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How Does Contagion Affect General Equilibrium Asset Prices?

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Abstract

This paper analyzes the equilibrium pricing implications of contagion risk in a Lucas-tree economy with recursive preferences and jumps. We introduce a new economic channel allowing for the possibility that endowment shocks simultaneously trigger a regime shift to a bad economic state. We document that these contagious jumps have far-reaching asset pricing implications. The risk premium for such shocks is superadditive, i.e. it is 2.5% larger than the sum of the risk premia for pure endowment shocks and regime switches. Moreover, contagion risk reduces the risk-free rate by around 0.5%. We also derive semiclosed-form solutions for the wealth-consumption ratio and the price-dividend ratios in an economy with two Lucas trees and analyze cross-sectional effects of contagion risk qualitatively. We find that heterogeneity among the assets with respect to contagion risk can increase risk premia disproportionately. In particular, big assets with a large exposure to contagious shocks carry significantly higher risk premia.

Keywords: Contagion, General Equilibrium, Asset Pricing, Recursive Preferences

JEL: G01, G12

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1 Introduction

There is ample empirical evidence that economic contractions cluster. For instance, during the 20th century US consumption per capita decreased in 21 out of 100 years.¹ However, the probability of seeing a subsequent decline was about 38% (8 out of 21 years). Declines clustered around and during World War I as well as during the Great Depression: Between 1907 and 1922 there were declines in 9 out of 15 years. From 1929 to 1933 consumption decreased in 4 consecutive years amounting to a total decline of about 16%. It thus appears that initial drops in aggregate consumption might increase the likelihood of subsequent declines. There are several approaches to capture this stylized fact. Barro (2006) suggests a so-called peak-to-trough method, which means that the consumption decrease over the first year of the disaster is calibrated to the measured cumulative consumption decrease from peak to trough of the disaster period. Constantinides (2008) discusses this calibration approach and indicates that it misses out on the dynamic nature of disasters which are usually characterized by a sequence of moderate declines instead of one large decline. To capture these dynamics of disasters, the literature essentially applies two approaches. Wachter (2012) proposes a time-varying probability of a consumption disaster. Veronesi (2004) assumes the drift of consumption to follow a Markov switching process, with a small probability of falling into a low state. While these approaches allow for a temporarily increased probability of consumption declines, a consumption shock itself does not trigger an increase of this probability. On the contrary, the present paper entertains a new economic channel allowing for consumption shocks that simultaneously trigger a regime shift to a bad economic state in which the probability of subsequent declines increases. In a general equilibrium setting with recursive preferences, we document that the risk premium of such a shock is significantly larger than the combined risk premia of a separate consumption shock and a pure regime shift.

Our model is silent on the economic reason of such a combined cash flow shock and regime shift ('contagious shock') and in this sense it is a reduced-form approach. We interpret these events as contagion effects that might spread over the economy. Initial moderate shocks in one sector of the economy might affect other sectors and thus cause subsequent shocks in these sectors leading to a cascade of shocks. Consequently, aggregate consumption might decrease over several years.

We analyze the impact of contagion risk on asset prices and asset price dynamics and consider a classic Lucas tree economy in which aggregate endowment is affected by contagion

¹The consumption per capita data is from Robert Barro's website.

risk. More precisely, the endowment dynamics are subject to diffusion risk and to downward jumps where the jump intensity follows a two-state Markov chain. It is moderate in normal times ('calm state'), but increases significantly when the economy enters the 'contagion state'. As mentioned above, a key feature of our model is that jumps from the calm state to the contagion state go together with a negative shock to aggregate endowment capturing the previously described effects. We document that this property is of first-order importance for the sizes of the equilibrium risk premium and risk-free rate. To illustrate these results, we compare our economy to a pure regime switching economy with the same local distribution of consumption, in which consumption jumps and regime switches are decoupled. We find that risk premia are superadditive. Contagious jumps carry a risk premium that is 2.5% higher than the sum of the risk premia for regime switches and for jumps in consumption in a pure regime switching framework. Besides, the equilibrium risk-free rate is reduced by 0.5% resulting from higher precautionary savings terms. In a pure regime switching economy, the agent can adjust his asset demand both after a pure consumption jump and after a regime change. In an economy with contagious shocks, this timing possibility is missing. As a result, the equilibrium rate of return on the risky asset is larger in the presence of contagion risk, and the risk-free rate is reduced, since an additional precautionary savings motive arises.

The interpretation of a combined cash-flow shock and regime shift as contagious shock in one part of the economy suggests to also study a Lucas tree economy with two trees. The trees can spread contagion across the economy or can be affected by it. This allows us to analyze how differences in contagion risk induce differences in prices and returns. As special cases, we analyze two types of assets: contagious assets that 'trigger' contagion and non-toxic, contagion-sensitive assets that are affected by, but are hardly causing contagion.²

To summarize, our paper makes three main contributions to the literature. First, we study quantitatively how the possibility of contagious shocks affects prices and price dynamics. We compare our economy with a pure regime switching economy, in which adverse regime shifts and shocks to consumption are disentangled. We find an economically significant extra risk premium for contagious shocks that is earned in normal times. In the calm state of our model, assets thus trade at a price discount if they are prone to contagion risk. Fur-

²Multi-tree models are analyzed in Dumas (1992) and Dumas, Harvey, and Ruiz (2003). Subsequently, Cochrane, Longstaff, and Santa-Clara (2008) consider a two tree economy with a log investor. Martin (2013) studies a Lucas orchard with several trees and CRRA preferences. Our equilibrium solution with recursive utility follows Branger, Dumitrescu, Ivanova, and Schlag (2012).

thermore, contagion gives the representative agent an additional motive for precautionary savings that lowers the equilibrium risk-free rate. Second, we derive semiclosed-form solutions for the wealth-consumption ratio and the price-dividend ratios (and thus all other key asset pricing figures) in a model with two Lucas trees, recursive preferences and contagious shocks. We document that the superadditive pricing effects of contagious shocks are particularly pronounced for assets written on big trees. Additionally, we show that assets written on a tree with zero consumption share also earn positive risk premia, since they are exposed to contagious shocks. Third, we qualitatively analyze possible cross-sectional effects of contagion. We consider two setups with heterogeneous assets: In the first setting, there is one asset that is predominantly affected by contagion ('contagion-sensitive asset'). In the second setting, there is one asset that mainly triggers contagion. We find that big assets, contagion-sensitive assets, and assets with a high propensity to trigger contagion carry the largest equilibrium risk premia. In these asymmetric cases, the superadditive effects of contagious shocks are further amplified.

Besides the growing research on asset pricing with multiple trees, our paper is related to several other strands of literature. There is a huge amount of research on the impact of jumps on asset prices. Naik and Lee (1990) derive the equilibrium in a one tree economy with utility from terminal wealth where the dividend follows a jump-diffusion process. Rietz (1988) and Barro (2006, 2009) show that rare but severe disasters help to explain the equity premium puzzle. Wachter (2012) analyzes the impact of a time-varying (exogenous) jump intensity on the variance of returns in a model with Epstein-Zin preferences. Gabaix (2012) focuses on time-varying jump intensities and time-varying recovery rates in a model with CRRA utility in order to solve several asset pricing puzzles. Backus, Chernov, and Martin (2011) draw a link between consumption-based asset pricing models and standard option pricing theory. They find that option prices imply more frequent, but also more modest jumps than the disaster literature suggests. Recently, Barro, Nakamura, Steinsson, and Ursua (2013) focussed on temporary versus permanent impacts of disasters. They also find that the temporal concentration of jump risk is of first-order importance for asset pricing. However, their model does not exhibit contagion.

Another strand of related literature studies contagion from an empirical point of view. The book of Claessens and Forbes (2001) provides a detailed synopsis of the different terms which are used in this literature. Longin and Solnik (2001) as well as Forbes and Rigobon (2002), among others, analyze the time-varying behavior of stock return correlations. Bae, Karolyi, and Stulz (2003) measure contagion via the coincidence of extreme return shocks of stock indices across several countries worldwide. They find significant evidence for

the existence of contagion. Ait-Sahalia, Cacho-Diaz, and Laeven (2013) study contagion effects using self-exciting processes.

The remainder of this paper is structured as follows. Section 2 explains the model setup and the equilibrium with one Lucas tree. Section 3 extends the economy to two trees. Section 4 analyzes the cross-sectional pricing implications of contagion risk. Section 5 concludes. All proofs can be found in the Appendix.

2 One Tree Economy

2.1 Consumption Dynamics

We consider a continuous-time Lucas tree economy with an infinite horizon. There is one tree producing a perishable consumption good which serves as numeraire. Section 3 will consider the extension to two trees. The economy can be in either of two states which we denote by 'calm' and 'contagion'. These states are formally captured by a Markov chain Z . Conditional on the state of the economy $Z_t \in \{calm, cont\}$, the outcome of the tree follows a jump-diffusion process. If the economy is in the calm state, then

$$\frac{dC_t}{C_{t-}} = \mu^{calm}dt + \sigma dW_t + L^{calm,calm}dN_t^{calm,calm} + L^{calm,cont}dN_t^{calm,cont}.$$

In the contagion state, the dynamics are

$$\frac{dC_t}{C_{t-}} = \mu^{cont}dt + \sigma dW_t + L^{cont,cont}dN_t^{cont,cont}, \quad (1)$$

where W is a Wiener process with constant, state-independent volatility σ . The different processes $N^{i,j}$ are jump processes. In particular, $N^{calm,cont}$ captures contagious shocks to consumption that can occur in normal times ('calm state') and go together with a regime shift to the contagion state. This type of shocks is a specific feature of our model, since it allows for a combined cash flow shock and regime shift.³ On the contrary, $N^{calm,calm}$ or $N^{cont,cont}$ refer to jumps that occur in the calm or contagion state, but do not go together with a regime shift. Consequently, they only have an impact on the level of consumption and are thus similar to the shocks analyzed in Rietz (1988) and Barro (2006), among others. There is also a fourth counting process in our model, $N^{cont,calm}$, that triggers

³Note that the economy can only enter the contagion state if there is a contagious shock to consumption. This facilitates the comparison with pure regime switching models. Additionally, we could also allow for transitions without an immediate shock to consumption.

regime switches from the contagion to the calm state, but does not directly affect the level of consumption.

The state of the Markov chain Z determines the drift rate μ and the intensities of the counting processes that are denoted by the constants $\lambda^{i,j}$. For instance, $\lambda^{calm,cont}$ captures the probability of a contagious shock in normal times, whereas $\lambda^{calm,calm}$ and $\lambda^{cont,cont}$ denote the intensities for jumps that do not trigger regime switches. In line with the interpretation of the states as 'good' and 'bad', we assume that consumption shocks are more frequent in the contagion state than in the calm state, i.e. $\lambda^{calm,calm} + \lambda^{calm,cont} < \lambda^{cont,cont}$. For simplicity, the respective jump sizes, $L^{calm,calm}$, $L^{calm,cont}$, and $L^{cont,cont}$, are negative constants.⁴

We compare our model to a 'pure regime switching economy' (see, e.g., Veronesi (2004) or Benzoni, Collin-Dufresne, and Goldstein (2011)), where shocks to consumption and regime switches cannot occur simultaneously. This is a special case of our setting where $L^{calm,cont} = 0$. The intensity for consumption drops in the calm state is then solely given by $\lambda^{calm,calm}$. To identify the equilibrium pricing effects of contagious shocks, we choose the parameters such that the local distributions of consumption growth are the same in both models. In particular, we make sure that the jump probabilities are the same in the calm state, i.e. we set the intensity for consumption jumps $\lambda^{calm,calm}$ in the pure regime switching model equal to the combined jump intensity $\lambda^{calm,calm} + \lambda^{calm,cont}$ in our contagion model.

Apart from the pure regime switching economy, we also study an 'economy without regimes'. This economy can formally be obtained as a special case of the pure regime switching economy where all consumption parameters are the same in both states. Again, we adjust the intensity for consumption jumps in the economy without regimes such that it is equal to the unconditional jump intensity in the other two economies. The model without regimes is closest to the setups in Rietz (1988) or Barro (2006).

To summarize, we compare three different models that allow for different temporal concentrations of consumption shocks. In the economy without regimes, jumps solely have an immediate effect on the level of consumption, but no after-effects.⁵ This model class can thus only match the size of empirical disasters if they are calibrated with the peak-to-trough method described by Barro (2006). In this sense, this model represents the highest

⁴Stochastic jump sizes would complicate the model and notation without adding much to our main results.

⁵Formerly, this is because shocks are modeled via ordinary Poisson processes that are memoryless.

possible degree of temporal concentration, since the total decline materializes in a single event. In contrast, a pure regime switching model induces a very low degree of temporal concentration. Regime switches solely change the future dynamics of consumption and thus only have a long-term effect: In the contagion state, consumption disasters occur with a higher probability and are thus more prone to clustering. Our contagion model lies between these two approaches. On the one hand, the state variable 'calm/contagion' has a long-term effect on the distribution of consumption. On the other hand, a regime change from calm to contagion ('contagious shock') goes together with a drop in consumption and thus also has an immediate effect.

2.2 Representative Investor

Our economy is populated by a representative investor with stochastic differential utility that was introduced by Duffie and Epstein (1992b). His subjective time preference rate is β , his relative risk aversion is γ , and his elasticity of intertemporal substitution is ψ . The investor has an infinite planning horizon, and his indirect utility function is

$$J_t = E_t \left[\int_t^\infty f(C_s, J_s) ds \right],$$

where the aggregator f is given by

$$f(C, J) = \frac{\beta C^{1-\frac{1}{\psi}}}{\left(1 - \frac{1}{\psi}\right) [(1-\gamma)J]^{\frac{1}{\theta}-1}} - \beta\theta J$$

and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. For $\psi = 1/\gamma$, the investor has time-additive CRRA preferences. In the following, we assume $\gamma > 1$ and $\psi > 1$. Therefore, the investor has a preference for early resolution of uncertainty.

2.3 Parametrization

Table 1 reports the calibrations of all parameters for the different settings. All values are annualized. We calibrate our parameters so that they are roughly in line with Bhamra, Kuehn, and Strebulaev (2010). These authors estimate the parameters of a regime switching model with two economic regimes using US consumption data from 1947 to 2005. Their estimate for the intensity of switches from the good to the bad state is 0.27, while the intensity of switches back to the good state is 0.49. We thus set $\lambda^{calm,cont} = 0.27$ and

$\lambda^{cont, calm} = 0.49$. Given a particular state the consumption process in their model is a geometric Brownian Motion. The volatility is almost state-independent and equal to 0.01. The expected consumption growth rates in the good and the bad state equal 0.042 and 0.014, respectively. On the contrary, we assume a jump-diffusion process and thus augment the dynamics by jumps in the level of consumption. Consequently, there are several parameter combinations that are in line with Bhamra, Kuehn, and Strebulaev (2010). We use $\sigma = 0.01$ and choose a constant drift rate $\mu^{calm} = \mu^{cont} = 0.058$. The jump size is set equal to $L = -0.03$ for all types of jumps. The moderate jump size reflects the intuition that our jumps are not 'disasters' in the sense of Barro (2006), but constitute more frequent and less severe consumption drops.⁶ The intensity $\lambda^{calm, calm}$ is set to 0.23, so that the intensity of a consumption drop in the calm state is $0.27 + 0.23 = 0.5$. The intensity in the contagion state is three times higher, i.e. $\lambda^{cont, cont} = 1.5$. Altogether, we closely match the expected consumption growth rates of Bhamra, Kuehn, and Strebulaev (2010) in both states. The resulting consumption volatility reported in the first line of Table 2 is

$$\sqrt{\sigma^2 + \lambda^{calm, calm}(L^{calm, calm})^2 + \lambda^{calm, cont}(L^{calm, cont})^2} = 0.0235$$

in the calm state and

$$\sqrt{\sigma^2 + \lambda^{cont, cont}(L^{cont, cont})^2} = 0.0381$$

in the contagion state. Given the unconditional probabilities of the two states

$$p^{calm} = \frac{\lambda^{cont, calm}}{\lambda^{cont, calm} + \lambda^{calm, cont}} = 0.6447 \quad \text{and} \quad p^{cont} = 0.3553,$$

the unconditional volatility of consumption equals

$$\sqrt{\sigma^2 + p^{calm} \sum_{k=calm, cont} \lambda^{calm, k}(L^{calm, k})^2 + p^{cont} \lambda^{cont, cont}(L^{cont, cont})^2} = 0.0295.$$

For the pure regime switching economy, we set $L^{calm, cont} = 0$, so that jumps in consumption and regime switches are no longer coupled. As argued in Section 2.1, the remaining parameters are chosen such that the local dynamics of consumption and the local dynamics of the Markov chain coincide in both models. This implies $\lambda^{calm, calm} = 0.5$, so that

⁶Since consumption declines are rare events, it makes sense to look at a longer time horizon. In the 20th century there were 21 years where US aggregate consumption declined with an average value of about 2.9%. If we only take declines of more than 1% into account and thus disregard jumps that might be attributed to diffusion risk, the average is about 3.3%. Therefore, our choice resembles these values.

consumption jumps in the calm state still have an intensity equal to 0.5, while all other parameters remain unchanged.

For the economy without regimes, we also set $L^{calm,cont}$ equal to 0. Moreover, the jump intensities $\lambda^{calm,calm}$ and $\lambda^{cont,cont}$ coincide, and we set them equal to the unconditional mean jump intensity in the economy with contagion:

$$\lambda^{uncond} = p^{calm} \cdot 0.5 + p^{cont} \cdot 1.5 = 0.855.$$

Finally, the preference parameters are chosen in line with Bansal and Yaron (2004) and other papers in the long run risk literature, i.e. $\gamma = 10$ and $\psi = 1.5$. Following Bhamra, Kuehn, and Strebulaev (2010), we set $\beta = 0.01$.

2.4 Asset Pricing Results

We solve for the pricing kernel following Duffie and Epstein (1992a,b) and Benzoni, Collin-Dufresne, and Goldstein (2011). Details of the derivation as well as the proofs of all following asset pricing results can be found in Appendix A. A summary of the numerical results discussed in this section is provided in Table 2.

We conjecture that the indirect utility J of the representative investor is

$$J_t = \frac{C_t^{1-\gamma}}{1-\gamma} \beta^\theta e^{\theta v^{Z_t}}.$$

The pricing kernel ξ is then given by

$$\xi_t = \beta^\theta C_t^{-\gamma} e^{-\beta\theta t + (\theta-1) \left(\int_0^t e^{-v^{Z_u}} du + v^{Z_t} \right)}. \quad (2)$$

The process $v_t = v^{Z_t}$ is equal to the logarithm of the wealth-consumption ratio (see, e.g., Campbell, Chacko, Rodriguez, and Viceira (2004) and Benzoni, Collin-Dufresne, and Goldstein (2011)).⁷ It depends on the state of the economy Z and can thus only take the

⁷Notice that v_t is a process and each v^j , $j \in \{calm, cont\}$, is a function that is constant in our one tree setting.

two values v^{calm} and v^{cont} that solve the following system of equations:

$$\begin{aligned}
0 &= e^{-v^{calm}} - \beta + \left(1 - \frac{1}{\psi}\right) \mu^{calm} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma^2 \\
&\quad + \frac{1}{\theta} \lambda^{calm, calm} \left[(1 + L^{calm, calm})^{1-\gamma} - 1\right] + \frac{1}{\theta} \lambda^{calm, cont} \left[(1 + L^{calm, cont})^{1-\gamma} e^{\theta(v^{cont} - v^{calm})} - 1\right] \\
0 &= e^{-v^{cont}} - \beta + \left(1 - \frac{1}{\psi}\right) \mu^{cont} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma^2 \\
&\quad + \frac{1}{\theta} \lambda^{cont, cont} \left[(1 + L^{cont, cont})^{1-\gamma} - 1\right] + \frac{1}{\theta} \lambda^{cont, calm} \left[e^{\theta(v^{calm} - v^{cont})} - 1\right].
\end{aligned}$$

2.4.1 Market prices of risk

From the pricing kernel, we directly get the market prices of risk.

Proposition 1 *The market price of diffusion risk in state $j \in \{calm, cont\}$ is*

$$\eta^{diff, j} = \gamma \sigma.$$

The market prices of jump risk are

$$\begin{aligned}
\eta^{calm, calm} &= (1 + L^{calm, calm})^{-\gamma} - 1 \\
\eta^{calm, cont} &= (1 + L^{calm, cont})^{-\gamma} e^{(\theta-1)(v^{cont} - v^{calm})} - 1 \\
\eta^{cont, cont} &= (1 + L^{cont, cont})^{-\gamma} - 1 \\
\eta^{cont, calm} &= e^{(\theta-1)(v^{calm} - v^{cont})} - 1.
\end{aligned}$$

The risk-neutral jump intensities follow from the physical intensities via the relation $\lambda^{\mathbb{Q}, j, k} = \lambda^{\mathbb{P}, j, k} (1 + \eta^{j, k})$.

