Impact of Compressor Failures on Gas Transmission Network Capability

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Abstract

National Grid, the gas operator in the United Kingdom, has experienced challenges in evaluating the capability of its gas transmission network to maintain function in the event of risks particularly to withstand the impact of compressor failures. We propose a mathematical programming model to support the operator in dealing with the problem. Several solution techniques are developed to solve the various versions of the problem efficiently. In the case of little data on compressor failure, an uncertainty theory is applied to solve this problem if the compressor failures are independent; while a robust optimisation technique is developed to solve it if the compressor failures are dependent. Otherwise, when there are data on compressor failure, Monte Carlo simulation is applied to find the expected capability of the gas transmission network. Computational experiments, carried out on a case study at National Grid, demonstrate the efficiency of the proposed model and solution techniques. A further analysis is performed to determine the impact of compressor failures and suggest efficient maintenance policies for National Grid.

Keywords: gas transmission network; capability evaluation; uncertainty; compressor failure.

1 Introduction

Gas currently plays an essential role in natural energy sources because of its low carbon dioxide emission and abundant reserves. It has a primary role in electricity generation. According to the International Energy Outlook 2016, world demand for energy will grow by 48% between 2012 and 2040, and fossil fuels are expected to account for more than three-quarters of this. Natural gas is the fastest-growing fossil fuel with global consumption increasing by 1.9% per year. Hence, efficient and effective gas transportation networks are a critical requirement for gas operators. Gas transportation networks involve three major subsystems: namely, the gathering system (from oil-shores to terminals), the transmission system (from terminals to off-takes), and the distribution

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system (from off-takes to customers). Unlike the gathering system and the distribution system which are characterised by low pressure, small diameter pipelines, the transmission system is characterised by long, large diameter pipelines operated at high pressures. The efficient performance of the gas transmission system thus poses a challenge in maintaining the safe regulation of pressure such that gas demands at off-takes are satisfied. Controlling pressure and flow in the gas transmission system depends on a number of compressor stations at which several compressors operate in serial and/or parallel. Compressor station/unit failures are extremely challenging for gas transmission. Evaluation of the impact of failures on gas transmission capability is a significant issue for gas operators.

The maximum flow problem can be used to evaluate network capability. It is one of the classic optimisation problems with many real applications in electrical power systems, computer networks, communication networks, logistic networks and transportation networks [1, 2, 3]. However, the uncertain maximum flow problem has not received as much attention by researchers. The few relevant works in the literature may be categorised into two approaches: uncertainty theory and robust optimisation. Uncertainty theory is first introduced by Liu [4] for solving project scheduling problem with uncertain duration times. Under the framework of uncertainty theory, Han et al. [5] investigate the maximum flow problem in an uncertain network. They introduce the concept of maximum flow function of network, and then use the so-called 99-method to give the uncertainty distribution and the expected value of the maximum flow of uncertain network. Ding [6] formulates an α-maximum flow model to find the distribution of the maximum flow for the problem with uncertain capacity on any arc, proving an equivalence relationship between the α-maximum flow model and the classic maximum flow model. A polynomial algorithm is developed based on properties of α-maximum flow model. Shi et al. [7] investigate two maximum flow models of an uncertain random network under the framework of chance theory. They consist of the expected value constrained maximum flow and the chance constrained maximum flow with uncertain random arc capacities. The authors propose two algorithms to solve these models, and prove that there exists an equivalence relationship between the models and the deterministic ones. Alipour and Mirnia [8] formulate uncertain dynamic network flow problems in which arc capacities are uncertain (may vary with time or not), and flow varies over time in each arc. They build an algorithm to solve the problems with independent uncertain factors. The algorithm cannot be applied for the problems with correlated uncertain factors or time-dependent distribution functions. Models built within the framework of uncertain or chance theory focus mainly on the maximum flow problem with uncertain capacity on arcs. A lack of models for the maximum flow problem with uncertain capacity on nodes exists. For the uncertain maximum flow problem solved by robust optimisation, readers can refer to [9] and [10]. Bertsimas and Sim [9] propose an approach to address data uncertainty (e.g., both the cost coefficients and the data in the constraints) for network flow problems that allows control of the degree of conservatism of the solution. In [10], the authors investigate uncertainty in the network structure (e.g, nodes and arcs) and assume that the network parameters (e.g., capacities) are known and deterministic. In particular, they study the robust and adaptive versions of the maximum flow problem in networks with node and arc failures. In general, the approaches have not considered impact of degeneration of node's capacity on maximum flow in network.

