# New Effective Power Terms And Right-Angled Triangle (RAT) Power Theory

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***Abstract*—This article presents two new electrical power terms called effective active ($P_{ef}$) power term and effective reactive ($Q_{ef}$) power term. These two terms, can approve the orthogonality relationship between fundamental and distorted components.**

**At the same time, these terms are necessary to complete the form of the right-angled power triangle which is compulsory condition to calculate the total apparent ($S_t$) in the non-sinusoidal systems.**

**This paper also, offers a new definition for the total apparent power $S_t$ or can be called effective apparent power ($S_{ef}$) in non-sinusoidal situation in order to avoid the misconception of the old definitions. Moreover, it shows a new power diagram represents all power components in one diagram consists of six right-angled triangles called the right-angled triangle (RAT) diagram.**

***Keywords*** - Harmonic distortion power, Power components definition, Power theory.

## I. Introduction

Power electronics researchers are conversant with the description of apparent (S), active (P) and reactive (Q) power and power factor (PF) in a single phase sinusoidal circuits and in balanced three phase circuits. Unfortunately, the generalization of this description into other types of circuits (e.g. nonlinear loads or non-sinusoidal systems) can lead to a misinterpretation of the understanding the physical meaning of these components.

A long dispute around the meaning of these power components in the non-sinusoidal conditions has been had along the electrical engineers since at least 1916 [1] when Steinmetz, discussed the problem of power components in sinusoidal and non-sinusoidal conditions. With the inescapable wide spread of power electronics devices, in the practical life of power engineering, and the problems which have been produced in a result of the harmonics, a new debate has been revived in the last 40 years [2] because of the efforts have been made in order to find new definitions for the power components with the increasing of frequency and new terms have been invented (e.g. nonactive power (N)).

According to [3], apparent power (S) is not a vector, but it is the product of the magnitude of two vectors : S $=|i||v|$ = I V .

So, when $i$ and $v$ are sinusoidal waveforms and the load is linear, it is possible to directly associate the real and reactive components of the current with those of the apparent power, namely real (active) power (P) and reactive power (Q). However, if $i$ or $v$ are non-sinusoidal or/and the load is nonlinear, a new term will present called the distortion power ($P_h$) or (D) [3]. This distortion power consists of active (watt) and nonactive (VAr) parts.

Some researchers have dedicated their entire academic lives to this purpose, even a number of schools of diverse elucidations have been launched for decades. The supporters of these schools abide to them frequently with a type of religious enthusiasm more than scientific evidences [4].

Generally, the author of every new definition shows the contradictions and limitations of other definitions, and claim that his proposal is finally right and will solve all the troubles related to the power components definitions in non-sinusoidal condition.

Unfortunately, there is still no consensus in the power community between the engineers on an interpretation of power phenomena of circuits with non-sinusoidal voltages and currents, in spite of this controversy has been continued for decades.

This article is a new attempt to describe the relationship between power components in non-sinusoidal system depending on the idea of orthogonality law and applying the geometric sum on all power categories, also suggests two new power terms called ($P_{ef}$) and ($Q_{ef}$).

This article has multiple contributions which can be summarized as follow:

- It proves the validity of right-angled triangle (RAT) power theory for all power components (total, fundamental, and distortion) and all power categories (S, P & Q) for non-sinusoidal systems.
- It refutes Budeanu's power theory and its three dimensional power diagram.
- Proves the ability to apply the orthogonality law on all power components in non-sinusoidal.
- suggests a new simple two dimensional power diagram can include all the power categories and components for non-sinusoidal system.
- presents two new power terms called ($P_{ef}$) and ($Q_{ef}$).
- Submit a new comprehensive definition for apparent power in non-sinusoidal system.

Next section emerges the justification points which prove the (RAT) theory. The third section describes the orthogonality law and new power terms in non-sinusoidal system. The fourth section presents a new definition of the apparent power (S) in non-sinusoidal condition. The fifth section explains the new (RAT) power diagram. The sixth section shows the mathematical proof of the RAT's equations. The seventh section provides examples to examine the new power diagram and theory.

## II. Justification of the Orthogonality Relationship Between Power Components

In the following, some important points which are either support the idea of geometric sum for all power components or deny the conventional (arithmetic sum) power theory, and these points are sufficient to justify the orthogonality law and the new power terms:

1) Budeanu's theory for non-sinusoidal condition [5] which is defining the reactive power $Q_t$ as the arithmetic sum of $Q_h$

$$Q_t = \sum_{h=1} V_h I_h \sin\theta_h$$

was strongly rejected by lots of researchers throughout 90 years of researches as mentioned in the following:

**a**. Fryze in 1931 [6], objected Budeanu's theory and he described the necessity to the voltage and current harmonic decomposition before the reactive power could be calculated and relied on the time domain approach.

**b**. Shepherd and Zakikhani in 1972 [2], assured that the Budeanu reactive power ($Q_B$) is not a real physical quantity and they suggested another quantity to be chosen as a reactive power.

**c**. Czarnecki in 1987 [7], objected strongly Budeanu's theory and proved that this theory does not possess the attributes which could be related to the power phenomena in the circuit and does not express any distinct energy phenomenon.
Moreover, Budeanu's values do not provide any information necessary for the design of compensating circuits. Also, the value of distortion power (D) is not related to the waveform distortion.
Also (D) is not equal to zero when the current and voltage waveforms are identical but shifted in time. However, D can be equal to zero even when the voltage and current waveforms are not identical.
In additioin, the author in 1997 [8], has concluded that, the distortion power (D) in single phase nonlinear circuits has nothing in common with waveform distortion, similarly as Budeanu's reactive power equation has nothing in common with energy oscillation. Thus, Budeanu's power theory misinterprets power phenomena in electrical circuits.

