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Consumption Habits and Humps

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Keywords: Consumption hump, life-cycle utility maximization, habit formation, impatience

JEL subject codes: D91, D11, D14

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1 Introduction

Empirical studies have documented that the consumption expenditures of individuals typically have an inverted U-shape over the life cycle by being increasing up to age 45-50 years and then decreasing over the remaining life. Standard, frictionless consumption-savings model cannot generate such a hump in consumption: if the subjective time preference rate of the individual is higher [lower] than the (risk-adjusted expected) return on investments, consumption is expected to decrease [increase] monotonically over the entire life time. Several plausible models that lead to a consumption hump have been suggested (see below), but these models are relatively complex and can only be solved with the help of numerical methods. This paper shows that, even in a very simple setting, the consumption hump can naturally emerge when the preferences of the individual exhibit habit formation instead of the time-additivity typically assumed.

The hump in life-cycle consumption emerges from a trade-off between impatience and the implications of habits. Suppose the individual has a high subjective time preference rate and thus, other things equal, would prefer to have a decreasing consumption pattern over life. However, if the individual forms enduring habits for consumption, she knows that a high initial consumption would lead to higher required consumption in the future. Consequently, she trades off the impatience regarding current consumption with the concerns about the future required consumption levels resulting from habit formation. The younger the individual, the more money has to be set aside to cover future required consumption. A sufficiently strong habit formation dominates the impatience in the early years, but as habit concerns decrease over life consumption gradually increases in the early years. At some age, the impatience outweighs the habit concerns so that consumption starts to decrease. Hence, the consumption pattern over the life cycle exhibits a hump.

The tradeoff between impatience and habit formation is fundamental, and we illustrate it in the simplest possible model with full certainty and no frictions. The individual is equipped with some initial wealth that can be invested at a constant risk-free rate. The individual can continuously withdraw funds for consumption. We assume that the individual's objective is to maximize utility of consumption over the remaining life. The utility at a given point in time is a concave power function of the difference between consumption and the habit level at that time, where the habit level is a scaled, exponentially weighted average of past consumption rates of the individual. This standard specification of habit-style preferences implies that the habit level is the minimum feasible consumption and, therefore, the individual must ensure to have sufficient funds to cover the minimum feasible consumption in the remaining life. This generates required savings which are, other things equal, decreasing as the remaining life-time shrinks.

The optimal consumption profile and the possibility of a hump in consumption depend on the values of various parameters of the model. We derive a set of sufficient conditions for the presence of a consumption hump. For a reasonable parametrization of the model, we show that the optimal consumption profile of the agent does exhibit a hump and that the consumption hump occurs at an age consistent with the empirical evidence. We illustrate how the location of the hump is affected by the various parameters in a way consistent with economic intuition. For example, we find that the bigger the impact of current consumption on future habit levels (via a higher scaling parameter and a lower decay rate of the habit level), the later the consumption hump occurs. Moreover, consumers who are very impatient or very risk-tolerant (i.e., consumers having a high elasticity of intertemporal substitution) have a consumption profile peaking at a relatively young age.

We calibrate our model to consumption data from the Consumer Expenditure Survey in the United States over the period 1980-2003. We find that our model matches very well the observed hump-shaped consumption pattern of both singles and couples.

The habit formation we model is sometimes referred to as *internal habit* formation to emphasize that the habit level entering the utility function is a result of the past consumption choices of the same individual. This can be contrasted with the case of *subsistence* consumption in which the individual derives utility from consumption above some exogenously given subsistence level and also to the case of *external habit* formation in which the individual's utility of consumption depends on some external factor, e.g., the consumption choices of peers ("keeping up with the Jones'es") or the per capita consumption in the economy.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model and provides a closed-form solution for the optimal consumption plan. Analytical results on the existence and location of the consumption hump are demonstrated in Section 4. Section 5 illustrates the optimal consumption profile for a set of benchmark parameter values and discusses the sensitivity with respect to the values of key parameters. Section 7 concludes. Proofs can be found in the appendices.

2 Literature review

Thurow (1969) appears to be the first to empirically document a hump in life-cycle consumption, i.e., that consumption over life has an inverted U-shape. Later studies using different data sources and periods confirm this pattern, see, e.g., Attanasio and Weber (1995), Attanasio, Banks, Meghir, and Weber (1999), Browning and Crossley (2001), and Gourinchas and Parker (2002).

The life-cycle consumption-saving theory builds on work by Ramsey (1928), Fisher (1930), Modigliani and Brumberg (1954), and Friedman (1957) and was extended to rigorously incorporate uncertainty by Samuelson (1969) and Merton (1969, 1971). Without frictions, such models produce an optimal life-cycle consumption pattern which is either monotonically increasing, monotonically decreasing, or flat depending on whether the subjective time preference rate of the individual is smaller than, greater than, or equal to the (risk-adjusted expected) return on investments.

Known explanations of the consumption hump include borrowing constraints (Thurow 1969), income uncertainty and precautionary savings (Nagatani 1972; Carroll 1997), endogenous labor supply with hump-shaped wages (Heckman 1974), variations in household size (Attanasio and Browning 1995; Browning and Ejrnæs 2009), mortality risk (Feigenbaum 2008; Hansen and İmrohoroğlu 2008), and consumer durables serving as collateral (Fernández-Villaverde and Krueger 2011). Our purpose is not to question any of these explanations, but rather to add a new, simple explanation of the hump. The formal models in the above-listed papers are all built on the maximization of time-additive utility of one good or two goods (with leisure or durables as the second good). The constraints, collateral, unspanned income risk, and mortality risk in these models make it difficult to derive the optimal consumption strategy in closed form. By relying on habit formation in preferences, we set up a simple model in which we derive a closed-form solution for optimal consumption and show that optimal consumption can exhibit a hump over the life cycle.

Economists have long recognized that preferences may not be intertemporally separable. According to Browning (1991), this idea dates back to the 1890 book "Principles of Economics" by Alfred Marshall. Ravina (2007) reports strong support of internal habit formation based on consumption decisions of a sample of U.S. credit card holders in the period 1999-2002. The consequences of habit formation have been studied formally at least since Ryder and Heal (1973). Regarding individual decision making, the literature has so far focused on the impact of habit formation on portfolio choice (Ingersoll 1992; Munk 2008), whereas we consider the implications for consumption.

Habit features in preferences have proven helpful in explaining stylized asset pricing facts that seem puzzling when agents are assumed to have time-separable power utility, see, e.g., Sundaresan (1989), Abel (1990), Constantinides (1990), Campbell and Cochrane (1999), and Menzly, Santos, and Veronesi (2004). Based on an endowment economy with identical agents, Grischenko (2010) concludes that an internal habit specification provides a better match with asset pricing data than an external habit specification. Heaton (1995) and Chen and Ludvigson (2009) report similar results. Habit formation is also used in other areas of macroeconomics, see, e.g., Carroll, Overland, and Weil (2000), Boldrin, Christiano, and Fisher (2001), and Del Negro, Schorfheide, Smets, and Wouters (2007). Fuhrer (2000) and Christiano, Eichenbaum, and Evans (2005) show that habit formation may explain the hump-shaped response over time of aggregate consumption to a monetary policy shock. In contrast, we show that habit formation may lead to (expected) consumption being hump-shaped over the life of an individual.

3 The model

We set up a deterministic, continuous-time model of an individual's life-cycle consumption and savings decisions. The individual enters the economy at time 0 with some initial wealth X_0 and lives until time $T > 0$, which we assume is a known constant. The individual consumes a single consumption good with $c(t)$ representing the consumption rate at time t, so that the number of goods consumed over a short interval $[t, t + dt]$ is approximately $c(t)\,dt$. The good is the numeraire in the economy. The wealth not spent on consumption is invested in a savings account that offers a constant, risk-free rate of r, continuously compounded. The individual receives no other income during life than the interest on savings. The dynamics of wealth $X(t)$ is then simply

$$dX(t) = rX(t)\,dt - c(t)\,dt. \tag{1}$$

The individual has to determine a consumption strategy $(c(t))_{t\in[0,T]}$, which in our deterministic setting is simply a function $c : [0, T] \mapsto \mathbb{R}$.