For literature reviews of optimisation problems related to gas networks, we refer to [11] and [12]. Zheng et al. [11] focus on three specific aspects, production, transportation and marketing, and consider six general problems: production scheduling, maximal recovery, network design, fuel cost minimisation, and regulated and deregulated market problems. Their survey discusses mathematical formulations and existing optimisation methods. Rios-Mercado and Borraz-Sanchez [12] present the relevant research works in the natural gas transport industry, studying short-term storage, pipeline resistance and gas quality satisfaction, and fuel cost minimisation. For the theoretical foundations and the applications of long-term basis storage, readers can refer to [13], [14], [15] and [16]. Studies on pipeline resistance and gas quality satisfaction can be found in [17], [18], [19] and [20]. Fuel cost minimisation is discussed in [21], [22], [23] and [24]. Although these surveys address applications of optimisation theory to the gas transmission and storage to satisfy contractual demands, there is a limited literature on the uncertain maximum flow problem in gas transmission network. Koch et al. [25] propose many mathematical programming models and algorithms to evaluate the gas network capability, but their models and algorithms can only solve deterministic problems. Recently, Praks and Kopustinskas [26, 27], Praks et al. [28] have developed models for determining the maximum network capability under impact of uncertainty. Praks and Kopustinskas [26] build a reliability model based using Monte Carlo methods to test various "what-if" scenarios. Their methods can be used not only for evaluating the current situation of security of supply, but also for testing effects of new network components (e.g., new pipelines) in various development strategies of the gas transmission network. Praks and Kopustinskas [27] and Praks et al. [28] develop a probabilistic gas network simulator (ProGasNet) software tool to estimate supply reliability, effect of time-dependent storage discharge, quantitative effects of new infrastructure, security of supply under different disruption scenarios. The tool is useful to compare and evaluate different supply options, new network development plans and analyse potential crisis situations. However, none of this work have not considered gas network capability under impact of compressor station uncertainty. Praks et al. [29] develop a Monte Carlo simulation-based approach to analyse disruptions of components (e.g., pipelines, terminals and compressor stations) in the European gas transmission network. They construct a vulnerability identification algorithm for determining a combination of component failures leading to the most significant security of supply disruptions. In the simulation, they do not consider the operational configuration of compressor units in stations (i.e., serial, parallel, or both). In addition, the Monte Carlo simulation-based approach is time-consuming as the number of components in the gas transmission network becomes significant.

Other works relevant to uncertainty in the gas network include [30], [31] and [32]. Carvalho et al. [30] introduce a model to deal with network congestion on various geographical scales. They propose a resilient response strategy to energy shortages and evaluate its effectiveness in a variety of scenarios. As a result, with the fair distribution strategy Europe's gas supply network can be robust even to major supply disruptions. Olanrewaju et al. [31] build a linear programming model to investigate the impact of the Ukraine transit capacity's loss on gas supply from Russia to Europe. The model is tested in a low-demand case and a high-demand case arising the winter of 2014/2015. The results show that gas sources from inter-connectors, storages and liquefied natural gas import terminals compensate for the supply shortfall. To mitigate the effect of supply shortage, the authors also consider increasing the capacities of selected pipelines within the Europe against enhancing the maximum storage withdrawal rates in southeast Europe. The comparison concludes that the high storage withdrawal rates can give lower demand curtailment than extending the inter-connector capacity in both scenarios. Wollega [32] propose a heuristic simulation and optimisation algorithm for large scale natural gas storage valuation under uncertainty.

In summary, some research has been devoted to various perspectives in the gas transmission network under uncertainty. However, the evaluation of the capability of gas transmission network to withstand the impact of compressor failures has not much received attention, especially considering the operational configuration of compressor units in stations (i.e., serial, parallel, or both). National Grid operates a complex and large-scale gas transmission network in the UK that includes pipelines, compressor stations, regulators, valves and other components. They have experienced challenges in evaluating network capability to withstand the impact of compressor failures. To address this issue, a network reduction technique is applied to reduce the original network by aggregating sets of demand nodes among compressor stations into demand zones. A mathematical programming model is built on this reduced network to find maximum network capability. The objective function is maximisation of gas flows in the network such that all constraints are satisfied. In the case of little data on compressor failure, we apply the uncertain theory of Ding [6] with an extension of uncertain capacity on nodes for solving the problem if compressor failures are independent, and develop a robust optimisation model to solve it when compressor failures are dependent. When there are data on compressor failure, we use Monte Carlo simulation to obtain the expected network capability. Computational experiments have carried out on a case study using actual data from National Grid to demonstrate the efficiency and effectiveness of our models. In addition, we provide a comprehensive analysis to find the most critical compressor stations for maintenance policies at National Grid.

The remaining of this paper is organised as follows. Section 2 describes the details of the UK gas transmission network, and the transformation of the original network into an associated reduced network. Section 3 presents a mathematical programming model for evaluating the gas transmission network capability under impact of compressor failures. The solution techniques for this problem, such as uncertain theory, robust optimisation and Monte Carlo simulation, are presented in the section as well. The case study at National Grid and the corresponding computational results of the proposed model and solution techniques are shown in Section 4. Finally, conclusions and future work are provided in Section 5.

2 The UK Gas Transmission Network

National Grid runs a complex UK gas transmission network that consists of about 7,000 km pipes, 24 compressor stations, each of which comprises several compressor units in serial and/or parallel

Table 1: List of compressor stations.

Label	Compressor	Label	Compressor	Label	Compressor
ABE	Aberdeen	CHU	Churchover	LOC	Lockerley
ALR	Alrewas	DIS	Diss	LON	Longtown
AVO	Avonbridge	FEL	Felindre	MOF	Moffat
AYL	Aylesbury	FER	St Fergus	PET	Peterborough
BIS	Bishop Auckland	HAT	Hatton	WAR	Warrington
CAM	Cambridge	HUN	Huntingdon	WIS	Wisbech
CAR/NEK	Carnforth/Nether Kellet	KIL	Kings Lynh	WOO	Wooler
CHE	Chelmsford	KIR	Kirriemuir	WOR	Wormington

operation, 6 major terminals, 8 storage sites, more than 200 exit points, and other components (e.g., regulators and valves). Figure 1 shows the pipeline network to transmit gas from terminals to exit points. The large-scale network poses many challenges to National Grid in meeting the demands of its customers, and requires much effort in modelling and optimisation. To reduce the modelling and computational effort, we apply a network reduction technique introduced by [33]. Sets of supply and/or demand nodes bounded by compressor stations are aggregated into zones. In this case, we obtain 36 zones. Based on historical data, we compute net flows for each zone, subtracting total supply and demand. We define a *supply* zone to be when the net flow is greater than 70 million cubic meter, a *demand* zone if the net flow is less than -70 million cubic meter, and a *transit* zone for the remaining cases. Table 1 shows the list of compressor stations and their label. The list of aggregated zones and the information of zonal type (i.e., supply, demand or transit) are provided in Table 2.

