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A review of probabilistic techniques: towards developing a probabilistic lifetime methodology in the creep regime

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Abstract

Lifetime assessment procedures for high temperature components incorporate conservatism to account for various uncertainties. These assessments are by nature deterministic, and do not provide advice on assessing their in-built conservatism. Building on conventional deterministic approaches for assessing plant components, this paper outlines the initial stages of developing a probabilistic methodology for high temperature assessments associated with creep and creep-fatigue. The principal aim is to identify the main sources of uncertainty through probabilistic analyses and suggest systematic approaches for estimating conservatism. Such a need is not currently addressed by conventional deterministic assessments. Selected probabilistic techniques are highlighted, and proposed as the building blocks of a prospective probabilistic creep methodology. These help build confidence in the underlying procedures, while guiding future work and areas of further investigation. A case-study is presented to demonstrate the utility of sensitivity analysis techniques in identifying and quantifying parameter uncertainties in the assessment results.

Keywords: Creep, creep-fatigue, life assessment, crack initiation, probabilistic techniques

1. Introduction

Various codes for high temperature structural integrity assessments have been developed in different countries. The most advanced of these consider creep-fatigue interactions during both the stages of crack initiation in defect free components, and crack growth of present defects. One such code is the R5 assessment procedure developed by EDF Energy [1], which is a descriptive procedure for assessing the lifetime of components at high temperature. In general, however, conventional deterministic assessments rely on being conservative in order to account for various sources of uncertainty, and still depend on the judgement of the assessor for key aspects. Conservatism is commonly achieved by using bounding values for the input parameters involved in the various analyses. As a result, some of the key uncertainties in the underlying assessment procedures can be neglected when opting for over conservatism, including [2]:

- The issue of insufficient data to characterise material properties for long-term conditions.
- The use of extrapolations beyond the short-term experimental testing ranges.
- The inherent large scatter in the material data, especially creep and creep-fatigue data.
- The existence of various approaches for calculating the same analysis input parameters.
- Difficulties associated with modelling complex, or in some cases unknown, loading histories.
- Lack of understanding of the underlying failure mechanisms and their interactions.

However, as a plant component ages over its service life, the focus shifts from lifetime assessment to lifetime extension, with the latter being more focused on reducing conservatism. This shift requires a parallel move from deterministic assessments to a more probabilistic mindset, as reliability and risk

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assessment become the main drivers. This paper aims to review a selection of probabilistic techniques which are applicable to current high temperature assessment procedures (with a key focus on R5 V2/3 for creep-fatigue initiation), paving the way towards formulating a probabilistic creep methodology for safe operation of high temperature power plants. Thus, the motivations for developing such a methodology are:

- Justifying a prospective shift from the currently followed deterministic procedures toward a probabilistic and risk management mind-set.
- Identification of various sources of uncertainty and their characterisation through probabilistic analyses (such need is not currently addressed by conventional deterministic techniques).
- The need for assessing the appropriateness of probabilistic techniques and their merits as compared with the currently used deterministic methods.

In addition, a case-study examining an R5 V2/3 assessment of a test specimen under uniaxial creep-fatigue loading is presented, the aim of which is to prove the utility of sensitivity analyses in identifying and quantifying parameter uncertainties in the underlying assessment procedure.

2. Probabilistic techniques

This section examines some of the techniques that have utility within the context of probabilistic creep assessments, which provide key building blocks for formalising a prospective methodology for creep and creep-fatigue assessments.

2.1. Data characterisation through distribution fitting

The main purpose of this is to firstly find the type of distribution that best fits the data trend (e.g. normal, log-normal or Weibull), and then to optimise the distribution parameters as to provide the best agreement with the data. There are two key approaches for fitting appropriate distributions to data samples: the *linear regression method* [3, 4] and the *maximum likelihood method* [4, 5].

