# Self-Tuned Descriptive Document Clustering using a Predictive Network 

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#### Abstract

Descriptive clustering consists of automatically organizing data instances into clusters and generating a descriptive summary for each cluster. The description should inform a user about the contents of each cluster without further examination of the specific instances, enabling a user to rapidly scan for relevant clusters. Selection of descriptions often relies on heuristic criteria. We model descriptive clustering as an auto-encoder network that predicts features from cluster assignments and predicts cluster assignments from a subset of features. The subset of features used for predicting a cluster serves as its description. For text documents, the occurrence or count of words, phrases, or other attributes provides a sparse feature representation with interpretable feature labels. In the proposed network, cluster predictions are made using logistic regression models, and feature predictions rely on logistic or multinomial regression models. Optimizing these models leads to a completely self-tuned descriptive clustering approach that automatically selects the number of clusters and the number of feature for each cluster. We applied the methodology to a variety of short text documents and showed that the selected clustering, as evidenced by the selected feature subsets, are associated with a meaningful topical organization.


Index Terms-Descriptive clustering, feature selection, logistic regression, model selection, sparse models

## 1 Introduction

EXPLORATORY data analysis techniques such as clustering can be used to identify subsets of data instances with common characteristics. Users can then explore the data by examining some instances in each cluster, rather than examining instances from the full dataset. This enables users to efficiently focus on relevant subsets of large datasets, especially for collections of documents [1]. In particular, descriptive clustering consists of automatically grouping sets of similar instances into clusters and automatically generating a human-interpretable description or summary for each cluster. Each cluster's description allows a user to ascertain the cluster's relevance without having to examine its contents. For text documents, a suitable description for each cluster may be a multi-word label, extracted title, or a list of characteristic words [2]. The quality of the clustering is important, such that it aligns with a user's idea of similarity, but it is equally important to provide a user with an informative and concise summary that accurately reflects the contents of the cluster. However, objective criteria for evaluating the descriptions as a whole, which do not resort to human evaluation, have been largely unexplored.

With the aim of defining an objective criterion, we consider a direct correspondence between description and prediction. We assume each instance is represented with sparse features (such as a bag of words), and each cluster will be described by a subset of features. A cluster's description

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should summarize its contents, such that the description alone should enable a user to predict whether an arbitrary instance belongs to a particular cluster. Likewise, a machine classifier trained using the features subset should also be predictive of the cluster membership. The classification accuracy provides an objective and quantitative criterion to compare among different feature subsets.

To serve as a concise description, the number of features used by the classifier must be limited (e.g., a linear classifier that uses all features is not easily interpretable). A relatively small set of predictive features can be identified using various feature selection methods [3], [4]. In particular, we identify features subsets by various statistical and information-theoretic criteria [5] and by training linear classifiers with additional sparsity-inducing regularizations [6], [7], [8], e.g., the $\ell_{1}$-norm for the Lasso [9] or a combination of $\ell_{1}$ and $\ell_{2}$-norms for the Elastic Net [10], such that only a small set of features have non-zero coefficients. In a similar spirit, Lasso has been used for selecting predictive features for explaining classification models [11].

In addition to the cardinality constraint on the number of features, we only permit features that are positively correlated with a given cluster, i.e., features whose presence are indicative of the cluster. This constraint ensures that no cluster is described by the absence of features, which are present in other clusters. For instance, given a corpus of book and movie reviews, the positivity constraint avoids a cluster consisting of mainly of book reviews from being described as $\neg$ movie, i.e., the absence of the word feature movie. In general, this constraint can be enforced by admitting only features that are positively correlated with a particular cluster; for linear classifiers, this can be done by enforcing the constraint that the coefficients are nonnegative [12].


Fig. 1. Proposed approach and network architecture for descriptive clustering (the notation for the decoupled auto-encoder is described in Table 1.).

Constraining the number of features and constraining the positivity will inevitably limit the performance of the classifier. The natural question is exactly how many features are needed to ensure a 'reasonable' classification performance. We use a model order selection criterion as a principled approach to answer this question. We model the probability of class membership given the different subsets of features using logistic regression and use the Bayesian information criterion (BIC) [13] to select the model corresponding to a feature subset that is both predictive and parsimonious.

Given a clustering of the instances, we apply the aforementioned approach to automatically generate an interpretable classifier for each cluster. The features of the classifier form the description of the cluster, and the network of classifiers is a compact approximation of the original cluster assignments. The two benefits of this paradigm versus previous descriptive clustering approaches [2], [14], [15], [16] are the ability to quantify the description accuracy as well as a principled and automatic way to select the size of feature subsets that serve as the descriptors for each cluster.

Similarly, predictive performance can be used to select the clustering itself and in particular, the number of clusters. Although a variety of criteria for selecting the number of clusters exist [17], they are often specific to the underlying objective of the clustering algorithm. Independent from the clustering algorithm, we propose to select the number of clusters that is most predictive of the original feature occurrences. For each clustering, we assume the sparse features are conditionally independent given the cluster assignment and model them by either a multinomial distribution, if the feature values are non-negative counts, or a multivariate Bernoulli, if the feature values are binary. The optimal clustering is then chosen by model selection.

Our approach is a self-tuned method for descriptive clustering, which we refer to as predictive-descriptive clustering (PDC), and is based on two predictive networks, as shown in Fig. 1. By choosing from among different network architectures, both the number of clusters and the feature subsets for describing each cluster are automatically optimized. Although we assume sparse count or binary occurrence-based features to use for the description and model selection, the clusterings themselves can be derived from arbitrary representations. We explore the approach in the context of short text documents with words and phrases
as features, but the approach is applicable to any data with sparse count-valued features.

Our main contributions are (1) a new quantitative evaluation for descriptive clustering; (2) PDC as a unified framework to automatically select both the clustering itself (across different numbers of clusters) and also select a set of features (including its size) to describe each cluster; (3) a comparison of feature selection algorithms including regularized and constrained logistic regression; (4) a direct comparison to an existing topic modeling approach [18], which shows that PDC both performs better and is more efficient; and (5) motivating examples of PDC on a variety of text datasets.

We begin by discussing related work in exploratory data analysis for text datasets in Section 2. We then present the proposed methodology in Section 3, detailing the selection of clustering, selection of candidate feature subsets, and selection of model size. In Section 4, we apply the proposed approach to publicly available text datasets (movie, book, and product reviews along with Usenet newsgroup posts, news summaries, academic grant abstracts, and recipe ingredient lists) and show that meaningful and descriptive features subsets are selected for the clusters. Furthermore, we show that the interpretable classifiers that use the feature subsets are accurate.

## 2 Relation to Existing Approaches

There has been extensive research on clustering and other unsupervised methods, namely topic modeling but also low-dimensional embeddings [19], [20], [21], [22], for exploring text datasets. Some particularly relevant work has focused on website search results [15]. We highlight approaches that have considered user interpretation of the results in the form of descriptive keywords, phrases, or titles that summarize the semantic content of clusters and topics for a user. For datasets with known ground-truth topic categories, document clustering can be evaluated using correspondence measures, but comparisons of descriptive labels have often relied on human evaluation. Comparisons among description mechanisms is especially challenging, since the datasets, clustering or modeling paradigms, and form of the description varies widely. Although some user evaluations have concentrated on which labels users prefer [23], evaluation should concentrate on whether the descriptions aid a user in predicting the most relevant
cluster or topic [24], [25]. To our knowledge, no previous approach has posed descriptive clustering as a prediction problem with objective quantification in terms of classification performance. The other unique contributions of our approach include a principled approach to select the number of features in the description, and an automatic approach for selecting the number of clusters that is independent of the clustering algorithm but dependent on the cluster assignments and the binary feature representation.

### 2.1 Descriptive Clustering for Text Datasets

The motivation for applying descriptive clustering to text datasets is that it can be used as an information retrieval mechanism. A user can efficiently scan the descriptions for relevancy versus having to determine which clusters are relevant by manually checking the document instances. Scatter-gather [2], [24], [26], [27] is an iterative procedure that uses multiple stages of descriptive clustering to help a user find relevant documents. An initial clustering is given along with some description or preview of each cluster to the users, who are then asked to select clusters of interest. Instances within the selected clusters are combined and clustered again. This continues until a user hones in on a relevant set of documents. The quality of the automatic description is crucial to enable a user to recognize which clusters are relevant.

This exploratory approach should be contrasted to classic query-based information retrieval systems. While querybased systems predominate web searches, exploratory analysis is useful when the user does not know what topics are within a corpus (which could vary between a set of full-text documents to a set of short summaries of each result returned by a search engine) or is unable to formulate a query to retrieve relevant instances. In particular, the exploratory approach is useful for a user that believes, "I will know it when I see it."

Descriptive clustering can be performed by firstly clustering and then finding the set of features associated with each cluster. This enables any applicable clustering algorithms to be used. Selecting features that best inform a user on the contents of a cluster (the purpose of this study) is the subsequent challenge. The most basic approach is to describe each cluster by the most likely words in the cluster [2], titles (if available) near the center of each cluster [2], or phrases with similar context as the most likely words [14]. However, these features may not be optimal for discriminating between different clusters. Other scoring criteria such as mutual information [28] (i.e., information gain [29] rather than pointwise mutual information [30]) may be used to select more discriminating features (e.g., keywords or phrases) for the clusters.

Another approach is to firstly, or simultaneously, group features such that the grouping of features affects the clustering. Grouping features [31] can mitigate issues with feature sparsity and noise [32], and the use of grouped features has been shown to improve clustering performance [33], [34]. A novel approach is to directly learn a joint vector space representing both features (descriptive phrases) and instances [16]. The vectors representing the features and instances are optimized such that nearby vectors are semantically similar. Given this joint vector space, each cluster of
instances can be labeled by some of the features residing within its boundaries.

With the aforementioned approaches it is not clear how to objectively measure the selected feature lists or labels that serve as descriptors. Our hypothesis is that any description is only useful if it would enable a user to accurately predict the contents of the cluster. In this case, finding sets of features to describe each cluster can be seen as a feature selection problem. For instance, one can train a decision tree [35] for each cluster to classify instances directly by the presence or absence of certain features. The boolean expression corresponding to the decision tree serves as a description of the set [36].

A decision tree and clustering can be formed simultaneously using hierarchical clustering based on the presence or absence of individual features [37] or phrases as in suffixtree clustering [38], [39]. These approaches explicitly link the description with the organization of the documents and by consequence cannot be applied to arbitrary clusterings.

### 2.2 Topic Models and Automatic Topic Labeling

Besides clustering, other models can be used for exploratory analysis of large text datasets. Latent semantic indexing [40] is a matrix decomposition technique related to principal component analysis that can be applied to a bag-of-words representation of documents to group together words used in similar contexts and also group together documents which contain these contexts. A user could browse the list of highly weighted words associated with each latent dimension, which serves as a sort of description, to choose relevant dimensions. Alternatively, instead of a list, a single phrase that best describes each dimension can be used and then documents can be assigned based on the presence of this phrase [41].

Topic models offer a probabilistic interpretation of the themes present in bag-of-words representation of document collections [42], [43]. Each document is modeled as a mixture of topics/themes, with each topic or theme associated with a distribution of words. Specifically, latent topic models [44], [45] assume the words in each document are drawn from a mixture of word distributions, where each word distribution is associated with a topic. Each document has its own mixture of topics, and each topic defines a distribution over all the words. Given a document, the topics are unknown (latent) and are inferred from the distribution of words appearing in the document or in each sentence [46].

Browsing lists of the most probable words for each topic is not an efficient method to interpret topic models. Instead, various approaches to automatically label the individual topics with descriptive phrases, or a subset of distinguishing features, are useful for annotating models with a large number of topics [18], [23], [47]. Thus, the problem of automatically labeling topic models is closely related to descriptive clustering. Clustering bag-of-word representations can be seen as an extreme form of topic modeling, where each document is associated with a single topic. Since a clustering can be treated as a topic model, automatic topic labeling techniques can be applied to descriptive clustering.

In particular, we compare our feature selection approach to the topic labeling method proposed by Mei et al. [18].

Their method most closely resembles ours in that it is based solely on the statistics of the corpus (this is in contrast to other topic labeling methods that rely on external resources, for instance on the alignment of topics to the titles of Wikipedia articles [23], [47] or finding a hypernym via WordNet [48] for a topic's common words [49]). In Mei et al.'s approach, features are scored based on how predictive they are of topic-related words. The score is adjusted to discriminate between topics, and further adjusted when multiple features are selected in order to ensure they are not redundant [50]. This ensures the selected features are truly comprehensive. The drawbacks of their approach are its computational complexity, since a multinomial model over the features is formed for each candidate descriptor, and the score adjustments are based on trade-off parameters that must be selected by a user. Furthermore, automatically selecting the number of terms per topic is not considered.

Our proposed PDC framework can also be applied to topic models, but topic models are only useful for document retrieval when each topic can be clearly associated with a subset of documents, and each topic can be succinctly described. Topics that appear uniformly throughout the dataset may be useful for a global summary but are not interesting for searching for a subset of relevant documents. One approach to achieve localized topics is to combine clustering and topic modeling [51]. Another approach is to aim for a parsimonious model [52] that uses the minimal number of topics each associated with topic specific words, where the number of topics and words are directly optimized via a model order selection criterion. Soleimani and Miller's parsimonious model approach [52] yields models where each document is associated with only a few topics. Together this means the parsimonious modeling more closely resembles descriptive clustering. A key difference with our approach is that many more features are associated with each topic in the parsimonious model. This is because their model seeks to predict the features with the topics, while ours seeks to predict the topics from a subset of features.

### 2.3 Feature Selection

Given a particular cluster, predicting whether instances belong to the cluster is a standard classification problem, and selecting the best subset of features for this task is a feature selection problem [3], [4], [29]. Choosing a small subset of maximally predictive features is a difficult task. One tractable approach is to use a greedy algorithm that adds (or removes) features until a desired cardinality is reached. For small feature sets, one can consider a stepwise approach, where at each step the feature that either improves (or contributes the least) to the classification performance is added (or removed). However, the stepwise approach, which must fit a model for each candidate feature, cannot scale to cases with tens of thousands of features as encountered with text data. Tractable greedy algorithms that can scale to large number of features can be divided into three groups:

- Filter approaches [3] that do not assume specific form to the classifier but use a specific criterion to judge the relevance of individual features or feature subsets. The simplest approach is to rank features by a
selection criterion (for instance, mutual information, which has been shown to perform well for document classification [28], weighted likelihood ratio [53], or other heuristics [20]) and select the top-ranked subset.
- Joint filter approaches that consider the dependency and (possible redundancy) among features [54]. In particular, the information theoretic criteria [5] that consider the pairwise dependence include CMIM [55], MRMR [56], and JMI [57], [58].
- Heuristic approaches that are specific to the classifiers. For instance, decision trees [35] can be trained by choosing the feature that reduces a metric such as the Gini impurity value. For logistic regression, the magnitude of gradient can be used as a criterion for feature inclusion. Methods using this approach include grafting [59], logistic regression based orthogonal matching pursuit [60], and greedy cardinalityconstrained optimization [61].

Another tractable approach for features selection is based on fitting a linear classifier with additional sparsity-inducing regularizations on the coefficients of features in the objective function. With certain choices of regularization parameters, many of the optimal coefficients are exactly zero and only a small set of features are associated with non-zero coefficients [6], [7], [8]. Convex regularizations of this sort are the $\ell_{1}$-norm (Lasso) [9] and the $\ell_{1}+\ell_{2}$-norm (Elastic Net) [10].

Any of these approaches can be adjusted to ensure that the features are positively correlated with a target class, i.e., a feature's occurrence rate given the class should be higher than its average rate. For the regularization-based approaches, the positivity constraints on the coefficients can be enforced during logistic regression model optimization.

## 3 Predictive-Descriptive Clustering (PDC)

The PDC framework consists of two prediction tasks: predicting the original feature occurrences based on the cluster assignments and predicting whether an instance belongs to a specific cluster using only a small set of feature dimensions that serve as the description for the cluster. The first task provides a quantitative objective to automatically select from clusterings with different numbers of clusters. Each cluster is associated with a certain distribution of the features, with some features occurring more (or less) frequently for instances within that cluster. If all the instances assigned to the same cluster have similar feature distributions, then the knowledge of the cluster assignment will be predictive of the feature occurrences. The second task of predicting the cluster membership is clearly dependent on the chosen clustering. The amount of information carried by the clustering increases with more clusters, but the difficulty of predicting cluster membership also increases with more, finer-grained clusters. Additionally, for a fixed number of clusters there is an inherent trade-off between prediction performance and the number of features.

As a tractable approach to choose the number of clusters and features, we use a multistage modeling process. At each stage, we estimate a set of candidate models and select the best model from the candidate set.

- Create candidate clusterings. A set of possible clusterings are formed. These clusterings may vary in the number of clusters, arise from different clustering algorithms, or use different data representations.
- Select the most predictive clustering. For each clustering, a model is trained to predict feature occurrences from the cluster assignments. The clustering associated with the most predictive model is selected.
- Create descriptive feature subsets. For each cluster, different subsets of features are chosen by a feature selection mechanism. In particular, candidate feature subsets are identified using logistic regression with positivity constraints and sparsity-inducing regularization by varying the amount of regularization.
- Select the most informative feature subset. A logistic regression model is trained for each candidate feature subset. The best model is selected by a model order selection criterion that balances cardinality with predictive performance.

Each stage is associated with a standard modeling process: clustering, regression, feature selection, and model selection. Before detailing each stage, we introduce an interpretation of the PDC framework as an approach for estimating an auto-encoder neural network with a binary hidden layer [62], [63], [64].

### 3.1 PDC as a Binary Auto-Encoder Network

The predictive architecture in the PDC framework can be interpreted as an auto-encoder network [64]. The network consists of an encoder that maps each instance to its cluster assignment vector, which serves as the hidden layer of the network, and a decoder that tries to reconstruct the original features based on the cluster assignment. With a standard auto-encoder, the output of the encoder is directly fed to the input, and both the encoder and decoder are adapted in order to minimize the reconstruction error of the decoder, subject to any additional constraints on the hidden layer.

Alternatively, in our stage-wise approach, we decouple the auto-encoder by using a clustering algorithm to provide the cluster assignments that serve both as the template for training the encoder and as the input to the decoder, as shown in Figure 1. We proceed to formulate both the coupled and decoupled forms of the auto-encoder. Table 1 contains the main notation.

We assume each data instance, denoted $\xi$, is associated with a sparse feature vector $\mathbf{x}=\left[x^{(1)}, \ldots, x^{(N)}\right] \in \mathbb{Z}_{+}^{N}$, where $N$ is the number of features and $x^{(i)}$ is the number of times the $i$-th feature occurs in this instance or a binary value indicating if the $i$-th feature was present in the instance. The encoder assigns each instance to one or more clusters based on the presence of features. A cluster assignment with $C$ clusters is represented by the vector $\mathbf{y}=\left[y^{(1)}, \ldots, y^{(C)}\right] \in\{0,1\}^{C}$, where $y^{(c)}=1$ if the instance is assigned to cluster $c$. The decoder predicts the feature occurrences from the cluster assignments; the prediction is denoted $\hat{\mathbf{x}}=\operatorname{Decode}(\mathbf{y}, \mathcal{U})$, where $\mathcal{U}$ denotes the parameters of the decoder. The cluster assignment vector $\mathbf{y}$ correspond to the hidden layer activations. The objective for training the auto-encoder is to minimize both the expected loss of

TABLE 1
Notation for the descriptive clustering auto-encoder network.

| Notation | Description |
| :--- | :--- |
| $\xi$ | Data instance |
| $\mathbf{x}$ | An instance's sparse feature vector |
| $x^{(i)}$ | Value of the $i$-th feature |
| $N$ | Number of features |
| $\mathbf{y}$ | An instance's cluster assignment vector |
| $y^{(c)}$ | Binary membership variable for $c$-th cluster |
| $C$ | Number of clusters |
| $\phi(\xi)$ | Clustering function, returns an assignment vector |
| $\Phi$ | Set of candidate clusterings |
| Decode $(\mathbf{y}, \mathcal{U})$ | Cluster-to-feature decoder with parameters $\mathcal{U}$ |
| $\hat{\mathbf{x}}$ | Decoder's predicted feature vector |
| Encode $(\mathbf{x}, \mathcal{W})$ | Cluster encoder with parameters $\mathcal{W}$ |
| $\hat{\mathbf{y}}$ | Encoder's predicted cluster assignment |
| $\mathcal{S}^{(c)}$ | Set of selected feature indices for $c$-th cluster |

the feature predictions and also the number of features used in the encoder, and can be written as

$$
\begin{array}{rl}
\min _{C, \mathcal{W}, \mathcal{U}} & \mathbb{E}[\operatorname{Loss}(\mathbf{x}, \hat{\mathbf{x}})]+\alpha \Omega(\mathcal{W})  \tag{1}\\
& \hat{\mathbf{x}}=\operatorname{Decode}(\mathbf{y}, \mathcal{U}), \quad \mathbf{y}=\operatorname{Encode}(\mathbf{x}, \mathcal{W}) \\
& \text { subject to } \mathbf{y} \in\{0,1\}^{C}
\end{array}
$$

where $\mathcal{W}$ is the set of parameters of the encoding model, $\Omega(\mathcal{W})$ is a penalty function based on the number of features used in the encoding model, and $\alpha$ is a trade-off parameter.

Even without the additional penalty on the number of features, finding the optimal auto-encoder is a difficult problem to solve [64]. Rather than attempt to jointly learn all of the parameters, we decouple the decoder from the encoder, using a separate clustering function $\phi: \xi \mapsto y$ to define the cluster assignments ( $\phi$ maps each instance $\xi$ to a cluster assignment vector $\mathbf{y}$ ). The clustering assignments defined by $\phi$ are used as the input to the decoder and also as the target for the descriptive encoder. The decoupled autoencoder optimization can be written as

$$
\begin{align*}
& \min _{\phi, \mathcal{U}, \mathcal{W}} \mathbb{E}[\operatorname{Loss}(\mathbf{x}, \hat{\mathbf{x}})+\beta \operatorname{Loss}(\mathbf{y}, \hat{\mathbf{y}})]+\alpha \Omega(\mathcal{W})  \tag{2}\\
& \hat{\mathbf{x}}=\operatorname{Decode}(\mathbf{y}, \mathcal{U}), \quad \mathbf{y}=\phi(\xi), \quad \hat{\mathbf{y}}=\operatorname{Encode}(\mathbf{x}, \mathcal{W})
\end{align*}
$$

where $\hat{\mathbf{y}}$ is the predicted cluster assignments from the descriptive encoder and $\beta$ is a trade-off parameter between the loss functions.

This joint optimization is still difficult; however, for a fixed clustering, the encoder and decoder can be independently optimized, and the resulting objective value can be used to select from a set of different clusterings $\Phi=$ $\left\{\phi_{1}, \ldots, \phi_{K}\right\}$. Moreover, as in the coupled auto-encoder, each candidate clustering can be evaluated solely on the decoder performance (this avoids training an encoder for each possible clustering). The optimization corresponding to selecting the most predictive clustering can be written as

$$
\begin{align*}
\phi^{\star}= & \underset{\phi \in \Phi}{\arg \min } \min _{\mathcal{U}} \mathbb{E}[\operatorname{Loss}(\mathbf{x}, \hat{\mathbf{x}})],  \tag{3}\\
& \hat{\mathbf{x}}=\operatorname{Decode}(\mathbf{y}, \mathcal{U}), \quad \mathbf{y}=\phi(\xi) .
\end{align*}
$$

Given the vector of cluster assignments for the best clustering $\mathbf{y}=\phi^{\star}(\xi)$, the feature selection problem is then

$$
\begin{gather*}
\min _{\mathcal{W}} \mathbb{E}[\operatorname{Loss}(\mathbf{y}, \hat{\mathbf{y}})]+\alpha \Omega(\mathcal{W}),  \tag{4}\\
\hat{\mathbf{y}}=\operatorname{Encode}(\mathbf{x}, \mathcal{W})
\end{gather*}
$$

and can be performed independently for each cluster:

$$
\begin{align*}
& \min _{\mathcal{S}^{(c)}, \mathcal{W}^{(c)}} \mathbb{E}\left[\operatorname{Loss}\left(y^{(c)}, \hat{y}^{(c)}\right)\right]+\alpha \Omega\left(\mathcal{W}^{(c)}\right)  \tag{5}\\
& \hat{y}^{(c)}=\operatorname{Encode}\left(\mathbf{x}^{(c)}, \mathcal{W}^{(c)}\right), \quad \mathbf{x}^{(c)}=\left[x^{(d)}\right]_{d \in \mathcal{S}^{(c)}}
\end{align*}
$$

where $\mathcal{S}^{(c)}$ is the set of selected feature indices. A complete solution to the descriptive clustering problem is the cluster assignment and the feature subsets $\left(\phi,\left\{\mathcal{S}^{(1)}, \ldots, \mathcal{S}^{(C)}\right\}\right)$. With this mathematical formulation we proceed to detail the optimizations involved in each step of the approach.

### 3.2 Predictive Clustering Selection

The PDC framework can be used with the cluster assignments of any clustering algorithm that produces flat clusterings. As a baseline algorithm we use spectral clustering [65], which can be efficiently implemented for sparse data when cosine similarity is used, and vary the number of clusters to create a set of candidate clusterings $\Phi=\left\{\phi_{1}, \ldots, \phi_{K}\right\}$. (An alternative is to use a coarser set of cluster sizes for less computation.) We automatically select from among different clusterings based on how well the feature vectors can be predicted from the cluster assignments.