The market prices of risk depend on the dynamics of consumption and on the dynamics of the wealth-consumption ratio v . The latter dependence vanishes in case of a CRRA agent with $\psi = 1/\gamma$, for whom state variables – here the state of the Markov chain Z – are not priced. In contrast, we assume a preference for early resolution of uncertainty and $\theta < 0$. Therefore, possible changes in future consumption opportunities which impact the wealth-consumption ratio enter the pricing kernel. Both the market prices of risk for contagious jumps and for regime switches back into the calm state involve the change of the wealth-consumption ratio upon a regime shift. In particular, $\eta^{cont, calm}$ is nonzero although these regime switches do not affect consumption directly, but only through the state variable Z .

Our economy differs from a pure regime switching economy since we allow for contagious shocks that combine a cash-flow shock and a regime shift. The market price of risk for these jumps can be decomposed as follows:

$$1 + \eta^{calm,cont} = \underbrace{(1 + L^{calm,cont})^{-\gamma}}_{1 + \eta^{jump}} \underbrace{e^{(\theta-1)(v^{cont}-v^{calm})}}_{1 + \eta^{RS}}. \quad (3)$$

The variable η^{jump} denotes the (hypothetical) market price of risk for isolated consumption jumps with size $L^{calm,cont}$, whereas η^{RS} denotes the (hypothetical) market price of risk for a pure regime switch from calm to contagion. In a pure regime switching economy, jumps in consumption and regime switches are disentangled and thus carry different market prices of risk, η^{jump} and η^{RS} . The decomposition (3) shows that the market price of risk for contagious shocks is not just the sum of its parts. In fact, it is superadditive. Risk-averse agents with a preference for early resolution of uncertainty demand an extra risk premium if the two adverse events 'consumption shock' and 'switch to the contagion state' occur simultaneously.

2.4.2 Risk-free interest rate

The risk-free rate follows from the (negative) expected growth rate of the pricing kernel.

Proposition 2 *The risk-free interest rate in state $j \in \{calm, cont\}$ equals*

$$r_f^j = \beta + \frac{1}{\psi} \frac{E_t[dC_t]}{C_t} - \Xi^{diff} - \Xi^{j,calm} - \Xi^{j,cont},$$

where the precautionary savings terms are given by

$$\begin{aligned} \Xi^{diff} &= \frac{1}{2} \gamma \left(1 + \frac{1}{\psi}\right) \sigma^2 \\ \Xi^{j,k} &= \lambda^{j,k} \left[\eta^{j,k} + \frac{1}{\psi} L^{j,k} + \frac{1-\theta}{\theta} \left((1 + L^{j,k})^{1-\gamma} e^{\theta(v^k - v^j)} - 1 \right) \right]. \end{aligned}$$

for $(j, k) \in \{(calm, calm), (calm, cont), (cont, cont)\}$ and

$$\Xi^{cont,calm} = \lambda^{cont,calm} \left[\eta^{cont,calm} + \frac{1-\theta}{\theta} \left(e^{\theta(v^{calm} - v^{cont})} - 1 \right) \right].$$

The risk-free interest rate comprises the subjective time preference rate β , the expected growth rate of consumption scaled by the inverse of the IES, and several precautionary savings terms for the different risk factors in our model. Table 3 reports the numerical

values and a decomposition of the risk-free rate into its components. The first three components (time preference rate, expected consumption growth, precautionary savings term for diffusive risk) are standard and identical across settings.⁸ A higher time preference rate or a higher expected consumption growth rate lower the incentives to save leading to a higher equilibrium risk-free rate. The precautionary savings term for diffusive risk, Ξ^{diff} , has the well-known form since the only diffusive risk in our economy is consumption risk.

Differences in the risk-free rate across settings are driven by differences in the precautionary savings terms for jump risk. In the economy with contagion, these terms reduce the risk-free rate by more than 1% in both states. This reduction is larger in the contagion state than in the calm state, since jump intensities are larger in the contagion state. Comparing the economy with contagion and the pure regime switching economy, we find significant differences for the precautionary savings terms in the calm state. In the economy with contagion, the precautionary savings term for contagious shocks is $\Xi^{calm,cont} = 0.0089$, whereas the term for ordinary consumption jumps is $\Xi^{calm,calm} = 0.0021$. Although both types of jumps have similar intensities (0.27 and 0.23), the effect of contagious shocks is four times bigger. In the pure regime switching economy, precautionary savings demand arises from the risk of pure consumption shocks and from the risk of regime switches. To match the jump probabilities in both models, we increase the intensity of ordinary consumption jumps from 0.23 to 0.50. The corresponding precautionary savings term increases proportionally from 0.0021 to 0.0045. Pure regime changes induce a precautionary savings term of $\Xi^{calm,cont} = 0.0013$. To summarize, the demand for precautionary savings due to jump risk has a larger impact in our economy than in the pure regime-switching economy and induces a difference of 52 basis points in the risk-free rate.⁹ This is because in a pure regime switching economy the representative agent waits to adjust his precautionary savings demand until the economy has entered the bad state. On the contrary, contagious shocks make this kind of timing impossible. As a consequence, the agent's precautionary savings motive in the calm state increases even more, which reduces the risk-free interest rate in equilibrium.

Finally, the risk-free interest rate in an economy without regimes equals 0.023 and is thus 45 basis points higher than the unconditional expected interest rate in the economy with contagion (0.0185 as reported in Table 2) and 13 basis points above the unconditional

⁸Notice that the expected consumption growth in the economy without regimes equals the unconditional expected consumption growth in the other two cases.

⁹Notice that, unconditionally, we find a difference of 33 basis points.

expected interest rate in the pure regime switching economy. This is because there is no precautionary savings motive stemming from regime changes, which leads to an increase of the risk-free rate by 13 basis points.

2.4.3 Price-Dividend Ratios

Following Bansal and Yaron (2004) and Wachter (2012), among others, dividends D_t are modeled as levered consumption. We assume a leverage parameter $\phi = 3$ for both diffusion risk and jump risk. The dynamics of dividends are

$$\begin{aligned} \frac{dD_t}{D_{t-}} = & \mu^{calm} dt + \phi \sigma dW_t + [(1 + L^{calm,calm})^\phi - 1] dN_t^{calm,calm} \\ & + [(1 + L^{calm,cont})^\phi - 1] dN_t^{calm,cont} \end{aligned}$$

in the calm state and

$$\frac{dD_t}{D_{t-}} = \mu^{cont} dt + \phi \sigma dW_t + [(1 + L^{cont,cont})^\phi - 1] dN_t^{cont,cont}$$

in the contagion state. Similarly to Longstaff and Piazzesi (2004) we assume the same drift rate, μ^{Z_t} , for consumption and dividends. The pricing equation for the dividend claim is

$$D_t e^{w_t} = E_t \left[\int_t^\infty \frac{\xi_\tau}{\xi_t} D_\tau d\tau \right],$$

where w is the logarithm of the price-dividend ratio. Analogously to the log wealth-consumption ratio, the log price-dividend ratio w depends on the state of the Markov chain. Its two possible values w^{calm} or w^{cont} satisfy a system of equations provided in Appendix A. For the parameters in Table 1, the price-dividend ratio equals 11.42 in the calm state and 10.47 in the contagion state. The sizes of the price jumps upon the different events are thus -8.7% for pure dividend jumps, -16.37% for contagious jumps and +9.1% for switches from the contagion to the calm state.

The valuation ratios in the pure regime switching economy are slightly larger than in the economy with contagion. Recall that the parameters of both models are chosen such that the local distribution of aggregate consumption is the same in both economies. In the economy with contagion, a transition into the contagion state is always accompanied by a simultaneous drop in consumption. In the pure regime switching economy, the prices in the calm state can be affected either by a drop in consumption or by a transition into the contagion state. As already pointed out in the discussion of the risk-free rate, the investor dislikes one large jump more than two small jumps. Consequently, the price-dividend ratio

in the regime-switching economy is larger than in the economy with contagion by around 8.7%. By a similar argument, the price-dividend ratio in the economy without regimes is larger than in the pure regime switching economy.

2.4.4 Risk Premia

The risk premium of an asset follows from its exposure to the various risk factors and the respective market prices of risk. The market prices of risk are given in Proposition 1. The exposures follow from the dynamics of the asset price P in the calm and in the contagion state, respectively:

$$\begin{aligned}\frac{dP_t}{P_{t-}} &= \mu^{calm} dt + \Upsilon^{diff} dW_t + \Upsilon^{calm,calm} dN_t^{calm,calm} + \Upsilon^{calm,cont} dN_t^{calm,cont} \\ \frac{dP_t}{P_{t-}} &= \mu^{cont} dt + \Upsilon^{diff} dW_t + \Upsilon^{cont,cont} dN_t^{cont,cont} + \Upsilon^{cont,calm} dN_t^{cont,calm}.\end{aligned}$$

Defining the instantaneous asset return by $dR = dP/P + D/Pdt$, we obtain the following proposition.

Proposition 3 *The sensitivities of the asset price P with respect to the different risk factors are*

$$\begin{aligned}\Upsilon^{diff} &= \phi\sigma \\ \Upsilon^{calm,calm} &= (1 + L^{calm,calm})^\phi - 1 \\ \Upsilon^{calm,cont} &= (1 + L^{calm,cont})^\phi e^{w^{cont} - w^{calm}} - 1 \\ \Upsilon^{cont,cont} &= (1 + L^{cont,cont})^\phi - 1 \\ \Upsilon^{cont,calm} &= e^{w^{calm} - w^{cont}} - 1\end{aligned}$$

The expected return on the dividend claim in the calm state equals

$$\frac{E_t[dR_t^{calm}]}{dt} = r_f^{calm} + \eta^{diff} \Upsilon^{diff} - \sum_{k=calm,cont} \lambda^{calm,k} \eta^{calm,k} \Upsilon^{calm,k}.$$

The expected return in the contagion state equals

$$\frac{E_t[dR_t^{cont}]}{dt} = r_f^{cont} + \eta^{diff} \Upsilon^{diff} - \sum_{k=calm,cont} \lambda^{cont,k} \eta^{cont,k} \Upsilon^{cont,k}.$$

The expected excess return over the risk-free rate (equity premium) is reported in Table 2. Table 4 decomposes the equity premium into its different components. In all settings,

the contribution stemming from diffusive risk is negligible and the premium is mainly driven by jump exposures and the possibility of regime switches. The premium in the (less risky) calm state is always smaller than the premium in the (riskier) contagion state. Furthermore, the unconditional equity risk premium in the (riskiest) economy with contagion is the largest, while the equity risk premium in the (least risky) economy without regimes is the smallest.

In the calm state of our economy, the total equity premium becomes sizeable 5.44%. More than 80% of the equity premium can be attributed to the risk of contagious jumps. In the pure regime switching economy, only ordinary consumption jumps can occur. To match the jump probabilities, they are assumed to be more than twice as likely as in the economy with contagion where ordinary jumps contribute about 72 basis points to the equity premium. Similarly as the precautionary savings terms in the risk-free rate, the corresponding risk premium increases linearly. Furthermore, the risk premium for pure regime switches is of moderate size, since a regime switch does not lead to an immediate drop in cash flows and thus the representative agent waits to adjust his asset demands until a regime switch has occurred. For our calibration, the total risk premium in the pure regime switching economy is more than 2% smaller than in the economy with contagion. To gain further intuition for these results, Table 5 decomposes the risk premium for contagious jumps into several components. As shown in Section 2.4.1, the market price of risk for contagious jumps is $(1 + \eta^{jump})(1 + \eta^{RS}) - 1$. Similarly, the exposure to contagious jumps can be decomposed into an exposure Υ^{jump} to a drop in consumption and an exposure Υ^{RS} to a regime switch. The risk premium for contagious jumps is

$$-\lambda^{calm,cont} \left[(1 + \eta^{jump})(1 + \eta^{RS}) - 1 \right] \left[(1 + \Upsilon^{jump})(1 + \Upsilon^{RS}) - 1 \right] = 0.0442.$$

In a pure regime switching model, a contagious jump is split up into two separate components, a pure consumption jump and a pure regime switch. The total premium for both risk factors is given by

$$-\lambda^{calm,cont} (\eta^{jump}\Upsilon^{jump} + \eta^{RS}\Upsilon^{RS}) = 0.0191,$$

where the premium for pure consumption jumps is 84 basis points and the premium for pure regime switches is 107 basis points. The premium for contagious jumps exceeds the sum of the premia for a separate jump and regime switch in a pure regime switching model by an interaction term of around 2.5%.

The equity premium in the contagion state, on the other hand, is larger in the pure regime switching economy. As can be seen from Table 4, this can be attributed to a

larger premium for the regime switch back to the calm state. The reason is the larger size $\Upsilon^{cont, calm}$ of the price jump upon this event, which induces a larger risk premium. Note that the premium for regime changes from the contagion state to the calm state is positive. This can be explained via the hedging demand of the representative agent. A switch back to the calm state improves the economic situation of the investor which, in general, gives rise to a negative hedging demand for assets that perform well upon this event. Since the asset price increases upon a regime shift back to the calm state, the asset must earn an additional positive risk premium in equilibrium.

Overall, the unconditional equity risk premium in the economy with contagious jumps is larger than in the pure regime-switching economy. The larger premium in the calm state resulting from the risk of contagious jumps overcompensates the smaller premium in the contagion state. Furthermore, the unconditional equity premium in our contagion model of 0.0578 is almost twice as high as the equity premium without regimes of 0.0296. To get a sensible level of the equity risk premium in the frameworks of Rietz (1988) or Barro (2006), one has to assume a much higher jump size, which Barro (2006) justifies by using a peak-to-trough calibration method. Constantinides (2008) critically discusses this approach. Altogether, our paper shows that the temporal concentration of (moderate) jump risk can change the risk premia in an economy significantly.

The premium for regime switches can also be seen as a version of the premium for a stochastic jump intensity as discussed by Wachter (2012) or Gabaix (2012). Our paper, however, shows that a direct link between state variables (in the present paper calm vs. contagion) and the consumption process itself has significant consequences for the risk premia. The investor in our model has a preference for early resolution of uncertainty and thus would like to hedge against state variable risk. If state variable risk is connected to consumption risk, this hedging motive increases disproportionately. As a result, the highest equilibrium equity premium is paid in a model with contagious jumps, although the local distribution of future consumption is the same across all analyzed economies here.

2.4.5 Return Volatilities

From the dynamics above, we can compute different local variances of the asset's excess return in closed form. Conditional on being in the calm state, the local variance equals¹⁰

$$\sigma^2\phi^2 + \lambda^{calm,calm}((1 + L^{calm,calm})^\phi - 1)^2 + \lambda^{calm,cont}(e^{w^{cont}-w^{calm}}(1 + L^{calm,cont})^\phi - 1)^2.$$

Conditional on being in the contagion state, we obtain

$$\sigma^2\phi^2 + \lambda^{cont,cont}((1 + L^{cont,cont})^\phi - 1)^2 + \lambda^{cont,calm}(e^{w^{calm}-w^{cont}} - 1)^2.$$

Table 2 reports the numerical results. In line with intuition, the unconditional volatility is the largest in an economy with contagious jumps and the smallest in an economy without regimes. For the conditional volatility, we again have to distinguish between the calm and the contagion state. In the calm state, the conditional volatility is larger in an economy with contagious jumps due to the presence of these jumps. On the other hand, the conditional volatility in the contagion state is larger in the pure regime-switching economy. The reason is the larger difference between the price-dividend ratios and thus the larger positive return when the economy switches to the calm state.

3 Two Tree Economy

The interpretation of a combined cash-flow shock and regime shift as contagious shock in one part of the economy suggests to extend the basic model and study a Lucas tree economy with two trees. Since the differences between an economy with contagion, a pure regime switching economy and an economy without regimes have been analyzed in Section 2, we solely focus on a two tree economy with contagion. As a first contribution, the present section provides analytical results for the risk premia and the risk-free rate in such an economy. Second, this section also discusses numerical results if the trees have identical parameters. We refer to this situation as the case with 'identical trees', although the trees might have different sizes. Cases with different jump parameters for the two trees are analyzed in Section 4 where we concentrate on the cross-sectional implications of contagion and study how differences in cash flows are reflected in prices and price dynamics.

¹⁰Formally, the local variance is defined by $d\langle P \rangle_t / (P_t^2 dt)$.

3.1 Consumption Dynamics

The basic assumptions in this section are the same as before, but now there are two Lucas trees, A and B , producing the same perishable consumption good. Total consumption comprises the outcome of the two trees, i.e. $C = C_A + C_B$. In the calm state, the dynamics are

$$\begin{aligned}\frac{dC_{A,t}}{C_{A,t-}} &= \mu_A^{calm} dt + \sigma_A dW_{A,t} + L_A^{calm,calm} dN_{A,t}^{calm,calm} + L_A^{calm,cont} dN_{A,t}^{calm,cont} \\ \frac{dC_{B,t}}{C_{B,t-}} &= \mu_B^{calm} dt + \sigma_B dW_{B,t} + L_B^{calm,calm} dN_{B,t}^{calm,calm} + L_B^{calm,cont} dN_{B,t}^{calm,cont},\end{aligned}$$

whereas, in the contagion state, they read

$$\frac{dC_{A,t}}{C_{A,t-}} = \mu_A^{cont} dt + \sigma_A dW_{A,t} + L_A^{cont,cont} dN_{A,t}^{cont,cont} \quad (4)$$

$$\frac{dC_{B,t}}{C_{B,t-}} = \mu_B^{cont} dt + \sigma_B dW_{B,t} + L_B^{cont,cont} dN_{B,t}^{cont,cont}. \quad (5)$$

We assume that the diffusion volatilities σ_i , $i \in \{A, B\}$, and the correlation ρ between the Brownian motions W_A and W_B are constant and state-independent. In general both trees are exposed to contagious jumps, i.e. each tree can 'spread' contagion. Since in multiple tree economies the equilibrium outcomes depend on the relative size of the two trees, we follow Cochrane, Longstaff, and Santa-Clara (2008) and introduce the consumption share $s_A = C_A/(C_A + C_B)$ of tree A . The share of tree B is given by $s_B = 1 - s_A$. Throughout the rest of the paper, the term 'size' refers to the consumption share of a tree. For instance, asset A is said to be 'big' if s_A is close to 1. In the calm state, the dynamics of aggregate consumption C are given by

$$\begin{aligned}\frac{dC_t}{C_{t-}} &= s_{A,t-} \frac{dC_{A,t}}{C_{A,t-}} + s_{B,t-} \frac{dC_{B,t}}{C_{B,t-}} \\ &= \sum_{i=A,B} s_{i,t} \mu_i^{calm} dt + \sum_{i=A,B} s_{i,t} \sigma_i dW_{i,t} + \sum_{\substack{i=A,B \\ k=calm,cont}} s_{i,t-} L_i^{calm,k} dN_{i,t}^{calm,k},\end{aligned} \quad (6)$$

while in the contagion state we have

$$\frac{dC_t}{C_{t-}} = \sum_{i=A,B} s_{i,t} \mu_i^{cont} dt + \sum_{i=A,B} s_{i,t} \sigma_i dW_{i,t} + \sum_{i=A,B} s_{i,t-} L_i^{cont,cont} dN_{i,t}^{cont,cont}. \quad (7)$$

Applying Ito's lemma, we obtain the dynamics of s_A in the calm state:

$$\begin{aligned} \frac{ds_{A,t}}{s_{A,t-}s_{B,t-}} &= \left[\mu_A^{calm} - \mu_B^{calm} - s_{A,t}\sigma_A^2 + s_{B,t}\sigma_B^2 + (s_{A,t} - s_{B,t})\rho\sigma_A\sigma_B \right] dt \\ &\quad + \sigma_A dW_{A,t} - \sigma_B dW_{B,t} \\ &\quad + \sum_{k=calm,cont} \frac{L_A^{calm,k}}{1 + L_A^{calm,k} s_{A,t-}} dN_{A,t}^{calm,k} - \sum_{k=calm,cont} \frac{L_B^{calm,k}}{1 + L_B^{calm,k} s_{B,t-}} dN_{B,t}^{calm,k}. \end{aligned} \quad (8)$$

In the contagion state, we have

$$\begin{aligned} \frac{ds_{A,t}}{s_{A,t-}s_{B,t-}} &= \left[\mu_A^{cont} - \mu_B^{cont} - s_{A,t}\sigma_A^2 + s_{B,t}\sigma_B^2 + (s_{A,t} - s_{B,t})\rho\sigma_A\sigma_B \right] dt + \sigma_A dW_{A,t} - \sigma_B dW_{B,t} \\ &\quad + \frac{L_A^{cont,cont}}{1 + L_A^{cont,cont} s_{A,t-}} dN_{A,t}^{cont,cont} - \frac{L_B^{cont,cont}}{1 + L_B^{cont,cont} s_{B,t-}} dN_{B,t}^{cont,cont}. \end{aligned}$$

Downward jumps in tree A reduce the consumption share of tree A , whereas downward jumps in tree B increase it. To shorten notations, we denote the consumption share of tree A after a jump in tree A or B , respectively, by

$$s_{A,t}^{A+} = s_{A,t-} \frac{1 + L_A^{j,k}}{1 + L_A^{j,k} s_{A,t-}}, \quad s_{A,t}^{B+} = s_{A,t-} \frac{1}{1 + L_B^{j,k} (1 - s_{A,t-})}$$

where $(j, k) \in \{(calm, calm), (calm, cont), (cont, cont)\}$. Moreover, we denote the drift rates of aggregate consumption and of the consumption share s_A by μ_C^Z and μ_s^Z , respectively. We also abbreviate the quadratic variation terms

$$\sigma_{CC}dt = \frac{d\langle C^c \rangle_t}{C_t^2}, \quad \sigma_{Cs}dt = \frac{d\langle C^c, s_A^c \rangle_t}{C_t}, \quad \sigma_{ss}dt = d\langle s_A^c \rangle_t$$

where the upper index c refers to the continuous part of the respective process.¹¹ Appendix B provides details. Conditional on the state, the local variance of consumption equals

$$\frac{d\langle C \rangle_t}{C_t^2} = \sigma_{CC}dt + \sum_{\substack{i=A,B \\ k=calm,cont}} s_i^2 (L_i^{calm,k})^2 \lambda_i^{calm,k} dt$$

in the calm state and

$$\frac{d\langle C \rangle_t}{C_t^2} = \sigma_{CC}dt + \sum_{i=A,B} s_i^2 (L_i^{cont,cont})^2 \lambda_i^{cont,cont} dt$$

in the contagion state. It depends on the consumption share s_A and on the state Z (via the jump parameters) and is thus stochastic. In the case of identically parameterized trees, it is the smallest if both trees have the same size.