2.2. Monte-Carlo Simulation (MCS)

A MCS strives to approximate the probability density function (PDF) of an output parameter based on the repeated computations of the input-output function (also called a *performance function*) using randomly generated combinations of input variables. With the samples going into these randomly generated combinations being fed from the PDFs of the input parameters [6]. To produce accurate estimations of the output PDF, a large number of iterations (typically $10^5 - 10^7$) must be computed. This puts a limitation on the applicability of MCS for computationally intensive calculations such as stress analyses using finite element (FE) models or time-dependent calculations. For such cases a sampling strategy such as *Latin-Hypercube Sampling* can be used to reduce the number of trials to a practical level.

2.3. Latin-Hypercube Sampling (LHS)

Latin-Hypercube Sampling is based on the principle that for each input parameter the samples going into the MCS must have equal probability. If the distribution of an input parameter is known, then the samples are determined by dividing the area under the PDF into portions of equal areas, which in fact represent equal probabilities. This ensures that even though there is a small number of samples, these are truly representative of the underlying distribution. A detailed account of how to implement a LHS approach can be found in [7]. But the general principle is that careful definition of the ranges of values (also called *bins*) for all input parameters is crucial to the outcome of a MCS using LHS as the sampling approach.

2.4. Sensitivity Analysis (SA)

The general purpose of conducting a sensitivity analysis is to identify the main sources of variability in the model output introduced by the input parameters. When a large number of possible input parameters is considered, SA provides a tool for identifying which should be considered with the most care (e.g. requiring further data acquisition), and which could be omitted from the probabilistic procedure all together. This is done by calculating the sensitivity indices, which are measures of the contributions of each input parameter toward the overall variability in the output. There are various approaches for calculating the sensitivities indices, four of which are summarised in Table 1. To demonstrate their utility, these four approaches have been implemented within a case-study which is reported in Section 4.

Table 1: Comparison of four sensitivity analysis approaches where R is the number of runs required, I is the number of input parameters and N is the number of trials in the probabilistic assessment.

Approach	R	Advantages	Disadvantages
Finite-difference approach [3, 8]	$2 \times I$	Simple and quick to implement.	Inputs are assumed to be normally distributed and have no interactions. Thus it examines local sensitivity only.
Variance-based approach [8]	$I \times N$	Conceptually simple to implement.	Not suitable for highly-skewed distributions (e.g. log-normal) as it assumes normal distributions.
Correlation-based approach [9]	N	Requires the same number of runs as the probabilistic assessment.	As the effect of each parameter is not isolated, it can overestimate the importance of the least influential parameters.
δ-sensitivity [10, 8]	$I \times N^N$	Measures global sensitivities.	Computationally taxing as it requires large numbers of runs.

Table 2: Comparison of statistical testing tools for normally and non-normally distributed data.

Purpose	Normally distributed data	Any distributed data
Significance and hypothesis testing	t-test	Mann-Whitney test.
	F-test	Levene's (using the mean), Brown–Forsythe (using the median) and Welch tests.
Error analysis	ANOVA	Mood's median, Kruskal-Wallis and Friedman tests.
Goodness-of-fit	χ^2 test	Kolmogorov-Smirnov, Anderson–Darling and Likelihood-ratio tests.
Correlations	Pearson coefficient	Spearman Rank and Kendall coefficients.

2.5. Statistical testing

Statistical testing tools serve a multitude of purposes, some of which are summarised in Table 2. For example, hypothesis testing is often used to assess whether two data sets are statistically similar [11]. Commonly used test statistics are the *t-test* and the *F-test*. However, a key underlying assumption when using these tests is that the data can be represented by a normal distribution. By comparison, the *Mann-*

Whitney test is a generalisation of the t-test (i.e. it is a non-parametric test) and does not impose any restrictions on the underlying distributions. A further use of hypothesis testing is assessing goodness-of-fit between a fitted distribution and a data set. The χ^2 test, which compares expected and observed frequencies, can be used for this purpose but it strictly does not apply for small samples ($N < 15$). The *Kolmogorov-Smirnov test*, which is based on comparing cumulative frequencies, can also be used for assessing goodness-of-fit and it does not impose restrictions on sample size [3].

