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When Do Jumps Matter for Portfolio Optimization?

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Abstract

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JEL-Classification: G11; C63

Keywords: Optimal investment, jumps, stochastic volatility, welfare loss

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1 Introduction

We consider the continuous-time portfolio optimization problem of an investor with constant relative risk aversion who maximizes the expected utility of terminal wealth. The investor has access to a risky and a risk-free asset.[1] The risky asset follows a generalized jump-diffusion process. We assume that the stock price is subject to jump risk, while the state variable follows a diffusion process. In general, this problem cannot be solved analytically.

We propose an approach to find approximating solutions for the optimal portfolio strategies. Our approximation is based on applying a Taylor approximation to the jump-related terms in the Hamilton-Jacobi-Bellman (HJB) equation. This is equivalent to matching the first two moments of the jumps by a diffusion and leads to an approximating portfolio problem in which the risky asset follows a diffusion process. This problem can be solved using a stochastic representation result. We refer to the resulting closed-form optimal strategies as approximating strategies. The crucial question is how good the approximating solution performs. To answer this question, we first give bounds for the difference between the (unknown) true strategy and the approximating strategy. Second, we find bounds for the relative wealth equivalent loss (RWEL), which is the proportion of wealth at time 0 that an investor using the optimal strategy can sacrifice in order to have the same indirect utility as an investor using the approximating strategy.

We apply our general results to an affine model with stochastic volatility and jumps in the stock price. The stochastic volatility of the stock is modeled as in Heston (1993), and the stock price is subject to jumps with a possibly stochastic jump size. Bates (1996) and Bates (2000) study these types of model in the context of option pricing. The jump size is assumed to be constant, beta-distributed, or log-normally distributed. The parameters are similar to the ones in Pan (2002) and Liu, Longstaff, and Pan (2003), who calibrate the model to S&P 500 option data. We provide an explicit solution for the approximating problem and use the

[1] This problem has been introduced in Merton (1969) and Merton (1971).

numerical procedure from Liu, Longstaff, and Pan (2003) to also solve the true problem. For risk-aversion levels of $\gamma > 7$, we find that the bounds on the optimal strategies are less than 18% apart from each other. We apply the numerical method from Liu, Longstaff, and Pan (2003) to calculate exact RWELs for a ten year horizon and find that our approximating strategy performs well. For the base calibration, all RWELs are below 1.16% if jumps are stochastic, and below 1% if the jump size is constant and $\gamma \geq 5$. In line with intuition, an increase in the expected jump size or in the jump intensity leads to larger RWELs. For realistic combinations of jump risk and risk-aversion, however, the RWELs are below 2%. Finally, we provide a second approximating strategy that implies RWELs of less than 5bps. This strategy is, however, not fully explicit, since we must determine a constant that is implicitly given.

The remainder of the paper is organized as follows: Section 2 formulates a general model for a market with jumps in the stock price, presents our approximation procedure and derives bounds on the optimal strategy and on the RWEL resulting from this approximation. Section 3 considers an affine specification of the general model calibrated to S&P 500 data. It provides a numerical solution to the true problem and a closed-form solution to the approximating problem. Furthermore, we study the optimal and the approximating strategies in different settings and report the respective RWELs. The settings include varying risk-aversions, jump sizes, and jump intensities. Section 4 concludes. Proofs that do not follow their statement can be found in the Appendix.

2 Model and Portfolio Optimization

We consider an economy where uncertainty is described by a complete filtered probability space denoted by $(\Omega, \mathcal{P}, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T^*]})$ with $\mathcal{F} = \mathcal{F}_{T^*}$. The dynamics of the stock price S are

$$dS = S\left(\mu\, dt + \sigma\, dW - L\, dN\right), \tag{1}$$

where W is a Browninan motion and N is a Cox process with intensity λ. The jump size L can be stochastic. The drift μ, the volatility $\sigma \geq 0$, and the jump intensity $\lambda \geq 0$ are functions of time t and a state variable Y. The dynamics of Y are given by

$$dY = \alpha\, dt + \beta\, (\rho\, dW + \sqrt{1-\rho^2}\, d\tilde{W}). \tag{2}$$

Its drift α and volatility $\beta \geq 0$ are also functions of t and Y. The correlation ρ is constant, and $\tilde{W}$ is a Brownian motion independent of W. Unless stated otherwise, Y is the local variance process of the stock, i.e. $\sigma^2 = Y$.

In general, the portfolio problem for models with jumps cannot be solved explicitly. As discussed later, this is due to additional terms in the HJB involving the portfolio strategy. Our objective is to derive an analytical optimal strategy in a simplified market. This strategy is then used as an approximating optimal strategy in the original market.

2.1 Portfolio Problem with Jumps

We consider an investor who maximizes expected utility from terminal wealth. He has constant relative risk aversion equal to $\gamma > 1$. His investment opportunities comprise a money market account with constant interest rate r and a stock with dynamics given by (1). Hence, his wealth dynamics are given by

$$dX = X[(r + \pi\chi)dt + \pi\sigma dW - \pi L dN],$$

where π denotes the proportion of wealth invested in stock, and $\chi \equiv \mu - r$. The stock's expected excess return is given by $\tilde{\chi} \equiv \mu - \bar{L}_1\lambda - r = \chi - \bar{L}_1\lambda$, where $\bar{L}_k$ denotes the k-th moment of L. The investor's indirect utility (value function) is $J(t, x, y) = \max_\pi \mathrm{E}^{t,x,y}[u(X_T)]$.

It satisfies the Hamilton-Jacobi-Bellman equation (HJB):

$$
\begin{aligned}
0 \;=\; \max_{\pi} \Big\{ & J_t + x(r + \pi\chi)J_x + 0.5x^2\pi^2\sigma^2 J_{xx} + \alpha J_y + 0.5\beta^2 J_{yy} \\
& + x\pi\sigma\beta\rho J_{xy} + \lambda\Big[\mathrm{E}[J(t, x(1 - \pi L), y)] - J(t, x, y)\Big]\Big\},
\end{aligned} \tag{3}
$$

where the expectation operator $\mathrm{E}[\cdot]$ represents the expectation over the jump size L. Since the marginal utility goes to infinity at zero wealth (Inada condition), the investor avoids strategies with a positive probability of instantaneous total losses triggered by large jumps. Admissible strategies must satisfy the condition

$$
1 - \pi L > 0. \tag{4}
$$

2.2 Approximating Portfolio Problem

To find an approximating solution to the portfolio problem, we use a second-order Taylor expansion for the jump term in the HJB-equation (3):

$$
J(t, x(1 - \pi L), y) \approx J(t, x, y) - J_x(t, x, y)x\pi L + 0.5J_{xx}(t, x, y)(x\pi L)^2. \tag{5}
$$

This implies $\mathrm{E}[J(t, x(1 - \pi L), y)] \approx J(t, x, y) - J_x(t, x, y)x\pi\bar{L}_1 + 0.5J_{xx}(t, x, y)(x\pi)^2\bar{L}_2$. Substituting this approximation into the HJB equation (3) yields

$$
\begin{aligned}
0 \;=\; \max_{\pi} \Big\{ & \tilde{J}_t + x[r + \pi(\underbrace{\chi - \lambda\bar{L}_1}_{\equiv\tilde{\chi}})]\tilde{J}_x + 0.5x^2\pi^2(\underbrace{\sigma^2 + \lambda\bar{L}_2}_{\equiv\tilde{\sigma}^2})\tilde{J}_{xx} \\
& + \alpha\tilde{J}_y + 0.5\beta^2\tilde{J}_{yy} + x\pi\sigma\beta\rho\tilde{J}_{xy}\Big\}.
\end{aligned} \tag{6}
$$

We use J to denote the solution of the HJB equation (3), while $\tilde{J}$ denotes the solution of the approximating HJB equation (6). Analogously, π^* and $\tilde{\pi}^*$ denote the optimal strategies corresponding to (3) and (6).

The approximating HJB-equation (6) also results from a model in which the jump-diffusion process for the stock price is replaced by a diffusion process with drift and volatility adjusted in such a way that the expected change and the local variance match those from the original process. For $L = 0$, the two equations are of course identical. The main point is that the approximating portfolio problem can be solved in closed-form. For problem (6), on the other hand, there is no closed-form solution available.

Now, the decisive question arises how good this approximation performs. How large is the utility loss from following the approximating strategy $\tilde{\pi}^*$ instead of the truly optimal strategy π^*? Furthermore, can we find an estimate for the approximation error *solely* by solving (6), i.e. *without knowing* the actual indirect utility J and the actual optimal stock proportion π^*? To address these points, we first solve the approximating problem (6). In the following, we make the assumption that the jump intensity is proportional to the diffusive variance, i.e. that there is a positive constant K such that

$$\lambda = K\sigma^2. \tag{7}$$

This assumption is for instance satisfied in the Heston model with jumps if we set $\lambda = KY_t$.