### 3.2.1 Predicting features from cluster assignments

For the expected loss, we explore two approaches: in the first approach, we model binary features using logistic regression and use the cross-validated average of the negative loglikelihood; as a more scalable alternative, we model countvalues features using multinomial regression.
3.2.1.1 Binary features (Bernoulli model): We model binary feature vectors based on the assumption that they are independent Bernoulli random variables given the cluster assignment. The probability ${ }^{1}$ of a particular feature $X$ being present given the cluster assignment $\mathbf{y}=\left[y^{(1)}, \ldots, y^{(C)}\right]$ is modeled as

$$
\begin{equation*}
\operatorname{Pr}(X=1 \mid \mathbf{y})=\frac{1}{1+e^{-u_{0}-\mathbf{u} \cdot \mathbf{y}}}=\frac{1}{1+e^{-\mathbf{u}^{\prime} \cdot \mathbf{y}^{\prime}}} \tag{6}
\end{equation*}
$$

where for compactness the bias $u_{0}$ is included with the coefficients in the parameter vector $\mathbf{u}^{\prime}=\left[u_{0}, \mathbf{u}\right]$ and a constant feature is added to the cluster assignments $\mathbf{y}^{\prime}=[1, \mathbf{y}]$. For a sample $\left\{\left(\mathbf{y}_{i}, x_{i}\right)\right\}_{i=1}^{n}$ of $n$ instances the log-likelihood is

$$
\begin{equation*}
\ln \mathcal{L}\left(\mathbf{u}^{\prime}, \mid\left\{\mathbf{y}, x_{i}\right\}_{i}\right)=\sum_{i=1}^{n} x_{i}\left(\mathbf{u}^{\prime} \cdot \mathbf{y}_{i}^{\prime}\right)-\ln \left(1+e^{\mathbf{u}^{\prime} \cdot \mathbf{y}_{i}^{\prime}}\right) \tag{7}
\end{equation*}
$$

In practice, if a feature only occurs in a subset of clusters, the coefficient for the feature can become arbitrarily large while maximizing the log-likelihood. This behavior can be avoided by constraining the norm of the coefficients to be less than some value. For any constraint value, an optimization with an equivalent solution is obtained by minimizing the negative log-likelihood function combined with a scaled penalty on the squared $\ell_{2}$-norm of the coefficients:

$$
\begin{equation*}
\underset{\mathbf{u}^{\prime}}{\arg \min }-\ln \mathcal{L}\left(\mathbf{u}^{\prime} \mid\left\{\mathbf{y}_{i}, x_{i}\right\}_{i}\right)+\frac{\tau}{2}\|\mathbf{u}\|_{2}^{2} . \tag{8}
\end{equation*}
$$

1. For a Bernoulli random variable $X$ with a mean frequency of $p$ the probability mass function can be written as $\operatorname{Pr}(X=x)=p^{x}(1-p)^{(1-x)}, x \in\{0,1\}$. Given a sample $\left\{x_{i}\right\}_{i}$ from this distribution, the $\log$-likelihood function is $\ln \mathcal{L}\left(\hat{p} \mid\left\{x_{i}\right\}\right)=$ $\sum_{i} x_{i} \ln \hat{p}+\left(1-x_{i}\right) \ln (1-\hat{p})$.

To select the best clustering among multiple clustering candidates, each clustering is evaluated in terms of the cross-validated log-likelihood for the optimized coefficients. For a particular clustering defined by the mapping $\phi: \xi \mapsto \mathbf{y}$, the parameters for each feature are optimized and fixed, and the negative log-likelihood on a separate test sample $\left\{\left(\phi\left(\tilde{\xi}_{i}\right), \tilde{\mathbf{x}}_{i}\right)\right\}_{i}$ is combined across all $N$ features:

$$
\begin{equation*}
\mathbb{E}[\operatorname{Loss}(\mathbf{x}, \text { Decode }(\mathbf{y}, \mathcal{U}))]=\sum_{j=1}^{N}-\ln \mathcal{L}\left(\mathbf{u}_{j}^{\prime},\left\{\tilde{y}_{i}, \tilde{x}_{i}^{(j)}\right\}_{i}\right) \tag{9}
\end{equation*}
$$

where $\tilde{y}_{i}=\phi\left(\tilde{\xi}_{i}\right)$ and $\mathcal{U}=\left\{\mathbf{u}_{j}^{\prime}\right\}_{j=1}^{N}$ is the set of coefficients.
In practice, some of the features may be extremely sparse (only a few non-zeros). Rather than attempt to predict these sparse features, we use only the subset of features that occur in at least $1 \%$ of the instances in the cross-validation sets.
3.2.1.2 Count-valued features (multinomial model): For sparse feature vectors corresponding to occurrence counts, such as bag-of-words, we assume the counts follow a multinomial distribution for each cluster. For a multinomial random vector $\mathbf{X}=\left[X^{(1)}, \ldots, X^{(N)}\right]$ described by the probability distribution $\mathbf{p}=\left[p_{(1)}, \ldots, p_{(N)}\right], p_{(j)} \geq 0$, $\sum_{j} p_{(j)}=1$, the probability mass function for observing the count vector $\mathbf{x}=\left[x^{(1)}, \ldots, x^{(N)}\right] \in \mathbb{Z}_{+}^{N}$ with a total count of $m=\sum_{j} x^{(j)}$ is

$$
\begin{equation*}
\operatorname{Pr}(\mathbf{X}=\mathbf{x})=\frac{m!}{x^{(1)}!\cdots x^{(N)!}} p_{(1)}^{x^{(1)}} p_{(2)}^{x^{(2)}} \cdots p_{(N)}^{x^{(N)}} \tag{10}
\end{equation*}
$$

Given a sample $\left\{\left(\mathbf{x}_{i}, m_{i}\right)\right\}_{i=1}^{n}$ from this distribution, the loglikelihood function is

$$
\begin{equation*}
\ln \mathcal{L}\left(\hat{\mathbf{p}} \mid\left\{\mathbf{x}_{i}\right\}\right)=\sum_{i} \sum_{j} \ln m_{i}!-\ln \left(x_{i}^{(j)}!\right)+x_{i}^{(j)} \ln \hat{p}_{(j)} \tag{11}
\end{equation*}
$$

Rather than use the maximum likelihood estimate of $\hat{\mathbf{p}}$, we adopt additive smoothing to estimate the multinomial parameters for each cluster as $\hat{p}_{(j)}^{(c)}=\frac{1+r_{(j)}^{(c)}}{N+\sum_{k=1}^{N} r_{(k)}^{(c)}}$, where $r_{(j)}^{(c)}=\sum_{i: y_{i}^{(c)}=1} x_{i}^{(j)}$ (the number of times the $j$-th word occurs in the $c$-th cluster) and $\mathcal{U}=\hat{\mathbf{p}}$. The empirical loglikelihood of the sample itself is used to estimate the quality of the clustering, and to compare clusterings of different size we use the Akaike information criterion (AIC) [66]:

$$
\begin{equation*}
\mathbb{E}[\operatorname{Loss}(\mathbf{x}, \operatorname{Decode}(\mathbf{y}, \mathcal{U}))] \approx-2 \ln \mathcal{L}\left(\hat{\mathbf{p}} \mid\left\{\mathbf{x}_{i}\right\}\right)+2 C \tag{12}
\end{equation*}
$$

Using AIC is more efficient than cross-validation and has the same asymptotic performance for model selection [67].

### 3.3 Descriptive Feature Selection

Given the clustering, training an interpretable encoder for each cluster is a supervised learning problem consisting of both feature selection and classifier training. Each cluster is treated as a class, and the corresponding dimension of the cluster membership vector $y^{(c)}$ is treated as the indicator for class membership. For each cluster $c \in\{1, \ldots, C\}$ the problem is to predict $y^{(c)}$ using a subset of features:

$$
\begin{equation*}
\underset{\mathcal{W}^{(c)}}{\arg \min } \mathbb{E}\left[\operatorname{Loss}\left(y^{(c)}, \hat{y}^{(c)}\right)\right]+\alpha \Omega\left(\mathcal{W}^{(c)}\right) \tag{13}
\end{equation*}
$$

subject to $w \geq 0 \quad \forall w \in \mathcal{W}^{(c)}$,

$$
\begin{gathered}
\mathbf{y}=\left[y^{(1)}, \ldots, y^{(C)}\right]=\phi(\xi), \quad \hat{y}^{(c)}=\operatorname{Encode}\left(\mathbf{x}, \mathcal{W}^{(c)}\right) \\
\Omega\left(\mathcal{W}^{(c)}\right)=\left|\left\{w \in \mathcal{W}^{(c)}: w>0\right\}\right|
\end{gathered}
$$

where $\mathcal{W}^{(c)}$ denotes the coefficients of the encoder model associated with the features, $y$ represents the original cluster assignment vector and $\hat{y}^{(c)}$ is encoder's assignment for cluster $c$. The constraint ensures the features are positively associated with a cluster.

Optimizing Eq. (13) is not tractable due to the integral nature of the feature count $\Omega\left(\mathcal{W}^{(c)}\right)$. Instead, for each cluster a candidate set of encoders using features subsets of varying cardinality are optimized to predict the cluster assignment, and the feature subset for the encoder that minimizes the cost is selected. Feature subsets can be generated by various algorithms or by optimizing a linear classifier with sparsityinducing regularization. We select $\alpha$ by equating Eq. (13) to BIC by using the negative log-likelihood for the expected loss and setting $\alpha=\ln \sqrt{n}$, where $n$ is the number of instances in the sample.

### 3.3.1 Regularized and constrained logistic regression for feature subset generation

Candidate feature subsets can be obtained by combining logistic regression with sparsity-inducing regularizations. For a given cluster, the probability that an instance is assigned to the cluster is modeled as a conditionally Bernoulli random variable:

$$
\begin{equation*}
\operatorname{Pr}(Y=1 \mid \mathbf{x})=f_{\mathbf{w}^{\prime}}(\mathbf{x})=\frac{1}{1+e^{-w_{0}-\mathbf{w} \cdot \mathbf{x}}} \tag{14}
\end{equation*}
$$

where for compactness the bias $w_{0}$ is combined with the coefficients $\mathbf{w}^{\prime}=\left[w_{0}, \mathbf{w}\right]$ and a constant is added to the feature vector $\mathbf{x}_{i}^{\prime}=[1, \mathbf{x}]$. Given a sample $\left\{\left(\mathbf{x}_{i}, y_{i}\right)\right\}_{i=1}^{n}$, the log-likelihood function is

$$
\begin{align*}
\ln L\left(\mathbf{w}^{\prime}\right) & =\sum_{i=1}^{n} y_{i} \ln f_{\mathbf{w}^{\prime}}\left(\mathbf{x}_{i}\right)+\left(1-y_{i}\right) \ln \left(1-f_{\mathbf{w}^{\prime}}\left(\mathbf{x}_{i}\right)\right) \\
& =\sum_{i=1}^{n} y_{i}\left(\mathbf{w}^{\prime} \cdot \mathbf{x}_{i}^{\prime}\right)-\ln \left(1+e^{\mathbf{w}^{\prime} \cdot \mathbf{x}_{i}^{\prime}}\right) \tag{15}
\end{align*}
$$

When instances from each class are linearly separable (which is likely to occur with clustering), coefficients may grow without bound to maximize the log-likelihood function. This can be remedied by adding the squared $\ell_{2}$-norm of the coefficients to the negative log-likelihood function to ensure a minimization problem with a finite solution. The regularized optimization is

$$
\begin{equation*}
\underset{\mathbf{w}^{\prime}}{\arg \min }-\ln L\left(\mathbf{w}^{\prime}\right)+\frac{\tau}{2}\|\mathbf{w}\|_{2}^{2} . \tag{16}
\end{equation*}
$$

This regularization reduces the magnitude of the coefficients, but in general, all of the features are associated with non-zero coefficients. Due to the geometry of the equivalent constraints, replacing the $\ell_{2}$-norm with an $\ell_{1}$-norm, i.e., Lasso [9], can yield solutions where many coefficients are exactly zero. The Lasso-regularized optimization is

$$
\begin{equation*}
\underset{\mathbf{w}^{\prime}}{\arg \min }-\ln L\left(\mathbf{w}^{\prime}\right)+\gamma\|\mathbf{w}\|_{1} . \tag{17}
\end{equation*}
$$

The choice of $\gamma$ affects the number of features with nonzero coefficients, as a relatively large value of $\gamma$ will yield a solution with few non-zero coefficients. The features with non-zero coefficients can be considered selected features. An entire suite of candidate feature subsets can be found by sweeping the value of $\gamma$ (known as the regularization path).

This regularization path can be computed as efficiently as solving for the optimization for a single choice of $\gamma$ [68]. The candidate feature subsets $\mathcal{S}_{1}, \ldots, \mathcal{S}_{J}$ are formed at the sequence of change-points $\gamma_{1}, \ldots, \gamma_{J}$ (parameter values where a new feature assumes a non-zero coefficient or a coefficient becomes zero) along the regularization path. A particular subset can be selected by using a model order selection criterion, which we discuss in Section 3.3.2.

It is straightforward to combine the positive constraints with regularized logistic regression. The positivelyconstrained version of logistic regression with Lasso regularization is

$$
\begin{equation*}
\underset{\mathbf{w}^{\prime}: w_{1} \geq 0, \ldots, w_{N} \geq 0}{\arg \min }-\ln L\left(\mathbf{w}^{\prime}\right)+\gamma \sum_{i=1}^{N} w_{i} . \tag{18}
\end{equation*}
$$

Instead of Lasso, an alternative is to use Elastic-Net regularization [10], which uses a combination of both the $\ell_{1}$ norm and $\ell_{2}$-norm, and is more stable for a large number of correlated features [10]. The optimization for Elastic-Net regularization is

$$
\begin{equation*}
\underset{\mathbf{w}^{\prime}: w_{1} \geq 0, \ldots, w_{N} \geq 0}{\arg \min }-\ln L\left(\mathbf{w}^{\prime}\right)+\gamma \sum_{i=1}^{N}\left(\mu w_{i}+\frac{1-\mu}{2} w_{i}^{2}\right) . \tag{19}
\end{equation*}
$$

Again, candidate feature subsets $\mathcal{S}_{1}, \ldots, \mathcal{S}_{J}$ correspond to features with non-zero coefficients for the sequence of parameter values $\gamma_{1}, \ldots, \gamma_{J}$ at which a new feature assumes a non-zero coefficient or an existing coefficient becomes zero along the regularization path.

Both optimizations (Eq. 18 and Eq. 19) are convex and smooth-the positivity constraints avoid the challenges of optimization with the non-smooth $\ell_{1}$-norm regularization. Although a generic optimization can be used to solve this problem, in order to scale to a large number of features, a solver specialized for Lasso and Elastic Net should be used to compute the regularization path. To further increase the scalability, implementations ${ }^{2}$ that apply rules to filter out many of the irrelevant features [71] are necessary.

### 3.3.2 Feature subset selection via BIC

To select a particular feature subset from among a candidate set, we form a logistic regression model for each feature subset and rank them using the Bayesian information criterion (BIC) [13]. Given the candidate feature subsets $\mathcal{S}_{1}, \ldots, \mathcal{S}_{J}$ for a particular cluster and their corresponding cardinalities $\left|\mathcal{S}_{1}\right|, \ldots,\left|\mathcal{S}_{J}\right|$, the optimal feature subset is $\mathcal{S}_{j^{\star}}$, where $j^{\star}$ is chosen as

$$
\begin{align*}
& j^{\star}=\underset{j \in\{1, \ldots, J\}}{\arg \min }-\ln L\left(\tilde{\mathbf{w}}_{j}^{\prime}\right)+\left|\mathcal{S}_{j}\right| \ln \sqrt{n}, \\
& \tilde{\mathbf{w}}_{j}^{\prime}=\underset{\mathbf{w}^{\prime}: w_{i}=0, i \in \overline{\mathcal{S}}_{j}}{\arg \min }-\ln L\left(\mathbf{w}^{\prime}\right)+\frac{\tau}{2}\|\mathbf{w}\|_{2}^{2}, \tag{20}
\end{align*}
$$

and $\overline{\mathcal{S}}_{j}$ denotes the set of features not included in the $j$-th features subset.

It is noteworthy that new coefficients are estimated for the feature subsets generated by Lasso or Elastic Net (a process known as debiasing [72], [73]), which is important since sparsity-inducing regularizations cause the non-zero

[^0]coefficients to be biased towards zero. This bias would otherwise affect comparisons between feature subsets chosen with different amounts of regularization. We also note that the choice of BIC instead of the AIC is motivated by the theoretical consistency of BIC [67], since we desire an interpretable number of features that should be stable across changes in sample size. Yet, in practical situations, the candidate feature sets may need to be limited in size: a user may want at most a dozen features to describe a cluster. While a limit may decrease the predictive performance, it will also decrease the computation in terms of the number of candidate feature sets that need to be evaluation.

## 4 Experimental Results

In this section, we empirically evaluate the PDC framework on publicly available datasets. Firstly, we evaluate the performance of various feature selection approaches for selecting predictive and descriptive feature labels. We use ground-truth class labels and evaluate how well the selected descriptive features can predict class membership while comparing against baseline linear classifiers, which use all features. Secondly, we evaluate descriptive clustering in terms of the information content of the descriptive features and the clustering itself.

### 4.1 Datasets

We use the 20-Newsgroup dataset [74], a compilation of online reviews [75], [76], the Reuters-21578 Distribution 1.0 newswire articles ${ }^{3}$, ingredient lists from Yummly's recipe dataset ${ }^{4}$, the NSF research award abstracts 1990-2003 data set [77], and news articles provided ${ }^{5}$ by Antonio Gulli. The 20-Newsgroup dataset ${ }^{6}$ consists of Usenet posts divided among 20 topics, with some of the topics categorized together (baseball and hockey are both under rec). The reviews cover 5 topics: ${ }^{7}$ movies, books, dvds, electronics, and kitchen; for each topic there are 2000 reviews with 1000 positive sentiment and 1000 negative sentiment reviews. Like previous usage of this datatset [53], we explore both topic and sentiment classification within a given topic to compare the feature selection algorithms, but for clustering we ignore the sentiment and only consider mixtures of different topics. For the Reuters dataset, we use the 8 largest categories of singlecategory articles. ${ }^{8}$ We use the training portion of the recipe
3. The corpus is on the UCI Machine Learning Repository [77].
4. The dataset is at https://www.kaggle.com/c/whats-cooking.
5. The dataset is available at http://www.di.unipi.it/~gulli/AG_ corpus_of_news_articles.html.
6. For the 20 Newsgroup corpus we use two versions: a preprocessed MATLAB/OCTAVE version of the bag-of-words representation provided by Jason Rennie, http://qwone.com/~jason/20Newsgroups/ that we divide into various subsets, and a tokenized "bydate" training-testing split of the full dataset http://ana.cachopo.org/ datasets-for-single-label-text-categorization [78].
7. Movie reviews are taken from the polarity dataset v2.0 available at http://www.cs.cornell.edu/people/pabo/movie-review-data/. Raw source is from the IMDb archive of the rec.arts.movies.reviews newsgroup http://reviews.imdb.com/Reviews. Book and product reviews crawled from the online retailer Amazon are available in XML format at http://www.cs.jhu.edu/~mdredze/datasets/ sentiment/index $2 . h t m l$. For the reviews, apostrophes and numbers are removed, and the text is tokenized by whitespace and punctuation.
8. A tokenized version of this subset with a trainingtesting split [78] is available at http://ana.cachopo.org/ datasets-for-single-label-text-categorization.
dataset that is categorized by cuisine. ${ }^{9}$ The NSF abstracts are classified by organization, we keep only abstracts from the 35 most common organizations. ${ }^{10}$ For AG's news dataset, we use a subset of the 4 largest categories [79]. ${ }^{11}$ The list of subsets and datasets are given in Table 2. Stop words ${ }^{12}$ are removed (excepting the $20-$ News subsets and recipes). For each subset/dataset, features that appear in only one instance are removed.

TABLE 2
Dataset/subset profiles: classes, instances $(n)$, and features $(N)$.

| Classes |  | $n$ | $N$ |
| :---: | :---: | :---: | :---: |
| Reviews (+/-): |  |  |  |
| movie | 2 | 2000 | 24093 |
| kitchen | 2 | 2000 | 4665 |
| dvd | 2 | 2000 | 10423 |
| electronics | 2 | 2000 | 5167 |
| books | 2 | 2000 | 10551 |
| Reviews (topic): |  |  |  |
| movie, kitchen (m k) | 2 | 4000 | 25543 |
| movie, books (mb) | 2 | 4000 | 27436 |
| movie, electronics (me) | 2 | 4000 | 25778 |
| kitchen, electronics (ke) | 2 | 4000 | 7786 |
| dvd, electronics (d e) | 2 | 4000 | 13048 |
| movie, books, electronics, kitchen (m b e k) | 4 | 8000 | 30079 |
| dvd, movie, electronics, kitchen (d m e k) | 4 | 8000 | 29552 |
| dvd, books, movie, kitchen ( d b m k ) | 4 | 8000 | 31101 |
| dvd, books, electronics, movie (d b e m) | 4 | 8000 | 31299 |
| dvd, books, electronics, movie, kitchen (d b e m k) | 5 | 10000 | 32375 |
| 20-News (subsets): |  |  |  |
| med guns | 2 | 1896 | 15204 |
| autos space hardware | 3 | 2951 | 16071 |
| rec: autos, motorcycles, baseball, hockey | 4 | 3968 | 18728 |
| sci: crypto, electronics, med, space | 4 | 3945 | 22198 |
| mix: ms-windows, forsale, baseball, space, politics.misc | 5 | 4677 | 23059 |
| comp: graphics, ms-windows, pc.hardware mac.hardware, x -windows | 5 | 4852 | 20382 |
| mix2: ms-windows, autos, baseball, med, space | 5 | 4913 | 24457 |
| sci comp: sci.* comp.* | 9 | 8797 | 33501 |
| sci comp rec: sci.* comp.* rec.* | 13 | 12765 | 42049 |
| 20-News (all) | 20 | 18820 | 41747 |
| Reuters | 8 | 7674 | 10250 |
| Recipe | 20 | 39774 | 10506 |
| NSF | 35 | 125730 | 119773 |
| AG's news | 4 | 127600 | 39763 |

### 4.2 Implementation Details

For representing text documents, we use a bag-of-words representation. Each feature dimension is weighted by the logarithm of the inverse occurrence rate, the standard termfrequency inverse-document frequency (TF-IDF), and instances are normalized to have unit-norm. For clustering text documents, we use spectral clustering applied to the
9. The Yummly recipes are provided as lists of ingredients. Each ingredient is a character string that is processed by splitting the string into tokens by whitespace, numerals, hyphens, periods, commas, and parentheses. Features are formed from all subsequences of consecutive tokens, and additional features are appended by mapping together possessive forms with missing apostrophes and by combining features with both singular and plural forms.
10. We retain the NSF abstracts that have an award number listed in the abstract file, are referenced in the list files (idnsfid.txt), and are longer than one line and have at least 17 tokens (after removing stop words), where tokenization is based on whitespace and punctuation (apostrophes and internal hyphens are retained).
11. https:/ / github.com/mhjabreel/CharCNN/tree/master/data/ ag_news_csv.
12. We use a list of 571 English stop words, http://members.unine. ch/jacques.savoy/clef/.
similarity matrix implicitly formed from the cosine similarity between the TF-IDF vectors [80], and the spectral clustering algorithm of Ng et al. [81] is applied to the similarity matrix. All instances are used by the clustering algorithm, but a subset of instances are used as training instances for selecting feature subsets and evaluating the predictive features. The number of clusters is varied between 2 and 26.

A logistic regression model is trained for each feature subset, with regularization of $\tau=\frac{1}{10}$ to ensure bounded coefficients, and the feature subset with minimal BIC is selected as in Eq. (20). To transform the selected logistic regression model to a binary classifier, the probability estimates are thresholded, with the threshold that maximizes the $F_{1}$-score (the harmonic mean of precision and recall) in the training data [82].

### 4.3 Evaluation

For evaluating the classifier and cluster predictions, we use the $\mathrm{F}_{1}$-score per cluster/class and summarize the results by using the macro-average of the $\mathrm{F}_{1}$ score. For computing the correspondence between ground-truth topics and known classes, we must take into consideration that an instance may be assigned to multiple clusters, or not assigned to any cluster. Each instance is given equal weight, and when an instance is assigned to multiple clusters, its weight is divided equally among its cluster assignments. Unassigned instances are considered to be grouped together into an additional outlier cluster. This ensures a valid contingency table for computing cluster correspondence metrics-in particular, normalized mutual information (the mutual information between two discrete variables divided by the maximum entropy of either variable [83])-in order to compare the correspondence between ground-truth categories and the classifier-based clusters assignments. We also use the normalized mutual information to compare the original cluster assignments to the ground-truth topic categories.

### 4.4 Comparison of Feature Selection Performance

The PDC framework is based on selecting an interpretable set of predictive features. For the purpose of interpretability we limit the number of features and constrain them to be positively correlated with the class/cluster of interest. We examine how much the positivity constraints limit the classification performance and how well the different feature selection approaches perform.

We compare a range of feature selection algorithms with and without positivity constraints. These include ranking methods that simply select top-ranked features for different criteria: weighted log-likelihood ratio (WLLR) [53] (which is only defined for selecting positively correlated features), mutual information (MI), and the chi-squared statistic ( $\mathrm{CHI}^{2}$ ); and forward-selection algorithms using information theoretic criteria [5]: JMI [57], [58], MRMR [56], and CMIM [55]. To estimate these quantities, co-occurrence statistics are computed after the input feature vectors are transformed to binary vectors (removing any information about counts, weightings, and instance normalization). We limit the forward-selection algorithms to 250 features with the highest mutual information for each class/cluster. For regularization-based feature selection we compare Lasso
and Elastic Net with positivity constraints. For the latter, the $\ell_{1}-\ell_{2}$-norm trade-off is fixed at $\mu=0.2$ in Eq. (19). For each feature selection algorithm, training consists of selecting feature subsets of varying size (up to a maximum of 50 features per class/cluster), then the selection process discussed in Section 4.2 is applied to select the number of features for each algorithm.

To give an upper bound on classification performance using unconstrained classifiers, we use the LIBLINEAR package to train linear classifiers using different loss functions ( $\ell_{1}$-loss or logistic loss) with $\ell_{2}$-norm regularization. The regularization trade-off parameter is chosen by LIBLINEAR's built-in cross-validation. We note that these classifiers do not yield readily interpretable feature subsets.

On datasets without predefined training and testing splits, 10 Monte Carlo runs of training and testing splits are used to assess feature selection performance. The results are included in Table 3. We note the classifiers that used feature selection are all outperformed by the baseline linear classifiers. In this classification task, feature selection does not improve prediction performance. Secondly, we note that the gap between the performance of the baseline classifiers and cardinality-constrained methods is smaller than the gap between the performance of the positively-constrained methods. Nonetheless, we maintain these constraints for interpretability such that the features are positively correlated with each class.

