Monetary system dynamics

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Abstract: Modern economies are prone to persistent increases in debt and recurrent financial crises. We present a systems and control perspective on two hypotheses which aim to explain these dynamics: 1. the *growth imperative*—which investigates the conditions under which capitalist economies necessarily exhibit growing debt; and 2. the *financial instability hypothesis*—which proposes mechanisms underlying the tendency of capitalist economies to experience financial crises.

Keywords: Economic models, financial dynamics, debt, growth, financial instability.

1. INTRODUCTION

Modern economies are prone to persistent increases in debt and recurrent financial crises. We present a systems and control perspective on two hypotheses which aim to explain these dynamics: the *growth imperative* and the *financial instability hypothesis*. These hypotheses indicate a need for greater regulation of the monetary system in order to constrain this growth in debt. As argued by Turner (2016), we expect this will require financial reforms which are more radical than those which have been implemented since the 2008 financial crisis. In the years since 2008 (commonly referred to as the *great recession*), there has been persistent low growth in most developed economies, despite record lows in interest rates and the attempts of quantitative easing and other measures to stimulate growth. These measures are considered necessary to prevent economic stagnation, yet they serve to encourage further increases in debt levels, inequality, and asset prices (such as real estate), and they support continued trade imbalances between countries. But rising debt, inequality, asset prices, and trade imbalances are among the key causes of the 2008 financial crisis (see Turner, 2016). Thus, despite an initial reduction in private debt levels in western economies following the crisis, there has been a significant increase in national debt levels, and in the debt levels of other economies (notably China). Moreover, private debt levels in the UK are projected to rise above pre-crisis level by 2020 (Turner, 2016, p. 86).

The application of control theory to the design of stabilising economic policies dates back to the 1950s, and a comprehensive survey of applications is provided in Neck (2009). A key point we wish to emphasise in this paper is the importance of the role of the lending decisions of financial institutions on the economy. Indeed, as argued in Graziani (1990), *"in any macroeconomic model banks and firms can never be merged into one single sector"*. In this paper, we analyse two hypotheses which indicate the destabilising effect of an unregulated financial sector: the growth imperative and financial instability hypotheses.

2. THE GROWTH IMPERATIVE

The *growth imperative hypothesis* seeks to explain the empirical evidence of increasing debts in capitalist economies. One proposed mechanism underlying a growth imperative is provided by the so-called *A plus B theorem* (see Douglas, 1933). This formalises the idea that, if all money is created with an equal amount of debt, and there is a persistent removal of money from circulation, then a non-contracting economy necessarily exhibits growing debts.

A more detailed model exhibiting this phenomenon appeared in Binswanger (2009). This provided a discrete time dynamical system model of a simple monetary economy in which households, firms and banks are assumed to spend their income once in every period. All money is created through loans, which is a reasonable assumption for modern economies as the printing of new paper money is a very small proportion of the total money supply. Some money is removed from circulation at each instance by banks retaining a proportion of the interest payments received. This last assumption is justified in Binswanger (2009) on empirical grounds, by comparing the increase in banks' reserves relative to the increase in loans in the US and Germany from 1979 to 2003.[1] Under these assumptions, Binswanger (2009) showed that a strictly positive growth rate is then necessary in order for firms to break even.

The model in Binswanger (2009) contains six parameters:

(i) r: fraction of profits reinvested by firms, $0<r<1$;
(ii) c: fraction of loans spent on investment, $0\leq c\leq 1$;
(iii) d: depreciation rate, $0<d\leq 1$;
(iv) b: fraction of interest paid as bankers' wages, $0\leq b<1$;

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[1] Note, however, that it is important to distinguish between banks' *reserves* (which is money held by banks and so not circulating in the economy), and banks' *capital* (which includes equity and certain bonds which enable the bank to absorb losses). An increase in banks' capital need not restrict lending, and there is a strong case for raising banks' capital to debt ratios to reduce the fragility of the monetary system (see Admati and Hellwig, 2013).

