A Review of Choice-based Revenue Management: Theory and Methods

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Abstract

Over the last fifteen years, the theory and practice of revenue management has experienced significant developments due to the need to incorporate customer choice behavior. In this paper, we portray these developments by reviewing the key literature on choice-based revenue management, specifically focusing on methodological publications of availability control over the years 2004-2017. For this purpose, we first state the choice-based network revenue management problem by formulating the underlying dynamic program, and structure the review according to its components and the resulting inherent challenges. In particular, we first focus on the demand modeling by giving an overview of popular choice models, discussing their properties, and describing estimation procedures relevant to choice-based revenue management. Second, we elaborate on assortment optimization, which is a fundamental component of the problem. Third, we describe recent developments on tackling the entire control problem. We also discuss the relation to dynamic pricing. Finally, we give directions for future research.

Keywords:

Revenue Management, Customer Choice, Availability Control, Capacity Control

1. Introduction

Revenue Management (RM) refers to the theory and practice of IT-supported management of demand by means such as prices or product availability based on demand models so as to maximize profits or revenue. As a discipline, it originated in the airline industry in the 1970s following the deregulation of the US airline market. The term RM was coined

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because, in the airline sector, low variable costs and high fixed costs lead to an environment where maximization of revenue is (almost) equivalent to maximizing profits. However, RM techniques are not limited to revenue optimization, and indeed are also used for maximization of other objectives such as profit. RM can be delineated from more general pricing practice by its use of often highly sophisticated computer systems that automatically process sales and other data to produce demand forecasts, which in turn are used to optimize demand management decisions. Since its conception, it has seen widespread adoption in many areas, including transportation (trains, car rental, ferries, cargo shipping), hospitality (hotels, casinos), broadcasting and advertising, and others.

A lot of research has been dedicated to this field over the past 40 years; for relevant literature reviews see McGill and Van Ryzin (1999) and Chiang et al. (2007), for relevant textbooks see Talluri and Van Ryzin (2004), Phillips (2005), or Bodea and Ferguson (2014). Until around 2004, most of it was based on the assumption that demand for any product is independent of the availability of other products. In other words, it was assumed that customers would never substitute one product for another, but instead would consider the purchase of a specific product only and, if this was not available, not to purchase at all. This so-called independent demand assumption was and still is reasonable in quasi-monopolistic situations where different product offerings are strongly fenced off. For example, in the airline industry, different types of fares used to be associated with specific restrictions (cancelation policy, advance purchase requirement, . . .) that were intended to appeal to a specific customer segment only. However, with the rise of low-cost carriers and the advent of online travel agencies like Expedia or Orbitz by the early 21st century, the independent demand assumption turned out to be problematic due to the increased competition and visibility of different fares.

Consequently, the focus of this review is on the design and estimation of discrete choice models for RM, and on the problem of how to control demand via product availability over a finite period of time while accounting for customer's choice behavior. The most recent comprehensive review in this field is by Shen and Su (2007), who provide an overview of research on customer behavior in RM and auctions. More specifically, they partition work in the area in those papers dedicated to inter-temporal substitution and strategic customer purchase behavior, and those focusing on multi-product substitution. We exclude the former from our review and only take multi-product substitution into account. The relevant section on multi-product substitution in their paper covers the period from the seminal paper of Talluri and Van Ryzin (2004) to 2007. We include these works and expand the review to developments in choice-based RM up to and including 2017. Note that Weatherford and Ratliff (2010) review the literature on RM under dependent demand; however, they focus on heuristics specifically designed for airlines' legacy booking systems. Our review has a wider scope and provides an overview of general methodological developments beyond specific applications.

Our review is structured as follows: In Section 2, we begin by presenting a general formulation of the choice-based network RM problem of availability control. Based upon this, we dedicate the following three sections to the three main challenges arising in the context of this problem. In particular, in Section 3, we discuss the demand modeling side by reviewing the literature on customer choice models that have been integrated in the model so far, along with appropriate estimation methods that have been developed. Second, in Section 4, we discuss different approaches of assortment optimization, which forms a significant part of the overall problem and which is closely related to the integrated choice model. Third, in Section 5, we focus on the literature regarding the entire dynamic control problem, focusing specifically on different approximations that have been developed. In Section 6 we conclude the review by giving an outlook on potential future research opportunities.

2. The choice-based network RM problem of availability control

In this section, we first provide a general description of the availability control problem in §2.1. We then present a formulation of the resulting choice-based network RM problem as a stochastic dynamic program in §2.2. Three main challenges in solving the problem are identified in §2.3. Each of these challenges is then addressed in Sections 3 to 5 in detail.

2.1. General description

During a selling horizon (also called booking horizon), a firm offers products out of a finite set to heterogeneous customers arriving over time. Most commonly, the products correspond to services that have to be delivered after the selling horizon. In general, prices (revenues) of the products and capacities for providing the services will be fixed in the short-term. Furthermore, capacities are perishable and often come with high fixed costs, whereas the marginal costs resulting from selling an additional product unit are rather low. With such a cost structure on hand, revenues can be used as proxies for profits.

Facing such a situation, the firm's goal consists in maximizing its overall total revenue by performing revenue management. Since heterogeneous customers will have different preferences concerning the products, they will also have a different willingness-to-pay and, in the end, will make different choices depending on what products they are offered. Therefore, the firm has to manage the sales process during the selling horizon appropriately, which is the task of availability control. It consists of deciding which products to offer at which point of time in the selling horizon.

To illustrate, in car rentals a product could be a specific car type to be rented out over a number of specified days at a certain price at a particular station. Each rental day represents a resource with capacity equivalent to the inventory of that car type, so a multi-day rental consumes multiple resources. In the airline industry, a product is usually a ticket for the desired itinerary in a certain compartment, linked to a specific fare and booking class which may come along with booking restrictions, e.g. minimum stay restrictions. Each compartment on each point-to-point flight (called a leg) in the airline's network on the considered day of service represents a resource with a certain seat capacity. Each product requires one (non-stop flight) or several (stop-over flight) of these resources. Obviously, in both examples, the resources are perishable and their provision comes with high fixed costs.

On the demand side, customers could have different preferences, e.g., depending on the purpose of their travel. A common distinction in the airline industry is among business travelers and leisure travelers. When these groups of customers are offered the same set of products by the firm, they will usually make different choices. Furthermore, if their preferred product is not available, they may show a different substitution behavior, i.e., switch to different products or not purchase at all.

To reflect the characteristics of the supply side, the term "network RM" is often used, whereas for the demand side the term "choice-based RM" is common. To conclude the general description, let us briefly provide some background on common terms in RM. When the practice of RM emerged in the airline industry of the 1970ies, price changes were difficult to implement and hence booking limits were introduced to control different fare classes. In other words, capacity (as opposed to prices) is controlled, and hence this is referred to as capacity or inventory control. In the literature, control problems of this type usually assume that we set booking limits or accept/reject individual demand requests.

In this review, we focus on "availability control", which refers to deciding on the set of available product alternatives from which the customer may choose. The main difference is that the terms capacity control/inventory control tend to be used in the context of independent demand (i.e. independent of available product alternatives), whereas the term availability control is usually used in the context of dependent demand. We regard both capacity/inventory control and availability control as being part of "quantity-based" RM, in contrast to dynamic pricing or "price-based" RM where the firm changes prices of products to manage the sales process. However, there is some ambiguity in the literature on the use of these terms, especially because there is some overlap (e.g. pricing on a discrete set of price points can be framed as special case of availability control).

Finally, the term RM may encompass further instruments which can be found in service industries. For instance, in the airline industry, customers may cancel their flights and, hence, overbooking may be applied which consists in selling more tickets than available capacity anticipating future cancelations.

