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Low performance on mathematical tasks in preschoolers: the importance of domain-general and domain-specific abilities.

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Conflict of interest statement

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Abstract

Background: Different domain-specific and domain-general cognitive precursors play a key role in the development of mathematical abilities. The contribution of these domains to mathematical ability changes during development. Primary school-aged children who show mathematical difficulties form a heterogeneous group but it is not clear whether this also holds for preschool low achievers (LAs) and how domain-specific and domain-general abilities contribute to mathematical difficulties at a young age. The aim of this study was to explore the cognitive characteristics of a sample of preschool LAs and identify sub-types of LAs.

Methods: Eighty-one children were identified as LAs from 283 preschoolers aged 3 to 5 years old and were assessed on a number of domain-general and domain-specific tasks.

Results: Cluster analysis revealed four subgroups of LAs in mathematics: 1) a weak processing sub-type, 2) a general mathematical LAs sub-type, 3) a mixed abilities sub-type, 4) a visuo-spatial deficit sub-type. Whilst two of the groups showed specific domain-general difficulties, none showed only domain-specific difficulties.

Conclusions: Current findings suggest that preschool LAs constitute a heterogeneous group and stress the importance of domain-general factors for the development of mathematical abilities during the preschool years.

Keywords: mathematics, low performing, preschoolers, sub-types, domain-general abilities, domain-specific abilities.

Studies show that between 5% and 10% of school-aged children experience a substantial deficit in mathematics (Desoete et al. 2004; Barbaresi et al. 2005) and that mathematical achievement in school years correlates with educational and financial success later in life (Geary et al. 2013; Parson & Bynner 2005). This stresses the importance of investigating the cognitive characteristics of preschool children who are low achievers (LAs) on mathematical tasks, in order to promote early identification of children at risk for mathematical learning disabilities and implement appropriate early educational interventions.

Mathematical learning disabilities and low achievers

The term Mathematical Learning Disabilities (MLD) is used to describe a developmental delay or deviance in the acquisition of one or more mathematical functions (Berch & Mazzocco 2007). Recent studies have explored the cognitive characteristics of MLD children in primary education. Some of these studies used a top-down approach, examining the cognitive profiles of MLD sub-groups specified beforehand based on a-priori assumptions (Jordan et al. 2003; Murphy et al. 2007). As a-priori specified criteria may bias outcomes, other studies have used a data-driven classification approach to describe the MLD profiles (Bartelet et al. 2014; Von Aster 2000). Both types of study emphasise the complexity of establishing the core deficits that constitute the MLD phenotype. In particular, previous data-driven studies in primary schools (Bartelet et al. 2014; Von Aster 2000) support the notion that MLD is a heterogeneous disorder with a multifactorial origin. These studies identified sub-types characterised by domain-specific deficits, such as weak ANS skills or number-line difficulties. In contrast, no clusters of primary school children were characterised by domain-general cognitive deficits only.

Although previous studies have examined mathematical difficulties in primary school children with MLD, little is known about the rate of mathematical difficulties in preschoolers.

There is currently some debate about what abilities drive mathematical competencies in typically developing (TD) preschool children, and it has been suggested that different abilities may be important at different developmental stages.

Domain-specific and domain-general precursors of mathematical learning

Research studies show that both domain-specific and domain-general abilities predict successful mathematical achievement outcomes in later life (Mazzocco & Thompson 2005; Passolunghi & Lanfranchi, 2012). However, the role played by these cognitive abilities in mathematical cognition most likely varies with expertise and development.

Domain-specific abilities that relate to mathematics include both verbal and non-verbal number-specific cognitive processes. Counting ability, and especially knowledge of the number word sequence, seems to be one of the most discriminating and efficient precursors of early mathematical learning (Passolunghi et al. 2007; Krajewski & Schneider 2009). A child who has achieved the cardinality principle (Gelman & Gallistel 1978) understands that the last word spoken in counting indicates the cardinality of the whole set. Studies focusing on preschool children indicate that understanding the cardinality principle is important for creating the link between non-symbolic skills and number symbols (Ansari et al. 2003; Le Corre & Carey 2007). Another building block for the development of mathematics is the non-verbal ability to discriminate approximate large numerosities, supported by the Approximate Number System (ANS) (Halberda et al. 2008). Specifically, some researchers propose that the acquisition of the meaning of symbolic numerals is done by mapping number words and Arabic digits onto the pre-existing approximate number representations (Dehaene 2001; Piazza 2010). Research has found a relationship between ANS and mathematical abilities in primary school and preschool children (Libertus et al. 2011; Mussolin et al. 2012). Moreover, longitudinal studies have shown that ANS abilities at age three predict general mathematical achievement in 6 year olds (Mazzocco et al. 2011a). A core hypothesis regarding MLD is that it originates from a core deficit in the innate ANS (Dehaene 2001). In line with this hypothesis, research on MLD has shown that impaired acuity of the ANS contributes to lower calculation skills and MLD in general (Mussolin et al. 2010; Mazzocco et al. 2011b). Still, other studies have failed to find a relationship between non-verbal ANS abilities and mathematical performance (Lyons et al. 2014; Passolunghi et al. 2014; Szücs et al. 2014; Vanbinst et al., 2016).