We assume that the preferences of the individual exhibit (internal) habit formation. We define the time t habit level to be

$$h(t) = h_0 e^{-\beta t} + \alpha \int_0^t e^{-\beta(t-s)} c(s)\,ds, \tag{2}$$

where h_0, α, and β are non-negative constants. The last term is proportional to a weighted average of past consumption where we can interpret β as a persistence parameter and α as a scaling parameter. Finally, $h_0 \geq 0$ is an initial habit level whose influence fades away over time provided that $\beta > 0$. Note that the habit level evolves as

$$dh(t) = (\alpha c(t) - \beta h(t))\, dt. \tag{3}$$

Given a wealth of x and a habit level of h at time t, the individual is assumed to evaluate a given consumption strategy $c = (c(s))_{s\in[t,T]}$ over the remaining life by

$$J^c(t, x, h) = \int_t^T e^{-\delta(s-t)} U\left(c(s) - h(s)\right)\, ds + \varepsilon e^{-\delta(T-t)} U\left(X(T)\right), \tag{4}$$

where δ is a constant subjective time preference rate, $\varepsilon \geq 0$ is a constant indicating the preference weight of the bequest X_T relative to consumption, and

$$U(z) = \frac{1}{1-\gamma}\, z^{1-\gamma} \tag{5}$$

where $\gamma > 1$ is a risk aversion parameter. The relative risk aversion with respect to consumption gambles is $-c\frac{\partial^2 U(c-h)}{\partial c^2} / \frac{\partial U(c-h)}{\partial c} = \gamma c/(c-h)$ so that γ is the minimal relative risk aversion possible. The indirect utility function is defined as

$$J(t, x, h) = \max_c J^c(t, x, h). \tag{6}$$

As we want to illustrate in the simplest possible setting that habit formation can induce a consumption habit, we deliberately assume full certainty in our model. Hence, the concept of risk aversion may seem misplaced, but we can alternatively think in terms of the elasticity of intertemporal substitution, which is the reciprocal of the relative risk aversion, i.e., $(c-h)/(\gamma c)$. A higher γ represents a lower elasticity of intertemporal substitution and $1/\gamma$ is the maximal level of the elasticity of intertemporal substitution.[1] For terminological convenience we will continue to refer to γ as a risk aversion parameter in the remainder of the paper.

The setup requires that consumption stays above the habit level, otherwise the marginal utility

[1] An alternative measure of the elasticity of intertemporal substitution is the derivative of the consumption growth rate with respect to the interest rate. The two measures are identical in the setting without habit formation, but not in the setting with habit formation as explained by Constantinides (1990) in the case of an infinite time horizon.

would be infinite. This implies that wealth at any point in time has to be sufficient to finance future minimum consumption. If the individual has a time t habit level of $h(t)$ and consumes at the minimum level (i.e., habit level) in all future so that $c(s) = h(s)$, the habit level evolves as $dh(s) = -(\beta - \alpha)h(s)\, ds$ which implies that

$$h(s) = e^{-(\beta-\alpha)(s-t)}h(t), \quad s \in [t, T].$$

The time t present value of this stream of minimum future consumption is

$$\int_t^T e^{-r(s-t)}h(s)\, ds = h(t)A(t),$$

where

$$A(t) = \begin{cases} T - t, & \text{if } r_A = 0, \\ \dfrac{1}{r_A}\left(1 - e^{-r_A(T-t)}\right), & \text{otherwise,} \end{cases} \tag{7}$$

with

$$r_A = r + \beta - \alpha.$$

Remark 1. It seems natural to expect that $\beta > \alpha$ since then the habit level decreases whenever the agent consumes at the minimum level as can be seen by applying $c(t) = h(t)$ in (3). If, furthermore, the interest rate is non-negative, the constant r_A is definitely positive.

Intuitively, the individual can split up her time t wealth into *tied-up wealth* $h(t)A(t)$, which covers the minimum future consumption, and the *free wealth* $X(t) - h(t)A(t)$ which can finance excess consumption and thus generates utility. This explains the form of the solution to the utility maximization problem. The following theorem gives the exact formulation. See Appendix A for a proof.

Theorem 1. *Assume $x > hA(t)$. Then the indirect utility function is given by*

$$J(t, x, h) = \frac{1}{1-\gamma}g(t)^\gamma(x - hA(t))^{1-\gamma}, \tag{8}$$

where A is given by (7) and

$$g(t) = \int_t^T e^{-r_g(s-t)}(1+\alpha A(s))^{\frac{\gamma-1}{\gamma}}\,ds + \varepsilon^{\frac{1}{\gamma}} e^{-r_g(T-t)}, \qquad (9)$$
$$r_g = \frac{\gamma-1}{\gamma}r + \frac{\delta}{\gamma}.$$

The optimal consumption strategy is

$$c(t) = h(t) + \frac{X(t) - h(t)A(t)}{g(t)}(1+\alpha A(t))^{-\frac{1}{\gamma}}. \qquad (10)$$

In the following, we are mainly interested in the shape of the function $t \mapsto c(t)$.

Let us briefly consider some special cases. First, consider the case with constant relative risk aversion which requires $h_0 = \alpha = \beta = 0$. In this case optimal consumption is simply $c(t) = X(t)/g(t)$ and, since $X'(t) = (r - g(t)^{-1})X(t)$ and $g'(t) = r_g g(t) - 1$, we obtain

$$c'(t) = \frac{(r - r_g)X(t)}{g(t)} = \frac{r-\delta}{\gamma}\frac{X(t)}{g(t)}. \qquad (11)$$

Hence, we see that the consumption function is increasing if $r > \delta$ (the return on savings exceeds the impatience), decreasing if $r < \delta$, and flat if $\delta = r$. Secondly, consider the case where the consumption benchmark $h(t)$ is an exogenously given function (an external habit or a subsistence consumption level) and, in particular, $\alpha = 0$ so that consumption does not affect future values of $h(t)$. In this case optimal consumption becomes

$$c(t) = h(t) + \frac{X(t) - F(t)}{g(t)}, \quad F(t) = \int_t^T e^{-r(s-t)}h(s)\,ds, \qquad (12)$$

where g is given by (9) albeit with $\alpha = 0$ and we must require $X(t) > F(t)$. Then

$$c'(t) = h'(t) + \frac{r-\delta}{\gamma}\frac{X(t) - F(t)}{g(t)}, \qquad (13)$$

which is decreasing if h is decreasing and $\delta > r$, but can be increasing under other assumptions. With a suitably specified benchmark function $h(t)$, the optimal consumption path can even exhibit a hump.

Now we return to the case with (internal) habit formation. Note that direct differentiation with respect to time in (10) would involve the derivatives of both $h(t)$ and $X(t)$, but the latter can be expressed in terms of $X(t)$ and $c(t)$ through (1) and then the $X(t)$ can be expressed in terms of $c(t)$ and $h(t)$ using (10). Hence, the derivative of $c(t)$ can be expressed in terms of $c(t)$ and $h(t)$. From (3), the derivative of $h(t)$ can be expressed in terms of $c(t)$ and $h(t)$. In fact, it turns out to be useful to rewrite the system of derivatives of $c(t)$ and $h(t)$ as a system of derivatives of $c(t)$ and the *surplus consumption* defined as

$$\Delta(t) = c(t) - h(t). \tag{14}$$

Note that the optimal consumption strategy is such that the surplus consumption is a time-dependent fraction of the free wealth. We summarize the derivatives in the following theorem, which is proved in Appendix B.

Theorem 2. *The dynamics of the optimal consumption and the associated surplus consumption is given by the system*

$$d\Delta(t) = \phi(t)\Delta(t)\,dt, \tag{15}$$

$$dc(t) = (\beta + \phi(t))\,\Delta(t)\,dt - (\beta - \alpha)\,c(t)\,dt, \tag{16}$$

where

$$\phi(t) = \frac{r - \delta + \alpha B(t)}{\gamma}, \tag{17}$$

$$B(t) = \frac{1 - r_A A(t)}{1 + \alpha A(t)} = \begin{cases} \dfrac{1}{1 + \alpha(T-t)}, & \textit{if } r_A = 0, \\ \dfrac{r_A}{(r_A + \alpha)e^{r_A(T-t)} - \alpha}, & \textit{otherwise.} \end{cases} \tag{18}$$

In the next section we provide analytical results on the existence and location of a hump in the consumption function $t \mapsto c(t)$. Section 5 illustrates the results by numerical examples.

4 Analytical results on the consumption hump

While the expression (10) for optimal consumption appears simple, note that it depends both on $h(t)$, which is determined by the entire consumption path up to time t via (2), and on wealth, which also is determined by past consumption via (1). In general it seems very difficult to analytically establish conditions under which consumption is hump-shaped.

For long horizons, however, we can obtain an analytical characterization of the consumption pattern and establish sufficient conditions for the existence of a consumption hump. As explained in the introduction, the hump emanates from a combination of habit formation and an impatience exceeding returns on savings. In light of Remark 1, the following assumption is therefore natural.

Assumption 1. *The parameters satisfy the conditions $\delta > r$, $\alpha > 0$, and $r_A = r + \beta - \alpha > 0$.*

Recall that the condition $\delta > r$ requires the agent to be sufficiently impatient so that the consumption in the absence of habit formation would be monotonically decreasing over life. The condition $\alpha > 0$ means that the habit level is increasing in past consumption so that preferences exhibit genuine habit formation.