Proposition 2.1 (Stochastic Representation of Approximated Indirect Utility) *If assumption* (7) *is satisfied, the following results hold: (i) The indirect utility $\tilde{J}$ of the approximating model solves* (6) *and is given by*

$$\tilde{J}(t,x,y) = \frac{1}{1-\gamma} x^{1-\gamma} \tilde{f}(t,y)^k \quad \textit{with} \quad k = \frac{\gamma(1+K\bar{L}_2)}{\gamma(1-\rho^2+K\bar{L}_2)+\rho^2} = \textit{const}, \tag{8}$$

where $\tilde{f}$ has the stochastic representation

$$\tilde{f}(t,y) = \tilde{\mathrm{E}}^{t,y}\left[e^{-\int_t^T \tilde{r}_u \, du}\right] \quad \textit{with} \quad -\tilde{r} = \tfrac{1-\gamma}{k}\left(r + 0.5\tfrac{1}{\gamma}\tfrac{\tilde{\chi}^2}{\tilde{\sigma}^2}\right). \tag{9}$$

The expectation $\tilde{\mathrm{E}}[\cdot]$ is calculated using the measure $\tilde{\mathcal{P}}$ under which Y has the drift $\tilde{\alpha}$ =

$\alpha + \frac{1-\gamma}{\gamma} \frac{\tilde{\chi}\sigma\beta\rho}{\tilde{\sigma}^2}$.

(ii) If $(\tilde{\chi}/\tilde{\sigma})^2$ *as a function of* Y *is differentiable and increasing (decreasing), then* $\tilde{f}_y$ *is negative (positive).*

2.3 Bounds on Optimal Strategy

In the following, we derive bounds on the optimal strategy π^*. The investor's indirect utility function is given by $J(t, x, y) = \frac{1}{1-\gamma} x^{1-\gamma} g(t, y)$. The optimal strategy π^* follows from the first-order condition

$$\gamma\sigma^2\pi = \chi + \sigma\beta\rho\frac{g_y}{g} - \lambda \mathrm{E}[L(1 - \pi L)^{-\gamma}]. \tag{10}$$

To compare (10) with the first-order condition of the approximating HJB-equation (6), we consider the first-order expansion $(1 - \pi L)^{-\gamma} = 1 + \gamma\pi L + R_1(-\pi L)$, where[2]

$$R_1(-\pi L) \equiv \frac{\gamma(1+\gamma)}{2} \frac{1}{(1+\xi)^{\gamma+2}} (\pi L)^2, \quad \xi \in < -\pi L, 0 >,$$

denotes the remainder term of the expansion. Therefore, the first-order condition (10) can be rewritten as

$$\gamma(\underbrace{\sigma^2 + \lambda\bar{L}_2}_{=\tilde{\sigma}^2})\pi = \underbrace{\chi - \lambda\bar{L}_1}_{=\tilde{\chi}} + \sigma\beta\rho\frac{g_y}{g} - \lambda \mathrm{E}[L R_1(-\pi L)].$$

We now make two assumptions to derive bounds on the optimal strategy. First, we assume that the jump size distribution is positive

$$L \geq 0, \tag{11}$$

i.e. jumps only have a negative effect. Second, we only consider positive strategies, i.e.

$$\pi \geq 0. \tag{12}$$

[2] We define $< a, b > \equiv [\min(a, b), \max(a, b)]$.

As we will show in Section 3, condition (12) is satisfied for realistic calibrations. Assumptions (11) and (12) together imply that the presence of jumps can never increase the utility of the investor. Furthermore, assumptions (4), (11), and (12) guarantee $|\pi L| < 1$, which ensures that $(1-\pi L)^{-\gamma}$ has a well-defined binomial series expansion.

We first consider the case $\rho = 0$. The first-order condition becomes

$$\pi^* = \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2} - \frac{\lambda \mathrm{E}[LR_1(-\pi^* L)]}{\gamma\tilde{\sigma}^2}. \tag{13}$$

The terms depending on g have vanished. With zero correlation between the stock price and the state variable, the investor can no longer hedge changes in the investment opportunity set, and the hedging demand is zero. By abstracting from hedge terms, we can perform a clean analysis of the effect of jumps. As we will see later on, the general case is more involved. Given the results of Larsen and Munk (2012), who study sub-optimal strategies with misspecified hedge terms, this is not surprising. The following proposition gives bounds on the optimal strategy π^*.

Proposition 2.2 (Bounds on Optimal Stock Demand (I)) *If assumptions* (4), (7), (11), (12) *hold and* $\rho = 0$, *then*

$$\tilde{\pi}^* \geq \pi^* \geq \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2} - \frac{(1+\gamma)\lambda}{2\tilde{\sigma}^2}(\tilde{\pi}^*)^2 C(\tilde{\pi}^*),$$

where $C(\pi) = \mathrm{E}\left[\frac{L^3}{(1-\pi L)^{\gamma+2}}\right]$.

Proposition 2.2 uses the approximating strategy $\tilde{\pi}^*$ as an upper bound. The next proposition shows that we can find tighter upper and also lower bounds using a quadratic inequality.

Proposition 2.3 (Bounds on Optimal Stock Demand (II)) *If assumptions* (4), (7), (11), (12) *hold and* $\rho = 0$, *then* $\pi_u \geq \pi^* \geq \pi_l$, *where for* $j \in \{u, l\}$

$$\pi_j = -\frac{1}{2A_j} + \sqrt{\frac{1}{(2A_j)^2} + \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2 A_j}}$$

with $A_u = \frac{(1+\gamma)\lambda}{2\tilde{\sigma}^2}\bar{L}_3$ *and* $A_l = \frac{(1+\gamma)\lambda}{2\tilde{\sigma}^2}E\left[\frac{L^3}{(1-\pi_u L)^{\gamma+2}}\right]$.

Next, we consider the general case $\rho \neq 0$. To derive bounds in this general case, we make the additional assumption

$$\rho g_y \geq 0, \tag{14}$$

which is satisfied for realistic calibrations of the Heston model. Without this assumption, the Merton-Breeden term in the first-order condition (10) is negative and in general unbounded from below, which makes it very hard to derive bounds.

Proposition 2.4 (Bounds on Optimal Stock Demand (III)) *If assumptions* (4), (7), (11), (12) *and* (14) *hold, then the bounds for the optimal stock demand are*

$$\overline{\pi} \geq \pi^* \geq \underline{\pi},$$

where $\overline{\pi}$ *is the optimal strategy in an auxiliary model where we set* $L = 0$*, and* $\underline{\pi}$ *is the optimal strategy in an auxiliary model where we set* $\rho = 0$*. The corresponding first-order conditions are*

$$\gamma\sigma^2\underline{\pi} = \chi - \lambda \mathrm{E}[L(1-\underline{\pi}L)^{-\gamma}], \text{ and} \tag{15}$$

$$\gamma\sigma^2\overline{\pi} = \chi + \sigma\beta\rho\frac{g_y^{nj}}{g^{nj}}, \tag{16}$$

where g^{nj} *denotes the non-wealth dependent part of the investor's indirect utility if the jump size is set to zero ("no jumps").*

The following proposition can be used to check assumption (14). Here χ_y, $(\sigma^2)_y$, and π_y denote the partial derivatives of χ, σ^2, and π with respect to y.

Proposition 2.5 (Sign of Hedge Term) *Assuming* (4), (7), *and sufficient differentiabil-*

ity, the derivative g_y is positive (negative) if

$$(1-\gamma)\left[\pi_y\chi + \pi\chi_y - \gamma\pi\pi_y\sigma^2 - 0.5\gamma\pi(\sigma^2)_y - \lambda\pi_y \mathrm{E}[L(1-\pi L)^{-\gamma}\right] + \lambda_y\left\{\mathrm{E}[(1-\pi L)^{1-\gamma}] - 1\right\} \tag{17}$$

is positive (negative). If π is deterministic, then (17) *simplifies to*

$$(1-\gamma)\left[\pi\chi_y - 0.5\pi(\sigma^2)_y\right] + \lambda_y\left\{\mathrm{E}[(1-\pi L)^{1-\gamma}] - 1\right\}.$$

2.4 Bounds on Relative Wealth Equivalent Loss

When the investor relies on the approximating strategy from solving (6), he suffers a utility loss. We measure this utility loss by the RWEL, that is the percentage of initial wealth that an investor using the optimal strategy can sacrifice and still have the same indirect utility as an investor using the approximating strategy. The investor's indirect utility in the original and the approximating model for *given* strategies π and $\tilde{\pi}$ can be represented by

$$G(t,x,y;\pi) = \frac{1}{1-\gamma}x^{1-\gamma}g(t,y;\pi) \quad \text{and} \quad \tilde{G}(t,x,y;\tilde{\pi}) = \frac{1}{1-\gamma}x^{1-\gamma}\tilde{g}(t,y;\tilde{\pi}),$$

The RWEL ℓ is defined by

$$\underbrace{G(t,x(1-\ell),y;\pi^*)}_{=J(t,x(1-\ell),y)} = G(t,x,y;\tilde{\pi}^*). \tag{18}$$

Using the functional form of G, the loss can be calculated as $\ell = 1 - \left(\frac{g(t,y;\tilde{\pi}^*)}{g(t,y;\pi^*)}\right)^{\frac{1}{1-\gamma}}$. To determine the exact loss, we thus need to know the truly optimal strategy π^* and the utility $G(t,x,\pi)$ in the original model. The challenge, however, is to find bounds for this loss that only depend on the optimal strategy $\tilde{\pi}^*$ and the utility $\tilde{G}$ in the approximating model.