2.2. Formulation as a stochastic, dynamic program

More formally, the choice-based network RM problem that we consider can be formulated as follows. A firm sells products over a finite time horizon consisting of T time periods, indexed by t, starting at the beginning of period $t = 1$ and ending after period $t = T$. The time periods are assumed to be sufficiently small such that the probability of more than one customer arriving within a single time period is negligible.

The set of products is denoted by $\mathcal{J} := \{1, \ldots, J\}$. Each product $j \in \mathcal{J}$ has a fixed revenue r_j associated with it. For the given set of resources $\mathcal{I} := \{1, \ldots, I\}$, the resources needed for any product are defined in a matrix $A \in \mathbb{N}_0^{I,J}$, where $a_{ij} = b$ if product j uses b units of resource $i \in \mathcal{I}$. Accordingly, the j-th column vector $\mathbf{A}_j$ represents the overall resource consumption of product j. At the beginning of period t, we have an available inventory of $\mathbf{c}_t \in \mathbb{N}_0^I$, so $\mathbf{c}_1$ denotes the vector of initial capacities. The set $\mathcal{J}(\boldsymbol{c}_t)$ contains all products that can be feasibly offered given the remaining available capacity: $\mathcal{J}(\boldsymbol{c}_t) := \{j \in \mathcal{J} | \boldsymbol{A}_j \leq \boldsymbol{c}_t\}$. The so-called offer set $S \subseteq \mathcal{J}(\boldsymbol{c}_t)$ consists of all products that the firm makes available to customers in period t.

The customer population is assumed to consist of $\mathcal{L} := \{1, \ldots, L\}$ segments. Each segment $l \in \mathcal{L}$ is characterized by an arrival rate, a consideration set, and a model that governs customers' product choices for a given offer set. We define these characteristics in turn, starting with the arrival rate. The probability that a customer arrives in time period t is assumed to be fixed at λ; this is to simplify notation, but our approach can be generalized to time-heterogeneous arrival rates. With probability p_l, a customer who has arrived belongs to segment l, so that we can define a segment-specific arrival rate $\lambda_l := p_l \lambda$. The consideration set $C_l \subseteq \mathcal{J}$ represents the set of all products that a customer of segment l would consider for purchase if they were to be offered. Two consideration sets l and l' are said to be overlapping, if $C_l \cap C_{l'} \neq \emptyset$, and disjoint, otherwise. For a given offer set S, a customer from segment l buys product $j \in S$ with probability $P_j^l(S)$ if $j \in S \cap C_l$. Overall, the firm sells product $j \in S$ when offering the set S with probability $P_j(S) = \sum_{l \in \mathcal{L}} p_l P_j^l(S_l)$. The probability that there is no sale is accordingly $P_0(S) = 1 - \sum_{j \in S} P_j(S)$. The values of the probability parameters are usually obtained from some theoretical choice model which is calibrated on real-word data, but which we will leave unspecified at this stage.

Within availability control, the firm has to determine the offer set S in every time period $t = 1, ..., T$ of the selling horizon so as to maximize expected revenue over the remaining booking horizon. The corresponding optimal policy for the entire network RM problem can, in theory, be obtained by a stochastic dynamic program. Although intractable, we do nevertheless state it (as is commonly done in the literature) so as to have a conceptual reference point. Let the value function $V_t(\boldsymbol{c}_t)$ denote the maximum expected revenue that can be earned over the remaining time horizon $[t, T]$, given remaining capacity $\boldsymbol{c}_t$ in period t. Then $V_t(\boldsymbol{c}_t)$ must satisfy the Bellman equation

$$V_t(\boldsymbol{c}_t) = \max_{S \subseteq \mathcal{J}(\boldsymbol{c}_t)} \left\{ \sum_{j \in S} \lambda P_j(S) \Big(r_j + V_{t+1}(\boldsymbol{c}_t - \boldsymbol{A}_j) \Big) + \Big(\lambda P_0(S) + 1 - \lambda \Big) V_{t+1}(\boldsymbol{c}_t) \right\}, \qquad \forall\, t, \forall\, \boldsymbol{c}_t, \tag{2.1}$$

with the boundary condition $V_{T+1}(\boldsymbol{c}_{T+1}) = 0$ for all $\boldsymbol{c}_{T+1}$. Let $V^{DP} := V_1(\boldsymbol{c}_1)$ denote the optimal value of this dynamic program from 1 to T, for the given initial capacity vector $\boldsymbol{c}_1$.

Equation (2.1) is the standard formulation of the choice-based network RM problem of availability control. Let $\Delta_j V_{t+1}(\boldsymbol{c}_t) = V_{t+1}(\boldsymbol{c}_t) - V_{t+1}(\boldsymbol{c}_t - \boldsymbol{A}_j)$ be the opportunity cost associated with selling product j, which represents the expected loss of future revenue due to selling product j. With these values on hand, we can re-write equation (2.1) to yield:

$$V_t(\boldsymbol{c}_t) = \max_{S \subseteq \mathcal{J}(\boldsymbol{c}_t)} \left\{ \sum_{j \in S} \lambda P_j(S) \Big(r_j - \Delta_j V_{t+1}(\boldsymbol{c}_t) \Big) \right\} + V_{t+1}(\boldsymbol{c}_t), \qquad \forall\, t, \forall\, \boldsymbol{c}_t. \tag{2.2}$$

2.3. Challenges in solving the choice-based network RM problem

By taking a closer look at equation (2.2), the challenges to solve the described choice-based network RM problem and to obtain an implementable policy can be elicited.

First of all, we have to determine the arrival rate λ and the probabilities $P_j(S)$ of choosing a certain product j depending on the offer set S. This requires specifying a suitable demand model as well as estimating its parameters from booking data observed in the past. Hence, in Section 3 we will present different demand models of the discrete choice type and discuss approaches for their estimation.

Next, for each state $\boldsymbol{c}_t$ in period t, all possible actions have to be evaluated. In our context, an action consists of specifying an offer set S. Since there are $2^{|\mathcal{J}(\boldsymbol{c}_t)|}$ possible sets in period t, this may not be done by brute force. Instead, in order to cope with the possibly large action space, an optimization problem has to be solved, which is closely related to assortment optimization for physical retail shelves, or for products dynamically generated and displayed on a web page. Indeed, if the opportunity costs $\Delta_j V_{t+1}(\boldsymbol{c}_t)$ in (2.2) are assumed to be known, each single availability control decision in a period t boils down to an assortment optimization problem, where a revenue maximizing assortment with revenues $\tilde{r}_j = r_j - \Delta_j V_{t+1}(\boldsymbol{c}_t)$ has to be determined. For that reason, we will also review some of the assortment optimization literature in Section 4. However, we concentrate on those methodological papers that are of importance to the RM setting of controlling demand over time.

Finally, the value function $V_t(\boldsymbol{c}_t)$ or the opportunity cost $\Delta_j V_{t+1}(\boldsymbol{c}_t)$ have to be computed and stored for all periods $t = 1, \ldots, T$ and possible combinations of remaining capacity $\boldsymbol{c}_t$, that is for all possible states. This may again be prohibitive due to the resulting size of the state space, which grows exponentially in the number of resources I. Indeed, we emphasize that the dynamic programming formulations (2.1) and (2.2) are intractable for all but the smallest problem instances even under the assumption of independent demand (see Section 1), which is the most simple demand model. Therefore, much of the literature is focused on how to approximate $V_{t+1}(\boldsymbol{c}_t)$ or $\Delta_j V_{t+1}(\boldsymbol{c}_t)$. We will review this literature in Section 5.

3. Discrete Choice Models: Design & Estimation

In this section, we discuss choice model design and estimation in RM applications. At the highest level, we categorize choice models as parametric (§3.1), non-parametric (§3.2), or multi-stage (§3.3), and review models that have received significant attention in each category. The aim is not to provide a detailed discussion of each choice model, but rather to give an overview of different approaches. In particular, much recent theoretical work has been devoted to exploiting structural properties of certain choice models that we seek to highlight.

