NOTE

3-5-MAN CHESS: MAXIMALS AND MZUGS

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ABSTRACT

This note reports the combined results of several initiatives in creating and surveying complete suites of endgame tables (EGTs) to the Depth to Mate (DTM) and Depth to Conversion (DTC) metrics. Data on percentage results, maximals and mutual zugzwangs, *mzugs*, has been filed and made available on the web, as have the DTM EGTs.

1. INTRODUCTION

Nalimov and Wirth independently, and essentially contemporaneously, have completed suites of 3-to-5-man EGTs respectively to the Depth to Mate (DTM) and Depth to Conversion (DTC) metrics (Wirth and Nievergelt, 1999; Nalimov, Haworth and Heinz, 2000, 2001; Wirth, 2000; Hyatt, 2001; Lincke, 2001a; Tamplin, 2001a).

Karrer (2000) has mined Nalimov's EGTs to produce complete lists of:

- maxDTM positions and data: 1-0 and 0-1, wtm and btm,
- maxDTM and all mutual zugzwangs: three types, positions and data.

Wirth (2000) produced the analogous DTC data and also calculated the percentage results, 1-0, draw and 0-1, wtm and btm. As he provided the number of positions won in a specific number of plies (Lincke, 2001b), quick wins based on tactical devices may be discounted as required from these percentages. Because Wirth eliminates from his EGTs duplicates of positions with two Kings on a long diagonal, his percentage statistics are marginally more accurate than Nalimov's. Both, for reasons of comparability, discount only those unreachable positions where the side-not-to-move is in check.

Tamplin and Haworth correlated the mzug data to confirm that the sets of positions had indeed been twin-sourced. Tamplin (2001a) provides, with the assistance of the Lincke (2001a) site, an excellent query service to both the DTC and DTM EGTs and files of endgame data, including the data discussed here.

2. MAXIMAL DATA

The large table of maximal results is published on the web (Tamplin, 2001a) rather than here. It includes for both the DTC and DTM metrics, the maxDT*x* figures (1-0 and 0-1, wtm and btm) and the %-win statistics derived from Wirth's data. Some observations follow.

Independent maxDTM results by Rasmussen (2000) agreed completely with Nalimov's and Karrer's data. Thompson's original and comprehensive set of 5-man EGTs (Thompson, 2000; Tamplin, 2001b) minimax the DTC of the next btm position rather than strictly optimising the next conversion to a subgame: the weaker side sometimes captures voluntarily as a human player might do. The inconsequential difference is that his maxDTC is just one less than Wirth's for KRKNP, KRRKN and KBNKP. Thompson (2001) now minimaxes the current DTC by minimaxing the number of men on the board first.

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Note that, when minimising DTC, the stronger side may unnaturally force-sacrifice surplus force. This occurs for example in KQQQK and KQBNK, and was seen in Game 4 of the DEEP FRITZ – DEEP JUNIOR match in 2001[5]. Wirth used the existing ETH(Zürich) software RETROENGINE which assumed that captures are made by the winner. Where this need not be so as in wKc3Rb4c2/bKa1+w, Wirth's depths in plies are one ply too great: some maxDTCs[6] and counts of maxDTC positions are affected (Tamplin, 2001c). Further, DTC measured in winner's moves can rate moves equi-optimal whose depths differ by one ply. An example is wKc5Rb4c2/bKa1Na7a8 (Thompson, 2001) where Ra4+ and Kd4/5/6 are rated alongside Ra2+.

3. MUTUAL ZUGZWANGS

A reciprocal or mutual zugzwang, *mzug*, in chess is a position where, ironically, each side could get a better result in theory if it were the other side's turn to move. There are three types of mzug:

ww	= /1-0	a 'White win' mzug ... the position is a wtm draw and a btm win for White
bw	0-1/ =	a 'Black win' mzug ... the position is a wtm win for Black and a btm draw
fp	0-1/1-0	a 'full point' mzug ... the side that has to move loses.

They are relatively rare and the mzug is a running theme in the composition of endgame studies (Roycroft, 1972; Nunn, 1992, 1994, 1995; Beasley and Whitworth, 1996; Elkies, 1998a; Beasley, 2000). Many counts of mzugs by Rasmussen (1991-2000) have already been published in the endgame quarterly *EG*: they confirm and are confirmed by the data here.

