3D beams reverse geometry analysis to optimise the Boundary Element Method mathematical model creation

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ABSTRACT: Among other benefits, the use of integrated CAD/CAM/CAE systems minimises the need to create and analyse physical models, thus saving valuable resources. On the other hand, many problems arise during the anticipated transformation of the CAD geometrical model into the mathematical one on which the numerical calculations are based. A novel methodology is introduced for optimising the distance between two consecutive cross-sections of any arbitrary 3D beam, in order to produce optimum stress results, when using the Boundary Element Method (BEM). The innovation introduced is the adaptive cross-sectioning process, which uses an error indicator as the criterion. The criterion takes account of a pre-specified difference of the max von Mises Stress between two consecutive cross sections of the beam. The methodology was validated within the boundaries of a modern 3D-CAD system by its software implementation through the use of the CAD Application Programming Interface.

INTRODUCTION

With the implementation of integrated CAD/CAE/CAM programs, engineers can test and optimise their designs. In addition, they have the ability to examine alternative scenarios and designs but also eliminate time-consuming and expensive alterations, especially at the production stage. The most positive feature of engineering analysis using computer programs; however, is that they provide the ability to make calculations for complicated geometries by using numerical methods. The Finite Element Method (FEM) and the Boundary Element Method (BEM) are two powerful numerical methods. In this article, a novel methodology is created to optimise the distance between two consecutive cross-sections of an arbitrary 3D beam, in order to lead to optimum stress results. Calculations are conducted on the boundary of each section along the length of the beams. The proposed adaptive procedure is based on ideas of adaptive meshing techniques used for Boundary Element Methods [1-3].

METHODOLOGY

In this article, a novel adaptive strategy is presented, based on adaptive meshing techniques of 2D cross sections [4]. The methodology is based on the use of a CAD geometrical model (Figure 1) for producing the BEM mathematical one, by creating the 2D cross section (Figure 2) along the 3D beam [5], and calculating the maximum equivalent stress $\sigma_v(x,y)$, as well as its position $P(x,y)$ on the boundary (Figure 3).
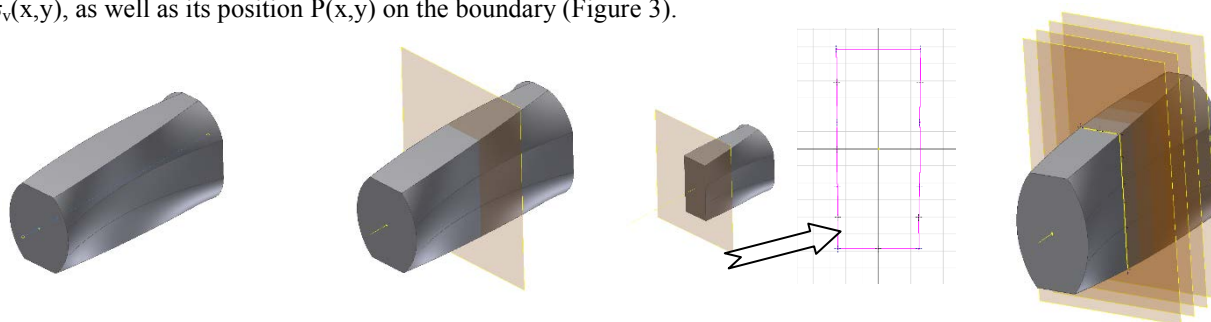

Figure 1: Geometric model.   Figure 2: Section on 3D model.   Figure 3: 2D cross section.   Figure 4: Parallel sections.

In order to have results along the 3D beam, parallel sections are created, each including a cross section. The presented methodology optimises the needed distance between every two parallel sections, by calculating each time an error is estimated ($\tau_{\varphi i}$) (Equation 1) and comparing it to an upper limit ($\overline{\tau_G}$). So, the rate of variation between two consecutive

parallel sections cannot exceed the percentage given. If this estimator is below a certain specified tolerance (3% to 5% [4]), then, the adaptive procedure is stopped:

$$\tau_{\varphi_{k+1}} = \frac{\sigma_{vm}^{k+1} - \sigma_{vm}^{k}}{\sigma_{vm}^{k}} \cdot 100 \leq \overline{\tau_G} \qquad (1)$$

where, $\tau_{\varphi k+1}$ is the error/difference between k and k+1 cross sections, the σ_{vm}^{k} and σ_{vm}^{k+1} are the maximum equivalent stress of k and k+1 cross section and $\overline{\tau_G}$ is the maximum acceptable error/difference of $\tau_{\varphi k+1}$.

