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DISCUSSION

A new method of measuring plastic limit of fine materials

V. SIVAKUMAR, D. GLYNN, P. CAIRNS and J. A. BLACK (2009). *Géotechnique* **59**, No. 10, pp. 813–823

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The authors rightly state that the strength gain ratio over the plastic range ($R = c_{\mathrm{u(PL)}}/c_{\mathrm{u(LL)}}$) is often about 100 for many soils, although there is no theoretical basis for a universal value of 100, and that other researchers have postulated $c_{\mathrm{u(PL)}} \approx 170$ kPa and R up to as high as 170. The LL, PL and R values largely depend on the grading and mineralogy, shape and surface texture of the constituent solids (Trauner *et al.* 2005); activity of the clay minerals (Wood, 1990); and test method (e.g. predefined $c_{\mathrm{u(LL)}}$ value assigned to determine the fall-cone LL value). PL values determined using Casagrande's thread-rolling method define the water content below which 3 mm diameter, saturated soil threads cannot be remoulded (stress and strain rate controlled; that is, soil-toughness dependent).

The discusser agrees that there is a need for a more consistent and accurate procedure to determine PL, and in this regard, the authors have developed a pneumatic loading system to apply predetermined forces in a fast-static ('almost instantaneous') manner on a standard cone in the fall-cone apparatus. Knowing the LL and PL values of Speswhite kaolin, the authors determined experimentally that a fast-static force of 54 N applied to a 0·8 N, 30° cone produced 20 mm penetration at the PL for the kaolin test material. Next, the affects of loading rate on penetration depth were studied close to the PL state, with fast-static and quasi-static scenarios under 54 N loading producing similar penetration depths h_{s} (Table 1). The fall-cone strength under the applied vertical cone force Q can be calculated as

$$c_{\mathrm{u}} = \frac{\alpha Q}{h^2} \tag{4}$$

where

$$\text{the cone factor } \alpha = \frac{1}{\pi N_{\mathrm{c}} \tan^2(\beta/2)} \tag{5}$$

β is the cone apex angle; and N_{c} is the cone bearing capacity factor.

Considering the affects of soil displacement/heave and surface roughness (noting the cone surface was not smeared with oil in the present study), Koumoto & Houlsby (2001) have shown that $\alpha \cong 1{\cdot}33$ for dynamic 'free fall' loading and $\alpha \cong 0{\cdot}654$ ($N_{\mathrm{c}} \cong 6{\cdot}79$) for quasi-static loading in the case of a 80 g, 30° cone with surface roughness $\chi = 0{\cdot}5$; where χ is the ratio of adhesion to undrained shear strength. Hence, given both fast-static and quasi-static loading produced similar penetrations for kaolin close to its PL, from equation (4), the calibration fast-static force of 54 N mobilises $c_{\mathrm{u(PL)}} = 88{\cdot}2$ kPa for kaolin. Note the measured dynamic penetration depths (d) and quasi-static penetration depths (h_{s}) in Table 1 are in good agreement with theory

$$h_{\mathrm{s}} = \frac{d}{\sqrt{3\zeta}} \text{ (Koumoto \& Houlsby 2001)} \tag{6}$$

where $\zeta = c_{\mathrm{u}}/c_{\mathrm{ud}} \cong 0{\cdot}71$; and c_{ud} is the dynamic undrained shear strength.

Considering the high strain rates in the fall-cone LL test (typically $1{\cdot}0 \times 10^6$ %/h), Koumoto & Houlsby (2001) calculated $c_{\mathrm{u(LL)}} = 2{\cdot}66$ kPa for the 30° dynamic fall-cone; hence $R \cong 33$ (88·2/2·66) for kaolin, in general agreement with Wood (1990). On the basis of $c_{\mathrm{u(LL)}} = 1{\cdot}7$ kPa, Wood (1990) reported 'R nearer 30 and of the order of 100' for mixtures (25–100% by dry mass) of kaolinite and montmorillonite respectively. In subsequent tests, the authors used the kaolin fast-static calibration load of 54 N to define the PL state for 16 different clays (Table 3). Hence, the PL values determined by the new method are based on the premise that $R \cong 33$ also applies to the other clays tested.