We compare the scaling performance of the top three performing feature selection algorithms with the linear classifiers across different training set sizes for the AG's news dataset (the largest dataset). Fig. 2 shows that the vocabulary size of the training set grows sub-linearly with the size of the corpus. The figure also shows the classification performance, which stabilizes at around 10000 training instances, and the computation time. ${ }^{13}$ The methods all scale linearly with the number of training instances with the $\ell_{1}$-loss support vector machines having the longest running times for training set sizes above 10000, whereas logistic regression and the CMIM-based feature selection algorithm are the fastest. Based on the marginal difference in performance among these three feature selection algorithms, and the efficiency of the CMIM-based algorithm, we choose CMIM+ for further comparisons.

### 4.5 Predictive and Descriptive Clustering

We apply the PDC framework to identify and label clusters of documents within each of the mixture-of-topics datasets listed in Table 2. Firstly, we compare the cluster prediction performance using the CMIM algorithm with positivity constraints versus an existing topic labeling approach [18], and qualitatively examine the selected features. Secondly, we evaluate the automatic selection of the number of clusters, testing whether the number of clusters correlates with the ground-truth number of topics or whether the number of clusters maximizes the information content.

### 4.5.1 Comparison with existing topic labeling approach

We compare against the method proposed by Mei, Shen, and Zhai [18] to select features for describing multinomial

[^1]TABLE 3
Comparison of classification performance of feature selection algorithms versus baseline linear classifiers. Columns correspond to average number of features $(|\mathcal{S}|)$, macro-averaged $\mathrm{F}_{1}$-score for different datasets (training set size is listed and *indicates a pre-defined training-testing split), and average rank based on the $F_{1}$-score within each algorithm group and overall.

|  | $\|\mathcal{S}\|$ | Reviews |  | 20-News <br> (subsets) <br> 1000 | $\begin{gathered} \hline \text { 20-News } \\ \text { (all) } \\ { }^{*} 11293 \end{gathered}$ | Reuters*5485 | $\begin{array}{r} \hline \text { Recipe } \\ 10000 \end{array}$ | $\begin{array}{r} \text { NSF } \\ 10000 \end{array}$ | AG'snews${ }^{1} 120000$ | Ave. rank |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | +/- | topic |  |  |  |  |  |  |  |  |
|  |  | 1000 | 1000 |  |  |  |  |  |  | group | overall |
| Positive constraints: |  |  |  |  |  |  |  |  |  |  |  |
| WLLR+ | 26.7 | 0.670 | 0.791 | 0.725 | 0.661 | 0.836 | 0.513 | 0.518 | 0.749 | 6.4 | 15.1 |
| MI+ | 26.9 | 0.671 | 0.791 | 0.723 | 0.665 | 0.833 | 0.519 | 0.521 | 0.751 | 5.9 | 14.5 |
| JMI+ | 26.6 | 0.671 | 0.791 | 0.727 | 0.668 | 0.831 | 0.520 | 0.525 | 0.753 | 5.4 | 13.1 |
| $\mathrm{CHI}^{2}+$ | 26.4 | 0.671 | 0.793 | 0.725 | 0.669 | 0.834 | 0.526 | 0.524 | 0.750 | 5.3 | 13.3 |
| MRMR+ | 26.2 | 0.668 | 0.792 | 0.731 | 0.674 | 0.824 | 0.544 | 0.530 | 0.758 | 4.3 | 11.0 |
| CMIM+ | 27.0 | 0.669 | 0.795 | 0.736 | 0.682 | 0.827 | 0.564 | 0.523 | 0.764 | 3.5 | 9.2 |
| Elastic Net+ | 28.1 | 0.673 | 0.802 | 0.733 | 0.683 | 0.841 | 0.536 | 0.536 | 0.754 | 2.7 | 8.7 |
| Lasso+ | 27.7 | 0.671 | 0.803 | 0.740 | 0.689 | 0.840 | 0.547 | 0.515 | 0.758 | 2.6 | 8.5 |
| Unconstrained: |  |  |  |  |  |  |  |  |  |  |  |
| MI | 28.3 | 0.728 | 0.799 | 0.734 | 0.667 | 0.853 | 0.516 | 0.523 | 0.751 | 5.7 | 11.0 |
| JMI | 28.1 | 0.729 | 0.801 | 0.737 | 0.667 | 0.838 | 0.519 | 0.527 | 0.752 | 5.4 | 10.4 |
| $\mathrm{CHI}^{2}$ | 27.6 | 0.730 | 0.803 | 0.733 | 0.668 | 0.838 | 0.527 | 0.525 | 0.753 | 5.3 | 10.1 |
| MRMR | 27.9 | 0.729 | 0.808 | 0.741 | 0.676 | 0.843 | 0.548 | 0.532 | 0.756 | 3.6 | 6.7 |
| Lasso | 29.0 | 0.734 | 0.825 | 0.748 | 0.692 | 0.844 | 0.532 | 0.499 | 0.763 | 2.9 | 6.4 |
| CMIM | 28.4 | 0.729 | 0.810 | 0.746 | 0.678 | 0.845 | 0.562 | 0.525 | 0.764 | 2.7 | 5.7 |
| Elastic Net | 29.3 | 0.732 | 0.818 | 0.741 | 0.681 | 0.857 | 0.538 | 0.537 | 0.761 | 2.4 | 5.4 |
| Baseline linear classifiers: |  |  |  |  |  |  |  |  |  |  |  |
| L-logit R- $\ell_{2}$ | - | 0.782 | 0.902 | 0.789 | 0.734 | 0.858 | 0.598 | 0.550 | 0.911 | 1.8 | 2.7 |
| $\mathrm{L}-\ell_{1} \mathrm{R}-\ell_{2}$ | - | 0.776 | 0.907 | 0.828 | 0.777 | 0.888 | 0.607 | 0.596 | 0.915 | 1.2 | 1.2 |



Fig. 2. Performance (macro-averaged $F_{1}$-score and running time) across different training set sizes for the AG's News dataset.
topic models (we refer to this method as MSZ). For each topic, the MSZ method selects features that-when used as conditioning variables-approximate the multinomial distribution of features within the topic and also discriminate the topic from other topics. The score is further adjusted to remove redundancy between selected features [50]. The method requires parameter choices for controlling the tradeoff between approximation and discrimination $(\mu)$ and for lowering the score of redundant features $(\lambda)$. The number of features to select for each topic is left to the user.

We use the MSZ scoring to generate candidate feature
subsets with up to 50 features per cluster and follow the same procedure used in the previous section to select the best performing subset using BIC. The main drawback of the MSZ method is that it estimates a multinomial distribution for each candidate feature, which has a computational complexity of $\mathcal{O}\left(N^{2}\right)$. This is further exacerbated with redundancy elimination, as each candidate feature must be compared to features already included in the set by the divergence between their multinomial distributions. For scalability, we firstly find features (from those with at least 5 occurrences) with the highest score for each target cluster and keep only the top 250 when performing the redundancy elimination (the same approach used for the forward selection models). We use the parameter choices of $\mu=1$ and $\lambda=0.5$. We also test without redundancy elimination $(\lambda=1)$ and with a higher level $(\lambda=0.2)$, but the performance is worse for both cases.

We assess how well the selected features subsets can predict cluster membership in terms of the $\mathrm{F}_{1}$ score, and normalized mutual information is used to assess the correspondence between the classifier predictions and the cluster assignments. The results for CMIM+ and MSZ are detailed in Table 4. Across the datasets, CMIM+ consistently outperforms MSZ in terms of both $\mathrm{F}_{1}$-score and normalized mutual information (one-tailed sign-test with significance threshold of 0.1 and Bonferroni's correction for multiple testing). As mentioned, the MSZ approach is more computationally demanding: per cluster, the average computation time for feature selection for the MSZ approach is 7.09 s , which is significantly slower than the 0.34 s for CMIM + .

An example of the descriptive clustering in terms of the selected features, classifier-topic contingency table, and cluster-topic contingency table is shown in Table 5. From the contingency table, there is a clear matching between the topics and the clusters, and the features chosen by CMIM + corroborate this. Assuming a user is interested in one of the

TABLE 4
Cluster classification performance: $\mathrm{F}_{1}(Y, \hat{Y})$ —macro-averaged $\mathrm{F}_{1}$ and $\mathrm{NMI}(Y, \hat{Y})$ —normalized mutual information between classifier output and clusters. Proposed approach (CMIM+) compared to (MSZ) [18].

|  |  | $\mathrm{F}_{1}(Y, \hat{Y})$ |  | NMI $(Y, \hat{Y})$ |  |
| :--- | ---: | ---: | ---: | ---: | :--- |
| Number of clusters $C$ |  | CMIM+ | MSZ | CMIM + | MSZ |
| Reviews (topic): |  |  |  |  |  |
| m k | $\mathbf{0 . 9 2}$ | 0.90 | $\mathbf{0 . 6 4}$ | $\mathbf{0 . 6 4}$ |  |
| m b | 6 | $\mathbf{0 . 5 4}$ | 0.52 | 0.26 | $\mathbf{0 . 2 7}$ |
| m e | 2 | $\mathbf{0 . 9 4}$ | $\mathbf{0 . 9 4}$ | 0.66 | $\mathbf{0 . 6 7}$ |
| k e | 9 | $\mathbf{0 . 6 4}$ | 0.61 | $\mathbf{0 . 3 5}$ | 0.33 |
| d e | 8 | $\mathbf{0 . 6 4}$ | 0.62 | $\mathbf{0 . 3 7}$ | $\mathbf{0 . 3 7}$ |
| m b e k | 4 | $\mathbf{0 . 8 0}$ | 0.79 | $\mathbf{0 . 5 2}$ | 0.49 |
| d m e k | 4 | $\mathbf{0 . 7 4}$ | 0.70 | $\mathbf{0 . 4 3}$ | 0.40 |
| d b m k | 4 | $\mathbf{0 . 7 3}$ | $\mathbf{0 . 7 3}$ | $\mathbf{0 . 4 0}$ | $\mathbf{0 . 4 0}$ |
| d b e m | 5 | 0.63 | $\mathbf{0 . 6 5}$ | 0.33 | $\mathbf{0 . 3 6}$ |
| d b e m k | 5 | $\mathbf{0 . 7 2}$ | 0.70 | $\mathbf{0 . 4 3}$ | 0.42 |
| 20-News (subsets): |  |  |  |  |  |
| med guns | 8 | $\mathbf{0 . 6 6}$ | 0.58 | $\mathbf{0 . 3 7}$ | 0.35 |
| autos space hardware | 8 | $\mathbf{0 . 6 2}$ | 0.54 | $\mathbf{0 . 3 7}$ | 0.16 |
| rec | 8 | $\mathbf{0 . 5 9}$ | 0.50 | $\mathbf{0 . 3 6}$ | 0.16 |
| sci | 10 | $\mathbf{0 . 6 3}$ | 0.56 | $\mathbf{0 . 3 5}$ | 0.16 |
| mix | 8 | $\mathbf{0 . 6 7}$ | 0.56 | $\mathbf{0 . 4 1}$ | 0.37 |
| comp | 11 | $\mathbf{0 . 5 0}$ | 0.42 | $\mathbf{0 . 2 5}$ | 0.07 |
| mix2 | 8 | $\mathbf{0 . 7 2}$ | 0.67 | $\mathbf{0 . 4 5}$ | 0.21 |
| sci comp | 8 | $\mathbf{0 . 6 7}$ | 0.56 | $\mathbf{0 . 3 5}$ | 0.09 |
| sci comp rec | 9 | $\mathbf{0 . 6 5}$ | 0.54 | $\mathbf{0 . 3 9}$ | 0.16 |
| 20-News (all) | 15 | $\mathbf{0 . 7 4}$ | 0.72 | $\mathbf{0 . 5 6}$ | 0.54 |
| Reuters | 13 | 0.74 | $\mathbf{0 . 7 6}$ | 0.60 | $\mathbf{0 . 6 6}$ |
| Recipe | 21 | $\mathbf{0 . 6 8}$ | 0.67 | $\mathbf{0 . 4 9}$ | 0.48 |
| NSF | 14 | $\mathbf{0 . 7 5}$ | 0.73 | $\mathbf{0 . 5 4}$ | 0.52 |
| AG's news | 26 | $\mathbf{0 . 7 4}$ | $\mathbf{0 . 7 4}$ | $\mathbf{0 . 5 5}$ | $\mathbf{0 . 5 5}$ |

original topics, the selected features appear to be sufficient to guide the user to a specific cluster.

### 4.5.2 Selecting the number of clusters

We turn our attention to evaluating the automatic selection of the number of clusters in the PDC framework. Ideally, the selected number of clusters would maximize the information carried by the clustering and classifiers about the topics. To test this, we run an experiment wherein we train a classifier for every possible number of clusters and evaluate the information between the classifier predictions and the ground-truth topics. We find that the selected number of clusters often maximizes the amount of information carried about the original clusters and that the selected number varies proportionally to the ideal number.

Three examples with varying number of ground-truth topics are show in Fig. 3. On these examples, the automatically selected number (based on Eq. 9) of clusters adapts to the data, matching the ground-truth number of clusters on the first two examples and choosing a reasonable but smaller number of clusters on the full 20 Newsgroup dataset. The optimal number of clusters may be different from the number of ground-truth categories, since the categories themselves may be too coarse or too fine.

As a surrogate baseline, we use an oracle to select $C$ that maximizes the normalized mutual information between the topics and the clusters and another oracle that maximizes the normalized mutual information between the cluster and the classifier output. The performance of the selection process (based on Eq. 9) compared to the two oracles is in Table 6. We test the hypotheses that the selected number of clusters using either Eq. (9) or Eq. (12)

TABLE 5
Selected features, classifier-topic contingency table, and cluster-topic contingency table for descriptive clustering on the dvd, movie, electronics, kitchen reviews dataset. The number of clusters was chosen automatically to maximize the prediction of the original feature occurrences in the training set. (A) Contingency table between topics and original clusters. (CMIM+) Features selected using CMIM with positivity constraints, and classifier-topic contingency table. (MSZ) Features selected using the MSZ approach [18]. (Features common between multiple clusters are underlined. The number of unassigned instances per topic are listed in the last column.)

| (A) | Cluster 1 | Cluster 2 | Cluster 3 | Cluster 4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| dvd | 164 | 986 | 596 | 23 |  |
| movie | 1 | 19 | 1760 | 0 |  |
| electronics | 487 | 45 | 4 | 1196 |  |
| kitchen | 1626 | 9 | 3 | 81 |  |
| (CMIM+) |  |  |  |  |  |
| amazon, baking, blender, clean, coffee, cooking, counter, cream, cuisinart, dishwasher, easy, grill, heat, item, kitchen, kitchenaid, knives, le, maker, oven, pan, plastic, pot, price, product, products, sheets, shipping, stainless |  | concert, documentary, dvd, dvds, edition, episode, fan, fantastic, loved, movie, videos, watch, watched, workout | action, actors, character, characters, director, film, james, life, movie, people, played, plays, plot, real, scene, story, world | adapter, batteries, battery, cable, headphones, ipod, laptop, logitech, mp, pc, plug, software, sound, speakers, support, unit, usb, volume, wireless |  |
| dvd | 37.7 | 435.7 | 858.2 | 66.5 | 371 |
| movie | 5.7 | 36.2 | 1671.7 | 14.5 | 52 |
| electronics | 810.8 | 57.3 | 13.6 | 413.4 | 437 |
| kitchen | 86.3 | 9.0 | 5.3 | 1144.3 | 474 |
| (MSZ) |  |  |  |  |  |
| amazon, bought, buy, clean, cleaning, coffee, cooking, counter, cuisinart, cup, dishwasher, easy, item, kitchen, large, lid, medium, months, oven, pan, plastic, pot, price, product, products, purchased, replacement, stainless, warranty, water |  | dvd, movie, watch | character, characters, comic, director, film, films, finds, played, plays, relationship, scene, sense, story, threatening, world, young | batteries, battery, bought, headphones, ipod, mp, pc, sound, speakers, unit, volume |  |
| dvd | 29.1 | 486.7 | 758.4 | 88.7 | 406 |
| movie | 2.0 | 71.3 | 1641.8 | 13.8 | 51 |
| electronics | 515.3 | 53.7 | 17.0 | 575.0 | 571 |
| kitchen | 75.0 | 3.5 | 3.0 | 1136.5 | 501 |

are positively correlated with the number selected by the oracles, and whether the optimized numbers for the two measures are correlated. We use a significance threshold of 0.05 with Bonferroni's correction, and find that the rank correlation is significant for both oracles and measures:

|  | Rank |  |
| :--- | :---: | ---: |
| correcelation | p-value |  |
| Eq. (9) - Oracle: NMI(Topic, Cluster) | 0.5943 | $\mathbf{0 . 0 0 5 5}$ |
| Eq. (9) - Oracle: NMI(Cluster, Classifier) | 0.6123 | $\mathbf{0 . 0 0 3 7}$ |
| Eq. (12) - Oracle: NMITTopic, Cluster) | 0.8092 | $\mathbf{4 . 2 1} \times \mathbf{1 0}^{-6}$ |
| Eq. (12) - Oracle: NMI(Cluster, Classifier) | 0.5932 | $\mathbf{0 . 0 0 5 6}$ |
| Eq. (9) - Eq. (12) | 0.8469 | $\mathbf{4 . 5 2 \times 1 \mathbf { 1 0 } ^ { - 6 }}$ |

Computing AIC (Eq. 12) across the clusterings of different size is much more efficient than the binary loss used Eq. (9). We report some example running time in Table 7. Running times and feature sets for all datasets are included in the supplementary material (available at http://ieeeexplore.ieee.org).


Fig. 3. Automatic selection of the number of clusters assessed in terms of the normalized mutual information between the topic categories and clusters; topics and classifier output; and clusters and classifier output. The selected number of clusters ( 2,5 , and 15 ) are chosen to minimize the feature prediction error Eq. (9), which correlates with Eq. (12)-the AIC for a multinomial model of features occurring in at least 5 instances.

TABLE 6
Normalized mutual information (NMI) between true topic categories and clusters, and between clusters and classifier predictions, for different numbers of clusters. The proposed prediction-based selection $C^{\star}$ (based on Eq. 9) is compared to an oracle that maximizes the NMI for each case. The selected number of clusters is in parentheses.

|  | NMI(Topic,Cluster) |  |  | NMI(Cluster,Classifier) |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
|  | Oracle | $C^{\star}$ | Oracle | $C^{\star}$ |  |
| Reviews (topic): |  |  |  |  |  |
| m k | $0.97(2)$ | $0.97(2)$ | $0.63(2)$ | $0.63(2)$ |  |
| m b | $0.83(2)$ | $0.31(6)$ | $0.51(2)$ | $0.25(6)$ |  |
| m e | $0.98(2)$ | $0.98(2)$ | $0.66(2)$ | $0.66(2)$ |  |
| k e | $0.41(2)$ | $0.14(9)$ | $0.35(17)$ | $0.33(9)$ |  |
| d e | $0.54(3)$ | $0.25(8)$ | $0.39(5)$ | $0.34(8)$ |  |
| m b e k | $0.73(4)$ | $0.73(4)$ | $0.53(4)$ | $0.53(4)$ |  |
| d m e k | $0.60(4)$ | $0.60(4)$ | $0.43(6)$ | $0.43(4)$ |  |
| d b m k | $0.64(4)$ | $0.64(4)$ | $0.40(5)$ | $0.39(4)$ |  |
| d b e m | $0.56(4)$ | $0.53(5)$ | $0.37(4)$ | $0.33(5)$ |  |
| d b e m k | $0.63(5)$ | $0.63(5)$ | $0.43(5)$ | $0.43(5)$ |  |
| 20-News (subsets): |  |  |  |  |  |
| med guns | $0.48(2)$ | $0.23(8)$ | $0.40(11)$ | $0.38(8)$ |  |
| autos space hardware | $0.63(3)$ | $0.32(8)$ | $0.41(3)$ | $0.34(8)$ |  |
| rec | $0.54(4)$ | $0.43(8)$ | $0.36(6)$ | $0.34(8)$ |  |
| sci | $0.40(5)$ | $0.33(10)$ | $0.35(11)$ | $0.35(10)$ |  |
| mix | $0.61(5)$ | $0.51(8)$ | $0.43(6)$ | $0.42(8)$ |  |
| comp | $0.32(5)$ | $0.20(11)$ | $0.29(7)$ | $0.22(11)$ |  |
| mix2 | $0.69(5)$ | $0.54(8)$ | $0.47(9)$ | $0.47(8)$ |  |
| sci comp | $0.41(8)$ | $0.41(8)$ | $0.34(8)$ | $0.34(8)$ |  |
| sci comp rec | $0.54(12)$ | $0.47(9)$ | $0.41(7)$ | $0.39(9)$ |  |
| 20-News (all) | $0.64(21)$ | $0.55(15)$ | $0.57(9)$ | $0.55(15)$ |  |
| Reuters | $0.62(4)$ | $0.46(13)$ | $0.64(4)$ | $0.50(13)$ |  |
| Recipe | $0.28(9)$ | $0.26(21)$ | $0.51(5)$ | $0.42(21)$ |  |
| NSF | $0.44(23)$ | $0.39(14)$ | $0.51(8)$ | $0.49(14)$ |  |
| AG's news | $0.44(5)$ | $0.22(26)$ | $0.51(23)$ | $0.49(26)$ |  |

TABLE 7
Computation time for evaluating clusterings of different sizes.

|  | Reuters | Recipe | NSF | AG's news |
| :--- | ---: | ---: | ---: | ---: |
| Eq. (9) | 180.1 s | 219.0 s | 968.4 s | 1832.3 s |
| Eq. (12) | 0.5 s | 4.3 s | 34.0 s | 13.9 s |

## 5 Discussion

The self-tuned nature of the proposed descriptive document clustering enables it to select both the number of clusters and the number of features used to describe each cluster; nonetheless, a user must choose a suitable range that is both computationally feasible and appropriate for the datasets. In our tests, the upper limit was only met once on the largest
dataset (AG's news). In such a case, a user may wish to increase the range to find a more optimal clustering.

In addition to text, the proposed descriptive clustering can be applied to any data with sparse count-valued features. Example experiments are included in the supplementary material (available at http:/ /ieeeexplore.ieee.org).

## 6 Conclusion

We posed descriptive clustering as two coupled prediction tasks: 1) choosing a clustering that is predictive of the features and 2) predicting the cluster assignment from a subset of features. Using predictive performance as the objective criterion, the parameters of descriptive clusteringthe number of clusters and the number of features per cluster-are chosen by model selection. With the resulting solution, each cluster is described by a minimal subset of features necessary to predict whether an instance belongs to the cluster. Our hypothesis is that a user will also be able to predict the cluster membership of documents using the descriptive features selected by the algorithm. Given some relevancy requirements, a user can then quickly locate clusters likely to contain relevant documents.

We evaluated this self-tuned approach on datasets with count-valued features. For feature selection we used both information theoretic feature selection and linear classifiers trained by logistic regression with sparsity-inducing regularizations and positivity constraints. The results showed that these feature selection approaches perform accurately and yield feature subsets that are indicative of the cluster content. Furthermore, the predictive approach selected a meaningful terms of number of clusters and number of features per cluster. This objective self-tuning distinguishes the proposed framework from previous descriptive clustering approaches that have not addressed parameter selection and have relied on subjective criterion for assessing descriptors.

Currently, we have begun using the PDC framework to generate descriptive clustering to help users screen large collections of abstracts to support the development of systematic reviews, especially in domains such as public health. ${ }^{14}$ For future work, we plan to investigate PDC using more complex features including multi-word expressions, named entities, and clusters of features themselves.

[^2]
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# Self-Tuned Descriptive Document Clustering using a Predictive Network 

Supplemental Material

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This supplement contains additional results on the datasets used in the main paper (Appendix 1) and results for other datasets not included in the main manuscript in Appendix 2.

## Appendix 1 Additional Results

We detail the descriptive clustering solutions and running time for the proposed predictive-descriptive clustering (PDC) algorithm across all datasets. The running times are given in Table 1 and the features selected for each cluster are given in the tables on the subsequent pages, followed by the contingency tables showing the correspondence between the clusters and the ground truth topics.

The running time includes generating candidate clusterings, cluster size selection, generating feature subsets for each cluster of the selected clustering, and selecting the best feature subset. The clusterings ranging from 2 to 26 clusters are generated using Ng , Jordan, and Weiss's spectral clustering algorithm [1] on an implicit similarity matrix formed via cosine similarity between normalized term-frequency inverse document frequency weighted vectors [2]. The algorithm firstly uses a truncated eigendecomposition of the normalized similarity matrix. As the eigendecomposition only requires matrix-vector multiplication, it can be performed as a series of multiplications between sparse or diagonal matrices and a vector-avoiding the explicit formation of the dense $n \times n$ similarity matrix. After the eigendecomposition, the first $C$ eigenvectors are used as input (after normalizing the vector for each instance) to the standard k-means algorithm with $k=C$. This is repeated for $C \in\{2,3, \ldots, 26\}$.

For cluster size selection, we use Akaike information criterion (AIC) to select the number of clusters based on a multinomial model of features that occur in at least 5 instances. The multinomial distribution is estimated with additive smoothing by adding 1 to each count.

For feature selection, we use the CMIM+ (conditional mutual information maximization) algorithm [3] with additional positivity constraints on binarized feature vectors (that is, feature occurrence is used instead of feature count). The number of features per cluster is chosen by the Bayesian
information criterion (BIC) where the likelihood is based on a $\ell_{2}$-regularized logistic regression model, which is fit in Liblinear [4] with input arguments -s 0 -c $10-\mathrm{B} 1$ (primal optimization, $\tau=\frac{1}{10}$, and included bias).