Domain-general precursors also play an important role in the development of mathematical abilities. This is especially so during the preschool years, with the importance of domain-general abilities usually diminishing as a consequence of a greater influence of domain-specific abilities during primary school education (Passolunghi & Lanfranchi 2012). The domain-general cognitive deficit hypothesis of MLD proposes that general cognitive impairments affect the development of mathematical skills (Geary & Hoard, 2005). Amongst general cognitive skills, working memory (WM) is considered a key domain-general predictor of mathematical learning. WM refers to a temporary memory system that allows short-term storage and manipulation of verbal and visuo-spatial information (Baddeley 1986). WM abilities and short-term memory (STM) skills are related both to early numeracy skills in preschool years and to later mathematical skills (Alloway & Alloway 2010; Friso-van den Bos et al. 2013). Visuo-spatial memory skills are especially strongly related to early numeracy ability in young children (Ansari et al. 2003; Kyttälä et al. 2003), while verbal WM abilities support mathematical performance to a greater extent from the age of seven onwards (Rasmussen & Bisanz, 2005). Speed of processing (SP), being the efficiency and speed with which cognitive tasks are executed (see Case 1985), is another domain-general cognitive precursor important for the development of mathematics (Gersten et al. 2005). A number of studies have shown children with poor mathematical abilities to have poor SP performance compared to TD peers (Bull & Johnston 1997; Geary et al. 2000).

The present study

Although previous studies have examined which cognitive abilities relate to mathematical difficulties in primary school children with MLD (Bartelet et al. 2014; Von Aster 2000), to our knowledge, no study has explored the cognitive sub-types of preschool children who show mathematical difficulties using a data-driven approach. As the developmental process needs to be considered, and it cannot be assumed that the deficit observed in older children is the same deficit in younger children (Ansari 2010), further studies are needed to identify potential different cognitive profiles in low achieving preschool children. The identification of these cognitive patterns is of great importance for the early identification and remediation of mathematical difficulties.

In the present study, we examined the cognitive characteristics of a sample of preschoolers with low mathematical abilities and identified sub-types of LAs using cluster analysis. Based on the literature discussed above (e.g. Bartelet et al. 2014; Geary et al. 2000; Kyttälä et al. 2003; Le Corre & Carey 2007; Mazzocco et al. 2011a) four cognitive factors were considered in the clustering process, two domain-general (cardinality and ANS) and two domain-specific (visuo-spatial memory and SP). This allowed greater insight into whether mathematical difficulties can be contributed to domain-general or domain-specific deficiencies in preschoolers.

Method

Participants

To identify LAs in mathematics, 283 preschool children aged 3 to 5 years old (M_{age}= 45.45 months. *SD* = 5.26. 141 girls) attending 14 preschool settings in Greater London and the South East, were included in the screening phase. All children were screened using a standardised assessment of mathematical abilities (Test of Early Mathematics Ability – Third Edition; Ginsburg & Baroody 2003) and on the Picture Similarities subtest of the British Ability Scales (BAS-3; Eliot & Smith 2011). Children considered as LAs if they performed at or below the 35th percentile on the Test of Early Mathematics Ability (TEMA-3)[1]. All children spoke English at home or performed within the normal rage on the Verbal Comprehension subtest of the BAS (*T* score > 37). Parents did not report any developmental delays or hearing and vision disabilities. Ten children were excluded from the screening sample as they failed to complete the assessment battery due to opting out (N= 2) and illness or long absence (N= 8).

Eighty- one children were identified as LAs (M_{age}= 44.38 months, *SD* = 5.47, 44 girls; $M_{TEMA\ percentile}$= 22.39, *SD* = 9.10). A control group of 56 (M_{age} 45.96 months, *SD* = 4.88, 31 girls; $M_{TEMA\ percentile}$= 71.21, *SD* = 17.45) children who obtained TEMA-3 scores at or above the 50th percentile was also identified. These children were randomly chosen from each preschool setting.

Socio-Economic Status (SES) was established using mothers' highest level of education, as parental education is considered to be one of the most stable aspects of SES (Sirin 2005).

Materials

Mathematical screening measure. Test of Early Mathematics Ability – Third Edition (TEMA-3) is normed for use with children aged 3 to 8 years old. TEMA-3 items assess both symbolic and non-symbolic knowledge and facts. Test-retest reliability is .93.

Our variable of interest was the percentile score to identify children as LAs, and raw scores were used to compare the different groups (see footnote 1 for a discussion).

Reasoning. The Picture Similarities non-verbal subtest of the BAS-3 was administered as a measure of general reasoning abilities. This task requires the child to match a sample card to one of four possible target pictures on the basis of perceptual similarities or semantic associations between items.

