Preface

Personal perspectives in nonlinear science: Looking back, looking forward

The International Conference **Perspectives in Nonlinear Dynamics** (PNLD 2016) was held between 25 and 29 July, 2016 at the Humboldt University, Berlin. PNLD 2016 continued the tradition of discussing and disseminating cutting-edge research in the field of nonlinear dynamics and was formally a satellite meeting to the 26th International Conference on Statistical Physics (STATPHYS26) of the International Union for Pure and Applied Physics (IUPAP) that was held in Lyon, France during July 18-22, 2016. Like the earlier meetings, PNLD 2004 (Chennai), PNLD 2007 (Trieste), PNLD 2010 (Bangalore), and PNLD 2013 (Hyderabad) whose proceedings were also published by the Indian Academy of Sciences, the articles in this volume are a record of PNLD 2016.

To mark the fact that this was the fifth conference in the series, we wrote to a number of the invited speakers at this as well as earlier PNLD conferences and requested them to share their own perspectives on how they see the field of nonlinear dynamics or more generally, nonlinear science evolving. What are the interesting questions at the present time? We also suggested that these "can be as informal as you like, with or without references, and with or without a signature, and as long or as short as you want." Happily, several colleagues responded, and these perspectives are presented below.

The Editors

1. Celso Grebogi, ABERDEEN

Nonlinear dynamics and complexity

With the advent of complexity, complex dynamical systems became a new area of research in nonlinear dynamics. But what is complexity? The mathematicians look suspiciously at it as the concept is filled with ambiguity and it is dependent on the context of what constitutes a mathematical definition. There is not a general consensus on a quantitative measure of complexity. However, there are general traits most agree a complex system should have. It is composed of many parts interrelated in a complicated manner, possessing both ordered and random behaviours, exhibiting a layered hierarchy of structures over a wide range of time and/or length scales. The essence of complexity was correctly expressed by the great French novelist Marcel Proust in his monumental work *À Recherche du Temps Perdu*[1] where I found the following passage:

> "But the absence of one part from a whole is not only that it is not simply a partial omission, it is a disturbance of all other parts, a new state which it was impossible to foresee from the old."

Indeed, the whole is made up of many parts, interacting among themselves, and resulting in the emergence of competing behaviours and structures.

To understand complexity, I like to follow the tradition of nonlinear dynamics, a tradition that started in the 19th century with a physicist, James Clerk Maxwell, and a mathematician, Henri Poincaré. Both understood the importance of systems having sensitive dependence on initial data. Their work led to a cautious, rigorous study of specific low dimensional systems by mathematicians working on the theory of dynamical systems during most of the 20th century. We find there none of the bold, intuitive philosophical generalisations that Maxwell and Poincaré

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[1]The English translation is *Remembrance of Things Past*, and the quote cited appears on page 396 of volume 1, *Swann's Way*, (Everyman Collection, 1906) in the translation by Scott Moncrieff. Proust was one of the greatest French novelists of the19th-20th century, and his novel is the longest in human history!

felt were justified. With the involvement of applied researchers in different fields in the second part of the 20th century, chaotic dynamics became the next major development in nonlinear dynamics. Finally, in the 21st century, complexity dynamics has become the latest stage in the evolution of nonlinear dynamics. As in chaotic dynamics, it presupposes no detailed knowledge of the equations, and fosters and promotes multidisciplinary interactions across both organisation lines and traditional disciplinary boundaries. Complexity in some sense is like life: It is both complicated and we need to learn from previous experiences, reaching out for lessons. There are no general simple laws. Ideas and techniques learned in one system can be applied to others.

Technically, an important issue is the question of mathematical modelling. In some sense, mathematical modellers of complex system need to understand and take seriously the question of their own limitations. Modelling of complex systems is often not amenable to an exact analytical description. A model uses simplifying assumptions and approximations. Its success depends on its ability to make predictions. But can the solutions of the models of complex systems approximate the dynamics in natural or man-made systems? Many technical difficulties may impose severe limits to modelling. The same difficulties impose severe limits on the validity of numerical solutions of the underlying equations.

