Six Centuries of British Economic Growth: a Time-Series Perspective

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Abstract

This paper provides a time-series analysis of recent annual estimates of real GDP and industrial output covering 1270 to 1913. We show that growth can be regarded as a segmented trend stationary process. On this basis, we find that trend growth of real GDP per person was zero prior to the 1660s but then experienced two significant accelerations, pre- and post-industrial revolution. We also find that the hallmark of the industrial revolution is a substantial increase in the trend rate of growth of industrial output rather than being an episode of difference stationary growth.

JEL Classification: N13; O47

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1 Introduction

The recent publication of the results of an ambitious project to provide estimates of long-run English economic growth (Broadberry et al., 2015) marks a major step forward in the analysis of pre-industrial economic performance. The book provides annual estimates of real GDP, population, and of real output in the agricultural, industrial and services sectors from 1270 to 1870 for England pre-1700 and Britain post-1700. On the basis of these numbers, the authors put forward a new analysis of British economic growth.

Our principal objective in this paper is to examine the validity of the description and analysis of economic growth provided by Broadberry et al. (2015), in particular by evaluating the statistical properties of the time series on which they are based. The discussion provided by the authors is informal and based on arithmetic rather than econometrics. We believe that time-series analysis can be helpful in several ways. Most obviously, it is valuable to consider explicitly the confidence intervals around estimated trends and to test formally for structural breaks. In addition, it is important to consider the nature of trend growth in a pre-industrial economy and whether it differs from that experienced under modern economic growth: for example, was a trend-stationary phase superseded by a difference-stationary era?[1] Time-series analysis of the new estimates also provides an opportunity to revisit the work of Mills and Crafts (1996a) on trends in industrial output growth during the industrial revolution to see how far it is now in need of revision.

For our purposes in this paper, we take the growth estimates provided by Broadberry et al. (2015) as reliable and we do not attempt any critique of or revisions to them. Of course, they may well be challenged or revised in future and already we know that the results of this project, based on building up estimates from the output side, contrast with those obtained by Clark (2010), which were derived from the income side. For example, Clark's estimates show incomes at the time of the Industrial Revolution to be no higher than 400 years earlier and thus seem consistent with a 'Malthusian economy'. Nevertheless, we think it useful to explore the implications of the Broadberry et al. estimates as they stand.

Our main findings are as follows. First, when Hodrick-Prescott (1997) trends are fitted to real GDP per person, we find that, on average, trend growth was 0.2% per year over the 500 years from 1270 to 1770. Nevertheless, growth performance before the Industrial Revolution differed between sub-periods, with trend growth approximately zero on average between

[1] A trend-stationary process is one for which all shocks to the series being analysed have only transitory effects and, in its simplest form, may be modelled as a linear time trend having random fluctuations about this trend, so that the series always reverts back to the trend line. For a difference-stationary process, in contrast, all shocks are permanent, so that in its simplest form the series evolves as a (possibly drifting) random walk, and thus becomes the accumulation of all past shocks, which must then remain permanently in the series. For more detailed discussion of the distinction from an economic history perspective see, for example, Mills (1996).

1400 and the mid-17th century, after which trend growth accelerated during the 18th century to about 0.3 per cent per year. Following the industrial revolution, trend growth in real GDP per person peaked at about 1.25 per cent per year in the mid-19th century.

Second, further results from fitting Hodrick-Prescott trends to real GDP and to industrial production highlight the impact of the industrial revolution and the difference between trend growth in the 19th century and all earlier periods. Here, in an era of rapid population growth, the increases in trend growth are much larger than for the per capita series, reaching peaks of well over 2 per cent per year and 3 per cent per year, respectively, compared with well below 1 per cent per year from the 16th through the 18th centuries.

Third, we believe that a segmented trend model is preferable to the estimates obtained using the Hodrick-Prescott methodology; naturally these have much smaller standard errors attached to the estimated trend growth rates. Fitting such a model using breakpoints indicated by the narrative account in Broadberry et al. (2015) yields estimates of trend growth of real GDP per person of 0.03 (0.00-0.06) per cent per year prior to 1663, 0.84 (0.68-1.00) per cent per year from 1663 to 1707, 0.27 (0.21-0.33) per cent per year from 1707 to 1822 and 1.03 (0.97-1.09) per cent per year thereafter.[2] This shows a statistically significant increase in growth during the early-modern period and distinctly before the industrial revolution and then a further, much larger, statistically significant increase as the impact of the industrial revolution was consolidated.

Fourth, we are able to clarify the idea that there was an absence of ‘growth reversals’. An analysis of the impact of the Black Death, which reduced population by nearly 50 per cent, shows that it can be seen as a levels shock which left the trend rate of growth unchanged. The trend rate of growth of real GDP per person is estimated as 0.03 per cent per year both pre-1350 and post-1400, but in the latter period output per person was sustained at a level about 40 per cent greater, quite unlike the predictions of a naive Malthusian model. We also reject the hypothesis of a unit root in favour of trend-stationarity in each of the segments of the trend that we identify, implying that following a shock the economy would fairly quickly revert to its trend path. From the second half of the 17th century trend growth of real GDP per person was significantly above zero, which suggests that the Malthusian era was over.

Fifth, trend growth in both GDP and industrial production follow an inverted U-shape if Hodrick-Prescott trends are estimated. We find that here the preferred model in each case is a cubic segmented trend. This reaches a maximum in 1857 of 2.27 (1.93-2.61) per cent per year for GDP and a maximum in 1842 of 2.99 (2.43-3.55) per cent per year for industrial production.

[2] Numbers in parentheses are 95% confidence intervals.

Sixth, it might be supposed that the properties of the time series changed as the economy moved from the Smithian growth characteristic of pre-industrial times to modern economic growth, in particular that trend stationarity was superseded by difference stationarity. In fact, this is generally not the case for the linear segmented trend models with the exception of failing to reject the unit-root null for industrial production in the interval 1782-1851, which includes the classic industrial revolution years. Closer examination reveals, however, that for the preferred cubic segmented trend model the unit root hypothesis is rejected in favour of cubic trend stationarity.

Overall, we think that these results are generally supportive of the interpretation of growth performance proposed by Broadberry et al. (2015).

2 Literature Review

The key features of the Broadberry et al. view can be summarized as follows. First, on average over the 500 or so years before the Industrial Revolution, the economy experienced a small positive rate of growth of real GDP per person (about 0.2 per cent per year) rather than stagnation or decline (p. 213). Second, there were two main components of this growth, namely, a big increase in the level of real GDP per person associated with the Black Death and then, for the first time, a move to a positive trend rate of growth of real GDP per person in the second half of the 17th century (pp. 210-211). Third, Britain did not experience major 'growth reversals'; on the contrary, stability and growth prevailed over decline of real GDP per person and this meant that gains in income levels were sustained over the long run, unlike in a Malthusian economy (p. 203). Fourth, the Industrial Revolution saw a transition from Smithian growth to 'modern economic growth' (Kuznets, 1966), characterized by a further acceleration in the trend rate of growth of real GDP per person combined with significant population growth and underpinned by technological progress (pp. 212, 408-409).[3]

By contrast, Clark (2010) explicitly rejects the idea that there was a long period of Smithian growth between 1300 and 1800, although he does see a period of slow growth of per capita GDP beginning around 1600 which makes the discontinuity of the industrial revolution less sharp. On the other hand, the suggestion that the growth of real GDP per person shifted to a modest positive trend in the latter part of the pre-industrial revolution period matches the hypothesis, advanced by Galor (2011) in his exposition of unified growth theory, that in the later phases of the Malthusian Epoch a positive but very small growth rate of output per capita is to be expected. The only attempt thus far to explore econometrically whether there

[3] The acceleration in growth during the Industrial Revolution is quite similar to that estimated by Crafts and Harley (1992) rather than the earlier, more dramatic, change proposed by Deane and Cole (1962).

are trend breaks in the Broadberry estimates suggests positive shifts in 1582 and 1853 (Greasley et al., 2013).[4]

Voigtländer and Voth (2013) propose that a high level of income in the Malthusian equilibrium is the platform from which subsequent take-off was launched. If the Black Death resulted in a positive levels-effect shock to real GDP per person which was sustained for a lengthy period of time, this would account for a high-level equilibrium. The consolidation of a higher income level after the Black Death saw a long period when population failed to recover – it took until the early 17th century to return to the level of the 1340s and a high level of mortality has been suggested as a key factor (Smith, 2012).

