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Structuring students' view of learning mathematics

Rösken, Bettina

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**Long-term Research
in the Didactics of
Mathematics and Science**

Proceedings of the FMSERA annual
symposium in Vaasa, October 27-28, 2006

*Lars Burman, Ole Björkqvist,
Ann-Sofi Røj-Lindberg (Eds.)*

Report from the Faculty of Education, Åbo Akademi University
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Cover photograph taken by Fredrik Åman

The photograph shows a long-term process in the Vaasa-region:
land rising from the sea, about one meter every hundred years.
The nearest stone shows the level of the water in the beginning of
the 16th century, the last stone in the beginning of the 20th century
and of course, the water itself shows the level of today.

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Preface

The Finnish Mathematics and Science Education Research Association (FMSERA) was formed at the national symposium on Research in Mathematics Education held in Turku, in August 1983. The first common symposium in the field of science and mathematics education research was organized in Jyväskylä in September 1987. The aims of FMSERA are to promote collaboration in mathematics and science education research between researchers in Finland as well as to strengthen links between researchers in Europe and similar communities elsewhere in the world.

The FMSERA annual symposium in 2006 was hosted by the Åbo Akademi University in Vaasa, Finland, from October 27 to October 28. The theme of the symposium was chosen to reflect the importance of research in mathematics and science education that has an impact making a long-term difference. This includes the way it influences educational practices as well as the theoretical fundamentals of mathematics and science education.

This volume contains two keynote presentations and nineteen papers from the conference in Vaasa, arranged in three sections, and within the sections in alphabetical order. The papers in sections two and three have been scientifically reviewed by peers. The review process was also chosen by one of the keynote speakers, although it was not required. Each author is responsible for the content of her/his text. The editors have only made corrections that increase clarity and minor adjustments in order to reach sufficient conformity in layout and language.

The editors, Lars Burman, Ole Björkqvist and Ann-Sofi Røj-Lindberg all work at the Faculty of Education, Åbo Akademi University, Vaasa, Finland.

We want to thank all participants for their valuable contributions and patient cooperation. We are also very grateful to lecturer emerita Lisen Häggblom for her contributions in the early stages of the editorial work. Last but not least, we thank Tarja Grahn-Björkqvist for her cooperation in making the manuscript ready for print.

We are particularly grateful to Stiftelsens för Åbo Akademi forskningsinstitut, Schildts Förlags AB and Söderströms & Co Förlags AB for their support, which made it possible to host the MALU conference at Åbo Akademi, Vaasa and to publish this volume.

The editors

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Keynotes

Classroom research: Impact and long term effect *versus* justice, liberation and empowerment?

Simon Goodchild
Agder University College

This paper considers the coherence between the research goals of making an impact and having a long-term effect with the researcher's concern to contribute towards justice, liberation and empowerment. The fundamental issue of how the researcher can be sure that her or his research will not result in injustice and oppression, and maybe worse, is considered and some criteria are suggested that might be used to guard against harmful effects. These criteria are then used to examine some key issues in classroom research, with illustrations from an example of the author's own classroom research into students goals.

Introduction

I believe it goes without saying that a basic principle of educational research is that it should make an impact. As Stenhouse (1979/1985) remarked:

Research may be broadly defined as systematic enquiry *made public*. (p. 120, my emphasis).

And more recently Bassey (1995) has asserted:

I believe a definition like this needs to be nailed to the door and printed on the letterhead of everyone who claims to be an educational researcher! **Educational research aims critically to inform educational judgements and decisions in order to improve educational action.** (p. 39, bold type in original)

But my title offers the suggestion of the possibility that making an impact might be contrary to the goal of contributing to justice, liberation and empowerment. The question that I want to address in this paper is: how can we ensure that our research into teaching and learning in classrooms makes a positive contribution to the development of mankind? My concern with the issues of justice, liberation and empowerment were stimulated recently whilst reading an account of the genocide that took place in Rwanda in the 1990s (Rusesabagina, 2006). Rusesabagina traces the interracial conflict in Rwanda back to some rather bad anthropology reported one hundred and fifty years earlier by the British explorer, John Hanning Speke, this is the man who is credited with identifying the source of the river Nile. I was struck by the train of events and injustices that

were linked to Speke's journal and I reflected on my own work. How can I be sure that my research activity will not eventually lead to dreadful consequences? How can I ensure that any impact from my research will be for good?

The simple answer to this question is that I must ensure that my research attains the *highest standards of scientific rigour*. In this paper I want to share my thoughts about what 'highest standards of scientific rigour' might mean in the context of classroom research.

Ethical characteristics of educational research

Before considering the scientific standards of classroom research I want to suggest some ethical characteristics of educational research which I believe are essential if any impact or long term effect will also contribute to justice, liberation and empowerment? [Note, I am using the word characteristics rather than 'standards' here because my intention is not to argue a set of ethical standards, this has been attempted elsewhere, (e.g., British Educational Research Association, 2004). My intention is merely to identify some of the things that I can apply to the key issues of classroom research.] These characteristics are, in no particular order: honesty, openness, critical reflection, rationality, impact on informants/participants, and voice.

- *Honesty*: in reporting data and evidence; in working with our informants and participants; in recognising the limitations of any claims to knowledge.
- *Openness*: in reporting all issues and being prepared to admit error and failure; to criticism and suggestions.
- *Critical reflection*: in examining everything done at each stage of the research, to try to expose better ways of doing things, alternative explanations and interpretations, and trying to expose weaknesses and limitations; to ensure awareness of how our own knowledge, experience values, attitudes and emotions shape the research process.
- *Rationality*: in identifying the reasons for what we do, in terms of our existing knowledge, existing theory, research questions, data collection, interpretation and style of reporting.
- *Impact on informants/participants*: this is normally the main focus of an ethical risk assessment; everything already mentioned is part of an embracing 'ethical framework' but there are other issues regarding the impact upon the well-being of our informants/participants.
- *Voice*: in making sure that the authentic voice of informants and participants is audible through the reporting of the research.

Classroom Research

I worked for 16 years in secondary schools before moving into higher education. I then worked for a further sixteen years training secondary teachers. My whole professional life has been focused on the activity that takes place in mathematics classrooms. It follows that if I have anything to offer the wider community it will surely be from my experience and knowledge of teaching and learning mathematics in school. Consequently, all that I present here is related primarily to mathematics classroom research. I hope that all I discuss is relevant to classroom research in general, irrespective of the subject, but I do accept that the subject matter is a significant focus of classroom research and this will inevitably mean differences in the research carried out.

I want to emphasise that I do not believe there is a ‘specialism’ that might be called ‘classroom research’. Educational research includes classroom research, good classroom research is good research and the principles of classroom research are the same as the principles of any other type of educational research. On the other hand I do think classrooms are very special places in which to do research and they present researchers with great challenges, not least because of the complexity of the classroom. As Ernest (2001) observes:

the mathematics classroom is a fiendishly difficult object to study. For the mathematics classroom involves the actualised relationships between a group of students ... and a teacher ... with a variety of material and semiotic resources in play within a set of temporally and geographically delimited spaces. Furthermore, each student, teacher, classroom, school and country has a life history with antecedent and concurrent events and experiences which impinge on the thin strand chosen for study within all this complexity: periodic mathematics lessons. (p. 7)

This complexity requires us to be very careful in researching classrooms, it is too easy to make mistakes and draw wrong conclusions, and make an impact that is contrary to the aims of justice, liberation and empowerment. Shulman (1987) summarises one concern:

When policymakers have sought “research-based” definitions of good teaching to serve as the basis for teacher tests or systems of classroom observation, the lists of teacher behaviours that had been identified as effective in the empirical research were translated into the desirable competencies for classroom teachers. They became items on tests or on classroom-observation scales. They were accorded legitimacy because they had been “confirmed by research.” While the researchers understood the findings to be simplified and incomplete, the policy community accepted them as sufficient for the definitions of standards. (p. 6)

I am not sure that we can ever protect our work from misuse; my first concern is that anything that I report is trustworthy. In the consideration of the foundations of trustworthiness, and the ethical characteristics that I have listed, I want to address the following issues in relation to classroom research: types of

classroom research, paradigm, theory, unit of analysis, methodology, operationalization, method, disturbance, and concern for informants.

In addressing these issues I will make reference to my own ethnographic style case study of a year ten mathematics classroom. Briefly, I joined the class for every mathematics lesson for very nearly one complete year; I had conversations with the students while they were engaged in the tasks given to them by their teacher. My purpose was to expose the “Students’ Goals” (Goodchild, 2001) in their classroom activity.

I start from the assumption that classroom research can be any inquiry into what happens in classrooms, into teaching and learning, and into teachers’ and students’ experiences, values, beliefs, attitudes etc. The data collection may take place wholly within classrooms – such as with observation, or wholly outside, as in the completion of questionnaires, or, indeed, part in and out as with design research. In my own experience I have engaged in action-research within my own classroom, I have explored teaching and learning through observation, and engaged in curriculum evaluation – using pre and post tests. Currently, I am working with teachers in a development-research project.

Paradigm

A researcher needs to be conscious of the paradigm within which he or she is working. It is necessary to be clear within oneself whether, for example one believes in unproblematic cause-effect relationships that can be modelled in generalisable laws as in a scientific/positivist paradigm, or conversely that human behaviour is so complex and dynamically related simultaneously to a range of social, historical, emotional and physical phenomena that such generalisable rules are unknowable, even if they exist. As Lincoln and Guba (1985) summarise:

The possibility of causal linkages

Positivist version: Every action can be explained as the result (effect) of a real cause that precedes the effect temporally (or is at least simultaneous with it).

Naturalist version: All entities are in a state of mutual simultaneous shaping so that it is impossible to distinguish causes from effects. (p. 38, italics in original)

It will become apparent that in my research into students goals I was heavily influenced by Lincoln and Guba’s work. One of the weaknesses that I now recognise in my own work was that I was not sufficiently critical of their work and allowed it to exert an undue influence in all that I did. Researchers have a responsibility to understand the philosophical foundation upon which they base their claims for knowledge and to reflect on these and consider the consequences

in practice. Researchers need to be clear about the paradigm within which they are working and that of any interlocutor because, the chances are that when one engages in dialogue with a researcher working in a different paradigm the likelihood is that a mutual understanding will be unattainable and any sense of agreement will be illusory.

Nevertheless, it is necessary also to act within a given paradigm in a critical fashion. Pring (2000) recognises that different social groups interpret the world differently, (I assert that this difference is a fundamental issue to be considered by classroom researchers). Nevertheless, Pring argues that we can only be aware of and understand these differences because of their 'enduring features' which 'enable generalizations to be made' (p. 56). He asserts:

The qualitative investigation can clear the ground for the quantitative – and the quantitative be suggestive of differences to be explored in a more interpretive mode.

Understanding human beings, and thus researching into what they do and how they behave, calls upon many different methods, each making complex assumptions about what it means to explain behaviours and personal and social activities. (pp. 56-57)

I find it reassuring that an eminent educational philosopher argues so clearly for a stance that I feel, from the perspective of research practice, is sensible.

Theory

Theory is critical to the production of research knowledge, and to work more generally. (Boaler, 2002, p. 4)

I place my own research in a naturalistic paradigm, and I pursued and critically reflected on my work in exploring students' goals within a framework set out by Lincoln and Guba. Ten years on from when I completed that work I am rather more critical of the framework, especially when related to classroom research. For example, Lincoln and Guba (1985) argue for a grounded theory approach asserting:

N (the naturalist) prefers to have the guiding substantive theory emerge from (be grounded in) the data because no a priori theory could possibly encompass the multiple realities that are likely to be encountered ... (p. 41, emphasis in original)

I certainly agree that theory must be grounded in data and tested empirically. Empirical evidence is essential in the verification of theory. However, I am sceptical of research based on 'the *discovery* of theory from the data' (Glaser & Strauss, 1967/2006, p. 1, my emphasis). I must emphasise that my critique here is focused on the notion of '*discovery*' that seems to ignore the existence of informal theory that resides subconsciously in the mind of the researcher. I believe that in classroom research it is fundamentally necessary to start from a

priori, guiding, substantive theory. It is impossible to come to classroom research as, in Lave's (1988) description of ethnographers, 'ignorant strangers':

Ethnographers are nonmembers of the cultures they study, being observant strangers whose ignorance they themselves take to be a condition for eliciting from informants explicit accounts of the obvious and basic aspects of culture and everyday practice. (p. 185)

Apart from the small number of people taught at home or by a private tutor everyone has a range of experiences in the classroom. If we do not come to the classroom with a consciously held and clearly articulated theory then we come with a subconscious theory – theory helps the researcher to 'see' through the filter of her or his own preconceptions.

Additionally, as we approach our inquiry into classrooms from a theoretical perspective we can be more sure of making a contribution to the development of scientific knowledge, that is, we avoid the creation of producing discrete pieces of information that are not connected to the wider body of knowledge. Theory also helps us to focus on the issues that concern our research and professional community.

A good theory should help researchers understand what is going on in the classroom, it is essential for framing the research, it is essential in interpreting the evidence, it is essential in the development of knowledge. Consequently, I treat with suspicion research reports that 'excuse' an apparent denial of existing relevant theory by claiming a 'grounded theory' approach. I want reports to be clear – if it is meant that no explicit theory has guided the research then perhaps my time will be better spent elsewhere. If on the other hand it means that the testing and development of theory is grounded within the data then I want to applaud – but I do want that theory to be clearly articulated in the report.

This raises questions about what theory is appropriate to classroom research. In my research into students' goals I took three separate theoretical perspectives. I argued that my familiarity with the classroom, and my own personal theory of teaching and learning needed to be enlightened and challenged by trying to view and explain things from different perspectives. I chose to take a social constructivist perspective, an activity theory perspective, and a situated cognition perspective. I acknowledged the inconsistencies between these perspectives and avoided any suggestion of trying to combine them into one 'super theory'. In a way the approach was successful, the different perspectives allowed me to focus on different levels within the classroom, the individual student, the student in socially mediated activity, the student as a member of a community of practice. These different foci led to complementary accounts of what was happening in the classroom. However, ten years on, I believe the approach was naïve and possibly a lazy way of engaging with the complexity of the classroom. I no longer believe that it is necessary to use several theories, no matter how complex the classroom, each theory is sufficient in itself to address the range of individual, social and cultural issues. If I had used just one theory,

perhaps my work would have been stronger and made a greater ‘scientific impact’, nevertheless, I think what I attempted is original and has some merit in that. However, Boaler (2002), in the same article from which the quotation at the head of this section is drawn, argues for the knowledge generating potential of drawing connections between theories – using Andrew Wiles approach to solving Fermat’s last theorem as an example. Perhaps my work allows such connections to be made and there is greater value in the use of complementary theories than I have argued.

Unit of analysis

The discussion of theory naturally leads on to the unit of analysis. A unit of analysis is ‘the minimal unit of “evidence” that preserves the properties of the whole’ (Davydov & Radzikhovskii, 1985, p. 50) object that is being studied. So if we are considering the properties of ‘classrooms’, that is, of teaching and learning mathematics in classrooms, it is necessary to have a unit of analysis that makes possible a study of the complexity of relationships between teacher, pupils, resources, history, culture, etc. The complexity of the classroom is a challenge to classroom researchers, and it challenges us in our choice of a suitable unit of analysis.

In my work on students’ goals I used three units of analysis, one relating to each of the theories I was using. Within the social constructivist theory I used Neisser’s perceptual cycle (Neisser, 1976) in this the individual’s mental schema directs his or her exploration which samples the object or available information (a mathematical task or problem, say) as a result of these processes the individual modifies his or her schema. Most components of this unit of analysis are hidden from view, in particular the individual’s schema and the processes of direction and modification. Those elements concealed from the direct exploration of the researcher have to be inferred by careful and critical examination of the range of evidence relating to the individual’s action. Within activity theory I used Engeström’s (1987) model of an extended activity system. In this it is the dialectical mediation of tools and signs, community, rules and division of labour that come between the individual and the object of his or her activity that are the focus. From the perspective of situated cognition I used Lave’s (1988) model of dialectical relations between students acting, the classroom arena, students in activity and the task setting. Each model characterises the student’s activity differently and draws attention to different features of the context of his or her work on a task. Nevertheless, it is the same data that is used, transcripts of conversations, student’s writing, copies of resources used within the classroom, etc. from these pieces of information a theoretical account of the classroom can be developed, and the theory can be challenged – when events and observations defy explanation. Each unit of analysis is a product of the theoretical perspective, and as with the theories, I did not try to combine the units of analysis.

It bothers me when I read in a research report, that is taking a *single* theoretical stance, of a number of different ‘units of analysis’. Either the unit of analysis is ‘the minimal unit of evidence’ or it is not.

Methodology

Methodology is much more than method. Ernest (1994) defines methodology as:

A theory of which methods and techniques are appropriate and valid to use to generate and justify knowledge, given the epistemology (of the research). (p. 21)

When one engages in research, it is necessary to make decisions about what data to collect and how to collect it and analyse it. These decisions will rest on a number of issues including: the paradigm and theoretical framework; the nature of the research question; what research has been done in this area before; our understanding of the nature of the subject – or our research participants or informants; what possible obstacles might exist for collecting trustworthy data in an ethically justifiable manner and so on.

I want to credit Leone Burton (2002) for opening my eyes to these issues, she writes:

I am asking for researchers to be clear to themselves about the values, beliefs, and attitudes that are driving the study that they propose to do and to make that clarity visible to the reader. ... Second, I am making the assumption that research in mathematics education is emancipatory in that its intentions are to empower – pupils, teachers, curriculum designers, policymakers – those who could be users or affected by use of the research. (p. 4)

... the choice of which method(s) is best, in order to gather the data necessary to the exploration of a particular question, is always a function of the theoretical stance adopted by the researcher(s) together with, of course, the research context and the related research questions, the informants, and so forth. This, in itself is a product of the attitudes, beliefs, and values underlying that stance. (pp. 7, 8)

Operationalization

By operationalization I mean the translation of theoretical constructs into phenomena that can be observed and thus, used as evidence within the inquiry. Here again, in my research into students’ goals, I followed Lincoln and Guba (1985) who assert that operationalization is:

not meaningful or satisfying ... too shallow – depending on sensations (not meanings or implications ... results in a meaningless splintering of the world (pp. 26, 27)

In ‘students’ goals I provided working definitions of the ‘goals’ I was seeking evidence of, but I did not operationalize these by articulating beforehand how they would emerge in the data I would collect, or later before starting analysis.

On the one hand, this meant that the account of students' goals was created by my informants (and my interpretation!) rather than constrained by preconception. Additionally, the approach avoided Brousseau's topaz effect of which Jaworski (1994) writes:

Beware the topaz effect – 'the more explicit you are about what you want, the more likely you are to get that because it's perceived that you want it, not because it is actually the case'. [(Jaworski's) paraphrasing of Brousseau, 1984]. (p. 139)

[It should be noted that for Jaworski the point is a didactic issue, that 'getting it turns 'it' into a mechanistic routine, rather than the deeply thoughtful learning process that is desired' (personal communication, October, 26, 2006)]. From my perspective, as researcher the issue is about trustworthiness, if I lead my informants/participants so carefully that they 'give' me what I want, then I am only getting my own 'theory' confirmed. However, when I eventually set about analysing the data I do not believe the working definitions were sufficient to overcome my own, mostly subconscious and maybe inconsistent, notions of what might constitute evidence. The working definitions were not sufficient to transform the utterances in the conversations into clearly identifiable evidence of students' goals, the transformation took place largely within my own reflective activity and it was only after the analysis that I felt able to provide sharp definitions. Thus, there is circularity in my interpretation of the data and I fall prey to the same criticism that I level at those who take a strictly grounded theory approach to their research.

In research we seek, as far as is possible, trustworthy accounts that balance the objectivity provided by theory with the acknowledged and known subjectivity of the researcher. The key issue for the researcher is to make his or her subjectivity explicit and avoid subconscious or unconscious application of preconceptions and prejudice. I now believe, in my research into students' goals, that my failure to operationalize, explicitly, the constructs I was seeking, undermines my claims for the trustworthiness of my interpretation.

Method

The next question, that I want to address, concerns whether some research methods are more appropriate to classroom research than others. The simple answer is 'no'! My remarks on methodology form the basis of my assertion that the method adopted will be determined by a range of factors including the theoretical framework, the question to be researched and the nature or characteristics of the informant. Nevertheless, my use of Lave's description of an ethnographer suggests to me that classroom 'ethnography' is not a reasonable option – even though I admit Woods (1990) ethnographic account of pupils in secondary school is powerful and informative. I describe my own study of students' goals as 'ethnographic style', and there are other notable research reports in mathematics education that can be similarly characterised, for example

Jaworski's *Investigating Mathematics Teaching* (1994), and Boaler's *Experiencing School Mathematics* (1997). It is evident that the techniques used in ethnography have the potential to answer the questions we ask about activity in classrooms.

We have to admit that we as researchers act as filters and make decisions that will impact upon the nature and quality of the evidence we use – even before we begin to analyse the data and make interpretations. In researching students' goals most of my data arose through conversations I had with students or from discussions between students when I was present. I can only wonder what the discussions between the students would have been like if I were not present. I made a decision to collect data from the students while they were engaged in their tasks because, I argue, if I had collected it outside the classroom it would be 'coloured' by their rationalisations – it seems that I am suggesting that my presence had a smaller impact on the goals I was able to expose than their own reasoning or rationality. In my present work, more of which later, I am using 'naturally occurring data', that is recordings of events that are made in the course of meetings and workshops without the intervention of activity done purely for the research element of the project. But there is a vast amount of data and I am making choices about which pieces to use and how it will be analysed, thus, my influence is still present.

The point I want to make is that, in classroom research it is necessary to consider the position and voice of the teacher and students, and, naturally, the subject content of lessons. Wagner (1997) identifies three types of relationship between researchers and teachers. Data extraction agreements, where the researcher goes into the field, collects data, and returns to her or his office to analyse and interpret. Clinical partnerships, in which teachers share in the research activity in both deciding what will be done and reflecting on the outcomes, but the emphasis is on learning about what happens in classrooms, about teaching and learning – the purpose is for the researcher to inform her or his scientific community and policy makers, and the teacher about practices in the classroom. The third type is a co-learning agreement in which the researcher enters the field alongside the teacher and acknowledges that the joint activity with the teacher has the potential to inform the research about her or his role as much as the teacher about classroom practices.

In my research into students' goals my relationship with the teacher was that of a data extraction agreement. I remain deeply grateful to the teacher and students for allowing me such freedom in their classroom but as far as I can recall, they showed little interest in what I was doing with the data I was collecting or what I was learning – and I did not try to interest them. The teacher did read the final report and made some comments but I am not aware that it had any impact upon her practice – I don't know! And that is the point.

In classroom research the place that, perhaps, the research has the potential to make an impact on are the classrooms, and the practices of the teachers, who

collaborate in the research. In this I mean a symmetrical relationship in which teacher and researcher come together, each with their own experience and specialised knowledge, each prepared to learn about their own practice in addition to learning about teaching and learning. That is, in a co-learning agreement, anything short of this is making ‘use’ of another person and their practice, and the researcher ends up taking away or changing the voice of the teacher. A clinical partnership has the potential to empower a teacher in her or his practice, albeit with the teacher and researcher in asymmetrical relationship and the teacher being given the researchers’ voice. A co-learning agreement has the potential to empower and the teacher’s voice to be heard.

As an example, I am currently working with a very experienced and talented primary school teacher. I am learning a great deal about his practice and the way that he manages teaching and learning in his classroom. We have written one paper together and I hope this will be the first of many. In the paper I reflect on what I am learning from this classroom, I describe events that I see and how I interpret these from my perspective as an experienced secondary teacher. However, the teacher I am working with has the opportunity to ensure that his own voice is heard and the balance of reporting fits with his view of his classroom. In fact he is in a powerful position because even though I have done the greatest part of the drafting of the paper he has translated it into Norwegian. I have a basic understanding of Norwegian but it is insufficient to judge the nuances of language that can make so much impact on meaning.

Disturbance

Is inevitable! Almost any form of data collection is likely to disturb the subject. I have remarked that I am now using naturally occurring data, this is available because we record, audio or video, every event that takes place within the project. Within our project we have become so accustomed to the presence of a recorder or camera that we forget it is there – I guess, up to the moment that we begin to think very carefully about what we say because we know there will be a recording!

Gallagher (1995) writes about an instance in her research when a student asked her for help, she struggled with her role as ‘detached observer’ and her feelings as person and teacher. She decided to help the student. Following the event she felt that she had violated the tenets of good research and resolved to remain detached in future. Gallagher goes on to regret that decision believing that she might have been able to have a positive impact on the teacher’s practice. In researching students’ goals it was inevitable that I would disturb the students, given that I was sitting next to them and talking to them during the course of their activity. I also found it very difficult to ‘switch off’ the teacher within me, so at times my ‘research conversation’ took the form of a ‘teaching conversation’ in which I seem as interested to expose to the student the

mathematics in which they are engaged as I am to expose for myself the goals they are working towards.

Wiske (1995) reports from research in which teachers and researchers worked together in clinical interviews of students and describes how the teacher's view of the interview differed from the researcher's.

The teacher regarded the clinical interview as an educational experience for the student. She wanted the child to be treated as the teacher would have treated her in class, not allowed to feel stupid or discouraged by a prolonged period of ignorance unlike anything the teacher would willingly sustain in class. (p. 203)

My sympathy here is with the teacher. I think it is an abuse of our informants or participants and the trust that they put in us when they agree to take part in our research, if our actions lead them to feel 'stupid' or 'discouraged'.

While researching students' goals I felt reasonably pleased with myself that the students did not view me as a teacher. I recall, for example, an occasion near the beginning of my field work when the class had a substitute teacher, we were relocated in an unfamiliar and uncomfortable room which unsettled the class and resulted in an unusual degree of poor behaviour. It seemed to make no difference to the students that I was sitting next to the paper aeroplane production and launch site. Later in the year, towards the end of my field work, a student asked me if I was intending to become a mathematics teacher. However, my feelings of satisfaction were diminished very near to the end of my work. I had analysed 90 % of the data and produced my account of students' goals. I then turned to the 10 % of data that I had selected randomly for archiving and later testing the theory I built. One of these archived conversations took place in the first lesson in which I started talking to the students. In this conversation I observed a new phenomenon. In all the conversations I had used to build the theoretical account of the classroom, I had been able to identify goals towards which students were working. However, in this archived conversation I could find no evidence of any goal, in fact it seemed that the student had no goal, because prior to the conversation the student had been inactive. One conclusion I drew from this was, that my presence in the classroom had sharpened the students' awareness to the possibility of me coming to them and asking them about what they were doing and why. I believe, part of the honesty that I assert underpins research that contributes to justice, empowerment and liberation is to admit the possibility that we may have it wrong! Unfortunately, this does not go down too well with the policymakers who want definite answers, hard facts and relationships. To admit to the possibility of error weakens our case and undermines the impact that our research might have.

Regard for informants and participants

The whole of this paper is, I believe, an ethical statement. But there are a few recognisably ethical considerations that I want to add. First, we take for granted

that we must seek permission from our informants before we collect data of any kind and we are careful to explain how the data might be used. I wonder how aware teachers and students are of the risk they are taking when they give permission for research to take place. Teachers in particular: the research might expose parts of their practice that make them feel uncomfortable and perhaps even a loss of confidence; I think this is almost inevitable when we lift the lid on a person's practice and examine carefully what is going on. Some research entails the teacher trying something new; this also puts the teacher at risk because it moves her or him from their established patterns of behaviour. When we seek permission for research from teachers, I wonder how careful we are to offer them a risk analysis – and if we did, whether they would still participate.

One of the realities of our lives as researchers is that our work is often determined by the time frames of short-term funded projects. Within a project we establish a working relationship with teachers, complete our research agenda and then have to walk away because the funds no longer exist to sustain the relationship, we have to turn our attention to the next project. I do not think this is being fair to the teachers and their students, unless we build into the project some mechanism by which any impact of the research may be sustained after the project ends.

Cooper and McIntyre (1996) offer three characteristics of the behaviour of researchers towards informants: being willing and able to empathise with informants; unconditional regard for participants, that is liking and being interested in informants as individuals; and congruence, that is the researcher is honest and authentic in his or her relationships with informants. I would like to believe that anyone involved in classroom research will have had experience as a teacher which will ensure empathy. I know that during my 16 years teaching in school I will have demonstrated both good and mediocre practice, things I am proud of and things I am ashamed of. And it is not the case that all the bad came at the beginning and the end was only good. The opportunity to reflect on one's own practice in the past should prevent one being judgemental of another teacher in the present. I think if one does not have a liking and interest in teachers and students then one should not even begin to research classrooms. Being honest and authentic also requires some humility, to be clear about one's own mistakes and weaknesses.

Voice

All that I have argued so far needs to be reported with the same care as the research is carried out, thus the report will be characterised in the same way by honesty, openness, critical reflection and 'voice'. I want to say something in particular about giving the teacher and students 'a voice'. Keitel (2004) writes about using my report of Students' Goals with a group of students on an initial teacher training course:

I confronted [a] group with some of the excerpts of students' interviews from Goodchild's study and asked them to recall their own experiences in school time and search for similar situations, or make notes about other students' experiences. As they usually are asked to write a biographical essay, they complemented their excerpts with examples of stories about getting stuck, complete lack of understanding, hard debates with other students about getting meaning and significance of a given task and so on. In short, the excerpts encouraged them to look for their own stories and remember their school mates' struggle as well as their own. They were mostly fascinated that a researcher and teacher are giving students a voice and really were interested to listen to them – they argued that this might be an almost necessary condition for becoming a teacher, but almost nobody among them had experienced such a 'listening' mathematics teacher. (p. 276)

Naturally, I am pleased to see the report being used, but I do wonder if I am really 'giving students a voice' as suggested. I think it is more likely that I am using their voice to convey my story. I made selections from my data, using those bits that I thought were interesting and 'useful' for my account. I presented these within the context of my account, with my interpretation alongside. I did the same with the teacher, presenting my account of the classroom using extracts of the teacher's explanations from an extended interview I had with her when I had completed the classroom observations. This has the appearance of giving the teacher and students a voice but in reality I am using their voice to speak my meaning. Perhaps this is more dishonest than giving no voice at all.

Sometimes we will be allowed into classrooms and observe a consistency of bad practice that causes us alarm. For example, we might observe students getting a severely impoverished experience of mathematics and a teacher struggling with subject content knowledge, or pedagogical content knowledge. What do we do in these cases? If our relationship with the teacher is of the data extraction type, as it was with students' goals, I think there is little that we can do. The teacher did not allow me into her classroom to criticise her practice and it would have been a violation of our agreement if I had. If I went past the teacher and described her poor practice publicly then the chances are I would never be allowed into another classroom. This provides me with one more reason for entering into at least a clinical partnership with a teacher and ideally a co-learning agreement, in which both I and the teacher engage in the activity prepared to learn about our own practices.

Conclusion

Returning to the title of this talk, in classroom research I do not believe it is possible to engage in respectable research that will contribute to justice, liberation and empowerment and produce easy recipes for teaching and learning that are likely to appeal to policy makers and thus make a substantial or long term impact. In this respect I can see that impact and long term effect are possibly contrary to the goals of justice, liberation and empowerment.

Nevertheless, I do think it is possible to have an impact, immediately and in the long term, not as I did with my research into students' goals, but rather by entering into co-learning agreements with teachers and working with them in developmental research as in the Learning Communities in Mathematics Project (LCM), in which I am now engaged. [LCM is supported by the Research Council of Norway (Norges Forskningsråd): Project number 157949/S20]

LCM is a development-research project in which the aim is for didacticians to enter co-learning agreements with teachers. The project is established on a notion of 'inquiry as a way of being' (Jaworski, 2004, p. 26). Inquiry is seen to be empowering. At the heart of the development part of the project the aim is for teachers, working in school teams to design tasks. The teachers' role in the design process respects their knowledge. A condition of joining the project was that at least three mathematics teachers in any school should participate; this is part of our attempt to ensure sustainability beyond the end of the project.

We aim to facilitate teachers expressing their own voice. One such instance is the joint article I have mentioned earlier, and at the moment we are busy preparing a project book which will include chapters written by school teams as well as didacticians – and some eminent mathematics educators who have made a special contribution to the project. At the beginning of September this year, as we came to the end of the second phase of the project we held a major conference with participants coming from all parts of Norway, teachers and didacticians made contributions. The conference helped to convince the didacticians that the project was having an impact on the professional practice of the teachers. It also gives us hope that the impact might be felt beyond our small project group.

We are now entering the third phase and final year of the project. One aim this year is to consolidate the developments that are taking place in schools, but we have been pleased to be given more funds from the Research Council of Norway for a new project which aims to widen the scope in two ways. First there is the possibility of more schools being drawn in, but also the new project is based on a consortium of mathematics educators in university colleges in different regions of Norway, who will pursue their own projects within the common theoretical framework of communities of inquiry that has been established within LCM.

In these projects I want to assert that it is possible to achieve impact and long term effect alongside the goals of justice, liberation and empowerment.

References

- Bassey, M. (1995). *Creating education through research: A global perspective of educational research for the 21st century*. Edinburgh, Scotland: British Educational Research Association.

- British Educational Research Association (2004). *Revised ethical guidelines for educational research*. Retrieved March, 01, 2006, from <http://www.bera.ac.uk/publications/pdfs/ETHICA1.PDF>
- Boaler, J. (1997). *Experiencing school mathematics: Teaching styles, sex and setting*. Buckingham, England: Open University Press.
- Boaler, J. (2002). Exploring the nature of mathematical activity: Using theory, research and 'working hypotheses' to broaden conceptions of mathematics knowing. *Educational Studies in Mathematics*, 51, 3-21.
- Brousseau, G. (1984). The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics. In *Theory of Mathematics Education Occasional Paper 54, November*, (pp. 110-119). Institut für Didaktik der Mathematik IDM, Universität Bielefeld.
- Burton, L. (2002). Methodology and methods in mathematics education research: Where is the why? In S. Goodchild & L. English (Eds.). *Researching mathematics classrooms: a critical examination of methodology*, (pp. 1-10). Westport, CT: Praeger.
- Cooper, P. & McIntyre, D. (1996). *Effective teaching and learning: Teachers' and students' perspectives*. Buckingham, England: Open University Press.
- Davydov, V. V. & Radzikhovskii, L. A. (1985). Vygotsky's theory and activity-oriented approach in psychology. In J. V. Wertsch (Ed.). *Culture, communication and cognition: Vygotskian perspectives*, (pp. 35-65). Cambridge, England: Cambridge University Press.
- Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki, Finland: Orienta-Konsultit.
- Ernest, P. (1994). *An introduction to research methodology and paradigms*. Exeter, England: University of Exeter School of Education Research Support Unit
- Ernest, P. (2001). Foreword. In S. Goodchild. *Students goals: a case study of activity in a mathematics classroom*, (pp. 7-8). Bergen, Norway: Caspar Forlag.
- Gallagher, D. J. (1995). In search of the rightful role of method: reflections on conducting a qualitative dissertation. In T. Tiller, A. Sparkes, S. Kårhus & F. Dowling Næss (Eds.). *The qualitative challenge: Reflections on educational research*, (pp. 17-35). Bergen, Norway: Caspar Forlag.
- Glaser, B. G. & Strauss, A. L. (2006). *The discovery of grounded theory: Strategies for qualitative research*. New Brunswick, NJ: Aldine Transaction. (Original work published 1967)
- Goodchild, S. (2001). *Students goals: A case study of activity in a mathematics classroom*. Bergen, Norway: Caspar Forlag.
- Jaworski, B. (1994). *Investigating mathematics teaching: A constructivist enquiry*. London: Falmer Press.
- Jaworski, B. (2004). Grappling with complexity: Co-learning in inquiry communities in mathematics teaching development. In M. Johnsen Høines & A. B. Fuglestad (Eds.). *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education: Vol. 1*, (pp. 17-36). Bergen, Norway: Bergen University College.

- Keitel, C. (2004). Book review. *Journal of Mathematics Teacher Education*, 7, 269-277.
- Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge, England: Cambridge University Press.
- Lincoln, Y. S. & Guba, E. G. (1985). *Naturalistic inquiry*. London: Sage.
- Neisser, U. (1976). *Cognition and reality: Principles and implications of cognitive psychology*. San Francisco: W. H. Freeman and Co.
- Pring, R. (2000). *Philosophy of educational research*. London: Continuum.
- Rusesabagina, P. (with Zoellner, T.) (2006). *An ordinary man: The true story behind 'Hotel Rwanda'*. London: Bloomsbury.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57, 1-22.
- Stenhouse, L. (1979). Research as a basis for teaching: inaugural lecture, University of East Anglia, February 1979. In L. Stenhouse (1983). *Authority education and emancipation*, pp. 177-195. London: Heinemann Educational. This print from J. Rudduck & D. Hopkins, (Eds.) (1985). *Research as a basis for teaching: readings from the work of Lawrence Stenhouse*, (pp. 113-128). London: Heinemann Educational.
- Wagner, J. (1997). The unavoidable intervention of educational research: A framework for reconsidering researcher-practitioner cooperation. *Educational Researcher*, 26, 13-22.
- Wiske, M. S. (1995). A cultural perspective on school-university collaboration. In D. N. Perkins, J. L. Schwartz, M. M. West & M. S. Wiske (Eds.). *Software goes to school: Teaching for understanding with new technologies*, (pp. 187-212). Oxford, England: Oxford University Press.
- Woods, P. (1990). *The happiest days? How pupils cope with school*. London: Falmer Press.

A reflection upon science education research in the Swedish context

Helge Strömdahl
Linköping University

With the essential question of communication in learning and teaching scientific knowledge as a background, some glimpses are made upon the situation of science education research in the Swedish context. The establishment of the National Graduate School in Science and Technology education (Forskarskolan i Naturvetenskapernas och teknikens didaktik, FontD) have meant a tremendous step forward for the research in this field in Sweden. The reception of research results in school practice is, as in other countries, a persistent issue to consider. Some comments on the implementation process in Swedish school practice are stated.

Introduction

It has been a great challenge for mankind to find out the grand scientific theories about motion, the atom, the genetic code, evolution, etc., theories that are not evident from a mere unguided inspection of Nature. If scientific knowledge were given by direct perception, the history of science would be void. But history shows us a painstaking struggle to make fortunate modelling of natural phenomena and objects; not at least due to the fact that the scientific description of Nature often is contradictory to common sense (Wolpert, 1993).

Under these circumstances it is not astonishing that pupils have problems to learn science during some few lessons and a short time of laboratory work for each science content area.

Additionally there is still a widespread pedagogical idea that students through own unguided investigations, learning by doing via direct contact with Nature, are able to re-discover scientific knowledge that have taken mankind thousands of years to attain.

However, this exaggerated belief have been criticised some twenty years ago by e.g. Driver (1983) in her book "The pupil as a scientist". The criticism is repeatedly confirmed by empirical investigations, e.g. in Sweden by Säljö and Bergqvist (1994) in their study of pupils' laboratory works in optics. Without guidance of the use of e.g. the 'optic bench', it is a concealed enigma. As we know from general results of research in science education, the importance of the teacher as an expert guiding the students into the theories of science is

indisputable. Science education is an enculturation of the learner into the great building of science and not primarily a general and free study of Nature. Sutton express this fact by saying “Science itself may be a study of nature, but science lessons should be the study of what people have said and thought about nature” (Sutton, 1992, p. 92). In other words, to learn science is more a question of interpreting scientific texts about Nature than consulting Nature itself (cf. Kuhn, 1979, p. 136). However, this statement should not be misinterpreted as dethroning laboratory work or excursions in school practice, but recognising how to use them in connection to the theoretical network of science. From this general perspective the importance of language use and communication in science education is evident.

Even if we talk the same language there are large variations in how we express ourselves and understand words and phrases. This applies to verbal information, pictures and physical experiences. In order to understand communication about scientific knowledge we need insights into pupils’ and teachers’ linguistic abilities and how they can be developed. These insights are especially important in the teaching of science since its language is often analytical and abstract. The abilities to understand science can in several contexts be more dependent on how information is formulated than by any inherent difficulty in the content-matter.

It is not unusual that simplifications or attempts to link scientific phenomena to everyday life lead to abstractions which only make them more difficult to understand. One such example is to be found in a Swedish textbook for the secondary level. This book states that a cheese sandwich contains solar energy. This could be very puzzling to the student if we bear in mind that with the same logic, petrol is to be seen as an example of a substance containing solar energy. Scientific concept-formation contains many examples of how linguistically complex descriptions of phenomena and objects in the school context are condensed into one or a few words. These words are then regarded as facts and are uncritically learned. It is also often the case that scientific *representation* is seen and communicated as being synonymous with what is to be represented. An impression is given that scientific representation directly depicts nature. Different types of images are often used in science and technology education as an aid to understand or to explain that which is difficult or impossible to explain only in words. Phenomena and objects can be visualised with the help of models, illustrations, graphs and tables. However, the uninitiated observer is sometimes lured into thinking that the image is a true illustration of reality. In this way scientific models and theories are often presented wrongly, not least in textbooks and in popular texts – as an enlarged, simplified and manipulated picture of reality, rather than an aid to interpret, understand and cope with the often very complex reality behind the model (cf. Öhman, 1993). This is perhaps particularly evident when we look at the approach to scale-issues in both time and space. Atoms, cells and planets are often illustrated in a similar fashion, but are essentially different in size and can hardly be illustrated using the same imagery. The movements of electrons, the metabolic activities of the cell and

changes in the universe have to be studied on different time-scales. If we, for example, study weather at a time-scale of some weeks, it behaves somewhat stochastically and is more or less unpredictable. If we, on the other hand, study weather over a 30-year period we can see a clear pattern obviously linked to seasons.

Out of this kind of reasoning it is quite clear that communication is at the core in science learning and teaching. This is also mirrored in science education research. Hence, when an anthology comprising contributions from fifteen Swedish science education researchers was published, it was published under the title “Kommunicera naturvetenskap i skolan” (“To communicate Science in School”) (Strömdahl, 2002). It was easy to set out the title since, independent of each other; all researchers more or less stressed the communicating aspect of science education.

Science education research in Sweden around the turn of the millennium

In 1999 Strömdahl (2000) was commissioned by the Swedish Agency for Higher Education within the so called NOT-project to make a survey of the status of science education research at the Swedish universities and university colleges at the turn of the millennium. The NOT- project was a joint attempt by the two Swedish Agencies for school and higher education to increase interest among young people to study science and technology.

In the report of the survey, Strömdahl (2000) concluded that there were a few developed research sites, first of all at Göteborg University but also at Uppsala University, the University College at Kristianstad and Lärarhögskolan in Stockholm. Single researchers were also located at several other universities and university colleges, especially at teacher training programs, but however, lacking a daily base to communicate research issues with colleagues. Research was foremost focussed towards students' conceptions of physical and biological concepts. Some few studies were also done within chemistry. Some 15 doctoral works were completed. The contacts with the international research community were quite developed, mostly by researchers participating in international conferences. But there was a lack of financial resources to develop and increase the volume of the research. As an answer to the Swedish governmental recurrent rhetoric of creating increased interest among young people for science studies, Strömdahl suggested some ideas to come to grips with the situation, among others, a national resource centre to strengthen the position for science education and research in Sweden. At the same time the Swedish Research Policy bill *Forskning för framtiden – en ny organisation för forskningsfinansiering* (1999/2000:81) proposed a new model for financing educational research. Additionally, and of essential importance, another Swedish Research Policy bill *Forskning och förnyelse* (2000/01:3) proposed a new organizational model for

research and doctoral education, *the national graduate school*. In this bill a national graduate school in Science and technology education research was suggested among 15 other graduate schools, all in strategic research areas.

The Swedish National graduate school in Science and Technology education

In 2001/02 the national graduate school in science and technology education research was inaugurated. Financial support, around 12 million SEK per year, was guaranteed by the Swedish Government. After a general application session among Swedish universities, Linköping University became the host university for this school, and together with seven partner universities and university colleges it was named The national Graduate School in Science and Technology Research (Forskarskolan i naturvetenskapernas och teknikens didaktik, with the Swedish acronym FontD).

The board of the University of Linköping appointed the committee of the philosophical faculty to be responsible for the formation of the graduate school, and two years later it was organized under the faculty of education. The school is geographically localised at the Norrköping campus of Linköping University.

FontD is, in accordance with the bill, a joint collaboration between University College Mälardalen, University College Malmö, The College of Education in Stockholm and the University of Karlstad. The joint partners are natural participants in the activities of the graduate school. The network should, however, also be able to accommodate research environments at other universities and colleges. Thus, Kalmar University College, Kristianstad University College and Umeå University are also, since the start 2002, full members of the network.

Doctoral students are placed at each participating university/university college and each doctoral student should carry out his/her duties at one of the participating institutions. At the same time the graduate school have a solid common kernel developing a knowledge base of national and international interest. Thus, the aim is that the graduate school should both contribute to the formation of didactic environments at the various universities/university colleges and also function as a national and international arena for didactic research as well as training researchers in the field of science and technology education.

Three main perspectives describe the content of the graduate school research:

- Learning and communicating science and technology
- Scientific and technological knowledge cultures in school and society
- Scientific and technological knowledge – general education, democracy, gender and ethnicity

The three perspectives do not mark clear-cut boundaries. It is easy to see that individual research projects can, and in many cases should, address issues which are approached from and inspired by all perspectives.

According to the bill 2000/01:3 the graduate school was assigned to make research with relevance to teacher training and examine at least 25 doctors up to the year 2008. In autumn 2006 FontD comprises 43 doctoral students. Those who entered their studies in 2002 are now ready for their dissertations in 2007. The first dissertation will be held in January 2007. Abstracts of all doctoral students' work can be found at www.isv.liu.se/fontd.

Other research sites

Apart from the National Graduate School there are considerable research done at the pedagogical department of Göteborg university (<http://www.ped.gu.se/amnesdidaktik/natek>) and also, however on a smaller scale, at Uppsala university (<http://www.did.uu.se/forskning.lasso>). Additionally, single research projects in science education are also done at other universities, often connected to teacher training departments.

The present situation for science education research

If we look at the international arena, Peter J. Fensham (2004) has, via interviews with a number of researchers all around the world, captured the development of the field of science education research. His book is a valuable source to find the roots of science education research and its multifaceted approaches. Among others, research has been done in the fields of

- Conceptions
- Visualisation
- Language
- Analysis of textbooks/ curricula
- Attitudes/Value/aesthetics
- Interest/motivation/emotion
- Teaching design
- Assessment/evaluation
- Processes of learning
- Genus, ethnicity, handicap
- PUST/Scientific literacy
- Science Centres
- Teacher education

In Sweden Gustav Helldén, Britt Lindahl and Andreas Redfors (2005), at the University College at Kristianstad, have recently published a report about the present situation for science education research in Sweden. Their review is done in relation to the international science education research, not at least leaning on the development described by Fensham (2004). It is evident that the situation for science education research in Sweden has had a tremendous development since

the turn of the millennium. This is mainly due to the establishment of FontD, but also according to better possibilities for financing research in the educational field. In a couple of years the number of doctors in science education research will be multiplied in Sweden. This will be a radical reinforcement of the personal research capacity at our universities and a quality increase in the teaching and learning of teacher training at all levels of the school and university systems.

Implementation of research results

Being aware of the fact that there is low preparedness to adopt scientific research results in school practice, how do we attain sustainable implementation of research results? Recurrent reports from teachers refer to lack of time and hard workload as reasons for not applying research results in everyday practice. This follows from the fact that traditional research results are seldom directly applicable to a complex practice in the classroom. It is a demanding task to assimilate research results in functioning educational design. Here is a big assignment for e.g. textbook authors and teacher training teachers. But research can also be done by aiming at concrete teaching design. This is something that is done e.g. at Göteborg University with a very favourable outcome (<http://www.ped.gu.se/amnesdidaktik/natek>).

Another entrance to reinforce the contact between research and practice is to engage in-service teachers and researchers in joint researcher/teachers research projects.

Generally, teacher education and in-service teacher training need to be intimately research-based to develop a professional knowledge base. Teacher professionalism ought to include skills in using research results in educational planning and design. Last but not least, the Swedish National Agency for School Improvement (http://www.skolutveckling.se/in_english/) has a considerable responsibility to be an active agent in these matters.

References

- Bergqvist, K. & Säljö, R. (1994). Conceptually blindfolded in the optics laboratory. Dilemmas of inductive learning. *European Journal of Psychology of Education*, 9(1), 149-158.
- Driver, R. (1983). *The pupil as a scientist?* Milton Keynes: Open University Press.
- Fensham, P. J. (2004). *Defining an identity – The evolution of science education as a field of research*. Dordrecht: Kluwer Academic Publishers.
- Helldén, G., Lindahl, B., & Redfors, A. (2005) *Lärande och undervisning i naturvetenskap – en forskningsöversikt*, Vetenskapsrådets rapportserie 2005:2. Available at www.vr.se
- Kuhn, T. (1979). *De vetenskapliga revolutionernas struktur*. Lund: Doxa.

- Strömdahl, H. (2000). *No-didaktisk forskning i Sverige – en lägesrapport och några förslag vid millenieskiftet 1999/2000*. Stockholm: Högskoleverket. Available at www.isv.liu.se/fontd
- Strömdahl, H. (Ed.) (2002). *Kommunicera naturvetenskap i skolan – några forskningsresultat* (To communicate science in school – some research results). Lund: Studentlitteratur.
- Sutton, C. (1992). *Words, Science and Learning*. Buckingham: Open University Press.
- Wolpert, L. (1993). *The unnatural nature of Science*. London: Faber & Faber Ltd.
- Öhman, S. (1993). *Svindlande perspektiv. En kritik av populärvetenskapen*. Stockholm: Wahlström & Widstrand.

Papers in Mathematics

Trends in the level of mathematical knowledge at the beginning of upper-secondary school

Lars Burman
Åbo Akademi University

In the years 1991-2005 the teachers in mathematics in all the 36 Swedish upper-secondary schools in Finland have been able to test their first year students in mathematics. The tests consisted of 12 multiple-choice tasks from six different areas of mathematics and the results, which are based on an average of about 500 student answers yearly, give a fifteen years long perspective of the general level of the mathematical knowledge of these first-year students. Some trends of the development concerning gender differences, the areas of mathematics and the level of mathematical knowledge in general are presented.

Introduction

Since the year 1991 every teacher in mathematics in the 36 upper-secondary schools with Swedish as the teaching language in Finland, has had the opportunity to test his/her students in the knowledge of six central areas of mathematics. The test has been conducted among the new students in the first grade preferably within their first weeks in the upper-secondary school.

Thus, the test is intended to give some information about the first-year students concerning their general level of knowledge and perhaps also about what the students remember of the mathematical contents and the mathematical methods after the summer holidays. At the time when the test is written the students have already chosen the long or the short course in mathematics, but nevertheless, they have more or less the same background as to what they have been taught in mathematics. Of course, those who have chosen the long course in mathematics are undoubtedly expected to receive higher scores in the test.

The test consists of twelve multiple-choice tasks, two from each of six central areas of mathematics. These areas are

Numbers and operations	(NO)	Equations and inequalities	(EI)
Algebraic expressions	(AE)	Geometry	(G)
Functions	(F)	Applied tasks	(A)

In the multiple-choice tests every task has always one correct answer and four distractors and every right answer is given 1 point. ¹

Aims of the study

From the very beginning, the tests were constructed and distributed with the intention to serve the teachers with an initial test for upper-secondary school. Gradually, and in the long run, it also became possible to receive some information about trends in the knowledge and use of mathematical methods among first-year students in upper-secondary school. These trends are the main interest of this study. The aims can be concentrated in the following question:

What trends can be found in the students' level of mathematical knowledge in the six different areas of mathematics at the beginning of upper-secondary school and seen from a 15 year long perspective?

In connection to this question, the number of students that have chosen long or short course in mathematics as well as possible gender variations in the scores of the students will be observed and commented.

About the circumstances connected to the collection of data

Three main circumstances about the collection of data will be described. Firstly, the tests were often carried out in the beginning of the first year in the upper-secondary school, in late August or in September, but sometimes also in October or November. The tests were carried out later when the students had no mathematics at all in the first period but a couple of times also due to a wish from teachers, who think that the results of the tests are suffering too much from the summer break. However, in the results of the tests that were conducted later no higher scores are to be identified, even if students had had a first course in (some part of) mathematics before the test.

Secondly, there are some variations from one year to another in the number of students taking part in the test. In the fifteen years included in this study, the number of student have been 837, 709, 239, 433, 185, 466, 350, 614, 342, 552, 405, 452, 647, 517 and 417, and thus the average is 478. Of course, the number of the students we have received the scores from, must be related to a total number of students in the first grade in upper-secondary schools with Swedish as the teaching language in Finland. The real total number is not so easy to estimate, but as it in these years has been nearly 2 000, it can be considered that we have got scores from about 25 % of all first-year students. This percentage is

¹ Every second year there has also been a second type of test, not included in this article. This test consists of six somewhat extended tasks, one from each area of mathematics, but still with rather short calculations and answers.

also credible as we have received scores from between 10 and 15 schools out of 36 schools every year and moreover, in this case a “school” does not necessarily mean that we have received scores from every group (or more precisely, from every possible teacher) in the school. Thus, as we have received scores from this amount of students and there are large schools and smaller schools, schools in the cities and on the countryside and schools in different regions, it can be considered reasonable to look at the results in order to find trends.

Thirdly, there is a special difficulty in choosing suitable tasks for a test of this kind. Of course, this was a well-known fact when we started the project and we decided not to choose two certain tasks from every area to use every year but to get eight tasks from every area and then if possible re-use them every fourth year. However, there are changes in the curriculum and a task that is central one year might be less central fifteen years later. There are also other changes than those connected to the curriculum which might prevent a task from being used again. The best example of this fact is provided by the change in currency from mark to euro. Of course, it is possible to make a change and choose a new task, but at the same time you lose the possibility to compare the students’ scores for the specific task from one year to another.

Gender aspects on the number of students and on the average of scores

At first, the number of answers received from girls and from boys gives some interesting information. There have been quite stable proportions of girls and boys but, of course, with some fluctuations. For long-course students, the majority has traditionally been boys. In the first four years the proportion of girls were 42, 38, 39 and 44 % with the average 41 %, but in the last four years the proportion of girls were 37, 45, 49 and 50 % with the average 45 %. Thus, we can notice a trend with a slight increasing number of girls reaching 50 % in the last year. It is perhaps no coincidence that the number in 2005 is going up to exactly 50 % for the first time.

For short-course students, there were in the first four years 67, 69, 75, 70 % girls with the average 70 %, and in the last four years 68, 68, 70 and 79 % girls with the average 71 %. Here we can notice no increasing trend but a fluctuation about 70 % all the time. Again, the highest notation 79 % occurs in 2005. The higher number of girls having chosen short course in mathematics may be related to a higher number of boys in the vocational schools.

Table 1 provides the average of the correct answers for long-course students, short-course students and all the students and in every section of the table for all students, girls and boys. Furthermore, the year and the number of students are noted. The fourth section of the table provides the difference between long-course students and short-course students for all students, girls and boys. It is to be remembered that 12 points is the maximum number of points.

Table 1. The average share of the number of correct answers for long-course students, short-course students and all the students, and in addition, the differences between long-course and short-course averages

Long Pupils	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	Mean
All pupils	404	269	105	193	79	189	230	389	181	352	209	283	360	295	224	251
Girls	5,68	5,98	6,42	5,66	6,76	6,83	5,40	5,43	5,35	6,51	5,77	5,49	4,53	5,41	5,11	5,76
Boys	5,63	5,84	6,66	5,92	6,62	6,81	5,57	5,58	5,29	6,65	5,79	5,78	4,45	5,30	4,96	5,79
	5,71	6,07	6,27	5,45	6,87	6,83	5,28	5,35	5,40	6,40	5,74	5,31	4,59	5,52	5,26	5,74
Short Pupils	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	Mean
All pupils	433	440	134	240	106	277	120	225	161	200	196	169	287	222	193	227
Girls	3,72	3,41	4,04	3,87	3,79	3,81	3,55	3,79	3,29	3,38	3,46	2,74	3,20	3,61	3,34	3,53
Boys	3,77	3,25	3,89	3,85	3,81	3,97	3,63	3,86	3,34	3,44	3,56	2,77	3,22	3,53	3,25	3,54
	3,61	3,77	4,48	3,92	3,73	3,46	3,34	3,69	3,20	3,21	3,19	2,72	3,14	3,82	3,68	3,53
All Pupils	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	Mean
All pupils	837	709	239	433	185	466	350	614	342	552	405	452	647	517	417	478
Girls	4,66	4,39	5,08	4,67	5,06	5,03	4,77	4,83	4,38	5,41	4,65	4,47	3,94	4,64	4,29	4,68
Boys	4,45	3,90	4,69	4,54	4,65	4,72	4,64	4,72	4,24	5,04	4,46	4,21	3,78	4,38	3,97	4,43
	4,92	5,03	5,66	4,84	5,72	5,41	4,90	4,95	4,53	5,67	4,92	4,71	4,13	5,00	4,84	5,02
DIFF L-S all p.	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	Mean
L-S girls	1,96	2,57	2,38	1,79	2,97	3,02	1,85	1,64	2,06	3,13	2,31	2,75	1,33	1,80	1,77	2,22
L-S boys	1,86	2,59	2,77	2,07	2,81	2,84	1,94	1,72	1,95	3,21	2,23	3,01	1,23	1,77	1,71	2,25
	2,10	2,30	1,79	1,53	3,14	3,37	1,94	1,66	2,20	3,19	2,55	2,59	1,45	1,70	1,58	2,21

Table 2. The percentage share of the number of correct answers for long-course students, short-course students and all the students in the six areas of mathematics in the years 1991-2005

	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	Mean
Long																
1 NO	64	51	77	86	61	42	58	61	35	57	61	66	30	33	52	56
2 AE	25	46	70	41	40	62	57	46	42	58	70	46	38	45	59	50
3 F	18	57	50	36	41	52	33	27	30	60	41	44	21	38	39	39
4 EI	64	59	54	49	66	75	47	56	43	47	38	35	32	62	35	51
5 G	44	43	39	50	43	57	23	47	52	49	32	49	54	41	30	44
6 A	72	46	39	22	66	53	52	36	57	59	44	33	54	53	40	48
Short																
1 NO	37	20	56	73	38	21	35	47	26	27	36	36	29	20	34	36
2 AE	19	19	41	18	33	36	31	24	21	36	37	17	22	37	35	28
3 F	15	28	27	28	22	31	30	20	20	20	30	29	14	24	32	25
4 EI	41	43	38	31	25	48	33	37	25	21	24	12	20	42	25	31
5 G	24	25	24	30	28	27	16	37	31	25	24	28	32	22	21	26
6 A	54	28	19	15	44	28	33	26	43	39	24	20	44	35	21	32
All																
1 NO	51	31	65	79	50	30	50	55	31	46	49	55	30	27	44	46
2 AE	22	36	54	28	47	46	48	37	31	50	54	36	31	41	48	41
3 F	16	39	37	31	32	39	32	25	25	45	36	38	18	32	35	32
4 EI	53	49	45	39	46	59	42	48	34	38	31	27	27	53	30	41
5 G	34	32	30	39	36	39	20	43	41	40	28	41	44	33	26	35
6 A	63	35	28	18	55	38	45	32	50	52	35	28	49	45	31	40

Surprisingly, girls and boys have extremely similar means as concerns the number of right answer almost every year and in both courses. The mean for boys is slightly higher one year and lower another year. In the long run, the mean of the mean values for girls is 0,05 higher for long-course students and 0,01 higher for short-course students. Consequently, table 1 gives no evidence for any differences between boys and girls.

When comparing the means for boys and girls in the two courses, the mean for the number of right answers seems always to be higher for the boys than for the girls and the mean of the mean values is even 0,61 higher for the boys. However, the explanation to this fact is not that the boys seem to achieve better results but very simple: the proportion of girls in the group with short course in mathematics is much higher than that of the boys and thus, lower scorers are more frequent among the girls.

Students' achievements in the six different areas of mathematics

A test for the first year students in upper-secondary school, and especially when the test is presented in the first month(s) of their studies, will most certainly be a test of the students' knowledge and skills after lower-secondary school or perhaps of what they remember some time after leaving lower-secondary school.

Table 2 presents the percentage of the right answers in the six areas of mathematics for all the fifteen years examined. The results are given in three sections: the results of the long-course students, the results of the short-course students and the results of all students. As there are twelve tasks in six areas, the results from two tasks are added in every area. The scores in different areas of mathematics depend very much on the tasks used in the tests and therefore, it is not possible to compare the scores in different areas.² Furthermore, it is perhaps not recommendable to compare consecutive years in the same area either, as the tasks are not comparable from one year to another. However, some tasks have been used every fourth year and these tasks enables an interesting comparison of the scores for the same task with an interval of four years, which will be seen in the following table.

Table 3a presents the percentage of the correct answers in the first three areas of mathematics: numbers and operations, algebraic expressions and functions. Sometimes there is only one task that has been used more than once in a series of four years. Sometimes there are two such tasks to be found, but in these cases,

² As examples, some tasks are included as an appendix in this article.

however, the second task is nearly always used only twice. Thus, in the overview below, not so much attention is given to tasks used only twice.

Table 3a. The percentage share of the number of correct answers for long-course students, short-course students and all the students in the areas numbers and operations, algebraic expressions and functions

NO, LONG COURSE				NO, SHORT COURSE				NO, ALL STUDENTS							
1991	1995	1999	2003	1991	1995	1999	2003	1991	1995	1999	2003				
	78	55	39		54	42	35		66	48	37				
1992	1996	2000	2004	1992	1996	2000	2004	1992	1996	2000	2004				
	52	52	47	45		21	28	17	27		32	38	36	37	
		32		20			14		14			21		18	
1993	1997	2001	2005	1993	1997	2001	2005	1993	1997	2001	2005				
	72	60	67	49		58	50	40	38		64	57	54	44	
			54	55				32	30				43	44	
1994	1998	2002						1994	1998	2002					
	76	78	75		60	45	44		67	65	64				
AE, LONG COURSE				AE, SHORT COURSE				AE, ALL STUDENTS							
1991	1995	1999	2003	1991	1995	1999	2003	1991	1995	1999	2003				
	77	63	45		38	29	18		58	46	33				
	31	43		32		19	27		26		25	35		29	
1992	1996	2000	2004	1992	1996	2000	2004	1992	1996	2000	2004				
	61	67	68	55		48	40	34	39		53	51	55	48	
			48	34				38	36				44	35	
1993	1997	2001	2005	1993	1997	2001	2005	1993	1997	2001	2005				
	80	86	66	57		56	46	33	34		67	72	50	46	
			75	62				40	36				58	50	
1994	1998	2002						1994	1998	2002					
	48	36	45		23	23	20		34	30	35				
F, LONG COURSE				F, SHORT COURSE				F, ALL STUDENTS							
1991	1995	1999	2003	1991	1995	1999	2003	1991	1995	1999	2003				
	41	29	24		28	21	18		35	25	21				
		30		18			19		9			24		14	
1992	1996	2000	2004	1992	1996	2000	2004	1992	1996	2000	2004				
	62	78	68	44		29	46	23	25		42	59	51	36	
			52	32				18	23				39	28	
1993	1997	2001	2005	1993	1997	2001	2005	1993	1997	2001	2005				
	42	55	54	55		28	51	44	44		34	54	49	50	
			28	23				15	19				22	21	
1994	1998	2002						1994	1998	2002					
	26	36	34		27	29	27		27	33	31				

As concerns number and operations (NO), two tasks introduced in the years 1993 and 1995 show decreasing percentages and in the second task, the tendency is very clear and substantial. There are also tasks with an almost stable development, but no tendency of increase is to be found.

In the area of algebraic expressions (AE) as well, two tasks that have been introduced in the years 1993 and 1995 show decreasing percentages. In both these tasks, the tendency is clear, at least if the years 2001 and 2005 are

compared with the year 1997 as concerns the first task. As in the case of number and operations, there are also tasks with no clear tendency at all.

Table 3b. The percentage share of the number of correct answers for long-course students, short-course students and all the students in the areas equations and inequalities, geometry and applied tasks

EI, LONG COURSE				EI, SHORT COURSE				EI, ALL STUDENTS			
1991	1995	1999	2003	1991	1995	1999	2003	1991	1995	1999	2003
	54	40	28		8	12	10		31	26	20
52	76		37	35	42		29	44	59		34
1992	1996	2000	2004	1992	1996	2000	2004	1992	1996	2000	2004
68	70	62	55	48	48	35	38	55	57	52	48
	79		69		49		46		61		59
1993	1997	2001	2005	1993	1997	2001	2005	1993	1997	2001	2005
64	53	48	45	51	38	36	35	56	48	42	40
		28	26			12	15			20	21
1994	1998	2002		1994	1998	2002		1994	1998	2002	
53	40	40		35	22	17		43	33	31	
G, LONG COURSE				G, SHORT COURSE				G, ALL STUDENTS			
1991	1995	1999	2003	1991	1995	1999	2003	1991	1995	1999	2003
	57	66	52	33	42	34	28		45	54	43
		51	56			28	36			39	47
1992	1996	2000	2004	1992	1996	2000	2004	1992	1996	2000	2004
68	69	61	58	35	37	36	36	48	50	52	49
		37	23			14	7			28	16
1993	1997	2001	2005	1993	1997	2001	2005	1993	1997	2001	2005
28	37	34	33	25	18	32	35	26	30	33	34
		30	27			15	6			23	18
1994	1998	2002		1994	1998	2002		1994	1998	2002	
61	58	65		44	40	40		52	51	56	
A, LONG COURSE				A, SHORT COURSE				A, ALL STUDENTS			
1991	1995	1999	2003	1991	1995	1999	2003	1991	1995	1999	2003
	64	76	51	42	47	45	43		54	62	48
		51	53			45	43			48	49
1992	1996	2000	2004	1992	1996	2000	2004	1992	1996	2000	2004
42	54	50	49	26	34	34	33	32	42	44	42
		50	49			34	33			44	42
1993	1997	2001	2005	1993	1997	2001	2005	1993	1997	2001	2005
63	65	59	51	29	33	29	18	44	54	44	36
		30	28			20	24			25	26
1994	1998	2002		1994	1998	2002		1994	1998	2002	
14	12	11		13	8	8		13	10	10	

As concerns functions (F), two tasks introduced in the years 1992 and 1995 show decreasing percentages. For both of them the tendency is clear, when for the first task, the years 2000 and 2004 are compared with the year 1996. There are also tasks without decreasing percentages and in one place even a tendency of a slight increase can be found. The increase occurs as a lower percentage in the year 1993 compared with the level of the years 1997, 2001 and 2005.

Table 3b presents the percentage of the correct answers in the other three areas of mathematics: equations and inequalities, geometry and applications. In the area of equations and inequalities (EI), almost every task shows decreasing percentages, but as seen in the areas AE and F, the top score is not always the

first score. In one task, the year 1995 gives a higher score than the year 1991, and in two other tasks, the year 1996 is the year with the highest score.

Geometry (G) provides a more heterogeneous picture. There are a couple of tasks where the last year gives the clearly lowest score, but in these tasks, only two different years are compared. Again, a task with the highest score in 1995 can be found, but mostly, there are tasks with quite stable scores, or scores with a slight tendency of increase.

In the area of applications (A), very few years can be compared due to the fact that in this area, the biggest need to change a task after an interval of four years has existed. The change of currency from marks to euro is the best example of this need to make changes but there might also be other reasons. Stable scores can often be found, but also some already well-known features: the last year 2005 in a series of four years gives the lowest score, a tendency of a slight increase from the year 1992 up to the level of the following three years and finally, the highest score occurs not in the beginning nor in the end of the years compared but for the years 1995 and 1997.

Discussion of the results

Before discussing possible trends, it can be pointed out that some changes in the background variables during the period 1991-2005 may have influenced the results of the tests. Firstly, in the decade 1990-2000, a recession in the country and the need to save money may have resulted in bigger classes in the secondary school. Secondly, the benefit of using calculators and computer programs may have increased, but in these tests such aids were not allowed. Thirdly, developments and changes in the Finnish education policy took place during the actual period, giving consequences that perhaps are the most difficult to evaluate. According to Pehkonen, Ahtee and Lavonen (2007), there are at least three relevant documents from this period. The first one, set by the Finnish government in 1996, presented the target in which the level of mathematical and scientific knowledge in Finland was to be raised up to an international standard. The other two are the two renewals of the national framework curriculum in 1994 and 2004, administered by the National Board of Education.

Finally, when looking at the situation in the Swedish upper-secondary schools in Finland, it must be noted that the two available series of textbooks in mathematics in the Swedish language were replaced with new series. "Matematikens värld" was completed in 2002 and "På tal om tal" in 2004. The word "completed" means that in this year, it was for the first time possible to find many classes who had used the whole series. However, this change affects only the very last few years examined. Nevertheless, in-service teachers may predict new trends in mathematics instruction and the tasks in the actual tests may reflect trends even before the change in textbook series. It is not easy to measure

the possible effects of these four aspects, nor it is the aim of this article, but it is important to keep them in mind when the results of the tests are discussed.

Gender

One interest of this study, perhaps in the sense of finding some kind of explanation to other results, is to find trends in the numbers of boys and girls who have chosen long or short course in mathematics, and in the students' scores comparing boys and girls. A possible state of affairs may be that there are girls who have the capacity to achieve the long course in mathematics but who do not choose the long course. This study does not give a direct argument to use in that discussion, but as the percentage of girls on the long course in mathematics, from which we have got scores, is gradually approaching 50 %, there are reasons to believe that at least, there is no large resource of this kind.

Another interest concerning gender is about differences in achievements between boys and girls. This study of tests conducted in the years 1991-2005 in upper-secondary schools gives no evidence of gender differences. These results are similar to the results in Björkqvist (1995), where he found few differences between boys and girls in the achievements in mathematics in a test for pupils in the ninth grade in Swedish schools in Finland. There was no significant difference in the mean values, which again was well in accordance with an earlier study (Björkqvist, 1995).

In Finland, the National Board of Education has evaluated the educational outcomes in mathematics at the end of the comprehensive school and also used multiple choice questions with a nearly similar division in different areas of mathematics. In the evaluation in the year 2000, Korhonen (2001) claims that girls have caught up with boys in mathematics and in the evaluation in the year 2004, Mattila (2004) finds no differences in the results of different genders in the test as a whole. Kupari, Reinikainen and Törnroos (2007) also state that generally, in Finland, gender differences have been almost negligible in all studies.