But what is the way out? Modelling must be expanded to include nonlinear time series analyses, to acquire knowledge from data. Of course, we have been doing that, developing more and more powerful techniques, such as the latest called compressive sensing, since the onset of chaotic dynamics. Instead of looking at modelling equations, we look at the solutions by observing the natural or man-made systems through a time series. We use nonlinear time series for the observation, analysis and understanding, prediction and control of the nonlinear dynamics, since the time-series trajectory reflects the true behaviour of the dynamics of the real system. In other words, the system itself is the best model, it contains all the imperfections and noisy inputs, including the correct initial and boundary conditions.

The idea behind the past and ongoing work and the evolution of nonlinear dynamics is that we do supplement the reductionist approach to science with an integrative agenda, not only to understand nonlinear dynamics, and now complexity, but use it to manipulate systems in science, engineering and medicine. Nonlinear dynamics is the cornerstone facilitating contacts and interactions among disciplines.

2. **Rama Govindarajan,** BANGALORE

Clouds

My view is of course coloured by what I now work on, but I do believe that the Earth system, and predictions of its future is among the most important nonlinear problems there is. I will discuss clouds in particular, where I believe there are many open questions waiting to be addressed.

We know that a majority of the Earth's surface is covered by clouds, which contribute the biggest uncertainties in climate predictions. How they are born, how they attain vast heights, how they evolve and die, how they organise themselves into continental scales and how these organised structures move, are just some of the open questions in this problem. We need to understand many of these basic things about clouds and their dynamics before we can fully address their response to and effect on a changing climate.

The monsoon is a special phenomenon occurring in only a few parts of the world. To understand the monsoon, we need to study deep convective clouds. One of the open questions in cloud physics is how raindrops form. The temperature falls as we move up in the Earth's atmosphere, and a rising parcel of air containing water vapour can become supersaturated at some height. The water vapour needs solid or liquid surfaces to condense onto. We know that aerosol particles act as cloud condensation nuclei. These are available in the upper reaches of the atmosphere, though at a concentration decreasing with height. By diffusion, water droplets, nucleating on aerosol particles, can grow to a size of about 10 micron in a short time. Cloud droplets which are a 50 micron in size or larger fall under gravity rapidly enough to collide with other droplets and coalesce into raindrops whose size is of the order of millimetres. How cloud droplets grow from 10 to 50 microns in a matter of a few minutes is an open question: this is termed the *droplet growth bottleneck*. Both diffusion and gravity act too slowly in this regime to explain the rapid growth.

Turbulence is thought to be an important reason for enhancing droplet growth rates in this size regime. Turbulence has a counter-intuitive effect on droplets. Imagine adding sugar to your tea, and stirring. In a short time the sugar, which was initially distributed inhomogenously, is caused by turbulence to mix evenly to achieve a uniform sugar concentration everywhere in the tea.

The opposite happens with droplets. Starting with a uniform concentration of droplets, turbulence causes them to leave regions of high vorticity and concentrate into regions of high strain. For droplets that have considerable inertia, most of the flow is rendered droplet-free, with droplets concentrate into evolving fractal-like shapes. Collision rates in these regions can be higher than would be expected *a priori*, and this can create large droplets quickly.

Our group has investigated how it is possible for tiny droplets to attain high enough inertia in order to arrive quickly in regions of high strain. They need to already be somewhat bigger to be able to to this. An interesting feature of the inertial dynamics of droplets is that they can form caustics: drops centrifuging out of two different vortical regions can collide at significant relative velocity. Only larger droplets are considered capable of participation in such caustics, and so this is not considered important for droplet growth. We have asked whether (a) caustic events can occur near a single vortex and (b) smaller droplets can participate in such events, so as to result in a few larger droplets that can then act as seeds for runaway growth. Note that one million ten-micron cloud droplets need to coalesce in order to make a single 1mm raindrop!