Approximate Number System (ANS). In this ANS task children were asked to identify which side of the screen showed more dots. They responded using a touch screen. There were 48 randomised trials of 0.5, 0.6, 0.7, and 0.8 ratios. The number of dots in each array ranged from 5 to 20 and both congruent and incongruent trials were used to control for continuous quantity variables (see Simms et al. 2015). In each of the experimental trials, children saw a fixation point followed by presentation of the stimuli for 1500ms. The ANS task was preceded by instructions and practice trials with dot displays of the ratio 1:3. A score of 1 was given for every trial performed correctly. The minimum score was 0 and the maximum was 48. Cronbach's α based on average inter-item correlation = .867.

Visuo-spatial short-term memory. The Pathway Recall task (Lanfranchi et al. 2004) was used to assess visuo-spatial STM abilities. The child was shown a path taken by a small toy frog on a 3 × 3 or 4 × 4 grid. The child had to recall the pathway immediately after the presentation by moving the frog from square to square, reproducing the experimenter's moves. The task is composed of eight trials and had four levels of difficulty, depending on the number of steps in the frog's path and dimensions of the chessboard (3×3 in the first level with two steps and 4×4 in the other levels with two, three, and four steps, respectively). Two trials for each difficulty level were presented. A score of 1 was given for every trial performed correctly. The minimum score was 0 and the maximum was 8. The test-retest reliability for this task is .70 (Passolunghi & Costa 2016).

Speed of processing (SP). The Naming Speed sub-test from the Phonological Assessment Battery 2 (PhAB2; Gibbs & Bodman 2014) was used to assess SP. A total of 50 stimuli were presented on one page, and the child was asked to name the stimuli (pictured objects) as quickly as possible without making errors. Non-standardised total response time (RT) measure was obtained. The reliability of this test is adequate with internal consistency alpha coefficients above .80.

Cardinality. To assess cardinality abilities children were assessed on a Give-a-Number task (Wynn 1992). In this task children were asked to give the experimenter exactly 1, 2, 3, 4, and 5 beads from a pile. The child was asked to provide each number three times in randomised order. A score of 1 was given for every trial performed correctly. The minimum score was 0 and the maximum was 15. Cronbach's alpha for reliability was .865.

Procedure

After the head teacher or manager had provided consent, parental written consent was obtained for all children as well as children's verbal assent before the start of the study. Trained research assistants carried out the assessments. Each child was assessed individually in a quiet area within the preschool setting. In order to encourage children and keep them motivated, they received stickers at the end of each session.

This study was assessed by the Ethics Committee for the Faculty of Arts and Social Sciences at Kingston University, London and allowed to proceed.

Data Analyses

We examined the existence of sub-groups within the group of LAs using cluster analysis. Cluster analysis is a descriptive, multivariate statistical technique that aims to group individuals that are close together. Performance on the ANS, SP, cardinality, and visuo-spatial STM tasks were entered as variables of interest in the cluster analysis. Standardisation into *z* scores was performed prior to the cluster analysis to ensure that differences in measurement scale did not influence the results. The variable scores entered in the analysis were standardised relative to the sample of LAs. Thus, the profile description represents performance relative to the average performance of the LAs group, not relative to the norm.

First, we used an agglomerative hierarchical clustering approach (Ward's method) to determine the number of optimal clusters. This approach combines the cases into clusters such that the variance within a cluster is minimised. To do so, clusters whose merger increases the overall within-cluster variance to the smallest possible degree are merged (Mooi & Sarstedt 2011). The result can be described with a dendogram (see Figure 1) and a plot of the agglomeration coefficients. To establish the initial clusters, the percentage change in the agglomeration coefficients was evaluated and a screen-plot was used to detect a point of inflection. Following the interpretation of the height of the different nodes in the dendogram, a four-cluster solution was judged as the optimal one for producing subgroups in LAs. As suggested by Milligan (1980), after the number of clusters and the cluster centroids has been determined with the hierarchical method, a K-means cluster analysis was used in order to reduce the overall within-cluster variation and optimise the results.

Figure 1 about here

To further describe the sub-types identified, we compared performance scores for the different clusters to a group of control children on the variables of interest entered in the cluster analysis, as well as age, mathematical abilities (TEMA-3) and reasoning abilities

(Picture Similarities). For this purpose, Univariate ANOVAs on the unstandardised scores for each of the clustering variables were used to describe differences between the clusters. The Bonferroni procedure was used for post-hoc comparisons of the means.