The distance between k and k+1 cross sections is adapted until the above criterion is met. This procedure is repeated until the final face/cross section is reached and thus the whole length is examined. So, similar to the stopping criterion used for the error estimation in the BEM's refinement procedure [4][6], the error/difference $\overline{\tau_G}$ together with the total length of the beam are used for the adaptive cross-sectioning scheme.

So, instead of conducting calculations on equally distanced cross sections, through this methodology the number of cross-sections adapts is based on the beam's length, the maximum equivalent stress of each cross section and the maximum acceptable error/difference between each two cross section.

IMPLEMENTATION

The evaluation of the methodology was conducted by implementing a program within the boundaries of a 3D parametric CAD modeller by using its Application Programming Interface (API). This direct access to the NURBS (Non Uniform Rational B-Splines) [7] geometrical model gives the advantage of minimising the idealisation error [8].
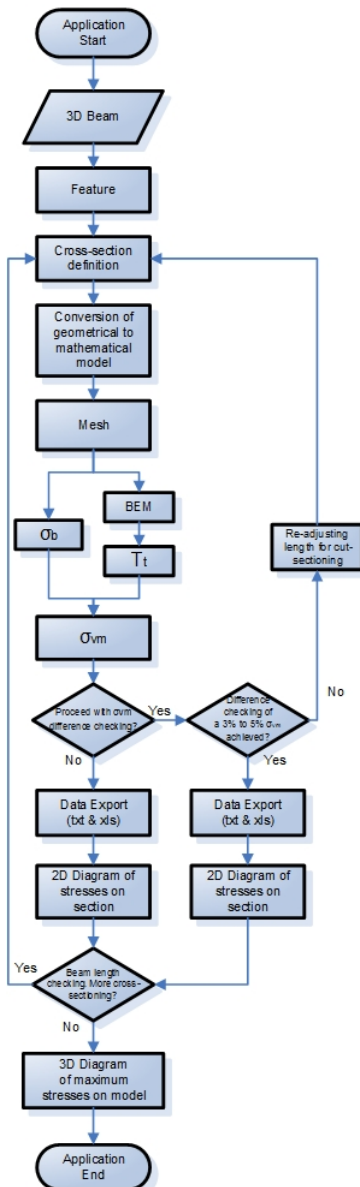

Figure 5: Flowchart of algorithm.

The program can handle Extrude and Loft features and takes into account Torsional and Bending moments. Torsional stresses are calculated though BEM and bending analytically [9][10]; thus, the implementation error is reduced. Every numerical method includes different type of errors as classified [8]. Due to the fact that idealisation errors are small and implementation error is reduced, discretisation errors are considered having important role to such calculations. The von Mises equation of combined fatigue in sidelong bending and torsion will be used. This equivalent stress for a point P(x,y) on the boundary Γ_i of the cross-section is calculated using the Equation (2) as follows:

$$\sigma_{vm}(x,y) = \sqrt{\sigma_b^2(x,y) + 3 \cdot \tau_t^2} \qquad (2)$$

Regardless of the feature type, however, the basic idea is that the subroutine begins to *cut* the model by setting work planes using API commands, in order to derive the boundary domain of the section. The initial reference plane that is used is the end face of the beam. So, the offset is initially set at zero length, for the first cross-section to be done right on the end face. Then, during the first iteration the offset is set at a trial distance of 10 mm in length, but that can change from iteration to iteration depending on the maximum σ_{vm} results of each section and the max σ_{vm} difference percentage tolerance.

After the program calculates all the stresses for the section under analysis, it uses an iterative procedure for defining the maximum equivalent σ_{vm} stress value for this particular section. It inserts the max σ_{vm} stress value in a collection memory and compares it with the previous max σ_{vm} stress value of the previous cross-section, if available. This algorithm works until the collection has two items in its collection memory. If the comparison of the max σ_{vm} stresses fulfils the max σ_{vm} difference percentage tolerance criterion defined by the user; then, the procedure continues by removing the first item - value of the collection (thus the second item becomes first) and by exporting all the related stress results to files.

After that, the procedure starts again defining a new cross-section at a new offset length. On the contrary, if the comparison of the max σ_{vm} stresses does not fulfil the criterion defined by the user, then, all the related results are erased from the memory and the algorithm decides on the appropriate offset length adjustments of cross-sectioning, in order to approximate the required accuracy of the criterion defined. The adjustments involve either subdivision to a half of the initial offset length or multiplication of a 50% of the initial offset length. The procedure starts again defining a new cross-section at an adjusted offset length and re-calculates everything again, reaching to a new improved max σ_{vm} stress.

