ON THE DEVELOPMENT OF A FAIR AND EFFICIENT SLOT SCHEDULING MECHANISM AT CONGESTED AIRPORTS

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INTRODUCTION

In many airports around the world, demand to use the airport infrastructure exceeds the available capacity, leading to congestion-related delays or infeasible slot scheduling. Outside of the US, capacity is managed according to an administrative scheme detailed in the IATA Worldwide slot guidelines (*1*). To use infrastructure at a congested airport, an airline must be assigned a *slot*. A slot is a time period during which an aircraft can use the airport for landing or take-off.

A number of optimization models have been developed to solve the problem of allocating slots to airlines. The problem was first explicitly modelled in the paper (*2*) which formulated the problem as an integer linear program. More recently, fairness has been incorporated into slot allocation models (*3, 4*). Fairness is a key component of a schedule which is acceptable to the participating airlines.

Another key component in constructing a schedule which is acceptable to airlines is the incorporation of airline's preferences as to which flights are displaced from their preferred slots. Although this has been addressed in the context of air traffic flow management (*5, 6*), as far as we are aware, no work has been done in this direction for the slot allocation problem.

The aim of the present work is to propose a mechanism for allocating slots which is both fair and takes into account airlines' preferences. The scheme consists of first constructing a fair baseline reference schedule, followed by using airline preferences modify this schedule in order to improve its acceptability.

METHODOLOGY

Overall Scheme

The overall scheme we propose is a two-stage approach. In this first stage, we construct a baseline which is fair in the sense that airlines are assigned total displacement costs in proportion to their *contribution to congestion*. In order to achieve this, we develop a new measure of fairness, and calculate the efficient frontier between the total schedule displacement and the level of fairness. A fair baseline schedule is then selected from this efficient frontier.

In the second stage, the airlines suggest improvements to the baseline schedule. In particular, for each of its requests, the airline indicates how much schedule displacement it would be willing to accept subject to its total preferred schedule displacements being at least equal to its total assigned schedule displacement in the baseline schedule. The proposed mechanism then attempts to meet as many requests for improvement as possible. The total schedule displacement assigned to the airline is referred to its budget and hence we call this the *budget displacement mechanism*.

The notation required to present our new mechanism is shown in Table 1. For sake of brevity, we do not present the full slot allocation model here, and refer the reader to (*2*) for further details.

Congestion-based fairness

By congestion, we mean excess of demand for slots in a given time period with respect to the airport capacity constraints. Note that it does not make sense to displace requests for movements in uncongested periods. However, using the previous fairness approaches based on assigning schedule displacement to airlines in proportion to the number of movement requests, airlines with lots of flights in off-peak periods may have their peak flights disproportionately displaced.

We therefore propose to modify the fairness index of (*7*) to take into account the number of requests an airline makes during congested periods. By v_t^d we denote a congestion indicator

Sets	
$\mathscr{T}$	set of time coordination intervals in a day
$\mathscr{A}$	set of airlines
$\mathscr{M}\,(\mathscr{M}_a)$	set of movement requests (by airline a)
$\mathscr{D}\;(\mathscr{D}_m)$	set of dates in scheduling season (for which movement m is requested)
Parameters	
f_m^t	displacement cost of assigning movement request m a slot at time t
v_d^t	indicates whether time period t is congested at time t on date d
σ_m	schedule displacement cost assigned to movement request m in fair baseline schedule
δ_m	airline's preferred schedule displacement cost for movement request m
Σ_a	schedule displacement cost assigned to airline a in fair baseline schedule
Decisions and functions	
x_m^t	indicating whether movement request m is assigned slot at time t
y_m	indicates whether request for improved displacement by airline m can be satisfied (in budget displacement mechanism)
$s_a = \sum_{m\in\mathscr{M}_a}\sum_{t\in\mathscr{T}} f_m^t x_m^t$	total schedule displacement cost assigned to airline a
$S = \sum_{a\in\mathscr{A}} s_a$	total schedule displacement
μ_a	fairness index for airline a (see (1))

TABLE 1 Notation

which takes the value 1 if the time interval t on date d is congested, and 0 otherwise. Congestion for a given time period does not just occur because of an excess of demand for that time interval in particular, but also by excess demand for adjacent time intervals whose requests may be displaced to other time periods in order to construct a feasible schedule. Hence, we construct our congestion indicator from a schedule which is optimal with respect to total displacement. Specifically, if an extra request for a slot would cause a capacity constraint to be broken in the optimal schedule, then we deem the time interval of the request to be congested.