Pomeranz (2000) triggered off a major debate on the timing of the so-called 'Great Divergence' between China and Europe by claiming that European overtaking in terms of real per capita income levels was much later than generally supposed and only completed during the industrial revolution. This view has subsequently been rejected by quantitative economic historians. Broadberry et al. (2015) contribute to this rejection and suggest that their new estimates imply that between the mid 16th and the mid-19th centuries Britain went from being a European laggard to a global leader (p.372) while economic decline in China meant it fell behind England by 1570 (p.375). British success was underpinned by the absence of growth reversals augmented by Smithian growth from the late 18th century.

Greasley and Oxley (1994, p. 762) argued that to identify the industrial revolution as a distinct macroeconomic epoch requires that output innovations were permanent (difference stationary) during the discontinuity but transient (trend stationary) at other times. They claimed that this criterion was met for industrial output for the years 1780-1851 during the period 1700 to 1913. Crafts and Mills (1994) rejected this argument for two reasons. First, they thought that the key criterion for the industrial revolution is in terms of a substantial rise in the trend rate of growth of output rather than the statistical properties of the time series and, second, they suggested that the appropriate alternative hypothesis was not linear but quadratic trend stationarity and rejected the unit root on this basis. In the absence then of a suitable time series neither of these papers was, however, able to analyze real GDP or to adopt a longer-run perspective including pre-1700 observations.

3 Trend Growth Rates: an Initial Overview

The underlying model for obtaining trend growth rates is that of an additive decomposition of the series x_t, the logarithm of the variable under consideration and which is observed over

[4] Greasley et al. (2013) impose a two-break model and consider the whole period from 1270 to 2011 based on augmenting the estimates of Broadberry et al. (2015) with modern data. They do not report the estimated trend growth rates for their three segments. This superficial analysis was not the main focus of their paper.

the years $t = 1, 2, \ldots, T$, into a trend, μ_t, and a cycle, ψ_t, typically assumed to be independent of each other, i.e.,

$$x_t = \mu_t + \psi_t \qquad E(\mu_t \psi_s) = 0 \text{ for all } t \text{ and } s \tag{1}$$

Various models may be assumed for the trend component. For example, μ_t may be defined as a deterministic segmented trend, typically linear in *t* but Mills and Crafts (1996a) have used a cubic polynomial; it may be defined as a stochastic 'structural' trend in which μ_t follows a random walk with, perhaps, a random walk drift (see Crafts, Leybourne and Mills, 1989, and Mills and Crafts, 1996b); or μ_t may be estimated using a filter, as in Crafts and Mills (2004). Mills (2009, 2016) provides detailed historical and technical development of these various approaches.

We employ in this section the last approach, and consequently fit Hodrick-Prescott (H-P, 1997) trends to GDP, GDP per person, industrial production and industrial production per person. H-P trends are chosen for a preliminary analysis because they are known to be robust to the shifting regimes that must naturally occur over a very long sample period of over 600 years, and are readily comparable across the alternative series. This comparison is facilitated by using the same smoothing parameter, set at $\lambda = 10000$, for each series, a setting that produces satisfactorily smooth, albeit evolving, trend components and readily interpretable trend growth rates, defined as $100\nabla\mu_t$, which gives the growth rate in percentages per annum. The use of a higher value for λ than is often employed in much macroeconomic modelling (for example, setting the smoothing parameter to 100 is common practice when using annual data) may be justified from the theoretical and simulation analyses of Harvey and Trimbur (2008) and Flaig (2015).

The data that we use for this analysis are taken from the estimates reported as continuous series in index number format for 1270 to 1870 by Broadberry et al. (2015, Appendix 5.3). These estimates are for England only prior to 1700 and then for Great Britain. We have extended these series to 1913 using the well-known estimates for the United Kingdom by Feinstein (1972, pp. T18, T111 and T120).

The logarithms of the four series with H-P trends superimposed are shown in Figures 1 and 2, with trend growth rates shown in Figures 3 and 4. Figure 3 shows that estimated trend growth of real GDP per person fluctuated around zero before the late 17th century, increases to a low positive rate prior to the industrial revolution and then rises to a new peak of about 1.25 per cent per year before falling back in the later 19th century. Trend growth averaged 0.2 per cent per year from 1270 to 1770 which matches the claim made by Broadberry et al. (2015). Trend growth in industrial production per person, displayed in Figure 4, is more volatile and, on average, higher, but follows a similar pattern over time.

In Figure 3, it is apparent that trend growth in real GDP shows a larger increase during the industrial revolution to reach a maximum of 2.41 per cent per year in 1851 (1.93% in 1853 for real GDP per person), and a notable feature of the 19th-century experience is the inverted U-shape in estimated trend growth. There are clear similarities with trend growth in industrial production, which is graphed in Figure 4. Here too there is an inverted U-shape in the 19th century with a peak at 3.22 per cent per year in 1834 (1.91% in 1838 for industrial production per person).

We also show one-standard error lower bounds for the trend growth rates in Figures 3 and 4, computed using the method outlined in Giles (2011). The lower bounds may be interpreted as providing approximately 16% level significance tests: one-standard error bounds are used here because of the extreme variability of the actual annual growth rates compared to trend growth rates, with the variance of actual growth rates being between 35 and 250 times the variance of trend growth rates. This leads to standard errors in the region of 0.8% apart from at the two ends of the sample period, where the standard errors increase rapidly because of end-point effects (Figures 3 and 4 are consequently truncated to the years 1300 to 1900). Unfortunately, this means that discussion of comparative growth performance over time using this technique is somewhat compromised since differences in trend growth are never statistically significant at conventional levels.

Periods in which the one-standard error lower bound of trend growth exceeds zero are shown as shaded intervals in Figures 3 and 4: for example, GDP per person trend growth is 'significantly' above zero on this criterion between 1347 and 1352, between 1662 and 1688, and from 1830 onwards. Even so, this implies that if a lower bound estimate is computed on a 2-standard error basis this will always be below zero, even after the industrial revolution. This seems to suggest that a forecaster could not rule out a 'growth reversal' even in the era of 'modern economic growth'. We believe that this would be an inappropriate conclusion to draw, as subsequent sections of the paper suggest.

4 Segmented linear trend models of GDP per person and industrial production per person

An alternative approach to estimating trend growth rates is to fit a segmented trend model which entails periodic breaks in trend. We report estimates which suggest that this procedure has some statistical justification, at least when compared with the alternative of a difference stationary model, and generally provides support for the view that the growth process was trend stationary. Viewing the long-run growth performance of the British economy through this lens allows a number of interesting inferences to be drawn, in particular because the standard errors associated with the estimated trends are much smaller, since the trend path is assumed to be deterministic, than those resulting from the Hodrick-Prescott methodology.