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TABLE 1
Dataset profiles, selected cluster size, and total running time as logged in MATLAB on Mac OS X with a 2.8 GHz Intel Core i7 and 16 GB RAM.

|  | Classes | Instances $n$ | Features $N$ | Clusters $C$ | Time (s) |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Reviews (+/-): |  |  |  |  |  |
| movie kitchen | 2 | 4000 | 25543 | 2 | 4.22 s |
| movie books | 2 | 4000 | 27436 | 2 | 4.70 s |
| movie electronics | 2 | 4000 | 25778 | 2 | 3.23 s |
| kitchen electronics | 2 | 4000 | 7786 | 4 | 3.24 s |
| dvd electronics | 2 | 4000 | 13048 | 4 | 3.23 s |
| movie books electronics kitchen | 4 | 8000 | 30079 | 4 | 5.49 s |
| dvd movie electronics kitchen | 4 | 8000 | 29552 | 3 | 5.95 s |
| dvd books movie kitchen | 4 | 8000 | 31101 | 3 | 7.32 s |
| dvd books electronics movie | 4 | 8000 | 31299 | 3 | 6.83 s |
| dvd books electronics movie kitchen | 5 | 10000 | 32375 | 4 | 8.25 s |
| 20-News (subsets): |  |  |  |  |  |
| med guns | 2 | 1896 | 61188 | 4 | 2.37 s |
| autos space hardware | 3 | 2951 | 61188 | 4 | 3.53 s |
| comp | 5 | 4852 | 61188 | 10 | 7.96 s |
| rec | 4 | 3968 | 61188 | 7 | 6.10 s |
| sci | 4 | 3945 | 61188 | 5 | 5.34 s |
| mix2 | 5 | 4913 | 61188 | 6 | 6.64 s |
| mix | 5 | 4677 | 61188 | 5 | 5.99 s |
| sci comp | 9 | 8797 | 61188 | 10 | 15.20 s |
| sci comp rec | 13 | 12765 | 61188 | 11 | 20.81 s |
| 20-News (all) | 20 | 18820 | 41747 | 12 | 31.36 s |
| Reuters | 8 | 7674 | 10250 | 10 | 9.65 s |
| Recipe | 20 | 39774 | 10506 | 21 | 118.53 s |
| NSF | 35 | 125730 | 119773 | 23 | 387.60 s |
| AG's news | 4 | 127600 | 39763 | 26 | 268.67 s |

TABLE 2
Reviews: movie kitchen. Descriptive features selected for each cluster sorted by coefficient value.
1: film, movie, story, films, life, characters, movies, director, character, scene, man, comedy, plot, young, action, world, scenes, funny, role, john, plays, script, audience, screen, played, acting, actors, end, people
2: bought, product, buy, clean, kitchen, pan, plastic, purchased, coffee, amazon, item, size, unit, price, dishwasher, ordered, sheets, kitchenaid, oven, sturdy, knives, lid, blender, pot, heat, cups, stainless, purchasing, purchase, cooking, toaster, dishes, rust, cuisinart, oxo, shipping, counter, cook, durable, warranty, grill, products, baking, cookware, pots, heating, calphalon

TABLE 3
Reviews: movie books. Descriptive features selected for each cluster sorted by coefficient value.
1: film, movie, films, director, comedy, scene, movies, script, cast, screen, watch, audience, scenes, plays, man, black, performance, actors, role, action, young, wife, character, played, performances, acting, john, moments, hollywood, actor, minutes, comic, head, big, star, funny, plot, watching, bad 2: book, read, books, reading, author, reader, quot, authors, recipes, pages, volume, page, readers

## TABLE 4

## Reviews: movie electronics. Descriptive features selected for each cluster sorted by coefficient value.

1: film, movie, story, films, man, character, director, characters, scene, comedy, plot, action, life, young, role, scenes, john, movies, cast, funny, script, world, audience, plays, acting, girl, sex, played, actors, hollywood, interesting, people, watch
2: bought, product, quality, buy, unit, price, ipod, purchased, headphones, battery, card, amazon, software, sony, purchase, button, cord, pc, volume, batteries, printer, cable, customer, cd, wireless, keyboard, item, laptop, products, speakers, usb, mp, plug, headset, router, install, speaker, hp, signal, gb, panasonic, cables, plugged, logitech, xp , adapter, defective, sandisk, warranty, charger

TABLE 5
Reviews: kitchen electronics. Descriptive features selected for each cluster sorted by coefficient value.
1: pan, knife, sheets, kitchen, oven, dishes, cooking, set, dishwasher, cutting, knives, cake, handle, thin, grill, rust, clean, sturdy, handles, pans, bowls, wash, food, beautiful, family, plates, love, pieces, baking, pizza, stick, cook, toaster, skillet, surface, cookware, heat, bread, evenly, chicken, steel, meat, heavy, cleans, utensils, vegetables, calphalon, bake, cooks
2: water, coffee, maker, ice, cup, machine, tea, filter, blender, grinder, grind, cooker, motor, broke, drink, liquid, plastic, fill, years, drinks, pour, milk, time, cuisinart, lid, hot, pot, cups
3: card, camera, printer, work, windows, hp, drive, amazon, memory, item, canon, data, garmin, information, failed, days, gb, product, software, computer, cards, service, network, router, photos, worked, support, link, sandisk, customer, firmware, gps, pictures, problems, connection, wireless, site, laptop, refund, usb, pc, device, linksys, company, working, drivers, mb, install, download, ethernet
4: sound, ipod, phone, headphones, ear, dvd, quality, speakers, logitech, reception, speaker, radio, cable, noise, headset, bass, hear, remote, audio, screen, low, cables, price, ears, batteries, stations, monster, volume, picture, music, cord, system, phones, player, fm, battery, listen, static, component, interference, microphone, station, power, tv, headphone, earphones, stereo, panasonic, listening, sony

TABLE 6

## Reviews: dvd electronics. Descriptive features selected for each cluster sorted by coefficient value.

1: movie, film, story, acting, man, characters, people, guy, script, comedy, men, plot, role, character, films, funny, young, movies, life, ending, girl, book, women, real, watch, cast, scene, interesting, sex, director, scenes, actor, actors, boring, predictable
2: dvd, season, video, collection, show, videos, episodes, music, elmo, workout, seasons, series, disc, dvds, song
3: ipod, price, sound, quality, headphones, speakers, printer, cable, remote, ink, noise, ears, bass, ear, monster, player, radio, product, difference, cd, stations, cables, cds, nano, bought, portable, expensive, cheaper, fm , mp , earphones, cost, sony, speaker, stereo, bose, listen
4: phone, card, mouse, keyboard, wireless, memory, router, worked, sandisk, work, data, customer, unit, support, laptop, works, software, batteries, item, computer, gps, windows, linksys, hardware, garmin, problem, pda, device, usb, buttons, months, products, computers, pc, netgear, cordless, working, battery, install, download, contacted, month, problems, product, bought

TABLE 7
Reviews: movie books electronics kitchen. Descriptive features selected for each cluster sorted by coefficient value.
1: film, movie, films, director, scene, comedy, movies, man, script, character, action, john, role, young, scenes, audience, plays, cast, funny, actors, acting, watch, played, hollywood, actor, plot, characters, story
2: book, read, books, reading, author, history, writing, authors, pages, text, information, readers, page, reader, quot, language, analysis, chapter, novels, library, bias, enjoyed, essay, chapters, illustrations
3: ipod, card, sound, phone, battery, headphones, product, printer, cable, worked, dvd, software, laptop, usb, gps, pc, sony, keyboard, wireless, mp, batteries, amazon, speakers, hp, sandisk, signal, router, logitech, unit, bought, price, fm, cables, adapter, gb, cord, quality, headset, button, speaker, customer, refund, plugged, xp , plug, charger, buy, purchased, purchase, warranty
4: clean, water, pan, kitchen, coffee, dishwasher, bowl, knife, sheets, handle, maker, pot, dishes, blender, heat, oven, toaster, stainless, knives, plastic, counter, cup, sturdy, lid, size, kitchenaid, cuisinart, pans, cups, cook, oxo, cooker, metal, cooking, mixer, grill, cleaning, cookware, bread, wash, container, pour, rust, baking, calphalon, evenly, cooks, pots, heating, bake

TABLE 8
Reviews: dvd movie electronics kitchen. Descriptive features selected for each cluster sorted by coefficient value.
1: movie, film, story, characters, films, funny, watch, character, comedy, man, director, young, scene, plot, show, scenes, acting, movies, world, life, guy, boring, role, series, shes, script, humor, people, action, interesting, men, girl, cast, woman, fun, played, plays, actors, audience, hollywood, actor, sex, watching, end 2: sound, ipod, card, software, battery, product, phone, headphones, printer, cable, worked, cd, laptop, usb, batteries, keyboard, pc, sony, amazon, gps, wireless, bought, mp, hp, quality, speakers, gb, router, signal, buy, sandisk, unit, logitech, price, cables, adapter, cord, fm, headset, volume, customer, speaker, xp, refund, purchased, plugged, purchase, plug, charger, item
3: water, pan, coffee, kitchen, clean, sheets, dishwasher, bowl, handle, knife, dishes, maker, pot, oven, blender, kitchenaid, easy, toaster, plastic, cook, stainless, heat, food, cooking, knives, size, lid, sturdy, cup, oxo, counter, metal, cuisinart, pans, bread, rust, cooker, cups, baking, cleaning, grill, cookware, pour, container, calphalon, evenly, pots, cooks, heating, bake

TABLE 9
Reviews: dvd books movie kitchen. Descriptive features selected for each cluster sorted by coefficient value.
1: book, read, books, reading, author, reader, authors, page, pages, text, writing, information, research, readers, quot, israel, analysis, chapter, published, bias, library, data, novels, illustrations, chapters
2: film, movie, films, director, movies, comedy, watch, dvd, scene, music, script, show, video, scenes, plays, acting, young, funny, cast, man, actors, performance, played, watched, camera, action, performances, role, guy, hollywood, screen, plot, hilarious, tv, special, star, character, actor, watching, play, story, characters 3: product, clean, kitchen, plastic, pan, water, item, handle, coffee, size, sheets, unit, knives, bowl, dishwasher, bought, purchased, maker, sturdy, toaster, kitchenaid, pot, oven, blender, dishes, price, amazon, ordered, lid, rust, cups, stainless, counter, pans, heat, replacement, cleaning, cuisinart, shipping, products, replace, cooking, purchase, cook, warranty, cookware, buy, baking, heating

TABLE 10
Reviews: dvd books electronics movie. Descriptive features selected for each cluster sorted by coefficient value.
1: book, read, books, author, reading, recipes, authors, writing, research, pages, text, readers, reader, practical, page, israel, analysis, quot, concepts, library, bias, novels, chapter, resource, illustrations, chapters
2: film, movie, films, director, comedy, cast, scene, watch, movies, script, acting, scenes, show, young, performances, plays, watched, man, funny, audience, action, played, actors, guy, role, hollywood, girl, hilarious, plot, actor, character, star, love, watching, characters, story
3: product, quality, unit, ipod, bought, price, card, sound, headphones, printer, battery, phone, cable, sony, worked, purchased, button, item, batteries, keyboard, laptop, software, purchase, buy, pc, wireless, usb, cord, customer, amazon, signal, brand, plug, mp, logitech, router, speakers, gb, install, sandisk, fm, cd, plugged, headset, adapter, panasonic, warranty, speaker, $x p$, charger

TABLE 11
Reviews: dvd books electronics movie kitchen. Descriptive features selected for each cluster sorted by coefficient value.
1: book, read, books, reading, author, history, writing, authors, pages, page, readers, research, literature, text, reader, analysis, israel, quot, chapter, novels, published, historical, library, bias, illustrations, chapters
2: film, movie, films, director, comedy, scene, watch, movies, acting, script, cast, show, scenes, performances, young, watched, funny, plays, man, audience, played, role, actors, action, guy, girl, hollywood, actor, woman, guys, humor, plot, hilarious, moments, watching, character, effects, end, fun, series, star, story, dialogue, people, characters
3: sound, ipod, card, battery, phone, product, cable, headphones, printer, software, worked, laptop, sony, usb, amazon, quality, gps, batteries, wireless, pc, keyboard, speakers, bought, mp, router, hp, signal, price, unit, cd, buttons, sandisk, cord, buy, gb, adapter, fm, logitech, headset, speaker, purchased, customer, plugged, xp , plug, purchase, refund, item, charger, warranty
4: water, pan, kitchen, coffee, clean, handle, knife, bowl, maker, dishwasher, pot, kitchenaid, dishes, sheets, blender, oven, stainless, toaster, plastic, heat, easy, knives, sturdy, size, lid, cup, cuisinart, counter, metal, pans, oxo, cook, cooker, cups, cleaning, grill, cookware, food, cooking, wash, rust, container, pour, baking, bread, calphalon, evenly, cooks, heating, bake

TABLE 12
20-News: med guns. Descriptive features selected for each cluster sorted by coefficient value.
1: msg, food, dyer, photography, stuff, uga, sauce, foods, amateur, leung, kirlian, urinals, allergic, taste, unlv, corona, linus, sol, aura, plates, processing 2: banks, medicine, treatment, thanks, disease, health, doctor, patients, information, infection, physician, pain, dr, surgery, patient, weight, russell, medical, symptoms, sci, condition, chronic, cancer, medication, diagnosed, treatments, hospital, doctors, diagnosis
3: gun, guns, killed, weapons, police, government, crime, waco, senate, ufl, federal, firearms, military, rights, veal, nra, state, laws, firearm, against, law, citizens, amendment
4: cdt, fbi, brent, fire, gas, handheld, koresh, survivors, chopin, inside, bastard, bd, atf

TABLE 13
20-News: autos space hardware. Descriptive features selected for each cluster sorted by coefficient value.
1: thanks, card, pc, please, dos, monitor, bios, modem, port, windows, dx, disk, gateway, cpu, drive, motherboard, computer, board, scsi, chip, mb, hi, machine, cards, video, drives, pin, cache, ide, floppy, controller, mhz, bus, meg, ram, vlb, isa, os
2: car, cars, engine, ford, oil, insurance, honda, magnus, auto, bmw, eliot, police, wheel, rear, nissan, front, tires, driving, toyota, vw, transmission, mustang, wagon, clutch, autos, gear, drew
3: jbh, mccall, nick, sky, calpoly, bby, wats, rights, pgf, world, theory, war, steinn, energy, known, human, pat, rays, oort, idea, miner, universe, nature, we, writes, they, given, things, bursts, see, earth, space
4: nasa, space, henry, shuttle, spacecraft, science, orbit, jupiter, solar, nsmca, satellite, astronomy, propulsion, project, zoo, pat, russian, mission, earth, moon, launch, sun, flight, military, launched, development, lunar, funding, high, mars, missions, orbital, satellites

TABLE 14
20-News: comp. Descriptive features selected for each cluster sorted by coefficient value.
1: ac, navy, group, uk, graphics, comp, xv, subscribe, proposed, chris
2: windows, andre, zip, cica, file, files, biochemistry, ftp, directory, alec, bmp, shareware, dos
3: windows, dos, win, it, mouse, crash, ms, installed, when, writes, manager, apps, microsoft, ndw, os, load, emm, autoexec, ini, my, run, exe, rmohns, problem, disk, loaded, sys
4: thanks, please, firstclass, mail, email, where, information, looking, utas, advance, hpgl, dxf, anybody, hi, pcx, gif, skiing, rpicas
5: card, bus, diamond, dx, video, eisa, vesa, ati, drivers, mode, driver, cards, vlb, isa, board, svga, mb, local, bios
6: image, points, objects, sphere, science, images, sun, source, motif, we, public, package, simple, code, convex, create, bezier, books, color, algorithm, surface,
hidden, human, looking, vector, convert, tar, postscript, form, tool, graphs, header, integrate, graphics, lines, ray, information, such
7: apple, they, edu, life, power, cnn, hacker, modem, just, modems, price, months, computers, re, macs, mac, powerbook, money, battery, dealer, years, day, that, what, duo, he, selling, buy, pluses, heat, electricity, cable
8: drive, scsi, drives, disk, hd, hard, controller, ide, slave, seagate, mb, quantum
9: monitor, simms, cpu, iisi, clock, nubus, mhz, pentium, processor, centris, fpu, performance, simm, lc, board, slot, chips, chip, motherboard, price, vram, lciii, video, quadra, apple, slots
10: window, server, xterm, widget, lib, int, usr, openwindows, motif, colormap, event, application, return, set, default, displayed, events, ld, function, xdm, following, foreground, cursor, null, button, sun, xlib, gc, error, endif, compile, string, pointer, false, pixmap, olwm, sunos, display, mit, trying, xt, problem, clients, undefined, override, running, code, when, using

TABLE 15
20-News: rec. Descriptive features selected for each cluster sorted by coefficient value.

| 1: car, cars, thanks, engine, ford, eliot, engines, callison, dealer, geico, price, toyota, nissan, autos, vw, auto, models, trunk, turbo, door, mustang, wagon, model, <br> sedan, transmission, audi |
| :--- |
| 2: my, shop, fit, bmw, police, interstate, gently, roads, vehicle, cop, lane, killed, tossing, bikes, lock, tank, driving, gear, gas, kawasaki, required, your, rear, |
| suggestions, bike, miles, mph, filters, oil, cops, truck, motorcycle, com, dod, ride |

TABLE 16
20-News: sci. Descriptive features selected for each cluster sorted by coefficient value.
1: key, clipper, encryption, government, nsa, security, sternlight, crypto, keys, pgp, law, des, privacy, com, eff, secret, chip, police, pmetzger, encrypted, trust, cryptography, court, netcom, algorithm, escrow, fbi, warrant, classified, rsa, secure, wiretap
2: radio, battery, car, michael, hp, plastic, georgia, tone, heat, cooling, wire, batteries, ai, use, bulb, wiring, metal, wd, electronics, amps, wires, covington, capacitor, phone, you, circuit, voltage, concrete
3: thanks, anyone, mb, mail, appreciated, please, am, info, looking, copy, pc, email, appreciate, video, circuit, dsp, hi, ic, motorola, screen, interface, signal, hello, mhz , inputs, advance, eprom, circuits, diode, sehari, suggestions, voltage, pin, audio, games, amp, display, rs, hc, controller, thanx, schematic, help, dos
4: geb, food, she, doctor, medicine, disease, treatment, body, skin, patients, my, blood, medical, dyer, diet, drugs, her, patient, he, eat, studies, yeast, health, symptoms, pain, cause, physician, cancer, infection, foods, effects, med, chronic, surgery, syndrome, brain, infections, doctors, severe, anecdotal, clinical, hospital, diagnosed
5: space, nasa, henry, launch, orbit, earth, flight, sky, moon, spacecraft, pat, gamma, shuttle, dseg, nsmca, cso, atmosphere, prb, rocket, solar, jupiter, man, vehicle, satellite, funding, lunar, mars, observatory, apollo, launched, hst, propulsion, landing, mission, planetary, pluto, payload, manned, orbital, missions

TABLE 17
20-News: mix2. Descriptive features selected for each cluster sorted by coefficient value.
1: windows, file, dos, os, printer, files, font, pc, card, ms, program, using, video, win, drivers, microsoft, driver, tried, disk, mouse, version, window, cica, screen, bmp, progman, machine, ftp, exe, icon, desktop, mb, ini, shareware, meg, vga, click
2: car, cars, engine, dealer, oil, ford, magnus, police, eliot, driving, rear, wheel, sc, price, toyota, auto, engines, honda, wagon, tires, bmw, gear, jeep, nissan, vw, turbo, miles, mustang, transmission, drive, autos, shifting, taurus, door, mazda, dodge, porsche, chevy, valve, drew, wheels, tranny
3: york, mail, philadelphia, thanks, thank, mailing, francisco, please, fax, appreciate, email, address, info, joseph, hello, interested, contact, rangers, angeles, regal, jtchern, ja
4: baseball, he, game, players, team, ball, season, pitcher, year, him, runs, player, his, hitter, roger, career, games, hit, pitching, teams, ted, pitchers, batting, mets, alomar, play, stats, batter, pitch, league, braves, hits, hitting, rbi, hitters, mss, sox, fans, obp, playing, pitches
5: geb, doctor, medicine, medical, treatment, effect, skin, blood, of, is, disease, food, she, body, drugs, patient, med, diet, msg, cause, patients, pain, effects, dyer, infection, drug, symptoms, health, studies, physician, allergic, foods, evidence, chronic, her, yeast, cancer, oral, severe, doctors, treat, therapy, brain, sugar, infections, diagnosed, medication, syndrome, clinical, anecdotal
6: space, pat, nasa, sky, earth, orbit, launch, henry, moon, flight, gamma, shuttle, spacecraft, atmosphere, nsmca, project, srl, satellite, solar, jupiter, government, cost, military, billion, mission, funding, propulsion, landing, zoo, lunar, mars, planetary, observatory, launched, payload, ssto, telescope, pluto, satellites, apollo, orbital, missions, altitude

TABLE 18
20-News: mix. Descriptive features selected for each cluster sorted by coefficient value.
1: windows, file, dos, files, driver, using, thanks, font, ftp, program, drivers, problem, microsoft, advance, win, anyone, tried, running, version, ms, cica, pc, window, ati, progman, utility, hi, exe, printer, ini, shareware, download
2: sale, shipping, offer, condition, asking, interested, sell, mail, email, please, price, brand, obo, contact, monitor, forsale, cd, tapes, stereo, cassette, mint, send, original, selling, external, manuals, offers, drive, sony, keyboard, controller, floppy, manual, warranty
3: baseball, he, players, team, ball, pitcher, season, mets, league, sox, cubs, game, phillies, his, fan, teams, hit, braves, jays, year, giants, career, stats, runs, fans, pitchers, hitter, pitching, yankees, last, alomar, pitch, batting, player, hitting, rbi, games, play
4: space, nasa, pat, earth, orbit, launch, shuttle, moon, henry, atmosphere, spacecraft, energy, srl, gamma, solar, flight, nsmca, sky, science, cost, jupiter, satellite, jbh, project, dseg, military, billion, astronomy, high, of, rockets, mission, lunar, gravity, vehicle, observatory, spencer, orbital, launched, landing, funding, pluto, payload, mars, planetary, satellites, propulsion, manned, altitude, that
5: people, kaldis, com, fbi, clinton, drugs, cramer, health, government, children, writes, rights, trial, you, state, men, koresh, tax, laws, fact, law, country, opinions, gay, their, sexual, not, crime, american, that, americans, themselves, population, we, liberal, by

## TABLE 19

20-News: sci comp. Descriptive features selected for each cluster sorted by coefficient value.


TABLE 20

## 20-News: sci comp rec. Descriptive features selected for each cluster sorted by coefficient value.

1: ftp, fax, format, graphics, package, program, image, convert, motif, create, code, comp, help, version, sparc, widget, print, thanks, tiff, domain, email, wireframe, pd, files, shareware, advance, packages, source, application, xlib, formats, pc, hi, postscript, viewer, software, looking, wuarchive, gif, images, unix, display, file, windows
2: windows, window, abpsoft, file, default, manager, lib, server, using, dos, sunos, ini, ms, font, exe, mouse, directory, menu, screen, works, problem, xterm, exit, use, app, os, button, program, error, config, cursor, text, apps, set, icon, microsoft, command, run, files, client, create, shell, unix, display, tried, motif, running, application, code, user
3: drive, mb, scsi, mhz, card, bus, cpu, drives, dx, mac, board, apple, disk, bios, controller, rom, cards, lc, motherboard, port, simms, external, quadra, processor, pin, eisa, pc, video, centris, cable, isa, monitor, cache, ide, system, adapter, ram, fpu, iisi, vlb, meg, jumpers, hd, memory
4: thanks, anyone, please, mail, looking, info, firstclass, email, information, appreciated, gandler, interested, thank, thanx, appreciate, anybody, hello, schematic, hi, post, arcade, advance, reply, address, am, pointers, fabled, newman, erini, any, bobc, stalnaker, responses, multisync, replies, help
5: key, clipper, encryption, sternlight, nsa, government, security, chip, keys, amendment, law, des, netcom, gtoal, crypto, eff, cryptography, privacy, pmetzger, algorithm, encrypted, pgp, speech, denning, secret, trust, clinton, phones, escrow, court, warrant, secure, classified, police, rsa, cryptographic, wiretap, feds, fbi, terrorists, public, enforcement, private, crypt
6: canada, sweden, usa, april, det, uk, germany, phone, pt, que, france, subscribe, se, university, europe, austria, nerone, fax, energieanwendung, de, internet, switzerland, munich, ga, pp, dortmund, contact, tone, list, sj , finland, la, min, czech, peter, email, ericsson, hc, mail, mtl, smail, address, petr
7: geb, amp, medicine, food, cause, that, skin, not, patient, heat, dyer, doctor, reaction, are, symptoms, usually, she, water, patients, years, treatment, about, physician, yeast, of, few, diet, people, they, doctors, blood, was, very, infection, pain, medical, effects, were, syndrome, health, as, disease, most, because 8: nasa, space, orbit, henry, launch, pat, shuttle, earth, spacecraft, energy, flight, moon, nsmca, nuclear, atmosphere, prb, sky, high, solar, fraering, project, lunar, satellite, jupiter, military, funding, rocket, propulsion, planets, billion, observatory, landing, gamma, zoo, mission, rockets, spencer, pluto, telescope, development, payload, ssto, mars, manned, orbital, satellites, planetary, missions, altitude
9: dod, car, bike, article, engine, cars, com, bmw, ride, eliot, bgardner, rear, wheel, rider, yamaha, oil, ranck, road, honda, my, gear, bikes, shaft, infante, tires, toyota, egreen, motorcycles, brake, driving, seat, front, gas, riding, gt, riders, motorcycle, gs, steering, miles, you, passenger, tire, wheels, mph, brakes, clutch
10: he, his, baseball, him, pitcher, ball, year, player, batting, career, hitting, baseman, players, pitch, pitchers, ted, hit, defensive, pitches, bonds, inning, braves, pitching, sherri, alomar, hitter, last, bat, singer, season, batters, rbi, obp, roger, stats, league, game, team, games
11: philadelphia, go, montreal, detroit, hockey, rangers, boston, division, game, blues, teams, buffalo, games, bruins, nhl, devils, play, pittsburgh, league, espn, wings, stanley, team, cup, leafs, jets, capitals, caps, goals, playoffs, fans, playoff, win, islanders, canucks, fan, pens, season