**d**. Budeanu's definitions has been refused by Slonim in 1988 [9]. The author has confirmed that $Q_B$ is just an arbitrary mathematical implication and it has no physical interpretation.

**e**. Filipski in 1993 [10], approved that Budeanu's definitions of reactive and deformation powers, do not reflect properly the energetic relation in non-sinusoidal conditions. Also, the author in [11] has proved with illustrated examples that Budeanu's definition of reactive power in non-sinusoidal system has many shortcomings.
Filipski proved that the calculated reactive power according to Budeanu's definition, may be zero even though there is a reciprocating energy flow between the source and the load at different frequencies.
Also, he showed that one can still completely compensate a reactive load to unity power factor even when Budeanu's reactive power is zero.
Moreover, Reactive power as defined according to Budeanu (arithmetic sum of Q) has no physical significance in non-sinusoidal circuits.
The use of $Q_B$ in non-sinusoidal situations is misleading, because merely reducing Q to zero does not generally result in optimum power factor compensation. Hence, compensation of the reactive power as defined according to Budeanu alone may be useless for power factor improvement.

2) Reactive power definition proposed by Fryze [6] in 1931 is based on the division of the current into two terms as the active current ($I_a$) and the reactive current ($I_r$), considering that these terms are orthogonal, then:

$\frac{1}{T}\int_0^T i_a i_r dt = 0$ (as the inner product is zero).

Similarly, ($P_{avg}$) for different frequencies is zero due to orthogonality ($P_{13}, P_{15}$ or $P_{35}$....etc.), as:

$P_{35} = \frac{1}{T}\int_0^T V_3 I_5 dt = 0$

Consequently, harmonic currents and voltages of different frequencies are all orthogonal with each other and effective active power can be expressed as following:

$P_{ef}^2 = P_{11}^2 + P_{33}^2 + P_{55}^2 + P_{77}^2 +$.......etc.

$$\therefore P_{ef}^2 = P_{11}^2 + \sum_{h=3} P_{hh}^2$$

3) Depenbrock in [12] [13], has supported Fryze's model which says: ($I = I_a + I_b$), and claimed that the total current is consisting of fundamental and harmonic parts: ($i = i_1 + i_h$).

Therefore, when $I_{a1} = I_1\cos\theta_1$ , $I_{ah} = I_h\cos\theta_h$ :

$\therefore I_a = \sqrt{I_{a1}^2 + I_{ah}^2} = I\cos\theta$

$\because P = VI_a = V\sqrt{I_{a1}^2 + I_{ah}^2}$

$\therefore P_{ef}^2 = P_{a1}^2 + P_{ah}^2 = P_{fundamental}^2 + P_{harmonics}^2$

Notice that this allegation supports the idea of geometric sum of power components for the active power.

4) Since the nonlinear load is always polluted, then the active part of the distortion power (D) which consists of $P_{33}$, $P_{55}$ and $P_{77}$ are negative values some times (depending on the load nature). However, the total $P_{hh}$ should be positive because, an amount of energy is being dissipated. Accordingly, the total result will be positive always when the geometric sum is using instead of the arithmetic sum.

thus: $P_{hh} = \sqrt{P_{33}^2 + P_{55}^2 + P_{77}^2 + .....etc.}$

5) Slonim, concluded in 1988 [9] that there is no accepted, clear definition of reactive and distortion power.

Also, he confirmed that there is no physical interpretation for reactive, distortion and apparent power.
Accordingly, he claimed that:

$$S_t^2 = \sum_{k=0}^{\infty} P_k^2 + \sum_{k\neq n}^{\infty} P_{kn}^2 + \sum_{n=0}^{\infty} Q_k^2 + \sum_{k\neq n}^{\infty} Q_{kn}^2$$

$$\therefore S^2 = \sum_{n=1}^{\infty} P_n^2 + \sum_{n=1}^{\infty} Q_n^2$$

Obviously, this equation confirms the idea of geometric sum because, $(P_{\sum}^2)$ and $(Q_{\sum}^2)$ are the geometric sum of active and reactive power components, respectively.

6) Czarnecki in [4], has presented a new power theory. His method was meant to improve on the limitation of Fryze's model. He has a collective reactive power, Unlike Budeanu's reactive power:
$Q_r = VI_r = V\sqrt{\sum(B_h V_h)^2}$.
This allegation supports the idea of geometric sum to be used for the total reactive power calculation because:
$Q_t = \sqrt{\sum Q_n^2}$.
This type of summation has the attribute of getting a positive result always (bigger than zero) and that's mathematically and practically truthful to the actual oscillation of energy.

7) Each of the current harmonics $I_h$ can be decomposed into two orthogonal component ($I_h \cos\theta_h$ & $I_h \sin\theta_h$), since all current harmonics are mutually orthogonal. Thus, the square of the rms current (geometric sum) is:

$$\sum_{n=1} I_n^2 = I_1^2 + \sum_{h=2} I_h^2$$

$$\sum_{n=1} I_n^2 = \Big[I_1^2\cos^2\theta_1 + I_1^2\sin^2\theta_1\Big] + \Big[\sum_{h=2} I_h^2\cos^2\theta_h + \sum_{h=2} I_h^2\sin^2\theta_h\Big]$$

Therefore, if there is a reciprocating energy transmission between the source and the load then the term $\sum Q_n^2$ (but not $\sum Q_n$) is responsible for the source apparent power's increase [14].

8) According to Shepherd and Zand in [14], the equation:

$$Q = \sum_{h=1} V_h I_h \sin\theta_h$$

does not correctly define the reactive power in non-sinusoidal system. Because, in non-sinusoidal supply situation, the fluctuations of the stored capacitive and inductive energies are not synchronous such that the pulsating power to be delivered by the source does not correspond to the difference of both energy components.
Consequently, there is no justification for simply adding (arithmetic sum) the reactive powers corresponding to different frequencies as its done in Budeanu's reactive power concept[14].