The authors also carried out dynamic and fast-static loading under 54 N on Ampthill clay, producing penetrations of 23·5 and 19·5 mm respectively, at the same water content close to the PL state. From equation (6) and on the basis of the 23·5 mm dynamic penetration, a quasi-static 54 N force would produce a penetration $h_{\mathrm{s}} = 16{\cdot}1$ mm ($< 19{\cdot}5$ mm measured for fast-static). The authors also report that for Ampthill clay, the penetration rate under fast-static loading reduced 'by as much as fivefold' compared with dynamic conditions: based on data reported by Koumoto & Houlsby (2001), this would suggest a strain rate of the order of 10^5 %/h for the fast-static scenario described by the authors. In the absence of collaborative data from calibration loading specific to other soils with well-defined PL, it is possible that the 54 N fast-static and quasi-static loading that produced similar penetrations for kaolin may have been an artefact of the apparatus calibration for kaolin, and in general, strain rate effects may also be significant in fall-cone tests close to the PL state.

An assessment/evaluation on selected materials by the authors indicated that the PL obtained by the new method 'may be slightly on the wet side' of Casagrande's thread-rolling PL values. Given the above, it is postulated that the relative accuracy of the PL values determined using the new method may arise from the fact that shear strength increases approximately exponentially with reducing water content nearer the PL state so that differences between the apparatus calibration of $c_{\mathrm{u(PL)}} \cong 88$ kPa (for kaolin) and the true (typically higher) $c_{\mathrm{u(PL)}}$ values for other test-materials manifest as relatively minor differences in water content, typically on the wet side of the true PL value, although still within the wide PL range determined using the thread-rolling method. Finally, in considering the trend lines for LL and PL in Fig. 9, it should be noted that the cone penetration depths, and hence c_{u} values, were determined by dynamic and fast-static loading, respectively, for which the cone mass/vertical force but also the cone factors are different.

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The Atterberg consistency limits, together with clay size fraction, are being used not only for classification of fine-grained soils, but also for empirical correlations with important engineering properties required for analyses and design of geotechnical construction (e.g. Terzaghi *et al.*, 1996). The authors' paper reminded the discusser of an important contribution by Feng (2000) in connection to determining the Atterberg plastic limit, w_p, using the fall-cone test. The present paper refers to Feng (2000); however, it does not attempt a comparison of the plastic limit determined by the Feng (2000) method and plastic limit determined by the proposed new method.

The Feng (2000) method is based on the assumptions of (*a*) a linear relationship between logarithm of cone penetration, *d*, against logarithm of water content, *w*, in the range of plastic limit to liquid limit, w_ℓ, and (*b*) cone penetration at the plastic limit 1/10 of the cone penetration at the liquid limit. Feng (2000) referenced the log *d* against log *w* relationship to the water content at 1 mm penetration. A better alternative is to reference the log *d* against log*w* relationship to the liquid limit which for the standard 80 g 30° cone corresponds to 20 mm penetration. This approach leads to

$$w_p = w_\ell/10^m \tag{7}$$

where

$$m = \frac{\Delta \log w}{\Delta \log d} \tag{8a}$$

or

$$m = \log \frac{w_\ell}{w_p} \tag{8b}$$

and plasticity index

$$I_p = w_\ell(1 - 1/10^m) \tag{9}$$

Table 1 of Feng (2000) includes values of *m* for 26 soils, as well as the values of w_p determined by the Feng (2000) method and the values of w_ℓ determined by the Casagrande procedure. The values of w_p determined by Feng (2000) were based entirely on the fall-cone measurements. The discusser computed w_p using equation (7) together with values of *m* reported by Feng (2000), however, with values of w_ℓ determined by the Casagrande procedure. The values of w_p computed by the discusser and w_p computed by Feng (2000) are compared in Fig. 12. As one would expect, there is excellent agreement between the two alternative interpretations of the Feng (2000) method, except for a few soils, especially the two bentonites. This means that the Feng (2000) fall-cone tests, with a few exceptions, accurately estimated the liquid limit according to the Casagrande procedure. Apparently, the current interpretation of the fall-cone test does not do a very good job of determining the liquid limit of very high plasticity soils such as the bentonites.

The liquid limit is the water content at which a 30° cone, weighing 80 g, penetrates the soil 20 mm. The current procedure for determining the liquid limit is to repeat the fall-cone test at about four water contents corresponding to penetration range of about 15 to 25 mm. By plotting these (*d*, *w*) data points in log*d* against log*w*, one can determine the liquid limit at $d = 20$ mm, and also determine *m* as defined by equation (8a), which is then used together with equation (7) to compute the plastic limit. In other words, if one has decided to determine the liquid limit by the fall-cone test, very little additional effort is required to compute the plastic limit as well.