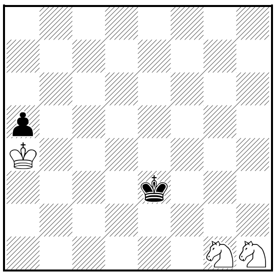

Figure 1: max mzug.

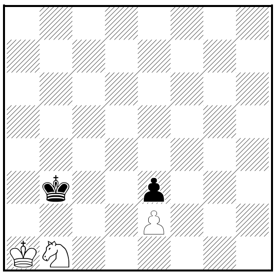

Figure 2: max fp mzug.

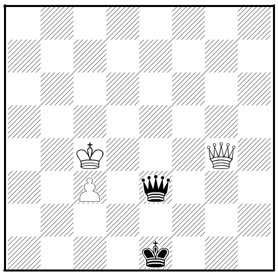

Figure 3: *dc* = 91.

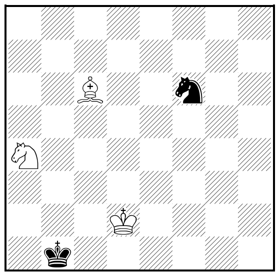

Figure 4: *dc* = 67.

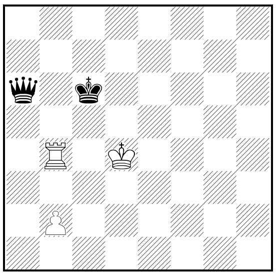

Figure 5: bw, *dc* = 63.

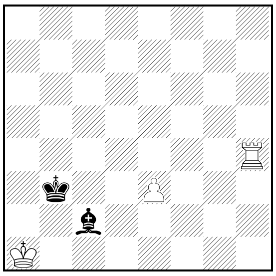

Figure 6: *dc* = 58.

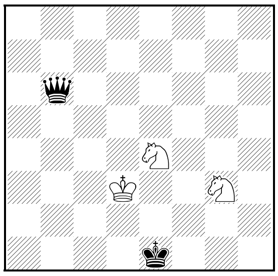

Figure 7: bw, *dc* = 53.

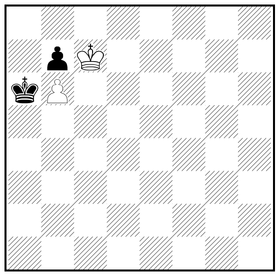

Figure 8: Le Trébuchet.

3.1 The Results

Karrer (2000) scanned Nalimov's 3-to-5-man EGTs (Hyatt, 2001; Tamplin, 2001a) for mzug positions, giving:

- a list of distinct mzugs together with statistics about counts and maxDTM depths
  n.b. for a full-point mzug, the *depth* is taken to be the sum of the wtm and btm depths
- a list of the distinct maxDTM mzugs

The lists were then passed via Haworth to workers in this field including Elkies, Rasmussen, Roycroft, Tamplin and Wirth. Haworth collated the resulting statistics, confirming full agreement between the data of Karrer, Wirth and Rasmussen, and identifying mzugs which were maximal in both DTC and DTM terms.

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[5] DJ-DF, r7/8/5R2/2k5/8/3R1K2/6P1/8 w: 70. Rc6+ Kb5 71. Rc5+ Kb4 72. Rb5+ Kc4 73. Rd4+ Kc3 74. Rc5+ K×d4

[6] To date, Tamplin (2001c) has identified this second inconsequential maxDTC '1 out' difference for KQ(B/N/Q/R)K, KRRK, KQQKN, KQRKR, KRRK(N/Q), KBBBK, KQ(BN/RB/RN)K and KR(BN/RR)K.