3.1. Parametric models

Parametric models are rooted in random utility theory, where we assume that consumers associate a certain utility with every choice alternative (product), and decide on the alternative that maximizes his/her utility. This utility for alternative j has a deterministic and a random part: the utility is expressed by $U_j = u_j + \epsilon_j$, where u_j is the deterministic component and ϵ_j the random component with zero mean. Therefore, u_j can be regarded as the mean utility of alternative j. Let $S \subseteq \mathcal{J}$ be the set of products offered to a customer, then the probability that product j is chosen is given by $P_j(S) = P(U_j = \max\{U_{j'} : j' \in S \cup \{0\}\})$, with U_0 representing the utility of not choosing anything or buying from a competitor, i.e., the utility of an aggregated non-purchase alternative that is always available. The deterministic component u_j is usually expressed as a linear function $\beta^T x_j$ of vector of attributes x_j that influence the purchase probabilities. Different choice models are obtained depending on the assumptions made on the distribution of the random utility part. For a comprehensive introduction to the (econometric) theory of parametric choice models, including derivation and estimation, we refer to the textbooks of Ben-Akiva and Lerman (1985), Train (2009), and Hensher et al. (2015).

3.1.1. Multinomial logit

One of the most widely used models is the Multinomial Logit (MNL). This assumes that the entire customer population can be described with the same set of parameters β. Furthermore, it is based on the assumption that the random utility components ϵ_j are independent and identically distributed random variables with a Gumbel distribution, that means

$P(\epsilon_j \leq x) = \exp(-\exp(-x))$ for all $x \in \mathbb{R}$. Under this assumption, it can be shown that the probability that product j is chosen from set S is given by:

$$P_j(S) = \frac{e^{u_j/\mu}}{\sum_{i \in S} e^{u_i/\mu} + e^{u_0/\mu}}. \tag{3.1}$$

Changing the scaling parameter μ affects the behavior of MNL: as $\mu \to 0$, MNL becomes purely deterministic (i.e., $P_j(S) = 1$ if $u_j = \max\{u_i : i \in S \cup \{0\}\}; 0$ otherwise). Conversely, when $\mu \to \infty$, the utility of each product becomes irrelevant and the probability is uniformly spread across all available products and the non-purchase alternative (i.e., $P_j(S) = 1/(|S|+1),\ \forall j \in S$). This shows that, despite its simplicity, MNL is a powerful and flexible choice model. Nonetheless, MNL is also based on a strong assumption known as Independence from Irrelevant Alternatives (IIA): For any two sets $S, S' \in \mathcal{J}$ and any two alternatives $i, j \in S \cap S'$, the choice probabilities satisfy $P_i(S)/P_j(S) = P_i(S')/P_j(S')$, which implies proportional substitution across alternatives and can lead to overestimation of choice probabilities of products considered similar by consumers.

Talluri and Van Ryzin (2004) were the first to introduce the MNL choice model in revenue management. They acknowledge the difficulty of dealing with unobservable no-purchase outcomes and propose an expectation-maximization (EM) approach to jointly estimate the homogeneous arrival rate λ and the utility parameters β from historical data. A similar estimation approach is used by Vulcano et al. (2010) and Gallego et al. (2015b). The EM method is also the basis for the estimation approach developed by Vulcano et al. (2012) that is also applicable when demand follows a nonhomogeneous Poisson arrival process.

Several researchers have investigated alternatives to using EM in MNL parameter estimation: Newman et al. (2014) formulate a method using marginal log-likelihood functions, resulting in an algorithm faster than the EM approach suggested by Talluri and Van Ryzin (2004). Abdallah and Vulcano (2016) likewise investigate how to overcome the slow convergence of EM; they introduce a minorization-maximization (MM) algorithm to estimate the MNL parameters. Rusmevichientong et al. (2010a) propose an estimation approach that can be applied under resource capacity limitations. Rusmevichientong and Topaloglu (2012) assume the MNL parameters to be unknown and provide a robust formulation based on a set of likely parameter values.

3.1.2. Finite-mixture logit

Revenue management typically seeks to exploit differences in customers' preferences and willingness to pay. For this purpose, a natural extension is to consider multiple customer segments, each assumed to follow a segment-specific MNL model. If we can observe each customer's membership in a segment, then we can continue to use the MNL model: one for each segment. However, if membership of a customer in a segment is unobservable, then the individual segment-level MNL models are linked in that we can only probabilistically attribute a segment to a customer. We need to jointly estimate a probability q_l representing the likelihood of this membership for every segment $l \in \mathcal{L}$, along with the MNL parameters β_l for all segments $l \in \mathcal{L}$. In other words, we mix a finite number of MNL models, hence the resulting choice model is called finite-mixture logit or latent class model.

Each segment l will have its own vector of coefficients β_l used to build utilities $u_{lj} = \beta_l^T x_j$ of customers belonging to that segment, where x_j is a vector containing the attribute values of product j. Every segment l also has a consideration set of products $C_l \subseteq \mathcal{J}$ associated with it, containing the products that consumers in the segment would consider for purchase. The probability of choosing product j is given by:

$$P_j(S) = \sum_{l \in \mathcal{L}} q_l \frac{e^{u_{lj}}}{\sum_{i \in S \cap C_l} e^{u_{li}} + e^{u_{l0}}}, \quad j \in S. \tag{3.2}$$

So-called Mixed MNL (MMNL) models represent a MNL model with random coefficients β drawn from a cumulative distribution function, and the finite-mixture model is obtained for the special case where this distribution has finite support. Under mild regularity conditions, McFadden and Train (2000) show that any discrete choice model derived from random utility maximization has choice probabilities that can be arbitrarily closely approximated by a MMNL model. Therefore, it is not surprising that the finite-mixture logit tends to represent choice behavior better than MNL, at the expense of a more challenging parameter estimation. The latter is also discussed by McFadden and Train (2000). To the best of our knowledge, while finite-mixture logit has been considered in assortment optimization (see Section 4), there have been no papers developing specific estimation procedures.

3.1.3. Nested logit

The Nested Logit (NL) model is appropriate if we can aggregate alternatives into nests in a way such that the IIA holds within each nest but not across nests. Each nest represents a set of substitutes. A customer decides which nest to buy from, or to leave without purchase. If a nest is selected, the customer chooses an alternative out of the nest. Let K be the set of nests and S_k the set of products offered from nest $k \in K$. The deterministic, observable part of the utility of product j is denoted by u_j, and we call $v_{kj} := \exp(u_j/\mu_k)$ the preference value for product j in nest k. The preference value of non-purchase is labeled v_0. The parameter μ_k is a measure of independence in unobserved utility among the alternatives in nest k (Train, 2009). The overall preference of a customer for nest k is $V_k(S_k) = \sum_{j \in S_k} v_{kj}$. If a customer has already chosen to buy from nest k (given S_k products offered), then the probability that he/she will purchase product $j \in S_k$ is given by $v_{kj}/V_k(S_k)$.

The probability that a customer will purchase from nest k can be computed as follows:

$$\frac{V_k(S_k)^{\mu_k}}{v_0 + \sum_{h \in K} V_h(S_h)^{\mu_h}},$$

Therefore, the choice probability for product j from nest k is given by:

$$P_j([S_h]_{h \in K}) = \frac{v_{kj} V_k(S_k)^{\mu_k - 1}}{v_0 + \sum_{h \in K} V_h(S_h)^{\mu_h}}.$$

The parameters can be either estimated by simultaneous maximum likelihood, or sequentially (first within nests, then over nests). Train (2009) recommends the former. A recent paper featuring NL parameter estimation using a maximum likelihood method in the context of RM is the work of Anderson and Xie (2012), who propose an opaque pricing approach in a hotel context.