The results of this complementary approach to constructing trend growth rates for GDP per person are shown in Figure 5, where a segmented linear trend is fitted with breaks for the arrival and departure of the Black Death, at 1348 and 1352, and further breaks at 1663, 1707 and 1822. Although there is a developing literature on automatically selecting multiple break dates using computational algorithms and structural break tests (see Perron, 2006, for a comprehensive survey), they nevertheless remain in a relatively early stage of development and their properties in dynamic regression models have not been completely established, an important proviso here given that autocorrelation is found to be an integral feature of the segmented trend models reported below.[5] Consequently, the break dates listed above have been chosen visually (i.e., exogenously), taking the description of growth outcomes given by Broadberry et al. (2015, p. 155-6, 158, 198, 211-2, 213) as a starting point.[6] However, alternative break points in the vicinity of those chosen were also investigated to assess the robustness of this selection, which was found to be best in terms of overall goodness of fit. We would, in any case, emphasise that such models should be regarded primarily as descriptive devices rather than as formal structural models of the growth process.

Nevertheless, the changes in trend growth in the late 17th and early 19th centuries can be linked to changes in the real economy, as has long been recognised in the historiography. The former probably represents an acceleration of Smithian growth associated with 'the commercial revolution' (Davis, 1967) reflected in expanding international trade and closely linked with urbanization, including the advance of London to become Europe's largest city by 1700 (Wrigley, 1987). The latter is the outcome of the technological changes of the industrial revolution whose impact on per capita growth only came through strongly by the second quarter of the 19th century (Crafts and Harley, 1992).

The general linear segmented model with *m* break dates can be written

$$x_t = \alpha + \beta t + \sum_{i=1}^{m} \theta_{T_i} D_{T_i,t} + u_t \qquad (2)$$

where

[5] Clearly, standard structural break tests of the Chow variety are not appropriate here as multiple breaks are being considered within which there is both autocorrelation and heterogeneity of variance.

[6] These are as follows. First, there is a description of negative growth of industrial production from the Black Death to the closing decades of the 15th century (pp. 155-6). Second, a phase of sustained growth of industrial production before a marked acceleration after 1780 is identified (pp. 158-9). Third, there is a description of similar trends in real GDP growth (p. 198). Fourth, there is statement that positive trend growth in real GDP per person began between the end of the Civil War and the Glorious Revolution and then moderated in the early 18th century before accelerating to over 1 per cent per year by the 1830s (pp. 211-2). Fifth, it is noted that a big increase in the level of real GDP per head resulted from the Black Death and that this was subsequently maintained until trend growth began in the late 17th century (p.213).

$$D_{T_i,t} = \begin{cases} t - T_i + 1270 & t > T_i \\ 0 & \text{otherwise} \end{cases}$$

Thus trend growth in the i-th segment is given by $\beta + \theta_{T_1} + \ldots + \theta_{T_i}$, from the estimate of which a standard error may readily be calculated using the formula for a variance of a linear combination of random variables. The form of (2) actually fitted here has $m = 5$ with $T_1 = 1348$, $T_2 = 1352$, … , $T_5 = 1822$, and takes the form

$$\begin{aligned} x_t = &- \underset{(0.020)}{0.765} + \underset{(0.00014)}{0.00026}\, t + \underset{(0.010)}{0.103}\left(D_{1348,t} - D_{1352,t}\right) + \underset{(0.00091)}{0.00816}\, D_{1663,t} \\ &- \underset{(0.00102)}{0.00570}\, D_{1707,t} + \underset{(0.00049)}{0.00757}\, D_{1822,t} + u_t \\ u_t = &\ \underset{(0.037)}{0.531}\, u_{t-1} + \underset{(0.043)}{0.159}\, u_{t-3} + a_t \qquad \hat{\sigma}_a = 0.0546 \qquad \bar{R}^2 = 0.988 \end{aligned} \tag{3}$$

The error process is a stationary third order autoregression and $\hat{\sigma}_a$ denotes the residual standard error. Estimation is carried out by autoregressive least squares with the error process estimated jointly with the deterministic part of the model. Figures in parentheses are HAC standard errors, used to capture any remaining and unmodelled autocorrelation and heteroskedasticity. The restriction $\theta_{1348} + \theta_{1352} = 0$ has been imposed, which forces the pre- and post-Black Death trend growth rates to be the same at 0.026 (0.014)%.[7] This restriction is statistically acceptable (the accompanying test statistic is $F(1,632) = 2.98$ with a marginal significance level of 0.08) and the unrestricted estimates are $\hat{\theta}_{1348} = 0.089\,(0.008)$ and $\hat{\theta}_{1352} = -0.091\,(0.008)$, so that $\hat{\theta}_{1348} + \hat{\theta}_{1352} = -0.002$.

Further statistical support for this segmented trend model is presented in Table 1(a), which reports Dickey-Fuller unit root tests for the segments defined in the model (3). For each segment a unit root is conclusively rejected in favour of a trend stationary process. Interestingly, the test statistic for the complete post-1663 period is just –1.11 with a marginal probability value of 0.92, demonstrating that ignoring the shifts at 1707 and 1822 leads to the erroneous conclusion that the process generating GDP per person is difference stationary so that all shocks are permanent: a segmented trend model asserts, on the contrary, that shocks are typically transitory except at a small number of break points, where the trend shifts permanently.

The picture that emerges from this analysis provides a number of interesting results. First, it is seen in Figure 5 that both before and after the Black Death trend growth was constant at

[7] The standard error accompanying the estimate is shown in parentheses.

0.03% per annum. An implication is that the Black Death produced a permanent upward shift of 36% in the level of GDP per person. This is mirrored in the estimates reported by Broadberry et al., (2015, Table 5.06), which show real GDP per person in 1700 prices to have been £6.07 in the early 14th century compared with £8.26 in the 1550s, at which point the population of England was 3.12 million compared with 4.72 million in the 1300s. Second, the break at 1663 increased trend growth to 0.84 (0.68-1.00); trend growth then decreased to 0.27 (0.21-0.33) after 1707 before increasing once again to 1.03 (0.97-1.09) from 1823 onwards.[8] These estimates indicate that there was a significant increase in trend growth of real GDP per person in the 17th century, which was partly sustained before and during the early industrial revolution period, before a further significant increase after 1823. Third, trend growth of real GDP per person is significantly above zero after 1663 even though population growth tended to accelerate. By 1700, according to Broadberry et al. (2015, Table 5.06), real GDP per head in England (Great Britain) was £12.68 (12.24) and population was 5.20 (6.20) million.

A similar model was fitted to industrial production per person.

$$\begin{aligned} x_t &= \underset{(0.071)}{-1.280} + \underset{(0.00032)}{0.00151}\,t + \underset{(0.0134)}{0.0634}\left(D_{1348,t} - D_{1352,t}\right) + \underset{(0.00083)}{0.00480}\,D_{1663,t} \\ &\quad + \underset{(0.00128)}{0.00990}\,D_{1822,t} + u_t \\ u_t &= \underset{(0.046)}{0.736}\,u_{t-1} + \underset{(0.045)}{0.154}\,u_{t-3} + a_t \qquad \hat{\sigma}_a = 0.0574 \qquad \bar{R}^2 = 0.995 \end{aligned} \tag{4}$$

Here the restriction $\theta_{1348} + \theta_{1352} = 0$ is satisfied with a marginal significance level of 0.29 ($F(1,633) = 1.11$), the unrestricted estimates being $\hat{\theta}_{1348} = 0.047\,(0.013)$ and $\hat{\theta}_{1352} = -0.050\,(0.012)$, so that $\hat{\theta}_{1348} + \hat{\theta}_{1352} = -0.003$.

