



## Designing aluminium friction stir welded joints against multiaxial fatigue

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**ABSTRACT.** The present paper investigates the accuracy of the Modified Wöhler Curve Method (MWCM) in estimating multiaxial fatigue strength of aluminium friction stir (FS) welded joints. Having developed a bespoke joining technology, circumferentially FS welded tubular specimens of Al 6082-T6 were tested under proportional and non-proportional tension and torsion, the effect of non-zero mean stresses being also investigated. The validation exercise carried out using the experimental results have demonstrated that the MWCM applied in terms of nominal stresses, notch stresses, and also the Point Method is accurate in predicting the fatigue lifetime of the tested FS welded joints, with its use resulting in life estimates that fall within the uniaxial and torsional calibration scatter bands.

**KEYWORDS.** Friction stir welding; Multiaxial fatigue; Critical plane.

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### INTRODUCTION

Friction stir (FS) welding is a joining process that allows joints with high mechanical performance to be manufactured at a relatively low cost. Thanks to its specific features, in recent years this joining technology has been employed successfully in different industrial sectors [1] which include, amongst others, ship building [2], transportation [3], and aircraft [4]. In terms of design against fatigue, whilst a tremendous effort has been made since the

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mid-90s to investigate the fatigue behaviour of aluminium FS welded joints subjected to uniaxial cyclic loading (see, for instance, Ref. [5] and references reported therein), no systematic research work has been carried out so far to formulate and validate specific methodologies suitable for performing the multiaxial fatigue assessment of this type of joints. In this context, this paper summarises a part of the outcomes from an International Network research project sponsored by the Leverhulme Trust ([www.leverhulme.ac.uk](http://www.leverhulme.ac.uk)) on multiaxial fatigue assessment of aluminium FS welded tubular connections.

## FUNDAMENTALS OF THE MODIFIED WÖHLER CURVE METHOD

The MWCM is a critical plane approach which assesses fatigue damage under uniaxial/multiaxial fatigue loading via the maximum shear stress amplitude,  $\tau_a$ , as well as via the mean value,  $\sigma_{n,m}$ , and the amplitude,  $\sigma_{n,a}$ , of the stress normal to that material plane experiencing the maximum shear stress amplitude [6]. The combined effect of the relevant stress components relative to the critical plane is quantified via the following stress index [7]:

$$\rho_{\text{eff}} = \frac{m \cdot \sigma_{n,m} + \sigma_{n,a}}{\tau_a} \quad (1)$$

where  $m$  is the so-called mean stress sensitivity index [6, 7]. Index  $m$  is a material fatigue property that ranges from zero (no mean stress sensitivity) to unity (full mean stress sensitivity) [6]. According to the way it is defined, ratio  $\rho_{\text{eff}}$  is sensitive not only to the presence of non-zero mean stresses [8], but also to the degree of multiaxiality and non-proportionality of the load history being assessed [6].

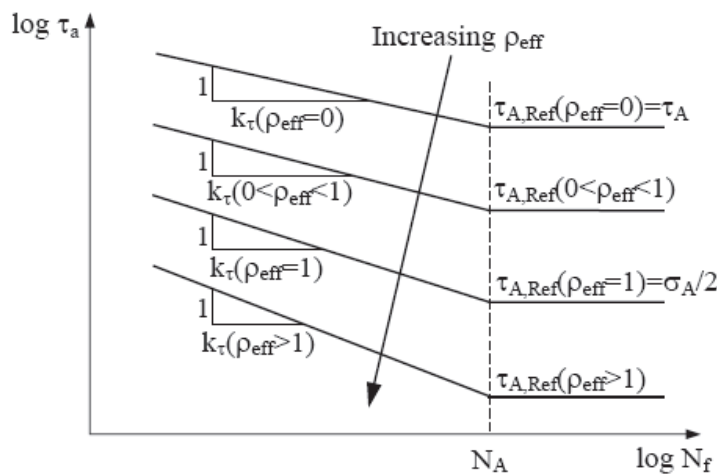


Figure 1: Modified Wöhler diagram.