(v) z: interest on loans, $z>0$;
(vi) w: growth rate of loans.

As will be shown, the strict inequalities are crucial to establishing the growth imperative. These parameters determine the behaviour of ten economic variables, which vary with the time instance t in accordance with the following ten relationships:

$$L_t = (1+w)L_{t-1}, \quad (1)$$
$$\Pi_t = C_t - WC_{t-1} - Z_{t-1} - dK_{t-1}, \quad (2)$$
$$K_t = (1-d)K_{t-1} + I_t, \quad (3)$$
$$D_t = (1-r)\Pi_{t-1}, \quad (4)$$
$$I_t = r\Pi_{t-1} + cL_t, \quad (5)$$
$$WC_t = (1-c)L_t, \quad (6)$$
$$Z_{t-1} = zL_{t-1}, \quad (7)$$
$$WI_t = I_t, \quad (8)$$
$$WB_t = bZ_{t-1}, \quad (9)$$
$$\text{and } C_t = D_t + WC_t + WI_t + WB_t. \quad (10)$$

Binswanger (2009) justifies these relationships as follows:

(a) Equation (1) represents a constant rate of change in the total amount of loans (L).
(b) Only firms in the consumption goods sector are explicitly modelled, and (2) equates these firms' profits (Π) to the difference between their income (equal to consumption in the present period, C) and their costs (due to the present levels of wages WC, interest payments Z, and capital depreciation dK).
(c) Capital (K) depreciates at a constant rate, and is replaced by investment (I) in accordance with (3).
(d) All loans are to firms in the consumption goods sector, whose expenditure is on dividends (D), wages, capital investment, and interest payments, in accordance with (4)–(7).
(e) Firms in the investment goods sector are neglected, with spending on investment being completely recirculated into the economy as wages from the investment goods sector (WI), in accordance with (8).
(f) A fraction of interest payments is recirculated into the economy as bankers' wages (WB) following (9).
(g) Given the assumption that households spend their income once in every period, then the consumption in the present period is given by (10) as the sum of firms' dividends and the wages from firms, the investment goods sector, and banks.

As will be discussed later, some issues with this model were raised in Gilányi (2015); Johnson (2015), but those papers did not fully resolve these issues (see Binswanger, 2015). Our main contribution in this section is to highlight a fundamental inconsistency in this model, and to propose and analyse a model which resolves this inconsistency. First, we analyse the growth imperative exhibited by the system in (1)–(10).

From (4)–(10), we find that D, I, WC, Z, WI, WB, and C are determined given Π, K and L, and their values in period t are non-negative whenever Π, K and L are non-negative in periods $t-1$ and t. Then, by eliminating C, WC, Z and I from (1)–(3), we obtain:

$$\begin{bmatrix}\Pi_t\\ K_t\\ L_t\end{bmatrix} = \begin{bmatrix}1 & -d & w+c-z(1-b)\\ r & 1-d & c(1+w)\\ 0 & 0 & 1+w\end{bmatrix}\begin{bmatrix}\Pi_{t-1}\\ K_{t-1}\\ L_{t-1}\end{bmatrix}. \quad (11)$$

With the notation

$$A_{11} := \begin{bmatrix}1 & -d\\ r & 1-d\end{bmatrix}, \text{ and } A_{12} := \begin{bmatrix}w+c-z(1-b)\\ c(1+w)\end{bmatrix}, \quad (12)$$

then $1+w$ is an eigenvalue of the matrix in (11), as are the eigenvalues of A_{11}. But it is easily shown that all of the eigenvalues of A_{11} lie inside the unit circle. Accordingly, if the economy is non-contracting, then $w \geq 0$. We then let

$$\mathbf{z}_t := \begin{bmatrix}\Pi_t\\ K_t\end{bmatrix} - \begin{bmatrix}\frac{w^2+w(d+c(1-d)-z(1-b))-dz(1-b)}{w^2+dw+dr}\\ \frac{w^2c+w(c+r)+r(c-z(1-b))}{w^2+dw+dr}\end{bmatrix} L_t,$$