Given the congestion indicator we propose the following fairness index for each airline $a \in \mathscr{A}$:

$$\mu_a := \frac{\frac{s_a}{S}}{\frac{\sum_{m\in\mathscr{M}_a}\sum_{d\in\mathscr{D}_m} v_{t_m}^d}{\sum_{m\in\mathscr{M}}\sum_{d\in\mathscr{D}_m} v_{t_m}^d}} \tag{1}$$

That is, μ_a is the proportion of schedule displacement allocated to airline a divided by the proportion of movement requests made during congested periods by airline a. We refer to this denominator as the *contribution to congestion* of an airline. Based on the principle that airlines should be allocated schedule displacement in proportion to contribution to congestion, it is desirable that this value should be as close as possible to 1 for all airlines. This can be ensured by adding the following fairness objective to our slot allocation optimization model, which we refer to as the maximum deviation from absolute (MDA) fairness:

$$\max_{a\in\mathscr{A}} |\mu_a - 1|,$$

A similar objective was proposed in (*7*).

Displacement Budget Mechanism
The incorporation of displacement preferences can be achieved through the modification of the slot allocation model of (*2*). These modifications depend on the following new sets of parameters:

$$\sigma_m = \text{baseline displacement cost for movement } m$$
$$\Sigma_a = \sum_{m\in\mathscr{M}_a} \sigma_a = \text{total baseline displacement cost for airline } a$$
$$\delta_m = \text{preferred displacement cost for movement } m$$

The baseline displacement cost for a movement m is calculated with respect to a reference schedule. The parameters $\delta_m \geq 0$ are displacement costs chosen by the operating airlines which they would prefer a movement did not exceed. To ensure each airline is assigned a fair amount of delay, we require the total displacement cost each airline uses to be at least equal to the total displacement cost for the baseline schedule, that is:

$$\sum_{m\in\mathscr{M}_a} \delta_m \geq \Sigma_a \qquad \text{for } a \in \mathscr{A}$$

Given these preferences, we add the following constraints to the base model:

$$\sum_{t\in\mathscr{T}} f_m^t x_m^t \leq \sigma_m - (\delta_m - \sigma_m) y_m \qquad \text{for } m \in \mathscr{M} \text{ if } \delta_m < \sigma_m$$

where y_m is a new decision variable that can be interpreted as follows:

$$y_m = \begin{cases} 1 & \text{if } m \text{ assigned slot such that schedule delay is less than } \delta_m, \\ 0 & \text{otherwise.} \end{cases}$$

For all other requests, we add the following constraints:

$$\sum_{t\in\mathscr{T}} f_m^t x_m^t \leq \delta_m \qquad \text{for } m \in \mathscr{M} \text{ if } \delta_m \geq \sigma_m.$$

These constraints ensure that all requests are assigned a delay at most $\max\{\delta_m, \sigma_m\}$. The objective of this model is then lexicographical optimize the number of satisfied the number of requests for improved schedule displacement and the total displacement:

$$\text{lexmin}\left(-\sum_{m\in\mathscr{M}:\delta_m<\sigma_m} y_m, \sum_{m\in M}\sum_{t\in T} f_m^t x_m^t\right)$$

FINDINGS
We test our overall slot allocation scheme using real data for a medium-sized airport. For simplicity, we solve the problem without priority classes. This problem has 898 requests. The integer linear programs are solved using Gurobi(*8*).

Reference schedule and simulation of preferences
The reference schedule is found by solving the total displacement-fairness problem for congestion-based fairness with $\varepsilon = 0.9$. This yields a total displacement cost of 8123, a relatively small increase as compared to the minimum total displacement cost of 8003 found by solving the problem without fairness constraints. Displacement preferences for the airlines are simulated assuming each

airline has a cost function for the displacement of each movement of the form:

$c_m^t = r_m|t - t_m|$

and then for each airline selecting preferred delays which minimize their displacement costs assuming all requests are satisfied. The base costs r_m are randomly generated from a uniform distribution on the interval $(0, 1)$.