¹¹For some process Y , the sharp bracket $\langle Y \rangle$ is the predictable quadratic variation of Y . $\langle Y \rangle$ is the compensator of the quadratic variation $[Y]$.

3.2 Parametrization

To illustrate the qualitative implications of a two tree economy numerically, we first study an economy with identical trees: We split up the calibrated tree from Section 2 as if the economy consists of two identical, initially equally big trees. The column labeled 'Identical Trees' of Table 1 reports the corresponding parameters. We choose the parameters of the two trees such that, for $s_A = 0.5$, the dynamics of aggregate consumption coincide with those of the single tree from Section 2. For both trees and both states, we assume drift rates of $\mu_i^Z = 0.058$. The diffusion volatilities σ_i equal $\sqrt{2} \cdot 0.01 = 0.014$ in both states, the diffusion correlation ρ is set to 0 for simplicity. We choose jump sizes $L_i^{j,k} = -0.06$ for $(j, k) \in \{(calm, calm), (calm, cont), (cont, cont)\}$. Upon a jump in one of the trees and for $s_A = 0.5$, total consumption thus drops by -0.03 as in the case with one tree. The intensities for consumption jumps are cut in half, i.e. we assume $\lambda_i^{calm, calm} = 0.115$, $\lambda_i^{calm, cont} = 0.135$ and $\lambda_i^{cont, cont} = 0.75$, so that the total intensity for jumps in consumption is again the same as before. Since regime switches from the contagion state back to the calm state do not affect the trees directly, we keep $\lambda^{cont, calm} = 0.49$.

Additionally, Section 4 studies two situations with heterogenous trees. In the first setting ('Robust versus Contagion-Sensitive Assets'), asset B is affected by contagion more heavily than asset A. In the second setting ('Propensity to Trigger Contagion'), asset A is the main source of contagion. Table 1 reports the parameters for these settings as well.

3.3 Asset Pricing Results

The pricing kernel ξ is still of the form (2). In contrast to the one tree case, the logarithm v of the wealth-consumption ratio now depends on two state variables: the state of the economy Z and the consumption share s_A . In models with recursive utility and affine dynamics for the state variables and consumption, the log wealth-consumption ratio is usually approximated by an affine function of the state variables (see Eraker and Shaliastovich (2008)). In an economy with multiple trees, this approach can be problematic, since aggregate consumption is the sum of exponentially affine processes and thus no longer affine. As our numerical results show, the log wealth-consumption ratio is indeed by no means an affine function of the state variable s_A . Therefore, we solve the differential

equation for v numerically. In the calm state, this equation is

$$\begin{aligned}
0 = & e^{-v^{calm}} - \beta + \left(1 - \frac{1}{\psi}\right) \mu_C^{calm} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma_{CC} \\
& + v_s^{calm} \mu_s^{calm} + (1 - \gamma) v_s^{calm} \sigma_{Cs} + \frac{1}{2} (v_{ss}^{calm} + \theta (v_s^{calm})^2) \sigma_{ss} \\
& + \sum_{\substack{k=calm, cont \\ i=A, B}} \frac{1}{\theta} \lambda_i^{calm, k} \left[(1 + s_i L_i^{calm, k})^{1-\gamma} e^{\theta v^k(s_A^{i+}) - \theta v^{calm}(s_A)} - 1 \right],
\end{aligned}$$

where v_s^{calm} and v_{ss}^{calm} denote the first and second order partial derivatives of v^{calm} with respect to s_A . An analogous differential equation holds in the contagion state. The boundary conditions follow from sending the consumption share s_A to 0 or 1. Both limits can be interpreted as one tree economies with an additional (exogenous) possibility of a regime shift.¹² Given the boundaries, we can solve the differential equations for v^{calm} and v^{cont} numerically. Appendix B provides the differential equations, the boundary conditions for $v^{calm}(0)$, $v^{cont}(0)$, $v^{calm}(1)$ and $v^{cont}(1)$ and the proofs of all other results in this section.

3.3.1 Market Prices of Risk

The market prices of risk follow directly from the dynamics of the pricing kernel. We omit the time dependence of all variables in the following for the sake of simplicity.

Proposition 4 *The market prices of diffusive risk in state $j \in \{calm, cont\}$ are*

$$\eta_A^{diff, j} = \left[\gamma s_A + (1 - \theta) v_s^j s_A s_B \right] \sigma_A, \quad \eta_B^{diff, j} = \left[\gamma s_B - (1 - \theta) v_s^j s_A s_B \right] \sigma_B.$$

The market prices of jump risk in the calm state are

$$\eta_i^{calm, k} = \left(1 + s_i L_i^{calm, k}\right)^{-\gamma} e^{(\theta-1) \left(v^k(s_A^{i+}) - v^{calm}(s_A)\right)} - 1, \quad (9)$$

where $i \in \{A, B\}$ and $k \in \{calm, cont\}$. The market prices of risk for pure consumption jumps in the contagion state are given by

$$\eta_i^{cont, cont} = \left(1 + s_i L_i^{cont, cont}\right)^{-\gamma} e^{(\theta-1) \left(v^{cont}(s_A^{i+}) - v^{cont}(s_A)\right)} - 1, \quad (10)$$

and the market price of risk for switches from the contagion state to the calm state is

$$\eta^{cont, calm} = e^{(\theta-1) \left(v^{calm}(s_A) - v^{cont}(s_A)\right)} - 1. \quad (11)$$

¹²Regime changes into the contagion state can be triggered by contagious jumps in the dominating tree, but also by contagious jumps in the tree with zero consumption share.

The jump intensities under the risk-neutral measure equal the physical intensities multiplied by the respective $1 + \eta$.

Different from the one tree case, the market prices of diffusive risk now consist of two terms. The first term depends on the contribution of the tree to aggregate consumption risk, while the second term captures the premium for the new state variable s_A . The consumption share s_A is most volatile for intermediate values around $s_A = 0.5$ (see equation (8)), and the market price of risk involves the square of s_A . Quantitatively, however, the second term is very small, which also implies that the market prices of diffusive risk are almost identical in the calm and contagion state.

The market prices of jump risk depend on the size of consumption jumps and on the impact of jumps on the wealth-consumption ratio. The first factors in Equations (9) and (10) are similar to those in the one tree case. The investor demands a compensation for the immediate impact of jumps on the consumption level. The second factors in Equations (9) and (10) (and the only factor in Equation (11)) reflect the impact of jumps on the wealth-consumption ratio via the consumption share s_A and the state of the economy Z . As in Section 2, the market prices of risk for contagious jumps exceed the sum of the market prices of risk for the components pure consumption jump and regime change. Decomposing the market price of risk for contagious jumps into its components yields

$$1 + \eta_i^{calm,cont} = \underbrace{\left(1 + s_i L_i^{calm,cont}\right)^{-\gamma} e^{(\theta-1)\left(v^{calm}(s_A^{i+}) - v^{calm}(s_A)\right)}}_{1+\eta^{jump}} \underbrace{e^{(\theta-1)\left(v^{cont}(s_A^{i+}) - v^{calm}(s_A^{i+})\right)}}_{1+\eta^{RS}}. \quad (12)$$

Here η^{jump} captures the (hypothetical) market price of risk for pure consumption jumps. η^{RS} gives the (hypothetical) market price of risk for a possible regime switch from calm to contagion. In contrast to the one tree case, η^{jump} now contains an additional factor reflecting the impact of pure consumption jumps in one of the trees on the state variable s_A . Jumps which move the consumption share s_A away from 0.5 and thus lead to a less balanced economy induce a larger market price of risk.¹³

3.3.2 Risk-free Rate

The risk-free rate follows from the (negative) expected growth rate of the pricing kernel.

¹³A more detailed analysis is provided by Branger, Dumitrescu, Ivanova, and Schlag (2012). In particular, note that the concavity of the wealth-consumption ratio depends on the elasticity of intertemporal substitution being larger than one.

Proposition 5 *The risk-free interest rate in the two states equals*

$$\begin{aligned}
r_f^{calm} &= \beta + \frac{1}{\psi} \frac{E_t[dC_t]}{C_t dt} - \Xi^{diff, calm} - \sum_{i=A,B} \Xi_i^{calm, calm} - \sum_{i=A,B} \Xi_i^{calm, cont} \\
r_f^{cont} &= \beta + \frac{1}{\psi} \frac{E_t[dC_t]}{C_t dt} - \Xi^{diff, cont} - \sum_{i=A,B} \Xi_i^{cont, cont} - \Xi^{cont, calm},
\end{aligned}$$

where the precautionary savings terms are given by

$$\begin{aligned}
\Xi^{diff, j} &= \frac{1}{2} \gamma \left(1 + \frac{1}{\psi} \right) \sigma_{CC} + (1 - \theta) v_s^j \sigma_{Cs} + \frac{1}{2} (1 - \theta) (v_s^j)^2 \sigma_{ss} \\
\Xi_i^{j, k} &= \lambda_i^{j, k} \left[\eta_i^{j, k} + \frac{1}{\psi} s_i L_i^{j, k} + \frac{1 - \theta}{\theta} \left((1 + s_i L_i^{j, k})^{1 - \gamma} e^{\theta(v^k(s_A^{i+}) - v^j(s_A))} - 1 \right) \right]
\end{aligned}$$

for $(j, k) \in \{(calm, calm), (calm, cont), (cont, cont)\}$ and

$$\Xi^{cont, calm} = \lambda^{cont, calm} \left[\eta^{cont, calm} + \frac{1 - \theta}{\theta} \left(e^{\theta(v^{calm}(s_A) - v^{cont}(s_A))} - 1 \right) \right].$$

The precautionary savings terms do not only depend on consumption risk and the risk of changes in the state of the economy. As in Cochrane, Longstaff, and Santa-Clara (2008), they are also driven by the additional risk of changes in the consumption share s_A . The upper right-hand graph of Figure 1 depicts the risk-free rate in the calm and contagion state as a function of the consumption share of tree A. With identical trees the risk-free rate is the largest if both trees have the same size ($s_A = 0.5$). Due to diversification, the risk of aggregate consumption is then the smallest. This reduces the demand for precautionary savings and thus also the precautionary savings terms. The effect is most pronounced for the jump-related terms. For $s_A = 0.5$ the size of all consumption jumps is -3%, independent of which tree actually jumps. If s_A approaches 0 or 1, consumption drops by either 0% (jumps of the small tree) or 6% (jumps of the big tree). While the average drop in consumption is still -3%, the risk is much larger, since the jump size of consumption is effectively stochastic. Therefore, a risk-averse agent has an additional precautionary savings motive.

Furthermore, the risk-free rate in the contagion state is 2% smaller than the risk-free rate in the calm state. As in the one tree case, this can be attributed to higher jump intensities, which lower the expected consumption growth and also induce an additional precautionary savings demand. For our parametrization, the first effect is the larger one.

3.3.3 Price-Dividend Ratios

Applying the pricing kernel (2) we can determine the values of the two equity claims with dividend dynamics

$$\begin{aligned} \frac{dD_{i,t}}{D_{i,t-}} = & \mu_i^{calm} dt + \phi \sigma dW_{i,t} + ((1 + L_i^{calm,calm})^\phi - 1) dN_{i,t}^{calm,calm} \\ & + ((1 + L_i^{calm,cont})^\phi - 1) dN_{i,t}^{calm,cont} \end{aligned}$$

in the calm state and

$$\frac{dD_{i,t}}{D_{i,t-}} = \mu_i^{cont} dt + \phi \sigma dW_{i,t} + ((1 + L_i^{cont,cont})^\phi - 1) dN_{i,t}^{cont,cont}$$

in the contagion state.¹⁴ The log price-dividend ratios of asset A in the calm and in the contagion state, w_A^{calm} and w_A^{cont} , depend on the consumption share s_A . They solve a system of two ordinary differential equations which is given in Appendix B. The log price-dividend ratios of asset B , w_B^{calm} and w_B^{cont} , satisfy similar differential equations. The boundary conditions follow by sending s_A to 0 or 1. In the limiting cases, the price-dividend ratios have the same values as in a one tree economy with (additional) exogenous regime shifts where aggregate consumption is either given by C_B (for $s_A \rightarrow 0$) or by C_A (for $s_A \rightarrow 1$).¹⁵

The upper left-hand graph of Figure 1 depicts the price-dividend ratios in the calm state. The wealth-consumption ratio (not shown here) is a concave function of the consumption share. It is largest if the trees are equally big ($s_A = 0.5$) and aggregate consumption is thus the least risky. On the other hand, the price-dividend ratios are monotonous functions of the consumption share. As pointed out by Cochrane, Longstaff, and Santa-Clara (2008), an asset with a small share is more valuable from a diversification perspective. The price-dividend ratios in the contagion state (not reported here) are similar to those in the calm state. They are however smaller than in the calm state by 6-9%, since jump intensities are higher.

¹⁴The equity claim with dividend D_i is a levered claim on tree i . Its cash flow is exposed to the same risk factors as the payment stream C_i . Including idiosyncratic components does not add to our main results.

¹⁵While the price-dividend ratio of asset A remains finite if s_A goes to 1, it can become infinite for the limit $s_A \rightarrow 0$. In this case, we disregard the boundary condition at $s_A = 0$ and solve the initial value problem with an initial condition at $s_A = 1$ instead.

3.3.4 Risk Premia

The risk premium of an asset follows from its exposure to the risk factors and the respective market prices of risk. The market prices of risk are given in Proposition 4. The exposures follow from the dynamics of the asset prices P_i , $i \in \{A, B\}$. In the calm state, the dynamics of P_A are given by

$$\begin{aligned} \frac{dP_{A,t}}{P_{A,t-}} &= \frac{E_t[dP_{A,t}]}{P_{A,t-}} + (w_{A,s}^{calm} s_{A,t} s_{B,t} \sigma_A + \phi \sigma_A) dW_{A,t} - w_{A,s}^{calm} s_{A,t} s_{B,t} \sigma_B dW_{B,t} \\ &\quad + \sum_{\substack{k=calm,cont \\ n=A,B}} \Upsilon_A^{n,calm,k} \left(dN_{n,t}^{calm,k} - \lambda_n^{calm,k} dt \right). \end{aligned}$$

An analogous equation holds for the price dynamics in the contagion state. In the following, we set

$$\begin{aligned} \Upsilon_A^{A,diff,Z} &= (w_{A,s}^Z s_A s_B + \phi) \sigma_A - w_{A,s}^Z s_A s_B \rho \sigma_B \\ \Upsilon_A^{B,diff,Z} &= (w_{A,s}^Z s_A s_B + \phi) \rho \sigma_A - w_{A,s}^Z s_A s_B \sigma_B, \end{aligned}$$

which can be interpreted as the total sensitivities of asset A with respect to the Brownian shocks A and B . The total sensitivities $\Upsilon_B^{A,diff,Z}$ and $\Upsilon_B^{B,diff,Z}$ are defined analogously. $\Upsilon_i^{n,j,k}$ is the sensitivity of asset i with respect to jumps in tree n ($i, n \in \{A, B\}$) that lead to a transition from state j to state k ($(j, k) \in \{(calm, calm), (calm, cont), (cont, cont)\}$). Finally, $\Upsilon_i^{cont,calm}$ denotes the sensitivity of asset i with respect to regime switches from the contagion state back to the calm state.

Proposition 6 *The sensitivities of asset A with respect to jumps in the consumption processes are*

$$\Upsilon_A^{A,j,k} = \left(1 + L_A^{j,k}\right)^\phi e^{w_A^k(s_A^+) - w_A^j(s_A)} - 1, \quad \Upsilon_A^{B,j,k} = e^{w_A^k(s_A^{B+}) - w_A^j(s_A)} - 1,$$

where $(j, k) \in \{(calm, calm), (calm, cont), (cont, cont)\}$. The sensitivity with respect to regime switches from the contagion to the calm state is

$$\Upsilon_A^{cont,calm} = e^{w_A^{calm}(s_A) - w_A^{cont}(s_A)} - 1.$$

The expressions for the sensitivities of asset B follow analogously.

The exposures of the asset prices to the risk factors in the economy are similar to the ones in the model of Cochrane, Longstaff, and Santa-Clara (2008) who provide a detailed

discussion. The exposure of an asset to a risk factor depends on the respective exposures of its dividend and of its price-dividend ratio. For a constant price-dividend ratio, the price sensitivities would be identical to those of the dividend (cash flow effect). As discussed in Section 3.3.3, the price-dividend ratios depend on the state variables Z and s_A , since the stochastic discount factor is driven by these state variables. The price-dividend ratios are thus stochastic, which drives a wedge between price and dividend sensitivities (sdf-effect). For our parametrization, Figure 2 depicts the exposures. The upper graphs show the diffusion exposures. For each asset $i \in \{A, B\}$, the cash flow channel generates an exposure $\phi\sigma_i$ to diffusion risk in its underlying tree and no exposure to the diffusion risk in the other tree (shown by the thin black lines in the figures). Deviations from these exposures are due to the sdf effect. For instance, the price-dividend ratio e^{w_A} is decreasing in the consumption share s_A . A positive shock to D_A increases s_A and thus lowers the price-dividend ratio e^{w_A} . Consequently, the exposure of the price P_A to diffusive shocks in tree A is smaller than $\phi\sigma_i$. The same effect leads to a positive cross exposure of P_A to innovations in tree B . The effect of pure consumption jumps is similar, which can be seen from the graphs in the second row of Figure 2. Finally, the exposure to contagious jumps is depicted in the lower row. The first graph in this row shows the (hypothetical) exposures to the risk of a regime change from calm to contagion, the second graph depicts the exposures to the opposite regime switch. The exposure to a contagious jump (depicted in the third and fourth graph in the lower row) again follows from the multiplicative structure of the exposure to a pure consumption jump and the exposure to a regime change. For our parameterization, a contagious jump induces a downward price adjustment of more than 20% in the corresponding asset.

Given the exposures and the market prices of risk, we can determine the expected returns of the assets. They are given in the following proposition.

Proposition 7 *The expected return on asset A in the calm state is*

$$\frac{E_t[dR_{A,t}^{calm}]}{dt} = r_f^{calm} + \sum_{i=A,B} \eta_i^{diff,calm} \Upsilon_A^{i,diff,calm} - \sum_{\substack{k=calm,cont \\ i=A,B}} \lambda_i^{calm,k} \eta_A^{i,calm,k} \Upsilon_A^{i,calm,k}.$$

In the contagion state, the expected return is

$$\begin{aligned} \frac{E_t[dR_{A,t}^{cont}]}{dt} &= r_f^{cont} + \sum_{i=A,B} \eta_i^{diff,cont} \Upsilon_A^{i,diff,cont} \\ &\quad - \sum_{i=A,B} \lambda_i^{cont,cont} \eta_A^{i,cont,cont} \Upsilon_A^{i,cont,cont} - \lambda^{cont,calm} \eta^{cont,calm} \Upsilon_A^{cont,calm}. \end{aligned}$$

The expressions for the expected return on asset B follow analogously.