Endgame			mzug count			DTC-maximal mzugs: counts			DTC-maximal mzugs: max DTC			DTC-maximal mzugs: max-DTC mzugs' max DTM			DTM-maximal mzugs: counts			DTM-maximal mzugs: max DTM			DTM-maximal mzugs: max-DTM mzugs' max-DTC			ZC ^ ZM counts		
Title		w-b	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp
KPK	3	2-1	80	0	0	1	0	0	18	—	—	27	—	—	1	0	0	27	—	—	18	—	—	1	—	—
KBKP	4	2-2	0	1	0	0	1	0	—	1	—	—	12	—	0	1	0	—	12	—	—	1	—	—	1	—
KBPK	4	3-1	6	0	0	1	0	0	14	—	—	20	—	—	1	0	0	20	—	—	14	—	—	1	—	—
KNKP	4	2-2	22	7	0	6	1	0	6	9	—	6	24	—	6	1	0	6	24	—	6	9	—	6	1	—
KNPK	4	3-1	75	0	0	1	0	0	14	—	—	19	—	—	2	0	0	19	—	—	14	—	—	1	—	—
KPKP	4	2-2	106	(106)	15	1	(1)	15	10	10	1+1	24	24	19+12	2	(2)	1	27	27	19+12	6	6	1+1	0	0	1
KPPK	4	3-1	43	0	0	6	0	0	11	—	—	26	—	—	1	0	0	26	—	—	11	—	—	1	—	—
KRKB	4	2-2	5	0	0	1	0	0	14	—	—	25	—	—	1	0	0	25	—	—	14	—	—	1	—	—
KRKN	4	2-2	18	0	0	2	0	0	10	—	—	24	—	—	2	0	0	24	—	—	10	—	—	2	—	—
KRKP	4	2-2	12	0	0	6	0	0	6	—	—	20	—	—	4	0	0	20	—	—	6	—	—	4	—	—
KBBKN	5	3-2	1	0	0	1	0	0	2	—	—	2	—	—	1	0	0	2	—	—	2	—	—	1	—	—
KBBKP	5	3-2	1	0	0	1	0	0	2	—	—	2	—	—	1	0	0	2	—	—	2	—	—	1	—	—
KBBKQ	5	3-2	0	1	0	0	1	0	—	5	—	—	17	—	0	1	0	—	17	—	—	5	—	—	1	—
KBBKR	5	3-2	3	0	0	3	0	0	1	—	—	19	—	—	1	0	0	19	—	—	1	—	—	1	—	—
KBNKB	5	3-2	45	0	0	1	0	0	10	—	—	36	—	—	1	0	0	36	—	—	10	—	—	1	—	—
KBNKN	5	3-2	922	0	0	1	0	0	67	—	—	97	—	—	1	0	0	97	—	—	67	—	—	1	—	—
KBNKP	5	3-2	61	1	0	1	1	0	13	1	—	36	45	—	4	1	0	37	45	—	8	1	—	0	1	—
KBNKQ	5	3-2	0	1	0	0	1	0	—	33	—	—	44	—	0	1	0	—	44	—	—	33	—	—	1	—
KBNKR	5	3-2	6	2	0	1	1	0	5	8	—	26	28	—	1	1	0	32	28	—	1	8	—	0	1	—
KBPKB	5	3-2	160	0	0	2	0	0	33	—	—	44	—	—	2	0	0	44	—	—	33	—	—	2	—	—
KBPKN	5	3-2	2112	13	0	2	12	0	37	2	—	47	2	—	2	12	0	50	2	—	19	2	—	0	12	—
KBPKP	5	3-2	403	2	1	1	1	1	20	2	2+8	31	11	16+22	1	1	1	31	14	16+22	20	1	2+8	1	0	1
KBPKQ	5	3-2	0	16	0	0	2	0	—	20	—	—	28	—	0	1	0	—	28	—	—	20	—	—	1	—
KBPKR	5	3-2	4	302	0	1	1	0	3	15	—	32	29	—	1	1	0	32	33	—	3	14	—	1	0	—
KBPPK	5	4-1	6	0	0	1	0	0	6	—	—	12	—	—	1	0	0	16	—	—	1	—	—	0	—	—
KNNKN	5	3-2	362	0	0	85	0	0	4	—	—	4	—	—	85	0	0	4	—	—	4	—	—	85	—	—
KNNKP	5	3-2	3124	19	0	1	2	0	105	3	—	105	39	—	1	1	0	105	53	—	105	1	—	1	0	—
KNNKQ	5	3-2	0	229	0	0	1	0	—	53	—	—	62	—	0	1	0	—	62	—	—	53	—	—	1	—