Figure 2 shows the associated reduced network for the UK gas transmission network. In the figure, green, red and blue nodes represent supply, demand and transit zones respectively. The compressor stations are represented by orange nodes. Our reduced network includes 4 supply zones (denoted by green nodes 1-4), 8 demand zones (denoted by red nodes 5-12), 24 transit zones (denoted by blue nodes 13-36), and 24 compressor stations (denoted by orange nodes with of compressor labels). The possible directions of gas flows among zones in the reduced network are shown.

In the gas transmission network, compressor stations manipulate pressure and gas flows from supply zones through transit zones to satisfy the customer's demand in demand zones. The network capability depends on the capacity of compressor stations. Therefore, if serious disruption at a compressor station occurs, the network capability is reduced, leading to unsatisfied customer demand. In the next section, we introduce a model to evaluate the impact of compressor station disruption to the network capability. The model can determine the most critical compressor stations to produce efficient maintenance policies for mitigation of the network capability loss.

Figure 1: The UK gas transmission network (National Grid source).

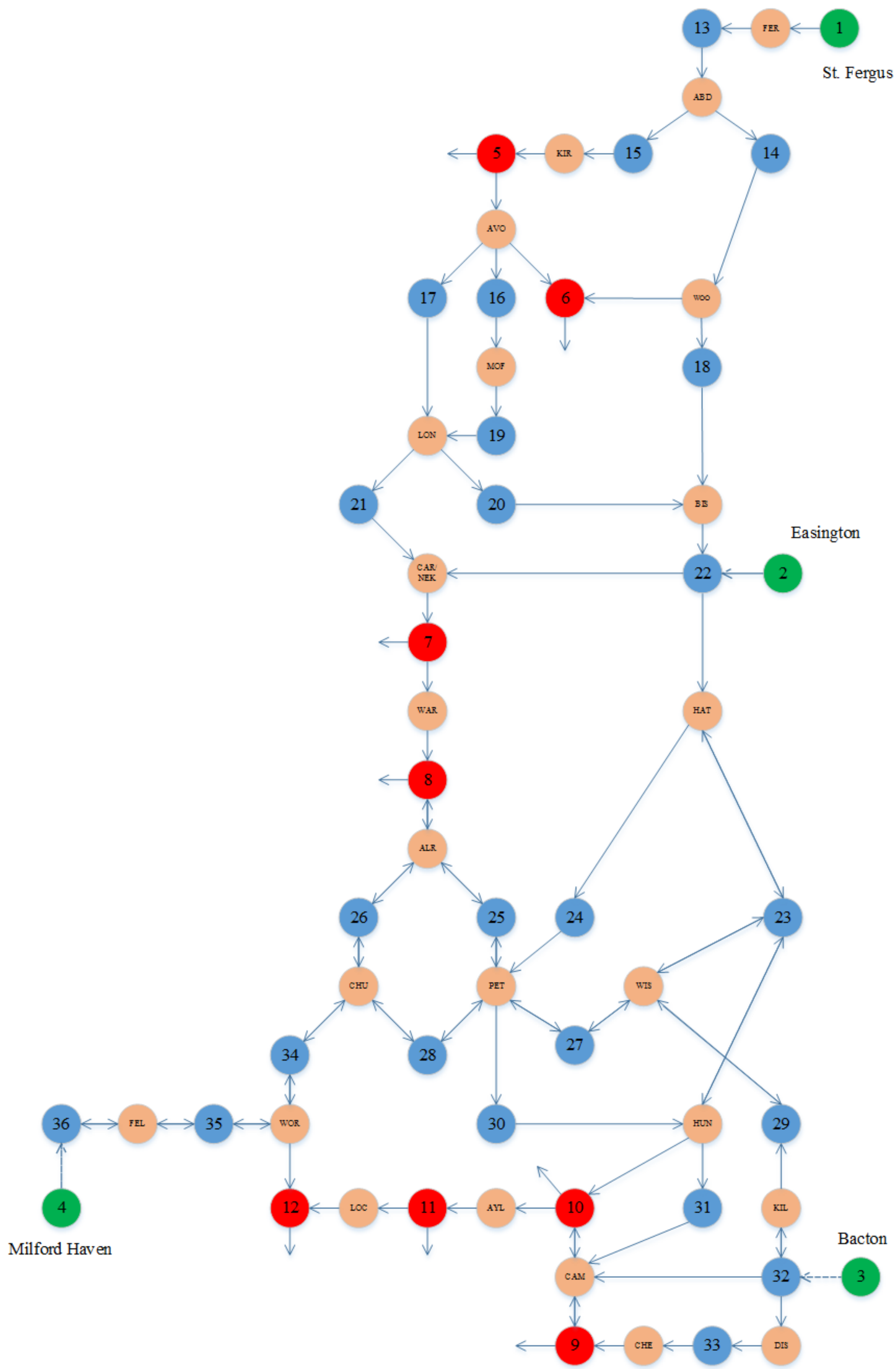

Figure 2: An associated reduced network for the UK gas transmission network.

Table 2: List of aggregated zones.