The MWCM's modus operandi is schematically explained through the modified Wöhler diagram of Figure 1. This log-log chart plots  $\tau_a$  against the number of cycles to failure,  $N_f$ . Much experimental evidence [6] suggests that, for a specific material, the modified Wöhler curves tend to move downward as  $\rho_{\text{eff}}$  increases (Fig. 1). In other words, for a given value of  $\tau_a$ , fatigue damage tends to increase as  $\rho_{\text{eff}}$  increases. According to the diagram in Figure 1, the position and the negative inverse slope of any modified Wöhler curve can be defined through the following linear relationships [6]:

$$k_{\tau}(\rho_{\text{eff}}) = \alpha \cdot \rho_{\text{eff}} + \beta \quad (2)$$

$$\tau_{A,\text{Ref}}(\rho_{\text{eff}}) = a \cdot \rho_{\text{eff}} + b \quad (3)$$



In these equations,  $k_{\tau}(\rho_{eff})$  is the negative inverse slope and  $\tau_{A,Ref}(\rho_{eff})$  is the reference shear stress amplitude extrapolated at  $N_A$  cycles to failure (Fig. 1). Constants  $\alpha$ ,  $\beta$ ,  $a$  and  $b$  are material fatigue parameters that have to be determined through appropriate experiments [6]. Observing that, in the absence of stress concentration phenomena,  $\rho_{eff}$  is equal to unity under fully-reversed uniaxial fatigue loading and to zero under torsional cyclic loading [6], the constants in Eqs 2 and 3 can be estimated from the fully-reversed uniaxial and torsional fatigue curves as follows [6]:

$$k_{\tau}(\rho_{eff}) = [k(\rho_{eff} = 1) - k(\rho_{eff} = 0)] \cdot \rho_{eff} + k(\rho_{eff} = 0) \quad (4)$$

$$\tau_{A,Ref}(\rho_{eff}) = \left( \frac{\sigma_A}{2} - \tau_A \right) \cdot \rho_{eff} + \tau_A \quad (5)$$

In Eq. 4  $k_{\tau}(\rho_{eff}=1)$  and  $k_{\tau}(\rho_{eff}=0)$  are the negative inverse slope of the uniaxial and torsional fatigue curve, respectively, whereas in Eq. 5  $\sigma_A$  and  $\tau_A$  are the endurance limits extrapolated at  $N_A$  cycles to failure under fully-reversed uniaxial and torsional fatigue loading, respectively. As to calibration relationships 4 and 5, it is important to point out that  $\tau_{A,Ref}(\rho_{eff})$  and  $k_{\tau}(\rho_{eff})$  are assumed to be constant and equal to  $\tau_{A,Ref}(\rho_{lim})$  and to  $k_{\tau}(\rho_{lim})$ , respectively, for  $\rho_{eff}$  values larger than an intrinsic material threshold denoted as  $\rho_{lim}$  [6, 8].

To estimate fatigue lifetime according to the MWCM, initially both  $\tau_a$  and  $\rho_{eff}$  have to be determined at the assumed critical location by adopting the appropriate algorithms [9, 10]. Subsequently, the corresponding modified Wöhler curve has to be derived from Eqs 2 and 3 through the estimated value for  $\rho_{eff}$ . Finally, the number of cycles to failure under the investigated load history can be predicted as follows [6]:

$$N_f = N_A \cdot \left[ \frac{\tau_{A,ref}(\rho_{eff})}{\tau_a} \right]^{k_{\tau}(\rho_{eff})} \quad (6)$$

To conclude, it can be recalled that, as far as conventional welded joints are concerned, the MWCM has proven [11] to be accurate in performing the multiaxial fatigue assessment when it is applied not only in terms of nominal and hot-spot stresses, but also along with the reference radius concept as well as the Theory of Critical Distances.