We have shown analytically, and by two-dimensional simulations of the Navier-Stokes equations that caustics can indeed form due to a single vortex [1], and that these single-vortex caustics can be an important cause of droplet growth. We find that the vicinity of a vortex can be divided into an inner (caustics) and outer (no caustics possible) region, with droplets vacating the inner region very rapidly. We showed [2] that caustics droplets have a far greater probability of participating in coalescence events and therefore displaying runaway growth, than droplets originating in the outer region. The structure of the equations is exploited to show that the divergence of the particle velocity field is large and negative in the inner, and small and positive in the outer region. This is an analytical way of showing that the inner region becomes droplet-free very rapidly. This is one way by which a few large droplets can form extremely quickly and these can then feel gravity. Whether this happens in a real cloud or not remains to be studied, by three-dimensional simulations and hopefully one day by direct measurements. We are also greatly interested in the interplay of fluid dynamics and thermodynamics in clouds, and study how it leads to altered turbulent dynamics. The interesting question here is how a dilute suspension of tiny droplets can change the life-times and behaviour of large vortices.

Fluid mechanics offers many more open problems in nonlinear dynamics. Happy studies!

3. **Thilo Gross,** BRISTOL

Nonlinear dynamics just got bigger!

Revolutions are events only in retrospect, but gradual processes when experienced. Presently nonlinear dynamics is undergoing a revolution that was triggered by the rise of the binary star of network and data science. The outcome of this revolution might well be a golden age of dynamics.

Consider the natural course of scientific revolutions. The advance of nuclear technology in the beginning of the 20th century or the rapid development in life sciences of the 1990's started with new technology leading to an explosion of available data. The momentum was eventually picked up and carried on by modelling and theory. Today we experience a rapid growth pervasive computing and digital connectivity which is leading to an explosion of new data in all areas of human activity. Harnessing this momentum we can create an age of "big models".

The direction in which nonlinear dynamics has grown is unexpected. Years ago one could have guessed that the growth would happen toward higher-dimensional dynamics and the goal of understanding high-dimensional chaos and turbulence. Instead much progress is now being made on low-dimensional dynamics in high-dimensional systems. In big models even the stability of stationary states [3] and the almost-stationary dynamics of phase oscillators hold new and fascinating secrets [4, 5].

If viewed through old lenses we may well ask, what is actually new? After all most of the questions in big models are old ones: When is a system stable? When is it controllable? However, to answer these old questions in big models properly, new challenges need to be overcome. It is not enough anymore to express the answers in terms of microscopic variables or parameters. In big models, we are mostly looking for answers that are on the scale of the model and not on the scale of its constituent parts. To formulate answers on the desired level of description additional mathematical thought is often necessary.

The language of networks has given us the vocabulary to talk about huge systems comprising thousands or millions of variables. While studies in networks were initially focused mostly on structure, dynamical questions increasingly come to the fore. Additionally network science is presently in the process of shedding its discrete

heritage, and increasingly embraces continuous variables and continuous time. Many network models now include dynamical rules governing dynamics on the network, dynamics of the network, or both. Thus many dynamic networks that are studied today are nothing but big dynamical models.

In the analysis of big models, questions about the dynamics of small models reappear, sometimes in new forms. Let me illustrate this with three examples from my own work. One approach to understanding the dynamics of huge networks are so-called moment expansions [6]. These approximations describe the abundance of certain motives in the network. Simple moment expansions lead to small systems of differential equations that can be studied with standard bifurcation theory [7]. When and why these approximations work is presently unclear, and resolving this issue will likely involve understanding the timescale separation between the dynamics of small and large motifs.

More sophisticated moment expansions keep track of more motifs, leading to large, and sometimes infinitely large, dynamical systems. However, using an exact transformation it is possible to translate those infinitely large differential equation systems into finite dimensional partial differential equations in an abstract space [8]. For instance common network models of epidemic spreading or political opinion formation are thus reduced to a two-dimensional PDE in two-dimensional space. Using PDE theory for instance to understand the information flow in these models could lead into deep insights into the nature of the underlying networks.

Finally, a persistent question in many networks concerns the localization of dynamics. When will dynamics of big models stay confined to a small part of the system? In recent work we showed mathematically that it is possible to pinpoint certain network motifs as sources of dynamical instability [9], create motifs that trap certain dynamical instabilities [10], and understand the emergence of localized patterns [11].

The recent progress in this field of network dynamics gives me hope that the big models of the future will not be monstrous simulation codes, incomprehensible to any human. Instead, they may turn out to be relatively small systems of equations, that describe the dynamics of big mathematical objects such as large operators or high-dimensional functions. While more complicated than dynamical systems acting on variables that are just numbers, these big models of the future will still be explorable using (to some degree rigorous) analytical thought.