in which the rightmost vector is equal to $((1+w)I - A_{11})^{-1}A_{12}$, and we obtain

$$\begin{bmatrix}\mathbf{z}_t\\ L_t\end{bmatrix} = \begin{bmatrix}A_{11} & 0\\ 0 & 1+w\end{bmatrix}\begin{bmatrix}\mathbf{z}_{t-1}\\ L_{t-1}\end{bmatrix} = \begin{bmatrix}A_{11}^t & 0\\ 0 & (1+w)^t\end{bmatrix}\begin{bmatrix}\mathbf{z}_0\\ L_0\end{bmatrix}.$$

Since both of the eigenvalues of A_{11} have modulus less than 1, then $A_{11}^t \to 0$ and $\mathbf{z}_t \to 0$ as $t \to \infty$. This implies

$$L_t = (1+w)^t L_0 \text{ for } t = 1, 2, \ldots; \text{ and}$$
$$\Pi_t \to \tfrac{w^2+w(d+c(1-d)-z(1-b))-dz(1-b)}{w^2+dw+dr}(1+w)^t L_0,$$
$$K_t \to \tfrac{w^2c+w(c+r)+r(c-z(1-b))}{w^2+dw+dr}(1+w)^t L_0,$$

as $t \to \infty$. Thus, in the long run, all variables grow at the rate $1+w$ (this was stated without formal proof in Binswanger (2009)). By noting that $K_t \to (r\Pi_t + c(1+w)L_t)/(w+d)$ as $t \to \infty$, it follows that, if $L_0 > 0$, then a necessary and sufficient condition for the economy to be non-contracting with non-negative long run values for all economic variables is for $w \geq 0$ to satisfy:

$$w^2 + w(d + c(1-d) - z(1-b)) - dz(1-b) \geq 0. \quad (13)$$

Since $-dz(1-b) < 0$, then $w = 0$ implies $\Pi_t < 0$ as $t \to \infty$. Thus, w must be greater than or equal to the (strictly) positive root of the quadratic expression in (13) for firms to make a non-negative profit in the long run.

It can be shown that some of the special cases $d = 0$, $z = 0$, $b = 1$, $r = 0$ or $r = 1$ do permit firms to break even in the long run without the requirement for a strictly positive growth rate of loans. The cases $b = 1$ and $z = 0$ were noted in Binswanger (2009), since in these cases the banks recirculate all interest payments as bankers' wages.

Johnson (2015) queried several of the relationships (1)–(10), and proposed an alternative set of relationships. Again, it was found that a strictly positive growth rate in loans was required in order for firms to break even in the long run. Many of the amendments proposed in Johnson (2015) related to the timings of various payments in (1)–(10). For example, it was proposed that (2) be replaced by $\Pi_t = C_t - WC_t - Z_t - dK_{t-1}$. But, as argued in Binswanger (2015), these timings are dependent on the nature of the business, and (2) is justified on an assumption that production takes one time instance. Also, altering the timing assumptions caused only a very small change to the growth imperative. It is concluded in Binswanger (2015) that there is little reason to prefer the timing assumptions in Johnson (2015) to those in Binswanger (2009).

A more fundamental query in Johnson (2015) concerns firms' expenditure. Equations (4)–(6) specify that the sum of dividend, investment and wage payments by firms in a given period is equal the sum of firms' profits from the previous period and the total amount of loans in the current period. It is claimed in Johnson (2015) that L_t in these equations should be replaced by the change in loans $L_t - L_{t-1}$. But, given the assumption in Binswanger (2009) that *during one period, households, firms, and banks spend their income once*, then the expenditure of firms should reflect the monetary balance of their bank accounts. As we discuss next, it is our contention that neither model is consistent with this assumption,

In response to Johnson (2015), Binswanger (2015) claims that his model is consistent with Keynes' notion of the *revolving fund of finance*, whereby a constant amount of loans finances a constant level of spending (this concept is formalised in Keen (2009)). But a key feature of the model in Binswanger (2009) is that money is consistently removed from circulation and replaced by new loans. In particular, there isn't a constant revolving fund of finance, and the total amount of loans exceeds the money supply.