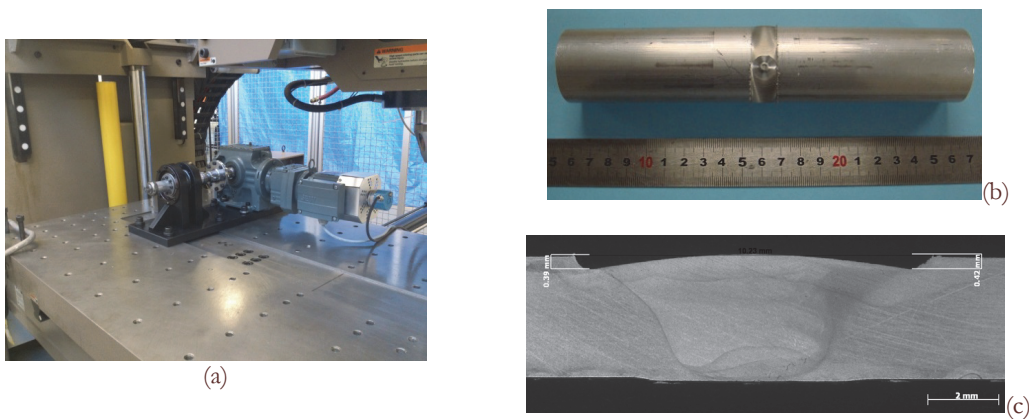


Figure 2: I-STIR FS welding platform equipped with a fourth axis (a); Al 6082-T6 FS welded tubular specimen (b); transverse macrosection of the weld region (c).

## EXPERIMENTAL DETAILS

The technology used to manufacture the FS welded tubular samples for testing was developed at the Nelson Mandela Metropolitan University, South Africa. Circumferential FS welds were manufactured by incorporating a fourth axis into a commercial I-STIR platform (Fig. 2a). In order to obtain high-quality welds, an ad hoc retracting

tool was designed and optimised. Figure 2b shows an example of a FS welded aluminium specimen manufactured using this technology. The parent material employed in the present investigation was Al 6082-T6 with ultimate tensile strength,  $\sigma_{UTS}$ , equal to 303 MPa. The tubular specimens had outer nominal diameter equal to  $\approx 38$  mm and inner nominal diameter to  $\approx 31$  mm. All the samples were tested in the as-welded condition.

The FS welded specimens were tested under axial fatigue loading at the University of Ferrara, Italy, using an MTS 810 Mod. 318.25 servo-hydraulic machine. The samples were tested under a load ratio,  $R$ , equal to 0.1 and to -1. The biaxial fatigue tests were carried out at the University of Sheffield, UK, using a SCHENCK servo-hydraulic axial/torsional testing machine equipped with two MTS hydraulic grips. The force/moment controlled tests were run under in-phase and  $90^\circ$  out-of-phase constant amplitude sinusoidal load histories with load ratios equal to -1 and 0.

The pictures seen in Figure 3 show some examples of the typical cracking behaviours displayed by the Al 6082-T6 FS welded joints tested under biaxial loading.

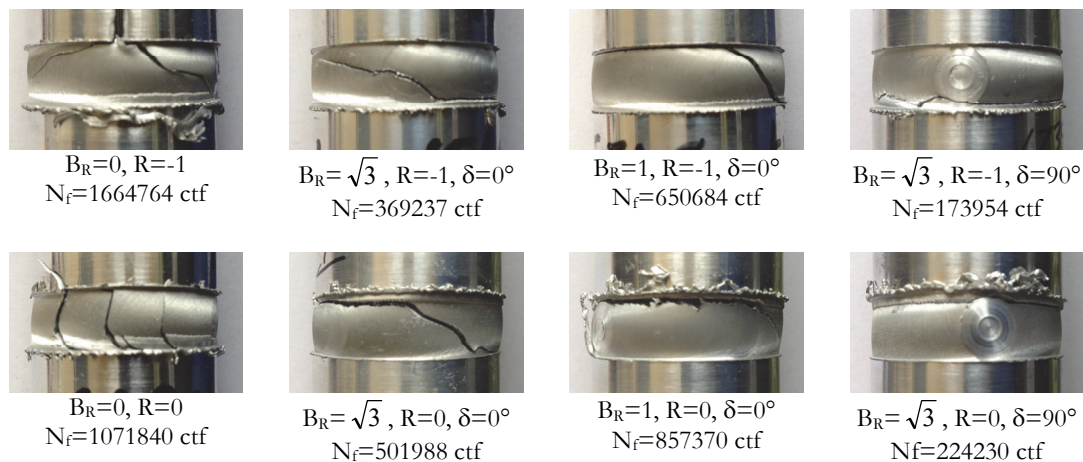


Figure 3: Examples of the observed macroscopic cracking behaviour under biaxial fatigue loading (ctf=cycles to failure).