With $\alpha > 0$, it is clear from (17) and (18) that both $B(t)$ and $\phi(t)$ smoothly approach constants as the terminal time T is increased. For T large enough, the graph of $\phi(t)$ is almost flat in the early years. With $r_A > 0$, the limit of $\phi(t)$ as $T \to \infty$ is $(r - \delta)/\gamma$. Define the constant

$$\kappa = \frac{\delta - r}{\gamma},$$

which can be interpreted as the product of a net impatience rate $\delta - r$ and the maximal elasticity of intertemporal substitution $1/\gamma$. By Assumption 1, we have $\kappa > 0$. We now replace $\phi(t)$ in the true dynamics (15) of the surplus consumption with its limit $(r - \delta)/\gamma = -\kappa$ and thus consider the time-*independent* dynamics

$$d\tilde{\Delta}(t) = -\kappa \tilde{\Delta}(t)\, dt, \tag{19}$$

$$d\tilde{c}(t) = (\beta - \kappa)\tilde{\Delta}(t)\, dt - (\beta - \alpha)\tilde{c}(t)\, dt \tag{20}$$

with initial values $\tilde{c}_0 > \tilde{\Delta}_0 > 0$. We demonstrate in numerical examples in Section 5 that the time-independent dynamics (19)–(20) is an accurate approximation of the true dynamics (15)–(16) except possibly for the final years. See also Lemma 1 below.

The next theorem establishes conditions under which the approximating dynamics produce a unique hump in the function $\tilde{c}(t)$. Appendix C provides the proof.

Theorem 3. *Suppose Assumption 1 is satisfied and that*

$$\beta > \alpha + \kappa, \tag{21}$$

$$(\alpha - \kappa)\tilde{c}_0 > (\beta - \kappa)\tilde{h}_0, \tag{22}$$

where $\tilde{h}_0 = \tilde{c}_0 - \tilde{\Delta}_0$*. Then the solution to the dynamic system* (19)–(20) *is such that the function* $\tilde{c}(t)$ *has a unique hump at* $t = t_H$*, where*

$$t_H = \frac{\ln(\beta - \alpha) - \ln(\kappa) + \ln\left(1 - \frac{\tilde{c}_0}{\lambda(\tilde{c}_0 - \tilde{h}_0)}\right)}{\beta - \alpha - \kappa}, \tag{23}$$

and where

$$\lambda = \frac{\beta - \kappa}{\beta - \alpha - \kappa}. \tag{24}$$

We emphasize that the parameter conditions stated in Theorem 3 are sufficient, not necessary, for the presence of a hump.

The condition (21) requires the decay rate β of the habit to be sufficiently high so that the habit is not too persistent. Note that $\kappa > 0$ since $\delta > r$, so the condition (21) sharpens the inequality $\beta > \alpha$ which is natural, cf. Remark 1. If we assume $\alpha > \kappa$ and $\tilde{c}_0 = c_0$, the inequality (22) can be rewritten, by applying (10) at $t = 0$, as

$$X_0 > \left(A(0) + \frac{\beta - \alpha}{\alpha - \kappa} g(0)(1 + \alpha A(0))^{1/\gamma}\right) \tilde{h}_0,$$

so that it is satisfied for a large enough initial wealth. Note that the parameter conditions stated in the theorem ensure that the log-terms in (23) are well-defined.

The hump derived from the approximate dynamics occurs at age t_H. Note that, for fixed initial values $\tilde{c}_0$ and $\tilde{h}_0$, t_H is independent of the horizon T of the agent. This observation highlights that the above theorem is relevant for the hump in the truly optimal consumption path if the horizon is sufficiently large and, in particular, larger than t_H. We find that t_H is increasing in $\tilde{c}_0$. Holding $\tilde{c}_0$ fixed, we find that t_H is increasing in r, γ, and α, but decreasing in $\tilde{h}_0$, δ, and β.

The following lemma shows that we can bound the difference between the true consumption function and the approximation over any interval $[0, t]$ by any margin $\eta > 0$ if the planning horizon T is long enough. The proof can be found in Appendix D.

Lemma 1. *Suppose Assumption 1 and the conditions* (21) *and* (22) *hold. Let* $\tilde{h}_0 = h_0$ *and* $\tilde{c}_0 = c_0$*. For any given* $t \geq 0$ *and any given* $\eta > 0$*, we can find* $\tilde{T} > 0$ *such that if* $T > \tilde{T}$ *then*

$$|c(s) - \tilde{c}(s)| < \eta, \quad s \in [0, t]. \tag{25}$$

By applying the lemma for $t > t_H$ and a small η, we see that the true dynamics also give rise to a consumption hump if the stated parameter conditions are satisfied and the horizon T is long enough.

In Section 5 we show in numerical examples that the approximate dynamics is very close to the true dynamics over most of life even for realistic horizons, and we therefore also see a consumption hump in such cases.

5 Numerical examples

In the following numerical examples we use the benchmark parameter values listed in Table 1 unless otherwise mentioned. We assume the agent has a remaining time horizon of 50 years, so the setting could represent the problem faced by an agent who is initially 30 years old and who lives until an age of 80. The agent has no utility of bequests.[2] The initial wealth is set to $X_0 = 20$ which is motivated by the observation that the median wealth for individuals of age 30-40 in the 2007 Survey of Consumer Finances is roughly USD 20,000. The benchmark values of the interest rate, the time preference rate, and the risk aversion parameter fall in the range considered in the literature. The initial value of the relative risk aversion $\gamma c_0/(c_0 - h_0)$ turns out to be around 9. The initial habit level is set at 0.25 which, compared to the initial consumption level that turns out to be 0.45, represents a significant but not extremely strong habit; the initial consumption is approximately 44% above the minimum consumption level which is identical to the habit level. The values of the habit parameters α and β are less clear given the limited literature. We consider values in the range also studied by Constantinides (1990) and Munk (2008), which seem to generate reasonable habit dynamics.

[Table 1 about here.]

Figure 1 illustrates the optimal consumption path with the benchmark parameter values. Consumption exhibits a distinct hump with a maximum after 20.3 years, which could correspond to an age of roughly 50 years, cf. the above discussion. This location of the hump matches well the empirically observed hump. Based on the analytical approximation the hump time is $t_H \approx 19.8$ years, very close to the actual time of the hump. We can see that the consumption pattern based on the approximate, long-horizon dynamics (thin curve) is virtually indistinguishable from the actual consumption pattern over the first 30 years. The consumption levels are unrealistically low as we

[2] This is in line with Hurd (1989) who shows empirically that bequest motives in various countries are close to zero.

do not take labor income into account, but a quick fix is to scale initial wealth appropriately to incorporate human capital, which leads to a similar scaling of consumption levels (provided that the initial habit level is scaled accordingly). This procedure does not affect the shape of the consumption path, nor the presence and location of the hump. The habit level tracks the consumption path closely.

Figure 1 further shows the monotonically decreasing consumption path of an agent not developing habits, but having a constant relative risk aversion (i.e., a constant elasticity of intertemporal substitution). As explained below Theorem 1, this is a consequence of δ exceeding r so that impatience beats returns on savings. Note the dramatic impact habit formation has on the optimal consumption path.

[Figure 1 about here.]

Next we investigate the sensitivity of the consumption path and the location of the consumption hump with respect to the values of key parameters. Figure 2 shows the optimal consumption profile for three different values of the habit scaling parameter α. Increasing α, current consumption has a bigger effect on future habit levels, which leads the agent to lower consumption in the early years. The increased savings are spent on higher consumption in the late years of life. Consequently, the consumption hump occurs later in life. For very small values of α (in our case around 0.1 and smaller), the consumption profile is monotonically decreasing over life (except for a small increase in the final couple of years), as in the case without habits, since then the habit level does not have a sufficient magnitude to subdue the impatience of the agent. Conversely, for very high values of α (in our case around 0.35 and higher), the consumption profile is monotonically increasing.

[Figure 2 about here.]

Figure 3 illustrates the importance of the habit persistence parameter β. A higher β means reduced influence of current consumption on future habit levels. Hence, the agent initially consumes more, which is naturally offset by lower consumption late in life. A higher β therefore also leads to an earlier consumption hump. If β is sufficiently high (around 0.85 and higher in our case), the hump disappears and consumption monotonically decreases over life except for the few final years where consumption increases. If β is sufficiently small (0.37 or lower), the optimal consumption is monotonically increasing over life.

[Figure 3 about here.]