The lower graphs of Figure 1 depict the expected excess returns. First, the assets earn a premium on their exposure to aggregate consumption risk. Since these exposures increase in the size of the underlying tree, the equity premium on an asset increases in the consumption share of its underlying tree. Second, the assets earn premia on their exposures to regime switches, i.e. on their exposure to the state variable Z . Analogously to the one tree economy, these risk premia are positive both for switches into the contagion state and for switches into the calm state. Therefore, assets on small trees also earn non-zero risk premia: despite their vanishing contribution to aggregate consumption risk, they carry risk premia of above 2% in both states. Finally, the lower graphs of Figure 1 show that risk premia are larger in the riskier contagion state than in the safer calm state.

Figure 3 decomposes the equity risk premium of asset A in the calm state into its various components. The left-hand graph shows that the dominant part of the equity premium is a compensation for the risk of contagious jumps. If the consumption share s_A approaches 1, the risk premium for contagious jumps in tree A exceeds 5%. The risk premium for contagious jumps in tree B , which do not have any direct impact on dividends of asset A , is around 1%. For most values of s_A , it even exceeds the premium for pure consumption jumps. Furthermore, the premium for diffusion risk is below 0.5%, so that the equity risk premium is mainly driven by jumps and priced state variables.

The right-hand graph of Figure 3 further decomposes the risk premium for contagious jumps into the premium for pure consumption jumps, the premium for regime changes, and an additional interaction term which captures the superadditivity of the premium. As in the one tree economy, the total premium for contagious jumps is superadditive, since both the market prices of risk and the exposures are multiplicatively connected:

$$\begin{aligned}
1 + \eta_A^{calm,cont} &= \underbrace{\left(1 + s_A L_A^{calm,cont}\right)^{-\gamma} e^{(\theta-1)\left(v^{calm}(s_A^{A+}) - v^{calm}(s_A)\right)}}_{1+\eta^{jump}} \underbrace{e^{(\theta-1)\left(v^{cont}(s_A^{A+}) - v^{calm}(s_A^{A+})\right)}}_{1+\eta^{RS}} \\
1 + \Upsilon_A^{A,calm,cont} &= \underbrace{\left(1 + L_A^{calm,cont}\right)^\phi e^{w_A^{calm}(s_A^{A+}) - w_A^{calm}(s_A)}}_{1+\Upsilon^{jump}} \underbrace{e^{w_A^{cont}(s_A^{A+}) - w_A^{calm}(s_A^{A+})}}_{1+\Upsilon^{RS}}
\end{aligned}$$

Stated differently, the premium for one large (contagious) jump is larger than the premia for its components since the investor is more averse to the risk of one large jump than to the risk of several small jumps, even if the overall loss were the same. The dashed black line depicts the difference between the premium for the contagious jumps and the sum

of the premia for pure consumption jumps and regime changes. It can reach almost 3% and is the largest component of the premium for contagious jumps. It is monotonically increasing in the consumption share s_A since the market price of risk for jumps in tree A is increasing in s_A as well.

4 Cross-sectional Pricing Effects of Contagion

This section studies the effects of contagion on the cross-section of asset prices when trees are heterogenous. Section 4.1 starts with the case where the two trees differ in the contagion state in the sense that one asset is more severely affected by contagion than the other one. In Section 4.2, we compare an asset that almost never induces contagion with an asset that induces contagion at almost every downward jump, i.e. the assets mainly differ in the calm state.

4.1 Robust versus Contagion-sensitive Assets

The column labeled 'Robust versus Contagion-sensitive Assets' of Table 1 reports the parameters for the first case. In the calm state, the jump intensity for each asset is 0.25 as in Section 3. In the contagion state, the jump intensity of asset A increases to 0.625, whereas the jump intensity of asset B equals 0.875. The overall intensity for a downward consumption jump in the contagion state is thus still 1.5. Intuitively, we can think of asset A as a 'robust' asset, while asset B is more severely affected by contagion and is called 'contagion-sensitive' in the following.

Figures 4 and 5 depict the corresponding results which are remarkably different from the case with identical assets. The price-dividend ratios shown in the upper left-hand graph of Figure 4 are asymmetric. The price of a small robust asset A is much larger than the price of an equally small contagion-sensitive asset B . As in the case of identical trees, small assets are more attractive than big assets, since they are less exposed to consumption risk. This drives the price-dividend ratio of small (big) assets up (down). Furthermore, the investor now prefers the safe haven A over the contagion-sensitive asset B , which induces larger price-dividend ratios for the robust asset than for the contagion-sensitive asset. If both trees have the same size, the robust asset is still the more valuable one. Price-dividend ratios are equal for $s_A \approx 0.65$ (instead of 0.5).

The risk-free rate depicted in the upper right-hand graph of Figure 4 is smaller in the

contagion state than in the calm state. While it is nearly symmetric in the calm state, it is asymmetric in the contagion state. If the contagion-sensitive asset B dominates ($s_A \rightarrow 0$), the economy is riskier than if the robust asset A dominates ($s_A \rightarrow 1$). Consequently, the risk-free rate is smaller for $s_A \rightarrow 0$ than for $s_A \rightarrow 1$. This also implies that the drop of the risk-free rate upon a contagious jump is larger for $s_A \rightarrow 0$ (around 3.5%) than for $s_A \rightarrow 1$ (around 2%).

The expected excess returns of the assets also become asymmetric. In line with intuition, the contagion-sensitive asset B earns a larger risk premium than an equally big robust asset A . Analogously to Figure 3, the equity premium of both assets is decomposed in Figure 5. The graphs reveal that the asymmetry is mainly driven by the risk premia for contagious jumps, which are larger for the contagion-sensitive asset than for the robust asset. This is because the exposures of the assets to regime switches and contagious jumps (not reported here) are asymmetric as well. If the economy switches to the contagion state, the price of the contagion-sensitive asset B drops by around 10% while the price of the robust asset A drops by only 5%. As a result, the risk premia for contagious jumps become asymmetric. For instance, the risk premium of a big robust asset A ($s_A \approx 1$) for contagious jumps in tree A (black solid line in the upper left-hand graph) is around 4%, while the risk premium of a big contagion-sensitive asset B ($s_A \approx 0$) for contagious jumps in tree B is around 5%. The asymmetric pattern of the risk premia for contagious jumps is further amplified by the extra interaction term (see Section 3.3.4). As the graphs on the right-hand side of Figure 5 show, this term is more pronounced for the contagion-sensitive asset B .

4.2 Propensity to Trigger Contagion

We next study the case where the trees are identical in the contagion state, but differ in the calm state. In particular, switches into the contagion state now mainly occur together with jumps in tree A , while tree B hardly exhibits contagious jumps. The column labeled 'Propensity to Trigger Contagion' of Table 1 reports the corresponding parameters. In the calm state, the total intensities for jumps in asset A or B are still 0.25, and the intensity for a regime switch from calm to contagion is still 0.27. But whereas both assets were equally likely to induce contagion in the previous cases, we now assume that all jumps of asset A are contagious jumps, i.e. we set $\lambda_A^{calm, calm} = 0$ and $\lambda_A^{calm, cont} = 0.25$. To keep the total jump intensities the same as in the previous sections, we further set $\lambda_B^{calm, calm} = 0.23$ and $\lambda_B^{calm, cont} = 0.02$. We refer to asset A as the 'contagion-triggering' asset and to asset

B as the 'non-toxic' asset for simplicity.

Figures 6 and 7 show that the differences to the case with identical trees are even larger than in the previous subsection. Differences between the assets with respect to their propensity to trigger contagion are thus more significant than differences with respect to the consequences of contagion. The price-dividend ratios in the calm state as depicted in Figure 6 show that a small non-toxic asset (asset B for s_A close to 1) is much more valuable than a small contagion-triggering asset (asset A for s_A close to 0). Its price-dividend ratio is around 11% higher. For the non-toxic asset B , the two channels 'regime switch' and 'jumps in consumption' are almost decoupled as in the pure regime switching economy. Since the representative agent tries to hedge against the risk of contagious jumps, he would like to increase his position in the 'safe haven', asset B . Due to the limited supply, this hedging demand drives up the equilibrium price. On the contrary, asset A is exposed to contagious jumps. Since these jumps are more likely than in all previous settings, the price-dividend ratio of asset A is lower than before.

The results for the risk-free rate are in line with intuition: If the contagion-triggering asset is big (s_A close to 1), the investor has a large demand for precautionary savings in the calm state, which drives the equilibrium risk-free rate down. In the contagion state, we hardly observe any effect on the risk-free rate. Upon a transition to the contagion state the risk-free rate thus decreases the most if the non-toxic asset B dominates the economy (around 3.5% reduction for s_A close to 0 as opposed to around 2.5% reduction for s_A close to 1).

The lower graphs of Figure 6 depict the risk premia of both assets. Apparently, the propensity to induce contagion has a significant impact. For the limiting cases where the assets become relatively big (asset A for s_A close to 1 and asset B for s_A close to 0), we find a difference in the risk premia of about 3%. The decomposition of the risk premia in Figure 7 shows that this asymmetry is again driven by the superadditivity of the risk premium for contagious jumps. In the previous sections, we have already documented that the premia for contagious jumps account for the largest part of the risk premia. The contagion-triggering asset A has a large exposure to these contagious jumps and thus carries a high risk premium. If the contagion-triggering asset is dominating the economy (s_A close to 1), the investor's reduced taste for asset A drives its equity premium up to 9%. On the other hand, the non-toxic asset B has a small exposure to these jumps, and thus its risk premium is smaller, too.

5 Conclusion

In this paper, we solve for the equilibrium in a Lucas tree model where the representative agent has recursive preferences and consumption is exposed to contagion risk. We propose a new economic channel by allowing for endowment shocks that simultaneously trigger a regime shift to a bad economic state. These contagious jumps do not only cause instantaneous, persistent losses in consumption, but also increase the probability of subsequent losses across the whole economy. We document that this new channel has far-reaching economic consequences. In the calm state, the risk premium for contagious shocks is superadditive, i.e. it is 2.5% larger than the sum of the risk premia for pure endowment shocks and for regime switches. Second, the possibility of contagious jumps increases the agent's precautionary savings demand, which reduces the equilibrium risk-free rate by around 0.5%.

We also study an economy with two Lucas trees. We derive semiclosed-form solutions for the equilibrium wealth-consumption ratio and the price-dividend ratios (and thus all other key asset pricing figures) and analyze the interplay between the agent's diversification motive and contagion risk. We find that the possibility of contagious shocks has superadditive pricing effects which are particularly pronounced for assets written on big trees. Besides, assets written on a tree with zero consumption share also carry positive risk premia, since they are exposed to contagious shocks in the other tree. Finally, we qualitatively analyze the cross-sectional effects of contagion risk. We find that heterogeneity among assets with respect to contagion risk further amplifies the documented nonlinearities. In particular, big assets with a high potential for contagious shocks earn significantly higher risk premia.

A Equilibrium in a One Tree Economy

A.1 Wealth-Consumption Ratio

Let $Z_t \in \{calm, cont\}$ denote the state of the economy at time t . Then the representative investor has two value functions, one for each state:

$$J_t^{Z_t} = E_t \left[\int_t^\infty f(C_s, J_s^{Z_s}) ds \right].$$

For the sake of readability, we will, however, suppress the dependence of the value function, the pricing kernel, the aggregate consumption and other variables on the state $Z_t \in \{calm, cont\}$ in the following. As usual, the aggregator f is defined as

$$f(C, J) = \frac{\beta C^{1-\frac{1}{\psi}}}{\left(1 - \frac{1}{\psi}\right) [(1-\gamma)J]^{\frac{1}{\theta}-1}} - \beta \theta J.$$

β denotes the subjective time discount rate, ψ the elasticity of intertemporal substitution, and γ the relative risk aversion. We also define $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. A Feynman-Kac-like computation then gives

$$0 = f(C_t, J_t) + DJ_t \quad (13)$$

i.e. one Bellman equation for each state.

The dynamics of consumption in the calm state are

$$\frac{dC_t}{C_{t-}} = \mu^{calm} dt + \sigma dW_t + L^{calm, calm} dN_t^{calm, calm} + L^{calm, cont} dN_t^{calm, cont}, \quad (14)$$

its dynamics in the contagion state are

$$\frac{dC_t}{C_{t-}} = \mu^{cont} dt + \sigma dW_t + L^{cont, cont} dN_t^{cont, cont}. \quad (15)$$

We apply the following conjecture for the functional form of the value function J :

$$J = \frac{C^{1-\gamma}}{1-\gamma} \beta^\theta e^{\theta v^Z} \quad (16)$$

where v^Z can take two values, one in each state. Campbell, Chacko, Rodriguez, and Viceira (2004) and Benzoni, Collin-Dufresne, and Goldstein (2011) show that, with this conjecture, v^Z is the log wealth-consumption ratio. Plugging the guess (16) for J into the aggregator function results in

$$f(C, J) = \theta J \left(e^{-v^Z} - \beta \right). \quad (17)$$

The infinitesimal generator DJ follows via Ito's Lemma:

$$\begin{aligned} DJ &= \left(1 - \frac{1}{\psi}\right) \theta J^{calm} \mu^{calm} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \theta J^{calm} \sigma^2 \\ &\quad + \lambda^{calm, calm} J^{calm} \left[(1 + L^{calm, calm})^{1-\gamma} e^{\theta v^{calm} - \theta v^{calm}} - 1 \right] \\ &\quad + \lambda^{calm, cont} J^{calm} \left[(1 + L^{calm, cont})^{1-\gamma} e^{\theta v^{cont} - \theta v^{calm}} - 1 \right] \end{aligned}$$

in the calm state and

$$DJ = \left(1 - \frac{1}{\psi}\right) \theta J^{cont} \mu^{cont} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \theta J^{cont} \sigma^2 \\ + \lambda^{cont,cont} J^{cont} \left[(1 + L^{cont,cont})^{1-\gamma} e^{\theta v^{cont} - \theta v^{cont}} - 1 \right] + \lambda^{cont,calm} J^{cont} \left[e^{\theta v^{calm} - \theta v^{cont}} - 1 \right]$$

in the contagion state. Plugging these expressions into (13), dividing by θJ^{calm} and θJ^{cont} respectively, and rearranging some terms gives the following two algebraic equations for the two unknowns v^{calm} and v^{cont} :

$$0 = e^{-v^{calm}} - \beta + \left(1 - \frac{1}{\psi}\right) \mu^{calm} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma^2 \\ + \frac{1}{\theta} \lambda^{calm,calm} \left[(1 + L^{calm,calm})^{1-\gamma} - 1 \right] + \frac{1}{\theta} \lambda^{calm,cont} \left[(1 + L^{calm,cont})^{1-\gamma} e^{\theta(v^{cont} - v^{calm})} - 1 \right] \\ 0 = e^{-v^{cont}} - \beta + \left(1 - \frac{1}{\psi}\right) \mu^{cont} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma^2 \\ + \frac{1}{\theta} \lambda^{cont,cont} \left[(1 + L^{cont,cont})^{1-\gamma} - 1 \right] + \frac{1}{\theta} \lambda^{cont,calm} \left[e^{\theta(v^{calm} - v^{cont})} - 1 \right].$$

A.2 Pricing Kernel

As Duffie and Epstein (1992a) and Benzoni, Collin-Dufresne, and Goldstein (2011) show, the pricing kernel is given by

$$\xi_t = \beta^\theta C_t^{-\gamma} e^{-\beta\theta t + (\theta-1) \left(\int_0^t e^{-v^{Z_u}} du + v_t^{Z_t} \right)}. \quad (18)$$

The dynamics of the pricing kernel can be computed via Ito's Lemma. The partial derivatives of ξ with respect to C and v follow from (18). The dynamics of C are given in (14) and (15). The dynamics of the pricing kernel are

$$\frac{d\xi_t}{\xi_{t-}} = \left[-\beta\theta + (\theta-1)e^{-v^{calm}} \right] dt - \gamma \mu^{calm} dt + \frac{1}{2} \gamma (1 + \gamma) \sigma^2 dt \\ - \eta^{diff,calm} dW_t + dN_t^{calm,calm} \eta^{calm,calm} + dN_t^{calm,cont} \eta^{calm,cont}$$

in the calm state and

$$\frac{d\xi_t}{\xi_{t-}} = \left[-\beta\theta + (\theta-1)e^{-v^{cont}} \right] dt - \gamma \mu^{cont} dt + \frac{1}{2} \gamma (1 + \gamma) \sigma^2 dt \\ - \eta^{diff,cont} dW_t + dN_t^{cont,cont} \eta^{cont,cont} + dN_t^{cont,calm} \eta^{cont,calm}$$

in the contagion state. For later use, we abbreviate the drift of the pricing kernel by

$$\mu_\xi^Z = -\beta\theta + (\theta-1)e^{-v^Z} - \gamma \mu^Z + \frac{1}{2} \gamma (1 + \gamma) \sigma^2.$$

The market price of diffusion risk is $\eta^{diff,Z} = \gamma\sigma$. The market prices of jump risk are

$$\eta^{calm,calm} = (1 + L^{calm,calm})^{-\gamma} - 1 \\ \eta^{calm,cont} = (1 + L^{calm,cont})^{-\gamma} e^{(\theta-1)(v^{cont} - v^{calm})} - 1 \\ \eta^{cont,cont} = (1 + L^{cont,cont})^{-\gamma} - 1 \\ \eta^{cont,calm} = e^{(\theta-1)(v^{calm} - v^{cont})} - 1.$$

The risk-neutral jump intensities are related to the physical intensities via the market prices of risk: $\lambda^{\mathbb{Q},j,k} = \lambda^{\mathbb{P},j,k}(1 + \eta^{j,k})$. The risk-free rate is equal to the negative expected growth rate of the pricing kernel ξ_t :

$$\begin{aligned}
r_f^{calm} &= \beta + \frac{1}{\psi} \left(\mu^{calm} + L^{calm,calm} \lambda^{calm,calm} + L^{calm,cont} \lambda^{calm,cont} \right) - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi} \right) \sigma^2 \\
&\quad - \lambda^{calm,calm} \left[\eta^{calm,calm} + \frac{1}{\psi} L^{calm,calm} + \frac{1-\theta}{\theta} \left((1 + L^{calm,calm})^{1-\gamma} - 1 \right) \right] \\
&\quad - \lambda^{calm,cont} \left[\eta^{calm,cont} + \frac{1}{\psi} L^{calm,cont} + \frac{1-\theta}{\theta} \left((1 + L^{calm,cont})^{1-\gamma} e^{\theta(v^{cont}-v^{calm})} - 1 \right) \right] \\
r_f^{cont} &= \beta + \frac{1}{\psi} \left(\mu^{cont} + L^{cont,cont} \lambda^{cont,cont} \right) - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi} \right) \sigma^2 \\
&\quad - \lambda^{cont,cont} \left[\eta^{cont,cont} + \frac{1}{\psi} L^{cont,cont} + \frac{1-\theta}{\theta} \left((1 + L^{cont,cont})^{1-\gamma} - 1 \right) \right] \\
&\quad - \lambda^{cont,calm} \left[\eta^{cont,calm} + \frac{1-\theta}{\theta} \left(e^{\theta(v^{calm}-v^{cont})} - 1 \right) \right].
\end{aligned}$$

A.3 Pricing the Dividend Claim

For the price-dividend ratio of the claim to dividends, we apply the Feynman-Kac formula. Let w denote the log price-dividend ratio. Defining $g(\xi, D, w) = \xi D e^w$ results in

$$g(\xi_t, D_t, w_t) = \xi_t D_t e^{w_t} = \mathbb{E}_t \left[\int_t^\infty \xi_\tau D_\tau d\tau \right] = \mathbb{E}_t \left[\int_t^\infty \frac{g(\xi_\tau, D_\tau, w_\tau)}{e^{w_\tau}} d\tau \right].$$

The Feynman-Kac formula yields

$$\mathcal{D}g(\xi, D, w) + \frac{g(\xi, D, w)}{e^w} = 0 \quad \Longleftrightarrow \quad \frac{\mathcal{D}g(\xi, D, w)}{g(\xi, D, w)} + e^{-w} = 0. \quad (19)$$

The dividend dynamics in our model are

$$\frac{dD_t}{D_{t-}} = \mu^{calm} dt + \phi \sigma dW_t + [(1 + L^{calm,calm})^\phi - 1] dN_t^{calm,calm} + [(1 + L^{calm,cont})^\phi - 1] dN_t^{calm,cont}$$

in the calm state and

$$\frac{dD_t}{D_{t-}} = \mu^{cont} dt + \phi \sigma dW_t + [(1 + L^{cont,cont})^\phi - 1] dN_t^{cont,cont}$$

in the contagion state. Ito's Lemma gives

$$\frac{\mathcal{D}g}{g} = \mu_\xi + \mu_D + \mu_w + \frac{1}{2} \frac{d\langle w^c \rangle}{dt} + \frac{d\langle \xi^c, D^c \rangle}{\xi D dt} + \frac{d\langle w^c, D^c \rangle}{D dt} + \frac{d\langle w^c, \xi^c \rangle}{\xi dt} + \text{Jump Terms}.$$