Theorem 2.6 (Approximation Error) *Assume that* (4) *holds.*

(i) The difference of the indirect utilities in the original and approximating model reads

$$G(t,x,y;\pi) - \tilde{G}(t,x,y;\tilde{\pi}) = \frac{1}{1-\gamma} x^{1-\gamma} D(t,y;\pi,\tilde{\pi}),$$

where $D(t,y;\pi,\tilde{\pi}) = g(t,y) - \tilde{g}(t,y) = \int_t^T \hat{\mathrm{E}}\Big[e^{-\int_t^s \hat{r}_u\,du}\hat{D}_s\Big]\,ds$ *with*

$$\begin{aligned}
\hat{D} &= \big\{\lambda \mathrm{E}[R_2(-\pi L)] - (1-\gamma)\Delta\pi\big[\chi - \lambda\bar{L}_1 + \sigma\beta\rho\tilde{g}_y/\tilde{g} - 0.5\gamma(2+\Delta)\pi(\sigma^2 + \lambda\bar{L}_2)\big]\big\}\,\tilde{g},\\
R_2(-\pi L) &= \frac{\gamma(1-\gamma^2)}{6}\frac{1}{(1+\xi)^{\gamma+2}}(-\pi L)^3, \quad \xi \in <-\pi L, 0>,
\end{aligned}$$

$-\hat{r} = (1-\gamma)(r + \pi\chi - 0.5\gamma\pi^2\sigma^2) + \lambda\big[\mathrm{E}[(1-\pi L)^{1-\gamma}] - 1\big]$, *and* $\hat{\alpha} = \alpha + (1-\gamma)\pi\sigma\beta\rho$. *The expectation* $\hat{\mathrm{E}}[\cdot]$ *is taken under a measure* $\hat{\mathcal{P}}$ *under which* Y *has the drift* $\hat{\alpha}$ *(instead of* α*). The variable* Δ *is defined by* $\tilde{\pi} = \pi(1+\Delta)$.

(ii) If $\tilde{\pi} = \tilde{\pi}^*$, *then* $\hat{D}$ *simplifies into*

$$\hat{D} = \big\{\lambda E\left[R_2(-\pi L)\right] - 0.5\gamma(1-\gamma)\left(\pi\Delta\right)^2\left(\sigma^2 + \lambda\bar{L}_2\right)\big\}\,\tilde{g}. \tag{19}$$

(iii) If assumption (7) holds and $\tilde{\pi} = \tilde{\pi}^*$, *then* $\tilde{g} = \tilde{f}^k$ *where* $\tilde{f}$ *and* k *are given in (8).*

Remarks. a) The representation of the utility difference D does not depend on the unknown function g, but only on the function $\tilde{g}$ calculated in the approximating model.

b) The theorem is not restricted to the optimal strategies π^* and $\tilde{\pi}^*$, but it holds for all admissible strategies π and $\tilde{\pi}$. It thus provides a general tool to find an estimate for the error the investor makes if he implements a misspecified strategy $\tilde{\pi}$ (instead of π^*). Of course, we are ultimately interested in the case where $\pi = \pi^*$ and $\tilde{\pi} = \tilde{\pi}^*$.

c) For parameterizations in which assumptions (11) and (12) hold, the remainder term R_2 is positive. Consequently, $\hat{D}$ and thus D are positive if $\tilde{\pi} = \tilde{\pi}^*$. This implies $G \leq \tilde{G}$, i.e. the solution to the approximating problem overestimates the investor's indirect utility.

We now apply Theorem 2.6 to two specific situations:[3]

$$g(\pi^*) = \tilde{g}(\tilde{\pi}^*) + D(t, y; \pi^*, \tilde{\pi}^*), \quad \text{and} \quad g(\tilde{\pi}^*) = \tilde{g}(\tilde{\pi}^*) + D(t, y; \tilde{\pi}^*, \tilde{\pi}^*)$$

$D(t, y; \pi^*, \tilde{\pi}^*)$ gives the difference between the original model and the approximating model when we use the respective optimal strategies π^* and $\tilde{\pi}^*$ in both models. $D(t, y; \tilde{\pi}^*, \tilde{\pi}^*)$ gives the difference between the original model and the approximating model when we use the approximating optimal strategy $\tilde{\pi}^*$ in both models.

Theorem 2.6 and the bounds on the optimal strategy π^* from Subsection 2.3 allow us to find bounds on these two functions. The bounds are denoted by

$$\underline{D} \le D(t, y; \pi^*, \tilde{\pi}^*) \le \overline{D}, \quad \text{and} \quad \underline{\tilde{D}} \le D(t, y; \tilde{\pi}^*, \tilde{\pi}^*) \le \overline{\tilde{D}}.$$

This also gives bounds on $g(\pi^*)$ and $g(\tilde{\pi}^*)$:

$$\underline{D} + \tilde{g}(\tilde{\pi}^*) \le g(\pi^*) \le \overline{D} + \tilde{g}(\tilde{\pi}^*), \quad \text{and} \quad \underline{\tilde{D}} + \tilde{g}(\tilde{\pi}^*) \le g(\tilde{\pi}^*) \le \overline{\tilde{D}} + \tilde{g}(\tilde{\pi}^*)$$

From these, we get lower and upper bounds for the ratio of the two functions:

$$\underbrace{\frac{\underline{\tilde{D}} + \tilde{g}(\tilde{\pi}^*)}{\overline{D} + \tilde{g}(\tilde{\pi}^*)}}_{\equiv \underline{B}} \le \frac{g(\tilde{\pi}^*)}{g(\pi^*)} \le \underbrace{\frac{\overline{\tilde{D}} + \tilde{g}(\tilde{\pi}^*)}{\underline{D} + \tilde{g}(\tilde{\pi}^*)}}_{\equiv \overline{B}}.$$

which then give upper and lower bounds on the RWEL: $1 - (\underline{B})^{\frac{1}{1-\gamma}} \le \ell \le 1 - (\overline{B})^{\frac{1}{1-\gamma}}$. Notice that $\underline{B}$ and $\overline{B}$ depend on $\tilde{g}(\tilde{\pi}^*)$ and on the bounds for D only. In particular, they can be calculated without knowing the solution to the portfolio problem in the original model.

[3] We use the short-hand notation $g(\pi)$ instead of $g(t, y; \pi)$.

3 An Affine Model

We now consider a specific (affine) parametrization of the model that is given by

$$\chi(t,y) = \bar{\chi} y, \quad \sigma(t,y) = \sqrt{y}, \quad \lambda(t,y) = \bar{\lambda} y, \quad \alpha(t,y) = \theta - \kappa y, \quad \beta(t,y) = \bar{\beta}\sqrt{y}. \tag{20}$$

The state variable Y is the local diffusion variance of the stock. It follows the square-root process from the model of Heston (1993). The stock price can jump, and with the jump intensity being proportional to the state variable Y, assumption (7) is satisfied. This model setup is widely used in the option pricing literature, including Bates (1996) and Bates (2000). Liu, Longstaff, and Pan (2003) study the corresponding portfolio problem. They derive three equations that implicitly define the optimal investment strategy and indirect utility function, which have to be solved numerically. The following proposition summarizes their results.

Proposition 3.1 (Optimal Strategy in the Affine Model)
Assume that the original model is given by the affine specification (20)*. The optimal indirect utility is given by*

$$J(t,x,y) = \frac{x^{1-\gamma}}{1-\gamma} e^{A(t)+B(t)y}, \tag{21}$$

where the following three equations implicitly define the optimal portfolio weight π_t^* *and the functions* B *and* A*:*

$$\bar{\chi} - \gamma\pi(t) + \bar{\beta}\rho B(t) - \bar{\lambda}\mathrm{E}\left[(1-\pi(t)L)^{-\gamma}L\right] = 0 \tag{22}$$

$$\Big\{ B_t(t) + B(t)\left[(1-\gamma)\bar{\beta}\rho\pi(t) - \kappa\right] + \bar{\lambda}\mathrm{E}\left[(1-\pi(t)L)^{1-\gamma}\right] + \frac{1}{2}\bar{\beta}^2 B(t)^2 + \left[\pi(t)^2\frac{\gamma(\gamma-1)}{2} + \pi(t)(1-\gamma)\bar{\chi} - \bar{\lambda}\right] \Big\} = 0 \tag{23}$$

$$A_t(t) + (1-\gamma)r + \theta B(t) = 0 \tag{24}$$

The boundary conditions are $A(T) = B(T) = 0$.

These equations can be solved by applying a backward differencing scheme. Furthermore, if we use a (not necessarily optimal) strategy π, equations (23) and (24) allow us to calculate the associated expected utility. When the correlation between the stock price and its volatility is zero, the optimal strategy and indirect utility become more explicit.

Proposition 3.2 (Uncorrelated Local Variance)

Assume that the original model is given by the affine specification (20) *and that* $\rho = 0$.