Zone	Compressors	Type	Zone	Compressors	Type
1	(•, FER)	Supply	19	(MOF, LON)	Transit
2	(BIS, CAR/NEK, HAT)	Supply	20	(LON, BIS)	Transit
3	(KIL, CAM, DIS)	Supply	21	(LON, CAR/NEK)	Transit
4	(•,FEL)	Supply	22	(BIS, CAR/NEK, HAT)	Transit
5	(KIR, AVO)	Demand	23	(HAT, WIS, HUN)	Transit
6	(AVO, WOO)	Demand	24	(HAT, PET)	Transit
7	(CAR/NEK, WAR)	Demand	25	(ALR, PET)	Transit
8	(WAR, ALR)	Demand	26	(ALR, CHU)	Transit
9	(CHE, CAM)	Demand	27	(PET, WIS)	Transit
10	(HUN, AYL, CAM)	Demand	28	(CHU, PET)	Transit
11	(AYL, LOC)	Demand	29	(WIS, KIL)	Transit
12	(LOC, WOR)	Demand	30	(PET, HUN)	Transit
13	(FER, ABD)	Transit	31	(HUN, CAM)	Transit
14	(ABD, WOO)	Transit	32	(KIL, CAM, DIS)	Transit
15	(ABD, KIR)	Transit	33	(DIS, CHE)	Transit
16	(AVO, MOF)	Transit	34	(CHU, WOR)	Transit
17	(AVO, LON)	Transit	35	(WOR, FEL)	Transit
18	(WOO, BIS)	Transit	36	(•,FEL)	Transit

3 Capability Evaluation of Gas Network under Disruption

To measure and evaluate capability of gas transmission network under disruption of compressor stations, we modify maximum flow problem by some additional constraints. Since the UK gas transmission network is complex and large-scale, we implement the uncertain maximum flow algorithm on the reduced network.

We introduce notations to formulate the maximum flow problem under disruption as follows.

Sets and parameters:
S = set of supply nodes
D = set of demand nodes
T = set of transit nodes
C = set of compressor station nodes
V = set of all nodes ($V = S \cup D \cup T \cup C$)
A = set of all arcs
A_b = set of bi-directional arcs (i.e., $A_b \subseteq A$)
V_j = set of nodes whose arc enters into compressor $j \in C$
s, d = dummy source and destination nodes, respectively
$\widetilde{b}_j$ = uncertain capacity of compressor station $j \in C$
Decision variables:
x_{ds} = flow rate from dummy destination to dummy source
x_{ij} = flow rate in arc $(i, j) \in A$

y_{ij} = binary decision variables for controlling flow direction

Figure 3 presents a graph representation of the maximum flow problem under disruption of compressor stations when we add dummy source and destination nodes. The dummy source node s is connected into the supply nodes, while the demand nodes are connected into the dummy destination node d. A flow is connected from node d to node s (denoted by x_{ds}). A mathematical programming model of the maximum flow problem under disruption of compressor stations (e.g., uncertain capacity $\widetilde{b}_j$ of compressor station) is then:

[UMFP]

$$\max x_{ds} \tag{1}$$

$$\text{s.t.} \sum_{\ell:(j,\ell)\in A} x_{j\ell} - \sum_{i:(i,j)\in A} x_{ij} = 0 \quad \forall j \in V, \tag{2}$$

$$\sum_{i\in S} x_{si} = x_{ds}, \tag{3}$$

$$\sum_{i\in D} x_{id} = x_{ds}, \tag{4}$$

$$y_{ij} + y_{ji} \leq 1 \quad \forall (i,j) \in A_b, \tag{5}$$

$$x_{ij} \leq \mathrm{M} y_{ij} \quad \forall (i,j) \in A_b, \tag{6}$$

$$\sum_{i\in V_j} x_{ij} \leq \widetilde{b}_j \quad \forall j \in C, \tag{7}$$

$$x_{ij} \geq 0, y_{ij} \in \{0,1\} \quad \forall (i,j) \in A, \tag{8}$$

where M is the maximum capacity of all compressor stations.

The objective (1) is to maximise the capability of gas transmission network. Constraints (2) represent the flow conservation law at nodes. Constraints (3) and (4) describe the flow conservation law at dummy source and destination nodes, respectively. Constraints (5)-(6) allow at most one flow to exist between supply, demand, transit and compressor station nodes at a time. In constraints (6), if $y_{ij} = 0$, $x_{ij} = 0$; otherwise, the constraints $x_{ij} \leq \mathrm{M}$ are always satisfied. Constraints (7) assure that flows through compressor stations cannot exceed the capacity of compressor stations in every scenario. Constraints (8) define non-negative variables of flow rate and binary variables of controlling flow direction.