3.1.4. Markov chain models

Blanchet et al. (2016) propose a choice model where the customer's choice process is represented by a Markov chain having $J + 1$ states, each corresponding to a product $j \in \mathcal{J}$ or to the non-purchase option. Let ν_i be the arrival probability at state i. A customer arriving at i either makes the purchase if i is available, or proceeds to a different state j, if i is unavailable. Therefore, ν_i can be interpreted as the probability that a customer chooses alternative i (which could be the non-purchase option $i = 0$) when all products are available. The transition probability from alternative i to j is denoted by ρ_{ij} and represents the probability of substituting alternative i with j, given the unavailability of product i. Every state is connected with state 0 to represent the no-purchase event. The model is entirely defined by the parameter vectors ν and ρ, which need to be estimated. Blanchet et al. (2016) show that this model is a good approximation to any random utility discrete choice model under mild assumptions. In fact, Berbeglia (2016) shows that the Markov chain model itself belongs to the class of discrete choice models based on random utility. Şimşek and Topaloglu (2017) propose an Expectation-Maximization algorithm to estimate the Markov chain model parameters.

3.1.5. Exponomial models

Alptekinoğlu and Semple (2016) propose a new choice model that incorporates negatively skewed distributions of consumer utilities, as opposed to the MNL or NL models that assume that the consumers' willingness to pay distribution is positively skewed. In this model, choice probabilities are expressed as a linear combination of exponential terms; hence the term 'exponomial'. It is designed for applications where consumers would not choose to overpay if they were well-informed about the products and prices. Let us assume that the utility of a product j is given by $U_j = u_j - z_j$, where u_j and z_j are the deterministic and random part of the utility, respectively. The random variables z_i are independent and identically distributed exponentially with mean $1/\xi$. Given a choice set S with n options, Alptekinoğlu and Semple (2016) show that the probability that a customer chooses product j is given in closed form:

$$P_j(S) = \frac{e^{-\xi \sum_{i=j}^{n}(u_i - u_j)}}{n-j-1} - \sum_{k=1}^{j-1} \frac{e^{-\xi \sum_{i=k}^{n}(u_i-u_k)}}{(n-k)(n-k+1)}, \tag{3.3}$$

under the assumption that the deterministic utilities are ordered $u_1 \leq u_2 \leq ... \leq u_n$. They also show that the loglikelihood function is concave and therefore use maximum likelihood estimation to obtain the unknown parameters that define the deterministic utilities u_i.

3.2. Non-parametric models

The main problem of parametric choice models is their assumption that the choice behavior can be captured in a certain functional form; while this brings advantages through the ability to exploit the structural properties of such models, they might be a poor representation of the actual choice behavior. In particular, it is necessary to specify the attributes that are assumed to drive the choice process; a source of specification errors. Furthermore, estimation becomes more challenging as we include more attributes, leading to increased estimation errors.

An alternative is to consider non-parametric, rank-based models. We assume that every consumer ranks all alternatives $j \in \mathcal{J}$ and the non-purchase option in a certain order, and chooses the highest-ranking available option (which may be the non-purchase option). Any such ranking list is referred to as a customer type, and demand is modeled through a probability mass function over these customer types. The model is very general, and various common choice models such as the MNL are special cases of a rank-based model as discussed by Mahajan and Van Ryzin (2001). Since there is a factorial number of potential customer types, the challenge of specifying the probability mass function lies in the identification of relevant customer types. As Van Ryzin and Vulcano (2015) describe, the trade-off between model specification errors and estimation errors in parametric models exists similarly in rank-based models: the trade-off is now between more customer types and a higher risk of overfitting versus fewer types with a higher estimation error.

Some recent work has been conducted on partial identification of customer types: Given a fixed assortment, Farias et al. (2013) employ a robust approach by identifying the distribution over customer types with the worst-case revenue using constraint sampling. This turns out to be essentially the sparsest-choice model, with sparsity defined in terms of the number of customer types with positive probability weight. Sher et al. (2011) propose a linear programming approach to obtain upper and lower bounds on probabilities of any customer type. Van Ryzin and Vulcano (2015) propose an approach of progressively building a set of customer types by starting from a simple set and extending it with new types that increase the likelihood. Van Ryzin and Vulcano (2017) develop an expectation-maximization method to jointly estimate the arrival rate of customers and the probability mass function, based only on sales transaction and product availability data. One of the main disadvantages of non-parametric models is their inability to make predictions on products not seen before (as opposed to parametric models).

3.3. Multi-stage choice models

In all the choice models discussed so far, the choice process is represented in a single stage: given a set of available alternatives, each choice model returns a purchase probability for each alternative. In the marketing and economics literature, much attention has been devoted to the study of consider-then-choose decision processes that involve two stages. First, consumers form a consideration set and then to choose from among the considered available alternatives. In a recent literature review, Hauser (2014) highlights that there is strong evidence for consumers choosing in this way.

Consideration sets may be observable or unobservable; in the latter case, the estimation process includes the additional challenge of predicting what the relevant consideration sets are. For example, Gensch and Soofi (1995) show that MNL is strong in predicting the consideration set, but not in predicting first choice demand. The model has also been extended to more than two stages: Masatlioglu and Nakajima (2013) propose a dynamic search process where consumers' consideration sets are formed iteratively.

In the retail and revenue management literature, this concept has received some (but relatively little) attention. Cachon et al. (2005) consider a retail assortment planning problem with search costs where consumers either choose a product from the retailer's assortment, or choose to continue searching for alternatives available elsewhere (incurring a certain search cost). Wang and Sahin (2014) likewise investigate the impact of consumer search cost on assortment planning and pricing, but explicitly building on a two-stage consider-then-choose logit model. Jagabathula and Rusmevichientong (2015) propose a tractable nonparametric expectation-maximization approach to estimate a two-stage choice model and design a solution approach to the joint assortment and price optimization problem.

Also Jagabathula and Vulcano (2017) propose a two-stage nonparametric estimation approach, but here the customers are represented by partial orders of preferences. When a customer arrives in the system, he/she samples a full preference list of the products consistent with the partial ordering, forms a consideration set, and chooses the preferred option among the considered ones.

4. Assortment optimization

There is a vast literature on how to optimize assortments; see Kök et al. (2008) and Hübner and Kuhn (2012) for recent reviews that focus on both assortment optimization and inventory planning. In our article, we focus on the dynamic availability control problem under customer choice behavior. It contains an assortment optimization problem in each time step of the Bellman equation (2.2), which we re-state for convenience:

$$\max_{S \subseteq \mathcal{J}(\boldsymbol{c}_t)} \left\{ \sum_{j \in S} \lambda P_j(S) \Big(r_j - \Delta_j V_{t+1}(\boldsymbol{c}_t) \Big) \right\} = \max_{S \subseteq \mathcal{J}(\boldsymbol{c}_t)} \left\{ \sum_{j \in S} \lambda P_j(S) \tilde{r}_j \right\},$$

where $\Delta_j V_{t+1}(\boldsymbol{c}_t) = V_{t+1}(\boldsymbol{c}_t) - V_{t+1}(\boldsymbol{c}_t - \boldsymbol{A}_j)$ is the opportunity cost associated with selling product j and $\tilde{r}_j = r_j - \Delta_j V_{t+1}(\boldsymbol{c}_t)$ represents the so-called displacement adjusted revenue.

This assortment optimization problem often needs to be solved in real-time (e.g. to decide on which products to display on a website of an airline or a car rental company), so the ability to solve it very quickly is crucial. The difficulty in finding an optimal solution depends on the choice model that underpins the purchase probabilities $P_j(S)$. In the following, we review recent literature on efficient optimization for the choice models described in the previous section. As described in §2.1, we assume that capacities are fixed in the short term. Hence, we do not concern ourselves with issues such as planning of optimal inventory levels; we are only concerned with the assortment itself.