Results

Description of the clusters

Analyses revealed four clusters (Figure 2):

Cluster 1 (n = 13) included LAs characterised by average performance in the visuo-spatial STM task (*M z* score = .00, *SD* = .72), low-average performance on cardinality (*M z* score = -.87, *SD* = 1.10) and ANS (*M z* score = -.88, *SD* = .97), and weak performance on the SP task (*M z* score = -1.71, *SD* = .93). On the basis of the described profile, this group was assigned the label *weak processing sub-type.*

Cluster 2 (n = 37) is the larger subgroup of LAs and consisted of children characterised by average performance on all tasks: visuo-spatial STM *M z* score = .26, *SD* = .61; cardinality *M z* score = .12. *SD* = .65; ANS *M z* score = -.09, *SD* = .68; SP *M z* score = -.38, SD = .53. As it is not possible to identify a specific cognitive strength of weaknesses in this group compared to the other LAs, this group was labelled *general mathematical LAs sub-type***.**

Cluster 3 (n = 15) included LAs with average performance on the SP task (*M z* score = -.55, *SD* = .59) and above average performance on the visuo-spatial STM task (*M z* score = .97, *SD* = .60). This group is characterised by strong performance on the domain-specific mathematical tasks considered in this study (cardinality: *M z* score = 1.17, *SD* = .59; ANS: *M z* score = 1.25, *SD* = .60) and was therefore labelled *mixed abilities sub-type.*

Cluster 4 (n = 16) comprised LAs with impaired visuo-spatial STM skills (*M z* score = -1.51. *SD* = .37) but average performance on the other tasks: cardinality *M z* score = -.66. *SD* = .70, ANS *M z* score = -.26. *SD* .88, SP *M z* score = .01. *SD* = .75. Scoring specifically low on the visuo-spatial STM tasks, this group was assigned the label *visuo-spatial deficit sub-type*.

Figure 2 about here

Group comparisons

The descriptive statistics for age and performance scores for each task for the LAs sub-types and the control group are presented in Table 1 and Table 2.

Table 1 about here

Table 2 about here

An ANOVA for *T*-scores on the Picture Similarities subtest, revealed no significant difference between the five groups, $F(4.132) = .80$, $p = .53$, $\eta^2_p = .02$). Group comparisons revealed a significant effect for chronological age, $F(4.132) = 2.971$, $p = .022$, $\eta^2_p = .083$, and thus age was included as a covariate in subsequent analyses.

As predicted, the ANCOVA for TEMA raw scores revealed a significant difference between groups, $F(4.131) = 45.10$, $p < .001$, $\eta^2_p = .58$. Post-hoc Bonferroni analyses showed that all four LAs sub-types differed significantly from the control group (all *p*s < .001) but not from one another (all *p*s = n.s.).

The ANCOVA for performance scores on the ANS task revealed a significant difference between groups; $F(4.131) = 16.20$, $p < .001$, $\square_p^2 = .33$. Bonferroni-adjusted post-hoc pairwise comparisons indicated that the performance of three of the sub-types of LAs was significantly lower compared to the control group (all *p*s < .001). However, children in Cluster 3 (mixed abilities sub-type) showed higher average ANS scores compared to the controls (M_{diff} = 1.83, p = .99) as well as all the other sub-types (all *p*s < .001). There were no significant differences between the other three LAs sub-types (all *p*s = n.s.).

There was a significant difference between the groups for SP performance: $F(4.111)$ = 48.16, $p < .001$, $\eta_p^2 = .59$. Bonferroni-adjusted post-hoc pairwise comparisons indicated that the control group performed significantly better compared to all LAs sub-types (Cluster 1: p < .001; Cluster 2: p = .030; Cluster 4: p = .001), except for Cluster 3 (mixed abilities sub-type) (M_{diff} = 12.50, p = .999). Considering the difference between the LAs sub-types, performance for children in Cluster 1 (weak processing sub-type) was significantly slower for the SP task compared to all the LAs sub-types (all *p*s < .001), but there were no significant differences between the other LAs sub-types (all *p*s = n.s.).

There was a significant difference between the groups for performance on the visuo-spatial STM task $F(4.111)$ = 25.80, $p < .001$, $\eta_p^2 = .44$. Bonferroni-adjusted post-hoc pairwise comparisons indicated that the control group performed significantly better compared to all the LAs sub-types (Cluster 1: p = .008; Cluster 2: p <.002; Cluster 4: p < .001), except for the Cluster 3 (mixed abilities sub-type) (M_{diff} = .04, p = .999). Comparisons between the LAs sub-types showed that children in Cluster 4 (*visuo-spatial deficit sub-type*) performed significantly lower on the visuo-spatial STM task compared to all the other LAs groups (all *p*s < .001) and that there were no differences between the remaining LAs sub-types (all *p*s = n.s.).

The ANCOVA for the cardinality scores revealed a significant difference between the groups, $F(4.111)$ = 37.99, $p < .001$, $\eta_p^2 = .54$). Bonferroni-adjusted post-hoc pairwise comparisons indicated that the control group performed significantly better compared to all the LAs sub-types (all *p*s < .001), except for the Cluster 3 (mixed abilities sub-type) (*Mdiff* = 1.08, $p = .999$). Considering the difference between the LAs sub-types on cardinality scores, children in Cluster 3 (mixed abilities sub-type) outperformed those in all other clusters (Cluster 1: $p < .001$; Cluster 2: $p = .001$; Cluster 4: $p < .001$). Cluster 2 (*general mathematical LAs sub-type*) performed better on the cardinality task compared to Cluster 1 (weak processing sub-type) ($M_{diff} = 3.45$, $p = .002$) and Cluster 4 (*visuo-spatial deficit sub-type*) ($M_{diff} = 2.64$, $p = .019$). There was no significant difference between Clusters 1 and 4, which both have the lowest cardinality scores (respectively $M = 4.69$ and $M = 5.50$, $p = .999$).