The role of the time preference rate δ can be seen in Figure 4. A higher δ means that the agent is more impatient and therefore increases consumption early in life with the consequence of reducing consumption late in life, which also causes the consumption hump to occur earlier in life. Because a high early consumption raises the minimum consumption level in the following years, it takes an extremely high value of δ (in our case 1.10 or higher) before the consumption path becomes downward-sloping right from the beginning. For low values of δ (around 0.035 and lower), the optimal consumption profile is monotonically increasing. Note that this happens also for cases in which optimal consumption in the absence of habit formation is monotonically decreasing; in our case this occurs for δ between 0.02 (the benchmark value of r) and 0.035.

[Figure 4 about here.]

Figure 5 shows that a higher value of the risk aversion parameter γ – or equivalently a lower value of the elasticity of intertemporal substitution $1/\gamma$ – leads to lower consumption early in life and higher consumption late in life with the consumption hump occurring later in life.

[Figure 5 about here.]

Figure 6 illustrates the optimal consumption profile for three different values of the time horizon T. Since we fix the initial wealth and disregard labor income, the agent with a longer horizon consumes at a lower level throughout life. The consumption hump occurs earlier for longer horizons. For a sufficiently short horizon (about 42 years or shorter, given the other parameter values) the consumption path is monotonically increasing over life.

[Figure 6 about here.]

Finally, we consider the relevance of the strength of the bequest motive as represented by the parameter ε, cf. the preference specification in (4). We have used a benchmark value of $\varepsilon = 0$ corresponding to no utility of bequest, which obviously implies that the agent consumes everything and ends up with zero wealth. In Figure 7 we compare the benchmark consumption profile with the consumption profile when ε is either one thousand or one million. In the first of these two cases, the agent leaves a bequest of 1.035 (compare with the initial wealth of 20) which corresponds to roughly the consumption in the final 1.5 years. In the latter case, the agent leaves a bequest of 5.294 corresponding to the consumption over the final 8-9 years. Naturally, we see that the stronger the bequest weight ε, the lower the consumption throughout life as more savings need to be generated. However, the shape of the consumption profile and the location of the consumption hump are only affected slightly. The hump occurs after 20.3 years without bequest, after 20.1 years when $\varepsilon = 1000$, and after 19.4 years when $\varepsilon = 1000000$.

[Figure 7 about here.]

An interesting observation from the figures presented above is that, after the mid-life hump and subsequent decline, optimal consumption starts to increase again in the final years of life. However, this behavior depends on the constellation of parameter values. Figure 8 shows an optimal consumption path with no increase near the end, based on a set of reasonable parameter values. In this particular case, the slope of the consumption path becomes less negative so that the consumption path flattens out near the end. It appears to be impossible to provide a simple analytical characterization of the shape of the optimal consumption path just before the terminal date. Intuitively, as the terminal date approaches, the agent becomes less concerned about the impact of current consumption on future habit levels as the future is becoming increasingly irrelevant. Therefore, the dampening effect of internal habit formation on consumption weakens as the final date approaches. Of course, even in the final years, the agent has to cope with the current minimum consumption level caused by past consumption decisions. Therefore, the optimal consumption decision of the agent with internal habit formation does not approach the optimal decision of an agent with constant relative risk aversion, but rather the optimal consumption of an agent with an external habit or subsistence consumption level. In the larger picture, the optimal consumption pattern (late) in retirement is heavily influenced by the increase in mortality risk ignored by our simple model.

[Figure 8 about here.]

The key empirical papers documenting the hump consider consumption only up to about age 65, and their graphs indicate that consumption flattens out in the years leading up to that age, cf., e.g., Figure 1 in Feigenbaum (2008). This appears consistent with the flattening of the consumption path in the final years in our Figure 8 or with the relatively flat part of the consumption path in, say, Figure 1 just before the final few years of increasing consumption. The limited existing empirical evidence on consumption in retirement is inconclusive with respect to how consumption varies with age late in retirement, cf., e.g., Fisher, Johnson, Marchand, Smeeding, and Torrey (2005).[3]

[3] In contrast, there is substantial empirical evidence that consumption typically falls at retirement, but this should be seen in relation to frequently occurring contemporaneous changes in leisure, housing, health conditions, mortality risk, and household composition, cf., e.g., the discussion in Browning and Crossley (2001).

6 Calibrating the model to consumption data

In this section we investigate the extent to which our model can match the observed consumption hump. We apply consumption data from the Consumer Expenditure Survey from the United States over the period 1980-2003. The data was originally processed and used by Krueger and Perri (2006) and it is made available online by the authors.[4] The consumption is deflated back to represent "1982-84 constant dollars." We apply their so-called ND+ consumption measure; we refer the reader to Krueger and Perri (2006) for details on the data. The consumption data is on a household basis, whereas our model is better suited for individuals. We focus on the consumption of singles and the per-person consumption of couples without children (household consumption divided by 1.7 as recommended by the OECD equivalence scale). The uneven curves in Figure 9 show the average consumption per year in thousands of US dollars for individuals at different ages who are living either as singles (upper panel) or in childless couples (lower panel). The consumption of singles is relatively flat over life, but still higher in mid-life than in the early and in the late years. The consumption of couples exhibits a more pronounced hump-shape.

[Figure 9 about here.]

We calibrate our model so that the optimal consumption path from the model best matches the observed age-profile of consumption of either singles or couples. Since the consumption data covers ages from 25 to 65, we let $t = 0$ and $T = 40$. We fix the risk-free rate at $r = 0.01$ and assume no utility from bequeathing wealth ($\varepsilon = 0$). We search for the remaining parameters with the objective of minimizing the sum of the squared differences between the model consumption and the observed average consumption at ages $25, 26, \dots, 65$. Table 2 shows the parameter values from the calibrations. The habit process parameters α and β have very reasonable values. We restrict α to be at least 0.1, which is a binding constraint in the calibration to the consumption of singles. As long as the difference $\beta - \alpha$ is fixed, we can also obtain an excellent fit to the data for higher values of α and β. The time preference rate δ is restricted to be at most 0.25. A slightly better fit can be obtained by increasing δ further (together with the risk aversion parameter γ). On the other hand, we also obtain a good fit to the data if we lower both δ and γ (results are available on request). When comparing the value of the initial wealth to real-life wealth levels, recall that the model wealth includes any human capital.

[4] Web-link: http://www.fperri.net/research_data.htm

[Table 2 about here.]

The smooth curves in Figure 9 depict the life-cycle consumption pattern from our calibrated model. The figure illustrates that our parsimonious model driven by impatience and habit formation nicely matches the observed consumption pattern over the life-cycle including the mid-life consumption hump. Our calibration is mostly challenged by the apparent substantial increase in the consumption of persons of an age between, say, 30 and 45 years who live in a childless couple. However, this pattern may be partially explained by a "survivorship" bias in the sample. Obviously, when the persons in a couple become parents, they leave this group of individuals and their subsequent consumption is not reflected by the data we use. If the less wealthy and therefore low-consuming couples are more inclined to become parents, the remaining sample of childless couples is tilted towards the more wealthy and high-consuming individuals.

7 Conclusion

This paper proposes a new potential explanation of the empirically observed hump in the consumption of individuals over their life cycle. If the preferences of the individual exhibit habit formation, the hump can naturally materialize from a tradeoff between impatience and concerns about the effects of current consumption on future habit levels and thus future minimum consumption. The habit concerns cause a large reduction in the otherwise very high consumption early in life, but a smaller reduction of the otherwise medium-sized consumption in mid life. In some circumstances, a hump-shaped consumption path emerges.

We present a set of sufficient conditions for the presence of a hump and characterize the age at which the hump occurs. Numerical examples illustrate the consumption hump and the sensitivity of the optimal consumption path to the values of key parameters of our model. We show that our parsimonious model provides a nice match with consumption patterns derived from the 1980-2003 Consumer Expenditure Surveys in the United States.

As the purpose of the paper is to demonstrate that habit formation can generate a consumption hump, we deliberately keep our model simple and, in particular, disregard uncertainty, labor income, portfolio constraints etc. However, the basic tradeoff identified in this paper carries over to more elaborate settings.