TABLE 21

## 20-News (all). Descriptive features selected for each cluster sorted by coefficient value.

1: windows, file, window, files, motif, program, server, font, xterm, widget, microsoft, comp, advance, ftp, postscript, application, openwindows, compile, code, xv, format, client, sunos, manager, gif, xview, create, xlib, directory, bitmap, unix, default, ms, version, shareware, libraries, bmp, graphics, sparc, package, image, user, menu, dos, display, running, app, screen, software
2: card, monitor, video, mhz, monitors, cpu, diamond, apple, board, dx, centris, upgrade, lc, vesa, cards, lciii, simms, vga, ati, irq, port, motherboard, memory, color, simm, mac, coprocessor, processor, iisi, mouse, mode, fpu, ram, pin, connector, pc, nubus, serial, quadra, svga, isa, hardware, bios, ports, computer, vlb, vram, slots, bus, mb
3: drive, scsi, drives, disk, hd, ide, rom, mb, controller, bios, harddisk, disks, hard, jumper, quantum, jumpers, pc, formatted, maxtor, seagate, floppy, mfm
4: sale, mail, email, price, offer, vhs, ink, condition, address, contact, interested, phone, catalog, shack, list, impedance, cassette, shipping, circuit, forsale, joysticks, wanted, sell, audio, analog, stereo, buy, modem, electronic, info, send, cd, obo, fax, oscilloscope, printer, mint, box, voltage, output, selling, cable, package, amp, software, cod, amplifier, reply, brand
5: dod, car, cars, bike, engine, bmw, riding, motorcycle, ford, bgardner, ride, road, oil, bikes, vw, gear, dealer, motorcycles, mph, nissan, driving, cb, eliot, exhaust, yamaha, wheel, porsche, brake, rider, egreen, mustang, zx , rear, countersteering, passenger, toyota, insurance, tire, seat, tires, riders, front, torque, honda, gs, clutch, wheels, steering, brakes, miles
6: team, season, game, baseball, nhl, hockey, players, year, braves, montreal, hitter, teams, ball, pitcher, league, play, mets, rangers, playoff, batting, wings, goalie, pitchers, detroit, jays, coach, playoffs, cup, phillies, pitching, player, leafs, stats, roger, sox, win, bruins, devils, games, fan, fans, scored, boston, espn, pittsburgh, penguins, scoring, pens, islanders, played
7: clipper, key, chip, algorithm, encryption, tapped, nsa, phone, security, crypto, cryptography, sternlight, keys, des, pgp, denning, secret, encrypted, wiretap, privacy, eff, cryptographic, rsa, government, secure, escrow, crypt
8: god, atheists, jesus, bible, christian, verse, morality, lord, church, religion, christians, athos, christ, christianity, argument, catholic, atheism, faith, sin, atheist, benedikt, heaven, moral, beliefs, rutgers, holy, clh, truth, doctrine, verses, sandvik, gospel, biblical, scripture, belief, testament, religions, religious
9: space, nasa, orbit, moon, distance, shuttle, project, launch, henry, research, spacecraft, high, flight, temperature, earth, nuclear, gamma, solar, sci, mission, sky, energy, pat, lunar, nsmca, planets, prb, images, science, propulsion, zoo, spencer, atmosphere, satellite, observatory, ray, mars, satellites, landing, funding, orbital, ssto, jbh, payload, fraering, pluto, planetary, orbiting, missions, altitude
10: geb, cramer, article, doctor, study, kaldis, food, symptoms, health, medicine, patients, foods, sexual, pregnancy, doctors, med, treatment, skin, infection, gay, patient, medication, sex, physician, chronic, yeast, dyer, effects, studies, disease, medical, women, disorder, infections, percentage, syndrome, homosexuals, people, drugs, diet, drug, cancer
11: israel, turkish, israeli, turks, muslim, jews, arab, bosnia, war, occupied, iran, jewish, policy, countries, soldiers, nazi, peace, anti, troops, muslims, propaganda, military, genocide, civilians, israelis, political, land, palestinian, arabs, army, forces, villages, terrorism, country, occupation, population, killed, inhabitants, ethnic, government, people
12: gun, atf, waco, cdt, guns, firearms, fire, clinton, weapons, batf, koresh, fbi, gas, tax, constitution, state, country, compound, crime, article, armed, people, police, utkvm, government, death, dscomsa, federal, murder, assault, reno, rights, bd, killed, jmd, deaths, amendment, children, tanks, davidians, citizens, raid, law, arms

TABLE 22
Reuters. Descriptive features selected for each cluster sorted by coefficient value.
1: split, declared, date, dividend, shareholders, distribution, sets, payable, board, stock, paid, holders, payout, quarterly, share, record
2: div, record, pay, payout, april, qtly, prior, cts
3: loss, shr, net, qtr, revs, note, cts, oper, jan, avg, profit, mths
4: sell, unit, company, terms, undisclosed, expects, completed, buys, division, acquisition, purchase, acquire, disclosed, acquired, earnings, completes, sale, agreed, acquires, sells, subsidiary, intent, expected, equipment, business, revenues, earned, buy, announced, transaction, privately, principle, units, makes, closing, agreement, services, based, products, manufacturing, signed, systems, cash, previously, corp, owned, subject, assets, definitive, dlrs
5: offer, bid, stake, takeover, shares, merger, control, airlines, court, commission, usair, tender, received, proposal, group, buyout, caesars, filing, buy, cyclops, taft, filed, comment, voting, board, rights, investor, owns, proposed, acquire, advisor, seeking, securities, tendered, directors, outstanding, unsolicited, bought, holds, investment, stock, common, management, shareholders, shareholder, exchange, share, led, pct, company
6: trade, grain, dispute, agriculture, japanese, japan, officials, foreign, states, told, senate, ships, work, congress, house, union, tariffs, reagan, official
7: stg, billion, fed, turnover, francs, profits, compared, marks, guilders, income, loans, parent, improvement, earnings, reported, customer, pretax, forecast, year, tax, earlier, system, january, pre, ag, total, results, reserves, deposits, unchanged, extraordinary, rose, offset, repurchase, fell, profit, mln, rise, net
8: rate, bank, pct, banks, rates, prime
9: oil, crude, prices, sea, barrels, refinery, shipping, bpd, port, opec, iranian, barrel, petroleum, gulf, minister, production, shell, kuwait, iran, output
10: monetary, baker, dollar, currency, currencies, finance, exchange, rates, germany, economic, accord, nations, paris, stability

## TABLE 23

## Recipe. Descriptive features selected for each cluster sorted by coefficient value.

1: seed(s), masala, dal, cumin seed(s), turmeric, red chili, cashew(s), green chilies, curry leaf(ves), cardamom, chili(es), curry, coriander, mustard seeds, basmati, clove, cinnamon sticks, gram, chili powder, paneer, tumeric, coconut, curds, ginger, amchur, ginger paste, chickpea(s), lentil(s), asafoetida, ghee, cumin, garlic paste, leaves, cilantro leaves, coriander powder, yoghurt, fenugreek, ground cumin, cinnamon, salt, red chili powder, powder, cilantro 2: part skim, mozzarella, cheese, italian, pizza, basil, lasagna, tomato(es), bread, ricotta, pasta sauce, marinara sauce, parmesan, pasta, pesto, dough(s), frozen chopped, spinach, cottage, ground beef, oregano, tomato, tomato sauce, sauce
3: self, vegetable oil, rising, eggs, garlic herb, large eggs, boudin, plain flour, chop(s), crumb(s), pink salmon, tapioca flour, breadcrumbs, grits, sharp, bread, catfish fillets, matzo, potatoes, pan drippings, black treacle, mustard, cream style corn, pickle(s), bread crumb(s), mayonaise, freshly ground, saltine(s), egg(s), ham, sausages, milk, seasoning salt, meal, bacon, green tomatoes, prepared mustard, russet, cornmeal, buttermilk, butter, crumbs, flour
4: all purpose, large eggs, baking powder, soda, flour, eggs, baking, chocolate candy, buttermilk, color food green, corn meal, biscuits, bran, unsalted butter, baking soda, drippings, butter, lard, shortening, extract, cornmeal, vanilla, melted, milk, white sugar, sugar, salt, large, large egg(s)
5: cheese, parmesan, bread slices, grated, grated parmesan, mushrooms, half \& half, butter, sage, broth, heavy, potato gnocchi, prosciutto, gorgonzola, bacon, fresh asparagus, fresh basil, basil leaves, whipping, polenta, fontina, grated parmesan cheese, fettucine, pasta, asparagus, grated nutmeg, penne, cream cheese, pinenuts, basil, mozzarella cheese
6: orange(s), brown sugar, sugar(s), tea(s), pie, syrup, ice, condensed milk, apples, whiskey, gelatin, bananas, vanilla, cardamom, cachaca, fruit, marshmallow(s), pecan(s), peach(es), strawberri(es), cinnamon, whipped, sweetened, rum, raspberries, coffee, firmly, cake, almond(s), blackberry(ies), cherries, liqueur, chocolate, packed, cookie(s), graham cracker(s), espresso, evaporated milk, mint sprigs, light, cream, brandy, powdered, light brown, cocoa, nutmeg, granulated sugar, whipping, milk, extract
7: red pepper(s), flakes, vinegar, red wine, tomatoes, dried tomatoes, crushed, collard(s), feta, onion, cherry, olives, dried, penne, crushed red, crushed red pepper flakes, caper(s), balsamic, kalamata, oregano, basil, olive(s), dried basil, linguine, crushed tomatoes, garlic cloves, pasta, red, pepper flakes, red pepper flakes 8: tortilla(s), cheddar cheese, beans, taco, old el, soup, mix, jack, cream, shredded, ground beef, pico de, sour cream, green chilies, chips, kernel, salsa, black olives, enchilada sauce, shredded monterey, sharp, diced green chilies, cheese, refried, cooked chicken, ground turkey, dressing, mexican cheese, mexican, chili powder, ripe olives, corn, green chile(s), colby, kernel(s), picante sauce, lettuce, ranch, shredded cheddar cheese, tomato(es), green, shredded lettuce, diced, corn tortilla(s), black bean(s), corn tortillas, guacamole, avocado(s), onion(s), chili(es)
9: jalapeno(s), best, lime, lime juice, cilantro, chipotle(s), avocado(s), tomatillos, crema, tequila, chile(s), cilantro fresh, ancho, onion, purple, queso, hominy, white onion, kernel(s), serrano, corn tortillas, epazote, pork shoulder, tomatoes, cotija, jicama, chopped cilantro, poblano, salt, mayonaise, tortilla(s), chili(es), coarse salt, mexican, corn, chiles, black beans, salsa, fresh, jack, cumin, cilantro leaves, onion(s)
10: extra virgin olive oil, olive oil, garlic cloves, olives, tuna, sea salt, sherry vinegar, pecorino, balsamic, blanched almonds, vinegar, ground pepper, arugula, freshly, basil leaves, cured, baguette, dijon mustard, pinenuts, fine sea salt, fine(s), basil, cheese, fresh lemon, lemon, pepper(s), white wine, pepper
11: lemon, lemon juice, yogurt, horseradish, mayonaise, feta cheese, cucumber, fillet(s), fresh, fresh chives, greek, pita(s), dijon, creole mustard, fresh lemon, dill, mint, salmon, fresh lemon juice, chopped fresh, grated lemon, olives, lemon zest, capers, parsley, fresh parsley, fresh mint, kalamata, chopped fresh mint 12: skinless, chicken, boneless, breast(s), skinless chicken, chicken breast(s), boneless skinless chicken, skinless chicken breast(s), breast, boneless skinless chicken breast(s), soup, chicken breasts, breasts, dressing, breast halves, chicken breast, chicken breast halves, biscuit(s), boneless chicken, poultry seasoning, cream of chicken soup, onions, onion(s), broth, skinless chicken breast, chicken broth, boneless chicken breast, shredded, green, sodium, pepper
13: black pepper, pork, ground, cider, cayenne, guajillo, dried, barbecue sauce, ground black pepper, beef, roast, garbanzo beans, garlic powder, lamb, paprika, allspice, chickpeas, stew, ketchup, cinnamon, chops, garlic, ground ginger, lentil(s), oregano, thyme, tomato, onion, chili powder, salt, shoulder, couscous, kosher, ground cumin, tomato(es), ground cinnamon, ground cloves, onion powder, diced tomatoes, dried thyme, cumin, ground coriander, pepper, dried oregano, onion(s) 14: fat, less, fat free, low, fat free less sodium, sodium, chicken broth, sodium chicken broth, broth, reduced, cooking, \%, chopped onion, cooking spray, garlic cloves, chopped, $\%$ low, $\%$ low fat milk, low sodium
15: bell pepper(s), green(s), celery, long, red beans, kidney, okra, green, cooked, sausage(s), seasoning, hot, cajun, smoked sausage, onions, shrimp, andouille, onion(s), green bell, creole
16: yeast, warm water, water, active dry yeast, unbleached flour, bread flour, masa, flour, barley, ghee, wheat flour, dry, dry yeast, semolina, instant, shortening, eggs 17: white, grain, wine, leaf parsley, broth, collard green(s), low salt chicken, thyme, dry, black peppercorns, red wine, snails, truffle(s), saffron, shallots, bay leaf(ves), heavy, arborio rice, leaf, tarragon, pearl onions, bacon, mushrooms, dry white, ham, veal, stock, porcini mushrooms, rosemary, vinegar, sprig(s), chopped fresh, clam(s), grain white, white wine vinegar, garlic cloves, leek(s), butter, parsley, white wine, celery, wine vinegar, white onion, dried thyme, olive oil, white pepper, white sugar
18: potato(es), half \& half, bay, butter, carrot(s), peas, ground pepper, pastry, turnip(s), beef, ham, grits, thyme, bacon, onions, cabbage, stock, sausage(s), rib(s), meat, leek(s), worcestershire sauce, dill, celery, half(ves), smoked, yellow onion, parsley, salt, sweet potato(es), beef stock, pepper
19: egg, yolk(s), puff, egg(s), large egg(s), chocolate, large egg, almond(s), mascarpone, cake, vanilla, cream, large egg yolks, of tartar, espresso, preserves, jam, large eggs, large egg whites, bittersweet chocolate, hazelnut(s), marnier, confectioners sugar, semisweet chocolate, whole milk, pistachio(s), cocoa, vanilla beans, ladyfingers, coffee beans, gelatin, powdered, heavy, whipping cream, raspberry, extract, strawberri(es), dark rum, sugar, unsalted butter, liqueur, whole, coffee, almond, milk, granulated sugar, butter, flour
20: lime, fish sauce, coconut milk, fish, thai, leaves, mint, coconut, chili pepper(s), prawn(s), rice stick(s), vermicelli, sauce(s), chili sauce, curry paste, ginger, basil, peanut(s), peanuts, shallots, spring, cucumber(s), cilantro, tofu, shrimp paste, brown, sriracha, shrimp, sugar, beansprouts, rice, noodle(s), rice paper, rice noodles, chile(s), jasmine, lime leaves, paste, lemongrass, peanut, chili, cilantro leaves, milk, soy sauce
21: soy sauce, sesame, five, gochujang, bean, chinese, wrappers, szechwan, sauce(s), scallion(s), oyster sauce, nori, pork, shiitake, ramen, sushi, miso, shaoxing mirin, water chestnuts, japanese, sake, tofu, cabbage, spring, seaweed, napa cabbage, rice wine, ginger, hoisin, green onions, udon, mushrooms, dashi, white pepper, wasabi, rice vinegar, starch, daikon, kimchi, noodles, rice, sesame oil, vinegar, seed(s), carrots, ground pork

## TABLE 24

## NSF. Descriptive features selected for each cluster sorted by coefficient value.

1: theory, conjecture, geometry, spaces, algebraic, algebras, numbers, manifolds, theorem, investigator, groups, questions, invariants, space, objects, polynomials, curves, functions, oldest, proved, representation, topology, lie, arithmetic, classification, topological, analytic, cohomology, finite, geometric, algebra, study, representations, euclidean, infinite, conjectures, operators, invariant, abstract, symmetries, classical, operator, combinatorics, polynomial, mathematics, dimensional, hilbert, mathematical, problems, physics
2: mantle, rocks, magma, tectonic, magmatic, terranes, crustal, constrain, crust, seismic, km, plate, deformation, orogenic, belt, geochemical, ridge, volcanic, subduction, spreading, uplift, dating, deposits, basin, faulting, continental, isotopic, zone, magmas, margin, tectonics, petrologic, faults, minerals, zones, arc, rock, evolution, east, volcanism, geologic, fault, mineral, magmatism, lithosphere, hydrothermal, lithospheric, geological, rift, geophysical
3: laboratory, courses, analysis, students, introductory, exercises, geology, experiments, lecture, instrument, lab, laboratories, equal, software, principles, modern, acquisition, macintosh, biology, spectrometer, equipment, undergraduate, concepts, majors, instructor, curriculum, instruments, instrumental, computers, taught, computer, student, instruction, modules, learning, equipped, instrumentation, spectrophotometer, purchase, workstations, teaching, instructional, teach, chemistry, hands-on, college, department, projects, experience, faculty
4: equations, solutions, problems, nonlinear, models, numerical, asymptotic, methods, equation, method, boundary, fluid, partial, flows, random, investigator, flow, mathematical, approximations, convergence, dynamical, stability, problem, stochastic, finite, phenomena, parameter, arise, behavior, singular, inverse, variational, linear, singularities, solution, dynamics, chaotic, applied, approximation, analysis, differential, arising, computational, complicated, mechanics, applications, physical, systems, algorithms, theory
5: reactions, molecules, professor, complexes, catalysts, compounds, chemistry, synthetic, reaction, reactivity, dr, studies, bond, metal, ions, radicals, catalytic, solvent, macromolecular, spectroscopic, chiral, intermediates, synthesis, ligands, organometallic, program, molecule, organic, bonds, products, chemical, supporting, university, reagents, spectroscopy, oxidation, reactive, catalysis, resonance, intramolecular, dynamics, electron, hydrogen, award, molecular, nmr, atoms, transition, department, inorganic
6: dissertation, political, interviews, people, social, cultural, empirical, economic, examines, archaeological, legal, literature, examine, labor, conflict, firms, outcomes, trade, historical, beliefs, capital, societies, policy, prices, data, individuals, survey, factors, institutional, goods, economy, household, public, decisions, income, implications, ethnographic, markets, households, case, census, risk, welfare, market, decision, organizational, government, organizations, policies, research
7: carbon, water, waters, concentrations, dissolved, ocean, biogeochemical, nitrogen, phytoplankton, atlantic, circulation, production, rates, quantify, sediments, isotopes, ecosystems, biomass, fluxes, anthropogenic, soil, microbial, nutrients, uptake, seasonal, co2, tracer, cycling, composition, sediment, benthic, flux, trace, matter, particulate, seawater, organic, measurements, global, coastal, processes, nutrient, variability, atmosphere, marine, ecosystem, atmospheric, productivity, climate, sea
8: feasibility, phase, conventional, high, demonstrated, cost, sbir, small, process, commercial, laser, business, low, compact, innovation, fiber, technique, efficiency, imaging, fabrication, device, detection, proposed, operating, ii, technology, sensor, demonstrate, fabricated, beam, operation, surface, instrument, prototype, sensitivity, material, fabricate, capability, sensors, optical, silicon, power, monitoring, manufacturing, processing, applications, devices, performance, semiconductor, design
9: species, populations, predators, fossil, diversity, habitat, habitats, dispersal, offspring, taxa, morphological, predation, evolutionary, traits, patterns, plants, insects, variation, history, females, phylogenetic, genus, genetic, test, taxonomic, relationships, population, ecological, birds, genera, reproductive, natural, plant, male, hypotheses, animals, fitness, breeding, characters, insect, reproduction, organisms, mating, phylogeny, selection, biodiversity, conservation, evolution, ecology, dna 10: brain, cells, neurons, nerve, perception, mechanisms, visual, humans, hormones, cell, stimuli, stimulus, muscle, nervous, animals, synaptic, normal, movements, secretion, cortex, stimulation, hormone, receptors, sensory, role, motor, electrophysiological, activity, receptor, tissue, neural, body, signals, physiological, neuronal, vertebrates, movement, neurotransmitters, responses, vertebrate, neurotransmitter, function, system, mammalian, animal, cellular, understanding, developmental, regulation, neuroscience
11: support, cooperative, university, dr, equipment, visit, oceanographic, operated, scientific, purchase, national, vessel, fellowship, expertise, funding, ship, award, months, funds, facilities, nsf, sciences, facility, center, director, installation, research, marine, recommends, acquisition, joint, oceanography, scientists, institute, shipboard, researchers, conduct, institution, qualified, contributing, postdoctoral, dedicated, instrumentation, calendar, awards, maximal, program
12: materials, properties, films, alloys, microstructure, microscopy, devices, polymer, temperature, microstructural, epitaxial, film, mechanical, interfacial, crystals, polymers, fundamental, nanoscale, diffraction, crystal, composites, defects, structures, optical, magnetic, crystalline, interfaces, thin, silicon, oxide, coatings, bulk, atomic, scanning, semiconductors, surface, technologically, liquid, thermal, electrical, surfaces, processing, electron, scattering, characterization, fabrication, semiconductor, metal, spectroscopy, electronic
13: protein, mutants, proteins, gene, mutant, genes, expression, pathway, dna, yeast, coli, binding, mutagenesis, rna, mechanism, vitro, cloned, mutations, enzymes, regulatory, biochemical, acid, sequence, identified, transcription, vivo, arabidopsis, isolated, specific, sequences, bacterial, mrna, plants, enzyme, metabolism, eukaryotic, genome, regulated, involved, amino, plant, function, regulation, cell, functional, cellular, molecular, genetic, mechanisms, cells
14: model, measurements, flow, ionosphere, wave, wind, particle, effects, waves, weather, ionospheric, thermosphere, magnetosphere, turbulence, observed, velocity, plasma, convective, precipitation, transport, dynamics, observations, storms, turbulent, radar, cloud, vorticity, numerical, convection, models, variability, heating, simulations, mesoscale, vertical, scale, heat, horizontal, atmosphere, layer, momentum, atmospheric, doppler, parameterizations, flows, boundary, winds, processes, forcing, circulation
15: workshop, conference, held, meeting, international, travel, researchers, forum, papers, future, bring, symposium, scientists, discuss, june, committee, speakers, organizers, published, society, proceedings, directions, leading, august, american, october, support, organized, countries, attendees, july, invited, september, attend, experts, participants, sessions, discussion, participation, exchange, topics, attendance, young, poster, presentations, discussions, session, plenary, ideas, talks 16: earthquake, design, control, loads, buildings, process, manufacturing, product, damage, objective, geotechnical, reinforced, develop, steel, loading, load, dynamic, vibration, planning, industry, concrete, methodology, failure, power, uncertainties, analytical, system, safety, fault, reliability, stiffness, actuators, civil, developed, controllers, industrial, modeling, performance, cost, intelligent, engineering, controller, engineers, models, feedback, robust, mechanical, systems, optimal, optimization
17: quantum, theoretical, physics, atoms, quark, states, violation, atomic, experimental, experiment, interactions, nuclei, nucleon, nuclear, quarks, particle, electrons, excitations, decay, collisions, photon, meson, fermilab, collider, magnetic, particles, matter, spin, electron, scattering, accelerator, polarized, superconductors, nucleons, energy, detector, excitation, charge, experiments, qcd, condensed, calculations, relativistic, theory, energies, proton, heavy, superconducting, excited 18: ice, climate, records, record, glacier, cores, climatic, antarctic, sea, paleoclimatic, glacial, late, dating, stratigraphic, paleoclimate, holocene, arctic, antarctica, chronology, lake, years, deposits, age, data, pleistocene, past, proxy, variability, high-resolution, stratigraphy, radiocarbon, paleoenvironmental, region, glaciation, northern, atmospheric, alaska, western, pollen, southern, variations, award, sediments, history, sediment, global, circulation
19: algorithms, techniques, languages, efficient, user, language, query, reasoning, complexity, automatic, tasks, framework, specification, algorithm, execution, parallel, code, automatically, software, compiler, architectures, logic, semantics, programming, performance, tools, architecture, processor, memory, application, processing, machine, scheduling, distributed, implementation, parallelism, computation, correctness, processors, optimization, implementations, machines, systems, applications, computational, hardware, computing, computer, problems, design
20: program, reu, graduate, students, university, engineering, faculty, undergraduates, scholars, minority, academic, entering, women, scholarship, summer, talented, institutions, career, scholarships, programs, ethics, participate, participation, recruitment, internships, undergraduate, mentoring, careers, seminars, recruited, retention, student, weekly, college, colleges, projects, seminar, opportunities, mentor, underrepresented, experiences, activities, recruiting, trips, minorities, mentors, year, education, participants, science
21: teachers, teacher, mathematics, science, school, schools, content, standards, systemic, grades, learning, project, children, middle, math, reform, girls, assessment, staff, parents, instructional, activities, achievement, dissemination, community, district, education, classrooms, public, educators, districts, technology, grade, educational, curriculum, leadership, classroom, workshops, elementary, professional, materials, programs, skills, standards-based, curricula, teaching, instruction, college, student, students
22: galaxies, stars, observations, telescope, star, observatory, ast, solar, stellar, radio, interferometer, detector, galaxy, cosmic, dr, sky, gravitational, mass, galactic, emission, pulsars, supernova, observational, spectra, interstellar, quasars, spectral, angular, wavelengths, universe, infrared, objects, sun, evolution, clouds, observatories, dust, astrophysical, radiation, supernovae, photometric, astronomical, observing, faint, extragalactic, telescopes, luminosity, astrophysics, astronomers, astronomy
23: network, internet, access, services, connection, resources, networks, wireless, bandwidth, users, information, nsfnet, management, service, file, multicast, digital, connect, vbns, connections, packet, remote, protocol, speed, libraries, traffic, routing, atm, servers, user, resource, protocols, supercomputers, infrastructure, performance, requests, qos, testbed, server, support, ip, requirements, backbone, scalable, networking, distributed, communication, multimedia, communications