Discussion

The current study builds on previous findings of mathematical abilities relying on a wide range of domain-specific and domain-general abilities. We examined the cognitive characteristics of a sample of preschool children with low mathematical abilities, identifying different sub-types of LAs using a data-driven approach. The analyses identified four different cognitive profiles: a general mathematical LAs sub-type, a weak processing sub-type, a visuo-spatial deficit sub-type and a mixed abilities sub-type.

Within the sub-types identified, two LAs groups were characterised by particularly weak domain-general abilities, supporting the domain-general cognitive deficit MLD hypothesis (Geary & Hoard, 2005). Children in the visuo-spatial deficit sub-type showed impaired performance on visuo-spatial abilities compared to the other LAs and the control group. Importantly, there were no differences in visuo-spatial skills between the other LAs sub-types, suggesting that mathematical difficulties in the visuo-spatial deficit sub-type could be driven by a specific deficit in this area. This finding is in line with Bartelet et al. (2014) who found a sub-group of primary school children with MLD that had spatial difficulties. Moreover, research has generally shown visuo-spatial memory skills to be strongly related to preschooler's mathematical skills (Kyttälä et al. 2003). Children in the weak processing sub-type were characterised by impaired SP compared to all other sub-types and the control group. Again, there were no differences between the other LAs sub-types. Children in the weak processing sub-type process information more slowly, which may explain their low mathematical performance. This result is in line with previous studies showing that SP is an important domain-general cognitive precursor for the development of mathematics (Bull & Johnston 1997; Geary et al. 2007).

The larger subgroup consisted of the general mathematical LAs sub-type and included children characterised by average performance on all tasks compared to the other LAs groups and impaired performance on all tasks compared to the control group. The sub-type did not differ from the other groups in terms of general reasoning abilities. These children are not characterised by a specific deficit in one area but their performance is generally low on both domain-general and domain-specific precursors. This result supports the idea of MLD as heterogeneous disorder with a multifactorial origin (Traff et al. 2017). Indeed, some children with mathematical difficulties might not have a single deficit but rather have multiple small deficits on different domains (Dowker 2005; Rubinsten & Henik 2009; Von Aster & Shalev 2007).

Finally, children in the mixed abilities sub-type outperformed the other LAs groups on all tasks but TEMA. This group did not differ from controls in terms of domain-general and domain-specific abilities, despite their low performance on TEMA. Interestingly, they demonstrated above-average performance on the ANS task compared to controls. Thus the difficulties in this group are likely caused by other cognitive or behavioural difficulties (e.g. anxiety) not currently measured. The current finding that one group of children did not show any numerical deficits, despite low performance on the TEMA, is in line with the 'no numerical cognitive deficit' group in the study by Bartelet et al. (2014).

In our study, three of the sub-types (65% of the LAs children) showed lower performance on all four of the variables considered (i.e. ANS, cardinality, SP and WM) compared to the control group. In contrast to previous studies of primary school children (Bartelet et al. 2014; Von Aster 2000), we did not find any groups that showed numerical deficits (ANS or cardinality) only. Instead, nearly half of the LAs preschool children included in the study are characterised by domain-general cognitive deficits. This suggests that in preschoolers, WM and SP deficits may impair performance on tasks assessing mathematical abilities (Geary & Hoard, 2005). This result has important implications for the early prevention of MLD and suggests the possibility to work on the improvement of domain-general abilities in preschool years (Passolunghi & Costa, 2016).

Some limitations of the current study include that, due to their young age, children were not assessed on all aspects of mathematical ability or domain-general abilities. Given the importance of domain-general abilities that emerged in this study, future studies should consider including an assessment of all components of WM, to further examine what sub-components are important precursors of mathematical learning (Passolunghi & Lanfranchi 2012). The inclusion of more variables could provide valuable insight into why some children in the mixed abilities sub-type performed low on TEMA. In addition, we did not examine the effects of SES in detail. Further research using larger samples is required to see how low SES affects the different LAs sub-types so that effective educational programmes can be developed.

Overall, our findings show that mathematical difficulties are associated with different patterns of cognitive abilities in preschool years compared to primary school age children, and that young low-achieving children may struggle with mathematics for different reasons. In contrast to previous studies with older children, we found two groups characterised by severe weaknesses in domain-general precursors, while no groups were characterised by domain-specific weaknesses only. This is in line with previous studies that highlight the importance of domain-general abilities in preschool years, and suggest that domain-specific abilities become more important during primary school (Passolunghi & Lanfranchi 2012). Yet, longitudinal studies are required to investigate the stability of the LAs sub-types identified in this study as well as to examine which children obtain a formal diagnosis for MLD later on (Ansari 2010).