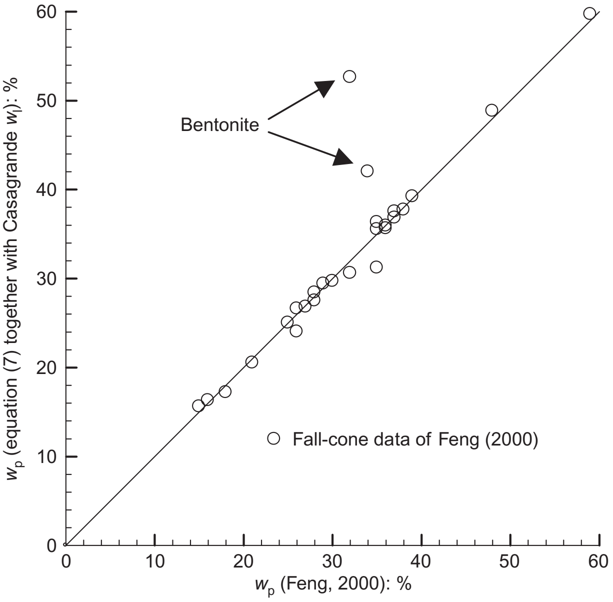

Fig. 12. Plastic limit computed using equation (7) together with *m* and the Casagrande liquid limit from Table 1 of Feng (2000), compared with the plastic limit determined by Feng (2000) based entirely on fall-cone tests

The authors did in fact carry out fall-cone tests, using the standard 30° cone, weighing 80 g, to determine the liquid limit of their 16 soils, reported in Fig. 9. The discusser plotted the (*d*, *w*) data points in log*d* against log*w*, and determined both w_ℓ and *m*, which were used in equation (7) to compute w_p. These values of w_p are compared in Fig. 13 with w_p determined by the thread rolling method and reported as the average in Table 3. The values of w_p by the authors' new method, reported in Table 3, are also compared with w_p by the thread rolling method in Fig. 14. There does not appear to be a particular advantage of the authors' new method over the Feng (2000) method.

The requirements for an acceptable index test and index property are: (*a*) property simple to express, (*b*) measurement quick, (*c*) measurement simple, (*d*) measurement reproducible and (*e*) index property significant. Unfortunately, the authors' new method does not satisfy requirements (*b*) and (*c*), and may not qualify as an acceptable index test.

The fall-cone tests for the liquid limit reported in Fig. 9, as well as those by Feng (2000), interpreted using equations (7) and (8), are summarised in Fig. 15, and the values of *m*, computed using equation (8) for soils with a wide range of plasticity, are shown in Fig. 16.

In summary, it is concluded that if it is decided to determine the liquid limit as the water content at 20 mm penetration of the 80 g, 30° standard cone, then the plastic limit can be readily computed using equations (7) and (8a). The reliability of such an approach may be judged by examining Fig. 15. Fig. 16 suggests that in general *m* increases with the increase in liquid limit; however, a unique empirical relationship for *m* does not appear to exist, and therefore, for each soil fall-cone tests need to be carried out at about four water contents in the range corresponding to

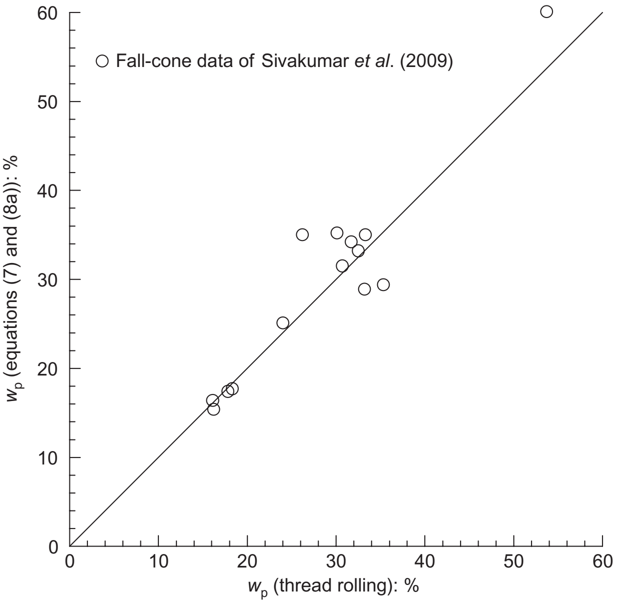

Fig. 13. **Plastic limit computed using equations (7) and (8a) compared with the plastic limit determined by thread rolling procedure**