KNNKR	5	3-2	0	25	0	0	1	0	—	5	—	—	28	—	0	1	0	—	35	—	—	2	—	—	0	—
KNPKB	5	3-2	640	2	0	1	1	0	28	2	—	40	2	—	1	1	0	40	2	—	28	2	—	1	1	—
KNPKN	5	3-2	4128	63	0	3	42	0	40	2	—	56	2	—	1	42	0	56	2	—	40	2	—	1	42	—
KNPKP	5	3-2	2281	14	8	1	6	1	21	5	1+13	29	24	17+28	1	1	1	33	24	17+28	19	5	1+13	0	1	1
KNPKQ	5	3-2	0	52	0	0	1	0	—	29	—	—	45	—	0	1	0	—	45	—	—	29	—	—	1	—
KNPKR	5	3-2	23	1158	0	2	1	0	4	34	—	19	58	—	1	1	0	28	58	—	3	34	—	0	1	—
KNPPK	5	4-1	93	0	0	6	0	0	5	—	—	11	—	—	1	0	0	16	—	—	1	—	—	0	—	—
KPPKB	5	3-2	211	1	0	2	1	0	14	2	—	25	2	—	1	1	0	28	2	—	12	2	—	0	1	—
KPPKN	5	3-2	920	157	0	1	11	0	22	12	—	36	12	—	2	15	0	36	12	—	22	12	—	1	11	—
KPPKP	5	3-2	4179	52	6	3	2	1	24	7	6+3	38	18	15+19	1	2	1	86	23	16+20	10	1	4+3	0	0	0
KPPKQ	5	3-2	0	2	0	0	1	0	—	8	—	—	13	—	0	1	0	—	13	—	—	8	—	—	1	—
KPPKR	5	3-2	18	99	2	1	1	1	11	21	1+4	39	36	18+17	1	1	1	39	36	15+21	11	21	3+1	1	1	0
KPPPK	5	4-1	11	0	0	2	0	0	7	—	—	17	—	—	1	0	0	17	—	—	7	—	—	1	—	—
KQBKQ	5	3-2	25	0	0	1	0	0	9	—	—	11	—	—	1	0	0	14	—	—	6	—	—	0	—	—
KQNKQ	5	3-2	38	0	0	1	0	0	24	—	—	30	—	—	1	0	0	30	—	—	24	—	—	1	—	—
KQPKQ	5	3-2	640	0	0	1	0	0	91	—	—	102	—	—	1	0	0	102	—	—	91	—	—	1	—	—
KQPKR	5	3-2	1	0	0	1	0	0	2	—	—	11	—	—	1	0	0	11	—	—	2	—	—	1	—	—
KQRKQ	5	3-2	1	0	0	1	0	0	11	—	—	18	—	—	1	0	0	18	—	—	11	—	—	1	—	—
KRBKP	5	3-2	1	0	0	1	0	0	1	—	—	8	—	—	1	0	0	8	—	—	1	—	—	1	—	—
KRBKQ	5	3-2	0	372	0	0	1	0	—	38	—	—	67	—	0	1	0	—	67	—	—	38	—	—	1	—
KRBKR	5	3-2	17	0	0	1	0	0	49	—	—	55	—	—	1	0	0	55	—	—	49	—	—	1	—	—
KRNKN	5	3-2	3	0	0	1	0	0	19	—	—	24	—	—	2	0	0	24	—	—	19	—	—	1	—	—
KRNKQ	5	3-2	0	455	0	0	1	0	—	42	—	—	63	—	0	1	0	—	63	—	—	42	—	—	1	—
KRNKR	5	3-2	10	0	0	2	0	0	22	—	—	23	—	—	2	0	0	23	—	—	22	—	—	2	—	—
KRPKB	5	3-2	225	0	0	1	0	0	58	—	—	67	—	—	1	0	0	67	—	—	58	—	—	1	—	—
KRPKN	5	3-2	413	0	0	4	0	0	37	—	—	47	—	—	1	0	0	47	—	—	37	—	—	1	—	—
KRPKP	5	3-2	0	2	1	0	2	1	—	1	1+4	—	15	15+15	0	1	1	—	15	15+15	—	1	1+4	—	1	1
KRPKQ	5	3-2	2	241	0	2	1	0	2	63	—	12	91	—	1	1	0	12	91	—	2	63	—	1	1	—
KRPKR	5	3-2	209	0	0	1	0	0	43	—	—	54	—	—	2	0	0	54	—	—	43	—	—	1	—	—
KRRKB	5	3-2	1	0	0	1	0	0	8	—	—	16	—	—	1	0	0	16	—	—	8	—	—	1	—	—
KRRKQ	5	3-2	10	0	0	1	0	0	7	—	—	20	—	—	1	0	0	20	—	—	7	—	—	1	—	—