9) In mathematical laws, quantities x(t) and y(t) are orthogonal if one of three cases has been applied, and one of these cases is when $x$ & $y$ are harmonics in different orders:
$x = x_r\sin(rw_1t - \alpha)$ , $\quad y = y_s\sin(sw_1t - \theta)$
(when $r \neq s$), and that's including all the harmonics components, thus:
$I_t^2 = I_1^2 + I_3^2 + I_5^2 + .............. + I_n^2$
$V_t^2 = V_1^2 + V_3^2 + V_5^2 + .......... + V_n^2$
$S_t^2 = S_1^2 + S_3^2 + S_5^2 + ........... + S_n^2$
$P_t^2 = P_1^2 + P_3^2 + P_5^2 + .......... + P_n^2$
$Q_t^2 = Q_1^2 + Q_3^2 + Q_5^2 + .......... + Q_n^2$
Which basically means the geometrical sum of power components

10) According to the orthogonality law, two components are orthogonal if the inner product of them is equal to zero.
In order to prove the geometric sum of active power, the orthogonality of $(P_1)$ with $(P_h)$ should be tested when, $P_1 = V_1I_1\cos\theta_1$ and $P_h = V_hI_h\cos\theta_h$:
The inner product = $\frac{1}{2\pi}\int_0^{2\pi} P_1P_h dwt$
= $\frac{1}{2\pi}\int_0^{2\pi} I_1V_1\cos wt \, . \, I_hV_h\cos(hwt - \theta_h)dwt$
= $\frac{I_1V_1I_hV_h}{2\pi}\int_0^{2\pi}\frac{1}{2}\big[\cos(wt - hwt + \theta_h) + \cos(wt + hwt - \theta_h)\big]dwt$
= $\frac{I_1V_1I_hV_h}{4\pi}\big[\sin(wt(1-h)+\theta_h) + \sin(wt(1+h)-\theta_h)\big]\Big|_0^{2\pi}$
= $\frac{I_1V_1I_hV_h}{4\pi}\big[\sin(2\pi(1-h)+\theta_h) + \sin(2\pi(1+h)-\theta_h) - \sin(0(1-h)+\theta_h) - \sin(0(1+h)-\theta_h)\big] = Zero$
Therefore $(P_1)$ & $(P_h)$ are orthogonal components and the geometric should be used in order to calculate the total active power.

11) The advantage of using the geometric sum instead of the arithmetic sum in active and reactive power calculation is increasing the dependency on the fundamental component and decreasing the effect of harmonics component, because (mostly) the fundamental component is originally bigger than the harmonics component, moreover, the squaring of the values make the difference huge between the fundamental and the harmonic component.
According to the author in [15], the philosophy of separating the main (fundamental) component from the pollution (non-fundamental) components and their cross terms is successful because:
**a**. Utilities generate and distribute nearly perfect fundamental sinusoidal voltage.
**b**. The consumer expects fundamental sinusoidal voltage.
**c**. Generally, more than 99% of the total active power

flowing in the network is fundamental active power ($P_1$). The author in [12] concluded that, its better to separate ($P_1$) and ($Q_1$) from the rest of the power components, because the power apparent, active, and reactive components are essential factors for the power system. A distribution system cannot perform without reactive power and the useful fundamental magnetizing flux in transformers and AC motors is separated by the fundamental current.

12) The author in 1987 [7], concludes that the phenomena of the reciprocating energy transmission at harmonic frequencies does not affect the source current RMS value and its apparent power (S) in the manner suggested by Budeanu's model. Namely, each of the current harmonics ($I_n$) can be decomposed into two orthogonal components:
$I_n^2 = I_n^2 \cos^2\theta_h + I_n^2 \sin^2\theta_h$
Since all current harmonics are mutually orthogonal, thus the square of the RMS value of the current is:

$$I_{rms}^2 = \sum_{n=1} I_n^2 = \sum (\frac{P_n}{V_n})^2 + \sum (\frac{Q_n}{V_n})^2$$

$$\therefore S_t^2 = \sum_{n=1} (P_n)^2 + \sum_{n=1} (Q_n)^2$$

Therefore, if there is a reciprocating energy transmission between the source and the load, then the terms ($\sum P_n^2$) and ($\sum Q_n^2$) not $(\sum P_n)^2$ and $(\sum Q_n)^2$ are responsible for the source apparent power increase.

13) Emanuel in 1990 [16], has approved that the total reactive power ($Q_t$) is composed of four distinctive types of elementary reactive powers: $S_t^2 = P_t^2 + Q_t^2$

$$Q_t^2 = \sum_{h=1} Q_{Bh}^2 + \sum_{h=1} Q_{Bmn}^2 + \sum_{m \neq n} Q_{Dh}^2 + \sum_{m \neq n} Q_{Dmn}^2$$

$$\therefore Q_t^2 = Q_1^2 + \sum_{h=1} Q_h^2$$

From these equations, it is obvious that the author has agreed indirectly with the idea of geometric sum for the reactive power components.

14) The last update of IEEE-standards 2010 [17] has been mentioned verbally in page (37) :"The fact that harmonic reactive powers of different orders oscillate with different frequencies reinforces the conclusion that the reactive powers should not be added arithmetically (as recommended by Budeanu)". However, the standards did not mention clearly (through equations) the geometric sum of reactive power components.

15) The idea of applying the geometric sum between the power components has been presented in [18] for single phase system. This theory has been proved mathematically and practical electrical circuits has been investigated by using the Matlab-simulink program. However, the author has limited his theory to the sinusoidal system.