(i) The optimal investment policy is constant over time, independent of the local variance Y *and implicitly given by*

$$\gamma\pi^* = \bar{\chi} - \bar{\lambda}\mathrm{E}\left[L\left(1-\pi^* L\right)^{-\gamma}\right]. \tag{25}$$

(ii) The indirect utility for a given constant strategy π *is*[4]

$$G_0\left(t,x,y,\pi\right) = \frac{1}{1-\gamma}x^{1-\gamma}g_0\left(t,y,\pi\right), \text{ with } g_0\left(t,y,\pi\right) = e^{(1-\gamma)r(T-t)+A_0(t,T)+B_0(t,T)y}, \tag{26}$$

where

$$\begin{aligned} A_0(t,T) &= \frac{2\theta}{\bar{\beta}^2}\ln\left(\frac{2ae^{\frac{1}{2}(a+\kappa)(T-t)}}{2a+(a+\kappa)\left(e^{a(T-t)}-1\right)}\right), \quad B_0(t,T) = \frac{2C_0\left(e^{a(T-t)}-1\right)}{2a+(a+\kappa)\left(e^{a(T-t)}-1\right)}, \\ C_0 &= (1-\gamma)\left(\pi\bar{\chi} - \frac{1}{2}\gamma(\pi)^2 + \frac{\bar{\lambda}}{1-\gamma}\left\{\mathrm{E}\left[(1-\pi L)^{1-\gamma}\right]-1\right\}\right), \end{aligned}$$

and $a = \sqrt{\kappa^2 - 2\bar{\beta}^2 C_0}$. *The indirect utility of the optimal strategy is thus* $J_0\left(t,x,y\right) = G_0\left(t,x,y;\pi^*\right)$, *with* π^* *implicitly defined by* (25).

(iii) The approximating strategy $\tilde{\pi}^*$ *is also constant and independent of the local variance with*

$$\tilde{\pi}^* = \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2} = \frac{\bar{\chi} - \bar{\lambda}\bar{L}_1}{\gamma\left(1+\bar{\lambda}\bar{L}_2\right)}. \tag{27}$$

[4] We use the subscript 0 to indicate the we assume $\rho = 0$.

(iv) The RWEL is

$$R = \left[\frac{g_0\left(t, y; \pi^*\right)}{g_0\left(t, y; \tilde{\pi}^*\right)} \right]^{\frac{1}{1-\gamma}} - 1.$$

We now turn to the general case $\rho \neq 0$. In the approximating market, we have $\tilde{\alpha} = \theta - \tilde{\kappa} y$ with $\tilde{\kappa} = \kappa + \frac{\gamma-1}{\gamma} \frac{\tilde{\bar{\chi}} \bar{\sigma} \bar{\beta} \rho}{\tilde{\bar{\sigma}}^2}$ where $\tilde{\bar{\chi}} = \bar{\chi} - \bar{\lambda} \bar{L}_1$ and $\tilde{\bar{\sigma}}^2 = \bar{\sigma}^2 + \bar{\lambda} \bar{L}_2$. We apply Proposition 2.1 to obtain an explicit solution of the approximating problem.

Proposition 3.3 (Approximating Problem in the Affine Model)
For the affine model (20), it holds that

$$\tilde{f}(t, y) = e^{-\tilde{A}(t,T) - \tilde{B}(t,T) y}.$$

The functions $\tilde{A}$ and $\tilde{B}$ are

$$\begin{aligned} \tilde{B}(t, T) &= 2b \frac{e^{a(T-t)} - 1}{e^{a(T-t)}(\tilde{\kappa} + a) - \tilde{\kappa} + a}, \text{ and} \\ \tilde{A}(t, T) &= \frac{2\theta}{\bar{\beta}^2} \ln\left(\frac{1 - q e^{-a(T-t)}}{1 - q} \right) + \left(\frac{2\theta b}{\tilde{\kappa} + a} - \frac{1-\gamma}{k} r \right)(T - t), \end{aligned}$$

where $\tilde{\bar{\eta}} \equiv \tilde{\bar{\chi}} / \tilde{\bar{\sigma}}$, $a \equiv \sqrt{\tilde{\kappa}^2 + 2b\bar{\beta}^2}$, $b \equiv 0.5 \frac{\gamma-1}{k\gamma} \tilde{\bar{\eta}}^2$, $q = (\tilde{\kappa} - a)/(\tilde{\kappa} + a)$, and the constant k is given by

$$k = \frac{1}{1 + \frac{1-\gamma}{\gamma} \frac{\rho^2}{1 + K\bar{L}_2}} \text{ with } K = \bar{\lambda}/\bar{\sigma}^2.$$

Remark. Obviously, $\tilde{B} \geq 0$ and thus $\tilde{f}_y(t, y) = -\tilde{B}(t, T) \tilde{f}(t, y) \leq 0$. This also follow from Proposition 2.1 (ii) since $\frac{\tilde{\chi}^2}{\tilde{\sigma}^2} = \frac{\tilde{\bar{\chi}}^2}{\tilde{\bar{\sigma}}^2} y$ is increasing in y.

3.1 Numerical Results

We use the following parameters that are similar to the ones in Liu, Longstaff, and Pan (2003):

$$\bar{\chi} = 5.363,\ \bar{\sigma} = 1.0,\ r = 0.028,\ \theta = 0.115,\ \kappa = 5.30,\ \bar{\beta} = 0.225,\ \rho = -0.57,\ \bar{\lambda} = 1.842. \quad (28)$$

Note that their estimates also include jumps in volatility. Jumps are assumed to be rare, but severe events. The expected jump size is 25% and jumps arrive on average about every 25 years. The jump intensity is linear in the local variance such that the frequency of jumps increases in volatile times. Empirical studies suggest that this is a feature observed in the data (see, e.g. Pan (2002)). We consider three different jump size distributions L:

(i) A constant jump size $L = 0.25$. Assumption (4) implies $\pi_t^* < 4$ for all $t \in [0, T]$.

(ii) A beta distributed jump size $L \sim K_L \mathcal{B}(\alpha_L, \beta_L)$. We set $\alpha_L = 18.5$, $\beta_L = 55.5$, and $K_L = 1.0$. This implies an expected stock jump size of $\mathrm{E}[L] = 0.25$ and a standard deviation of $\mathrm{SD}[L] = 0.05$. The support of the jump size distribution is $[0, K_L]$. Assumption (4) implies $\pi_t^* < \frac{1}{K_L} = 1$ for all $t \in [0, T]$.

(iii) A shifted log-normal jump size $L = 1 - e^{\mu_L + \sigma_L Z}$, where Z is a standard normal random variable. We set $\mu_L = -0.2965$ and $\sigma_L = 0.1327$, which imply $\mathrm{E}[L] = 0.25$ and $\mathrm{SD}[L] = 0.10$. The support of the jump size distribution is $(-\infty, 1]$. Assumption (4) implies $0 < \pi_t^* < 1$ for all $t \in [0, T]$. Note that this jump size distribution violates condition (11), which is needed to calculate bounds on the optimal strategy.[5] We still report the bounds even though the condition is not satisfied and check whether they give sensible results also in this case.

3.2 Optimal Strategies and Bounds for Zero Correlation

We first consider the case $\rho = 0$. The optimal strategy and the approximating strategy follow from Theorem 3.2. The bounds on the optimal strategy follow from Propositions 2.2 and 2.3. Figure 1 illustrates the optimal investment strategy π^*, the approximating investment strategy $\tilde{\pi}^*$ and the bounds on the optimal investment strategy for a constant jump size as a function of relative risk aversion γ. In line with intuition, the optimal and the approximating portfolio weight decrease in γ. This also holds true for the distance between the bounds and the optimal strategy π^* as well as for the distance between the approximating strategy $\tilde{\pi}^*$ and the optimal strategy π^*. By construction, the bounds II are narrower than the bounds I. They

[5] The assumption $L \geq 0$ can be relaxed, which is, however, beyond the scope of this paper.

particularly improve on the lower bound for low levels of risk aversion, for which the lower bound I is sometimes even negative. For risk-aversion levels of $\gamma > 7$, the difference between the upper and lower bound II is below 10%. The difference between the approximating and optimal strategy is below 5% for $\gamma > 7$.

Figure 2 depicts the corresponding results if the jump size is a beta-distributed. The upper bound $\pi < 1$ which follows from condition (4) is now actually binding for low levels of risk aversion. The lower and upper bounds are weaker than for a constant jump size. For $\gamma > 7$, the upper and lower bounds II are less than 12% apart from each other. The approximating and optimal strategies differ by less than 6% for $\gamma > 7$. Figure 3 depicts the same results for log-normal jumps.[6] For $\gamma > 7$, the upper and lower bounds II differ by less than 18%. The approximating and optimal strategies are less than 8% apart for $\gamma > 7$. To summarize, both the approximating strategy and the bounds are reasonable close to the optimal strategy.

3.3 Bounds for Negative Correlation

We now consider a model with $\rho = -0.57$ and analyze the bounds III implied by Proposition 2.4. We do not report the other bounds as they require $\rho = 0$. The upper bound does not depend on the type of jumps and is thus identical for all three jump types. For risk-aversion levels between 2 and 10 this upper bound is very large and is thus not reported. Figure 4 depicts the lower bounds. We observe little differences between the lower bounds for deterministic and stochastic jump sizes. The lower bounds for different stochastic jump size distributions are also very close to each other. Assumption (4) is binding for low risk-aversion levels ($\gamma < 4$). To summarize, we find that the bounds on the optimal strategy π^* for $\rho = -0.57$ are not very tight.

[6] Notice that the results of Section 2.3 require $L \geq 0$. This assumption can however be relaxed, which is beyond the scope of this paper.