Since households spend their income once in each period (see assumption (g) on page 2), then changes in the money supply can be identified with changes in the monetary balance of firms' bank accounts (M), which in one period is equal to the change in loans plus the difference between firms' income and expenditure in that period, i.e.:

$$M_t = M_{t-1} + L_t - L_{t-1} + C_t - D_t - I_t - WC_t - Z_{t-1}. \tag{14}$$

Then, rather than the sum of dividend, investment and wage payments by firms being equal to the sum of firms' profit and the total amount of loans, it is more consistent to set these equal to the sum of firms' profit and the monetary balance of firms' bank accounts. In other words, (5)–(6) should be replaced by the relationships:

$$I_t = r\Pi_{t-1} + cM_t, \tag{15}$$

$$WC_t = (1-c)M_t. \tag{16}$$

From (4), (7)–(10), and (15)–(16), D, I, WC, Z, WI, WB, and C are determined given Π, K, M and L, and their values in period t are non-negative whenever Π, K, M and L are non-negative in periods $t-1$ and t. Then, eliminating C, WC, Z, I and D from (1)–(3) and (14) gives:

$$\begin{bmatrix} \Pi_t \\ K_t \\ M_t \\ L_t \end{bmatrix} = \begin{bmatrix} 1 & -d & c & w-2z(1-b) \\ r & 1-d & c & c(w-z(1-b)) \\ 0 & 0 & 1 & w-z(1-b) \\ 0 & 0 & 0 & 1+w \end{bmatrix} \begin{bmatrix} \Pi_{t-1} \\ K_{t-1} \\ M_{t-1} \\ L_{t-1} \end{bmatrix}. \tag{17}$$

We note initially that, if $w = 0$, then from the final two rows of (17), we obtain $L_t = L_0$ and $M_t = M_0 - tz(1-b)L_0$. In this case, for $M_t \geq 0$ as $t \to \infty$ we require $L_t = 0$ for all t. There is then a constant money supply in the economy equal to M_0. If, on the other hand, $w \neq 0$, then

$$\begin{bmatrix} 1 & w-z(1-b) \\ 0 & 1+w \end{bmatrix}^t = \begin{bmatrix} 1 & \frac{w-z(1-b)}{w}((1+w)^t - 1) \\ 0 & (1+w)^t, \end{bmatrix}$$

so, from the final two rows of (17), we obtain

$$M_t = M_0 - \tfrac{w-z(1-b)}{w} L_0 + \tfrac{w-z(1-b)}{w}(1+w)^t L_0, \tag{18}$$

$$\text{and } L_t = (1+w)^t L_0. \tag{19}$$

Thus, if $w < 0$, then $L_t \to 0$ and $M_t \to M_0 - (w - z(1-b))L_0/w$ as $t \to \infty$, so for a non-negative long run money supply we require $M_0 \geq (w - z(1-b))L_0/w$. It can then be shown that, if the initial money supply M_0 is sufficiently large, then no growth imperative exists. If, however, all money is created through loans (i.e., $M_0 = 0$ and $L_0 > 0$), then for M_t to be non-negative as $t \to \infty$ we require $w \geq z(1-b)$. In fact, this growth imperative was established in Gilányi (2015) using a different method which did not seek to resolve the inconsistencies in Binswanger (2009). However, as we will now show, the model we propose here results in an even higher growth imperative.