## III. Description of the Orthogonality Law and New Power Terms

Depending on the outcomes has been concluded from the previewed literature in the previous section, the power components in different categories and frequencies are all orthogonal and should be calculated using the geometric (not arithmetic) sum, then a new power terms called effective active ($P_{ef}$) and reactive ($Q_{ef}$) power terms can be invented in order to understand the characteristics and relations between different power components and to calculate the total apparent power in non-sinusoidal situation.

Lets consider:

$$V_t^2 = V_1^2 + V_h^2 \quad \& \quad I_t^2 = I_1^2 + I_h^2 \quad \Rightarrow \quad S_t^2 = V_t^2 I_t^2$$

$$\therefore S_t^2 = I_1^2 V_1^2 + V_1^2 \sum_{n=3} I_n^2 + I_1^2 \sum_{m=3} V_m^2 + \sum_{m=n=3} V_m^2 I_n^2 + \sum_{m \neq n} V_m^2 I_n^2 \tag{1}$$

$$D_I^2 = V_1^2 \sum_{n=3} I_n^2 \tag{2}$$

$$D_V^2 = I_1^2 \sum_{m=3} V_m^2 \tag{3}$$

$$D_{mn}^2 = \sum_{m \neq n} I_m^2 V_n^2 \tag{4}$$

$$S_h^2 = \sum_{m=n=3} I_m^2 V_n^2 = \sum_{h=3} I_h^2 V_h^2 \tag{5}$$

$$S_t^2 = S_1^2 + S_h^2 + D_I^2 + D_V^2 + D_{mn}^2 \tag{6}$$

$$S_t^2 = P_1^2 + Q_1^2 + P_h^2 + Q_h^2 + D_I^2 + D_V^2 + D_{mn}^2 \tag{7}$$

This result shows that ($S_t$) can be represented either as a many-dimensional vector, or as a two dimensional vector.

$$S_t^2 = S_1^2 + S_N^2 \tag{8}$$

$$S_1^2 = P_1^2 + Q_1^2 \tag{9}$$

$$S_N^2 = P_h^2 + D_h^2 \tag{10}$$

$$D_h^2 = D^2 + Q_h^2 \tag{11}$$

$$D^2 = D_I^2 + D_V^2 + D_{mn}^2 \tag{12}$$

$$S_h^2 = P_h^2 + Q_h^2 \tag{13}$$

$$P_h^2 = \sum_{h=3} V_h^2 I_h^2 \cos^2\theta_h \tag{14}$$

$$Q_h^2 = \sum_{h=3} V_h^2 I_h^2 \sin^2\theta_h \tag{15}$$

Budeanu's distortion power ($D_B$) definition represents ($S_t$) as three dimensional vector: $S_t = iP + jQ_B + kD_B$ . This definition creates different problems, one of them is the necessity to use a new power unit for the distortion power ($D_B$) [19].

However, the distortion power has the same physical nature as reactive power, then the power unit of (D) has to be VAR [20]. This allows us to conclude that all the components of apparent power ($S_t$) in frequency domain may contains active power ($P_t$) and reactive power ($Q_t$) components [9]. The orthogonality law allows us to use only standard units, VA, W and VAR without needing to invent extra units.

$$\because S_t^2 = S_1^2(\cos^2\theta_1 + \sin^2\theta_1) + S_N^2(\cos^2\theta_N + \sin^2\theta_N) \quad (16)$$

$$\& \quad \because S_t^2 = P_1^2 + Q_1^2 + P_h^2 + D_h^2 \quad (17)$$

$$\& \quad \because S_t^2 = P_{ef}^2 + Q_{ef}^2 = S_{ef}^2 \quad (18)$$

$$\therefore P_{ef}^2 = P_1^2 + P_h^2 \quad (19)$$

$$\& \quad Q_{ef}^2 = Q_1^2 + D_h^2 \quad (20)$$

Owing to the dependence of the phase shift of current on frequency domain, the sinusoidal association cannot be extended by algebraic summation. At the same time, current and voltage remain vectors in non-sinusoidal systems permitting a vector summation of the harmonics.

## IV. New definition of the apparent power in non-sinusoidal system

The apparent power (S) in non-sinusoidal system has been described in 1988 [21], as "it is numerically equal to the maximum active power that exist at given points of entry with the given effective value of the sinusoidal current and the potential difference and hence is directly related to the size of the required equipment and to the generation and transmission losses".

Mathematically this can be described as: $S = P_{max}$ ,
$I_{rms}^2$= constant, $V_{rms}^2$= constant, $S = I_{rms}V_{rms}$

Consider a nonlinear load supplied by a sinusoidal voltage $v(t) = \sqrt{2}V_1$ , and the current is $i(t) = \sum \sqrt{2}I_h \sin(hwt + \theta)$.

The active power is equal to $P = V_1 I_1 \cos\theta_1$.

Its maximum value is equal to $P_{max} = V_1 I_1$ rather than ($V_{rms}I_{rms}$).

Postulate that (S) is equal to the maximum active power, in this case, (S) is equivalent to the replacement of the actual load by an equivalent resistive load, drawing a sinusoidal current rather than non-sinusoidal current. The limitation of this definition, is that its describes the power for equivalent load rather than the actual load.

Filipski asserts in [10], that the non-sinusoidal apparent power is an artificial quantity without any physical meaning and that there is neither theoretical nor practical justification for the electrical power application. The author has depended in his claims on the current definitions of the apparent power.

However, even in the definition of apparent power for non-sinusoidal system according to [17] in 2010, which define as: "its the amount of active power that can be supplied to a load or a cluster of loads under ideal conditions (the ideal condition may assume sinusoidal supply voltage and current with linear loads)" has got a serious limitation. This (ideal) condition constitute a big limitation for the apparent power interpretation, because in the practical life most of the loads are nonlinear and draw non-sinusoidal current. In addition, this condition is mathematically wrong because the inductive or capacitive loads are based on the frequency, even the skin effect of resistors is dependent on the frequency.

