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MATEMAATTISET OPPIMISVAIKEUDET – KATSAUS EUROOPPALAISEEN TUTKIMUKSEEN

Pirjo Aunio, Riikka Mononen ja Anu Laine

Helsingin yliopiston opettajankoulutuslaitoksella järjestettiin 6.11.2014 Helsingin yliopiston Opettajan Akatemian tuella henkilökunnalle, opiskelijoille ja muiden yliopistojen tutkijoille seminaari matemaattisista oppimisvaikeuksista. Seminaariin meillä oli mahdollisuus kutsua puhujiksi oppimisvaikeustutkijoita Euroopan eri yliopistoista. Tämän erikoisnumeron tarkoituksena on lyhyesti esitellä seminaarin teemoja ja johdattaa kiinnostuneet tutustumaan aiheeseen lisää. Erikoisnumero on rakennettu siten, että alussa on johdanto suomeksi, missä lyhyesti esittelemme seminaarin aiheita. Sitä seuraa lyhyt johdanto englanniksi. Tämän lisäksi seminaaripäivän aikana esiintyneet asiantuntijat ovat tehneet puheistaan yhteenvedot, jotka julkaistaan englanniksi. Olemme koonneet kirjoittajien käyttämät lähteet yhteenvetojen lopuksi yhdeksi lähdeluetteloksi. Asiantuntijoiden lyhyet esittelyt on sijoitettu erikoisnumeron loppuun.

1 Matemaattiset oppimisvaikeudet: määrittelyn haasteet, kehitystä ennustavia tekijöitä ja interventio-ohjelmien mahdollisuudet

1.1 Katsaus matemaattisten oppimisvaikeuksien tutkimukseen

Pekka Räsänen Niilo Mäki Instituutista Jyväskylästä esitti katsauksen matemaattisten oppimisvaikeuksien tutkimuksen tilaan. Räsänen käyttää termejä kehityksellinen dyskalkulia ja matemaattiset oppimisvaikeudet (Developmental Dyscalculia [DD], Mathematical learning disabilities) viitatessaan henkilöihin, joilla on perustavanlaatuisia kognitiivisia tekijöitä estämässä matematiikan perustaitojen, etenkin aritmetiikan, oppimista. Räsäsen esityksen tärkeimpänä tehtävänä oli kuvata sitä, miksi on niin vaikeaa määritellä kehityksellinen dyskalkulia yhdellä ja yleisesti hyväksyttävällä tavalla. Hän esitteli kuusi keskeistä syytä sille, miksi määrittely on niin vaikeaa: (1) tutkimuksen puuttuminen, (2) vertailun ja pirstaloitumisen ongelmat, (3) oppimisympäristön vaikutukseen liittyvät haasteet (4) kulttuuririippuvaisten taitojen monimutkaisen oppimisen ongelmallisuus (5) katkaisuraja-arvoon liittyvät ongelmat ja lopuksi (6) lyhenteiden käyttämiseen liittyvä ongelma.

1.2 Matemaattiset oppimisvaikeudet ja kielen kehittyminen

Annemie Desoete (University of Ghent & Hogeschool Artevelde, Belgia) esitteli oman ryhmänsä tutkimusta koskien kielen oppimista ja matemaattisia oppimisvaikeuksia. Yhdessä tutkimuksessaan Desoete ja hänen ryhmänsä ovat seuranneet esikouluikäisten lasten matemaattisten taitojen (laskemisen taidot, matemaattiset suhdetaidot, lukujonon arvioinnin taidot ja aritmeettiset perustaidot) ja kielellisten taitojen kehitystä ja ennustusvoimaa ensimmäiselle luokalle asti. Tässä tutkimuksessa esikoulussa mitatut kielelliset taidot olivat yhteydessä ensimmäisen luokan aritmeettisiin taitoihin senkin jälkeen, kun lasten esikoulussa mitattu laskemisen, lukumäärän arvioinnin ja matemaattisten suhdetaitojen osaaminen oli kontrolloitu. Etenkin tuottava kielen osaaminen oli merkityksellistä. Toisessa pitkittäistutkimuksessa esikoulusta toiselle luokalle keskiössä olivat matemaattisista taidoista lukujonon arvioinnin taidot ja aritmeettiset taidot. Tämä tutkimus paljasti, että kielen osaaminen esikoulussa oli yhteydessä vain matemaattisten taitojen osaamisen tasoon, mutta ei kehitykseen esikoulusta toiselle luokalle. Desoeten tutkimuksista opitaan, että kun ennustetaan matemaattisten taitojen kehitystä, on hyvä huomioida myös muita kuin matemaattisia taitoja.

1.3 Matemaattiset taidot ja niiden kehitykselliset yhteydet

Evelyn Kroesbergen (Utrecht University, Alankomaat) tarkasteli lapsilla kehittyviä matemaattisia taitoja ja niiden keskinäisiä suhteita. Kroesbergenin mukaan matemaattisten taitojen kehityksen osana ovat ei-symboliset (esim. pisteiden lukumäärän vertailu ja ei-verbaalinen lukujonotehtävä) ja symboliset matemaattiset taidot (esim. lukumäärän määrittämisen tehtävät), esi-matemaattiset taidot, numeeriset taidot ja taito yhdistää eri lailla ilmaistuja (esim. ei-symbolinen, symbolinen, konkreettiset esineet) lukumääriä. Kroesbergen tiivistää, että kehityksellisesti symboliset ja ei-symboliset taidot tukevat toisiaan eri kehitysvaiheissa. Erityisen tärkeä matemaattisen osaamisen kehittymiselle näyttäisi olevan lasten ymmärrys siitä, miten numerosymbolit, lukusanat ja konkreetit lukumäärät ovat yhteydessä toisiinsa.

1.4 Varhaisten matemaattisten taitojen kehityksen ennustaminen ja interventiot kehityksen tukemisessa

Maria Chiara Passolunghi (University of Triste, Italia) kertoi oman ryhmänsä tutkimuksista liittyen ensinnäkin niihin kognitiivisiin tekijöihin, jotka ennustavat varhaisten matemaattisten taitojen kehitystä. Passolunghin ja hänen ryhmänsä tulokset osoittivat, että esikouluikäisten lasten matemaattiset taidot olivat suoraan yhteydessä verbaaliseen älykkyyteen, fonologisiin taitoihin, prosessoinnin nopeuteen ja työmuistiin. Tämän lisäksi Passolunghi raportoi ryhmänsä tekemästä interventiotutkimuksesta esikoululaisilla. He havaitsivat, että niiden lasten, joiden kanssa harjoiteltiin tehostetusti matemaattisia taitoja, matemaattiset taidot paranivat, mutta kehitystä ei näkynyt työmuistin tehtävissä. Sen sijaan ryhmä, jonka kanssa harjoitettiin työmuistia, paransi osaamistaan työmuistin tehtävissä, mutta myös matemaattisten taitojen tehtävissä. Passolunghin ja hänen ryhmänsä tulokset tukevat ajatusta, että sen lisäksi, että matemaattisesti heikkojen lasten kanssa harjoitellaan matemaattisia taitoja, on potentiaalisesti järkevää tukea myös työmuistin kehitystä.