TABLE 25
AG's news. Descriptive features selected for each cluster sorted by coefficient value.
1: sales, quarter, profit, earnings, percent, research, profits, shares, stores, year, loss, mart, wall, results, outlook, net, maker, quote, update, expectations, chain, fiscal, retailer, forecast, forecasts, rose, increase, growth, increased, reported, revenue, demand, drop, fall, income, largest, posted, sees, lower, rise, estimates, posts, company, fell, strong, higher, corp, quarterly
2: company, drug, executive, motors, financial, group, research, billion, buy, jobs, ceo, accounting, amp, companies, investment, corp, million, stock, quote, buys, firm, announced, plans, kmart, cash, insurance, shareholders, maker, business, bank, plc, nortel, bn, stake, largest, based, holdings, sale, acquire, sell, management, unit, cut, shares, merger, update, biggest, giant, deal, agreed
3: bankruptcy, million, airlines, pay, airways, airline, securities, deal, air, workers, hollinger, accounting, executive, insurance, agreed, plan, contract, sec, executives, fund, enron, agreement, cuts, boeing, employees, company, yesterday, federal, filed, union, lawsuit, settlement, unions, bankrupt, delta, alitalia, financial, debt, fraud, billion, charges, settle, pilots, carrier, attendants, cost, update, pension, cut, restructuring
4: oil, prices, crude, price, fuel, opec, gas, futures, winter, supplies, energy, barrel, supply
5: reuters, dollar, rates, rate, jobless, economy, figures, economic, euro, average, survey, tokyo, treasury, month, august, growth, investors, report, showed, reserve, year, september, july, consumer, index, market, inflation, data, gains, fed, economists, pace, bank, interest, week, nikkei, lower, fell, rise, federal, low, slightly, higher, expected, yen, jobs, rose, stocks, percent, prices
6: oracle, peoplesoft, bid, takeover, offer, abbey, woolworths, software, wmc, ellison, chilcott, battle, enterprise, rival, applications, shareholders, bn, rejects, company, monti, billion, rejected, hostile, psft, orcl
7: wireless, phone, mobile, internet, fi, service, phones, cell, nokia, services, broadband, sprint, communications, network, blackberry, docomo, cellphone, spam, messaging, networks, verizon, vodafone, ip, voip, net, technology, siemens, telecom, access, vonage, customers, cellular, device, provider, users, voice, subscribers, connections, mail, telephone, companies, handsets, fcc, telephony, speed, motorola, based, providers, telecommunications, cable
8: search, google, web, engine, yahoo, ipo, rss, pages, beta, site, click, tool, links, blog, initial, page, sites, online, content, information, amazon, desktop, application, ads, offering, tools, mail, advertising, rsquo, toolbar, browser, popular, searchable, searches, internet, service, users, goog
9: ibm, apple, dvd, hp, pc, computer, format, digital, intel, storage, sony, ipod, power, technology, sun, hardware, music, portable, chip, gb, computing, memory, design, amd, graphics, dell, systems, toshiba, processor, product, announced, processors, notebook, electronics, unveils, servers, devices, linux, based, system, chips, dual, products, computers, version, market, server, desktop, company
10: microsoft, software, windows, security, worm, flaw, vulnerability, xp, code, linux, system, spam, firewall, browsers, application, source, firefox, spyware, browser, novell, virus, systems, applications, mozilla, versions, products, users, version, updates, explorer, tools, mail, os, hackers, sp, enterprise, release, malicious, aol, vulnerabilities, server, features, web, flaws, service, released, operating, open, internet, computer
11: gt, lt, fullquote, http, investor
12: space, nasa, earth, scientists, mars, astronomers, moon, prize, astronaut, genesis, spacecraft, saturn, flight, rocket, stars, cassini, lunar, sky, planet, capsule, surface, rutan, universe, launch, fly, solar, shuttle, science, jupiter, mission, telescope, spaceshipone, canaveral, titan, astronauts, planets, moons, craft, planetary, atmosphere, mojave, propulsion, satellite, orbit, miles, probe, desert, exploration, unmanned, orbiting
13: court, federal, drug, trial, law, pinochet, online, sex, rape, equatorial, years, sexual, industry, movie, case, file, copyright, press, cp, alleged, martha, prison, peer, attorney, murder, jail, guilty, convicted, charges, british, child, canadian, justice, lawyers, legal, sharing, prosecutors, recording, sentence, piracy, filed, charged, accused, judge, illegally, fraud, criminal, internet, music
14: quot, writes, coach, stewart, halo, president, minister
15: china, japan, scientists, researchers, bird, cambodia, chinese, disease, humans, south, study, fossil, animals, afp, sea, north, hong, death, island, children, human, species, beijing, typhoon, arctic, taiwan, submarine, ancient, earthquake, communist, state, decades, explosion, quake, global, climate, found, pacific, years, asian, asia, decade, country, korean, warming, missing, northern, people, toll, korea
16: gold, olympic, red, athens, sox, yankees, pedro, olympics, boston, medal, games, phelps, series, doping, women, marathon, men, meters, greece, gymnastics, schilling, hamm, american, silver, sprinter, marion, athletes, radcliffe, medals, sprinters, team, athletics, bronze, meter, greek, relay, world, night, win
17: season, quarterback, game, sports, yards, touchdown, nl, nba, miami, offense, bowl, coach, points, houston, dodgers, cleveland, field, night, giants, atlanta, kansas, ranked, defensive, college, receiver, career, nfl, left, eagles, bay, mike, pittsburgh, seattle, minnesota, back, redskins, florida, games, manning, football, teams, patriots, inning, play, victory, team, st, win, league, beat
18: cup, open, grand, round, masters, golf, championship, woods, champion, world, schumacher, final, tour, seed, par, race, henman, classic, tennis, title, vijay, sharapova, williams, hewitt, seeded, spain, els, lead, agassi, federer, event, finals, mauresmo, win, top, qualifying, championships, tournament, roddick, wins, atp, shot, pga, wta, sunday, semifinals, davenport, match, stroke, victory
19: striker, manchester, arsenal, league, manager, chelsea, champions, england, club, liverpool, united, premiership, celtic, newcastle, football, side, draw, boss, beckham, real, milan, team, players, coach, sir, wales, barcelona, rooney, english, goal, fa, portsmouth, defender, tottenham, trafford, midfielder, uefa, madrid, blackburn, captain, soccer, bolton, defeat, match, everton, goals, southampton, fulham, owen, cup 20: ap, coach, quarterback, california, helens, bowl, nfl, bay, st, miss, bryant, defensive, night, points, pittsburgh, colorado, receiver, career, touchdown, nba, scored, oklahoma, season, game, democratic, sen
21: india, test, australia, cricket, minister, pakistan, sri, indian, day, delhi, pm, zealand, pervez, africa, bangladesh, wickets, wicket, captain, tendulkar, kashmir, batting, islamabad, ganguly, south, aziz, pakistani, australian, nagpur, manmohan, icc, match, indies, afp, series, trophy
22: president, bush, john, election, presidential, susilo, vote, administration, voters, republican, kerry, ukraine, house, voting, debate, karzai, political, campaign, presidency, afghanistan, electoral, opposition, democratic, senate, democracy, party, war, terror, bill, elections, afghan, leader, polls, poll, candidate, votes, sen, democrats, republicans, parliament, congress, afp, incumbent, country, cheney, candidates
23: sudan, nuclear, darfur, talks, iran, nations, african, government, eu, states, ivory, peace, region, congo, treaty, uranium, union, european, wto, council, sudanese, rebels, libya, crisis, turkey, atomic, united, aid, countries, leaders, africa, secretary, end, resolution, reuters, south, country, negotiations, trade, rebel, foreign, sanctions, security, diplomats, afp, urged, meeting, summit, discuss, arms
24: people, police, hurricane, killed, bomb, car, storm, capital, attack, plane, haiti, crash, dead, school, killing, victims, blast, suspected, muslim, separatist, chechen, afghan, injured, authorities, died, officials, bodies, russia, deadly, suicide, security, blasts, arrested, bombs, attacks, embassy, kills, death, city, bombing, southern, explosion, wounded, exploded, forces, militants, kill, soldiers, wounding, baghdad
25: iraq, iraqi, british, al, hostage, najaf, blair, sadr, hostages, troops, kidnapped, saddam, minister, kidnappers, hassan, prisoners, fallujah, abu, falluja, army, release, bigley, italian, freed, military, muslim, sunni, allawi, shi, shiite, forces, cleric, aid, soldiers, government, insurgents, baghdad, captors, held, rebel, iraqis, militants, stronghold, militant, interim, fighting, zarqawi, insurgent, city, kill
26: palestinian, arafat, gaza, israeli, israel, sharon, palestinians, leader, west, hamas, jewish, israelis, jerusalem, bank, egypt

TABLE 26
Reviews: movie kitchen. Contingency table between the clusters and true topic categories.

|  | ¢ ¢ a | \% |
| :---: | :---: | :---: |
| 1: film, movie, story, films, life,... | 2000 | 23 |
| 2: bought, product, buy, clean, kitchen, pan,. | 0 | 1977 |

TABLE 27
Reviews: movie books. Contingency table between the clusters and true topic categories.

|  | 0.0 | $\frac{0}{0}$ |
| :--- | ---: | ---: |
|  | 0.0 | 0 |
| 0 |  |  |
| 1: film, movie, films, director, comedy,... | 2000 | 156 |
| 2: book, read, books, reading, author, reader, ... | 0 | $\mathbf{1 8 4 4}$ |

TABLE 28
Reviews: movie electronics. Contingency table between the clusters and true topic categories.

|  | \% | O |
| :---: | :---: | :---: |
| 1: film, movie, story, films, man, character,... | 2000 | 21 |
| 2: bought, product, quality, buy, unit, price,... | 0 | 1979 |

TABLE 29
Reviews: kitchen electronics. Contingency table between the clusters and true topic categories.

|  | 気 | 苞 |
| :---: | :---: | :---: |
| 1: pan, knife, sheets, kitchen, oven, dishes,... | 1059 | 38 |
| 2: water, coffee, maker, ice, cup, machine,... | 671 | 42 |
| 3: card, camera, printer, work, windows, $\mathrm{hp}, \ldots$ | 175 | 883 |
| 4: sound, ipod, phone, headphones, ear, dvd,... | 95 | 1037 |

TABLE 30
Reviews: dvd electronics. Contingency table between the clusters and true topic categories.


TABLE 31
Reviews: movie books electronics kitchen. Contingency table between the clusters and true topic categories.

|  | \% | \% | O | \% |
| :---: | :---: | :---: | :---: | :---: |
| 1: film, movie, films, director, scene,... | 1999 | 182 | 9 | 15 |
| 2: book, read, books, reading, author,... | 1 | 1776 | 11 | 9 |
| 3: ipod, card, sound, phone, battery,... | 0 | 19 | 1923 | 313 |
| 4: clean, water, pan, kitchen, coffee,... | 0 | 23 | 57 | 1663 |

TABLE 32
Reviews：dvd movie electronics kitchen．Contingency table between the clusters and true topic categories．

|  |  |  | .0 |  |
| :--- | ---: | ---: | ---: | ---: |
|  |  | 0 | 0 | 0 |
|  | 0 | 0 | 0 | 0 |

TABLE 33
Reviews：dvd books movie kitchen．Contingency table between the clusters and true topic categories．

|  | $\begin{aligned} & J \\ & \vec{D} \end{aligned}$ | \％ | \％ | 矿 |
| :---: | :---: | :---: | :---: | :---: |
| 1：book，read，books，reading，author，reader，．． | 70 | 1792 | 1 | 10 |
| 2：film，movie，films，director，movies，．．． | 1834 | 178 | 1999 | 21 |
| 3：product，clean，kitchen，plastic，pan，．．． | 96 | 30 | 0 | 1969 |

TABLE 34
Reviews：dvd books electronics movie．Contingency table between the clusters and true topic categories．

|  | － | \％ | \％ | \％ |
| :---: | :---: | :---: | :---: | :---: |
| 1：book，read，books，author，reading，．． | 67 | 1800 | 10 | 1 |
| 2：film，movie，films，director，comedy，cast，．．． | 1699 | 175 | 17 | 1999 |
| 3：product，quality，unit，ipod，bought，．．． | 234 | 25 | 1973 | 0 |

TABLE 35
Reviews：dvd books electronics movie kitchen．Contingency table between the clusters and true topic categories．

|  |  |  | 0 |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  |  |  | 0 | 0 |  |

TABLE 36
20－News：med guns．Contingency table between the clusters and true topic categories．

|  | 『 | $\stackrel{\square}{5}$ |
| :---: | :---: | :---: |
| 1：msg，food，dyer，photography，stuff，uga，．．． | 229 | 5 |
| 2：banks，medicine，treatment，thanks，．．． | 730 | 17 |
| 3：gun，guns，killed，weapons，police，．．． | 23 | 597 |
| 4：cdt，fbi，brent，fire，gas，handheld，．．． | 5 | 290 |

TABLE 37
20－News：autos space hardware．Contingency table between the clusters and true topic categories．

|  | 递 | \％ | \％ |
| :---: | :---: | :---: | :---: |
| 1：thanks，card，pc，please，dos，monitor，．．． | 941 | 38 | 34 |
| 2：car，cars，engine，ford，oil，insurance，．．． | 28 | 926 | 13 |
| 3：jbh，mccall，nick，sky，calpoly，bby，wats，．．． | 8 | 22 | 419 |
| 4：nasa，space，henry，shuttle，spacecraft，．．． | 2 | 1 | 519 |

TABLE 38
20-News: comp. Contingency table between the clusters and true topic categories.

|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1: ac, navy, group, uk, graphics, comp, xv, . . | 126 | 25 | 12 | 11 | 49 |
| 2: windows, andre, zip, cica, file, files,... | 85 | 209 | 24 | 6 | 55 |
| 3: windows, dos, win, it, mouse, crash, ms,... | 20 | 386 | 102 | 15 | 28 |
| 4: thanks, please, firstclass, mail, email,... | 266 | 72 | 100 | 108 | 95 |
| 5: card, bus, diamond, dx, video, eisa, vesa,... | 65 | 97 | 232 | 20 | 13 |
| 6: image, points, objects, sphere, science,... | 263 | 22 | 12 | 13 | 178 |
| 7: apple, they, edu, life, power, cnn, hacker,... | 96 | 77 | 133 | 397 | 25 |
| 8: drive, scsi, drives, disk, hd, hard,... | 0 | 23 | 237 | 126 | 1 |
| 9: monitor, simms, cpu, iisi, clock, nubus,... | 26 | 9 | 118 | 253 | 7 |
| 10: window, server, xterm, widget, lib, int,... | 23 | 43 | 9 | 9 | 531 |

TABLE 39
20-News: rec. Contingency table between the clusters and true topic categories.

|  | ¢ | $\begin{aligned} & \mathscr{Q} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \ddot{q} \end{aligned}$ |  | \} |
| :---: | :---: | :---: | :---: | :---: |
| 1: car, cars, thanks, engine, ford, eliot,... | 723 | 26 | 41 | 13 |
| 2: my, shop, fit, bmw, police, interstate,. . | 221 | 284 | 16 | 13 |
| 3: dod, com, uk, ranck, winona, shaft, nec,... | 14 | 668 | 7 | 12 |
| 4: he, baseball, runs, ball, pitching, sherri,. . . | 9 | 9 | 802 | 181 |
| 5: gm, hockey, rochester, mailing, baltimore,... | 18 | 5 | 74 | 141 |
| 6: det, bos, la, nyi, pit, stl, nyr, sj,... | 0 | 0 | 5 | 46 |
| 7: buffalo, detroit, montreal, go, pittsburgh,... | 2 | 1 | 46 | 591 |

TABLE 40
20-News: sci. Contingency table between the clusters and true topic categories.

|  | $\stackrel{\circ}{2}$ | $\begin{aligned} & .0 .0 \\ & \tilde{U} \\ & 0 \\ & \text { U } \\ & \text { U } \end{aligned}$ | ® | \% \% \% |
| :---: | :---: | :---: | :---: | :---: |
| 1: key, clipper, encryption, government, nsa,... | 834 | 16 | 3 | 4 |
| 2: radio, battery, car, michael, hp, plastic,... | 10 | 371 | 46 | 26 |
| 3: thanks, anyone, mb, mail, appreciated,... | 90 | 560 | 138 | 85 |
| 4: geb, food, she, doctor, medicine, disease,... | 50 | 16 | 792 | 39 |
| 5: space, nasa, henry, launch, orbit, earth,. . | 5 | 21 | 8 | 831 |

TABLE 41
20-News: mix2. Contingency table between the clusters and true topic categories.

|  |  | ¢ |  | ס | ¢ \% \% |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1: windows, file, dos, os, printer, files,... | 825 | 5 | 1 | 3 | 5 |
| 2: car, cars, engine, dealer, oil, ford,... | 18 | 878 | 5 | 11 | 14 |
| 3: york, mail, philadelphia, thanks, thank,... | 82 | 65 | 131 | 107 | 50 |
| 4: baseball, he, game, players, team, ball,... | 10 | 12 | 841 | 18 | 14 |
| 5: geb, doctor, medicine, medical, treatment,... | 17 | 21 | 10 | 829 | 25 |
| 6: space, pat, nasa, sky, earth, orbit,. . . | 11 | 6 | 3 | 19 | 877 |

TABLE 42
20-News: mix. Contingency table between the clusters and true topic categories.

|  |  |  |  | \% \% \% | 苟 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1: windows, file, dos, files, driver, using,. . . | 878 | 33 | 17 | 15 | 6 |
| 2: sale, shipping, offer, condition, asking, ... | 26 | 865 | 17 | 14 | 4 |
| 3: baseball, he, players, team, ball, pitcher,... | 7 | 16 | 931 | 13 | 21 |
| 4: space, nasa, pat, earth, orbit, launch,... | 12 | 12 | 0 | 876 | 44 |
| 5: people, kaldis, com, fbi, clinton, drugs,... | 40 | 38 | 26 | 67 | 699 |

TABLE 43
20-News: sci comp. Contingency table between the clusters and true topic categories.

|  |  |  |  |  |  | $\frac{8}{2}$ |  | ت | \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1: windows, dos, win, ms , file, os, exe,... | 62 | 564 | 112 | 9 | 64 | 11 | 11 | 2 | 1 |
| 2: ftp, fax, mail, de, domain, visualization,... | 380 | 117 | 33 | 24 | 216 | 29 | 34 | 15 | 50 |
| 3: card, mhz, dx, video, board, cards, lc,... | 75 | 77 | 307 | 268 | 16 | 1 | 53 | 1 | 0 |
| 4: drive, scsi, drives, disk, controller, hd,... | 2 | 31 | 263 | 168 | 1 | 1 | 34 | 0 | 1 |
| 5: window, widget, server, xterm, int, lib,... | 110 | 70 | 15 | 29 | 592 | 5 | 14 | 3 | 0 |
| 6: thanks, anyone, please, newsgroup, mail,... | 204 | 39 | 121 | 171 | 44 | 23 | 247 | 112 | 19 |
| 7: tube, solder, power, led, pulse, ink,... | 68 | 21 | 95 | 241 | 12 | 31 | 369 | 11 | 19 |
| 8: clipper, key, encryption, sternlight,... | 3 | 3 | 9 | 5 | 7 | 817 | 33 | 2 | 4 |
| 9: geb, doctor, food, covington, article, she,... | 50 | 34 | 19 | 37 | 15 | 64 | 138 | 830 | 77 |
| 10: space, nasa, henry, orbit, earth, launch,... | 16 | 7 | 5 | 6 | 15 | 7 | 51 | 11 | 814 |

TABLE 44
20-News: sci comp rec. Contingency table between the clusters and true topic categories.

|  |  | $\begin{aligned} & \text { n } \\ & \text { s } \\ & \text { 豆 } \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ |  |  |  |  |  | $\begin{aligned} & \text { ت̈ } \\ & \text { O } \end{aligned}$ | $\begin{aligned} & \stackrel{0}{0} \\ & \stackrel{0}{n} \end{aligned}$ | $\begin{aligned} & \stackrel{\pi}{2} \\ & \stackrel{7}{7} \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{0}{0} \\ & 0.0 \\ & \stackrel{0}{0} \\ & 0 \\ & \ddot{a} \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1: ftp, fax, format, graphics, package,... | 341 | 138 | 47 | 33 | 277 | 18 | 21 | 7 | 10 | 1 | 0 | 1 | 1 |
| 2: windows, window, abpsoft, file, default,... | 115 | 565 | 84 | 39 | 491 | 9 | 24 | 3 | 0 | 4 | 1 | 0 | 0 |
| 3: drive, mb, scsi, mhz, card, bus, cpu,... | 58 | 100 | 586 | 508 | 15 | 4 | 108 | 0 | 2 | 1 | 1 | 0 | 0 |
| 4: thanks, anyone, please, mail, looking,... | 254 | 79 | 169 | 229 | 72 | 27 | 263 | 125 | 39 | 92 | 21 | 61 | 26 |
| 5: key, clipper, encryption, sternlight, nsa,... | 10 | 1 | 10 | 12 | 16 | 843 | 45 | 3 | 3 | 3 | 0 | 1 | 1 |
| 6: canada, sweden, usa, april, det, uk,... | 71 | 23 | 23 | 26 | 66 | 18 | 92 | 40 | 18 | 38 | 23 | 14 | 81 |
| 7: geb, amp, medicine, food, cause, that,... | 65 | 28 | 41 | 68 | 14 | 41 | 260 | 639 | 85 | 168 | 44 | 29 | 8 |
| 8: nasa, space, orbit, henry, launch, pat,... | 42 | 11 | 10 | 13 | 23 | 3 | 92 | 73 | 810 | 12 | 4 | 1 | 1 |
| 9: dod, car, bike, article, engine, cars, com,... | 5 | 10 | 7 | 13 | 4 | 14 | 72 | 46 | 7 | 646 | 873 | 13 | 8 |
| 10: he, his, baseball, him, pitcher, ball,... | 7 | 5 | 2 | 12 | 4 | 12 | 5 | 50 | 10 | 21 | 25 | 657 | 221 |
| 11: philadelphia, go, montreal, detroit,... | 2 | 3 | 0 | 5 | 0 | 0 | 2 | 1 | 1 | 1 | 1 | 214 | 650 |

TABLE 45
20-News (all). Contingency table between the clusters and true topic categories.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1: windows, file, window, files, motif,... | 5 | 507 | 738 | 105 | 47 | 828 | 21 | 8 | 4 | 9 | 1 | 30 | 57 | 11 | 11 | 7 | 3 | 0 | 3 | 4 |
| 2: card, monitor, video, mhz, monitors, $\mathrm{cpu}, \ldots$ | 1 | 137 | 137 | 419 | 514 | 21 | 75 | 4 | 1 | 1 | 0 | 15 | 103 | 0 | 0 | 0 | 8 | 1 | 0 | 0 |
| 3: drive, scsi, drives, disk, hd, ide, rom,... | 0 | 6 | 39 | 318 | 182 | 1 | 84 | 10 | 12 | 2 | 0 | 2 | 55 | 0 | 4 | 2 | 0 | 0 | 1 | 1 |
| 4: sale, mail, email, price, offer, vhs, ink,... | 3 | 103 | 22 | 96 | 156 | 48 | 715 | 49 | 22 | 27 | 5 | 21 | 282 | 54 | 15 | 12 | 11 | 4 | 8 | 6 |
| 5: dod, car, cars, bike, engine, bmw, riding,... | 6 | 2 | 7 | 4 | 15 | 4 | 39 | 864 | 918 | 10 | 2 | 0 | 145 | 37 | 6 | 2 | 8 | 4 | 7 | 5 |
| 6: team, season, game, baseball, nhl, hockey,... | 2 | 4 | 2 | 3 | 5 | 1 | 14 | 2 | 3 | 900 | 980 | 0 | 13 | 4 |  | 2 | 3 | 1 | 5 | 2 |
| 7: clipper, key, chip, algorithm, encryption,... | 1 | 26 | 2 | 11 | 14 | 22 | 3 | 1 | 0 | 1 | 0 | 867 | 82 | 3 | 5 | 0 | 7 | 6 |  | , |
| 8: god, atheists, jesus, bible, christian,... | 566 | 2 | 2 | 0 | 0 | 1 | 1 | 1 | 5 | 2 | 2 | 0 | 5 | 13 | , | 903 | 2 | 6 | 13 | 397 |
| 9: space, nasa, orbit, moon, distance,... | 9 | 152 | 11 | 17 | 15 | 49 | 9 | 8 | 6 | 5 | 6 | 7 | 200 | 195 | 890 | 5 | 3 | 3 | 14 | 8 |
| 10: geb, cramer, article, doctor, study,... | 114 | 31 | 4 | 7 | 12 | 9 | 12 | 19 | 15 | 23 | 2 | 12 | 35 | 658 | 43 | 35 | 29 | 32 | 333 | 84 |
| 11: israel, turkish, israeli, turks, muslim,... | 63 | 3 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 11 | 0 | 1 | 2 | 1 | 1 | 20 | 11 | 868 | 32 | 19 |
| 12: gun, atf, waco, cdt, guns, firearms, fire,... | 29 | 0 | 1 | 2 | 2 | 0 | 2 | 23 | 10 | 3 | 1 | 36 | 5 | 14 | 8 | 8 | 824 | 15 | 350 | 98 |

TABLE 46
Reuters．Contingency table between the clusters and true topic categories．

|  | E | 耳্ণ | ت్ت゙ | $\stackrel{\cdots}{\approx}$ | $\begin{aligned} & \text { F } \\ & \text { 気 } \\ & \hline \end{aligned}$ | \％ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1：split，declared，date，dividend，．．． | 222 | 7 | 0 | 1 | 24 | 0 | 1 | 1 |
| 2：div，record，pay，payout，april，qtly，．．． | 490 | 0 | 0 | 0 | 73 | 0 | 0 | 0 |
| 3：loss，shr，net，qtr，revs，note，cts，oper，．．． | 1406 | 1 | 0 | 1 | 746 | 0 | 0 | 0 |
| 4：sell，unit，company，terms，undisclosed，．．． | 253 | 752 | 1 | 354 | 43 | 2 | 0 | 0 |
| 5：offer，bid，stake，takeover，shares，．．． | 40 | 687 | 0 | 303 | 10 | 1 | 0 | 0 |
| 6：trade，grain，dispute，agriculture，．．． | 71 | 42 | 228 | 56 | 29 | 5 | 1 | 9 |
| 7：stg，billion，fed，turnover，francs，．．． | 365 | 37 | 30 | 23 | 176 | 15 | 27 | 80 |
| 8：rate，bank，pct，banks，rates，prime，．．． | 39 | 44 | 0 | 13 | 12 | 11 | 212 | 32 |
| 9：oil，crude，prices，sea，barrels，refinery，．．． | 26 | 26 | 17 | 51 | 9 | 244 | 0 | 120 |
| 10：monetary，baker，dollar，currency，．．． | 3 | 10 | 11 | 2 | 2 | 62 | 30 | 85 |