Obviously, there is no unanimously accepted Apparent power definition until now, it's useful to find a new definition covers the general aspects for power systems and all practical (not just the resistive) loads. Therefore, the apparent power can be defined as **the geometric sum of the active powers $P_t$ (in all frequencies) and the geometric sum of the non active powers $Q_t$ in all (sinusoidal and non-sinusoidal) conditions**.

The physical interpretation of the definition shows that the apparent power (S) is an effective value (RMS); the effective values are calculate using the magnitudes of harmonics terms. The harmonic currents are mutually orthogonal, therefore the geometric sum has been used to get the total apparent power in non-sinusoidal conditions.

Six equations (9, 19, 20, 10, 8 and 18) respectively represent six right-angled triangles will form the new diagram which illustrate the relation between different power components, as it is shown in Fig.(1).

Power triangle (S-P-Q) has been introduced first time for time domain by Fryze [6] in 1931, when he defined $i_{aF}$ as the active current and $i_{rF}$ as the reactive current as a part of the time domain description. However, the right-angled triangle power theory is the first attempt to apply (Total-Fundamental-Harmonics) power triangle in the frequency domain in combination with the (S-P-Q) Power triangle.

Due to the non-active power $D_h$ is the geometric sum of $D_I, D_v, D_{mn}$ & $Q_h$, then:

$Q_{ef}^2 = Q_1^2 + D_h^2$ , ($D_h$ is mutually orthogonal on $Q_1$).

Simultaneously, ($P_h$) is orthogonal on ($D_h$) but not in-phase with $Q_1$ because they represent different frequencies.

In the same way, ($P_1$) is orthogonal on ($Q_1$) but not in-phase with ($D_h$) because it is in different frequency. Consequently, ($P_h$) is orthogonal on ($P_1$).

## V. The explanation of new power RAT diagram

The new power diagram can be form by gathering six equations which are (9, 19, 20, 10, 8 and 18) respectively, these equations represent six right-angled triangles as shown in Fig.(1).

These triangles are formed based on the notion of the orthogonality law. The distortion power component is always perpendicular with the fundamental power component.

In sum, the aggregation of these six triangles produced a new power diagram called right-angled triangle (RAT) diagram, this diagram is shown in Fig.(2).

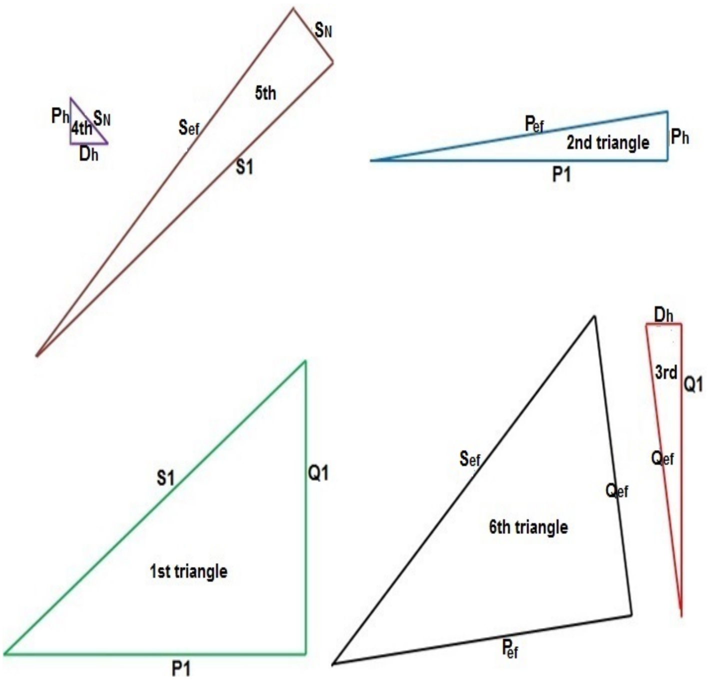


Fig. 1. Six right-angled triangles

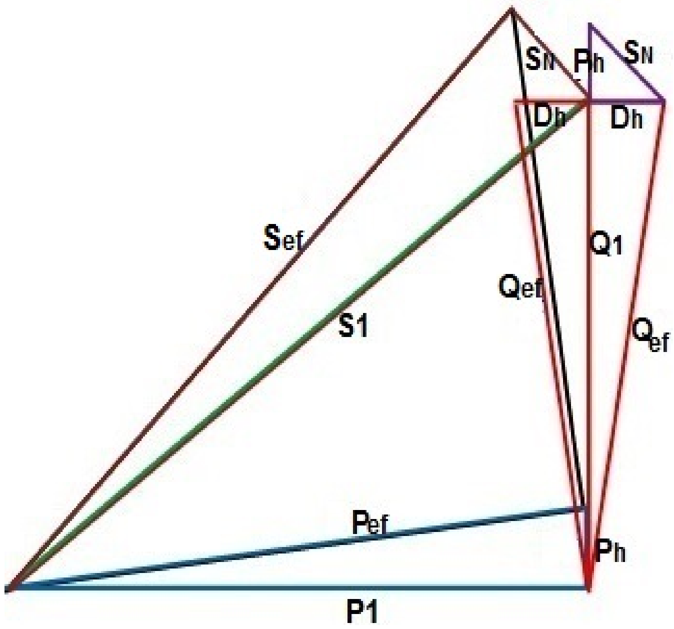


Fig. 2. New power diagram

In the following, three steps explain and simplify the understanding of Right-angled triangle diagram formation.