1.5 Luonnontieteellisen ajattelun kehittäminen matematiikan tehtävien avulla

Geerdina van Aalsvoort (Saxion, Hogeschool Deventer, Alankomaat) esitteli oman tutkimusryhmänsä interventiotutkimuksia kohdentuen ensimmäisen luokan lasten luonnontieteellisen ajattelun kehittämiseen. Van Aalsvoort ja hänen ryhmänsä tarkastelivat interventio-ohjelman vaikuttavuutta mitaten muutosta lasten ja opettajien toiminnassa verrattuna kontrolliryhmien toimintaan. Van Aalsvoort raportoi, että interventio-ohjelmassa mukana olleiden lasten matemaattiset taidot kehittyivät enemmän ja lasten välttämissuuntautuneisuus tehtävissä väheni. Sen sijaan muutosta ei näkynyt lasten taidoissa ymmärtää mittaamista ja tasapainoa. Opettajien toiminnassa havaittiin kehitystä siinä, miten he käyttivät kieltä, päättelyä ja matematiikkaa opetuksessaan.

1.6 Matematiikan oppimisvaikeudet ja interventiotutkimus

Pirjo Aunio (Helsingin yliopisto) puhui matematiikan oppimisvaikeuksiin liittyvän käsitteistön ongelmallisuudesta ja interventiotutkimuksista. Aunio ja Räsänen (2015) ovat muodostaneet tutkimuskirjallisuuden perusteella mallin niistä matemaattisista taidoista, joita lapsille kehittyy noin 5-8 vuoden iässä. Tämän mallin avulla on mahdollista suunnitella oppimisen arviointi, erityisesti heikkojen osaajien tunnistamisen ja tukemisen näkökulmista. Aunio kertoi myös oman ryhmänsä tutkimuskehitystyöstä, jonka tuloksena on julkaistu ThinkMath-verkkopalvelu, josta saa ilmaiseksi käyttöön pienryhmäharjoitteita heikkojen osaajien matemaattisten taitojen kehityksen tukemiseen. Aunion ja hänen ryhmänsä interventiotutkimuksissa on saatu positiivisia tuloksia ThinkMath-harjoitusohjelmien vaikuttavuudesta. Kuten muidenkin interventiotutkimusta tekevien ryhmien, myös Aunion ryhmän haasteena on saada näkyviin pitkäaikaiset ja laajat oppimisefektit.

1.7 Yhteenveto

Matemaattisiin oppimisvaikeuksiin liittyvä tutkimus on edistynyt ja lisääntynyt viimeisen kymmenen vuoden aikana selkeästi, mutta tarvitaan yhä lisää tutkimusta siitä, mitkä tekijät selittävät matemaattisten oppimisvaikeuksien syntymistä ja millä keinoilla matemaattisten taitojen oppimista voidaan tukea. Tämän teemanumeron kirjoituksista näkyy, että matematiikan oppimisvaikeuksiin liittyvää tutkimusta tehdään monella eri tavalla. Eri näkökulmat tuottavat hyvää ja tarvittavaa tietoa ilmiön ymmärtämiselle. Pitkittäistutkimukset keskittyvät taitojen kehitykseen. Etenkin tällä hetkellä tietoa on tulossa kielen oppimisen, työmuistikomponenttien ja muiden muistitoimintojen roolista matematiikan oppimisvaikeuksien ilmenemisessä. Matematiikan oppimisvaikeuksien tutkimuksessa interventiotutkimukset ovat toinen merkittävä osa uuden tieteellisen tiedon tuottamisessa. Tässä tutkimustraditiossa haasteina ovat muun muassa tutkimuksellisen asetelman kehittäminen, laboratorio-oloissa löydettyjen positiivisten tulosten siirtäminen päiväkoti- ja kouluympäristöön tukemaan heikkojen osaajien oppimista, sekä harjoitteluohjelmilla saatujen positiivisten tulosten saaminen pysyviksi ja siirtyminen muuhun oppimiseen. Tämän lisäksi tarvitsemme lisää tutkimusta, mikä toistaisi muiden tutkijaryhmien tuloksia, jotta voimme olla varmoja löydettyjen ilmiöiden yleisyydestä ja todenpitävyydestä. Matematiikan oppimisvaikeuksiin liittyvä tutkimus tarjoaa meille runsaasti mielenkiintoisia tutkimuksellisia haasteita, joita yhdessä ratkomassa ovat muun muassa tämän teemanumeron kirjoittajien eurooppalaiset tutkimusryhmät.

2 Mathematical learning difficulties – snapshots of current European research

Pirjo Aunio, Riikka Mononen ja Anu Laine

In November 2014 we had a wonderful possibility to organize a seminar *International Seminar on Mathematical Learning Difficulties* with our international colleagues in the field of mathematical learning difficulties. One of the main aims was to provide open lectures for the staff members and students in University of Helsinki. The meeting was supported by the Teachers' Academy in University of Helsinki.

We have collected extensive summaries of the presentations to form this special issue. Pekka Räsänen from Niilo Mäki Institute (Jyväskylä, Finland) demonstrated why it is so hard to find one definition to developmental dyscalculia. Annemie Desoete (University of Ghent, Belgium) presented her research group's studies into mathematical skills development and language skills. Evelyn Kroesbergen (University of Utrecht, the Netherlands) discussed about the variation of the mathematical skills developing in early childhood and the developmental connections between mathematical skills. Maria Chiara Passolunghi (University of Trieste, Italy) reported her group's results about the developmental links between cognitive components and early mathematical skills. In addition, she showed results from her group's early numeracy and working memory interventions. Geerdina van der Aalsvoort (Saxion University of Applied Sciences Deventer, The Netherlands) showed results from her group's intervention study on young children's scientific thinking skills. Pirjo Aunio (University of Helsinki, Finland) showed the model of developing core mathematical skills and results from intervention studies with mathematically low performing children. Extensive summaries have been listed in Table 1 and they can be found in following sections. In the end of the individual extensive summaries there is one joint reference lists and short introductions of the authors.

To sum up the main ideas from the presentations. Firstly, although mathematical learning difficulties are common, we do need more research to be able to understand the possible cognitive precursors or environmental issues affecting learning and causing problems. Secondly, we need more studies about intervention programmes designed to support the mathematical skills development in children having problems in learning mathematics. Thirdly, we also need more studies validating the positive findings in individual studies, using the same assessment and intervention tools.