This procedure of adaptive cross-sectioning, which optimises the max σ_{vm} stress results, during the procedure of generating the BEM mathematical model constitutes the objective which was set for the present dissertation. When the max σ_{vm} difference percentage tolerance criterion defined by the user has been fulfilled, then, the procedure continues by exporting all the related results to an MS Excel file and, then, starts again defining a new cross-section at a new offset length. The adaptive cross-sectioning procedure stops only, if the full length of the 3D beam has been reached or if an unexpected error has occurred.

The program ends with a visualisation of the 2D and 3D diagrams on the model beam. The 2D diagrams are as many as the cross-sections that have been made and represent the σ_{vm}, T_t or σ_b stresses according to the user's desire. The 3D diagram is a line segment connecting all the maximum σ_{vm} values together. Concluding, Lagrange iso-parametric elements have been used for the implementation and calculations.

IMPLEMENTATION TESTS

As mentioned, the methodology was evaluated by implementing a program and various case studies/tests were conducted. In this article, three case studies are presented. An even rectangular cross section beam (extrude feature), a non-even rectangular cross section beam (taper extrude of loft feature) and a non-even I cross section beam.

Even Rectangular Cross Section Beam

The rectangular-section 3D beam (Figure 6) is a simple example for demonstrating the program's capabilities. All the input data for the simulation of the beam are presented in Table 1 below.
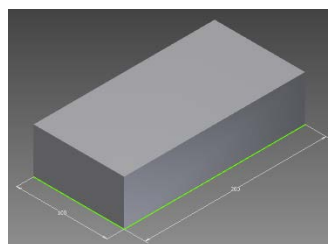

Figure 6: Rectangular beam.

Moments (N·mm)			Tolerance (%)	Dimensions
Torsional	Bending (x)	Bending (y)		(mm)
900.000	-650.000	250.000	5	100 x 200

Table 1: Input data for the even rectangular cross section.

The final output of the program can be seen in Figures 7-12.

According to Roark's formulas for stress and strain manual for engineering [11], the maximum torsional stress τ_t results for the solid rectangular section, should theoretically be found in the midpoint of each of the larger side of the section. The max shear stress value using the manual is found to be 1.833 N/mm^2. All the maximum stress results calculated by the program for the same section are presented in the following Table 2.

max σ_b	1.599 N/mm^2
max τ_t	1.839 N/mm^2
max σ_{vm}	3.287 N/mm^2

Table 2: Maximum values of stresses for the rectangular section.

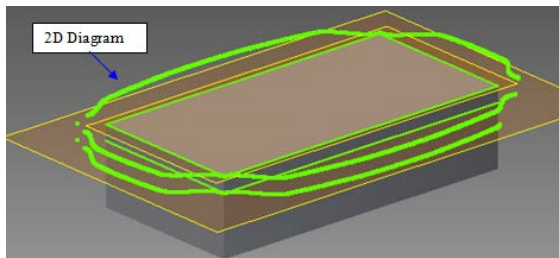The maximum torsional stress results calculated by the program proved to be very similar (0.3% difference).

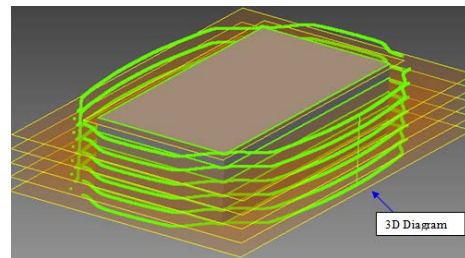Figure 7: The 2D visualisation diagram process of the σ_{vm} stresses per section.