This restricts the pre- and post-Black Death trend growth rates to be 0.15% (0.09-0.21), so that the Black Death thus produced a permanent increase in the level of industrial production per person of 30% with no alteration in trend growth. In contrast to the model (3) for GDP per person, no significant break is found at 1707, so that from 1664 to 1822, industrial production per person trend growth was 0.63% (0.51-0.75) and after 1822 trend growth more than doubled to 1.62% (1.44-1.80), as seen in Figure 6.

Table 1(b) reports unit root tests for the segments defined by model (4). All the segments apart from the pre-Black Death period conclusively reject a unit root, with this first segment

[8] Numbers in parentheses are 95% confidence intervals.

rejecting at a marginal significance level of 0.08. There is thus strong statistical support for the segmented trend model (4).

5 Segmented trend models of GDP and industrial production

Segmented trend models for actual GDP and industrial production were also fitted. For GDP the fitted model was

$$\begin{aligned} x_t &= \underset{(0.024)}{3.719} - \underset{(0.008)}{0.079}\, D_{1348,t} + \underset{(0.008)}{0.077}\, D_{1352,t} + \underset{(0.0003)}{0.0075}\, D_{1476,t} + \underset{(0.0003)}{0.0144}\, D_{1781,t} + u_t \\ u_t &= \underset{(0.039)}{0.528}\, u_{t-1} + \underset{(0.042)}{0.168}\, u_{t-3} + a_t \qquad \hat{\sigma}_a = 0.0545 \qquad \bar{R}^2 = 0.998 \end{aligned} \tag{5}$$

This provides a rather different picture of trend growth than that of GDP per person: see Figure 7. Trend growth is now zero pre-Black Death, which itself led to a permanent fall in GDP of 27%, followed by declining trend growth of 0.15% (0.11-0.19) per annum until 1476. From 1477 to 1781 trend growth returned to a positive value of 0.60% (0.56-0.64), and from 1782 onwards the rate increased to 2.04% (2.00-2.08). Unit root tests for the segments of this model are reported in Table 1(c). All conclusively reject a unit root, even when the final segment is itself split at 1851.

The model fitted to industrial production was

$$\begin{aligned} x_t &= \underset{(0.070)}{3.286} - \underset{(0.018)}{0.078}\, D_{1348,t} + \underset{(0.018)}{0.076}\, D_{1352,t} + \underset{(0.0010)}{0.0087}\, D_{1446,t} + \underset{(0.0006)}{0.0199}\, D_{1781,t} + u_t \\ u_t &= \underset{(0.154)}{0.718}\, u_{t-1} + \underset{(0.043)}{0.154}\, u_{t-3} + a_t \qquad \hat{\sigma}_a = 0.0569 \qquad \bar{R}^2 = 0.999 \end{aligned} \tag{6}$$

A similar pattern as for GDP emerges. As can be seen from Figure 8, trend growth before the Black Death is zero, with trend industrial production then falling continuously until 1446, the trend growth rate being –0.20% (–0.13-–0.27). From 1447 to 1781 trend growth returned to a positive value of 0.67% (0.65-0.69), and from 1782 onwards the rate increased to 2.66% (2.61-2.71). Unit root tests for the segments of this model are reported in Table 1(d). Industrial production is found to be difference stationary before 1446, in contrast to GDP. The final segment was again broken at 1851 and, unlike GDP, industrial production was found to be difference stationary for the sub-period 1782 to 1851.

The constant trend growth rates for GDP and industrial production estimated from the final segment from 1782 are clearly at odds with the 'inverted U-shaped' trend growth found for this period using an H-P filter shown in Figures 3 and 4. Consequently, cubic segmented trend models of the type considered by Mills and Crafts (1996a) for an earlier industrial production series were also fitted. For GDP this model took the form

$$\begin{aligned} x_t &= \underset{(0.023)}{3.718} - \underset{(0.007)}{0.077}\, D_{1348,t} + \underset{(0.007)}{0.075}\, D_{1352,t} + \underset{(0.0003)}{0.0078}\, D_{1472,t} \\ &\quad + \underset{(0.0016)}{0.0076}\, D_{1781,t} + \underset{(0.00003)}{0.00012}\, D^2_{1781,t} - \underset{(1.62\times 10^{-7})}{5.35\times 10^{-7}}\, D^3_{1781,t} + u_t \\ u_t &= \underset{(0.041)}{0.496}\, u_{t-1} + \underset{(0.038)}{0.140}\, u_{t-3} + a_t \qquad \hat{\sigma}_a = 0.0539 \qquad \overline{R}^2 = 0.999 \end{aligned} \tag{7}$$

while for industrial production, it took the form

$$\begin{aligned} x_t &= \underset{(0.144)}{3.070} + \underset{(0.0022)}{0.0042}\, t - \underset{(0.011)}{0.100}\, D_{1348,t} + \underset{(0.011)}{0.093}\, D_{1352,t} + \underset{(0.0007)}{0.0094}\, D_{1446,t} \\ &\quad + \underset{(0.0028)}{0.0124}\, D_{1781,t} + \underset{(0.00005)}{0.00017}\, D^2_{1781,t} - \underset{(2.39\times 10^{-7})}{943\times 10^{-7}}\, D^3_{1781,t} + u_t \\ u_t &= \underset{(0.043)}{0.703}\, u_{t-1} + \underset{(0.044)}{0.145}\, u_{t-3} + a_t \qquad \hat{\sigma}_a = 0.0566 \qquad \overline{R}^2 = 0.999 \end{aligned} \tag{8}$$

where

$$D^2_{1781,t} = \begin{cases} (t-1781+1270)^2 & t > 1781 \\ 0 & \text{otherwise} \end{cases} \qquad D^3_{1781,t} = \begin{cases} (t-1781+1270)^3 & t > 1781 \\ 0 & \text{otherwise} \end{cases}$$

These non-linear segmented trends provide better statistical fits than the linear forms (5) and (6), with both the additional variables required to model a cubic trend highly significant. Interestingly, a unit root test for industrial production with a cubic trend incorporated yields a test statistic of –5.90 for the period 1782-1913, considerably smaller, and hence more in favour of (cubic) trend stationarity, than those reported in Table 1(d) for this period.[9]

Figures 9 and 10 show trend growth from models (7) and (8) respectively along with H-P trend growth and the similarities between the two in the post-1782 period can now be clearly seen. For GDP, segmented trend growth reaches a maximum of 2.27% (1.93-2.61) in 1857 compared with 2.41% (0.81-4.01) for maximum H-P trend growth in 1851, while for

[9] Cubic segmented trends were also fitted to the two per person series for the final segment beginning in 1822. In both cases there was no evidence of an improved fit, with the higher order terms being insignificant in each equation.

industrial production segmented trend growth reaches a maximum of 2.99% (2.43-3.55) in 1842 compared with 3.22% (1.38-5.06) in 1834 for maximum H-P trend growth.

The outcome of this exercise is that the industrial revolution stands out as a transitory period when trend growth in industrial output was very high compared with both before and after. It seems to us that this is the distinctive feature of the early 19th century rather than the notion that industrial production was a difference-stationary process uniquely in the period from 1782 to 1851.

6 Conclusions

Our results have implications not only for the quantification of claims made by Broadberry et al. (2015) but also for the wider historiography reviewed in section 2. From the perspective of a segmented trend stationary view of the world and on the basis of the output estimates made by Broadberry et al. (2015), we have found the following.

First, from the Black Death through the English Civil War, trend growth of real GDP per person was approximately zero, but from then until the industrial revolution trend growth was positive at a modest rate; the industrial revolution itself saw a significant further increase after the Napoleonic Wars to a rate consistent with modern economic growth. This two-stage acceleration matches the predictions of a unified growth model as in Galor (2011) and the account of Broadberry et al. (2015), but the timing of the breaks in trend is quite different from that suggested by Greasley et al. (2013).