4. M. Lakshmanan, Tiruchirappalli

Outlook

It is extremely hazardous to speculate how a research field will evolve as time goes by. Particularly so for an interdisciplinary field like nonlinear dynamics. I still wish to present a few potential problems and possible developments in the next decade or so in this field. This is purely from a personal perspective.

4.1 *Integrability/nonintegrability*

One problem that remains not completely clarified for a long time is what is integrability, when does it arise, what distinguishes these systems with near-integrable ones and chaotic systems and so on. One possible definition is integrability in the complex time plane/manifold of independent variables. Solutions of nonlinear differential equations (ordinary, partial, difference, differential-difference, etc.) in general admit movable singular points/manifolds, besides fixed singular points/manifolds (dictated by the form of the equations as in the case of linear systems). Of these, movable poles generically lead to integrable systems while more complicated singular points like movable branch points (both algebraic and logarithmic) and essential singularities seem to indicate nonintegrability and possibly chaos. While such movable singular points are arbitrary and can be placed anywhere in the plane, they can be related to initial conditions. The crucial question is that the concepts such as integrability, nonintegrability, bifurcations and chaos which are related to insensitivity or sensitivity to initial conditions on the real time axis, how can they be related to the type of movable singular points which occur in the complex time plane/manifold. A clear understanding of these aspects can lead to much clarity about nonlinear dynamical systems and will help in the classification of nonlinear systems.

4.2 *Basic nonlinear excitations in higher spatial dimensions*

While integrable nonlinear dynamical systems in one spatial and one time dimensions are often characterized by localized structures, namely solitons, the important questions is what are their counterparts in higher spatial

dimensions, namely (2+1) and (3+1) dimensions? Does one have similar stable structures in physically interesting models like sine-Gordon equation, nonlinear Schrödinger equation, Heisenberg ferromagnetic spin system, ϕ^4 system and so on? Are these stable or metastable? What are their collision properties? How do additional forces affect these structures? Does any characteristic spatiotemporal pattern arise as the solution of the Cauchy initial value problem? Can one obtain sufficient information about the nature of general excitation through numerical analysis? One is familiar with certain basic excitations like vortices, monopoles, plane wave solutions, etc. in higher dimensions. Similarly, a class of nonlocal generalizations are known to admit exponentially localized soliton-like structures (termed dromions) in (2+1) dimensions. Do such structures survive in physically interesting (2+1) dimensional systems? These are some of the challenging problems in higher spatial dimensions.

4.3 *Network dynamics*

To improve our understanding on complex systems and the observed intricate dynamics in these systems, it is essential to move beyond simple networks and to investigate multi-layered networks of nonidentical systems. The dependence of collective dynamical states over various connection topologies are central towards this investigation. A control-theoretic perspective of these networks is also useful and it enables one to find efficient mechanisms that makes stabilization of desired states or destabilization of the undesired ones. To determine the control strategies, the roles of feedback, delay and external stimulus over these networks have to be studied.

4.4 *Brain dynamics*

A long standing question on complex networks that persist over years is on the brain dynamics and how the information processing is done efficiently in the intricate network of neurons. How the informations of different tasks are coded in terms of the neural synchrony and how the brain shows faster switching between different states or synchrony patterns with respect to different events or tasks are the important problems which have to be dealt with. The observed chaotic dynamics during the transitions to different states put forth the question on the role or requirement of chaos in information processing. The need of dynamic coupling or synaptic plasticity over the neural dynamics and its faster switching between states have to be understood.