The present study has practical implications for early identification and prevention of mathematical difficulties and disabilities. The identification of four different sub-types suggests that intervention programmes for low-achieving preschoolers need to be specific, based on children's cognitive profiles and focussed on domain-general abilities as well as domain-specific number skills.

Reference List

Alloway T. P. & Alloway R. G. (2010) Investigating the predictive roles of working memory and IQ in academic attainment. *Journal of Experimental Child Psychology* **106(1)**, 20-29. doi: http://dx.doi.org/10.1016/j.jecp.2009.11.003.

Ansari D. (2010) Neurocognitive approaches to developmental disorders of numerical and mathematical cognition: The perils of neglecting the role of development. *Learning and Individual Differences* **20(2)**, 123-129. doi: http://dx.doi.org/10.1016/j.lindif.2009.06.001.

Ansari D., Donlan C., Thomas M. S. C., Ewing S. A., Peen T. & Karmiloff-Smith A. (2003) What makes counting count? Verbal and visuo-spatial contributions to typical and atypical number development. *Journal of Experimental Child Psychology* **85**, 50-62. doi: http://dx.doi.org/10.1016/S0022-0965(03)00026-2.

Baddeley A. D. (1986) Working memory. Oxford University Press, Oxford.

Barbaresi W. J., Katusic S. K., Colligan R. C., Weaver A. L. & Jacobsen S. J. (2005) Math learning disorder: Incidence in a population-based birth cohort, 1976–82, Rochester, Minn. *Ambulatory Pediatrics* **5(5)**, 281-289. doi: 10.1367/A04-209R.1.

Bartelet D., Ansari D., Vaessen A. & Blomert L. (2014) Cognitive sub-types of mathematics learning difficulties in primary education. *Research in Developmental Disabilities* **35(3)**, 657-670. doi:http://dx.doi.org/10.1016/j.ridd.2013.12.010.

Berch D. B. & Mazzocco M. M. M. (2007) Defining and differentiating mathematical learning disabilities and difficulties. in Why is Math So Hard for Some Children: The Nature and Origins of Mathematics Learning Difficulties and Disabilities. Paul H. Brookes, Baltimore.

Bull R. & Johnston R. S. (1997) Children's arithmetical difficulties: Contributions from processing speed. item identification. and short-term memory. *Journal of Experimental Child Psychology* **65**, 1–24. doi:http://dx.doi.org/10.1006/jecp.1996.2358.

Case, R. (1985) Intellectual development. Birth to adulthood. Academic Press, New York.

Dehaene S. (2001) Précis of The Number Sense. *Mind and Language* **16(1)**, 16–36. doi:10.1111/1468-0017.00154.

Desoete A., Roeyers H. & De Clercq A. (2004) Children with mathematics learning disabilities in Belgium. *Journal of Learning Disabilities* **37**, 50–61. doi: 10.1177/00222194040370010601.

Dowker A. (2005) Individual differences in arithmetic: Implications for psychology, neuroscience and education. Psychology Press, New York.

Eliot C. D. & Smith P. (2011) British Ability Scales: Third Edition. GL Assessment.

Friso-van den Bos I., van der Ven S. H., Kroesbergen E. H. & Van Luit J. E. (2013) Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review* **10**, 29-44. doi: http://dx.doi.org/10.1016/j.edurev.2013.05.003.

Geary D. C., Hoard M. K., Nugent L. & Bailey D. H. (2013) Adolescents' functional numeracy is predicted by their school entry number system knowledge. *PloS One* ***8*****(1)**, e54651. doi:10.1371/journal.pone.0054651.

Geary D. & Hoard M. (2005) Learning disabilities in arithmetic and mathematics: Theoretical and empirical perspectives. In: *Handbook of Mathematical Cognition* (ed J. I. D. Campbell), pp 253–267. Psychology Press, New York, NY.

Geary D. C., Hoard M. K., Byrd-Craven J., Nugent L. & Numtee C. (2007) Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development* **78(4)**, 1343-1359. doi: 10.1111/j.1467-8624.2007.01069.x.

Geary D. C., Hamson C. O. & Hoard M. K. (2000) Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of experimental child psychology* **77(3)**, 236-263. doi: http://dx.doi.org/10.1006/jecp.2000.2561.

Gelman, R. & Gallistel, C. R. (1978) The child's understanding of number. Harvard University Press,Cambridge.

Gersten R., Jordan N. C. & Flojo J. R. (2005) Early identification and interventions for students with mathematics difficulties. *Journal of learning disabilities* **38(4)**, 293-304. doi: 10.1177/00222194050380040301.

Gibbs. S. & Bodman. S. (2014) Phonological Assessment Battery 2nd Edition Primary (PhAB 2 Primary). Swindon. GL Assessment.

Ginsburg. H. P. & Baroody. A. J. (2003) Test of Early Mathematics Ability: Third Edition. TX: PRO-ED, Austin.