Table 1. Papers presented in the special issue

Authors	Title of the paper
Aunio, P., Mononen, R. & Laine, A.	Matemaattiset oppimisvaikeudet – katsaus Eurooppalaiseen tutkimukseen; Mathematical learning difficulties – snapshots of current European research (introduction chapter)
Räsänen, P.	Longitudinal studies on dyscalculia
Desoete, A.	Language and math
Kroesbergen, E.	The structure of number sense and it's relation to mathematical (dis-)ability
Passolunghi, M.C.	Precursors of early mathematics and early intervention programs
Van der Aalsvoort, G.M., de Wit-Meijer, T., Compagnie, C. & van Schaik, M.	Improving science skills during mathematical activities in grade 1
Aunio, P. & Tapola, A.	Children with low performance in mathematics and interventions

2.1 Longitudinal studies on dyscalculia

Written by: Pekka Räsänen

Cite section as: Räsänen, P. (2015). Longitudinal studies on dyscalculia. In Aunio, P., Mononen, R., & Laine, A. (Eds). *Mathematical learning difficulties – snapshots of current European research*. (pp. 651-656). LUMAT, 3(5).

The current understanding on Developmental Dyscalculia/Mathematical learning disabilities (DD) shares a view that there are some fundamental cognitions related to the failure to learn basic mathematical (especially arithmetic) skills. There is also a shared understanding that a person can fail in learning mathematics even though s/he would have had an opportunity to learn in an optimal learning environment and would not have additional cognitive (e.g., low IQ), perceptual (e.g., vision impairment) or emotional problems to hinder learning. We also agree that a typical feature of the DD is persistency; difficulties in learning do no appear suddenly and do not disappear suddenly – even in circumstances of proper learning environment.

However, there is a disagreement what these genotypic features are, which cause the specific failure in learning, and whether there are only one or many genotypes of the DD. Some studies point to the direction of specific early developing numerical skills, while other studies claim that more general cognitions (e.g., components of working memory processing or spatial skills) can lead to the pathway of persistent difficulties in learning basic mathematical skills. We still lack a common understanding of the definition.

The skeleton of my presentation was the list of reasons why finding a common definition for the DD is so difficult (Mazzocco & Räsänen, 2013, see Box 1). Secondly, I went through some of the newest longitudinal analyses of our lab and collaborators to see, if we could get some flesh on the bones. In this extended summary of my presentation, I will only shortly describe these six reasons for the definition problem, which are

1. The 'Lack of research' problem
2. The 'Comparability and fragmentation' problem
3. The 'Learning environment effect' problem
4. The 'Complex nature of learning cultural skills' problem
5. The 'Cut score' problem
6. The 'Acronym' problem

The 'Lack of research' problem

The amount of research on the DD is increasing. Based on a database search we found (Räsänen & Koponen, 2011) that it had doubled during the last consecutive decades, but the ratio of studies on the DD had not changed compared to studies on dyslexia or dysgraphia. Still research on reading skills is dominating the field of learning disability research.

A graph from a review by Butterworth and Kovas (2013, their Table 1) illustrates the underrepresentation of studies and research funding on the DD compared to other types of learning difficulties.

Table 1. The estimated prevalence and the amount of funding by the US National Institute of Health on different learning difficulties in 2000–2009 (Butterworth & Kovas, 2013)

Table 1. Estimated prevalences of five specific learning difficulties. NIH research funding for these SLDs in 2000–2009 varied widely (*1*, *32*)

SLD	Estimated prevalence (%)	NIH research funding in U.S. $1000s
Dyslexia	4–8	27,283
Dyscalculia	3.5–6.5	1,574
Attention-deficit/hyperactivity disorder	3–6	532,800
Autism spectrum disorder	1	851,270
Specific language impairment	7	28,611

These two reports tell the same story: there are about twenty research publications on dyslexia against one on dyscalculia, not mentioning research on ADHD or Autism. This is all about the accumulation of knowledge. With a limited knowledge base there isn't enough understanding about the core features of the DD –yet.

The 'Comparability and fragmentation' problem

Researchers do not like repeating what other researchers have done. Producing new knowledge is good aim, but without comparable measurement tools, the research ends up to be fragmented. There are no universally accepted screening tools for the DD or validated "core deficits", so researchers develop and use a range of measures in their studies. These measures vary even when addressing the same construct (such as "counting" or "magnitude comparison"); even standardized tests and their norms vary across countries.

This fragmentation of research leads to problems to compare results from one study to another. The reasons for different research results may stem from small differences in tasks used, and not from the phenomena itself. And we will newer know whether it was the tasks or not, because in new studies, again, newly developed tasks are used. Studies replicating previous findings using the same measures have been rare exceptions.

Especially analysing intervention effectiveness using the same educational programs has been extremely rare. Some may think that the increased amount of using computer-assisted intervention (CAI) methods could have changed the situation. Unfortunately this is not the case. Researchers tend to develop new intervention applications for their experiments, and not use the old ideas implemented in applications done few years ago. A new research project lasts, from design to publications, several years. So, the application programmed at the beginning of the project is technically old when the project ends, leading to a new cycle of development projects, and to a fragmented research field (Räsänen, in press).

The 'Learning environment effect' problem

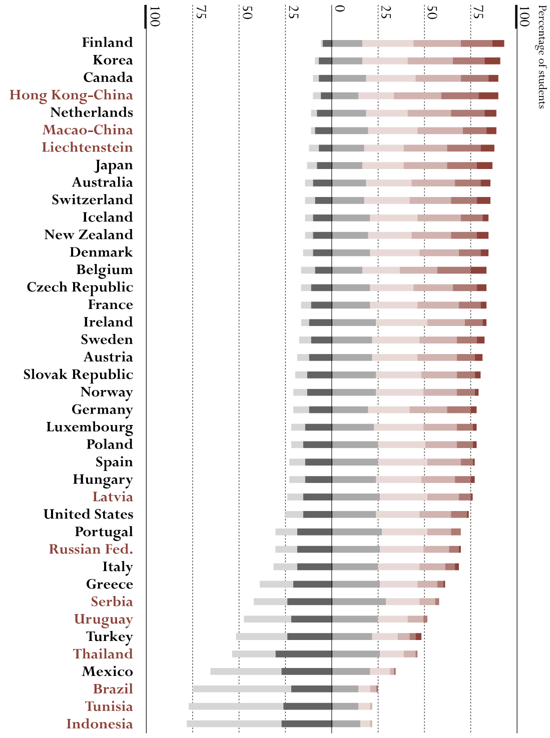
The term "developmental dyscalculia" (DD) does not refer to all forms of mathematics difficulty seen in childhood. Some children phenotypically show features of DD at some point of development, but their difficulties are not linked to a DD genotype; this is common among children with inadequate home or school learning environments linked to poverty. This is well illustrated in the international comparison studies (see table 2). The within and between countries variation is large, and the skill levels of children vary from country to another. Therefore the mathematical skills of the lowest 10 to 15 percentages in different countries show a different phenotype. It is difficult to differentiate the factors of educational and home environment, which shape the phenotype, from genotypic factors. Many children show a poor performance in mathematics without learning difficulties.