Table 1: 3-5-man data on mutual zugzwangs.

Tamplin (2001a) confirmed that the scans of DTC and DTM tables had yielded exactly the same sets of mzugs of each type, further comprehensive evidence in itself of the integrity of the EGTs.

There are 21,677 *ww*, 3,395 *bw* and 33 *fp* 3-to-5-man-mzugs. These occur in 59 of the 146 endgames. The full point mzugs occur in just six endgames, namely KBPKP, KNPKP, KPKP, KPPKP, KPPKR and KRPKP: each features at least two Pawns. The complete data, statistics and various sets of positions, are available (Tamplin, 2001a). Here, the statistics are in Table 1 and the examples of maximal mzugs are in Table 2.

Endgame			type		dc	dm	a maximal mzug
KPK	3	2-1	cm	ww	18	27	8/8/8/1k6/8/1K6/6P1/8
KBKP	4	2-2	cm	bw	1	12	8/8/8/8/8/8/1pK5/kB6
KBPK	4	3-1	cm	ww	14	20	1B1K4/8/8/k7/8/P7/8/8
KNKP	4	2-2	cm	ww	6	6	8/8/8/8/8/p7/k2N4/2K5
			cm	bw	9	24	8/K7/N1k5/p7/8/8/8/8
KNPK	4	3-1	cm	ww	14	19	8/8/8/8/8/1k1P4/8/KN6
KPKP	4	2-2	c	ww	10	24	8/k7/6p1/K5P1/8/8/8/8
			m	ww	6	27	8/8/k7/8/K5p1/8/5P2/8
			c	bw	10	24	8/8/8/8/k5p1/6P1/K7/8
			m	bw	6	27	8/5p2/8/k5P1/8/K7/8/8
			cm	fp	1+1	19+12	8/1pK5/kP6/8/8/8/8/8
KPPK	4	3-1	cm	ww	11	26	8/8/8/8/1k6/1P6/1PK5/8
KRKB	4	2-2	cm	ww	14	25	8/8/1b6/5R2/8/3K4/8/2k5
KRKN	4	2-2	cm	ww	10	24	8/8/8/6n1/3K4/4R3/3k4/8
KRKP	4	2-2	cm	ww	6	20	8/K7/8/k7/1p6/8/8/1R6
KBBKN	5	3-2	cm	ww	2	2	8/8/8/8/8/6n1/2K4B/kB6
KBBKP	5	3-2	cm	ww	2	2	B1k5/1pB5/3K4/8/8/8/8/8
KBBKQ	5	3-2	cm	bw	5	17	8/8/8/8/q7/2BB4/1K6/3k4
KBBKR	5	3-2	cm	ww	1	19	8/8/8/B7/8/3k4/2r5/KB6
KBNKB	5	3-2	cm	ww	10	36	8/8/8/8/1b6/8/2K5/k1BN4
KBNKN	5	3-2	cm	ww	67	97	8/8/2B2n2/8/N7/8/3K4/1k6
KBNKP	5	3-2	c	ww	13	36	8/8/2K5/8/2k5/5p2/8/2N1B3
			m	ww	8	37	8/8/8/8/8/B7/p3N3/k2K4
			cm	bw	1	45	8/8/8/1N6/3K4/B7/5p2/k7
KBNKQ	5	3-2	cm	bw	33	44	8/8/q7/8/3K4/2N5/8/k1B5
KBNKR	5	3-2	c	ww	5	26	3r4/8/2B5/8/1N6/8/8/k1K5
			m	ww	1	32	8/8/8/8/B7/1r6/N1k5/K7
			cm	bw	8	28	8/r7/8/B7/8/8/N1k5/K7
KBPKB	5	3-2	cm	ww	33	44	8/8/8/2P5/4b3/1B6/8/k1K5
KBPKN	5	3-2	c	ww	37	47	8/8/8/8/8/K5n1/B5P1/k7
			m	ww	19	50	k7/8/Kn1BP3/8/8/8/8/8
			cm	bw	2	2	K7/P1k5/8/8/8/1B6/1n6/8
KBPKP	5	3-2	cm	ww	20	31	2K5/8/3k4/8/7p/5P2/8/7B
			c	bw	2	11	8/8/8/8/8/1k6/1P1p4/KB6
			m	bw	1	14	8/8/8/8/8/2p5/2P5/kBK5
			cm	fp	2+8	16+22	8/8/8/8/8/k1p5/2P5/1BK5
KBPKQ	5	3-2	cm	bw	20	28	5k2/1P1K4/1qB5/8/8/8/8/8
KBPKR	5	3-2	cm	ww	3	32	K7/1rB1P3/k7/8/8/8/8/8
			c	bw	15	29	8/8/4r3/8/2k5/K6P/5B2/8
			m	bw	14	33	8/8/8/8/k7/r1P5/1K6/B7
KBPPK	5	4-1	c	ww	6	12	8/B1k5/K7/P7/P7/8/8/8
			m	ww	1	16	kB6/8/1PK5/1P6/8/8/8/8
KNNKN	5	3-2	cm	ww	4	4	8/8/8/8/n7/8/2KN4/kN6
KNNKP	5	3-2	cm	ww	105	105	8/8/8/p7/K7/4k3/8/6NN
			c	bw	3	39	7N/8/K1N5/8/8/1pk5/8/8
			m	bw	1	53	1N6/8/8/N7/K7/8/kp6/8
KNNKQ	5	3-2	cm	bw	53	62	8/8/1q6/8/4N3/3K2N1/8/4k3
KNNKR	5	3-2	c	bw	5	28	6rN/5N2/8/8/8/2k5/8/1K6
			m	bw	2	35	5N2/1N6/8/3r4/8/2k5/8/2K5
KNPKB	5	3-2	cm	ww	28	40	8/6b1/8/8/1N5P/8/2K5/k7
			cm	bw	2	2	K5b1/P1k2N2/8/8/8/8/8/8