Achievements in different areas of mathematics

The aim of this study is to find out what trends can be found in the students' level of mathematical knowledge in the six different areas of mathematics at the beginning of upper-secondary school, seen from a 15 year long perspective.

In the three areas number and operations (NO), algebraic expressions (AE) and functions (F), this study points at two tasks in each area, introduced in the years 1992, 1993 and 1995, in which a considerably lower percentages of correct answers is to be found four years and even to a greater extent eight years later. There are also tasks with almost stable percentages or tasks with no clear tendency at all, but almost nowhere any tendency of increase is to be found.

In Finland, the National Board of Education has evaluated the educational outcomes in mathematics at the end of the comprehensive school and also used multiple choice questions from different areas of mathematics in a nearly similar way as in the tests of this study. The evaluation has been performed every second year but since new tasks have been used in every test it is not possible to find trends. However, good results can be found in the year 2000 (Korhonen, 2001), but in algebra also deficiencies, affecting the study and applications of mathematics. In the year 2002, functions was found to be the most difficult area (Mattila, 2002), and in 2004, the largest variations in pupils' results were found in the areas functions and algebra (Mattila, 2004).

Of the three areas equations and inequalities (EI), geometry (G) and applications (A), the tasks from the first one show reasonable decreases in percentages and sometimes, a slight increase in percentages was noted between the first four years. In the NBE evaluations, not much is said about equations and inequalities as separate areas, which makes a comparison difficult. In the second area, geometry, this study provides a heterogeneous picture with quite stable scores but also both slight decreases and slight increases in percentages. However, in the NBE evaluation in the year 2002 and the year 2004, geometry proved to be one of the most difficult areas (Mattila, 2002 and 2004). As concerns the third area, applications, there are stable scores as well as changes in both directions, but it is very hard to recognize trends, because the tasks may differ considerably from one year to another. In the NBE evaluation in the year 2000 (Korhonen, 2001), it is noted that about one tenth was not able to solve problems requiring the applications of mathematics in a satisfactory manner.

In "How Finns Learn Mathematics", Kupari, Reinikainen and Törnroos (2007) point at ambivalent results for Finnish students relative to traditional mathematics. On one hand, there have been very good results in TIMSS 1999 with regard to numbers and mathematical operations and in PISA 2003 as concerns quantity, but on the other hand, only average results were to be found in TIMSS in tasks involving operations with fractions and decimal numbers. In mechanical calculus Finland has been even among the lowest performing countries. In some specific areas of mathematics, there have been both good and bad results: Finnish students have performed well in units of measurement and outlining three-dimensional objects, but not so well in tasks with perimeters, areas and other features of geometric figures. In algebra they have been good at functions but not so good at handling equations and formulas or other such expressions.

The conclusion in "How Finns Learn Mathematics", that Finnish students can solve verbal problems with calculators (like in PISA, for instance) but without calculators their technical skills leave room for improvement, is well in line with the results in the tests of this study, where calculators have not been permitted. Moreover, because there are tasks used every fourth year in this study, it has

been possible to find some trends and there are results that to some extent indicate decreasing achievements in some areas of mathematics.

Conclusion

The results of this study concerning gender differences are that the number of girls choosing long course in mathematics seems to be catching up with the corresponding number of boys and that there seem to be no differences between boys and girls in the achievements in mathematics, at least not in the ninth grade in Swedish schools in Finland. The main results about students' achievements in the six different areas of mathematics show both good and stable results, but also a clear trend with decreasing achievements in several areas of mathematics. These results are in accordance with other Finnish studies and the conclusion that without calculators some rather basic operations may cause difficulties to a surprising degree (Kupari, Reinikainen & Törnroos, 2007). Because the differences between the two language groups in Finland can be assumed to be quite small (Kupari, Reinikainen & Törnroos, 2007), it is not excluded that the same decreasing trend could occur in Finnish schools in general. The results in this study raise questions which could serve as the basis for follow-up studies. Which subareas of mathematics are represented by the tasks? Are there possible connections with some background variables? Can similar results be found from schools more generally, in Finland and in other countries?

References

- Björkqvist, O. (1995). *Utvärdering av matematikkunskaperna i årskurs 9 i grundskolan, del II*. Publikationer vid Pedagogiska fakulteten vid Åbo Akademi nr 16. Vasa: Institutionen för lärarutbildning.
- Korhonen, H. (2001). *Perusopetuksen päättövaiheen matematiikan oppimistulosten kansallinen arviointi 2000*. Oppimistulosten arviointi 3/2001. Helsinki: Opetushallitus.
- Kupari, P., Reinikainen, P. & Törnroos, J. (2007). Finnish students' mathematics and science results in recent international assessment studies. In Pehkonen, E., Ahtee, M. and Lavonen, J. (Eds.) *How Finns Learn Mathematics and Sciences*, pp. 11-34. Rotterdam: SensePublishers.
- Mattila, L. (2002). *Perusopetuksen matematiikan oppimistulosten kansallinen arviointi 9. vuosiluokalla 2002*. Oppimistulosten arviointi 8/2002. Helsinki: Opetushallitus.
- Mattila, L. (2004). *Perusopetuksen matematiikan kansalliset oppimistulokset 9. vuosiluokalla 2004*. Oppimistulosten arviointi 2/2005. Helsinki: Opetushallitus.
- Pehkonen, E., Ahtee, M. & Lavonen, J. (Eds.) (2007). *How Finns Learn Mathematics and Sciences*. Rotterdam: SensePublishers.

Appendix

Examples of tasks in tests from different areas of mathematics

(Numbers and operations)

How many three-digit numbers are divisible by 5 ?

- a) 171 b) 179 c) 180 d) 200
- e) none of the answers above is correct

(Algebraic expressions)

The expression $\frac{x-1}{2} + \frac{1}{3}$ can also be written

- a) $\frac{3x-1}{6}$ b) $\frac{x-1}{6}$ c) $\frac{x-1}{5}$ d) $\frac{x}{5}$

- e) none of the answers above is correct

(Functions)

The line $y = 4x + 1$

- a) intersects the x -axis in the point $(0, 1)$
- b) is decreasing for negative values of x
- c) passes through the point $(4, 1)$
- d) has got the x -intercept $x = -5$
- e) none of the answers above is correct

(Equations and inequalities)

The equation $5(x - 20) = 0,5$

- a) can be simplified to $x - 20 = 0,1$
- b) can be simplified to $5x = 20,5$
- c) has the solution $x = 19,9$
- d) has the solution $x = 20,5$
- e) none of the answers above is correct

(Geometry, extended task)

A crossroad is rebuilt to a roundabout, in which the cars drive 20 m from the center of the roundabout. How many meters longer do they drive compared to the former straight forward passing?

Shifting of mathematical profiles among elementary teacher students and mathematics students

Lasse Eronen, Lenni Haapasalo
University of Joensuu

A long-term study of the history of mathematics reveals eight main activities, which have been proved to be sustainable in the history of human thinking processes and for the generation of new mathematics. Those activities were used as a framework for the three kinds of mathematical profiles represented in this article. It turned out that those profiles are quite degenerated among elementary teacher students as well as among mathematics students. The study suggests that shifting of those profiles gives new kinds of challenges for mathematics education, including pedagogical studies.

Introduction

Zimmermann (2003) suggests that the following eighth activities have been sustainable and viable for more than 5000 years when making new mathematics at different times and in different cultures: ordering, finding, playing, constructing, applying, calculating, evaluating, and arguing. After interpreting the term ‘evaluate’ in very broad sense to include also ‘estimation’, we used his idea to model these activities as an octagon and to quantify each activity as follows: the distance of the activity from the centre tells how strongly a student thinks this activity is represented by each of the following three profiles:

- Math-profile: How strong does each activity appear to the student when using the term ‘mathematics’;
- Identity-profile: How a good student thinks he or she is performing each of the activities;
- Techno-profile: ‘How suitable a computer is in performing each of the activities’.

Background

Motivated by our findings (see Eronen & Haapasalo, 2006), that a voluntary working with modern ClassPad calculator during the summer holiday shifted 8th grade students’ profiles in a creative direction (i.e. increased the role of the first four activities above), we decided to use the same instrument for two kinds of

student groups who started their university studies. The first one consisted of elementary teacher students, who made their educational studies at the Faculty of Education. The other group of students had just started to study mathematics at the Mathematical Faculty, without any educational studies. There were no students belonging to both of these categories. The study was designed and administrated during the study year 2005/2006 at the Pedagogical Faculty of the University of Joensuu, when both authors were teaching pedagogical studies for the first target group.

Aims and methods

The research aimed to find out (1) how the profiles of the two groups differ from each other, and (2) how the profiles change during the first study year. To measure the profiles, the same web-based questionnaire was used in September 2005 and in April 2006. The first test was made by 66 mathematics students, and 116 elementary teacher students. The number of students in the samples was, at the second stage, 22 and 50, respectively. For each of the three profiles and each of the eighth activities there were questions with 4-step Likert type scaling (altogether 3 x 8 questions; see Eronen & Haapasalo, 2005). For every question student also had to express (within the same Likert- scale) how sure he/she was about his/her opinion. From a student's answer and the certainty of this answer, a new weighted variable was defined to be used to span the student profiles. Because of asymmetric and discontinuous data, Mann-Whitney Test and Sign test (2-tailed) were used for statistical analysis. To find out how student understood the questions, sub-samples from both of the samples were also interviewed.

Student profiles in 2005

Math-profiles of both of the target groups were very similar, emphasizing calculating and arguing (see Figure 1, left). There is a significant difference only in applying, which appears stronger among elementary teacher students than among mathematics students. The identity-profiles followed the shape of math-profiles, being throughout the components a little bit smaller, especially in finding and arguing. A plausible explanation might be that first year students do not think they are good at mathematics, especially concerning those two activities. Evaluation appears weakly within every of the three profiles, which might be because of a quite general definition of this term. Interviews support this explanation. Techno-profiles did not differ significantly (Figure 1, right). Students saw that computers mainly support calculating, but very weakly arguing. This might, for example, be interpreted as an implication of degenerated school teaching, which over-appreciates paper-and-pencil work and even denies the use of modern technology in matriculation examination.

Concerning playing, perhaps this term should have been explained in more detail in the test questions.

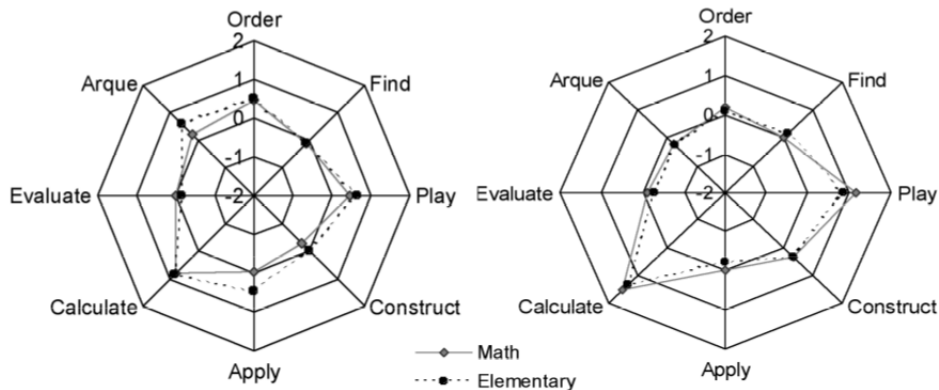


Figure 1. Students’ math-profiles (left) and techno-profiles (right) in 2005

Profile shifts among student elementary teachers

Figure 2 (left) shows shifts of the math-profiles among elementary teacher students. A significant shift appeared in playing (growth) and in calculating (decrease), the latter being the only decreasing dimension. In 2006, students emphasized applying, arguing and playing, whereas evaluating appeared weakly. Students are more sure about their answers than in 2005. The figure in the middle also shows a shift in the identity-profiles. A significant growth happened in finding, constructing, and evaluating. The most remarkable shifts happened in techno-profiles, especially significant changes in finding, playing, constructing and arguing.

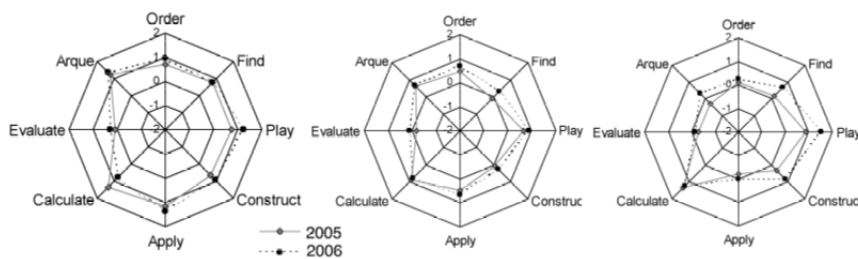


Figure 2. Profile shifts among student elementary teachers: math-profile to the left, identity-profile in the middle, and techno-profile to the right

Profile shifts among mathematics students

There are no significant shifts in any of the components, neither of the math-profiles nor of the techno-profiles among mathematics students. Also in the identity-profile the only significant change appears in calculating, which surprisingly showed an increasing trend. These findings can be interpreted to tell about very old-fashioned "papermedia-culture" in the department of mathematics, and for example, stressing pure mathematics without any great interest to put emphasis on applications or making of own hypotheses. This appears even through students' expressions in everyday language, when they speak about "calculating exercises" even though the question would be about a course in abstract topology.

Comparing the profiles in 2006

When taking a closer look on students' profiles in 2006, elementary teacher students show more all-round math-profiles and identity-profiles than mathematics students, especially concerning the creative components on the right hand side of the octagon (see Figure 3). A significant difference appears in playing, constructing, applying and calculating (the latter appearing stronger, as mentioned above). In techno-profile the trend above appears in finding, constructing, arguing, playing and ordering. In the first three, the difference is significant.

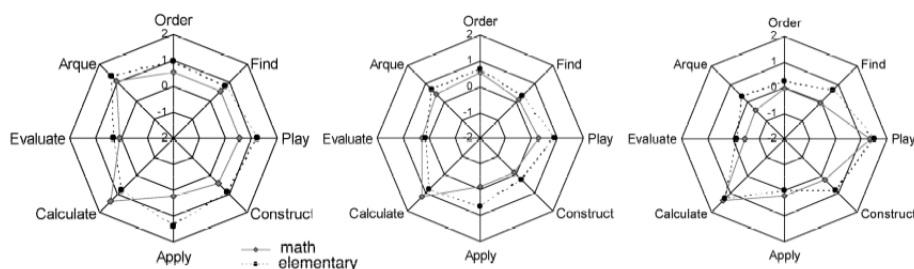


Figure 3. Profiles of both groups in 2006. Math-profile to the left, identity-profile in the middle, and techno-profile to the right.

Looking back

This study is an effort to develop an instrument based on Zimmermann's activity components in the guidelines of the criteria for relevant and viable educational research, highlighted by Sierpiska (1993). One of our many challenges is to develop the test to measure not only the eighth Zimmermann dimensions but

also links between them. The analysis revealed a need for improvement of the questionnaire, especially concerning the definition of the terms 'evaluate' and 'play'. Furthermore, the Likert-scaling and combining of students' answers with their certainty needs a thorough re-evaluation. After that, our next plan is to measure the test reliability by using bigger samples. In this study, less than 50 % of the original samples answered in the second test. Also the great amount of extra, so-called control questions in our test could have decreased the test reliability, if students got bored when answering.

Discussion

For anybody who wants to get a solid view of mathematics, it is important to know how and from where mathematical knowledge and mathematical thinking might appear and to come into life and action. Based on "human laboratory within 5000 years", Zimmermann activities can be considered as a relevant framework, not only for the teaching and learning of mathematics, but also for what could be called 'mathematical education' in a more general sense. Besides that, our results reveal a very stereotypic view of mathematics among both sample groups. The finding of most concern is that the answer to the provocative question "does mathematics education contaminate the mathematical education?" might be affirmative. Conventional mathematics education (in the sense of mathematics teaching) seems to act in a contra-productive way when thinking about extending students' views of mathematics. However, a progressive shift to promote creative activities within a constructivist framework using modern technology – as we did within our teaching of pedagogical courses for student elementary teachers (see also Haapasalo & Eronen, 2007) - seems not only to change students' view of mathematics, but also to increase their self-confidence in making mathematics and utilizing technology.

References

- Eronen, L. & Haapasalo, L. (2005). *WWW-based questionnaire for measuring of student profiles*. <http://jnor.joensuu.fi/eronen/ztest>
- Eronen, L. & Haapasalo, L. (2006). Shifting from textbook tasks to mathematics making. In T. Asunta & J. Viiri (Eds.). *Pathways into research-based teaching and learning in mathematics and science education, Research Report 84*, 75-92. Jyväskylä: University of Jyväskylä.
- Haapasalo, L. & Eronen, L. (In this volume). Integrating pedagogical studies of mathematics teacher students within socio-constructivist technology-based environments.
- Sierpinska, A. (1993). Criteria for scientific quality and relevance in the didactics of mathematics. In G. Nissen & M. Blomhøj (Eds.). *Criteria for scientific quality and relevance in the didactics of mathematics*, 35-74. Roskilde: IMFUFA.

Zimmermann, B. (2003). On the genesis of mathematics and mathematical thinking - a network of motives and activities drawn from the history of mathematics. In L. Haapasalo & K. Sormunen (Eds.). *Towards meaningful mathematics and science education*, 29-47. Joensuu: University of Joensuu.

Socio-constructivist collaborative group processes in teacher education

Pasi Eskelinen

Hatsala Comprehensive School, City of Kuopio

Lenni Haapasalo

University of Eastern Finland

This article focuses on features of group dynamics when applying the Learning by Design principle within a collaborative socio-constructivist approach for two student groups in teacher education. The objective of the elementary teacher students' design process was a hypermedia, whereas the mathematics teacher students planned and realised microteaching lessons within their peer group. Apart from other studies of design processes the study stresses the impact of knowledge structure, pedagogical philosophy and support for reflective communication.

Introduction

In the year 2000 the first author carried out the course “Introduction to ICT in education” for primary-school teacher students at the Faculty of Education. Students designed hypermedia for the learning of the conceptual field decimals-measurement-accuracy. In his dissertation, Eskelinen (2005) uncovered how different kinds of pedagogical approaches and support for reflective communication affect students' conceptions of teaching and learning, interest in ICT support, and group dynamics. Motivated by interesting findings in this study we decided to find out if the same kinds of features would appear, when mathematics teacher students planned and realised, within their peer-teams, microteaching lessons, aiming to focus on the basic features of socio-constructivist theories. This part of our study was carried out in 2006 within the second author's course “The basics for teaching and learning of mathematics”. We start by introducing the pedagogical philosophy of our courses, staying, however, with a compact description, because this framework is introduced elsewhere (see Haapasalo & Eronen in this volume). After that, we represent the most interesting results concerning group dynamics, and we also give a short summary of how students' conceptions of teaching and learning shifted.

Background

For our framework concerning group development we utilized the famous classification of Tuckman (1965): *forming, storming, norming, and performing*. We do not illustrate these phases, because the typing of “Tuckman group process” in Google (08.08.2011) gave more than 25000 appropriate hits with interesting links. Note that even though Tuckman in 1977 refined and developed his model by adding the fifth stage ‘adjourning’, we stayed with his original model. There are numerous studies of group dynamics in problem solving. However, very important aspects, such as the impacts of knowledge structure, pedagogical philosophy and support for reflective communication on the learning process, have been neglected. In relation to this philosophy, learning may be based upon a developmental approach, assuming that there is a dependence of procedural on conceptual knowledge, or then an educational approach, that proposes the opposite, the appropriateness of which is to be clarified. This clarification, based upon contextual realism within a socio-constructivist framework, may benefit from a deep theoretical understanding of mathematical knowledge construction obtained from the large Finnish project “Model construction for didactic and empirical problems of mathematics education” (MODEM). We do not introduce this framework and the two above-mentioned approaches, which are described by Haapasalo and Eronen (2007) in this volume. Here we emphasize the concepts “collaborative” and “socio-constructivist”.

By *collaborativity* we mean that student teams define own goals for their work, and maintain active processes aiming to this goal, profiting from diversity of perspectives and opinions, and free flow of information (cf. Alessi & Trollip, 2001). In an ideal collaborative learning many features of minimalist instruction can also be identified (cf. Haapasalo & Eronen, in this volume). Concerning our socio-constructivist framework, we apply the famous *pragmatic theory of truth* emphasized by Peirce and modified by Niiniluoto (1987) (see Figure 1). When an open problem is given, the teams work in causal interaction with this problem within collaboration. After testing the viability of radical ideas (von Glasersfeld, 1991) within the teams and between the teams, finally only those ideas remain, which are viable in the whole social group consisting of those teams.

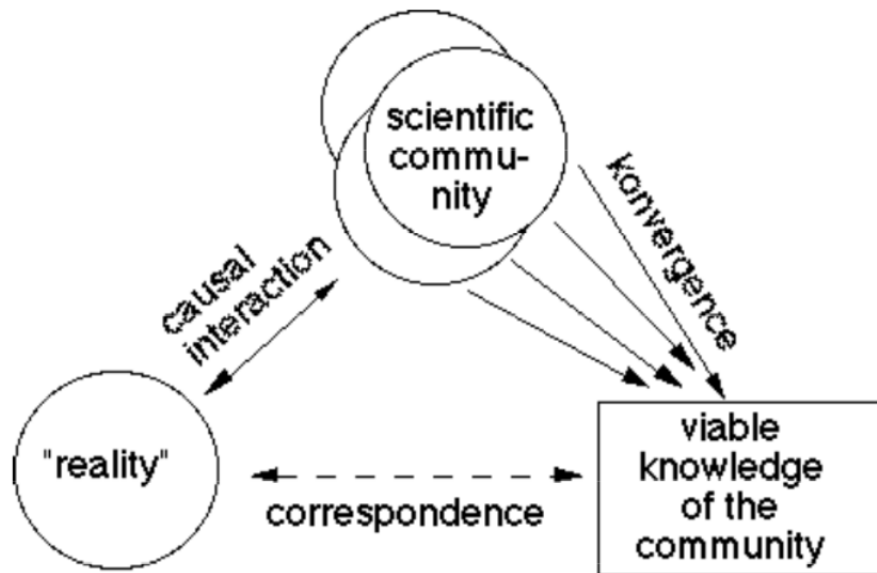


Figure 1. Pragmatic theory of truth: viable knowledge as a result of social construction (see Niiniluoto, 1987, 46)

In his courses, the second author used to apply this framework in interaction with MODEM-framework as illustrated in Figure 2. At first, an authentic pedagogical problem is given for each of the student teams. At the beginning students have to interpret this on the basis of their own conceptions and learning history. This phase is based on a developmental approach utilizing more or less spontaneous procedural knowledge. Through social communication inside the teams and between the teams, students test the viability of their ideas. Usually the diversity of those ideas motivates the students to find out what would be the viable scientific interpretation and explanation. This gives the teacher an ‘excuse’ to give lessons on pedagogical theories. The aim of the lectures is to scaffold students with appropriate conceptual knowledge, which means applying an educational approach. After that the students experienced the phases of forming, storming, and norming within those activities, the next step for each of the student teams is to plan a microteaching lesson concerning the very first pedagogical problem, to be realized later in the whole group. This means the group dynamical phase of performing. The team in charge takes the role of a teacher, whereas the other teams play the role of pupils. After carrying out the microteaching lesson, all the students discuss about appropriateness of the mathematical and pedagogical content. General educational goals are not necessarily in focus at this phase. An ideal case would be if the pedagogical solutions realized in microteaching lessons could be developed and tested immediately in authentic school teaching (which, unfortunately, seldom is the

case). This would as well give students opportunities to focus on general educational aspects.

OBJECTIVE	INTER- PRETATION	RESULT OF THE CONSTRUCTION	ARGUING	APPROACH
authentic problem	spontaneous knowledge	viable conception among peer groups	own experiences and conceptions	developmental
analysis of the process	modern theory of teaching and learning	accommodation of own conceptions to those of theory/educators	accommodation of scientific explanations	educational
posing of learning task	own learning history	“quasi-script” for own teaching	own experiences and conceptions	developmental
basis of assessment	modern theory of teaching and learning	accommodation of own conceptualization and new theories	accommodation of scientific framework	educational
posing of learning task	own learning history	constructing of viable knowledge of modern teaching and learning	accommodation of scientific framework	educational
microteaching	accommodation of theories	authentic situation among own peer group	own experiences vs. theories	developmental & educational
authentic school teaching	accommodation of theories	realisation of own authentic teaching	own experiences vs. theories	developmental & educational
general educational dimensions	behavioral observations assisted by an expert	cognitive conflicts between observations and own educational conceptions	accommodation of own explanations to general peda- gogical theories	developmental & educational

Figure 2. Allocating of microteaching within the educational approach (see Haapasalo, 2004, 179)

Samples and methods

In our work with elementary teacher students, we applied educational and developmental approaches as follows. The sample group (n = 84) was divided into four sub-groups according to the two pedagogical approaches and

communicational tutoring. The research was based on a quantitative analysis of the follow-along measurements through questionnaires administered at different phases of the design process (Figure 3). The design of the hypermedia itself was based on socio-constructivist collaborative activities. In addition to that, two ‘educational groups’ got a two hours lesson on the *MODEM* framework, which means that they had basically a basis to apply ‘constructivism on constructivism’, i.e. to base their hypermedia product on cognitive conflicts (see Figure 2). Students in the two ‘developmental groups’ worked without any pedagogical framework, coming up with their own pedagogical ideas. Both of these groups were divided into two sub-groups according to if they had support for reflective communication or not. The five consecutive follow-along measurements of students’ conceptions on teaching and learning were administrated as illustrated in Figure 3 (on the right) by using statements within 7-step Likert type scale from -3 to +3 (strongly disagree, disagree, slightly disagree, neutral, slightly agree, agree, strongly agree; see <http://wanda.uef.fi/lenni/grpdyn/test.html>). The reliability and the significance tests connected to this instrument are discussed in Eskelinen (2005, 112-119). Note that in this article we concentrate on comparing the two approaches and therefore, we neglect the sub-classification according to reflective communication.

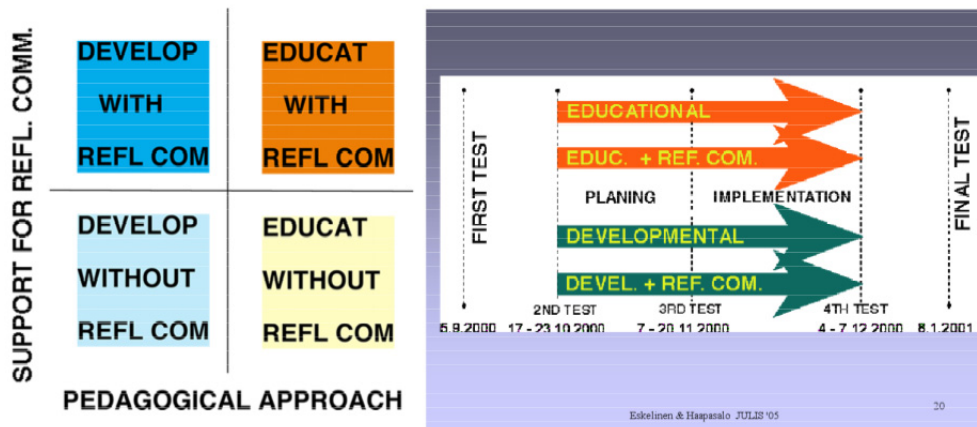


Figure 3. Sub-groups and follow-along measurements during the design process among elementary teacher students

The task of mathematics teacher students ($n = 27$) was to plan and realize microteaching lessons among own peer-teams (for the conceptions of vector, place value system, quadrilaterals, derivative, symmetry). This task was given after about 8 lessons on modern constructivist views on teaching and learning mathematics, emphasizing the philosophy beyond Figure 1 and the left-hand box of the *MODEM*-framework (see Figure 1 in Haapasalo & Eronen, in this volume). For this reason this student group represented a sample group of an educational approach, and because the teaching professor helped students to maintain argumentation and profiting from contradictory opinions, this group

can be considered to receive tutoring for reflective communication. Students had to answer the above-mentioned questionnaire concerning group dynamical variables. Figure 4 illustrates how this test was repeated three times: before the planning of microteaching, after the planning, and after microteaching-lessons had been carried out. In the sense of Tuckman framework, the measurements could be interpreted to be located before norming, during norming/performing, and after performing.

The data gained from questionnaires was analysed quantitatively. A sub-sample of the mathematics teacher students was interviewed by the peer-students who were making their pedagogical studies (see Haapasalo & Eronen, in this volume).

The two samples differed radically from each other. While the educational approach was represented for the elementary teacher students only by giving one lesson (2 hours) for one half of the sample group, the mathematics teacher students had had several hours time to get oriented into this approach, including also reflective discussions.

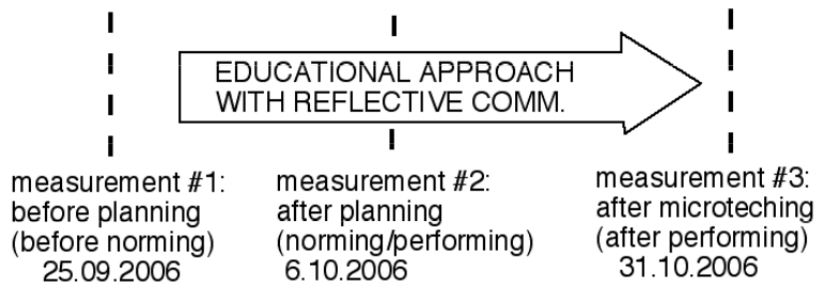


Figure 4. Measurements during the work of mathematics teacher students

Aims

Our meta-level goals were to develop teacher education and to generate hypotheses for future studies. The study aimed to find answers to the following research questions:

1. Which significant changes can be found in affective variables within Tuckman phases?
2. How does the pedagogical approach affect on group dynamics?
3. What can be said about collaborativity in the planning phase?
4. Which kinds of differences appear between the two sample groups?
5. How did the pedagogical approach affect student's conceptions on teaching and learning?

Results

Affective variable shifts and effect of the pedagogical approach

In the whole sample of the elementary teacher students the group dynamics varied in the design process as anticipated in the light of earlier studies: in the planning phase the amount of stress and aggression was increased ($p = 0.00$; $ES = 0.63$ between measurements 2 and 3), whereas in the phase of implementation, the self confidence was increased ($p = 0.05$; $ES = 0.21$ between measurements 3 and 4). The right-hand side of Figure 5 illustrates that increasing of stress and aggression was even stronger among students of the educational approach ($p=0.05$; $ES=0.78$) than in the developmental approach. We can also conclude that there might be some shift also in their self-confidence (see Eskelinen, 2005, 156-157). The left-hand side of Figure 5 shows that social skills increased significantly in the developmental approach group ($p = 0.00$; $ES = 0.81$ between measurements 1 and 4), whereas this shift is not significant in the educational group.

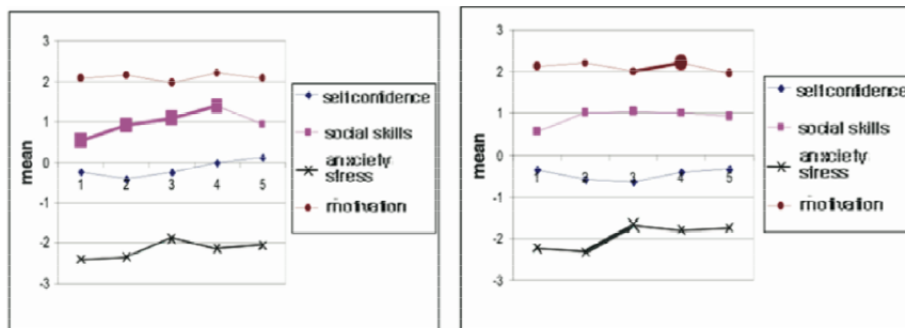


Figure 5. Shifts in Tuckman phases among elementary teacher students, developmental group on the left and educational group on the right

Figure 6 illustrates the shifts in the variables 'anxiety and stress' and 'social skills' among mathematics teacher students. The statistical analysis revealed that these shifts are no significant. On the other hand, social skills finally reached about the same level as among elementary teacher students. For the first group, the starting level of anxiety and stress was higher than for the latter one, finishing onto the same level. The reason for differences between the two groups will be discussed later.

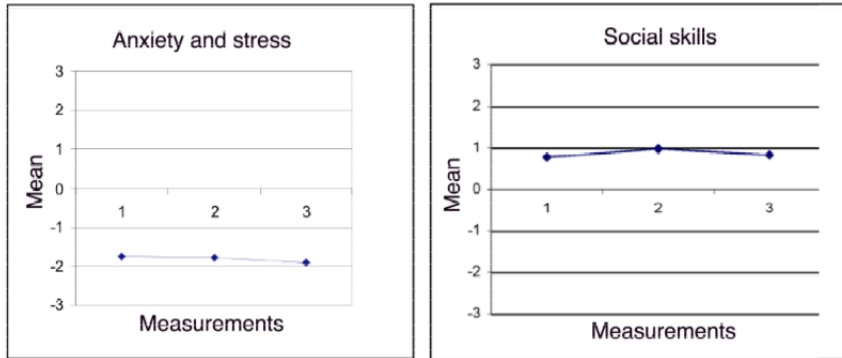


Figure 6. Shifts in the variables 'anxiety and stress' and 'social skills' among mathematics teacher students

Collaborativity in the planning phase

The planning of microteaching lessons gave us an opportunity to evaluate the collaborativity among mathematics teacher students. On the other hand, we also had data from the year 2000, when elementary teacher students planned their hypermedia, also according to two pedagogical approaches having been applied. Figure 7 (left) illustrates students' time consumption in each sample.

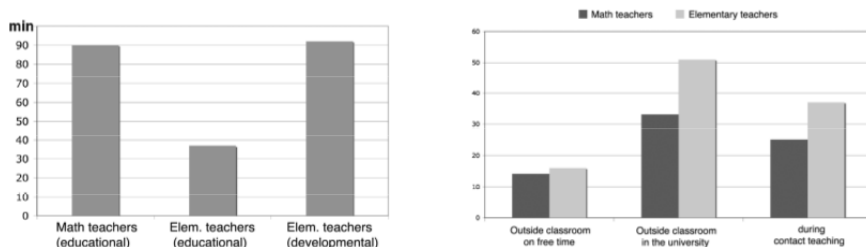


Figure 7. Time consumption (left), and information change (right) among students

We also wanted to find out how and where students shared knowledge within teams and between the teams. The finding is that this kind of communication occurred at most on free time in the university or elsewhere (Figure 7, right).

Differences between the two teacher student groups

To compare the two groups of objects we could only use qualitative methods. Furthermore, it is the educational sub-group among elementary teacher students that could be used for this comparison. This group can be considered very suitable to analyse with the Tuckman process, because the students met each other for first time, and in addition to that, the students had had an extra demand to apply new kind of socio-constructivist philosophy connected to the educational MODEM-framework. Moreover, students had had to use new kind of software to make a hypermedia application. Perhaps those two demands cumulated to cause extra stress. Mathematics teacher students had studied

mathematics at least one year at the university. In addition to that, they got at least two courses in the pedagogy of mathematics, where constructivist working methods were applied. Hence, they got more support and time to understand the MODEM-framework during their lessons, which also contained reflective discussions. Furthermore, concerning the group dynamics, they had some knowledge of each other. The right-hand side of Figure 5 does not reveal the cognitive results gained from the educational approach for elementary teacher students. Figure 8 illustrates that students understood what is needed to trigger a cognitive conflict for the construction process by the learner.



Figure 8. Causing a cognitive conflict (students' work)

The hypermedia of those teams, namely, produced quite sophisticated material showing that students understood basic components of constructivist learning (see Eskelinen, 2005, 87-94). Contrary to that, it seems that the students in the developmental group mainly used their time for more or less entertainment, which strengthened their social skills and had a small impact on their understanding of the appropriate goals of modern teaching and learning.

Impact of the pedagogical approach on student's conceptions on teaching and learning

Our conjectures above find support from the fact that in both samples the educational approach very significantly shifted students' conception on teaching and learning from objectivist-behaviourist view to a constructivist one. At the same time their behaviourist views weakened very significantly (Figure 9). In the developmental group this shift was not significant.

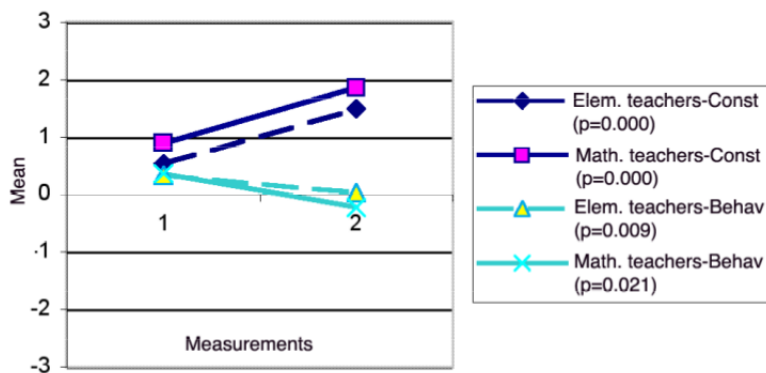


Figure 9. Shifts in the conceptions on teaching and learning

Conclusions

Today, not only experts, but also the work of ordinary citizens utilizes the idea of shared knowledge and distributive methods that are independent of time, place and formal modes. Thus, the design of technology-based learning environments within an adequate constructivist theory, linked to the knowledge structure, might be a proper framework to respond to the main challenge of teacher education: to get students to understand the basic components of modern constructivist theories on teaching, and maintaining the learning through cognitive conflicts. The developmental approach based on spontaneous procedural knowledge seems to be appropriate in relation to both cognitive and affective variables. In order to apply the educational approach so as to stress the importance of conceptual knowledge, an educator needs to be sensitive to the cognitive and emotional variables present in the learning process. Our results suggest that well-tailored learning environments – actually multi-disciplined investigation spaces – can not only produce good cognitive results, but also promote dynamical computer skills among the students without being taught separately from information structures and pedagogical thought (see Eskelinen, 2005, 150-155).

Even though the small amount of objects among mathematics teacher students allows us to make only suggestive conclusions, our study gives many hypotheses, which are worth testing within large-scale research. It would be interesting, for example, to find out if the smaller amount of students' time consumption could be due to appropriate conceptual knowledge, which releases students from unnecessary procedure-based activities. Furthermore, the fact that students seem to use most of their study time outside the classroom, earns serious efforts from the educators, to consider if the focus from well-prepared classroom lessons should be put on reflective tutoring inside and outside the classroom, and even on students' free time. If the answer to these conjectures would be affirmative as Haapasalo and Eronen (in this volume) suggest,

institutions could be some kinds of pit stops to scaffold students' activities outside the classroom.

References

- Alessi, S. M. & Trollip, S. R. (2001). *Multimedia for Learning. Methods and Development*. Boston: Allyn & Bacon.
- Eskelinen, P. (2005). *Luokanopettajaopiskelijoiden tieto- ja oppimiskäsityksen muuttuminen kollaboratiivisessa design-prosessissa*. Joensuu: Joensuun yliopisto.
- von Glasersfeld, E. (1991). *Radical constructivism in mathematics education*. Dordrecht: Kluwer.
- Haapasalo, L. (2003). The conflict between conceptual and procedural knowledge: should we need to understand in order to be able to do, or vice versa? In L. Haapasalo & K. Sormunen (Eds.). *Towards meaningful mathematics and science education*, pp. 1-20. Joensuu: University of Joensuu.
- Haapasalo, L. (2004). Opettajankoulutuksen dilemma: Tekemällä oppimaan vai oppimalla tekemään? In P. Atjonen & P. Väisänen (Eds.). *Osaava opettaja*, pp. 171-190. Joensuu: Joensuun yliopisto.
- Niiniluoto, I. (1987). Pragmatismi. In I. Niiniluoto & E. Saarinen (Eds.). *Vuosisatamme filosofia*, pp. 40-73. Helsinki: WSOY.
- Tuckman, B. (1965). Developmental sequence in small groups. *Psychological Bulletin*, 63, 384-399.

Integrating pedagogical studies of mathematics teacher students within socio-constructivist technology-based environments

Lenni Haapasalo, Lasse Eronen
University of Eastern Finland

The article represents a cavalcade of empirical studies, which mathematics teacher students made as a part of their pedagogical studies in the authors' home faculty when involved in the so-called ClassPad-project. This technology-based project emphasizes informal more than formal mathematics within the 'minimalist instruction' approach and in an eight-fold framework of activities that, in the history of human thought, have proved to be sources of sustained new ideas in mathematics.

Introduction

A modern pocket computer, such as ClassPad made by Casio (see <http://www.classpad.org>), allows the use of the drag-and-drop technology, where the student can easily manipulate mathematical objects between two windows, illustrating two different forms of mathematical representation. To illustrate this, we pick up the following example from Eronen & Haapasalo (2006): A child can just play harmlessly by typing zero, for example, in the Main window (see Figure 1). Maybe (s)he gets an idea to find out how this zero would look like in the geometry window. After drag-and-dropping it into that window, a line appears (a). When lifting the line and drag-and-dropping it into the algebra window, zero had changed to 9 (b). The student can repeat these kinds of activities by lifting and lowering the line and watching the impact by utilizing drag-and-drop. When choosing the 'Rotate' tool (c), the computer asks for a number, as, for example, 45° . The line rotates and a drag-and-dropping gives something very strange, an "unknown" x appears (d). The student can add a coordinate system by clicking its icon (e), and then continue manipulating the coefficient of x (the slope of the line) and the constant term (f) to notice the connection between each number in the algebraic window and the position of the line in the geometry window.

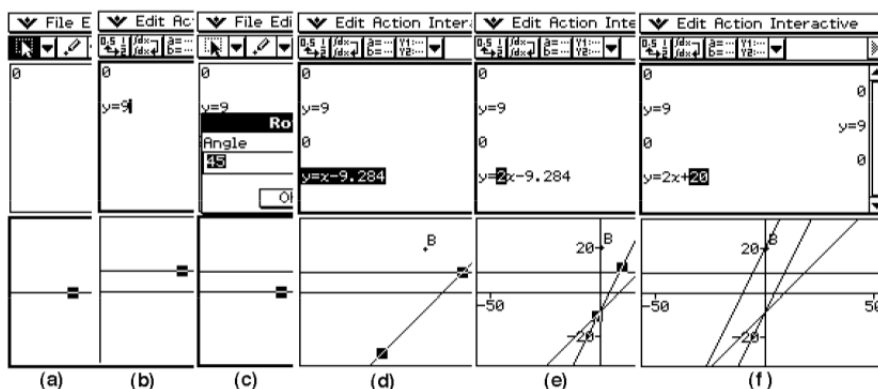


Figure 1. To get oriented to the equation of a straight line by using ClassPad

These maneuvers above are intended to form links between *conceptual* and *procedural knowledge* by utilizing the so-called *simultaneous activation principle*¹. The Geometry Link –option of the tool carries students’ manipulation between the two windows even without any drag-and-drop maneuver. The so-called ClassPad project began at the end of May 2005 when Casio Europe donated 25 tools for research purpose. Because there were only a few days before the pupils at 8th grade would go for their summer holiday, we got the idea to ask if they would like to take the tool with them home to play with it voluntarily. The only wish was that they should write a portfolio about what they eventually had done. Almost all students accepted this opportunity and thus, a spontaneous compact problem set was prepared for pupils on the topic “Equation of a Straight Line”, which is the most important topic to come at the 9th grade. In case pupils would play with the tool and learn something, we made a cognitive test on this topic. We also measured all three Zimmermann-profiles among the pupils, i.e. how strong the following eighth activities appear to the pupil (1) when using the term ‘mathematics’, (2) when evaluating their own mathematical performance, and (3) when evaluating how suitable a computer is to perform each of the activities: *ordering*, *finding*, *playing*, *constructing*, *applying*, *calculating*, *evaluating*, and *arguing*. The basis of these profiles is explained in our second article in this volume (see Eronen & Haapasalo, in this volume).

¹ We adopt the following characterizations of Haapasalo and Kadjevich (2000):

- Procedural knowledge denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representational forms. This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them.
- *Conceptual knowledge* denotes knowledge of particular networks and a skilful “drive” along them. The elements of these networks can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representational forms.

The following portfolio example shows how a pupil who usually kept showing a poor motivation in the classroom applied the simultaneous activation principle in a sophisticated way without any tutoring from the teacher's side. Just to mention that she made this in the middle of the most beautiful Finnish summer night at 00:27 a clock!

- The equation of a line is $y = 1.613x - 0.5992$
 - By changing the equation to $y = 2x - 0.5992$ the angle between the line and y-axis is getting smaller
 - By changing the equation to $y = 1x - 0.5992$, the angle between the line and y-axis is getting bigger.
 -
 - The equation is $y = 1.613x - 0.5992$.
 - I change the equation to $y = 1.613x - 0.4$.
I don't see any changes.
 - I change the equation to $y = 1.613x - 4$, the line moves to the same direction away from origin.
 - When changing the equation to $y = 1.613x + 4$, the line moves in the same way, but to another direction on x-axis with equal distance from the origin.
 - I will continue tomorrow.
- Time is now 01.42 (Duration of session, 1h 15 min)

When measuring the Zimmermann-profile after the summer holiday, we found that doing mathematics with ClassPad shifted her mathematical and identity profiles within the Zimmermann activities (cf. Eronen & Haapasalo, in this volume) into a positive direction (i.e. promoted playing and finding, for example), opening up new progressive ways to arrange teaching and learning. Encouraged by these kinds of findings we decided to involve all the mathematics teacher students in this project during the study year 2005-2006. In this article we represent some of the empirical studies, which students planned and realized in order to link theoretical studies and practical teaching. Our starting point was to emphasize the following categories concerning what modern technology can maintain and promote (see Haapasalo, 2007): links between conceptual and procedural knowledge; interplay between systematic approaches and minimalist instruction; sustainable components of mathematics making; meta-cognitions and problem-solving skills; and learning by design. We will describe why these categories are relevant concerning teacher education. At the end of each chapter, we shortly represent the sub-studies, including some interesting cognitive and affective results (gained by using quantitative vs. qualitative methods, respectively). To avoid mismatch, we use the term pupil when speaking about the students of 9th grade. When speaking about 'research teams' we mean mathematics teacher students.

Promoting conceptual and procedural knowledge

There is a basic conflict between conceptual and procedural knowledge: how much students should understand before they are able to do, and vice versa. Concerning ICT-based learning, the first challenge arises from the structure of the topic to be learned, whereas the other is caused by the instructional variables required for technology use. These two knowledge types seem to develop iteratively, where a change of problem representation influences their relation. Relating different representations can not only support the development of conceptual knowledge, but also relate procedural and conceptual knowledge (cf. Haapasalo, 2003). To coordinate the process and object features of mathematical knowledge, multiple forms of representation are to be utilized and connected, especially with the aid of modern technological tools. Such a development was assumed in the pedagogical model of the MODEM-project². This model for the interplay between the two knowledge types makes use of spontaneous procedural knowledge as well as the simultaneous activation of conceptual and procedural knowledge (see Figure 2).

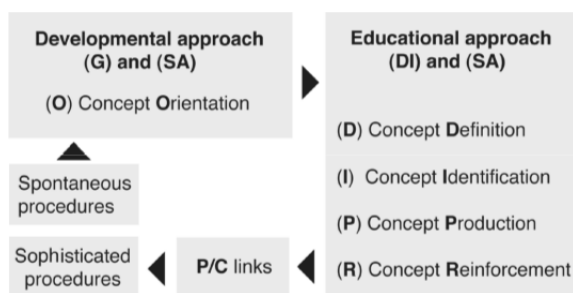


Figure 2. MODEM framework as sophisticated interplay between developmental and educational approach

Examples of studies belonging to this category

One research team (Ruokolainen & Kukkonen, 2006) used the MODEM-framework to plan learning and assessment material for basic concepts of statistics (e.g. organizing and representing data, mean, standard deviation, mode, median). Another research team (Hurskainen et al., 2006) did the same concerning the basic features of a linear function. Figure 3 represents an example of an orientation task which utilizes a developmental approach: the interpretations of the situation can be based on mental models of the pupils,

² See Haapasalo (2003) and <http://www.joensuu.fi/lenni/modemfin.html>. To see the systematic approach of the MODEM-framework, the CAL-software is freely downloadable at <http://www.joensuu.fi/lenni/programs.html>.

coming, more or less, from their naïve procedural ideas. These act like a wake-up voltage in an electric circuit that triggers another, much more powerful current to be amplified again. The procedural and conceptual knowledge types start to support each other.

Draw in the Geometric Window a line, which looks something like in each of these pictures. After that, by using *Drag-and-Drop* or *eActivity*, find the line equation.

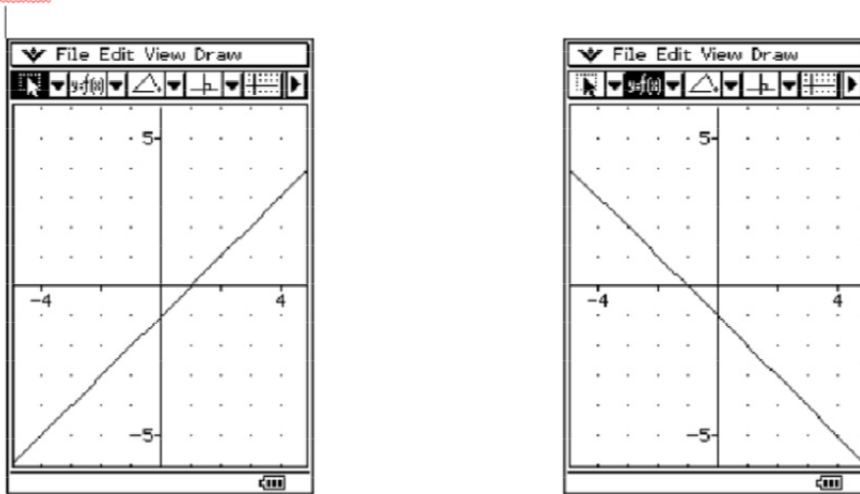


Figure 3. An orientation task with ClassPad

The third research team (Tilli & Vauhkonen, 2006) measured pupils' understanding of this concept by using the same test as in MODEM 1 study, fitting the levels of mathematical concept building illustrated in Figure 2 (cf. Haapasalo, 2003). Because the main results are represented in Eronen & Haapasalo (in this volume), we just summarize here: students' scores of the ClassPad test group (grey) were significantly higher than those of students in conventional school teaching. Furthermore, students also showed high procedural skills. A postponed test after five months revealed that this scoring level remained, and by many students even improved (see Figure 4). This happened especially among those students who preferred to go for procedural activities but also later on understanding what they had done (the so-called procedurally-oriented students defined by Haapasalo and Järvelä, 2005).

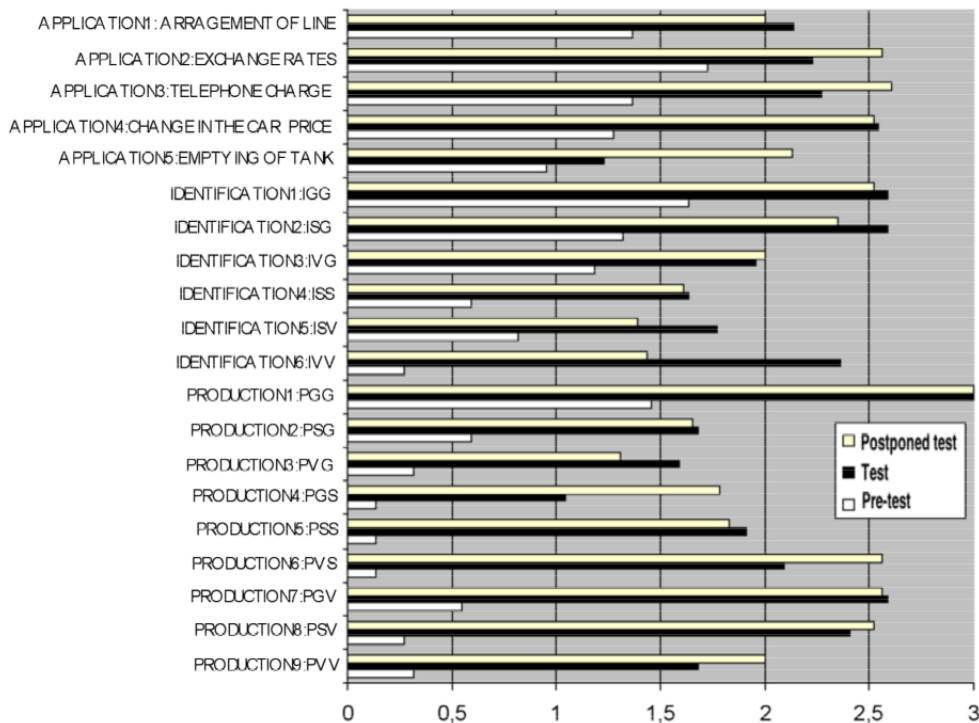


Figure 4. Students' cognitive results before (white), after (black), and five months after (grey) the ClassPad working period

Interplay between systematic approaches and minimalist instruction

Students very often neglect the teacher's tutoring or they feel they do not have time to learn how to use technical tools. Teachers similarly feel they do not have time to teach how these tools should be used. This problem becomes even more severe when the versatility of advanced technology cannot be accessed without first reading heavy manuals. The term *minimalist instruction* was introduced by Carroll (1990), who observed that learners often tend to "jump the gun". They avoid careful planning, resist detailed systems of instructional steps, tend to be subject to learning interference from similar tasks, and have difficulties in recognizing, diagnosing, and recovering from their errors. Modifying Lambrecht (1999), we pick up the following characteristics of minimalist instruction: specific content and outcomes cannot be pre-specified, although a core knowledge domain may be specified; learning is modeled and coached for students with unscripted teacher responses; learning goals are determined from authentic tasks stressing doing and exploring; errors are not avoided but used for instruction; learners construct multiple perspectives or solutions through discussion and collaboration; learning focuses on the process of knowledge construction and development of reflexive awareness of that process; criterion

for success is the transfer of learning and a change in students' action potential; the assessment is ongoing and based on learner needs.

The features of minimalism include several varieties of constructivism, offering also instructional assumptions. However, to trig and maintain successful problem-solving processes inside and outside the classroom, a thorough planning of the problem to be posed and studied is needed. For this, empirically tested more or less systematic pedagogical models (as MODEM described above) can be helpful. When planning a constructivist approach to the mathematical concepts under consideration, the focus is on the left-hand side of Figure 2. On the other hand, when offering students opportunities to construct links between representation forms of the concept, the focus is on the right-hand box, which describes the stages of mathematical concept building. In learning situations, however, students must have freedom to “jump the gun”. They must be able to choose the problems that they want to learn within continuous self-evaluation instead of relying on express guidance from teachers.

Examples of studies belonging to this category

The above-mentioned research teams, who designed the learning environments for the learning of linear function and statistics, combined the features of systematic approach and minimalism. Team Karttunen et al. (2006) observed and videotaped pupils' learning processes and analyzed the thinking processes, which appeared among pupils when studying linear functions. Team Ruokolainen and Kukkonen (2006) made the same concerning the learning of basic statistical concepts. Both of these studies offer interesting findings concerning not only the interplay between minimalist instruction approach and systematic approach but affective variables and group dynamics within student pairs. We pick up an example, which shows how students chose tasks from the problem buffet. To go for linear function, one pupil team, for example, initially picked the following problem series on optimizing mobile phone costs: GSM operator A offers free calls for a total fixed price 17,90 € pro month. Operator B does not charge any basic fee whereas cost for every call is 0,10 €/min. Can you make a graphic representation to compare which of these offers might be the most suitable one?

After realizing that the (partly linear) cost models appeared too difficult for them, they then chose a new, much easier, problem set. This happened to consist of ‘identification tasks’ – the first and lowest level of the concept building within the systematic MODEM framework, which was on the basis of the planning of the learning environments. This example shows that a sophisticated interplay between a systematic and minimalist approach can be achieved even by simple pedagogical solutions. Note this important feature of minimalism: we did not want to regulate students' work by recommending them an easier sub-problem, for example. Instead of that it was students' internal motivation that regulated their task choice. Karttunen et al. (2006) found that students used interesting strategies when choosing tasks. Figure 5 represents the path of a student team,

how students selected different kinds of problem types from the buffet. Recalling the abbreviations referred to in Figure 2, we notice that this team did not utilize the MODEM framework in an optimal way but went directly to PSG tasks and also selected from that list those eleven tasks (#1 - #11) more or less randomly. Students evidently liked the amazing ClassPad drag-and-drop function, which automatically performed the PSG action when selected twelve PSG tasks in quite strange order.

→	PSG (#1)	→	PSG (#25)	→	PSG (#3)	→	PSG (#47)	→	O (#13)
→	O (#21)	→	O (#32)	→	PSG (#52)	→	I VG(all)	→	PSG (#6)
→	I VS(all)	→	I SS(all)	→	R (#1)	→	R (#1)	→	R (#1)
→	PSG (#74)	→	PSG (#89)	→	PSG (#98)	→	PSG (#1011)	→	PSG (#1110)
→	P VV(all)	→	P GV(all)	→	PSG (#12)	→	P GG(all)	→	P VV(all)

Figure 5. An example of a “classpath” when picking up selecting tasks

Sustainable components of mathematics making

Zimmermann’s (2003) long-term study of the history of mathematics reveals eight main activities, which very often proved to lead to new mathematical results at different times and in different cultures for more than 5000 years. We took this network of activities as an element in our theoretical framework for the structuring of learning environments and for analyzing student’s cognitive and affective variables. We refer to Eronen and Haapasalo (in this volume) to see how the so-called Zimmermann-profiles were defined.

Examples of studies belonging to this category

Two research teams were involved in studying the students’ Zimmermann-profiles. The first one (Sormunen et al., 2006) measured the profiles among elementary teacher students and first year mathematics students. Those results were used in the above-mentioned article in this volume. The second team (Hakkarainen et al., 2006) analyzed those profiles among students of 9th class. We would like to pick up the profiles of two students who worked as peers with ClassPad and differed from their learning styles radically (see Järvelä & Haapasalo, 2005). In Figure 6, the first one who classified herself to be a conceptually-oriented person (i.e. prefers understanding; see Haapasalo and Järvelä, 2005) acted as peer-teacher, whereas the second one, who mainly listened, was classified as procedurally-bounded learner (i.e. stayed with concrete working procedures). At the beginning the mathematical identity of the learner was even larger than that of the teacher, especially concerning finding and arguing. The more the peer-teacher explained, it was only her own mathematical profile that showed some tendency to expand but only to play-direction. This, however, is not as interesting as what happens to both students’ self-confidence to make mathematics. After the working the learner seemed to

believe he could do more: find, apply and argue. However, surprisingly the self-confidence of the peer-teacher seemed to collapse in almost all dimensions. Let this example be an invitation to the reader to find his or her own explanations and reasons why the spirit of this particular example might be important to understand for every teacher.

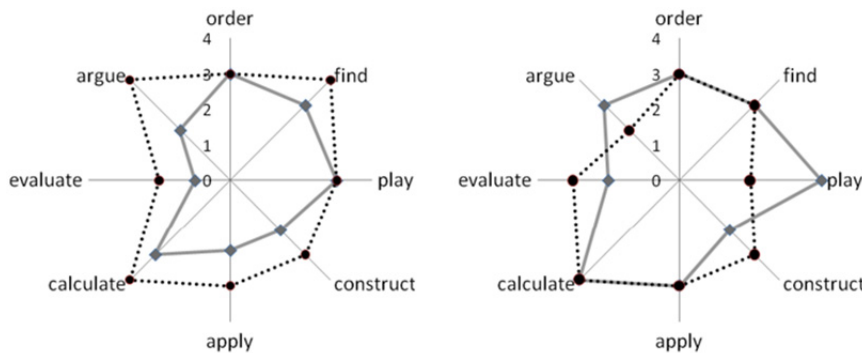


Figure 6a. Mathematical profiles of the peer-teacher (on the right) and her classmate (on the left) at the beginning (dashed) and at the end of the working period

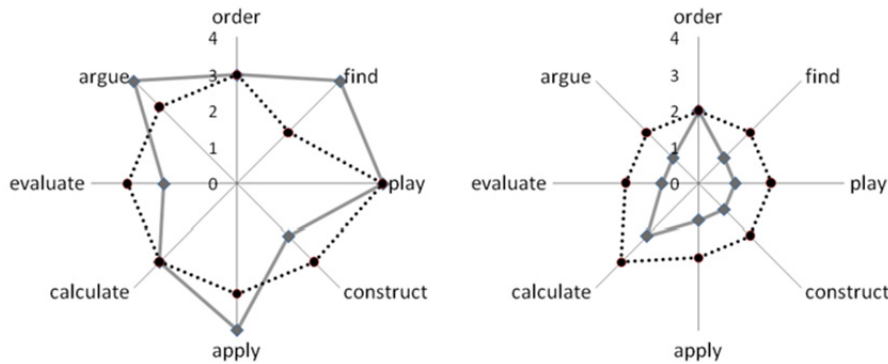


Figure 6b. Identity profiles of the peer-teacher (on the right) and her classmate (on the left) at the beginning (dashed) and at the end of the working period

Learning by Design

Studies of design processes have produced useful information concerning problem solving and group dynamics, for example. Eskelinen and Haapasalo (2006, in this volume) uncover how different kinds of approaches and support for reflective communication affect students' conceptions of teaching and learning, group dynamics and interest in ICT support. The results clearly show that design of a technology-based learning environment within an adequate constructivist theory linked to the knowledge structure offers a promising response to the main challenge of teacher education: to get students to

understand the basic components for teaching and learning. The findings also give strong evidence to the position that technology should have in teacher education programs. Furthermore, the findings do not support the conception that computer skills in teacher education should be taught separately from the information structures and pedagogical thinking. We would like to share the famous view of Jonassen (2000) that those who learn more from the instructional materials are their developers, not the users. Therefore, teachers and students should design ICT-based lessons and thus, become knowledge constructors rather than knowledge users.

Most of the above-mentioned sub-studies profit from the pedagogical advantages of Learning by Design principle. In their reports, our research teams showed an ability to use solid theoretical frameworks for the planning, orchestrating and assessment of the learning processes. Perhaps the most important aspect is that teacher students became aware of the potential to use modern technology inside and outside the classroom (cf. Eronen & Haapasalo, in this volume).

Promoting meta-cognitions and problem-solving skills

This category is also a leading idea in the work of many of our research teams. Findings support for the view that technology-based learning promotes metacognitions and problem-solving skills (cf. Haapasalo, 2007; Eronen & Haapasalo, in this volume). Accepting constructivist views of teaching and learning mathematics means both emphasizing the genesis of heuristic processes and the ability of students to develop intuition and mathematical ideas. The ClassPad project emphasizes informal more than formal mathematics within the ‘minimalist instruction’ approach and in an eight-fold framework of activities. Within those activities it focuses on “changing representation” which is not only a powerful thinking tool to promote links between procedural and conceptual knowledge, but it might also enhance problem-solving abilities.

Closing remarks

The discussion about the relationship between theoretical studies and teaching practice is still going on. The implementing of technology makes the discussion more complicated, but at the same time, opens new, maybe progressive approaches, especially when learning through design is utilized. Our task is to uncover and explore these paths contributing to a better education for both students and their teachers. When considering technology-based learning to reinforce and implement creative thinking, the focus has been shifted from a technology-oriented viewpoint to a humanistic view, stressing cognitive, affective and social variables involved in the learning process. Mathematics educators should be aware of the way citizens use technology in the modern

society and how this affects those variables. What happens in institutions should have some reasonable equivalent to what happens outside the classrooms.

One of the most remarkable findings of our ClassPad project is that the answer to the future question “Does the allocation of learning shift from the classroom into leisure time?” might be affirmative and, moreover, the role of school and therefore also teacher education needs a thorough re-consideration. If we accept the assumption that the main task of education is to promote a skilful ‘drive’ along knowledge networks so as to scaffold students to utilize their rich activities outside school, it seems appropriate to look for an appropriate educational approach. On the other hand, the fact that students seem to learn effectively many kind of skills – even mathematical ones – outside the school, forces us to ask if there is something wrong inside the school and university as far as the question “how to learn” concerns. We did not choose a progressive way to integrate students’ pedagogical studies because this question is easy, but because it is hard. After having got used to ready-planned materials and objectivist-behaviorist teaching during their own learning history, it was the demand for our teacher students of continuous improvising that seemed to cause uncertainty especially at the beginning of the project. However, at the end students expressed mainly positive opinions on this effort, which they described as progressive, challenging and worth of risks. What students’ own research concerns, the most important aspect is, that they got a touch of doing relevant, valid, original and rigorous educational research, where mathematics is not just a place holder (cf. Sierpiska, 1993). Even though there are lacks in their data analysis and interpreting of the results, we also learned a lot as supervisors. The project gave us great amount of new research ideas and with students’ kind permission, we can also use their data and pre-analysis to improve our methods.

As teacher educators we would like to avoid repeating the quite general mistake of our scientific community, namely, to consider just one or two component of the teaching and learning process at a time. We decided to open up a cavalcade of research components instead of writing a report in a rigid form of meta-study, occupying perhaps more than 100 pages of these proceedings. We apologize if some concepts do not open up fully for the reader. Hence, we invite him or her to take a closer look at the references.

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References

- Carroll, J. M. (1990). *The Nurnberg funnel: designing minimalist instruction for practical computer skill*. Cambridge, Massachusetts: The MIT Press.
- Eronen, L. & Haapasalo, L. (2006). Shifting from textbook tasks to mathematics making. In T. Asunta & J. Viiri (Eds.). *Pathways into research-based teaching and learning in mathematics and science education*, pp. 75-92. Jyväskylä: University of Jyväskylä.
- Eronen, L. & Haapasalo, L. (in this volume). *Shifting of mathematical profiles among elementary teacher students and mathematics students*.
- Eskelinen, P. & Haapasalo, L. (2006). Teachers and students as designers of their learning environments. In T. Asunta & J. Viiri (Eds.). *Pathways into research-based teaching and learning in mathematics and science education*, pp. 107-115. Jyväskylä: University of Jyväskylä.
- Eskelinen, P. & Haapasalo, L. (in this volume). *Socio-constructivist collaborative group processes in teacher education*.
- Haapasalo, L. (2003). The conflict between conceptual and procedural knowledge: should we need to understand in order to be able to do, or vice versa? In L. Haapasalo and K. Sormunen (Eds.). *Towards meaningful mathematics and science education*, pp. 1-20. Joensuu: University of Joensuu.
- Haapasalo, L. (2007). Adapting mathematics education to the needs of ICT. *The Electronic Journal of Mathematics and Technology* 1 (1). <http://www.radford.edu/ejmt>.
- Haapasalo, L. & Järvelä, J. (2005). Conceptual and procedural features by designing of technology-based learning environments. In M. Julkunen (Ed.). *Learning and instruction on multiple context and settings III*, pp. 189-204. Joensuu: University of Joensuu.
- Haapasalo, L. & Kadijevich, Dj. (2000). Two types of mathematical knowledge and their relation. *Journal für Mathematik-Didaktik*, 21 (2), 139-157.
- Jonassen, D. H. (2000). *Computers as mindtools for schools*. Upper Saddle River, NJ: Prentice Hall.
- Järvelä, J. & Haapasalo, L. (2005). Three types of orientations by learning basic routines in ICT. In M. Julkunen (Ed.). *Learning and instruction on multiple context and settings III*, pp. 205-217. Joensuu: University of Joensuu.
- Lambrecht, J. J. (1999). *Developing employment-related office technology skills*. Berkeley: University of California. <http://www.tc.umn.edu/~jlambrec/technology/index.htm>.
- Sierpinska, A. (1993). Criteria for scientific quality and relevance in the didactics of mathematics. In G. Nissen & M. Blomhoj (Eds.). *Criteria for scientific quality and relevance in the didactics of mathematics*, pp. 35-74. Roskilde: IMFUFA.
- Zimmermann, B. (2003). On the genesis of mathematics and mathematical thinking - a network of motives and activities drawn from the history of mathematics. In L. Haapasalo & K. Sormunen (Eds.). *Towards meaningful mathematics and science education*, pp. 29-47. Joensuu: University of Joensuu.

**Students' study reports (in Finnish) at the
Department of Applied Education of the University of
Joensuu:**

- Hakkarainen, M., Sivonen, S. & Pellikka, M. (2006). *Oppilaiden affektiivisen ajattelun ja matematiikkakuvan kehittyminen*. [Affections and beliefs among 9th grade students when studying with ClassPad].
- Hurskainen, O., Tanskanen, H., Lampela, E. & Pylkkö, S. (2006). *Empiirinen tutkimus käsitteenmuodostusprosessin mukaisen materiaalin kehittämisestä lineaarisen funktion peruskäsitteiden opiskeluun*. [Designing MODEM-material for the learning of linear function].
- Hyttinen, M. & Juntunen, J. (2006). *Oppilaiden kognitiivisten taitojen kehittyminen käsitteessä taulukkolaskennan ja tilastomuuttujien perusteet*. [Cognitive development of 9th grade students when studying spreadsheets and statistical concepts with ClassPad].
- Karttunen, K., Nevalainen, M., Pirkonen, K. & Tillonen, J. (2006). *Empiirinen tutkimus dynaamisesta opiskeluseurannasta opiskeltaessa lineaarisen funktion peruskäsitteitä*. [Dynamical assessment of 9th grade students when learning basic concepts of linear function].
- Ruokolainen, A. & Kukkonen, T. (2006). *Empiirinen tutkimus progressiivisen opiskelumateriaalin toimivuudesta eräiden tilastollisten peruskäsitteiden opiskelussa*. [Testing and evaluating progressive learning material when 9th grade students learn linear function].
- Sormunen, M., Turkulainen, J. & Ylinen, T. (2006). *Luokanopettajaopiskelijoiden ja matematiikan opiskelijoiden matematiikkakäsityksistä*. [Mathematical profiles among student elementary teachers and student mathematics teachers].
- Tilli, K. & Vauhkonen, S. (2006). *Oppilaiden kognitiivisten taitojen kehittyminen lineaarisen funktion käsitteessä*. [Cognitive development of 9th grade students when studying basic concepts of linear function with ClassPad].