This is not a traditional maximum flow problem due to constraints (7), uncertain capacity of compressor stations. These constraints generate huge numbers of scenarios for the problem, leading to

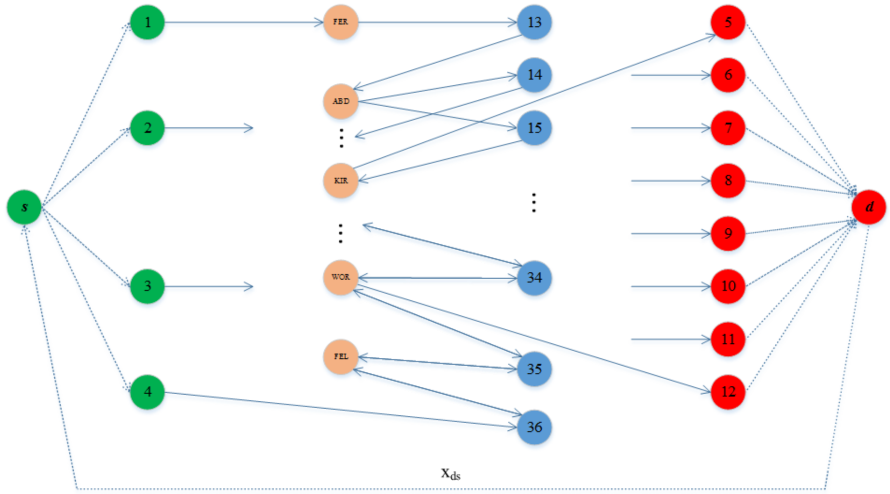

Figure 3: A graph representation of the uncertain maximum flow problem.

an NP-hard problem. Therefore, we develop specific solution techniques for specific scenarios. If the data on a compressor unit's failure is unknown, we apply uncertainty theory and robust optimisation for independent and dependent failures of compressor stations, respectively. Otherwise, we implement Monte Carlo simulation to determine the expected capability of the gas transmission network. The solution techniques are discussed in next subsections.

3.1 Individual Chance Constraint Programming

Assume that failures of compressor stations are independent, we can determine the capacity of each compressor station based on the uncertainty theory of Ding [6]. Let $z_j = \sum_{i \in V_j} x_{ij}$, constraints (7) become $z_j \leq \widetilde{b}_j \ \forall j \in C$. In uncertainty theory, $\mathrm{M}\{\widetilde{b}_j \leq z_j\} = \mathrm{P}\{\widetilde{b}_j \leq z_j\} \leq \alpha \ \forall j \in C$ can be derived into $z_j \leq \Phi_j^{-1}(\alpha) \ \forall j \in C$ where $\Phi_j(\alpha)$ is a function with belief degree $\alpha \in [0, 1]$. The function might, for instance, be linear, zigzag, normal, or log-normal distribution over random uncertainty variable ξ. Figure 4 shows an illustration of linear belief degree function. Then, constraints (7) can be written as

$$\sum_{i \in V_j} x_{ij} \leq \Phi_j^{-1}(\alpha) \quad \forall j \in C. \tag{9}$$

These are linear constraints. Hence, we can solve the problem by mixed-integer linear programming (MILP) solvers. Given that a belief degree α, we can determine the capacity of corresponding compressor station by $\Phi_j^{-1}(\alpha)$. For other belief degree functions, readers should consult [6].

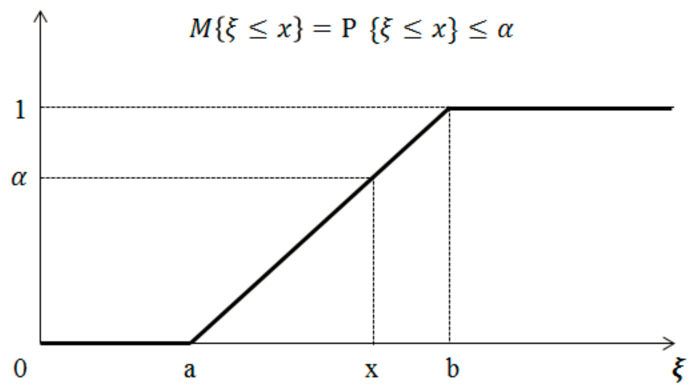

Figure 4: A belief degree function.

3.2 Joint Chance Constraint Programming

Assume that the failures of compressor stations are dependent, we cannot apply the uncertainty theory of Ding [6]. For this case, we develop a robust optimisation technique to handle the constraints (7). Given that a confidence level $\alpha \in [0,1]$, the minimum probability of occurring the event that $z_j \leq \tilde{b}_j \forall j \in C$, we have a joint chance constraint programming as follows:

$$P\{z_j \leq \tilde{b}_j, \forall j \in C\} \geq \alpha;$$

corresponding to

$$\text{Inf}_{P \in \mathbf{P}} P\{z_j \leq \tilde{b}_j, \forall j \in C\} \geq \alpha,$$

where $\mathbf{P}$ is the set of all probability distributions for random variable $\tilde{b}_j$ with known mean and variance (μ_j, σ_j^2).

Bonferroni's inequality leads to

$$\text{Sup}_{P \in \mathbf{P}} P\{\cup_{j \in C} z_j > \tilde{b}_j\} \leq 1 - \alpha.$$

In addition, we have

$$P\{\cup_{j \in C} z_j > \tilde{b}_j\} \leq \sum_{j \in C} P\{z_j > \tilde{b}_j\} \ \forall P \in \mathbf{P}.$$

Set

$$\sum_{j \in C} P\{z_j > \tilde{b}_j\} \leq 1 - \alpha.$$

Let $1 - \alpha = \epsilon$ (risk level), we have

$$\sum_{j \in C} P\{z_j > \tilde{b}_j\} \leq \epsilon.$$

Let $\epsilon = \sum_{j \in C} \epsilon_j$, we get

$$P\{z_j > \tilde{b}_j\} \leq \epsilon_j \ \forall j \in C$$

$$\Longleftrightarrow P\{z_j - \tilde{b}_j > 0\} \leq \epsilon_j \ \forall j \in C$$

$$\Longleftrightarrow P\{z_j \leq \tilde{b}_j\} \geq 1 - \epsilon_j \ \forall j \in C$$

$$\Longleftrightarrow \mathrm{Inf}_{P \in \mathbf{P}} P\{z_j \leq \tilde{b}_j\} \geq 1 - \epsilon_j \ \forall j \in C$$

where

$$\sum_{j \in C} \epsilon_j \leq 1 - \alpha.$$

We can set $\epsilon_j = \frac{1-\alpha}{|C|}$, then the joint chance constraint can be derived into

$$z_j \leq \mu_j + \sigma_j \sqrt{\frac{|C|}{1-\alpha} - 1} \ \forall j \in C.$$

Then, constraints (7) can be written by

$$\sum_{i \in V_j} x_{ij} \leq \mu_j + \sigma_j \sqrt{\frac{|C|}{1-\alpha} - 1} \ \forall j \in C. \tag{10}$$