Second, these estimates imply that the English economy was surprisingly resilient between the Black Death and the industrial revolution. The Black Death itself emerges as a big positive levels shock to real GDP per person which moved the pre-industrial economy to a ‘high-level equilibrium’ in the sense of Voigtländer and Voth (2013). From the second half of the 17th century we not only see positive trend growth in real GDP per person but this is based on a trend stationary process which implies that following a shock the economy would revert to its trend path. This configuration underpins the emphasis placed by Broadberry et al. (2015) on the absence of ‘growth reversals’ and reinforces their interpretation of the Great Divergence.

Third, we can clarify the nature of the ‘industrial revolution’. It entailed an economically and statistically significant increase in trend growth of real GDP per person and industrial output per person but this was accompanied by an even more impressive increase in trend growth of industrial output. We do not, however, agree with the suggestion made by Greasley and

Oxley (1994) that the industrial revolution should be characterized as a distinct macroeconomic epoch during which the growth of output was a difference stationary process. Trend growth in both GDP and industrial production exhibits an inverted-U shape during the 19th century which appears to be best described as a cubic segmented trend. This finding is of great interest since it relates to the British experience of the first industrial revolution. Since this was the beginning of 'modern economic growth' it is important to understand what it entailed as clearly as possible. This point is given added weight by the advent of unified growth theory for which the escape from the Malthusian era is a focal point.

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	Test Statistic [prob value]
1270–1348	–5.25 [0.00]
1353–1663	–8.42 [0.00]
1664–1707	–5.64 [0.00]
1708–1822	–5.99 [0.00]
1823–1913	–3.99 [0.01]

(a) GDP per person

	Test Statistic [prob value]
1270–1348	–3.28 [0.08]
1353–1663	–4.52 [0.00]
1664–1822	–3.77 [0.02]
1823–1913	–3.50 [0.04]

(b) Industrial production per person

	Test Statistic [prob value]
1270–1348	–4.26 [0.01]
1353–1476	–7.16 [0.00]
1477–1775	–9.74 [0.02]
1782-1851	–3.48 [0.05]
1852-1913	–3.54 [0.04]
1782–1913	–4.02 [0.01]

(c) GDP

	Test Statistic [prob value]
1270–1348	–2.38 [0.39]
1353–1476	–1.97 [0.61]
1447–1781	–6.61 [0.00]
1782-1851	–2.75 [0.22]
1852-1913	–3.83 [0.02]
1782–1913	–3.35 [0.06]

(d) Industrial production

Table 1 Dickey-Fuller test statistics for a unit root (constant and trend included; lag augmentation automatically selected using AIC)

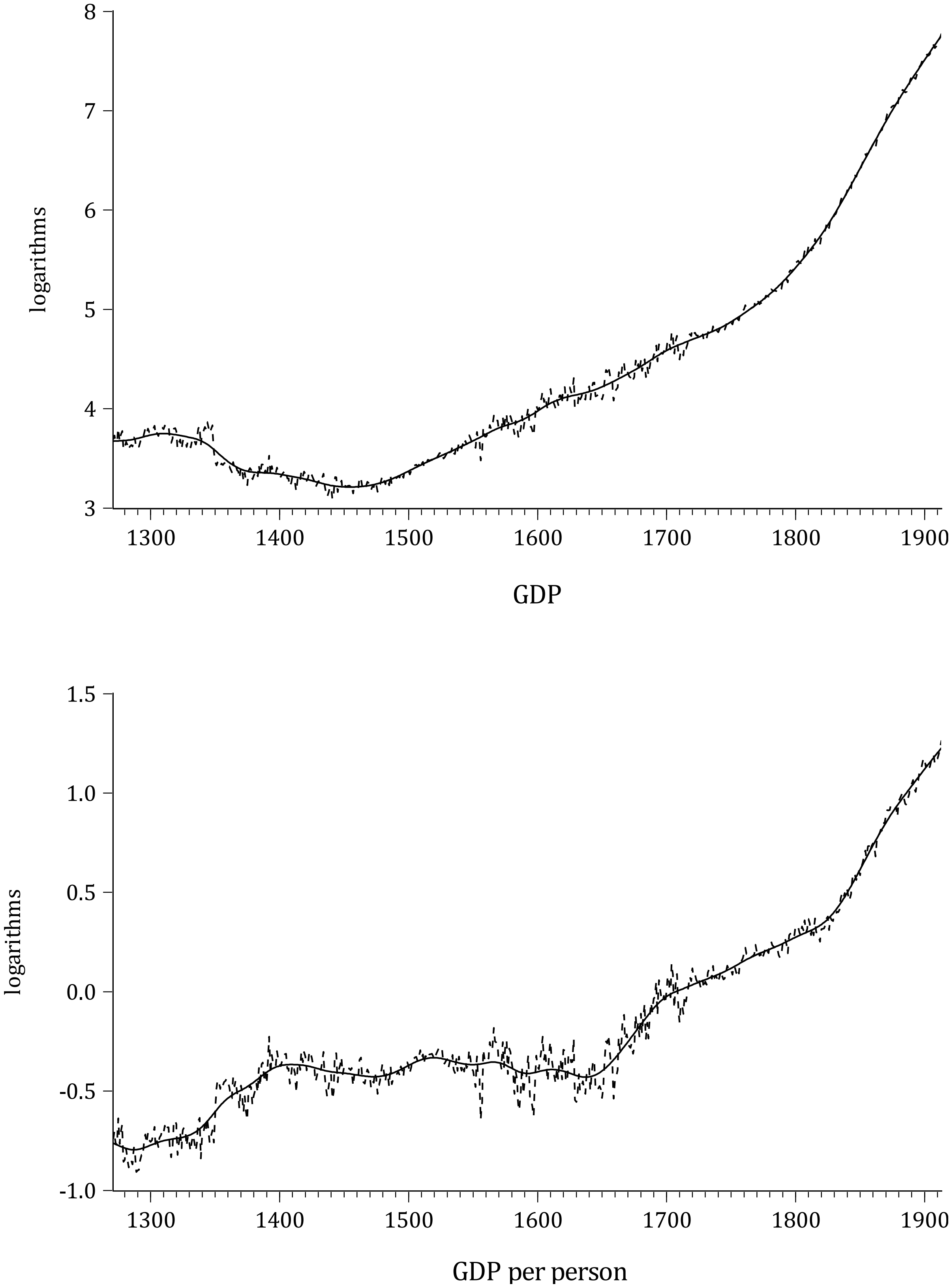

Figure 1 GDP and GDP per person with H-P trend, 1270 – 1913.