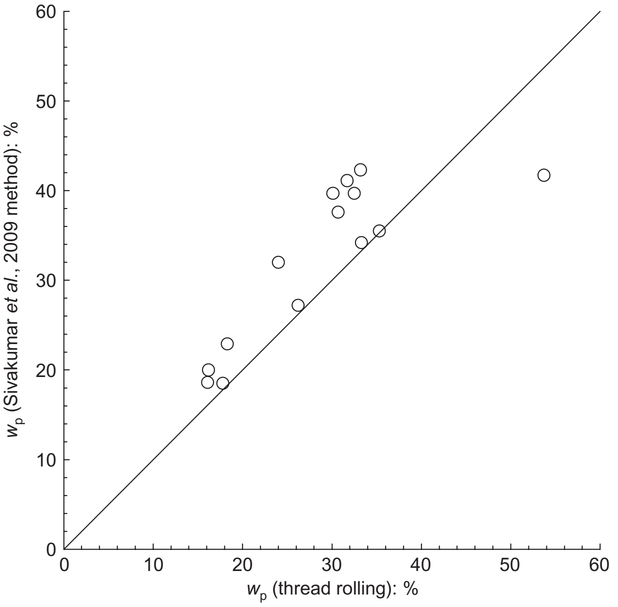

Fig. 14. **Plastic limit determined by Sivakumar *et al.* (2009) method compared with the plastic limit determined by thread rolling procedure**

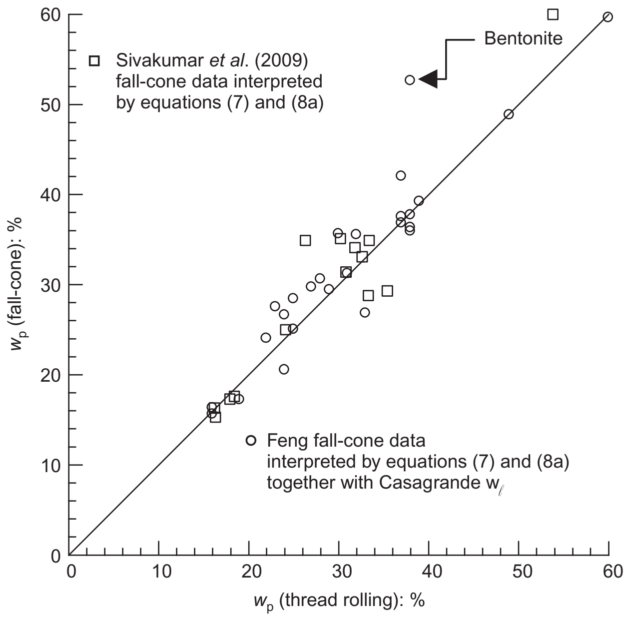

Fig. 15. **Plastic limit by fall-cone tests interpreted by the Feng (2000) method compared with the plastic limit determined by thread rolling procedure**

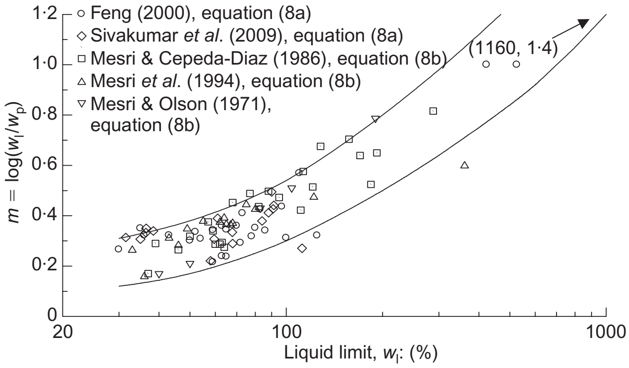

Fig. 16. **Values of $m = \log\ (w_\ell/w_p)$ plotted against liquid limit**

penetration of about 15 to 25 mm in order to plot log*d* against log*w* and compute *m*.