4.5 *Nonlinear $\mathcal{PT}$-symmetric systems*

Controllable and robust confinement of light is central to many applications including diffractionless long distance light propagation, imaging and so on. In this connection, the newly explored nonlinear $\mathcal{PT}$-symmetric systems enable unusual and elusive light propagation features including non-reciprocal transport, unidirectional invisibility, double refraction and band merging. Due to these facts, the light propagation and control in $\mathcal{PT}$-symmetric lattices have to be explored in detail. For this purpose of localization of light, defects also play a vital role and so the effects of $\mathcal{PT}$-symmetric or non-Hermitian defects in Hermitian lattices or effects of Hermitian/non-Hermitian defects in non-Hermitian lattices have to be analyzed. The recently explored flat band systems show interesting compact localization of light and enable dispersionless propagation of light and so it is also of great interest to combine the features of $\mathcal{PT}$-symmetric systems with the flat band structures to enable versatile control over light propagation in photonic lattices or waveguide arrays.

4.6 *Nonlinear dynamics of ferromagnetism*

An area of great potential application is the study of influence of nonlinearity in ferromagnetism. The underlying Landau-Lifshitz-Gilbert-Slonczewski nonlinear evolution equation is effectively a master equation which encompasses many of the integrable and nonintegrable nonlinear dynamical systems. The study of the above equation at different levels, macro, nano and micro levels, can not only lead to great understanding of the basic aspects of nonlinear dynamics but also can lead to the design of new devices such as spin transfer nano oscillators (STNOs) useful for magnetic switching and microwave generation. Dynamics of networks/arrays of such devices through collective states such as synchronization can lead to the development of useful devices.

I expect considerable developments in each of the above lines of investigation in the next decades, leading to substantial understanding and application of nonlinear dynamics.

5. Gabriel B. Mindlin, Buenos Aires

Where and how is the field of nonlinear science evolving

Biologically inspired problems open great windows of opportunity to a dynamicist, as well as posing deep challenges. A biological problem has to be framed within the theory of evolution, what entails a profound level of complexity. This makes it hard to identify the basic mechanisms behind the phenomenon under consideration, the core strategy followed by dynamicists to study a problem.

In fact when a dynamicist starts interacting with biologists, it rapidly gets complicated. And many times, frustrating. A first thought is that what lies at the root of the difficulty is the biologist's lack of mathematical background, or the dynamicists lack of knowledge of the biology. In fact, it is even more difficult: what it means to be rigorous is what is different in the two communities. A dynamicist searches for depth in the identification of the minimal mechanisms at play in a given problem. A biologist, on the other hand, needs to frame the problem under study within its evolutionary context. In this way a biologist will be by default suspicious of simplifications, just as a dynamicist will lose interest in a problem as "details" enter the picture.

Having warned of all these complications, biology is full of problems, which not only require good dynamics for their understanding: they can be a source of extremely deep questions as well. How to establish the connections between the different description scales of out-of-equilibrium systems? This question, dynamical and statistical in nature, is at the core of the endless collection of phenomenological models that attempt describing a living system. Understanding these changes of scales is particularly relevant in neuroscience, where the central nervous system is in charge of the physiological instructions that control peripheral systems and, therefore, of the interaction of the organism with the macroscopic world. As there is no comprehensive theory to deal with out-of-equilibrium statistical mechanics, macroscopic models of nervous systems are usually built phenomenologically and not statistically. Having said that, there have been very important advances recently [12].

Besides these ambitious questions, it is important to notice that since behaviour emerges when a nervous system interacts with a peripheral biomechanical device and the environment, there is in biology a place specifically saved for dynamics: how much of the complexity observed in a give behaviour is due to the complex instructions originating it, and how much due to the complex response that a (nonlinear) biomechanical device might display after driven by eventually simple instructions?

Even if biologically inspired problems can trigger pure dynamical or statistical questions (which require real good old fashion "theory"), it is difficult to advance in the analysis of a biological problem without a continuous dialogue between dynamics and biology. Modelling without a strong link to experiments or biological background runs the risk of sinking in irrelevance. Or in plain nonsense: since different biological mechanisms can be described in terms of the same dynamical elements, modelling will always require a continuous dialogue between dynamics and experiments.

The time of ultra-simplified models is long gone, and the candour of the pioneer modellers has to evolve into a respectful attitude towards other disciplines like biology, which have other standard and expectations rooted in their culture.