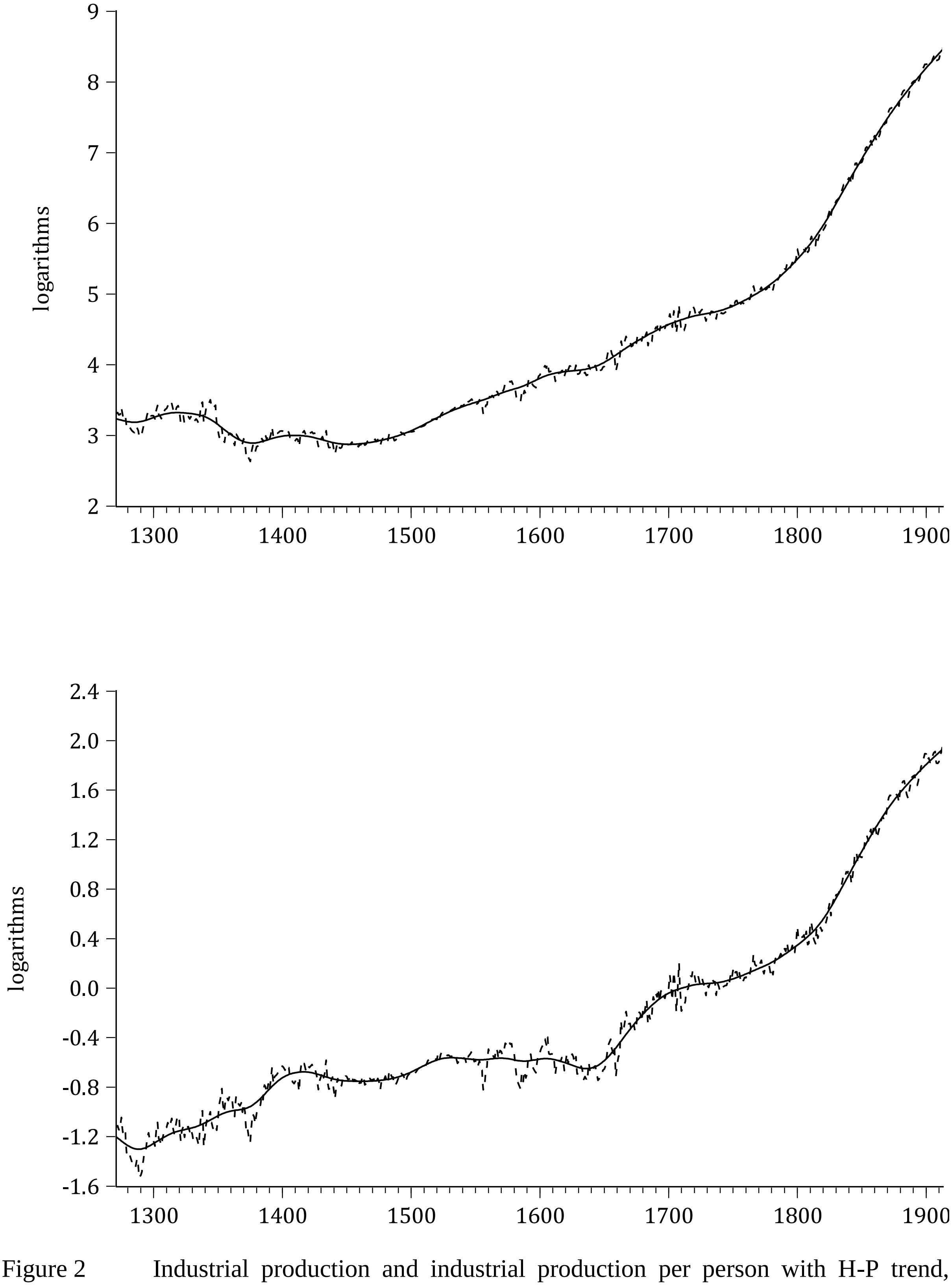

Figure 2 Industrial production and industrial production per person with H-P trend, 1270 – 1913.

Figure 3 GDP and GDP per person trend growth with one-standard error lower bounds (vertical scale decimalised percentages).

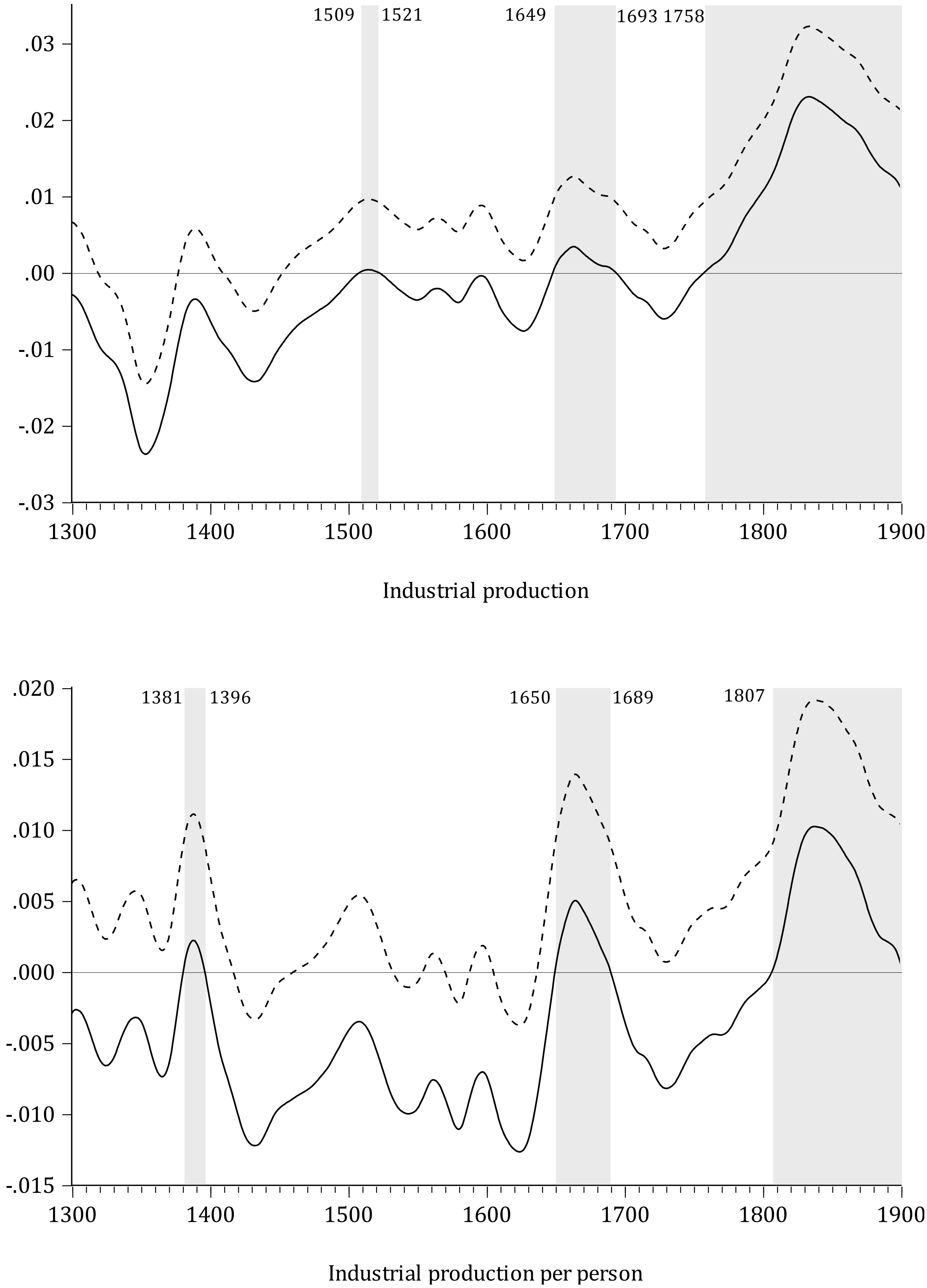

Figure 4 Industrial production and industrial production per person trend growth with one-standard error lower bounds (vertical scale decimalised percentages).