4.1. General choice model

Some authors considered the assortment optimization problem without exploiting the specific structure adherent to a particular choice model. This has the advantage that the solution approach is portable, regardless of the specific choice models being used. To that end, more general heuristics are derived, either using a *revenue-ordered approach* or by a *greedy approach* that adds products iteratively to an initially empty set. The former approach involves searching the revenue maximizing set among the sets $S_1, , \ldots, S_J$ which are nested by (displacement adjusted) revenues such that each set S_k contains the k products with the highest revenues. This heuristic can even be optimal (see MNL discussion below). Berbeglia and Joret (2017) analyze revenue-ordered strategies and their ability to approximate the optimal solution for a broad class of models including all random utility discrete choice models. The greedy approach has been used by Jagabathula (2014) who further extends the heuristic by allowing it to delete and exchange products in each iteration.

4.2. MNL

Talluri and Van Ryzin (2004) show that the revenue-ordered approach is optimal under the MNL with a single customer segment and without further constraints. Various versions of assortment problems under MNL have been considered: optimization under a capacity constraint (Rusmevichientong et al., 2010a; Desir and Goyal, 2014), constraints that have a totally unimodular structure (Davis et al., 2013), robust assortment optimization where the true MNL parameters are unknown (Rusmevichientong and Topaloglu, 2012), an extension applicable to the so-called generalized attraction model (Wang, 2013), as well as MNL with nested consideration sets but identical MNL parameters for every segment (Feldman and Topaloglu, 2015a).

4.3. Finite-mixture logit

The assortment optimization problem becomes very challenging when moving from the single-segment MNL model to the multi-segment case. Miranda Bront et al. (2009), Rusmevichientong et al. (2010b), and Rusmevichientong et al. (2014) established that the problem is NP-complete when the consideration sets of the segments overlap, even with just two customer segments. Rusmevichientong et al. (2010b) provide a polynomial approximation scheme, and both they and Rusmevichientong et al. (2014) provide approximation guarantees for using the revenue-ordered heuristic. Upper bounds on the optimal expected revenue are proposed by Feldman and Topaloglu (2015b) that can be used as a proxy for the optimality gap. Extensions include cardinality-constrained assortments, which are considered by Méndez-Díaz et al. (2014) and Şen et al. (2015). The latter propose a conic quadratic mixed integer programming formulation that can be solved by commercial software even for large instances.

4.4. Nested logit

For the two-level NL, Li and Rusmevichientong (2014) present a necessary and sufficient condition for the optimal assortment. Furthermore, they identify a certain structure of the optimal solution that can be exploited in a greedy algorithm running in $O(nm \log m)$ time, where m denotes the number of nests, each having n products. The more general case of having d levels is tackled by Li et al. (2015). Davis et al. (2014) provide an upper bound on the optimal expected revenue. Furthermore, they find that nested-by-revenue assortments can be expected to perform well when the revenues or the attractiveness of the products within a particular nest are not too different from each other (assuming the customers always purchase within the selected nest). Also, the case that the latter assumption is violated is considered. Feldman and Topaloglu (2015c) extend the results to the case where there are capacity constraints on the total capacity consumption of all products offered in all nests. Cardinality and space constraints are investigated by Gallego and Topaloglu (2014), who find that the assortment problem under cardinality constraints can be solved efficiently via a linear program. Space constraints render the problem NP-hard, and the authors develop performance guarantees for this case.

4.5. Exponomial, Markov and rank-based models

Alptekinoğlu and Semple (2016) provide structural results on the assortment optimization under the exponomial choice model. The Markov chain based choice model was proposed in Blanchet et al. (2016), who also provide polynomial-time solution algorithms. Feldman and Topaloglu (2017) give a linear program to obtain the optimal solution to the assortment problem (and solution approaches to the single-resource and network RM problem) for the Markov chain choice model. Désir et al. (2015) investigate the same choice model in the assortment problem under cardinality and capacity constraints and present constant-factor approximations.

Aouad et al. (2015b) provide best-possible approximability bounds for the assortment problem without capacity constraints when a rank-based choice model is considered. Goyal et al. (2016) show that the capacitated assortment planning problem is NP-hard to approximate within a factor better than $1 - 1/e$.

For the special case that customers only consider purchasing one of two substitutable products, Paul et al. (2016) propose a 2-approximation algorithm for the rank-based choice model. Also Bertsimas and Mišić (2015) use a rank-based approach: they propose a two-step procedure of estimation and optimization via a mixed-integer programming formulation that can accommodate various constraints.

4.6. Multi-stage models

Several authors have recently investigated assortment optimization under consider-then-choose choice models. Wang and Sahin (2014) focus on a joint assortment planning and pricing problem where customers initially balance utility uncertainty and search costs to form a consideration set, before evaluating these options and choosing one option. Revenue-ordered assortments fail to be optimal; instead, they propose so-called attractiveness ordered assortments that are shown to exhibit good performance. Aouad et al. (2015a) develop a dynamic programming approach that they show to be efficient under certain assumptions on how customers consider and choose. In particular, they present polynomial running time guarantees assuming that customers consider arbitrary subsets of products, but that their relative ranking preferences are derived from a common permutation.

5. Availability control over time

In this section, we review the literature that focuses on how to solve/approximate the choice-based network RM problem given as a general dynamic program in (2.1). We begin with the independent demand case (§5.1), and then discuss availability control using a single resource (§5.2) and a network of resources (§5.3), respectively. Finally, we discuss the relation to dynamic pricing (§5.4).

5.1. The independent demand case

Note that the traditional independent demand model as described in Section 1 obviously is a special case of (2.1). More precisely, as Talluri and Van Ryzin (2004) point out, it can be obtained from defining the choice probabilities by $P_j(S) = p_j$ if $j \in S$ and 0 otherwise, with p_j denoting the probability that the incoming customer requests product j, regardless of what alternatives are available to him/her. Even though the independent demand model is somewhat outdated for the reasons described earlier, it still has some following in the airline industry, as airlines often still rely on independent demand systems. That is the reason why since the early 2000s, attempts have been taken to find approaches that allow for incorporating choice behavior to some extent while still using the legacy systems. In this context, Fiig et al. (2010) describe a theory around the so-called fare transformation, also known as marginal revenue transformation, which technically goes back to Kincaid and Darling (1963). The main idea is to feed an independent demand system with modified fare values, such that buy-down choice behavior is adequately reflected. A buy-down occurs when a customer decides not to buy a higher fare product (even though it is less than – or equal to – the customers willingness-to-pay) in favor of a lower available fare. The value by which the original fare is reduced is also denoted as the fare modifier, reflecting the cost of customers' price elasticity. The fare transformation can basically be applied to deterministic as well as stochastic models, ranging from dynamic programming and dynamic programming decomposition, to EMSR-based methods (also see Walczak et al. (2010)), and it is also possible to model a hybrid setting with a mix of price-sensitive demand with complete buy-down choice behavior (called "priceable demand" in the industry) and independent demand (called "yieldable demand"). However, the transformation is technically only exact in the case that the so-called efficient frontier is nested with regard to the products contained in the corresponding offer sets (see the following subsection for details). Moreover, in the case of network availability control, the transformation requires that the choice behavior is non-overlapping with regard to different itineraries, which can be a rather strong limitation.

