at the fixed point and  $N_{F3} = 40$ ,  $N_{F2} = 24$ . There exist four different kinds of phases denoted as I, II, III, IV dependent on the initial value of  $\alpha_h$ . The red point denotes the ultraviolet critical value of  $\alpha_h$  which determines whether we could have a UV safe fixed point with positive or negative  $\alpha_h$  value.

ultraviolet critical value of  $\alpha_h$ . We distinguish the ultraviolet critical value with the initial critical value of  $\alpha_h$ , discussed later. The former quantity is scale dependent; thus, the ultraviolet critical  $\alpha_h$  is at a scale close to the UV fixed point. The latter quantity is an IR quantity, above which the Higgs self-coupling flows to an UV fixed point; we shall take this initial critical  $\alpha_h$  to be at 2 TeV. The plot shows that for the Higgs self-coupling to be asymptotically safe it must run towards the ultraviolet to values within region III, where the other couplings have already reached their fixed point values. If, however, the dynamics is such that it will run towards ultraviolet values immediately below the critical one the ultimate fate, dictated by phase II, is vacuum instability.

Figure 3 also provides a few insights for constraining viable vectorlike fermion models. The expression of  $\beta_{\alpha_h}^{2\text{loop}}$  in Eq. (1) shows the new vectorlike fermions will only provide negative contributions to  $\beta_{\alpha_h}^{2\text{loop}}$  when  $N_{F2}$  is order of 10. In conjunction with Fig. 3, we expect that the smaller the negative contribution of these new vectorlike fermions, the smaller will be the critical value of  $\alpha_h$ , and the easier to enter phase III. Actually, we find that the pure SM RG function of  $\alpha_h$  (without new vectorlike fermion contributions to  $\beta_{\alpha_h}$  only) provides the smallest critical value of  $\alpha_h$ , commensurate with the above expectation. Alternatively, if these negative contributions are too large, the cubic curve of  $\beta_{\alpha_h}$  will never intersect the  $\alpha_h$  axis and we will never achieve an asymptotically safe solution (only two phases remain in this limiting case). We learn that the smaller the hypercharge and dimension of the representation, the smaller the critical value of  $\alpha_h$  will be (making it easier to realize asymptotic safety for the Higgs quartic). Following this criterion, model (ii) should have the smallest critical value of  $\alpha_h$ .

We obtain the same results for the gauge and Yukawa couplings as before, taking their initial values to be the SM ones at 2 TeV as in Eq. (6). We find that to obtain an asymptotically safe solution for  $\alpha_h$  we must choose its initial value to be (at least)  $\alpha_h(t=3) = 0.0034$ , about 6 times the SM value at that scale. For the SM initial value  $\alpha_h(t_0) = 0.00054$  the theory achieves the negative value  $\alpha_h = -0.06$  at the UV fixed point, yielding an unstable vacuum. The results for model (ii) (again using  $N_{F3} = 40$ ,  $N_{F2} = 24$ ) are shown in Fig. 2, with the Higgs quartic coupling in orange.

We thus attain UV completion for the whole gauge-Yukawa-Higgs system with gauge and Higgs quartic couplings ( $\alpha_1, \alpha_2, \alpha_3, \alpha_h$ ) asymptotically safe and top Yukawa coupling  $\alpha_t$  asymptotically free. The UV fixed point occurs at  $3.6 \times 10^{14}$  GeV—well below the Planck scale and so gravity contributions can be safely ignored. The unique feature in Fig. 2 occurs because when  $\alpha_2$  reaches its fixed point value  $\beta_{\alpha_h}$  almost vanishes. However, when  $\alpha_1$  increases to its final value the almost fixed point in the scalar coupling settles to its true fixed point value. In

addition this feature, for fixed  $N_{F3} = 40$ , disappears gradually when increasing  $N_{F2}$  from 18 to 25. This is because the larger  $N_{F2}$ , the smaller  $\alpha_2$  is; consequently, the self-coupling is more sensitive to the change in  $\alpha_1$ .