Halberda J., Mazzocco M. M. M. & Feigenson L. (2008) Individual differences in non-verbal number acuity correlate with maths achievement. *Nature* **455(7213)**, 665-668. doi:10.1038/nature07246.

Jordan N. C., Hanich L. B. & Kaplan D. (2003) A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development* **74(3)**, 834-850. doi: 10.1111/1467-8624.00571.

Krajewski K. & Schneider W. (2009) Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction* **19(6)**, 513–526. doi: http://dx.doi.org/10.1016/j.learninstruc.2008.10.002.

Kyttälä M., Aunio P., Lehto J. E., Van Luit. J. & Hautamäki J. (2003) Visuospatial working memory and early numeracy. *Educational and Child Psychology* **20(3)**, 65–76.

Lanfranchi S., Cornoldi C. & Vianello R. (2004) Verbal and visuospatial working memory deficits in children with Down syndrome. *American Journal of Mental Retardation* **109(6)**, 456–466. doi:10.1352/0895- 8017(2004)109<456:VAVWMD>2.0.CO;2.

Le Corre M. & Carey S. (2007) One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition* **105(2)**, 395-438. doi: 10.1016/j.cognition.2006.10.005.

Libertus M. E. Feigenson L. & Halberda J. (2011) Preschool acuity of the approximate number system correlates with school math ability. *Developmental Science* **14(6)**, 1292–1300. doi:10.1111/j.1467-7687.2011.01080.x.

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L. & Ansari, D. (2014) Numerical predictors of arithmetic success in grades 1-6. *Developmental Science* **17**, 714-26. doi: 10.1111/desc.12152.

Mazzocco M. M. M., Feigenson L. & Halberda J. (2011a) Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PloS One* **6(9)**, E23749. doi: http://dx.doi.org/10.1371/journal.pone.0023749.

Mazzocco M. M. M., Feigenson L. & Halberda J. (2011b) Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Development* **82(4)**, 1224-1237. doi:10.1111/j.1467-8624.2011.01608.x.

Mazzocco M. M. M. & Thompson R. E. (2005) Kindergarten Predictors of Math Learning Disability. *Learning Disabilities Research and Practice* **20(3)**, 142–155. doi:10.1111/j.1540-5826.2005.00129.x.

Milligan G.W. (1980) An examination of the effect of six types of error perturbation on fifteen clustering algorithms. *Psychometrika* **45(3)**, 325-342.

Mooi E. & Sarstedt M. (2011) A concise guide to market research: The process, data and methods using IBM SPSS Statistics. Springer, London.

Murphy M. M., Mazzocco M. M. M., Hanich L. B. & Early M. C. (2007) Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities* **40(5)**, 458-478. doi:10.1177/00222194070400050901.

Mussolin C., Mejias S., & Noël M.P. (2010) Symbolic and nonsymbolic number comparison in children with and without dyscalculia. *Cognition* **115**, 10–25. doi: 10.1016/j.cognition.2009.10.006.

Mussolin C., Nys J., Leybaert J. & Content A. (2012) Relationships between approximate number system acuity and early symbolic number abilities. *Trends in Neuroscience and Education* **1(1),** 21-31. doi: http://dx.doi.org/10.1016/j.tine.2012.09.003.

Parsons S. & Bynner J. (2005) Does Numeracy Matter More?. National Research and Development Centre for adult literacy and numeracy, London.

Passolunghi M. C. & Costa H. M. (2016) Working memory and early numeracy training in preschool children. *Child Neuropsychology* **22(1)**, 81-98. doi: http://dx.doi.org/10.1080/09297049.2014.971726.

Passolunghi M. C. & Lanfranchi S. (2012) Domain-specific and domain-general precursors of mathematical achievement: a longitudinal study from kindergarten to first grade. *The British Journal of Educational Psychology* **82**, 42–63. doi: 10.1111/j.2044-8279.2011.02039.x.

Passolunghi M. C., Cargnelutti, E. & Pastore M. (2014) The contribution of general cognitive abilities and approximate number system to early mathematics. *British Journal of Educational Psychology* **84(4)**, 631-649. doi: 10.1111/bjep.12054.

Passolunghi M. C., Vercelloni B. & Schadee H. (2007) The precursors of mathematics learning: Working memory. phonological ability and numerical competence. *Cognitive Development* **22**, 165–184. doi: http://dx.doi.org/10.1016/j.cogdev.2006.09.001.

Piazza M. (2010) Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences* **14(12)**, 542–51. doi:10.1016/j.tics.2010.09.008.

Rasmussen C. & Bisanz J. (2005) Representation and working memory in early arithmetic. *Journal of experimental child psychology* **91(2)**, 137-157. doi: https://doi.org/10.1016/j.jecp.2005.01.004.

Rubinsten O. & Henik A. (2009) Developmental dyscalculia: heterogeneity might not mean different mechanisms. *Trends in Cognitive Sciences* **13(2)**, 92-99. doi: http://dx.doi.org/10.1016/j.tics.2008.11.002.