Actual infinity: a complex concept to be learnt?

Markku S. Hannula
University of Helsinki, Tallinn University

Erkki Pehkonen
University of Helsinki

The paper is based on survey results, and will focus on the development of students' understanding of infinity. The same tasks were answered by different age groups of students: grades 5, 7, 11, and teacher students. The results show that most of the students did not have a proper view of infinity, even not at the teacher education program.

Introduction

Infinity awakes curiosity in children already before they enter school: “*Preschool and young elementary school children show intuitions of infinity*” (Wheeler, 1987). In the primary curriculum, the concept ‘infinity’ is implicitly present in many of the topics, e.g. in arithmetic, when dealing with fractions, or in geometry, e.g. when introducing the concept ‘straight line’. However, this early interest is not often met by school mathematics curriculum and not discussed in school, and infinity remains mysterious for most students throughout the school years.

Actual and potential infinity

Consider the sequence of natural numbers 1, 2, 3, ... and think of continuing it on and on. There is no limit to the process of counting; it has no endpoint. Such ongoing processes without an end are usually the first examples of infinity for children; such processes are called *potentially infinite*. In mathematics, such unlimited processes are quite common.

However, the interesting cases in mathematics are cases, when infinity is conceptualised as a realised “thing” – the so-called *actual infinity*. It requires us to conceptualise the potentially infinite process as if it was somehow finished (Lakoff & Núñez, 2000). The transition from potential to actual infinity includes a transition from (an irreversible) process to a mathematical object. In the history of mathematics, the exact definition of and dealing with actual infinity is

something more than one hundred years old (e.g. Boyer, 1985; Moreno & Waldegg, 1991).

We may distinguish different kinds of infinities in mathematical objects. For example, the set of natural numbers has infinitely many elements, and it has no upper bounds. Therefore, the numbers may become bigger and bigger. Whereas, some sub-sets of rational numbers are different. For example, the set of rational numbers between zero and one has also infinitely many elements, but it is bounded. Furthermore, between any two rational numbers (however close they are to each other) there are infinitely many rational numbers. This property of rational numbers is called *density*.

Tsamir and Dreyfus (2002) summarise the problems mathematicians have had with actual infinity, as follows:

“Actual infinity, a central concept in philosophy and mathematics, has profoundly contributed to the foundation of mathematics and to the theoretical basis of various mathematical systems. It has long and persistently been rejected by mathematicians and philosophers alike, and was highly controversial even in the last century in spite of the comprehensive framework provided for it by Cantorian set theory.”

Infinity has been an inspiring, but difficult concept for mathematicians. It is no wonder, that also students have had difficulties with it – especially with actual infinity and density. Previous research has identified typical problems and constructive teaching approaches to cardinality of infinite sets (e.g. Tsamir & Dreyfus, 2002). Fishbein, Tirosh and Hess (1979) inquired students’ view of infinite partitioning through using successive halvings of a number segment. They concluded that students on grades 5-9 seem to have a finitist rather than a nonfinitist or an infinitist point of view in questions of infinity. Even at the university level, the concept of infinity of real numbers is not clear for all students (cf. Merenluoto & Pehkonen, 2002). For example, Wheeler (1987) points out that university students distinguished between $0.999\dots$ and 1, because “the three dots tell you the first number is an infinite decimal”.

In this paper we want to find out what the level of students’ understanding on infinity in Finnish schools is, and how this understanding develops as students progress through different levels of education: grade 5, grade 7 and grade 11. Furthermore, we compare the results with those of elementary teacher students.

Methods

The paper combines some partial results of two research projects implemented in Finland (Hannula et al, in press). In grades 5 and 7 of the Finnish comprehensive school, the representative random sample of Finnish students consisted of 1154 fifth-graders (11 to 12 years of age) and 1902 seventh-graders (13 to 14 years of age). In the sample of elementary teacher students, we had all first-year students

(altogether 269) from three Finnish universities (Helsinki, Turku, Lapland). A reference sample from school (grade 11) was selected at random ($n = 1200$). This last sample is somewhat biased towards students who study advanced syllabus, which need to be taken into account when interpreting the results.

In both research projects, there was a questionnaire inquiring students' mathematical understanding, and a part of the tasks singled out students' conceptions and skills in infinity.

Here we focus on the two following infinity tasks:

Task A: Write the largest number that exists. How do you know that it is the largest?

Task B: How many numbers are there between the numbers 0.8 and 1.1?

About the results

To each question, we can find answers that remain on the level of finite numbers, answers that describe processes that do not end (potential infinity) as well as some answers that indicate that the student has an understanding of the final state of the infinite process (actual infinity). (Figures 1 & 2)

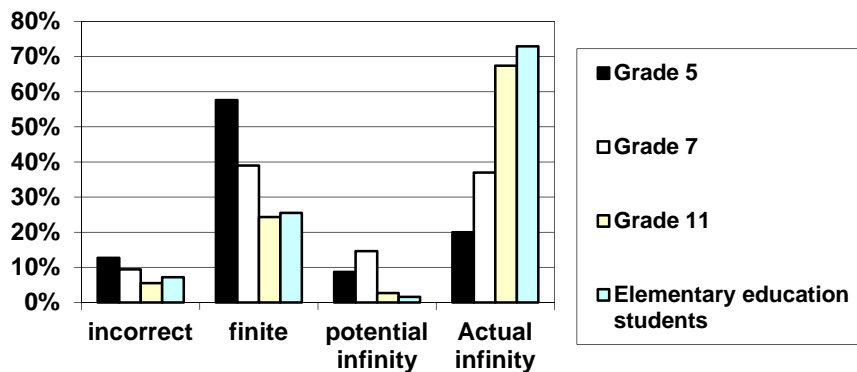


Figure 1. Student responses to the task A on infinitely large numbers: “Write the largest number that exists. How do you know that it is the largest?”

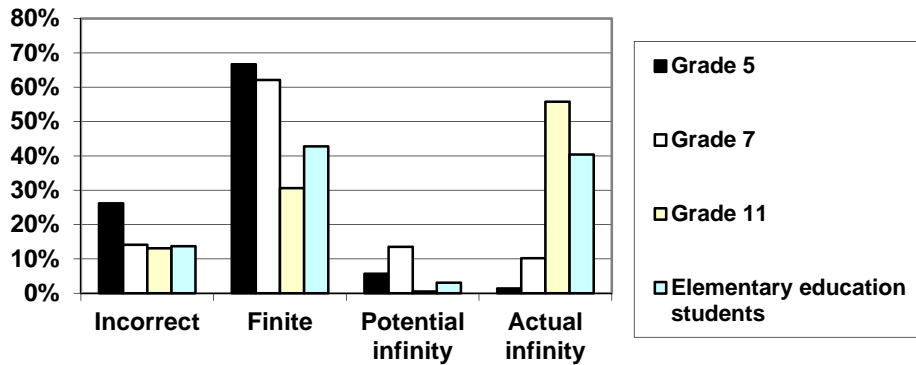


Figure 2. Student responses to the task B on density of numbers: “How many numbers are there between the numbers 0.8 and 1.1?”

In the fifth grade, 20 percent of the students have some understanding of the infinity of natural numbers, but only few have any understanding of density of rational numbers. The situation improves, as the students get older (and selected). Infinity of natural numbers is understood earlier than density of rational numbers, and potential infinity is understood earlier than actual infinity. It is somewhat worrying that even in the selected group of grade 11 students barely half of the students understand density of numbers. As students get older, the potential infinity becomes less frequent; it seems as if it was an intermediate stage that leads to an understanding of actual infinity (at least in these contexts).

Discussion

In our opinion the results are not satisfactory. Too many of even the older students seemed to think purely on the level of finite processes. It should be noted that elementary education is very popular and highly competed field of study in Finland. Furthermore, the grade 11 sample was biased in holding larger proportion of students with advanced syllabus than the case is for the general population. Hence, the results are likely to provide a picture that is brighter than reality.

The basic idea of potential infinity is not difficult to introduce to students. Usually children are very interested about these questions. Are teachers afraid of this topic as too difficult? In school we should teach mathematics and not only to master the routine tasks of the textbook. This means that the main mathematical ideas should be discussed in the class, too. Infinity is one of the mathematical ‘Fundamental Ideas’ (Schweiger, 1992). Our view is that these topics ought to be introduced to children early on. Infinity is introduced relatively late in the curriculum, which may be especially harmful for female students, who tend to rely more on the ideas taught in school. Students need experiences that allow

them to develop rich images of the topic, which will function as the basis for a formalisation at a later stage (Pirie & Kieren, 1994).

Mathematics is often considered to form a hierarchical structure where all new concepts logically follow from prior ones, which allows students to enrich their knowledge step by step. Such a structural way of learning mathematics can, however, be dangerous, since in some cases old structures may disturb the adaptation of new and more general ones. The very fundamental idea of a successor, for example, is necessary for learning the notion of natural numbers. The idea is, however, later on seriously conflicting with the understanding of the character of both rational and real numbers. According to the theories on conceptual change (cf. Carey, 1985; Vosniadou, 1994; Merenluoto, 2005), the relationship between learners' prior knowledge and new information to be learned is one of the most crucial factors in determining the quality of learning.

A radical restructuring is necessary for grasping the density of rational (real) numbers. It requires moving beyond the logic of natural numbers, where each number has a successor, and this seems to be more difficult for students. In another analysis of the longitudinal development of student competence in number concept, we noticed that proper understanding of fractions as numbers is an important predictor of learning the density of numbers (Hannula, Maijala & Pehkonen, 2004). This suggests that learning fractions is an important opportunity for this challenging conceptual change.

References

- Boyer, C.B. (1985). *A history of mathematics*. Princeton (NJ): Princeton University Press.
- Carey, S. (1985). *Conceptual change in childhood*. Cambridge, MA: MIT Press.
- Fishbein, E., Tirosh, D. & Hess, P. (1979). The intuition of infinity. *Educational Studies in Mathematics 1*, 3-40.
- Hannula, M. S., Maijala, H., & Pehkonen, E. (2004). Development of understanding and self-confidence in mathematics; grades 5-8. In M. J. Høines & A. B. Fuglestad (Eds.). *Proceedings of the 28th conference of the international group for the psychology of mathematics education*, vol. 3, pp. 17-24. Bergen: Bergen University College.
- Hannula, M. S., Pehkonen, E., Maijala, H., & Soro, R. (in press). Levels of students' understanding of infinity. *Teaching Mathematics and Computer Science*.
- Lakoff, G. & Núñez, R. E. (2000). *Where mathematics comes from; How the embodied mind brings mathematics into being*. New York: Basic Books.
- Merenluoto, K. (2005). Discussion about conceptual change in mathematics. *Nordic Studies in Mathematics Education*, 10 (2), 17-33.
- Merenluoto, K. & Pehkonen, E. (2002). Elementary teacher students' mathematical understanding explained via conceptual change. In: D. Mewborne, P. Sztajn, D.Y.

- White, H.G. Wiegel, R.L. Bryant & K. Nooney (Eds.). *Proceedings of the PME-NA XXIV*, pp. 1936-1939. Columbus, OH: ERIC.
- Moreno, A. L. E. & Waldegg, G. (1991). The conceptual evolution of actual mathematical infinity. *Educational Studies in Mathematics*, 22 (3), 211-231.
- Pirie, S. & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics*, 26 (2-3), 165-190.
- Schweiger, F. (1992). Fundamentale Ideen. Eine geistesgeschichtliche Studie zur Mathematikdidaktik. *Journal für Mathematik-Didaktik*, 13, 199-214.
- Tsamir, P. & Dreyfus, T. (2002). Comparing infinite sets – a process of abstraction. The case of Ben. *The Journal of Mathematical Behavior*, 21 (1), 1-23.
- Vosniadou, S. (1994). Capturing and modeling the process of conceptual change. *Learning and Instruction*, 4, 45-69.
- Wheeler, M. M. (1987). Children's understanding of zero and infinity. *Arithmetic teacher*, Nov. 1987, 42-44.

A study on proof in a community of mathematical practice

Kirsti Hemmi
Stockholm University

I present parts of my doctoral thesis, the aim of which is to describe how students encounter proof in a community of mathematical practice at a mathematics department and how they are drawn to share mathematicians' views and knowledge of proof. The data consists of transcripts of interviews with mathematicians and students as well as survey responses of university entrants, protocols of observations of lectures, textbooks and other instructional material. Both qualitative and quantitative methods were applied in the data analysis. A theoretical model with three different teaching styles with respect to proof could be constructed from the data. The study shows that the students related positively to proof when they entered the practice. Though the mathematicians had not an explicit intention of dealing so much with proof in the basic course, students felt that they were confronted with proof from the very beginning of their studies. A lot of aspects of proof remained invisible as experienced by students when they struggled to find out what proof is and to understand its role and meaning in the practice. The first oral examination in proof seems to be significant in drawing students to the practice of proof.

Background

Proof is a method of getting acceptance for and generating new mathematical knowledge. Proof is a complex notion, difficult to define, and on which different persons have different views. Proof is not a specific topic in the curriculum but can be seen to be permeating all mathematics. Proof is also a part of mathematics that has been considered difficult to teach and learn (e.g. Bell, 1976b; Moore, 1994; Selden & Selden, 1995; Weber, 2001). The status of proof in school mathematics has changed during the last decades and proof has had a diminished place in the secondary school mathematics curriculum in many countries (Hanna, 1995; Niss, 2001). There is no research about such changes in Sweden. However, in the national curriculum 1994 for upper secondary school, the word proof was not mentioned (Grevholm, 2003). There are signs that proof is coming back to the school curriculum in many countries, also in Sweden (Knuth, 2002; Waring, 2001; Skolverket, 2006). Lately, there has been an explosion of articles and research papers on various topics concerning proof in mathematics education. Most of these studies deal with cognitive aspects of learning. The main topics of the contemporary research are presented in my

doctoral thesis and I use the results in the conceptual frame to analyse how students meet proof in Sweden today.

The purpose of my doctoral thesis is to describe and characterise the culture of proof in a mathematical practice at a mathematics department in Sweden and how newcomers are engaged in proof and proving. The main issue is what possibilities there are for students to learn proof and how students are drawn to share the knowledge and the views of mathematicians on proof.

Theoretical framework

I created a theoretical framework for the study by combining a socio-cultural perspective and the social practice theories of Lave and Wenger (1991) and Wenger (1998) with theories about proof obtained from didactical research. The notion of *practice* is defined as “doing in a historical and social context that gives structure and meaning to what we do.” (*ibid.*, p. 47) For example, mathematical practice is doing in a historical and social context and includes its special language, symbols, tools, documents and specified criteria, that give structure and meaning to what people in that practice do. Learning is defined as increasing participation in *communities of practice* that leads to changing identities (*ibid.*).

The community of practice of mathematics at a mathematics department

I use Wenger’s (1998) theory to give structure to the practice I am studying. As a unit/level of analysis I use *a community of practice of mathematics at a mathematics department*. In this community I include all people exercising and learning mathematics at the department which is the focus of my study. There are mathematicians, doctoral students, teaching assistants and students. It is a dynamic practice and the joint enterprise for all participants is the learning of mathematics in a broad sense. Also researching new mathematics is seen to be learning. There is a richness of competence, and learning in this community occurs on different levels. Many students learn mathematics on a basic level but there are also doctoral students who are learning to carry on research in mathematics. Mathematicians are researching and obtaining new mathematical knowledge, teaching, examining and supervising students, improving teaching etc. But not only mathematicians are teaching mathematics to students, also doctoral students and teaching assistants take part of this enterprise. There are also pedagogical and didactical seminars, discussions and activities that aim to develop the teaching of mathematics. There are lectures, lessons, seminars and other kinds of meetings for the participants where teaching and learning of mathematics takes place. All these activities are included in the exercising of mathematics and are important for maintaining and developing the community of mathematical practice at the academic department.

There is a diversity of experience about mathematics among those who participate in this practice; there are *old-timers* and *newcomers*. However, it is not possible to exactly define when a newcomer becomes an old-timer as the character of these notions is relative. My thesis gives a contribution to knowledge in this area by describing how the enculturation of newcomers to the practice takes place with a special focus on their access to proof.

Proof as an artefact

An important theoretical aspect that I put forward in my thesis is that proof can be considered as an *artefact* in mathematical practice, not only in the community of practice of mathematics at the mathematics department that is the object of my study, but in mathematics as whole. I found support for this view from both the didactical literature and from the data. Artefact is a central concept within all socio-cultural theories even if there are slightly different interpretations of the notion in different research projects. Artefacts are the concrete and abstract tools that *mediate* between the social and the individual (Säljö, 2005). Proofs have mediating character about the mathematical knowledge and how it is connected and can therefore be considered as an artefact in mathematical practice.

Säljö (2005) divides artefacts into two groups, *intellectual tools* like discourses and systems of ideas and *physical tools* like texts, maps and computers. Further, he classifies them as *primary tools* (for example a hammer) and *symbolic tools* (used for communicating ideas). As proof is a system of ideas and used for communicating ideas, it can be seen as an intellectual and symbolic tool in mathematical practice. Säljö states that artefacts are carriers of information, stabilise, co-ordinate and discipline the practice and facilitate continuities across generations. Proof stabilises the practice of mathematics and facilitates continuities across generations because axiomatic deductive way of organising mathematics makes it easier for new generations to move further to new problems in mathematical practice. Proof also co-ordinates and disciplines mathematical reasoning by offering criteria for accepted mathematical knowledge.

Lave and Wenger (1991) introduce the concept of *transparency* of artefacts. They use it in connection to technology but I have examined its strength for describing the treatment of proof (in the teaching of newcomers) in the mathematical practice. There is an intricate balance between how much we focus on different aspects of proof at a meta-level and how much we use proof invisibly in the teaching of mathematics (Hemmi, 2006, p. 54). My thesis sheds light on this dilemma.

The conceptual frame about various aspects of proof

I created a conceptual frame from the main themes and controversies within the research on proof and the teaching and learning of proof, in order to link my study and the data to the previous studies. I sum up the main themes and issues

in mathematics education research on proof along the following aspects. All of them had an important role in the data analysis:

- Conviction/Explanation
- Induction/Deduction
- Intuition/Formality
- Invisibility/Visibility

These aspects involve two interacting components. In Figure 1, I illustrate these pairs with hints and examples of what I mean with them.

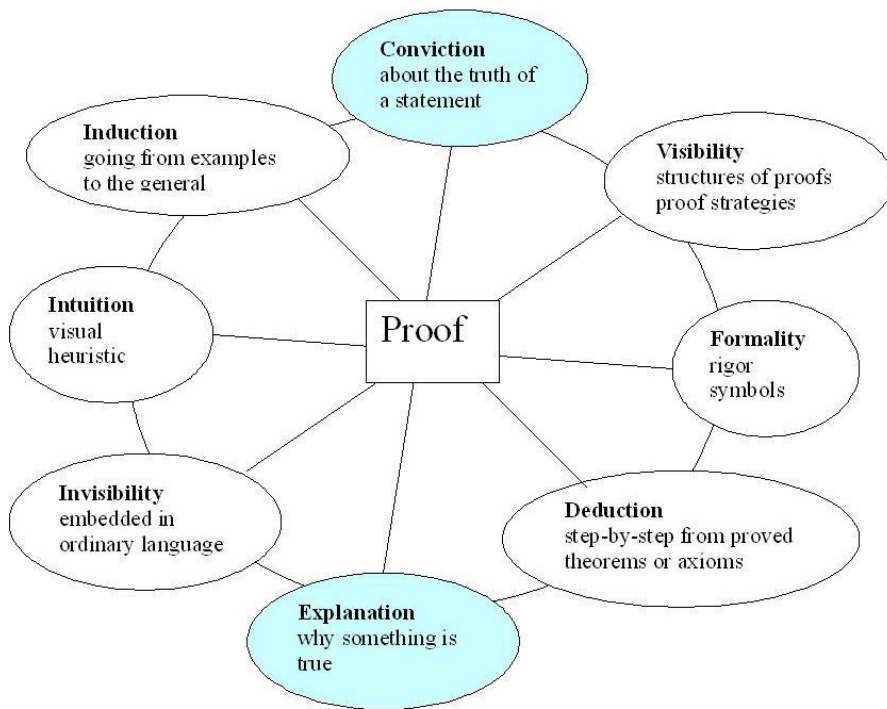


Figure 1. The interacting aspects of proof

Conviction/Explanation has a different color from the other aspects in the figure, since this pair is different from the other aspects in the model in a sense that the others deal with properties of proof and how to approach proof whereas Conviction/Explanation refers to the functions of proof. There are other functions of proof (e.g. de Villiers, 1990; Hemmi, 2006), which I included in the conceptual frame. They are Communication, Aesthetics, Systematisation, Intellectual challenge and Transfer.

All the aspects in the frame are partly overlapping and intertwined. In my thesis, I provide examples both from literature concerning mathematical practice, from mathematics education research and from some empirical studies illuminating these aspects and the concerns in the pedagogical debates.

The research questions and the methodology

The purpose of my study is to describe and characterise the culture of proof in a community of mathematical practice and how students are engaged in proof. Structuring resources for learning come from a variety of sources, not only the pedagogical activity (Wenger, 1998). Pedagogical intentions create a context in which learning can take place. Teachers, lectures, lessons and instructional materials, like textbooks, become resources for learning in complex ways. In my study, I approach the issue from different directions (Figure 2).

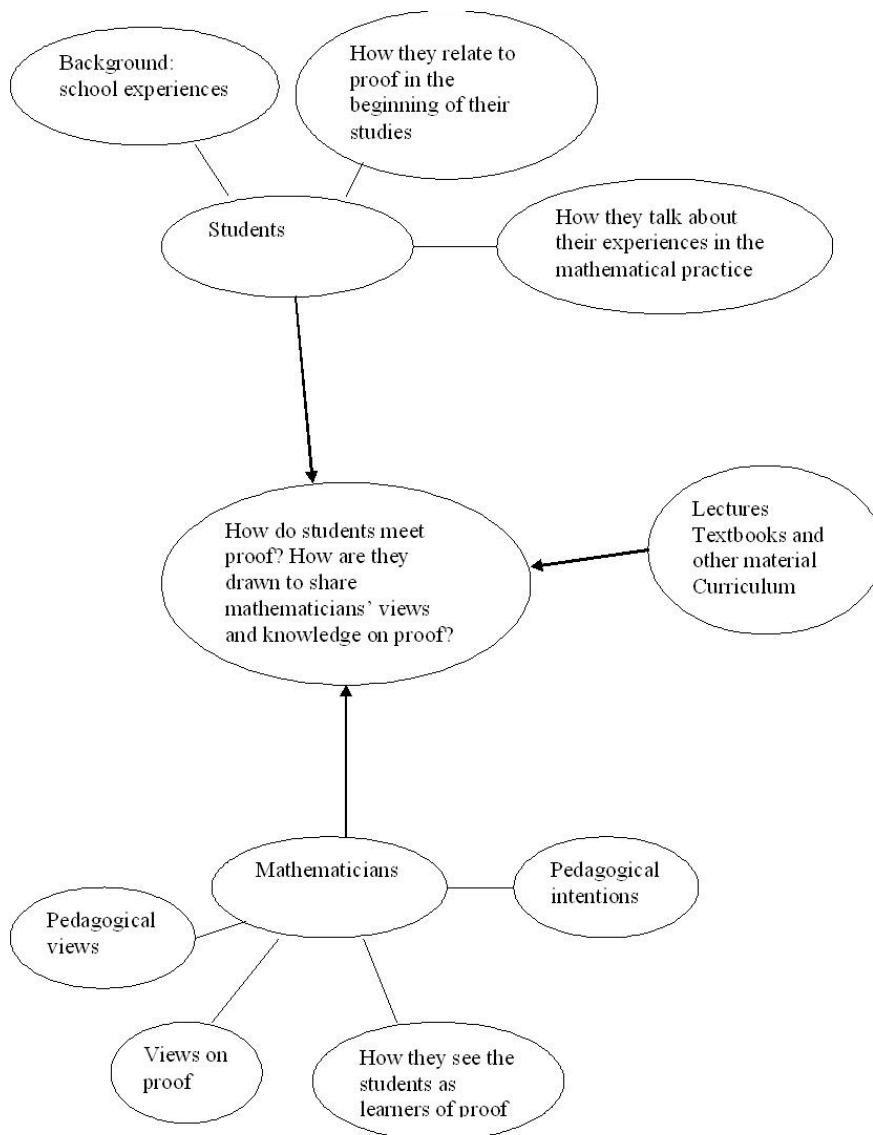


Figure 2. Approaching the issue from different directions

I explore proof in mathematicians' views and intentions, their pedagogical perspectives, textbooks and lectures and the organisation of teaching on the one hand, and how students experience proof in the mathematical practice, on the other hand. The general research questions were formulated in the following way:

- How do students meet proof in the community of mathematical practice at a mathematics department?
- How are students drawn to share mathematicians' views and knowledge of proof?

In Table 1, I present the specific research questions and the associated methods through which I will shed light on the general research questions. The table provides an overall picture about the design of my study. I combined both quantitative and qualitative methods. At last, I contrasted the results concerning the mathematicians' practice and the results concerning the students' practice in order to shed light on how the structuring resources and mathematicians' intentions became resources for learning.

According to the social practice theory of Lave and Wenger (1991) and Wenger (1998) that I apply in my study, the world is seen to consist of objective forms and systems of activity, on the one hand, and agents' subjective and inter-subjective understanding of them, on the other hand. These mutually constitute both the world and its experienced forms. Furthermore, cognition and communication in, and with, the social world are situated in the historical development of ongoing activity (Lave and Wenger, 1991, p. 51). Knowledge about proof and the teaching and learning of proof is not simply in individual teachers' minds: it is tied to their identities and evolves in and through co-participation in the practices of the community. Hence, I consider the mathematicians and the students as participants in the community of mathematical practice and interpret their utterances, not entirely as their own opinions but to some extent as reproduction of views belonging to the community, utterances that are influenced by the social, cultural and historical context of the same mathematics environment but also from other possible environments they are members of.

Table 1. Design of the methods

	MAIN DATA			COMPLEMENTARY DATA	
Methods Research questions	Interviews with mathematicians	Surveys with university entrants	Focus group interviews with students	Observations of lectures	Document analysis and interviews with experts
Mathematicians' views and pedagogical perspectives	X			X	textbooks curriculum examinations other material
Students' upper secondary school background		X	X		upper secondary school textbooks
How students relate to proof		X	X	X	
What kind of participation in proof is there available in the practice?	X	X	X	X	curriculum textbooks other material examinations interviews with experts
How students talk about their experiences			X		
How do students meet proof? How are they drawn to share mathematicians' views and knowledge of proof?	Results of the analysis of interviews with mathematicians	Results of the survey analysis	Results of the analysis of the focus group interviews with students	X	X

Results

The centrality of proof in the mathematical practice was obvious in all of the interviews with mathematicians. The data also supports the idea of considering proof as an artefact in mathematical practice. The mathematicians talked about proof as a tool in their practice, for example for communication, explanation and deriving new results in mathematics.

A *theoretical model* with three different teaching styles was constructed from the data. The styles are idealised - no individual mathematician could perfectly fit into one of them. As main criteria for different categories, I used pedagogical intentions, the views on students and the aspects in the conceptual frame.

The first style, which I call *progressive*, can be characterised by the following cite: *"I don't want to foist the proofs on them"*. There is a kind of sensitivity towards students within this style, for example the word proof is avoided to not frighten the students. Formal, long and technical proofs are avoided because *"students are not interested in them"*. Inductive rather than deductive approaches are preferred in the teaching of mathematics. According to the progressive style, only few students can value and understand proof; most of the students do not need to learn proof. There are no discussions on proof or proof techniques, the students who are interested, matured or/and had self-confidence can by themselves find out what is accepted as proof in the practice.

The second style, which I call *deductive*, can be characterised by the following cite: *"It's high time for them to see real mathematics"*. Within this style, the word proof, abstractions or mathematical symbols, long or technical proofs are not avoided, rather the opposite; students should get used to them in the very beginning of their studies. A deductive approach in the teaching of mathematics and proof is preferred. The demands of mathematical practice have to be made clear to the students so there is a willingness to discuss proof and proving techniques (but no time). There is a view on students that they are capable to learn abstract thinking and proof but the learning to prove statements demands time and a lot of practice.

The third style, which I call *classical*, can be characterised by the following cite: *"I can't help giving some nice proofs"*. There is a great appreciation towards proof and a desire to convey this to students within this style. Yet, proof is avoided for external circumstances like students' lacking of prior knowledge, lack of time and students' interest. There is a desire to inspire the capable students, so sometimes "nice proofs" are given if there is time for that. There is not the same kind of sensitivity visible towards students as in the utterances categorised into the progressive style.

Hence, there are different styles that mathematicians mix and apply in their personal ways in the teaching of newcomers, but also students are individuals with various backgrounds. They have different goals with their studies as well as different tastes regarding the presentation of mathematics. Some students may prefer a careful presentation of mathematical contents with definitions, theorems and proofs whereas some others get bored when listening to that kind of presentation.

Q: *...I think that I'm kind of a structure person and I totally lose the appetite for learning maths if they just stand there and prattle and don't even finish the examples.*

P: *It's as if I fall to sleep if they like go through a proof extremely carefully*
(Student studying advanced courses, 2004)

The student Q might prefer a deductive teaching style, whereas the student P could be more satisfied with the classical style:

“No rigorous proofs, too formalised proofs are unbearable. A piece of poetry, (proof) can be as attractive as the entire theorem.”
(Mathematician, 2003)

Most of the mathematicians conveyed a careful position concerning the teaching of proof and claimed that they did not deal so much with proof in the basic courses. On the other hand, students in the focus groups talked about an experience of proof from the very beginning of their studies.

There can be various reasons for this discrepancy between the mathematicians’ declared intentions and how many students experienced the lectures. I suggest two possible ones, which I base on the classroom observations and the analyses of the interviews:

- 1) Mathematicians cannot help giving some “nice proofs” now and then, even if they state that they do not prove so many statements.
- 2) The mathematicians’ and the students’ views on proof were similar in many aspects. Yet, proving statements can mean different things for different persons. It is natural for mathematicians to present mathematics in a deductive way starting with definitions and proceeding in a deductive manner, justifying the most steps they take. They might not always think of this as proving, even if many of them whom I interviewed talked a lot about derivations of formulas as proving and also stated that proof somehow existed in all mathematics. Students may conceive this as different from the way mathematics was presented to them in upper-secondary school and thus, as proving.

It is also possible that students experienced the lectures at the beginning of their studies as containing a lot of proof due to both of the reasons presented above but this issue would need further examinations. However, this inconsistency shows that what is intended to be in focus by mathematicians is not necessarily in focus for all students. There were differences between the three teaching styles regarding what aspects of proof were aimed to be focused in the teaching.

Over 80 percent of the university entrants stated that they wanted to learn more about proof and showed positive attitudes towards proof when they started to study mathematics. Students had various school backgrounds regarding their experiences with proof, and hence, they were in very different positions as regards to how they could participate in the negotiation of meaning concerning proof. Since proving tasks are occasional in the examinations, it is possible to study the basic course in mathematics without much participation in proof. The condition of transparency proved to be an important issue for students’ access to proof. Many students seem to struggle with the very notion of proof. What is proof, how do one construct a proof, why is proof important? Students lacked discussions on the issue. After the first oral examination of proof during the course *Mathematical Analysis 3*, in the end of the first year, students talk about an aha-experience and strongly connect proof to ‘real mathematics’ and understanding.

Discussion and implications to the educational practice

The results bring about the following reflections. Students related positively to proof and they wanted to learn more about proof when they entered the practice. How, then, could mathematicians, in the best way, take care of students' positive relation and expectations regarding proof and help them to proceed in their mathematical practice? Both the students and the mathematicians agreed on the fundamental role of proof in mathematics. Hence, a focus on proof as a dynamic notion could serve as a source of inspiration for both teachers and students. Yet, students had various backgrounds and some of them had very little experience of proof from upper-secondary school. I hope my thesis will rouse a debate about the role of proof in mathematics curricula, both in school and at university, because in the end, it is a question of value whether proof is included in the curriculum, a question that has to do with how mathematics is seen and what aspects of mathematics are in the focus of teaching. I also hope that the thesis with both the empirical findings and the theoretical insights about the teaching of proof will enhance consciousness among mathematicians, upper-secondary school teachers, the authors of mathematics textbooks and teacher educators about the role and functions of proof in the teaching of mathematics, as well as the problems in drawing students to the practice of proof.

References

- Bell, A. W. (1976). A study of pupils' proof-explanations in mathematical situations. *Educational Studies in Mathematics*, 7, 23-40.
- de Villiers, M. (1990). The role and function of proof in mathematics. *Pythagoras, South Africa*, 23, 17-24.
- Grevholm, B. (2003). Personal communication in August. Kristiansand.
- Hanna, G. (1995). Challenges to the importance of proof. *For the Learning of Mathematics*, 15(3), 42-49.
- Hemmi, K. (2006). *Approaching Proof in a Community of Mathematical Practice*. Doctoral Thesis, Stockholm University, Stockholm.
- Knuth, E. J. (2002). Teachers' conceptions of proof in the context of secondary school mathematics. *Journal of Mathematics Teacher Education*, 5, 61-82.
- Lave, J., & Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. Cambridge: Cambridge University Press.
- Moore, R. (1994). Making the transition to formal proof. *Educational Studies in Mathematics*, 27, 249-266.
- Niss, M. (2001). Den matematikdidaktiska forskningen karaktär och status. In B. Grevholm (Ed.), *Matematikdidaktik - Ett Nordiskt Perspektiv*. Lund: Studentlitteratur.

- Selden, J., & Selden, A. (1995). Unpacking the logic of mathematical statements. *Educational Studies in Mathematics*, 29, 93-95.
- Skolverket. (2006). *Nya kursplaner för gymnasieskolan*, from http://www.skolverket.se/content/1/c4/37/82/Matematik_060201.pdf
- Säljö, R. (2005). Cultural tools and human knowing: A sociocultural perspective on learning and the study of learning. Summer school presentation. Jyväskylä.
- Waring, S. (2001). Proof is back (A proof-oriented approach to school mathematics). *Mathematics in School*, January 2001.
- Weber, K. (2001). Students' difficulty in constructing proofs. *Educational studies in mathematics*, 48, 101-119.
- Wenger, E. (1998). *Communities of Practice*. Cambridge: Cambridge University Press.

Some aspects of comprehensive school mathematics: A follow-up study

Kauko Hihnala
University of Jyväskylä

This paper presents some results of a four-year follow-up study of the mathematical skills of pupils from the beginning of the sixth grade to the end of the ninth grade ($n = 89$). Some aspects concerning proportionality and pre-algebra proved to be more important than general mathematical skills. The procedure the pupils used in fraction exercises and percentage calculations did not seem to be based on proportionality. The result goes against the structure of mathematics, but is in line with the national core curriculum for basic education (2004).

Introduction

The idea of this paper derives from my dissertation (Hihnala, 2005) concerning the didactics of mathematics in the Finnish comprehensive school. The study started by surveying the mathematical skills of pupils from the sixth to the ninth grade at a general level. Initially, data collection focused chiefly on procedural knowledge, very much as in Finnish school surveys (e.g. Kupari, 1996). During the four-year follow-up it became apparent that, by analyzing pupils' ways of answering and solving problems, it was possible to obtain conceptual knowledge (c.f. Haapasalo, 2003) and to assess their mathematical thinking. Mathematical thinking and understanding mathematics are here considered from the point of view of mathematical processes (e.g. Pehkonen, 2000; Joutsenlahti, 2005).

The division of the mathematics teaching curriculum into arithmetic, algebra and geometry in Finnish schools dates back to the former parallel school system in Finland and hitherto has defined the framework of mathematics teaching in comprehensive school (Hihnala, 2005). It has been thought that arithmetic and, as its most demanding areas, proportionality and percentage calculation are necessary pre-requisites for the studying of algebra. In practice this has meant that the studying of algebra has been limited to the two highest grades of comprehensive school (e.g. Hassinen, 2006). The aim of my dissertation was to study whether it is possible to start learning algebra earlier, for instance in the sixth grade.

The aim of this paper is to highlight some of the points made in my dissertation and provide some details of instances where a longitudinal study gave more

information than a cross-study. One such issue is the predictability of pupils' mathematical skills until their fourth year, and another is the use of multiple choice tests.

Theoretical framework

Mathematical thinking in comprehensive school

The national core curriculum for basic education (2004) lays down guidelines for mathematics teaching in Finnish comprehensive schools today. The national core curriculum underlines mathematical thinking from first grades up. It is proposed as the central task of elementary instruction. In the upper grades mathematical thinking will be developed. Nevertheless, the national core curriculum does not precisely define the meaning of mathematical thinking. In real school-life many teachers use the textbooks as a curriculum (Perkkilä, 2002; Törnroos, 2005).

Some researchers have seen mathematical thinking as belonging to problem solving (Schoenfeld, 1992; Pehkonen, 2000; Joutsenlahti, 2005), where pupils' metacognitive skills play an important role. Some other researchers underline the use of methods of reasoning (Niiniluoto, 1983; Malinen, 1993).

Traditionally, in the shift from arithmetic to algebra, literal terms have been used instead of numbers. Küchemann (1981) posed a six-step classification for interpretations of literal terms in mathematical expressions. The three lowest steps were: literal terms are rated as a number, overruled, or used as concrete objects (c.f. Hihnala, 2005; Hassinen, 2006).

Hihnala (2005) assessed mathematical thinking from the point of view of algebra and proportionality. Mathematical thinking will occur in the ways mathematical processes are handled and alternative answers are chosen. For instance proportionality and percentage calculations provide some insights into pupils' mathematical thinking.

Proportional reasoning

Many research studies underline proportionality as a central relation in school mathematics (e.g. Langrall & Swafford, 2000; Riddle & Rodzwell, 2000; de Bock, van Dooren, Janssens & Verschaffel, 2002). Langrall et al. (2000) wrote: "Students who fail to develop proportional reasoning are likely to encounter obstacles in understanding higher-level mathematics, particularly algebra."

Langrall et al. (2000) identified a system of four different levels (numbered 0, 1, 2, and 3) for proportional reasoning. Strategies at level 0 are characterized by additive rather than multiplicative comparisons. At level 1 students can think productively about problems, using concrete models to make sense of situations. At level 2 students can use quantitative reasoning without concrete models. At

level 3 students can set up a proportion using a variable and solve it using the cross-product rule or equivalent fractions. For instance, percentage tasks consisting of divisible whole numbers are situated on level 2. Level 2 is here called the “digital level”. More precise results were obtained by following five pupils with the highest test scores and five pupils with the lowest test scores for four years.

Method

Data collection

A four-year follow-up study was a case study targeted at a single-age group of 89 pupils. The measurements were made in the autumn from 1996 to 1999, and in 2000, in the spring of the ninth grade, with tests lasting one hour. The supposition was that pupils would have forgotten most of their superficial knowledge during the most recent summer. The autumn test would then measure “permanent knowledge”. Initially the study surveyed the mathematical skills of pupils from the sixth to the ninth grade at a general level. Later arose certain interesting aspects concerning proportionality and pre-algebra. For this reason a cross-study (autumn 2000) focused on these issues (Hihnala, 2005).

When the follow-up study began with sixth graders, it was not clear in what school they would be the following year. Fortunately 89 of the 114 pupils transferred to the same upper level of the comprehensive school. The pupils were mixed into four classes. Because they had signed their test-papers it was easy to continue the study on an individual level. Nevertheless, it was difficult to assess if certain learning results were due to the previous teacher, present teacher, new classmates or some other factor.

Data analysis

At first the data consisted of 19 items including eight basic calculations and two pre-algebra tasks. From the seventh grade on there were two percentage tasks and one literal task concerning proportionality. Other fraction tasks and literal tasks are not considered here.

It is quite popular to use multiple choice items in mathematical research studies. It makes marking and grading easy. In this study about one half of the tasks were multiple choice items and the other half production items.

The data were assessed by scoring pupils’ answers with 1 point (correct answer) or 0 points (wrong answer) in multiple choice items, and with 2, 1 or 0 points in production items. The sums of the points were converted into solving percentages, making it possible to compare the results of different grades. The numbers of test items, and hence, the maximum test scores, were not the same in different grades.

The progress of every pupil in mathematics was followed over about four years through mathematics tests. In the beginning of the sixth grade the group of pupils (N = 89) was divided into five equal hierarchical groups according to their test scores. At the end of the ninth grade the results were compared. It appeared that the gap between the highest and the lowest fifths had narrowed. When, however, a new division into fifths was made at the end of the follow-up, it was found that differences between the highest and lowest fifths had actually grown.

Results

Predictability of mathematical skills

The study reveals that the level of mathematical skills deteriorated from the sixth grade to the eighth grade every year, figure 1. The mathematics school reports confirm this finding. On the other hand, basic calculation skills, which were measured by eight common items, improved steadily during the four-year monitoring period. It seems that moving from primary school to secondary school causes pupils problems. The teaching styles of subject teachers may differ considerably from the teaching styles of class teachers. One point is that in the sixth grade the main topics related to basic calculations are revised.

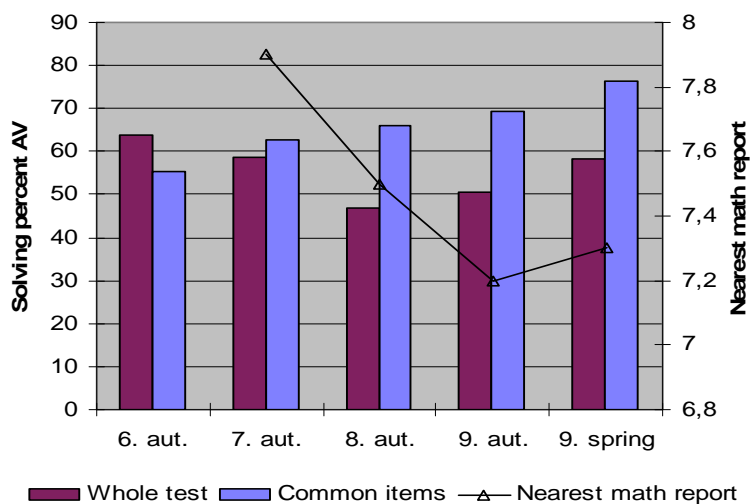


Figure 1. The development of mathematical skills during the follow-up and the most recent mathematics report mark

The difference in test score averages between the highest fifth and the lowest fifth, which were initially 49 percentage units, had decreased during the follow-up to 38 percentage units (Table 1). But, when the sample was divided into new fifths on the basis of the latest test scores, the difference had increased from 49 percentage units to 61 percentage units (Table 1).

Table 1. Average solving percents in the groups with highest performance and lowest performance (n = 17)

	Division into equivalent fifths, grade 6, autumn		Division into equivalent fifths, grade 9, spring
	Solving percentage, grade 6, autumn	Solving percentage, grade 9, spring	Solving percentage, grade 9, spring
Highest fifth	86	70	82
Lowest fifth	37	32	21
Difference	49	38	61

Further results were found by following five pupils with the highest test scores and five pupils with the lowest test scores over four years (Figure 2). It seemed that at the beginning of the sixth grade and at the end of the ninth grade pupils' scores coincided in the same groups: the highest ones in a group of their own and the lowest ones in a group of their own. Nevertheless, considerable variation had taken place over four years.

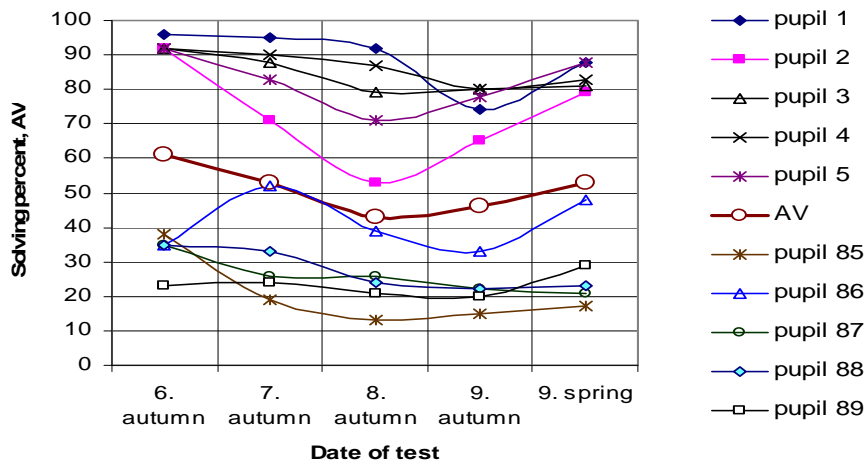


Figure 2. Change in solving percentages of the five highest scoring and five lowest scoring pupils in the follow-up tests

Fractional skills and percentage calculations

All tests from the seventh to the ninth grade included one literal task concerning proportionality and another concerning percentage calculation. Proportionality

and percentage calculation tasks were equally represented by nearly the same equation:

$$\text{Proportionality task } \frac{8}{10} = \frac{x}{35} \text{ and percentage task } \frac{9}{9+16} = \frac{x}{100}$$

From the point of view of mathematical structure the tasks seemed to be very similar. However, about 45 - 55 % of the pupils solved the proportional task, but only 10 - 30 % solved the percentage task (Figure 2). The procedure the pupils used in the percentage calculation did not seem to be based on proportionality. A special point was that percentage calculations had been practised extensively in the seventh grade but the results deteriorated slightly.

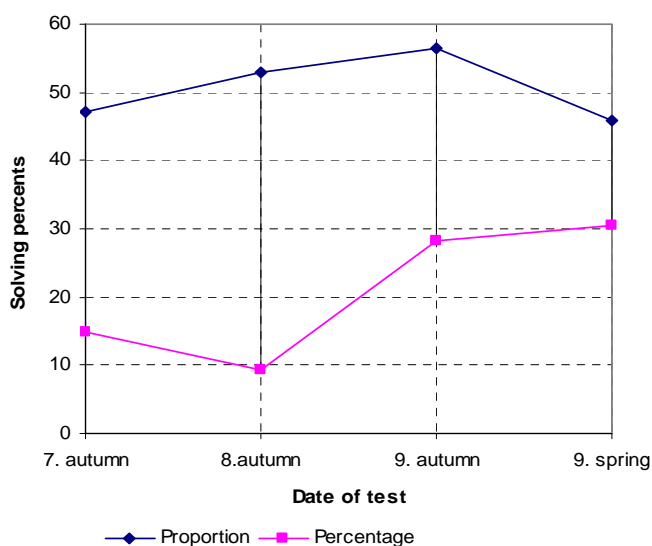


Figure 3. Average solving percentages in a literal proportion task and a corresponding percentage task in grades 7-9 (n = 85) in the follow-up study

Fractions as decimals

It was found that sixth-graders and seventh-graders had a rather weak command of symbols of fractions and decimals compared with eighth-graders and ninth-graders. How to change a fraction into decimal form became a significant problem for some pupils.

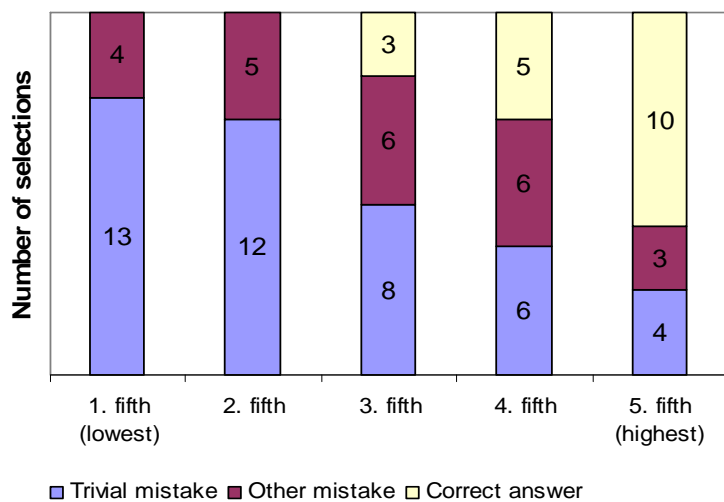


Figure 4. Distribution of alternative answers among the fifths of sixth graders in the fraction to decimal task (n = 17)

Especially younger pupils chose the alternative $\frac{6}{5} = 6,5$ from the five

alternatives. The choice was called “a trivial mistake”. The sample was divided into equivalent fifths (n = 17) according to the test scores. At the beginning of the sixth grade trivial mistakes were common in every fifth but the right answer occurred only in the highest fifths (Figure 4).

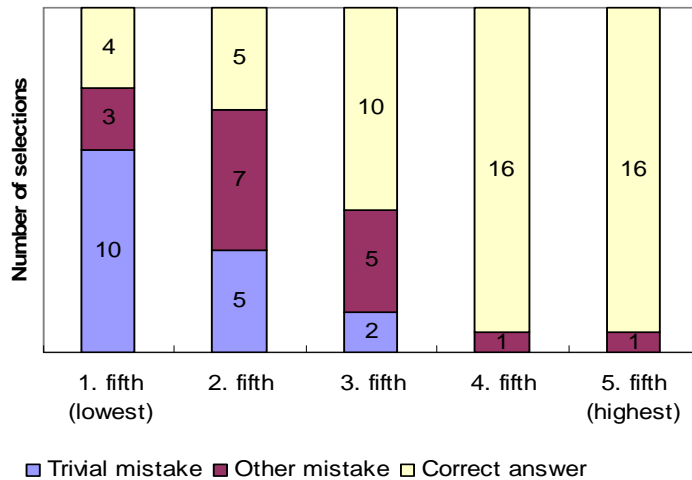


Figure 5. Distribution of alternative answers among the fifths of eighth-graders in the fraction to decimal task (n = 17)

After two years a new division into fifths was made. Then, it was found that trivial mistakes occurred only in the lowest fifths but that the right answer occurred in every fifth (Figure 5). It appeared that the “from fraction to decimal” task distinguished “good mathematicians” in the sixth grade and “bad mathematicians” in the eighth grade. It should be remembered that one item does not decide very much. Nevertheless, it was found later that those who made a trivial mistake on changing a fraction into a decimal had difficulties in many areas of mathematics, especially in percentage tasks (Hihnala, 2005).

Pre-algebra skills

Pupils’ skills in pre-algebra were studied with just two items. The items were the same during the entire follow-up. The results (test scores) were very much in line with expectations: pre-algebra skills changed very much like fractional task skills and were better than percentage task skills (Figure 6).

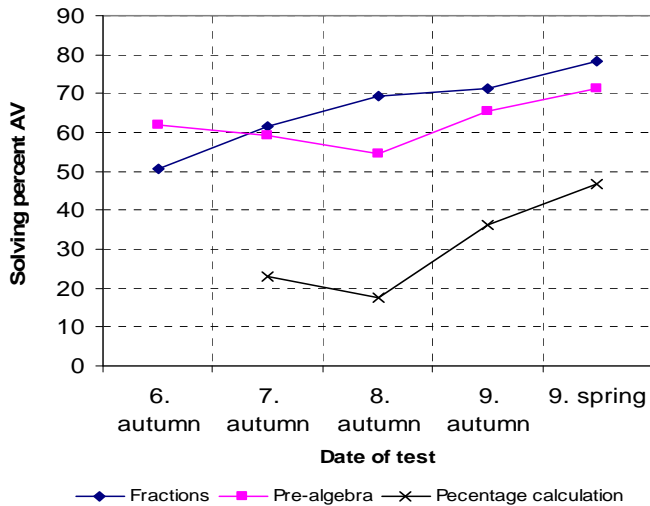


Figure 6. Average solving percentages according to grades in fractions, pre-algebra and percentage tasks (n = 85)

Statistical analysis revealed some interesting aspects. For instance, in a multiple choice item where pupils had to find the successor of a natural number a , there was a very low level of reliability (Example 1).

Example 1

Let a be one of the natural numbers 1, 2, 3,... What is then the next number?

- A $a + 1$ B $a - 1$ C $a - 10$ D $a + 10$ E $10a$

The statistics indicated that pupils’ answers were inconsistent. In the sixth grade about one half of the pupils were able to choose the right alternative but in the following year a half of “this half” chose the wrong alternative. The reader may ask him/herself where their knowledge had vanished. Later a classroom test was administered by the author. The test showed that only two pupils out of a sample of 40 were able to write the successor of a natural number a . Many pupils wrote for instance: “If $a = 5$, then $a + 1 = 6$ ”. They could not see a as a number but as the name of a number or as a concrete object.

Conclusions

Proportionality seemed to be a model of thought that nearly every other pupil could intuitively apply to a written task, whereas less than one third of the pupils could solve a percentage calculation based on the same mathematical model. The procedure the pupils used in fraction tasks and percentage calculations did not seem to be based on proportionality.

The results support the idea presented by several researchers that the understanding of proportionality is a slowly progressing skill. According to the

study carried out, this skill did not change very much during secondary school. The national core curriculum for basic education (2004) emphasises the constructivist model of learning and a problem-centred approach as the central feature of studying. This is a clear incentive to move in the direction of teaching fractions and percentage calculations based on proportionality, of which pupils already have an intuitive command.

In order to apply proportionality it is essential to train pupils to use proportionality on numbers divisible by each other before formal solving methods (Hihnala, 2005). According to the theory of Langrall et al. (2000), it means operating at level 2 first and then shifting to level 3. An excessively rapid shift to formal operations, for instance, to cross-production in proportions, leaves a gap in the sequence of learning at level 2 or the “digital level” (Hihnala, 2005). When a pupil cannot solve a problem at level 3, it may happen that he or she begins to test different calculations and so falls to level 0.

The results showed that many pupils had difficulties with percentage calculations. Even extensive practice with percentage calculation in the seventh grade did not seem to improve understanding of the concept of percentage. However, by the beginning of the ninth grade, there had been a significant improvement in these skills, which suggests that the thinking skills of the pupils had developed. It was interesting that in the eighth grade some aspects of proportionality, but not percentage calculations, were an important part of the curriculum. It seems that understanding of percentage calculations has a close connection to proportionality, which is absent from the national core curriculum for basic education (2004).

During the follow-up pre-algebra skills changed slightly, very much like proportional reasoning. Because of low reliability it is not possible to draw any firm conclusions about pre-algebra issues. Some items, however, indicated that in the sixth and the seventh grade pupils saw literal terms as names or concrete objects at Küchemann’s (1981) levels 1-3. It was also found that a trivial interpretation of a fraction as a decimal $\frac{6}{5} = 6,5$ was a crucial mistake connected with many areas of school mathematics.

References

- de Bock, D., van Dooren, W., Janssens, D. & Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of secondary school students’ errors. *Educational studies in mathematics*, 50, 311-334.
- Haapasalo, L. (2003). The conflict between conceptual and procedural knowledge: Should we need to understand in order to be able to do, or vice versa? In L. Haapasalo & K. Sormunen (Eds.). *Towards meaningful mathematics and science education. Proceedings of the IXX symposium of the Finnish Mathematics and Science Research Association*, pp. 1-20. Joensuu: University of Joensuu.

- Hassinen, S. (2006). Idealähtöistä koulualgebraa. IDEAA-opetusmallin kehittäminen algebran opetukseen peruskoulu 7. luokalla [Idea-based school algebra]. Helsinki: University of Helsinki.
- Hihnala, K. (2005). *Laskutehtävien suorittamisesta käsitteiden ymmärtämiseen. Peruskoululaisen matemaattisen ajattelun kehittyminen aritmetiikasta algebraan siirryttäessä*. [Transition from performing of arithmetic tasks to the understanding of concepts. The development of pupils' mathematical thinking when shifting from arithmetic to algebra in comprehensive school]. Jyväskylä: University of Jyväskylä.
- Joutsenlahti, J. (2005). *Lukiolaisen tehtäväorientoiuneen matemaattisen ajattelun piirteitä 1900-luvun pitkän matematiikan opiskelijoiden matemaattisen osaamisen ja uskomusten ilmentämänä* [Characteristics of task-oriented mathematical thinking among students in upper-secondary school]. Acta universitatis Tamperensis 1061.
- Kupari, P. (1996). Miten peruskoulun matematiikan opiskelulle on käynyt säästöjen kourissa? [What is the state of mathematics learning under pressures of economical recession?] In R. Jakku-Sihvonen, A. Lindström & S. Lipsanen (Eds.). *Toteuttaako peruskoulu tasa-arvoa?*, pp. 436-450. Helsinki: Opetushallitus.
- Küchemann, D. (1981). Algebra. In K. M. Hart (Ed.) *Children's understanding of mathematics, 11-16*, pp. 102-119.. London: John Murray.
- Langrall, C. W. & Swafford, J. (2000). Three balloons for two dollars: Developing proportional reasoning. *Mathematics teaching in the middle school*, 6(4), 254-261.
- Malinen, P. (1993). Can Finnish elementary school pupils think logically? In P. Kupari & L. Haapasalo (Eds.). *Constructivist and curriculum issues in school mathematics education. Mathematics education in Finland: yearbook 1992-1993*, pp. 35-41. Jyväskylä: University of Jyväskylä.
- Perkkilä, P. (2002). *Opettajien matematiikkauskomukset ja matematiikan oppikirjan merkitys alkuopetuksessa* [Teachers' mathematical beliefs and the significance of the maths textbook]. Jyväskylä: Jyväskylän yliopisto.
- Perusopetuksen opetussuunnitelman perusteet* (2004). [Comprehensive School Framework Curriculum]. http://www.oph.fi/info/ops/pops_web.pdf. MSN 14.9.2004.
- Riddle, M. & Rodzwell, B. (2000). Fractions: What happens between kindergarten and the army? *Teaching children mathematics*, 7(4), 202-206.
- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Eds.). *Handbook of research on mathematics teaching and learning*, pp. 334-370. New York: Macmillan.
- Törnroos, J. (2005). *Opetussuunnitelma, oppikirjat ja oppimistulokset. Seitsemännen luokan matematiikan osaaminen arvioitavana*. [Curriculum, textbooks, and achievement – Grade 7 mathematics achievement under assessment]. Jyväskylä: University of Jyväskylä.

Language and learning in mathematics

Lisen Häggblom
Åbo Akademi, Vasa

The linguistic dimension in the mathematical development of pupils is highly present in various learning processes. In this article, the multifaceted function of language is discussed, based on learning theoretical aspects, curricular perspectives, and a longitudinal study of pupils' knowledge of mathematics from the age of 6 to the age of 15. Variations in pupils' language use are presented and new visions for developing the learning of pupils through new forms of assessment are discussed. Pupils' language use is multifaceted and analyses show the need of more extensive focus on the significance of language and the way language is used to support pupils' learning of mathematics throughout basic education.

Introduction

The significance of language for pupils' learning of mathematics has been raised and discussed from different perspectives by a large number of researchers during the last 20 years. The specific characteristics of mathematics is symbolised by a powerful symbolic language that the pupils need to learn to use (e.g. Morgan, 1998; Winsløw, 2004). The pupils' oral and written language use is encouraged from a learning perspective and in order to help pupils develop and show their understanding of mathematical terminology (e.g. Brissenden, 1988; Johnsen Høines, 2002). Connections between mathematics and reading and writing disabilities have been given an increasing amount of attention during the last decade (e.g. Malmer & Adler, 1996; Sterner & Lundberg, 2002). Linguistic communication in the classroom has been made visible (Alrø & Skovsmose, 1993; Löwing, 2006; Wyndhamn, 1990) and assessment in mathematics is seeking new forms via active usage of language in order to support pupils' learning (e.g. Häggblom, 2006; Kulm, 1993; Romberg, 2004; Wyndhamn, 1990). Language use is stressed also in order to develop pupils' argumentation and critical examination of their results (e.g. Fischbein, 1990; Keranto, 1993). Winsløw (1998, 38) describes mathematical language as a didactic potential where there has been development towards greater focus on linguistic competence.

Different perspectives on language and mathematics thus show a multitude of dimensions of linguistic use. The significance and function of language in pupils' mathematical learning can be analysed partly based on the specific

characteristics of mathematics and partly based on the individual's linguistic competence and ability to express his/her thinking strategies both orally and in writing. Part of the characteristics of mathematics is its language which is exact, concise and related to a specific terminology. This means that pupils need to develop a mathematical linguistic understanding that helps them understand both the instructions of the teacher and the texts in the text books. Furthermore, they also need to be able to use this language themselves in the solving and presentation of mathematical problems.

According to Löwing (2006, 145), the teacher cannot make language accessible to the pupils through using everyday language, but should successively strive to develop the language of the pupils in order to make them communicate and deal with formal mathematical language. Löwing also considers the lack of adequate language for teaching to be one of the reasons for pupils' shortcomings in mathematics.

In a longitudinal study on pupils' mathematical development from the age of 6 to the age of 15 (Hägglom, 2000), I have among other things noted that there are significant differences in pupils' encounters with mathematical structures and that there is a gradual elimination with age. Two forms of assessment were used in the study above, one competence-oriented and one communication-oriented (Imsen, 1992). The choice was determined by the view of learning and assessment that is to be found in the school. In the communication-oriented assessment the children's written accounts were used as a basis for describing the variations in the children's ways of calculating. The analysis of individual tasks in the study shows great variation in the pupils' ability to deal with the mathematical symbolic language and in their skills to describe their solutions. There are many mathematical items of knowledge that are strongly related to the pupil's language. The study of pupils' cognitive development has inspired me to focus on language in pupils' learning of mathematics from a communicative perspective. The overall purpose of this article is to describe how language directs and affects pupils' knowledge in mathematics. I attempt to map out variations in pupils' learning of mathematics from a linguistically orientated perspective. The multifaceted use of language will be discussed based on the characteristics of the subject and with reference to theories on pupils' learning of mathematics. I will also discuss how the pupil's use of language can be used to assess their learning. The content focuses on the specific terminology and symbolic system of mathematics and on the pupils' usage to make their solutions visible. The language use of the teachers has not been mapped out in this study.

The empirical analyses are comprised by written pupil response that forms part of a communication oriented assessment (Hägglom, 2000) with both an informative and descriptive research approach. The survey group consists of 113 pupils whose knowledge has been mapped out at the ages of 9, 12, and 15. This corresponds to school years 3, 6 and 9. Based on a few specific tasks the pupils' responses are interpreted from a language related perspective.

Theoretical framework

Within the theoretical framework I will discuss learning in mathematics as a process where language constitutes the basis in the field of tension existing between the subject of mathematics and the mathematical development of the pupils. Language is a mediating tool (Vygotsky, 1962) in the cognitive development of pupils. The pupils learning in a language oriented learning environment should be seen towards the background of the social interaction created in a teaching situation. The social constructivist view on learning stresses a learning environment with active communication between teacher and pupil, a communication where the pupil is given the opportunity to verbalise his or her thoughts, and where the teacher is perceptive not only towards the pupils' argumentation, but also has the capability to assess the qualities in the patterns of thought of the pupils.

Concept building and mathematical knowledge

The understanding of concepts is central in mathematics. It is developed as relationships between the *schedule of the individual* and both the experienced *situation* and a *numeric and symbolic language* (Vergnaud 1988, 177). It is thus connected to the individual meeting with different *forms of representation* (Lesh, Post & Behr 1987, 34). The various forms of representation that help the pupils to build their understanding are related to each other like a network. It means that the pupil's linguistic use has to be looked at from several different perspectives and in close connections with concrete models, visual images and symbolic use. Spoken language is a meta-language for the concept building of the individual.

Based on Vygotsky and Stieg Mellin-Olsen, Marit Johnsen Høines (1990, 66) has developed a model for concept building which describes concept as a relationship between practical circumstances, a so called conceptual content, and a written form of expression, a so called conceptual expression. The model relates to the reality that surrounds us and to the practical phenomena that affect us. The conceptual content is the significance and interpretation that we assign to objects and situations around us. It can be seen as pupils' perceptions, associations, thoughts and views on a phenomenon which in this case can be seen as a concept. How something is perceived can depend on the context and the occasion. The formal symbolic language symbolises the conceptual expression and reflects the pupils' thought about the conceptual content itself and is used to describe the reality perceived by the pupils.

The models described above show that a concept is not purely a definition, but the concept is related to an abundance of situations based on individual perceptions and relationships between forms of linguistic use.

Curricular intentions and competencies

In the curriculum for basic education (Utbildningsstyrelsen, 2004) concept building in mathematics is stressed and pupils are to be given opportunities to relate their experiences and cognitive systems to the abstract system of mathematics. Starting from the first years at school the pupils are supposed to develop their ability to *present solutions orally and in writing* to other pupils and to the teacher. The solutions should also be carried out with concrete models and images. Later, the use of mathematical models is stressed along with the ability to *express their thoughts unambiguously* and the ability to *interpret and produce mathematical texts*. Thus, the intentions of the curriculum prescribe a *multifaceted use of language* which starts out from simple conceptual models with a high level of linguistic activity and progresses into more advanced skills to *use the mathematical language orally and in writing*.

In the Danish KOM - project (KOM-arbejdsgruppen, 2002), Kompetencer och matematikl ring (i.e. Competencies and the Learning of Mathematics), a model based on focusing on mathematical competencies, has been developed. By competencies is meant having knowledge about, understanding, practising, using and being acquainted with the use of mathematics in different contexts, where mathematics form, or will form an integral part. The eight competencies defined have been divided into two main groups (ibid, 2002, 43-47). In the first main group, the ability to answer and ask questions in and with mathematics, *the competency of thinking mathematically, the competency of posing and solving mathematical problems, the competency of modelling and the competency of reasoning*, are included. These four competencies are focused on the pupil's ability to embrace all the varying problems and the reasoning where mathematics is used to describe, explain and uncover everyday and abstract phenomena. The second main group, the ability to manage the language and tools of mathematics, comprises *representational competency, symbolic and formalist competency, communicational competency and the competency of using aids and tools*. Communicational competency implies not only the ability to communicate with and about mathematics, but also the ability to acquaint oneself with and interpret other people's oral and written reasoning. The competency of reasoning, which implies being able to follow and participate in a mathematical reasoning, is closely connected to the competency of language and formalism which comprises the use of elementary symbolic language and formal rules. Even if these eight competencies are divided into two main groups there is a close connection between the groups. The eight competences present interesting possibilities to uncover *qualities in mathematical learning* and several of them are directly linked to language.

There is a close connection between the competencies described above and the intentions of the Finnish core national curriculum for basic education (NBE, 2004). Both stress linguistic competence which is very much in line with the view on pupils' learning in social interaction.

Pupils' encounters with mathematical terminology

The terminology which belongs to mathematical language contains terms that are often exact and that supply specific information. Many words have been borrowed from everyday language and have their own meaning in mathematics. Words like e.g. *volume*, *the legs of an angle*, *the root of*, *the side of a square* are a few examples of linguistic concepts that belong to mathematical terminology. These words are strange to the pupils if they do not get the possibility to use the words in an appropriate mathematical context.

A great deal of attention is required, both from the perspective of the pupil and the teacher, to learn to understand and to use mathematical terminology in the right context. From the pupil's perspective the mathematical terminology must be used in several different situations. Pupils need to understand the *teachers' explanations* as well as the *texts in the schoolbooks*, in other words they need to have an *extensive vocabulary*. Sterner and Lundberg (2002, 41) see an extensive vocabulary as a prerequisite for understanding what you read and point out the importance of not taking the words out of their context, but rather making use of the situation and focusing on the new words and uncovering them. Thus, the new words can be learnt in a meaningful context and can be connected to the pupils' own experiences. According to Lesh, Post and Behr (1987), this means, among other things, looking at terminology based on the different representational forms of the concept

From a teacher perspective, language use means e.g. systematically uncovering both the meaning of the new words, as well as helping the pupils to remember words that have been dealt with earlier, i.e. establishing and maintaining what I would call a *linguistic awareness in mathematics*. This linguistic awareness of pupils can be developed as soon as they have learnt to listen to and compare spoken and written language. E.g. a *square metre* (m^2) means the size of a *square* with a side measuring one *metre*. Language simultaneously gives an image of what a square metre means. Then you can bridge the discrepancy between everyday language and mathematical language.

Many words in mathematics are linguistically closely related and belong to the same conceptual area but are used differently. When such words are used in texts, pupils are expected to know their mathematical meaning. One example is the number units *tens* and *tenths*. Pupils in year 6 solved the following task:

You have the number 456,123. Which numeral shows

- a. the tens*
- b. the hundreds ?*

In year 6 the solution percentage was 70 % for task a and 53 % for task b. In year 9 the corresponding result was 83 % and 47 % respectively. Consequently, it is more difficult to identify the hundreds than the tens. There was a marginal

improvement from year 6 to year 9 in the task a while the result in task b was somewhat poorer. The following task led to similar results:

Which number is

a. 3 thousands larger than 191 468?

b. 7 tenths smaller than 17,6?

The solution percentage in year 6 is 68 % for task a and 55 % for task b. In year 9 the corresponding result was 73 % and 60 % respectively. The improvement is marginal. The most common cause for getting the wrong answer was getting the names of the number units mixed up. An occurring misconception is that the expression *larger than* is interpreted as multiplication rather than addition. Since the numeral units of the decimals were introduced already in year 4, you can observe that misinterpretations that have come about early can easily become permanent.

Pupils' use of formal symbolic language

Because of the specific characteristics of the subject, the formal symbolic language stands out as the most typical for mathematics. It is a language with its own *semiotics* which can be used as an *analytical tool* (Winsløw, 2004). It is an exact and unambiguous symbolic language which functions as a bridge between individuals' thoughts and descriptions of phenomena in the surrounding world that are related to a mathematical model. This result in the quality of pupils' understanding of mathematical concepts is often being noted via the pupils' ability to deal with the mathematical symbolic language. Sterner and Lundberg (2002, 18) draw parallels between the development of written language and the development of mathematical symbols by noting that it takes the pupils a long time to transfer concrete experiences into mental representations and corresponding mathematical symbols.

Mathematical forms of thinking are expressed with the aid of symbols. Skemp (1989, 90) sees the symbols as a bridge between our thoughts and the thoughts of others and as a bridge between the different levels in our thinking that are difficult to reach and those that are reached more easily. In order to profit from information expressed with the mathematical symbolic language the individual needs to learn this language. The symbols are related to specific concepts that form networks within the mathematical conceptual framework. The symbols have been developed over a long period of time (Weaver & Smith, 2007) and create a language which is internationally usable. *Symbolic competency* includes the knowledge of what the symbols stand for, how they are read and how they are used. From the perspective of the pupil it means linguistic use that is developed from a simple writing of numbers into a more complex expression where numbers and signs should be combined in the correct way.