To establish the growth imperative for this model, we let

$$\hat{A}_{11} := \begin{bmatrix} 1 & -d & c \\ r & 1-d & c \\ 0 & 0 & 1 \end{bmatrix}, \text{ and } \hat{A}_{12} := \begin{bmatrix} w-2z(1-b) \\ c(w-z(1-b)) \\ w-z(1-b) \end{bmatrix}.$$

Here, $\hat{A}_{11}$ has one eigenvalue at 1, and the remaining two eigenvalues lie inside the unit circle. We let

$$\hat{\mathbf{z}}_t := \begin{bmatrix} \Pi_t \\ K_t \\ M_t \end{bmatrix} - \begin{bmatrix} \frac{w^2+(d+c(1-d)-2z(1-b))w-z(1-b)(2d+c(1-d))}{w^2+dw+dr} \\ \frac{rw(w-2z(1-b))+c(w^2+w+r)(w-z(1-b))}{w(w^2+dw+dr)} \\ \frac{w-z(1-b)}{w} \end{bmatrix} L_t,$$

in which the rightmost vector is equal to $((1+w)I - \hat{A}_{11})^{-1}\hat{A}_{12}$, and we obtain

$$\begin{bmatrix} \hat{\mathbf{z}}_t \\ L_t \end{bmatrix} = \begin{bmatrix} \hat{A}_{11} & 0 \\ 0 & 1+w \end{bmatrix} \begin{bmatrix} \hat{\mathbf{z}}_{t-1} \\ L_{t-1} \end{bmatrix} = \begin{bmatrix} \hat{A}_{11}^t & 0 \\ 0 & (1+w)^t \end{bmatrix} \begin{bmatrix} \hat{\mathbf{z}}_0 \\ L_0 \end{bmatrix}.$$

We recall that M_t, L_t are as in (18)–(19), and we find that

$$\hat{A}_{11}^t \to \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{c}{d} \\ 0 & 0 & 1 \end{bmatrix} \text{ as } t \to \infty; \text{ so}$$

$$\Pi_t \to \tfrac{w^2+(d+c(1-d)-2z(1-b))w-z(1-b)(2d+c(1-d))}{w^2+dw+dr}(1+w)^t L_0,$$

$$K_t \to \tfrac{rw(w-2z(1-b))+c(w^2+w+r)(w-z(1-b))}{w(w^2+dw+dr)}(1+w)^t L_0 + \tfrac{c}{d}\left(M_0 - \tfrac{w-z(1-b)}{w} L_0\right),$$

as $t \to \infty$. In particular,

$$K_t \to \tfrac{d(r\Pi_t + c(1+w)M_t) + c(1-d)(wM_0 - (w-z(1-b))L_0)}{d(w+d)},$$

as $t \to \infty$, so K_t is non-negative for large t whenever Π_t and M_t are. Thus, for firms to make a non-negative profit in the long-run, the following inequality must hold:

$$w^2 + (d + c(1-d) - 2z(1-b))w - z(1-b)(2d + c(1-d)) \geq 0. \tag{20}$$

By letting $w = z(1-b)$ in the quadratic expression in (20), we obtain $-z(1-b)(d + z(1-b))$, which is negative. It follows that, if the initial money supply M_0 is zero (but $L_0 > 0$), then a necessary and sufficient condition for the economy to be non-contracting with non-negative values for all economic variables in the long run is for w to be greater than or equal to the positive root of the quadratic expression in (20). For the values $c = 0.4$, $d = 0.1$, $z = 0.1$ and $b = 0.8$ used in Binswanger (2009), this gives a growth imperative of 2.5%, in contrast to the figure of 0.45% obtained in that paper (see equation (13)).

As emphasised in Binswanger (2015), there are many important aspects of modern economies which are not captured in this model. In particular, it does not include the effects of household savings (e.g., for retirement), household borrowing (e.g., for the purchase of real estate), and inequality. Also, the precise timings proposed in the model can be disputed, although it appears the growth imperative is robust to such timing assumptions (see Binswanger, 2015, p. 658). As argued in Keen (2009), continuous time modelling is a better framework for resolving such timing issues. Nevertheless, there is a compelling intuitive argument that if all money is created through interest-bearing loans, and some money is removed from circulation on a continual basis, then there will be a tendency for loans (and consequently debts) to grow in time. That said, it is important to note that this growth imperative follows from the assumption that money is continually removed from circulation. The mechanism proposed in Binswanger (2009) which removes money from circulation is due to banks retaining a proportion of interest payments received, and the growth imperative is not present in the model if banks recirculate all interest payments as bankers' wages. Thus, the mechanism for removing money from circulation enters as an (empirically justified) assumption in the model in Binswanger (2009), but is likely to occur if it is in the interest of the institutions who control this mechanism. This could be explored further using dynamic game theory, as is advocated for other applications to economic policy in Neck (2006, 2009). For example it would be interesting to investigate if growing debts are a feature of a Stackelberg equilibrium solution (see Neck, 2006) for a hierarchical dynamic game in which households aim to maximise utility (balancing leisure with consumption) in response to firms (who set salaries, prices, etc. to maximise profit); and where both households and firms borrow in response to the lending decisions of banks (who set the interest rate and the proportion of money recirculated to maximise profits given the optimal response of firms and households).