The log price-dividend ratio w can take the two values w^{calm} and w^{cont} only, i.e. μ_w is 0. From (19), we get the following two algebraic equations:

$$\begin{aligned}
0 &= e^{-w^{calm}} + \mu_\xi^{calm} + \mu^{calm} - \eta^{diff,calm} \phi \sigma \\
&\quad + \lambda^{calm,calm} [(1 + \eta^{calm,calm})(1 + L^{calm,calm})^\phi - 1] \\
&\quad + \lambda^{calm,cont} [(1 + \eta^{calm,cont})(1 + L^{calm,cont})^\phi e^{w^{cont}-w^{calm}} - 1] \\
0 &= e^{-w^{cont}} + \mu_\xi^{cont} + \mu^{cont} - \eta^{diff,cont} \phi \sigma_A \\
&\quad + \lambda^{cont,cont} [(1 + \eta^{cont,cont})(1 + L^{cont,cont})^\phi - 1] \\
&\quad + \lambda^{cont,calm} [(1 + \eta^{cont,calm})e^{w^{calm}-w^{cont}} - 1].
\end{aligned}$$

A.4 Exposures and Moments

Conditional on the state, the dynamics of the asset price $P = e^w D$ follow via Ito's Lemma. In the calm state, we have

$$\begin{aligned}
\frac{dP_t}{P_{t-}} &= \mu^{calm} dt + \phi \sigma dW_t + [(1 + L^{calm,calm})^\phi - 1] dN_t^{calm,calm} \\
&\quad + [(1 + L^{calm,cont})^\phi e^{w^{cont}-w^{calm}} - 1] dN_t^{calm,cont}.
\end{aligned}$$

In the contagion state, the dynamics are

$$\begin{aligned}
\frac{dP_t}{P_{t-}} &= \mu^{cont} dt + \phi \sigma dW_t + [(1 + L^{cont,cont})^\phi - 1] dN_t^{cont,cont} \\
&\quad + [e^{w^{calm}-w^{cont}} - 1] dN_t^{cont,calm}.
\end{aligned}$$

In the following, we abbreviate the sensitivities of the asset price to the different risk factors as

$$\begin{aligned}
\Upsilon^{diff} &= \phi \sigma \\
\Upsilon^{calm,calm} &= (1 + L^{calm,calm})^\phi - 1 \\
\Upsilon^{calm,cont} &= (1 + L^{calm,cont})^\phi e^{w^{cont}-w^{calm}} - 1 \\
\Upsilon^{cont,cont} &= (1 + L^{cont,cont})^\phi - 1 \\
\Upsilon^{cont,calm} &= e^{w^{calm}-w^{cont}} - 1
\end{aligned}$$

The expected excess return on the dividend claim, i.e. the equity risk premium, follows from these exposures and the respective market prices of risk. In the calm state, it is equal to

$$\begin{aligned}
\gamma \phi \sigma^2 &+ \lambda^{calm,calm} [1 + L^{calm,calm})^\phi - 1] [1 - (1 + L^{calm,calm})^{-\gamma}] \\
&+ \lambda^{calm,cont} [(1 + L^{calm,cont})^\phi e^{w^{cont}-w^{calm}} - 1] [1 - (1 + L^{calm,cont})^{-\gamma} e^{(\theta-1)(v^{cont}-v^{calm})}].
\end{aligned}$$

The equity risk premium in the contagion state is

$$\begin{aligned}
\gamma \phi \sigma^2 &+ \lambda^{cont,cont} [1 + L^{cont,cont})^\phi - 1] [1 - (1 + L^{cont,cont})^{-\gamma}] \\
&+ \lambda^{cont,calm} [e^{w^{calm}-w^{cont}} - 1] [1 - e^{(\theta-1)(v^{calm}-v^{cont})}]
\end{aligned}$$

Conditional on being in the calm state, the local variance equals

$$\sigma^2 \phi^2 + \lambda^{calm, calm} ((1 + L^{calm, calm})^\phi - 1)^2 + \lambda^{calm, cont} (e^{w^{cont} - w^{calm}} (1 + L^{calm, cont})^\phi - 1)^2.$$

Conditional on being in the contagion state, we obtain

$$\sigma^2 \phi^2 + \lambda^{cont, cont} ((1 + L^{cont, cont})^\phi - 1)^2 + \lambda^{cont, calm} (e^{w^{calm} - w^{cont}} - 1)^2.$$

The unconditional variance is given by

$$\begin{aligned} & \sigma^2 \phi^2 + p^{calm} \lambda^{calm, calm} ((1 + L^{calm, calm})^\phi - 1)^2 \\ & + p^{calm} \lambda^{calm, cont} (e^{w^{cont} - w^{calm}} (1 + L^{calm, cont})^\phi - 1)^2 \\ & + p^{cont} \lambda^{cont, cont} ((1 + L^{cont, cont})^\phi - 1)^2 \\ & + p^{cont} \lambda^{cont, calm} (e^{w^{calm} - w^{cont}} - 1)^2. \end{aligned}$$

B Equilibrium in a Two Tree Economy

B.1 Wealth-Consumption Ratio

In order to express the equilibrium prices in the two tree economy, we introduce the variable 'consumption share of tree A':

$$s_{A,t} = \frac{C_{A,t}}{C_{A,t} + C_{B,t}}$$

and define $s_B = 1 - s_A$. The dynamics of s_A in the calm and contagion state are given by

$$\begin{aligned} \frac{ds_{A,t}}{s_{A,t-} s_{B,t-}} &= \left[\mu_A^{calm} - \mu_B^{calm} - s_{A,t} \sigma_A^2 + s_{B,t} \sigma_B^2 + (s_{A,t} - s_{B,t}) \rho \sigma_A \sigma_B \right] dt + \sigma_A dW_{A,t} - \sigma_B dW_{B,t} \\ &+ \frac{L_A^{calm, calm}}{1 + L_A^{calm, calm} s_{A,t-}} dN_{A,t}^{calm, calm} + \frac{L_A^{calm, cont}}{1 + L_A^{calm, cont} s_{A,t-}} dN_{A,t}^{calm, cont} \\ &- \frac{L_B^{calm, calm}}{1 + L_B^{calm, calm} s_{B,t-}} dN_{B,t}^{calm, calm} - \frac{L_B^{calm, cont}}{1 + L_B^{calm, cont} s_{B,t-}} dN_{B,t}^{calm, cont} \\ \frac{ds_{A,t}}{s_{A,t-} s_{B,t-}} &= \left[\mu_A^{cont} - \mu_B^{cont} - s_{A,t} \sigma_A^2 + s_{B,t} \sigma_B^2 + (s_{A,t} - s_{B,t}) \rho \sigma_A \sigma_B \right] dt + \sigma_A dW_{A,t} - \sigma_B dW_{B,t} \\ &+ \frac{L_A^{cont, cont}}{1 + L_A^{cont, cont} s_{A,t-}} dN_{A,t}^{cont, cont} - \frac{L_B^{cont, cont}}{1 + L_B^{cont, cont} s_{B,t-}} dN_{B,t}^{cont, cont}. \end{aligned}$$

The dynamics of consumption in the calm state are

$$\begin{aligned} \frac{dC_t}{C_{t-}} &= s_{A,t-} \frac{dC_{A,t}}{C_{A,t-}} + s_{B,t-} \frac{dC_{B,t}}{C_{B,t-}} \\ &= [s_{A,t} \mu_A^{calm} + s_{B,t} \mu_B^{calm}] dt + s_{A,t} \sigma_A dW_{A,t} + s_{B,t} \sigma_B dW_{B,t} \\ &+ s_{A,t-} \left[L_A^{calm, calm} dN_{A,t}^{calm, calm} + L_A^{calm, cont} dN_{A,t}^{calm, cont} \right] \\ &+ s_{B,t-} \left[L_B^{calm, calm} dN_{B,t}^{calm, calm} + L_B^{calm, cont} dN_{B,t}^{calm, cont} \right], \end{aligned} \tag{20}$$

its dynamics in the contagion state are

$$\begin{aligned} \frac{dC_t}{C_{t-}} &= [s_{A,t}\mu_A^{cont} + s_{B,t}\mu_B^{cont}] dt + s_{A,t}\sigma_A dW_{A,t} + s_{B,t}\sigma_B dW_{B,t} \\ &\quad + s_{A,t-}L_A^{cont,cont} dN_{A,t}^{cont,cont} + s_{B,t-}L_B^{cont,cont} dN_{B,t}^{cont,cont}. \end{aligned} \quad (21)$$

In order to abbreviate all following equations, we additionally define the consumption share of asset A after a jump in tree A or B , respectively, as

$$s_{A,t}^{A+} = s_{A,t-} \frac{1 + L_A^{j,k}}{1 + L_A^{j,k} s_{A,t-}}, \quad s_{A,t}^{B+} = s_{A,t-} \frac{1}{1 + L_B^{j,k} (1 - s_{A,t-})},$$

where $(j, k) \in \{(calm, calm), (calm, cont), (cont, cont)\}$. Moreover, we denote the drift rates of consumption C and of the consumption share s_A in state Z by

$$\begin{aligned} \mu_C^Z &= s_A \mu_A^Z + s_B \mu_B^Z \\ \mu_s^Z &= s_A s_B \left(\mu_A^Z - \mu_B^Z - s_A \sigma_A^2 + s_B \sigma_B^2 + (s_A - s_B) \rho \sigma_A \sigma_B \right). \end{aligned}$$

We also abbreviate the quadratic variation terms

$$\begin{aligned} \sigma_{CC} dt &= \frac{d\langle C^c \rangle_t}{C_t^2} = (s_A^2 \sigma_A^2 + s_B^2 \sigma_B^2 + 2s_A s_B \rho \sigma_A \sigma_B) dt \\ \sigma_{Cs} dt &= \frac{d\langle C^c, s_A^c \rangle_t}{C_t} = s_A s_B \left[s_A \sigma_A^2 - s_B \sigma_B^2 - (s_A - s_B) \rho \sigma_A \sigma_B \right] dt \\ \sigma_{ss} dt &= d\langle s_A^c \rangle_t = s_A^2 s_B^2 \left[\sigma_A^2 + \sigma_B^2 - 2\rho \sigma_A \sigma_B \right] dt \end{aligned}$$

where the upper index c refers to the continuous part of the respective process. Similar to the one tree case, we apply the following conjecture for J :

$$J = \frac{C^{1-\gamma}}{1-\gamma} \beta^\theta e^{\theta v^Z} \quad (22)$$

where now v^Z is a twice differentiable function of the consumption share s_A .

In models with recursive utility and affine dynamics for the state variables and consumption, the log wealth-consumption ratio is usually approximated by an affine function of the state variables (see Eraker and Shaliastovich (2008)). In an economy with multiple trees, this approach can be problematic, since aggregate consumption is the sum of exponentially affine processes and thus no longer affine. As our numerical results show, the log wealth-consumption ratio v is by no means an affine function of the state variable s_A . Therefore, we solve the differential equation for v numerically.

Plugging the guess (22) for J into the aggregator function results in

$$f(C, J) = \theta J \left(e^{-v^Z} - \beta \right). \quad (23)$$

The infinitesimal generator DJ follows via Ito's Lemma:

$$\begin{aligned}
DJ = & \left(1 - \frac{1}{\psi}\right) \theta J^{calm} \mu_C^{calm} + v_s^{calm} \theta J^{calm} \mu_s^{calm} \\
& - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \theta J^{calm} \sigma_{CC} + \frac{1}{2} \theta J^{calm} (v_{ss}^{calm} + \theta (v_s^{calm})^2) \sigma_{ss} + (1 - \gamma) \theta J^{calm} v_s^{calm} \sigma_{Cs} \\
& + \lambda_A^{calm, calm} J^{calm} \left[(1 + s_A L_A^{calm, calm})^{1-\gamma} e^{\theta v^{calm}(s_A^{A+}) - \theta v^{calm}(s_A)} - 1 \right] \\
& + \lambda_A^{calm, cont} J^{calm} \left[(1 + s_A L_A^{calm, cont})^{1-\gamma} e^{\theta v^{cont}(s_A^{A+}) - \theta v^{calm}(s_A)} - 1 \right] \\
& + \lambda_B^{calm, calm} J^{calm} \left[(1 + s_B L_B^{calm, calm})^{1-\gamma} e^{\theta v^{calm}(s_A^{B+}) - \theta v^{calm}(s_A)} - 1 \right] \\
& + \lambda_B^{calm, cont} J^{calm} \left[(1 + s_B L_B^{calm, cont})^{1-\gamma} e^{\theta v^{cont}(s_A^{B+}) - \theta v^{calm}(s_A)} - 1 \right]
\end{aligned}$$

in the calm state and

$$\begin{aligned}
DJ = & \left(1 - \frac{1}{\psi}\right) \theta J^{cont} \mu_C^{cont} + v_s^{cont} \theta J^{cont} \mu_s^{cont} \\
& - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \theta J^{cont} \sigma_{CC} + \frac{1}{2} \theta J^{cont} (v_{ss}^{cont} + \theta (v_s^{cont})^2) \sigma_{ss} + (1 - \gamma) \theta J^{cont} v_s^{cont} \sigma_{Cs} \\
& + \lambda_A^{cont, cont} J^{cont} \left[(1 + s_A L_A^{cont, cont})^{1-\gamma} e^{\theta v^{cont}(s_A^{A+}) - \theta v^{cont}(s_A)} - 1 \right] \\
& + \lambda_B^{cont, cont} J^{cont} \left[(1 + s_B L_B^{cont, cont})^{1-\gamma} e^{\theta v^{cont}(s_A^{B+}) - \theta v^{cont}(s_A)} - 1 \right] \\
& + \lambda^{cont, calm} J^{cont} \left[e^{\theta v^{calm}(s_A) - \theta v^{cont}(s_A)} - 1 \right]
\end{aligned}$$

in the contagion state. The subscripts s and ss denote the first and second derivatives with respect to s_A . Plugging these expressions into the Bellman equation

$$0 = f(C_t, J_t) + DJ_t,$$

dividing by θJ^{calm} and θJ^{cont} respectively, and rearranging some terms gives the following two ODEs for the two unknown functions v^{calm} and v^{cont} :

$$\begin{aligned}
0 = & e^{-v^{calm}} - \beta + \left(1 - \frac{1}{\psi}\right) \mu_C^{calm} + v_s^{calm} \mu_s^{calm} \\
& - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma_{CC} + (1 - \gamma) v_s^{calm} \sigma_{Cs} + \frac{1}{2} (v_{ss}^{calm} + \theta (v_s^{calm})^2) \sigma_{ss} \\
& + \frac{1}{\theta} \lambda_A^{calm, calm} \left[(1 + s_A L_A^{calm, calm})^{1-\gamma} e^{\theta v^{calm}(s_A^{A+}) - \theta v^{calm}(s_A)} - 1 \right] \\
& + \frac{1}{\theta} \lambda_A^{calm, cont} \left[(1 + s_A L_A^{calm, cont})^{1-\gamma} e^{\theta v^{cont}(s_A^{A+}) - \theta v^{calm}(s_A)} - 1 \right] \\
& + \frac{1}{\theta} \lambda_B^{calm, calm} \left[(1 + s_B L_B^{calm, calm})^{1-\gamma} e^{\theta v^{calm}(s_A^{B+}) - \theta v^{calm}(s_A)} - 1 \right] \\
& + \frac{1}{\theta} \lambda_B^{calm, cont} \left[(1 + s_B L_B^{calm, cont})^{1-\gamma} e^{\theta v^{cont}(s_A^{B+}) - \theta v^{calm}(s_A)} - 1 \right]
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
0 = & e^{-v^{cont}} - \beta + \left(1 - \frac{1}{\psi}\right) \mu_C^{cont} + v_s^{cont} \mu_s^{cont} \\
& - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma_{CC} + (1 - \gamma) v_s^{cont} \sigma_{Cs} + \frac{1}{2} (v_{ss}^{cont} + \theta (v_s^{cont})^2) \sigma_{ss} \\
& + \frac{1}{\theta} \lambda_A^{cont, cont} \left[(1 + s_A L_A^{cont, cont})^{1-\gamma} e^{\theta v^{cont}(s_A^+) - \theta v^{cont}(s_A)} - 1 \right] \\
& + \frac{1}{\theta} \lambda_B^{cont, cont} \left[(1 + s_B L_B^{cont, cont})^{1-\gamma} e^{\theta v^{cont}(s_B^+) - \theta v^{cont}(s_A)} - 1 \right] \\
& + \frac{1}{\theta} \lambda^{cont, calm} \left[e^{\theta v^{calm}(s_A) - \theta v^{cont}(s_A)} - 1 \right].
\end{aligned} \tag{25}$$

To derive boundary conditions, we study the behavior of the differential equations if the consumption share s_A goes to 0 or 1. Both limits can be interpreted as one tree economies with an additional (exogenous) possibility of a regime shift, for which solutions exist under suitable conditions.¹⁶ This gives

$$\begin{aligned}
0 = & e^{-v^{calm}(0)} - \beta + \left(1 - \frac{1}{\psi}\right) \mu_B^{calm} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma_B^2 \\
& + \frac{1}{\theta} \lambda_A^{calm, cont} \left[e^{\theta v^{cont}(0) - \theta v^{calm}(0)} - 1 \right] \\
& + \frac{1}{\theta} \lambda_B^{calm, calm} \left[(1 + L_B^{calm, calm})^{1-\gamma} - 1 \right] + \frac{1}{\theta} \lambda_B^{calm, cont} \left[(1 + L_B^{calm, cont})^{1-\gamma} e^{\theta v^{cont}(0) - \theta v^{calm}(0)} - 1 \right] \\
0 = & e^{-v^{calm}(1)} - \beta + \left(1 - \frac{1}{\psi}\right) \mu_A^{calm} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma_A^2 \\
& + \frac{1}{\theta} \lambda_A^{calm, calm} \left[(1 + L_A^{calm, calm})^{1-\gamma} - 1 \right] + \frac{1}{\theta} \lambda_A^{calm, cont} \left[(1 + L_A^{calm, cont})^{1-\gamma} e^{\theta v^{cont}(1) - \theta v^{calm}(1)} - 1 \right] \\
& + \frac{1}{\theta} \lambda_B^{calm, cont} \left[e^{\theta v^{cont}(1) - \theta v^{calm}(1)} - 1 \right] \\
0 = & e^{-v^{cont}(0)} - \beta + \left(1 - \frac{1}{\psi}\right) \mu_B^{cont} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma_B^2 \\
& + \frac{1}{\theta} \lambda_B^{cont, cont} \left[(1 + L_B^{cont, cont})^{1-\gamma} - 1 \right] + \frac{1}{\theta} \lambda^{cont, calm} \left[e^{\theta v^{calm}(0) - \theta v^{cont}(0)} - 1 \right] \\
0 = & e^{-v^{cont}(1)} - \beta + \left(1 - \frac{1}{\psi}\right) \mu_A^{cont} - \frac{1}{2} \gamma \left(1 - \frac{1}{\psi}\right) \sigma_A^2 \\
& + \frac{1}{\theta} \lambda_A^{cont, cont} \left[(1 + L_A^{cont, cont})^{1-\gamma} - 1 \right] + \frac{1}{\theta} \lambda^{cont, calm} \left[e^{\theta v^{calm}(1) - \theta v^{cont}(1)} - 1 \right].
\end{aligned}$$

This system of equations determines the four boundary values $v^{calm}(0)$, $v^{cont}(0)$, $v^{calm}(1)$ and $v^{cont}(1)$. The resulting boundary problem for v^{calm} and v^{cont} can then be solved using finite differences.

B.2 Pricing Kernel

The dynamics of the pricing kernel can be computed via Ito's Lemma. The partial derivatives of ξ w.r.t. C and v follow from (18). The dynamics of C are given in (20) and (21). The dynamics of v follow from

¹⁶See, e.g., Duffie and Epstein (1992a). Note that in the limit there can still be contagious jumps in the tree with zero consumption share; these jumps influence the state of the economy and thus prices even if the particular tree has a relative size of 0.