Table 2. Percentage of students at each level of proficiency on the mathematics scale 'Mathematical literacy at the age of 15' (OECD PISA, 2003)

The percentage of adolescents with poor performance (Level 1 or below level 1) in mathematical literacy in PISA 2003

- Finland, 6%
- Netherlands, 10%
- England and Germany, 20%
- USA, 25 %
- Italy, 35%
- Serbia, 40%
- Bulgaria, 55%
- Tunis, 70%
- Kirgisia, 90%

Source: OECD PISA 2003 database, Table 2.5a



The 'Complex nature of learning cultural skills' problem

The DD is considered a mathematics disorder, and mathematics encompasses a very broad range of cognitive abilities, skills, and strategies influenced further by innate, environmental, cognitive, and social factors.

Mathematics is like any other culturally mediated skill: it requires an opportunity to learn offered by the culture in general and in the home/school environment in particular. Learning happens within a social context incorporating the interaction with adults and peers, each having their own ways of interacting, values and learning experiences.

In the modern societies the main responsibility of the cultural mediation of formal mathematical skills has been given to the school systems, where the curriculum, organization of supportive education, the systems sensitivity to recognize failures to learn, the teacher education and individual teacher's skills all shape the development of each individual child's mathematical skills.

The genotype(s) of the DD are only a small part of the big picture in the development of mathematical skills, and even the genotype(s) are shaped by individual variation in other cognitive skills not directly related to the core problems. Likewise, the learning experiences shape the development of skills: motivation to practice and the efficiency of learning strategies are depended on experiences and guidance, e.g. the learning environment is the key element in the development of math related anxiety, not the severity of the learning difficulties.

The 'Cut score' problem

The DD or some components of the DD are likely to represent an extreme on a continuum of skills and abilities; therefore, it may be difficult to establish boundaries between typical development and the DD, and how much knowledge of typical mathematics development and function can inform studies of the DD.

When we are analysing a skill on a continuum, it is always arbitrary where we place a 'cut score' between typical and atypical levels of performance. The estimated prevalence of the DD is typically placed close to 5–7 percentiles. However, in empirical research the cut score has varied from two to twenty five. Therefore, in different studies we have analysed different subpopulations (Figure 1).

However, the DD or some components of the DD may appear qualitatively distinct from other forms of low mathematics achievement, limiting the extent to which we can generalize findings from studies of typical mathematics development to the study of the DD. However, the number of studies that would have entangled on this complex design comparing children close to both sides of the cut score is very limited. The question whether there are qualitative differences between children with the DD from those with otherwise low performance is one of the hot topics in the current research on the DD.

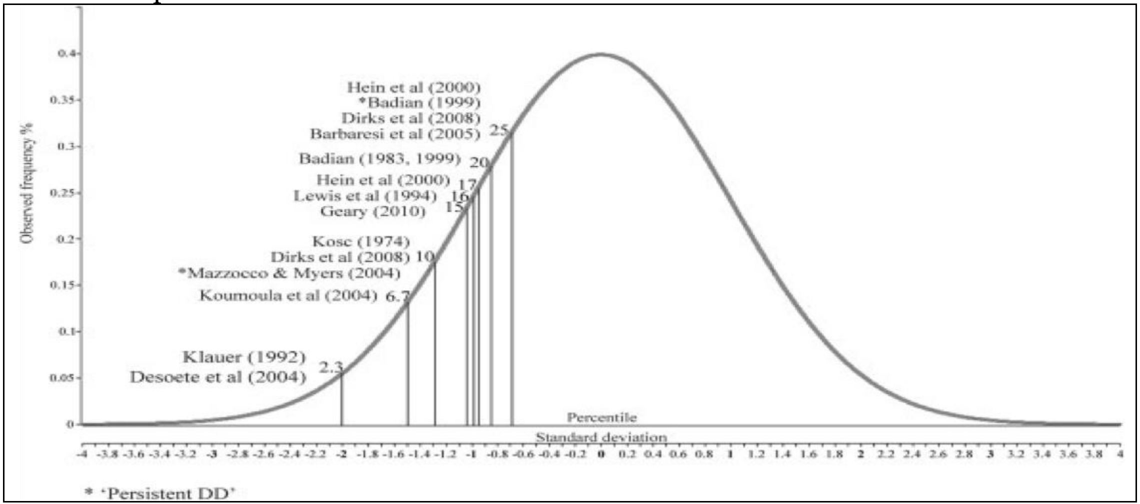


Figure 1. Different cut scores used in studies on DD (Devine et al., 2013)

The 'Acronym' problem

Across research studies, educational media, and government reports, the terminology used when referring to DD is inconsistent. Math learning disability (MLD) has been used as synonymous with DD, but also as distinct from DD when MLD is used to refer to the larger category of mathematics difficulties (MD), it is intentionally referring to all children who struggle with math. The emphasis on MD is understandable, given that all such children need our research and educational attention. However, not all these children have the severe, specific disability in math that we refer to as DD.

The increasing amount of studies comparing children from the extreme left tail of the distribution to those with low achievement (LA) will give us important understanding whether the children with DD differ qualitatively from others who fail in acquiring proper skills in numeracy. Especially, we should be interested in the question whether there are children who do not respond to well-designed intervention programs. And when they fail, we need to know whether the reasons are connected to mathematical cognition, more general cognition or to other reasons (emotional, social interaction). This kind or studies will help us to find labels and acronyms which will guide us to support each child in a way they need.

2.2 Language and math

By Annemie Desoete

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There has been extensive research on counting skills (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Hannula, Räsänen, & Lehtinen, 2007; Praet & Desoete, 2014) in predicting arithmetic skills in the primary grades. Moreover, 87% of the children with mathematical learning disabilities in grade 2 (at age 7 to 8) can be correctly diagnosed in kindergarten by a combination of counting and magnitude estimation tasks (Stock, Desoete, & Roeyers, 2010). In addition to numerical abilities, the value of including logical thinking abilities (e.g., Nunes et al., 2006; Stock, Desoete, & Roeyers, 2009) and language skills as predictors for arithmetic skills have been stressed (e.g., Purpura, Hume, Sims, & Lonigan, 2011; Vukovic & Lesaux, 2013). However, surprisingly few studies have been conducted to explore the combined effect of these predictors on arithmetic knowledge in kindergarten and the primary grades. The present paper expands previous findings, by presenting two studies on the value of language in addition to other known predictors for arithmetic skills in preschool and grade 1 and 2.