3.4 Relative Wealth Equivalent Losses

We now examine how the approximating strategy performs compared to the true optimal strategy. The comparison is done via the RWEL. This loss measure is independent of the current level of wealth x, but depends on the local variance Y. In the following, we set the local variance equal to its mean reversion level. Furthermore, we assume a time horizon of $T = 10$ years.

For a correlation of $\rho = 0$, we can use Theorem 3.2 to calculate the RWEL explicitly. Figure 5 shows the RWEL for different assumptions about the jump size as a function of the relative risk-aversion γ. For a constant jump size, the losses decrease monotonically in γ, and they are below 1% if $\gamma > 4$. For beta-distributed and log-normally distributed jumps, the losses are below 1% and 2%, respectively. In both cases, the upper bound from (4) is binding for the optimal and approximating strategies for small values of γ. The optimal and the approximating strategy then coincide, which of course brings the RWEL to zero. When γ increases and the restrictions are not binding any more, RWELs first increase and then decrease again.

To calculate RWELs for the case $\rho = -0.57$, we can use the numerical procedure of Proposition 3.1. Table 1 reports the RWELs for risk-aversion levels γ between 2 and 10. For low risk-aversion levels, there are high losses only when the jump size is constant, while the losses are zero for stochastic jump sizes. This is again due to condition (4) that implies an upper bound on the admissible strategies if jump size is stochastic. This upper bound becomes binding if the relative risk aversion is small. As a result, the RWELs are all below 1.16% if jumps are stochastic. They are below 1% if the jump size is constant and $\gamma \geq 5$.

Next, we study the dependence of RWELs on the expected stock jump size. In our model, the expected excess return on the stock is $\left(\bar{\chi} - \mathrm{E}\left[L\right]\bar{\lambda}\right) y$. When we change the expected jump size $E[L]$, we offset its impact on the expected excess return by simultaneously changing $\bar{\chi}$. For constant jumps we simply set the jump size equal to the new expected jump size. For beta-distributed jumps, we adjust the support of the jump size distribution.[7] For log-normally distributed jump sizes, we only change the expected jump size and leave the volatility fixed at 10%.

Tables 2 reports the RWELs for different expected jump sizes ranging from 0.05 to 0.30 for a constant jump size, a beta-distributed jump size, and a log-normally distributed jump size, respectively. In line with intuition, higher expected jump sizes result in higher RWELs across all risk-aversion levels. If the expected jump size is below 15%, the RWELs are below 21bps (constant), 10bps (beta distribution), and 2bps (log-normal distribution). The highest RWELs are obtained for an expected jump size of 30%: 14.38% (constant, $\gamma = 2$), 2.55% (beta distribution, $\gamma = 5$), and 2.03% (log-normal distribution, $\gamma = 5$). To summarize, our approximation performs well if the jump size is stochastic or if it is constant and below 20%.

Next, we study the dependence of RWELs on the jump intensity. As before, we fix the expected excess return on the stock and use $\bar{\chi}$ to offset the change in the jump intensity parameter $\bar{\lambda}$.[8] Table 3 reports the RWELs for different jump intensity parameters $\bar{\lambda}$ ranging from 0.1 to 4.0 (the benchmark value is $\bar{\lambda} = 1.84156$). Results for all three jump size specifications are reported. Larger jump intensities result in higher RWELs across all risk-aversion levels. If $\gamma \geq 5$, the RWELs are below 2% for all jump types and $\bar{\lambda} \in [0.1, 4.0]$. The highest RWELs are 8.41% (constant, $\gamma = 2.0$, $\bar{\lambda} = 4.0$), 2.66% (beta distribution, $\gamma = 4.0$, $\bar{\lambda} = 4.0$), and 1.83% (beta distribution, $\gamma = 5.0$, $\bar{\lambda} = 4.0$).

3.5 An Alternative Approximate Strategy

Our previous results suggest that our approximating strategy performs well in most cases. Nevertheless, approximating the jump component by adjusting the diffusion components im-

[7] We set K_L to be twice the expected stock jump size. Additionally, we linearly scale the volatility of the stock jump size with respect to the maximal stock jump size K_L according to $\mathrm{Vol}\,(L) = 0.01\frac{K_L}{0.5}$. This procedure avoids unrealistic or even non well-defined distributions. If the volatility would be left at 0.1 for an expected jump size of 0.05 with a maximal jump size of 0.1, we cannot obtain a fit with a beta distribution.

[8] Note that the resulting jump intensity is $\bar{\lambda}Y$. If Y is equal to its mean reversion level, the jump intensity varies from 22bps to 8.69%.

plies that a typical risk characteristic of jumps is lost. In particular, we ignore the illiquidity character of jumps, which is discussed in Liu, Longstaff, and Pan (2003).

We thus construct an alternative approximating strategy. For $\rho = 0$, we solve for the optimal and approximating strategies π_0^* and $\tilde{\pi}_0^*$ by using Theorem 3.2. Both strategies are constant over time. They ignore the impact of correlation, but the difference between them accounts for the characteristics of jumps. We now use this difference as a proxy for the missing component in the approximating strategy. If $\tilde{\pi}^*$ is the approximating strategy for general $\rho \neq 0$, we define the new approximating strategy by

$$\tilde{\pi}_2 = \tilde{\pi}^* + (\pi_0^* - \tilde{\pi}_0^*) \,. \tag{29}$$

The strategy in (29) requires only the approximating strategy $\tilde{\pi}^*$, the approximating strategy $\tilde{\pi}_0^*$ for $\rho = 0$, and the optimal strategy π_0^* for $\rho = 0$. The first two are known explicitly, the last is constant over time and can be computed from (25).

We calculate RWELs using the numerical procedure of Proposition 3.1. As before, we run robustness checks with respect to risk aversion, the expected jump size, and the jump intensity for all three jump types. Overall, the adjusted strategies perform very well. For the same parameters intervals as considered above, the largest RWEL is below 5 basis points. It is obtained for a small risk aversion of $\gamma = 2$ and a large constant jump size of 30%.

4 Conclusion

We consider the continuous-time portfolio optimization problem of an investor with constant relative risk aversion who maximizes expected utility of terminal wealth in a model in which the risky asset follows a generalized (not necessarily affine) jump-diffusion model with a stochastic state process. This problem cannot be solved analytically. We use a Taylor approximation to obtain an approximating portfolio problem and provide an analytical stochastic representation of its solution. Furthermore, we derive bounds on the deviation between the optimal and the approximating strategy and bounds on the RWELs. These bounds do not depend on the solution of the portfolio planning problem in the original market and can thus be calculated even when this solution is unknown.

Furthermore, we study a specific affine parametrization of the portfolio problem in which the risky asset follows a Heston (1993) process with jumps. We provide an explicit solution to the approximating problem and use the numerical procedure from Liu, Longstaff, and Pan (2003) to solve the true problem. We find that the bounds on the optimal strategies are less than 18% apart from each other for risk-aversion levels $\gamma > 7$ and for $\rho = 0$. For a non-zero correlation, the upper bounds on the optimal strategy are larger. For the base calibration, all RWELs are below 1.16% if jumps are stochastic and below 1% if the jump size is constant and $\gamma \geq 5$. If we increase the expected jump size or jump intensity, then the RWELs increase as well. However, they are below 2% for realistic combinations of jump risk and risk aversion. Finally, we propose a second approximating strategy that leads to very low RWELs of less than 5bps.

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A Proofs

Proof of Proposition 2.1. (i) We conjecture $\tilde{J}(t,x,y) = \frac{1}{1-\gamma}x^{1-\gamma}\tilde{f}(t,y)^k$, $\tilde{f}(T,y) = 1$ with k chosen as in (8). Substituting into (6) yields the optimal stock demand $\tilde{\pi}^* = \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2} + \frac{k\sigma\beta\rho}{\gamma\tilde{\sigma}^2}\frac{\tilde{f}_y}{\tilde{f}}$, where

$$0 = \tilde{f}_t + \underbrace{\tfrac{1-\gamma}{k}\left(r + 0.5\tfrac{1}{\gamma}\tfrac{\tilde{\chi}^2}{\tilde{\sigma}^2}\right)}_{\equiv -\tilde{r}}\tilde{f} + \underbrace{\left(\alpha + \tfrac{1-\gamma}{\gamma}\tfrac{\tilde{\chi}\sigma\beta\rho}{\tilde{\sigma}^2}\right)}_{\tilde{\alpha}}\tilde{f}_y + 0.5\beta^2\tilde{f}_{yy}. \tag{30}$$

Applying Feynman-Kac to (30) gives the representation (9) of $\tilde{f}$.

(ii) We set $\tilde{R}(t,Y_t) \equiv \tilde{r}_t$. $\tilde{R}_y = \partial\tilde{R}/\partial y$ denotes the partial derivative of the function $\tilde{R}$ with respect to its second argument. Then $\tilde{f}_y(t,y) = \tilde{\mathrm{E}}^{t,y}\left[e^{-\int_t^T \tilde{r}_u\,du}\int_t^T -\tilde{R}_y(u,Y_u)P_u\,du\right]$, where $P_u \equiv \frac{\partial}{\partial y}Y_u$ denotes the derivative of the process Y with respect to its initial value $Y(t) = y$. P satisfies the stochastic differential equation[9] $dP_s = P_s[\tilde{\alpha}_y(s,Y_s)dt + \beta_y(s,Y_s)d\tilde{\tilde{\tilde{W}}}]$, where $\tilde{\tilde{\tilde{W}}}$ is a $\tilde{\mathcal{P}}$-Brownian motion. Since $P \geq 0$, the sign of f_y is negative if $-\tilde{R}_y = \frac{1-\gamma}{2k\gamma}\frac{\partial(\tilde{\chi}/\tilde{\sigma})^2}{\partial y} \leq 0$ and positive if $-\tilde{R}_y \geq 0$. Notice that, by assumption, $\gamma > 1$. Therefore, the claim follows. □

Proof of Proposition 2.2. Our assumption yields $0 \leq \pi L \leq 1$ and thus $\xi \in [-\pi L, 0] \subset$

[9] See Protter (2005), pp. 311ff.