Results

A summary of the results from the displacement budget mechanism over all airlines and for the airline with the largest displacement costs are given in Table 2. These show that about a fifth of requests for improved displacements were satisfied. However, this has led to the total displacement costs increasing by around a third. Averaged over all airlines, the additional displacement cost per satisfied improvement request is $\frac{11946-8123}{43} = 88.9$, Similar observations also hold for the airline with the largest displacement costs.

All airlines	
Initial displacement costs	8123
Preferred displacement costs	8472
New displacement costs	11946
Number improvement requests	214
Satisfied improvement requests	43
Airline with largest displacement costs	
Initial displacement costs	1463
Preferred displacement costs	1463
New displacement costs	1839
Number of improvement requests	50
Satisfied improvement requests	9

TABLE 2 Summary of prioritization results

In Figure 1 are plotted the baseline delay costs with preferred delay costs and the new delay costs after the displacement budget mechanism. Note that all requests which had no initial displacement and for which the airline requested no displacement have been omitted from this plot. The plot demonstrates that when an airline allows a larger displacement cost than the baseline, this flexibility is often not used, or only partially used. This is because the mechanism will not assign requests a greater displacement unless it will help another request for improved displacement to be fulfilled.

CONCLUSION

In this paper we have proposed a slot allocation mechanism which incorporates efficiency, fairness, and takes into account the airlines' priorities. The scheme consists of first constructing a fair reference schedule, and then using airlines preferences to make adjustments to this.

The method used to construct a fair reference schedule improves over previous schemes by using the principle that the schedule delay an airline receives should be proportional to its contribution to congestion rather than simply the number of requests it makes.

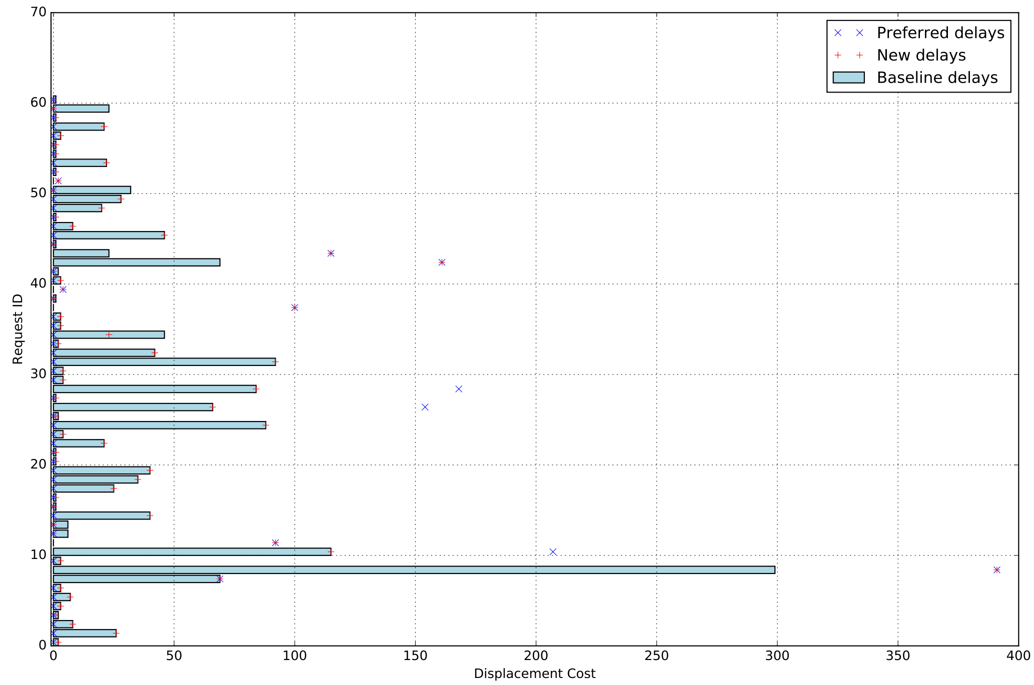

FIGURE 1 Displacement costs for an airline before and after displacement budget mechanism

The mechanism for taking into account the airlines' priorities consisted of allowing the airlines specify how the displacement costs assigned to them in the reference schedule should be distributed among their flights, and then meeting as many prioritized requests as possible. In a numerical experiment, we demonstrated that a significant number of these requests could be met, but at the expense of some increase in the total schedule displacement.

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