These are linear constraints. Once gain, we can solve the maximum flow problem under disruption of compressor failures by MILP solvers.

3.3 Monte Carlo Simulation

Compressor stations comprise a set of serial and/or parallel compressor units. Their capacity is thus affected by the failures of compressor units. In the case that we know failure data for each compressor unit, we can apply Monte Carlo simulation to evaluate the gas transmission network capability under impact of compressor failures instead of using the approximations of uncertainty theory and robust optimisation.

Assume that the disruption event on compressor unit i follows Binomial distribution with failure probability q_i. Since the disruptions may occur simultaneously at many compressor units of compressor stations, we generate a number of scenarios for the failures of compressor units based on a Binomial distribution and their failure probabilities. Based on the operational configuration of compressor stations, we can determine their capacity under the scenarios. The model is then applied iteratively for solving all the scenarios to find the corresponding network capabilities. From the results, we can determine the expected network capability.

4 Computational Experiments

In the section, we first describe the case study at National Grid used for evaluating the different solution approaches before reporting computational results to compare the quality of the solutions.

4.1 National Grid Case Study

As we described, we reduced National Grid's complex, large-scale gas transmission network to an aggregated network of 4 supply zones, 8 demand zones, 24 transit zones and 24 compressor stations: see Figure 2.

Table 4 shows the data of supplies and compressor stations' capacity levels. In this table, there are two capacity levels (a, b) for compressor stations which use linear belief degree functions, and three capacity levels (a, b, c) for compressor station which use zigzag belief degree functions. These capacity levels are based on the operational configuration of compressor units in the stations (e.g., serial, parallel, or both) and the capacity of compressor units. Since there is not enough data to extract normal (or log-normal) distribution information for compressor station's capacity, we could not test solutions using the assumption of normal (or log-normal) distribution in uncertainty theory. Gas volumes are given in million cubic meter - mcm.

Table 5 describes mean and variance of capacity for each compressor station. These data are used to test the robust optimisation approach for solving the case study. In particular, they are input into constraints (10) to approximate the capacity of compressor stations.

For Monte Carlo simulation, we compute the failure probability for compressor units based on 2009-2013 data. Let

H_i = event that compressor unit i starts successful

$\bar{H}_i$ = event that compressor unit i fails to start

K_i = event that compressor unit i starts successful, and does not fail during process

$\bar{K}_i$ = event that compressor unit i starts successful, but fails at a moment during process.

The failure probability of compressor unit i is defined as follows:

$$P_i = P(\bar{H}_i) + P(\bar{K}_i).$$

Table 4: Data of supplies and compressor station's capacity levels.

Supply zone	Capacity (mcm)	Compressor	Capacity (mcm)			Compressor	Capacity (mcm)		
			a	b	c		a	b	c
1	154.22	FER	62	73	135	CHU	0	50	60
2	170.98	ABD	75	150	-	PET	0	73	140
3	164.62	KIR	0	90	109.5	WIS	0	31	34
4	87.69	AVO	35	70	140	HUN	0	55	105
		WOO	0	60	-	KIL	42	56	84
		MOF	0	62	-	DIS	0	44.5	-
		LON	1.1	76.32	-	CHE	0	43	-
		CAR/NEK	62	70	120	CAM	0	48	-
		BIS	0	100	-	AYL	0	60	-
		WAR	0	80	-	LOC	0	18	30
		ALR	30	50	60	WOR	40	50	80
		HAT	0	65	130	FEL	39	78	100

Table 5: Data of mean and variance of compressor station's capacity.

Compressor	Capacity (mcm)		Compressor	Capacity (mcm)	
	μ	σ^2		μ	σ^2
FER	80.00	1.00	CHU	36.67	1.00
ABD	125.00	1.00	PET	71.00	1.00
KIR	66.50	1.00	WIS	21.67	1.00
AVO	81.67	1.00	HUN	53.33	1.00
WOO	40.00	1.00	KIL	60.67	1.00
MOF	41.33	1.00	DIS	29.67	1.00
LON	51.25	1.00	CHE	28.67	1.00
CAR/NEK	84.00	1.00	CAM	32.00	1.00
BIS	66.67	1.00	AYL	40.00	1.00
WAR	53.33	1.00	LOC	16.00	1.00
ALR	46.67	1.00	WOR	56.67	1.00
HAT	65.00	1.00	FEL	72.33	1.00

Table 6 describes the derived failure probability for each compressor unit in compressor station. Based on the failure probability of compressor units and the operational configuration of compressor units in stations (i.e., serial, parallel, or both), we can compute the capacity of compressor stations under a certain scenario. Monte Carlo simulation is then applied to find the expected network capability.