6. Rajarshi Roy, College Park

Complexity, photons and randomness: A personal perspective

The waxing and waning of research areas and topics has been observed by historians of science and impacted the lives of scientists. As a young scientist learning about nonlinear dynamics and chaos in the early eighties (1980s, that is) I was often exhorted by my senior colleague Joe Ford at Georgia Tech to put my knowledge of optics to some use in basic science, and not to restrict my efforts in research to try and invent the next version of a better "mouse-trap". Joe was tireless in his efforts to convince people that a new area of science was in the process of discovery, development and formation. He was preoccupied with trying to understand how quantum mechanics and nonlinear dynamics were related — to find model systems to study that helped us to understand the roots of randomness, complexity and determinism — and their relationship to each other. I was inspired by an article that

he wrote for Physics Today, titled "How Random is a Coin Toss?" [13]. Joe passed away in 1995[2] and I moved from Tech to Maryland in 1999, where a few years later Atsushi Uchida, a young postdoctoral associate from Japan joined our laboratory.

Of the many twists and turns that my own research has taken over the years (and that is primarily all that I am able to comment about coherently, with the myopia that is typical of scientists sometimes), one topic that I am currently fascinated by is the generation of random numbers by physical dynamical systems. It brings the origins of chaos, quantum phenomena and information theory into perspective through the use of modern optical and electronics technology, with applications to Monte Carlo simulations and cryptography, in a very unique way. We have tried to explore this area — that was brought to my attention almost a decade ago by (now) Professor Uchida who is a close friend and colleague. We met at breakfast in Brussels[3] where he told me about recent work on using chaotic semiconductor lasers for generating random numbers at speeds that were three orders of magnitude faster than possible at the time [14, 15].

Fast forward to 2017: we have just collaborated on a paper that examines different techniques for generating random numbers and utilizing algorithms and techniques based on information theory to study entropy generation by dynamical systems. These are real experimental systems — that produce reams of data — and our interest at this time is in learning how much entropy we can harvest from dynamical systems. The very nature of experimental measurements, the influence of microscopic quantum noise on macroscopic chaotic systems, photon counting, time-delayed dynamics and embedding . . . so many of the basic ideas that I learned about all through life seemed to converge in this area of study [16, 17]. Then I began to learn about how the human brain evaluates and assesses randomness. So I have to confess that nonlinear dynamics continues to captivate and draw me in with new problems, possible applications to technology, biology and living organisms, new techniques of measurement and numerical simulation . . . life is never boring or stagnant when one studies nonlinear dynamical systems that are constantly changing, evolving and reinventing themselves.

7. K.R. Sreenivasan, NEW YORK

My journey through nonlinear science

Professor Ramaswamy asked me to write a personal account of nonlinear science. I agreed because I have known and liked Ram for many years and could very well not say "no" to him. He charmingly stated that my account could be "as informal as you like, with or without references, and with or without a signature, and as long or as short as you want". I never had this much license from any editor—so I thought, why not? I will sign my note, indeed add some references, and keep it short—but it will be definitely personal and informal.

My advanced education was in fluid dynamics, so most problems I studied were inherently nonlinear. I learnt similarity methods and their approximate solutions by regular and singular expansions, as well as numerical methods (with, of course, experimental insight). The only way one builds up one's intuition in nonlinear problems, I thought, was to solve a number of specific problems and sensibly extrapolate from one's accumulated knowledge. I emphasize the word "sensibly" because, unfortunately, extrapolations in nonlinear domain are not always successful, and the trick is usually to linearize the problem around a "right" linear state. Using these traditional skills acquired, I did some interesting work here and there.

I also learnt, as a student, a moderate amount about turbulence. In particular, I learnt that initial conditions have sensitive effect on flow details (especially of the transitional ones), but the conventional wisdom was that small changes in initial conditions would not affect the average properties of a turbulent flow (but large changes could do that too). I learnt about scaling exponents and scaling functions via Kolmogorov's phenomenology of turbulence [18]—though Kolmogorov himself didn't use those terms, nor did the people from whom I learnt the subject. By the way, very similar arguments as Kolmogorov's had been used a few years earlier [19] in deriving the well-known logarithmic law in turbulent wall flows, but this work did not blaze a trail of the sort that Kolmogorov's theory did.