A sample of 63 children was tested in kindergarten on counting, logical thinking, estimation, language and arithmetic skills (Praet, Titeca, Ceulemans, & Desoete, 2013). These children were tested again on arithmetic skills in grade 1. To get a picture of the language skills, the children were tested with the Clinical Evaluation of Language Fundamentals or the CELF-$_4$Nl (Semel, Wiig, & Secord 2008). Counting skills were measured with the Tedi-Math (Grégoire et al., 2004) and number estimation skills were assessed with a 0-100 Number Line Estimation (NLE) test. The study replicated previous research on the relationship between counting and arithmetic. Moreover, our findings underlined the value of number estimation skills in kindergarten. In addition, in this study logical thinking skills assessed in kindergarten correlated signficantly with both arithmetic assessed in kindergarten and grade 1. However, when other variables were added to explore the combined effect of these predictors, logical thinking no longer explained a significant amount of variance in arithmetic skills among young children. Finally, language explained variance in arithmetic skills among children. The core language index in kindergarten was significantly correlated with arithmetic skills even when controlling for counting, estimation and logical thinking skills. Expressive language skills in kindergarten explained about one fifth of the variance in arithmetic skills among kindergarteners. Moreover, expressive language skills predicted about 4% of the variance of grade 1 arithmetic skills when controlling for kindergarten arithmetic knowledge.

In our second study we investigated whether language is important for all arithmetic tasks and if the prediction power remained important in grade 2. Therefor 132 children were tested with the CELF-$_4$Nl and the TEDI-MATH in kindergarten. Since we know that there are two subtypes of children with mathematical learning disabilities (Pieters et al., 2013) stressing the importance to assess timed and an untimed arithmetic tasks, both kind of test were used. In addition, a number line estimation task with a 0-100 interval was completed at 5 time points (from kindergarten till grade 2) on an iPad. Our data revealed the predictive power of language skills for untimed arithmetic calculation in grade 1. This was also the case in grade 2 and the prediction of language was significantly over and above the prediction of counting and estimation skills. However, language did not add to the prediction of untimed arithmetic (fact retrieval skills) in grade 2. A latent growth curve model revealed that language skills influenced the starting point (in kindergarten) but not the development or evolution of the number line estimation accuracy (in grade 1 and 2).

To conclude, preschool language should not be ignored as predictor for the development of arithmetic skills in elementary school. Moreover an intensified stimulation of language as head-start or buffer against poor arithmetic outcome might enhance young children's development. In addition, arithmetic seems no unitary construct and different predictors can be found for different aspects of arithmetic skills, with preschool language especially predicting calculation accuracy and not number fact retrieval speed.

2.3 The structure of number sense and it's relation to mathematical (dis-) ability

By Evelyn Kroesbergen

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Number sense is defined in many different ways, and - as a consequence - is used to describe many different skills (cf., Berch, 2005; Mazzocco, Feigenson, & Halberda, 2011; Jordan, Glutting, Ramineni, & Watkins, 2010). One of the most influencing definitions of number sense comes from Dehaene (1992): number sense is the ability to quickly understand and manipulate numerical quantities. In his triple-code model, Dehaene distinguishes between three different 'codes', in which numerical information can be processed: (1) the analogue code, in which numerosities are represented without symbols; (2) the verbal code, or the number-words; and (3) the Arabic-visual code, or the number symbols. Others, however, have used a broader definition of number sense, in which also other skills like the understanding of whole numbers, number operations, and counting knowledge are included (e.g., Aunio, Hautamäki, & Van Luit, 2005; Jordan, et al., 2010). Some use an even broader definition of number sense, including the understanding of number and operations along with the ability to use this understanding in flexible ways to make mathematical judgements and develop useful strategies for handling numbers and operations (McIntosh, Reys, & Reys, 1992).

These different views of number sense are visually presented in Figure 1. The triple-code model (Dehaene, 1992) could be seen as representing the most basic skills of number sense, namely the non-symbolic skills ("analogue code") and symbolic number sense skills ("verbal code" and "visual code"). The non-symbolic and the symbolic skills together form the basis for further numerical skills: pre-math skills (e.g., number and counting knowledge, Aunio et. al, 2005; Jordan et al., 2010) and the more advanced numeracy (understanding and flexible use of number and operations; McIntosh et al., 1992). However, the most important skills children have to learn is to combine the different representations of numerical information, here called 'mapping skills'. These mapping skills are especially important to give meaning to the number words and Arabic numerals, by associating them with the quantities they represent (Geary, 2013; Mazzocco et al., 2011).

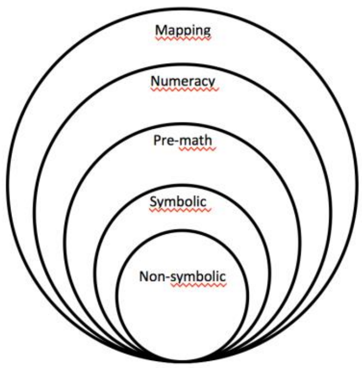

Figure 1. A model of number sense skills

Although the triple-code model is not meant as a developmental model, the 'analogue code', referring to the approximate number system (ANS), is often seen as prerequisite for the verbal and visual code, which in turn precede further mathematical development (e.g. Libertus, Feigenson, & Halberda, 2011; Star, Libertus, & Brannon, 2013). However, our recent studies show that the ANS is also influenced by children's learning of symbolic and (pre-) math skills (Kolkman, Kroesbergen, & Leseman, 2013; Toll, Van Viersen, Kroesbergen, & Van Luit, 2015). In the study by Kolkman and colleagues, 69 4-year old children were followed for two years. Non-symbolic skills were measured with a dot comparison and a non-verbal number line task. Symbolic skills were measured with a number naming task and a counting task (Early Numeracy Test; Van Luit & Van de Rijt, 2009). It was found that the symbolic skills at time 1 predicted non-symbolic skills at time 2, but not vice versa. In the second study (Toll et al., 2015), 671 4-year olds were followed for 2.5 years. A dot comparison task and a number comparison task were used to measure non-symbolic and symbolic skills. Multivariate latent growth curve models showed that both constructs were related to each other, and a mutual influence was found on the development of non-symbolic and symbolic comparison skills. It should be noted that other environmental factors also influence children's non-symbolic skills, such as teacher's math talk at school (Boonen, Kolkman, & Kroesbergen, 2011), and parent's math talk at home (Kroesbergen, Kolkman, & Van der Ven, 2009).

More importantly, number sense has been found to be a good predictor of later math skills (e.g., De Smedt, Verschaffel, & Ghesquière, 2009; LeFevre et al, 2010; Libertus et al., 2013; Locuniak & Jordan, 2008; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). In our own studies, we found that symbolic skills are better predictors of later math skills than non-symbolic skills (Toll et al., 2015), but that mapping skills even better predicted later math skills (Kolkman et al., 2013). Mapping skills in the study by Kolkman et al. (2013) were studied with a numberline task (number-to-position) and a comparison task (number comparison). In a recent study by Friso-van den Bos (2014), 442 5-year-old-children were followed for three years. Their (development in) performance on the numberline task predicted later mathematical performance, in specific whether children were at-risk for mathematical learning disabilities (MLD). Other studies also show that mapping skills, as measured with numberline taks, are impaired in children with MLD (e.g., Geary, Hoard, & Bailey, 2012). Some recent eye-tracking studies have shown that children with MLD do not only differ in their accuracy on a numberline task, but also in strategy use (Van 't Noordende & Kolkman, 2013; Van Viersen, Slot, Kroesbergen, Van 't Noordende, & Leseman (2013).