- First step: The basis of the diagram is the first triangle which is the fundamental power $(S_1, P_1, Q_1)$, and then the second triangle is the active power triangle $(P_{ef}, P_1, P_h)$ should be placed and $P_1$ is identical in both triangles. Then the third triangle is the reactive power triangle $(Q_{ef}, Q_1, D_h)$ will be placed in reverse and perpendicular way to the second triangle. In order to simplify the proposed theory, the third triangle should be placed twice and in symmetry with $Q_1$.
- Second step: The fourth triangle, the distortion power triangle $(S_N, P_h, D_h)$, should be in line above the third triangle and $D_h$ is identical in both triangles. the fifth triangle,the apparent power triangle $(S_{ef}, S_1, S_N)$, has been placed on the $S_1$ line of the first triangle and same $S_N$ of the fourth triangle and thats will produce $S_{ef}$ component.
- Third step: Finally, the total power triangle $(S_{ef}, P_{ef}, Q_{ef})$, will be formed automatically by gathering $(S_{ef})$ line of the fifth triangle with $(P_{ef})$ line of the second triangle and $(Q_{ef})$ from the third triangle.

This diagram has the attribute of the simplicity in comparing with the Budeanu's 3-dimension diagram because it has only two dimensions and that gives it the ability to be easily applicable to find the relationships between the different components with there angles $(\theta_t, \theta_1 \& \theta_h)$ and even the calculation of the amplitude value of the power parameters.

## VI. The mathematical proof of the effective power terms $(P_{ef})$ & $(Q_{ef})$

This section submit the mathematical evidence to the validity of the effective power terms $(P_{ef})$ & $(Q_{ef})$ and shows the algebraic relation between arithmetical and geometrical summation of power components in non-sinusoidal systems.

### A. For average power:

In algebra equations, Lagrange's identity was described in [22] and [23] as :

$$(\sum_{k=1}^{n} a_k b_k)^2 = (\sum_{k=1}^{n} a_k^2)(\sum_{k=1}^{n} b_k^2) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (a_i b_j - a_j b_i)^2 \quad (21)$$

or :

$$(\sum_{k=1}^{n} a_k b_k)^2 = (\sum_{k=1}^{n} a_k^2)(\sum_{k=1}^{n} b_k^2) - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} (a_i b_j - a_j b_i)^2 \quad (22)$$

when: $P_h = V_h I_h \cos\theta_h$ & $P_h = a_k b_k$

$$a_k = V_h = (V_1^2 + \sum_{n=1} V_n^2)$$

$$b_k = I_h \cos\theta_h = (I_1^2 \cos^2\theta_1 + \sum_{n=1} I_n^2 \cos^2\theta_n)$$

$$\therefore (\sum_{h=1} V_h I_h \cos\theta_h)^2 = (\sum_{h=1} V_h^2)(\sum_{h=1} I_h^2 \cos^2\theta_h) - \frac{1}{2} \sum_{m=1} \sum_{n=1, n\neq m} (V_m I_n \cos\theta_n - V_n I_m \cos\theta_m)^2$$

$$\therefore (\sum_{h=1} V_h I_h \cos\theta_h)^2 = (V_1^2 + \sum_{n=1} V_n^2)(I_1^2 \cos^2\theta_1 + \sum_{n=1} I_n^2 \cos^2\theta_n) - \frac{1}{2}\sum_{m=1}\sum_{n=1,n\neq m}(V_m I_n \cos\theta_n)^2 - (V_n I_m \cos\theta_m)^2 - 2V_m V_n I_m \cos\theta_m I_n \cos\theta_n$$

When : h=1,2,3,4,5 , m=1,2,3,4,5 & n=1,2,3,4,5

$$(\sum_{h=1} V_h I_h \cos\theta_h)^2 = (V_1^2 + V_2^2 + V_3^2)(I_1^2\cos^2\theta_1 + I_2^2\cos^2\theta_2 + I_3^2\cos^2\theta_3) - \frac{1}{2}\sum_{m=1}\sum_{n=1,n\neq m}(V_m I_n \cos\theta_n)^2 - (V_n I_m \cos\theta_m)^2 - 2V_m I_m \cos\theta_m V_n I_n \cos\theta_n$$

After some steps of simplification, the above equation can be represented as following:

$$(\sum_{h=1} P_h)^2 = \sum_{h=1}(P_h)^2 + 2(P_1P_2 + P_1P_3 + P_1P_4 + P_1P_5 + P_2P_3 + P_2P_4 + P_2P_5 + P_3P_4 + P_3P_5 + P_4P_5)$$

The algebraic relation between arithmetical and geometrical sum is shown as:

$$(\sum_{h=1} P_h)^2 = \sum_{h=1}(P_h)^2 + 2\sum_{m=1}\sum_{n=1,n\neq m} P_m P_n \tag{23}$$

When : $P_h = V_h I_h \cos\theta_h$

$P_m = V_m I_m \cos\theta_m$ & $P_n = V_n I_n \cos\theta_n$

$$\therefore \sum_{h=1}(P_h)^2 = (\sum_{h=1} P_h)^2 - 2\sum_{m=1}\sum_{n=1,n\neq m} P_m P_n \tag{24}$$

$\sum_{h=1}(P_h)^2$ is the geometrical sum of power components

$(\sum_{h=1} P_h)^2$ is the square of arithmetic sum of power components

*B. For instantaneous power:*

For Single phase non-sinusoidal supply voltage with a nonlinear RLC-load:

$$V(t) = \sum_{k=1}^{\infty} \sqrt{2} V_k \sin(kwt)$$

$V_k$ : The rms value of the $k_{th}$ harmonic of the source voltage.