Figure 8: The final 3D diagram connecting the max σ_{vm} stresses of all cross-sections.
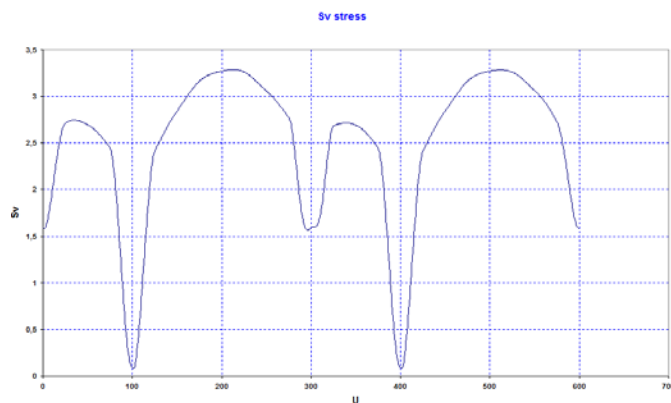

Figure 9: Graph of the σ_{vm} stresses along the boundary of the final cross section.
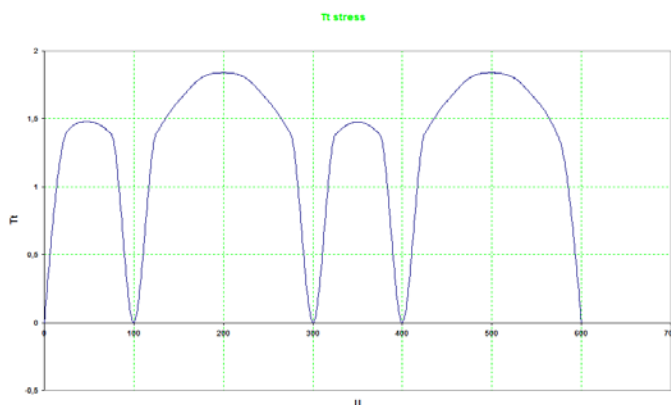

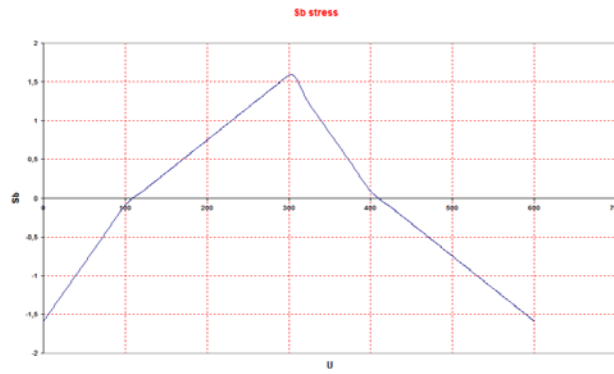Figure 10: Graph of the torsional τ_t stresses along the boundary of the final cross section.

Figure 11: Graph of the bending σ_b stresses along the boundary of the final cross section.

Non-even Rectangular Cross Section Beam

The non-even rectangular cross section beam is shown in Figure 12 and all input data (moments, tolerance, etc) are presented in Table 1.
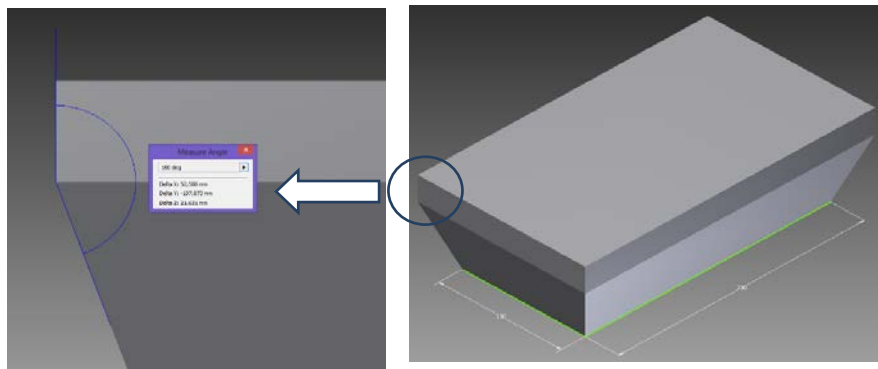

Figure 12: Non-even rectangular cross section beam.

In this case study, the methodology worked and the final output was twelve (12) section along the beam. The adaptive procedure worked as shown in Figure 14.
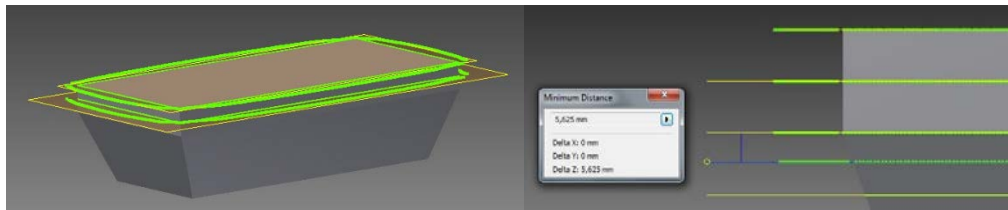

Figure 13: First two sections along the beam.   Figure 14: Detail view of different distance sections.