Endgame			type		dc	dm	a maximal mzug
KNPKN	5	3-2	cm	ww	40	56	2n5/8/8/1N6/8/6P1/8/k1K5
			cm	bw	2	2	K7/P2n4/1k6/N7/8/8/8/8
KNPKP	5	3-2	c	ww	21	29	8/8/8/8/8/2k2p2/P7/1KN5
			m	ww	19	33	K7/N6p/k7/8/8/8/7P/8
			cm	bw	5	24	8/8/8/8/6P1/3k2p1/8/2KN4
			cm	fp	1+13	17+28	8/8/8/8/8/1k2p3/4P3/KN6
KNPKQ	5	3-2	cm	bw	29	45	4N3/3P4/2K5/q7/1k6/8/8/8
KNPKR	5	3-2	c	ww	4	19	k6r/5P2/K7/3N4/8/8/8/8
			m	ww	3	28	3k4/r1N5/2KP4/8/8/8/8/8
			cm	bw	34	58	8/8/8/5N2/K7/2k5/P2r4/8
KNPPK	5	4-1	c	ww	5	11	K7/P1k5/P7/8/8/8/8/N7
			m	ww	1	16	kN6/8/1PK5/1P6/8/8/8/8
KPPKB	5	3-2	c	ww	14	25	1b6/8/3P4/4P3/8/8/8/3K1k2
			m	ww	12	28	8/8/5b1P/8/1K6/1P6/1k6/8
			cm	bw	2	2	8/8/8/8/8/b2k4/P2P4/1K6
KPPKN	5	3-2	cm	ww	22	36	8/5n2/8/K6P/3P4/k7/8/8
			cm	bw	12	12	K7/2k5/8/P7/P7/5n2/8/8
KPPKP	5	3-2	c	ww	24	38	1K3k2/8/7p/8/8/7P/6P1/8
			m	ww	10	86	8/2p5/8/8/8/2k1P3/P7/3K4
			c	bw	7	18	K1k5/8/7p/P7/2P5/8/8/8
			m	bw	1	23	8/2p5/8/8/1P1P4/2k5/8/2K5
			c	fp	6+3	15+19	8/8/8/2k5/K1p5/P3P3/8/8
			m	fp	4+3	16+20	8/8/8/8/5k2/3K1p2/3P3P/8
KPPKQ	5	3-2	cm	bw	8	13	8/2KP3q/8/2P3k1/8/8/8/8
KPPKR	5	3-2	cm	ww	11	39	2k5/K6P/6Pr/8/8/8/8/8
			cm	bw	21	36	8/8/8/8/2P5/2K1P3/4r3/2k5
			c	fp	1+4	18+17	1r1k4/1P6/1PK5/8/8/8/8/8
			m	fp	3+1	15+21	8/8/8/8/k7/r1P5/1KP5/8
KPPPK	5	4-1	cm	ww	7	17	8/8/8/1k1P4/8/PK6/P7/8
KQBKQ	5	3-2	c	ww	9	11	8/3K4/3B4/8/k7/3Q4/8/2q5
			m	ww	6	14	1q6/8/2Q5/B7/8/1k6/8/1K6
KQNKQ	5	3-2	cm	ww	24	30	8/3q4/1Q1N4/8/k7/8/3K4/8
KQPKQ	5	3-2	cm	ww	91	102	8/8/8/8/2K3Q1/2P1q3/8/4k3
KQPKR	5	3-2	cm	ww	2	11	k7/8/KQ1r4/P7/8/8/8/8
KQRKQ	5	3-2	cm	ww	11	18	8/8/8/8/1R6/k4q2/8/1K2Q3
KRBKP	5	3-2	cm	ww	1	8	1k1K4/7R/8/8/8/8/6p1/7B
KRBKQ	5	3-2	cm	bw	38	67	1q6/8/1B3R2/8/k7/8/8/1K6
KRBKR	5	3-2	cm	ww	49	55	5R2/8/8/8/8/3K4/5Br1/2k5
KRNKN	5	3-2	cm	ww	19	24	8/8/8/8/8/3n4/N2k4/RK6
KRNKQ	5	3-2	cm	bw	42	63	1Nk5/8/8/8/8/1R6/6q1/2K5
KRNKR	5	3-2	cm	ww	22	23	8/8/8/8/8/2KRN3/8/2k1r3
KRPKB	5	3-2	cm	ww	58	67	8/8/8/8/7R/1k2P3/2b5/K7
KRPKN	5	3-2	cm	ww	37	47	K7/1R6/2n5/4k3/8/4P3/8/8
KRPKP	5	3-2	cm	bw	1	15	8/8/8/8/8/1p6/kP6/1RK5
			cm	fp	1+4	15+15	8/8/8/8/8/2p5/1kP5/2RK4
KRPKQ	5	3-2	cm	ww	2	12	7k/2K2P1q/8/8/8/8/5R2/8
			cm	bw	63	91	8/8/q1k5/8/1R1K4/8/1P6/8
KRPKR	5	3-2	cm	ww	43	54	8/8/8/8/8/2RPr3/8/2K1k3
KRRKB	5	3-2	cm	ww	8	16	8/8/8/8/8/b1k5/1R6/1RK5
KRRKQ	5	3-2	cm	ww	6	20	6R1/8/8/8/6R1/7q/1K5k/8