$[-1,0]$. Then $\mathrm{E}[LR_1(-\pi^* L)] = \frac{\gamma(1+\gamma)}{2}(\pi^*)^2 \mathrm{E}\left[\frac{L^3}{(1+\xi)^{\gamma+2}}\right] > 0$ and the upper bound $\pi^* \leq \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2} = \tilde{\pi}^*$ follows from (13). We also have $1+\xi \geq 1-\pi L > 0$ and thus $\frac{1}{1+\xi} \leq \frac{1}{1-\pi L}$. Consequently,

$$\mathrm{E}[LR_1(-\pi L)] = \frac{\gamma(1+\gamma)}{2}\pi^2 \mathrm{E}\left[\frac{L^3}{(1+\xi)^{\gamma+2}}\right] \leq \frac{\gamma(1+\gamma)}{2}\pi^2 C(\pi) \tag{31}$$

holds for any $\pi \geq 0$ satisfying (4). Next, we use the upper bound $\tilde{\pi}^* = \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2}$ (adjusted to satisfy (4) if needed) and obtain $\mathrm{E}[LR_1(-\pi^* L)] \leq \frac{\gamma(1+\gamma)}{2}(\tilde{\pi}^*)^2 C(\tilde{\pi}^*)$. Substituting into (13) yields $\pi^* \geq \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2} - \frac{(1+\gamma)\lambda}{2\tilde{\sigma}^2}(\tilde{\pi}^*)^2 C(\tilde{\pi}^*)$, which is a lower bound. □

Proof of Proposition 2.3. First, we derive a sharper upper bound on π^*. The first-order condition (13) is equivalent to

$$\pi^* = \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2} - \frac{\lambda(1+\gamma)}{2\tilde{\sigma}^2}(\pi^*)^2 \mathrm{E}\left[\frac{L^3}{(1+\xi)^{\gamma+2}}\right]. \tag{32}$$

From (4), (11), and (12), it follows that $-1 < \xi \leq 0$. Therefore, $\frac{1}{1+\xi} \geq 1$ and thus $\mathrm{E}\left[\frac{L^3}{(1+\xi)^{\gamma+2}}\right] \geq \mathrm{E}[L^3] = \bar{L}_3$. Plugging this bound into (32) then gives $\pi^* \leq \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2} - A_u(\pi^*)^2$ with $A_u > 0$. The solution of this quadratic inequality yields a lower and an upper bound on π^*. The lower bound is negative and is thus redundant due to (12). The upper bound is given by π_u. Next, we derive a non-trivial lower bound. From (31), we have that $\pi^* \geq \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2} - \frac{\lambda(1+\gamma)}{2\tilde{\sigma}^2}C(\pi^*)(\pi^*)^2$. Because $C(\pi)$ is monotone, we use π_u as upper bound in $C(\pi^*)$ to obtain the quadratic inequality $\pi^* \geq \frac{\tilde{\chi}}{\gamma\tilde{\sigma}^2} - A_l(\pi^*)^2$. The solution of this inequality yields π_l. The corresponding upper bound is redundant because it is below zero. □

Proof of Proposition 2.4. The first-order condition of the problem is stated in (10). Without loss of generality, we can assume $\sigma > 0$ and $\beta > 0$. Then, (14) implies that the Merton-Breeden term is positive. The myopic demand in (15) thus gives a lower bound for π^*. Furthermore, jumps always decrease the stock price due to assumption (11). If we set $L = 0$, the stock in this market is more attractive than the stock with jumps in the original market. Hence, the strategy given in (16) is an upper bound on the optimal strategy π^*. □

Proof of Proposition 2.5. The investor's indirect utility for a *given* strategy π can be represented by $G(t,x,y;\pi) = \frac{1}{1-\gamma}x^{1-\gamma}g(t,y;\pi)$, where, by Ito's lemma, g satisfies

$$\begin{aligned} 0 &= g_t + (1-\gamma)(r + \pi\chi - 0.5\gamma\pi^2\sigma^2)g + [\alpha + (1-\gamma)\pi\sigma\beta\rho]g_y + 0.5\beta^2 g_{yy} \\ &\quad + \lambda\Big\{\mathrm{E}[(1-\pi L)^{1-\gamma}] - 1\Big\}g. \end{aligned} \tag{33}$$

Notice that $J(t,x,y) = G(t,x,y;\pi^*)$ for the optimal strategy π^*. Differentiating (33) with respect to the state variable y gives

$$\begin{aligned} 0 &= g_{ty} + (1-\gamma)(\pi_y\chi + \pi\chi_y - \gamma\pi\pi_y\sigma^2 - 0.5\gamma\pi^2(\sigma^2)_y)g + (1-\gamma)(r + \pi\chi - 0.5\gamma\pi^2\sigma^2)g_y \\ &\quad + [\alpha_y + (1-\gamma)(\pi_y\sigma\beta + \pi(\sigma\beta)_y)\rho]g_y + [\alpha + (1-\gamma)\pi\sigma\beta\rho]g_{yy} + 0.5(\beta^2)_y g_{yy} + 0.5\beta^2 g_{yyy} \\ &\quad + \lambda_y\Big\{\mathrm{E}[(1-\pi L)^{1-\gamma}] - 1\Big\}g - \lambda(1-\gamma)\pi_y\mathrm{E}[L(1-\pi L)^{-\gamma}]g + \lambda\Big\{\mathrm{E}[(1-\pi L)^{1-\gamma}] - 1\Big\}g_y. \end{aligned}$$

Sorting terms and denoting the coefficient of g by $\check{D}$ shows that D is equal to (17). Applying the Feynman-Kac theorem gives $g_y(t,y) = \int_t^T \check{\mathrm{E}}^{t,y}[e^{-\int_t^s \check{r}_u\,du}\check{D}_s g(s,Y_s)]\,ds$, where $-\check{r} = (1-\gamma)(r + \pi\chi - 0.5\gamma\pi^2\sigma^2) + [\alpha_y + (1-\gamma)(\pi_y\sigma\beta + \pi_y(\sigma\beta)_y)\rho] + \lambda\big\{\mathrm{E}[(1-\pi L)^{1-\gamma}] - 1\big\}$ and the expectation $\check{\mathrm{E}}[\cdot]$ is taken under a measure under which Y has the drift $\alpha + (1-\gamma)\pi\sigma\beta\rho + 0.5(\beta^2)_y$. Since $g > 0$, the derivative g_y is positive (negative) if $\check{D}$ is positive (negative). □

Proof of Theorem 2.6. (i) The investor's indirect utility in the original and the approximating economy for *given* strategies π and $\tilde{\pi}$ can be represented by $G(t,x,y;\pi) = \frac{1}{1-\gamma}x^{1-\gamma}g(t,y;\pi)$ and $\tilde{G}(t,x,y;\tilde{\pi}) = \frac{1}{1-\gamma}x^{1-\gamma}\tilde{g}(t,y;\tilde{\pi})$, where, by Ito's lemma, g and $\tilde{g}$ satisfy

$$\begin{aligned} 0 &= g_t + (1-\gamma)(r + \pi\chi - 0.5\gamma\pi^2\sigma^2)g + [\alpha + (1-\gamma)\pi\sigma\beta\rho]g_y + 0.5\beta^2 g_{yy} \\ &\quad + \lambda\Big\{\mathrm{E}[(1-\pi L)^{1-\gamma}] - 1\Big\}g, \end{aligned} \tag{34}$$

$$\begin{aligned} 0 &= \tilde{g}_t + (1-\gamma)(r + \tilde{\pi}\chi - 0.5\gamma\tilde{\pi}^2\sigma^2)\tilde{g} + [\alpha + (1-\gamma)\tilde{\pi}\sigma\beta\rho]\tilde{g}_y + 0.5\beta^2\tilde{g}_{yy} \\ &\quad - \lambda\tilde{\pi}(1-\gamma)(\bar{L}_1 + 0.5\gamma\tilde{\pi}\bar{L}_2)\tilde{g}. \end{aligned} \tag{35}$$