Ito's Lemma:

$$dv_t^{Z_t} = v_s \mu_s^{Z_t} dt + \frac{1}{2} v_{ss} \sigma_{ss} dt + v_s s_{A,t} s_{B,t} \left(\sigma_A dW_{A,t} - \sigma_B dW_{B,t} \right) + \text{Jump Terms.}$$

Plugging everything together and reallocating some terms gives

$$\begin{aligned} \frac{d\xi_t}{\xi_{t-}} = & \left[-\beta\theta + (\theta - 1)e^{-v_t^{calm}} \right] dt - \gamma \mu_C^{calm} dt + (\theta - 1)v_s^{calm} \mu_s^{calm} dt \\ & - (\theta - 1)v_s^{calm} \gamma \sigma_{Cs} dt + \frac{1}{2}(\theta - 1) \left(v_{ss}^{calm} + (\theta - 1)(v_s^{calm})^2 \right) \sigma_{ss} dt + \frac{1}{2}\gamma(1 + \gamma)\sigma_{CC} dt \\ & - \eta_A^{diff, calm} dW_{A,t} - \eta_B^{diff, calm} dW_{B,t} \\ & + dN_{A,t}^{calm, calm} \eta_A^{calm, calm} + dN_{A,t}^{calm, cont} \eta_A^{calm, cont} + dN_{B,t}^{calm, calm} \eta_B^{calm, calm} + dN_{B,t}^{calm, cont} \eta_B^{calm, cont} \end{aligned}$$

in the calm state and

$$\begin{aligned} \frac{d\xi_t}{\xi_{t-}} = & \left[-\beta\theta + (\theta - 1)e^{-v_t^{cont}} \right] dt - \gamma \mu_C^{cont} dt + (\theta - 1)v_s^{cont} \mu_s^{cont} dt \\ & - (\theta - 1)v_s^{cont} \gamma \sigma_{Cs} dt + \frac{1}{2}(\theta - 1) \left(v_{ss}^{cont} + (\theta - 1)(v_s^{cont})^2 \right) \sigma_{ss} dt + \frac{1}{2}\gamma(1 + \gamma)\sigma_{CC} dt \\ & - \eta_A^{diff, cont} dW_{A,t} - \eta_B^{diff, cont} dW_{B,t} \\ & + dN_{A,t}^{cont, cont} \eta_A^{cont, cont} + dN_{B,t}^{cont, cont} \eta_B^{cont, cont} + dN_t^{cont, calm} \eta^{cont, calm} \end{aligned}$$

in the contagion state. The market prices of diffusion risk are given by

$$\eta_A^{diff, Z} = \left[\gamma s_A - (\theta - 1)v_s^Z s_A s_B \right] \sigma_A, \quad \eta_B^{diff, Z} = \left[\gamma s_B + (\theta - 1)v_s^Z s_A s_B \right] \sigma_B$$

and the market prices of jump risk are given by:

$$\begin{aligned} \eta_A^{calm, calm} &= \left(1 + s_A L_A^{calm, calm} \right)^{-\gamma} e^{(\theta-1) \left(v^{calm}(s_A^{A+}) - v^{calm}(s_A) \right)} - 1 \\ \eta_A^{calm, cont} &= \left(1 + s_A L_A^{calm, cont} \right)^{-\gamma} e^{(\theta-1) \left(v^{cont}(s_A^{A+}) - v^{calm}(s_A) \right)} - 1 \\ \eta_B^{calm, calm} &= \left(1 + s_B L_B^{calm, calm} \right)^{-\gamma} e^{(\theta-1) \left(v^{calm}(s_A^{B+}) - v^{calm}(s_A) \right)} - 1 \\ \eta_B^{calm, cont} &= \left(1 + s_B L_B^{calm, cont} \right)^{-\gamma} e^{(\theta-1) \left(v^{cont}(s_A^{B+}) - v^{calm}(s_A) \right)} - 1 \\ \eta_A^{cont, cont} &= \left(1 + s_A L_A^{cont, cont} \right)^{-\gamma} e^{(\theta-1) \left(v^{cont}(s_A^{A+}) - v^{cont}(s_A) \right)} - 1 \\ \eta_B^{cont, cont} &= \left(1 + s_B L_B^{cont, cont} \right)^{-\gamma} e^{(\theta-1) \left(v^{cont}(s_A^{B+}) - v^{cont}(s_A) \right)} - 1 \\ \eta^{cont, calm} &= e^{(\theta-1) \left(v^{calm}(s_A) - v^{cont}(s_A) \right)} - 1. \end{aligned}$$

The market prices of jump risk η from above lead to risk-neutral jump intensities of the form $\lambda_i^{\mathbb{Q}, j, k} = \lambda_i^{\mathbb{P}, j, k} (1 + \eta_i^{j, k})$. For later use, we abbreviate the drift of the pricing kernel by

$$\begin{aligned} \mu_\xi^Z &= -\beta\theta + (\theta - 1)e^{-v^Z} - \gamma \mu_C^Z + (\theta - 1)v_s^Z \mu_s^Z \\ &\quad - (\theta - 1)v_s^Z \gamma \sigma_{Cs} + \frac{1}{2}(\theta - 1) \left(v_{ss}^Z + (\theta - 1)(v_s^Z)^2 \right) \sigma_{ss} dt + \frac{1}{2}\gamma(1 + \gamma)\sigma_{CC}. \end{aligned}$$

Replacing e^{-v^Z} using the ODEs (24) and (25) and then computing the expectation of the pricing kernel results in the following expressions for the risk-free interest rate:

$$\begin{aligned}
r_f^{calm} &= \beta + \frac{1}{\psi} \left(\mu_C^{calm} + s_A \sum_{k=calm,cont} L_A^{calm,k} \lambda_A^{calm,k} + s_B \sum_{k=calm,cont} L_B^{calm,k} \lambda_B^{calm,k} \right) \\
&- \frac{1}{2} \gamma \left(1 + \frac{1}{\psi} \right) \sigma_{CC} - (1 - \theta) v_s^{calm} \sigma_{Cs} - \frac{1}{2} (1 - \theta) (v_s^{calm})^2 \sigma_{ss} \\
&- \sum_{k=calm,cont} \lambda_A^{calm,k} \left[\eta_A^{calm,k} + \frac{1}{\psi} s_A L_A^{calm,k} + \frac{1 - \theta}{\theta} \left((1 + s_A L_A^{calm,k})^{1-\gamma} e^{\theta(v^k(s_A^{A+}) - v^{calm}(s_A))} - 1 \right) \right] \\
&- \sum_{k=calm,cont} \lambda_B^{calm,k} \left[\eta_B^{calm,k} + \frac{1}{\psi} s_B L_B^{calm,k} + \frac{1 - \theta}{\theta} \left((1 + s_B L_B^{calm,k})^{1-\gamma} e^{\theta(v^k(s_A^{B+}) - v^{calm}(s_A))} - 1 \right) \right]
\end{aligned}$$

and

$$\begin{aligned}
r_f^{cont} &= \beta + \frac{1}{\psi} \left(\mu_C^{cont} + s_A L_A^{cont,cont} \lambda_A^{cont,cont} + s_B L_B^{cont,cont} \lambda_B^{cont,cont} \right) \\
&- \frac{1}{2} \gamma \left(1 + \frac{1}{\psi} \right) \sigma_{CC} - (1 - \theta) v_s^{cont} \sigma_{Cs} - \frac{1}{2} (1 - \theta) (v_s^{cont})^2 \sigma_{ss} \\
&- \lambda_A^{cont,cont} \left[\eta_A^{cont,cont} + \frac{1}{\psi} s_A L_A^{cont,cont} + \frac{1 - \theta}{\theta} \left((1 + s_A L_A^{cont,cont})^{1-\gamma} e^{\theta(v^{cont}(s_A^{A+}) - v^{cont}(s_A))} - 1 \right) \right] \\
&- \lambda_B^{cont,cont} \left[\eta_B^{cont,cont} + \frac{1}{\psi} s_B L_B^{cont,cont} + \frac{1 - \theta}{\theta} \left((1 + s_B L_B^{cont,cont})^{1-\gamma} e^{\theta(v^{cont}(s_A^{B+}) - v^{cont}(s_A))} - 1 \right) \right] \\
&- \lambda^{cont,calm} \left[\eta^{cont,calm} + \frac{1 - \theta}{\theta} \left(e^{\theta(v^{calm}(s_A) - v^{cont}(s_A))} - 1 \right) \right].
\end{aligned}$$

B.3 Pricing the Dividend Claims

The price-dividend ratio of the dividend claims can be obtained from a Feynman-Kac argument as in the one tree case. The dividends follow

$$\frac{dD_{i,t}}{D_{i,t-}} = \mu_i^{calm} dt + \phi \sigma dW_{i,t} + ((1 + L_i^{calm,calm})^\phi - 1) dN_{i,t}^{calm,calm} + ((1 + L_i^{calm,cont})^\phi - 1) dN_{i,t}^{calm,cont}$$

in the calm state and

$$\frac{dD_{i,t}}{D_{i,t-}} = \mu_i^{cont} dt + \phi \sigma dW_{i,t} + ((1 + L_i^{cont,cont})^\phi - 1) dN_{i,t}^{cont,cont}$$

in the contagion state. Let w_A denote the log price-dividend ratio of asset A . For $g(\xi, D_A, w_A) = \xi D_A e^{w_A}$, the Feynman-Kac formula yields

$$\frac{Dg(\xi, D_A, w_A)}{g(\xi, D_A, w_A)} + e^{-w_A} = 0. \tag{26}$$

Applied to our model specification, Ito's Lemma gives

$$\frac{Dg}{g} = \mu_\xi + \mu_A + \mu_w + \frac{1}{2} \frac{d \langle w_A^c \rangle}{dt} + \frac{d \langle \xi^c, D_A^c \rangle}{\xi D_A dt} + \frac{d \langle w_A^c, D_A^c \rangle}{D_A dt} + \frac{d \langle w_A^c, \xi^c \rangle}{\xi dt} + \text{Jump Terms.}$$

Another application of Ito's Lemma leads to

$$\begin{aligned}
dw_A^{calm} &= w_{A,s}^{calm} \mu_s^{calm} dt + \frac{1}{2} w_{A,ss}^{calm} \sigma_{ss} dt \\
&\quad + w_{A,s}^{calm} s_A s_B \sigma_A dW_A - w_{A,s}^{calm} s_A s_B \sigma_B dW_B \\
&\quad + (w_A^{calm}(s_A^{A+}) - w_A^{calm}(s_A)) dN_A^{calm,calm} + (w_A^{calm}(s_A^{B+}) - w_A^{calm}(s_A)) dN_B^{calm,calm} \\
&\quad + (w_A^{cont}(s_A^{A+}) - w_A^{calm}(s_A)) dN_A^{calm,cont} + (w_A^{cont}(s_A^{B+}) - w_A^{calm}(s_A)) dN_B^{calm,cont} \\
dw_A^{cont} &= w_{A,s}^{cont} \mu_s^{cont} dt + \frac{1}{2} w_{A,ss}^{cont} \sigma_{ss} dt \\
&\quad + w_{A,s}^{cont} s_A s_B \sigma_A dW_A - w_{A,s}^{cont} s_A s_B \sigma_B dW_B \\
&\quad + (w_A^{cont}(s_A^{A+}) - w_A^{cont}(s_A)) dN_A^{cont,cont} + (w_A^{cont}(s_A^{B+}) - w_A^{cont}(s_A)) dN_B^{cont,cont} \\
&\quad + (w_A^{calm}(s_A) - w_A^{cont}(s_A)) dN^{cont,calm},
\end{aligned}$$

where, again, the subscripts s and ss denote the first and second derivatives with respect to the consumption share s_A . Plugging everything into Equation (26) leads to two ODEs for w_A^{calm} and w_A^{cont} :

$$0 = e^{-w_A^{calm}} + \mu_\xi^{calm} + \mu_A^{calm} + \frac{1}{2} (w_{A,ss}^{calm} + (w_{A,s}^{calm})^2) \sigma_{ss} \quad (27)$$

$$\begin{aligned}
&- \eta_A^{diff,calm} (\phi \sigma_A + w_{A,s}^{calm} s_A s_B \sigma_A - w_{A,s}^{calm} s_A s_B \rho \sigma_B) \\
&- \eta_B^{diff,calm} (\phi \rho \sigma_A - w_{A,s}^{calm} s_A s_B \sigma_B + w_{A,s}^{calm} s_A s_B \rho \sigma_A) \\
&+ w_{A,s}^{calm} \mu_s^{calm} + w_{A,s}^{calm} s_A s_B \phi \sigma_A^2 - w_{A,s}^{calm} s_A s_B \phi \rho \sigma_A \sigma_B \\
&+ \lambda_A^{calm,calm} \left[(1 + \eta_A^{calm,calm}) (1 + L_A^{calm,calm})^\phi e^{w_A^{calm}(s_A^{A+}) - w_A^{calm}(s_A)} - 1 \right] \\
&+ \lambda_B^{calm,calm} \left[(1 + \eta_B^{calm,calm}) e^{w_A^{calm}(s_A^{B+}) - w_A^{calm}(s_A)} - 1 \right] \\
&+ \lambda_A^{calm,cont} \left[(1 + \eta_A^{calm,cont}) (1 + L_A^{calm,cont})^\phi e^{w_A^{cont}(s_A^{A+}) - w_A^{calm}(s_A)} - 1 \right] \\
&+ \lambda_B^{calm,cont} \left[(1 + \eta_B^{calm,cont}) e^{w_A^{cont}(s_A^{B+}) - w_A^{calm}(s_A)} - 1 \right]
\end{aligned}$$

$$0 = +e^{-w_A^{cont}} + \mu_\xi^{cont} + \mu_A^{cont} + \frac{1}{2} (w_{A,ss}^{cont} + (w_{A,s}^{cont})^2) \sigma_{ss} \quad (28)$$

$$\begin{aligned}
&- \eta_A^{diff,cont} (\phi \sigma_A + w_{A,s}^{cont} s_A s_B \sigma_A - w_{A,s}^{cont} s_A s_B \rho \sigma_B) \\
&- \eta_B^{diff,cont} (\phi \rho \sigma_A - w_{A,s}^{cont} s_A s_B \sigma_B + w_{A,s}^{cont} s_A s_B \rho \sigma_A) \\
&+ w_{A,s}^{cont} \mu_s^{cont} + w_{A,s}^{cont} s_A s_B \phi \sigma_A^2 - w_{A,s}^{cont} s_A s_B \phi \rho \sigma_A \sigma_B \\
&+ \lambda_A^{cont,cont} \left[(1 + \eta_A^{cont,cont}) (1 + L_A^{cont,cont})^\phi e^{w_A^{cont}(s_A^{A+}) - w_A^{cont}(s_A)} - 1 \right] \\
&+ \lambda_B^{cont,cont} \left[(1 + \eta_B^{cont,cont}) e^{w_A^{cont}(s_A^{B+}) - w_A^{cont}(s_A)} - 1 \right] \\
&+ \lambda^{cont,calm} \left[(1 + \eta^{cont,calm}) e^{w_A^{calm}(s_A) - w_A^{cont}(s_A)} - 1 \right].
\end{aligned}$$

Boundary conditions for the ODEs can be found as for the log wealth-consumption ratio v by studying

the behavior in the limit as the consumption share s_A goes to zero or one. This results in

$$\begin{aligned}
0 &= -\beta\theta + (\theta - 1)e^{-v^{calm}(0)} + e^{-w_A^{calm}(0)} + \mu_A^{calm} - \gamma\mu_B^{calm} + \frac{1}{2}\gamma(1 + \gamma)\sigma_B^2 - \gamma\phi\rho\sigma_A\sigma_B \\
&\quad + \lambda_A^{calm, calm} \left[\left(1 + L_A^{calm, calm}\right)^\phi - 1 \right] + \lambda_B^{calm, calm} \left[\left(1 + L_B^{calm, calm}\right)^{-\gamma} - 1 \right] \\
&\quad + \lambda_A^{calm, cont} \left[\left(1 + L_A^{calm, cont}\right)^\phi e^{(\theta-1)\left(v^{cont}(0) - v^{calm}(0)\right)} e^{w_A^{cont}(0) - w_A^{calm}(0)} - 1 \right] \\
&\quad + \lambda_B^{calm, cont} \left[\left(1 + L_B^{calm, cont}\right)^{-\gamma} e^{(\theta-1)\left(v^{cont}(0) - v^{calm}(0)\right)} e^{w_A^{cont}(0) - w_A^{calm}(0)} - 1 \right] \\
0 &= -\beta\theta + (\theta - 1)e^{-v^{cont}(0)} + e^{-w_A^{cont}(0)} + \mu_A^{cont} - \gamma\mu_B^{cont} + \frac{1}{2}\gamma(1 + \gamma)\sigma_B^2 - \gamma\phi\rho\sigma_A\sigma_B \\
&\quad + \lambda_A^{cont, cont} \left[\left(1 + L_A^{cont, cont}\right)^\phi - 1 \right] + \lambda_B^{cont, cont} \left[\left(1 + L_B^{cont, cont}\right)^{-\gamma} - 1 \right] \\
&\quad + \lambda^{cont, calm} \left[e^{(\theta-1)\left(v^{calm}(0) - v^{cont}(0)\right)} e^{w_A^{calm}(0) - w_A^{cont}(0)} - 1 \right]
\end{aligned}$$

for $s_A = 0$ and

$$\begin{aligned}
0 &= -\beta\theta + (\theta - 1)e^{-v^{calm}(1)} + e^{-w_A^{calm}(1)} + (1 - \gamma)\mu_A^{calm} + \frac{1}{2}\gamma(1 + \gamma)\sigma_A^2 - \gamma\phi\sigma_A^2 \\
&\quad + \lambda_A^{calm, calm} \left[\left(1 + L_A^{calm, calm}\right)^{\phi-\gamma} - 1 \right] \\
&\quad + \lambda_A^{calm, cont} \left[\left(1 + L_A^{calm, cont}\right)^{\phi-\gamma} e^{(\theta-1)\left(v^{cont}(1) - v^{calm}(1)\right)} e^{w_A^{cont}(1) - w_A^{calm}(1)} - 1 \right] \\
&\quad + \lambda_B^{calm, cont} \left[e^{(\theta-1)\left(v^{cont}(1) - v^{calm}(1)\right)} e^{w_A^{cont}(1) - w_A^{calm}(1)} - 1 \right] \\
0 &= -\beta\theta + (\theta - 1)e^{-v^{cont}(1)} + e^{-w_A^{cont}(1)} + (1 - \gamma)\mu_A^{cont} + \frac{1}{2}\gamma(1 + \gamma)\sigma_A^2 - \gamma\phi\sigma_A^2 \\
&\quad + \lambda_A^{cont, cont} \left[\left(1 + L_A^{cont, cont}\right)^{\phi-\gamma} - 1 \right] \\
&\quad + \lambda^{cont, calm} \left[e^{(\theta-1)\left(v^{calm}(1) - v^{cont}(1)\right)} e^{w_A^{calm}(1) - w_A^{cont}(1)} - 1 \right]
\end{aligned}$$

for $s_A = 1$.

However, the price-dividend ratio of tree A can become infinitely large if its consumption share s_A tends to zero. If w_A becomes infinitely large in the limit, we omit the boundary condition at $s_A = 0$ in the numerical computation and, instead, solve the initial value problem with the remaining initial condition at $s_A = 1$. The numerical results are not affected by this methodology. We emphasize again that the boundary conditions at $s_A = 1$ are not equal to the conditions for the price-dividend ratios in a single-tree economy. The reason is that there can still be contagious jumps in tree B even if s_A tends to 1. Even if tree B has a relative size of zero, its jumps can influence the state of the economy and thus asset prices. The price-dividend ratio of tree B can be computed symmetrically to the price-dividend ratio of tree A .

B.4 Exposures and Moments

Conditional on the state, the dynamics of the asset price $P_A = e^{w_A} D_A$ follow via Ito's Lemma. In the calm state, we have

$$\begin{aligned} \frac{dP_{A,t}}{P_{A,t-}} &= \frac{E_t[dP_{A,t}]}{P_{A,t-}} + (w_{A,s}^{calm} s_{A,t} s_{B,t} \sigma_A + \phi \sigma_A) dW_{A,t} - w_{A,s}^{calm} s_{A,t} s_{B,t} \sigma_B dW_{B,t} \\ &+ \Upsilon_A^{A,calm,calm} \left(dN_{A,t}^{calm,calm} - \lambda_A^{calm,calm} dt \right) + \Upsilon_A^{A,calm,cont} \left(dN_{A,t}^{calm,cont} - \lambda_A^{calm,cont} dt \right) \\ &+ \Upsilon_A^{B,calm,calm} \left(dN_{B,t}^{calm,calm} - \lambda_B^{calm,calm} dt \right) + \Upsilon_A^{B,calm,cont} \left(dN_{B,t}^{calm,cont} - \lambda_B^{calm,cont} dt \right). \end{aligned}$$

In the contagion state, the dynamics are

$$\begin{aligned} \frac{dP_{A,t}}{P_{A,t-}} &= \frac{E_t[dP_{A,t}]}{P_{A,t-}} + (w_{A,s}^{cont} s_{A,t} s_{B,t} \sigma_A + \phi \sigma_A) dW_{A,t} - w_{A,s}^{cont} s_{A,t} s_{B,t} \sigma_B dW_{B,t} \\ &+ \Upsilon_A^{A,cont,cont} \left(dN_{A,t}^{cont,cont} - \lambda_A^{cont,cont} dt \right) + \Upsilon_A^{B,cont,cont} \left(dN_{B,t}^{cont,cont} - \lambda_B^{cont,cont} dt \right) \\ &+ \Upsilon_A^{cont,calm} \left(dN_t^{cont,calm} - \lambda^{cont,calm} dt \right). \end{aligned}$$

In the following, we set

$$\begin{aligned} \Upsilon_A^{A,diff,Z} &= (w_{A,s}^Z s_A s_B + \phi) \sigma_A - w_{A,s}^Z s_A s_B \rho \sigma_B \\ \Upsilon_A^{B,diff,Z} &= (w_{A,s}^Z s_A s_B + \phi) \rho \sigma_A - w_{A,s}^Z s_A s_B \sigma_B \end{aligned}$$

for $Z \in \{calm, cont\}$, which can be interpreted as the total sensitivities of asset A with respect to the Brownian shocks A and B . The total sensitivities $\Upsilon_B^{A,diff,Z}$ and $\Upsilon_B^{B,diff,Z}$ are defined analogously. $\Upsilon_i^{n,j,k}$ is the sensitivity of asset i with respect to jumps in dividend n ($i, n \in \{A, B\}$) and from state j to state k ($(j, k) \in \{(calm, calm), (calm, cont), (cont, cont)\}$). Finally, $\Upsilon_i^{cont,calm}$ denotes the sensitivity of asset i with respect to regime switches from the contagion state back to the calm state.