Sample preparation for the index tests on fine-grained soils, especially for stiff clays, shales and residual soils, is an important aspect of the index testing procedure (e.g. Mesri & Cepeda-Diaz, 1986). A detailed treatment of this topic is beyond the scope of the present discussion; however, it needs to be mentioned that the discusser is not in favour of '... initially oven-drying [the soil] at 105°C ...'.

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Authors' reply

The authors wish to thank the discussers for their interest in the paper, and for their constructive remarks which provide additional confidence in the newly proposed technique. There are essentially two important observations that require a response.

Professor Mesri presents (in Fig. 15 of his discussion) an interesting comparison of plastic limits predicted using the Feng (2000; 2001) approach and those measured by the authors (Sivakumar *et al.*, 2009). The Feng (2000; 2001) approach, also suggested by Koumoto & Houlsby (2001), is based on a linear log-log relationship between cone penetration depth and water content expressed by the following equation

$$\log w = \log c + m \log d \tag{10}$$

where w is the water content, c is water content at $d = 1$ mm and m is the slope of the (linear) log-log relationship. The water content at the plastic limit is determined by regression and extrapolation of corresponding curves to a penetration depth of 2 mm. It is evident that predicted and measured plastic limits are in reasonable agreement; however, it must be noted that there is inconsistency in some soils, where the predicted plastic limit using the Feng (2000) approach was significantly less well correlated. In response to this graph, the authors conducted an additional independent investigation of two soils used as part of the original study in order to evaluate both approaches. Index properties of Ampthill and kaolin clay were assessed at four leading geotechnical testing laboratories in Northern Ireland and the plastic limits measured using the rolling thread method are reported in Table 5. As per the Feng (2000) approach, cone penetration tests were carried out to obtain multiple cone penetrations in the range 8–23 mm at various water contents. On the assumption that the average plastic limit measured using the rolling thread technique reported by the laboratories is a benchmark for the actual plastic limit, it appears that the Feng (2000) approach overestimates plastic limit in the case of Ampthill clay and underestimates the plastic limit in the case of kaolin. As per the Sivakumar *et al.* (2009) method, the plastic limit of Ampthill clay is overestimated by 3% whereas kaolin is in good agreement, although one should note the fact that the variation in the plastic limit of kaolin reported by the four laboratories is relatively high.

While it is evident that the two approaches delivered a good approximation of the plastic limit, is must be noted that the method proposed by the authors offers a repeatable mechanical process that does not rely on extrapolation of cone penetration–water content correlations. Furthermore, the Feng (2000) approach relies heavily on a strong linear correlation established using between eight and 14 penetration results in the range 3–25 mm to provide confidence in curve fitting. Such rigour is unlikely to be applied in commercial tests. The discusser suggests that a linear correlation could be established using as little as four water contents obtained for penetrations in the range 15–25 mm as specified for the liquid limit evaluation. In the authors' opinion, determining plastic limit based on a limited number of cone penetration–water content measurement points over a reduced range (particularly when plotted on a log scale) is unlikely to result in consistent and accurate predictions of the plastic limit.

Second, Dr O'Kelly in his discussion refers to the strength ratio R defined as

$$R = \frac{c_{\mathrm{u(PL)}}}{c_{\mathrm{u(LL)}}} \tag{11}$$

where $c_{\mathrm{u(PL)}}$ and $c_{\mathrm{u(LL)}}$ are defined as the undrained shear strength at the plastic limit and liquid limit respectively. This ratio has been the subject of debate over many years and it is widely accepted that $R \approx 100$ is a reasonable approximation (Skempton & Northey, 1953; Hansbo, 1957; Wood & Wroth, 1978; Stone & Phan 1995). But as the discusser identifies, values for R as high as 170 have been reported occasionally in the literature. The wide range in R reported is largely atttributed to the fact that measuring undrained shear strength at the liquid limit is not straightforward.