Table 2: Sample maximal mutual zugzwangs of the three types.

Some explanatory notes, which also apply to the associated website (Tamplin, 2001a) are appropriate:

- White has at least as many men as Black: the men are listed in the standard K-Q-R-B-N-P order and endgames are listed in alphabetical order.
- with one exception, all positions are essentially unique, i.e., they cannot be transformed into another listed position by board transformation or by switching colours. The exception is that in symmetric endings, only KPKP here, the set of *bw* mzugs is acknowledged even though it is transformed by colour-reversal into the set of *ww* mzugs. The brackets in Table 1 highlight this equivalence.
- depths are in winner's moves. In Table 2, *c*, *m* and *cm* denote a maxDTC, maxDTM and maxDTC-&-maxDTM position respectively. *ww*, *bw* and *fp* denote the three types of mzug as above.

- a/the maxDTC mzug with the greatest DTM depth has been cited. A maxDTM mzug with the greatest DTC depth has also been cited if different, as it is in 18 cases. *dx* is the depth in metric *DTX*.the positions are in *canonical form* in the sense that:

  the wK is confined to a-d for endgames with Pawns, and to a1-d1-d4 for endgames without Pawns
  if there are no Pawns and the wK is on a1-d4, the bK is confined to a1-h1-h8
  if both Kings are on a1-h8, only one position is counted where there are two equivalent ones.