The real pleasure I derived through my work in fluid dynamics was the opportunity it afforded me in getting to know first-rate people in fluid dynamics, applied mathematics and stochastic problems. Listed in roughly the

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[2]For a short biography of Joseph Ford, see http://www.cns.gatech. edu/people/ford/index.html
[3]This was at a Solvay Workshop on *Bits, Quanta, and Complex Systems* held in Brussels, April 30 - May 3, 2008

order in which I got to know them, some of them are: Roddam Narasimha, Leslie Kovasznay, Stanley Corrsin, Stephen Davis, George Veronis, Ira Bernstein, Hans Liepmann, Daniel Joseph, Steven Orszag, Martin Kruskal, Russell Donnelly, Harry Swinney, Uriel Frisch, Willem Malkus, Philip Saffman, Andrew Majda, Joseph Keller, Peter Lax, Akiva Yaglom, Keith Moffatt, Lawrence Sirovich, Grisha Barenblatt and quite a few others besides—not to mention several highly talented younger people with whom I had the privilege to work.

All this is way of some background. And don't be put off by the fact that I have dropped a number of names; indeed, I will drop more of them below but you will see towards the end that I have a purpose.

The revolution that Ed Lorenz created in nonlinear dynamics [20] took almost twenty years to reach me. That I had just gotten out of high school in 1963 was one reason; but another important reason was that much of my fluid dynamics circle at the time was essentially oblivious to the importance of Lorenz' and related work, of which I became aware in early 1980's when I had already been teaching for a few years at Yale. In fact, I came across an important body of work via an experimental paper by Jerry Gollub & Harry Swinney [21]. Thereafter, I quickly read with fascination the papers by David Ruelle and Floris Takens [22], Mitchell Feigenbaum [23], T.-Y. Li & James Yorke [24], Jean-Pierre Eckmann & David Ruelle [25], etc. Thanks to later papers by Yorke, Edward Ott and Celso Grebogi [26, 27], and my interactions with Barry Saltzman at Yale, made my entry into the subject much easier. I got to know most of these people well, and [28, 29, 30] are examples of my work at the time.

I came to fractals [31] via chaos. A blob of initial conditions on an attractor gets stretched in some directions and contracted in others by chaotic dynamics, leading to the formation of filaments. Because the trajectories are bounded, the filaments have to fold on themselves eventually. As is well known, a repeated sequence of stretching and folding yields a fractal attractor. There are many fractal attractors in fluid dynamics [32], and several geometrical objects of turbulence in real space are fractals (with appropriate cut-offs). I spent a few years trying to understand the relevance of fractals and multifractals to turbulence in fluid flows [33, 34]. This endeavor too brought me close to a group of extraordinary people such as Benoit Mandelbrot, Leo Kadanoff, Michael Fisher, Pierre Hohenberg, Joel Lebowitz, Itamar Procaccia, Albert Libchaber, and many others.

What my foray into these topics did was to get me connected to first-rate people, some of whom I have listed already. These people did not necessarily push me in new directions, and I hardly pursued anyone else's dreams; their influence was subtler. I began to think about science a little differently: they brought me back closer to statistical physics and mathematics (topics I had studied during my graduate and post-doctoral days), and lent strength to my belief of earlier days that pure thought could be powerful. It was somehow exciting to learn that systems that were unpredictable possessed much richer structure than random systems. Some of the meetings organized by Robert Helleman on nonlinear dynamics were quite exciting: super-specialization was being shattered in scientifically delightful ways. Robert would himself chair all the sessions, as I recall, and, if a talk was interesting, allow it to continue well beyond the scheduled time—but he would stand up early during a boring talk and obnoxiously ask, "How much more stuff do you have?" I learnt a lot during those years.

Yet, I don't think that nonlinear dynamics changed my view of the physical world in fundamental ways because I never took Laplacian determinism too seriously. I had once, ca. 1972, given a talk on determinism and free will to my fellow graduate students (and one or two faculty members), and had at that time concluded, quite unambiguously in my mind, that determinism was a limited world-view. It took little by way of contemplation to conclude that statements such as "the flap of a butterfly's wings in Brazil can set off a tornado in Texas" [35] and "chaos was the third great revolution in the physical sciences, besides relativity and quantum mechanics", [36] were likely the result of overabundant enthusiasm than substance. But I was open-minded enough to know that there was much to explore in huge areas of nonlinear science with this new outlook on dynamics, certain types of transitional flows included.