5.2. Single resource availability control

The seminal publication on choice-based revenue management is Talluri and Van Ryzin (2004), who focus on a special case of the network problem (2.1) which is obtained if $m = 1$, that is, if only one single resource ist considered ($\mathbf{c}_t$ is a scalar in this case). Note that in the early days of revenue management, in practice as well as in theory, the focus was usually on such single-leg problems only, before there was a shift towards more sophisticated network-based methods in the 1990s. In their paper, Talluri and Van Ryzin (2004) develop a whole new theory of availability control, thereby adapting techniques from portfolio theory. In particular, they show that an optimal offer set S is always from an ordered family of subsets of $\mathcal{J}$ that are efficient with regard to the trade-off between expected revenue and purchase probability, while inefficient subsets of $\mathcal{J}$ can be ignored. The efficient subsets can be visualized two-dimensionally to form a so-called efficient frontier in their order, and it can be shown that the more capacity is available or the less time remains, the further an optimal set lies on this frontier, i.e., the lower the quotient of revenue and purchase probability of the currently optimal offer set is. Moreover, in the case that the efficient subsets are nested with regard to the contained products along the frontier, a nested allocation policy is optimal. For the case of MNL as the underlying choice model (as well as for independent demand), the authors show that the optimal policy is nested by the order of the product's revenues (so called "nesting by fare-class order," see Section 4).

Given the thoroughness of Talluri and Van Ryzin (2004), in recent years, only a few publications can be observed that continue to focus specifically on the single-leg choice-based setting. Cooper and Li (2012) study an airline setting relying on an alternative modeling approach based on a buy-up based choice model that estimates the probability that a customer who cannot buy the fare he/she is aiming to, will purchase a higher fare (also denoted as upsell or sell-up). They compare the performance of their approach with a modified version of Littlewood's rule incorporating choice, and investigate the long-run behavior of their approach when flights take place repeatedly and the parameters of their choice model are updated accordingly over time. They show that their approach works well in most cases in as far as it can eliminate the so-called spiral-down effect to a certain extent (even when the buy-up model is misspecified). The spiral-down effect refers to steadily reducing revenues due to the use of demand models that assume demand for any individual product is independent of available alternative products, although actual demand does depend on them (see Cooper et al. (2006)).

Gallego et al. (2009) propose new extensions of the traditional EMSR-based approaches for the single-leg problem, relying on buy-up-probabilities that are consistently derived from an MNL model.

The single-leg problem is also investigated, among other problems, by Feldman and Topaloglu (2017) with a Markov chain as the underlying choice model. They show that the optimal policy can be represented by protection levels. Furthermore, they prove that the nesting properties from Talluri and Van Ryzin (2004) regarding the optimal policy with regard to time and capacity monotonicity, also hold under the Markov chain model.

5.3. Network availability control

The network availability control problem has been formally defined by the dynamic program (2.1). Since it is intractable for realistically-sized problem instances, the primary concern of research on network availability control is on deriving approximations of the value function $V_t(\boldsymbol{c}_t)$. Our ultimate goal is to use these approximations to estimate the opportunity costs $\Delta_j V_{t+1}(\boldsymbol{c}_t)$, so that we can use them in a control policy in the form of the assortment optimization problem discussed in Section 4.

A first step towards network availability control is to consider management of products that use different resources, however, no two products use the same resource. In the airline context, this corresponds to customers choosing between parallel flights. Zhang and Cooper (2005) investigate this problem and propose upper and lower bounds on the value function. They use a weighted average of these bounds to obtain an approximation to the value function.

For the more general network revenue management problem, Gallego et al. (2004) extend a standard linear programming model under independent demand to a model that incorporates dependent demand, i.e. that accounts for customer choice behavior. It is called the Choice-based Deterministic Linear Program, CDLP for short. We state it here because much of the literature over the past ten years is related to it. Let $h(S)$ represent the number of time periods that a certain offer set S shall be offered (not necessarily integer). Given the arrival of a customer, the expected revenue $R(S)$ from offering S for one time period is given by $R(S) := \sum_{j \in S} P_j(S) r_j$, and the expected consumption of resource i from offering S is defined by $Q_i(S) := \sum_{j \in S} P_j(S) a_{ij}$, $\forall i$. Let c_i denote the available capacity of resource i. CDLP assumes that demand is deterministic and equal to its expected value and it is given by:

$$
\begin{aligned}
(\mathbf{CDLP}) \quad z_{\text{CDLP}} = \max_h & \sum_{S \subseteq \mathcal{J}} \lambda R(S) h(S) \\
& \sum_{S \subseteq \mathcal{J}} \lambda Q_i(S) h(S) \leq c_i, && \forall i \in \mathcal{I}, \\
& \sum_{S \subseteq \mathcal{J}} h(S) = T, \\
& h(S) \geq 0, && \forall S \subseteq \mathcal{J}.
\end{aligned}
$$

CDLP maximizes expected revenue subject to resource capacity constraints and a fixed time horizon. The CDLP's optimal dual values associated with the capacity constraints can be used as estimates of the marginal value of capacity and thereby to estimate opportunity costs. Liu and Van Ryzin (2008) build on this work and propose to solve this linear program by column generation (since it has a number of variables that grow exponentially in the number of products in the network). They use the dual solution of CDLP to decompose the network dynamic program (2.1) into resource-level dynamic programs, and use the resulting value function approximation to obtain opportunity cost estimates in the control policy. However, in general, the column generation is computationally inefficient and, in fact, intractable for even moderate-sized applications. Various researchers have focused on proposing better formulations, i.e. either delivering better revenue results at the expense of more computational challenges, or the same or similar results in shorter computational time under exploitation of certain properties of the choice model. CDLP provides an upper bound z_{CDLP} on the optimal expected revenue $V_1(\boldsymbol{c}_1)$, and much attention has been devoted to establishing alternative bounds on the optimal expected revenue, and whether bounds dominate each other. The motivation behind this is that they can be used as benchmarks and, furthermore, that tighter bounds usually (but not necessarily) imply better policy performance in terms of expected revenue.

A string of papers has been devoted to the development of efficient solution techniques for variants of the CDLP: Kunnumkal and Topaloglu (2008) improve on the work of Liu and Van Ryzin (2008) by making their approximation of the dynamic program time-dependent. The resulting formulation provides tighter upper bounds on the optimal value of the dynamic program than CDLP, and gives better revenue results in simulation studies. Furthermore, Kunnumkal and Topaloglu (2010) propose a dynamic programming decomposition similar to Liu and Van Ryzin (2008) but with the main difference that they view revenue allocations to individual resources as decision variables (as opposed to using the dual values of CDLP), and they show that this results in upper bounds that are tighter than that of CDLP.

Upper bounds that are looser than those given by CDLP are provided by Talluri (2014) for the finite-mixture MNL: he proposes a mathematical program that is based on decomposing the CDLP by customer segment. The sub-problems are loosely linked and overall form an upper bound on CDLP. It is typically a fairly loose upper bound, and indeed its revenue performance in simulation studies is not always good, but it can be solved by orders of magnitude faster than CDLP. Meissner et al. (2013) build on this by devising valid inequalities that can be added to this more efficient mathematical program without losing its tractability. Their results are promising in that the observed revenue performance in simulation studies is often nearly equivalent to that of CDLP, despite being much simpler to solve. This approach is particularly interesting against the background of Bront et al. (2009) who show that CDLP (specifically, the column generation subproblem) is NP-hard for the finite-mixture logit model with products that are being considered for purchase by more than one segment (in other words, if the consideration sets overlap), while it can only efficiently be solved in the non-overlapping (disjoint) case, where the problem can be decomposed by segment each following a standard MNL (Liu and Van Ryzin (2008)). The approach of Meissner et al. (2013), however, also provides a tractable formulation for general choice models where there may be an overlap of the consideration sets of the different customer segments. In fact, their formulation often returned bounds equivalent to those given by CDLP. This observation motivated research by Strauss and Talluri (2017) who provide structural insights as to when CDLP and the approach of Meissner et al. (2013) are equivalent.