A Proof of Theorem 1

The Hamilton-Jacobi-Bellman (HJB) equation associated with the utility maximization problem (6) is

$$0 = \max_c \left\{ \frac{1}{1-\gamma} (c-h)^{1-\gamma} + J_t + rxJ_x - cJ_x - \delta J + (\alpha c - \beta h) J_h \right\}, \tag{26}$$

where we have suppressed the arguments of the functions and where subscripts on J indicate partial derivatives. The terminal condition is

$$J(T, x, h) = \varepsilon U(x) = \frac{\varepsilon}{1-\gamma} x^{1-\gamma}. \tag{27}$$

The first-order condition is

$$-J_x + (c-h)^{-\gamma} + \alpha J_h = 0 \qquad \Leftrightarrow \qquad c = h + (J_x - \alpha J_h)^{-\frac{1}{\gamma}}. \tag{28}$$

The second-order condition is satisfied by concavity of the utility function. After substituting the first-order condition back into the HJB equation and simplifying, we see that J should satisfy the partial differential equation (PDE)

$$0 = \frac{\gamma}{1-\gamma} (J_x - \alpha J_h)^{1-\frac{1}{\gamma}} + J_t + rxJ_x - hJ_x - \delta J + (\alpha - \beta) hJ_h. \tag{29}$$

We conjecture that

$$J(t, x, h) = \frac{1}{1-\gamma} g(t)^\gamma (x - hA(t))^{1-\gamma}$$

for some deterministic functions g and A. The relevant derivatives are

$$J_t = -A_t h (x - hA)^{-\gamma} g^\gamma + \frac{\gamma}{1-\gamma} (x-hA)^{1-\gamma} g^{\gamma-1} g_t,$$
$$J_x = (x-hA)^{-\gamma} g^\gamma, \qquad J_h = -A(x-hA)^{-\gamma} g^\gamma.$$

By substituting the derivatives into the first-order condition (28), we obtain

$$c = h + \left((x-hA)^{-\gamma} g^\gamma + \alpha A (x-hA)^{-\gamma} g^\gamma \right)^{-1/\gamma} = h + \frac{x-hA}{g} (1 + \alpha A)^{-1/\gamma}. \tag{30}$$

After substitution of the derivatives, the PDE (29) can be written as

$$
\begin{aligned}
0 &= hg^{\gamma}(x-hA)^{-\gamma}\left[-A_t+(r+\beta-\alpha)A-1\right] \\
&\quad + \frac{\gamma}{1-\gamma}g^{\gamma-1}(x-hA)^{1-\gamma}\left[g_t-\frac{1}{\gamma}\left(\delta+(\gamma-1)r\right)g+(1+\alpha A)^{1-\frac{1}{\gamma}}\right],
\end{aligned}
\tag{31}
$$

which is satisfied if A and g satisfy the ordinary differential equations (ODEs)

$$
A_t=(r+\beta-\alpha)A-1, \qquad g_t=\frac{1}{\gamma}\left(\delta+(\gamma-1)r\right)g-(1+\alpha A)^{1-\frac{1}{\gamma}}.
$$

Because of the terminal condition (27), we also need $A(T)=0$ and $g(T)=\varepsilon^{1/\gamma}$. It is straightforward to verify that these conditions and the above ODEs are indeed satisfied when the functions A and g are given by (7) and (9), respectively.

B Proof of Theorem 2

From (10), we can write

$$
\Delta(t)=\left(X(t)-h(t)A(t)\right)H(t), \qquad H(t)=\frac{(1+\alpha A(t))^{-\frac{1}{\gamma}}}{g(t)}.
$$

Straightforward differentiation leads to

$$
\begin{aligned}
\Delta'(t) &= \big(X'(t)-h'(t)A(t)-h(t)A'(t)\big)H(t)+(X(t)-h(t)A(t))H'(t) \\
&= \big(rX(t)-c(t)-[\alpha c(t)-\beta h(t)]A(t)-h(t)[r_A A(t)-1]\big)H(t)+\Delta(t)\frac{H'(t)}{H(t)} \\
&= \big(rX(t)-c(t)-\alpha c(t)A(t)-h(t)[(r-\alpha)A(t)-1]\big)H(t)+\Delta(t)\frac{H'(t)}{H(t)} \\
&= r\big(X(t)-h(t)A(t)\big)H(t)-\left(c(t)-h(t)\right)(1+\alpha A(t))H(t)+\Delta(t)\frac{H'(t)}{H(t)} \\
&= \Delta(t)\left(r-(1+\alpha A(t))H(t)+\frac{H'(t)}{H(t)}\right),
\end{aligned}
$$

where we have used $X'(t) = rX(t) - c(t)$ and $h'(t) = \alpha c(t) - \beta h(t)$, as well as $A'(t) = r_A A(t) - 1$. By further applying that $g'(t) = r_g g(t) - (1 + \alpha A(t))^{1-\frac{1}{\gamma}}$, we obtain

$$
\begin{aligned}
H'(t) &= -\frac{\alpha}{\gamma}\frac{(1+\alpha A(t))^{-\frac{1}{\gamma}-1}A'(t)}{g(t)} - (1+\alpha A(t))^{-\frac{1}{\gamma}}\frac{g'(t)}{g(t)^2} \\
&= -\frac{\alpha}{\gamma}\frac{r_A A(t) - 1}{1+\alpha A(t)}H(t) - r_g\frac{(1+\alpha A(t))^{-\frac{1}{\gamma}}}{g(t)} + (1+\alpha A(t))\left(\frac{(1+\alpha A(t))^{-\frac{1}{\gamma}}}{g(t)}\right)^2 \\
&= \frac{\alpha}{\gamma}B(t)H(t) - r_g H(t) + (1+\alpha A(t))H(t)^2,
\end{aligned}
$$

where we have introduced

$$
B(t) = \frac{1 - r_A A(t)}{1 + \alpha A(t)}.
$$

Going back to the derivative of the surplus consumption, we get

$$
\begin{aligned}
\Delta'(t) &= \Delta(t)\left(r - (1+\alpha A(t))H(t) + \frac{\alpha}{\gamma}B(t) - r_g + (1+\alpha A(t))H(t)\right) \\
&= \Delta(t)\left(r - r_g + \frac{\alpha}{\gamma}B(t)\right) \\
&= \Delta(t)\frac{1}{\gamma}\left(r - \delta + \alpha B(t)\right) \\
&= \Delta(t)\phi(t),
\end{aligned}
$$

where

$$
\phi(t) = \frac{r - \delta + \alpha B(t)}{\gamma}.
$$

Since $c(t) = h(t) + \Delta(t)$ by definition, we obtain

$$
\begin{aligned}
c'(t) &= h'(t) + \Delta'(t) \\
&= (\alpha c(t) - \beta h(t)) + \Delta(t)\phi(t) \\
&= (\beta + \phi(t))\,\Delta(t) + (\alpha - \beta)c(t),
\end{aligned}
$$

which completes the proof.

C Proof of Theorem 3

Note that when Assumption 1 and (21) hold, we have $\kappa > 0$ (since $\gamma > 0$), $\beta > \alpha$, and $\lambda > 1$.

The solution to (19) is clearly

$$\tilde{\Delta}(t) = \tilde{\Delta}_0 e^{-\kappa t},$$

and the solution to (20) is

$$\tilde{c}(t) = (\tilde{c}_0 - \lambda\tilde{\Delta}_0)e^{-(\beta-\alpha)t} + \lambda\tilde{\Delta}_0 e^{-\kappa t}$$

as can be verified by straightforward differentiation.

Observe that

$$\tilde{c}'(0) = (\beta - \kappa)\tilde{\Delta}_0 + (\alpha - \beta)\tilde{c}_0 = (\alpha - \kappa)\tilde{c}_0 - (\beta - \kappa)\tilde{h}_0,$$

which is positive because of the assumption in (22). On the other hand, we can write

$$\tilde{c}'(t) = e^{-\kappa t}\left((\beta - \alpha)(\lambda\tilde{\Delta}_0 - \tilde{c}_0)e^{-(\beta-\alpha-\kappa)t} - \kappa\lambda\tilde{\Delta}_0\right). \tag{32}$$

Because of Assumption 1 and (21), the first term in the brackets approaches zero as $t \to \infty$, since $\beta - \alpha - \kappa > 0$, and the second term $\kappa\lambda\tilde{\Delta}_0$ is positive. Therefore, $\tilde{c}'(t) < 0$ for large enough t.

Because $\tilde{c}'(t)$ is a smooth function with $\tilde{c}'(0) > 0$ and $\tilde{c}'(t) < 0$ for large enough t, there must be at least one point t_H for which $\tilde{c}'(t_H) = 0$. The condition $\tilde{c}'(t_H) = 0$ implies that

$$(\beta - \alpha)(\lambda\tilde{\Delta}_0 - \tilde{c}_0)e^{-(\beta-\alpha)t_H} = \kappa\lambda\tilde{\Delta}_0 e^{-\kappa t_H},$$

where both sides of the equality are positive due to the parameter conditions. Hence, the solution is

$$t_H = \frac{1}{\beta - \alpha - \kappa}\ln\left(\frac{(\beta - \alpha)(\lambda\tilde{\Delta}_0 - \tilde{c}_0)}{\kappa\lambda\tilde{\Delta}_0}\right) = \frac{\ln(\beta - \alpha) - \ln(\kappa) + \ln\left(1 - \frac{\tilde{c}_0}{\lambda(\tilde{c}_0 - \tilde{h}_0)}\right)}{\beta - \alpha - \kappa}.$$

This is the *only* solution to $\tilde{c}'(t_H) = 0$ since the term in the brackets in (32) is a decreasing function of t. Combining this with the above observation that $\tilde{c}'(t) < 0$ for large enough t, it becomes clear that $\tilde{c}(t)$ is hump-shaped, i.e., increasing from $t = 0$ up to $t = t_H$ where it attains its maximum and then decreasing for $t > t_H$.