Also, the model in Binswanger (2009) is linear, and so it can only exhibit a narrow range of behaviours. In particular, it is incapable of modelling phenomena such as financial crises, which are the concern of the next section.

3. THE FINANCIAL INSTABILITY HYPOTHESIS

In Keen (1995), a nonlinear dynamical model of financial instability was presented, which was inspired by an earlier verbal model described by the economist Hyman Minsky. This model includes an exponentially growing population (N) with exponentially growing productivity (a):

$$a = a_0 e^{\alpha t} \text{ and } N = N_0 e^{\beta t},$$

The labour force (L) is then equal to the product of the population and the employment rate (λ); national income (Y) is the product of the labour force and productivity; and capital (K) is a fixed ratio (ν) of national income. I.e.,

$$L = \lambda N, \;\; Y = aL \text{ and } K = \nu Y.$$

The average wage (w), debt level (D), and aggregated profits (Π) satisfy

$$\Pi = Y - wL - rD,$$

where r is the interest rate. These variables evolve in accordance with the following three differential equations:
$\frac{dw}{dt} = g_1(\lambda) w, \;\; \frac{dK}{dt} = h_1\big(\frac{\Pi}{K}\big) Y - \gamma K$ and $\frac{dD}{dt} = rD + \frac{dK}{dt} - \Pi$,
where g_1 and h_1 are monotonically increasing functions, with $g_1(\lambda) \to \infty$ as $\lambda \to 1$; and γ is the rate of capital depreciation. The first equation represents the greater ability of workers to demand higher wages as the employment rate rises. The second represents the role of expectations in determining investment, as a higher current profit level encourages more investment in the expectation of future profits. The final equation assumes that debt (together with profits) is used only to finance investment and make interest payments.

Keen (1995) also defined the non-dimensional parameters:

$$\pi = \frac{\Pi}{Y} \text{ and } d = \frac{D}{Y},$$

whereupon it is straightforward to show that all variables are determined given λ, π, and d (together with the constants $a_0, N_0, \alpha, \beta, \nu, \gamma$ and r, and the functions g_1 and h_1). Also, with the notation $g_2(\pi) := h_1(\pi/\nu)/\nu$; and

$$\begin{aligned} f_1(\lambda, d, \pi) &:= \lambda(g_2(\pi) - \alpha - \beta - \gamma), \\ f_2(\lambda, d, \pi) &:= rd - \pi + (g_2(\pi) - \gamma)(\nu - d), \\ \text{and } f_3(\lambda, d, \pi) &:= (1 - rd - \pi)(\alpha - g_1(\lambda)) \\ &\quad - r(rd - \pi + (g_2(\pi) - \gamma)(\nu - d)); \end{aligned}$$

then $\frac{d\lambda}{dt} = f_1(\lambda, d, \pi)$, $\frac{dd}{dt} = f_2(\lambda, d, \pi)$ and $\frac{d\pi}{dt} = f_3(\lambda, d, \pi)$.