Simms V., Gilmore C., Cragg L., Clayton S., Marlow N., & Johnson S. (2015) Nature and origins of mathematics difficulties in very preterm children: a different etiology than developmental dyscalculia. *Pediatric Research* **77(2)**, pp. 389-395. doi:10.1038/pr.2014.184.

Sirin S. R. (2005) Socioeconomic Status and Academic Achievement: A Meta-Analytic Review of Research. *Review of Educational Research* **75(3)**, 417–453. doi:10.3102/00346543075003417.

Szűcs D., Devine A., Soltes F., Nobes A. & Gabriel F. (2014) Cognitive components of a mathematical processing network in 9-year-old children. *Developmental Science* **17(4)**, 506-524. doi: 10.1111/desc.12144.

Träff U., Olsson L., Östergren R., & Skagerlund K. (2017) Heterogeneity of developmental dyscalculia: Cases with different deficit profiles. *Frontiers in psychology* **7**, 2000. https://doi.org/10.3389/fpsyg.2016.02000.

Vanbinst K., Ansari D., Ghesquière P. & De Smedt B. (2016) Symbolic numerical magnitude processing is as important to arithmetic as phonological awareness is to reading. *PloS one* **11(3)**, e0151045. doi: https://doi.org/10.1371/journal.pone.0151045.

Von Aster M. G. & Shalev R. S. (2007) Number development and developmental dyscalculia. *Developmental Medicine and Child Neurology* **49(11)**, 868-873. doi:10.1111/j.14698749.2007.00868.x.

Von Aster M. G. (2000) Developmental cognitive neuropsychology of number processing and calculation: varieties of developmental dyscalculia. *European Child and Adolescent Psychiatry* **9(2)**, S41-S57. doi:10.1007/s007870070008.

Wynn K. (1992) Children's acquisition of the number words and the counting system. *Cognitive Psychology* **24**., 220-251. doi:10.1016/0010-0285(92)90008-P.

Endnotes

[1] Although previous studies usually consider children who perform below 25th centile on TEMA-3 as LA (e.g., Murphy et al. 2007), these studies included primary school-aged children. The TEMA-3 scores from all the children screened in the current study showed that TEMA-3 is insensitive at the lower range and that even a high percentile score on TEMA-3 is based on getting just a few items correctly. Based upon the fact that 45% of the total sample of children in the current study scored a raw score of 6 or less on the TEMA-3, where the score range for the age group is 0-32, we opted for a higher percentile cut-off for TEMA-3 (below 35th percentile) to define LA in mathematics.

Table 1

Chronological Age in months and Performance scores for reasoning ability (Picture Similarities), mathematical abilities (TEMA), non-symbolic comparison (ANS), Speed of processing, Visuo-spatial STM and Cardinality.

	Cluster 1 N = 13 (10 Females)		Cluster 2 N = 37 (19 Females)		Cluster 3 N = 15 (8 Females)		Cluster 4 N = 16 (7 Females)		Control Group N=56 (31 Females)	
	Mean	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*
Chronological Age	42.77	5.31	44.49	5.04	47.47	5.21	42.56	5.93	45.96	4.88
Picture Similarities	46.85	6.89	44.59	8.38	45.33	6.23	43.63	5.80	47.09	10.25
TEMA	2.85	1.77	4.38	1.66	6.00	2.67	3.25	1.73	13.21	5.64
ANS	23.77	5.07	27.89	3.56	34.87	3.11	27.00	4.59	32.80	5.23
Speed of Processing	227.46	49.36	116.16	28.26	107.20	26.07	136.94	39.87	95.68	23.40
Visuo-Spatial STM	2.77	1.17	3.19	1.00	4.33	.98	.31	.60	4.34	1.82
Cardinality	4.69	4.23	8.51	2.50	12.53	2.26	5.50	2.71	13.32	3.05

Note: Cluster 1= weak processing sub-type; Cluster 2 = general mathematical LAs sub-type; Cluster 3 = mixed abilities sub-type, Cluster 4= visuo-spatial deficit sub-type.

Table 2

Overview of Socio-Economic Status (SES) per group as a percentage

Group	No formal qualification	Finished secondary school	Vocational degree	Undergraduate degree	Post-graduate degree	Missing data
Cluster 1	14.3 %	28.6 %	42.9 %	14.3 %	0%	42.9%
Cluster 2	16.7 %	10.0 %	16.7 %	56.7 %	0%	10%
Cluster 3	0.0 %	20.0 %	20.0 %	40.0 %	20%	20%
Cluster 4	50.0 %	0.0 %	20.0 %	20.0 %	10%	40%
Control	8.9 %	11.1 %	11.1 %	35.6 %	33.3%	4.44%

Note: Cluster 1= weak processing sub-type; Cluster 2 = general mathematical LA sub-type; Cluster 3 = mixed abilities sub-type, Cluster 4= visuo-spatial deficit sub-type