4.2 Computational Results

Solution algorithms based on uncertainty theory, robust optimisation and Monte Carlo simulation were implemented in Visual Studio C++, and the mathematical programming models were solved using IBM ILOG CPLEX version 12.5 callable library. All the computational experiments were run on an Microsoft Windows 7 Enterprise PC with an Intel Core i7-3770 processor (3.40 GHz per chip) and 24 GB of RAM.

Table 6: Data of the failure probability for compressor unit in compressor station.

Compressor	Unit	Failure probability
ABD	A	23.03
	B	7.07
	C	21.68
ALR	A	33.57
	B	20.39
	C	16.62
AYL	A	28.75
	B	50.29
AVO	A	29.67
	B	70.00
	C	18.03
	D	27.78
BIS	A	10.75
	B	9.93
CAM	A	4.76
	B	14.29
	C	32.65
CAR	A	38.91
	B	30.52
	C	36.69
CHE	A	12.50
	B	7.14
CHU	A	25.00
	B	23.40
DIS	A	32.21
	B	30.95
	C	27.72
FEL	A	1.00
	B	1.00
	C	1.00
HAT	A	16.17
	B	17.32
	C	15.38
	D	1.00
HUN	A	28.64
	B	16.03
	C	22.68

Compressor	Unit	Failure probability
KIL	A	39.09
	B	16.67
	C	19.29
	D	11.10
KIR	A	11.68
	B	12.12
	C	17.11
	D	12.62
	E	1.00
LOC	A	20.13
	B	72.40
MOF	A	45.83
	B	12.45
NEK	A	30.13
	B	30.65
PET	A	5.80
	B	6.67
	C	2.78
FER	1A	7.22
	1B	14.78
	1C	14.62
	1D	12.00
	2A	19.75
	2B	7.89
	2C	23.71
	3A	1.00
	3B	1.00
WAR	A	12.89
	B	1.00
WIS	A	5.63
	B	20.96
WOO	A	18.15
	B	33.06
WOR	A	25.81
	B	4.35
	C	11.07
LON	A	1.00

Table 7: Evaluation of the UK gas transmission network capability by uncertainty theory.

Belief level (%)	Network capability (mcm)	Time (s)
100	510.98	0.21
95	496.93	0.10
90	482.88	0.10
85	468.83	0.11
80	454.78	0.14
75	440.73	0.16
70	426.68	0.15

In the first computational experiment, we used uncertainty theory (i.e., individual chance constraint programming), solving for a range of belief levels $\alpha = 0.7, 0.75, .., 1.00$. The results are shown in Table 7. This suggests that if managers' uncertainty in the availability of compressors is, e.g., at least 90% they may reasonably assume that network capability will be better than 482.88mcm. Comparing with the peak national demand in 2005-2015 historical data (i.e., 465.50 mcm), it can be seen that National Grid can satisfy all the cases of national demand with belief level $\alpha \geq 0.85$ for each compressor station's capacity. If belief level $\alpha < 0.85$, there exist some cases of national demand that National Grid cannot meet. In practice, National Grid satisfied all cases of national demand from 2005-2015 with given the gas operator's belief level on compressor station's capacity $\alpha = 0.90$. While not offering a complete validation of our model, this suggests that its results are sensible and in line with experience.

To identify the most critical compressor station, we conduct a sensitivity analysis in which each compressor station is assumed to fail completely (i.e., its capacity is set up zero). We then solve the corresponding problems with various belief levels to determine the network capacity (see Figure 5). It is apparent that compressor stations St Fergus (FER) and Aberdeen (ABD) play critical roles in the UK gas transmission network, since their failure makes the most significant impact on the network capability. These results suggest that an efficient maintenance policy would mitigate the loss of network capability by prioritising St Fergus and Aberdeen to keep maximum capacity at these, so reducing the maximum loss of network capability in the case that one compressor station failure.

In addition, this evaluation supports National Grid in forecasting national demand scenario that risk not being satisfied. For example, any national demand higher than 510.98 mcm (the maximum capability of our network) would certainly be of concern. Furthermore, adopting a belief level (or operational probability) of compressor stations of $\alpha \leq 0.90$, our concern starts from national demand forecasts higher than 482.88 mcm.

We now turn to solution by robust optimisation (i.e., joint chance constraint programming). The approach is applied if compressor failures are dependent, and we only obtain information of mean and variance of compressor station's capacity. Table 8 presents computational results with a range of various confidence levels $\alpha = 0.7, 0.75, .., 0.99$. We did not solve the case study with $\alpha = 1.00$ to