D Proof of Lemma 1

Define $f(t) = c(t) - \tilde{c}(t)$ and note that $f(0) = 0$. From (16) and (20), we get

$$\begin{aligned} df(t) &= [(\beta + \phi(t))\Delta(t) - (\beta - \kappa)\tilde{\Delta}(t)]\, dt + [(\alpha - \beta)c(t) - (\alpha - \beta)\tilde{c}(t)]\, dt \\ &= [(\beta + \phi(t))(\Delta(t) - \tilde{\Delta}(t)) + (\phi(t) + \kappa)\tilde{\Delta}(t)]\, dt + (\alpha - \beta) f(t)\, dt. \end{aligned}$$

Noting that $\tilde{\Delta}(s) = \tilde{\Delta}_0 e^{-\kappa s}$, we can write the solution as

$$\begin{aligned} f(t) &= \int_0^t e^{-(\beta-\alpha)(t-s)} \left[(\beta + \phi(s))(\Delta(s) - \tilde{\Delta}(s)) + (\phi(s) + \kappa)\tilde{\Delta}(s) \right] ds \\ &= \tilde{\Delta}_0 \int_0^t e^{-(\beta-\alpha)(t-s)} e^{-\kappa s} \left[(\beta + \phi(s)) \left(\frac{\Delta(s)}{\tilde{\Delta}(s)} - 1 \right) + (\phi(s) + \kappa) \right] ds. \end{aligned}$$

From (21) and Assumption 1, we know that $\beta > \alpha$ and $\kappa > 0$. Furthermore,

$$\frac{\Delta(s)}{\tilde{\Delta}(s)} = \frac{\Delta_0 e^{\int_0^t \phi(s)\, ds}}{\tilde{\Delta}_0 e^{-\kappa s}} = e^{\int_0^t (\phi(s)+\kappa)\, ds},$$

where we apply $\tilde{\Delta}_0 = \Delta_0$ which follows from assuming $\tilde{h}_0 = h_0$ and $\tilde{c}_0 = c_0$. For any $\nu > 0$ we can find a $\tilde{T} > 0$ big enough that $|\phi(s) + \kappa| < \nu$ for all $s \in [0, t]$ if $T > \tilde{T}$. Moreover, since

$$\beta + \phi(s) = \beta - \kappa + \frac{\alpha}{\gamma} B(s),$$

it follows from (21) and Assumption 1 that $\beta + \phi(s) > 0$ and

$$\beta + \phi(s) \leq \beta - \kappa + \frac{\alpha}{\gamma} B(t), \quad s \in [0, t].$$

Putting this together, we find that

$$|f(t)| \leq \tilde{\Delta}_0 \int_0^t \left[(\beta - \kappa + \frac{\alpha}{\gamma} B(t)) \left(e^{\nu t} - 1 \right) + \nu \right] ds = t\tilde{\Delta}_0 \left[(\beta - \kappa + \frac{\alpha}{\gamma} B(t)) \left(e^{\nu t} - 1 \right) + \nu \right].$$

Note that the right-hand side is increasing in t, which implies that

$$|f(s)| \leq t\tilde{\Delta}_0 \left[(\beta - \kappa + \frac{\alpha}{\gamma} B(t)) \left(e^{\nu t} - 1 \right) + \nu \right], \quad s \in [0, T].$$

If we decrease ν from positive values towards zero, the right-hand side in this inequality decreases towards zero. Hence, for any given $\eta > 0$, we can find a small enough $\nu > 0$ and therefore a corresponding big enough $\tilde{T}$ so that

$$|f(s)| < \eta, \quad s \in [0, T].$$

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Parameter	Description	Value
δ	time preference rate	0.05
γ	risk aversion parameter	4
ε	preference weight of bequest	0
α	habit scaling parameter	0.3
β	habit persistence parameter	0.4
X_0	financial wealth	20
h_0	initial habit level	0.25
r	risk-free rate	0.02
T	remaining life time	50

Table 1: Benchmark parameter values. The table lists the parameter values used in the numerical examples unless otherwise noted.

Parameter	Description	Consumption data	
		Singles	Couples
δ	time preference rate	0.250	0.250
γ	risk aversion parameter	4.423	5.945
α	habit scaling parameter	0.100	0.124
β	habit persistence parameter	0.124	0.163
X_0	financial wealth (kUSD)	438.2	442.2
h_0	initial habit level (kUSD)	1.049	0.000

Table 2: Parameter values from calibration. The table shows the set of parameter values giving the best fit to the consumption data considered, both for the consumption of singles and the per-person consumption of couples. The data is taken from the webpage `http://www.fperri.net/research_data.htm` of Fabrizio Perri and generated by Krueger and Perri (2006) from the Consumer Expenditure Survey over the period 1980-2003. The calibration objective is to minimize the sum of the squared differences between the model consumption and the observed average consumption at ages $25, 26, \ldots, 65$. We impose the restrictions $\delta \leq 0.25$ and $\alpha \geq 0.1$.

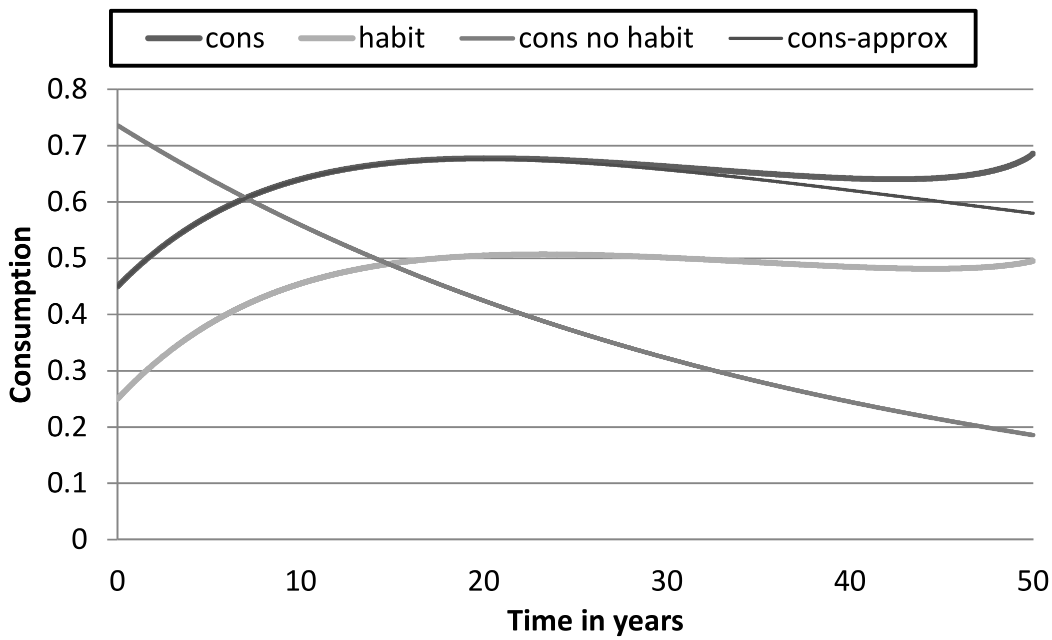

Figure 1: Consumption in the benchmark case. The dark, thick curve shows the optimal consumption path. The pale, thick curve shows the corresponding path of the habit level. The thin curve depicts the consumption path based on the approximate, long-horizon dynamics. These curves are generated using the benchmark parameter values listed in Table 1. The downward-sloping curve shows the optimal consumption path for the case without habit formation. This curve is drawn using the same parameters, except that $h_0 = \alpha = \beta = 0$.