Gallego et al. (2015b) consider an efficient solution of CDLP under the so-called general attraction choice model, of which MNL is a special case. They show that CDLP can equivalently be re-formulated in a linear program (called Sales-Based Linear Program, SBLP for short) with many fewer decision variables and a fairly small number of constraints. This equivalence also holds for the finite mixture MNL model under the assumption that the segments' consideration sets do not overlap. This strong result motivated further research by Kunnumkal and Talluri (2015) who develop similarly tractable linear programs that provide a tighter upper bound than SBLP. They also demonstrate that the valid inequalities proposed by Meissner et al. (2013) can be adapted to be used in their linear programming formulation so as to also deal with the case of overlapping consideration sets. Cheung and Simchi-Levi (2016) propose an algorithm that solves CDLP to near optimality with provable efficiency under the assumption that the underlying assortment optimization can be solved approximately. Similarly, Gallego et al. (2015a) propose an algorithm that can solve CDLP to near-optimality in a setting that allows for different customer types (and therefore can be used in personalised assortment optimization over time). Their algorithms generate ϵ-optimal solutions for any $\epsilon > 0$ and is applicable to various choice models including MMNL.

A number of works build on the seminal paper of Adelman (2007) who proposes to use a linear programming (LP) formulation that equivalently corresponds to the DP to construct value function approximations. He assumes independent demand, but Zhang and Adelman (2009) extend Adelman's work to the customer choice setting. We briefly state the LP formulation here because various authors have used this approach of constructing approximations over recent years. As stated e.g. in the book of Powell (2007), the DP (2.1) can be equivalently expressed by the following LP:

$$\min_{v(\cdot)} v_1(\boldsymbol{c}_1)$$

$$v_t(\boldsymbol{c}_t) \geq \lambda \sum_{j \in S} P_j(S) \left[r_j - (v_{t+1}(\boldsymbol{c}_{t+1}) - v_{t+1}(\boldsymbol{c}_{t+1} - \mathbf{A}_j)) \right] + v_{t+1}(\boldsymbol{c}_{t+1}), \quad \forall \boldsymbol{c}_t, t, S \subset \mathcal{J}.$$

Clearly, this formulation is as intractable as the DP due to the large number of decision variables $v_t(x)$ and constraints. However, the formulation is helpful for deriving value function approximations and in establishing upper bound relationships. For example, Zhang and Adelman (2009) reduce the number of decision variables $v_t(\boldsymbol{c}_t)$ by replacing them with the time-dependent, affine approximation $v_t(\boldsymbol{c}_t) \approx \theta_t + \sum_{i=1}^{m} V_{t,i} c_{ti}$, $\forall\, t, c$, where the values $V_{t,i}$ can be interpreted as the time-dependent value of a unit of resource i, and θ_t is an adjusting constant. In the same paper, they establish an upper bound on $V_t(\boldsymbol{c}_t)$, namely $V_t(\boldsymbol{c}_t) \leq \min_i\{v_{t,i}(x_i) + \sum_{k \neq i} \pi_k x_k\}$, where π_i is the optimal dual value associated with the capacity constraint on resource i in CDLP. The function $v_{t,i}(\cdot)$ can be obtained by solving a dynamic program with a single resource as proposed by Liu and Van Ryzin (2008).

This bound motivated Zhang (2011) to use it directly as a value function approximation in the LP shown above. He solves the resulting nonlinear approximation with a simultaneous dynamic programming procedure. Meissner and Strauss (2012b) also build on Zhang and Adelman (2009) and extend their approach by also including dependence of the approximation on the inventory levels (in addition to time), which results in further tightened bounds. Kunnumkal and Talluri (2016b) derive analytic results on how much the affine approximation and that of Meissner and Strauss (2012b) can tighten the CDLP bound z_{CDLP}. In their previous work, Kunnumkal and Talluri (2015), they show that their proposed tractable LP-formulations provide upper bounds on the optimal expected revenue provably between z_{CDLP} and the bound of the affine approximation. Vossen and Zhang (2014) improve on the affine approximation by developing a new dynamic aggregation/disaggregation algorithm that allows the problem to be solved far quicker than with the affine formulation of Zhang and Adelman (2009). A tractable piecewise-linear approximation is proposed by Kunnumkal and Talluri (2016a), who also show that this approach is equivalent to the Lagrangian relaxation that has been used, e.g. by Topaloglu (2009) to relax certain linking constraints in order to decompose the problem into simpler ones. Vossen and Zhang (2015) propose a way of reducing the size of the resulting LP for both the affine and the piecewise-linear approximations.

The CDLP has the disadvantage of assuming that demand is deterministic and equal to its expected value. One way of overcoming this limitation to some extent is to consider random samples of demand; Kunnumkal and Topaloglu (2011) and Kunnumkal (2014) use such an approach. Also Van Ryzin and Vulcano (2008) and Koch (2017) propose a network availability control mechanism that is based on simulation. A common approach to overcome the static nature of the deterministic demand assumption is to re-solve the CDLP several times over the booking horizon. Jasin and Kumar (2012) investigate such re-solving schedules for a deterministic linear programming formulation under customer choice and give performance guarantees in the form of a bound on expected revenue loss relative to the optimal expected revenue.

The network availability control problem is discussed specifically for the Markov chain choice model by Feldman and Topaloglu (2017), and specifically for a rank-based choice model by Chen and Homem-de Mello (2010). In a network RM setting where availability control is implemented by bid prices, Meissner and Strauss (2012a) propose a simple greedy heuristic to solve the assortment problem in the control policy with good results. Dai et al. (2014) present a study on an airline in which they build and test a number of choice models (MNL, NL, mixture logit) and develop a deterministic fluid optimization formulation for parallel flights. They find that MNL performs better than the more sophisticated alternatives.

5.4. The relation to dynamic pricing

As the origins of revenue management are closely linked to the airline industry, revenue management is often equated with availability control. However, given the current developments regarding airline reservation systems, as well as the application of revenue management techniques in many other industries that do not necessarily depend on the limitations of reservation systems, directly controlling product prices in a dynamic fashion has become much more relevant. In this section, we specifically focus on the relationship between multi-product dynamic pricing and availability control from a technical perspective.

Mainly motivated by the retail industry, e.g., the selling of seasonal goods, there is a large body of dynamic pricing literature under the assumption of independent demand (see e.g. Bitran and Caldentey (2003) for a review), that means we assume that customers do not substitute products but rather only decide whether or not to buy a given product at the given price. There are much less publications to be found that consider dynamic pricing in the presence of customer choice behavior.

Discrete prices

If we include customer choice behavior and assume that product prices are selected from a finite set, one can think of the resulting problem as a variant of the availability control problem (2.1) with an additional side constraint for the maximization. More precisely, for each real-world product, here denoted by k, we define a set of virtual products $\mathcal{J}_k$ which are all similar except for the price. The sets are defined as mutually exclusive and completely exhaustive, i.e., $\mathcal{J}_i \cap \mathcal{J}_k = \emptyset$ for all products i, k with $i \neq k$ and $\bigcup_{\forall k} \mathcal{J}_k = \mathcal{J}$. Then, we can optimize prices by adding constraints

$$|S \cap \mathcal{J}_k| \leq 1 \;\forall\; k \tag{5.1}$$

to the maximization at each stage of (2.1), which ensures that for each real-world product (at most) one price is selected. Note that selecting no price for a product k can virtually be interpreted as setting such a high price that demand for k is driven to zero, which e.g. allows handling stock-out situations.