To summarize, number sense skills are very important basic skills children have to acquire early in life. Especially children's knowledge of symbols, number words, and counting, in relation to the quantities they represent, are good predictors of later mathematical ability. If children don't adequately develop these basic number sense skills, they will be at risk for later mathematical learning disabilities.

2.4 Precursors of early mathematics and early intervention programs

By Maria Chiara Passolunghi

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Several recent studies investigated the precursors of mathematical learning in children attending preschool or first grades of primary school. Competencies that specifically predict mathematical abilities may be considered domain-specific precursors, such as early numeracy, whereas general cognitive abilities, such as working memory, that may predict performance not only in mathematics but also in other school subjects may be considered domain-general precursors. Investigating the abilities linked to mathematics learning is therefore important both from a theoretical standpoint and from a social and educational point of view, with a view to the early identification of individuals at risk of mathematical learning disability and to the development of appropriate enhancement training.

Studies on preschool children found a relationship between early numerical abilities and later mathematical skills (e.g., also Aunio & Niemivirta, 2010; Mazzocco & Thompson, 2005; Jordan, Kaplan, Ramineni, & Locuniak, 2009). Conversely, several studies demonstrated that working memory is a key predictor of mathematical competence (Alloway & Alloway, 2010; De Smedt et al., 2009; Friso-Van den Bos et al., 2013; Gathercole & Pickering, 2000; Szucs, Devine, Soltesz, Nobes, & Gabriel, 2014). Indeed, even the simplest mathematics calculations require working memory processes: temporary storage of information, retrieval of relevant procedures, and processing operations to convert the information into numerical output. These same processes are needed even for simple number comparison tasks: the child need to map the different number symbols onto the corresponding quantities, store them into memory and then integrate this with the incoming information to performing the task (Kroesbergen, Van't Noordende, & Kolkman, 2014).

However, only a few studies have considered the role of both general cognitive skills and more specific abilities, in predicting early numerical abilities or later mathematical achievement in primary school (Geary, 2011; Locuniak & Jordan, 2008; Ostergren & Traff, 2013). One our recent study (Passolunghi, Lanfranchi, Altoè, & Sollazzo, 2015) verified a comprehensive and unitary model that includes general cognitive variables such as working memory, processing speed, phonological ability, intelligence, and early numerical abilities in children attending their last year of preschool. The results showed that both general and specific abilities are related to early mathematics learning in preschool-age children. In particular, children's early numerical abilities were found directly related to their verbal intelligence, phonological abilities, processing speed and working memory (for similar findings in first graders see Passolunghi & Lanfranchi, 2012).

Although some efforts have been made to improve precursors of mathematical learning, different effects of training focused on the enhancement of either domain-general or domain-specific precursors are still unclear. Moreover, the debate regarding the effects of working memory training is still open: some studies show a positive effects of working memory training on arithmetic abilities in primary school children using computerized or school-based training procedures (Alloway, Bibile, & Lau, 2013; Holmes & Gathercole, 2013; Holmes, Gathercole, & Dunning, 2009; Kuhn & Holling, 2014; St. Clair-Thompson, Stevens, Hunt, & Bolder, 2010; Witt, 2011). Other authors questioned the effectiveness of working memory training concluding that there is no convincing evidence of the generalization of working memory training to other skills (Melby-Lervåg & Hulme, 2013). However, it should be considered the possibility that cognitive training applied to younger individuals tends to lead to significantly more widespread transfer of training effects (Wass, Scerif, & Johnson, 2012).

In one study, our aim was to verify and to compare the effects on early numerical competence of two types of training in a sample of preschool children (Passolunghi & Costa, 2014). One type of training focused on the enhancement of domain-general precursors, working memory abilities, and the other focused on the enhancement of domain-specific precursors, early numeracy abilities.

We expect that early numeracy training will have a specific effect only on early numeracy abilities. On the other hand, we expected that working memory training will improve not only working memory but will also produce a far transfer effect on early numeracy in line with previous studies in primary and preschool children (Holmes at al., 2009; Kroesbergen et al., 2014).

The results supported our hypotheses. The group of children that received the early numeracy training exhibited a significant enhancement of early numeracy abilities compared to the control group, but did not significantly improve their working memory abilities. More importantly, this study showed that the group that received the working memory training exhibited a significant enhancement of both working memory abilities and early numeracy abilities. The gain obtained in the working memory training group did not differ significantly from the gain obtained in the early numeracy training group.

These findings stress the importance of performing activities designed to train working memory abilities, in addition to activities aimed to enhance more specific skills, in the early prevention of learning difficulties. Moreover, these results highlighted the key role of working memory in a range of cognitive skills including mathematics (see Cowan & Alloway, 2008).

2.5 Improving science skills during mathematical activities in grade 1

Written by: Geerdina M. van der Aalsvoort, Tjarda de Wit- Meijer, Carla Compagnie, Martine van Schaik

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Theoretical considerations

Studies over the years have revealed that numeracy is related to contextual factors, such as parenting (Aunola Nurmi, Lerkkanen & Rasku-Puttonen, 2004; Linnell & Fluck, 2001), instruction time (Boonen, Kolkman & Kroesbergen, 2011; Klibanoff, Levine & Huttenlocher, 2006), relationship with teacher and peers (Buyse, Verschueren, Doumen, van Damme, & Maas, 2008; O'Connor & McCartney, 2007; Verachtert, Gadeyne, Onghena & Ghesquière, 2008) and the dynamics that underlie the complexity of talk in the classroom especially during science conversations (Justice, McGinty, Zucker, Cabell & Piasta, 2013).

We started an intervention study aiming to improve science skills during mathematic activities using small group activities including some of these contextual factors (Durden & Dangel, 2008; Kilday & Kinzie, 2009). This also allows teachers to notice and accept variability in emergent numeracy (Aunio & Niemivirta, 2010) and science skills (Meindertsma, 2014; Van der Steen, 2014). As teachers in early grades have poor ways to teach science (Van Aalderen-Smeets & Walma van der Molen, 2013), we combined science and numeracy teaching (SLO, 2010; Van Keulen, & Sol, 2012). We also applied opportunities to playfully explore and experience scientific phenomena (Fisher, Hirsh-Pasek, Newcombe & Golinkoff, 2013).