Notice that $J(t,x,y) = G(t,x,y;\pi^*)$ and $\tilde{J}(t,x,y) = \tilde{G}(t,x,y;\tilde{\pi}^*)$ for the optimal strategies π^* and $\tilde{\pi}^*$. We set $D \equiv g - \tilde{g}$ and define the process Δ such that $\tilde{\pi} = \pi(1+\Delta)$. Furthermore, the Taylor series of $(1-L\pi)^{1-\gamma}$ converges, since this is in fact a Binomial series and (4) holds. Its second-order expansion reads $(1-\pi L)^{1-\gamma} = 1-(1-\gamma)\pi L - 0.5\gamma(1-\gamma)\pi^2 L^2 + R_2(-\pi L)$, where $R_2(-\pi L) \equiv \frac{\gamma(1-\gamma^2)}{6}\frac{1}{(1+\xi)^{\gamma+2}}(-\pi L)^3$, $\xi \in < -\pi L, 0 >$, denotes the remainder term. Substracting (35) from (34) and rearranging terms yields

$$0 = D_t - \hat{r}D + \hat{\alpha}D_y + 0.5\beta^2 D_{yy} + \hat{D}, \tag{36}$$

where $-\hat{r} \equiv (1-\gamma)(r + \pi\chi - 0.5\gamma\pi^2\sigma^2) + \lambda\big[\mathrm{E}[(1-\pi L)^{1-\gamma}] - 1\big]$, $\hat{\alpha} \equiv \alpha + (1-\gamma)\pi\sigma\beta\rho$, and

$$\hat{D} = \big\{\lambda \mathrm{E}[R_2(-\pi L)] - (1-\gamma)\Delta\pi\big[\chi - \lambda\bar{L}_1 + \sigma\beta\rho\tilde{g}_y/\tilde{g} - 0.5\gamma(2+\Delta)\pi(\sigma^2 + \lambda\bar{L}_2)\big]\big\}\,\tilde{g}.$$

(ii) follows because $\tilde{\pi}^*$ satisfies the first-order condition $\chi - \lambda\bar{L}_1 - \gamma\tilde{\pi}(\sigma^2 + \lambda\bar{L}_2) + \sigma\beta\rho\tilde{g}_y/\tilde{g} = 0$.
(iii) follows from Proposition 2.1. □

Proof of Theorem 3.1. The three equations (22), (23), (24) follow if we insert the affine model (20) and the separation (21) into the HJB (3). (22) follows from the first-order condition of this HJB. (23) and (24) follow from inserting (22) into the HJB and collecting terms that involve the state variable and terms that do not involve the state variable. □

Proof of Theorem 3.2. (i) The first-order condition of (3) reads

$$\gamma\sigma^2\pi = \chi + \sigma\beta\rho\frac{g_y}{g} - \lambda \mathrm{E}[L(1-\pi L)^{-\gamma}]. \tag{37}$$

Using (20) the local variance cancels and we obtain $\gamma\pi = \bar{\chi} + \bar{\beta}\rho\frac{g_y}{g} - \bar{\lambda}\mathrm{E}\left[L\left(1-\pi L\right)^{-\gamma}\right]$. Equation (25) follows for $\rho = 0$.

(ii) Using the affine model (20) in (34) with $\rho = 0$ yields the following pde for $g_0(t,y,\pi)$:

$$0 = g_{0,t} + (1-\gamma)rg_0 + \theta g_{0,y} - \kappa y g_{0,y} + \frac{1}{2}\bar{\beta}^2 y g_{0,yy} + C_0 y g_0 \tag{38}$$

with $C_0 \equiv (1-\gamma)\left(\pi\bar{\chi} - \frac{1}{2}\gamma\pi^2 + \frac{\bar{\lambda}}{1-\gamma}\left\{\mathrm{E}\left[(1-\pi L)^{1-\gamma}\right] - 1\right\}\right)$. $J_0(t,x,y) = \frac{1}{1-\gamma}x^{1-\gamma}g_0(t,x,\pi)$ is the indirect utility if we use the optimal strategy $\pi = \pi^*$, which is implicitly defined by (25). To solve (38) we use the Feynman-Kac Theorem: $g_0(t,y) = e^{(1-\gamma)r(T-t)}\mathrm{E}^{t,y}\left[e^{C_0\int_t^T Y(u)\mathrm{d}u}\right]$ with $\mathrm{d}Y_t = (\theta - \kappa Y_t)\,\mathrm{d}t + \bar{\beta}\sqrt{Y_t}\mathrm{d}\tilde{W}_t$. This representation has a closed-form solution. Equation (26) is obtained analogous to the pricing of a bond in a model in which the short rate follows a Cox-Ingersoll-Ross process.

(iii) follows from (20) and $\rho = 0$ in (37). (iv) follows from the definition of ℓ in (18). □

Figure 1: Investment strategies for the affine model (20) **with constant jump size for different risk-aversion parameters γ.** The correlation parameter ρ is set to zero and thus the optimal strategy and the bounds are constant over time. The solid line is the optimal strategy. The dashed lines are the lower and upper bounds I (Proposition 2.2). The dotted lines are the bounds II (Proposition 2.3). Note that the upper bound I is the approximating strategy.

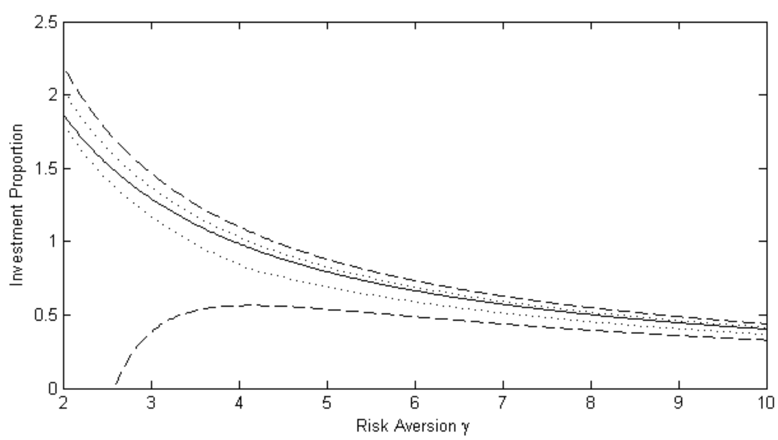

Figure 2: Investment strategies for the affine model (20) **with beta-distributed jump size for different risk-aversion parameters** γ**.** The correlation parameter ρ is set to zero and thus the optimal strategy and the bounds are constant over time. The solid line is the optimal strategy. The dashed lines are the lower and upper bounds I (Proposition 2.2). The dotted lines are the bounds II (Proposition 2.3). The investment cannot be higher than 1 due to (4). Note that the upper bound I is the approximating strategy.

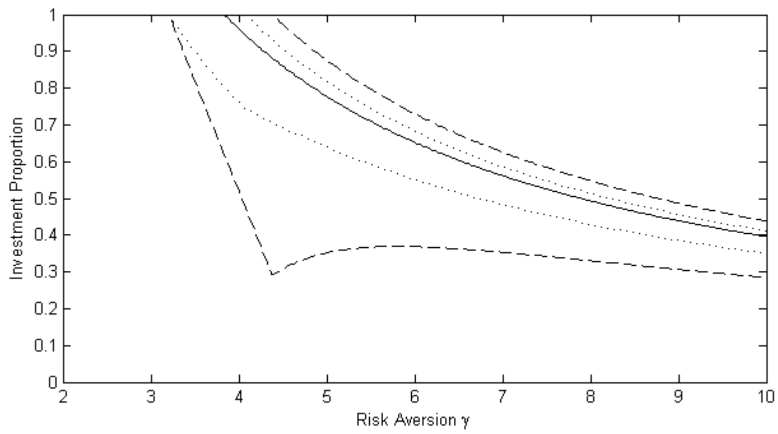

Figure 3: Investment strategies for the affine model (20) **with log-normally distributed jump size for different risk-aversion parameters** γ**.** The correlation parameter ρ is set to zero and thus the optimal strategy and the bounds are constant over time. The solid line is the optimal strategy. The dashed lines are the lower and upper bounds I (Proposition 2.2). The dotted lines are the bounds II (Proposition 2.3). The investment cannot be higher than 1 due to (4). The lower dashed line has its smallest value of -0.7246 at $\gamma = 4.325$. Note that the upper bound I is the approximating strategy.

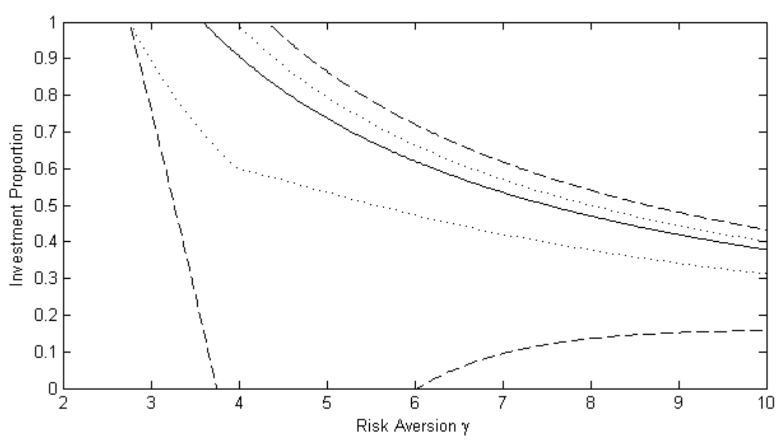

Figure 4: Lower bounds III on optimal investment π_t^* implied by Proposition 2.4 for the affine model (20) **for different risk-aversion parameters γ.** The remaining parameters are from (28). The solid line is the lower bound for a deterministic jump size, the dashed line for beta-distributed jump size and the dotted line for log-normally distributed jump size. The upper bounds are large and thus omitted.