In the model for pupils' development of a concept by Johnsen Høines (1998, 78-79), the conceptual expression is about the language, the symbols that express

thought. In lower years, the pupil can use a word, an image or a symbol to express a concept.

In the following, based on given tasks, I will illustrate how the pupils' handling of symbols shows a great variation. The tasks have been chosen not only to demonstrate the variation, but with the purpose to catch their misconceptions based on an insufficiently developed conceptual understanding.

In children's early understanding of numerals the handling of symbols takes place in close connection to the name of the number and the way it is pronounced. Before the writing of numerals is fully developed, a lot of children write the number *one hundred and seven* as *1007*. The direction of writing also varies. As the numbers increase, so do the difficulties when it comes to writing numbers and there is a great uncertainty. From the ages of 12 to 15, many pupils show no development in the writing of numerals when it comes to big numbers. For example, the number *twelve million thirteen thousand* is written as *1213000* both in years 6 and 9. The difficulties with applying the positioning system, as well as the use of zeros, lead to an incorrect conceptual expression.

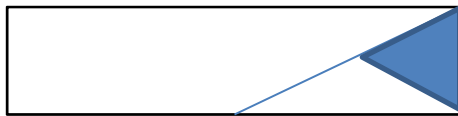
For decimal structures, similar variations apply. An example:

Write a decimal number which lies between 9,5 and 9,6.

The solution percentage is 64 % in year 6 and a third of the pupils leave the task unanswered or note that they do not understand the task. Among the incorrect answers there are five answers of $9,5 \frac{1}{2}$. The answer opens up for the interpretation that the pupil has developed an understanding for the conceptual content but lacks competence in using the correct symbolic form. Out of these five pupils there are still two pupils in year nine who answer $9,5 \frac{1}{2}$. Thus, there is a tendency that the formal writing of numerals does not develop substantially during school years 6-9.

Even when it comes to fractional numbers you can observe that the mastering of symbols can take different forms and leave room for different interpretations (Behr et al., 1992). Fractions can be illustrated with numerous representational models which all demand good perceptive skills and ability to use the models in relevant situations. The following example reflects the variation in pupils' perceptions of the fractional model:

What fraction of the figure is shaded?



Of the pupils in year 6, 38 % answered $\frac{1}{8}$. In most of the incorrect answer the result was $\frac{1}{4}$, which is explained by the picture being interpreted in the way that one part out of four is shaded. In these interpretations the pupils lack the concept

of *equal sharing* which is central for the concept of *fraction*. In some constructions you can find traces of pupil's conceptual content being developed (equal sharing), but the symbolic mastery is still seeking its forms. The pupil answers:

$\frac{1}{2} 4$ or "half a four".

The above-mentioned example further show that the combination of weak conceptual understanding leads to great variations in answers including differing ways of mastering symbols:

$\frac{1}{3}$, $\frac{1}{6}$, "half of a third", $\frac{1}{2}$ of 3 or $\frac{1}{2} 3$

Thus, a picture model of a fraction can be used to assess pupils' understanding of fractions and in this analysis you can also analyse the pupil's conceptual expression within the framework of their symbolic competence. I would like to call this type of assignment a "conceptually rich" assignment.

The analyses of the pupils' mastering of symbols show that writing of symbols gives extensive possibilities for pupils to construct their own interpretations and there is a tendency that the more possibilities there are for combinations of numbers, the more extensive are the variations. Especially when the pupils are supposed to be able to switch from one symbolic form to another, many different interpretations may arise. When the pupils in year 9 were to write the number 30 000 with a tenth power the solution percentage was 46 %. There were 24 different answers among a total of 48 incorrect answers! There is a tendency among pupils to construct new numbers by *manipulating with given numbers*.

Symbolic writing is a process where the pupils learn how to follow given rules based on the laws of mathematics. On the one hand, one can ask oneself if mathematics teaching sufficiently focuses on symbolic writing as an important part of the conceptual development belonging to mathematical competence. On the other hand, the learning process of the pupils should be supported in order to help them analyse their answers, even if the result may be incorrect. This process where the pupil develops a conceptual content should also support the development of a conceptual expression and ought to be studied on the basis of that context.

Spoken and written communication

According to the curriculum (NBE, 2004) the pupils should learn how to communicate and to develop mathematical thinking. In the following, I will illustrate the significance and characteristics of spoken discussion in order to analyse the more traditional linguistic communication.

Mathematical discussion

Spoken language is a metalanguage for the whole symbolic system (Vergnaud, 1988, 177). Through spoken communication pupils learn how to reason and at

the same time to listen to their own reasoning. Johnsen Høines (2002, 166-167) focuses on the importance of letting the pupils practice how to communicate and to use their knowledge with others. This forms part of the didactic competence that the pupils need to develop.

When the pupils use their language to describe a phenomenon, a concept or a strategy for counting, their thoughts become visible both to them and to their teacher (Ahlström et al, 1996, 45-47). The pupils' self-reflection is developed and knowledge is uncovered in the form of the language. If the pupils get to practice how to use mathematical terminology, it will also become part of their usage. The discussion does not just follow the traditional pattern of questions and answers, but involves deeper reasoning. Such a mathematical discussion is important to help pupils

- to express their thinking
- to learn how to reason mathematically
- to learn how to make assumptions, to formulate and solve problems
- to search for connections and applications.

All pupils should be given the possibility to develop mathematical reasoning. In this process the role of the teacher becomes very central, as spoken communication is very demanding for the teacher. In order to understand the variation in pupils' reasoning, subject knowledge is required and it is important to be able to ask questions that prompt discussion. At the same time, pupils need to learn to listen to each other. In order to uncover the knowledge used in the mathematical discussion and to make the other pupils perceive the variations, the structure can be written down. Then the spoken communication is transformed into a written form which enables more pupils to grasp the content. At the same time, the pupils receive a model for how spoken communication can be expressed in written form. Teaching, where the thinking processes of the pupils are in focus and where the aim of teaching is to teach pupils how to think mathematically and to focus on the awareness of their thinking, is given the term the "Thinking Curriculum" by Ginsburg, Jacobs and Lopez (1993, 127).

The communication taking place in the classroom between teacher and pupils should help the pupils to develop their mathematical proficiency. Alrø and Skovsmose (1993) point out that this is not always a self-evident and effective process, but it presupposes that pupils are aware of their goals for learning and that they are involved in the process themselves.

The purpose of mathematical discussion is to accustom pupils to mathematical usage as well as support their written communication.

Written communication

Through spoken communication pupils have developed a language to express their thoughts. The next step is to practise their skills in writing these thoughts down. The written representation then becomes significant in two different

senses. On the one hand, it teaches the pupil to handle mathematical language in written form; on the other hand the knowledge is then assessable by several stakeholders. Furthermore, it enables many more pupils to express their thoughts.

When the pupils are given the possibility to describe their thinking process in numerical tasks (Häggbloom, 2000), many differences in the way they express their thinking arise. These variations show that there are several ways of thinking and reasoning. The use of for instance symbols in mathematics to describe thinking is an ability that exhibits extensive variation among pupils within the same age group. Björkqvist (1992, 114-117) calls attention to the fact that the thinking of one pupil is not an image of anybody else's way of thinking but has originality and possesses its own function.

In my study of pupils' development of mathematical knowledge, I have examined the variations in pupil's knowledge, by looking at their counting strategies based on their abilities to explain how they have counted in writing. The result shows not only great differences in the ways of using different strategies for counting, but also different abilities and interest in describing their counting. Some pupils do not describe their thinking but answer that "it is drummed into me" or that "it comes automatically". Some answer that they do not know or they repeat the question.

Already at the age of 9, pupils use different methods for counting the same task; mental arithmetic or written arithmetic. In the pupils' account of their counting strategies using mental arithmetic, there are great variations when it comes both to the choice of methods and to the form of expression. Often the pupils use an account which is a combination of a formal use of numbers and signs and linguistic reasoning. Some pupils describe their solutions verbally and in this linguistic description they use a narrative linguistic game consisting of the story that can be made out from an observable narrative discourse.

Already in year 3, some pupils have developed written description. This narrative way of expression can sound like this in the task $697 + 4 = 701$:

*I take away 3 from 4 and add 3 and 7 and 690 and then I get 700.
But then I have another one left from 4 so I add 1 and 700.*

Another pupil gives the following explanation to the same task: $697 + 3 + 1 = 701$.

In year 6, a pupil reasons in the following way when she writes a decimal number between 9,5 and 9,6:

My thinking is that I would put a zero after 5 and 6. Then it becomes 9,50 and 9,60. Then I think what is between 50 and 60 and then I get e.g. 55. That is why my answer is 9,55.

Another pupil reasons about tenths and hundredths based on the same task:

The 5 is a tenth; you just need to write a hundredth, half of a tenth is five hundredths.

Maybe the most valuable in the description of counting strategies are the incorrect answers containing an explanation. The fact that half of the pupils, who have given either a correct or an incorrect answer, accounted for their strategies, indicates that the ability to communicate in writing has been developed also in weak pupils. They show a will and a commitment to describe their mathematical solutions. A pupil in year 3 describes the solution for $805 - 798$ in the following way:

First I take away a hundred from 800 then I get 100 and then I do $98 - 5$ and I get 193.

By analysing the counting strategy behind an incorrect answer you can retrieve information about the reasoning of the pupil and thereby open up new possibilities for supporting the pupil's learning. Häggblom (2000, 267) also states that you can understand an incorrect answer by studying the counting strategy behind another incorrect solution where the account is missing. The pupils develop qualitatively better counting strategies with increasing age, but some types of counting errors consistently occur throughout primary school.

The written solution for a mathematical task has always been a form of expression for mathematical knowledge. All too seldom there are comparisons between different solutions and forms of thinking. Mostly, it has been considered enough to get the correct answer. If the focus increasingly is put upon the variations in the development of knowledge, the differences between pupils could become an asset in the learning process.

By tradition, the written solution of the mathematical task is the most common form of expression. You can see it as a traditional use of mathematics. Let us see the written communication from the perspective that it can offer possibilities to get acquainted with the pupil's underlying forms of thinking.

Sterner and Lundberg (2002, 16-17) point out the complexity in learning that arises when a pupil has linguistic difficulties and stress the importance of concrete actions or visual images to create inner conceptions where inner speech is included. This is a process, according to Brissenden (1988, 25), where pupils develop a mathematical model which is coded with symbols and is tested. The pupils' spoken communication forms the bridge between the action and the coding with symbols. This means that the pupils' use of language is integrated with many activities.

When pupils in year 6 were solving the task $2,5 - 2/5$, the solution percentage was 23 %, and 27 % left the task unsolved. Out of the incorrect answers (50 %), 68 % accounted for their solution which made it possible to follow their reasoning and to assess their counting strategies. The most common solution was to write the second term in a decimal form and then perform the subtraction. One pupil solved the task with the support of a picture model and reasoned according to the following:

I drew 3 circles. I divided them into fifths. Two of the circles I left like that. They corresponded to the two in 2,5. The third circle I

divided in half that means that a fifth is 2/10. (Correct answer 2 1/10)

The pupil has the ability to make a connection between a concrete action and the written language, which supports learning. This incorrect answer shows great difficulties in seeing the connection between fractions and decimal form.

Reading comprehension and mathematics

Solving text-based tasks demands a vast arsenal of different linguistic skills and already in the lowest years, pupils show great variation when they solve text-based tasks (Ahlberg, 1992; Häggblom, 2000; Romberg & Collis, 1987). The variation in the solutions often depends on the semantics of the text, numerical values, computations and the cognitive skills of the pupil. Pupils with difficulties in solving text-based tasks (Romberg & Collis, 1987) had low cognitive ability which could be observed in their low memory capacity, their lack of systematic ability to count and ability to solve more complex problems. The number units have more complex structures in higher years. According to Cobb (1986), pupils perceive tasks with higher number units as more abstract, as they cannot clearly see the connection between number and solution. In the corresponding way there is a progression in the semantics of a text.

In the following, there is a summary of some of the causes of variation in the solving of textual tasks based on my longitudinal study.

The pupils' ability to solve text-based tasks varies depending on the *semantic understanding* of the meaning of the text and the *mathematical representation of number units*. When the pupils are to solve a text-based task it is not only their reading comprehension that is put to the test, but they also need good arithmetic skills. In the study at hand there is a *higher correlation between arithmetic skills and ability to solve text-based tasks than there is between reading comprehension and ability to solve text-based tasks*. The result probably depends on the characteristics of the task but also shows the importance of sound counting skills when it comes to solving text-based tasks.

Bilingual pupils show a very positive development when it comes to solving text-based tasks. The statistically significant difference of the mean value for the variable text-based tasks that existed at the ages of 6 and 7 between the Swedish-speaking (59 %) and the bilingual (41 %) children, in favour of the Swedish-speaking, did exist neither at the age of 12 nor at the age of 15. There are, however, statistically significant differences in reading comprehension between the age groups at the ages of 9, 12 and 15.

The pupils who have participated in teaching with *systematic focus on both the solving of text-based tasks and on math stories produced by themselves* (c.f. Häggblom, 1994) achieved a statistically significantly better result in years 3 and 6 than the pupils who had not participated in the project. In year 9, the

corresponding comparison could not be made because of the change of school. The systematic effort was focused on the pupils working on the oral text-based tasks in the form of stories and math stories, with images and concrete materials, already during the first year of school. As soon as their writing was developed, stress was put on their own creation of text-based tasks. The text-based tasks formed an integral part of the teaching of mathematics.

Language as a tool for assessment

When assessing pupils' knowledge in maths, it is not enough to state that a pupil has counted correctly or incorrectly, neither is it enough to establish individual variations in results. One should rather *uncover the underlying strategies for thinking* in the learning of mathematics. Mathematics is a mental activity and an intellectually demanding subject. Especially the fact that an individual's thoughts are invisible makes it all the more important to uncover the thinking process in order to assess the *quality in the mathematical proficiency* of the pupil. Thus, an active use of language becomes a support in the learning process of the pupils (Bauersfeld, 1998, 59). When the pupils use their language to describe a phenomenon, a concept or a strategy for counting, their self-reflection develops (Ernest, 1998) and their knowledge becomes visible in the form of language. When the pupils are to formulate their thoughts, they stop and reflect on their knowledge. At the same time, they develop their proficiency in using the correct terminology and the language of mathematics becomes the pupil's knowledge.

If I, as a teacher, know how a pupil reasons, I can help him/her to develop further. I can even use one pupil's thinking mistakes to understand another pupils' way of thinking or to make another pupils aware of his/her thinking and solution. In this process the pupil's competencies of reasoning, thinking and communication become extremely central. By interpreting the pupil's handling of symbols, I can assess the quality in the pupil's learning and in some cases notice a logically and conceptually correct way of expressing a phenomenon, even if the formal exactness is lacking. The pupil is then about to develop an understanding of the concept, but has not yet been able to develop all the representational forms of the concept. From a longitudinal perspective, it is possible to find patterns that recur in the pupils' answers and show individual misconceptions which stem from a long time back.

Within a learning culture founded on the above-mentioned view on learning, assessment forms an integral part of teaching. Romberg (2004, 589) characterises a classroom assessment with principles where the purpose is to increase learning by giving pupils the opportunity to show what they know rather than what they do not know. The assessment includes multifaceted and varying forms in which the pupils explains and documents their achievements. Assessment becomes a kind of tool for helping the pupils to develop their knowledge in a varied way. The assessment is constructed to become a support

for the pupils to reflect on their learning as well as to become more conscious of their progression, their possibilities and their needs.

Concluding discussion

The linguistic dimension in mathematics is extremely multifaceted. On the basis of the specific characteristics of mathematics and the differences in pupils' learning, I have chosen to focus several components that form pupils' learning. The empirical starting point has been a longitudinal study of pupils' development of knowledge of mathematics. When you analyse pupils' answers, you meet variations showing individual solutions. The terminology of mathematics together with the specific symbolic language forms the foundations of learning. Small variations in the meaning and usage of the words in specific contexts lead to varying interpretations. The exactness of the language is a great challenge and cause pupils to give up, unless their language develops to enable the exact usage that mathematics require. The interpretations and use of language can cause personal interpretations and incorrect answers that can even become permanent.

The basis of linguistic mathematical development lies in pupils' meetings between their own linguistic development, the different representations of the mathematical concepts and the culture of learning developed in the classroom. It is a meeting between the development of personal strategies for counting and formal school mathematics. Conscious attention on the significance of language, when it comes to success in mathematics, can give a deeper understanding of individual variations.

The use of conscious language orientated mathematical teaching, causes assessment to become part of the learning process. Above all, a pupil's description of a thinking strategy can guide a teacher's understanding of a pupils' thinking. The ability to understand and use specific mathematical terminology is an important part of pupils' learning. The national core curriculum (NBE, 2004) stresses the development of the pupils' reflective skills and the awareness of their own learning. Maybe we need a more language orientated classroom culture in mathematics where a metacognitive attitude is given a lot of space.

The Finnish national core curriculum for basic education (NBE, 2004) offers guidelines to look at assessment from a process-orientated perspective. As assessment seeks new forms, it is important to elucidate the purpose of assessment. The kind of assessment that is carried out continuously throughout the learning process needs to support the pupil's learning and help the pupil to develop clear aims. The pupils can only assess themselves if they have a clear view of what the goals are. This means that pupils gradually should develop their awareness and ability to assess their learning. For this, the pupils need the help of a teacher who can analyze their forms of thinking. In social interaction

the thoughts of pupils are uncovered and thus become objects of assessment. Based on the competencies of the KOM-project, assessment can be developed in the direction of clearer criteria for what learning of mathematics includes. Such a model of assessment coincides with both the view of pupils' learning in social interaction and the specific characteristics of mathematics. At the same time, it becomes a quality assurance and helps the teachers to assess their own work.

All pupils should be given the possibility to develop mathematical reasoning and in this process the role of the teacher becomes very central. The task of the teacher is to interpret and to uncover the pupil's language in written form both to the pupil him/herself and to other pupils in class. Simultaneously, there is an assessment of the pupil's perceptions and of the quality of the thinking process. Many pupils need help with expressing their own thoughts in written form. This kind of teaching, where pupils' thinking processes and strategies are in focus, aims at teaching the pupils to think mathematically and to focus their awareness of their thinking, a metacognitive perspective.

References

- Ahlberg, A. (1992). *Att möta matematiska problem. En belysning av barns lärande.* Göteborg studies in educational sciences 87. Göteborg: Acta Universitatis Gothoburgensis.
- Alrø, H. & Skovsmose, O. (1993). Det var icke meningen! – om kommunikation i matematikundervisningen. *NOMAD*, 1 (2),6-29.
- Bauersfeld, H. (1998). Radikalkonstruktivism, interaktionism och matematikundervisning. I A. Engström (red.), *Matematik och reflektion.* En introduktion till konstruktivismen inom matematikdidaktiken (s. 54-81). Lund: Studentlitteratur.
- Behr, M., Harel, G., Post, T. & Lesh, R. (1992). Rational number, ratio and proportion. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 296-333). New York: Macmillan.
- Björkqvist, O. (1992). Socialkonstruktivism som grund för matematikundervisning. I O. Björkqvist (red.) *Forsknings- och utvecklingsarbete i de matematiska ämnernas didaktik* (ss. 109-118). Rapporter från Pedagogiska fakulteten vid Åbo Akademi, nr 2.
- Brissenden, T. (1988). *Talk about mathematics.* Oxford: basil Blackwell.
- Cobb, P. (1986). Context, goals, beliefs and learning mathematics. *For the learning of mathematics.* 6 (2), (2-9).
- Ernest, P. (1998). Vad är konstruktivism? I A. Engström (red.), *Matematik och reflektion. En introduktion till konstruktivismen inom matematikdidaktiken* (s. 54-81). Lund: Studentlitteratur.
- Fischbein, E. (1990). Introduction. In P. Neshier & J. Kilpatrick (eds.) *Mathematics and cognition: A research Synthesis by the International Group for the Psychology of Mathematics Education* (pp. 1-13). Cambridge: Cambridge University Press.

- Ginsburg, H. P., Jacobs, S. & Lopez, L. S. (1993). Assessing mathematical thinking and learning potential in primary grade children. In M. Niss (Ed.), *Investigations into assessment in mathematics education. An ICMI study* (pp. 157-167). Dordrecht: Kluwer.
- Häggbloom, L. (2000). *Räknespår. Barns matematiska utveckling från 6 till 15 års ålder*. Åbo: Åbo Akademis förlag.
- Häggbloom, L. (2006). Utvärdering I lärandeprocessen. I L. Häggbloom, L. Burman & A-S. Røj-Lindberg (red.) *Perspektiv på Kunskapens och lärandets villkor. Festskrift tillägnad professor Ole Björkqvist*. Specialutgåva från Pedagogiska fakulteten vid Åbo Akademi, nr 1/2006. Vasa: Pedagogiska fakulteten.
- Johnsen Høines, M. (1990). *Matematik som språk*. Stockholm: Utbildningsförlaget.
- Johnsen Høines, M. (2002). *Fleksible språkrom*. Matematikkläring som tekstutvikling. Bergen: Universitetet i Bergen.
- Lesh, R., Post, T. & Behr, M. (1987). Representation and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics*. London: Lawrence Erlbaum Associates.
- Keranto, T. (1998). Kriittinen ajattelu ja tieteentuntemus matematiikan opetuksessa. In P. Räsänen, P. Kupari, T. Ahonen & P. Malinen (Eds.), *Matematiikka – nälökulmia opettamiseen ja oppimiseen* (p. 18-38). Jyväskylä: Niilo Mäki Instituutti & Koulutuksen tutkimuslaitos.
- KOM-arbejdsgruppen (2002): *Kompetencer og matematiklæring – Idéer og inspiration til udvikling af matematikundervisning i Danmark*. Uddannelsesstyrelsens temahæfteserie nr. 18, Undervisningsministeriet, Danmark.
- Kulm, G. (1993). *Mathematics Assessment. What works in the classroom*. San Fransisco: Jossey – Bass Publishers
- Löwing, M. (2006). *Matematikundervisningens dilemman*. Lund: Studentlitteratur.
- Malmer, G. & Adler, B. (1996). *Matematiksvårigheter och dyslexi*. Lund: Studentlitteratur.
- Morgan, C. (1998). *Writing Mathematically*. The discourse of Investigation. Studies in Mathematics Education Series: 9. London: Falmer Press.
- xxxxxxNämnamn-Tema. (1996). *Matematik – ett kommunikationsämne*. Göteborg: Göteborgs universitet.
- Romberg, T. A. (2004). Classroom Assessment Studies. In B. Clarke, D.M. Clarke, G. Emanuelsson, B. Johansson, D.V. Lamdin, F.K. Lester, A. Wallby & K. Wallby (Eds.) *International Perspectives on Learning and Teaching Mathematics*. Göteborg: Göteborg university. National Center for Mathematics Education.
- Romberg, T. A. & Collis, K. F. (1987). Learning to add and subtract. *Journal for Research in Mathematics Education*. National Council of Teachers of Mathematics.
- Skemp, R. (1989). *Mathematics in the Primary School*. London: Routledge.
- Sterner, G. & Lundberg, I. (2002). *Läs- och skrivsvårigheter och lärande i matematik*. NCM-Rapport 20002: 2. Göteborg: NCM.

- Utbildningsstyrelsen (2004). *Grunderna för läroplanen för den grundläggande utbildningen 2004*. Helsingfors.
- Weaver, D & Smith, A. D (2007.) The History of mathematical Symbols. <http://www.roma.unisa.edu.au/07305/symbols.htm>. 8. 3. 2007
- Vergnaud, G. (1998). A comprehensive Theory of representation for mathematics Education. In *Journal of Mathematical Behavior*, 17 (2), 167-181. Ablex Publishing Corp.
- Winsløw, C. (1998). Matematikkens sproglighet som didaktisk potentiale. *NOMAD*, 6 (2), 1988, 29-40.
- Winsløw, C. (2004). Semiotics as an analytic tool for the didactics of mathematics. *NOMAD*, 9 (2), 81-99.
- Vygotsky, L. S. (1962). *Thought and Language*. Cambridge: The M.I.T. Press.
- Wyndhamn, J. (1990). *Fyra matematikdidaktiska satser*. Skapande vetande. Rapport nr 15. Linköping: Linköpings universitet.

Finnish and Turkish national core curriculum for basic education in mathematics

Jorma Joutsenlahti
University of Tampere

Nihan Sahinkaya
Gazi University, Ankara

We have analyzed how the curricula support the development of a pupil's mathematical thinking. We have chosen the five features of mathematical proficiency in school mathematics to describe central processes of mathematical thinking and factors that influence or express mathematical thinking. The features in our study are: procedural fluency, conceptual understanding, strategic competence, adaptive reasoning and affective domain.

Introduction

Finland and Turkey have perhaps more differences than similarities if they are compared by their history, culture, geography, economy, population etc. Mathematics is a global (universal) language, which has the same application all over the world. That's why it is interesting to study the structures which direct mathematics teaching in classrooms in both countries. A curriculum is an official directive, determined by national and political policies, which takes the form of a document, setting out the structures guiding classroom teaching, and based on perceptions of what children need to know about mathematics. Recent developments in the education systems of both Finland and Turkey have been reported on e.g. in the OECD reports (PISA, 2003; OECD, 2004).

Table 1. Mathematics lesson hours in the curricula.

Finland	6		12			14			32	
Grades	1	2	3	4	5	6	7	8	9	Sum
Turkey	4	4	4	4	4	4	4	4		32

Comprehensive schooling in Finland lasts nine years and is intended for children between the ages of 7 and 16 years. During the first six years in basic education instruction is usually given by class teachers, who teach all or most subjects. Classes in the last three grades of comprehensive school are taught by teachers who are specialised in a particular subject. In the Finnish comprehensive school pupils study mathematics for a minimum of 3-4 lessons per week. The Finnish

mathematics curriculum is made up of three main domains: objectives, core contents and descriptions of good performance. Those three domains are represented in three periods (Table 1): the first period is from the 1st grade to the 2nd grade, the second from the 3rd grade to the 5th grade and the third from the 6th grade to the 9th grade. The conception of learning is based on the constructivist approach, which takes into account learning environment and social factors. (Joutsenlahti & Sahinkaya, 2006).

The basic education period in Turkey is eight years and is intended for children between the ages of 6 and 14. The primary school is divided into two sections: the first is from the 1st grade to the 5th grade and the second is from the 6th grade to the 8th grade. Class teachers teach almost all subjects during the first five grades. Subject teachers are responsible for teaching subjects in the remaining three grades. There is a weekly minimum of four mathematics lessons in the basic education period (Table 1). The Turkish curriculum is based on constructivist philosophy and pupils' instructional needs are the main focus. In the new Turkish curriculum, mathematics is taught on the basis of "every pupil is able to learn mathematics". Evaluation of learning is based on process evaluation rather than product evaluation. (Joutsenlahti & Sahinkaya, 2006).

Finnish and Turkish core curricula

Finnish mathematics curriculum

The task of instruction in mathematics is described in the Finnish curriculum as the task of offering opportunities for the development of mathematical thinking, and of learning mathematical concepts and the most widely used problem-solving methods. The Finnish mathematics curriculum for basic education is divided into three sections: grades 1-2, grades 3-5 and grades 6-9. Each section describes objectives, core contents and description of good performance at the end of the section (2nd grade, 5th grade and 9th grade). The curriculum is normative and it is the national framework on the basis of which the local curriculum is formulated.

Description of good performance is an important part of pupil's evaluation. The description shows what a pupil has to know and be able to do in order to achieve the mark 8. This means that the mark 8 is defined by description of performance in the curriculum. Other marks are not defined.

The Finnish mathematics curriculum is divided into eight areas: numbers and calculation, geometry, measurement, algebra, data processing and statistics, probability and statistics, functions and thinking skills and methods. There are also defined objectives, core contents and descriptions of good performance for every domain.

Table 2. Divisions of the core contents of mathematics

Domain of mathematics Domain of Mathematics	FINLAND			TURKEY	
	1-2	3-5	6-9	1-5	6-8
Numbers and calculations	■	■	■	■	■
Geometry	■	■	■	■	■
Measurement	■	□	□	■	■
Algebra	■	■	■	□	■
Data processing and statistics	■	■	■	■	□
Probability and statistics	□	□	■	□	■
Functions	□	□	■	□	□
Thinking skills and methods	□	□	■	□	□

Turkish mathematics curriculum

In the new Turkish program (Turkish National Board of Education Council, 2005), the mathematics curriculum has been formed on the basis of “every pupil is able to learn mathematics”. The Turkish mathematics curriculum is grounded on a conceptual approach and aims to develop conceptions and relationships between conceptual and procedural knowledge. It focuses on learning domains formed by concepts and relations. With the adopted conceptual approach, its aim is to help pupils to make abstraction and obtain mathematical understanding from their actual experiences and intuitions. In addition to developing mathematical concepts, its aim is to develop some important skills, which are problem solving, reasoning, making connections and communications.

The most important skill emphasized in the program is the problem-solving skill. Whilst acquiring problem-solving skills, the aim is also to acquire the following:

1. The use of problem-solving skills to evaluate and understand mathematical concepts
2. Setting problems using mathematical and daily life situations
3. Interpretation and evaluation of the solutions of problems
4. Developing self confidence and positive attitudes to use mathematics in a reasoning way
5. The use of different problem-solving strategies to solve different problems.

There are four learning domains in the program, which are: numbers, geometry, measurement and data (Table 2). In these domains, there are also sub-learning domains e.g. natural numbers, four operations with natural numbers in numbers domain. Also, the grade at which these domains and sub-domains are to be applied, and the acquirements in each sub-domain, are clearly stated in the program. The program presents plan and activity examples related with learning domains to the teachers.

The evaluation in the program is based on the learning process and it aims to follow the development of students. In the evaluation, the ability of using mathematics in their daily life, the extent of development in their problem-

solving skills, their reasoning skills, attitudes directed at mathematics, the development of self-confidence and, the development of social skills and aesthetics should be considered. Babadogan and Olkun (2006) have analyzed more precisely the Turkish new and old curriculum in their article.

Method

We have analyzed the contents of both mathematics curricula (content analysis). We have chosen five features of mathematical proficiency in school mathematics to describe central processes of mathematical thinking and factors that influence or express mathematical thinking. Mathematical thinking is a concept that is emphasized in both curricula, but it is not defined. We define mathematical thinking as the information process monitored by one's meta-cognition (Joutsenlahti, 2005).

Kilpatrick, Swafford and Findell (2001, 5) have constructed the concept *mathematical proficiency*, which has five strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition.

Conceptual understanding (CU) means the comprehension of mathematical concepts, operations and relations. Knowing what mathematical symbols, diagrams and procedures mean is also included in conceptual understanding. Students who have conceptual understanding know more than just isolated facts and methods. They are able to learn new ideas by connecting them to what they already know (Kilpatrick et al., 2001, 118). We can describe the essence of this feature by the word "understand".

Procedural fluency (PF) means skills in carrying out mathematical procedures (such as addition, subtraction, multiplication, and division of numbers) flexibly, accurately, efficiently and appropriately. Typical for this feature are mechanical counting, managing algorithms and procedures etc. (Kilpatrick et al., 2001, 121). We can describe the essence of this feature by the words "can do".

Strategic competence (SC) refers to the ability to formulate problems mathematically and to devise strategies for solving those using concepts and procedures appropriately (Kilpatrick et al., 2001, 121). Polya's problem solving is the essence of this feature.

Adaptive reasoning (AR) refers to the use of logic to explain and justify a solution to a problem or to extend from something known to something not yet known (Kilpatrick et al., 2001, 129). For example applying mathematical knowledge and "languaging" mathematics – meaning to express or communicate mathematical thinking – are typical for adaptive reasoning. We can describe the essence of this feature by the word "apply".

Productive disposition refers to seeing mathematics as sensible, useful, and doable (Kilpatrick et al., 2001, 131). We use *affective domain* (AF) instead of

productive disposition. Affective domain is a broader concept and it includes for example a pupil's beliefs, attitudes and views of mathematics.

We can combine the four features (excluding the affective domain) to types of knowledge. Conceptual understanding is related to conceptual knowledge and procedural fluency to procedural knowledge. Strategic competence is related to strategic knowledge and adaptive reasoning needs strategic knowledge and procedural knowledge. (Hiebert & Lefevre, 1986; Joutsenlahti, 2005, 98).

It is obvious that the five features of mathematical proficiency are interwoven and interdependent, but they are useful in categorization. We used those five categories in our analysis. We analyzed the mathematics curricula as to how they emphasize the five features of mathematical proficiency. We categorized expressions in the curricula to the five categories in terms of semantics: what the expression in the curriculum is supposed to mean in the working environment of classroom.

Results

During the process of analysis we noticed that some particular words were good indicators for categorization of expressions in the curricula. For example the verbs "calculate", "use", "master", "become practiced" and "be able to" often indicate that the expression belongs to procedural fluency. The verbs "understand" and "think" often reveal that the expression describes conceptual understanding and the verbs "solve", "transform" and "classify" belong to the area of strategic competence. Typical verbs for expressions in adaptive reasoning are "apply", "explain", "evaluate", "estimate", "formulate", "justify", "communicate", "present", "express" and "describe". Affective domain includes the expressions "derive satisfaction", "gain experience" and "learn to trust".

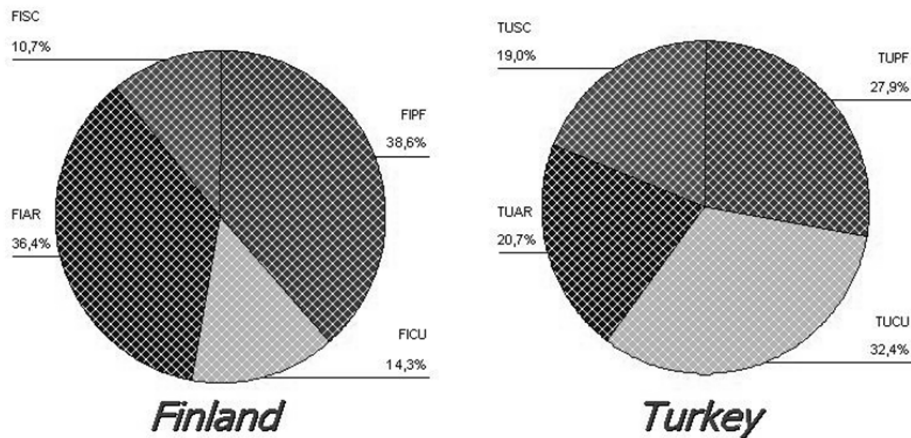


Figure 1. Distributions of the features of mathematical proficiency (except affective domain) in grades 1-9 (8) in the Finnish and Turkish mathematics curricula

After categorizing the expressions in the mathematics curricula we calculated part proportions for each feature of mathematical proficiency except affective domain (Figure 1). The Finnish curriculum has emphasis on procedural fluency (FIPF) and adaptive reasoning (FIAR). The Turkish curriculum has emphasis on conceptual understanding (TUCU) and procedural fluency (TUPF).

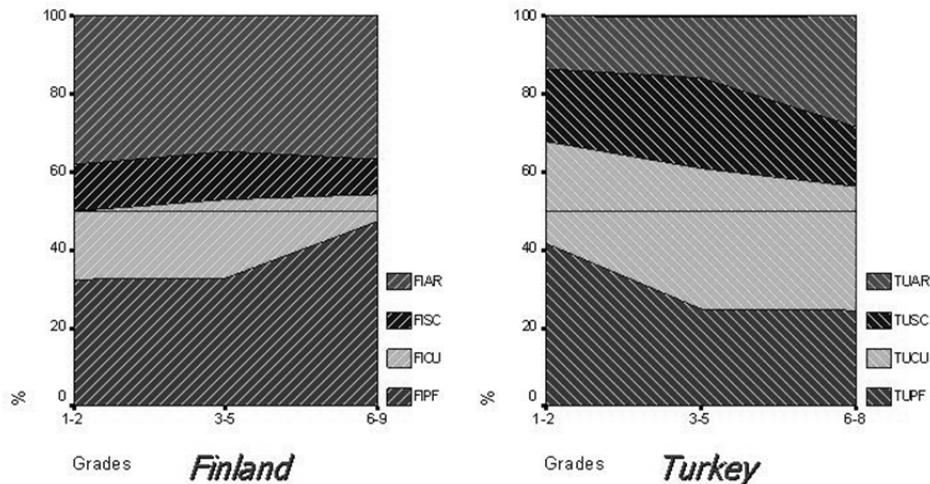


Figure 2. Distributions of the features of mathematical proficiency (except affective domain) in grades 1-2, 3-5 and 6-9(8) in the Finnish and Turkish mathematics curricula

When we study the distribution of the four features (Figure 2) in both curricula we notice that in the Finnish curriculum the proportion of procedural fluency (FIPF) increases from grades 1-2 to grades 6-9, but in the Turkish curriculum (TUPF) it decreases. On the other hand conceptual understanding (FICU) decreases in the Finnish curriculum from grades 1-2 to 6-9.

Adaptive reasoning is quite a strong feature of the Finnish curriculum during all grades and it increases in the Turkish curriculum during grades 6-8. An important part of adaptive reasoning is “languaging” mathematics in the Finnish and Turkish curricula. “Languaging” belongs to the main ideas of social constructivism, which emphasizes the importance of mother tongue and social environment. In the Finnish curriculum the first reference to “languaging” mathematics is in objectives and in description of good performance. For example the Finnish curriculum for grades 1-2 states: “The pupils are able to ... explain what they have done, and know how to present their solutions by means of pictures and concrete models and tools orally and in writing.” (Finnish National Board of Education, 2004, 160).

Strategic competence plays a central role in the Turkish curriculum and the concept of problem-solving is emphasized: “Problem solving is an important part of mathematics lesson and of mathematical activities ...” and “Pupils are able to develop high-level thinking skills using problem-solving.” (Turkish National Board of Education Council, 2005).

Traditionally the concept of “mathematical thinking” has an emphasis in the Finnish curriculum throughout grades 1-9. For example, the core task for grades 3-5 states: “The core tasks of mathematics instruction in the third through fifth grades are to develop mathematical thinking, introduce the learning of

mathematical models of thinking...” (Finnish National Board of Education, 2004, 161).

Affective domain has been taken into account in both curricula. This is an important innovation in both mathematics curricula, because in the earlier curricula affective domain expressions were either included in the general part of curricula or they didn't exist at all. For example, the Finnish objectives in grades 1-2 state: "...they will derive satisfaction and pleasure from understanding and solving problems” and in grades 3-5 "The pupils will gain experience in succeeding with mathematics” (Finnish National Board of Education, 2004, 158, 161). The affective domain is important also in the Turkish objectives for grades 1-8:” Pupils are able to enjoy mathematics” and ”Pupils are able to develop positive attitudes towards mathematics.” (Turkish National Board of Education Council, 2005).

Discussion

The analyzing method we used has uncertainty factors. The expressions in the curricula could be interpreted as belonging to several features of mathematical proficiency. For each expression we choose only one feature which we thought describes it best according to our experience and knowledge. Nevertheless, we believe that our analysis gives a reliable description of the Finnish and Turkish curricula from our point of view.

Procedural fluency receives emphasis in both Finnish and Turkish core curricula. It is deemed necessary that both Finnish and Turkish pupils have good basic skills in operating with numbers and algorithms, because pupils need those skills in mathematical problem solving and when they apply mathematics to everyday situations. Problem solving is an important part of the curricula since it develops the pupil's conceptual understanding and strategic competence.

Pupils' communication skills and characteristics of pupils' affective domain concerning mathematics are also considered important in both curricula. The objective is that pupils can express their own mathematical thinking using their mother tongue effectively. In the Finnish curriculum adaptive reasoning is emphasized (Figure 1). Finnish pupils were on top in mathematics in the PISA survey (PISA, 2003) and the survey measures especially how pupils are able to apply their mathematics skills and knowledge.

Learning materials in mathematics should be made according to the curricula, but at least in Finland there is some discrepancy between the written curriculum and learning materials. Learning materials emphasize more procedural fluency than the curricula. A survey of Finnish mathematics learning materials for grades 1-6 is expected to be published during 2007 by the Department of Teacher Training (Hämeenlinna) at the University of Tampere.

One important difference between the Finnish and Turkish curricula is that the Finnish curriculum does not include any instructions on how to teach mathematical concepts and methods (e.g. using manipulatives). The Turkish curriculum, on the other hand, has instructions how to teach mathematical concepts and methods, but they are not standards. Anyway, the mathematics curricula are mostly coherent in main streams. Local circumstances in Finland and in Turkey are very different, but the kind of mathematics our children will need is the same in both countries.

References

- Babadogan, C. & Olkun, S. (2006). Program development models and reform in Turkish primary school mathematics curriculum. *International Journal for Mathematics Teaching and Learning*. <http://www.cimt.plymouth.ac.uk/journal/default.htm> (5.1.2007)
- Finnish National Board of Education (2004). *National core curriculum for basic education*. Helsinki: Finnish National Board of Education.
- Hiebert, J. & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: an introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: the case of mathematics*, pp. 1-27. Hillsdale (NJ): Lawrence Erlbaum.
- Joutsenlahti, J. (2005). *Lukiolaisen tehtävääorientoituneen matemaattisen ajattelun piirteitä: 1990-luvun pitkän matematiikan opiskelijoiden matemaattisen osaamisen ja uskomusten ilmentämänä*. (Characteristics of task-oriented mathematical thinking among students in upper-secondary school. English abstract). Tampere: University of Tampere.
- Joutsenlahti, J. & Sahinkaya, N. (2006). *Finnish and Turkish national core curriculum for basic education in mathematics: comparing study*. In the Programme and abstracts of the Second World Curriculum Studies Conference WCSC, pp. 26-27. Tampere.
- Kilpatrick, J., Swafford, J. & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.
- OECD (2004). *Small and medium enterprises in Turkey: Issues and policies*. http://www.oecd.org/LongAbstract/0,2546,en_2649_33956792_31932174_119663_1_1_1,00.html (5.1.2007)
- PISA (Programme for International Student Assessment) (2003). *Learning for tomorrow's world: First results from PISA 2003*. <http://www.pisa.oecd.org> (5.1.2007)
- Turkish National Board of Education Council, (2005). *Elementary school mathematics curriculum*. Ankara: National Board of Education Council.

Some genetic considerations on the development of the theory of meaningful learning as a conceptual framework for teaching mathematics

Rauno Koskinen
Helsingin yliopisto

The purpose of my theoretical study is to investigate the concept of meaningful learning, and in more detail, the development of the theory of meaningful learning as a conceptual framework for teaching mathematics. Meaningful learning has been one of the main goals in mathematics education, and the theory of meaningful learning a psychological base for teaching mathematics for a very long time. But we have not reached the goal. One reason might be that the main concept, meaningful learning, has been understood in very different ways in different paradigms and traditions of educational research. The aim of my study is to find the different main dimensions of the concept and bring together the different parts and integrate the conceptual framework as a whole – for a better understanding of meaningful learning and the implications for teaching. The study method is a systematic analysis applying meta-analysis and hermeneutic approach. The main study materials include articles from *Journal for Research in Mathematics Education*, from year 1970 to 2005. The focus in this article is on the genetic history of the main concept of meaningful learning.

The problem of meaning in learning and teaching mathematics

Meaningful learning has been one of the main goals in mathematics education for a very long time. Still we have not reached the goal. Mathematics is a very abstract subject and therefore difficult to approach. "Where do we need this?", "What does it mean?", "How have we come to this?", are very often repeated questions in mathematics lessons. If students don't get answers to questions like these, the feeling "Why should we study this?" will stay in their minds and they lose their motivation of studying mathematics. I call this "A problem of meaning". In a broader sense, on an international stage, and not only in Finland, the problem is that children tend to enjoy mathematics in the primary grades, but the enjoyment and motivation tend to fall dramatically when children proceed into secondary school, and the number of students who want to study more

mathematics in and after school is declining all the way (e.g. Middleton & Spanias, 1999; Kupari & Törnroos, 2005).

The theory of meaningful learning has been a psychological base for teaching mathematics for a very long time. The general view (e.g. Kilpatrick & Weaver, 1977; Kieran, 1994) of meaningful learning goes back to the beginning of last century and to William A. Brownell (1895-1977), who was one of the pioneers in creating the theory of meaningful learning. But the concept of meaningful learning has been understood in very different ways in different paradigms and traditions of educational research. The purpose of my study is to reconstruct and expand the conceptual framework of meaningful learning and discuss the implications for the model of teaching-learning process in teaching mathematics. The study method is a systematic analysis (Scriven, 1988; Jussila et al., 1993) applying meta-analysis (Haig, 1991) and hermeneutic approach (Gadamer, 1979). The main study materials include articles selected from Journal for Research in Mathematics Education (JRME). By analyzing the texts, I try to find the different dimensions of the concept of meaningful learning, bring together the different parts and integrate the conceptual framework as a whole – for a better understanding of meaningful learning. The focus in this article is on the main concept of meaningful learning and its genetic history in the context of teaching mathematics. Referred articles are selected from the journal volumes, including the years 1970-2005, and in this article some of them are discussed.

But why take a look into the history? The genetic history of the concept of meaningful learning, give us the view how the conceptual framework has developed, what key concepts have been involved in the description of meaningful learning through the years, and we get a better understanding of the meaning they get today in the context of teaching mathematics. The view we get from the journal goes all the way back to the beginning of last century.

The cognitive view of meaningful learning: Learning with understanding approach in the light of JRME

Two great pioneers: William Brownell and David Ausubel

William A. Brownell (1895-1977) was one of the pioneers in creating the theory of meaningful learning at the beginning of the 1900 century. Brownell's writings, ideas and statements have had a great influence on the development of the theory of meaningful learning and on other researchers – as we can see, for example, in writings in JRME. In the beginning of his career, he was studying at the University of Chicago, and Brownell came under the influence of Charles H. Judd. In his lectures in educational psychology Judd stressed the importance of meaning and understanding in learning (Kilpatrick & Weaver, 1977). In 1930s -

1940s, Brownell was very productive, and one of his main interests was the case of meaningful arithmetic – understanding vs. drill, or meaningful vs. mechanical learning in arithmetic (e.g. Brownell, 1935; 1945; 1947; 1954; Brownell & Moser, 1949). Brownell pointed out that research ought to focus on the processes of learning, and the classroom situation offers unusual opportunities to investigate learning which involves the higher mental processes (Kilpatrick & Weaver, 1977).

According to Brownell (1935), although drill and practice should not be ignored, arithmetic should be made less a challenge to the pupils' memory and more a challenge to the intelligence. Brownell stressed the importance of meaning for each child, and both the mathematical and social (perhaps the humanistic) dimension of it: "To be intelligent in quantitative situations children must see sense in the arithmetic they learn. Hence, instruction must be meaningful and must be organized around the ideas and relations inherent in arithmetic as mathematics. But they must also have experiences in using the arithmetic they learn, in ways that are significant to them at the time of learning, and this requirement makes it necessary to build arithmetic into the structure of living itself." (Brownell, 1954; Kilpatrick & Weaver, 1977, p. 384).

One of the most fundamental theories in educational psychology that was influenced by Brownell, was Ausubel's theory of meaningful learning (Ausubel, 1963, 1968; Ausubel & Robinson, 1969). Essential to Ausubel's theory is the postulation of two dimensions of learning: reception – discovery learning, and meaningful – rote learning. Meaningful reception learning takes place when the teacher presents the generalization in its final form, and the learner relates it to his existing ideas in some fashionable way. Rote reception learning will occur if the teacher presents the generalization, and the learner merely memorized it. Meaningful discovery learning will occur if the learner formulates the generalization himself and subsequently relates it in a sensible way to his existing ideas. Rote discovery learning could occur if the learner, having arrived at the generalization himself (typically by trial and error), subsequently commits it to memory without relating it to other relevant ideas in his cognitive structure. (Ausubel & Robinson, 1969, p. 45)

There are obvious connections between Brownell's works and Ausubel's theory (see for example Ausubel, 1963, 1968). The situation is the same in the section of mathematics education, like we can see in JRME: Brownell's influence on mathematics education is manifested in many ways (see also Kilpatrick & Weaver, 1977; Kilpatrick, 1992; Kieran 1994) – "including an unforgettable impact on those who had the privilege of being his students" (Kilpatrick & Weaver, 1977). The central issue in meaningful mathematics learning and mathematics instruction is the point Brownell stressed in arithmetic – that: "children see sense in arithmetic they study" (Brownell & Moser, 1949, p. 147). This "seeing sense"-statement was to be one key note in meaningful mathematics learning and instruction, and the development of the theory.

General outlining of the development of meaningful learning

In the past it was quite usual that understanding was separated from learning and learning without understanding was possible. In behaviouristic and positivistic view of research in psychology and education it was denied to make any comments on human mind at all. In our days it is more believable that at least some level of understanding is involved in real learning.

According to Kieran (1994, pp. 591-593), up until the late 1970s, much of the research had focused on the effects of “this or that”, on the achievement of students, and on the correlation of both mathematical and nonmathematical measures with mathematics achievement. Speaking about learning was usually speaking about the conditions of learning. The situation in the beginning of 1970s was, that very often no definition for learning was provided (e.g. Heimer, 1969), and it was quite same with the definition of understanding (e.g. Rector & Henderson, 1970).

But the times are usually changing. In 1974, Wittrock introduced a model of learning with understanding, and pointed out that understanding is a generative process (Wittrock, 1974, will be discussed later in more detail). In 1976, Skemp introduced new dimensions to perspectives of understanding: Relational – Instrumental understanding (Skemp, 1976). Skemp stressed that it is important for students to “know both what to do and why” – not only “rules without reason” (see also Kieran, 1994). These works led to an increase of mathematics education researchers who were interested in building models of mathematical understanding (e.g. Herscovics & Bergeron, 1983; Pirie & Kieren, 1994). Furthermore, a movement via “situated cognitive theory” (e.g. Greeno, 1991) to socio-constructivist view (e.g. Cobb et al. 1992) of meaningful learning can be found in JRME.

But the development of the general view of the conceptual framework of meaningful learning was not “linear”, and the notions of what was important in learning and teaching mathematics were sometimes in opposition to each other. In 1983, the so called “Gagné paper” appeared. Gagné postulated his theses about teaching mathematics in this article, “Some issues in the psychology of mathematics instruction”.

Gagné (1983a, pp. 10-17) describes a model of pupils’ performance of mathematical task in three major phases: 1) translating phase (translation of a concrete situation, or statement, into mathematics), 2) computation phase, (numerals are written down, followed by rules for calculating to achieve a product) and 3) validating phase (the abstract product must be related to the concrete situation with which the problem started to be understood). He discusses the human learning and teaching mathematics and bases his arguments on the cognitive learning theory (or information processing theory). He stresses the importance of the phases 1 and 3, and in phase 2, the automatization of the intellectual skills is a prerequisite for the understanding of mathematics.

In his article, Gagné (1983a) stimulated a renewed discussion of the cognitive phenomena, “understanding and skills”, involved in learning and teaching mathematics. One of the critiques of this paper was written by Wachsmuch (1983). Wachsmuch pointed out that the learning-with-understanding approach of contemporary mathematics education had led to the view that the learner should be put in the position of doing mathematics by deriving it meaningfully from declarative knowledge. In a critique of Steffe and Blake (1983), they lamented that Gagné’s application of information-processing theory distorts what it means to learn mathematics. Gagné (1983) refers to information processing theory and stresses the automatization of the correct operations for the computation phase by “drilling” and “practising” (speed drill, games, computers). Steffe and Blake (1983) pointed out the question: what is the goal of the computation phase in learning and teaching mathematics? They gave an example with an equation system: $x + y = 15$ and $x - y = 3$ (Steffe & Blake, 1983, p. 212). They argue that Gagné would want the student to be able to solve the pair of equations automatically. With their own words Steffe and Blake depart radically from Gagné. They wanted the mathematics student initially to view solving the equations like a genuine mathematical problem. They saw that the “Gagné paper” leads to a mechanic and behaviouristic view of learning.

These sentences from this discussion have been taken out from their context and they do not give us the whole truth, but some ideas of the content of the discussion. (See also Kieran (1994), and in more detail, “the discussion” between Gagné (1983a) - Wachsmuch (1983) - Steffe and Blake (1983) - Gagné (reply, 1983b) in JRME.)

As we see in JRME, some of the research in mathematics education has questioned the strategy “teaching for skills (in mechanic way) part of the time and teaching for understanding part of the time”, and has shown that this strategy is not as effective as teaching for understanding all the way (e.g. Wearne & Hiebert, 1988; Pesek & Kirshner, 2000). Understanding and skills should not be considered as dichotomies, but as necessary aspects of mathematical thinking and learning. Operating with problems, like Steffe and Blake stated the understanding of the mathematics lying under the equations and executions, is prerequisite for finding and controlling over the solution. If some operations have been automatized before, there is always a need for control of the execution, and when you enter a new problem for which you do not have automatized operations yet, your only way to deal with it is by understanding the mathematics and the mathematical situation you are dealing with. If we turn back to Brownell, the central issue in mathematics instruction is that “students see sense in what they learn” – we must remember that, after all, students should not be treated as machines!

Learning and understanding as an active, constructive and social action

Lovell (1972) introduces a view of active constructive learning in his article: “Intellectual growth and understanding mathematics” with some assumptions concerning the conceptual framework of understanding mathematics and meaningful learning. Lovell bases his arguments on the cognitive-development theory of Piaget and reviews of research, and then he makes some implications for teaching mathematics in the classroom. Lovell also refers to educational psychologists as Ausubel and Galperin, and to their theories of meaningful learning.

Here I present a brief summary of the six assumptions Lovell pointed out, based on Piaget’s theory (Lovell, 1972, pp. 164-166):

1. The general ways of knowing have to be actively constructed by the child through interaction with the environment.
2. At the core of the central mechanism of intelligence are the basic operations of uniting, seriating, equalizing, putting into one-to-one correspondence, and so on. These are internalized (in the first 21 months of the life) with the help of language, but not deriving it from it. Knowledge comes not from the objects themselves, but from the actions performed on objects.
3. Thinking is an action that transforms one reality state into another, thereby leading to knowledge of the state. To understand a state one must understand the transformations from which the state results. In mathematics these implicit mental actions or covert transformations are important.
4. The concept of stage is a key concept (for Piaget). It indicates successive developmental periods of intelligence, such as sensorimotor, preoperational, concrete, and formal operational.
5. Language plays a role in the growth of thought (and we are in the beginning to understand the role of language – Lovell in 1972).
6. When a mathematical idea to be learned depends on a level of logical thought beyond that which the child possesses, the idea is either partially learned or learned with much difficulty.

Lovell (1972, pp. 175-178) outlines some implications for teaching mathematics:

1. A move from a formal classroom atmosphere with much talk by the teacher directed to the whole class, to the position where the pupils work in small groups or individually on tasks that have been provided.
2. The opportunity for the pupils to act on physical materials and to use games. It is the abstractions from actions performed on objects that aid further knowledge of mathematical ideas (not the objects themselves).

3. Social interaction using verbal language is an important influence in the development of concrete-operational thought. Cooperative aspects of exchange are important. Language helps the child organize experiences and thoughts into a coherent structure, and the child is also forced to elicit the strategies of thinking available to him or her.
4. Since mathematics is a structured and interlocked system of relations expressed in symbols and governed by firm rules, the initiative and the direction of the work must be the teacher's responsibility. This does not mean that pupils should never have a choice of activities, and it does not imply that teachers should ignore naturally occurring, but relevant situations.
5. Alongside the abstraction of the mathematical idea from the physical situation, there must be the instruction of the relevant symbolization and working examples, involving drill and practice and problems, on paper.

Lovell describes learning and understanding as an active, constructive and social action. Here we can see an early version of the socio-constructive approach, which twenty years later was going to be a very popular view of learning in mathematics education. But in the 1970s, the focus in most of the studies was, at first, on the individual process of learning with understanding.

Learning and understanding as an individual process

Wittrock (1974) introduced a model of learning with understanding in his article "A Generative model of mathematics learning". Wittrock mentions the theory of meaningful learning by Ausubel (Wittrock, 1974, p. 186), which has been the basis for further development of the concepts of meaningful learning and understanding.

The main hypothesis (Wittrock, 1974, p. 182) is that learning with understanding is a generative process, involving the construction of

- a) organizational structures for storing and retrieving information, and
- b) processes for relating new information to the stored information.

According to Wittrock (1974, p. 182) one should not construe learning, not even so called reception learning, as a passive reception of someone else's organizations. Learning with understanding can occur with discovery learning or with reception treatments. The important point is what these treatments cause the learner to do. Generation and understanding is very closely related. For example, a student may not understand sentences spoken to him/her by his teacher, but it is very likely that a student understands sentences that he/she generates himself.

Meaningful learning requires well organized cognitive structures and enough of relevant background knowledge (Wittrock, 1974, p. 191). Knowledge that is well organized and meaningful to the learner, makes it possible to discover and generate the new ideas from the content presented to him or her. According to

Wittrock (1974), the very basic nature of meaningful learning is always generative and a kind of discovery learning. Whether or not the new learning material and its organization have been made explicit, the new ideas must be discovered.

Wearne and Hiebert (1988) published an article “Cognitive approach to meaningful mathematics instruction: Testing a local theory using decimal numbers”. Here they introduce a model where four main cognitive processes are involved in meaningful mathematics learning (Wearne & Hiebert, 1988, pp. 372-373):

- 1) The connecting process is the construction of links between individual symbols with referents.
- 2) The developing process builds the procedures used to manipulate symbols.
- 3) The elaborating/routinizing process, where elaborating means extending syntactic procedures to other appropriate contexts, and routinizing syntactic procedures memorizing and practicing rules, until they become automatic.
- 4) The abstracting process continues the separation by using symbols and rules for a familiar system as referents in constructing more abstract systems.

In this model the connecting process is very important in meaningful mathematics learning. It is focused on building bridges and on crossing over them mentally, which gives meaning to written numbers, and symbols. The connecting and developing processes are not simple, usually they are very complex. These processes can be thought of as a semantic analysis: “Tasks are solved by reflecting the symbol expressions of the problem into referent world and selecting strategies based on the meanings associated with the expressions. The arguments of the reasoning processes are the external referents rather than the symbols (Greeno, 1983). The cognitive processes and solution strategies may not be efficient, but they are meaningful to their user.” (Wearne & Hiebert, 1988, p. 373)

According to Wearne and Hiebert (1988, p. 371), further advancements are being made by shifting the focus from static descriptions of cognitive processes to descriptions of cognitive change. Secondly, initial theories of cognitive change are likely to be local rather than global, and they need to be developed in the context of particular subject matters.

Here, we have seen dichotomies as “relational understanding – instrumental understanding”, “conceptual knowledge – procedural knowledge” and “understanding – skills”, but these can be considered as necessary aspects of mathematical understanding (Wearne & Hiebert, 1988; Kieran 1994, pp. 597-598). This kind of integration (of different dichotomies) was going to be the main direction, until we come up to the 1990s, as we can see in next chapter.

Learning and understanding as a social process

Cobb, Yackel and Wood (1992) describe meaningful learning as an integration of individual and social processes in their article “A Constructivist alternative to the representational view of mind in mathematics education”. They criticize the representational view of mind (RVM). In such an approach, the mathematics to be learned is often said to be a salient property of external, instructional representations. Learning is characterized as a process in which students gradually construct mental representations that accurately “mirror” the mathematical features of external representations. RVM fails to consider the social and cultural nature of mathematical activity and leads to recommendations that are at odds with the espoused goal of encouraging “learning with understanding”. Cobb, Yackel and Wood also introduce an alternative view, and some implications for mathematics education.

According to Cobb, Yackel and Wood (1992), learning with understanding is taking place in a social environment. Students’ individual constructive activity occurs in interaction between pupils and teacher, when students attempt to resolve problems that arise, as they participate in the mathematical practises of the classroom. The interactive teaching-learning process causes mathematical meanings to emerge in students’ minds. The teacher and students elaborate the “taken-as-shared” mathematical reality that constitutes the basis for their ongoing communication. Instructional materials – or so called instructional representations – are symbolic in their nature (Cobb et al., 1992, pp. 21-25). They are typically developed to symbolize the taken-as-shared mathematical interpretations of wider society. We can view them as possible means that students might use to symbolize their developing mathematical activity. Here these are called “pedagogical symbol systems”, instead of representations, to emphasize their symbolizing role in individual and collective mathematical activity.

Pirie and Kieren (e.g. 1994) have also integrated different philosophical and psychological aspects (like Cobb et al. (1992) before) and they introduce a synthesis, a complex and very detailed model of “growth in mathematical understanding”. The paradigm here is socio-constructivist, and the theory includes the individual (Piaget) and the social (Vygotsky) aspects of learning. According to Kieren (Kieran, 1994, Carolyn Kieran’s interview with Tom Kieren), meaningful learning always requires understanding, at least on some level. In Pirie’s and Kieren’s (1994) model, understanding is an ongoing activity with no end. This is something very different from a traditional view of understanding, with correct and incorrect answers. “Growth in mathematical understanding” includes “potential levels” as follows: primitive knowing (PK), image making (IM), image having (IH), property noticing (PN), formalising (F), observing (O), structuring (S), inventising (I). (In more detail see e.g. Pirie & Kieren, 1994)

On the one hand, much more attention should be paid to the process, by which each student try to make sense of their worlds. On the other hand, to be successful, students' meaningful learning process requires that students' alternative frameworks form the fundamental basis for teaching mathematics – when the goal of teaching in mathematics is understanding, conceptual change and meaningful learning. (Kieran, 1994)

Conclusions of the studies in JRME

It is very difficult to “cut off” the conceptual development, in both ends. Firstly, the development in this tradition, called “learning with understanding”, is still going on. Secondly, we can go back all the way to Dewey, or Pestalozzi. In this article I have started with William Brownell, because he and his works have been the explicit background for many researchers and their arguments, as we can see in JRME (see e.g. Kieran, 1994).

In the 1970s, Skemp and Wittrock introduced their ideas of meaningful learning and understanding in mathematics, and they had a great influence on research in mathematics education and the following development. For example, researchers who began with modelling of mathematical understanding, as Pirie and Kieren (1994), have expanded our cognitive view of understanding and learning mathematics.

If we think in a conceptual way, the concept of understanding is an inseparable part of the concept of learning, and the whole concept of meaningful learning has expanded from an individual stage to a social one – like we have more commonly seen to happen with learning in this so called socio-constructivist paradigm.

The development of this framework of meaningful learning, as it shows in the tradition of learning with understanding approach (LWU, see also Hiebert & Carpenter, 1992; Kilpatrick, 1992; Kieran, 1994), can be drawn together and stressed in the following phases:

- Learning with no understanding, or learning and understanding are separated.
- Learning with understanding, but (also) as an achievement (e.g. correct answers).
- Learning with understanding, (also) as an individual mental process.
- Learning with understanding, (also) as a social process.

Here I have introduced only the main phases. They are a very “abstract”, and not “historical”, because there are no years pointed out (the years are very difficult to point out exactly). But this may deserve here as a conclusion of the main phases that was involved in the conceptual development of the theory of meaningful learning, as it appears in the “learning with understanding”-tradition.

Meaningful learning requires well organized cognitive structures and enough of relevant background knowledge (see e.g. Ausubel, 1968). Knowledge that is

well organized and meaningful to the learner makes it possible to discover and generate the new ideas from the content presented to him or her. According to Wittrock (1974), the very basic nature of meaningful learning is generative and discovery learning and the new material must be presented in a way that makes it possible to reach the meanings meant to be learned. Therefore, a variety of different teaching methods and approaches is needed for each student.

Understanding can be defined as “an achievement” and as “a process”, and it depends on the specific situation which of the definitions is more usable. For example, Hiebert and Carpenter (1992) define understanding as “an achievement”: “A mathematical idea or procedure or fact is understood if it is part of an internal network. More specifically, the mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and the strength of the connections. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections.” (“defining understanding”, Hiebert & Carpenter, 1992, pp. 67-69). They use the concept “building understanding”, when they are speaking of the process which leads to understanding: “Using our definition of understanding, we can now describe, at least in a metaphoric way, the structuring process that produces understanding. Networks of mental representations are built gradually as new information is connected to existing networks or as new relationships are constructed between previously disconnected information. Understanding grows as the networks become larger and more organized.” (“building understanding”, Hiebert & Carpenter, 1992, pp. 69-74)

If we try to understand the concept of meaningful learning from the perspective of Brownell (as the situation is in JRME and the “learning with understanding” approach), we can see how important in teaching mathematics it is to take into account the “inner nature” of mathematics – the connections between different concepts and make it possible to understand how we operate in a “mathematical system”. According to Brownell: “Meaning is to be sought in the structure, the organization, the inner relationships of the subject itself.” (Brownell, 1945, p. 481)

In Brownell’s time, the situation in mathematics teaching was, that teachers by meaningful learning meant different things, like they also do in our time. Very often this concept was related to the usefulness of mathematics, outside the classroom, and at the same time teachers taught the subject without these inner meanings and connections. That is, perhaps, one main reason why Brownell went into this direction, to stress the meaning of this “inner nature” of mathematics. But he also saw this “outer nature” of meaningful learning. He wanted to construe clear definitions by naming these different dimensions of meaningful learning: “Failure to recognize the difference between meanings *of* and meanings *for* makes it difficult for those of us who are interested in the improvement of arithmetic instruction to agree on procedures. We use the same

words but in different senses. The third usage, namely, that children have meaningful arithmetic experiences when they use arithmetic in connection with real life needs, relates to meanings *for*. On this account some prefer to call such arithmetic experiences ‘significant’ rather than ‘meaningful’.” (Brownell, 1947, p. 256).

Discussion and critiques of the conceptual framework of meaningful learning

It is obvious that the explicit content of the conceptual framework of meaningful learning is very, very “cognitive”, and there are other than the cognitive and social dimensions, at least implicitly involved. If we want to understand the real and whole nature of meaningful learning – the conative (e.g. motivation) and affective (e.g. feelings) dimensions must come into account.

If we look back at those questions introduced in the beginning of this article, we must make a conclusion that “meaningful learning” includes both inner and outer dimensions, if we are trying to understand “the problem of meaning” in a broader sense – and if we try to answer questions like “What does it mean?” and “Where do we need this?”. Ernest (1989) introduces a model of teacher knowledge in mathematics instruction, which consists of knowledge of mathematics and knowledge of teaching mathematics. Knowledge of mathematics is consisting of 1) mathematical knowledge – knowledge of concepts, knowledge of procedures and strategies, etc., and 2) knowledge about mathematics – knowledge about mathematics as a whole, knowledge about the history of mathematics, etc. (Ernest, 1989, pp. 16-18). These dimensions of knowledge of mathematics are quite similar to those in Brownell’s theory. The conclusion of these discussions (with “Brownell” and “Ernest”) is, that meaningful mathematics learning involves both dimensions: we need 1) mathematical knowledge (“inner”) and 2) knowledge about mathematics (“outer”), to answer the questions presented above.

There is also another very difficult question in “the problem of meaning” – the question of dimensions: the “general” (as introduced here so far) and the “personal” dimension of meaningful learning. Usually, in front of new situations and new content, it requires actions like orientation, manipulating concrete materials and use of language, by which we formulate and construct the new ideas in our minds. This socio-constructivist view of learning has taken the leading place of paradigmatic conceptions that occur very broadly in recent research of mathematics teaching and learning (Cobb et. al. 1992; Kieran, 1994; Lerman et al., 2002). But we are usually interested in actual actions, and the main question here is, what makes the learner try to understand the new learning materials and discover the new ideas and meanings. In Ausubel’s theory of meaningful learning (Ausubel, 1963, 1968), it is the decision or will that makes meaningful learning to occur: “Since meaningfulness is largely a personal

phenomenon, it can be achieved only if the individual is willing to expend the active effort required to integrate new conceptual material into his unique frame of reference.” (Ausubel, 1968, p. 366) Therefore it is very important to take the concept of motivation into account when we are formulating a framework of meaningful learning.

In the “learning with understanding”-tradition (in the light of JRME), the concept of motivation has been quite clearly separated from the concept of meaningful learning (see also McLeod, 1994; Schiefele & Csikszentmihalyi, 1995). There is no place (or only marginal place) for motivation or affective considerations, when understanding of mathematics or meaningful learning is the focus of the article or one of the main concepts in the text. These questions of motivation and affect domain are usually treated as “marginal” questions. At the same time students are asking: “Where do *I* need this?” (a “personal version” of the question: “Where do we need this?”).

The framework of meaningful learning, and the psychological basis of mathematics instruction, as we can see in JRME, includes elements from learning theories of Ausubel, Piaget and Vygotsky (and Galperin) – and these are still alive today (for example Piaget in Simon et al. (2004)). Vygotsky seems to be very “popular”, and it is a little bit strange that western world do not follow the footsteps from Vygotsky to the works of Galperin (e.g. 1957), where he expands Vygotsky’s paradigmatic ideas to a theory of meaningful learning, and outlines the teaching-learning-process (this is the situation in JRME, except with some researchers, e.g. Lovell (1972)). In Finland this theory has been known quite well (see Koskinen, 2005a, 2005b).

A different paradigm in common use (in JRME) is the humanistic paradigm, that may give us some more answers to (perhaps “help to solve”) “the problem of meaning”. For example the psychologist Carl Rogers have outlined a theory of meaningful learning from a humanistic view. This view has been published in his very famous book – “Freedom to learn” (Rogers, 1969).

Rogers (1969, pp. 3-5) says that learning can be divided in two general types of learning – meaningful and meaningless learning. Much of the material presented to students in the classrooms has, for the student, the same meaningless quality as the list of nonsense syllabus, like: “baz”, “ent”, “nep”, “arl”, “lud” (the reader can try to remember these), and nearly every student finds that large portions of his curriculum are meaningless for him/her. Meaningful learning is something quite different. In contrast, according to Rogers (1969, pp. 3-5), meaningful learning is significant and experiential learning. It has a quality of personal involvement (the whole person in both his feeling and cognitive aspects being in the learning event), it is self-initiated (the sense of discovery, of reaching out, of grasping and comprehending, etc.), it is pervasive (it makes a difference in the behaviour, attitudes, etc.), and it is evaluated by the learner (the learner knows if something is meeting his/her needs, what he/she wants to know, etc.).