$$I(t) = \sum_{n=1}^{\infty} \sqrt{2} I_n \sin(nwt - \theta_n)$$

$I_n$ : The rms value of the $n_{th}$ harmonic of the source current.

$$P(t) = 2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} V_k I_n \sin(kwt)\sin(nwt - \theta_n) \tag{25}$$

$P(t) = 2\big[V_1 \sin(wt) + V_2 \sin(2wt) + V_3 \sin(3wt)\big]\big[I_1 \sin(wt - \theta_1) + I_2 \sin(2wt - \theta_2) + I_3 \sin(3wt - \theta_3)\big]$

For: k = 1,2,3 and n = 1,2,3

$P(t) = 2\big[V_1 \sin(wt) I_1 \sin(wt - \theta_1) + V_1 \sin(wt) I_2 \sin(2wt - \theta_2) + V_1 \sin(wt) I_3 \sin(3wt - \theta_3) + V_2 \sin(2wt) I_1 \sin(wt - \theta_1) + V_2 \sin(2wt) I_2 \sin(2wt - \theta_2) + V_2 \sin(2wt) I_3 \sin(3wt - \theta_3) + V_3 \sin(3wt) I_1 \sin(wt - \theta_1) + V_3 \sin(3wt) I_2 \sin(2wt - \theta_2) + V_3 \sin(3wt) I_3 \sin(3wt - \theta_3)\big]$

When (k=n) :

$$P(t) = 2\sum_{n=1}^{\infty} V_n I_n \sin(nwt)\sin(nwt - \theta_n)$$

$$P(t) = \sum_{n=1}^{\infty} V_n I_n [\cos\theta_n - \cos(2nwt - \theta_n)] \tag{26}$$

The effective value of active power is the root mean square of the active power for all harmonics (the geometric sum).

$P_{ef}^2 = P_{rms}^2 = \frac{1}{T}\int_0^T P^2(wt)dwt$

$$P_{ef}^2 = \frac{1}{2\pi}\int_0^{2\pi}\sum_{n=1}\Big[V_n I_n[\cos\theta_n - \cos(2nwt - \theta_n)]\Big]^2 dwt$$

From (24):

$$P_{ef}^2 = \frac{1}{2\pi}\int_0^{2\pi}\sum_{n=1}\Big[V_n^2 I_n^2[\cos^2\theta_n - \cos^2(2nwt - \theta_n) + 2\sum_{n=1} V_n I_n(\cos\theta_n - \cos(2nwt - \theta_n)) V_m I_m(\cos\theta_m - \cos(2mwt - \theta_m))]\Big] dwt$$

$$P_{ef}^2 = \frac{1}{2\pi}\int_0^{2\pi}\sum_{n=1}\Big[V_n^2 I_n^2[\cos^2\theta_n - \frac{1}{2}(1 + \cos(4nwt - 2\theta_n) + 2\sum_{n=1} V_n I_n(\cos\theta_n - \cos(2nwt - \theta_n)) V_m I_m(\cos\theta_m - \cos(2mwt - \theta_m))]\Big] dwt$$

$$P_{ef}^2 = \frac{1}{2\pi}\int_0^{2\pi}\sum_{n=1}\Big[V_n^2 I_n^2[\cos^2\theta_n - \frac{1}{2} - \frac{\cos(4nwt - 2\theta_n)}{2} + 2\sum_{n=1} V_n I_n V_m I_m(\cos\theta_n\cos\theta_m - \cos\theta_n\cos(2mwt - \theta_m) - \cos\theta_m\cos(2nwt - \theta_n) + \cos(2nwt - \theta_n)\cos(2mwt - \theta_m))\Big] dwt$$

$$P_{ef}^2 = \frac{1}{2\pi}\sum_{n=1}\Big[V_n^2 I_n^2[wt\cos^2\theta_n - \frac{wt}{2} - \frac{\sin(4nwt-2\theta_n)}{8n} +$$
$$2\sum_{n=1} V_n I_n V_m I_m(wt\cos\theta_n\cos\theta_m - \cos\theta_n\frac{\cos(2mwt-\theta_m)}{2m}$$
$$-\cos\theta_m\frac{\sin(2nwt-\theta_n)}{2n} + \frac{\sin(2nwt-\theta_n+2mwt-\theta_m)}{4(n+m)}$$
$$+\frac{\sin(2nwt-\theta_n-2mwt+\theta_m)}{4(n-m)})\Big]\Big|_0^{2\pi}$$

$$\therefore P_{ef} = \sqrt{(\sum_{h=1} P_h)^2 - 2\sum_{m=1}\sum_{n=1,n\neq m} P_m P_n} \quad (27)$$

## VII. Evidence Examples

This section presents some examples of simple electrical circuits in order to investigate and prove the orthogonality law and the validity of (RAT) diagram.

1) **Example 1**: In this example, randomly chosen values has been applied in order to examine the validity of the new proposed RAT power diagram:

   Let: $P_1 = 10 \quad \& \quad Q_1 = 5 \Rightarrow \therefore S_1$ = 11.18 (equ. 9)

   Let: $P_h = 2, \because P_1 = 10 \Rightarrow \therefore P_t$ = 10.2 (equ. 19)

   Let: $D_h = 1, \because Q_1 = 5 \Rightarrow \therefore Q_t$ = 5.1 (equ. 20)

   $\because P_h = 2 \quad \& \quad D_h = 1 \Rightarrow \therefore S_N$ = 2.236 (equ. 10)

   $\because S_1 = 11.18 \quad \& \quad S_N = 2.236 \Rightarrow \therefore S_t$ = 11.4 (equ. 8)

   $\because P_t = 10.2 \quad \& \quad Q_t = 5.1 \Rightarrow \therefore S_t$ = 11.4 (equ. 18)

   This example shows the coherence of the equations and the consistency of the values of power components through one diagram and prove the validity of the new RAT diagram.