2.6. Response Surface Method (RSM)

The Response Surface Method [6, 12, 13] implements a set of statistical and mathematical modelling techniques in order to approximate an output response using an explicit, multivariate closed-form expression. RSM finds its main use when the performance function (a closed-form expression relating the inputs to the output) cannot be defined. RSM thus assumes that the performance function can be approximated by a polynomial. The steps involved as part of RSM can be summarised as follows:

1) Selection of the independent input valuables that have major influence on the output using a screening strategy (either based on sensitivity analysis or design of experiments).
2) A design of experiments (DOE) approach is used to define the parameter designs which are used to conduct numerical experiments to test the output response.
3) Finding the regression coefficients by fitting a polynomial to the response given by the experimental observations.
4) Conducting statistical characterisation of the error between observed and predicted responses using an analysis of variance (ANOVA) approach.

3. Review of previous work on probabilistic creep assessments

This section examines some of the available literature on employing probabilistic techniques with creep and creep-fatigue assessments. It is clear that work in this area is rather limited. The general consensus, however, is that conventional deterministic assessments using worst-case values for input parameters introduce various degrees of conservatism. They also fail to make full use of the statistical information that could be inferred from available data.

Most previous work has concentrated on creep rupture (e.g. using simple Time-Fraction rules for creep damage) and creep-fatigue crack growth (e.g. R5 V4/5) assessments [9, 13, 14, 15, 16, 17, 18]. However, the subject of probabilistic creep-fatigue crack initiation assessments (the R5 V2/3 type, which is the focus of this work) seems to be under-explored, with the exception of [7, 14, 19, 20, 21, 22]. Relevant to AGR applications, [21] includes an R5 V2/3 assessment of a plant component (tube bifurcations) for which the effects of tube flow restrictions (which can cause overheating) and environmental degradation (carburisation) were examined.

The starting point for the most basic probabilistic assessment is to treat input parameters as random variables and then use a MCS in order to infer the variability of the output parameter. A recurring approach in the literature has been the statistical treatment of creep rupture, strain and crack growth data to account for scatter (e.g. [15, 17]). This was commonly achieved through the incorporation of statistical error terms (commonly treated as normally distributed random variables) in the various physical models used. Furthermore, least mean squares (LMS) linear regression can be used to characterise the error terms. Bayesian regression has also been used to characterise variability in creep-fatigue rupture predictions using creep extrapolation models (e.g. the Larson-Miller model) [16].

Scatter in test data may be attributed to a number of sources including: test procedures and equipment, data analysis methods and interactions between failure modes [9]. Interestingly, in [17] a distinction is made between scatter due to variations within a specific material cast (random variations attributed to the failure processes) and variations between different casts (e.g. due to differences in chemical compositions or manufacturing processes). These variabilities were quantified by dividing the available data into appropriate subsets from which statistical measures were inferred.

Log-normal distributions have been commonly adopted to statistically characterise various material data [7, 9, 14, 15, 17, 20, 21], especially for creep models where power laws are used. For operating loads and temperatures other distributions may be more appropriate, for which some advice can be found in [21]. Furthermore, in the BS-PD6605 procedure a range of models were fitted to creep rupture data using the maximum likelihood method. Two key characteristics of this procedure were the use of Weibull and log-logistic error distributions to model the stochastic nature of the data and the inclusion of unfailed test results using a survival function [26, 27].

Following the statistical data characterisation, some previous work [15] has adopted simple sensitivity analysis using deterministic calculations. Similarly, in [9] sensitivity was assessed by correlating the output to the input parameters to establish the parameters of most importance (i.e. the ones which introduced the most variability in the output parameters).

Moreover, a challenging aspect of any creep assessment is the approximation of stress states. For instance, [13] and [18] acknowledged the issue of incorporating computationally intensive FE analyses into the probabilistic framework and suggested the adoption of RSM in conjunction with DOE as an alternative for reducing computational efforts. Finally, the subject of incorporating correlations between the input parameters (e.g. average creep strain rate and creep ductility [21]) is generally acknowledged for its importance, but not widely treated. For example, in [17] it is suggested that joint probability distributions can be used to sample correlated parameters, but this was expected to pose difficulties arising from the absence of rigorous statistical data treatment [9].