The key word in Rogers' theory is "freedom" and the starting point for learning and teaching is the learner's own "system of values" (in cognitive psychology the starting point is the "cognitive structure", or "conceptual framework"). The teacher's task is to give freedom (time, space, opportunities etc.) to the learner and help him/her in his/her learning-process, and organize and control the learning environment, providing all kinds of resources (books, concrete materials, videos, games, etc.) which will make his/her students' experiential learning relevant to their needs, and make such resources available. According to Rogers (1969), the teacher is the most important resource in the learning environment – he/she is there for emphatic listening and understanding, helping and guiding the students – to find and feel their own values, and to find and feel the real-world around them, and in general, open their minds for the learning-process.

When we are coming to the beginning of a new century, there has been a movement to make synthesis of different paradigms (e.g. Cobb et. al. 1992; Battista, 1999; see also Kirshner, 2002). Kirshner (2002) outlines a view of the paradigmatic development and needs in educational research and teaching mathematics – "A cross-disciplinary strategy for relating psychological theory to pedagogical practice". According to Kirshner (2002), it is important to see the value of integrating the different dimensions of our knowledge of learning and its implementations to teaching. If we want to define meaningful learning in a way that describes its nature not only as potential, we need a model of teaching and learning and a framework of meaningful learning that integrates the different paradigms of cognitive, socio-cultural and humanistic psychology.

References

- Ausubel, D. P. (1963). *The Psychology of Meaningful Verbal Learning*. New York: Grune & Stratton.
- Ausubel, D. P. (1968). *Educational psychology: A cognitive view*. New York: Holt, Rinehart & Winston.
- Ausubel, D. P. & Robinson, F. G. (1969). *School learning. An introduction to Educational Psychology*. New York: Holt, Rinehart & Winston.
- Battista, M. T. (1999). Fifth graders' enumeration of cubes in 3D arrays: conceptual progress in an inquiry-based classroom. *Journal for Research in Mathematics Education*, 30 (4), 417-448.
- Brownell, W. A. (1935). Psychological considerations in the learning and teaching of arithmetic. In W. D. Reeve (Ed.). *The teaching of arithmetic: Tenth yearbook of the National Council of Teachers of Mathematics* (pp. 1-31). New York: Columbia University, Teachers College, Bureau of Publications.
- Brownell, W. A. (1945). When is arithmetic meaningful? *Journal of Educational Research*, 38 (7), 481-498.
- Brownell, W. A. (1947). The place of meaning in the teaching of arithmetic. *The Elementary School Journal*, 47 (5), 256-265.

- Brownell, W. A. (1954). The revolution in arithmetic. *Arithmetic Teacher*, 1, 1-5.
- Brownell, W. A. & Moser H. E. (1949). *Meaningful vs. mechanical learning: A study in grade III subtraction?* Research studies in education, 8. Durham, North Carolina: Duke University Press.
- Cobb, P., Yackel, E. & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education*, 23 (1), 2-33.
- Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: a model. *Journal of Education for Teaching*, 15 (1), 13-32.
- Gadamer, H.-G. (1979). *Truth and method*. London: Sheed and Ward.
- Gagné, R. M. (1983a). Some issues in the psychology of mathematics instruction. *Journal for Research in Mathematics Education*, 14 (1), 7-18.
- Gagné, R. M. (1983b). A reply to critiques of some issues in the psychology of mathematics instruction. *Journal for Research in Mathematics Education*, 14 (3), 214-216.
- Galperin, P. Ya. (1957). An experimental study in the formation of mental actions. In B. Simon (Ed.). *Psychology in the Soviet Union* (pp. 213-225). London: Routledge and Kegan Paul.
- Greeno, J. G. (1983). Conceptual entities. In D. Gentner & A. L. Stevens (Eds.). *Mental models* (pp. 227-252). Hillsdale, NJ: Erlbaum.
- Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education*, 22 (3), 170-218.
- Haig, B. D. (1991). Meta-analysis, methodology and research integration. In G. Lakomski (Ed.). *Beyond paradigms: Coherentism and holism in educational research*. Special Issue. *International Journal of Educational Research*, 15 (6), 569-586.
- Heimer, R. T. (1969). Conditions of learning in mathematics: Sequence theory development. *Review of Educational Research*, 39, 493-508.
- Herscovics, N. & Bergeron, J. C. (1983). Models of understanding. *Zentralblatt für didaktik der mathematik*, 2, 75-83.
- Hiebert, J. & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.). *Handbook of research on mathematics teaching and learning. A project of the National Council of Teachers of Mathematics* (pp. 65-97). New York: MacMillan Publishing Company.
- Jussila, J., Montonen, K. & Nurmi, K. E. (1993). Systemaattinen analyysi kasvatustieteiden tutkimusmenetelmänä. In T. Gröhn & J. Jussila (Eds.). *Laadullisia lähestymistapoja koulutuksen tutkimuksessa* (pp. 157-208). Helsinki: Yliopistopaino.
- Kieran, C. (1994). Doing and seeing things differently: A 25-year retrospective of mathematics education research on learning. *Journal for Research in Mathematics Education*, 25 (6), 583-607.
- Kilpatrick, J. (1992). A history of research in mathematics education. In D. A. Grouws (Ed.). *Handbook of research on mathematics teaching and learning. A project of the*

- National Council of Teachers of Mathematics* (pp. 3-38). New York: MacMillan Publishing Company.
- Kilpatrick, J. & Weaver, J. F. (1977). The Place of William A. Brownell in Mathematics Education. *Journal for Research in Mathematics Education*, 8 (5), 382-384.
- Kirshner, D. (2002). Untangling teachers' diverse aspirations for student learning: A crossdisciplinary strategy for relating psychological theory to pedagogical practice. *Journal for Research in Mathematics Education*, 33 (1), 46-58.
- Koskinen, R. (2005a). Orientaatiokäsité Pjotr Galperinin oppimisen teoriassa ja sen merkitys matematiikan opetuksessa. In L. Jalonen, T. Keranto & K. Kaila (Eds.). *Matemaattisten aineiden opettajan taitotieto – haaste vai mahdollisuus? Matematiikan ja luonnontieteiden opetuksen tutkimuspäivät Oulussa 25.-26.11.2004* (pp. 111-122). Acta Universitatis Ouluensis E, Scientiae Rerum Socialium 80. University of Oulu.
- Koskinen, R. (2005b). Mielekäs oppiminen matematiikan opetuksen käsitteellisenä lähtökohtana: Pjotr Galperinin merkitys. In A. Virta, K. Merenluoto & P. Pöyhönen (Eds.). *Ainedidaktiikan ja oppimistutkimuksen haasteet opettajankoulutukselle. Ainedidaktinen symposium 11.2.2005* (pp. 214-222). Turun opettajankoulutuslaitos. Turun yliopiston kasvatustieteiden tiedekunnan julkaisuja B:75. University of Turku.
- Kupari, P. & Törnroos, J. (2005). Matematiikan opiskelua tukevat asenteet ja oppimisstrategiat. In P. Kupari & J. Välijärvi (Eds.). *Osaaminen kestäväällä pohjalla. PISA 2003 Suomessa* (pp. 151-172). Jyväskylä: Koulutuksen tutkimuslaitos.
- Lerman, S., Xu, G. & Tsatsaroni, A. (2002). Developing theories of mathematics education research: The ESM story. *Educational Studies in Mathematics*, 51 (1-2), 23-40.
- Lovell, K. R. (1972). Intellectual growth and understanding mathematics. *Journal for Research in Mathematics Education*, 3 (3), 164-182.
- McLeod, D. B. (1994). Research on affect and mathematics learning in the JRME: 1970 to the presents. *Journal for Research in Mathematics Education*, 25 (6), 637-647.
- Middleton, J. A. & Spanias P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education*, 30 (1), 65-88.
- Pesek, D. D. & Kirshner, D. (2000). Interference of instrumental instruction in subsequent relational learning. *Journal for Research in Mathematics Education*, 31 (5), 524-540.
- Pirie, S & Kieren, T. (1994). Growth in mathematical understanding: how we can characterise it and how can we represent it? *Educational Studies in Mathematics*, 26 (2-3), 165-190.
- Rector, R. E. & Henderson, K. B. (1970). The relative effectiveness of four strategies for teaching mathematical concepts. *Journal for Research in Mathematics Education*, 1 (2), 69-75.
- Rogers, C. R. (1969). *Freedom to learn. A View of What Education Might Become*. Columbus, Ohio: Merrill.

- Schiefele, U. & Csikszentmihalyi, M. (1995). Motivation and ability as factors in mathematics experience and achievement. *Journal for Research in Mathematics Education*, 26 (2), 163-181.
- Scriven, M. (1988). Philosophical inquiry methods in education. In R. M. Jaeger (Ed.). *Complementary methods for research in education* (pp. 131-149). Washington, D. C.: American Educational Research Association.
- Simon, M. A., Tzur, R., Heinz, K. & Kinzel, M. (2004). Explicating a Mechanism for conceptual learning: Elaborating the construct of reflective abstraction. *Journal for Research in Mathematics Education*, 35 (5), 305-329.
- Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.
- Steffe, L. P. & Blake, R. N. (1983). Seeking meaning in mathematics instruction: A response to Gagné. *Journal for Research in Mathematics Education*, 14 (3), 210-213.
- Wachsmuch, I. (1983). Skill automaticity in mathematics instruction: A response to Gagné. *Journal for Research in Mathematics Education*, 14 (3), 204-209.
- Wearne, D. & Hiebert, J. (1988). A cognitive approach to meaningful mathematics instruction: Testing a local theory using decimal numbers. *Journal for Research in Mathematics Education*, 19 (5), 371-384.
- Wittrock, M. C. (1974). A generative model of mathematics learning. *Journal for Research in Mathematics Education*, 5 (4), 181-196.

Designing a Learning Environment for the Mathematical Problem Solving

Henry Leppäaho
University of Jyväskylä

In this study the teaching of mathematical problem solving was integrated in thirty lessons at grade 6 in mathematics, mother tongue, natural sciences, art and craft. The experimental group ($n = 17$) and the control group ($n = 35$) took part in a pre-test, post-test and delayed test, which consisted of problem tasks. In the teaching for the experimental group, in addition to integration, the role of the problem solving map was emphasized. The problem solving map supports pupils in looking for the route toward the solution and checking the stages they have used in their attempts to solve the problem. In the pre-test and in the delayed test there were no statistical differences between the groups, except in one task in the delayed test. But in the post-test there was a clear statistical difference in the whole test: according to the results, the experimental group was 25% better than the control group. Problems in the tests were such that pupils could solve them using the information given in their mathematics book.

Introduction

Problem solving is a popular term and it has been used in different kinds of contexts. According to the Lester and Kehle (2003) *mathematical problem solving* is a thinking process in which a solver tries to make sense of a problem situation using mathematical knowledge she/he has and attempts to obtain new information about that situation till she/he can “resolve the tension or ambiguity”.

Problem solving strategy, a previously learnt way to solve problems, helps learners to solve new tasks (see e.g. Schoenfeld 1985, 109–110). Problem solving strategies can be learnt and practised using examples. But when pupils meet a new problem, they have to find out themselves what kind of strategy can be applied in solving a certain problem. It is difficult to teach this skill, but pupils can be guided to choose a possible way to start and this, in turn, helps them to control their uncertainty.

Fairly often solvers cannot solve the problem in spite of their persistent attempts. They feel uncertainty and failure. According to De Hoyos, Gray & Simpson (2004), students start the problem solving activity with some degree of uncertainty that may vary from high to low. This degree of uncertainty may affect students' decisions at early stages of the problem solving process. Op't

Eynde, De Corte & Verschaffel (2001) found out that pupils often had negative emotions like frustration and anger at the beginning of the solving process. Absence of an obvious procedure frustrates those who really want to solve the problem.

In the early studies of mathematical problem solving, attempts were made to show the usefulness of teaching general problem solving strategies. But, for example Schoenfeld (1992), has concluded that prescriptive use of heuristics is not particularly helpful for improving problem solving performance or its transfer to a new situation. Pupils need, however, examples of strategies in order to learn to apply procedures to new problems. Therefore, practising of problem solving strategies has the potential to serve as a powerful descriptor of problem solving behaviour (Schoenfeld, 1992). In this way pupils' uncertainty can be reduced and their attitude towards mathematical problem solving may improve.

The importance of communications has been stressed in learning mathematics (Elliot & Kenney, 1996). Teachers agree widely on the importance of oral communication in learning mathematics. Speaking and listening are the primary means by which pupils learn to participate in mathematical communities, explaining, questioning, justifying, clarifying their ideas, listening to the ideas of others, and working with others to construct mathematical arguments. According to Morgan (2001, 233–234), writing has some useful characteristics that are not shared with spoken language: 1) Writing produces a lasting record. During the writing process, writers can look back at what they have already written, reflect on whether it really transmits the intention, revise and redraft it. 2) Writing does not take place face-to-face with its audience. The writer generally has more time to think about what they are writing and hence to clarify and refine their thinking. It also means that what is written has to be more complete and precise than spoken communications because the reader cannot ask for clarification or more information. Completeness and precision are particularly important to mathematical communication and to the development of mathematical thinking. 3) Writing and mathematics are similar activities. The processes of writing and mathematical problem solving are similar, as both of them involve recursive development of clarity about the nature of the problem and its solution.

Pugalee (2004) has compared writing and the verbal (talk aloud) description of the mathematical problem solving process. He noticed that problem solving and writing have a better connection to the right answers than problem solving and verbal expression. Students who wrote descriptions of their thinking were significantly more successful in problem solving tasks than students who verbalized their thinking. Recent interest in writing as a communicative tool indicates that this form of communication has the potential to promote mathematical understanding (Sierpinska 1998; Morgan 1998).

The aim of this study is the find out:

- 1) How to integrate the teaching of the mathematical problem solving to the other school subjects in practice?

- 2) How the Problem Solving Map -method works as pupil's tool in mathematical problem solving?

The problem solving map (PSM) method

It is widely known that hypothesis and testing form the cycle of a problem solving model. Already Pólya (1948) has given systematic instructions on how to solve a problem. First of all, pupils have to perceive and understand the problem in order to extract from it the relevant information. It is necessary to keep the information in the working memory so that they can try different approaches in their mind in order to make a plan before they can move on to the next step, the actual solving process, i.e. carrying out the plan. I think that pupils need systematic guidance as well as knowledge about different problem solving strategies in order to be able to manage in problem solving. Therefore, in this study pupils were first introduced to the use of a problem solving map (PSM) as a helping tool.

The main idea of the PSM is that pupils will learn to collect notes that will help to solve the problem. Thus, the PSM acts as a map to support pupils when they look for the route toward the solution and they can always come back and check the stages they have passed through in their attempt to solve the problem (Leppäaho, 2004; 2005). The PSM emphasizes metacognitive thinking. The application of metacognitive techniques has two important mathematical purposes: 1) It allows pupils to keep track of what they have done and are planning to do next, and 2) It allows pupils to make connections between their problem solving work and their knowledge of subject matter and mathematical procedures. (Finkel, 1996). Figure 1 shows an example of problem solving map designed by a pupil in the experimental group.

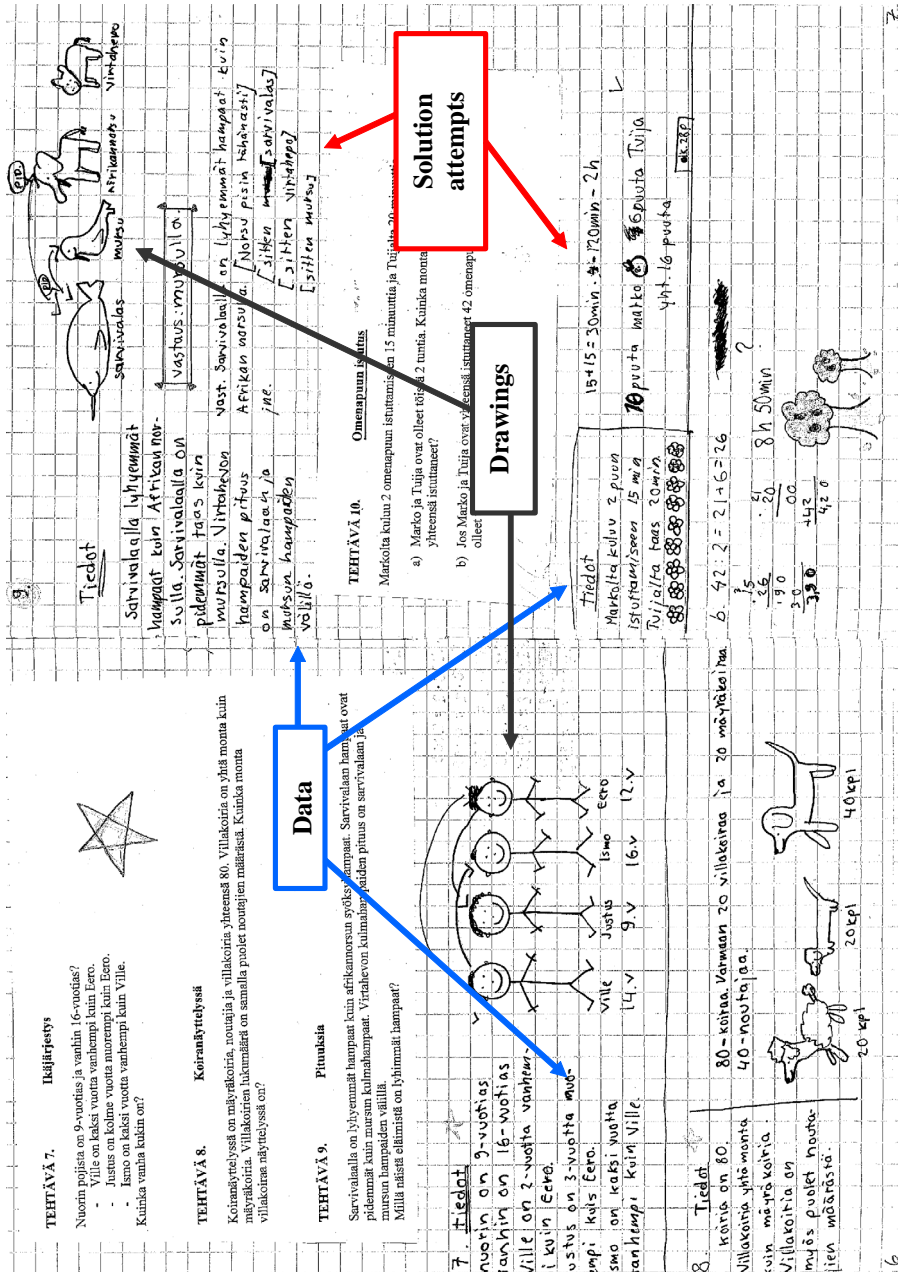


Figure 1. An example of a problem solving map

Method

Design research (Edelson, 2002) is used as a guidance method in this study. One purpose of the study is to develop teaching methods and artefacts (such as teaching materials). This study was carried out in the 6th grade of a small Finnish country school using a quasi-experimental design. One study group of

17 pupils (9 girls, 8 boys), the experimental group, was taught problem solving over six weeks in 30 lessons integrated into their regular school days, mainly in mathematics but also in mother tongue (Finnish), science, art and craft.

The researcher taught the experimental group during these 30 lessons, whereas during the other lessons their own teacher taught the pupils. The control group consisted of two study groups with 35 pupils (16 girls, 19 boys) in total. The control group studied mathematics and other school subjects in their normal way. The problem solving map (PSM) method played a central role in the course. In teaching problem solving Schroeder's and Lester's (1989) ideas on the three components – about, for and via problem solving – were used. During the first lessons the idea of the PSM and some other problem solving strategies was explained to the experimental group for the purpose of teaching about problem solving. With the meaning of teaching for problem solving, the experimental group was taught, for example, how to calculate volume and how to process problems by using calculation rules. The need to construct the PSM for any problem whenever possible was emphasized. During the course altogether about 30 problems were solved in this manner, which was used with the intention of teaching via problem solving.

In the beginning of the course the pupils were given instructions how to construct the PSM: 1) Read the task, 2) Pick out information about the task and write it down, 3) Choose a solving strategy, 4) Write down your thinking and solution, 5) Always make a drawing or diagram of the task if it is possible, 6) Evaluate and check your solution, 7) If you find a lot of errors, make a new solution after the wrong solution and 8) Don't delete the wrong solution; it is part of the solving process, too!

In the course, pupils were given open-ended problems that were integrated in different school subjects. Integration was carried out in teaching, for example in the following way: In the art lessons they designed their own boat. One requirement for the boat was that it should have a capacity of one litre. In the science lessons they made experiments on floating and sinking related to Archimedes' Principle (figure 2).



Figure 2. Testing the boat in the science lesson

Finally, in craft lessons they built the boat according to their drawings. In the mathematics lesson pupils calculated the volume needed for a boat to carry a cargo of one litre. The concrete and self-made product was an effective way to characterize the concept of volume. At the same time the pupils noticed that it is very important to write down the plan and make careful drawings. (A more detailed report on the integration was given in Leppäaho, 2007).

Pupils' problem solving performance was measured in a pre-test and a post-test, both of which consisted of 14 tasks and took 90 minutes. They contained three numerical, five geometric, four procedural, one open and one matchstick problem. In selecting the tasks the following criteria were set: 1) a diverse group of different problem types should be presented, 2) the problems should fulfil the definition of the problem i.e. they should not be routine tasks, and 3) the problems could be solved according to the 6th grade curriculum. The corresponding tasks were not dealt with during the lessons. The tasks in the post-test were designed on the basis of the problems in the pre-test in order to find out any changes. In some cases the numerical values were changed, in other cases the setting was changed but the structure and the type of the task were kept the same. Pupils were also requested to explain their solutions using words, equations and drawings. In the delayed test pupils had 45 minutes to solve six problems: two numerical, one geometric, two procedural and one open problem. The structures of the best tasks according the pre- and post-test results were chosen. The problems in the delayed test were slightly more demanding so the pupils also faced real problems in the delayed test. In the following I present an example of the variety of tasks in the pre-test, post-test and delayed test. The instructions for the tasks were the same in all tests:

The formation of every number sequence can be described by some rule. Fill in possible missing numbers in the boxes and justify your choice by giving your rule for each number sequence.

Pre-test: A1c)	$1 \rightarrow 4 \rightarrow 9 \rightarrow 16 \rightarrow \underline{\quad} \rightarrow \underline{\quad}$
Post-test: C1c)	$1 \rightarrow 5 \rightarrow 11 \rightarrow 19 \rightarrow \underline{\quad} \rightarrow \underline{\quad}$
Delayed test: E1c)	$1 \rightarrow 2 \rightarrow 2 \rightarrow 4 \rightarrow 8 \rightarrow \underline{\quad} \rightarrow \underline{\quad}$

The reliability coefficients (Cronbach's alpha) were as follows: in the pre-test 0.884, in the post-test 0.885 and in the delayed test 0.785. The tests can therefore be considered reliable.

Results

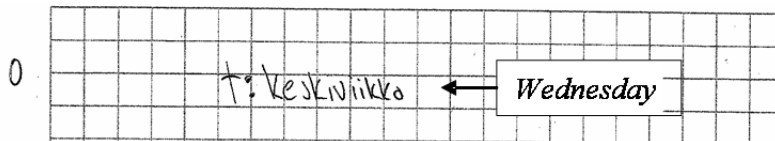
According to the teaching intervention, it was possible to use in addition to the mathematics lessons also Finnish, science, art and craft lessons for the teaching of the mathematical problem solving.

In the problem solving tests the scoring of the problem solving tasks was divided in two parts: 1) the solving part including the solution (the answer) to the task and 2) the reasoning process part including clarifying drawings, tables and written reasoning. The reason for this was to ensure that the pupils' ability to solve problems was tested and not the training of the experimental group.

Figure 3 shows the answers of a pupil in the experimental group to the corresponding tasks. We can notice the changes in the answers in the pre- and post-test and in the delayed test.

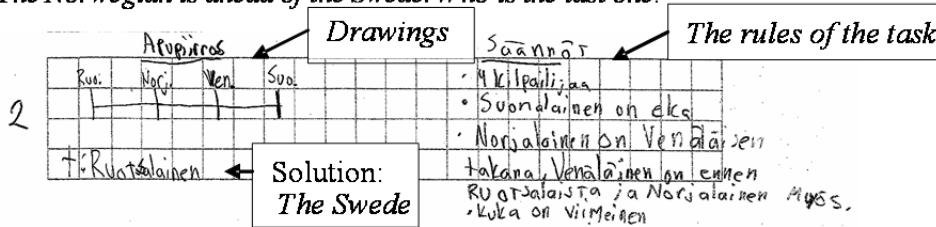
An example of a pre -test problem B7:

Ville bought a cat on 13th of March. It was Thursday. What day of the week was the 1st of March?



The corresponding post -test problem D6:

In the skiing competition four skiers are approaching the finishing line. The Finnish skier is leading. The Norwegian is behind the Russian. The Russian is in front of the Swede. The Norwegian is ahead of the Swede. Who is the last one?



The corresponding delayed test problem E3:

In the F1 race the fastest cars are approaching the finish: Schumacher is behind Webber. Webber is ahead of Alonso. Räikkönen overtakes Webber and Alonso overtakes Coulthard. Schumacher is ahead of Alonso. What is the order of the competitors?

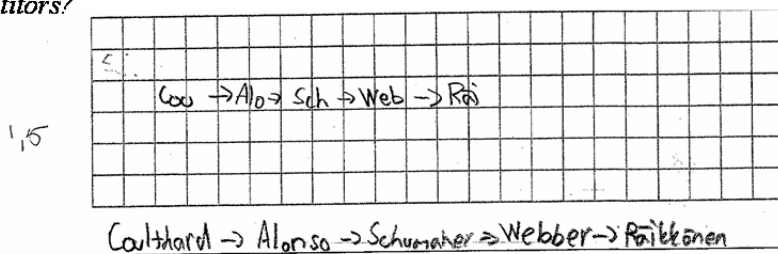


Figure 3. The answers of a pupil in the experimental group on the pre- and post-test and on the delayed test

In pre-test task B7 the boy in the experimental group has only written the wrong answer: Wednesday. There is no justification for the solution, so he got zero points on the solving part and the reasoning process part of the task.

In post-test task D6 we can see that the same boy has used the PSM method successfully. The pupil has written the essential information about the task on the right-hand side of the paper, and he has designed a helpful drawing, which helps him to discover the correct solution. So, he gets full marks on this task:

one point on the solving part and one point on the reasoning process part of the task.

In delayed test task E3 the boy has sketched out the right order of the drivers using the abbreviated form of the names (figure 3). But it is not clear who is first and who is last. So, this inaccuracy lowered his score by 0.5 point on the reasoning process part.

To summarize the results on the solving of the presented tasks, it seems that this boy has learned to apply the PSM method in the exam situation in the post-test. He also remembered something of the PSM method in the delayed test 18 months later.

The improvement of the experimental group compared to the control group in the total scores between the pre- and post-test (table 1) was significant (analysis of variance $p = 0.000$; $F = 26.63$; $df = 1$). An important finding was also that all the pupils in the experimental group improved their scores in the post-test within a range of 2.25 to 21.25 points.

Table 1. The total scores in the pre- and post-test of the groups

Total scores	Pre-test	Post-test
Experimental group (N = 17)	32.26	40.84
Control group (N = 35)	33.01	32.60
Difference between groups	-0.75	+8.24***
<i>Exp – Cont</i>	(-2.3%)	(25.3%)

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

The results of the reasoning process and the solving part are shown in figure 4.

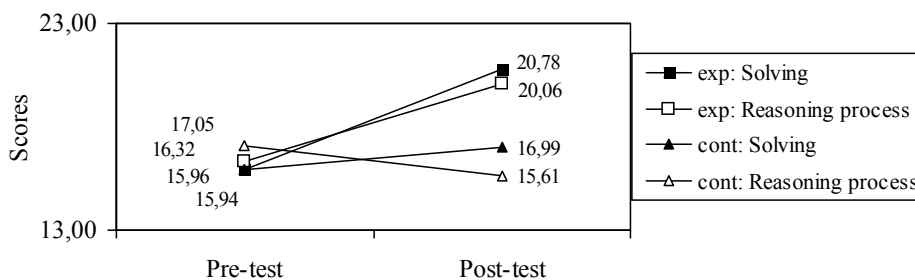


Figure 4. The scores of the solving part and the reasoning process in the pre- and post-test of the experimental (exp) and control (cont) groups

The improvement of the experimental group in the solving part in the post-test was significant (analysis of variance; $p = 0.000$; $F = 13.71$; $df = 1$), as was the improvement in the reasoning process part ($p = 0.000$; $F = 31.62$; $df = 1$). The results of the solving part improved in both groups, but only the experimental

group also improved the reasoning process. It seems that improvement in the reasoning process has a positive influence on the results of the solving part.

The purpose of the delayed test was to find out how stable the differences between the groups are. The differences were not statistically significant, but in percentage terms the experimental group still got better results than the control group on both parts of the test (table 2).

Table 2. The scores of the experimental and control groups on the delayed test

Delayed test	Experimental group (N = 17)	Control group (N = 35)	Difference between groups
			Exp – Cont
Solving part	7,76	6,97	0,79 (11,3 %)
Reasoning process	5,57	4,66	0,91 (19,5 %)
Total scores	13,34	11,63	1,71 (14,7 %)

All the pupils were studying in different and mixed study groups at grade 7 in the same school, so that they had studied a whole school year in similar teaching and learning environments. Therefore it was expected that the results of the experimental and control group would approach each other. The difference in the scores was not significant in the whole of the delayed test. However, in problem E1, which consisted of six subtasks (subtask E1c was presented above), the experimental group was statistically better in the solving part (t-test, $p = 0,027$), in the reasoning process (t-test, $p = 0,002$) and in the total scores (t-test, $p = 0,004$).

In figure 5 the performance of the experimental group and the control group are compared when the results of the control group are standardized to 100 % in all the three tests.

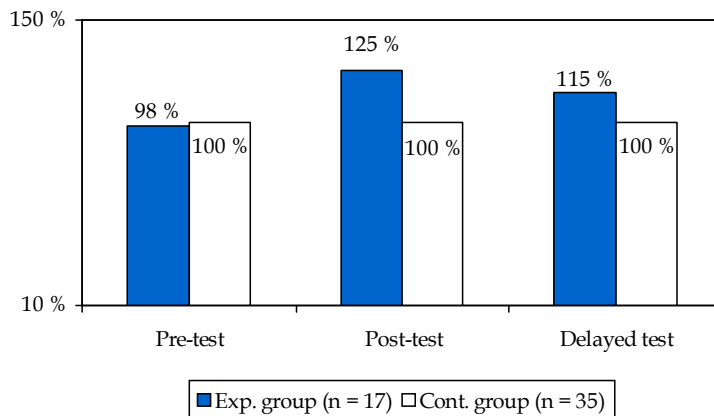


Figure 5. Percentage differences between the experimental group and the control group in the pre-, post- and delayed tests

In the post-test there is a clear statistical difference in the overall results between the groups in the post-test. The experimental group was 25,3 % better than the

control group. There is no statistical difference in the overall results in the pre-test and in the delayed test. Only in percentage terms the experimental group was still 14,7 % better in the delayed test, even if it was 2,3 % weaker than control group in the pre-test.

Conclusions

This study shows that it was possible to create a workable learning environment for the mathematical problem solving in practice by integrating the teaching to the mother tongue, natural sciences, art, craft and mathematics lessons.

Pugalee (2005, 39-40) gives fifty activities for writing in mathematics, for example: 1) making a list of characteristics or steps, 2) creating a drawing or an illustration for a problem and describing how they are related. The aim of the problem solving map method is to help pupils to illustrate and process mathematical problems in writing and drawing. As the results show, the pupils' performance has improved. The PSM method supports the pupils' memory so that it is easier for them to go back to the basic information or to look at the drawing to see the structure of the problem. Also Kotovsky, Hayes & Simon (1985) have noticed that external memory tools help the solution process.

The pre-test showed that the pupils had difficulties to express their thinking in mathematical language. The results of the control group in the post-test confirm this interpretation. The scores of the experimental group on both the solving part and the reasoning process clearly improved. Furthermore, the written explanations showed development in clarity of thinking. The use of this kind of systematic method like PSM in starting to work with a problem as well as the "permission" to make all kinds of attempts to find the solution probably reduces pupils' feelings of uncertainty.

In the delayed test, 18 months after the post-test, the experimental group was statistically still better in one of the six tasks than the control group. In percentage terms the difference was clearly in favour of the experimental group, although the control group was a little better in the pre-test.

Of course, we should treat the conclusions cautiously because the groups are quite small and many factors might have influenced the results, as is always the case in studies of teaching and learning. But, in this case, the results suggest that using integration of school subjects together with the PSM method could be a useful way to teach and learn mathematical problem solving.

Hohn & Frey (2002) have obtained similar results when they used a corresponding heuristic strategy, similar to the PSM method, in teaching 3rd to 5th grade pupils in problem solving. Lorenzo (2005) has noticed that the problem solving heuristic she used in her study helped chemistry students to understand the stages of the problem solving process. Van Garderen & Montague (2003) have found out that sixth graders' visual-spatial representations related significantly to the way they solved problems.

Awareness that wrong answers are accepted as a step towards the right answer may help pupils to try different kinds of attempts to solve the problem. Too often it is forgotten that wrong answers are also a part of studying problem solving and mathematics. Skilful mathematicians also make numerous attempts when they try to solve difficult problems (Stylianou, 2002).

Many pupils believe that mathematics is only doing calculations (Grigutsch, 1998). But mathematics includes a lot of problem solving, just as everyday life sometimes does. Therefore, it would be worthwhile to teach and to learn a method to solve problems.

References

- Edelson, D. (2002). Design research: What we learn when we engage in design. *The Journal of the Learning Sciences*, 11(1), 105-121.
- Elliot, P. & Kenney, M. (Eds.). (1996). *Communication in mathematics, K-12 and beyond*. Reston, VA: NCTM.
- Finkel, E. (1996). Making sense of genetics: Students' knowledge use during problem solving in a high school genetics class. *Journal of Research in Science Teaching*, 33(4), 345-368.
- van Garderen, D. & Montague, M. (2003). Visual-spatial representation, mathematical problem solving, and students of varying abilities. *Learning disabilities Research & Practice*, 18(4), 246-254.
- Grigutsch, S. (1998). On pupils' views of mathematics and self-concepts: developments, structures and factors of influence. In E. Pehkonen & G. Törner (Eds.). *The State-of-Art in Mathematics-Related Belief Research. Results of the MAVI activities*, 169-197. Helsinki: University of Helsinki. Department of Teacher Education.
- Hohn, R & Frey, B. (2002). Heuristic training and performance in elementary mathematical problem solving. *The Journal of Educational Research*, 6 (95) 374-380.
- de Hoyos, M. Gray, E & Simpson, A. (2004). Uncertainty during the early stages of problem solving. In M. J. Høines & A. B. Fuglestad (Eds.). *Proceedings of the 28th conference of the International Group for the Psychology of Mathematics Education*, 2, 255-262. Bergen: Bergen University College.
- Kotovsky, K., Hayes, J & Simon, H. (1985). Why are some problems hard? Evidence from tower of Hanoi. *Cognitive Psychology*, 17, 248-294.
- Leppäaho, H. (2004). Development of problem solving at comprehensive schools. In H. Rehlich & B. Zimmermann (Eds.). *ProMath Jena 2003. Problem solving in mathematics education*, 71-86. Hildesheim: Verlag Franzbecker.
- Leppäaho, H. (2005). Integration of mathematical problem solving at comprehensive school. In E. Pehkonen (Ed.). *Problem Solving in Mathematics Education, Proceedings of the ProMath meeting June 30-July 2, 2004 in Lahti*, 105-114. Helsinki: University of Helsinki.
- Leppäaho, H. (2007). *Matemaattisen ongelmanratkaisutaidon opettaminen peruskoulussa. Ongelmanratkaisukurssin kehittäminen ja arviointi*. [Teaching

- mathematical problem solving skill in the Finnish comprehensive school. Designing and assessment of a problem solving course.]. Jyväskylä: University of Jyväskylä.
- Lester, F., Jr., & Kehle, P. (2003). From problem solving to modeling: The evolution of thinking about research on complex mathematical activity. In R. Lesh & H. M. Doerr (Eds.). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*, 501-517. Mahwah, NJ: Lawrence Erlbaum.
- Lorenzo, M. (2005). The development, implementation, and evaluation of a problem solving heuristic. *International Journal of Science and Mathematics Education*, 3, 33-58.
- Morgan, C. (1998). *Writing mathematically: The discourse of investigation*. London: Falmer Press.
- Morgan, C. (2001). The place of pupil writing in learning, teaching and assessing mathematics. In P. Gates (Ed.). *Issues in mathematics teaching*, 232-244. London: Routledge Falmer.
- Op't Eynde, P., De Corte, E. & Verschaffel, L. (2001). Problem solving in the mathematics classroom: A socioconstructivist account of the role of students' emotions. In M. van den Heuvel-Panhuizen (Eds.). *Proceedings of the 25th conference of the International Group for the Psychology of Mathematics Education*, 4, 25-32. Utrecht: Utrecht University.
- Pugalee, D. (2004). A Comparison of verbal and written descriptions of students' problem solving processes. *Educational Studies in Mathematics*, 55(1), 27-47.
- Pugalee, D. (2005). *Writing to develop mathematical understanding*. Norwood: Christopher-Gordon Publisher, Inc.
- Pólya, G. (1948). *How to solve it? A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.
- Schoenfeld, A. (1985). *Mathematical problem solving*. London: Academic Press.
- Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.). *Handbook for Research on Mathematics Teaching and Learning*, 334-370. New York: Mac Millan.
- Schroeder, T. & Lester, F. (1989). Developing understanding in mathematics via problem solving. In P. Trafton & A. Shulte (Eds.). *New directions for elementary school mathematics, NCTM Yearbook*, 31-42. Reston, VA: NCTM.
- Sierpiska, A. (1998). Three epistemologies, three views of classroom communication: Constructivism, sociocultural approaches, interactionism. In H. Steinbring, M.G. Bartolini Bussi & A. Sierpiska (Eds.). *Language and Communication in the Mathematics Classroom, NCTM Yearbook*, 30-62. Reston, VA: NCTM.
- Stylianou, D. A. (2002). On the interaction of visualization and analysis: The negotiation of a visual representation in expert problem solving. *Journal of Mathematical Behavior*, 21, 303-317.

In the process of becoming a mathematics teacher: A case study of the first year experiences

Päivi Portaankorva-Koivisto
University of Tampere

The purpose of this paper is to present some first findings concerning the process of becoming a mathematics teacher. In my study I followed a group of six mathematics student teachers during their first year in teacher education and met the challenges that arise from these experiences.

The method I used was narrative research and the analysis described in this particular paper was a narrative analysis with the help of Greimas's actant model. The target group of the study comprised students who began their studies on a master's programme at the University of Tampere in autumn 2005 ($n = 6$). They were students who majored in education and had mathematics as their minor subject. The data I present here include students' reflective essays and the individual interviews conducted in November-December 2005 and May 2006. The data analysed this far is very rich and colourful. The participants' stories, which were compressed into two so called fictional stories, were eloquent and they challenge both teacher educators and researchers.

Introduction

Traditional mathematics teaching is still going strong. Many preservice teachers believe strongly in the way of teaching that has become familiar to them throughout their school years and they want to preserve it. This is one of the challenges teacher educators face every day.

During my years in the profession I have tried out various ways of working with my pupils or students and sought the golden apples of mathematics teaching. As a teacher educator I have found that there are still many left to be found. Because of my research I have developed a mathematics teaching approach which I call 'phenomenological mathematics teaching' after the ideas enlightened to me after reading Tony Brown (2001). I have called it 'phenomenological' because in the phenomenological tradition the researchers emphasize individual experiences and meanings, which I too want to stress in mathematics teaching. In the early stages of my research my 'phenomenological mathematics teaching' consisted of six components: interactive, communal or cooperative, experiential, explorative, illustrative or using illustrations, and

mathematics as a language. When all went well these components contributed to the formation of a positive atmosphere in the classroom, helped in tempting pupils to study and consider learning mathematics not just learning technical procedures but learning mathematical concepts as well.

In Finland, during the recent few years, there has been a trend to combine the primary schools and the lower secondary schools to form an integrated comprehensive school. For many years these schools have situated in different buildings and the teachers have not had the possibility to 'cross the border'. The teacher education has also been separate. While primary school teachers are very much involved in educational sciences in their education having their master's degree in education, subject matter teachers for lower secondary schools study their subject matter mostly at their departments of mathematics, history and so on. To become a teacher they have to take their pedagogical studies at a faculty of education. This takes a year or two and forms only a small part of their education. As primary school teachers usually teach pupils aged 7 to 12 and subject matter teachers teach pupils older than that, there will be many challenges if the schools unite. In mathematics teacher education we have to take into consideration that the prospective teachers must have experience from the lower grades and also knowledge about how pupils learn the basics of mathematics and information about teaching methods suitable for pupils of that age. I believe that all mathematics teaching, like 'phenomenological mathematics teaching' as I call it in my present research, accepts these challenges.

Research Question and Methods

In this paper I will concentrate on my research methods and analysis. The analysis on which I focus here is only part of the whole research analysis. In this particular paper I start for answering question "how does the phenomenological approach to mathematics teaching influence the student teachers' process of developing a teacher identity?" In this chapter I will shortly review my research method and the model I used to analyse the data.

Research Method

My research method is narrative although, during my study, I will make many interventions and meet each of my students or informants both as a teacher and as a researcher. So, my study will have some features of action research. There are three cycles in my research. The first cycle is now coming to its close. After this analysis I will present my findings to my informants and we will decide together how to continue to work on the agreed-on themes of phenomenological mathematics teaching and the process of becoming a mathematics teacher. In the second cycle there might be some changes in the components of phenomenological mathematics teaching as well as in collecting data. As those engaging in action research are recommended to include in their research reports

more personal and narrative material and innovative background (Heikkinen, 2002b, p. 124), I endeavour to write out the way I work and my thoughts throughout the research. The approach is hermeneutic and also partly phenomenological. In my research I try to collect original and unique experiences from both the students and myself (see among others Huttunen & Kakkori, 2002).

Data Collection Method

The target group of my study comprises all the students who began their studies on a master's programme at the university of Tampere ($n = 6$) in autumn 2005, which means that it is the whole group not a sample. They are students majoring in education at the Teacher Education Department with mathematics as a compulsory minor subject. This group is very different from the groups normally attending our courses in the teacher education department who do their pedagogical studies (60 credit points) during one year or two years and who study at their own department, e.g. at a mathematics department, most of the time. The research group and I have more time to work together (143 credit points) which might have some influence on their thoughts on being a teacher. I see it as richness from the perspective of my study that five of the participants had already studied at university before they started these studies being well-advanced in their early studies. A member of the group had commenced university studies in our department right after leaving upper secondary school. Though the group is very small I am pleased that they come from very different parts of Finland.

The data corpus I will focus on in this paper includes reflective essays by the students, the first individual interviews supplemented in November-December 2005 and the second individual interviews in May 2006. The essays were collected during a basic study course I was lecturing to this group. In these essays the students were asked to write about their school memories, about teachers and events that seemed to be important or had had influence on their decision to become a teacher. They were also asked to write about their thoughts of phenomenological mathematics teaching. In the first interview I asked them to tell me more about the experiences they had told me about in their essays. I noticed that some of my informants preferred talking instead of writing and I was happy to hear their stories. The second interviews focused on the changes in the students' thoughts; I asked them to explain what had influenced them during their first year of teacher education. What kind of challenges they had faced, what had been delightful, what had been annoying, what do they think about teaching and learning now after the first year? I also asked a question that seemed to be very difficult for them to answer: How do the children learn mathematics? I recorded the interviews and transcribed them. I got 101 pages of material from the first interviews (some 8 hours) and 90 pages of data from the second interviews (over 7 hours). From the essays I got some 55 pages of written material.

My interviews were conducted in a qualitative way. This is how Rubin and Rubin (1995) describe qualitative interviews:

In qualitative interviews the interviewees share the work of the interviewer, sometimes guiding it in channels of their own choosing. They are treated as partners rather than as objects of research. (Rubin & Rubin, 1995, p. 10)

Rubin and Rubin (1995) keep calling the informants as conversational partners. They pay attention to the fact that some conversational partners are self-revelatory, others more restrained and formalistic. Some know a lot about the topic and some do not. In their opinion it makes little sense to present the same questions to everyone in qualitative interviews (p. 11).

In narrative research, the aim is that a dialogue emerges between the narrator and the researcher in which there is an endeavour towards mutual understanding. For this reason it would be ideal for the researcher and the narrator to work together for a longer period of time and hold numerous discussions (see among others Hatch & Wisniewski, 1995 in Heikkinen, 2002a, pp. 10-20). Recurrent interviews will provide the researcher with an opportunity to consider the theme from several angles and much more profoundly and actually bring more sensitive issues into the discussion (Erkkilä & Mäkelä, 2002, p. 49). Different research methods also have different effects on participants and change people in different ways (Moilanen, 2002, p. 93). Leaning on this I shall compile material over a period of more than three years of study during which some of the students will have completed the bachelor's degree and some of them may already be in the final phases of their master's degrees.

About Making My Study Credible

Rubin and Rubin (1995, p. 85) suggest that most indicators of validity and reliability do not fit qualitative research. Instead, researchers judge the credibility of a research by its transparency, consistency-coherence and communicability. As to my data collection the transparency means that I have saved the original recordings and my transcripts were made directly from tape and they include the whole recording with only some nonverbal indications of what occurred like "laughing together", "coughing" etc. The coherence of the themes means that if there are contradictory explanations of what has happened the researcher has to offer evidence why s/he prefers one version to the other (Rubin & Rubin, 1995, pp. 87-88). Because of my method of analysis I have the possibility to include different views that my interviewees offer and thus to increase the consistency of my research this way. The consistency of individuals means that the interviewees say things that mesh i.e. they do not change their opinions or hold contradictory views simultaneously (Rubin & Rubin, 1995, pp. 88-89). It is very natural that my interviewees change their minds during the research. I try to explain these changes as thoroughly as possible so that the reader can follow the process of developing a teacher identity. The consistency across cases means that the researcher can show that the core concepts and

themes consistently occur in a variety of cases and in different settings (Rubin & Rubin, 1995, p. 90). In my research report this variation is a richness and I will present it very carefully. Rubin and Rubin (1995) explain that the informants, conversational partners, should see themselves in the researcher's descriptions although they may not agree with every detail or interpretation. Other researchers should understand the text and readers who have never been in this research setting should feel confident that they now can find their way around this arena. The richness of detail, abundance of evidence, and vividness of the text increases the communicability of the research (Rubin & Rubin, 1995, p. 91). I find it important that when describing a process and following it for three years it is necessary to select such a method of collecting data and such a method for analysing it that it respects the phenomenon.

Method of Analysis

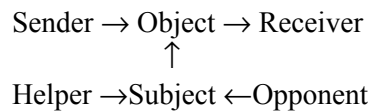
Narrative research does not pursue objectivity and generalisability of knowledge but rather generation of personal, subjective knowledge (Heikkinen, 2002a, pp. 17-18). I have collected my data according to the principles of narrative research and I shall use two different methods to analyse it: analysis of narratives and also the narrative analysis. Polkinghorne (1995) makes a distinction between the analysis of narratives and narrative analysis, the result of which is a narrative (Polkinghorne, 1995, pp. 6-8). Analysis of narratives occurs mainly through classification when various types, comparative images and categories are identified (Heikkinen, 2002a, p. 20). In narrative analysis the researcher collects descriptions of events and synthesizes or configures them into a new narrative or story or stories (Polkinghorne 1995, p. 12).

At first in my analysis I wrote two fictional narratives on the students. They were fictional because they were some sort of syntheses of the participants' stories, presenting their thoughts and experiences before the time my students began their studies. The hypothetical main characters in these two stories I discuss here are Sara and Laura. I tried to make them as real 'persons' as possible so that, through their experiences, those who read my research paper will find it easy to identify with the growth of a prospective mathematics teacher. I also endeavour to preserve the anonymity of my informants in these narratives: both Sara's and Laura's narrative is composed of the experiences of five informants. The stories are based on the reflective essays where I asked the participants to write down their school memories, their thoughts of becoming a mathematics teacher, about the people who had influenced them most, and how they were taught mathematics and so on.

Why did I form two narratives? After reading the essays and the written transcriptions over and over again, I noticed that there were excerpts or moments that were encouraging the prospective teacher towards a future primary school teacher or a becoming mathematics teacher. So, I decided that Sara's future profession will be a mathematics teacher and Laura's aim is to become a primary school teacher. I will validate the stories by letting the informants read them and

make comments on them. I have also decided that they can suggest the names they want me to use in my final report. I chose Sara and Laura for this occasion only because of the international sound of the names.

The stories themselves will inform readers about the professional growth process, but to help the reader to see the crucial points of the narratives I used Greimas's actant model which is based on roles and relations. According to Greimas this model suits well to analysing autobiographies and all kinds of stories. In Greimas's model there are six actants: the Subject is the one who seeks; the Object is that which is sought; the Sender sends the Subject and the Receiver is its destination; the Helper assists the action and the Opponent blocks it (Chandler, 2001, pp. 95-96). Eladhari and Lindley (2004) present the model like this:



They use a simple story scenario from the computer game world to describe the different characters: Sender can be a Wizard who gives a quest to Subject, the Player character, Object will be a Dragon which is to be killed and Receiver will be the Wizard who then gives the reward. Eladhari and Lindley describe the actant theory as a conceptualization that breaks down the parts of the story into force fields and makes it possible for the narrative to come into existence (Eladhari & Lindley, 2004, pp. 8-9). In my stories The Subject is the becoming teacher, Sara or Laura. The Sender is the person or the incident that 'sent' my story character on her way to becoming a prospective teacher. The Object is then to become a teacher. The Receiver is either a primary school or a secondary school depending on the future profession of my story character. The Opponent is what is against this plan and the Helper describes what is encouraging my story character to go on with this plan.

I continued the narratives after the second interviews and writing about the students' first-year experiences. The story of Sara continued as if she had been the one who had already started her teacher training, and the story of Laura proceeded as if she had been a student who did not have any experience as a teacher yet. Greimas's idea is that the researcher forms the text corpus from the data by selecting or eliminating, then she objectivates the text and decides which pronoun she uses, what time she tells about and then eliminates the references referring to places, persons and so on (Greimas, 1980, pp. 168, 176). I have written the stories using the pronoun I and formulating the text as if it had been an autobiography. When referring to informants I have used square brackets after the sequences. My narratives are of course my interpretations. Riessman (1993, p. 13), like Greimas, reminds us: there are decisions to be made about form, ordering, style of presentation, and how the fragments of lives that have been given in interviews will be housed, what gets included and what excluded.

To strengthen the research validation I will let my informants read the stories I have written and also suggest the names for the fictive characters in my final report. They can also change my plans for the next action research cycle and tell me how they would like to go on next year.

Research Analysis

In this paper I am not presenting the narratives themselves instead I will present the four Greimas's actant models I have formed from those narratives: Sara's models 1 and 2 in figures 1 and 2 and Laura's models 1 and 2 in figures 3 and 4, and then open them to readers. The numbers in brackets refer anonymously to the six informants.

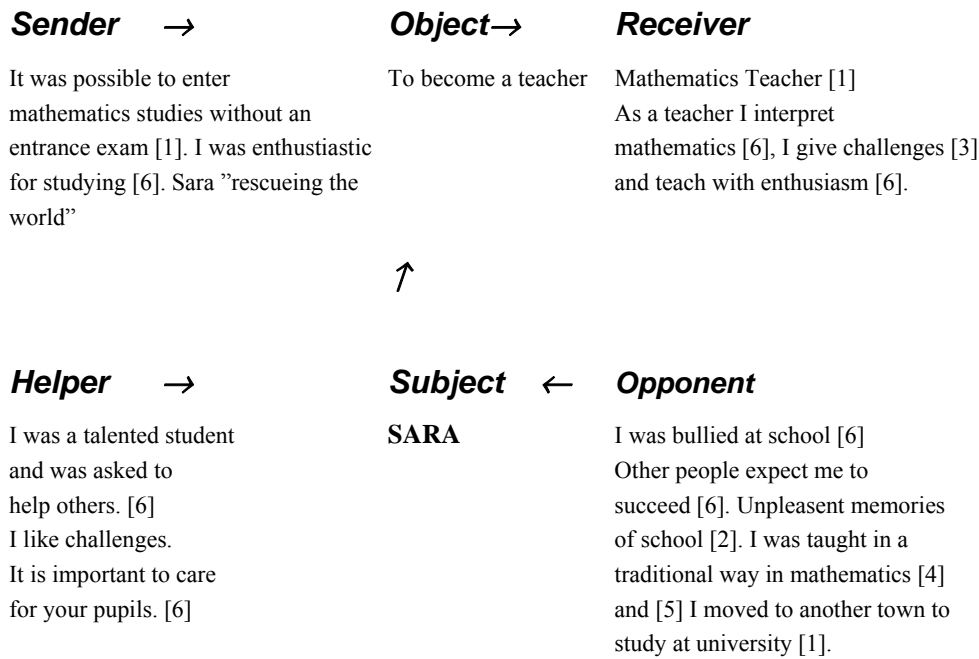
Sara's story

Sara (see figure 1) has always been a talented student who has enjoyed studying and learning. Because of her giftedness people seem to expect her to succeed in her studies and they might wonder why she chose to be a teacher and not something else. She started to study mathematics because it was possible for her to enter university because of her good marks. It was a good choice and she is now heading to become a mathematics teacher.

Sara was bullied at school because of her giftedness and she has bad memories of school. Why go back there? She is, however, the type of a character who wants to show other people that she can do things differently and keep her own mind. What is encouraging is that she really wants to take care of her pupils and show them her enthusiasm. She uses the verb 'interpret' when she talks about teaching mathematics. She seems to believe that she could open up some mathematical concepts and she might perhaps want to try out her "wings" and not just continue teaching mathematics in a traditional way which is something she had got used to during her own school time. She conjectures that loneliness or perhaps homesickness might have some effect on her studies.

FIGURE 1

Sara’s narrative set in the Greimas’s actant model when she started her studies to become a mathematics teacher.



Sara’ story continues after the interviews I had in the spring of 2006. I again collected extracts from four interviews. During the winter Sara had been doing her teaching practice in school. In the curriculum, the first teacher training period consists of twelve school lessons and the teacher student is in charge of these lessons all by herself. The mentoring teacher is only following the work that is going on on the sidelines. The first teacher training period also consists of 60 lessons, under the guidance of the mentor, during which the student gets to know the school and the work being done there.

Sara’s story (see figure 2) continues as her first year in teacher education is over. She has been working as a teacher and also doing her teacher training so she is telling a lot about her experiences at school. She is still demanding a lot from herself, she wants to do things differently. She has experience from the lower secondary school and she is happy when she tells me that it was not so terrible an experience at all compared to the memories she had of her own time in lower secondary school. She is interested in school affairs, teacher’s profession and she is beginning to see her learning processes through ‘the teacher’s lenses’. This can be seen from the way she talks about the pupils and their differences, about the articles she has read about school and the teacher’s profession, and about mentioning the colleagues many times. She sometimes tells in a positive way how important they are and sometimes in not so positive way how they suspect

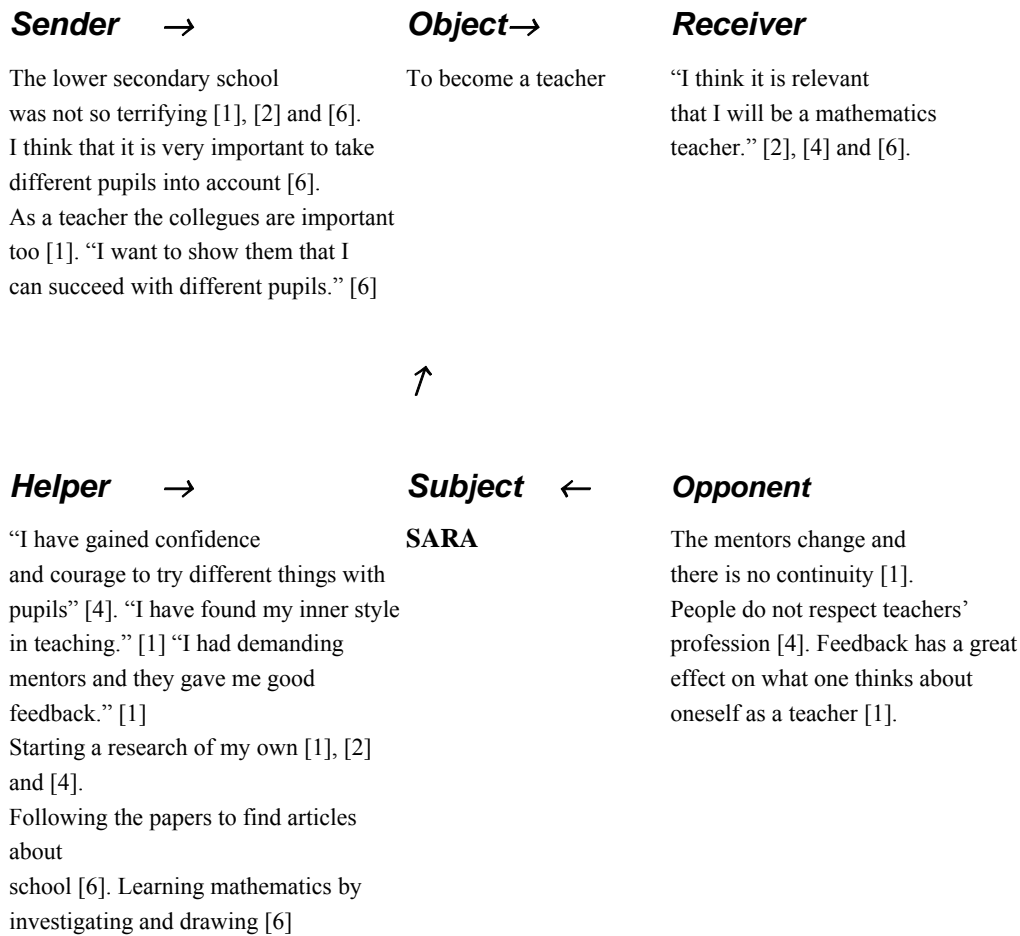
her or comment on her when she wants to take into account learners of all kind in her class.

She feels that she had found her way of teaching and she would like to try out methods she has found effective in her own learning process. This comes out when she tells about researching. She has started her own study and her candidate thesis. Another learning experience has been a didactic course in mathematics she had attended to. During that course they have had many exercises where the students have to do investigations and geometrical drawing. She talks about it enthusiastically and wonders if she could manage to teach like that in school.

It seems that she is going to be a mathematics teacher. What worries her is that her mentors change very often and there is no continuity with the feedback. If one mentor tells her to do something in a certain way and she is willing to try it, the mentor may change and the other mentor may not be too interested in that kind of experiment. The feedback she has received seems to have made a great impact on her and it affects what she thinks about herself as a teacher.

FIGURE 2

Sara's narrative set in the Greimas's actant model when she has studied a year to become a mathematics teacher.



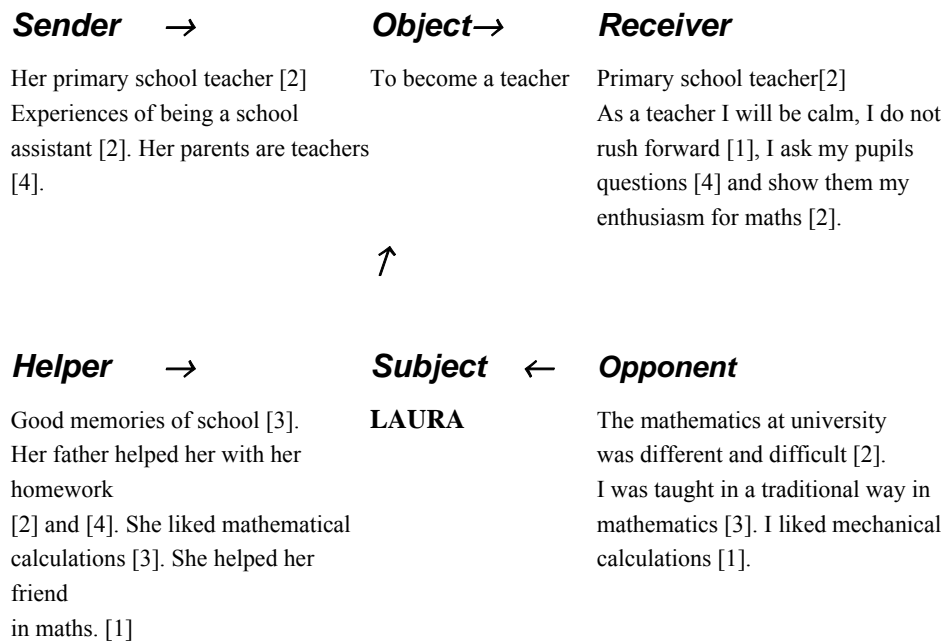
Laura's story

Laura instead (see figure 3) liked her primary school teacher a lot. She enjoyed school mathematics and to become a teacher was a relevant option for her because her parents were teachers too. She has good experiences of school, and after upper secondary school she worked as a school assistant. Laura, like Sara, was taught mathematics in a very traditional way. She liked mechanical calculations at school and the first courses of university mathematics seemed to be very different and difficult for her. There might be a possibility that she will drop out of university mathematics and, after her master's degree, quit mathematics and become a primary school teacher. It is possible though that she uses the option to become a primary school teacher and selects other minor

subjects instead of mathematics. She mentions that she still likes mathematics or at least she thinks she will like to teach it.

FIGURE 3

Laura’s narrative set in the Greimas’s actant model when she started her studies to become a mathematics teacher.



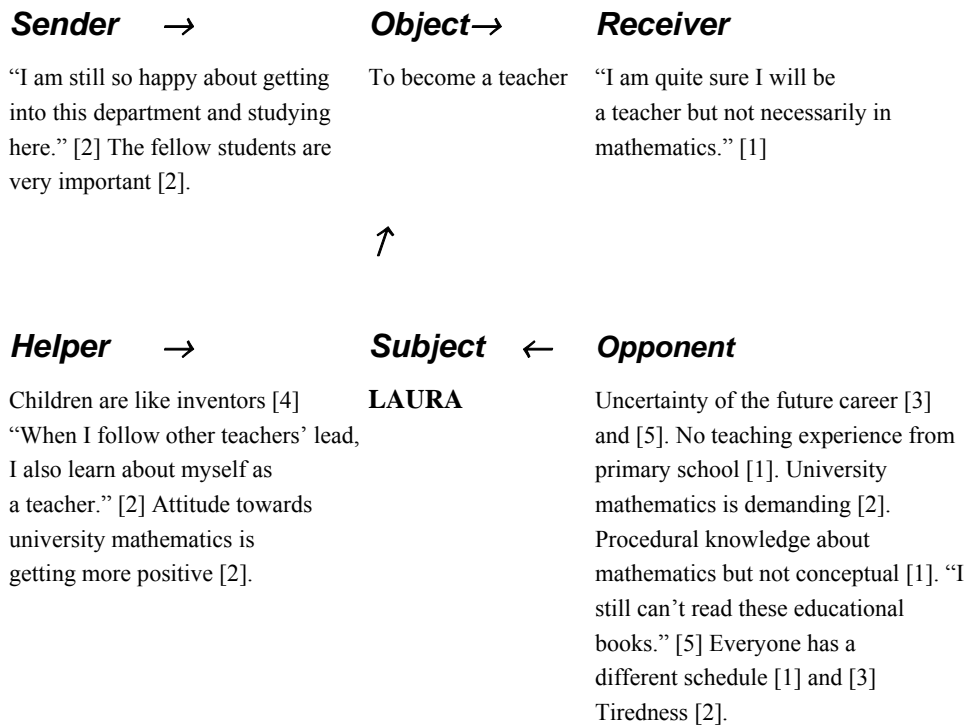
After the students’ first year in our programme I conducted the second interview. From the transcribed data I collected extracts to continue Laura’s story. In figure 4 the reader notices that this story is composed of five interviews. While Sara has already started her teacher training and has had a lot of new kind of experiences of school, Laura has not taught at all. She has been observing other teachers teaching and this has made an impact on her and her thoughts.

After the first year Laura (see figure 4) is wondering about what would be her future career. Her attitudes towards university mathematics are getting more positive but it is still hard for her, and educational books are also demanding. She has been observing some small children and teachers’ teaching, but she has not herself been teaching. Now she has a better idea of what she will be like as a teacher. For her it was amazing how clever the small children can be when they are learning mathematics. She also tells about one lesson she was attending. There was one pupil who asked about powers: “why is it so that any number (not zero) raised to the number 0 is 1?” Laura was astonished. This was something she could not answer. She knew that this is the rule but not the reason why. She understood that she had the procedural knowledge but to work as a teacher she should also have conceptual knowledge.

Laura also tells about her studies. Every one of her fellow students has a different schedule. They seldom see each other. She calls for a more communal way of studying. At least for Laura it is necessary to have fellow students and encouragement from them. The tiredness Laura mentions was connected with her studying on weekdays and working on holidays and being a bit uncertain what would be enough of studying this year and what courses she should take next year and in which order. She felt that there is nobody to help her with these questions.

FIGURE 4

Laura’s narrative set in the Greimas’s actant model when she has studied a year to become a mathematics teacher.



Concluding remarks

These models seem to show that teacher training experiences strengthen the motivation to become a mathematics teacher and they even encourage working at the lower secondary school [2], [4], [6]. (The numbers in the square brackets stand for the informants 1-6.) If the teacher students follow the news from school and teaching journals it builds up the picture of what it is like to be a teacher [6], and that to feel oneself as a younger colleague helps the identification process [1]. Following other teachers’ work can help the student teacher to understand his/her own beliefs and attitudes [2]. To have an opportunity to acquire first hand learning experiences like learning mathematics

by investigating [6], or starting one's own research study as part of your studies [1], [2], [4] is a powerful way of building one's teacherhood. Fellow students are important [2] and their support can be necessary [1], [3]. It is important to organize the studies so that these resources are provided. We should not forget in our education, firstly, that procedural knowledge (how?) does not necessarily connect to conceptual knowledge (what? and why?) but together they could help the teaching process [1] and they are necessary skills for the teachers, and, secondly, that it takes time to learn to read educational books [5] and that is also a skill worth practicing.

Discussion

What kind of challenges do my research findings present to our teacher education? We have many very powerful methods in our hands already but the question is how to help the students to test or try different teaching methods in their teacher training? How to promote discussions and interaction during lectures? How to organize meetings where the students have an opportunity to talk with experts? And how pose new questions to the students to reflect on and write about in their essays?

There are also challenges that are not so easy to meet: should we encourage our students to take temporary jobs as mathematics teachers during their studies? Or should we worry about the practices in the school where the students do their teacher training? And how can we be sure that the students gain experiences of different classes and grades? How can we help the students to take different positions when they are observing other teachers' lessons? And how can we help the students to see what the real profession of a teacher is like?

There are also attitudes worth fighting against. "A teacher teaches what is written in the book"-attitude and "you do not have to keep your knowledge fresh if you are a mathematics teacher"-attitude are kinds of attitudes that do not promote any major changes in the school. "I do my homework only because the teacher will ask for it"-attitude which at university level means that our students only complete courses. "The upper secondary school is preparing only for the finals"-attitude is also worth fighting against, because it promotes the procedural knowledge rather than conceptual knowledge and guides our teachers towards 'curling-teachers' who only smooth the way for their pupils.

Greimas suggests that semantics is an effort to describe the sensible world (Greimas, 1980, p. 17). I believe that through narratives I can search for a better understanding of the process of becoming a mathematics teacher, and I hope to crystallize it for the readers through modelling the narratives with Greimas's actant model. This model is not just a list of things but it is a net of connections and relations. According to Bridges (2002) some educationalists have adopted a narrative style of writing because they want to tell their research stories without tying them tightly to what actually happened in reality. They want to use their

imagination, which indeed they would use anyway when conducting research, in order to fill up gaps in their data and possibly to replace the research environment with some imaginary environment or their research objects with some imaginary figures (Bridges, 2002, pp. 30-31). Clough (2002) reports that he always began from the data, but that it was more important to describe the fascinating thoughts and visions underlying the data. Frequently, when using all the information he had on the object of his research, the resulting story resembled the informant much more closely than what the individual herself could ever have described to him (Clough, 2002, pp. 16-18). Whatever will happen during the next two years when my informants continue their studies, I am sure that through this little piece of analysis I got at least a glimpse of the process they are living through.

References

- Becker, H. J. (2001). *How are teachers using computers in instruction?* Paper presented at AERA 11.4.2001. Retrieved from http://www.crito.uci.edu/tlc/findings/conferences-pdf/how_are_teachers_using.pdf.
- Bridges, D. (2002). Narratives in history, fiction and educational research. In R. Huttunen, H. L. T. Heikkinen & L. Syrjälä (Eds.). *Narrative research, Voices of teachers and philosophers*, 29-44. Jyväskylä: Jyväskylän yliopisto.
- Brown, T. (2001). *Mathematics education and language, interpreting hermeneutics and post-structuralism*. Dordrecht: Kluwer Academic Publishers.
- Chandler, D. (2001). *Semiotics: the basics*. New York: Routledge.
- Clough, P. (2002). *Narratives and fictions in educational research. Doing qualitative research in educational settings*. Buckingham: Open University Press.
- Eladhari, M. and Lindley, C. (2004). *Story construction and expressive agents in virtual game worlds*. Paper presented at the Other Players conference, center of Computer Games Research, IT University of Copenhagen, Denmark, 6-8.12. 2004. Proceedings available online: http://itu.dk/op/papers/eladhari_lindley.pdf.
- Erkkilä R. & Mäkelä M. (2002). Face to face – Human dimensions in biographical interviews. In R. Huttunen, H. L. T. Heikkinen & L. Syrjälä (Eds.). *Narrative research, Voices of teachers and philosophers*, 45-56. Jyväskylä: Jyväskylän yliopisto.
- Greimas, A. J. (1980). *Strukturaalista semantiikkaa*. Tampere: Tammerpaino Oy.
- Heikkinen, H. L.T. (2002a). Whatever is narrative research? In R. Huttunen, H. L. T. Heikkinen & L. Syrjälä (Eds.). *Narrative research, Voices of teachers and philosophers*, 13-28. Jyväskylä: Jyväskylän yliopisto.
- Heikkinen, H. L.T. (2002b). Telling stories in teacher education. A narrative – biographical view of portfolio work. In R. Huttunen, H. L. T. Heikkinen & L. Syrjälä (Eds.). *Narrative research, Voices of teachers and philosophers*, 123-142. Jyväskylä: Jyväskylän yliopisto.
- Huttunen, R. & Kakkori, L. (2002). The hermeneutics of truth and selfhood. Heidegger's, Gadamer's and Ricoeur's significance in the autobiographical research.