This model was simulated in Keen (1995) for different interest rate regimes. Our main contribution in this section is to provide a local stability analysis to explain the results of those simulations. It is easily verified that this model has an equilibrium point (λ_e, d_e, π_e) where $g_1(\lambda_e) = \alpha$, $g_2(\pi_e) = \alpha + \beta + \gamma$ and $d_e = ((\alpha+\beta)\nu - \pi_e)/(\alpha+\beta-r)$. The local stability of this equilibrium point can be determined from the characteristic polynomial $p(s)$ of the Jacobian matrix for this system at (λ_e, d_e, π_e). We note that $\frac{\partial f_1}{\partial \lambda} = \frac{\partial f_1}{\partial d} = \frac{\partial f_2}{\partial \lambda} = 0$ and $\frac{\partial f_2}{\partial d}\frac{\partial f_3}{\partial \pi} = \frac{\partial f_2}{\partial \pi}\frac{\partial f_3}{\partial d}$ at (λ_e, d_e, π_e), so $p(s) = s^3 + p_1 s^2 + p_2 s + p_3$ where

$$\begin{aligned} p_1 &= -\Big(\frac{\partial f_2}{\partial d} + \frac{\partial f_3}{\partial \pi}\Big)_{\lambda_e, d_e, \pi_e} = \alpha + \beta - r + r\Big(\frac{\pi_e - r\nu}{\alpha+\beta-r}\frac{dg_2}{d\pi}_{\pi_e} - 1\Big) \\ p_2 &= -\Big(\frac{\partial f_1}{\partial \pi}\frac{\partial f_3}{\partial \lambda}\Big)_{\lambda_e, d_e, \pi_e} \\ &= \frac{\lambda_e}{\nu}\frac{dg_2}{d\pi}_{\pi_e}\frac{dg_1}{d\lambda}_{\lambda_e}\frac{(\alpha+\beta)(1-\pi_e-\nu r) - r(1-2\pi_e)}{\alpha+\beta-r}, \\ p_3 &= \Big(\frac{\partial f_1}{\partial \pi}\frac{\partial f_2}{\partial d}\frac{\partial f_3}{\partial \lambda}\Big)_{\lambda_e, d_e, \pi_e} \\ &= \frac{\lambda_e}{\nu}\frac{dg_2}{d\pi}_{\pi_e}\frac{dg_1}{d\lambda}_{\lambda_e}((\alpha+\beta)(1-\pi_e-\nu r) - r(1 - 2\pi_e)). \end{aligned}$$

By the Routh Hurwitz test, the equilibrium (λ_e, d_e, π_e) is locally stable if $p_2, p_3 > 0$ and $p_1 > p_3/p_2$. Since $\lambda_e > 0$ and g_1 and g_2 are monotonically increasing functions, then the local stability of the equilibrium holds if $r > 0$ satisfies

$$\begin{aligned} r &< \alpha + \beta, \\ r(1 - 2\pi_e + \nu(\alpha+\beta)) &< (1 - \pi_e)(\alpha + \beta), \\ \text{and } r\Big(\nu\frac{dg_2}{d\pi}_{\pi_e} - 1\Big) &< \pi_e \frac{dg_2}{d\pi}_{\pi_e} - (\alpha + \beta). \end{aligned}$$

For the values in Keen (1995) ($\alpha = 0.015, \beta = 0.035, \gamma = 0.02, \nu = 3, g_1(\lambda) = 0.0000641/(1-\lambda)^2 - 0.0400641$ $(0 \leq \lambda < 1)$, and $g_2(\pi) = 0.0175/3(0.53 - 2\pi)^2 - 0.065/3$ $(\pi < 0.265)$), we obtain $\lambda_e = 0.966$ and $\pi_e = 0.139$. As the interest rate r is increased from zero, all three inequalities initially hold, and the third inequality is the first to fail, at $r = 0.0452$. The most striking feature of the model is the behaviour of the system above this threshold (see Fig. 1). After some initial large oscillations, λ and π both exhibit relatively small oscillations about the equilibrium values λ_e and π_e for an extended period of time, while the debt to output ratio rises at a relatively constant rate. However, this is then followed by oscillations of rapidly increasing amplitude, accompanied by an acceleration in the rate of growth of the debt to output ratio. A similar behaviour is also observed if there is a variable interest rate of the form $r = \zeta + \phi d$, for sufficiently large ϕ (see Keen, 1995). The model thus demonstrates that a period of relatively benign economic behaviour, accompanied by a growing debt to output ratio, can be followed by financial crisis, even in the absence of external shocks to the system. Keen (1995) goes on to show that the introduction of a government sector, which employs a countercyclical taxation and spending policy, can prevent instability. The resulting economic behaviour instead exhibits persistent cycles. However, Keen