The experimental fatigue data were re-analysed using the hypothesis of a log-normal distribution of the number of cycles to failure for each stress level with a confidence level equal to 95% [12]. The results of the statistical reanalysis are listed in Tab. 1 in terms of nominal stresses referred to the annular section of the parent tube, where:  $B_R = \sigma_{nom,a} / \tau_{nom,a}$  is the ratio between the amplitudes of the axial and torsional nominal stress,  $R$  is the nominal load ratio,  $\delta$  is the out-of-phase angle,  $k$  is the negative inverse slope,  $\sigma_A$  and  $\tau_A$  are the amplitudes of the axial and torsional endurance limits extrapolated at  $N_A = 2 \cdot 10^6$  cycles to failure, and  $T_\sigma$  is the scatter ratio of the amplitude of the endurance limit for 90% and 10% probabilities of survival.

## VALIDATION BY EXPERIMENTAL RESULTS

As far as conventional aluminium welded joints are concerned, the MWCM can be applied not only in terms of nominal [11, 13] and notch stresses [14], but also using the Theory of Critical Distances (in the form of the Point Method) [15].

Owing to the high level of accuracy which was obtained with standard welded connections [6], in the present investigation the above three stress analysis strategies were used, with the MWCM being applied to post-process the experimental results summarised in Tab. 1.

Independently from the definition adopted to calculate the relevant stress states, the MWCM was applied using multiaxial fatigue software Multi-FEAST© ([www.multi-feast.com](http://www.multi-feast.com)).

Initially, the accuracy of the MWCM in estimating the fatigue lifetime of the tested FS welded joints was checked by applying this approach in terms of nominal stresses. The calibration constants in Eqs 4 and 5 were estimated using the fully-reversed uniaxial and torsional fatigue curves reported in Tab. 1, obtaining:



$$k_{\tau}(\rho_{\text{eff}}) = -4.3 \cdot \rho_{\text{eff}} + 10.8 \quad (7)$$

$$\tau_{\text{Ref}}(\rho_{\text{eff}}) = -22.2 \cdot \rho_{\text{eff}} + 38.9 \text{ MPa} \quad (8)$$

As shown by the SN curves summarised in Tab. 1, the fatigue strength of these FS welded joints was seen to be sensitive to presence of non-zero mean stresses, and this holds true even though the specimens were tested in the as-welded condition. According to this experimental evidence, the mean stress sensitivity index,  $m$ , was taken equal to unity, with  $\rho_{\text{lim}}$  being set equal to 1.3.

$B_R$	$R$	$\delta$ [°]	N. of data	$k$	$\sigma_A$ [MPa]	$\tau_A$ [MPa]	$T_{\sigma}$
$\infty$	-1	-	9	6.5	33.5	-	1.58
$\infty$	0.1	-	10	4.4	18.6	-	1.82
0	-1	-	11	10.8	-	38.9	1.49
0	0	-	10	9.5	-	32.9	1.52
$\sqrt{3}$	-1	0	8	5.3	26.1	15.1	1.55
$\sqrt{3}$	0	0	7	4.2	17.2	9.9	1.73
$\sqrt{3}$	-1	90	10	5.3	21.1	12.2	2.97
$\sqrt{3}$	0	90	7	10.4	23.4	13.5	1.38
1	-1	0	7	5.4	23.2	23.2	2.12
1	0	0	7	3.2	12.8	12.8	2.00
1	-1	90	9	3.9	11.3	11.3	1.66
1	0	90	7	15.8	22.6	22.6	1.35

Table 1: Summary of the generated experimental results.