4. Case-study: sensitivity analysis of an R5 V2/3 assessment of uniaxial creep-fatigue tests

4.1. Case-study definition

The subject of this case-study is the R5 V2/3 assessment procedure for a uniaxial specimen under creep-fatigue conditions. The test was conducted using displacement control, with a hold period (i.e. creep dwell) at peak strain. The material was 316H tested at 550°C. Four sensitivity analysis approaches were applied with the aim of assessing which assessment input parameters were of most influence towards the output variability.

4.2. R5 V2/3 creep-fatigue assessment procedure for uniaxial case

This section provides a detailed account of the R5 assessment procedure [1, 23] for a uniaxial test specimen subjected to displacement controlled. As shown in Figure 1, the hysteresis cycle is divided into two portions:

- Portion CA: half-cycle without creep dwell.
- Portion ABC: half-cycle with creep dwell. This is divided into AB (monotonic loading) followed by BC (the creep dwell).
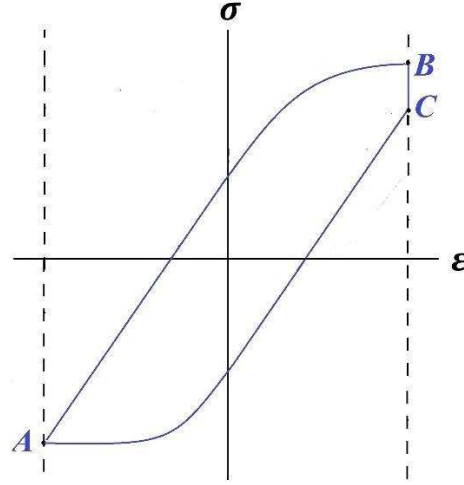
Figure 1: Stress-strain ($\sigma - \varepsilon$) hysteresis cycle for a displacement controlled creep-fatigue test with creep dwell at peak cycle with elastic follow-up factor $Z = 1$.

Starting with portion CA, and noting that the creep-fatigue test was in displacement control, the total strain range is the elastic-plastic strain range ($\Delta\varepsilon_T = \Delta\varepsilon_{ep}^{CA}$). Thus the elastic-plastic stress range ($\Delta\sigma_{ep}^{CA}$) can be found using a *Ramberg-Osgood* expression:

$$\Delta\varepsilon_T = \Delta\varepsilon_{ep}^{CA} = \frac{\Delta\sigma_{ep}^{CA}}{\bar{E}} + \left[\frac{\Delta\sigma_{ep}^{CA}}{A}\right]^{1/\beta} \tag{1a}$$

where $\bar{E}$ is the modified Young's modulus:

$$\bar{E} = \frac{3E}{2(1+\nu)} \tag{1b}$$

while A and β are the Ramberg-Osgood parameters. Thus using *symmetrisation* to find the *reverse stress datum* (σ_{RD}), and assuming isothermal conditions during the creep-fatigue test:

$$\sigma_{RD} = \frac{\Delta\sigma_{ep}^{CA}}{2} \tag{2}$$

For portion AB a *modified* Ramberg-Osgood expression is used:

$$\Delta\varepsilon_T = \Delta\varepsilon_{ep}^{AB} = \frac{\sigma_B + \sigma_{RD}}{\bar{E}} + \left[\frac{2\sigma_B}{A}\right]^{1/\beta} \tag{3}$$

which gives the stress at the beginning of the creep dwell, σ_B. It is worth noting that under displacement control with $Z = 1$ it follows that $\sigma_B = \sigma_{RD}$, and that the elastic-plastic strain ranges for portion CA and portion AB are identical. To estimate the stress relaxation during the creep dwell (portion BC), a time stepping scheme was used to calculate the stress drop and the creep damage for the first creep dwell. Thus using the following relaxation equation:

$$\frac{Z}{\bar{E}}\frac{d\sigma}{dt} = -\left[\dot{\varepsilon}_c(\varepsilon_c, \sigma) - \dot{\varepsilon}_c(\varepsilon_c, \sigma_{ref})\right] \tag{4}$$

where $\sigma_{ref} = 0$ because the creep-fatigue test was in displacement control (i.e. only secondary loads apply), Z is the elastic follow-up factor and $\dot{\varepsilon}_c$ is calculated instantaneously as a function of current creep strain and the instantaneous relaxing stress using:

$$\dot{\varepsilon}_c = Max[\dot{\varepsilon}_p, \dot{\varepsilon}_s] \tag{5}$$

where $\dot{\varepsilon}_p$ and $\dot{\varepsilon}_s$ are the instantaneous primary and secondary creep strain rates respectively, which are defined in the *strain hardening* version of the RCC-MR creep deformation model [25]:

$$\dot{\varepsilon}_p = K \, \varepsilon_c^X \, \sigma^Y \tag{6a}$$

$$\dot{\varepsilon}_s = C\sigma^n \tag{6b}$$

$$K = C_1^{1/C_2} C_2 \tag{6c}$$

$$X = \frac{C_2 - 1}{C_2} \tag{6d}$$

$$Y = \frac{n_1}{C_2} \tag{6e}$$

where $C, C_1, C_2, n, n_1, K, X$ and Y are creep constants. Because Eq 6a is singular at $\varepsilon_c = 0$, and for the first time step only, the initial creep strain can be calculated using a *time hardening* expression:

$$\varepsilon_c = \varepsilon_p = C_1 \, t^{C_2} \, \sigma^{n_1} \tag{7}$$

Thus the rate of change in stress is calculated using a discretised form of Eq 4:

$$(\sigma)_{i+1} = (\sigma)_i + (\Delta\sigma_c)_i = (\sigma)_i - \frac{\bar{E}}{Z}(\dot{\varepsilon}_c \, \delta t)_i \tag{8}$$

from which the stress drop ($\Delta\sigma_c$) and creep strain ($\Delta\varepsilon_c$) per cycle can be obtained. The creep strain for successive time increments is found using:

$$\varepsilon_{i+1} = \varepsilon_i + \dot{\varepsilon}_c \, \delta t \tag{9}$$

The creep damage per cycle (d_c) is determined using a ductility exhaustion model:

$$(d_c)_{i+1} = \frac{\varepsilon_{i+1}}{\varepsilon_f} \tag{10}$$

where ε_f is the uniaxial ductility. The total strain range including the effect of the creep dwell for this half cycle is:

$$\Delta\varepsilon_T' = \Delta\varepsilon_T + \left(\Delta\varepsilon_c - \frac{\Delta\sigma_c}{\bar{E}}\right) \tag{11}$$

This increased strain range is the one used for calculating the fatigue endurance. However, it is worth noting that if $Z = 1$ then the total strain range is virtually unchanged as $\Delta\varepsilon_c = \Delta\sigma_c \bar{E}$. By comparison, if $Z > 1$ then the total strain range is increased (i.e. $\Delta\varepsilon_T' > \Delta\varepsilon_T$).

The following stage of the assessment is to calculate the fatigue damage per cycle. Fatigue endurance data obtained from [24] was fitted to the following expression [22]:

$$\log_{10}\left(\frac{N_f}{15}\right) = C_f \, (\Delta\varepsilon)^{n_f} \tag{12}$$

where N_f is the number of cycles to fatigue failure, $\Delta\varepsilon$ is the strain range in percentages and C_f and n_f are the coefficient and exponent of the fitted power law respectively. The fatigue damage per cycle is found as follows:

$$N_i = N_f \, \exp(-8.06 \, N_f^{-0.28}) \qquad (13a)$$

$$N_g = N_f - N_i \qquad (13b)$$

$$N_g' = N_g \, M \qquad (13c)$$

using $a_0 = 0.2$ mm as the initiated crack depth, the adjustment factor was found as $M = 0.187$. Thus the fatigue damage (d_f) is:

$$d_f = \frac{1}{N_0} \qquad (13d)$$

where N_0 is given by:

$$N_0 = N_g' + N_i \qquad (13e)$$

and the total damage (d) per cycle is:

$$d = d_c + d_f \qquad (14)$$

Finally, multiplying d by the number of cycles to uniaxial creep-fatigue failure (determined experimentally) gives the expected damage at failure, D, which is the output of this analysis.