- In R. Huttunen, H. L. T. Heikkinen & L. Syrjälä (Eds.). *Narrative research, Voices of teachers and philosophers*, 77-90. Jyväskylä: Jyväskylän yliopisto.
- Moilanen, P. (2002). Narrative, truth and correspondence. A defence. In R. Huttunen, H. L. T. Heikkinen & L. Syrjälä (Eds.). *Narrative research, Voices of teachers and philosophers*, 91-104. Jyväskylä: Jyväskylän yliopisto.
- Polkinghorne, D. (1995). Narrative configuration in qualitative analysis. In J. A. Hatch & R. Wisniewski (Eds.). *Life history and narrative*, 5-23. London: Falmer.
- Rubin, H. J. & Rubin, I. S. (1995). *Qualitative interviewing: the art of hearing data*. Thousand Oaks: Sage Publication.
- Riessman, C. K. (1993). *Narrative analysis*. Newbury Park: Sage University Press.

Structure and closure of school mathematical practice - The experiences of Kristina

Ann-Sofi Røj-Lindberg
Åbo Akademi University

I present some preliminary results from a story-telling case study, the main focus of which is to give an account of the school mathematical practice at one particular Finland-Swedish lower-secondary school. In the paper, the school mathematical practice is looked upon through the voice of the student Kristina. Kristina told me about her experiences of participation in school mathematics in several qualitative interviews during the three school years 7, 8 and 9. The analysis of the interviews has so far resulted in the creation of several preliminary themes indicating her experiences. In this paper I limit the presentation to one theme, *Structure and Closure*. The paper meets the requests for more studies where researchers and teachers listen to the voices of students in order to understand and learn from their school mathematical experiences and expectations (see e.g. Burton, 1994; Corbett & Wilson, 1995).

School mathematics as an individually experienced socio-cultural practice

Many researchers within the field of mathematics education nowadays accept a relational view of learning within which the knowing and identity of persons can be seen as social constructions and as results of the person's experiences in and contributions to social and cultural contexts. The processes of creating, teaching and learning mathematics and the outcomes of the processes can from this viewpoint be considered as social, cultural and interdependent phenomena (see e.g. Atweh, Forgasz & Nebres, 2001; Lerman, 2000; Nunes, 1995). Hence, school mathematical practice as well as mathematics as a scientific subject is seen as existing within social systems of thought and culture (Ernest, 1998).

In this paper the level of analysis is the school mathematical practice in a particular school where some mathematics teachers intended to change their teaching practices with the support of colleagues. Elsewhere I have pointed at constraints the teachers experienced in trying to free themselves from the safety of habitual social and cognitive patterns (Røj-Lindberg, 2003, 2006). Practice is conceptualized in different ways by different researchers within social constructivist and socio-cultural perspectives. Paul Cobb defines 'practice' as an emergent aspect of the communication in the classroom. Within a lesson design

that focuses negotiation in communicative interactions, Cobb is researching how students' mathematical conceptions emerge and become established as a *taken-as-shared mathematical practice* in the classroom (Cobb, 2000). Jeppe Skott describes *classrooms as communities of mathematical practice* and discusses practice as an emergent phenomena influenced by the teacher's school mathematical images, which are unique for each teacher. School mathematical images are in Skott's study defined as teachers' personal interpretations of and priorities in relation to mathematics, mathematics as a school subject and the teaching and learning of mathematics in schools. Skott proposes that the school mathematical images contribute to the development of students' learning opportunities (Skott, 2000). Marilyn Goos and colleagues also discuss the mathematics classroom as a community of practice, but with a focus on interactions of teachers and students to describe the social conduct in a collaborative classroom. Taken together, these interactions reveal the emerging classroom culture and can be taken as indications of whether the mathematical practice in the classroom makes sense to students. Goos and her colleagues further argue that both the teacher's and the students experiences of schooling can act as potential barriers to reform of school mathematical practice (Goos, Galbraith & Renshaw, 1999). In researching students' goals, Simon Goodchild approaches 'practice' as *the context of routine tasks* that students are engaged in, in a mathematics classroom (Goodchild, 2001). From a philosophical viewpoint, Anna Sfard discusses meta-rules as her lens into 'mathematical practice' and she uses Wittgenstein to illuminate her points of view. According to Wittgenstein, "the person who follows a rule has been trained to react in a given way. Through this training the person learns to respond in conventional ways and thus *enters into practice*" (Sfard, 2000). Miller and Goodnow propose an epistemological definition of 'practice' as *actions that are repeated, shared with others in a social group and invested with normative expectations and with meanings or significance that go beyond the immediate goals of the action* (Miller & Goodnow, 1995, 7). I find their definition of practice satisfactory if 'action' is considered to comprise both cognitive and socio-cultural activities and 'meaning' as referring to individually experienced as well as socio-culturally constructed and valued meanings.

The word 'practice' is used with different connotation in common language and also in this paper, it is difficult to avoid a certain ambiguity in its use. I sometimes refer to practice as the actions of teacher and students in the classroom. Admittedly, most of the individually experienced and socially valued meanings of school mathematics are related to actions in the classroom. However, when I use 'school mathematical practice' my intention is to cast the conceptual net more widely and as well include the images and pedagogical intentions of the mathematics teachers as a social group within the collaborative reform work.

My main focus in the paper is on the student Kristina and her experienced meanings of school mathematical practice during lower-secondary school. A

basic assumption is that the story she tells in interviews is the meaning she imposes on her world. Her story is the truths of her experiences, not an ‘objective’ reflection of an ontologically real reality (von Glasersfeld, 1991). Thus, it is important to note that her experienced meanings might differ from meanings constructed by other students and by the teachers who were participants in the same mathematics classroom. For example, Ben-Chaim, Fresko and Carmeli (1990) write as one conclusion from a survey given to teachers and students that the teachers saw the classroom environment as more diverse than the students. Another remark of importance is the use of the concepts ‘knowing’ and ‘knowledge’. ‘Knowing’ refers to personal meanings of socially and culturally constructed ‘knowledge’, like for instance, of mathematics in textbooks, of activities in school, of what it is to do mathematics and be a learner in a classroom.

Theoretical considerations

In the theoretical frame I am inspired by socio-cultural perspectives on human learning and development and a cultural-psychological view on the human mind (Bruner, 1990, 1996/2002; Rogoff, 2003).

Within the socio-cultural perspective that I adapt, learning is participation and development is a function of ongoing transformation of roles and understandings in the socio-cultural activities of the communities in which the individual participates. I argue that the students and teachers within the school mathematical practice that is the focus of my study can be interpreted as forming developing communities of learners in this sense. An essential theoretical argument in this perspective is that culture, including mathematics and school mathematical practice, is seen neither as something static that adds on or is gradually internalized by the individual mind nor as a surrounding to the individual which is gradually changed to fit the developing mind. Culture is the *common ways* that participants in a community share even if they may contest them (Rogoff, 2003). From a cultural-psychological viewpoint, Bruner argues that we should think of culture as something that is in the mind of persons (Bruner, 1996/2002, 200).

This perspective on learning and development is radically different from one that describes learning as transmission of knowledge from authorities outside the individual or as acquisition or discovery of knowledge by the individual. Moreover, it challenges the idea of a boundary between internal and external phenomena, as for example between a students’ knowing of school mathematical practice and the cultural tools used in this practice. Among cultural tools Roger Säljö includes intellectual tools like systems of ideas and discourses and physical tools like textbooks, mathematical symbols and diagrams (Säljö, 2005).

If this perspective is accepted, a researcher may place an individual student’s knowing or the school mathematical practice in the foreground without assuming that they are actually separate elements. It also makes sense to discuss

an individual's school mathematical experiences in terms of *situated knowing* or *situated understanding* and to acknowledge the importance of context for the development of these experiences. The research studies by for instance Boaler (1997), Stigler and Hiebert (1999), Boaler and Greeno (2000) and Nardi and Steward (2003) are well grounded examples of the situated nature of school mathematical experiences: it is impossible to separate the knowing of individuals from the social and cultural practices in which the individual participates.

Barbara Rogoff (2003) uses the concept *participatory appropriation* instead of internalization or appropriation to refer to the change and development in knowing, resulting from a person's *guided participation* in an activity. As people participate they are making a process their own, they are not taking something from other persons like you take a thing and add it on to something you already possess. But they are creatively taking for their own use and changing how they may treat future situations that they see as related. Processes of guided participation are defined to incorporate guidance in *the sense of direction* of a shared endeavour. It is participation in meaning that is the key issue, not necessarily in shared actions of the moment. A person who is observing and following without direct contribution to the decisions made by other people is also participating in the activity. Moreover, a person who acts alone is also participating in a shared endeavour as he or she follows and builds on community traditions for the activity. For instance, a student doing homework is participating with guidance provided as well by the teacher and textbook writers, who may have set the tasks and approaches to be used, as by classmates, and family members, who may support some approaches and suppress others. Also, a mathematician like Andrew Wiles, who grappled eight years with proving the Last Theorem of Fermat and without direct involvement from peers, was building on socially validated traditions for mathematical activities.

Within a theory of development as transformation of participation, people are seen as contributing to the creation of socio-cultural processes and socio-cultural processes as contributing to the creation of people (Rogoff, 2003, 51). An example of how this kind of mutuality and interdependence might operate within school mathematical practices can for example be seen in what *homework* means to a student. The students might perhaps imagine it as a questioning activity in the classroom where *teacher wants me to show that I know the correct answer*. Then by displaying the correct answer, she is participating in the creation of this activity as an activity where the student is expected to give the one answer that is correct. When the teacher evaluates the answer as *a good answer* the teacher fulfils her expectations and they both contribute to the emerging implicit agreements about 'homework actions', about responsibilities for actions, about when to do what and how to do it in this particular classroom. In this process the students and the teacher together create normative expectations for the kind of mathematical competence valued, what it means to be a good student and an effective teacher in relation to homework actions. Or the student might imagine *homework* as questioning activity where *teacher wants me to argue*

mathematically for my answers. If this is the case, a homework review in the classroom is an activity constituted by a very different participation structure with different expectations and implicit agreements. At the level of formal engagement, the student may be doing the same thing in both examples, i.e. answering questions set by a teacher about homework. These examples indicate that the socially valued meaning of homework review might be constructed very differently in different school mathematical practices. They also indicate that a participation perspective on learning and development requires considerations of not only *what* it is that the individual is participating in but also the experienced *meanings* of these activities.

Jerome Bruner states that a culturally sensitive psychology should be based not only upon what people say caused them to do what they did. It is also concerned with what people say others did and why. And above all, it is concerned with *what people say their worlds are like* (Bruner, 1990, 16). Bruner further states that cultural psychology seeks out the rules that human beings bring to bear in creating meanings in cultural contexts of practice (p. 118). Thus, I argue, it becomes important to foreground school mathematical practices as seen through the eyes of the students. Moreover, this research perspective is needed if students are considered as *legitimate participants in*, not only as *beneficiaries of* school mathematical practice (Corbett & Wilson, 1995).

Methodological considerations

Methodology is interpreted by Wellington as the activity when the researcher chooses, reflects upon, evaluates and justifies the research methods she uses to answer the research questions (Wellington, 2000, 22). As already indicated, the topic of this paper has emerged from a case study which involved a group of mathematics teachers who participated in reform work to develop their mathematics teaching. I took part in the process as research-assistant within a commissioned research work including a complex set of tasks, among them to bring the voices of students into the reform work. I approached the research work from the viewpoint of being a mathematics teacher myself and I used a mixture of research methods to build up a case record. I got to know the teaching traditions and the teachers' pedagogical intentions from interviews with teachers and students, from surveys and from being a participant as well as a non-participant observer and note-taker at action research meetings and observer of lessons. Thus, it is obvious that I cannot act as an outsider detached from the story of school mathematical experiences that is the focus of this paper. As the research process started with a very open *bottom-up* approach, it is natural that a retrospective format is used for reporting the research. Bruner (1990, 119) considers retrospective reporting as viable when personal meanings of activities is the focus for research inquiry.

The interest in gaining insights into the views of students was developed as more specific research questions of which I in this paper will discuss the following:

How does Kristina experience her participation in school mathematical practice during school years 7, 8 and 9?

To answer the research question I will rely on interpretations of interviews as my main method to provide evidence. The interview format offered possibilities to gather students' accounts of their experiences of school mathematical practice, and make inquiries into their values, feelings, expectations and views, issues that are difficult to reach with other methods, like classroom observations or surveys. The research interview gave the students a voice and provided them with a platform and a chance to make their viewpoints heard and read (Wellington, 2000). With a psychological constructivist view of knowing in mind, I conjectured that every student would conceive the school mathematical practice differently and thus, each one of them would have a unique story to tell. The interview format seemed as a reasonable way for me to monitor the relationships between the students' experiences of school mathematical practice and their emerging meanings of these experiences. As interviewing all students was impossible, and to be assured of variation in perspectives, I chose four to five students as key informants from each reform class. As instruments for selection of students I used a test for mathematical achievement and a self-concept questionnaire (see Linnanmäki, 2002 for a discussion of these instruments). The key informants represent as big variation in level of mathematical achievement and self-concept as possible. Kristina is one of these key informants.

Semi-structured interviews with Kristina were conducted five times: in September and December school year 7, in December school year 8 and in January and May school year 9. The interviews with her comprise a total of 2 hours and 26 minutes, transcribed into approximately 45 pages of text (12pt, sp 1). With the double aim of both validating findings from the lower-secondary interviews and shedding more light upon Kristina's relation to school mathematical practice, I had a conversation with Kristina where she as a grown up looked back on her school mathematical experiences. Implications from this interview are included in the final coda of the paper.

The theme and aspects discussed in the next section have emerged out of an open coding approach with attention both to theories regarding school mathematical practice in literature and to new aspects emerging out of repeated listening to the interviews and reading of the transcripts (Bryman, 2002). The transcripts were imported into a computer program for qualitative research (NVivo), which I used as a help when I wanted to focus particular aspects of the emerging themes. Reading research literature in parallel with coding made me more theoretically sensitive, and at the same time I tried to stay as close to the voice of the students as possible by reading and listening to the interviews over and over again. The analytic approach I take falls within the interpretivist tradition. The interpretivist researcher's aim can be to explore people's perspectives and to develop insight in situations, e.g. schools and classrooms (Wellington, 2000, 16). In the following, I describe the contextual frame which is essential for the

reader's emerging understanding of Kristina's school mathematical experiences (Lincoln & Guba, 1985, 360).

The context for Kristina's story

The very same autumn as Kristina came to her first day at lower-secondary school, five mathematics teachers at the school committed themselves to reform¹ work and to a redesign of their pedagogy. The format for the reform work was inspired by collaborative action research (see e.g. Elliott, 1991; see also Røj-Lindberg, 2003, 2006). The teachers came together regularly to discuss and evaluate aspects of their pedagogical approaches. The driving force for the reform was of both a theoretical and a pragmatic nature.

Firstly, during early 1990ies, constructivist theories of learning had gained a strong foothold in pedagogy and in the theoretical rhetoric of reform in mathematics education (Björkqvist, 1993; Hansén & Myrskog, 1994; Kupari & Haapasalo, 1993). In their pedagogical intentions the teachers were strongly inspired by constructivism, which they also saw as a new theoretical norm for their teaching of mathematics. As persons representing the culture of mathematical knowledge in the classroom, they felt troubled by the difficulties of communicating this knowledge to their students. Thus, the rhetoric of each student constructing his or her 'own mathematics' felt exactly like the situation they met in their classrooms each day. The epistemological dilemma of talking about constructivism as if it was a teaching practice was of no concern at the time.

Secondly, they involved themselves in the reform work because they wanted to leave unproductive teaching patterns and create new traditions (Røj-Lindberg, 2003, 2006). The statements of one of the teachers can serve as an example. In an interview halfway into the first year of reform, this teacher describes how he had become more and more unsatisfied with the formalistic nature of school mathematics and with his own pedagogy. In his opinion school mathematical practice was "too theoretical and too much of formulas bandied about". In many ways the teachers expressed a conviction of the superiority of more process-based and student-centred environments for the learning of mathematics. Statements similar to the pedagogical intentions articulated by the teachers were expressed in the national mathematics curriculum which stated that in comprehensive school such "learning situations should be organized where you discuss, experiment and as often as possible solve problems that originates from the students' own everyday experiences" and that "problem solving and the

¹ In this paper the concept "reform" denotes the intentions by the teachers to change their teaching practices and not necessarily the outcomes. Kristina's story and the "traditional" school mathematical practice as experienced by her that this paper reveal can however be interpreted as an outcome. Nevertheless, it is not my intention nor is it possible to relate Kristina's story to a discussion of success or failure of the reform.

internal logic of mathematics are the most important principles for mathematics teaching". (1994, my translation)

Kristina was all three years of lower-secondary schooling in a group of 15 students, 8 girls and 7 boys. The school mathematical practice was tuition in this mixed-ability group up to the second half of school year 9. At the action-research meetings the group was referred to as "a group of high-achievers". Out of the 15 students 11 were positioned as high-achievers in the achievement test used for selecting key informants. During the three school years the average test results of the group was usually above the average results in the other two reform groups. At the beginning of school year 7 the teacher describes how the students in the group willingly explain to each other and help each other. Especially boys classified by the teacher as 'smart' are described as very active, while the girls are described as silent and cautious. There are students in the group that the teacher classifies as 'weak'. The teacher complains about lack of time for individual students and that he doesn't manage to keep up with all students. As a solution some students are supported with remedial teaching out of the classroom. The teacher describes pedagogy where he monitors the social interaction in plenary session by setting simple questions first and by trying to find positive aspects also in answers classified as wrong answers.

The main tools the teachers used for pedagogical planning was the textbook but also the national curriculum and the school's own mathematics curriculum. The school's curriculum stated discovery of mathematical structures as the most important aim in relation to students' with qualifications and personal interest to study mathematics. This striving for structural clarity is indicated in the curriculum as a norm for the mathematics teaching at all stages in school. Among the new learning activities the teachers introduced was project work as homework and different types of mathematical problems in combination with and a stronger focus on written explanations by the students' mathematical work.

Textbook tasks and teacher made tasks were used in classroom activities on a regular basis. One type of teacher made task was the so-called 'mini-problems'. These problems the students solved individually during lessons within 10-15 minutes and handed in to the teacher for assessment. 'Mini-problems' were in most frequent use in Kristina's class during school year 8.

Once or twice each term the students worked with project work as homework, mostly on an individual basis. During school year 9 the students' responsibility for developing the mathematical content of the projects increased and the students were organized into smaller groups to cooperate in the planning and execution of the projects.

The written tests used for assessment of mathematical achievement in the reform groups consisted of both traditional tasks and less traditional tasks of multi-choice type. Special types of task introduced only in tests used in the reform groups were called 'explanation tasks'. The process of doing explanation tasks

demanded the students to explain in detail each step in the written solution. During year 9 the use of explanation tasks diminished.

During the autumn term in year 9 the students in the reform groups were regrouped and new groups were formed according to the student's plans for his or her academic future after lower-secondary school. Three groups of students were formed: students planning for vocational school, students planning for upper secondary school and a long course in mathematics, students planning for upper secondary school and a short course in mathematics.

Kristina

In all interviews Kristina showed a narrative zest and expressed her views easily. In comparison to interviews with the other key informants from the same year group, the five interviews with Kristina were the longest. A comparison of the average amount of words spoken in the five interviews show that the interviews with her comprise 60 % more words than an average of the other interviews. At the first interview she played a little bit of a waiting game, a usual reaction among the key informants. But already then, her answers were more narrative and reflective than the answers of most of the other key informants. A general feature in all interviews is that her answers to my questions include statements and argumentative as well as emotional and spontaneous utterances.

Kristina began lower-secondary school as a high-achieving student in mathematics but with a low self-concept in relation to mathematics. In the achievement test at the beginning of school year 7, she belonged to the highest achieving third of the group. In the midst of high school she was evaluated by her teacher with the grade '9' in mathematics when '10' is the best possible. Her self-concept was also generally classified as good, disregarding the aspects in the survey that related specifically to mathematics and mathematics teaching. All students in the reform classes were included in a self-concept survey in the beginning of school year 7. Kristina participated in the same questionnaire in May school year 9. According to Bruner (1996/2002, 54) a person's self-concept is a question of empowerment and sense of agency: it originates in a sense of being able to initiate and carry out actions on one's own or by one's own effort. At the end of school year 9, Kristina is still distinguished by the same strong general self-concept. In fact she seems to identify even more positively with school practice than as a newcomer. But the renewed survey of her self-concept also indicated that her self-concept in relation to mathematics and mathematics teaching did not develop positively during lower-secondary school. At the end of school year 9 it was at the same low level as at the beginning of school year 7.

In the first interview Kristina describes mathematics as a *useful but dull* school subject. After one and a half year in lower-secondary school she again indicates a negative identification with school mathematics and states that mathematics has never been a favourite subject of hers. She occasionally compares her

negative identification with school mathematics to that of her mother, but more often she relates her negative identification to sources within herself, to her insufficient personal competencies, to her *lack of energy* to listen actively enough to the teacher and to her *deficient ability to concentrate*. She even says that she most of all would like to *avoid mathematics* in school all together because it often is such a *boring activity*. But on the other hand she is focused on learning and tells about how *important* it is to know mathematics. In December school year 9 she declares: "You cannot just think of those things that are amusing, mathematics is important you know". Kristina's negative identification with school mathematics has not emerged from any general resentment towards engagement in school mathematical practice. She does *like to engage* herself mathematically and she has, at least in the beginning of lower-secondary school, confidence in the explanation that school *mathematical knowledge is valuable* in the long run. At the first interview she expresses a hope that engagement in school mathematical practice will provide her with a knowledge base that can support her during all her life. However, talking with her later on, I got an expression that she has started to doubt the argument of lifelong usefulness as a sufficient motivational basis for school mathematics. At the third interview, she is more doubtful: "I don't really know, everybody you ask, if you ask someone, they say that it is important to know (mathematics) in professions, you have never got any real explanation, I don't know myself what I think, everybody just tells me it is for your profession you need it, and then I say so myself too". She seems to have developed awareness that the argumentation about mathematics as a useful subject, the learning of which gives her rewards in the future, may have the form of a persuasive ritual with an unclear connection to the school mathematical practice she has experienced so far. Thus, you could ask whether the argument of usefulness the teachers use over and over again is seen by Kristina as part of the learning process as a whole and which social function the argument is serving.

Kristina achieved good results in mathematics during all the three years in lower-secondary school. Nevertheless, she occasionally figured that a good grade in a test situation was a surprise to her and expected it would fall as a result of next graded evaluation. Interestingly she showed more confidence in being able to get good grades from project work. The dilemma was that she considered project work as outside school mathematical practice. Moreover, she did not experience herself as ever becoming a member of a community where school mathematical knowing is playing any significant role. This is not unusual for successful students who relates negatively to mathematics in school (Boaler & Greeno, 2000, 184). During school year 9, she is unsure about her choice of school after lower-secondary school and goes into the group for students planning for a short course in mathematics. She has applied for studies in upper secondary school but refers to *insufficient competence* and energy for any deeper studies in mathematics. She argues that there is a significant difference between those that understand mathematics and those that don't understand. She labels

those students that plan for a long course in mathematics as “fairly smart” students and declares that she needs more time to think of what she wants to do as a grown-up. The change in grouping resulted in change of teacher twice during lower-secondary school. Kristina experienced extended teaching periods with two teachers: slightly more than two school years with the first, some weeks with the second and about three-fourth of school year 9 with the third teacher. The changes in teacher obviously implied some changes in the classroom discourse, but behind the changes Kristina sees a structure, a pedagogical system, which from her viewpoint lies outside the boundaries of socially valued school mathematics. The design of mathematics teaching at school is simply something the students have to cope with, not something they can transform. “If all students would say how they like their lessons, it would not serve any good end, I do think it is best that the teachers have like some own system they follow”. In the last interview Kristina indicates clearly how she conceptualizes such a system.

Well, surely from the beginning it is rather like, if we start with something new, it shall be something rather simple. Or that you start from something simple, that he looks to that we all from the beginning, and we say how we think ourselves, then, while it is rather simple. Then it gets more and more difficult so that something small is added all the time, so that we then, when we have got that most simple, not that anything is really simple, but, in the simpler, that we can, that we have understood everything, so that from there, some little more can be added all the time, so then it is easier to understand those more difficult tasks. (Kristina, Int5)

Structure and closure – one theme created from Kristina’s story

The preliminary themes created so far out of Kristina’s story can be organized under four headlines. The first, Structure and Closure, together with the second, Tempo and Linearity, capture Kristina’s reports of aspects of the classroom as a teaching and learning environment. The third, Alienation and Marginality, focus on her emotional reactions and identifications toward participation and engagement in school mathematics practice. Under the fourth headline, Tensions and Dilemmas, I note issues that penetrate the other three themes. Due to the paper format I limit myself to the five aspects of the classroom as a teaching and learning environment which comprise the theme *Structure and Closure*. All themes will be presented in my forthcoming doctoral thesis. The included extracts and utterances are chosen either because they typify Kristina’s descriptions or views on different occasions within or across interviews, or because they stand out as significant, critical or contradictory in some sense.

Stability of the lesson design

At each interview Kristina described the pedagogical practice of recent lessons or of lessons she conceived as ordinary mathematics lessons. Mostly her

description focused on what the teacher did and what the students did or were expected to do and was a response to a direct question like in the following extract from the first interview. Sometimes it was spontaneously included in a conversation about some related issue.

I: Can you tell me something about the last lesson?

Kristina: Do you mean the last one we had?

I: Yes. Was it an ordinary lesson?

Kristina: Yes, it was ordinary. What did we do? We learnt about powers. Isn't that the name of it? I think so. An ordinary lesson.

There are activities like project work or problem-solving tasks that occasionally break the routine but through all interviews she describes the ordinary lessons as following roughly the same pattern of activities. The activities Kristina describes are very similar to the “traditional” design for mathematics teaching at the lower-secondary level (see Bodin & Capponi, 1996; Hiebert & Stigler, 1999; Røj-Lindberg, 1999) including review of homework or mathematical contents from earlier lessons; answering questions set by the teacher at plenary sessions; listening to teacher explaining mathematical methods and solution procedures at plenary sessions; writing down methods and solution procedures; working with exercises or problems set by teacher on an individual basis and teacher assigning homework for next lesson.

In the third interview she described an ordinary lesson in the following utterance:

We go through the homework, or first we come into class and then if we have had homework, we do it, or he just asks if there is something someone has not understood, then he can take it on the board, if someone has not understood you can copy it. Then, if we don't continue training the old stuff and calculate from the book, we usually take a new thing and write some points in the theory booklet about how it is all right to calculate it and then we can try ourselves or he says, he asks, if we are going to calculate, you can raise your hand if you have some good idea about how to calculate the exercise, and then, if it is correct he writes it down. There can be several alternatives then and we write down all the alternatives in the theory booklet. Then we are supposed to solve some tasks. We usually get homework, if we don't have a project work going, but anyhow, next lesson we go through that homework. Or we have a written test on the homework.
(Int3)

After a change of teacher in year 9 Kristina describes a move to a more accentuated discovery approach to teaching with more independent student work. It is “less telling of answers to the whole class, more finding out yourself what is important”, she says. But nevertheless, Kristina doesn't hesitate when asserting that as she has got used to the new teacher she has noticed that “it is pretty much the same”. From her descriptions of classroom practice it is

obvious, that the lesson design includes similar features also after the change of teacher in grade 9.

Teacher does a task on the board, it is of course like to continue what we had before, and then he asks us to find a solution by ourselves or asks how we could do to calculate it. Like think forward from what we have had before and then we can go through some examples on the board and write something in the theory booklet or exercise booklet and then we will do a review the next day and look up if we have understood (Int4)

The textbook and especially the “theory booklet” stand out as very important tools within the ordinary lessons. Kristina describes the theory booklet as “a smaller book”. This booklet parallels the textbook as both contain “rules and examples”, the difference is in authoring. Kristina herself is the author of the booklet even though she doesn’t choose herself what to write in it and is left unsure why certain things are more important to write down than others. Kristina experiences a need to legitimize the content of the booklet by the authority of the teacher as can be seen in her utterance on p. 12. The theory booklet seems to act like a ‘security system’ where she ‘catches’ the mathematics she is supposed to learn and which she can return to between lessons, as she does homework or prepares herself to an assessment activity. She likes lessons where she is explicitly told which rules and examples that have to go into the booklet, when the teacher “not only tells things, but you write it up in the theory booklet” (Int3). The telling of important mathematics has, she thinks, to be accompanied by writing it up on the board in order to secure the chance of copying into the theory booklet. As the use of the theory booklet diminished during school year nine it meant a lack of sense of order to Kristina. ”You always had order in your theory booklet and you new exactly, mathematics was not easier, but you new where you were and that was a good thing” (Int5).

Purpose of and division of responsibility within school mathematical practice

To Kristina learning mathematics in school is about learning ”rules and systems”, “what we are busy with in the theory booklet” (Int3), ”things to remember, different formulas and ways of calculating” (Int5).

In the theory booklet Kristina writes down all those things that the teacher during plenary session indicates to her as important. ”There can be different alternatives and then we write all the alternatives in the theory booklet” (Int3). The indication of importance is mainly of three types: (1) the *teacher is explicitly telling* what is important “write down this”, ”this belongs to the theory”; (2) the indication emerges in *discovery* plenary sessions where the teacher guides a dialogue with the students. Kristina describes the dialogue as a guessing game where the students can win by discovering the mathematical content of the teacher’s thoughts: ”everybody can guess and the teacher tells you if you are closer to or further away from the correct answer and then you write a rule” (Int2). If the students can win this game without understanding the content

and conclusions of the dialogue, it shows a remarkable resemblance with dialogues characterized by the so called 'Jourdain-effect'. The Jourdain-effect is described by Brousseau as a situation where the teacher acts as if he recognized evidence of knowledge in an answer despite that the response may actually be motivated by very trivial causes and meanings (Brousseau, 1984); (3) the indication of importance is established *from patterns in particular examples and solution models* that are discussed at plenary sessions and written on the board.

We write something on the board and then you yourself have to, yes, this is maybe important and the you have to write it ... not as much rules and systems, it is only examples, examples, examples, and then when you go through the theory booklet, you have to figure out yourself how you have done it. And if we ask him he says: Yes, those belong to the theory. (Int4)

The division of responsibility between the teacher and Kristina within the ordinary lesson design is such that the teacher presents and explains procedures; she memorizes the procedures, retrieves the right procedure(s) for solving a task and shows that she knows how to perform the procedure to find the answer to the task. She relates learning school mathematics to a person's ability to memorize rules and to discover procedural similarities in tasks. Thus, she accepts it as natural that she shows the teacher how she solves tasks because the teacher "has to know how you do it to be able to teach you in a manner that you understand". If Kristina knows 'how to do it', it means that she is able to perform the correct steps in a solution process. But Kristina finds it difficult to grasp the mathematical logic behind these steps unless the teacher has given a clear explanation. On the one hand, she emphasizes her need for careful and non-ambiguous explanations by the teacher to maintain some order in the growing amount of "things to remember like rules, formulas, ways of calculating". On the other hand, she finds the acts of active listening, memorizing and rule following as demanding an ever-increasing level of concentration, self-discipline and acceptance of dullness and boredom. Kristina's earlier positive experiences of participating in school mathematical practice, because she then felt that she "knew everything", develops during the school years into a negative experience where she, because of perceived personal shortcomings, have a sense of not gaining access to the understanding she strives for. To begin with, she secures her mathematical understanding relying both on her thinking and intuition and on the teacher's explanations. Later she withdraws more and more from her own cognitive capabilities and shows a stronger dependency on outside resources; teacher, textbook, theory booklet, classmates. To learn the "rules and systems" of school mathematics she has to be very good at following and catching up, however, her learning goal is impeded by the rules and systems of the classroom interaction of which her own actions are a constitutive part.

/.../ I do also remember a time in primary school, when I found mathematics to be super easy and then it was fun to be at lessons, because then you knew everything and you was able to raise your hand when they asked something and there was

not the same need to be alert all the time, to be, like, you could take it a little bit easy. Not that you should do it then, but it just gets like ... So then it is fun. But, now it is like, you have to listen all the time to what he is saying and you get so tired. Like, you cannot always go on any longer. Directly as you talk just a little bit with your seatmate then it goes, oh, now I don't understand anything again. It is rather strenuous. You should have a better ability to concentrate. (Int5)

Nature of classroom interaction

Social constructivist studies have shown the significance of social interaction for the meanings individuals develop in school mathematical practice (see e.g. Cobb, Wood, Yackel, & McNeal, 1992; Yackel, 2001). On the other hand, student-teacher or student-student interaction in the classroom is no guarantee for interactions to be mathematical or for significant mathematical knowing to emerge, as the student might appropriate other aspects of the mathematical practice than the teacher intended (Boaler, 1997; see e.g. Moschkovich, 2004; Wood, 1998). Thus, one way to understand Kristina's experiences of participating in school mathematical practice is through her utterances about the social conduct and interaction in the classroom.

Kristina states that collaborative discussions in groups are rare within the ordinary lesson design. She further declares that the school mathematical practice is different from the practice in other school subjects in this regard.

/.../ we never are in groups as in other school subjects or, like, whole-class discussions about what we shall calculate. You just ask those you sit next to, you don't do any greater collaboration. (Int3)

When prompted about the effects of more student collaboration, she visualizes problem-solving activities as a possible setting for explaining and comparing solutions in groups. This is a feature of the interaction commonly found in 'inquiry' classrooms (Cobb et al., 1992). But, referring to constraints of the usual task setting, Kristina asserts that working on an individual basis has to be the normal feature of social conduct. In another interview she mentions the shame she feels when she gives a wrong answer in public. Thus, a more collaborative involvement in whole-class interaction besides public exchanges between individual students and the teacher, might perhaps prevent this to happen. However, utterances in all interviews show that the individualist feature of school mathematical practice prevails during lower-secondary school.

According to Kristina, cooperation with other students is generally a matter of informally checking procedures and answers, a usual mode of working in mathematics classrooms (see e.g. Goos et al., 1999). The students are not responsible for explaining solutions to each other but Kristina finds it easier to follow a class-mate who explains why her answers are wrong, than to follow the teacher explaining the same thing. In every interview she mentions one classmate who is especially important to her. Also, to resolve issues by themselves seem perfectly natural to Kristina as they otherwise, at worst,

wouldn't be doing anything as the teacher's time to intervene with each student is limited. But the final helper and dispenser of knowledge is anyhow the teacher, to whom Kristina turns as he walks around in the classroom, and when classmates have not been able to give her the support she needs to get the answer she is reaching for.

We do work alone all of us. But it automatically gets like if you don't understand something you ask the one who sits besides you. So it usually becomes pair-work or group-work anyway, like that you ask for some help from everybody and then it is like that if you don't understand, haven't yet got an answer, then it is (teacher) you ask. It is anyhow like that, that he has to come to everybody, so it is the fastest way to ask the one you sit next to, so you talk to everybody. (Int5)

Hand raising is a silent and visible sign dynamically used as a tool by both students and teachers in school practice (Sahlström, 2001). The meaning of "to discuss" in the mathematics classroom is for Kristina first of all to raise her hand and deliver an answer to a question set by the teacher. In year 8, Kristina explains, "when he (the teacher) asks then you can raise your hand and tell how you think you could calculate it". And she continues with a remark that the possibilities to participate in this type of discussion are higher if the tasks allow quick calculating in the head:

.../ but we haven't had as much as in 7th grade those tasks where you like only tell the answer, calculate in your head, but in 7th grade we had more such that we kind of circled, everybody in class got a task from the textbook and was supposed to calculate in the head and tell, but we haven't had so much of that now, it is because we haven't had so much that we can calculate in the head, so that's why. (Int3)

From Kristina's utterances I can conclude that hand raising serves different functions depending on how Kristina experiences the situation. Usually hand raising is a sign of willingness to contribute in public with a suggestion for a solution or a valuable idea. Sometimes it is a signal from Kristina to the teacher that help is needed. But it is also an activity marker and a way to please the teacher. Being an active hand-raiser is the same as being a good student which may pay off immediately in public praise from the teacher or later on in better grades. Hand raising is definitely a visible indication of appropriated understanding from Kristina's point of view.

I just think that before I understood much more and then I raised my hand much more. But now I don't understand really well and I have not raised my hand that much and then maybe I haven't done homework as much so my grade will drop I am sure (Int3)

When prompted about reasons for a drop in grades and how she thinks she can prevent this to happen, she refers to the complex relationship between hand raising, understanding, being able to do homework and getting better test results.

Well, I have thought that I will sharpen up and do my homework real properly and in that way you are of course more active during lessons and then you can

raise your hand and say and try and understand. Then you don't need to read and swot up so much to the test, because you can manage better then. (Int4)

In addition to the discovery type of activity already discussed, there is very little indication in Kristina's utterances of communicative activities where students are responsible for developing the mathematical content of the lessons until year 9 when there seems to be a shift. An indication of how Kristina experiences this shift was focused above (p. 14-15). Participation in public interaction in the classroom is monitored by the teacher while the students are given the role of answering the teacher's questions. The goal of the interaction is as a rule set by the teacher and with reference to delivery of answers or solutions to set tasks or discovering mathematical structures from examples. The dilemma from Kristina's point of view is, that she often seems to find herself in a position where she doesn't have the ability to participate fully, and remains in the periphery of the communicative process where the mathematical content of the interaction emerges. Thus, she experiences the school mathematical practice as problematic because it pushes her into a position where her strategy of participation becomes cognitive disengagement.

Sometimes you can hear from the teacher that you should know this from primary school and then you sit there and you should figure out the answer or the solution. But as you can't find it, you aren't able to concentrate and then you sit and wait until somebody says something and when they start to talk, you don't understand what they are talking about (Int4).

Another aspect of the classroom interaction that creates problems for Kristina is the taskrelated meta-rule that the teacher starts a plenary session with explaining tasks at the board when sufficiently many students have individually finished a certain amount of tasks. This creates a dilemma for Kristina as she experiences a lack of possibility to understanding and possibly a feeling of being ignored by the teacher.

Dominance of 'tasks'

Students working with set tasks, mostly with textbook tasks, is a widely spread and contested approach in school mathematical practice (Boaler, 1998; Johansson, 2006; Røj-Lindberg, 1999; Törnroos, 2001). Students usually work with set tasks in order to display knowledge, not because they have a personal interest in getting the answers or making inquiries into mathematical ideas. Traditionally, each task forms a restricted whole and the student does one task after the other. The choices of methods to do the tasks are usually limited and set by the subject area presented in the book. A solved task, finding an answer, leads the student to the next task or to the next subject in the textbook. Tasks seldom invite the students to formulate own problems and questions (Mellin-Olsen, 1991). Tasks can function like a Russian-doll where a solved task includes solving sub-tasks. In the following paragraph, taken from the second interview in year 7, Kristina describes features of so called 'text tasks' in a test.

Text tasks I found fairly easy, it was just some subtraction and addition, but otherwise you have to read the task very carefully and then sometimes it might come like several times that you have to calculate the first part this and this and then you have to subtract this from that and so on. And then of course, you have to write the answer down there with a unit because it is a text task and you have to know what it is all about. (Int2)

According to Stieg Mellin-Olsen (1990, 1991), the influence of the didactical use of different types of tasks is so deeply rooted in school mathematics practice, and in the teachers' images of school mathematics, that the teaching tradition of school mathematics can be conceptualized as a 'task discourse'. Thus, transformation of participation in school mathematical practice can be seen as an issue of re-negotiating the meta-rules of the 'task discourse'.

In the five interviews with Kristina I found significant traces of her participation in a school mathematical practice that continued to be very much dominated by a similar type of task-discourse. In fact, substantial parts of all interviews, and especially of the second to fifth interviews, expose aspects of Kristina's experiences in task related actions and activities. This is no surprise, as one considers the more than thousand textbook tasks that was part of the school mathematical practice each school year. Generally, Kristina starts to talk about tasks in her descriptions of activities that occur within the frame of ordinary lesson design, but also in descriptions of homework, project work and of written evaluations as individual tests and problem-solving activities.

An indication of the dominance of tasks is that Kristina in all interviews introduced 'task'-related issues into the conversation without being prompted. Out of 65 paragraphs with task-related utterances from Kristina, only 15 are answers to task-related questions. In the following extract from the first interview, Kristina describes monitoring of affordance to cognitive challenge as a task related issue. With the help of appropriate tasks the teacher can set the cognitive challenge on a level that Kristina finds comfortable.

I: Do you think that you get enough demanding questions and tasks?

Kristina: Yes. Or, I think that they are not too difficult, but you kind of have to start with something easy so you can manage the difficult things later when you get those tasks. And not start directly with the difficult.

An interesting aspect in the first interview is that Kristina experiences such a high confidence in her ability to do tasks that she usually ignores the answer key (see e.g. Erlwanger, 1973, on the impact of answer key on students' knowing). She explains that she is used to manage without an answer key because of the lack of textbook in primary school. Nevertheless, having a textbook is the most positive experience next to learning mathematics that she can think of in the beginning of lower-secondary school.

I guess it is what you learn... Yes! And then that we have got a mathematics textbook. That is good. I didn't like it when we got papers before. It is much better to have a textbook.

However, Kristina's positive identification with the textbook might not primarily be related to the answer key, but more to the book as an essential sign of order for her in a school mathematical practice so heavily dependent on tasks for learning 'rules and systems'. In such a practice, "by default the book has epistemic authority: teachers explain assignments to pupils by saying this is what *they* want you to do here, and the right answers are found in the answer key" (Boaler & Greeno, 2000, 181)

The influence of textbook on classroom interaction is explained by Kristina when she relates the necessity of individual work in the mathematics classroom to the nature of the textbook tasks. As the solution procedures of these tasks are already set in advance, there is no need for any real collaboration with others. In other types of tasks, in project work and in problems set by the teacher, she sees the potential for learning from other students and to adopt solutions developed by others for her own use. But, she explains, "if you have ordinary textbook tasks you must try to manage them by yourself" (Int3).

To sum up, from Kristina's task related utterances, I conclude that she expects the school mathematical content to emerge from tasks, to be practiced within set tasks and to be assessed with tasks. Taken together, there are indications in the five interviews with Kristina that she continued to see

- tasks as a significant feature of the ordinary lesson design,
- tasks as tools for regulating the tempo of lessons, the social interaction and the cognitive challenge,
- tasks as a significant feature of homework. She should do a certain amount of tasks before she meets the next field of mathematical knowledge,
- tasks as a significant feature of project work characterized as 'school mathematical', i.e. "you know directly when you see them that they are school work" (Int2),
- tasks as a significant feature of tests,
- tasks as 'containers' for mathematical procedures. By following the steps in a solution she learns the rules for doing a particular task. Kristina can retrieve forgotten procedures from memory with the help of repetition with tasks set by the teacher in plenary sessions and more individual practice in doing tasks,
- tasks as a rationale for written solutions. A written solution displays the cognitive processes of the solver. If she copies solutions from the board

she has caught a written trajectory of the cognitive process of the teacher.

- tasks as catalysts for personal satisfaction and as indicator of understanding. The quicker she manages to solve a task the better she understands.

Invisibility of school mathematics

At several occasions during the interviews, Kristina spontaneously in her utterances compared her experiences of school mathematical practices with her experiences of practices in other school subjects. In the following utterance she compares mathematics with textile work. The statement was given in the third interview, at a stage when Kristina expressed a negative emotional relation to mathematics. The only possible way out from the boredom and dullness she experienced in mathematics as well as in physics and chemistry was, as she saw it, to change her own behaviour: to motivate herself into being more alert, to accept that feeling bad about mathematics and that lack of understanding is part of the game. At the time, I was quite astonished by her very strong negative relation to school mathematics and asked her in one of the follow up questions to look back and reflect on whether there had been some changes during lower-secondary school. No, she said

mathematics has never been any fun subject ... like textile work is fun, you can do a lot and you kind of see what you are doing, but mathematics is dull, you cannot do anything about it. And there are those that do like mathematics, you have to have those subjects anyhow. (Int3)

The difference between the practices of textile work and the practices of school mathematics is a matter of visibility. In textile work she has experienced participation in doing things that are visible. This has given her a sense of satisfaction that she has not gained from participating in school mathematics. Mathematics at school is from Kristina's point of view closed into a "rules and systems" – practice where she is occupied with learning "things to remember, different formulas and ways of calculating" (Int5), cognitive and internal activities that indeed may be very far from externalised activities where you can "see what you are doing".

Coda

On the basis of considerations that are included in the theoretical frame for this paper, I argue, that the development of Kristina's experiences is situated in and cannot be separated from the development of the school mathematical practice. Even though I in this paper have chosen to focus on Kristina and her story, it is strictly a research based decision. I separated the experiences of Kristina but nevertheless, I see her experiences as integrated in and emerging out of the particular school mathematical practice where she was a member. Thus, I argue,

that her story can be seen as traces of a trajectory of participation in this practice. In relation to school mathematics her expectations and attitudes have developed precisely because she has participated in a particular school mathematical practice. Not only did she learn mathematics by participating in this practice, she also developed her image of what it takes to learn mathematics in this particular practice. Taken together her school mathematical knowing can be described as a kind of school mathematical disposition. Thus, participating and engagement in school mathematical practice can be seen as a process that both produced and transformed her identity as learner and school mathematician (Boaler & Greeno, 2000; Lerman, 2000).

This opens up for the questions how participation in school mathematical practice affected her identity as learner of mathematics and why participation in school mathematical practice did not boost Kristina's confidence in her ability to learn mathematics even though she was considered to be "a good student".

One of my hypothesis is that the her "rules and systems" - experiences coalesced into a very limited view on the purpose of mathematical activities as finding a precise procedure for each task or type of tasks. Tasks were indeed, as she saw it, used over and over again in school to check her ability to quickly remember and whether she "understood" correctly which procedure to apply. Along with the expansion of the mathematical content, she, in short, became totally overwhelmed by the load on her memory and presumably, as a result, her feeling of being alienated from the whole business of doing mathematics was growing constantly. She experienced the practice as closed and the practice closed her out.

The statement below about mathematics given by Kristina as a grown-up strengthens my hypothesis. In the conversation I had with her about her school mathematical experiences in lower-secondary school, she argued against my suggestion that mathematical activities and problems perhaps can be seen as having a quite open character. On the contrary, she said, there are a lot of similar problems, the difficulty is to define what type of problem it is, and then to pick the right procedure. Kristina argued that

there are many similar problems in mathematics, you have to be able to define the type of problem and which procedure to use to solve it. Every problem belongs to a certain group of problems. It is difficult to know which procedure to combine with a particular problem if you cannot define the type of problem. And as the amount of procedures increases this dilemma grows, and is worsened by the high pace of teaching. You have to know the types you have gone through before and then they just grow and grow and it gets into some sort of chaos. It becomes too much.

In one of her last statements in the fifth interview, Kristina referred to her decision to enter upper-secondary school where she reasoned she would meet a tough time with mathematics. This expectation of hers turned out to be partly fulfilled. When I met Kristina again, she told me how she had struggled with

mathematics the first year in upper-secondary school. But then a decision to study biology at the university level completely changed the scenario. Then she became motivated to study mathematics in school. Voluntarily she even repeated some courses, and she had no problem to pass the examination with a high grade in mathematics. However, after secondary school she eventually chose pedagogy as her field of study, because, as she said, “*in pedagogy there are no right answers*”; a statement that brings with it a clear, albeit implicit, question: why is there such a strong focus on right answers in school mathematics?

References

- Alexandersson, M. (1994). Den fenomenografiske forskningsansatsens fokus. (Focus on the phenomenographic research design) In P. Svensson (Ed.), *Kvalitativ metod och vetenskapsteori* (pp. 111-136). Lund: Studentlitteratur.
- Atweh, B., Forgasz, H., & Nebres, B. (Eds.). (2001). *Sociocultural research on mathematics education*. London: Lawrence Erlbaum.
- Ben-Chaim, D., Fresko, B., & Carmeli, M. (1990). Comparison of teacher and pupil perceptions of the learning environment in mathematics classes. *Educational Studies in Mathematics*, 21, 415-429.
- Björkqvist, O. (1993). Social konstruktivism som grund för matematikundervisning. *Nordisk Matematikdidaktik*, 1(1), 8-17.
- Boaler, J. (1997). *Experiencing school mathematics. Teaching styles, sex and setting*. Buckingham: Open University Press.
- Boaler, J. (1998). Open and closed mathematics: Students experiences and understandings. *Journal for Research in Mathematics Education*, 29(1), 41-62.
- Boaler, J., & Greeno, J. G. (2000). Identity, agency and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171-200). London: Ablex Publishing.
- Bodin, A., & Capponi, B. (1996). Junior secondary school practices. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International Handbook of Mathematics Education*. Dordrecht: Kluwer.
- Brousseau, G. (1984). The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics. In H. Steiner (Ed.), *Theory of Mathematics Education, Occasional Paper 54* (pp. 110-119). Bielefeld: IDM.
- Bruner, J. (1990). *Acts of meaning*. London: Harvard University Press.
- Bruner, J. (1996/2002). *Kulturens väv. Utbildning i kulturpsykologisk belysning*. Göteborg: Daidalos.
- Bryman, A. (2002). *Samhällsvetenskapliga metoder*. Stockholm: Liber.
- Burton, L. (1994). Whose culture includes mathematics? In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 69-83). Dordrecht: Kluwer.

- Cobb, P. (2000). Constructivism in social context. In L. P. Steffe & P. W. Thompson (Eds.), *Radical constructivism in action. Building on the pioneering work of Ernst von Glasersfeld*. London: RoutledgeFalmer.
- Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal*, 29(3), 573-604.
- Corbett, D., & Wilson, B. (1995). Make a difference with, not for, students: A plea to researchers and reformers. *Educational Researcher*(June/July), 12-17.
- Elliott, J. (1991). *Action research for educational change*. Milton Keynes: Open University Press.
- Erlwanger, S. H. (1973). Benny's conception of rules and answers in IPI mathematics. *Journal of Children's Mathematical Behavior*, 1(2).
- Ernest, P. (1998). *Social constructivism as a philosophy of mathematics*. Albany: State University of New York Press.
- Goodchild, S. (2001). Students' goals. A case study of activity in a mathematics classroom. Oslo: Caspar Forlag.
- Goos, M., Galbraith, P., & Renshaw, P. (1999). Establishing a community of practice in a secondary mathematics classroom. In L. Burton (Ed.), *Learning mathematics. From hierarchies to networks* (pp. 36-61). London: Falmer Press.
- Hansén, S.-E., & Myrskog, G. (1994). Läroplansreform och inläringssyn. In A. Wickman-Skult (Ed.), *Grundskolan förändras. Läroplansarbetet i blickpunkten*. Helsingfors: Utbildningsstyrelsen.
- Hiebert, J., & Stigler, J. W. (1999). *The teaching gap*. New York: The Free Press.
- Johansson, M. (2006). Textbooks as instruments. Three teachers' way to organize their mathematics lessons. *Nordisk Matematikk Didaktikk*, 11(3), 5-30.
- Kupari, P., & Haapasalo, L. (Eds.). (1993). Constructivist and curriculum issues in school mathematics education. Yearbook 1992-1993. Jyväskylä: University of Jyväskylä.
- Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19-44). London: Ablex Publishing.
- Lincoln, Y. S., & Guba, E. G. (1985). *The naturalistic inquiry*. London: Sage.
- Linnanmäki, K. (2002). *Matematikprestationer och självuppfattning (Mathematical achievement and self-concept (diss.))*. Åbo: Åbo Akademis Förlag.
- Mellin-Olsen, S. (1990). Oppgavediskursen. In J. Björneboe (Ed.), *Matematikundervisning og demokrati* (pp. 47-64). Roskilde: IMFUFA Roskilde Universitetscenter.
- Mellin-Olsen, S. (1991). The double bind as a didactical trap. In A. J. Bishop, S. Mellin-Olsen & J. von Dormolen (Eds.), *Mathematical knowledge: Its growth through teaching* (pp. 39-59). Dordrecht: Kluwer.
- Miller, P. J., & Goodnow, J. J. (1995). Cultural practices: Toward an integration of culture and development. In J. J. Goodnow, P. J. Miller & F. Kessel (Eds.), *Cultural practices as contexts for development* (pp. 5-16). San Fransisco: Jossey-Bass.

- Moschkovich, J. N. (2004). Appropriating mathematical practices: A case study of learning to use and explore functions through interaction with a tutor. *Educational Studies in Mathematics*, 55, 49-80.
- Nardi, E., & Steward, S. (2003). Is mathematics T.I.R.E.D? A profile of quiet disaffection in the secondary mathematics classroom. *British Educational Research Journal*, 29(3).
- Nunes, T. (1995). Cultural practices and the conception of individual differences: Theoretical and empirical considerations. In J. J. Goodnow, P. J. Miller & F. Kessel (Eds.), *Cultural practices as contexts for development* (pp. 91-103). San Fransisco: Jossey-Bass.
- Rogoff, B. (2003). *The cultural nature of human development*. New York: Oxford University Press.
- Røj-Lindberg, A.-S. (1999). *Läromedel och undervisning i matematik på högstadiet. En kartläggning av läget i Svenskfinland*. Vasa: Svenskfinlands läromedelscenter.
- Røj-Lindberg, A.-S. (2003). "I was trapped in a pattern": The dilemma of change. *S MDF Medlemsblad*, December (8), 6-23.
- Røj-Lindberg, A.-S. (2006). "Jag satt fast i mönster" – Metaforer i lärares berättelser om matematikundervisning i förändring. ("I was trapped in a pattern" – Metaphors in teachers' stories about mathematics teaching in transition In A.-S. Røj-Lindberg (Ed.), *Perspektiv på kunskapens och lärandets villkor* (pp. 113-124). Vasa: Pedagogiska fakulteten vid Åbo Akademi.
- Sahlström, F. (2001). The interactional organization of hand raising in classroom interaction. *Journal of Classroom Interaction*, 37(2).
- Sfard, A. (2000). On reform movement and the limits of mathematical discourse. *Mathematical thinking and learning*, 2(3), 157-189.
- Skott, J. (2000). *The images and practice of mathematics teachers*. Unpublished Ph. D. Diss., The Royal Danish School of Educational Studies.
- Stigler, J. W., & Hiebert, J. (1999). *The teaching gap*. New York: The Free Press.
- Säljö, R. (2005). *Lärande & kulturella redskap. Om lärprocesser och det kollektiva minnet*. Falun: Norstedts Akademiska Förlag.
- Törnroos, J. (2001). *Finnish mathematics textbooks in grades 5-7*. Paper presented at the Second Scandinavian Symposium on Research Methods in Science and Mathematics Education, June 18-19, Helsinki.
- Wellington, J. (2000). *Educational research*. London: Continuum.
- von Glasersfeld, E. (Ed.). (1991). *Radical constructivism in mathematics education*. London: Kluwer.
- Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In A. Sierpiska (Ed.), *Language and communication in the mathematics classroom* (pp. 167-178). Reston: NCTM.
- Yackel, E. (2001). Explanation, justification and argumentation. In v. d. Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the international group for the PME* (Vol. 1, pp. 9-24). Utrecht: Freudenthal Institute.

Structuring students' view of learning mathematics

Bettina Rösken
University of Duisburg-Essen

Markku S. Hannula^{1,2}, Erkki Pehkonen¹, Raimo Kaasila³, Anu Laine¹

¹University of Helsinki, ²Tallinn University,

³University of Lapland

Students' mathematics-related beliefs are a decisive parameter for engagement and success in school. Particularly, we are interested in studying the complex structure of students' view of learning mathematics. We analyzed our data by means of exploratory factor analysis and carefully tested different factor solutions. We finally obtained seven dimensions described by reliable scales structuring this construct. Thereby, three factors relate to personal beliefs regarding competence, effort and confidence, two factors primarily to social context variables like teacher quality and family encouragement, one to emotional expressions concerning enjoyment of mathematics and one to perceived difficulty of mathematics. Participants in our study were 1436 grade 11 students from randomly chosen secondary schools from all over Finland.

Introduction

Students' mathematics-related beliefs are a decisive parameter for engagement, motivation and success in school. Particularly, motivation for achievement is influenced by personal beliefs about the reasons events turn out the way they do (Zimbardo, McDermott, Jansz and Metaal, 1995). The study of students' beliefs about mathematics and its teaching and learning has received much attention in recent years. A lot of results have been presented examining beliefs for different groups (students, teachers) under diverse conditions. These studies are in most cases descriptive, for example reporting typical beliefs held by students (e.g. Ma & Kishor, 1997). Some of the studies compare student beliefs in different countries (e.g. Pehkonen, 1994) or according to background variables such as gender (e.g. Frost, Hyde and Fennema, 1994). Furthermore, most of the studies of beliefs have been carried out with a separate focus on cognitive, motivational or affective aspects and only few contributions address explicitly belief systems (Op 't Eynde & De Corte, 2003). Green (1971) introduced this term and although the importance of the systematic nature of beliefs is widely

acknowledged (e.g. Schoenfeld, 1985), there is a clear lack of studies elaborating on belief systems.

We, however, focus explicitly on studying the structure of students' mathematical beliefs, and to emphasize this, we use the term 'view of learning mathematics' (Schoenfeld, 1985; Pehkonen, 1995; Pehkonen & Törner, 1996). We are not only interested in students' view of mathematics as a subject but in their view of learning mathematics. Therefore, we consider this view as composed of several but related dimensions pertaining to aspects of both internal variables like learning ability or confidence and external variables like experiences in class or at home.

Hitherto, we have explored the relational structure of teacher students' view of mathematics (Hannula, Kaasila, Laine & Pehkonen, 2005a,b). This study led to eight scales describing these students' view of mathematics, and particularly three dimensions that were closely related.

Now, we have used a modified questionnaire to collect and analyze data from a sample of secondary school students. We will explore what dimensions describe their view of learning mathematics, how they are related and what structure they generate. In this paper we will extensively report about the major methodological decisions concerning our factor analysis; further results can be found in Rösken, Hannula, Pehkonen, Kaasila and Laine (forthcoming).

Aspects of students' view of learning mathematics

Human learning in general can be described by three components, namely cognition, motivation, and emotion. Most of research addressing these psychological categories of the mind has been carried out separately by elaborating on one of those (Meyer & Turner, 2002; Hannula, 2004). We, however, pay attention to students' cognition, motivation and emotions as crucial elements of their view of learning mathematics. In all these areas beliefs play an important role, for example, as attributions about personal success or failure, and they also provide a base for related affective reactions. Furthermore, motivation for achievement is affected by personal beliefs about the reasons for events (Zimbardo et al., 1995). Current research on motivation focuses on interpreting the dimensions of expectancy-value models as dispositional components in terms of generalized beliefs (Weiner, 1992; Buehl & Alexander, 2005). These beliefs represent one's subjective estimation of competencies and abilities relevant for learning.

In the literature, beliefs have been described as a messy construct with different meanings and accentuations (Pajares, 1992). The term belief has not yet been clearly defined and there are many variations of this concept and of belief systems as well (Furinghetti & Pehkonen, 2002). However, we consider

mathematical beliefs as personal philosophies or conceptions about the nature of mathematics and its teaching and learning (Thompson, 1992). Beliefs cannot be regarded in isolation; they must always be seen as part of a belief system (Green, 1971). These beliefs systems can be characterized by three dimensions as there are quasi-logicalness, psychological centrality, and cluster structure. Op 't Eynde, De Corte and Verschaffel (2002) as well, consider explicitly the structure of beliefs about mathematics but with a different focus. They provide a framework of students' mathematics-related beliefs that is based on a review of research on this construct. Constitutive dimensions are object (mathematics education), self, and context (class), which further lead to several sub-categories:

1. Beliefs about mathematics education
 - a) beliefs about mathematics as a subject
 - b) beliefs about mathematical learning and problem solving
 - c) beliefs about mathematics teaching in general
2. Beliefs about self
 - a) self-efficacy
 - b) control beliefs
 - c) task-value beliefs
 - d) goal-orientation beliefs
3. Beliefs about social context
 - a) beliefs about the social norms in their own class (- the role and functioning of the teacher - the role and functioning of the students)
 - b) beliefs about socio-mathematical norms in their own class (Op 't Eynde et al., 2002, p. 28)

In our previous study (Hannula et al., 2005a) we found eight scales describing teacher students' view of learning mathematics. Two of these dimensions relate primarily to the student teachers' past learning experiences with their teacher and family, three to personal beliefs regarding talent, effort and difficulty of mathematics, one to emotions and two to the person's expectations about future success. All dimensions, except for the emotional one, could be assigned to the main categories and most of the subcategories suggested by Op 't Eynde and De Corte (2002).

Methodology

Instrument and Participants

The view of mathematics indicator has been developed in 2003 as part of the research project "Elementary teachers' mathematics", financed by the Academy of Finland (project #8201695). It has been applied to and tested on a sample of student teachers and was slightly modified for the present sample. That is, items addressing specifically aspects of teaching mathematics like *View of oneself as mathematics teacher* (D1-D6) and *Experiences as teacher of mathematics* (E1-

E7) were removed. More information about the development of the instrument can be found e.g. in Hannula et al. (2005a). The statements in the questionnaire are grouped around the following topics:

- Experiences as mathematics learner (A1-A29)
- Image of oneself as a mathematics learner (B1-B16)
- View of mathematics and its teaching and learning (C1-C12).

Further, the B-items comprise a self-confidence scale containing ten items (B1 – B10) from the Fennema-Sherman mathematics attitude scales (Fennema & Sherman, 1976). The students were asked to respond on a Likert scale (5 point, agree to disagree) to statements such as the following:

- I have worked hard to do mathematics
- My family has encouraged me to study mathematics
- I can get good grades in mathematics
- I would have needed a better teacher.

The participants in our study came from fifty randomly chosen schools from all over Finland, including classes for both advanced and general mathematics. The respondents were in their second year course of mathematics in grade 11. Altogether, 1436 students filled in the questionnaire and returned it.

Process of factor analysis

The basic assumption of factor analysis is that underlying dimensions can be used to describe a complex phenomenon (Norusis, 1985). Although our previous study provided insight into the beliefs structure of a specific sample, we still aimed at exploring the field of structuring students' view of learning mathematics and therefore, we employed an exploratory factor analysis instead of a confirmatory one. We used maximum likelihood factor analysis with oblique rotation for determining useful and statistically robust dimensions regarding this construct. As one advantage, this extraction method allows making inferences from sample to population (Kline, 1994). Therefore, the sample of 1436 students is large and adequate enough. Maximum likelihood analysis requires multivariate normality and we found all distributions of the measured variables fulfilling this request (skew < 2, kurtosis < 7). Further, we chose to use an oblique rotation. We do not perceive the dimensions of the view of mathematics to be independent from each other. Therefore, an oblique rotation will lead to a better estimate of factors, because it derives factor loadings based on the assumption that they are correlated (Fabrigar, Wegener & MacCallum, 1999); as rotation method we employed direct oblimin.

Besides these methodological considerations, the next choice to take after extraction is to retain for rotation. We based this decision on passing through the following procedure. First, a principal component analysis was calculated to determine eigenvalues and to conduct a scree test. The Kaiser "eigenvalue greater than one"-rule led to an initially extraction of 8 factors and the scree plot suggested the extraction of 5 to 8 factors. Then, we used maximum likelihood

factor analysis to obtain by successive factoring an appropriate set of factors where the number differed from 6 to 8. In the first extractions we obtained items with low communality; those with very low values were removed. Even when communality of an item is low, it can be reliable, provided that a factor is measuring a variable very broadly. We therefore chose to only exclude items with very low communality ($h^2 < 1$). The remaining items had good communality and factor analysis was repeated. For comparing the different factor solutions we considered the statistic tests offered by maximum likelihood factor analysis. Additionally, we based our decision on the following criteria (Gorsuch, 1983; Fabrigar et al., 1999): the factors should be homogenous and meaningful regarding content, internal consistency estimated by Cronbach's alpha has to be sufficiently high, each factor should contain enough items with high loading and a factor must clarify a sufficiently high amount of variance. As the best fit for the data we looked for item loadings above .30 with no or few crossloadings and a minimum of three items per factor.

Maximum likelihood factor analysis provides statistical tests for the significance of each factor. As also observed in our previous study (Hannula et al., 2005a), the goodness-of-fit for all models are poor. We refer to Kline (1994, p. 160) and his argumentation that this problem can be overcome by using the ratio of χ^2 to degrees of freedom whereby a ratio of between 2 and 3 is said to be acceptable and the smaller the ratio the better is the fit. The ratios of the χ^2 to degrees of freedom for the 7 and 8 factor solutions are reasonable good ($\chi^2 = 3.18$ and $\chi^2 = 2.71$), whereas the ratio for the 6-factor solution is 3.75; the best ratio is obtained for the 8-factor solution.

We found 5 factors that were consistently presented in all tested solutions with a sufficient amount of high loading items; only in the 8-factor solution the last factor consisted of three items with loadings from .36 to .49. Since Cronbach's alpha for this factor also was low, .484, we rejected this solution. For the 7-factor as well as the 6-factor solution reliability (Cronbach's alpha) was satisfactory with coefficients between 0.800 and 0.912. Here, factors 2 to 6 were stable and factor 7 coincided with factor 1 when an extraction of six factors was forced. Reliability for separating this first factor was high and we found good theoretical argumentation to favor the 7-factor solution, which clarifies 59 % of total variance. In the 7-factor solution the first factor clarifies 30 % of variance, the factors 2 to 4 between 5.1 % and 7.5 % while the contributions of factors 6 and 7 are clearly lower than 5.0 %. Despite this, and with respect to the other criteria, we obtained the best fit to the data by accepting the 7-factor solution.

Results and discussion

Factor analysis led us to seven dimensions describing students' view of learning mathematics. In the following we present the factors, the related items as well as the factor loadings and Cronbach's alpha.

F1 Competence (Cronbach's alpha = .91)

- B 8 I am no good in math -.77
- B 6 I am not the type to do well in math -.70
- B 3 Math has been my worst subject -.43
- A15 I have made it well in mathematics .43
- B 4 Math is hard for me -.40

F2 Effort (Cronbach's alpha = .83)

- B13 I am hard-working by nature .86
- B12 I have not worked hard enough -.85
- A 4 I have worked hard to learn mathematics .64
- B15 I always prepare myself carefully for exams .62
- B11 My attitude is wrong -.55
- B16 It is important for me to get good grades in mathematics .30

F3 Teacher Quality (Cronbach's alpha = .81)

- A27 I would have needed a better teacher -.75
- A 3 The teacher has not been able to explain the things we were studying -.73
- A21 My teacher has not inspired me to study mathematics -.59
- A26 My teacher has been a positive example .57
- A 6 The teacher has not explained what for we need the things we were learning -.47
- A 5 I have not understood the teacher's explanations -.45
- A24 The teacher has hurried ahead -.43
- C10 If the teacher is too good in mathematics he or she cannot explain clearly -.41

F4 Family Encouragement (Cronbach's alpha = .80)

- A17 The importance of competence in mathematics has been emphasized at my home .83
- A23 My family has encouraged me to study mathematics .83
- A18 The example of my parent(s) has had a positive influence on my motivation .67

F5 Enjoyment of Mathematics (Cronbach's alpha = .91)

- A 7 It has been boring to study mathematics -.71
- A 8 Doing exercises has been pleasant .70
- C 1 Mathematics is a mechanical and boring subject -.70
- A13 To study mathematics has been something of a chore -.65
- A25 I have enjoyed pondering mathematical exercises .62
- A12 Mathematics has been my favorite subject .60
- A22 Mathematics has been the most unpleasant part of studying -.57

F6 Difficulty of Mathematics (Cronbach's alpha = .82)

- C 4 Mathematics is difficult .563
- C 2 Learning mathematics requires a lot of effort .56
- A11 Mathematics has been difficult in high school .53

F7 Confidence (Cronbach's alpha = .87)

- B 2 I can get good grades in math .81
- B 1 I am sure that I can learn math .79
- B10 I know I can do well in math .67
- B 5 I think I could handle more difficult math .38
- B 9 I am sure I could do advanced work in math .37

We obtained seven dimensions for students' view of themselves as learners of mathematics. Three factors relate to personal beliefs since a clear self-relation aspect regarding *competence* (F1), *effort* (F2) and *confidence* (F7) can be found. Two factors relate primarily to social context variables, namely *teacher quality* (F3) and *family encouragement* (F4), one to more emotional expressions concerning *enjoyment of mathematics* (F5) and one to mathematics as a subject; that is, *difficulty of mathematics* (F6).

As mentioned in the methodology section, we carefully searched for the proper number of factors and finally decided to retain seven factors. When the extraction was forced to six factors, factor 7 coincided with factor 1. All items of these factors, except for one, come from the self-confidence scale of Fennema-Sherman (1976). Although both factors are clearly related to students' self-confidence in mathematics, we found them to reflect different aspects. We therefore decided to distinguish a more stable competence dimension (*I am not good in math, I am not the type to do well in math*) and a more dynamic confidence dimension (*I can get good grades in math, I am sure that I can learn math*). This differentiation has also been found in the analysis of our teacher students' data (Hannula et al., 2005a) and was strongly confirmed by methodological considerations.

Conclusions

The present study supports previous research on mathematics-related belief structure. That is, we found evidence for an extended conceptualization of beliefs as a system. By means of factor analysis we determined useful and statistically robust dimensions of students' view of mathematics, which could be assigned to all main categories and several subcategories of the framework provided by Op 't Eynde et al. (2002). However, the factor enjoyment of mathematics (F5) consists of more emotional expressions and is not covered by any of these categories.

Considering the correlations between dimensions, we additionally employed a second order analysis to test whether the correlations among the first-order factors could be accounted for in terms of second-order factors. This analysis led to a first second-factor containing F1 *Competence*, F6 *Difficulty of mathematics* and F7 *Confidence* and showed that beliefs about self are highly correlated with beliefs about the difficulty of mathematics. The second factor covered the remaining scales but could not be explained in a theoretically plausible way.