The experimental,  $N_f$ , vs. estimated,  $N_{f,e}$ , number of cycles to failure chart shown in Figure 4a summarises the overall accuracy which was obtained by using the MWCM in terms of nominal stresses to predict the lifetime of the FS welded tubular samples being tested. This graph makes clear that the use of the MWCM resulted in life estimates falling within the wider scatter band between the two that characterise the fully-reversed uniaxial and torsional fatigue curves used to calibrate the constants in the MWCM's governing equations.

Subsequently, the MWCM was applied in terms of notch stresses [14, 16]. The average notch root radius both on the retreating and the advancing side was measured to approach 0.5 mm (Fig. 2c). The required notch stresses were determined by solving axisymmetric linear-elastic finite element (FE) models done using commercial software ANSYS©. The stress analysis returned the following values for the gross stress concentration factors:  $K_{t,ax}=2.4$  (axial stress),  $K_{t,hs}=0.48$  (hoop stress),  $K_{t,t}=1.7$  (torsional stress). The fully-reversed uniaxial and torsional fatigue curves were used in terms of notch stresses to determine the constants in the MWCM's calibration function, obtaining:

$$k_{\tau}(\rho_{\text{eff}}) = -4.3 \cdot \rho_{\text{eff}} + 10.8 \quad (9)$$

$$\tau_{\text{Ref}}(\rho_{\text{eff}}) = -24.9 \cdot \rho_{\text{eff}} + 64.9 \text{ MPa} \quad (10)$$

The uniaxial fatigue curve for a load ratio,  $R$ , equal to 0.1 was used to estimate both the mean stress sensitivity index and the limit value for  $\rho_{\text{eff}}$  (i.e.,  $m=1$  and  $\rho_{\text{lim}}=2$ ). The error chart of Figure 4b confirms that the MWCM applied in terms of

notch stresses resulted in estimates falling within the two calibration scatter bands, with only a few data points being on the non-conservative side (i.e., series  $\delta=90^\circ$ ,  $B_R=\sqrt{3}$ ,  $R=-1$ ).

Finally, an attempt was made to apply the MWCM in conjunction with the Point Method to estimate the fatigue lifetime of the FS welded specimens. The relevant linear-elastic stress states were determined at a distance from the crack initiation locations equal to 0.075 mm [15]. The stress analysis was performed by solving axisymmetric linear-elastic FE models done with commercial FE code ANSYS® [6]. The fully-reversed uniaxial and torsional experimental fatigue curves post-processed according to the Point Method were used to calibrate Eqs 2 and 3, i.e.:

$$k_\tau(\rho_{\text{eff}}) = -3.7 \cdot \rho_{\text{eff}} + 10.8 \tag{11}$$

$$\tau_{\text{Ref}}(\rho_{\text{eff}}) = -28.8 \cdot \rho_{\text{eff}} + 58.0 \text{ MPa} \tag{12}$$

The uniaxial fatigue curve with  $R=0.1$  was then used to estimate both the mean stress sensitivity index and the limit value for  $\rho_{\text{eff}}$ , obtaining:  $m=1$  and  $\rho_{\text{eff}}=1.6$ .