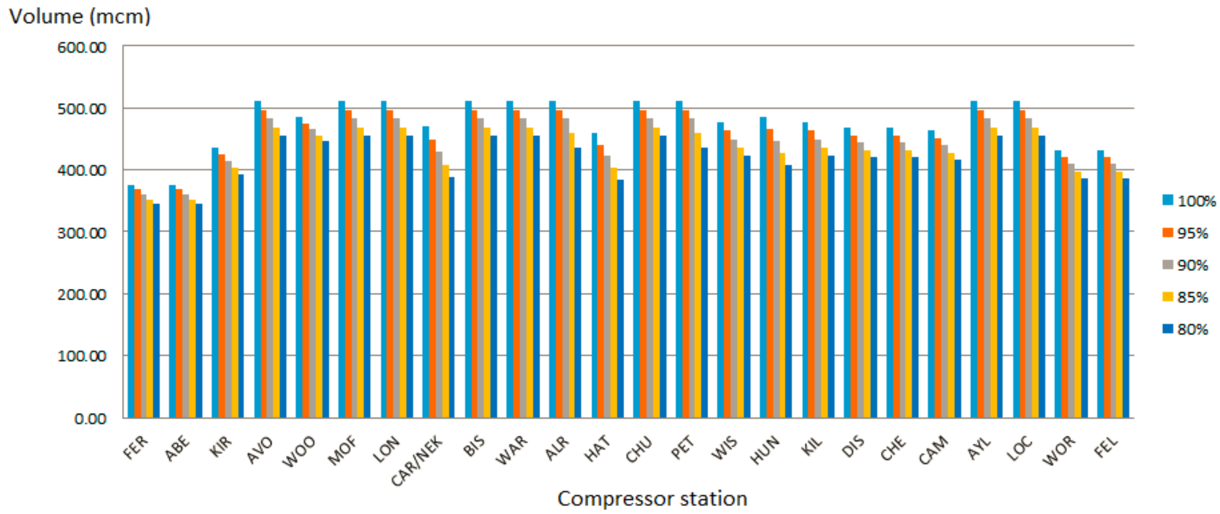

Figure 5: Sensitivity analysis for one compressor station's complete failure with $\alpha = [0.80, 1.00]$.

Table 8: Evaluation of the UK gas transmission network capability by robust optimisation.

Confidence level (%)	Network capability (mcm)	Time (s)
99	552.27	0.11
95	499.42	0.12
90	467.29	0.13
85	453.04	0.15
80	444.37	0.12
75	436.24	0.12
70	430.23	0.12

avoid overflow issues with $(1-\alpha)$ in the denominator of constraints (10). There are, not surprisingly, differences with results obtained using uncertainty theory; dependencies increase the probability of simultaneous failures reducing capacity. Moreover the methods use different means of approximating and bounding the uncertainties. In particular, the results from robust optimisation suggest that National Grid may not satisfy some cases of national demand observed in 2005-2015 (e.g., higher 453.04 mcm) at $\alpha = 0.85$. However, at $\alpha = 0.90$ they could. The average computation time using uncertain theory is a little slower than that using robust optimisation (0.14 vs. 0.12 seconds).

Finally, we apply Monte Carlo simulation (10,000 runs), taking as known the failure probability of compressor units. Solving the case study under various scenarios of compressor failures provides the expected network capability. Figure 6 shows the minimum (301.00 mcm), the expected (490.03 mcm), the maximum (510.98 mcm) and the standard deviation (29.37 mcm) values of the UK gas transmission network capability. Confidence levels of 5% and 95% on the gas network capability are 431.00 mcm and 510.98 mcm respectively. Once again, the results suggest that National Grid can satisfy all the cases of national demand from 2005-2015 (peak demand 465.50 mcm). The results obtained by the simulation are closer to those of uncertainty theory than robust optimisation.

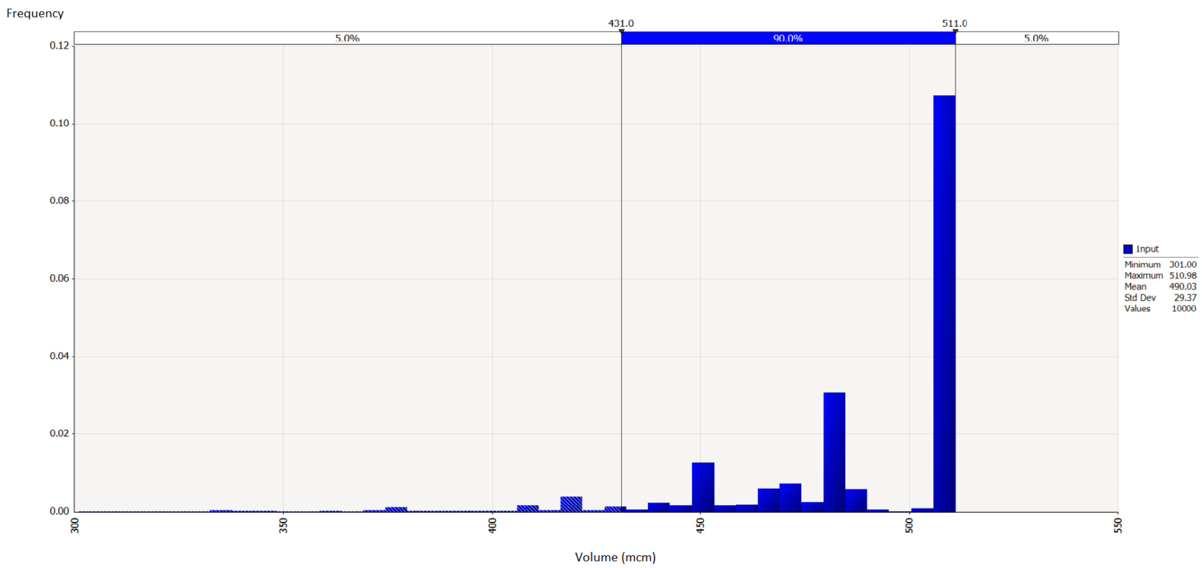

Figure 6: Evaluation of the UK gas transmission network capability by Monte Carlo simulation.

5 Conclusions and Future Work

In summary, we have developed three approaches to determine the capability of the UK gas transmission network. The results, carried out on the case study, demonstrate these methods are computationally practicable and give sensible results in line with current experiences. The methods can inform National Grids planning for forecast national demands in the future and also to build an efficient maintenance policy. We believe that these methods can be extended to solve similar uncertain network capability problems in other fields. Possible future work would be to consider other uncertainties, such as pipeline failure or supply loss.

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