2) The arithmetic sum $Q_h$ can be equal to zero at nonzero values of terms $Q_h$ despite the reciprocating energy transmission between the source and the load.

   **Example 2**: Suppose the circuit, shown in Fig.(3):

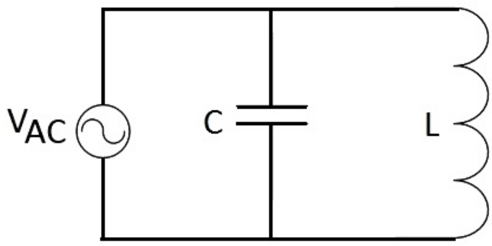


Fig. 3. LC - circuit

   The load has impedance $Z_1$ = j 20 in the fundamental component and for the 3rd order harmonic $Z_3$ = - j 1.25 , $w$ = 1 rad / sec . If the supply voltage is :

   $v(t) = (200\sin wt + 50\sin 3wt)V$

   Then the load current is equal to :

   $i(t) = [10\sin(wt-90) + 40\sin(3wt+90)]A.$

   The arithmetic sum of reactive power is :

   $Q_t = Q_1 + Q_3$ = 2000 - 2000 = 0 VAr

   However, there is energy oscillation in this circuit and the total reactive power should be more than zero because instantaneous power P(t) has a negative part in Fig. (3.4) thats mean the energy flows back to the source when it is negative and it causes losses in the transmission line.
   While, the geometric sum is :

   $Q_t = \sqrt{(2000)^2 + (2000)^2}$ = 2825.4 VAr

   By using the geometric sum in reactive power calculation, its guaranteed to get a positive reactive power as long as there is reciprocating energy in the circuit.

3) **Example 3**: This following example has been proposed previously in [17] (pp. 36-38) : $P_1$ = 8660 w, $P_3$ = -13.94 w, $P_5$ = -11.78 w and $P_7$ = -1.74 w
   In order to test equation (27):

$$\sum_{h=1}^{7} P_h^2 = P_1^2 + P_3^2 + P_5^2 + P_7^2 = 74995936.12W$$

$\therefore P_{ef} = \sqrt{P_h^2} = 8660.02$ watt

$$\sum_{h=1}^{7}(P_h)^2 = (P_1 + P_3 + P_5 + P_7)^2 = 74520746.85$$

$$\sum_{h=1}^{7} P_m P_n = 2(P_1P_3 + P_1P_5 + P_1P_7 + P_3P_5 + P_3P_7 + P_5P_7)$$

= -237594.64

$$\therefore \sum_{h=1}^{7}(P_h)^2 = 74520746.85 + 475189.28 = P_{ef}^2$$

$\therefore P_{ef}$ = 8660.02 watt

The total value of apparent power which has been calculated in this example from this equation: $S_t^2 = S_1^2 + S_N^2$ was 10517.55 VA.
By applying equation (18), the power triangle should give the same result of $S_{ef}^2$. The geometric sum of $P_t$ which is already $P_{rms} = P_{ef}$ = 8660.02 W
In order to find $Q_{ef}$ :

$Q_{ef} = \sqrt{Q_1^2 + Q_3^2 + Q_5^2 + Q_7^2 + D_I^2 + D_v^2 + D_{mn}^2}$

$Q_t$ = 5968 VAr = $Q_{ef}$

From (18): $S_t^2 = (5968)^2 + (8660.02)^2 = 10517.3 = S_{ef}^2$

However, the arithmetic sum of $P_t$ is :

$P_t = P_1 + P_3 + P_5 + P_7$ = 8632.54 watt

If: $S_t = \sqrt{P_t^2 + Q_t^2}$ = 10494.65 VA

This value is different from $S_t$ resulted from $S_1^2 + S_N^2$ in the same example. Consequently, the arithmetic sum does not investigate the power triangle.

## VIII. Conclusion

This article presented two new power terms called effective active ($P_{ef}$) and effective reactive ($Q_{ef}$) power terms. These two terms useful because they show the true relationship between all power components (total, fundamental & distortion) in all power categories (S, P & Q) in non-sinusoidal system.

These two terms are useful to investigate and prove the ability to apply the principle of orthogonality law and the right-angled power triangle theory. This theory shows the orthogonality between fundamental and distorted components for all power categories (S, P and Q).

The right-angled power triangle is a compulsory condition to calculate the total apparent ($S_t$) in the non-sinusoidal system in the right way and its compatible with the equations of IEEE-2010 standards.

A comprehensive literature review has been presented in section II in order to justify the use of the geometric sum of power components and deny the arithmetic sum of power components. This section also, shows that Budeanu's power definition has been refuted by a big number of valuable researchers and its no longer can be used.

This article also, offers a new definition for the apparent power as ($S_t$ or $S_{ef}$) in the non-sinusoidal situation and compares it with the previous power definitions which have serious limitations which has been shown in section IV.

Moreover, this article shows a new power diagram representing all power components in a single diagram consisting of six right-angled triangles called the right-angled triangle (RAT) diagram.

The notion of new power terms and new power right-angled triangle (RAT) diagram defined above, have the following interesting advantages:

1) The expressions and symbols which have been used in this article, are the same symbols and units have been used in IEEE standards-2010.
2) The vectors mentioned in the power diagram can be represented in phasor diagram.
3) The conventional units was used are the traditional units (W, VAr and VA), then no new power units have been added.
4) Defining the power components in two dimensional diagram without the need of sophisticated mathematical equations or three dimensional shapes as used before in budeanu's theory.
5) Finally, proving the (RAT) diagram, investigates the power right-angled triangle (S-P-Q) which bases on the orthogonality principle between different fundamental and harmonic components.

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