- As is usual, the statistics of Table 1 may include unreachable positions in which the side-not-to-move is not in check. Elkies (2000b) and van der Heijden (2000) have both pointed out such positions:

  KBPKP: 8/8/8/8/1p6/1k6/1P6/BK6
  KBPPK: kB6/8/1PK5/1P6/8/8/8/8, kB6/8/KP6/1P6/8/8/8/8 ... 2 of 6
  KNPPK: kN6/8/KP6/1P6/8/8/8/8, kN6/8/1PK5/1P6/8/8/8/8, K1k5/P1PN4/8/8/8/8/8/8 ... 3 of 93

Figures 1-8 illustrate the deepest mzugs of different types. Some facts, snapshots and curiosities:

KNNKP	q.v. Figure 1: a maxDTC/M 3-to-5-man mzug with *dc* = *dm* = 105 moves.
KNPKP	q.v. Figure 2: max fp mzug with (wtm) *dc* = 1 and *dm* = 17, (btm) *dc* = 13 and *dm* = 28.
KQPKQ	q.v. Figure 3: max KQPKQ mzug with *dc* = 91 and *dm* = 102.
KBNKN	q.v. Figure 4: max KBNKN mzug with *dc* = 67 and *dm* = 97.
KRPKQ	q.v. Figure 5: the maxDTC/M type-2 mzug with *dc* = 63 and *dm* = 91.
KBNKQ	8/8/q7/8/3K4/2N5/8/k1B5: the only maximal P-less mzug with both Kings on a1-h8.
KPPKR	only fp mzugs not having Pawns on both sides: one maxDTC, the other maxDTM.
KPKP	q.v. Figure 8; 'Le Trébuchet', the KPKP model for 15 of the 33 4-5-man full-point mzugs
KRKN	8/8/8/4k3/3R4/2K5/1n6/8: a diagonally symmetric mzug.

All the 4-5-man full-point mzugs feature at least two Pawns: the 6-man rk1N4/n2K4/P7/8/8/8/8/8 (Elkies, 2000b) needs only one and the 7-man 8/8/8/8/2N5/1N6/r1n5/1b1k1K2 needs none[7] (Elkies, 1998b). The likelihood is that there are no pawnless 6-man fp mzugs (Elkies, 2000a).

For a given endgame and type of mzug, *ZC* and *ZM* are the sets of maxDTC and maxDTM mzugs respectively. Curiously, ZC and ZM are either disjoint, identical or one is a subset of the other. There is no end-game for which ZC - ZM ≠ ∅ and ZM - ZC ≠ ∅:

ZC ∩ ZM = ∅	KBNKP ww, KBNKR ww, KBPKN ww, KBPKP bw, KBPKR bw, KBPPK ww, KNNKP bw, KNNKR bw, KNPKP ww, KNPKR ww, KNPPK, KPKP ww (and so bw), KPPKB ww, KPPKP ww, bw and fp, KPPKR fp and KQBKQ ww.
ZC ⊂ ZM	KNPK ww (1-2), KPPKN ww (1-2) and bw (11-15), KRNKN ww (1-2) and KRPKR ww (1-2).
ZC ⊃ ZM	KBBKR ww (3-1), KBPKQ bw (2-1), KNPKN ww (3-1), KNPKP bw (6-1), KPKP fp (15-1), KPPK ww (6-1), KRKP ww (6-4), KRPKN ww (4-1), KRPKP bw (2-1) and KRPKQ ww (2-1).

The following 55 2-to-4-man and 3-2 endgames have no mzugs:

KBBK, KBBKB, KBK, KBKB/N, KBNK, KK, KNK, KNKN, KNNK, KNNKB, KQBK, KQBKB/N/P/R, KQK, KQKB/N/P/Q/R, KQNK, KQNKB/N/P/R, KQPK, KQPB/N/P, KQQK, KQQKB/N/P/Q/R, KQRK, KQRKB/N/P/R, KRBK, KRBKB/N, KRK, KRKR, KRNK, KRNKB/P, KRPK, KRRK, KRRKN/P/R.

The only 4-1 endgames with mzugs are KBPPK, KNPPK and KPPPK, echoing the fact that the only 3-1 endgames with mzugs are KBPK, KNPK and KPPK.

4. SUMMARY

This note records the achievement of Karrer, Nalimov and Wirth whose combined work has created a definitive survey of the 3-5-man chess domain. Their work sets an example and provides a basis for further workers to respond to Van der Heijden's challenge (2001) to mine this data for the interesting study-like positions.

[7] White (to move) can do no better than mobilise Black with 1. Kf2, Kg1/2 or Nc5. Black (to move) allows mates in 1.

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