I Cross Section Beam

The I non-even cross section beam (Figure 15) is a simple example for demonstrating the program's capabilities. All the input data for the simulation of the beam are presented in Table 1. In this case study, the methodology worked and as shown in Figures 16-18.
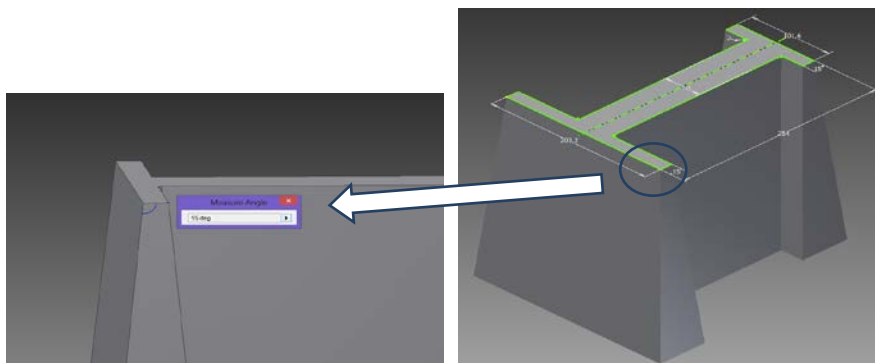

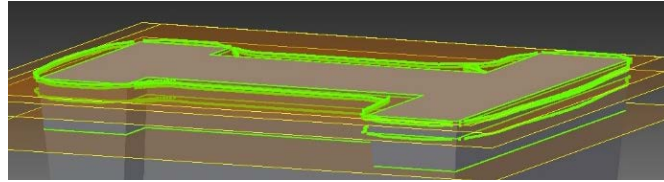Figure 15: I non-even cross section beam.

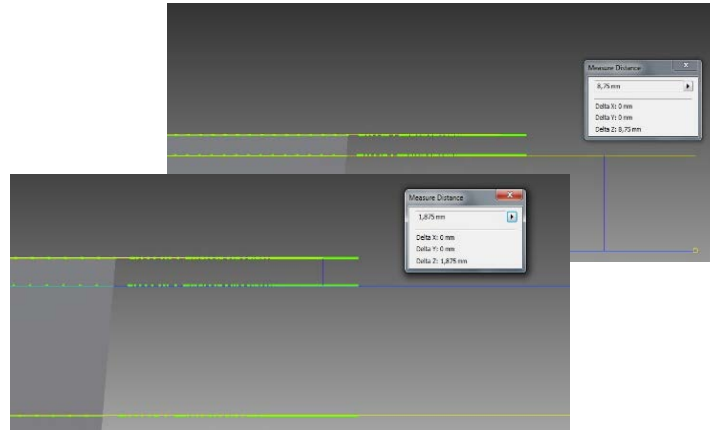Figure 16: First three sections along the beam.

Figure 17: Detail view of different distance sections.

IMPACT ON ENGINEERING AND TECHNOLOGY EDUCATION

The methodology presented here was implemented through the Application Programming Interface (API) in the CAD/CAE system of the CAD-laboratory at the Mechanical Engineering Department of TEI-Piraeus. Overall student feedback was very positive. Both postgraduate and undergraduate students were able to learn how mechanical parts behave under certain boundary and loading conditions, as well to gain an understanding of the methodology on how to develop such mechanical simulation tools. All parties to this work, researchers, as well as students, developed their cooperation and team working abilities, while the undergraduate students in particular had the opportunity to gain a deeper understanding of the way engineers should think to solve real life problems.

CONCLUSIONS

A novel methodology was presented for optimising the distance between two consecutive cross-sections of an arbitrary 3D beam, in order to produce optimum stress results, when using the Boundary Element Method (BEM). The adaptive cross-sectioning is based on the accuracy of the stress results calculated for each section. The method uses the maximum von Mises Stresses of two consecutive cross-sections of the beam in a comparative procedure, using the error criterion, in order to decide on the appropriate course of action, which optimise the accuracy of the stress results.

Another advantage of the methodology is that simplifies the calculations by reducing the model by one dimension (3D to 2D). The methodology is promising and can be extended to future work by obtaining not only the maximum equivalent stress of each cross section, but also the equivalent stress on every respective node of all sections.

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