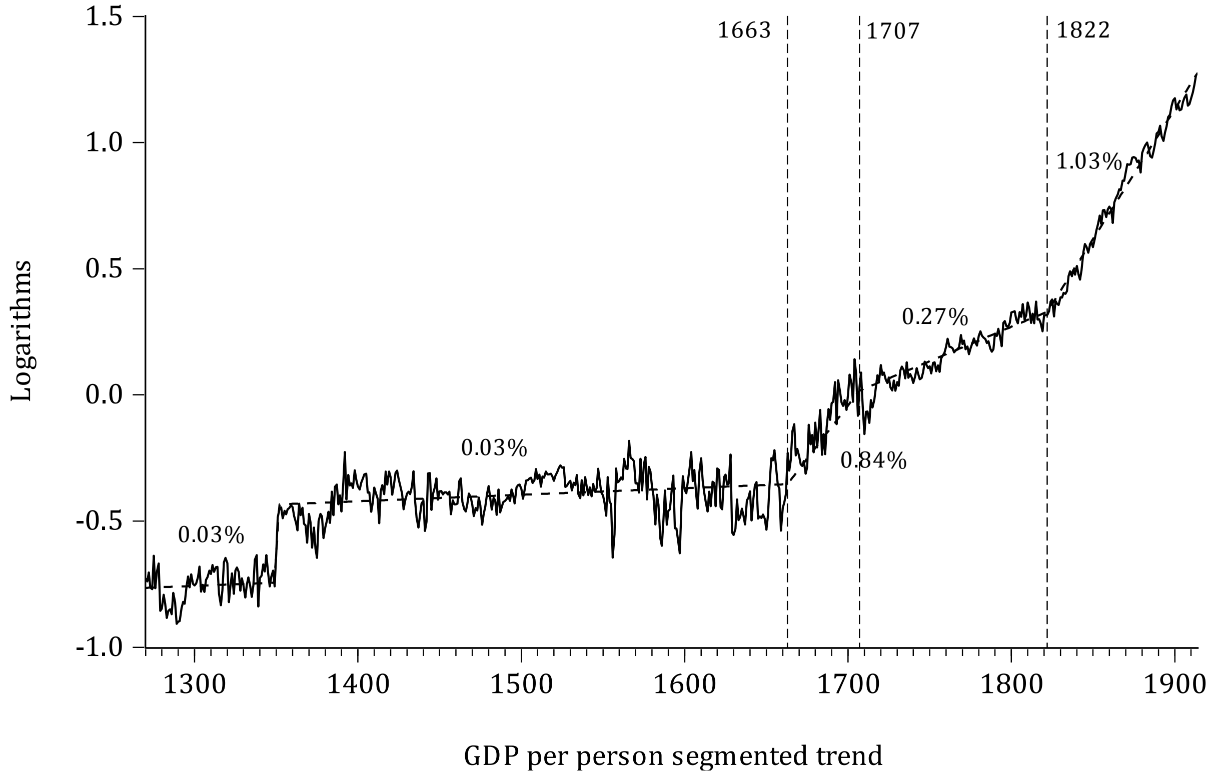

Figure 5 Segmented trend fitted to GDP per person.

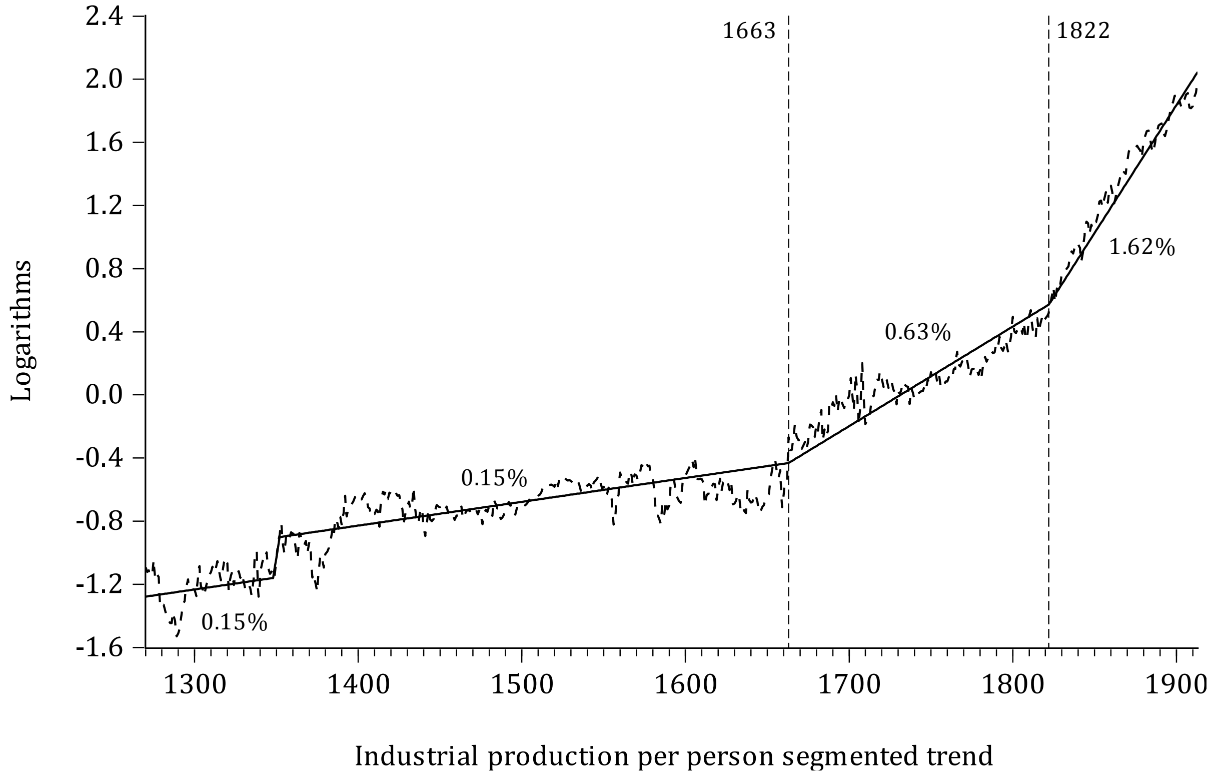

Figure 6 Segmented trend fitted to industrial production per person.