Fundamentally, the resulting problem that has to be solved at each stage of (2.1) is a variant of the assortment optimization problem discussed in Section 4, known as Assortment Pricing or Product Line Pricing in the literature. For the special case of parallel flights, Zhang and Cooper (2009) propose a pricing approach that is rooted in approximating the value function with a weighted average of an upper and lower bound similar to their earlier work (see Zhang and Cooper (2005)). In the case that probabilities are modeled by the MNL, it can be shown that the resulting fractional program has a total unimodular constraint matrix and a quasi-convex objective function (see Chen and Hausman (2000)), such that the binary variables for selecting a price out of $\mathcal{J}_k$ can be LP-relaxed. As a consequence, standard Charnes-Cooper-transformation can be applied (see Charnes and Cooper (1962)), leading to an efficiently solvable linear program. A corresponding linearization is proposed by Davis et al. (2013). The dynamic program of the overall discrete dynamic pricing problem is analyzed in more detail by Schön (2010b).

In the situation where more customer segments, each with its own MNL, need to be modeled, Schön (2010a) considers segment-specific pricing with common setup cost constraints, and shows that the problem can be decomposed such that the above-mentioned linearization is applicable. However, where prices are not segment-specific, total demand follows a finite-mixture logit model (see Sections 3.1.2 and 4) such that the pricing problem is NP-hard to solve. In this case, at least a mixed integer linear programming formulation can be obtained by applying adequate techniques (see e.g. Li (1994), Wu (1997)).

Continuous prices

In a situation where prices are continuous, the original formulation (2.1) needs to be modified. At each stage, $\boldsymbol{r} \in \mathbb{R}_+^J$ is a decision vector with one entry for each product's price.

$\mathbb{R}^J_+$ is the n-dimension non-negative real numbers and represents the set of allowable prices. A customer buys product $j \in \mathcal{J}$ with probability $P_j(\boldsymbol{r})$. Probabilities have to be defined such that setting a dummy price at infinity will drive demand for the corresponding product to zero. Then, the resulting problem is given by

$$V_t(\boldsymbol{c}_t) = \max_{\boldsymbol{r}\in\mathbb{R}^J_+} \left\{ \sum_{j\in\mathcal{J}} \lambda P_j(\boldsymbol{r})\Big(r_j + V_{t+1}(\boldsymbol{c}_t - \boldsymbol{A}_j)\Big) + \Big(\lambda P_0(\boldsymbol{r}) + 1 - \lambda\Big)V_{t+1}(\boldsymbol{c}_t) \right\}, \qquad \forall\, t, \forall\, \boldsymbol{c}_t \tag{5.2}$$

with the boundary conditions $V_{T+1}(\boldsymbol{c}_{T+1}) = 0$ if $\boldsymbol{c}_{T+1} \geq \boldsymbol{0}$ and $V_{T+1}(\boldsymbol{c}_{T+1}) = \infty$ otherwise.

Where demand follows the MNL, the resulting multidimensional and continuous maximization problem that we need to solve at each stage of (5.2) has been shown to be non-concave and thus difficult to solve (Hanson and Martin (1996)). However, more recently, Dong et al. (2009) present a strong result by proving that while this sub-problem is not quasi-concave in the price vector, after a reformulation it is concave in the choice probabilities. They give an analytic solution to the pricing problem that boils down to a simple Newton root search. The result is useful for solving pricing sub-problems quickly if we are working with the MNL model.

A related work by Akcay et al. (2010) likewise looks at solving the dynamic pricing model under MNL. As opposed to Dong et al. (2009), who argued via concavity of the MNL profit function in the choice probabilities, they derive their efficient solution method from proving the unimodality of the MNL profit function. The concavity of the profit function in the choice probabilities breaks down under the finite mixture MNL model as shown by Li et al. (2017), even for entirely symmetric price sensitivities across all segments and all products. Likewise, the equal markup pricing structure identified for the MNL does no longer hold. In contrast, the nested logit model has more structure: Li and Huh (2011) show that concavity still holds for this choice model, and Gallego and Wang (2014) prove that the adjusted markup (defined as price minus cost minus the reciprocal of the price variable per nest) is constant across all nests at optimiality. They exploit that latter feature to formulate the multiproduct pricing problem as a problem with a singe variable per nest; in fact, they can simplify the overall problem further and reduce it to maximization of a unimodal function with a single variable under mild conditions.

Li et al. (2015) look into assortment and price optimization problems under the d-level nested logit model. They consider pricing under fixed assortment and assortment optimization under fixed prices, but not joint control. As with the MNL model, the profit function is not concave in the product prices. Under this more sophisticated choice model, they propose an iterative algorithm to generate a sequence of prices that converge to a stationary point. This is related to work by Rayfield et al. (2015) who focus on price optimization under the nested logit model. They consider the specific angle where there are price constraints (upper and lower bounds) that need to be accounted for. Approximation methods with performance guarantees are being developed.

Alptekinoğlu and Semple (2016) take a different path in that they propose a new choice model, the so-called exponomial choice model that we discussed above. They present structural results on optimal assortments for both exogenous and endogenous prices. The pricing problem given a fixed assortment has a certain structure that can be exploited; in particular, Alptekinoğlu and Semple (2016) show that the resulting problem can be solved using a piecewise linear approximation and linear programming. However, it is more involved than using the simpler MNL model as shown by Dong et al. (2009) and Akcay et al. (2010).

In addition to the publications mentioned, there are a couple of other relevant papers that specifically consider dynamic pricing with multiple products under customer choice and discrete/continuous prices, and that have been published in the last decade. An excellent overview and categorization is given in Section 2 of the recent survey paper by Chen and Chen (2015), which is specifically dedicated to dynamic pricing.

6. Conclusion and Outlook

Choice-based revenue management has received a considerable boost in attention over the past decade, in particular assortment optimization has recently been intensively worked on. Various choice models have been investigated with the aim of establishing efficient solution approaches to the assortment optimization problem, which is important for RM control policies since these problems need to be solved in real-time in order to dynamically generate offer sets. Usually, a particular choice model was first analyzed on its own, and subsequently in conjunction with certain constraints on capacity, allowable prices, consideration set structures, etc. We expect that more work will appear in the coming years on this, using less common choice models such as the paired combinatorial logit model (Zhang et al. (2017)). In particular, we see that there is scope for future research on multi-stage choice models that have seen a lot of attention in the marketing literature but almost none in revenue management. As discussed above, in marketing, two-stage models (also called consider-then-choose models) are common, and there is also some work on multi-stage models that address the situation where customers' consideration sets may change over several stages.

Modeling of choice processes that incorporate several stages are also of interest in the context of incorporating ancillary revenues into the optimization. Customers can be modeled to first choose a product, and in a second stage to choose ancillary services; Bockelie and Belobaba (2017) propose a sequential consumer choice process along these lines. The increasing importance of ancillary revenues in traditional RM industries like airlines, car rentals, casinos, or cruise lines has so far not been adequately reflected in the academic literature.

As far as the network availability control problem as a whole is concerned, published work to date has focused on various approximations of the underpinning dynamic program. Most of these papers offer provable bounds on the optimal expected revenue as opposed to performance guarantees on the optimality gap. Furthermore, we expect future work to focus increasingly on learning problems that deal with the exploration-exploitation trade off faced when we drop the assumption of known parameters of the demand model, such as the paper of Agrawal et al. (2016).

Learning is particularly important in the context of personalization; this topic was recently investigated by Chen et al. (2015). Also in industry, there is a need for personalized approaches, as recent developments in airline distribution technology offer completely new possibilities with regard to making much more flexible and customized offers. Personalization may be approached in various ways, such as identifying which customer segment an incoming request could originate from, and then offering a segment-specific pre-assembled set of products. Gallego et al. (2015a) propose an optimization approach where each customer's segment is assumed to become known at the time of booking. As pointed out by Wittman and Belobaba (2017), one challenge in personalized RM is to develop sophisticated approaches that handle the potential negative effects on revenue of the new personalized control and pricing possibilities, especially in a competitive environment, where a competitive spiral down effect has been shown to exist (Isler and Imhof (2008)).

Overall, choice modeling has developed into one of the most active research areas in RM, and we expect that the advances in theory will lead to wider adoption of these techniques in practice as well.

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