Through all this, the notion that captured my imagination was universality. I came across this notion first in Kolmogorov's work and more of it via Kadanoff's papers [37] on critical exponents (which led me to back-track and learn about Onsager's work, Gibbsian statistical mechanics, the renormalization work of Michael Fisher and Ken Wilson, etc). It was obvious that the scaling discussed in critical phenomena, multifractals, nonlinear dynamics were all connected entities, and several seemingly disparate subjects could be seen as parts of a larger perspective. The real issue became one of understanding how the scaling exponents and scaling functions behave (if they exist) in intermittent systems with strong fluctuations that drive a system almost always from equilibrium. This problem has occupied my time for many years now; these days, I think about them primarily with Victor Yakhot [38], and through him have had some connections with extraordinary people like Alexander Polyakhov.

My interests have shifted in recent years [39, 40, 41], but the excitement that nonlinear dynamics brought into my scientific life is still strongly alive. In recent years, I keep up with aspects of nonlinear science partly through *Journal of Nonlinear Science* that I edit with my colleague Paul Newton. Where the field of nonlinear science is going is a question I ask myself constantly; although it is posed concretely in the immediate context of JNLS, it often transcends that constraint. There are, of course, the bread and butter aspects of "modern" nonlinear dynamics: low-dimensional maps, ODEs, stability, control of chaos, predictability, pattern forming systems, etc. There are also the traditional areas of mechanics, particularly geometric mechanics (of systems whose configurational space possesses certain group structure) and the control theory for such systems—and, of course, fluid transport, nonlinear waves and turbulence. A great variety of subjects in material science, such as nanostructures, thin films, optical materials, crystals, and smart structures can be studied from the point of view of nonlinear science. Coming from another direction, there is interesting work in non-equilibrium statistical mechanics and stochastic models of nonlinear phenomena (including nonlinear optics). But it appears to me that the field has exploded and moved on past the frame of reference of some 40 years ago when I entered nonlinear dynamics. Besides the topics just mentioned, more and more of nonlinear science is being filled by applications to biology and medicine, at both cellular and systems level, especially in areas connected to neuroscience and network theory; atmospheric dynamics and climate modelling; graph theory; game theory; finance systems, etc.—all of that somehow under the ubiquitous title of complexity. All of them use tools and concepts from the early years of nonlinear dynamics. This ubiquitous nature of nonlinear science makes it difficult to contain it within artificial boundaries, but it also gives it less identity.

I dropped many names along the way and want to explain why. My point is that it takes many people to build one broad-based scientific career. I have listed some of the people who created mine. I also came to know many more illustrious people in the last 15 years or so, and consider myself very fortunate: many first-rate people who knew parts of my work saw something positive in it. To the young readers of this article, I would thus say that it is important to get to know positive people, most of whom in my experience are successful themselves, and avoid the disillusioned few who think that negativity towards others is the necessary ingredient in maintaining high standards. I know a few such people myself (fortunately not many) and they have generally been a drain on my time, energy and outlook. Such people forget that progress in science takes many forms and needs many types of people: some begin new areas, some say the last word; some are encyclopædic and some make hitherto unseen linkages. Regretfully, great and original ideas do not appear every day—but even wrong ideas can sometimes provoke useful science.

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[35] The story I have heard is that when Lorenz failed to provide a title in time for the talk he was to deliver at the 139th meeting in 1972 of the American Association for the Advancement of Science, Philip Merilees concocted the title, "Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?" I have read Lorenz's abstract since then, and found it to be illuminating.

[36] In his book *Chaos: making a new science* (Penguin Books New York, 1987), James Gleick recalls a statement by an unnamed physicist: "Relativity eliminated the Newtonian illusion of absolute space and time; quantum theory eliminated the Newtonian dream of a controllable measurement process; and chaos eliminates the Laplacian fantasy of deterministic predictability". I heard this first from Joseph Ford who regarded himself as an evangelist of chaos, well before his article "What is chaos that we should be mindful of it?" appeared in *The New Physics*, ed. P W Davies (Cambridge University Press, 1989).

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