The sensitivities of asset A with respect to the jump processes are

$$\begin{aligned} \Upsilon_A^{A,calm,calm} &= (1 + L_A^{calm,calm})^\phi e^{w_{A,t}^{calm}(s_A^+) - w_{A,t}^{calm}(s_A)} - 1 \\ \Upsilon_A^{A,calm,cont} &= (1 + L_A^{calm,cont})^\phi e^{w_{A,t}^{cont}(s_A^+) - w_{A,t}^{calm}(s_A)} - 1 \\ \Upsilon_A^{A,cont,cont} &= (1 + L_A^{cont,cont})^\phi e^{w_{A,t}^{cont}(s_A^+) - w_{A,t}^{cont}(s_A)} - 1 \\ \Upsilon_A^{B,calm,calm} &= e^{w_{A,t}^{calm}(s_B^+) - w_{A,t}^{calm}(s_A)} - 1 \\ \Upsilon_A^{B,calm,cont} &= e^{w_{A,t}^{cont}(s_B^+) - w_{A,t}^{calm}(s_A)} - 1 \\ \Upsilon_A^{B,cont,cont} &= e^{w_{A,t}^{cont}(s_B^+) - w_{A,t}^{cont}(s_A)} - 1 \\ \Upsilon_A^{cont,calm} &= e^{w_{A,t}^{calm}(s_A) - w_{A,t}^{cont}(s_A)} - 1. \end{aligned}$$

For the exposures of asset B, one has to switch 'A' and 'B' (also on the left-hand side of the equations) and replace every derivative w_s by $w_{1-s} = -w_s$.

The expected return of asset A can be computed as the sum of expected price change and dividend yield:

$$\frac{E_t[dR_{A,t}]}{dt} = \frac{E_t[dP_{A,t}]}{P_{A,t} dt} + e^{-w_{A,t}}.$$

Replacing e^{-w_A} using the differential equations (27) and (28), computing the expectation of dP_A , rearranging some terms and finally using the expression for the risk-free rate, the expected excess return in the calm state becomes

$$\begin{aligned}
& \left[(w_{A,s}^{calm} s_A s_B + \phi) \sigma_A - w_{A,s}^{calm} s_A s_B \rho \sigma_B \right] \left[\gamma s_A - (\theta - 1) v_s^{calm} s_A s_B \right] \sigma_A \\
& + \left[(w_{A,s}^{calm} s_A s_B + \phi) \rho \sigma_A - w_{A,s}^{calm} s_A s_B \sigma_B \right] \left[\gamma s_B + (\theta - 1) v_s^{calm} s_A s_B \right] \sigma_B \\
& + \lambda_A^{calm, calm} \left[\left(1 + L_A^{calm, calm} \right)^\phi e^{w_A^{calm}(s_A^+) - w_A^{calm}(s_A)} - 1 \right] \left[1 - \left(1 + s_A L_A^{calm, calm} \right)^{-\gamma} e^{(\theta-1)(v^{calm}(s_A^+) - v^{calm}(s_A))} \right] \\
& + \lambda_B^{calm, calm} \left[e^{w_A^{calm}(s_A^{B+}) - w_A^{calm}(s_A)} - 1 \right] \left[1 - \left(1 + s_B L_B^{calm, calm} \right)^{-\gamma} e^{(\theta-1)(v^{calm}(s_A^{B+}) - v^{calm}(s_A))} \right] \\
& + \lambda_A^{calm, cont} \left[\left(1 + L_A^{calm, cont} \right)^\phi e^{w_A^{cont}(s_A^+) - w_A^{calm}(s_A)} - 1 \right] \left[1 - \left(1 + s_A L_A^{calm, cont} \right)^{-\gamma} e^{(\theta-1)(v^{cont}(s_A^+) - v^{calm}(s_A))} \right] \\
& + \lambda_B^{calm, cont} \left[e^{w_A^{cont}(s_A^{B+}) - w_A^{calm}(s_A)} - 1 \right] \left[1 - \left(1 + s_B L_B^{calm, cont} \right)^{-\gamma} e^{(\theta-1)(v^{cont}(s_A^{B+}) - v^{calm}(s_A))} \right]
\end{aligned}$$

and the expected excess return in the contagion state equals

$$\begin{aligned}
& \left[(w_{A,s}^{cont} s_A s_B + \phi) \sigma_A - w_{A,s}^{cont} s_A s_B \rho \sigma_B \right] \left[\gamma s_A - (\theta - 1) v_s^{calm} s_A s_B \right] \sigma_A \\
& + \left[(w_{A,s}^{cont} s_A s_B + \phi) \rho \sigma_A - w_{A,s}^{cont} s_A s_B \sigma_B \right] \left[\gamma s_B + (\theta - 1) v_s^{calm} s_A s_B \right] \sigma_B \\
& + \lambda_A^{cont, cont} \left[\left(1 + L_A^{cont, cont} \right)^\phi e^{w_A^{cont}(s_A^+) - w_A^{cont}(s_A)} - 1 \right] \left[1 - \left(1 + s_A L_A^{cont, cont} \right)^{-\gamma} e^{(\theta-1)(v^{cont}(s_A^+) - v^{cont}(s_A))} \right] \\
& + \lambda_B^{cont, cont} \left[e^{w_A^{cont}(s_A^{B+}) - w_A^{cont}(s_A)} - 1 \right] \left[1 - \left(1 + s_B L_B^{cont, cont} \right)^{-\gamma} e^{(\theta-1)(v^{cont}(s_A^{B+}) - v^{cont}(s_A))} \right] \\
& + \lambda^{cont, calm} \left[e^{w_A^{calm}(s_A) - w_A^{cont}(s_A)} - 1 \right] \left[1 - e^{(\theta-1)(v^{calm}(s_A) - v^{cont}(s_A))} \right].
\end{aligned}$$

This formulation gives rise to the interpretation of the expected excess return as the sum over 'exposure' times 'market price of risk' for all priced risk factors as noted in Proposition 7.

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	One Tree Economies (Section 2)			Two Tree Economies (Sections 3 and 4)		
	Economy with Contagious Jumps	Pure Regime Switching Economy	Economy without Regimes	Identical Trees	Robust vs. Contagion- sensitive Assets	Propensity to Trigger Contagion
μ_A^{calm}	0.058	0.058	0.058	0.058	0.058	0.058
μ_A^{cont}	0.058	0.058	0.058	0.058	0.058	0.058
μ_B^{calm}	—	—	—	0.058	0.058	0.058
μ_B^{cont}	—	—	—	0.058	0.058	0.058
σ_A	0.01	0.01	0.01	0.014	0.014	0.014
σ_B	—	—	—	0.014	0.014	0.014
ρ	—	—	—	0	0	0
$L_A^{calm,calm}$	-0.03	-0.03	-0.03	-0.06	-0.06	-0.06
$L_A^{calm,cont}$	-0.03	0	0	-0.06	-0.06	-0.06
$L_B^{calm,calm}$	—	—	—	-0.06	-0.06	-0.06
$L_B^{calm,cont}$	—	—	—	-0.06	-0.06	-0.06
$L_A^{cont,cont}$	-0.03	-0.03	-0.03	-0.06	-0.06	-0.06
$L_B^{cont,cont}$	—	—	—	-0.06	-0.06	-0.06
$\lambda_A^{calm,calm}$	0.23	0.5	0.855	0.115	0.115	0
$\lambda_A^{calm,cont}$	0.27	0.27	0.27	0.135	0.135	0.25
$\lambda_B^{calm,calm}$	—	—	—	0.115	0.115	0.23
$\lambda_B^{calm,cont}$	—	—	—	0.135	0.135	0.02
$\lambda_A^{cont,cont}$	1.5	1.5	0.855	0.75	0.625	0.75
$\lambda_B^{cont,cont}$	—	—	—	0.75	0.875	0.75
$\lambda^{cont,calm}$	0.49	0.49	0.49	0.49	0.49	0.49

Table 1: Parameters of Consumption Processes

The table reports the parameters of the consumption processes. The first three columns refer to the different one tree economies discussed in Section 2: the economy with contagious jumps, the pure regime switching economy, and the economy without regimes. For simplicity, the single tree corresponds to tree A in these columns. The fourth column refers to the two tree economy with identical trees which is studied in Section 3. The last two columns give the parameters if contagion mainly affects asset B ('Robust versus Contagion-sensitive Assets') or if contagion is mainly induced by asset A ('Propensity to Trigger Contagion'). These settings are discussed in Section 4. All parameters are annualized.

	Economy with				Pure Regime		Economy		Data	
	Contagious Jumps				Switching Economy		without		1890-2004	1947-2010
	Calm	Contagion	Uncond.		Calm	Contagion	Uncond.	Regimes		
	state	state	state		state	state	state			
$\sigma[C]$	0.0235	0.0381	0.0295		0.0235	0.0381	0.0295	0.0295	0.0356	0.0134
$\sigma[D]$	0.0687	0.1111	0.0862		0.0685	0.1111	0.0862	0.0861	0.1151	0.0659
$E[r_f]$	0.0269	0.0033	0.0185		0.0321	0.0030	0.0217	0.0230	0.0201	0.0134
$\sigma[r_f]$	0	0	0.0113		0	0	0.0140	0	0.0591	0.0266
$E[R - r_f]$	0.0544	0.0641	0.0578		0.0342	0.0700	0.0469	0.0296	0.0597	0.0706
$\sigma[R - r_f]$	0.0995	0.1282	0.1105		0.0873	0.1375	0.1079	0.0861	0.1848	0.1772
Sharpe Ratio	0.5467	0.5000	0.5231		0.3918	0.5091	0.4347	0.3438	0.32	0.40
$E[P/D]$	11.4224	10.4655	11.0824		12.5068	11.2079	12.0453	14.4325	—	—

Table 2: Asset Pricing Results (One Tree Economies)

The table reports the conditional and unconditional cash flow dynamics and asset pricing results in the one tree economies discussed in Section 2. The columns labeled 'Data' refer to US data and are taken from Wachter (2012).

	Total risk-free rate	β	$\frac{1}{\psi} \frac{E_t[dC_t]}{C_t dt}$	Precautionary savings terms			
				Diffusion risk	Pure cons. jumps	Contagious jumps	Regime switches
Economy with Contagious Jumps							
Calm state	0.0269	0.01	0.0287	−0.0008	−0.0021	−0.0089	—
Contagion state	0.0033	0.01	0.0087	−0.0008	−0.0135	—	−0.0010
Pure Regime Switching Economy							
Calm state	0.0321	0.01	0.0287	−0.0008	−0.0045	—	−0.0013
Contagion state	0.0030	0.01	0.0087	−0.0008	−0.0135	—	−0.0013
Economy without Regimes							
Steady state	0.0230	0.01	0.0216	−0.0008	−0.0077	—	—

Table 3: Decomposition of the Risk-free Rate (One Tree Economies)

The table reports the risk-free rate in the different one tree economies discussed in Section 2. The decomposition into its various components is based on Proposition 2.

	Total premium	Diffusion risk	Pure cons. jumps	Contagious jumps	Regime switches
Economy with Contagious Jumps					
Calm state	0.0544	0.003	0.0072	0.0442	—
Contagion state	0.0641	0.003	0.0466	—	0.0144
Pure Regime Switching Economy					
Calm state	0.0342	0.003	0.0155	—	0.0156
Contagion state	0.0700	0.003	0.0466	—	0.0203
Economy without Regimes					
Steady state	0.0296	0.003	0.0266	—	—

Table 4: Decomposition of the Risk Premium (One Tree Economies)

The table reports the risk premium in the different one tree economies discussed in Section 2. The decomposition into its various components is based on Proposition 3. The premium for contagious jumps in the first line is further decomposed in Table 5.

Total	Premium for pure cons. jumps	Premium for pure regime switches	Interaction term
0.0442	0.0084	0.0107	0.0251

Table 5: Decomposition of the Risk Premium for Contagious Jumps

The table decomposes the risk premium for contagious jumps in a one tree economy (see first line of Table 4) into its components as discussed in Section 2.

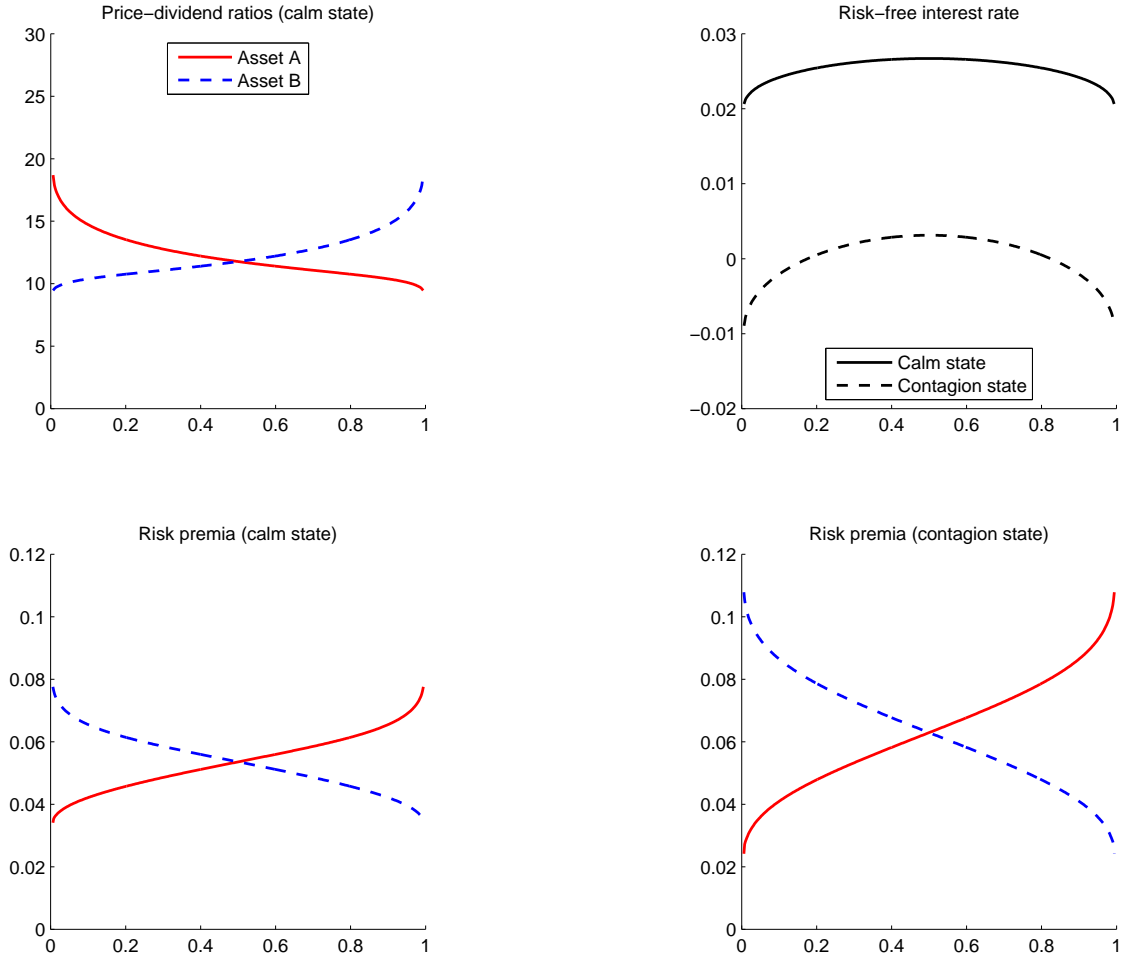


Figure 1: Asset Prices and Returns (Identical Trees)

The figure depicts the price-dividend ratios in the calm state, the risk-free interest rate, and the risk premia in both states as a function of the consumption share s_A of tree A . Price-dividend ratios and risk premia are given for asset A (red solid line) and asset B (blue dashed line). The parameters for this case are given in column 'Identical Trees' of Table 1.

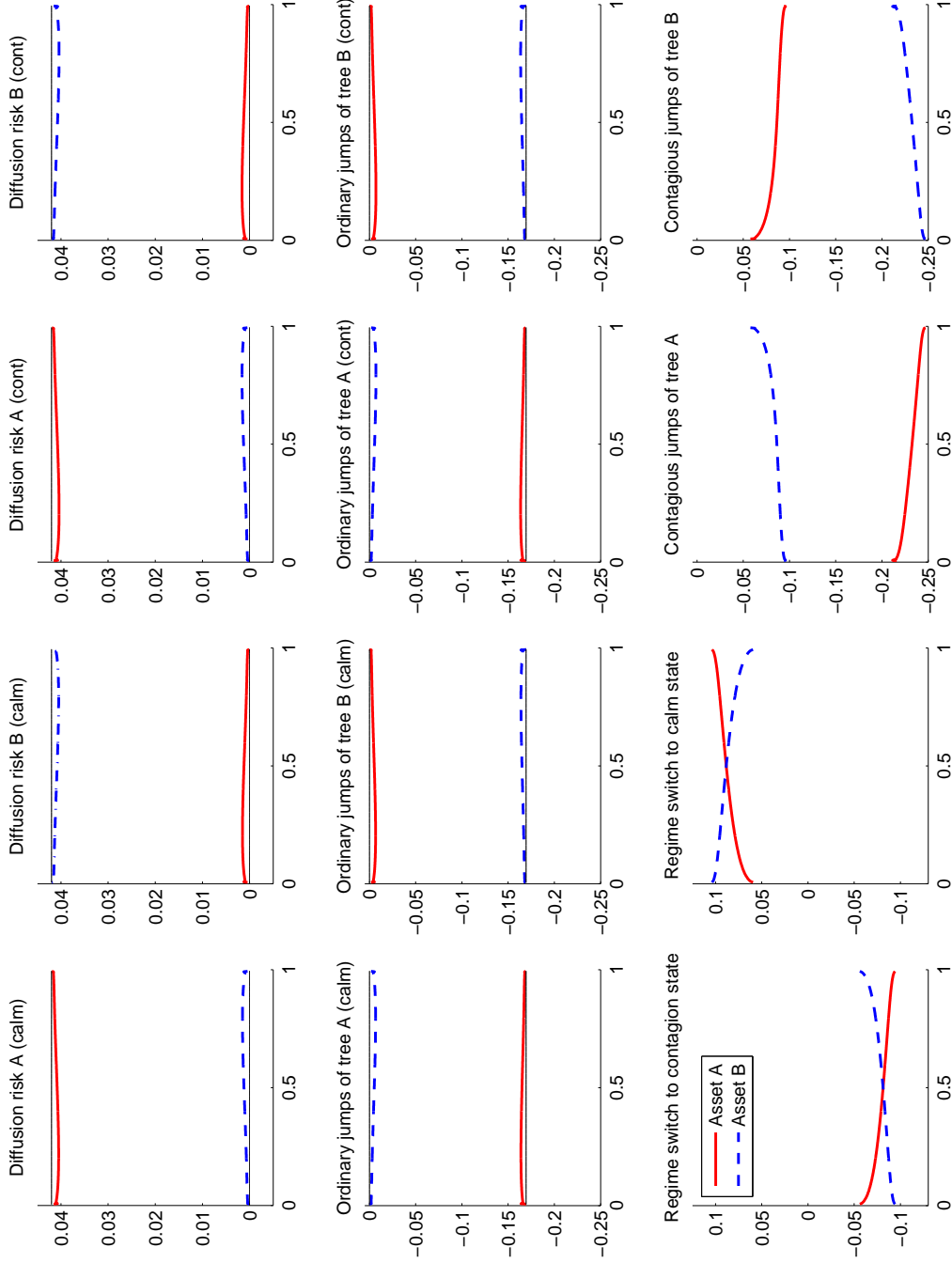


Figure 2: Exposures (Identical Trees)

The figure depicts the total price sensitivities to diffusion risk of tree A and tree B (upper row), to ordinary jumps that do not change the state (middle row) as well as to contagious jumps and to pure regime switches (lower row) as a function of the consumption share s_A of tree A . The sensitivities are given for asset A (red solid line) and asset B (blue dashed line). For comparison, we also give the dividend exposures (thin black lines). The parameters for this case are given in column 'Identical Trees' of Table 1.