TABLE 47
Recipe．Contingency table between the clusters and true topic categories．

|  |  |  |  | : |  |  | : |  |  |  | $\begin{aligned} & \text { च్జ } \\ & \underset{\sim}{n} \end{aligned}$ | $\ddot{0}$ 苋 $\frac{\pi}{2}$ $\stackrel{2}{5}$ |  |  | $\begin{aligned} & \text { J. } \\ & \text { B } \end{aligned}$ | $$ | $\frac{\tilde{n}}{: B}$ | $\begin{aligned} & \text { ฐ్ँ } \\ & \text { ジ } \end{aligned}$ |  | ． |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1：seed（s），masala，dal，cumin seed（s），．．． | 7 | 17 | 4 | 1712 | 7 | 2 | 12 | 87 | 23 | 10 | 47 | 12 | 5 | 10 | 8 | 91 | 3 | 8 | 58 | 5 |
| 2：part skim，mozzarella，cheese，italian，．．． | 49 | 13 | 1 | 4 | 0 | 8 | 1331 | 30 | 1 | 3 | 2 | 0 | 9 | 5 | 21 | 3 | 2 | 0 | 3 | 2 |
| 3：self，vegetable oil，rising，eggs，．．． | 22 | 418 | 27 | 38 | 15 | 49 | 137 | 75 | 38 | 97 | 9 | 5 | 45 | 20 | 66 | 41 | 31 | 13 | 0 | 41 |
| 4：all purpose，large eggs，baking powder，．．． | 36 | 893 | 21 | 58 | 19 | 14 | 261 | 149 | 31 | 168 | 5 | 6 | 47 | 6 | 183 | 39 | 161 | 15 | 8 | 68 |
| 5：cheese，parmesan，bread slices，grated，．．． | 13 | 76 | 1 | 3 | 2 | 6 | 835 | 16 | 2 | 7 | 1 | 1 | 23 | 12 | 79 | 5 | 1 | 0 | 0 | 2 |
| 6：orange（s），brown sugar，sugar（s），tea（s），．．． | 45 | 878 | 95 | 121 | 76 | 75 | 266 | 189 | 72 | 132 | 37 | 20 | 57 | 154 | 262 | 61 | 91 | 18 | 48 | 44 |
| 7：red pepper（s），flakes，vinegar，red wine，．．． | 83 | 122 | 10 | 35 | 15 | 45 | 612 | 99 | 61 | 3 | 43 | 19 | 51 | 12 | 28 | 42 | 3 | 54 | 11 | 7 |
| 8：tortilla（s），cheddar cheese，beans，taco，．．． | 7 | 160 | 4 | 3 | 2 | 3 | 55 | 1940 | 9 | 12 | 0 | 2 | 14 | 2 | 18 | 6 | 15 | 5 | 0 | 7 |
| 9：jalapeno（s），best，lime，lime juice，．．． | 4 | 55 | 5 | 66 | 20 | 27 | 22 | 1787 | 5 | 0 | 26 | 21 | 12 | 21 | 6 | 6 | 4 | 5 | 10 | 1 |
| 10：extra virgin olive oil，olive oil，．．． | 164 | 69 | 4 | 35 | 6 | 218 | 1145 | 129 | 11 | 13 | 4 | ， | 35 | 8 | 209 | 24 | 13 | 5 | 94 | 5 |
| 11：lemon，lemon juice，yogurt，horseradish，．．． | 362 | 170 | 10 | 155 | 3 | 37 | 402 | 74 | 13 | 29 | 21 | 10 | 96 | 7 | 207 | 45 | 23 | 5 | 143 | 33 |
| 12：skinless，chicken，boneless，breast（s），．．． | 61 | 106 | 28 | 163 | 29 | 23 | 361 | 474 | 290 | 2 | 170 | 41 | 96 | 11 | 55 | 47 | ， | 7 | 41 | 8 |
| 13：black pepper，pork，ground，cider，cayenne，．．． | 95 | 247 | 83 | 142 | 142 | 53 | 270 | 416 | 35 | 32 | 3 | 30 | 144 | 21 | 80 | 18 | 16 | 7 | 217 | 16 |
| 14：fat，less，fat free，low，．．． | 59 | 133 | 3 | 122 | 4 | 51 | 411 | 226 | 60 | 11 | 32 | 14 | 49 | 10 | 155 | 20 | 42 | 4 | 54 | 8 |
| 15：bell pepper（s），green（s），celery，long，．．． | 57 | 224 | 50 | 60 | 49 | 140 | 341 | 397 | 97 | 2 | 82 | 16 | 562 | 50 | 58 | 35 | 5 | 14 | 39 | 15 |
| 16：yeast，warm water，water，active dry yeast，．．． | 17 | 39 | 19 | 97 | 16 | 9 | 230 | 53 | 36 | 37 | 1 | 9 | 44 | 6 | 67 | 20 | 6 | 4 | 22 | 51 |
| 17：white，grain，wine，leaf parsley，broth，．．． | 45 | 156 | 37 | 24 | 18 | 111 | 619 | 142 | 44 | 36 | 14 | 13 | 73 | 31 | 400 | 41 | 12 | 10 | 19 | 28 |
| 18：potato（es），half \＆half，bay，butter，．．． | 20 | 334 | 97 | 62 | 65 | 57 | 162 | 56 | 20 | 138 | ， | 20 | 153 | 26 | 269 | 41 | 205 | 19 | 42 | 103 |
| 19：egg，yolk（s），puff，egg（s），large egg（s），．．． | 28 | 167 | 19 | 7 | 3 | 56 | 340 | 64 | 39 | 69 | 5 | 11 | 26 | 20 | 467 | 47 | 29 | 5 | 7 | 45 |
| 20：lime，fish sauce，coconut milk，fish，thai，．．． | 1 | 18 | 99 | 87 | 27 | 4 | 19 | 23 | 58 | 1 | 898 | 440 | 2 | 33 | 3 | 20 | 1 | 21 | 5 | 0 |
| 21：soy sauce，sesame，five，gochujang，bean，．．． | 0 | 25 | 138 | 9 | 8 | 1 | 7 | 12 | 1728 | 2 | 135 | 130 | 3 | 2 | 5 | 771 | 0 | 611 | 0 | 0 |

TABLE 48
NSF．Contingency table between the clusters and true topic categories．

|  | $\sum_{a}^{n}$ | $\begin{aligned} & \hline \text { 号 } \\ & \hline \end{aligned}$ | 晢 | $\sum_{0}^{E}$ | $\begin{aligned} & \hline 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{\|c\|} \hline 0 \\ 0 \\ \omega \end{array}$ | $\begin{aligned} & \hline \text { 思 } \\ & \hline \end{aligned}$ | $\underset{~}{z}$ | $乌$ | $\sum_{n}^{\prime \prime}$ | 号 | $\begin{array}{l\|} \hline \text { y } \\ \text { n } \end{array}$ | $\begin{aligned} & \hline 0 \\ & 2 \\ & \hline \end{aligned}$ | $\overline{\sum_{4}}$ | $$ | $\sum_{0}^{n}$ | "吅 | un | $\begin{aligned} & \text { Z } \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline 0 \\ u \end{array}$ | $\hat{0}$ | 可 | $\cong$ | $\begin{gathered} 0 \\ 0 \\ 0 \end{gathered}$ | $\begin{aligned} & 5 \\ & 4 \\ & \hline \end{aligned}$ | $\underset{y}{4}$ | $\overline{4}$ | $\begin{aligned} & \hline \stackrel{\rightharpoonup}{\mathrm{I}} \\ & \hline \end{aligned}$ | $\begin{array}{\|c} \hline u \\ \text { u } \\ \hline \end{array}$ | $\begin{aligned} & \hline \mathrm{u} \\ & \underset{\sim}{u} \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0 \\ & 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \mathbb{1} \\ & 0 \end{aligned}$ | $\begin{aligned} & \hline \text { 足 } \\ & \hline \end{aligned}$ | 䍃 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1：theory，conjecture， geometry，spaces，． | 4044 | 12 | 0 | 12 | 0 | 25 | 0 | 1 | 85 | 2 | 0 | 10 | 0 | 2 | 127 | 9 | 8 | 0 | 16 | 10 | 0 | 0 | 25 | 1 | 0 | 3 | 0 | 9 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2：mantle，rocks，magma， tectonic，magmatic，．．．． | 5 | 7 | 4039 | 2 | 1051 | 1 | 3 | 1 | 132 | 7 | 9 | 34 | 0 | 2 | 1 | 35 | 0 | 0 | 0 | 1 | 180 | 1 | 1 | 10 | 0 | 0 | 0 | 9 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3：laboratory，courses， analysis，students，．． | 28 | 5135 | 122 | 50 | 10 | 9 | 43 | 8 | 0 | 50 | 9 | 22 | 20 | 50 | 13 | 19 | 251 | 28 | 27 | 16 | 0 | 31 | 14 | 25 | 5 | 82 | 0 | 25 | 62 | 79 | 0 | 58 | 8 | 0 | 2 |
| 4：equations，solutions， problems，nonlinear，． | 2896 | 42 | 56 | 82 | 23 | 67 | 30 | 3 | 146 | 87 | 5 | 5 | 3 | 34 | 180 | 204 | 8 | 197 | 77 | 136 | 0 | 1 | 24 | 33 | 2 | 9 | 4 | 15 | 12 | 2 | 75 | 0 | 0 | 0 | 0 |
| 5：reactions，molecules， professor，complexes，．．． | 5 | 145 | 80 | 45 | 23 | 7 | 4306 | 8 | 323 | 179 | 2 | 1 | 196 | 98 | 1 | 10 | 90 | 369 | 31 | 4 | 2 | 3 | 1 | 68 | 8 | 1 | 0 | 26 | 2 | 2 | 5 | 17 | 3 | 1 | 4 |
| 6：dissertation，political， interviews，．． | 184 | 37 | 22 | 82 | 12 | 4897 | 0 | 45 | 124 | 0 | 52 | 2674 | 4 | 20 | 10 | 105 | 84 | 2 | 3 | 9 | 153 | 45 | 143 | 12 | 3 | 89 | 10 | 45 | 4 | 59 | 2 | 8 | 12 | 4 | 3 |
| 7：carbon，water，waters， concentrations，．．． | 10 | 23 | 600 | 63 | 2357 | 18 | 21 | 162 | 116 | ${ }^{8}$ | 675 | 82 | 78 | 526 | ${ }^{3}$ | 40 | 87 | 62 | 3 | 2 | 493 | 7 | 0 | 218 | 5 | 1 | 0 | 16 | ${ }^{9}$ | 0 | 5 | 4 | 3 | 0 | 1 |
| 8：feasibility，phase， conventional，high，．．． | 10 | 43 | 64 | 3035 | 39 | ${ }^{6}$ | 31 | ${ }^{4}$ | 13 | 120 | 2 | 4 | 5 | 54 | 58 | 188 | 125 | 359 | 37 | 626 | 6 | 3 | 18 | 154 | 30 | 17 | 6 | 8 | 53 | 0 | 2 | 2 | 0 | 3 | 6 |
| 9：species，populations， predators，fossil，．．． | 52 | 20 | 206 | 29 | 434 | 44 | 5 | 1019 | 272 | 3 | 3641 | 561 | 112 | 11 | 2 | 1 | 382 | 6 | 5 | 1 | 135 | 14 | 3 | 13 | 0 | 5 | 0 | 16 | 0 | 1 | 1 | 9 | 6 | 0 | 3 |
| 10：brain，cells，neurons， nerve，perception，．．． | 82 | 28 | ${ }^{6}$ | 44 | 35 | 39 | 22 | 2589 | 119 | 3 | 19 | 405 | 425 | 0 | 16 | 14 | 222 | 30 | 5 | 28 | 13 | 14 | 74 | 311 | 0 | 36 | 0 | 29 | 3 | 7 | 2 | 5 | 1 | 0 | 1 |
| 11：support，cooperative， university，dr，．．． | 426 | 31 | 448 | 33 | 924 | 51 | 41 | 19 | 1377 | 52 | 87 | 107 | 11 | 100 | 6 | 151 | 505 | 73 | 51 | 59 | 74 | 46 | 16 | 79 | 8 | 104 | 10 | 11 | 318 | 7 | 11 | 94 | 6 | 18 | 45 |
| 12：materials，properties， films，alloys，．．． | 88 | 82 | 158 | 507 | 4 | 9 | 395 | 2 | 368 | 3257 | ${ }^{4}$ | 18 | 22 | 1 | 6 | 544 | 26 | 637 | 22 | 570 | 2 | 12 | 2 | 71 | 1 | 6 | 0 | 35 | 71 | 0 | 6 | 12 | 3 | 1 | 14 |
| 13：protein，mutants， proteins，gene， mutant，．．． | 44 | 48 | 10 | 106 | 38 | 6 | 71 | 1205 | 145 | 22 | 150 | 12 | 3258 | 0 | 13 | 0 | 648 | 52 | 7 | 7 | 19 | 6 | 15 | 283 | 0 | 32 | 1 | 31 | ${ }^{7}$ | 0 | 7 | ${ }^{3}$ | 5 | 0 | 1 |
| 14：model， measurements，flow， ionosphere，．．． | 150 | 13 | 266 | 45 | 478 | 3 | 11 | 9 | 60 | 77 | 19 | 31 | 19 | 1778 | 4 | 183 | 18 | 569 | 76 | 64 | 70 | 2 | 3 | 64 | 38 | 1 | 1 | 8 | 3 | 1 | 5 | 1 | 0 | 0 | 1 |
| 15：workshop， conference，held， meeting，．．． | 557 | 282 | 189 | 163 | 118 | 231 | 100 | 286 | 749 | 283 | 135 | 159 | 361 | 189 | 191 | 451 | 127 | 350 | 254 | 300 | 112 | 92 | 214 | 210 | 49 | 105 | 91 | 38 | 41 | 40 | 47 | 10 | 2 | 21 | 12 |
| 16：earthquake，design， control，loads，．．． | 97 | 296 | 228 | 1314 | 9 | 100 | 1 | 7 | 228 | 13 | 1 | 11 | 1 | 8 | 216 | 1823 | 4 | 222 | 5 | 542 | 4 | 7 | 183 | 221 | 1 | 58 | 11 | 20 | 249 | 15 | 14 | 4 | 8 | 3 | 5 |
| 17：quantum，theoretical， physics，atoms，．．． | 172 | 85 | 37 | 11 | 3 | 35 | 222 | 7 | 543 | 967 | 1 | ${ }^{0}$ | 29 | 76 | 19 | 18 | 22 | 64 | 1878 | 176 | ${ }^{4}$ | 10 | 5 | ${ }^{3}$ | 14 | 29 | 0 | 18 | 4 | 1 | 19 | 10 | 3 | 0 | 2 |
| 18：ice，climate，records， record，glacier，．．． | 15 | 4 | 444 | 11 | 197 | 6 | 14 | 5 | 76 | 5 | 53 | 158 | 3 | 660 | 1 | 27 | 7 | 10 | 6 | 2 | 1437 | 1 | 1 | 2 | 6 | 0 | 0 | 7 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 19：algorithms， techniques，languages，．． | 481 | 174 | 24 | 562 | 17 | 69 | 27 | 7 | 190 | 20 | 14 | 292 | 13 | 22 | 2909 | 79 | 164 | 27 | 19 | 235 | 3 | 5 | 1362 | 53 | 13 | 614 | 55 | 41 | 14 | 32 | 450 | 1 | 5 | 0 | 1 |
| 20：program，reu， graduate，students，．． | 261 | 870 | 102 | 34 | 120 | 159 | 328 | 17 | 90 | 170 | 18 | 58 | 20 | 49 | 13 | 60 | 448 | 55 | 206 | 48 | 13 | 345 | 13 | 119 | 35 | 252 | 2 | 628 | 480 | 41 | 4 | 225 | 188 | 26 | 32 |
| 21：teachers，teacher， mathematics，science，．． | 38 | 1085 | 41 | 85 | 23 | 71 | 7 | 5 | 13 | 21 | 7 | 31 | 2 | 10 | 2 | 0 | 6 | 5 | 18 | 5 | 7 | 1986 | 15 | ${ }^{5}$ | ${ }^{6}$ | 53 | 2 | 406 | 44 | 543 | 5 | 6 | 126 | 238 | 4 |
| 22：galaxies，stars， observations， telescope，．．． | 20 | 58 | 21 | 18 | 3 | 12 | 8 | ${ }^{1}$ | 179 | 4 | ${ }^{0}$ | 2 | 1 | 480 | 2 | 0 | 15 | 6 | 299 | 9 | 87 | 24 | 6 | 6 | 1620 | ${ }^{0}$ | ${ }^{1}$ | 18 | 2 | 1 | 2 | 2 | 1 | ${ }^{0}$ | 0 |
| 23：network，internet， access，services，．．． | 40 | 187 | 38 | 173 | 20 | 65 | 5 | 5 | 41 | 6 | 37 | 19 | 5 | 42 | 355 | 40 | 129 | 7 | 11 | 121 | 20 | 15 | 176 | 7 | 4 | 233 | 1423 | 15 | 29 | 36 | 53 | 2 | 1 | 6 | 14 |

TABLE 49
AG＇s news．Contingency table between the clusters and true topic categories．

|  |  |  | 部 | T |
| :---: | :---: | :---: | :---: | :---: |
| 1：sales，quarter，profit，earnings，percent，．．． | 3874 | 558 | 24 | 102 |
| 2：company，drug，executive，motors，．．． | 4236 | 657 | 68 | 100 |
| 3：bankruptcy，million，airlines，pay，．．． | 4121 | 358 | 320 | 266 |
| 4：oil，prices，crude，price，fuel，opec，gas，．．． | 3436 | 116 | 19 | 429 |
| 5：reuters，dollar，rates，rate，jobless，．．． | 3509 | 480 | 35 | 344 |
| 6：oracle，peoplesoft，bid，takeover，offer，．．． | 776 | 607 | 19 | 17 |
| 7：wireless，phone，mobile，internet，fi，．．． | 1030 | 3832 | 35 | 82 |
| 8：search，google，web，engine，yahoo，ipo，．．． | 425 | 1742 | 45 | 65 |
| 9：ibm，apple，dvd，hp，pc，computer，format，．．． | 811 | 6172 | 30 | 97 |
| 10：microsoft，software，windows，security，．．． | 414 | 4469 | 7 | 33 |
| 11：gt，lt，fullquote，http，investor，．．． | 2236 | 1359 | 581 | 735 |
| 12：space，nasa，earth，scientists，mars，．．． | 80 | 2775 | 11 | 107 |
| 13：court，federal，drug，trial，law，pinochet，．． | 2032 | 1870 | 574 | 1922 |
| 14：quot，writes，coach，stewart，halo，．．． | 897 | 1060 | 1033 | 936 |
| 15：china，japan，scientists，researchers，．．． | 785 | 1330 | 182 | 2132 |
| 16：gold，olympic，red，athens，sox，yankees，．．． | 327 | 294 | 4302 | 384 |
| 17：season，quarterback，game，sports，yards，．．． | 282 | 635 | 10260 | 246 |
| 18：cup，open，grand，round，masters，golf，．．． | 117 | 320 | 5233 | 276 |
| 19：striker，manchester，arsenal，league，．．． | 113 | 137 | 4405 | 237 |
| 20：ap，coach，quarterback，california，helens，．．． | 209 | 1466 | 2937 | 1949 |
| 21：india，test，australia，cricket，minister，．．． | 436 | 240 | 1259 | 1875 |
| 22：president，bush，john，election，．．． | 731 | 576 | 139 | 3883 |
| 23：sudan，nuclear，darfur，talks，iran，．．． | 613 | 297 | 110 | 4145 |
| 24：people，police，hurricane，killed，bomb，．．． | 290 | 470 | 207 | 4280 |
| 25：iraq，iraqi，british，al，hostage，najaf，．．． | 103 | 62 | 60 | 4643 |
| 26：palestinian，arafat，gaza，israeli，israel，．．． | 17 | 18 | 5 | 2615 |

## Appendix 2 Other Datasets

In order to test the generality of descriptive clustering, we applied the PDC algorithm on three datasets that were not formed from text documents.

## Appendix 2.1 Mushrooms

We applied the descriptive clustering algorithm to the Mushroom dataset from the UCI Machine Learning Repository [5]. To do this we converted the categorical features into 112 binary features. There are 8,124 instances in this dataset. When applying the descriptive clustering algorithm, the algorithm selected 26 clusters, which is the upper limit of the range used, so we increased the range of cluster sizes to 50, and the algorithm selected 47 clusters. The selected features for each cluster are in Table 50. Interestingly, the clusters were very pure in terms of edible and poisonous, with only 4 of the 47 clusters having a mixture of both edible and poisonous instances as shown in Table 51. However, using only the selected features, the predictive classifier makes some mistakes; there are poisonous mushrooms that the classifier assigns to an edible cluster. Furthermore, the list of features is not sufficient to make the cluster assignment since the classifier is using a weighted combination of features to make the cluster assignment. Nonetheless, the descriptive clustering is able to succinctly describe many of the clusters using only a few features.

TABLE 50

## Mushroom. Descriptive features selected for each cluster sorted by coefficient value.



TABLE 51
Mushroom. Contingency table between the clusters and true topic categories.

|  | \% | 0 0 0 0 0 0 0 0 0 |
| :---: | :---: | :---: |
| 1: population=numerous, habitat=grasses,... | 208 | 0 |
| 2: gill-color=black, population=scattered,... | 120 | 0 |
| 3: gill-color=chocolate, population=abundant | 96 | 0 |
| 4: stalk-color-below-ring=gray, $\ldots$ | 384 | 0 |
| 5: gill-color=purple,... | 288 | 0 |
| 6: stalk-color-above-ring=orange | 192 | 0 |
| 7: population=abundant, gill-color=pink,... | 288 | 0 |
| 8: stalk-color-above-ring=gray, $\ldots$. | 288 | 0 |
| 9: habitat=waste | 192 | 0 |
| 10: gill-color=brown, habitat=woods,... | 80 | 0 |
| 11: gill-color=pink, spore-print-color=black,... | 96 | 0 |
| 12: cap-surface=fibrous, habitat=urban,... | 96 | 0 |
| 13: gill-color=white, habitat=woods, odor=none,. . . | 192 | 0 |
| 14: habitat=meadows, odor=anise, odor=almond,... | 256 | 0 |
| 15: gill-spacing=crowded, odor=almond,... | 96 | 0 |
| 16: population=scattered, ... | 288 | 0 |
| 17: cap-color=gray, gill-color=brown,... | 64 | 0 |
| 18: cap-color=red, gill-color=brown,... | 80 | 0 |
| 19: gill-color=pink, spore-print-color=brown,... | 64 | 0 |
| 20: habitat=grasses, odor=anise,... | 120 | 0 |
| 21: stalk-color-above-ring=gray,... | 192 | 0 |
| 22: gill-color=white, stalk-shape=enlarging, $\ldots$. | 72 | 8 |
| 23: stalk-surface-below-ring=scaly | 208 | 40 |
| 24: ring-type=flaring,. . | 192 | 48 |
| 25: spore-print-color=green, cap-color=pink,... | 56 | 72 |
| 26: odor=spicy, habitat=woods,... | 0 | 128 |
| 27: stalk-color-below-ring=pink,... | 0 | 144 |
| 28: odor=foul, habitat=leaves | 0 | 192 |
| 29: stalk-color-below-ring=buff,... | 0 | 240 |
| 30: habitat=urban, odor=pungent | 0 | 128 |
| 31: odor=musty | 0 | 36 |
| 32: cap-shape=knobbed, odor=spicy,... | 0 | 128 |
| 33: stalk-color-below-ring=brown,... | 0 | 192 |
| 34: odor=fishy, habitat=woods,. . | 0 | 256 |
| 35: odor=foul, habitat=woods, gill-color=buff,... | 0 | 384 |
| 36: habitat=leaves, odor=fishy | 0 | 192 |
| 37: odor=spicy, habitat=woods, . . | 0 | 128 |
| 38: habitat=leaves, odor=spicy | 0 | 192 |
| 39: habitat=grasses, odor=pungent | 0 | 128 |
| 40: spore-print-color=chocolate,... | 0 | 240 |
| 41: odor=creosote | 0 | 192 |
| 42: cap-shape=knobbed, odor=fishy, $\ldots$. | 0 | 128 |
| 43: stalk-color-below-ring=buff,... | 0 | 192 |
| 44: stalk-color-below-ring=pink,... | 0 | 216 |
| 45: gill-color=gray,... | 0 | 96 |
| 46: stalk-color-below-ring=brown,... | 0 | 144 |
| 47: ring-type=large, cap-color=gray, $\ldots$. | 0 | 72 |

TABLE 52
Mushroom. Contingency table between predicted clusters and true topic categories.

|  | \% | 0 0 0 0 0 0 0 0 |
| :---: | :---: | :---: |
| 1: population=numerous, habitat=grasses,... | 214 | 0 |
| 2: gill-color=black, population=scattered,... | 112 | 10 |
| 3: gill-color=chocolate, population=abundant | 97 | 2 |
| 4: stalk-color-below-ring=gray,. . . | 379 | 0 |
| 5: gill-color=purple,... | 283 | 0 |
| 6: stalk-color-above-ring=orange | 192 | 0 |
| 7: population=abundant, gill-color=pink,. . | 293 | 0 |
| 8: stalk-color-above-ring=gray,. . . | 275 | 0 |
| 9: habitat=waste | 191 | 0 |
| 10: gill-color=brown, habitat=woods,. . | 80 | 0 |
| 11: gill-color=pink, spore-print-color=black,. . . | 82 | 0 |
| 12: cap-surface=fibrous, habitat=urban,. . . | 96 | 21 |
| 13: gill-color=white, habitat=woods, odor=none,... | 201 | 0 |
| 14: habitat=meadows, odor=anise, odor=almond,.. . | 248 | 0 |
| 15: gill-spacing=crowded, odor=almond,... | 93 | 0 |
| 16: population=scattered,... | 270 | 0 |
| 17: cap-color=gray, gill-color=brown,... | 80 | 0 |
| 18: cap-color=red, gill-color=brown,... | 61 | 0 |
| 19: gill-color=pink, spore-print-color=brown,... | 82 | 0 |
| 20: habitat=grasses, odor=anise,... | 143 | 0 |
| 21: stalk-color-above-ring=gray,. . . | 195 | 0 |
| 22: gill-color=white, stalk-shape=enlarging, . . | 60 | 8 |
| 23: stalk-surface-below-ring=scaly | 208 | 33 |
| 24: ring-type=flaring, $\ldots$ | 184 | 24 |
| 25: spore-print-color=green, cap-color=pink,. . . | 69 | 72 |
| 26: odor=spicy, habitat=woods,... | 0 | 149 |
| 27: stalk-color-below-ring=pink,... | 0 | 154 |
| 28: odor=foul, habitat=leaves | 0 | 189 |
| 29: stalk-color-below-ring=buff,... | 0 | 233 |
| 30: habitat=urban, odor=pungent | 0 | 126 |
| 31: odor=musty | 0 | 36 |
| 32: cap-shape=knobbed, odor=spicy,. . | 0 | 134 |
| 33: stalk-color-below-ring=brown,. . . | 0 | 187 |
| 34: odor=fishy, habitat=woods,. . | 0 | 268 |
| 35: odor=foul, habitat=woods, gill-color=buff,... | 0 | 370 |
| 36: habitat=leaves, odor=fishy | 0 | 196 |
| 37: odor=spicy, habitat=woods,... | 0 | 153 |
| 38: habitat=leaves, odor=spicy | 0 | 196 |
| 39: habitat=grasses, odor=pungent | 0 | 125 |
| 40: spore-print-color=chocolate,... | 0 | 266 |
| 41: odor=creosote | 0 | 191 |
| 42: cap-shape=knobbed, odor=fishy, . . | 0 | 134 |
| 43: stalk-color-below-ring=buff,... | 0 | 179 |
| 44: stalk-color-below-ring=pink,. . | 0 | 217 |
| 45: gill-color=gray,. . | 0 | 85 |
| 46: stalk-color-below-ring=brown,. . . | 2 | 144 |
| 47: ring-type=large, cap-color=gray,. . . | 4 | 78 |

## Appendix 2.2 USDA Plant database

The USDA plant database [6] from the UCI Machine Learning Repository [5] contains a list of 34,781 plant varieties that occur across 70 locations in the USA, Canada, and surrounding islands. We applied the descriptive clustering to the locations and use the plants as the features. The algorithm found 3 clusters of locations each described by a single species of plant. The clusters of locations are given in Table 53. Figure 1 shows the layout of most of each cluster's locations: Greenland (DENGL), Puerto Rico (PR), and the Saint Pierre and Miquelon (FRASPM) are not shown. The distributions for each of the plants selected to describe the clusters are shown in Figure 2. Most of the cluster locations match the corresponding plant distributions. Only four US states (IA, MN, WI, MI) and one Canadian province (YK) are not covered by the selected plants.