We have further explored which regions of parameter space ( $\alpha_h, N_{F3}, N_{F2}$ ) can yield asymptotic safety. We find that  $\alpha_h$  reaches its lowest critical value of 0.0027 when  $N_{F2} = 18$  and  $32 \leq N_{F3} \leq 220$  (insensitive to  $N_{F3}$  and the bounds of  $N_{F3}$  are discussed below). This critical  $\alpha_h$  value can be further decreased by considering large  $N_F$  of order a few hundred. Interestingly, there exists an upper value of  $N_F$  above which  $A$  in Eq. (4) goes beyond the first UV pole, moving therefore to the second branch of  $F_1$  and  $H_1$ . Within the first branch, the smallest critical  $\alpha_h$  with large  $N_F$  occurs for  $\alpha_h = 0.002$  with  $N_{F2}$  near and slightly below the boundary (say  $N_{F2} = 590$ ) above which one needs to move to the second branch. The result is insensitive to  $N_{F3}$  as well and  $32 \leq N_{F3} \leq 220$  where the upper bound  $N_{F3} = 220$  is due to the second branch of  $\alpha_3$  while the lower bound  $N_{F3} = 32$  is to satisfy leading  $1/N_F$  expansion. The UV fixed point occurs below but near the Planck scale. An initial investigation of these other branches suggests that a SM Higgs self-coupling value might be reached, but we leave in-depth investigations for future studies.

Comparing models (i) and (ii), we find that the critical value of  $\alpha_h$  is overall a much higher for model (i). However, similar to model (ii), at very large  $N_F$  one can decrease  $\alpha_h$  below  $\alpha_h(t_0) = 0.0049$ , corresponding to the lowest critical value one can achieve for small  $N_F$ . For example, for an initial value of  $\alpha_h = 0.0035$  one encounters a UV fixed point provided  $N_F \geq 105$ . It is possible to further decrease  $\alpha_h$  with increasing  $N_F$ .

For model (iii), we have a similar trend as the previous models. For simplicity, we consider the case where  $N_{F1} = N_{F2}$  and note that to achieve  $\alpha_h = 0.0035$  (still quite large compared to the SM), one needs  $N_{F3} = 40$  and  $N_{F1} = N_{F2} \geq 131$ . Here we find the smallest critical Higgs self-coupling occurs for  $\alpha_h = 0.00176$  with  $N_{F1} = 2200$ ,  $N_{F2} = 147$ ,  $N_{F3} = 138$ . These values correspond to the uppermost values allowed by the first branches of the corresponding  $F_1$  and  $H_1$  functions. This Higgs quartic value is, however, still three times its SM one at 2 TeV, which is roughly two times the value at the electroweak scale. We expect that the critical  $\alpha_h$  further decreases in the second branch when considering even larger  $N_F$ . We have checked that our results are stable against the introduction of known higher order terms in  $1/N_F$  proportional to the  $F_{2-4}$  and  $H_{2-4}$  functions.

Summarizing, for all three vector-like-fermion models, with SM gauge and top Yukawa couplings values as initial conditions at IR, we are able to realize UV completion of the gauge-Yukawa subsystem (gauge couplings asymptotically safe and Top Yukawa coupling asymptotically free). Upon including the Higgs quartic coupling, we find that its initial low energy value must attain a certain threshold for a

given choice of the number of vectorlike fermions. Above this critical value, we attain a UV asymptotically safe completion, whereas below this value the system is UV unstable. For the three vector-like-fermion models we studied, model (ii) possesses the lowest critical value of  $\alpha_h = 0.0027$  for a relatively small number of flavors  $N_F$ . This value is still larger than the (as yet unmeasured) SM Higgs quartic coupling. If at future colliders the Higgs quartic coupling is found to be 5–6 times larger (predicted in some studies without altering the SM RG functions, e.g., Ref. [32]), model (ii) could realize asymptotic safety for the whole gauge-Yukawa-Higgs system. Intriguingly, an  $\alpha_h$  close to the SM value, say around 2 times at the electroweak scale, can be achieved for very large values of  $N_{F1}$  in model (iii) within the first branch of the  $F_1$  and  $H_1$  functions. This allows complete asymptotic safety at energies below but near the Planck scale.

Our results pave the way to new approaches for making the SM fully asymptotically safe. Indeed, building on the present approach in Ref. [33] it has been shown that one can construct related asymptotically safe SM extensions in which the Higgs quartic coupling matches the SM value. In Ref. [34] instead, asymptotic safety is achieved via dynamical symmetry breaking of a calculable UV fixed point.

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