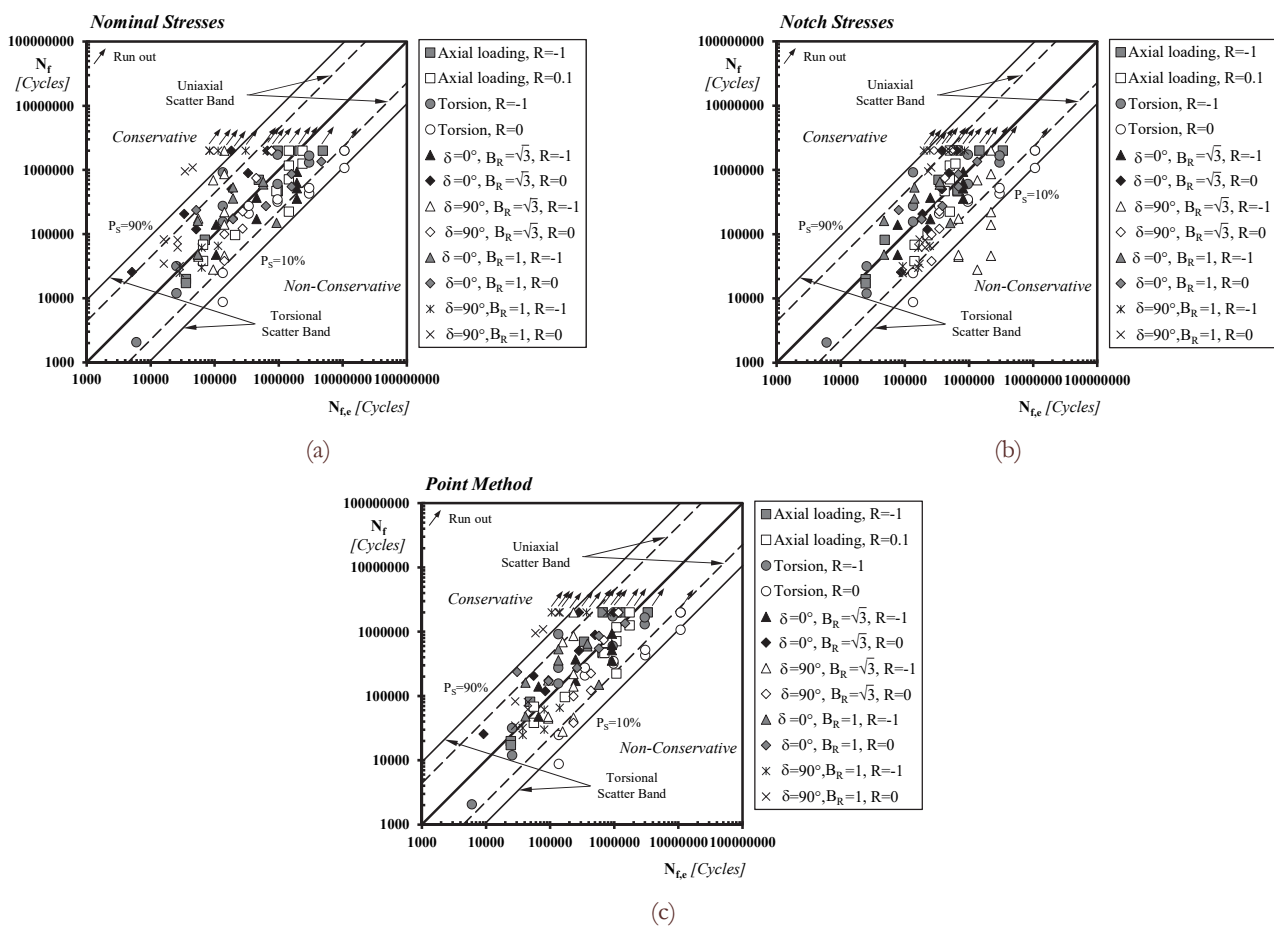


Figure 4: Accuracy of the MWCM applied in terms of nominal (a) and notch stresses (b) as well as along with the Point Method (c).

The error bands in Figure 4c summarise the overall accuracy that was obtained by applying the MWCM in conjunction with the Point Method. This diagram makes it evident that this design methodology was accurate, resulting in predictions falling within the scatter bands associated with the experimental calibration fatigue curves.





It can be concluded from these results that, independently from the adopted stress analysis strategy, the resulting level of accuracy is certainly satisfactory (see Figure 4), since, from a statistical point of view, we cannot expect that a predictive method will be more accurate than the experimental information used to calibrate the method itself.

## CONCLUSIONS

- For the specific profile of the FS welded fatigue specimens, the fatigue behaviour of these tubular joints of Al 6082-T6 could successfully be modelled using notch mechanics concepts.
- The MWCM was applied not only in terms of nominal and notch stresses, but also in conjunction with the Point Method and was seen to be highly accurate in estimating the fatigue lifetime of the FS welded joints.
- For the investigated FS welded connections, the MWCM was seen to be capable of correctly modelling not only the presence of non-zero mean stresses, but also the degree of multiaxiality and non-proportionality of the applied load history.

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