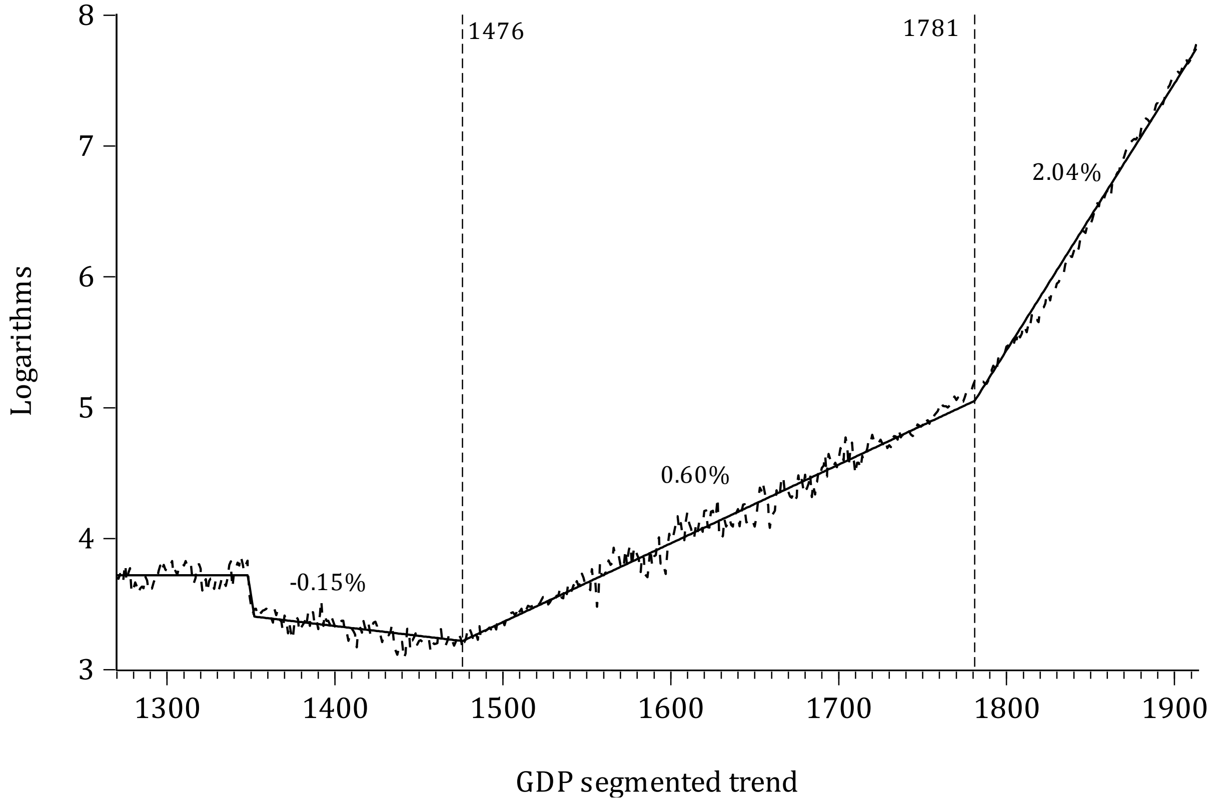

Figure 7 Segmented trend fitted to GDP.

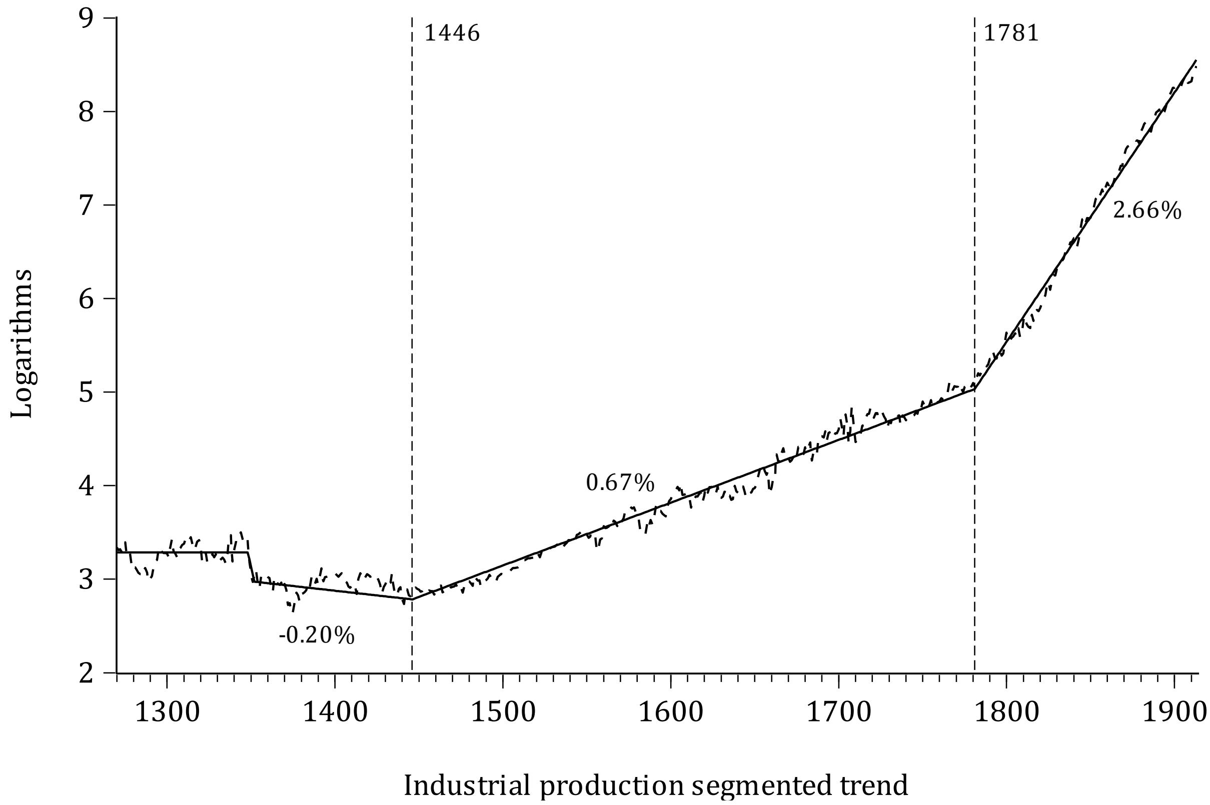

Figure 8 Segmented trend fitted to industrial production.

Figure 9 Cubic segmented and H-P trend growth rates for GDP.

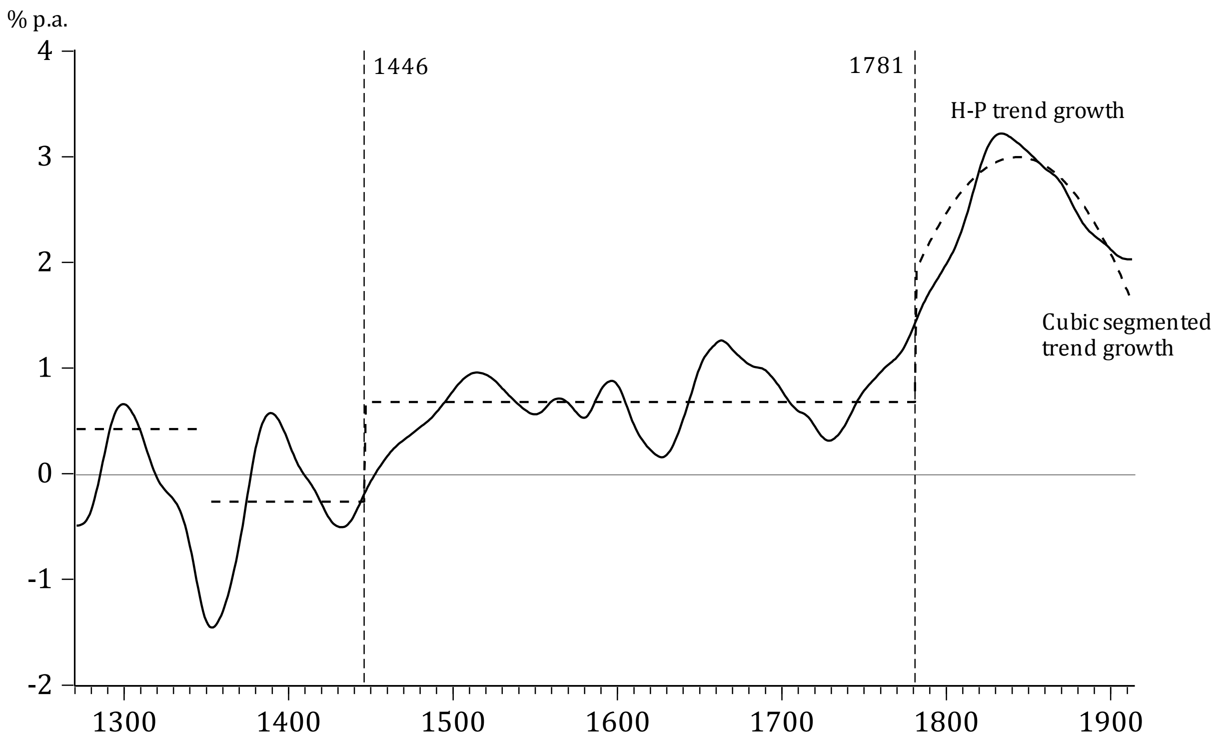

Figure 10 Cubic segmented and H-P trend growth rates for industrial production.