4.3. Statistical treatment of input data

Six input parameters were treated stochastically as part of the R5 probabilistic assessment. These are summarised in Table 3 below. Log-normal distributions were used for all parameters following the advice in the available literature [7, 9, 14, 15, 17, 20, 21]. Using log-normal distributions implies that the logarithms of the parameters are normally distributed with means and standard deviations calculated as:

$$SD = -\frac{1}{1.6445} \log_{10}\left(\frac{LB}{BE}\right) \qquad (15a)$$

$$\mu = \log_{10}(BE) \qquad (15b)$$

were BE and LB are the best estimate and lower bound values respectively. It was assumed that the range between the lower and upper bounds covers 95% of all possible values.

Table 3: Summary of the parameters treated stochastically in the probabilistic R5 assessment.

Parameter	Description	Units
C_1	Primary creep coefficient in the RCC-MR model [25].	$MPa^{-n_1} hr^{-C_2}$
C	Secondary creep coefficient in the RCC-MR model [25].	$MPa^{-n} hr^{-1}$
ε_f	Uniaxial creep ductility (creep strain at failure) [24].	%
A	Ramberg-Osgood coefficient [24].	MPa
C_f	Coefficient in fatigue endurance data fit [24].	--
E	Young's modulus [24].	MPa

4.4. Sensitivity analysis

The main objective of conducting a SA is to establish the input parameters of most dominance towards the output variability. Four SA approaches were highlighted in Section 2.4: the finite difference approach, a variance based approach, a correlations approach and the δ-sensitivity approach. The four approaches used to calculate the percentage contribution of each input parameter towards the output variability (also known as the *sensitivity indices*) are defined as follows:

1. The finite difference approach [3] defines the sensitivity indices (S_i) as the weighted percentage contribution of each distributed input variable to the variance (not the standard deviation) of the output variable:

$$S_i = \frac{\left(\frac{\partial \phi}{\partial x_i}\right)_k^2 \sigma_{x_i}^2}{\left[\sum_{i=1}^{I}\left(\frac{\partial \phi}{\partial x_i}\right)_k^2 \sigma_{x_i}^2\right]} \tag{16a}$$

where I is the total number of input parameters, σ_{x_i} is the standard deviation of the x_i input parameter, and ϕ is the output. The partial derivate can be estimated by using finite differences:

$$\left(\frac{\partial \phi}{\partial x_i}\right)_k = \frac{(\phi_{k+1} - \phi_{k-1})}{2\,\Delta x_i} \tag{16b}$$

$$\Delta x_i = 2\sigma_{x_i} \tag{16c}$$

where ϕ_{k+1} and ϕ_{k-1} are evaluations of the output at the extremities of each input:

$$(x_i)_{k\pm1} = \mu_{x_i} \pm \Delta x_i \tag{16d}$$

where μ_{x_i} is the mean value of the x_i parameter.

2. The variance based approach [8] quantifies the sensitivity indices as the reduction in output variance introduced by fixing one parameter at a time, while treating the reset of the input parameters stochastically. The simplest expression for estimating the first-order sensitivity indices is:

$$S_i = \frac{V(\phi) - E[V(\phi|x_i)]}{V(\phi)} \tag{17}$$

where $V(\phi)$ is the variance of the output and $E[V(\phi|x_i)]$ is the conditional expected value of $V(\phi)$ given x_i has a fixed value (e.g. $x_i = \mu_{x_i}$).

3. Correlation based SA [9] quantifies the relative importance of each parameter by computing the correlation coefficients between each input and the out:

$$S_i = \frac{corr(x_i, \phi)}{\sum_{i=i}^{I} corr(x_i, \phi)} \tag{18}$$

where *corr* refers to a correlation function (e.g. the Pearson or Spearman correlations). For this work the Spearman correlation was used.