Our findings support the model for describing students' view of mathematics found in a previous study when analyzing data of elementary teacher students (Hannula et al., 2005a,b). For identical items in both populations we found the same factor structure and reliability analysis confirmed excellent internal consistency of factors. We are now able to compare both samples by means of a valid and reliable instrument.

References

- Buehl, M. M. & Alexander, P. A. (2005). Motivation and performance differences in students' domain-specific epistemological beliefs profiles. *American Educational Research Journal*, 42(4), 697-726.
- Fabrigar, L. R., Wegener, D. T. & MacCallum, R. C. (1999). Evaluating the use of exploratory factor analysis in psychological research. *Psychological Methods*, 4(3), 272-299
- Frost, L. A., Hyde, J. S. & Fennema, E. (1994). Gender, mathematics performance, and mathematics related attitudes and affect: a meta-analytic synthesis. *International Journal of Educational Research*, 21, 373-385.
- Furinghetti, F. & Pehkonen, E. (2002). Rethinking characterizations of beliefs. In G. C. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 39-57). Netherlands: Kluwer Academic Publishers.
- Gorsuch, R. L. (1983). *Factor analysis*. Hillsdale, New Jersey: Lawrence Erlbaum Associate.
- Green, T. F. (1971). *The activities of teaching*. New York: McGrawhill.
- Hannula, M. S. (2004). *Affect in mathematics education*. Annales Universitatis Turkuensis B 273. University of Turku, Finland.
- Hannula, M. S., Kaasila, R., Laine, A. & Pehkonen, E. (2005a). The structure of student teacher's view of mathematics at the beginning of their studies. In *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education*. 17. – 21.2. 2005 in Sant Feliu de Guíxols, Spain. <http://cerme4.crm.es/Papers%20definitius/2/wg2listofpapers.htm>.
- Hannula, M. S., Kaasila, R., Laine A. & Pehkonen, E. (2005b). Structure and typical profiles of elementary teacher students' view of mathematics. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*. Vol 3, 41-48. Melbourne: PME.
- Kline, P. (1994). *An easy guide to factor analysis*. London: Routledge.
- Ma, X. & Kishor, N. (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics, a meta-analysis. *Journal for Research in Mathematics Education*, 28 (1), 26-47.
- Meyer, D. K. & Turner J. C. (2002). Discovering emotion in classroom motivation research. *Educational Psychologist*, 37 (2), 107-114.
- Op 't Eynde, P., De Corte, E. & Verschaffel, L. (2002). Framing students' mathematics-related beliefs. A quest for conceptual clarity and a comprehensive categorization. In

- G. C. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 13-37). Netherlands: Kluwer.
- Op 't Eynde, P. & De Corte, E. (2003). Students' mathematics-related belief systems: design and analysis of a questionnaire. Paper presented at the *2003 Annual Meeting of the American Educational Research Association*, April 21-25, Chicago.
- Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, 62 (3), 307-332.
- Pehkonen, E. (1994). On differences in pupils' conceptions about mathematics teaching. *The Mathematics Educator*, 5(1), 3-10.
- Pehkonen, E. (1995). *Pupils' view of mathematics. Initial report for an international comparison project*. Helsinki: University of Helsinki
- Pehkonen, E. & Törner, G. (1996). Mathematical beliefs and different aspects of their meaning. *Zentralblatt für Didaktik der Mathematik*, 28(4), 101-108.
- Rösken, B., Pehkonen, E., Hannula, M. S., Kaasila, R. & Laine, A. (forthcoming). Identifying dimensions of students' view of mathematics. *Accepted for proceedings of CERME 5*.
- Schoenfeld, A. (1985). *Mathematical problem solving*. Orlando: Academic press.
- Schoenfeld, A. (1998). Toward a theory of teaching-in-context. *Issues in Education*, 4(1), 1-94.
- Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.
- Weiner, B. (1992). *Human motivation: Metaphors, theories, and research*. Newbury Park, CA: Sage.
- Zimbardo, P., McDermott, M., Jansz, J. & Metaal, N. (1995). *Psychology: A European text*. London: HarperCollins Publishers.

Papers in Science

Interpretation of pupils' conceptions in the pulley demonstrations using the variation theory

Olavi Hakkarainen
University of Jyväskylä

Maija Ahtee
University of Helsinki

The teacher has to be aware of the various alternative conceptions on which pupils are relying in their reasoning. Variation theory gives guidance to teachers how to design a successful intervention by taking into account the critical aspects in teaching certain concepts. We have applied variation theory in analyzing the changes in the ninth graders' conceptions induced by three successive demonstrations in which the positions of the two equally heavy objects hanging in the pulley were varied. Based on the well known misconceptions, pupils made the wrong conclusion about the weight of the objects. The successive demonstrations brought the misconceptions as well as the critical detail simultaneously into pupils' mind. After getting rid of the old notion that the hanging objects had different weights they were able to solve the cognitive conflict brought up by their misconceptions.

Introduction

Very often, teachers assume that pupils see and understand the basic idea of the demonstration that has been shown to them. However, it is well known that pupils have a lot of various misconceptions that cause them to apply explanations that are not consistent with the scientifically acceptable one (see e.g. Driver, Squires, Rushworth & Wood-Robinson, 1994; Duit, 2005; Champange, Klopper & Anderson, 1980; Ioannides & Vosniadou, 2002). One common instructional strategy to foster conceptual change is to confront students with discrepant events that contradict their existing conceptions (see e.g. Stavy & Berkovitz, 1980; Dreyfus, Jungwirth & Elovitch, 1990; Kang, Scharmann & Noh, 2004). This is intended to invoke a cognitive conflict that induces students to reflect on their conceptions as they try to resolve the conflict. According to the variation theory (see Marton & Booth, 1997; Marton & Tsui, 2004) learning implies to experience and to discern something in a different

way. Thus the variation theory provides a way to describe the conditions necessary for learning. Ling, Chik and Pang (2006) have used three types of variation in their Lesson Study, namely, variation in students' ways of experiencing what is to be taught/learned; variation in teachers' ways of dealing with the "object of learning", and the use of "pattern of variation" as a guiding principle of pedagogical design to enhance students' learning.

In an earlier paper (Ahtee & Hakkarainen, 2005) we asked ninth graders to compare the weights of two objects suspended in a pulley in balance in three different positions. After the last demonstration 78 % of the ninth graders stated the both objects to have the same weight. Later on, we carried out a delayed paper-and-pencil test (Hakkarainen & Ahtee, 2006) in order to find out whether the series of pulley demonstrations have any permanent effect on pupils' conceptions towards the scientific explanation without the teacher telling the scientific explanation. In both cases pupils' conclusions and arguments were classified for each demonstration. In the demonstrations the pulley served as a prosthetic device for thinking, helping pupils to pay attention to the behaviour of the whole system instead of looking only at the separate objects.

For the present paper, we have analysed on the basis of the variation theory the changes in the ninth graders' conceptions induced by the three successive demonstrations.

Variation theory

According to the variation theory of learning, learning involves experiencing a phenomenon in a new light (Marton & Trigwell, 2000). Marton and Booth (1997) argue that the key feature of experiencing something is "the set of different aspects of the phenomenon as experienced that are simultaneously present in focal awareness". They state further that there is no learning without discernment, and no discernment without variation. According to Marton and Trigwell (2000), the most important constraints of learning are what variation can be possibly experienced in a certain situation. By experiencing variation, people discern certain aspects of their environment. A way of seeing something can be defined by the aspects discerned at the same time, i.e. the critical features of what is seen. The only way to experience certain features is to experience how they vary. In addition to discerning features and values of features, one has to be able to discern parts within wholes as well as wholes from their context. This is summarized in Figure 1.

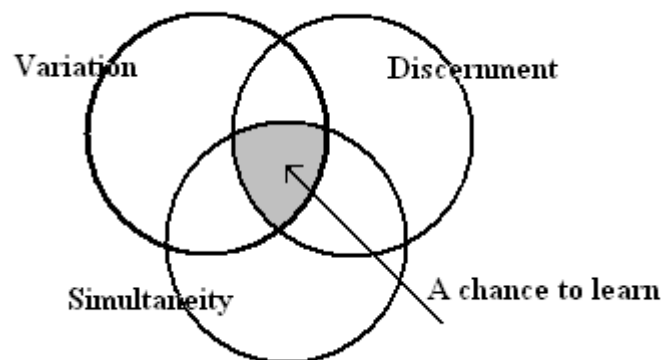


Figure 1. The conditions of learning according to the variation theory

These ideas can be applied to the cases when pupils have misconceptions. It is the task of the teacher to try to change pupils' misconceptions towards the scientific explanation. Thus, the question is how to bring a pupil's misconception and the correct scientific explanation simultaneously into the pupil's focal awareness so that s/he can realize the difference between her/his misconception and the scientific conception. The starting point has to be in the fact that every phenomenon has critical features that distinguish it from other phenomena. Thus, one way is to vary the features or the contexts in which the misconception occurs because according to the variation theory the pupil can then discern the misconception.

Changing pupils' conceptions using the pulley in balance

We have studied how pupils understand the conception of weight when two bodies (of equal weight) are hanging in a pulley in balance (Hakkarainen & Ahtee, 2005). We also tried to find out how pupils' thinking works when they try to figure out an explanation and to find factors that promote or hinder learning. The ninth graders were asked to write down what they thought the weight of the standard mass and the bag would be compared to each other, and also the reasons why they thought so. Despite the fact that we used a paper-and-pencil test, all the pupils were shown the proper equipment, and also the free motion of the flywheel was demonstrated.

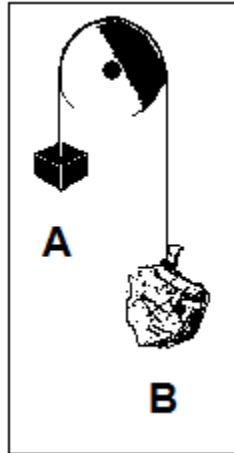


Figure 2. A standard mass and a bag are hanging in a pulley. The pulley can move freely and the string is very light.

Many pupils based their reasoning on their misconceptions like the larger the object is or the higher it hangs the heavier it is. The misconceptions are implicit in the five categories we found, as can be seen from the examples of pupils' reasoning (*in italics*) to their choices (given first):

1. **Motion.** The scientifically correct argument is based on the movement of the bag, the standard mass or the flywheel. This idea includes some notion of the idea of effects of gravity i.e. of the force concept.

Bag and standard mass weigh the same. The bag and the standard mass stay at their positions.

2. **Position.** The argument is based on the positions of the bag and the standard mass.

Bag is heavier. The bag hangs lower.

3. **Appearance.** The pupils pay attention to the concrete appearance of the bag and the standard mass.

Bag is heavier. *The bag is larger.*

4. **Material.** The pupils give concrete properties to the bag and the standard mass.

Standard mass is heavier. *The standard mass is of metal and the bag is of plastic.*

5. **No argument or confusing idea.** In most cases the pupils only stated their thought about the weights of the bag and of the standard mass compared to each other.

This pulley study gives information about pupils' misconceptions but it does not give any hints how to change these conceptions. One common instructional strategy to foster a conceptual change is to confront students with discrepant events that contradict their existing conceptions. This is intended to invoke a cognitive conflict that induces students to reflect on their conceptions as they try to resolve the conflict. In the study (Ahtee & Hakkarainen, 2005) we tried that by changing the positions of the standard mass and the bag in demonstrations 1 to 3 (see Figure 3).

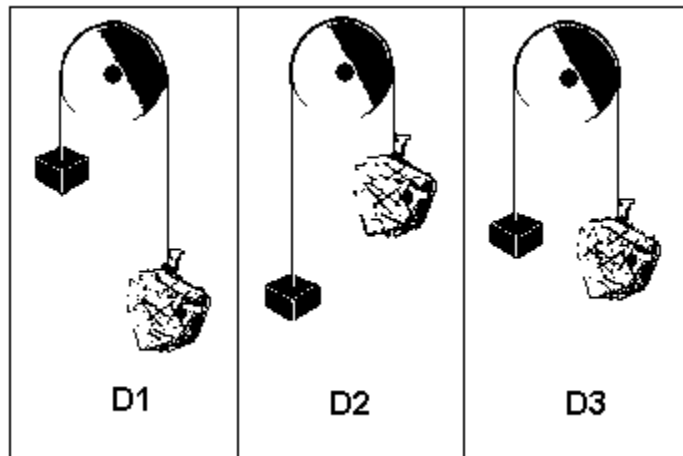


Figure 3. Three different positions of the pulley in balance in the three successive demonstrations

During a science lesson these three successive demonstrations were introduced. The changes in the choices as well as in the arguments of each pupil were followed through the demonstrations (see Ahtee & Hakkarainen, 2005). In Figure 4 the changes in the ninth graders' arguments are shown, both for the pupils who had made the correct choice and for the pupils who thought that the bag and the standard mass had different weights. It can be seen that after the second demonstration with the pulley in balance already 77 % of the ninth graders come to the conclusion that the bag and the standard mass must have the same weight and the total amount of the position model as the argument has dropped to 15 %.

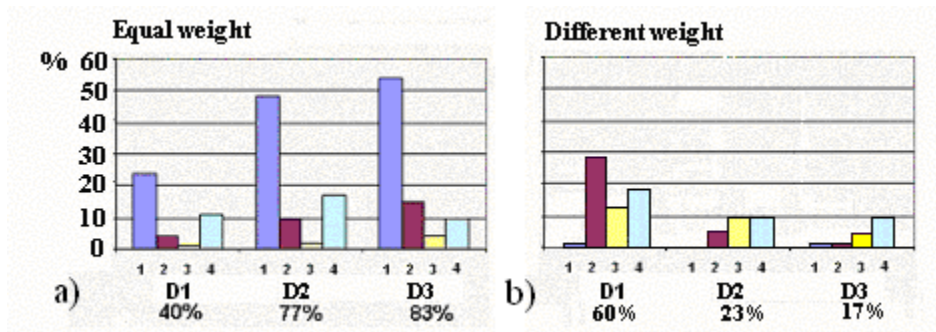


Figure 4. The changes in the ninth graders' arguments 1. Motion, 2. Position, 3. Appearance and Material, 4. None. Pupils had made a) the correct choice, b) the wrong choice after the demonstrations D1, D2, D3. The percentage underneath indicates the amount of the choice after each demonstration.

In order to get the pupils to pay attention to the concept of weight they were, however, first asked to compare with their hands the weight of the small standard mass and the bigger bag. The results of this work are reported by Hakkarainen (2005). This pre-activity had an effect on the results. After the first successive demonstration D1 (see Figure 4) 40 % of the ninth graders made the correct choice (the standard mass and the bag having the same weight) and 24 % of them justified their choice according to the motion model. However, in the earlier study (Hakkarainen & Ahtee, 2005), when the pupils just compared the weights of the standard mass and the bag, only 13 % had chosen the correct alternative and only 9 % used the motion model in justifying this choice.

Interpretation compatible with the variation theory

In the following we pay attention to the aspects that we consider the critical ones in the learning intervention with which we try to get pupils to perceive the scientific concept of gravitational force acting in the pulley demonstrations. According to the variation theory we have applied "the pattern of variation" as a guiding principle in the pedagogical design to enhance pupils' learning in the pulley context. We shall give an explanation as to what happens in the pupils' minds when they, on the basis of the different demonstrations shown to them, try to understand firstly, that the different objects hanging in the pulley are equally heavy, and secondly why they have to be equally heavy. For students it is important what comes to the fore of their attention, i.e. what aspects of the situation they discern and focus on (Marton, Runesson & Tsui, 2005). In Figure 5, the space of learning is divided into two parts. On the left, there are two pre-activities carried out with the pupils. One is the manual comparison of the weight of the two objects; the other is the set-up of the pulley system and the presentation that the flywheel is moving freely. On the right is the learning intervention during which the three successive demonstrations with the different positions of the hanging objects are shown.

After the first demonstration with the pulley 24 % of the ninth graders had reached the scientific conception (see Figure 4a) whereas only 9 % could give the corresponding argument when not shown the manual comparison of the objects. So we interpret that these pupils, when recognizing the cognitive conflict produced by the disturbing conclusion from the manual comparison – that the standard mass is heavier based on the material of the standard mass or the sensory perception, feeling – and the new conclusion from the first pulley demonstration – that the lower hanging big bag is heavier – were able to pay attention to the working of the pulley. Atkinson and Peijnenburg (2004) describe the hallmark of a remarkable thought experiment and that “it is simultaneously destructive and constructive, it destroys an old theory and at the same time establishes a new one”. Respectively here, when the pupils had these ideas simultaneously in their minds, they had first to get rid of the old notion that the hanging objects had different weights. After that they could solve the cognitive conflict by noticing that because there is no motion in the pulley system, the hanging objects have to be equally heavy. According to Schoultz, Säljö and Wyndhamn (2001) the pulley system acts here as a prosthetic device for thinking. Viennot, Chauvet, Colin and Rebmann (2005) speak about the importance of critical details. After finding the satisfactory explanation these pupils applied this conception in the next demonstrations.

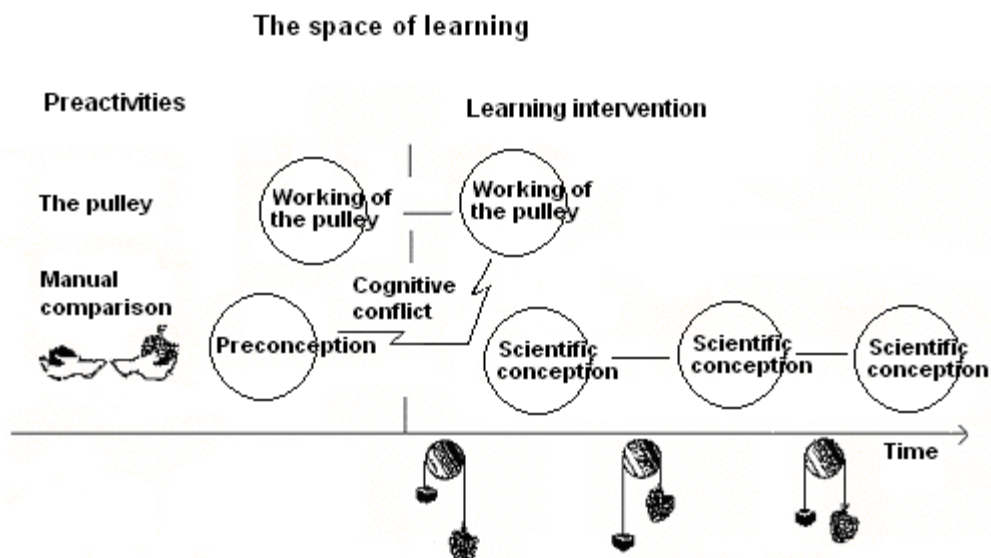


Figure 5. The space of learning showing how some of the ninth graders end up with the scientific conception after the first pulley demonstration

After the first pulley demonstration, the majority (60 %) of the pupils state that the objects have different weights. About 28 % of these pupils solve the cognitive conflict between the manual comparison and the first pulley demonstration using the position model (see Figure 4b). When in the next

demonstration the bag is moved up and the standard mass down most of these pupils recognize their erroneous thinking and they abandon the position model. The decrease in the use of the position model among the pupils stating that the objects have different weights, is about the same as the increase in the use of the motion model, 22 % and 24 % (Figure 4). As they give up their earlier wrong conclusion about the different weights they are open to look for new explanations, and they take up the notion about the immobility of the system (see Figure 6). It is important that the pupils have the possibility to check their new prediction with the working of the pulley in the next demonstration. On the delayed study (Hakkarainen & Ahtee, 2006), about half a year after the learning intervention study, the proportion of the ninth graders using the motion model stayed at the same level. These pupils had changed their thinking.

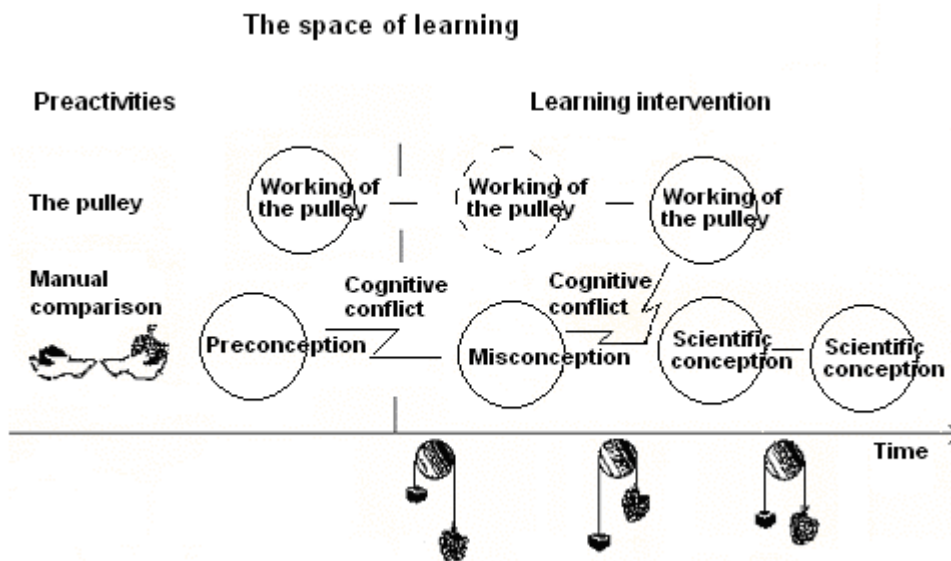


Figure 6. The space of learning showing how some of the ninth graders end up with the scientific conception after the second pulley demonstration

As seen from Figure 4a, still 15 % of the pupils base their prediction after the last demonstration D3, that the objects have the same weight, on the Position model. Obviously, these pupils did not recognize during the demonstrations the critical detail that the system is not moving. Now the third demonstration in which the objects were at the same level strengthened their notion. In the earlier demonstrations it was easier to observe the immobility as most of the pupils assumed the objects to have different weights and then it is natural to expect that they start to move. Those pupils who based their arguments on the appearance or the material of the objects, i.e. on the concrete models, could not notice any variation in these demonstrations. Therefore, it is understandable that their proportion stayed more or less the same throughout the demonstrations.

References

- Ahtee, M. & Hakkarainen, O. (2005). Importance of the order of demonstrations in changing pupils' conceptions. *NORDINA, Nordic Studies in Science Education*, 1, 31-42.
- Atkinson, D. & Peijnenburg, J. (2004). Galileo and prior philosophy. *Studies in History and Philosophy of Science*, 35, 115-136.
- Champagne, A.B., Klopfer, L.E. & Anderson, J.H. (1980). Factors influencing the learning of classical mechanics. *American Journal of Physics*, 48, 1074-1079.
- Dreyfus, A., Jungwirth, E., & Eliovitch, R. (1990). Applying the "cognitive conflict" strategy for conceptual change. Some implications, difficulties and problems. *Science Education*, 74, 555-569.
- Driver, R., Squires, A., Rushworth, P. & Wood-Robinson, V. (1994). *Making sense of secondary science. Research into children's ideas*. London: Routledge.
- Duit, R. (2005). Bibliography – STCSE. Students' and Teachers' Conceptions and Science Education. Accessed on 19.12.2006 from <http://www.ipn.uni-kiel.de/aktuell/stcse/stcse.html>
- Hakkarainen, O. (2005). Mental models in manual weight comparisons between two objects of different size. *Themes in Education*, 6(2), 151-167.
- Hakkarainen, O. & Ahtee, M. (2005). Pupils' mental models of a pulley in balance. *Journal of Baltic Science Education*, 4, 26-34.
- Hakkarainen, O. & Ahtee, M. (2006). The durability of conceptual change in learning the concept of weight in the case of a pulley in balance. *International Journal of Science and Mathematics Education*. Accessed on 19.12.2006 from <http://springerlink.metapress.com/content//1573-1774/?Content+Status=Accepted&sort=p-OnlineData&sortorder=desc&v=expanded&o=10>
- Ioannides, C. & Vosniadou, S. (2002). The changing meaning of force. *Cognitive Science Quarterly*, 2, 5-62.
- Kang, S., Scharmann, L.C. & Noh, T. 2004. Reexamining the role of cognitive conflict in science concept learning. *Research in Science Education*, 34, 71-96.
- Ling, L. M., Chik, P. & Pang, M. F. (2006). Patterns of variation in teaching the colour of light to primary 3 students. *Instructional Science*, 34, 1-19.
- Marton, F. & Booth, S. (1997). *Learning and awareness*. Mahwah, NJ: Lawrence Erlbaum.
- Marton, F., Runesson, U. & Tsui, A. B. M. (2004). The space of learning. In F. Marton & A. B. M. Tsui, (Eds.). *Classroom discourse and the space of learning*, 1-31. Mahwah, NJ: Lawrence Erlbaum.
- Marton, F. & Trigwell, K. (2000). Variatio est mater studiorum. *High Education Research & Development*, 19(3), 381-395.
- Marton, F. & Tsui, A. B. M. (2004). *Classroom discourse and the space of learning*. Mahwah, NJ: Lawrence Erlbaum.

- Schoultz, J., Säljö, R. & Wyndhamn, J. (2001). Heavenly talk: Discourse, artifacts, and children's understanding of elementary astronomy. *Human Development*, 44, 103-118.
- Stavy, R. & Berkovitz, B. (1980). Cognitive conflict as a basis for teaching quantitative aspects of the concept of temperature. *Science Education*, 64, 679-692.
- Viennot, L., Chauvet, F., Colin, P. & Rebmann, G. (2005). Designing strategies and tools for teacher training: The role of critical details, examples in optics. *Science Education*, 89(1), 13-27.

Lower-secondary school students' personal and situational interest towards physics and chemistry related themes in Finland

Jari Lavonen¹, Reijo Byman¹, Anna Uitto¹, Kalle Juuti¹, Veijo Meisalo¹ and Janis Gedrovics²

¹Department of Applied Sciences of Education,
University of Helsinki

²Riga Teacher Training and Educational
Management Academy, Latvia; research fellow of
the Academy of Finland

There is an increasing need for interest research in science education as interest has been recognised as one of the key factors having an effect on learning. Interest in physics and chemistry phenomena and out-of-school experiences of Finnish secondary school students (n = 3626, median age 15) were surveyed in spring 2003 using the international ROSE questionnaire. Likert-scaled items were categorized with explorative factor analysis. The scores of eight interestingness factors (situational interest or actualised personal interest) and six out-of-school experience (personal interest) factors were studied. The students have had a lot of out-of-school experiences in simple measuring and observing and in ICT use, but they have had few science and technology related hobbies, activities and experience of camping. For students the most interesting physics and chemistry related phenomena were astronomy and cosmology; chemicals and radiation that are dangerous for human beings. Less interesting for them were phenomena dealing with technical devices. Astronomy and cosmology, environmental issues and physical and chemical phenomena in relation to human beings were as interesting for boys as for girls. *Observing natural phenomena and collecting objects* was the most important factor correlating with the interestingness factors. Factors measuring experiences of ICT use and the use of mechanical tools had the lowest correlations between the interestingness factors.

Introduction

There is a long tradition from the 1960s onwards of studying students' interest, behaviour, achievement, and attitudes ('student voice') towards science,

especially towards physics and chemistry (Osborne, Simon, & Collins, 2003). Research has shown that students will study and learn physics and chemistry better and, moreover, choose science courses in upper secondary school if they are interested in learning. Interest research (for a review, see Hidi, Renninger, & Krapp, 2004) has also shown that interest-based motivation has positive effects on studying processes, and on the quantity and quality of learning outcomes. Thus, because students' interest is so important to learning and future involvements in the subjects, it is useful to know about the structure of students' interest. In the following we, firstly, review the literature of interest and secondly, we highlight aspects of interest in physics and chemistry. We have already conducted similar research concerning students' interest in biology and environmental issues (Uitto, Juuti, Lavonen & Meisalo, 2006).

Interest is typically approached from two major points of view (Krapp, 2003). One is interest as a characteristic of a person and the other is interest as a psychological state aroused by specific characteristics of the learning environment. Traditionally, the former approach has been termed topic interest or personal interest and the latter has been called situational interest. *Situational interest* is assumed to be spontaneous, fleeting, and shared among individuals. It is an emotional state that is evoked suddenly by something in the immediate environment and it may have only a short-term effect on an individual's knowledge and values. Situational interest is aroused as a function of the interestingness of the topic and is also changeable and partially under the control of teachers (Schraw, Flowerday & Lehman, 2001). According to Hoffman (2002) and Lewalter and Krapp (2004), an appropriate context or learning activity (teaching method) might have an influence on the quality of emotional experience, which they assume to be of central importance for the development of interest-based motivational dispositions (situational interest). *Personal interest* is topic specific, persists over time, develops slowly and tends to have long-lasting effects on a person's knowledge and values (Hidi, 1990).

Personal interest can be subdivided into latent and actualized interest (Schiefele, 1991; 1999). Latent personal interest guides student's cognitive engagement. Thus, when students are given the choice of what they should learn, they tend to choose familiar things that interest them or novel and complex things that they are curious about. Intrinsically motivated activities are activities which students do naturally and spontaneously when they follow their inner interests. Moreover, Schiefele has suggested that interest is a content-specific concept as well as a directive force and a facilitator of learning, and that it consists of two kinds of valences: feeling-related and value-related valences. Feeling-related valences are feelings that are associated with a topic or an object, for instance, feelings of enjoyment and involvement. Value-related valences refer to the attribution of personal significance to an object. Thus, some objects of interest are preferred because involvement with them creates, for instance, strong feelings of excitement, whereas other objects of interest are preferred because they may have high personal relevance. According to his valence distinction, Schiefele (1992, p. 154) reinterpreted interest "as a domain-specific or topic-specific

motivational characteristic of personality, which is composed of intrinsic feeling-related and value-related valences.” Thus, personal interest guides also such students’ out-of-school activities where they have a choice to do what they want: students are doing what they appreciate or what gives them a good feeling.

There is some empirical research evidence which sheds light on how disposition-like individual interest develops in young adults (see e.g. Lewalter & Krapp, 2004). However, relatively little is known about how interest in school subjects develops, especially for younger students who do not have vocational aims. Several researchers (e.g. Krapp, 2002) have made a distinction between catching and holding situational interest. Catching or triggering refers to variables that initially stimulate students to become interested in a specific topic. Holding interest refers to variables that empower students with a clear goal or purpose. Essential to the shift from catching to holding a student’s situational interest can be a support by appropriate learning conditions (e.g., context, teaching method, use of computers or the Internet or a textbook) that make a topic meaningful and personally relevant to students (Schraw et al., 2001; Lavonen, Byman, Juuti, Meisalo & Uitto, 2005). Mitchell (1993) has suggested that in the shift from “catching” to “holding” a student’s situational interest requires a learning environment which makes the topic meaningful to a student according to his or her personal goals. Consequently, students can self-regulate interest and, moreover, in certain conditions situational interest can transform into personal interest and, therefore, it is partially under the control of teachers. Students’ affective response to the topic forms the link between interest and learning (Ainley, Hidi & Berndorff, 2002). Furthermore, Krapp (2003) has assumed that the fulfilment of basic psychological needs for *competence*, *autonomy*, and *social relatedness* are also important for the development of interest.

Hidi and Renninger (2006) have recently suggested instead of the two-phase model (catch and hold) a four-phase model of interest development. This model integrates two conceptualizations of interest which both have two phases. First, it is described that *situational interest* has two phases. These phases are the phase in which the interest is triggered and the subsequent phase in which interest is maintained (see e.g. Mitchell, 1993). In *personal interest*, the two phases are emerging personal interest and well-developed personal interest (see e.g. Renninger, 2000). In this division emerging personal interest refers to a psychological state of interest as well as to the beginning phase of a relatively enduring predisposition to seek repeated reengagement with particular classes of content over time. Emerging individual interest is characterized by positive feelings, stored knowledge and value. It is typically, but not exclusively, self-generated and instructional conditions or the learning environment that can enable the development of it.

Instructional conditions or the learning environment can enable the development of individual interest. Thus, in the recent research on interest, the role of context variables has gained increasing attention (see e.g. Lewalter & Krapp, 2004; Turner & Meyer, 2000). According to Hidi and Renninger (2006, p. 112), “the

potential for interest is in the person but the content and the environment define the direction of interest and contribute to its development.” Volet and Jarvela (2001), for instance, have named their context emphasizing view as a “person-in-context perspective”. According to Lewalter and Krapp (2004), context variables have an influence on the quality of emotional experience, which they assume to be of central importance for the development of interest-based motivational dispositions. Based on a longitudinal study, they suggest that physics should be taught within a context that provides possibilities to transfer the acquired scientific knowledge to practical day-to-day life problems.

One important goal in the development of physics and chemistry education has been to bridge the gender gap in physics and chemistry. One possible approach is to change girls’ attitudes, interests, or behaviours. An example of this would be to give them examples of women with careers in this area. Another would be to conduct a marketing campaign advertising the technology industry to increase the perceived attractiveness of the field or to change the context or teaching method, the idea being that learning should be made more interesting for girls (cf. Biklen & Pollard, 2001).

Consequently, the general view of school education is that students’ knowledge of a school subject is acquired in the classroom within varying educational settings constructed by the teachers. In the present study, we focus on Finnish lower secondary school students’ (median age 15) personal and situational interests in physics and chemistry. Students’ physics and chemistry related out-of-school experiences are considered as an indication of their personal interests. Out-of-school activities may occur for example at home in everyday situations like interaction with friends, watching TV, reading books or magazines, in various hobbies, junior organizations and in institutions like museums and zoos. Interestingness of students’ to physics and chemistry related phenomena is considered as an indication of their situational interests or actualized personal interest. The research questions are:

1. What kind of out-of-school experiences of physics and chemistry phenomena are there in Finland among 9th grade students (~ personal interest)?
2. How interesting are physics and chemistry phenomena for 9th grade students in Finland (~ situational interest or actualized personal interest)?
3. Are there gender differences in the out-of-school experiences and interest in physics and chemistry phenomena?
4. What correlations are there between students’ physics and chemistry related out-of-school experiences and interestingness of physics and chemistry related phenomena?

Method

The data which were used for answering the research questions were gathered alongside the Finnish contribution to the international comparative research

project ROSE (The Relevance Of Science Education) aiming to shed light on factors of importance to the learning of science and technology in comprehensive school (Schreiner & Sjøberg, 2004). The ROSE questionnaire has been prepared through international co-operation so that the findings could help teachers and researchers to make science education and learning more interesting. We have reported other parts of the Finnish survey elsewhere (e.g. Uitto, Juuti, Lavonen & Meisalo, 2004) and we will further continue our analysis of the survey data.

In the ROSE questionnaire, there were altogether 61 items measuring students' out-of-school experiences (~ personal interest). In this research only 36 physics and chemistry related items are used. The students were asked: "*How often have you done this (e.g. used binoculars) outside school?*" Students answered by ticking the appropriate box on a four-point Likert type scale, the extreme categories being *Never* and *Often*. The responses were scored 1, 2, 3, and 4. Respectively, there were altogether 108 items measuring how interesting physics and chemistry phenomena are for students (~ situational interest or actualized personal interest). Here 46 physics and chemistry related items were used. The students were asked: "*How interested are you in learning about the following (e.g. stars, planets and the universe)?*" Students answered by ticking the appropriate box on a four-point Likert type scale, the extreme categories being *Not interested* and *Very interested*. Because of large skewness and kurtosis of the score distributions of some physics and chemistry related items, like items measuring students' experience of mobile phone use (skewness -2.1 and kurtosis 3.9), some items were not included in the analysis.

Altogether, 75 lower-secondary schools were randomly selected from the list of Finnish-language comprehensive schools in Finland. In each of these schools, about 65 students were asked to answer the survey, which meant about three classes from each school. In total, we had 4954 students participating in the survey. The questionnaire was sent to the schools in spring 2003, and the school principals were asked to organise the survey and return the completed questionnaires. The national and international purposes of the survey were carefully explained on the cover sheet. Altogether, 26 reminders (37 % of the selected schools) were sent to those principals who had not returned the survey in time. The survey was answered by 3626 students (1772 girls) in 61 schools representing 81 % of the selected schools. The number of students answering the survey was 7 % of the whole age cohort. Thus the sample represents the population quite well. The teachers or headmasters reported no problems in organising the survey.

The students' answers were read by optic scanners and data were saved to SPSS. The data were cleaned in SPSS by looking carefully at all lines and running frequency tables for all variables to search for values outside the 'legal' range.

Even if the scales are ordinal, it is easier to discern the findings when they are presented as means to each item. If the mean for a question "*How often have you*

done this outside school?” was over the middle of the scale (2.5) the activity was interpreted as happening often. If the mean for a question “*How interested are you in learning about the following (e.g. stars, planets and the universe)?*” was over the middle of the scale (2.5) the interest was interpreted as positive.

An exploratory factor analysis (EFA) was used to reduce the number of original variables, measuring students’ out-of-school experiences and interestingness of physics and chemistry phenomena, to a smaller number of factors describing personal and situational interest (or actualised personal interest). In the EFA analysis, maximum likelihood was used as the extraction method rotation being Promax ($\kappa = 4$) with Kaiser Normalization. All factors whose eigenvalues exceeded 1 before the rotation were accepted to the solution. The calculated Kaiser-Meyer-Olkin Measure of sampling adequacy and the Bartlett’s test of sphericity for the factor analysis showed that the data were adequate for EFA analysis. For the out-of-school experiences factor analyses the KMO of Sampling Adequacy was 0.931 and in the Bartlett’s Test the approximate Chi-Square was 43780 ($df = 630, p = 0.000$). For interestingness factor analyses the corresponding values were for KMO 0.953 and for the Bartlett’s Test 79863 ($df = 1035, p = 0.000$).

To compare girls’ and boys’ out-of-school experiences and interestingness of physics and chemistry related phenomena, factor related sum-variables were calculated of the items having main loading for each factor. The means of the boys’ and girls’ sum-variable distributions were compared using the Independent-Samples *t*-test (two-tailed). As an additional check we tested the power of the difference using Cohen’s *d* ($d = M_g - M_b / S.D._{pooled}$, where $S.D._{pooled} = \sqrt{[(S.D._g^2 + S.D._b^2)/2]}$, (Cohen, 1988). The Independent-Samples *t*-test procedure compares means for two groups of cases. Cohen’s *d* measures the effect size for the difference between boys and girls: no effect at $d < 0.2$, small effect at $0.2 \leq d < 0.5$, moderate effect at $0.5 \leq d < 0.8$, and large effect at $d \geq 0.8$.

The two-way Pearson’s correlation analysis was used to find out if there were any relations between students’ out-of-school experience factors and interestingness factors.

Results

The six factors that described students’ physics and chemistry related personal interest (FE1-FE6) and which were reduced by the EFA on students’ out-of-school activities items explained 47 % of the extraction sums of squared loadings (Table 1). Goodness-of-fit Test gave a high value: $\chi^2 = 3307^{***}$. Thus, the statistical fit of the six-factor solution was not acceptable. However, the examination of the residuals indicated no hidden variation, thus supporting the six-factor solution. Gorsuch (1983) has also stated that the ML significance test in EFA can overestimate the factor number. The students had a lot of physics and chemistry related out-of-school experiences in using a camera, basic

electronic devices and performing information searches on the Internet. They had little experiences in using a wind- or watermill, in making small devices like an instrument and in making compost from grass.

Eight interestingness factors describing interestingness of physics and chemistry related phenomena (FI1-FI8) reduced by the EFA on students' opinions of interestingness of single items explained 57 % of the extraction sums of squared loadings (Table 2). Goodness-of-fit Test gave a high value: $\chi^2 = 5794^{***}$. However, the examination of the residuals indicated no hidden variation, thus supporting the eight-factor solution. Each factor was named according to the loaded items, emphasizing the highest loadings and contents common for factor items. Factors are named as presented in Tables 3 and 4.

Table 1. Loadings of interestingness factors measuring students' physics and chemistry related personal interest (FE1-FE6) reduced by the EFA on students' out-of-school activities items. The loadings < 0.3 are not included.

I have ...	Factor							
	M	S.D.	FE1	FE2	FE3	FE4	FE5	FE6
used a windmill, watermill, waterwheel, etc	1.6	.83	.76					
made a model such as toy plane or boat etc	2.0	.98	.75					
used a water pump or siphon	2.1	.98	.66					
mended a bicycle tube	2.1	1.1	.63					
made a bow and arrow, slingshot or catapult	2.3	.98	.62					
used a rope and pulley for lifting heavy things	2.1	1.0	.59					.41
opened a device to find out how it works	2.7	1.1	.44					
made an instrument (like a flute) from natural materials	1.7	.84	.41		.35			
changed or fixed electric bulbs or fuses	2.8	1.0	.38	.36				
measured the temperature with a thermometer.	3.3	.83		.89				
used a measuring ruler, tape or stick	3.3	.83		.84				
used a stopwatch	3.3	.82		.79				
used a camera	3.4	.79		.60				
connected an electric lead to a plug etc.	3.5	.82		.59				
recorded on video, DVD or tape recorder	3.3	.89		.46				
used binoculars	2.9	.82		.42				
read about nature or science in books or magazines	2.5	.96			.60			
collected different stones or seashells	2.1	.90			.57			
collected edible berries, fruits or plants	3.0	.90	-.30		.55			
watched nature programmes on TV or cinema	2.9	.88			.55			
tried to find the star constellations in the sky	2.8	.97			.44			
sorted garbage for recycling or for appropriate disposal	2.5	1.0			.44			
made compost of grass, leaves or garbage	1.7	.88	.37		.44			
visited a science centre or science museum	2.3	.82			.33			
downloaded music from the internet	2.8	1.2				.69		
sent or received e-mail	3.3	.92				.65		
played computer games	3.3	.86				.58		
used a word processor on the computer	3.4	.82				.56		
searched the internet for information	3.6	.71				.54		

Table 1 (cont)

			Factor					
	M	S.D.	FE1	FE2	FE3	FE4	FE5	FE6
used a dictionary, encyclopaedia, etc. on a computer	2.5	.99	.34			.43		
made a fire from charcoal or wood	2.5	1.1					.72	
prepared food over a campfire, open fire or stove burner	2.4	.99					.70	
put up a tent or shelter	2.5	.88			.30		.46	
used a wheelbarrow	2.9	.88						.73
used a crowbar	2.5	1.0	.40					.66
used tools like a saw, screwdriver or hammer	3.1	.89						.31
Eigenvalue			8.3	4.0	2.3	.9	.7	.6
% of total variance			23.1	11.1	6.5	2.6	1.9	1.7

Table2. Loadings of factors measuring interestingness of physics and chemistry related phenomena (FI1-FI8) reduced by the EFA on students' opinions how interested they are in learning about physics and chemistry related phenomena. The loadings < 0.3 are not included.

			Factor							
	M	S.D.	FI1	FI 2	FI 3	FI 4	FI 5	FI 6	FI 7	FI 8
How things like radios and televisions work	2.4	.96	.96							
How cassette tapes, CDs and DVDs store and play sound and music	2.4	.99	.91							
How mobile phones can send and receive messages	2.5	.95	.82							
The use of lasers for technical purposes (CD-players, etc.)	2.3	1.0	.74							
How computers work	2.5	1.0	.70							
Optical instruments and how they work (telescope, camera, etc.)	2.0	.91	.40							
Explosive chemicals	2.4	1.1		.94						
Biological and chemical weapons and what they do to the human body	2.5	1.0		.92						
How the atom bomb functions	2.6	1.1		.89						
The effect of strong electric shocks and lightning on the human body	2.7	.95		.66						
Deadly poisons and what they do to the human body	2.7	.97		.63						
How a nuclear power plant functions	2.1	1.0		.43						
Chemicals, their properties and how they react	2.0	.88		.34						
Black holes, supernovas and other spectacular objects in outer space	2.7	1.1			.88					
How meteors, comets or asteroids may cause disasters on earth	2.7	1.0			.81					
Stars, planets and the universe	2.6	.99			.80					

Table 2 (cont.)

	M	S.D.	Factor							
			FI 1	FI 2	FI 3	FI 4	FI 5	FI 6	FI 7	FI 8
Rockets, satellites and space travel	2.4	1.0			.76					
The first landing on the moon and the history of space exploration	2.3	1.0			.63			.40		
How it feels to be weightless in space	3.0	1.0			.57					
The use of satellites for communication and other purposes	2.1	.95			.48					
How to find my way and navigate by the stars	2.3	.98			.39		.36			
The greenhouse effect and how it may be changed by humans	2.2	.94				.97				
The ozone layer and how it may be affected by humans	2.2	.93				.89				
What can be done to ensure clean air and safe drinking water	2.6	.94				.81				
How technology helps us to handle waste, garbage and sewage	2.2	.92				.66				
How energy can be saved or used in a more effective way	2.4	.97				.53				
New sources of energy from the sun, wind, tides, waves, etc.	2.3	.96				.47				
How the ear can hear different sounds	2.1	.87					.88			
How the eye can see light and colours	2.2	.90					.78			
How radiation from solariums and the sun might affect the skin	2.2	.96					.60			
How X-rays, ultrasound, etc. are used in medicine	2.2	.93					.60			
How different musical instruments produce different sounds	2.1	.97					.49			
Light around us that we cannot see (infrared, ultraviolet)	2.1	.91					.41			
How radioactivity affects the human body	1.9	.91					.41			
Electricity, how it is produced and used in the home	2.1	.90						.90		
How to use and repair everyday electrical and mechanical equipment	2.1	.95						.88		
How electricity has affected the development of our society	2.2	.90						.66		
Detergents, soaps and how they work	1.9	.84						.45		
How petrol and diesel engines work	2.1	1.1						.40		
How different narcotics might affect the body	2.7	.97							.88	
How alcohol and tobacco might affect the body	2.6	.94							.86	
The possible radiation dangers of mobile phones and computers	2.3	.95							.36	
How loud sound and noise may damage my hearing	2.4	.91							.33	

Table 2 (cont.)

	Factor									
	M	S.D.	FI 1	FI 2	FI 3	FI 4	FI 5	FI 6	FI 7	FI 8
Why the stars twinkle and the sky is blue	2.3	.99								.85
Why we can see the rainbow	2.2	.93								.83
How the sunset colours the sky	2.3	.97			.33					.41
Eigenvalue			14.4	3.6	2.4	1.7	1.4	1.0	.8	.9
% of total variance			31.3	7.9	5.2	3.7	3.1	2.2	1.8	1.9

For students the most interesting physics and chemistry related phenomena deal with astronomy and phenomena which are in one way or another connected to the human being. Less interesting phenomena deal with technical devices and matter.

Comparison of boys' and girls' out-of-school experiences (~ physics and chemistry related personal interest) and interest in physics and chemistry phenomena or interestingness factors (~ physics and chemistry related situational interest or actualized personal interest) are presented in Table 3 and Table 4. In both tables, means (*M*) and Standard deviations (*S.D.*) for factor related sum variables of item scores for boys and girls are also presented.

Table 3. Factors measuring students' physics and chemistry related personal interest, measured as their out-of-school experiences and comparison of boys and girls

	Girls		Boys		<i>t</i> 1)	<i>d</i> 2)
	<i>M_g</i>	<i>S.D._g</i>	<i>M_b</i>	<i>S.D._b</i>		
FE1 Science and technology related hobbies or activities	1.8	.54	<u>2.5</u>	.61	971***	0.69 ^C
FE2 Measuring and observing with simple tools	3.4	.48	3.2	.67	142***	0.34 ^B
FE3 Observing natural phenomena and collecting objects	<u>2.6</u>	.51	2.3	.57	177***	0.55 ^C
FE4 ICT use	3.1	.57	3.2	.69	2.7 ^{ns}	0.16 ^A
FE5 Camping	2.4	.79	2.5	.79	13***	0.13 ^A
FE6 Use of mechanical tools	2.7	.74	3.0	.76	213***	0.40 ^B

1) ^{ns} $p > 0.05$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

2) ^A no effect ($d < 0.2$), ^B small effect ($0.2 \leq d < 0.5$), ^C moderate effect ($0.5 \leq d < 0.8$), ^D large effect ($d \geq 0.8$)

The effect size measure did not indicate gender difference for the experiences of ICT use, camping and use of mechanical tools. The power of gender difference was 'small' in the factor *Measuring and observing with simple tools*. Boys had more *Science and technology related hobbies or activities* than girls and girls more experiences in *Observing natural phenomena and collecting objects* than boys (moderate effect).

Table 4. Factors measuring students' interest in physics and chemistry related phenomena (Interestingness) and comparison of boys and girls

	Girls		Boys		<i>t</i> 1)	<i>d</i> 2)
	<i>M_g</i>	<i>S.D_g</i>	<i>M_b</i>	<i>S.D_b</i>		
FI1: How technological devices work	2.1	.70	<u>2.6</u>	.78	489***	0.67 ^C
FI2: Effects of explosive and poisonous objects on the human body	2.1	.67	<u>2.7</u>	.74	476***	0.85 ^D
FI3: Astronomy and cosmology	<u>2.5</u>	.72	<u>2.6</u>	.75	12***	0.14 ^A
FI4: Environmental issues	2.3	.74	2.3	.75	.7 ^{ns}	0.00 ^A
FI5: Physics and chemistry phenomena in human beings	2.2	.61	2.1	.64	9.4**	0.16 ^A
FI6: Technology in society and in everyday life	1.8	.60	<u>2.4</u>	.68	575***	0.94 ^D
FI7: Dangerous chemicals and radiation for human beings	<u>2.6</u>	.70	2.4	.77	85***	0.27 ^B
FI8: Physics and chemistry phenomena in the environment	<u>2.4</u>	.83	2.1	.78	190***	0.37 ^B

1) ^{ns} $p > 0.05$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

2) ^A no effect ($d < 0.2$), ^B small effect ($0.2 \leq d < 0.5$), ^C moderate effect ($0.5 \leq d < 0.8$),

^D large effect ($d \geq 0.8$)

The effect size measure did not indicate gender difference for the student interest in astronomy and cosmology, environmental issues and physics and chemistry phenomena in human beings. The power of gender difference was 'small' in the dangerous chemicals and radiation for human beings and physics and chemistry phenomena in the environment. Boys were much more interested in explosive and poisonous objects and their effect on the human body as well as in technological themes (large effect).

Correlations between factors measuring students' physics and chemistry related out-of-school experiences and Interestingness of physics and chemistry related phenomena are presented in Table 5.

Table 5. Pearson correlation coefficients between factors measuring students' physics and chemistry related out-of-school experiences and Interestingness of physics and chemistry related phenomena

	F11: How technological devices work	F12: Effect of explosive and poisonous objects to human body	F13: Astronomy and cosmology	F14: Environmental issues	F15: Physics and chemistry phenomena in human being	F16: Technology in society and in everyday life	F17: Dangerous chemicals and radiation for human being	F18: Physics and chemistry phenomena in environment
FE1 Science and technology related hobbies or activities	.35***	.37***	.16***	.11***	.13***	.44***	-.17***	-.11***
FE2 Measuring and observing with simple tools	.11***	.16***	.26***	.23***	.24***	.10***	.21***	.23***
FE3 Observing natural phenomena and collecting objects	.08***	.14***	.36***	.45***	.38***	.16***	.28***	.41***
FE4 ICT use	.19***	.22***	.20***	.08***	.11***	.14***	.03	.00
FE5 Camping	.21***	.26***	.20***	.19***	.19***	.26***	.07***	.06**
FE6 Use of mechanical tools	.15***	.22***	.11***	.08***	.10***	.18***	.01	.01

Discussion and conclusions

According to our literature review, a teacher can partly regulate the catch component of situational interest or actualised personal interest (interestingness) and also students can be themselves encouraged to self-regulate interest. Moreover, a teacher can promote the change of situational interest to personal interest by systematically choosing appropriate contexts or teaching methods when new topics are learned (Schraw, Flowerday & Lehman, 2001). Once a teacher succeeds in catching the situational interest of his or her students the critical phase is then how to hold it long enough so that it leads to personal interest to study physics and chemistry. Our aim in this paper is to analyse what kind of out-of-school experiences students have had frequently and what they want to learn eagerly by their free will. Students' physics and chemistry related out-of-school experiences reflect areas of their personal interest in those fields or tell what they are eager to do or what topics are personally interesting for them. What students want to learn spontaneously reflect their situational interest or what topics are interesting for them. Teachers can use this information, when they are planning physics and chemistry lessons.

Based on Tables 1-4 we know what physics and chemistry related out-of-school experiences are popular and what topics are interesting for boys and girls. Single items and factors were shortly commented on already in the previous chapter, and below some more general comments are made, based on all the tables.

Students' physics and chemistry related out-of-school activities

It seems that both genders have a lot of experiences of ICT use and the use of simple mechanical tools. Therefore, classroom activities could remind these activities from the point of view of interest. Although there are also statistically significant correlations between these factors and several interest factors, the correlations are not as high as many others. The “*Camping*” factor was gender neutral, indicated rather high popularity and it had relatively high correlations with several interestingness factors.

Boys and girls had different out-of-school experiences having loadings to the factor “*Observing natural phenomena and collecting objects*”. These activities were more popular with girls than with boys. This factor had also very high correlation with interestingness factors: “*Astronomy and cosmology*”; “*Environmental issues*”; “*Physics and chemistry phenomena in human beings*”; and “*Physics and chemistry phenomena in the environment*”. On the other hand, boys' factor “*Science and technology related hobbies or activities*” had very high correlation with interestingness factors: “*How technological devices work*”; “*Effects of explosive and poisonous objects on the human body*”; “*Technology in society and in everyday life*”.

Based on our literature review, students' out-of-school activities, which are frequently occurring, reflect their personal interest. Therefore activities, similar to students' popular out-of-school activities, might help students to catch or hold interest. Some suggestions are presented in the conclusions below.

How interested students are in learning about physics and chemistry related phenomena

Phenomena described in items, like “*Why we can see the rainbow*” and “*How the sunset colours the sky*”, and which had high loadings to the factor “*Physics and chemistry phenomena in the environment*” are impressive and can be seen in nature. However, they were not so interesting for students, especially for boys. On the other hand, phenomena described in items, like “*How it feels to be weightless in space*” and “*Black holes, supernovas and other spectacular objects in outer space*” and which had high loadings to the factor “*Astronomy and cosmology*” were similarly interesting for both genders. Astronomy and cosmology, in general, seems to be very interesting for both boys and girls.

Girls lowest interest was towards phenomena described in items, like “*Detergents, soaps and how they work*” and “*Electricity, how it is produced and used in the home*”, and which had high loadings to the factor “*Technology in*

society and in everyday life.” However, these phenomena were rather interesting for boys and, therefore, there was one of the highest gender differences for these items. Boys had highest interest in phenomena described in items which had high loading to factors “*How technological devices work*” and “*Effect of explosive and poisonous objects on the human body*”. There was also one of the highest gender differences among these items (e.g. “*How mobile phones can send and receive messages*” and “*The effect of strong electric shocks and lightning on the human body*”). These results indicate that Science – Technology – Society (STS) context was felt not so interesting by students, especially by girls. Therefore, it is no wonder that technology is no longer so important in STS approach (Aikenhead, 1994, pp. 52-53; Hoffmann, 2002).

The most interesting phenomena for girls were in some way connected to human beings: “*How different narcotics might affect the body*” and “*How radiation from solariums and the sun might affect the skin*”. These items had high loadings in factors: “*Dangerous chemicals and radiation for human beings*” or “*Physics and chemistry phenomena in human beings*”. The gender difference of these two factors was small. It seems that the phenomena which are interesting for girls are also rather interesting for boys. However, this is not working vice versa as it was claimed in the previous paragraph. The results are in accordance with the findings of Uitto, Juuti, Lavonen and Meisalo (2005) from the same ROSE data, that more than boys, girls were interested in the context of human biology and health education and in general in living nature, such as animals.

The smallest gender difference was in students interest in phenomena described in items, like “*How energy can be saved or used in a more effective way*” and “*The ozone layer and how it may be affected by humans*”, having loadings to the factor “*Environmental issues*”. Therefore environmental issues are suitable for school science from the point of view of gender neutrality. However, these phenomena are not very interesting for students. Furthermore, in the same data Uitto, Juuti, Lavonen & Meisalo (2004) found that girls had a more positive attitude towards environmental responsibility than boys, even if in general, pupils found global, large-scale environmental problems distant from themselves. It is possible that if learning of environmental issues could be connected to pupils’ every-day life and the state and health of the nearby environment, the learning of chemistry and physics as well as the environmental ecology could become more interesting. It is important to know how environmental data are gathered, how the expert knowledge is used in society, and how an individual can contribute to sustainable development in individual and societal levels in science. Science-related environmental issues could be integrated to the education of physics, chemistry, biology and geography.

Conclusions

Based on our survey results, the first challenge is to develop learning activities to physics and chemistry teaching similar to activities students have spontaneously out-of-school. How can physics and chemistry teaching be like

camping giving experiences of nature, collecting, measuring, observing, and ICT use? To answer this problem, a simple solution would be to increase different kinds of site visits, but more generally, different kinds of research-based development projects are needed.

The second challenge is to clarify how the study of astronomy can be increased in school physics and chemistry. This is in parallel with findings of Osborne and Collins (2001). They indicated also that students would like to learn more about the solar system and the universe. The third challenge is to increase the role of human-being context, health education and examples of life sciences in physics and chemistry teaching. For example, ergonomics or anatomy and functioning of the human body might be possible areas. In chemistry, physiological effects of several chemicals can be discussed in the context of the human being.

The fourth challenge is technology teaching. Boys like to know how technical applications work. This kind of technical knowledge does not interest most girls. On the other hand, considering youngsters' future, technical applications play a core role in further studies in schools of technology (vocational schools, polytechnics or universities). Thus, it is important to find more versatile approaches to show technical applications' interestingness and importance for all students. Everyone uses technical applications; therefore, usability testing and user-centred design could be interesting study contexts. One interesting idea might be to develop this area so that technological applications are looked at more from the point of view of human beings. Several items where a human being was active or it was a target, was interesting for both boys and girls.

Based on the findings of the present study, it is worthwhile being aware of the fact that boys and girls as groups have different out-of-school experiences and interests in specific topics and contexts. From the viewpoint of physics and chemistry teaching and learning it is important to know how to hold part of situational interest long enough to lead to a motivation to study and the activities of studying. Results of the international ROSE survey indicate that Finnish results may be general in countries having the human development index at the same level as Finland does. We suggest to developers of curricula and textbook authors, who have a lot of freedom in choosing different approaches and especially contexts for certain topics, to take the result seriously and implement certain contents and contexts. In practice, classroom situations and learning materials can form the link between interest and learning.

It is important to understand that the findings of the survey are not absolute; they reflect present-day general trends among young people's perceptions and experiences. Results reflect the objects of interest of Finnish boys and girls and the world they experience in relation to science as measured with the questionnaire.

Results of this research do not explain how situational interest develops into a long-standing personal interest (see e.g., Alexander & Jetton, 1996; Krapp, 2002). It seems self-evident that teachers must first think how they can "catch"

and “hold” situational interest during their physics lessons. One interesting new approach to physics education could be developed by combining technological and human or astronomical contexts.

Quantitative measurements have been criticised in general, and when based on surveys, it is difficult to understand in detail the structure of students’ interests (Osborne, Simon & Collins, 2003). On the other hand, surveys like this research yield information about the significance of studied phenomena.

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References

- Aikenhead, G. (1994). What is STS teaching? In J. Solomon, G. Aikenhead (Eds.), *STS Education: International Perspectives On Reform* (pp. 52-53). New York, NY: Teachers College Press.
- Ainley, M., Hidi, S. & Berndorff, D. (2002). Interest, learning and the psychological processes that mediate their relationship. *Journal of Educational Psychology, 94*, 545-561.
- Alexander, P. A. & Jetton, T. L. (1996). The role of importance and interest in the processing of text. *Educational Psychology Review, 8*, 89-12.
- Biklen, S. K. & Pollard, D. (2001). Feminist Perspectives on Gender in Classrooms. In V. Richardson (Ed.), *Handbook of research on teaching* (pp. 723-747). Washington, DC: American Educational Research Association.
- Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Lawrence Erlbaum Associates
- Hidi, S. (1990). Interest and its contribution as a mental resource for learning. *Review of Educational Research, 60*, 549-571.
- Hidi, S., Renninger, A. & Krapp, A. (2004). Interest, a motivational variable that combines affective and cognitive functioning. In D. Y. Dai & R. J. Sternberg (Eds.), *Motivation, emotion, and cognition* (pp. 89-115). Mahwah, NJ: Lawrence Erlbaum.
- Hoffman, L. (2002). Promoting girls’ interest and achievement in physics classes for beginners. *Learning and Instruction, 12*, 447-465.
- Krapp, A. (2002). Structural and dynamic aspects of interest development: Theoretical considerations from an ontogenetic perspective. *Learning and Instruction, 12*, 383-409.
- Krapp, A. (2003). Interest and human development: An educational-psychological perspective. *Development and Motivation, British Journal of Educational Psychology Monograph Series II, 2*, 57-84.

- Lavonen, J., Byman, R., Juuti, K., Meisalo, V. & Uitto, A. (2005). Pupil Interest in Physics: A Survey in Finland. *Nordina* 2(1), 72-85.
- Lewalter, D. & Krapp, A. (2004). The role of contextual conditions of vocational education for motivational orientations and emotional experiences. *European Psychologist*, 9, 210-221.
- Mitchell, M. (1993). Situational interest: Its multifaceted structure in the secondary school mathematics classroom. *Journal of Educational Psychology*, 85, 424-436.
- Osborne, J., Simon, S. & Collins, S. (2003). Attitudes towards science: a review of the literature and its implications. *International Journal of Science Education*, 25(9), 1049-1079.
- Schiefele, U. (1991). Interest, learning, and motivation. *Educational Psychologist*, 26, 299-323.
- Schiefele, U. (1999). Interest and learning from text. *Scientific Studies of Reading*, 3, 257-279.
- Schraw, G. & Lehman, S. (2001). Situational interest: a review of the literature and directions for future research. *Educational Psychology Review*, 13, 23-52.
- Schreiner, C. & Sjøberg, S. (2004). *Sowing the seeds of ROSE. Background, Rationale, Questionnaire Development and Data Collection for ROSE (The Relevance of Science Education) - a comparative study of students' views of science and science education. Acta Didactica*, 4. Oslo: Dept. of Teacher Education and School Development, University of Oslo.
- Uitto, A., Juuti, K., Lavonen, J. & Meisalo, V. (2004). Who is responsible for sustainable development? Attitudes to environmental challenges: A survey of Finnish 9th grade comprehensive school students. In A. Laine, J. Lavonen & V. Meisalo (Eds.), *Current research on mathematics and science education. Proceedings of the XXI annual symposium of the Finnish Association of Mathematics and Science Education Research*. University of Helsinki, Department of Applied Sciences of Education. Research Report 253, 80-102.
- Uitto, A., Juuti, K., Lavonen, J. & Meisalo, V. (2005). Is pupils' interest in biology related to their out-of-school experiences? In V. Zogza (Ed.), *Proceedings of the 5th ERIDOB (European Researchers in Didactics of Biology) Conference*. Patras, Greece.
- Uitto, A., Juuti, K., Lavonen, J. & Meisalo, V. (2006). Is pupils' interest in biology related to their out-of-school experiences? *Journal of Biology Education* 40 (3), 124-129.

The importance of teaching models in physics education

Anssi Lindell
University of Jyväskylä

Jouni Viiri
University of Jyväskylä

Students' ideas of models are usually studied with questionnaires or paper-pencil tests. We approached the problem by a concrete demonstration. A sample of 103 student teachers and engineering students were asked to model a balloon in an upward air flow from a hairdryer before and after seeing the demonstration. Qualitative methods were used to study what kind of models the students use, how they represent them and how they change their models after seeing the demonstration. We found that great majority of students (84 %) used model only as an illustrative copy of reality. None of the primary school student teachers and only very few of engineering students (3 %) used mathematical forms to represent their models. Most of the primary school student teachers (62 %) but only 20 % engineering students were not able to see and model the essential phenomena, having complex components of the balloons movement in the post demonstration models.

Introduction

There are thousands of studies about students' (mis)conceptions in science (see e.g. the bibliography of Duit 2006). It has been found that for instance students' conceptions of force and motion are similar to those of Aristotle (Boeha, 1990) and medieval scientists' (Hestenes, Wells, & Swackhamer, 1992). The similar situation seems to be when we consider students' ideas of science and especially modelling in science. Students want the science in the secure form of indisputable set of eternal truths (Lakin & Wellington, 1994). They don't want to find out that science is not a set of facts, that theories change, and that science cannot give them all the answers. This attitude follows closely Aristotelian naive realism; in which only direct perception have the power to grasp actual reality. (Chalmers, 1999) Everything in the nature has been made for a purpose and the purpose of man is to understand the nature through science. Thus, it would have been a contradiction for nature to have fashioned the man and his organs in such a way that the knowledge and science would, from its inception, be false. In this point of view no models are needed, maybe even allowed. Galileo Galilei had

another idea (Hestenes, 1987a). He wrote about the grand book of universe. The grand book of universe has been written in the language of mathematics without which it is humanly impossible to understand a single word of it. This Galilei's modest and critical scientific realism suggests and justifies the use of models in describing and understanding nature by science. Rene Descartes (Holton & Brush, 2001) went a step further and considered scientific constructs about the nature as models to deduce or predict consequences in agreement with observations.

These scientific constructs are scientists' visions of the nature, developed using the scientific method. In the scientific method an experiment is the test for all knowledge (Feynman, 1995). To analyse experiments scientists search for uniform patterns and simplifications from the perceptions and construct their data into models. These models may then be used to explain the observations and make predictions of what happens if something is changed. Limitations and the predictive power of the models are tested by new experiments. Improved models are discussed and finally agreed or disagreed by the scientific society. Thus, scientific practise includes much of making and evaluating scientific models which are descriptive, predictive and simplified versions of perceptions. Hestenes (1987b) defined a model as "a surrogate object, a conceptual representation of a real thing".

Since experimentation and modelling are such essential tasks in doing science, they have sometimes been radically given the same role in teaching and learning science too. However, learning science is not just doing science. According to Hodson (1993) education of science includes three main aspects: learning science, learning about science and doing science. In addition to *doing science*, which is to know how to create and test models, also learning science and learning about science have their implications to modelling (Justi and Gilbert 2003):

- Learning science is to know the major models as the products of science
- Learning about science is to understand the role of models in the scientific outlook.

The role of models in the Hodson's view of education of science is sketched in Figure 1.

As this short introduction points out, the central role of models in science, history of science and teaching of science, it is clear that models and modelling should be emphasized also in school science. This brings out the importance of teaching models and modelling in science teacher education. It is not enough for teachers to know the major models, but they also need to know what scientific models are, how they are constructed, approved and taught. To teach these aspects we need to study student teachers' preconceptions of models and to make their ideas explicit.

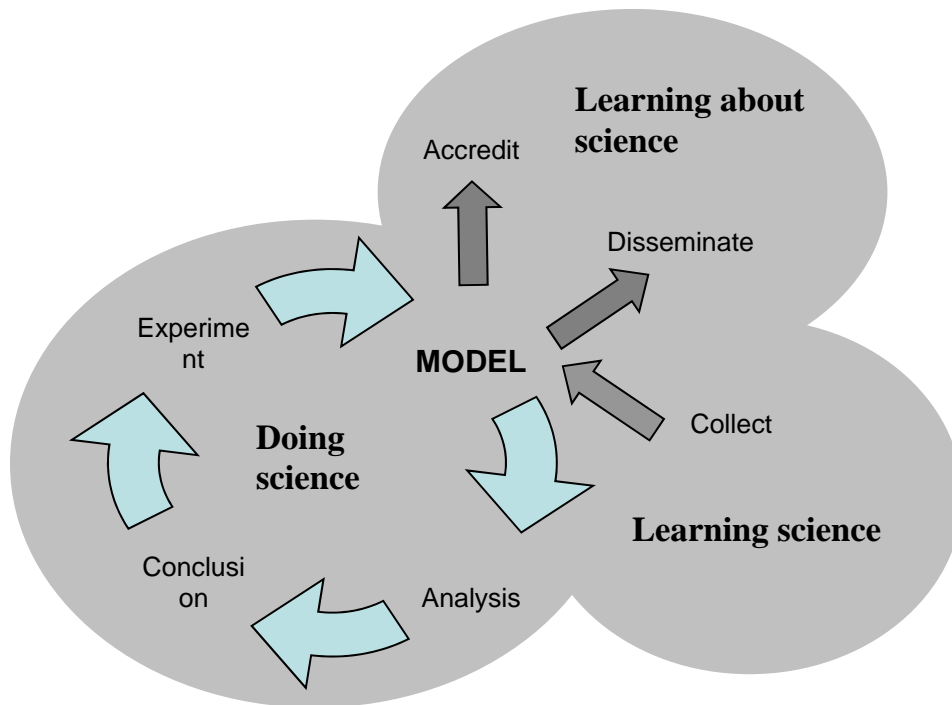


Figure 1. The role of models in Hodson’s three main aspects of education of science

Classification of models

Interpretation, use and meaning of models in science and science education can be classified in different ways (Harrison & Treagust, 2000). For example, models in science may be classified into four main categories according to their use (Gilbert, 1995; Gilbert & Boulter, 2000)

1. *Mental model* is a personal internal representation to account reasoning when one is trying to understand, explain or predict the physical world (Greca & Moreira, 2001).
2. *Expressed model* is a functionally, verbally or literally expressed version of mental model.
3. *Consensus model* is an expressed model, tested and accepted by a scientific society.
4. *Teaching model* has been designed and used by teachers to ease learning of a consensus model.

Students' and the teachers' relations to Nature linked by models are represented schematically in the Fig. 2. Learning to do science develops students' skills to create mental models to communicate with the nature directly by perceptions and experiments, while learning science deals with the consensus models of the nature, taught or learned with help of teaching models.