In this regard the discusser estimates R for the current work to be 33 based on a calculated $c_{\mathrm{u(LL)}} = 2{\cdot}66$ kPa from the fall-cone test reported by Koumoto & Houlsby (2001). As part of ongoing work the authors have recently conducted a series of tests in order to assess the strengths of kaolin prepared at the liquid and plastic limits. The soil strength at the liquid limit was assessed by inserting a smooth aluminium open channel rectangular section, (Π shape) of width 81·5 mm and thickness 1 mm, into a large chamber of kaolin slurry prepared at the liquid limit. The section was driven at a constant rate of 0·5 mm/min over a distance of 10 mm, during which time the load required for penetration was measured using a proving ring capable of measuring load accurately to 0·0071 N. The load–displacement behaviour is shown in Fig. 17. The slope of the line was used to determine the undrained shear strength assuming an adhesion factor of 1. The undrained shear strength was calculated to be 0·76 kPa. The undrained shear strength reported using this approach is lower than that reported by Koumoto & Houlsby (2001) indicating that the discusser's determination of R for the present study is under-predicted.

The undrained shear strength at the plastic limit was assessed by way of a standard unconsolidated undrained triaxial test on a 38 mm diameter by 76 mm high specimen prepared at a water content of 34% (the average value reported in Table 5). The sample was prepared by compaction using the technique described in the paper (Sivakumar *et al.*, 2009). The sample was sheared under undrained conditions at a rate of 0·5 mm/min. The measured undrained shear strength was approximately 77 kPa. From these tests the ratio R is evaluated to be approximately 101.

The above procedure shows that the strength ratio R is close to 100 for kaolin tested in a remoulded condition. A similar value was obtained for reconstituted kaolin. The

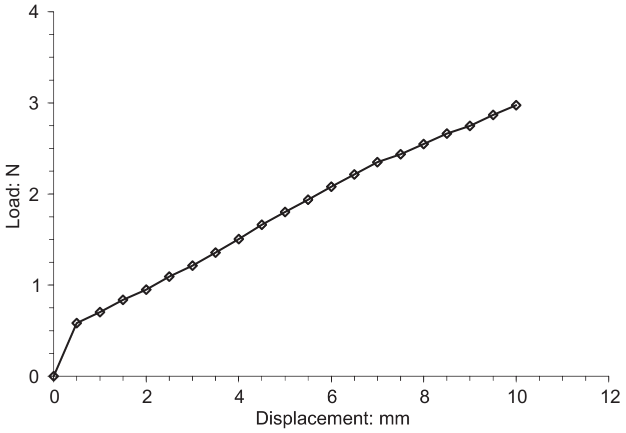

Fig. 17. Load–displacement of a channel section driven at a penetration rate of 0·5 mm/min into kaolin slurry prepared at the liquid limit

Table 5. Plastic limit measured using four site investigation laboratories

Soil type	Lab 1	Lab 2	Lab 3	Lab 4	Average	Feng (2000)	Sivakumar *et al.* (2009)
Ampthill	31	32	33	30	32	34	35
Kaolin	33	36	37	29	34	32	34

strength ratio R is given by the following relationship (Wood, 1990)

$$R = e^{G_s \mathrm{PI}/\lambda} \tag{12}$$

where λ is the slope of the critical state line (as defined in $q{:}p'$ plane), PI is the plastic index and G_s is the specific gravity. Substituting $\lambda = 0{\cdot}21$, $\mathrm{PI} = 36\%$ and $G_s = 2{\cdot}65$ for the kaolin used in the present study leads to a ratio $R = 94$.

The new plastic limit device was calibrated using artificial kaolin clay. However, the authors agree that further investigations considering a wider range of soil type with different characteristics need to be undertaken to validate fully the approach. In this respect the authors are currently involved in conducting additional calibration tests to determine the necessary cone force for a variety of natural soils in order to gain an enhanced correlation and confidence in the use of device.

In conclusion, the authors have responded to the need, as highlighted previously by several researchers (e.g. Wood & Wroth, 1978), for a more rigorous and robust method of measuring plastic limit directly. The authors have developed a mechanical device for measuring the plastic limit directly. This offers a viable alternative to the traditional method of rolling threads (BS 1377, BSI, 1990) or more recent extrapolation and predictive techniques (Feng, 2000; 2001). This system has the added benefit of not requesting direct contact between the operator and the soil, which is particularly relevant when assessing the index properties of contaminated soils, which may pose a health and safety hazard if evaluated using the traditional rolling thread approach. The correlation between the plastic limit obtained by the rolling method and this fall-cone method is very encouraging and further research is on-going to assess its validity for a wider range of soils. While the authors agree that empirical correlations (example, Feng (2000)) are useful, it would be unwise to rely solely on them for routinely determining plastic limit, as they are unlikely to be valid for all natural materials.

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