Intervention study

Based upon the results of two pilot studies we aimed at eliciting progress in science skills with first graders (aged 5-6 years) in nine regular primary schools divided into four conditions. To test the hypotheses four children were selected based on gender and low performance on a Dutch standard arithmetic test. In all conditions working in small teaching groups was carried out. Moreover special developed activities were offered that elicit practice in measuring and geometry skills for 10 minutes at a time. In condition 1 personal guidance every two weeks was added based upon the video registration of an activity. Condition 2 included Lesson study in that the teachers from two schools sat together with an expert and discussed video clips of an activity and discussed both outcomes and plans for the next two weeks. Condition 3 included the special developed activities eliciting practice in measuring and geometry skills only. Condition 4 served as control condition. It was hypothesized that the students and teachers from conditions 1 to 3 would outperform those from condition 4 with regard to several.

The pretest measurement took place in November 2013 and the posttest measures were collected in June 2014. In the pretest with children we used Citotoets Rekenen (Cito, 2010), Motivational tendencies (Van der Aalsvoort, Lepola, Overoom & Laitinen, 2013) Raven CPM, and Skill level of answers that reveal student progress on understanding balance (Fischer & Bidell, 2006; Meindertsma, 2014; Van der Steen, 2014). The teachers skills were investigated with Primary teachers' attitudes towards teaching science (Van Aalderen & Walma van der Molen, 2013), Early Childhood Environment Rating Scales-Revised (Harms, Clifford & Cryer, 1998), Early Childhood Environment Rating Scale Curricular Extension to ECERS-R (Sylva, Siraj-Blatchford & Taggart, 2010), Small group teaching quality (Van der Aalsvoort, Bootsma & De Wit, 2013), Scientific level of questions that elicit student thinking (Furtak, Seidel, Iverson & Briggs, 2012), Epistemic and procedural, Conceptual, and Social domain expertise in measuring and using the balance (Van der Aalsvoort, Slot & Van Keulen, 2014).

Intervention program focused on various numeracy skills such as number sense, measurement and geometry skills, and emergent science skills such living systems, technology and physics. The intervention took three rounds of four weeks between January and May 2014 in which a new activity was offered every week. There were three sessions per week, one session would last 10-15 minutes to schedule freely during the week. Each activity (one written page) included aim of the activity, instruction and feedback suggestions and tasks to complete after 5 minutes without the teacher present. There were for instance six measurement activities, such as, sharing lemonade between children, measuring size of a shoe, and six geometry skills such as, mapping the classroom and working with mirrors. During each intervention round video registrations were made weekly in condition 1, 2 and 3. The video registrations were planned in advance.

Findings

The pretest measures revealed that the mean age in months differed significantly between the intervention and control condition (70 months, intervention condition, and 66 months, control condition). No differences were found with regard to IQ, language score, motivational attitudes, SES (education and work of mother and father), and time in school. Moreover no differences were found regard the context (ECERS-R and ECERS-CE), teachers' attitude towards teaching science, and years of teaching experience.

We tested the hypothesis that the students in the intervention conditions would outperform those from the control condition. In mathematics performance there was significant improvement. In motivational tendencies we found decrease in task avoidance. In task investigating the performance on measuring and using the balance there was no significant improvement. We also found no significant improvement in skill level of answers that reveal student progress on understanding balance. With regard to the teachers' knowledge we found the following. In teachers' attitude towards teaching science there was significantly less perceived dependency on context factors. In scales measuring classroom quality (ECERS-R and ECERS CE) there were significant improvements in subscales Language and Reasoning, and Mathematics. In scale measuring Small group teaching quality there were no significant changes. In the scale for the Scientific level of questions that elicit student thinking in small group there was significant increase of questions regarding conceptual domain.

Discussion

The condition Lesson study did elicit more changes with regard to the teachers and the condition Personal guidance revealed higher progress with regard to student measures. However, the condition is very time consuming. Long before the start of Lesson study teachers need to be familiar with each other's teaching styles in order to allow meaningful suggestions; professional knowledge was sometimes lacking. Findings with regard to knowledge about emergent science in early childhood are few, so lots of developing work to do!

Emergent numeracy and science can be connected in instruction. Our activities allow discovery of measuring and geometry as part of teaching content and mathematics in particular. In addition, our activities allow students to discover and work together. In these kind of activities teachers can profit from using different kinds of questions more effectively whereas counting skills only does not allow this. Video analysis allows reflection over time and time again as discussing how student behavior can be interpreted within a session (personal guidance and Lesson study).

2.6 Children with low performance in mathematics and interventions

Written by: Pirjo Aunio & Anna Tapola

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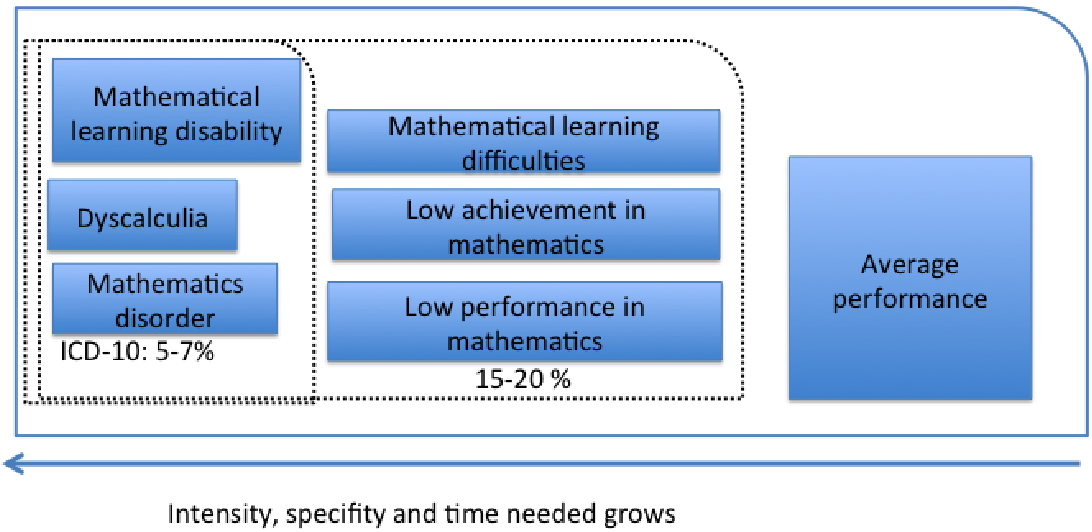
The main focus of this paper is on early mathematics development and low performance in mathematics. Second, we aim to clarify the methodology and goals of educational interventions. Finally, we introduce some preliminary results from our ThinkMath-intervention studies with low performing first- graders. When we started to design the LukiMat-webservice (www.lukimat.fi) with the team in Niilo Mäki Institute in 2006 we faced an interesting challenge. Our goal in LukiMat-project was to provide knowledge and tools mainly for educators working with 5-8 years-old children who had low performance in mathematical basic skills, so that it would be possible to prevent or at least diminish mathematical learning difficulties. The challenge was that, at the time, there was no general agreement on the definition of core skills in mathematics in early years of primary school. We conducted a literature review including early numeracy assessment batteries and longitudinal studies (Aunio, 2008; Aunio & Räsänen, 2015; see also www.lukimat.fi/matematiikka/tietopalvelu/taitojen kehitys) to identify the most important mathematical skills for later mathematics learning in early childhood and early primary grades. We were able to form four factors that included the core math skills in age group 5-8 years. These four factors are: Nonverbal and symbolic number sense, Number knowledge, Basic skills in counting and arithmetic, and Understanding mathematical relations (Table 1). For most children, learning these core factor skills is easy and happens without tremendous effort. The child needs all these skills to be able to learn and understand mathematics instruction at school.