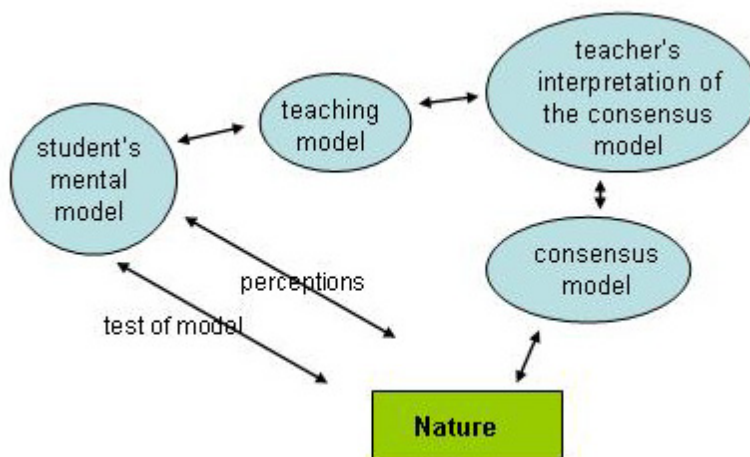


Figure 2. Student's and teacher's relations to the nature and learning are linked by different models

A scientist - or a student doing science - uses their mental models to understand what is going on in their experiments. Working in groups forces people to express their mental models in such a way that the members of the group share a common vision of the system and they can work as a team. Thus, while doing science, ones mental models debate with the perceptions of nature. The discussion takes the language of expressed models while working in groups. Experienced scientists typically do not need any assistance to understand the models expressed by the others. Nevertheless, ability to use effective teaching models might considerably improve discussion and shorten the time consumed in weekly meetings of scientific groups.

By the time working groups forming a scientific society agree on a common consensus model. Learning science is gaining understanding of these consensus models. To make the learning more effective teacher may construct specific teaching models tailored according to the preconceptions and skills of the group to be taught.

To be able to understand the role of models in science and education, it is important to learn ways and principles how consensus models have been developed in the history of science among scientific societies. This concludes Hodson's aspect of learning about science.

The purpose and use of models is apparent to experts, but novices may not have a clear idea of them. Grosslight, Unger, Jay and Smith (1991) studied students' and experts' ideas of models. They divided students' developing understanding of models into three steps. At the lowest level, level 1 students consider models as toys or simple copies of reality. Level 2 students understand that models are created for a certain purpose, and that is why the emphasis of certain components is altered, but the template of reality still predominates. In their opinion a model is tested solely in terms of its fitness for the predominated purpose. At the highest level, level 3 students are able to see that models are created to test ideas, rather than copies of reality. They accept that models are experiment dependent and that also the modeller has an active role in constructing a model for a certain purpose. They also understand that a model can be tested and changed in order to update and inform the development of ideas and that there may even be several different models on simultaneously.

Justi and Gilbert (2003) studied teachers' ideas of models and found that 'the notion of model' demonstrated by teachers could be described by seven aspects. But the researchers were not able to identify 'profiles of understanding' for individuals that cut completely across the seven aspects. The profiles of teachers' notions of 'model' in terms of the aspects and categories were complex, providing no support for the notion of levels in understanding which Grosslight et al. (1991) had found. Justi and Gilbert conclude that the absence of profiles in the teachers' thinking suggests that they probably do not hold coherent ontological and epistemological views. Therefore, it is important to include modelling ideas in teacher training courses.

There is not very much research about science student teachers' ideas of models in Finland, which is the aim of this study. Since most of the existing studies have investigated students' ideas of models either with questionnaires or interviews we decided to use a different method and asked student teachers' ideas of models in a real world phenomena.

Method

Subjects

The subjects of this study consisted of 41 primary school student teachers, 30 secondary school physics student teachers and 32 engineering (information and communication technology) students. Some of the primary school student teachers had a few years of teaching experience but the majority of them and every other student had minor teaching experience. Roughly half of the primary school student teachers had had volunteer high school courses in science; the other half had chosen only the minimum amount of science in their pre graduate studies. The secondary school physics student teachers were at their second year level in the university on the average. Every engineering student had had science

courses in their earlier studies in a high school and/or in a lower level engineering school (finished by about one third of them).

Data collection

Before data collection we did not give any lesson about models and modelling in science. Students were given an empty sheet of paper. A hair dryer and a balloon were shown to the subjects and they were asked to

1) Model a situation where an object such as a balloon is located into an air flow such as from a hair dryer.

We did not say what kind of model we are looking for. Students were free to decide themselves how to model the situation. After 5 minutes, a demonstration where the balloon was floating in the air flow from the dryer was shown and the students were asked to

2) Model the same situation after observations.

Finally they were asked to write down the main subject they study and the amount of science courses they had after the junior high school.

This demonstration was chosen since there are a lot of things to consider in that simple set up: balloon's size, shape, position, velocity, acceleration, vibration and rotation etc. In the demonstration all these can be seen and, thus, included to the model. However, the essential phenomena in the set up is the Venturi effect maintaining the balloon up at constant height in the air flow column and all the extra motions might be ignored from the post demonstration model. By this method we studied how student teachers master the role, the purpose and use of models. We also wanted to know what kind of models they use and whether and how they change their models after perceptions.

Data analysis

We analyzed the purpose and use of the students' models by their distribution on the levels of understanding presented by Grosslight et al. (1991). We defined the copy of the reality (level 1) as if the students just draw pictures of what they saw. Descriptive models for certain purposes (level 2) were considered as something between the models with flow lines and forces. These two cases were separated from each others since a model with flow lines is still kind of sensory picture of reality, even though it has invisible elements in it. One might argue that flow lines have also a strong descriptive and even a predictive power, but the case is not that clear and we positioned the model with flow lines just below the level 2. On the other hand, model with force vectors are clearly made for a purpose to explain why the balloon moves as it does. Predictive models (level 3) were defined as flexible constructions of the situation including symbolic parameters answering the question "what happens if something is changed?"

Students' models were classified also based on the representation that they had used. Gilbert et al. (2000) divided these into six modes from the most concrete to symbolic: concrete mode, verbal mode, mathematical mode, visual mode, gestural mode and symbolic mode. It should be noted that every two dimensional model represented on a piece of paper belongs more or less to the symbolic mode in this division and that our method was not meant to study concrete or gestural modes of models. Instead, we divided the symbolic mode into six sub modes according to the symbols used: pictures, arrows, text, equations, quantities and graphs.

To further study whether the students could understand the descriptive role of models, we divided their models into two categories. A model was categorized as simple when just the Venturi effect was considered. A model with any extra phenomena taken into account was categorized as complicated. In order to construct an effective teaching model only the essential phenomena are to be taken into account and at least the post demonstration model should be as simple as possible.

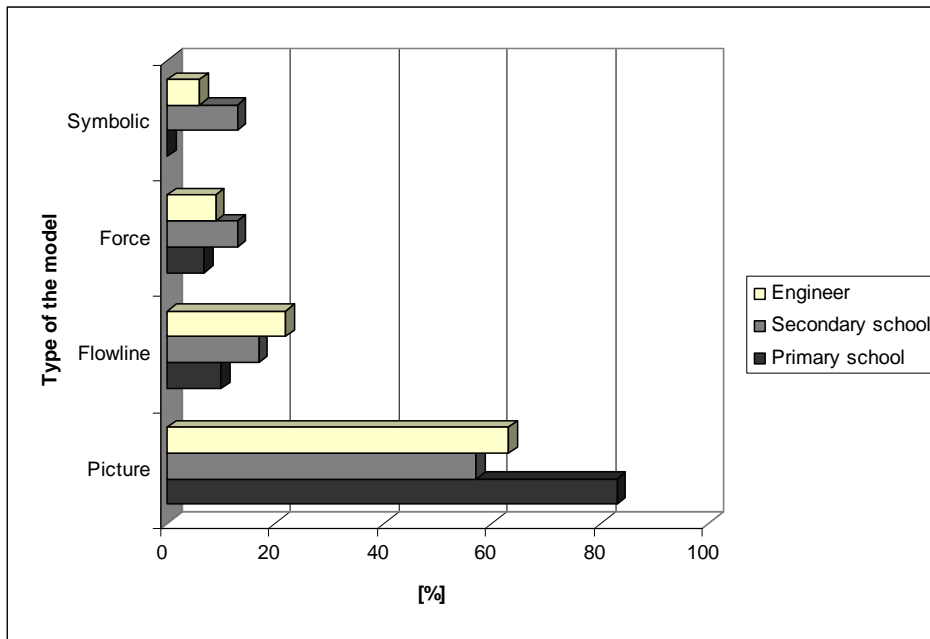


Figure 3. Relative proportions of teacher and engineering students using different kind of models. The lines between the types of the models on the vertical axis refer roughly to the levels (1-3) of understanding in Grosslight et al. (1991).

Results

Figure 3 shows the relative parts of the students using different kind of models. The levels 1-3 of understanding in Grosslight et al. (1991) can approximately be positioned between the vertical groups from down to up of that figure respectively. From the figure we can see that 83 % of primary school student teachers, 57 % of secondary school student teachers and 63 % of engineering students modelled the balloon in air flow by just drawing what they expected to see before the demonstration, and by what they saw after the demonstration, thus, hardly reaching the lowest level of understanding. If we consider flow lines only as sensory pictures of something invisible, 93 % of primary school student teachers could not understand the descriptive and predictive role of models. For secondary school student teachers and engineering students these numbers are 74 % and 85 %, respectively. In this research, none of the primary school student teachers could make a model able to predict anything and only 7 % of them were able to make models trying to explain how a balloon interacts with air flow. Secondary school student teachers were supposed to be most familiar with the use of models since they have had many opportunities to work with them in their earlier studies. However, less than one third of them used descriptive (13 %) or predictive (13 %) models. For engineering students these proportions were 9 % and 6 %, respectively.

The image shows a student's handwritten work on grid paper. On the left, there are two rows of three diagrams each, showing a balloon being inflated by a pump. In the center, there are two diagrams of a balloon with arrows pointing outwards, representing air resistance. To the right of these diagrams are several equations and text notes:

$F = ma$ $ma = F$
 No resistive forces $a = \frac{F}{m}$

$a = \frac{F}{m} - Bg - AF$
 $\frac{F}{m} - Bg - AF = 0$

m is mass of balloon + air
 F is the pressure exerted by the air from the hair
 B = mass of the balloon dryer.
 AF = air resistance

equilibrium, balloon doesn't move anymore.

+ ilmapaine on jaksilla on suurempi kuin pallon ja kuivaajan välillä

Figure 4. Examples of different sub modes of representation of symbolic modes. From left to right and top to bottom: pictures, arrows, text, equations and quantities. None of the student used any graphs to model the phenomena.

Examples of the students' models of the five first sub modes are represented in Figure 4. Surprisingly, none of the students used any graphical models, which would be the first choice of an expert scientist to our mind. Despite of their education, practically every student favored pictorial and verbal forms for modeling. Most of the models consisted of a picture with clarifying text. Only one engineer student and one secondary school student teacher used models without pictures (equations and plain text respectively). The proportions of different representations can be seen in Figure 5.

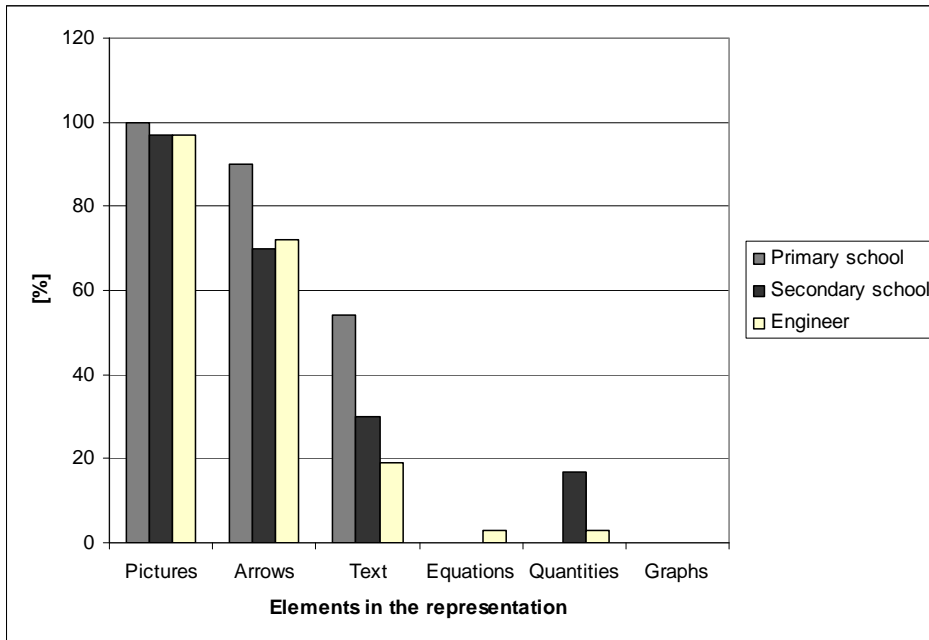


Figure 5. Six sub modes of symbolic representation of models and their usage by teacher and engineering students

The result of the analysis of models turning from simple to complicated or vice versa can be seen in Figure 6. As many as one third of secondary school physics student teachers turned their models from simple to complicated after seeing all the shakes, rattles and vibrations the balloon manoeuvres in slightly turbulent air flow. Amazingly, the major part of the engineering students, which would be stereotypically considered as the most pedant group, kept their model simple. From the graph we can see that altogether 62 % of the primary school student teachers, half of the secondary school student teachers and only 20 % of engineering students used complicated post demonstration modes.

Conclusions

The results of this work suggest that in addition to learn science and how to do science, the students should also learn the role of models in science: they are neither copies nor applications of reality, but rather they are logical patterns used to intermediate and predict observations (Figure 1). The results of how student teachers understanding of scientific models is distributed according to the levels of understanding presented in Grosslight et al. (1991) are quite different from Harrison's (2001) results for experienced high school teachers where 20 % of them were in the transition phase between the levels 1 and 2, 20 % were at the level 2, 40 % were in the transition phase between the levels 2 and 3 and 20 %

were at level 3. As great part as 84 % of all the students that participated in this research, considered models just as a copies of reality hardly reaching level 1. To reach the higher levels they should learn the descriptive and predictive power of models; they should be able to answer the questions “Why...?” and “What happens if...?”

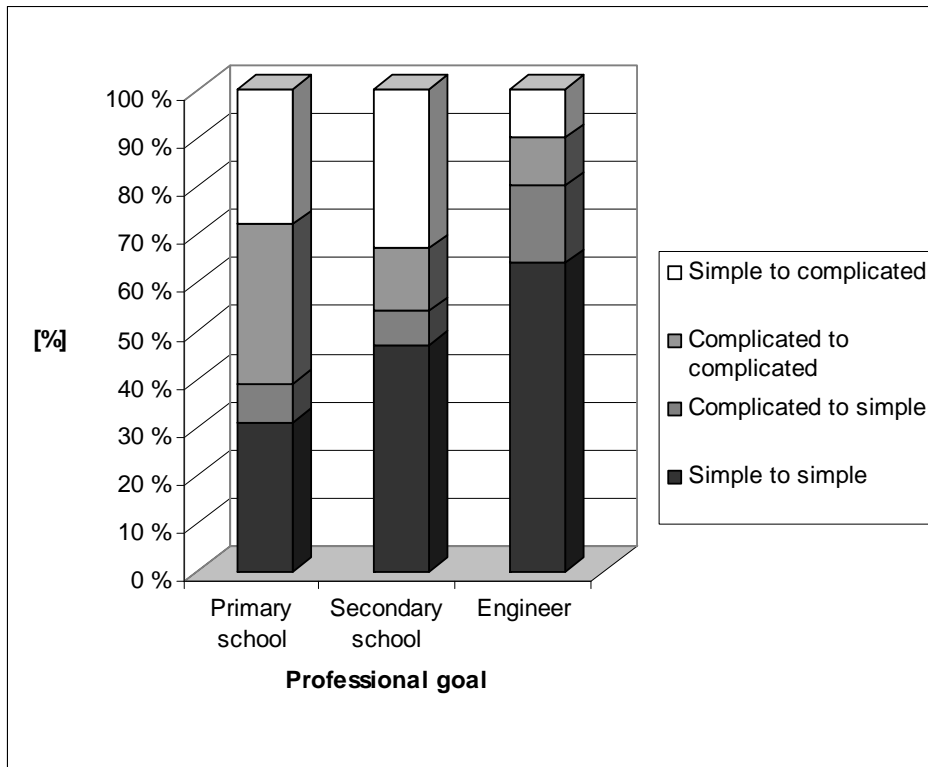


Figure 6. The relative proportions of teacher and engineering students turning their models from simple to complicated or vice versa.

The major part of students shares the Aristotelian direct vision of nature believing that the models and scientific theories are some kind of absolute knowledge or eternal truths and that learning science is learning a bunch of facts. They cannot understand that theories are changing all the time and science is merely a group of best approximations of the situation for the moment being.

None of the primary school student teachers and only very few of engineering students (3 %) used mathematical forms, such that equations quantities or graphs to represent their models. To the authors' minds the easiest way to model the Venturi effect when a balloon is hanging at a constant height in an upward cylindrical laminar air flow, would be a constant curve in a height-time coordinate system. However, none of the students used any graphical ways to represent their models. The favour of pictorial ways to represent models supports the conclusion that most of students consider models as copies of

reality. Another reason for pictorial representations might be a risk of failure in mathematical representations, even though the students were told that their papers should be handled anonymously.

Students representing a complicated post demonstration model are, once again, trying to make their model as exact a copy of reality as possible. They are not able to see that the describing or teaching power of the model suffers the extra details in the representation. Students using only simple models may be able to understand the role of models in teacher's interpretation between nature and students. Or then they are just lazy drawers. Students changing their models from complicated to simple, consider the situation from several points of views, and are able to leave out extra phenomena and pick up the relevant one after seeing what is going on in the experiment. They were very few especially in student teachers' groups.

It is important to teach modelling in teacher education courses since teachers then might teach modelling ideas also in schools. By this Venturi effect demonstration with simple equipment of just a balloon and a hairdryer we can teach that models in physics are simplifications to clarify and predict the behaviour of nature.

References

- Boeha, B. B. (1990). Aristotle, alive and well in Papua New Guinea science classrooms. *Physics Education*, 25, 280-283.
- Chalmers, A. F. (1999). *What is this thing called science*. Indianapolis,IN: Hackett Publishing Company, Inc.
- Duit, R. (2006). *Bibliography – STCSE Students' and Teachers' Conceptions and Science Education*. <http://www.ipn.uni-kiel.de/aktuell/stcse/stcse.html> (Retrieved 2.12.2006).
- Feynman, R. P. (1995). *Six easy pieces. Essentials of physics explained by its most brilliant teacher*. Reading, Mass.: Addison-Wesley.
- Gilbert, J. K. & Boulter, C. (2000). (Eds.). *Developing models in science education*. New York: Springer.
- Greca, I. M. & Moreira, M. A. (2001). Mental, physical and mathematical models in the teaching and learning of physics. Hoboken, NJ: John Wiley & Sons, Inc.
- Grosslight, L., Unger, C., Jay, E. & Smith, C. (1991). Understanding models and their use in science: conceptions of middle and high school students and experts. *Journal of Research in Science Teaching*, 28, 799-822.
- Harrison, A. G. (2001). How do teachers and textbook writers model scientific ideas for students? *Research in Science Education*, 31, 401-435.
- Harrison, A. G. & Treagust, D. F. (2000). A typology of school science models. *International Journal of science education* 22(9), 1011-1026.
- Hestenes, D. (1987a). *New foundations for classical mechanics*. New York: Springer.

- Hestenes, D. (1987b). Toward a modeling theory of physics instruction. *American Journal of Physics*, 55, 440-454.
- Hestenes, D., Wells, M. & Swackhamer, G. (1992). Force Concept Inventory. *The Physics Teacher*, 30(3), 141-158.
- Hodson, D. (1993). Re-thinking old ways: towards a more critical approach to practical work in school science. *Studies in Science Education*, 83, 163-177.
- Holton, G. & Brush, S. G. (2001). *Physics the human adventure: from Copernicus to Einstein and beyond*. Xxx: Rutgers University Press, NJ.
- Justi R. S. & Gilbert J. K. (2003). Teachers' views on the nature of models. *International Journal of Science Education*, 25, 1369-1386.
- Lakin, S. & Wellington, J. J. (1994). Who will teach the 'nature of science?': teachers' views of science and their implications for science education, *International Journal of Science Education*, 16(2), 175-190.

Innovative approaches in school science – an analysis based on the GRID project data

Veijo Meisalo, Jari Lavonen, Kalle Juuti
University of Helsinki

We analyse innovative approaches in science teaching in Finnish schools on the basis of data collected by Web questionnaires and teacher interviews for the international GRID project. In the background of innovative initiatives seems often to be a long history of participation in national projects and pedagogical experimentation. Implementation of modern information and communication technologies appears as an important feature in many of these initiatives, as well. However, the networking of active teachers both locally and nationally emerges as a most interesting factor in explaining success of innovative approaches. Resources, like lack of financial resources, technical support personnel in school science laboratories and in ICT applications are the main obstacles in the implementation of new ideas. Moreover, missing incentives for appropriate teaching methods, like project work, restrict the implementation of new ideas.

Introduction

It is often seen that a school is a very traditional system, and there are many obstacles if one wishes to implement innovations in teaching practice. We have lots of experience on difficulties of changing the habitual teaching methods of science teachers. Researchers have discussed, for instance, the ways teachers use ICT (Information and Communication Technologies) in their teaching (Willis, 1997), but also, more broadly pedagogical approaches promoting creative problem solving, e.g. in school science (Kuitunen & Meisalo, 1996) or in technology education (Lavonen, Meisalo & Lattu, 2001) etc. In the present study, the focus in our theoretical analysis is on the innovativeness of the initiatives often linked with novel types of ICT use. In the GRID (Growing Interest in the Development of Teaching science) project, and consequently in the present paper, *initiative* is used as a synonym for a school level project of implementing new ideas in science teaching.

The GRID project was a SOCRATES project identifying initiatives of good science teaching in schools. It urged European teachers to fill in, on the Internet,

a questionnaire identifying basic parameters of innovative projects to facilitate improving science teaching and learning. The project collected and published on the Internet (<http://www.grid-network.eu>) information from over 500 European initiatives. The database includes 23 entries from Finland. There were eight school visits collecting detailed information on the initiatives using another GRID questionnaire and interviewing contact teachers. While the database includes mainly factual information on the initiatives, in the present paper we are interested in finding more detailed information on the background of innovativeness in schools and how innovative projects started. In the following, we first discuss the adoption of innovative or creative approaches in schools, and then the innovative characters of school projects, before presenting and analyzing our empirical data. On the Internet pages of the GRID project there are addresses of contact persons in all contributing schools with the idea that they can be contacted for further information. However, in this paper we use anonymous reporting and analysis with code names for persons and schools. The general research question in this study on innovative projects is: what kind of general or specific features related to creative aspects can we find in the projects. Identifying these features may offer some help in promoting creative aspects in school science.

Strengths and weaknesses in Finnish school science

Since there are large differences in the school systems of European countries, it appears advisable to discuss here strengths and weaknesses of Finnish school science, although the good PISA outcomes have already advanced the knowledge of Finnish science education in other countries. These positive and negative features have an effect on creation, diffusion, and adoption of innovations in school science.

Science is compulsory in general education on primary and secondary levels. In vocational education its status depends on the vocation concerned. In the recent renewal of the primary school core curriculum, it was decided to have physics and chemistry as separate subjects in the grades 5 and 6. This was considered important since many primary-school teachers had previously interpreted school science as including biological sciences only. This new situation also brings up an urgent need for science courses for in-service primary teachers whose pedagogical content knowledge in physics and chemistry is deficient.

Another problem in Finnish school practice is that there is a poor interaction of science and technology teaching in school. Physics and chemistry are usually well integrated, but often biology is somewhat separated, and in technology education, the school culture is rather isolated from the previous ones.

In principle, schools have freedom to launch special programmes and projects, but local resources seldom allow major efforts. The school system in Finland is based on financing of school by local municipalities and many of these have had serious financial difficulties. On the other hand, there are also examples of good

local networking, where schools, industries and institutions of higher education have co-operated to promote science teaching. The financial problems of municipalities have an effect even on in-service education of science teachers, since it should be financed by the employer.

Pre-service teacher education (TE) in Finland is generally of adequate standard, and over several years, there have been visitors from all over the globe to acquaint themselves with our approach. It is important, even for science teachers, that the idea of *teacher as researcher and developer of her own work* is supported by opening doctoral programmes to practicing teachers in departments of teacher education. However, the professional development programmes have had many problems. One positive example has been the LUMA programme for science teachers (Lavonen, Meisalo & Juuti, 2004), which, unfortunately, has been discontinued. There is on-going discussion about how to organise adequate professional development programmes for all teachers. Many science teachers are reaching retirement age and extra measures have been taken to make it possible to recruit high numbers of students in science TE programmes.

All extra incentives etc. are decided on the local level although there is a national general salary system for teachers. It is mainly the local headmaster who has most influence, but even local politicians have much say, especially in funding major projects. It is problematic that usually, teachers do not have proper perspectives for career development. For instance, it is very seldom that there is a (mathematics and) science department in a school offering the possibility of a department head position for progressive teachers.

Allocating resources for initiatives is based on local decision-making. However, schools may be eligible for national support under special programmes like the information society programme Strategy for Education, Training and Research in the Information Society (SETRIS, 2000). A major problem on school level is the lack of skilled technical personnel to assist teachers in school laboratories. There are, indeed, very few laboratory technicians in Finnish schools. This is problematic not only for school science laboratories, but also for ICT use in schools more generally. Allocating some extra paid hours for ICT maintenance to science teachers is no solution to the problem.

Allocation of municipal resources and giving private support is decided mainly on local level as mentioned above. However, e.g. industrial associations have urged their members to be active in school-industry collaboration programs. All technical universities and polytechnics as well as science faculties at other universities have at least some kind of collaboration programs with schools. The LUMA Centre at University of Helsinki is upgrading its activities offering versatile collaboration programs for school teachers. There is also fruitful co-operation between schools and science centers and museums. Another nice example of co-operation with mutual benefits is the collaboration of University of Helsinki and the science centre Heureka in organizing teaching practice for student teachers.

Diffusion and adoption of innovations in schools - ICT use in school science as a case

There is a long tradition of research on diffusion of innovations (Rogers, 1995). Even important innovations are, however, welcomed in few schools only, and often in a simplified form. As an emblematic example we can take the use of ICT in school science since all our cases included some type of ICT use, although we do not want to particularly emphasise ICT in our empirical analysis. Often, it has been limited to specific tasks that computers can perform with the aid of digital cameras and different types of sensors connected to them (Thomas, 2001). This kind of technology-oriented approach becomes problematic when the focus of the discussion is on 'how to help teachers learn to integrate ICT into science education in a versatile way and help their students learn science'. Therefore, the use of ICT should be discussed in terms of alternative teaching methods and focusing on how ICT can positively influence studying and learning. These ideas offer an opportunity to help teachers discuss learning, analyze beliefs about learning and find out how they can facilitate it (Dadds, 1997).

Rogers (1995) defines 'adoption' as an individual's mental process through which he or she passes from first hearing about an innovation to final adoption or rejection. According to Knezek and Christensen's (2002) review, the beliefs of teachers regarding the usefulness of ICT in education appear to be surprisingly consistent across several nations and cultures. These beliefs have an effect on the adoption processes. Different approaches have been suggested to facilitate teachers in adopting innovations: new educational policy, curriculum design, professional development and the development of pedagogical study materials, any of which would emphasise the adoption of a new pedagogical innovation, such as ICT in science education (Fullan, 1991, p. 37). In the national strategy from the Ministry of Education (SETRIS, 2000), the level of teacher adoption and their competence in ICT were described in some detail.

Following Fullan (1991, p. 68), we categorised the properties of educational innovations that affect their acceptance or adoption into three groups:

(1) the properties of the innovation, in this case, the properties of the 'use of ICT in school science' itself (e.g. different ways in which ICT could be used in school science, and the usability of ICT). For example, if an innovation is too complicated for beginners, it is not adopted;

(2) local characteristics of the innovation. These include the teacher's pedagogical orientation and beliefs about the use of educational technology, and the administrative leadership and support available to teachers, both of which have an effect on the adoption of ICT. In addition, a teacher's previous ICT knowledge and skills have a strong effect (Willis, 1997). Zhao and Cziko (2001) suggested that the adoption of ICT in science education requires the teachers to be convinced that: (i) ICT can effectively help students to achieve or maintain high-level goals ('effectiveness'); (ii) ICT will not disturb other high-level goals

that teachers consider more important than the one being maintained ('disturbances'); and (iii) teachers have the ability and resources to use ICT ('control'). This last point may be interpreted to mean that teachers may need training and other guidance which should be contextual, connected to real teaching and learning situations. Training should also support co-operation between teachers and those who are developing national curricula or in-service programmes and providing guidance. The training of teachers has to focus on the development of ICT for pedagogical use, not on the training of ICT skills only; and

(3) external factors, such as a national ICT strategy, curriculum, financing and public hype, influencing the innovation. Different kinds of networking, such as cooperation between (i) schools, (ii) teaching and research, or (iii) schools and their environment or the workplace, may foster the integration of ICT into education (Moonen & Voogt, 1998).

Innovativeness of the initiatives and creativity

We interpret here innovativeness in school initiatives as *originality by virtue of introducing new ideas* and see that it is rather close to creativity. *Creativity* (or *creativity*) is commonly considered to be a mental process involving the generation of new ideas or concepts, or new associations between existing ideas or concepts. Unfortunately, in the school context, creativity is mostly sought in arts and seldom in sciences, and it has an everyday interpretation of meaning as the act of making something new. From a more scientific point of view, the products of creative thought are usually considered to have both *originality* and *appropriateness*, and creativity, particularly creative problem solving (*cf.* Lavonen et al., 2001), should be associated with all science studies. We regret that there is no consensus on the proper definition of these concepts and we do not want to analyse these in more detail in the present context.

After passing the first phase of the project where teachers had given information on their projects to the database, it was necessary to find criteria for selecting a number of initiatives where more information was collected using another questionnaire and teacher interviews. Representatives of GRID partners including the authors of the present paper listed qualities of interesting projects. As the outcome of these discussions the following qualities of school science initiatives were used to select the ones for further analysis. We can see that this list includes even aspects not closely related to creativity or innovativeness:

The GRID Project agreed on the qualities of innovative projects: a project can be said to be innovative and accepted as a good practice if it:

- creates a creative and innovative learning environment
- motivates young people to learn science in particular but also to learn in general
- invites young people to be creative (team skills)

- focuses on study skills and learning, and not on teaching, but on facilitating
- focuses on both teachers' (teacher as a researcher) and pupils' reflective skills
- builds on concrete co-operation with the local community (e.g. research institute, university, science museum, local companies, zoo, etc.)
- builds on research outcomes (possibly in co-operation with a university)
- observes suggestions made in educational policy papers
- integrates science learning into concrete contexts and life situations of pupils or students
- focuses on networking of teachers within the school and beyond
- focuses on specific groups that are difficult to be addressed as to science (girls, immigrants, ...)
- uses ICT and multimedia not only to gather information but also to create a creative learning environment (smartschool)
- focuses also on human and environmental (sustainable) contexts of sciences
- combines science learning with the acquisition of more generic competencies and skills such as teamwork, problem solving, communication skills, entrepreneurial skills etc.
- strengthens cross-disciplinary, interdisciplinary, multidisciplinary or transdisciplinary approaches (e.g. combining science and arts; or science and languages; or science and cultural heritage, etc.)
- has specific approaches as to the level of education; e.g. the use of audio materials and narrative approaches in learning science in primary school
- has a specific focus on training teachers for innovative science teaching, either in initial teacher education or during in-service training activities
- benefits from support for the teachers while implementing the project
- combines top-down with bottom-up (grassroots) approaches
- has potential to be transferred (or valorised) into other similar school situations;
- has a wide dissemination of the outcomes as essential part of the project
- has a thorough monitoring system (possibly in cooperation with a university!)
- has a thorough evaluation mechanism
- promotes European co-operation and mobility
- promotes active citizenship (e.g. youngsters taking steps to protect their environment)

It is only natural that each good practice project doesn't have all those characteristics, but some of them only, according to the nature of the project. This list *was used in the GRID project* as a tool when choosing the initiatives for school visits or teacher interviews. However, it appeared that practical reasons related to the school visits had a major role in decisions on choosing the school where to interview teachers.

Collecting empirical data and the method of analyzing

The European database collected by the GRID project includes 23 entries from Finland. In the first round the data were collected using a Web questionnaire QE1. This questionnaire was formulated by the project co-ordinator in co-operation with the partners. The original English questionnaire was translated in local languages and slightly modified by each partner to be better applicable in different school systems. The Finnish version was produced by the authors of this paper. There were ten items of multiple-choice character collecting factual information on the projects and the 11th item asking for a short general summary of the initiative. There were items asking about co-operation within the project, about outcomes, results and products, as well as about the use of information technology. However, there was no direct question about possibly innovative character of the initiative.

The most important channel for marketing this questionnaire to active science teachers seemed to be the mailing list of the LUMA Centre of the University of Helsinki. Through that list we reached more than one thousand active science teachers, it means about one third of all science teachers in Finland. Data from the registered questionnaire QE1 were qualitatively analysed in digital form. Each 'good practice project' was evaluated watching for the characteristics in the previously presented list. Any of them doesn't actually have all those good characteristics but it will have some of them according to the nature of the project. This list was used, in principle, as the main tool when choosing the initiatives for school visits. However, it appeared that practical reasons had a major role in decisions on choosing the schools where to visit and interview teachers. The most general reasons for not choosing a project for collecting further information was, that the project did not concern science education (but e.g. mathematics) or that it was not a school-level project (but concerned e.g. teacher education). In some cases a school visit was decided using general information available at the LUMA Centre on creative projects before the QE1 questionnaire data were collected.

Altogether, eight Finnish schools were visited collecting detailed information on the initiatives using another GRID questionnaire (QE2) and interviewing teachers active in the innovative initiatives. The majority of schools were in southern Finland, but not all in the National Capital region and there were also schools in western and central Finland. A short outline plan for the school visit ("Pedagogical Grid") was prepared to facilitate observations and practical collection of data. In most cases the interview happened in the school premises, but in one case it was decided to have the lengthy interviewing session at the university. In four schools only our contact teacher was interviewed, and in these cases this teacher had apparently almost sole responsibility for promoting the initiative. In three schools two teachers and in one school three teachers were interviewed and this indicated often close co-operation not only of these teachers

but there were some indications on wider co-operation of teachers in those schools. Altogether, there were seven female and six male teachers interviewed. In one school we had a possibility to interview a student and in most cases it was possible to ask students some short questions. In some cases the school visit started with filling in the basic QE1 questionnaire before proceeding with the interview. The researcher as interviewer used the QE2 questionnaire as a guide in the structured interview and collected the factual data by writing down notes during the session. In most cases the interview was also recorded on videotape, but in two cases it was not possible and therefore, only the written notes are available. In one case two of the teachers were interviewed simultaneously.

While the databases based on the two questionnaires include mainly factual information on the initiatives, in the present paper we are interested in finding more detailed information about the background of innovativeness in schools and how innovative projects started. The QE2 questionnaire included, on top of factual closed items, open questions on *New pedagogical or didactical approaches*, *Use of ICT*, *Co-operation with the local community*, *New learning environment for the pupils*, and *How does the team work relate to the innovative and creative work in the project*. Furthermore, it was interesting to know *Who decided to implement this initiative*. The written data are available on the GRID Web pages. It was possible to make further question on the creative aspects of the initiative during the interview. Most interviews took about 45 minutes to one hour. However, when two or three teachers from the same school were interviewed, the second/third interview was substantially shorter. The total time of the recording is about eight hours including some classroom observation sessions. Three teachers were interviewed with no video recording but notes were made during the session. The short case study reports with “Pedagogical Grid” outlines are available over the Internet. These include short video clips yielding little information about the nature of our research approach, but may help in giving some idea of the innovativeness of the interviewed teacher or of the nature of science study in the school concerned.

The research method used in this study is of the type of qualitative evaluation (Patton, 2002). It is also related to the ‘inverse critical incident method’ as used by Lappalainen, Lavonen and Meisalo (2000), and by Lavonen et al. (2001, 2002). In this study it means that more than eight hours of interview videotapes were repeatedly searched for comments indicating primarily not bottlenecks, but critical aspects, in the positive sense, for creating and identifying or adoption and implementation of new ideas. After identifying such a critical incident, contextual data were included in the core description of the case concerned. A second member of the research team validated a few cases. No essential discrepancies were found in the interpretations. After that, different cases were compared and an overall view emerged on innovative aspects of the initiatives and of science studies in schools was constructed.

Outcomes of the interviews

It is interesting that more than half of the visited schools were ones with no specialization in sciences. Actually, there was only one school where science specialization was the paramount character of the school. In three other cases there was also some other specialty like international (language and culture) education or arts (drama, rhetoric) etc. The interview data indicate that in the background of innovative approaches, there often seems to be a long history of participation in projects and pedagogical experimentation. In several cases the interviewed teacher noted spontaneously her/his long experience either in the LUMA project, in the Virtual School of Physics and Chemistry, or long-time cooperation with researchers at the university initiated during pre-service TE. Implementation of modern information and communication technologies appeared also as an important feature in many innovative projects.

If we summarise the general atmosphere in the discussions and the interviews, we may say that all teachers were enthusiastic about promoting science teaching in their schools. The key issue seemed to be how to motivate students and pupils. For instance, one teacher (CC) commented that “students are so much more motivated when I am not using chalk-and-blackboard chemistry” and another (CB): “I have been so glad when more students have chosen elective courses in physics”. Most often they wanted to test and introduce something new in their teaching. The initial idea might have come from international contacts (e.g. International Baccalaureate school network in school B) projects (LUMA mailing list, school E) or finding something new in the Internet (a powerful modelling program, school D). In school G, the starting point was to try to find out how physics could be studied over the Internet. It would mean promoting ICT skills, but also finding a practical way of organising elective physics courses in a small isolated school. Secondly, the teacher (GA) in charge of the project wanted to show his students concretely the experimental nature of scientific research. The third goal emerged along the running of the project: “Could we get those students, who are not active in school, to ponder science problems even outside school lessons? It is much better if students comment each other’s work than if the teacher is doing that”.

In one case the need to find an innovative approach was national. The national core curriculum introduced physics and chemistry as separate subject areas on grades five and six. A large number of primary school teachers had the problem of teaching their pupils physics and chemistry with very short training for themselves. Teacher AA in school A had already a long experience of teaching in these grades, and he had noticed that even if children are generally interested in natural phenomena they are enthusiastic about stories and fairytales. Thus, when asked by a group of developers of learning materials, how to approach basic physical phenomena on these grades, he had the creative idea of presenting mechanics in the form of an imaginary story. Space characters with experiences from space with no gravitation and no friction appeared to meet children and

compare how the phenomena changed in the Newtonian world we live in. Internet materials developed on the basis of this idea became a success. They are widely used in Finnish schools and while they have been partly translated in Swedish, they have aroused interest even in Sweden. Here the obvious national need made it possible that the original ideation was followed with substantial allocation of resources to the project and forming a multiprofessional project team for designing and producing the Internet materials. In schools E and G, the key idea of the initiative was to find a local solution to the same problem by letting upper-secondary school students teach physics to primary-school pupils. Each of these initiatives has important innovative aspects, but the latter ones are more of a local character.

The case in school A described above is of the type that there was a single teacher, who was supported by other teachers and school administration, but the project team was working mainly outside the school. Also teachers BA, DA, GA and HA were working rather individually in their schools. Only the last one indicated being a member of a project team where the other team members were working outside his school. However, even the other three teachers had good contacts with colleagues in other schools, with researchers etc. One of the teachers not mentioned above, FB, commented that “other teachers are not active in this project, but I have a good team with my students”. There was active co-operation in school F in the context of other projects: teacher FC commented that “my colleague (FB) persuaded me to join the project team as soon as it was possible for me, and all the time she is urging the younger colleagues to participate”. There was at least some reported contact with university researchers in all the visited schools, although not necessarily in the context of the reported initiative. It may indicate more the selective way of picking up the cases than a generalizable outcome that most contacts were with University of Helsinki. However, contacts were reported also with University of Joensuu, University of Tampere and with Tampere University of Technology. If we had had a larger number of contacted schools, certainly co-operation with other institutions of higher education had shown up.

In school E the innovation was obviously a product of pairwork. Two teachers had worked closely together after one of them, EB, had picked up the idea, EA immediately had accepted and supported it and them both co-operatively advanced to implementation with strong support of other teachers and the school administration. Teachers EC and ED joined the team to broaden the subject area competency and the initiative was a good example of the success of a co-operative teacher team doing creative implementation work in their school.

Discussion and conclusions

The high frequency of ICT-related comments is probably at least partly due to straightforward questions in the QE2 questionnaire, which often invited to further related comments and reflections. Lack of computer resources was not

mentioned as an obstacle, but often the need of flexibility and co-operation in the use of the equipment was acknowledged. The use of Internet as an essential part of the learning environment was observed in most cases. However, ICT use seemed to be very versatile including experimenting in microcomputer-based laboratory and three-dimensional modelling of organic molecules. It was also interesting to see some efficient combinations of virtual and real components of the learning environment in some initiatives.

We may also interpret the reasons for a wide ICT use in the framework of Fullan's (1991) categories of the properties of innovations (cf. Introduction). On the one hand, activated ICT applications are of several different types fitting local needs, and on the other hand, innovative teachers seemed to have a good ICT competence. Thus, the 'properties of ICT use' did not appear as limiting factors. Also the local characteristics seem to be favourable. It appears that innovative teachers considered new pedagogical ICT applications as positive as they considered any other possibilities for motivating their students. ICT offers powerful new tools for science teaching in schools. Teachers mentioned specifically that new approaches in science studies (e.g. in analysing experimental data or modelling) would be impossible without using ICT appropriately. Another group of prominent positive factors were external. There has been a widely publicized national ICT strategy implemented in all educational institutions so that all schools have individual local ICT strategies. Financing up-to-date hardware and software has been supported by both national and local projects. All the above positive factors contribute apparently to the wide use of ICT in the reported innovative initiatives.

However, networking of active teachers, both locally and nationally, emerges as a most interesting factor in explaining success of innovative approaches. Local networking with other educational institutions and with laboratories, industries etc. seemed to be strongest in the western part of the country (schools C and H). In all the visited schools the general atmosphere was open, promoting contacts over physical as well as subject area boundaries. Creative projects appeared to be found not only in sciences but in other areas as well, including integrative and international projects. In the beginning of the GRID project the hypothesis was that most valuable projects can be found hidden within individual schools. This importance of networking is in contrast with this starting point.

While we have found several favourable aspects of innovative initiatives, it is also worthwhile to try to look for negative ones. The first problem seems to be in the identifying of worthwhile new approaches. Indeed, we did find a few completely new ideas only in the studied initiatives. However, there was much creativity in the way ideas which were picked up in national or international contexts were modified and implemented in forms fitting local needs and local school cultures. Teachers themselves did not want to present their own initiatives as something special, in several cases it was some outsider who had apparently urged the teacher to fill in the QE1 questionnaire. This type of

shyness may be a national feature in the Finnish culture and be a reason of causing that many good initiatives are not presented in the database. Major obstacles in the adoption of new ideas seem to be not so much caused by a lack of financial resources as such, as by missing incentives for project work and lack of technical support personnel in school science laboratories and even more generally, in ICT applications.

This type of a study is explorative in nature. Thus, it can not be expected to find any final overall answers to our research question. However, we find for instance, that a single innovative teacher can promote creative ideas in his/her school, but it is an asset if there are innovative teachers also in other subject areas in his/her school, or if there is a living contact network with colleagues in other schools and with university researchers. Internet and e-mail facilitate networking, but also at least occasional face-to-face contacts are valuable. Teachers did not mention the 'best practices' approach when telling about launching their initiatives. The starting point was rather a pretty abstract and general idea and much concretization and creative modification was needed in the process of implementation in school practice.

References

- Dadds, M. (1997). Continuing professional development: Nurturing the expert within, *British Journal of In-service Education* **23** (1), 31-38.
- Fullan, M. (1991). *The New Meaning of Educational Change*. London: Cassell.
- Knezek, G. & Christensen, R. (2002). Impact of new information technologies on teachers and students. *Education and Information Technologies*, 7(4), 369-376.
- Kuitunen, H. & Meisalo, V. (1996). Luovan ongelmanratkaisun leviäminen koulun työtapanana: Tapaustutkimus LOTTO-projektin toiminnasta. *Helsingin yliopiston opettajankoulutuslaitoksen tutkimuksia* 144, 69 p.
- Lappalainen, A., Lavonen, J. & Meisalo, V. (2000). Environmental projects in primary school teacher training: River Vantaa is flowing -project as an example. In L. Aho & J. Viiri (Eds.). *Undervisning i naturvetenskap ur kultur-, teknologi- och miljöperspektiv. Rapport från det sjätte nordiska forskarsymposiet om undervisning i naturvetenskap i skolan*, 327-338. Joensuu: Joensuun yliopisto.
- Lavonen, J., Meisalo, V. & Juuti, K. (2004). The Role of Researchers in the Implementation of Educational Policies: The Finnish LUMA Programme (1996-2002) as a Case Study. *Journal of Baltic Science Education*, 2(6), 34-42.
- Lavonen, J., Meisalo, V. & Lattu, M. (2001). Problem Solving with an Icon Oriented Programming Tool: A Case Study in Technology Education. *Journal of Technology Education*, 12 (2), 21-34.
- Lavonen, J., Meisalo, V. & Lattu, M. (2002). Collaborative Problem Solving in a Control Technology Learning Environment, A Pilot Study. *International Journal of Technology and Design Education*, 12, 139-160.
- Moonen, B. H. & Voogt, J. M. (1998). Using networks to support professional development of teachers, *Journal of In-Service Education*, **24** (1), 99-110.

- Patton, M. Q. (2002). *Qualitative evaluation and research methods*. Thousand Oaks CA: Sage.
- Rogers, E. M. (1995). *Diffusion of innovations*. New York: Free Press.
- Roth, W.-M. (2005). *Doing Qualitative Research: Praxis of Method*. Rotterdam: Sense.
- SETRIS 2000. *Strategy for Education, Training and Research in the Information Society: A National Strategy for 2000-2004*. Helsinki: Ministry of Education. <http://www.minedu.fi/julkaisut/information/englishU/index.html> (Accessed 11.12.2005).
- Thomas, G. P. (2001). Toward effective computer use in high school science education: where to from here? *Education and Information Technologies*, 6 (1), 29-41.
- Willis, E. (1997). Technology: Integrated into, Not Added onto, the Curriculum Experiences in Pre-Service Teacher Education. *Computers in the Schools*, 13, 141-153.
- Zhao, Y. & Cziko, G. A. (2001). Teacher adoption of technology: a perceptual control theory perspective. *Journal of Technology and Teacher Education*, 9, 5-30.

Getting students interested in physics and changing their attitude by using a toy (car)

Anu Tuominen
University of Turku

In this case study the main topic is to find out if we can effect or even change ones attitudes, interest and conceptions about physics by using a toy. Participants are pre-service teacher students (N = 85). There were two questionnaires, pre-test and post-test, to see if there had been any change in students attitudes, interest and conceptions about physics. After the pre-test students were asked to measure and calculate speed and acceleration of a toy car on a slope. The sample was divided into three different groups depending on how the students had answered in the pre-test. The biggest change in ones attitudes and conceptions about physics to the positive direction was in the group, where all students were female and they had mostly succeeded poorly in physics before.

Introduction

It is internationally known that pupils do not learn the conceptions in physics that are required in curriculum and they do not choose physics in upper-secondary school or further education that requires knowledge in natural sciences. It has been proposed that physics should be taught in a context that pupils are interested in. One of the purposes of elementary school curriculum in Finland is to awake an interest and to inspire pupils to study science and hopefully, choose physics in high school in the future. The problem is the lack of interest that pupils, especially girls, have towards physics (Lavonen, 2006).

There is gender gap in science interest. Boys are more interested in technology and physics, while girls are more interested in biology than boys are (Baram-Tsabari & Yarden, 2011). The gap between genders grows as the students get older. While there is no statistically significant difference among science field interests in the kindergarten and lower grades 1-3, the gap gets 20-fold when we are talking about interest among 10-12 grade students. Boys ask more self-generated questions (show more interest) about physics and girls ask more about biology (Baram-Tsabari & Yarden, 2011). Cultural explanations include the masculine image of physics and lack of female role models in science media (Baram-Tsabari & Yarden, 2011).

This lack of interest seems to continue even in university. Teacher students feel that physics (and chemistry) is (are) less meaningful, boring and disliked compared to more interesting natural sciences like biology and geography (Ahtee & Rikkinen, 1995).

Elementary teachers are reluctant to teach science due to not having a scientific background (Victor, 1962). Now, over forty years later, we still have the same problem. After participating in an elementary activity-based science course the pre-service teacher students showed a positive shift in attitude when comparing measurements that were taken at the beginning and at the end of the semester (Minger & Simpson, 2006).

Pre-service teacher students have scientifically acceptable conceptions or alternative conceptions about subjects in physics. This will create problems in a classroom, if the pre-service teachers do not understand the difference between these two. With a familiar tool we might change the attitudes and conceptions that students might have about physics itself and we might re-develop alternative conceptions that students could have about physical phenomena.

According to Ausubel (1963), the most important tool for teaching is to ascertain students' prior knowledge of a topic. Teachers usually use specifically designed tools and equipment to teach physics and phenomena in a classroom. Maybe by using a familiar tool (a toy) we can concentrate on the phenomenon itself and there will be no misunderstandings like "this phenomenon happens only with this special tool (and in a classroom)". If the tool is familiar to students (pupils), they already know how it should behave in different situations and they can concentrate on the phenomenon.

Usually pupils just calculate acceleration for a given velocity and time but they rarely do actual required measuring. The aim of this case study is to stimulate students' interest in science and to bring physics closer to the students.

Concepts

Attitude is a complex concept. Attitude can be considered to have three underlying components. These components are cognitive, affective and behavioral component, which however, do not work separately. Most attitudes are probably based on interdependent and correlated cognitive, affective and behavioral responses (Eagly & Chaiken, 1993, 666-667).

A negative attitude towards certain school subjects necessitates lack of interest and, given opportunity, students do not choose more courses of such a subject (Oliver & Simpson, 1988). If something is interesting, it attracts our attention, and working with it is pleasant (Veermans & Tapola, 2006). For example, creating links between science, technology and society approaches has an important role to play in developing pupil's science literacy. Context-based

approaches provide an effective way to interest and motivate pupils (11-18 years) in their science lessons (Bennett, Hogarth & Lubben, 2005).

“Physics” here refers to physics as a school subject on elementary or on lower-secondary school level.

Research questions for the case study:

- 1) Is it possible to change one's attitude with a teaching experiment?
- 2) Boys like physics, girls do not. Is there a gender gap to be seen among pre-service teacher students?
- 3) Do students find it meaningful to play with a toy or is it considered childish?

Method

The subjects of the study were mainly 85 second year pre-service teacher students on a science course, 70 females and 15 males. The study took place in the autumn of 2006.

Quantitative and qualitative methods were used to evaluate students' answers. In order to get some general opinions and attitudes, students were asked to write a few sentences beginning with “I think physics is...” or “Physics is like...”. A few questions about their previous history were asked, e.g. had they studied physics in upper-secondary school or just in lower-secondary school, what grades had they had in physics, and so on. The students were asked to think of all the previous physics they had studied. The open answers were divided into five categories: “negative”, “partially negative”, “neutral”, “partially positive” and “positive”.

1) Pre-test open answers:

“I feel that physics is difficult and boring”, (Female, number 3). (*negative*)

“Physics is just formulas that are not explained to me”, (Female, number 22). (*partially negative*)

“Challenging, time consuming, rewarding”, (Female, number 31). (*partially positive*)

“Physics is challenging, practical, based on everyday phenomena”, (Male, number 29). (*partially positive*)

Answers with “interesting”, “fun”, “nice” have been considered “positive”.

Because most of the female students seemed to answer more negatively than male students, answers were classified according to gender to find categories of subjects that female and male students considered as “easy” in physics (Figure 1, Figure 2).

Female

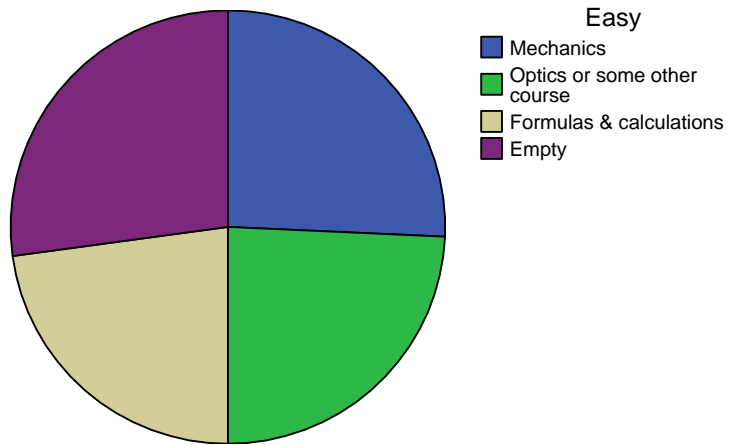


Figure 1.

Male

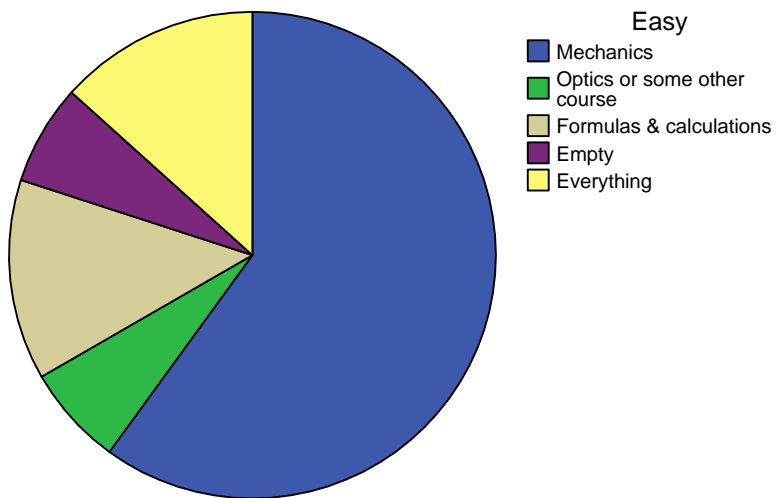


Figure 2.

The graphs for female students and male students look very different. Over 25 % of female could not come up with anything for “easy”. There were some male students who felt that everything was “easy”, whereas none of the female felt the

same. Mechanics was considered “easy” in both groups. Many female felt that everything is “difficult”, while none of the males felt the same. Electricity and electromagnetism were named “difficult” (Figure 3 and Figure 4).

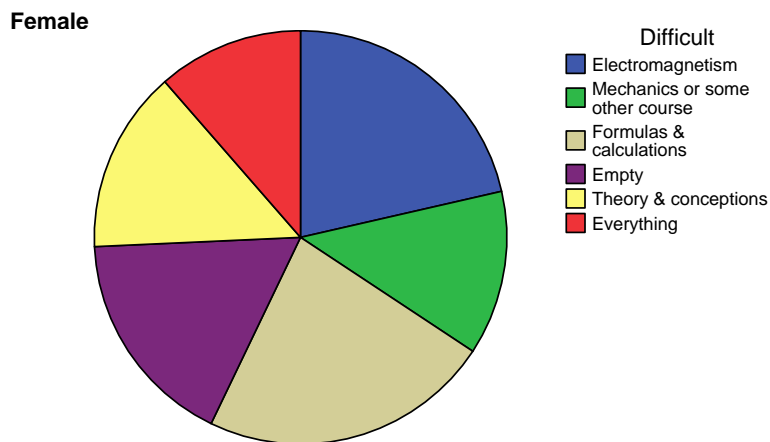


Figure 3.

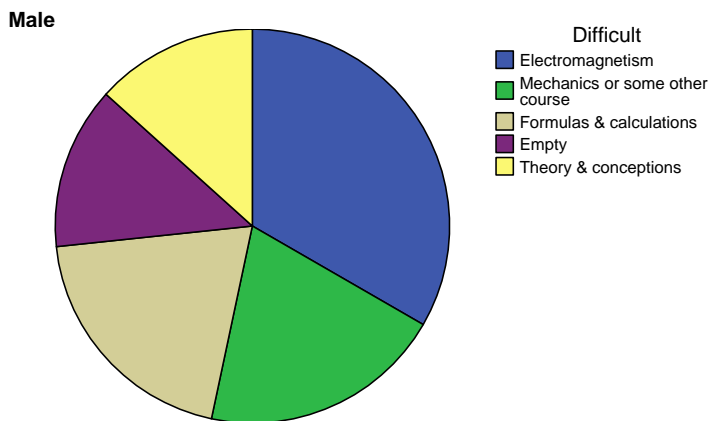


Figure 4.

- 2) After the pre-test, students measured and calculated the speed and the acceleration of a toy car.

Toy car

Students were in small groups of three or four. The equipments were a measuring tape, a clock, a toy car and a slope. Students had a report sheet and filled in their measurements (Appendix 1). Students were asked to come up with an appropriate name for the car like “The Red Dragon” so the cars became a

little more individual. After the measurements the “Team results” of acceleration were written on the black board. At the end of the session we had a little discussion about why all the accelerations were different and what is the effect of acceleration to motion. The fact that there was no “correct” answer for the acceleration of each car and that every group had a “correct” answer was somewhat difficult for the students to understand.

3) Post-test open answers:

In the post-test, students were asked to think about how they feel and what their attitudes to and conceptions about physics are, when they considered the exercise we did with the toy car. The questions about the attitudes and conceptions were the same in the pre- and the post-test. The attitudes changed considerably after the exercise.

“...interesting, nice and practical.”, (Female, number 4).

“So physics can be quite understandable, physics is accurate, physics is fun.”, (Female, number 10).

“More fun with funny equipments”, (Female, number 64).

“...sometimes boring because of repeating simple tasks”, (Male, number 70).

In the pre-test, students were asked to indicate their conceptions and attitudes (15 statements) about physics on a Likert-scale from 1 to 5. It was possible to summarize different attitudes in three sum categories, *nice* (pleasing, fun, nice, reversed boring, reversed obligatory), Cronbach $\alpha = 0,811$, *oppressive* (frightening, difficult, not easy, discouraging, unclear), Cronbach $\alpha = 0,860$, and *useful* (useful, practical, consistent and important), Cronbach $\alpha = 0,798$.

Students were divided into three different groups, Group 1, Group 2 and Group 3, according to the way they had answered in the pre-test. Then the groups were compared within three sum categories in pre- and post-test with ANOVA, (Figure 5). The biggest changes in a positive direction was found when comparing “oppressive” and “nice” in pre- and post-tests in Group 1. In Group 2 there was very little change and the results in Group 3 was in between.

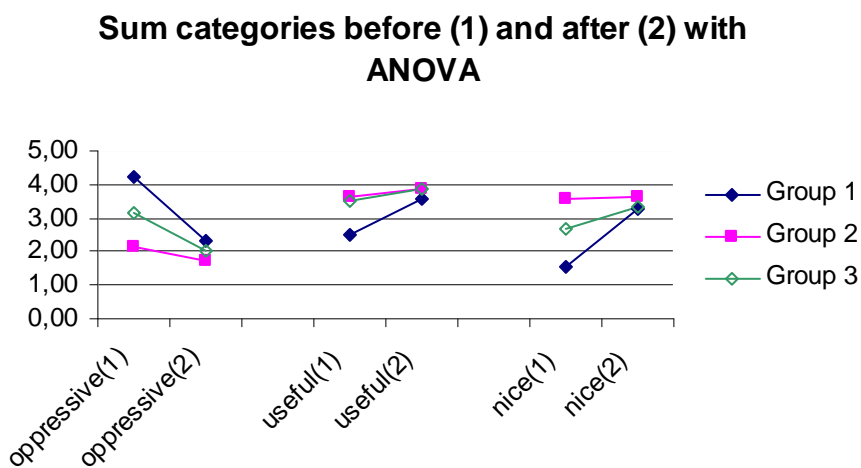


Figure 5.

Because of some background knowledge, it was possible to try to find out similarities between the students within each group.

Group 1 (n = 24)

All students were female. About 63 % of these students had studied a minimum amount of physics in upper-secondary school and 29 % had not studied physics in upper-secondary school at all. Grades were either average or good, for those who had a grade, but still 79 % of students in this group had negative attitude towards physics before (17 % were neutral and 0 % positive, 4 % could not tell).

Group 2 (n = 27)

There were almost as many students who had studied the minimum (44 %) and the maximum (52 %) studies of physics in upper-secondary school. Most of the male students were in this group. None of the students had poor grades, in fact 85 % had a good grade from upper-secondary school, and attitudes before were mainly positive 55 % (30 % neutral, 8 % negative, 7 % could not tell). There was very little change in attitudes.

Group 3 (n = 34)

About 82 % of the students were female. Only 6 % of the students had not studied physics in upper-secondary school. Most of those who had studied physics, had studied the minimum amount with a good or an average grade. The attitudes before were mainly positive (44 %), (38 % were neutral and 18 % negative).

Because of the positive change in attitudes in Group 1, it was interesting to see if we would get similar results in classifying answers according to gender (t-test, Table 1).

Table 1. The sum categories by gender before - after

Female	Mean	Std. Deviation	t	df	Sig. (2-tailed)
Oppressive	1.23	0.97	10.70	69.00	0.00
Useful	-0.61	0.76	-6.71	69.00	0.00
Nice	-0.93	1.07	-7.26	69.00	0.00
Male	Mean	Std. Deviation	t	df	Sig. (2-tailed)
Oppressive	0.64	0.97	2.54	14.00	0.02
Useful	-0.05	0.58	-0.34	14.00	0.74
Nice	-0.03	0.64	-0.16	14.00	0.87

* $p < 0,05$

** $p < 0,01$

Again female students had a considerable change towards more positive attitudes, while male students had little change. Because of the significant change in female attitudes, we took a closer look to see which of the attitudes had changed and which had not (Table 2).

Table 2. Female attitudes before - after

	Mean	Std. Deviation	t	df	Sig. (2-tailed)
Boring	0.66	1.51	3.64	69	0.00
Practical	-1.26	1.19	-8.85	69	0.00
Fun	-1.27	1.20	-8.84	69	0.00
Unclear	1.16	1.53	6.33	69	0.00
Pleasing	-1.04	1.44	-6.06	69	0.00
Obligatory	0.70	1.80	3.25	69	0.00
Discouraging	1.33	1.40	7.93	69	0.00
Useful	-0.31	1.08	-2.43	69	0.02
Easy	-1.29	1.59	-6.77	69	0.00
Consistent	-0.70	1.09	-5.35	69	0.00
Frightening	1.03	1.62	5.30	69	0.00
Difficult	1.26	1.30	8.06	69	0.00
Nice	-0.99	1.45	-5.69	69	0.00
Important	-0.17	0.87	-1.65	69	0.10

* $p < 0,05$

** $p < 0,01$

The only one with no significant change between pre- and post-test was “important”. For the comparison we had a similar table for the male students as well (Table 3).

Table 3. Male attitudes before - after

	Mean	Std. Deviation	t	df	Sig. (2-tailed)
Boring	0.20	1.01	0.76	14	0.46
Practical	-0.27	1.16	-0.89	14	0.39
Fun	-0.20	0.86	-0.90	14	0.38
Unclear	1.13	2.20	2.00	14	0.07
Pleasing	-0.07	0.80	-0.32	14	0.75
Obligatory	-0.40	1.06	-1.47	14	0.16
Discouraging	1.07	2.15	1.92	14	0.08
Useful	-0.13	1.51	-0.34	14	0.74
Easy	-0.80	1.08	-2.86	14	0.01
Consistent	-0.07	0.59	-0.43	14	0.67
Frightening	-0.67	2.26	-1.14	14	0.27
Difficult	0.87	0.83	4.03	14	0.00
Nice	-0.07	0.96	-0.27	14	0.79
Important	0.27	0.80	1.29	14	0.22

* $p < 0,05$

** $p < 0,01$

There were only two cases that had a significant change, “easy” and “difficult”. For all the others there was no significant change in attitudes.

Reliability

In order to minimize the effect that doing a research might have on participants' attitudes, students were told that their answers would not affect their grades for this course.

To ask people about their attitudes causes some limitations concerning the results. “These limitations follow from respondents' awareness that their attitudes are being assessed and their attendant desire to present themselves positively to others (often to a researcher or an experimenter).” (Eagly & Chaiken, 1993, 667). Thus, by doing the research, the researcher may affect the result itself.

Students answered the questionnaire individually, but because of the size of the group (five groups, each about 20), they could have copied answers from someone else or whispered answers to each other. Time to fill the form was 10 - 20 minutes. Thus, because the questionnaire was quite long, the last questions may have got less attention from the students. This effect could have been avoided by rotating the appearance order of the questions.

There was no postponed measuring so we cannot say how permanent the attitude change will be.

Discussion

It seems possible to change the students' attitudes and conceptions about physics by using a toy. Especially this seems to be the case with those female students who have little prior studies in physics and who have a negative attitude towards physics. It was amusing to see how excited students were about a simple toy. In the future, to get even more positive responses, it might be a good idea to use their own toy cars. I suggest that physics lessons can be more interesting and meaningful for the students (pupils) if they could bring their own toy for the experiment.

There are still more male physics teachers in lower- and upper-secondary school than female teachers in Finland. The lack of female role models could be one reason why girls are not interested in physics. It could be just a coincidence, but all students in Group 1 had had a male physics teacher in lower-secondary school.

Even though many of the females had a negative attitude towards physics in the pre-test, they still thought that physics is important. There is a gender gap between these two groups in how they think about the subject categories in physics. Both genders seem to feel that the subject categories that one can see (mechanics, optics) are considered "easy", and the categories that one cannot see, or that are hard to visualize (electricity and electromagnetism) are considered "difficult". Here is a challenge for every secondary school teacher: how to teach physics so that phenomena like electricity and electromagnetism becomes visible to pupils?

Because the biggest change in ones attitudes and conceptions about physics in the positive direction was in Group 1, it seems that this kind of playing benefits mostly those females who have not had so much prior education in physics or/and have a negative attitude. In other groups, those who already knew the theory "by heart" did not feel like they were learning anything new and felt that playing with a toy was a waste of time. If the physics part is easy for them, they should consider this experiment as something that they could do with their pupils in a classroom.

Since most of the pre-service teacher students are female, familiar tools (toys) could be one way to get them interested in physics.

References

- Ahtee, M. & Rikkinen, H. (1995). Luokanopettajaksi opiskelevien mielikuvia fysiikasta, kemiasta, biologiasta ja maantieteestä. *Dimensio*, 2, 54-58.
- Ausubel, D. (1963). *The Psychology of Meaningful Verbal Learning*. New York: Grune & Stratton.

- Baram-Tsabari, A. & Yarden, A. (2011). Quantifying the gender gap in science interests. *International Journal of Science and Mathematics Education*, 9, 523-550.
- Bennett, J., Hogarth, S. & Lubben, F. (2005). A systematic review of the effects of context-based and Science-Technology-Society (STS) approaches in the teaching of secondary science. York: University of York.
- Eagly, A. & Chaiken, S. (1993). *The Psychology of Attitudes*. Fort Worth: *Harcourt Brace Jovanovich College Publishers*.
- Lavonen, J. (2006). Uusia tuulia fysiikan opetuksessa. *Dimensio*, 6, 9.
- Minger, M. & Simpson, P. (2006). The Impact of a Standard-Based Science Course for Preservice Elementary Teachers on Teacher Attitudes Toward Science Teaching. *Journal of Elementary Science Education*, 18(2), 49-60.
- Oliver, J. S. & Simpson, R. D. (1988). Influences of attitude toward science, achievement motivation, and science self concept on achievement in science: A longitudinal study. *Science Education*, 72(2), 143-155.
- Victor, E. (1962). Why are our elementary school teachers reluctant to teach science? *Science Education*, 46(2), 185-192.
- Veermans, M. & Tapola, A. (2006). Motivaatio ja kiinnostuneisuus. In S. Järvelä, P. Häkkinen & E. Lehtinen (Eds.), *Oppimisen teoria ja teknologian opetuskäyttö*. Helsinki: WSOY.

Appendix 1.

The Acceleration of a Toy Car on a Slope

Name:

Date:



1. What is the purpose of this study?
2. Experimental setting. What kind of equipment will you need? Sketch your apparatus below.

3. Fill in your data to the table below. Repeat your measurement for each distance three times and then calculate the average of the measurements. How does this effect on possible errors occurring in the measurement? Write down your calculation for at least one distance (the average of the measurements).

Distance, Cm	Distance, m	Time 1, s	Time 2, s	Time 3, s	Average of the measurements	Coordinates
5	0,05					(, 0,05)
10	0,1					(, 0,1)
20						(,)
30						(,)
40						(,)
50						(,)
60						(,)
70						(,)
80						(,)
90						(,)

4. Draw (time, distance)-graph on a graph paper based on your measurements. Use a reasonable scale! Observe the figure. Is it possible to deduce if time and distance are related?

The graph looks like:

Conclusion:

Calculate the *average speed during various time intervals* by using the formula below:

$$\text{Speed} = \text{distance} / \text{time}, \quad v = \frac{s}{t}$$



Where **v** = speed, **s** = distance and **t** = time.

Distance, m	Time, s	Average speed, m/s
0,05		
0,1		
0,2		
0,3		
0,4		
0,5		
0,6		
0,7		
0,8		
0,9		

Extra: Find a match

Flying speed of a butterfly	40 km/h
Race car	10 m/s
Sprinter	250 km/h
Car	1 m/s
Scooter	80 km/h

While comparing average speed calculations above, what do you observe? What kind of information does it tell you about the motion of the toy car?



Copy the values from the previous table to the table below.

Calculate instantaneous acceleration from the **consecutive** measurements by using the formula given below.

$$a = \frac{\Delta v}{\Delta t} \text{ , where } a = \text{acceleration, } \Delta v = \text{change in speed and } \Delta t = \text{change in time.}$$

For example the instantaneous acceleration for the first two observations would be calculated as follows

$$\text{Acceleration}_1 = (\text{speed}_1 - 0) / (\text{time}_1 - 0), \quad \text{Acceleration}_2 = (\text{speed}_2 - \text{speed}_1) / (\text{time}_2 - \text{time}_1),$$

and so on.

Observation	Time, s	Speed, m/s	Acceleration, m/s ²
	0	0	0
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			

The average of the accelerations:

5. Error analysis

What do you consider are the biggest sources of error? Give an example. Are there significant differences between the measurements?

6. Other observations and conclusions

7. Literature

Compare your measurements with literature (if possible).