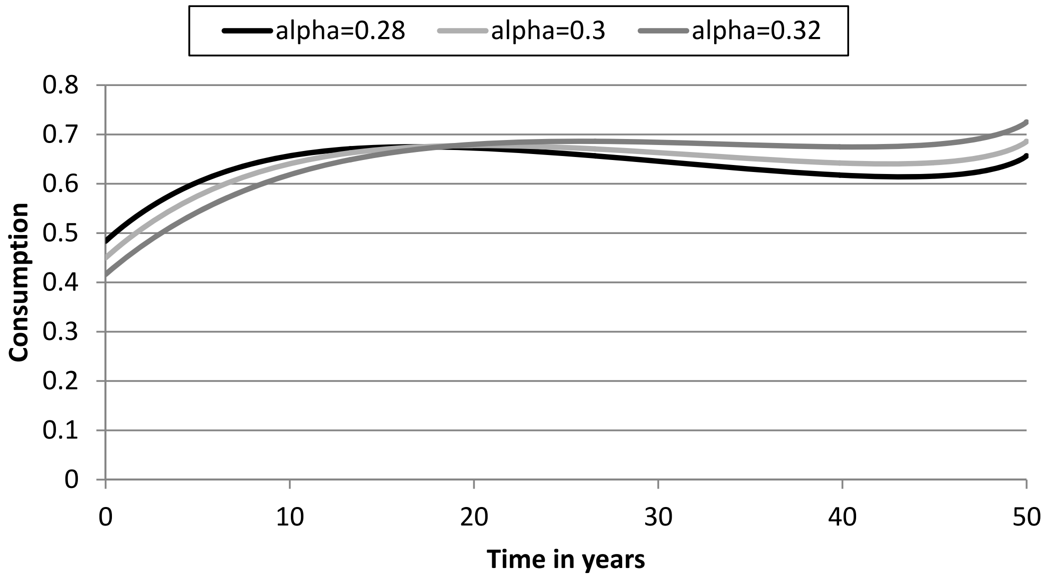

Figure 2: Consumption for different values of the habit scaling parameter α. For all other parameters the benchmark values listed in Table 1 are used.

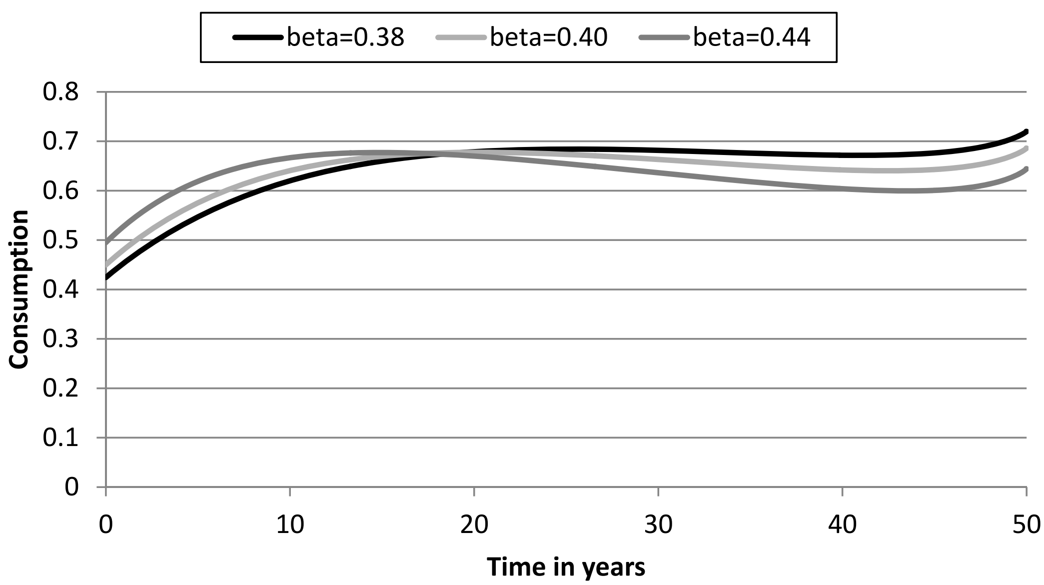

Figure 3: Consumption for different values of the habit persistence parameter β. For all other parameters the benchmark values listed in Table 1 are used.

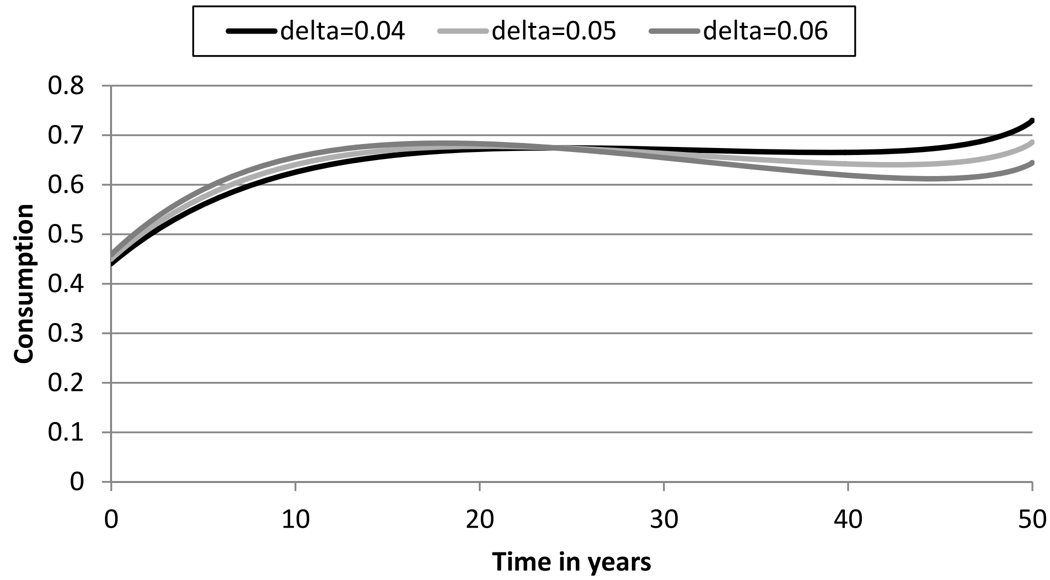

Figure 4: Consumption for different values of the time preference rate δ. For all other parameters the benchmark values listed in Table 1 are used.

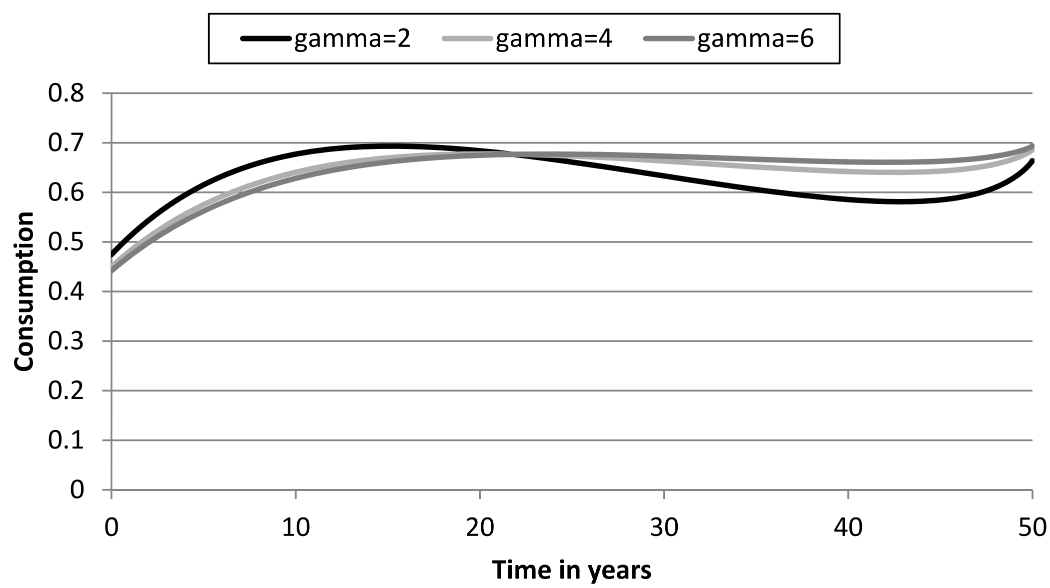

Figure 5: Consumption for different values of the risk aversion parameter γ. Recall that the elasticity of intertemporal substitution is proportional to $1/\gamma$. For all other parameters the benchmark values listed in Table 1 are used.