TABLE 53
Plants. Correspondence between the clusters (with a plant species as a descriptive feature) and locations.

| 1: elephantopus | AL, AR, DC, DE, FL, GA, HI, IA, IL, IN, KS, KY, LA, MD, MO, MS, NC, NJ, OH, OK, PA, PR, SC, TN, TX, VA, VI, WV |
| :--- | :--- |
| 2: crepis occidentalis | AB, AZ, BC, CA, CO, ID, MT, ND, NE, NM, NV, OR, SD, SK, UT, WA, WY |
| 3: puccinellia tenella | AK, CT, DENGL, FRASPM, GL, LB, MA, MB, ME, MI, MN, NB, NF, NH, NS, NT, NU, NY, ON, PE, QC, RI, VT, WI, YT |



Fig. 1. Locations colored by cluster membership (some island locations are missing from the map).


Fig. 2. Distribution of plants selected as descriptors of location clusters. (Left to right) elephantopus, crepis occidentalis, puccinellia tenella.

## Appendix 2.3 Edinburgh Associative Thesaurus

The Edinburgh Associative Thesaurus (EAT) [7] is a dataset consisting of empirical word associations. When the dataset was collected, participants were given a sheet with stimulus words or phrases and were instructed to write a single response word or phrase for each stimulus. Each participant was given 100 stimuli. See http://vlado.fmf.uni-lj.si/pub/ networks/data/dic/eat/Eat.htm for more details. We use the MATLAB version from the SuiteSparse Matrix Collection https:/ /sparse.tamu.ed [8] with a total count of 788,828 associations between the 23,219 entries: 8,210 unique entries were used as stimuli and 22,765 were given as responses.

## TABLE 54

Edinburgh Associative Thesaurus: Response Stimulus. Descriptive features (stimuli) selected for each cluster of responses.
1: 40, 22, psalm, 18 , route, 30 , unready, life-span, million, century, teen, twelve, 10, 1968, decade, year, 13 , twenty-one, seconds, thirty, ages, fortnight, fifteen, years, 9,20 th, thousand, month, 19 , lifetime, eighteen, hundred, five, ninety, calendar, 24 , six, forty, seven, minutes, 100 , seventy, age, nine, o'clock, twentieth, twenty, ten 2: david, alan, auntie, martin, hardy, ed, elegy, ronnie, godard, goodbye, saint, walter, dell, henry, doc, dr, inter, russell, william, john, cole, al, charles, emile, george, liz, darling, ken, mick, mister, glen, arthur, washington, mike, bye, orphan, bond, blimey, fie, beadle, wilson, aunt, author, dolittle, dee, elizabeth, dale, philosopher, ned, marx
3: humph, persists, choosers, constance, la, prolongs, hillock, izzy, lope, andrews, mount, louis, jack, anthill, chinaman, doubles, prolong, continuum, ho, caper, continual, twice, conquered, volcano, earthquake, shares, descending, flux, chinese, persevere, follows, semibreves, leonard, ling, remains, cone, encore, dizzy, atoll, larva, non, plurals, shrews, waltzers, hoo, grabs, et, lo, creepers
4: degrade, letting, debts, deficit, troubles, bills, redress, emitted, provider, owned, excluded, obligations, unpaid, exclude, recompense, repay, remit, send out, distribute, reprimanded, fund, repudiate, tax, credit, merit, return, joined, abandon, restore, distributed, payment, abdicate, debt, give out, hired, remittance, admission, more money, collect, refund, pay, allow, owing, replace, dispense, owed, concessive, wage, due, increase
5: proof, teaching, solving, review, effects, unequal, specialize, qualify, clause, plan, deduce, imprecise, selection, method, personal, observations, objective, means, theory, systematic, of a kind, decisions, words, cognition, difference, objects, instructions, diverse, analogy, process, values, determine, scientific, comparison, information, dative, particular, origin, basic, interpolate, progress, analysis, procedure, option, analyse, fact, critical, data, stipulation, arrange
6: classes, catalogue, commerce, posts, meetings, sports, manager, goods, aircraft, players, dealer, bureau, furniture, barrier, posture, theatre, hobbies, card, fare, retail, pose, ticket, warehouse, seats, post office, supermarket, coach, shop, stationery, studio, advertising, crossing, shops, sales, hall, status, production, directors, architects, freight, crowded, airport
7: tower, pier, county, marchioness, busby, jutland, mountains, midlands, halifax, continents, castle, maisonettes, wales, sussex, isles, vale, bristol, castles, arches, dukedom, isle, inland, buildings, suburbs, flagstones, outboard, inca, peninsula, via, newyork, brussels, ville, lakes, moorland, cherubs, mainland, colonies, whales, peasants, hills, chalet, docks, devon, moor, germany, spain, urban, lands, rome, canals
8: contents, diffraction, element, magnetism, submarine, mended, scientist, forceps, apparatus, wad, lens, flexible, jotting, dislocated, mal, lined, ion, valve, synthetic, expansion, spectacles, ganglia, mechanics, chemical, dot, magnetic, tilt, weights, fusion, edges, reactor, micro, connect, physical, muscles, atomic, structure, solder, electricity, fluids, compound, refill, wire, circuit, weld, atom, chloride, organic, plastic, bipolar
9: charlady, cider, corn, apple-tree, hen, prickly, pans, reddish, butter, toot, bud, meadows, roe, sipping, mop, veal, sprout, blueberry, chops, buds, vegetation, sedge, cheese, crab, hacks, olives, slime, plants, pickles, hawthorn, dandelion, fry, geese, fruit, planting, boiled, pigs, onion, roots, insects, lard, nuts, mildew, seeds, rotting, kippers, petals, drainage, buttercups, ponds
10: glycerine, potash, sugars, absorb, salts, acids, gland, choler, spuds, digestion, magnesium, kidneys, cough, stodge, indigestion, antiseptic, enzymes, fattening, vitamin c, jam, sweets, cereal, rice pudding, flour, gravy, vim, gastric, liver, potatoes, slurp, seasoning, oesophagus, essence, vegetables, protein, colic, cooker, tasty, chocolate, pies, pancreas, beef, creamy, flavour, savoury, bitters, sherbet, grapefruit, veg, sauce
11: bird, eery, doves, rake, jokes, witches, bitch, ghosts, smart, mighty, dad, graceful, nanny, dog, jeep, graveyard, dame, unicorn, woods, monster, gorilla, randy, wolf, wail, effeminate, eagle, lass, hairy, tigress, silly bitch, incense, hawk, blonde, fluffy, glade, aspidistra, witch, sexless, shrew, cub, peacock, tawdry, horns, witchcraft, brassy, prey, predator, grandma, feminine, scraggy
12: tribe, terrorist, russia, slavery, immigration, overlord, c. of e., waster, hungary, defender, dictatorship, townsman, foreigners, settlement, protestant, immigrants, immigrant, regime, mayoral, officials, alabama, india, capitalist, heraldry, customs, monarchy, governors, home rule, oppressor, hostage, communists, aggressor, warlike, wog, commandant, islam, national, servility, racialism, bushman, republic, indians, capitalism, warrior, wogs, indian, anarchist, american, liege, colonies 13: pi, variables, lb, $3 / 4$, per, frequency, rounded, quantities, rectangle, sums, triad, circle, shillings, rang, computer, equation, cubic, angle, bipolar, measure, centimetre, geometrical, coordinate, vector, dimension, series, pound, gram, metric, 2001, wireless, decimal, curves, sixpence, longitudinal, gramme, ellipse, rhombus, axis, hexagonal, stereo, geometry, diagonal, division, remainder, percentage, fraction, calculate, per cent, 100
14: realised, freely, persuaded, thoughtlessly, hopeful, apt, accepted, forgave, consider, in need, understood, deliberate, unfounded, ignored, intention, positively, unnecessary, reasonable, asked, forgot, disprove, proposition, obliged, gladly, certainty, repent, assured, surmise, never mind, couldn't, irrelevant, carefully, thanks, ability, unproven, explain, acknowledged, objection, ascertain, decide, doesn't, promised, deny, necessary, please, unable, prove, agree, reason, pertain
15: thugs, gangsters, hideaway, basher, thug, knifing, foundered, swords, hides, cosh, gangster, hideout, aground, lawless, perishers, castaway, violence, asylum, contraband, mob, thieves, chicago, rifleman, knives, cops, crew cuts, rifles, forces, riot, burglary, fences, loot, headquarters, anarchy, gang, rabble, trespass, outlaw, break in, bandit, fist-fight, rampage, detective, robber, criminology, rifle, burglar, raid, armed, crime
16: speaker, infamous, liar, humanism, manners, respect, bigotry, atheist, phoney, fame, nonconformist, frank, egomaniac, haughty, sincerity, honest, unholy, creed, representative, politic, ideals, moral, faithless, liberal, heckler, moderate, realistic, honour, hypocrisy, immoral, decent, spiritual, idealism, truth, righteousness, dogma, morality, polite, preaching, sinner, goodness, honor, religious, pious, righteous, methodist, related, beloved
17: attach, flagpole, pow, orders, about turn, draft, flags, prestige, cadet, convention, baton, welsh, flag, standard, actions, enlist, regiment, onward, colonel, wham, procession, corps, general, coup, enrol, backward, armies, cavalry, lieutenant
18: thereafter, therein, besides, forthwith, whatsoever, otherwise, penultimate, thereupon, as before, anyway, et cetera, whilst, began, whence, wherein, despite, wherefore, to what, however, finally, meanwhile, whatever, were, recently, nevertheless, soon, moreover, than, all the time, thus, unless, was, notwithstanding, though, happened, somehow, unto, perhaps, except, therefore, accordingly, also, eventually, yet, although, but, as, again, especially, since
19: beguilingly, aghast, distraught, quietly, coaxing, relaxing, slowness, paranoia, crying, hysteria, glee, laughter, impatient, fright, tantrum, easiness, grief, boredom, morbid, terribly, heartily, grieving, monstrously, leer, arousal, downcast, sad, shy, fervour, stupidly, old age, confused, peaceful, enthusiasm, insipid, liveliness, restless, annoyed, frown, anguish, morose, frenzy, depressed, woe, furtive, uneasy, temperament, wary, sadness
20: sensibly, stand out, watched, talkative, highly, distinguished, vocative, diligently, tested, arranged, fate, looked, pleased, said, mentioned, stated, exceptionally, properly, clearly, optimism, trance, beholder, admirable, foremost, insight, vision, genuinely, amaze, sisal, clever, really, files, perceived, beautifully, anonymous, delightful, heard, listener, visual, exceptional, faithfully, incognito, intelligent, seen, genius, nicely, listened, appeared
21: skirts, lacy, tiles, spindle, bracelet, horses, jewel, jumpers, woven, bikes, diamond, silver paper, boots, pants, rucksack, mending, shirts, hoist, togs, holes, handles, pelmet, knickers, tractor, tapestry, railings, pavement, covering, floors, manhole, weave, torn, silk, curtain, floor, jacket, balls, tacks, floats, hooves, buttons, cloth, lining, string, tracks, drawer, vest, pegs, braid, scissors
22: priestess, parson, singing, virginals, shrine, poems, grandeur, aries, saga, souvenirs, beethoven, planets, rites, jupiter, musician, glamour, viola, splendour, opera, earl, prophet, leo, brahms, classical, gazer, serenade, priest, mythology, ballet, mozart, poet, pagan, venus, ludwig, angels, sculpture, gazers, bishop, rhymes, stars, cathedral, songs, concerto, mars, symphony, bach, trio, statesman, mythical, recital
23: restrained, unrest, heaved, handicap, grabbed, clamped, eradicate, broken, harmed, headed, gripped, destroyed, released, tightly, hit, disturbed, immobile, destroy, fight, mania, repress, blunder, avoidance, agitate, hindered, tried, annoy, feinted, accidentally, retaliate, poked, casualty, killed, annihilate, annoying, disturbance, menace, malfunction, mismanaged, tackled, attack, disrupt, offend, convicted, disturb, fixed, prevent, abducted, tense, hinder
24: shake, cushion, snakes, totter, shiver, caress, repellent, snake, pains, flea, belting, pincer, gasp, fasten, fester, repulsive, withered, flimsy, tooth, itch, squirm, throbbing, pinching, bruise, trotted, clinging, mender, lounging, jerk, bouts, uncomfy, wriggling, flinch, immunity, bitten, smack, stiff, pant, nausea, convulse, crawling, stifle, injection, germs, hay fever, lithe, rasp, sore, painful
25: idiots, fairies, elf, legend, pansies, mac, goblin, shrunken, grotesque, fairy tale, choirs, dozy, dwarf, toadstool, fantasies, pure, portrait, dwarfs, causeway, wee, smallest, ladder, conjurors, livery, shorter, fairy, mime, tales, incredible, idiot, bad luck, fables, bewitched, mirrors, tiny, fable, wand, enchantment, weird, myth, fantasy, mythology

TABLE 55

## Edinburgh Associative Thesaurus: Response Stimulus. Correspondence between the clusters (described by stimuli) and a random subset of their instances (responses).

1: 40, 22, psalm, 18, route, 30, unready, life-span, million, century, teen,. 2: david, alan, auntie, martin, hardy, ed, elegy, ronnie, godard, goodbye,. 3: humph, persists, choosers, constance, la, prolongs, hillock, izzy, lope,. . 4: degrade, letting, debts, deficit, troubles, bills, redress, emitted,. 5: proof, teaching, solving, review, effects, unequal, specialize, qualify,. . 6: classes, catalogue, commerce, posts, meetings, sports, manager, goods,.. . 7: tower, pier, county, marchioness, busby, jutland, mountains, midlands,. 8: contents, diffraction, element, magnetism, submarine, mended, scientist,. 9: charlady, cider, corn, apple-tree, hen, prickly, pans, reddish, butter,. . 10: glycerine, potash, sugars, absorb, salts, acids, gland, choler, spuds,. . 11: bird, eery, doves, rake, jokes, witches, bitch, ghosts, smart, mighty, dad,. 12: tribe, terrorist, russia, slavery, immigration, overlord, c. of e., waster,. 13: pi, variables, $\mathrm{lb}, 3 / 4$, per, frequency, rounded, quantities, rectangle,. . . 14: realised, freely, persuaded, thoughtlessly, hopeful, apt, accepted, forgave,. 15: thugs, gangsters, hideaway, basher, thug, knifing, foundered, swords, hides,. . 16: speaker, infamous, liar, humanism, manners, respect, bigotry, atheist,. . . 17: attach, flagpole, pow, orders, about turn, draft, flags, prestige, cadet,. . 18: thereafter, therein, besides, forthwith, whatsoever, otherwise, penultimate,. . 19: beguilingly, aghast, distraught, quietly, coaxing, relaxing, slowness,... 20: sensibly, stand out, watched, talkative, highly, distinguished, vocative,. 21: skirts, lacy, tiles, spindle, bracelet, horses, jewel, jumpers, woven,. 22: priestess, parson, singing, virginals, shrine, poems, grandeur, aries, saga,. 23: restrained, unrest, heaved, handicap, grabbed, clamped, eradicate, broken,. . . 24: shake, cushion, snakes, totter, shiver, caress, repellent, snake, pains,. 25: idiots, fairies, elf, legend, pansies, mac, goblin, shrunken, grotesque,. .

20, feeder, chests, youth, weeps, jack bruce, aggregate, basie, proverbs annual, ph.d., d.c., cher, kith, tiddlywinks, matthew, karl, ding-dong
stirs, an, last one, chinese, bricky, staggering, scoring, funfair, oracle
allot, pretending, creditors, characteristic, istanbul, separated, married schedule, adjective, legacy, coalition, broadcast, same as, q.c., fend reproduction, trams, travel, afar, housing, business, bob, receptacle steamboat, and clyde, londoner, by way of, bognor, stalactites, schooner volt, apparatus, micro, coma, carved, urinate, taxonomy, h-bomb, rapier of time, wafer, supping, scotch, yardley, stic, scrub, amateurs, fowls porton, veal, squeezer, tumbler, steak, alcohol, granulated, toothbrush sheep, wearer, weird, apfel, invincible, carnation, brolly, sewing-machine white collar, amazons, of ireland, claude, capitalists, wars, town council 4th, pound, coordinates, cape kennedy, zebedee, lateral, sum, milestone obliged, wanting, thinking, revealed, tangible, rewarded, accuse assassination, trooper, demonstrator, cluster, portrait, sabre, pillage radical, tolerant, town crier, advisor, infidelity, christianity, boyfriend call up, services, norm, vietnam, girl guides, inward, charger, send away all under, precisely, dedication, brethren, is, therefore, morrow, instance raving, terrifying, thin air, amazement, idiocy, withdrawal, weakness careful, hypnotise, feelingly, wellknown, very good, knighted, doom of cancer, tin, spokes, rockets, tweed, socks, rods, armpit, wooden, gauze angels, pictures, crosses, churchy, victorious, lord, soc, miracles ative, quicken, destructive, corsets, whirlpool, kept back, stopped sickly, trenchant, tug, and judy, kip, writhe, censored, breath, shrug cinderella, magnitude, homosexual, oscillator, fraser, gnomes, unsightly

## TABLE 56

## Edinburgh Associative Thesaurus: Stimulus Response. Descriptive features (responses) selected for each cluster (of stimuli) sorted by coefficient value.

1: stupid, bad, love, happy, child, laugh, sex, funny, wrong, fool, mother, me, friend, woman, false, joy, nice, good, male, man, hate, daft, fun, person, boy, kind, girl, mad, women, friendly, smile, happiness, fear, youth, loving, no, people, pleasure, yes, sad
2: church, god, religion, priest
3: tree, flower, flowers, green, trees, plant, yellow, garden, grass, leaves, red, rose, roses, grow, hedge, bush, leaf, purple, plants, blossom, woods, summer, fruit
4: ship, army, war, soldier, police, america, law, ships, boat, officer, judge, prisoner, violence, criminal, prison, navy, uniform, general, castle, king, guns, illegal, court, lawyer, captain, chief, vietnam
5: correct, vote, look, blank, degree, difficult, word, people, make, talk, number, committee, questions, years, idea, twenty, see, first, clever, subject, new, theory, find, rubbish, psychology, maths, politics, computer, things, order, of, facts, time, study, exam, thought, answer, useless, part, work, one, test, life, class, words, observe, think, question
6: paper, school, book, picture, king, magazine, t.v., news, story, letter, teacher, work, art, board, maths, write, charles, numbers, science, newspaper, writing, drawing, read, words, ink, pen, pictures, learn, office, desk, rubbish, lesson, books, notes, computer, lines, page, letters, reading, journal, learning, study
7: now, not, yes, to, all, never, them, ever, you, why, finish, gone, must, again, think, here, then, no, that, near, more, often, other, nothing, go, so, soon, else, it, do, time, i , when, is, where, something, anything, there, yet, will, if, but, this, what, for
8: food, drink, eggs, bread, orange, cheese, soup, fat, eat, beer, cake, meat, tea, sugar, sweet, pudding, cooking, breakfast, chocolate, butter, jam, beef, stew, sweets, steak, fruit, cream, pie, sour, apple, milk, wine, dinner, cakes, sauce, meal, lemon, juice
9: car, train, fire, cars, stop, bus, bang, lorry, slow, road, fast, run, wheel, gun, speed, traffic, crash, motor, engine, wheels, travel, transport, vehicle
10: money, rich, buy, steal, thief, give, price, pounds, gain, bank, wealth, pound, goods, loot, shop, dollar, broke, miser, debt, poverty, theft, coins, pay, savings, spend, pennies, cost, cash
11: bird, dog, horse, animal, sheep, lion, cat, fly, animals, mice, cow, fox, cows, dogs, tail, tiger, farm, horses, hound, cattle
12: pain, death, blood, hurt, sad, doctor, hospital, anger, fight, fear, medicine, ill, sick, disease, dead, fright, cry, stomach, illness, knife, wound, scream, sore, hate 13: door, town, house, round, straight, carpet, houses, up, london, mountain, stone, wall, lift, wide, top, building, wood, bridge, circle, football, north, iron, climb, ball, out, ladder, high, roof, hill, stairs, game, flat, tall, floor, earth, down, brick, road, square, height, flats, hole, concrete, fall
14: water, sea, fish, river, bath, wet, beach, swim, wind, flow, deep, salt, rain, sand, stream, seas, shore, surf, sink, boat
15: light, cold, snow, smoke, sun, stars, glass, bright, heat, cancer, ice, windows, star, dirt, gold, mist, air, chemistry, shine, sky, soap, hot, clear, rain, dark, white, moon, winter, weather, glow, bulb, water
16: hair, shirt, foot, arm, trousers, legs, sharp, sew, smooth, dress, cloth, socks, clothes, face, nose, knees, head, shoes, brush, leg, holes, toe, skirt, coat, nail, cut, cotton, silk, wool, needle, shoe, hand, scissors, skin, clean, scarf, thread, stocking, lace, vest, sock, material
17: music, noise, dance, band, song, play, piano, record, ear, loud, beethoven, sound, violin, guitar, theatre, radio, voice, hear, player
18: sleep, bed, tired, lazy, chair, awake, quiet, rest, baby, night, comfort, easy, ease, comfortable, lie, soft, peace, sofa, relax, asleep, settee, sit, armchair, sleepy, pillow

TABLE 57
Edinburgh Associative Thesaurus: Stimulus Response. Correspondence between the clusters (described by responses) and a random subset of their instances (stimuli).

| 1: stupid, bad, love, happy, child, laugh, sex, funny, wrong, fool, mother, me,... | amuse, obstinate, corrupt, chuckle, humour, thankful, kindliness, honest |
| :--- | :--- |
| 2: church, god, religion, priest | catholic, prayer, lord, satan, fervour, hall, ceremony, praise, sins, abbey |
| 3: tree, flower, flowers, green, trees, plant, yellow, garden, grass, leaves,. . . | bile, hideout, flower-pot, lemon, danger, enlarge, sap, plant, birds |
| 4: ship, army, war, soldier, police, america, law, ships, boat, officer, judge,. . | barrister, hero, elizabeth, leader, cox, decker, rifles, powerful, bayonet |
| 5: correct, vote, look, blank, degree, difficult, word, people, make, talk,... | addition, therein, distribution, convention, organise, quaint, observe |
| 6: paper, school, book, picture, king, magazine, t.v., news, story, letter,... | bench, feature, digits, rhombus, tales, reagents, graph, copy, seller |
| 7: now, not, yes, to, all, never, them, ever, you, why, finish, gone, must,... | after, presences, compulsion, belonging, knowing, they, nearer, is |
| 8: food, drink, eggs, bread, orange, cheese, soup, fat, eat, beer, cake, meat,... | bars, swede, desert, cooked, peeler, rice pudding, oxo, empty, cellar |
| 9: car, train, fire, cars, stop, bus, bang, lorry, slow, road, fast, run,.. . | bang, fume, drivers, tongs, stuck, hire, conductor, triumph, dangerous |
| 10: money, rich, buy, steal, thief, give, price, pounds, gain, bank, wealth,... | beneficent, inherited, executive, yen, unemployed, mean, conned, decrease |
| 11: bird, dog, horse, animal, sheep, lion, cat, fly, animals, mice, cow, fox,... | biting, foul, egg, shepherds, tame, scraggy, hack, conjurors, soar |
| 12: pain, death, blood, hurt, sad, doctor, hospital, anger, fight, fear,. . | annoyed, war, fate, despair, prescription, skull, pincer, frightened |
| 13: door, town, house, round, straight, carpet, houses, up, london, mountain,... | base, ton, elevate, county, plan, semi-detached, path, floor, chisel |
| 14: water, sea, fish, river, bath, wet, beach, swim, wind, flow, deep, salt,... | boil, ice, fire, wind, mersey, depth, douse, seascape, resort, borne, slosh |
| 15: light, cold, snow, smoke, sun, stars, glass, bright, heat, cancer, ice,... | breaking, haze, fan, weight, vim, lenses, cough, detergent, shadows, polish |
| 16: hair, shirt, foot, arm, trousers, legs, sharp, sew, smooth, dress, cloth,... | blanco, yards, finger, dermis, scuff, snob, samson, grips, coarse, shirts |
| 17: music, noise, dance, band, song, play, piano, record, ear, loud, beethoven,... | brass, quietly, mike, song, grand, hearing, bugle, tuba, puppet |
| 18: sleep, bed, tired, lazy, chair, awake, quiet, rest, baby, night, comfort,. . | breathless, reverie, mistress, slothful, fatigued, homely, camp |


[^0]:    2. We use LIBLINEAR [69] for logistic regression with $\ell_{2}$-norm regularization, and GLMNET [70] for logistic regression with Lasso and Elastic-Net regularization.
[^1]:    13. Time for both feature subset generation and selection is logged in MATLAB on Mac OS X with a 2.8 GHz Intel Core i7 and 16 GB RAM.
[^2]:    14. http://www.nactem.ac.uk/robotanalyst