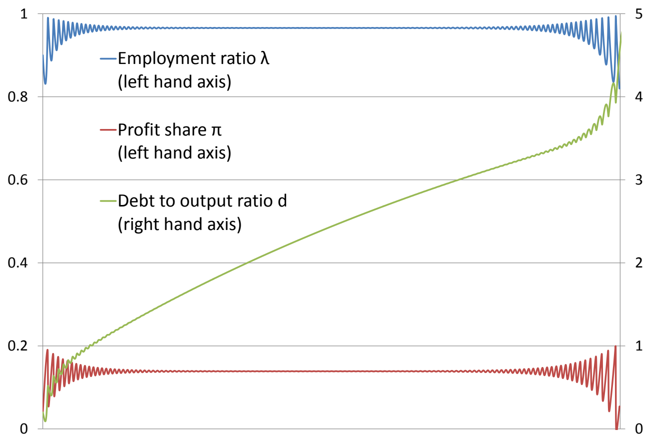

Fig. 1. Simulation of financial instability. Here, $r = 0.048$, $\lambda(0) = 0.9$, $d(0) = 0.2$ and $\pi(0) = 0.04$.

(1995) also queried whether Western governments were taking sufficient measures in this regard.

4. SYNTHESIS, IMPLICATIONS AND CONCLUSIONS

The focus of this paper was the monetary dynamics of capitalist economies: the proliferation of debt and the tendency towards financial crises.

The growth imperative hypothesis formalised the intuitive notion that, if all money is created by loans, and there is a persistent removal of money from circulation, then a non-contracting economy necessarily exhibits growing debts. Our main original contribution was to resolve a modelling inconsistency in Binswanger (2009), and to establish a higher growth imperative than that paper (which was noted in Binswanger (2009) to be considerably below the average growth rate of the world economy in recent decades). The growth imperative relies crucially on the consistent removal of money from circulation: it remains to establish what mechanisms are principally responsible for this. Empirical evidence was provided in Binswanger (2009) to support the mechanism proposed in that paper (banks retaining a proportion of interest payments), but it may be the case that factors which are not included in that model are more important (e.g., rising house prices). Regulation of these factors provides one approach to constraining the growth in debt. To transition to lower debt levels, overt money creation (creation of money without a corresponding debt) may also be required (see Turner, 2016). By directly considering the money supply, our model can easily be extended to consider this.

The financial instability hypothesis indicated the possibility for the debt to output ratio to increase considerably despite otherwise benign economic conditions (as in the years prior to the 2008 financial crisis). Financial fragility was seen to increase with the debt to output ratio, which ultimately led to crisis. Here, our main original contribution was a local stability analysis which explained the simulation results in Keen (1995). The hypothesis indicates a need for greater control over the debt to output ratio as opposed to other metrics (for example, the prevalence of inflation targeting by central banks).

While this paper has focused on closed economies (i.e., with no imports or exports), it is important to recognise that the most damaging effects of debt proliferation are on open peripheral economies. As emphasised in Wolf (2015), the most vulnerable economies are those which are unable to raise finance denominated in their own currency, due to either insufficient investor confidence or participation in a currency union (e.g., the Eurozone). Such economies are unable to devalue or overtly create their own currency to alleviate their debt, and can be forced to focus on exporting at the expense of the domestic economy. There are considerable opportunities for the systems and control community to contribute to the analysis and modelling of these issues. The importance of the effective control of the monetary system to the functioning of the economy, and the persistent features of rising debt and financial crises in modern economies, provide ample motivation for this.

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