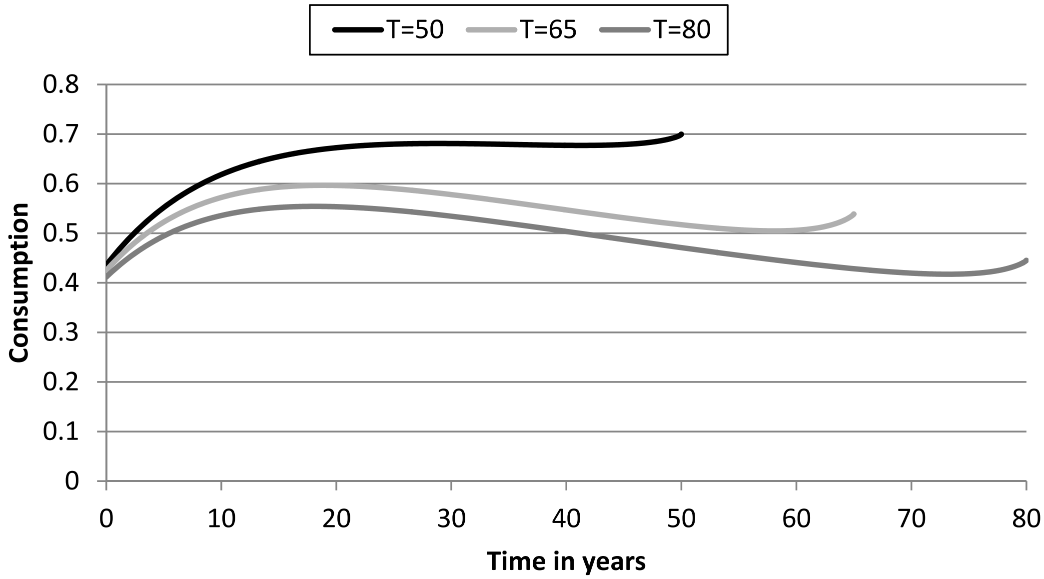

Figure 6: Consumption for different values of the time horizon T. For all other parameters the benchmark values listed in Table 1 are used.

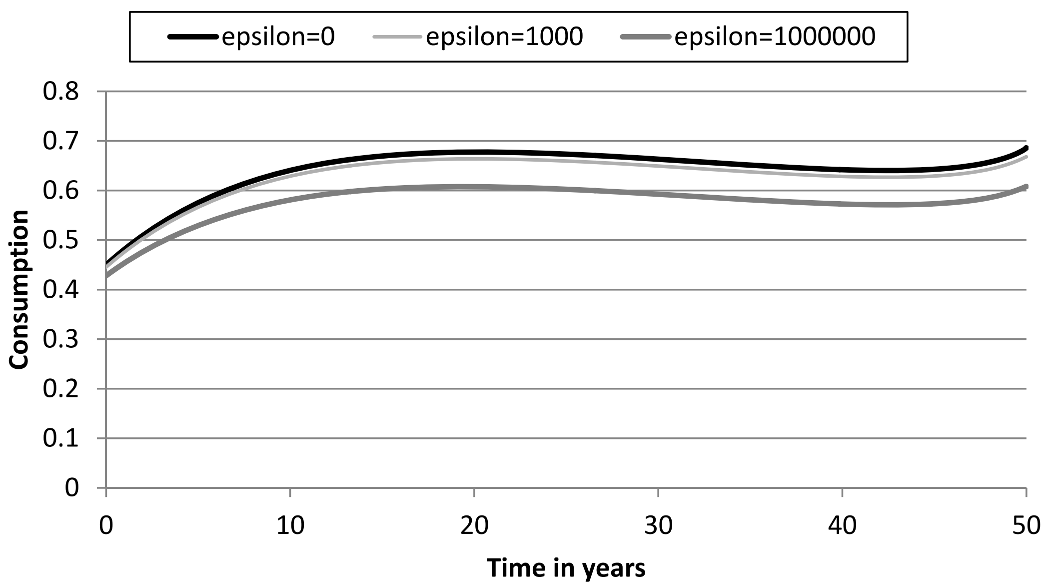

Figure 7: Consumption for different values of the bequest parameter ε. For all other parameters the benchmark values listed in Table 1 are used.

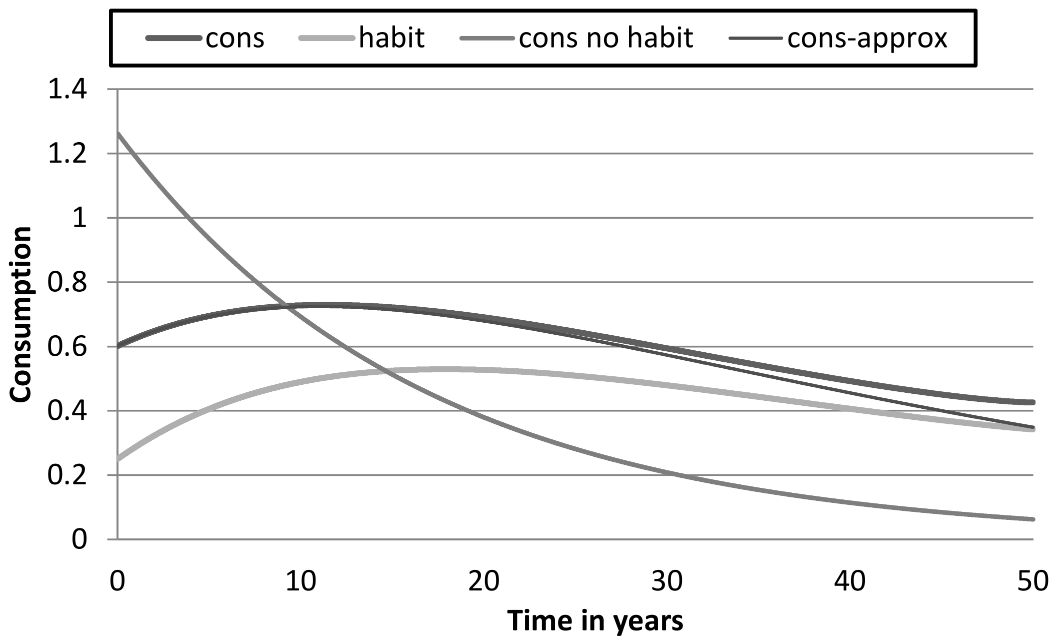

Figure 8: Consumption with no increase in the final years. The dark, thick curve shows the optimal consumption path when $\delta = 0.1$, $\gamma = 2$, $\alpha = 0.15$, $\beta = 0.2$, whereas benchmark values are used for the remaining parameter values as listed in Table 1.

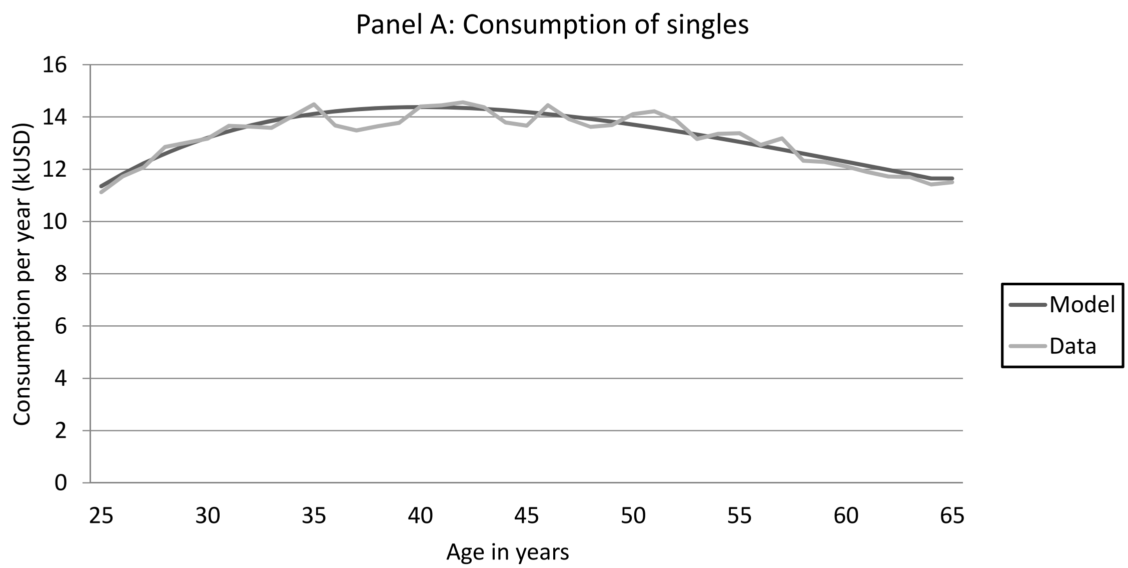

Figure 9: Consumption path from model calibrated to consumption data. The graphs show annual consumption per person in thousands of dollars deflated to reflect 1982-84 constant dollars. The uneven curve in the upper panel reflects average consumption per year of singles at different ages. The uneven curve in the lower panel shows the average per-person consumption per year of childless couples at different ages. Data is taken from the webpage `http://www.fperri.net/research_data.htm` of Fabrizio Perri and generated by Krueger and Perri (2006) from the Consumer Expenditure Survey over the period 1980-2003. The smooth curve in each panel is the consumption pattern in our model calibrated to the data in the way explained in the text.

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