Table 1. Four core factors of mathematical skills at age of 5-8 years

Core factor	Sub skills
Nonverbal and symbolic number sense	
Number knowledge	Number word sequence Enumeration Knowledge of number symbols
Basic skills in counting and arithemetic	Addition and subtraction with number symbols Arithmetic combinations
Understaning mathematical relations	Early mathematical-logical principles Arithmetic principles Mathematical symbols Place-value and base-ten system

One important thing to be aware of is think to whom we refer to when we talk about persons with mathematical learning difficulties. In literature there are several different terms used with different definitions (Figure 1). When reading the research literature it is beneficial to understand that there are different definitions and different origins of the problems ranging from neurological dysfunctions to not relevant opportunities to learn and practice mathematical skills (e.g., SES). For instance Geary (2013) suggests that children who score at or below the 10th percentile on standardized mathematics achievement tests for at least two consecutive academic years are categorized as MLD (Mathematical learning disability) in research studies and all children scoring between the 11th and 25th percentiles, inclusive, across two consecutive years, as LA (Low Achievers). In addition, it is important to understand that mathematics performance, also in the lower end, is a continuum; there is no strict point where the problem starts. From educational point of view the vital message from educational intervention studies is that all these persons benefit of good quality education, for some persons it means more intense and longer period of extra educational support, but it is still beneficial.

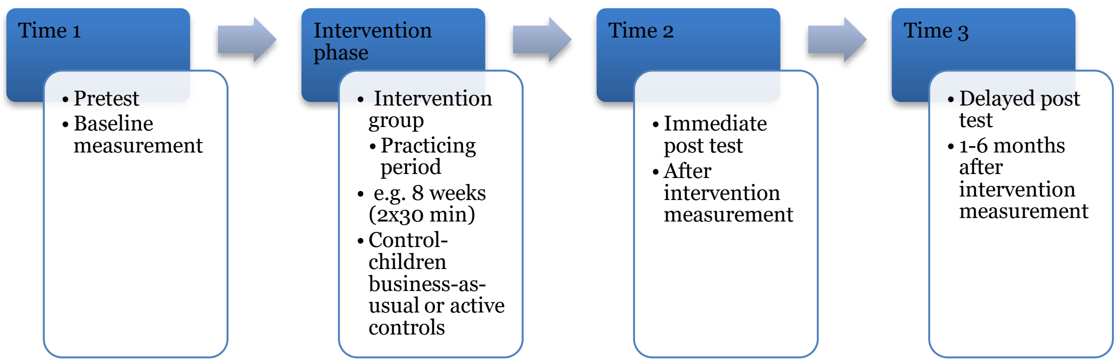
Figure 1. Different terms related to low performance in math



An educational intervention and especially a special educational intervention is a planned modification of the learning environment made for the purpose of altering behavior or learning in a prespecified way (Riley-Tillman & Burns, 2009). The effects of evidence-based (i.e., research-based) intervention programs are most often based on observed outcome data of children's and young people's learning (Hammersley 2013). The intervention programs and research can be implemented in the individual, small group, whole group or whole school or kindergarten level. There can be different components practiced in the intervention programs, for instance cognitive components (e.g., working memory) and academic skills (e.g., mathematics skills). The intervention programs are often developed by using intervention research design. Thus the concept ”intervention” can be used in reference to intervention program or research methodology, which can cause some confusion among students and researchers.

The intervention research design that is recommended includes pretest (i.e. baseline measurement), and immediate and delayed post-test measurements (Figure 2). The intervention and control group design allows the researchers and teachers to compare whether the children receiving the planned instruction (i.e., intervention program) develop faster than their peers not receiving extra support.

Figure 2. Example of special educational intervention research design



When can we say that an intervention program is effective? This is again an issue where researcher's conceptions differ from each other. Very seldom researches make a statement on what kind of intervention effects they are aiming at. In general, we can say that an intervention program is effective if the children with low performance (or learning difficulties) will progress better than their performance control-peers. Even better still would be a result showing children with low performance to be able to maintain their head start compared to the control group even after the intervention phase has ended. Finally, most optimal result would – in addition to the aforementioned effects – be achieved when the low-performing children could be demonstrated to close the gap with their average performing peers.

Development work with ThinkMath-webservice

The main focus in our second project the ThinkMath-webservice was to produce teaching materials for educators to support children's thinking and mathematical skills. The material is designed for kindergarten, first and second grade children (aged approximately 5-8 years) with weaknesses in thinking or mathematical skills. The aim is also to provide educators with knowledge on the theoretical framework of thinking and mathematical skills, executive functions, motivation, and interventions. Our material needs to meet the requirement of research-based evidence. The project was funded by the Finnish Ministry of Education and Culture (2011-2015) and the website can be found in http://blogs.helsinki.fi/thinkmath/

The mathematical skills intervention material features originate from the research showing that explicit teaching, visual representations of mathematical concepts at the concrete-representational-abstract levels (CRA) and structuring numbers (e.g., ten frame) are good elements to support learning in a low performing group. The materials are designed to be used with small group of children (2-6 children). The packages include about 15 intervention sessions. Each session takes about 30-45 minutes. However, it is also possible to arrange individual sessions if needed.

In one of our studies we investigated the effects of our math materials with second graders (Mage = 8 years, 2 months) with low performance in mathematics (Mononen and Aunio, 2014). We used intervention and control group setting and measured the intervention effects on children's performance in pre- and postests (immediate and delayed). A group of low-performing second graders (n = 11) was taught twice a week for eight weeks on number word sequence skills, counting skills and conceptual place value knowledge. We compared the mathematics performance of the intervention group with groups consisting of low-performing (n = 13) and typically performing children (n = 64), who followed their business-as-usual mathematics instruction. The intervention group made significant improvements in mathematics but did not show significantly better gains, compared to the low performing and typically performing control groups, immediately and three months after the intervention.

To sum up the main ideas of this paper; there are several important dimensions of mathematical skills developing already in early childhood and the early school years, but we need to know more about the developmental dynamics in these skills and especially about the dynamics of low performance (incl. skills, cognitive and motivational elements). About 20% of student population has problems in mathematics learning and about 4-7% of population has serious difficulties in math (i.e., mathematics disorder, dyscalculia). Mathematical skills interventions in school and kindergarten setting have promising results in supporting the learning in low performing student groups, but we need to investigate more which elements support the transfer-effects of learning and long-lasting increase in performance.

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