4. For the δ-sensitivity approach [10] the δ_i factor can be defined for each input parameter which uses the shift in area under the output PDF as a measure to quantify sensitivity. Suppose I input parameters are considered $(x_1, x_2, \ldots x_I)$ and that an output parameter PDF, $f_\phi(\phi)$, can be obtained through a MCS. By fixing one input parameter at a predefined value x_i^n $(n = 1,2,\ldots N)$

the output PDF can be evaluated for N possible values of x_i. These are conditional output PDFs, $f_{\phi|x_i^n}(\phi)$. A simple formulation for calculating the δ factors can be defined if the N possible values of each input parameter all have equal probability (e.g. if LHS is used):

$$\delta_i = \frac{1}{2N} \sum_{n=1}^{N} \left[\int \left| f_\phi(\phi) - f_{\phi|x_i^n}(\phi) \right| d\phi \right] \qquad (19a)$$

Thus, the associated sensitivity indices can be calculated as:

$$S_i = \frac{\delta_i}{\sum_{i=1}^{I} \delta_i} \qquad (19b)$$

For the creep-fatigue test under consideration, these SA techniques were applied to the assessment procedure detailed in Section 4.2, with the input parameters outlined in Table 3, whilst the output was the calculated total damage at the experimentally determined number of cycles to failure:

$$\phi = d \, N_f = (d_c + d_F) \, N_f \qquad (20)$$

4.5. Results

The results from the SA approaches are shown in Figure 2. Overall, the four approaches produced similar results as they agree that ductility and the primary-creep constant in the RCC-MR deformation model (C_1) were of most dominance. It is perhaps not surprising that the Young's modulus is the least influential parameter, as it exhibits small variability when compared with the other parameters, especially the creep related ones. The secondary creep constant (C) also seems to be non-influential, which can be explained by two factors. Firstly the hardening assumption that was used was to re-prime at the start of each dwell, which implies that every cycle will start in the primary creep regime. Secondly, the creep-fatigue test under consideration had 1 hour dwells, which may well be too brief a period for secondary creep to substantially develop. The fatigue constant (C_f) and the Ramberg-Osgood constant (A) had modest degrees of influence. These observations were not surprising since the lifetime is partially influenced by fatigue (although creep dominates in this case) while A determines the stress at the beginning of the dwell, which has a direct influence on creep damage.

It is worth nothing that all results presented in this section only apply to the situation at hand (i.e. a uniaxial creep-fatigue test with specific testing and material conditions) and for the level of complexity that was allowed in the probabilistic assessment (in this case only 6 parameters were treated stochastically). If more probabilistic input parameters were included in the assessment (e.g. variations in the temperature) the SA may have yielded completely different results, as the relative importance of each input parameter may have been affected by the inclusion of further, perhaps more important, input parameters. Furthermore, the failure of the specimen considered was dominated by creep (i.e. the calculated creep damage was far larger than its fatigue counterpart), which explains the modest influence of the fatigue parameter. However, the SA results would have been markedly different had the situation been fatigue dominated. Therefore, the results shown in Figure 2 must not be interpreted as general results, but rather examples demonstrating the utilities of the SA approaches considered in this work.
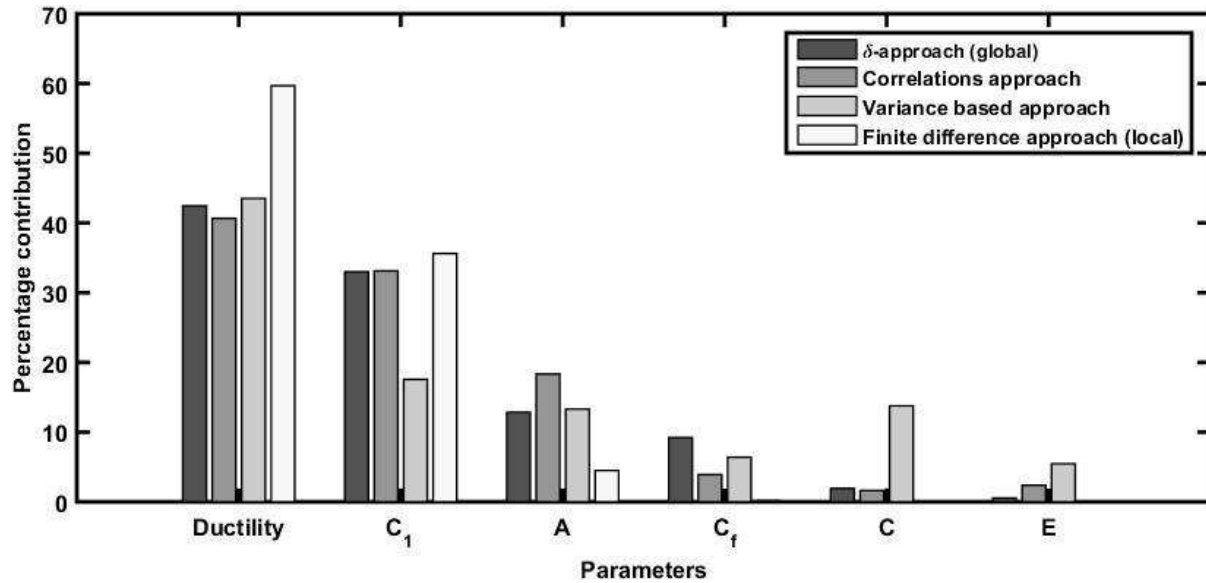
Figure 2: Comparison of SA results using four approaches.