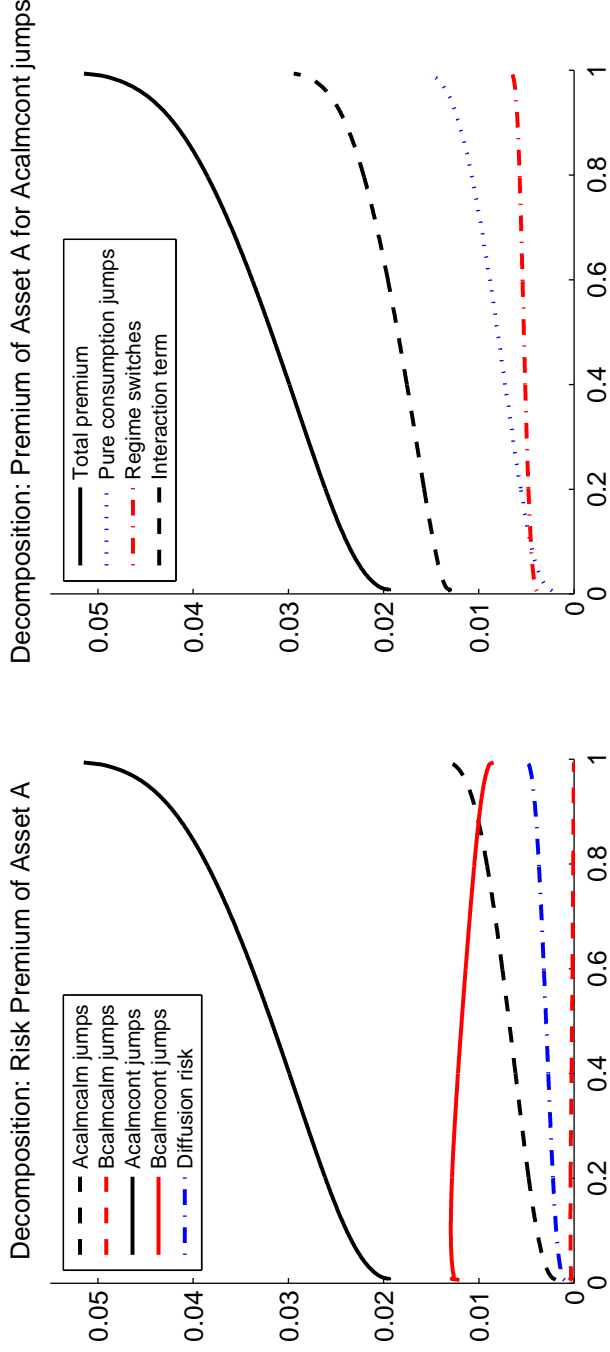


Figure 3: Decomposition of the Risk Premium (Identical Trees)

The left-hand graph decomposes the risk premium of asset A into the risk premia for ordinary jumps of tree A (black dashed line), ordinary jumps of tree B (red dashed line), contagious jumps of tree A (black solid line), contagious jumps of tree B (red solid line), and diffusion risk (blue dash-dotted line). The right-hand graph decomposes the premium for contagious jumps of tree A further into the (hypothetical) premium for pure consumption jumps (blue dotted line), the (hypothetical) premium for pure regime switches (red dash-dotted line) and the additional interaction term arising from our model structure (black dashed line). The independent variable in both graphs is the consumption share s_A of tree A . The parameters for this case are given in column 'Identical Trees' of Table 1.

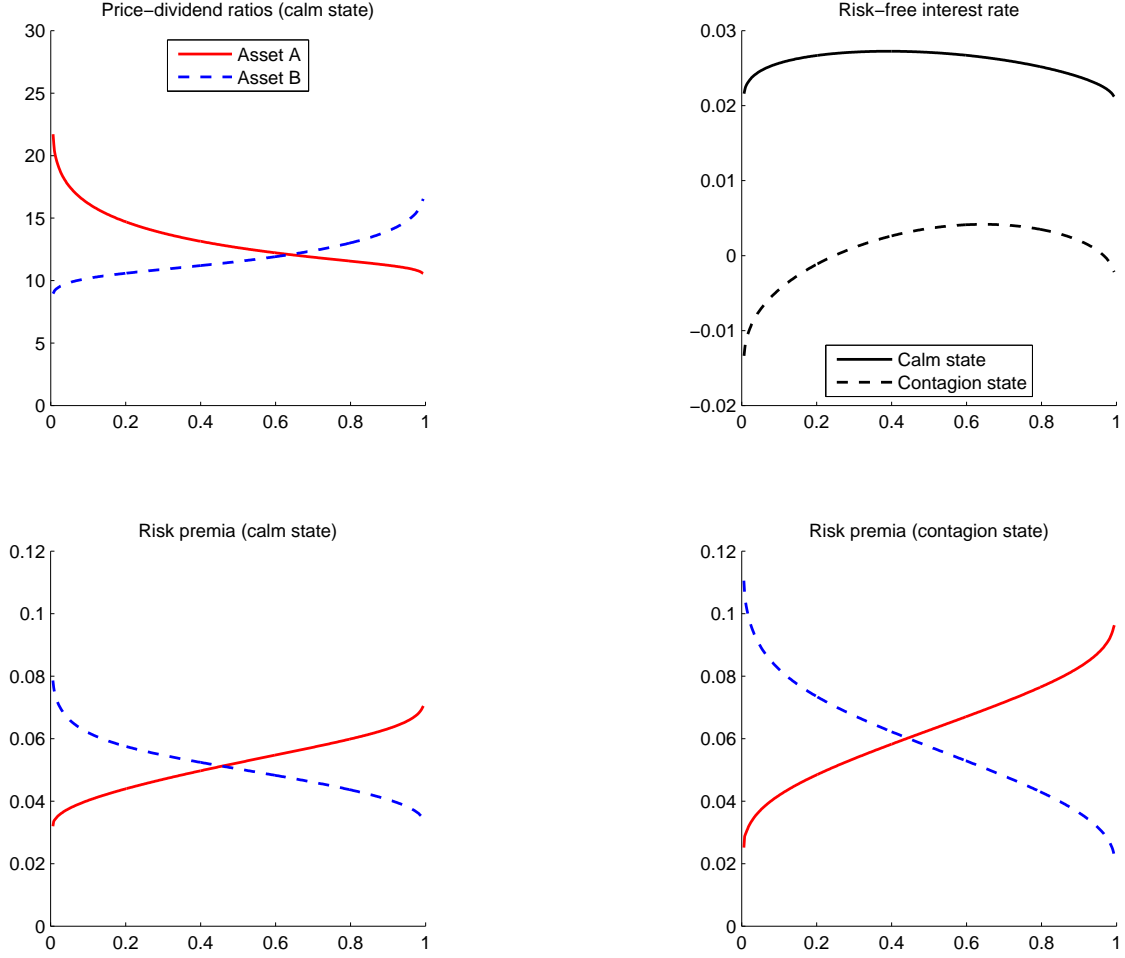


Figure 4: Asset Prices and Returns (Robust versus Contagion-sensitive Assets)

The figure depicts the price-dividend ratios in the calm state, the risk-free interest rate, and the risk premia in both states as a function of the consumption share s_A of tree A . Price-dividend ratios and risk premia are given for the robust asset A (red solid line) and the contagion-sensitive asset B (blue dashed line). The parameters for this case are given in column 'Robust versus Contagion-sensitive Assets' of Table 1.

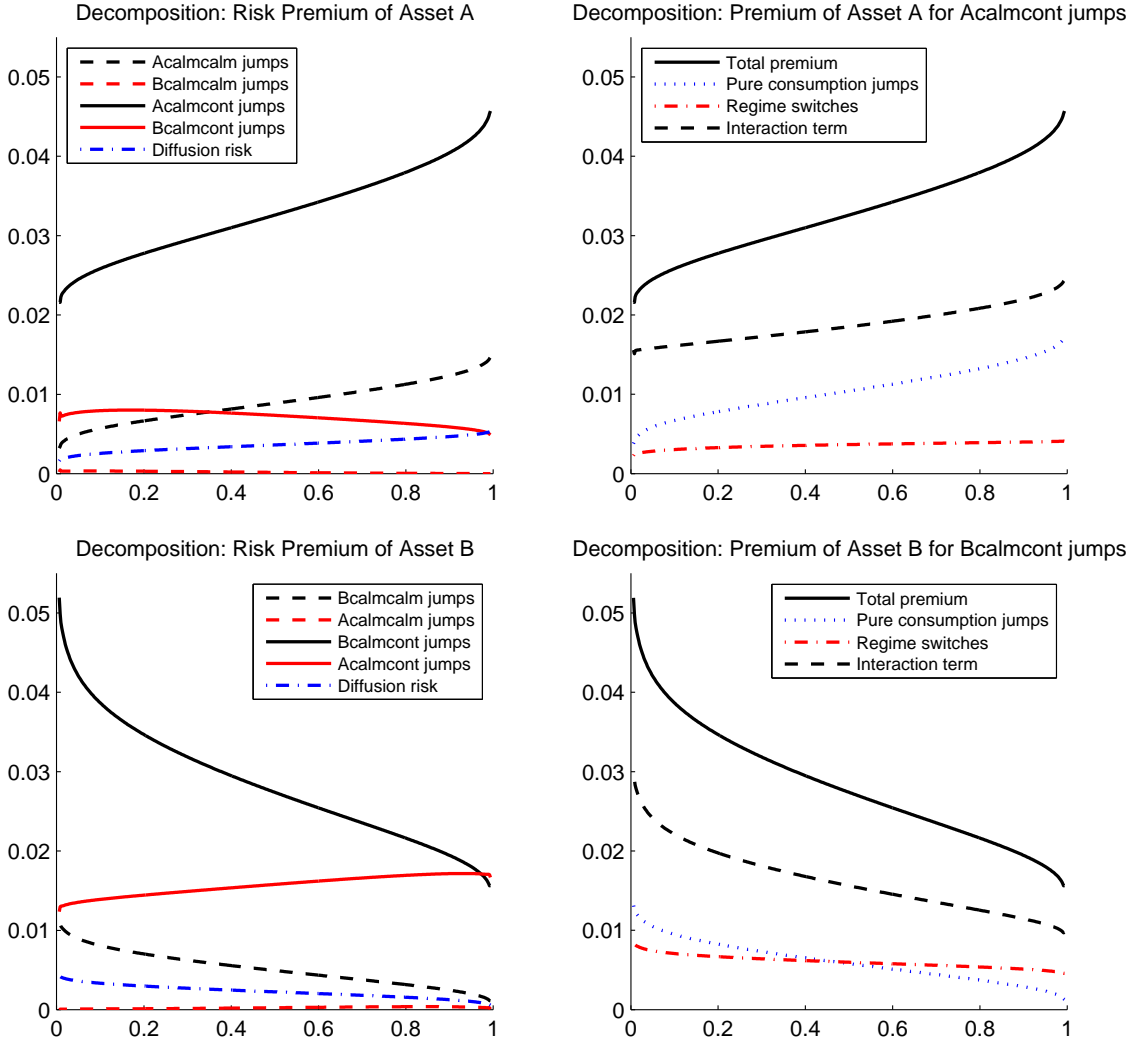


Figure 5: Decomposition of Risk Premia (Robust versus Contagion-sensitive Assets)

The figure depicts the various components of the risk premia of asset A and asset B as a function of the consumption share s_A of tree A . The upper left-hand graph decomposes the risk premium of the robust asset A into the risk premium for ordinary jumps of tree A (black dashed line), ordinary jumps of tree B (red dashed line), contagious jumps of tree A (black solid line), contagious jumps of tree B (red solid line), and diffusion risk (blue dash-dotted line). The upper right-hand graph decomposes the premium for contagious jumps of tree A further into the (hypothetical) premium for pure consumption jumps (blue dotted line), the (hypothetical) premium for pure regime switches (red dash-dotted line) and the additional interaction term arising from our model structure (black dashed line). The lower graphs depict similar decompositions for the risk premium of the contagion-sensitive asset B . The parameters for this case are given in column 'Robust versus Contagion-sensitive Assets' of Table 1.

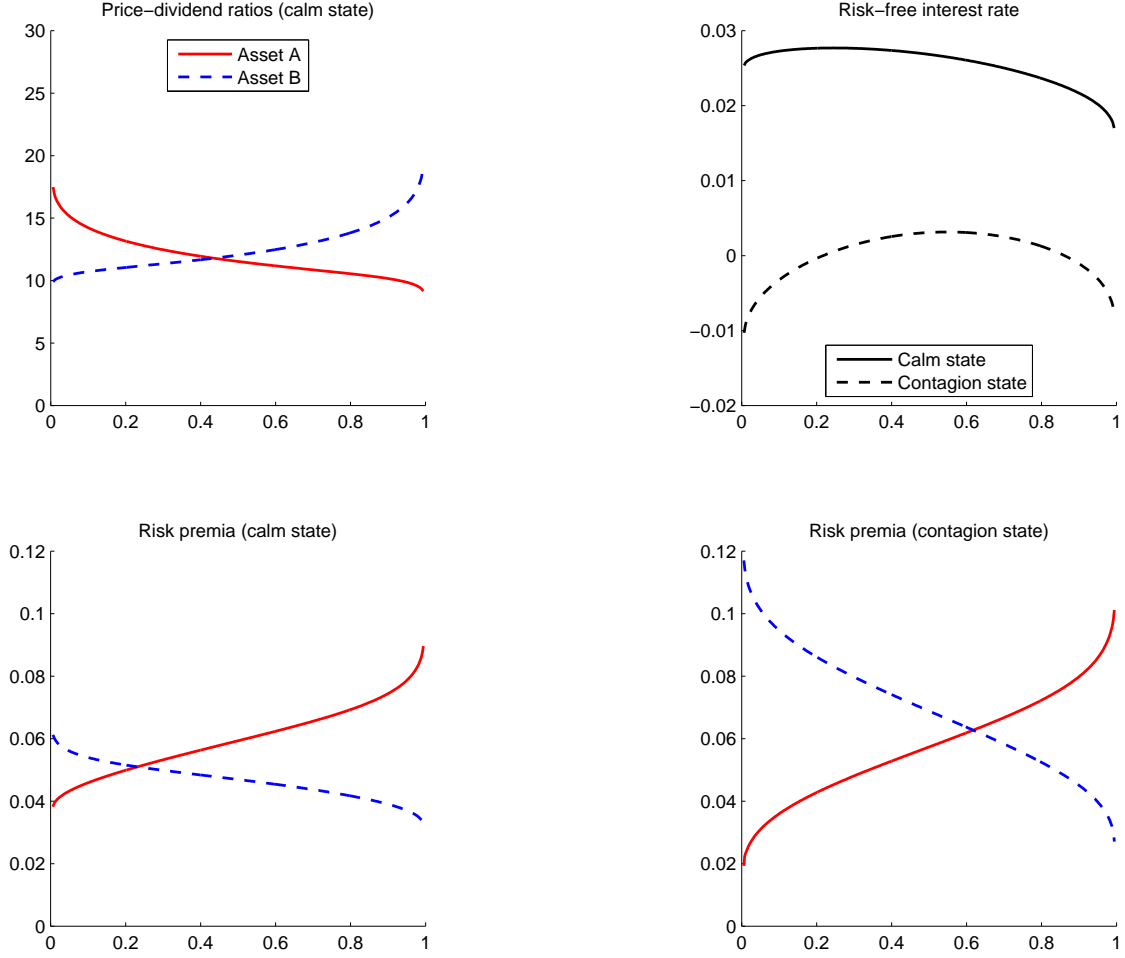


Figure 6: Asset Prices and Returns (Propensity to Trigger Contagion)

The figure depicts the price-dividend ratios in the calm state, the risk-free interest rate, and the risk premia in both states as a function of the consumption share s_A of tree A . Price-dividend ratios and risk premia are given for the contagion-triggering asset A (red solid line) and the non-toxic asset B (blue dashed line). The parameters for this case are given in column 'Propensity to Trigger Contagion' of Table 1.

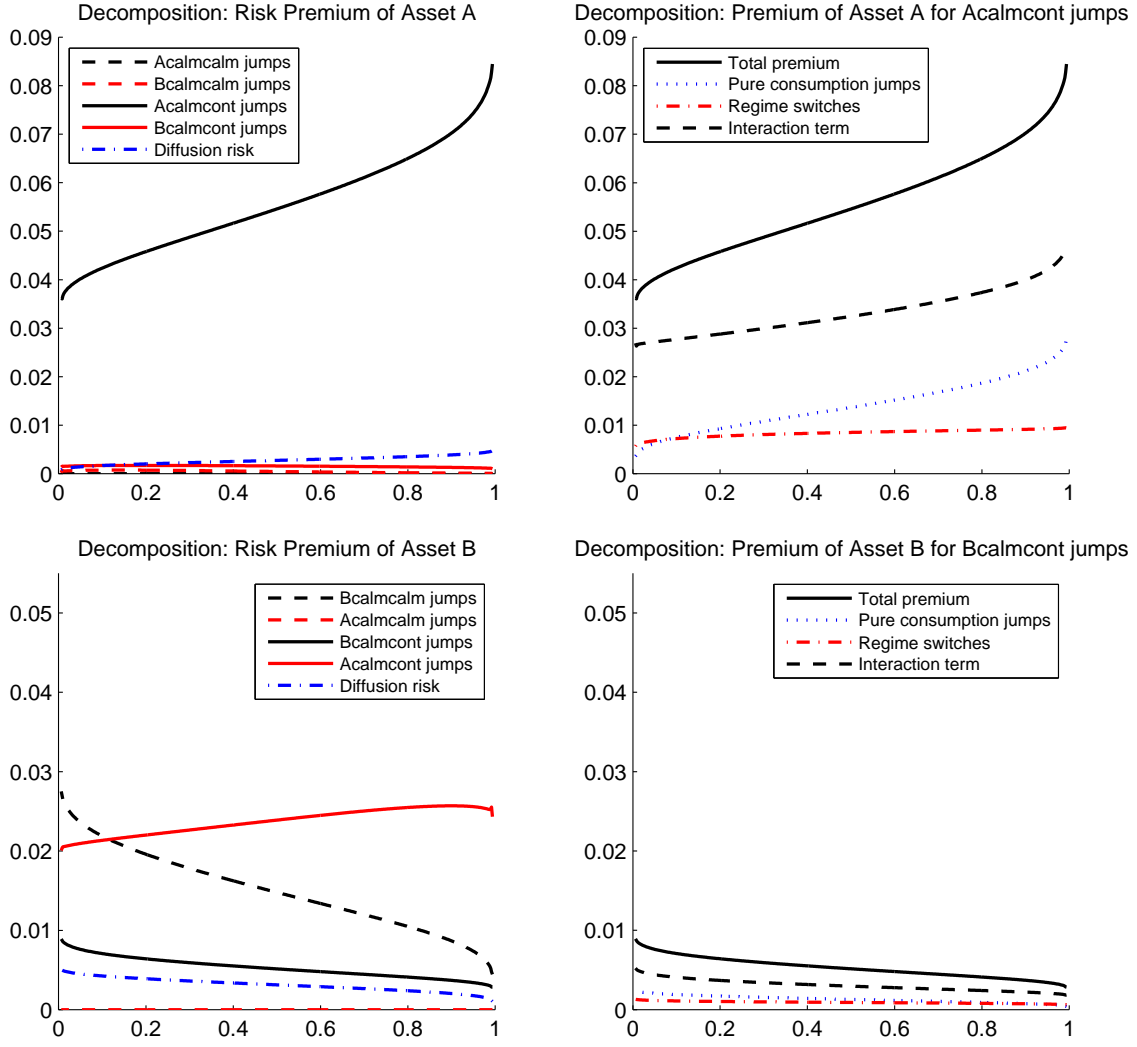


Figure 7: Decomposition of Risk Premia (Propensity to Trigger Contagion)

The figure depicts the various components of the risk premia of asset A and asset B as a function of the consumption share s_A of tree A . The upper left-hand graph decomposes the risk premium of the contagion-triggering asset A into the risk premia for ordinary jumps of tree A (black dashed line), ordinary jumps of tree B (red dashed line), contagious jumps of tree A (black solid line), contagious jumps of tree B (red solid line), and diffusion risk (blue dash-dotted line). The upper right-hand graph decomposes the premium for contagious jumps of tree A further into the (hypothetical) premium for pure consumption jumps (blue dotted line), the (hypothetical) premium for pure regime switches (red dash-dotted line) and the additional interaction term arising from our model structure (black dashed line). The lower graphs depict similar decompositions for the risk premium of the non-toxic asset B . The parameters for this case are given in column 'Propensity to Trigger Contagion' of Table 1.

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