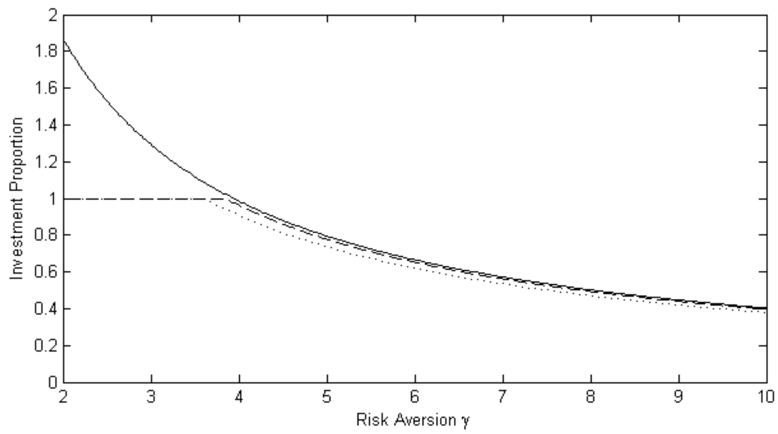

Figure 5: Relative wealth equivalent losses in the affine model (20) **for different risk-aversion parameters γ.** The correlation is set to $\rho = 0$, the remaining parameters are from (28). The solid line uses a deterministic jump size, the dashed line beta-distributed jump size and the dotted line log-normally distributed jump size.

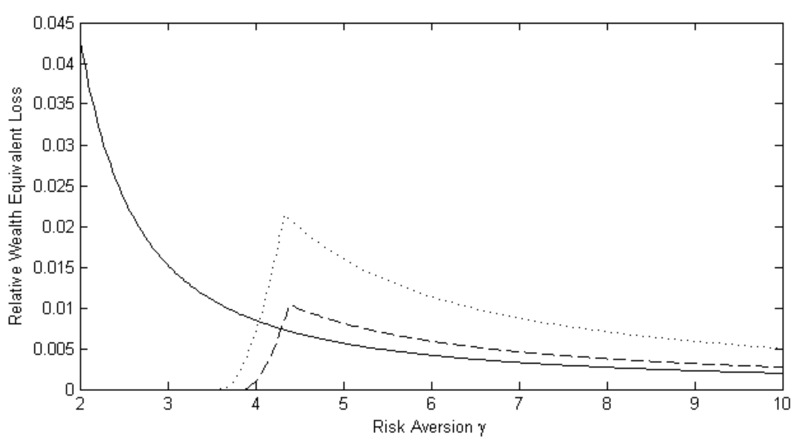

Table 1: RWELs for various risk aversions γ and different jump specifications in the affine model (20)**.** This table reports the RWELs that an investor using the optimal strategy would sacrifice in order to have the same indirect utility as an investor using the approximating strategy. The indirect utilities of the optimal strategy and approximating strategy in the true market are obtained using Proposition 3.1. Subsection 3.1 describes the three jump specifications. The reported values are in percentages.

Risk Aversion	RWEL		
γ	Constant	Beta	Log-normal
2.0	5.45	0.00	0.00
3.0	2.06	0.00	0.00
4.0	1.18	0.01	0.00
5.0	0.80	1.16	0.73
6.0	0.60	0.85	0.53
7.0	0.48	0.67	0.41
8.0	0.40	0.55	0.33
9.0	0.34	0.47	0.28
10.0	0.29	0.40	0.24

Table 2: RWELs for different constant jump sizes L and risk aversions γ. This table reports the RWELs that an investor using the optimal strategy would sacrifice in order to have the same indirect utility as an investor using the approximating strategy. The indirect utilities of the optimal strategy and approximating strategy in the true market are obtained using Proposition 3.1. When we vary the constant (expected) jump size, we adjust the parameters such that the expected excess return on stock remains the same as in the benchmark calibration (28). This procedure is described in Subsection 3.4. The quoted values are in percentages.

Risk Aversion	Constant Jump Size					
γ	0.05	0.10	0.15	0.20	0.25	0.30
2.0	0.00	0.01	0.21	1.41	5.45	14.38
3.0	0.00	0.01	0.09	0.57	2.06	5.20
4.0	0.00	0.00	0.06	0.34	1.18	2.93
5.0	0.00	0.00	0.04	0.23	0.80	1.98
6.0	0.00	0.00	0.03	0.18	0.60	1.47
7.0	0.00	0.00	0.03	0.14	0.48	1.17
8.0	0.00	0.00	0.02	0.12	0.40	0.96
9.0	0.00	0.00	0.02	0.10	0.34	0.82
10.0	0.00	0.00	0.02	0.09	0.29	0.71
Risk Aversion	Expected Stock Jump Size (Beta)					
γ	0.05	0.10	0.15	0.20	0.25	0.30
2.0	0.00	0.00	0.00	0.00	0.00	0.00
3.0	0.00	0.00	0.00	0.00	0.00	0.00
4.0	0.00	0.00	0.00	0.00	0.01	1.14
5.0	0.00	0.01	0.10	0.40	1.16	2.55
6.0	0.00	0.01	0.07	0.30	0.85	1.88
7.0	0.00	0.01	0.06	0.23	0.67	1.48
8.0	0.00	0.01	0.05	0.19	0.55	1.21
9.0	0.00	0.01	0.04	0.16	0.47	1.02
10.0	0.00	0.01	0.04	0.14	0.40	0.88
Risk Aversion	Expected Stock Jump Size (Log-normal)					
γ	0.05	0.10	0.15	0.20	0.25	0.30
2.0	0.00	0.00	0.00	0.00	0.00	0.00
3.0	0.00	0.00	0.00	0.00	0.00	0.00
4.0	0.00	0.00	0.00	0.00	0.00	0.06
5.0	0.00	0.00	0.00	0.12	0.72	2.03
6.0	0.00	0.00	0.02	0.13	0.53	1.47
7.0	0.00	0.00	0.02	0.10	0.41	1.14
8.0	0.00	0.00	0.01	0.08	0.33	0.92
9.0	0.00	0.00	0.01	0.07	0.28	0.78
10.0	0.00	0.00	0.01	0.06	0.24	0.67

Table 3: RWELs for different jump intensity parameters $\bar{\lambda}$ and risk aversions γ. This table reports the RWELs that an investor using the optimal strategy would sacrifice in order to have the same indirect utility as an investor using the approximating strategy. The indirect utilities of the optimal strategy and approximating strategy in the true market are obtained using Proposition 3.1. When we vary the jump intensity parameter, we adjust the parameters such that the expected excess return on stock remains the same as in the benchmark calibration (28). This procedure is described in Subsection 3.4. For the constant jump size we use $L = 0.25 = const$. For the beta-distributed and log-normally distributed jump sizes we use $\mathrm{E}(L) = 0.25$ and $\mathrm{Var}(L) = 0.01$. The quoted values are in percentages.

Risk Aversion	Jump Intensity Parameter $\bar{\lambda}$ (Constant)								
γ	0.1	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
2.0	0.08	1.20	2.97	4.55	5.82	6.80	7.53	8.05	8.41
3.0	0.02	0.36	1.01	1.66	2.23	2.71	3.10	3.40	3.63
4.0	0.01	0.19	0.55	0.94	1.29	1.59	1.83	2.03	2.19
5.0	0.01	0.13	0.37	0.63	0.88	1.09	1.27	1.42	1.53
6.0	0.00	0.09	0.27	0.47	0.66	0.82	0.96	1.08	1.17
7.0	0.00	0.07	0.22	0.37	0.53	0.66	0.77	0.86	0.94
8.0	0.00	0.06	0.18	0.31	0.43	0.55	0.64	0.72	0.78
9.0	0.00	0.05	0.15	0.26	0.37	0.47	0.55	0.61	0.67
10.0	0.00	0.04	0.13	0.23	0.32	0.41	0.48	0.54	0.58
Risk Aversion	Jump Intensity Parameter $\bar{\lambda}$ (Beta)								
γ	0.1	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
2.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.0	0.00	0.00	0.00	0.00	0.05	0.37	0.95	1.73	2.66
5.0	0.00	0.20	0.56	0.93	1.26	1.53	1.74	1.91	2.04
6.0	0.01	0.14	0.41	0.68	0.93	1.14	1.31	1.44	1.54
7.0	0.01	0.11	0.32	0.53	0.73	0.90	1.04	1.15	1.23
8.0	0.00	0.09	0.26	0.44	0.60	0.74	0.86	0.95	1.02
9.0	0.00	0.07	0.22	0.37	0.51	0.63	0.73	0.81	0.87
10.0	0.00	0.06	0.19	0.32	0.44	0.55	0.63	0.70	0.76
Risk Aversion	Jump Intensity Parameter $\bar{\lambda}$ (Log-normal)								
γ	0.1	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
2.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.10
5.0	0.00	0.03	0.29	0.54	0.81	1.08	1.34	1.59	1.83
6.0	0.00	0.06	0.21	0.39	0.59	0.79	0.99	1.18	1.35
7.0	0.00	0.05	0.16	0.30	0.46	0.62	0.78	0.93	1.07
8.0	0.00	0.04	0.13	0.25	0.37	0.51	0.64	0.76	0.88
9.0	0.00	0.03	0.11	0.21	0.32	0.43	0.54	0.64	0.75
10.0	0.00	0.03	0.09	0.18	0.27	0.37	0.46	0.56	0.65

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