4.6. Discussion

Out of the four SA approaches used, the one which is believed to provide the results with most confidence is the δ-approach, as it is a global, fully probabilistic technique which does not assume any a priori distributions for the input parameters. Thus, the results produced by this approach will be used as the baseline for the discussion below.

The correlations approach has produced similar results to the δ-approach but only required a fraction of the model runs, which makes it an appealing choice. The finite-difference approach provided results in agreement with the δ-approach, however, it seemed to underestimate the influence of C_f and A. Similarly, the variance approach overestimated the importance of E and C. The main conclusion from comparing the four approaches was that a global approach should be implemented if the computational capability is available. This is believed to provide a more realistic estimate of the importance of all input parameters. The finite-difference approach still provides a rapid assessment tool for identifying the least dominant parameters. Thus, it provides the first screening stage before more computationally taxing SA approaches are adopted, for which the required number of model runs may increase substantially with the number of input parameters. The variance approach provided somewhat misleading results, even though it required a considerable number of model runs. It is advised that this approach only be used with input parameters that are strictly accepted as normally distributed.

5. Concluding remarks

This paper highlighted the main probabilistic approaches that were deemed applicable to creep assessments. The key aim was to lay the groundwork upon which a prospective probabilistic methodology for high temperature lifetime assessments will be developed. A case-study examining the sensitivity of an R5 V2/3 crack initiation assessment to variabilities in six key input parameters was presented. This was intended to demonstrate the utility of sensitivity analysis techniques for guiding future work on reducing conservatism in the assessment procedure by addressing the dominant input parameters. Furthermore, this has also demonstrated the superiority of these techniques when compared with deterministic approaches as they provide key insight into the underlying uncertainties. Finally, it is believed that the natural progression for this work is to conduct a probabilistic assessment of a plant component. This will be crucially important for future probabilistic assessments as it will provide further advice on the statistical treatment of input data and the treatment of complex temperature and stress histories. Thus, the purpose of current and future work is to promote a shift from over-conservative

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deterministic assessments towards a probabilistic framework. The aim of which is to identify, quantify and incorporate real-life uncertainties in high temperature assessment procedures.

6. Further work

Moving towards developing a probabilistic methodology for creep assessments, a key requirement has been identified to be the applicability to AGR plant components. Therefore, it is believed that conducting a probabilistic assessment of a plant component is vital to demonstrate the utility of probabilistic techniques. Three of the envisaged challenges are:

1) The acquisition of material data, as these form the backbone of any probabilistic assessment.
2) Modelling of complex temperature histories and related stress states.
3) Assessing prospective probabilistic tools in terms of their utility for specific plant applications.

The validation of all parts of the probabilistic assessment is a further key challenge, especially since experimental validation is impractical. To gain confidence in the probabilistic assessment results verification/validation steps can be adopted. These can include the validation of the core deterministic creep-fatigue assessment of the AGR plant component, which can be done rigorously through comparison with an independent assessment. The various probabilistic tools must also be individually validated through trialling with known test cases to ensure their robustness.

Not only will this exercise guide the development of the sought methodology, but it will also provide insight into the main uncertainties associated with assessing a real-life component, which are issues not formally addressed by current deterministic assessments.

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