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f_B and f_{B_s} with maximally twisted Wilson fermions

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We present a lattice QCD calculation of the heavy-light decay constants f_B and f_{B_s} performed with $N_f = 2$ maximally twisted Wilson fermions, at four values of the lattice spacing. The decay constants have been also computed in the static limit and the results are used to interpolate the observables between the charm and the infinite-mass sectors, thus obtaining the value of the decay constants at the physical b quark mass. Our preliminary results are $f_B = 191(14)\text{MeV}$, $f_{B_s} = 243(14)\text{MeV}$, $f_{B_s}/f_B = 1.27(5)$. They are in good agreement with those obtained with a novel approach, recently proposed by our Collaboration (ETMC), based on the use of suitable ratios having an exactly known static limit.

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1. Introduction

The study of B-physics plays a fundamental role within flavour physics both in accurately testing the Standard Model and in the search of New Physics effects. To this aim it is crucial to have theoretical uncertainties under control, in particular those of the hadronic parameters computed on the lattice.

With the available computer power it is not possible to simulate quark masses in the range of the physical b mass keeping, at the same time, finite volume and discretisation effects under control. In order to circumvent these problems, many different methods have been proposed so far (see ref. [1] for an up to date collection of results).

The approach that we have adopted and that we discuss below consists in using lattice QCD data with the heavy quark mass ranging from the charm region up to $\sim 4/5$ of the physical b quark mass, together with the information coming from the static limit point. In order to deal with the simulated light quark mass and finite lattice spacing, a careful extrapolation to the chiral and continuum limits has been performed. An alternative method, based on the introduction of suitable ratios having an exactly known static limit, has been recently proposed and investigated by our Collaboration (ETMC) [2].

In section 2 we describe the computation of the decay constants in the static limit; in section 3 we present the interpolation between the charm and infinite-mass sectors and compare the results with those obtained in ref. [2].

2. Heavy-light decay constant in the static limit of HQET

We have combined a light doublet of twisted-mass fermions ($\psi^T = (u, d)$) defined at maximal twist with a static quark described by the HYP2 action [3] to improve the signal-to-noise ratio [4]:

$$S^{\text{stat}} = a^4 \sum_x \bar{\psi}_h(x) \nabla_0^* \psi_h(x), \quad \nabla_0^* \psi_h(x) = \frac{1}{a} \left[\psi_h(x) - U_{\text{HYP2}}^\dagger(x - a\hat{0}) \psi_h(x - a\hat{0}) \right]. \quad (2.1)$$

In order to extract the decay constant using maximally twisted lattice QCD, we need to evaluate the matrix element of the static-light local current. At maximal twist the pseudoscalar current $(\mathcal{P}^{\text{stat}})_R$ in the physical basis, in terms of the twisted basis used in the numerical simulations (light quark fields $\chi^T = (\chi_u, \chi_d)$), is given by

$$(\mathcal{P}^{\text{stat}})_R(x) = (\bar{\psi}_h(x) \gamma_5 u(x))_R = \frac{1}{\sqrt{2}} (Z_P^{\text{stat}} P(x) + i Z_S^{\text{stat}} S(x)) \quad (2.2)$$

where $P = \bar{\psi}_h \gamma_5 \chi_u$ and $S = \bar{\psi}_h \chi_u$ are the pseudoscalar and scalar densities which renormalise with the Z_P^{stat} and Z_S^{stat} appropriate to the static-light framework.

We define $c_1 = i \langle 0 | \bar{\psi}_h \chi_u | B \rangle$ and $c_5 = \langle 0 | \bar{\psi}_h \gamma_5 \chi_u | B \rangle$ where $|B\rangle$ is the lattice ground state. At maximal twist, the amplitude we need to compute is $\Phi = f_B \sqrt{M_B} = (Z_S^{\text{stat}} c_1 + Z_P^{\text{stat}} c_5)$. The (bare) matrix elements c_1 and c_5 have been measured from an analysis following the static HQET spectrum study with twisted-mass fermions [5]. The ETMC ensembles $B_{1,2,3,4}$ and $C_{1,2}$ [6, 7] have so

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far been considered (i.e. two lattice spacings). Here we concentrate on the lightest heavy-light meson state, the pseudoscalar meson which we call here the B meson (or B_s with a strange valence quark). We take the value of m_q for the strange quark from the ETMC studies of the strange-light mesons [8, 9] which used the same gauge configurations as used here, namely $am_s = 0.022$ at $\beta = 3.9$ and 0.017 at $\beta = 4.05$. We measure the correlation of operators at source and sink with a large choice of operators: local and smeared; parity conserving and non-conserving. We then make a simultaneous fit to a sub-matrix (typically 6×6) in a given Euclidean time t interval. We chose this t -interval to have similar physical extent at different lattice spacings. We find that the non-local operators have weaker coupling to excited states, as expected. Such non-local operators can give a good determination of the energy levels but to extract the required matrix element (related to f_B) we need to include local operators in the fit. At $\beta = 3.9$ we use a 4 state fit with t/a range 4 – 10 but with the correlations that have local operators (at sink and/or source) we restrict to t/a range to 6 – 10. This choice gives acceptable values of χ^2 using correlated fits. We then make uncorrelated fits to determine the required energies and matrix elements with statistical errors determined by bootstrap. At $\beta = 4.05$ the appropriate t/a range is found to be 5 – 12 for smeared correlators and 7 – 12 for local ones. We have checked by making many different fits that the fit parameters are stable, within the statistical error assigned. For the correlations of B_s mesons, we make similar fits but find that the minimum t/a value has to be increased by 1 unit to preserve an acceptable (correlated) χ^2 .

Then one computes Z_p^{stat} and Z_s^{stat} in order to get the matrix element renormalised in HQET at a specific scale μ . We have chosen to renormalise it in the \overline{MS} scheme at $\mu = 1/a$ and for this preliminary account of our work the renormalisation is done perturbatively at 1 loop order. \overline{MS} is a continuum-like scheme defined within dimensional regularisation, while the regulator of our bare quantities is the inverse lattice spacing. So one needs a matching between both regularisations. It can be written as

$$\begin{aligned} \langle O(p, \mu) \rangle^{\text{DR}, \overline{MS}} &= \left[1 - \frac{\alpha_s}{4\pi} (-\gamma_0 \ln a^2 \mu^2 + C^O) \right] \langle O(p, a) \rangle^{\text{lat}} \\ &\equiv Z_O(a\mu) \langle O(p, a) \rangle^{\text{lat}}, \end{aligned} \quad (2.3)$$

where the renormalisation scheme and scale of the coupling constant α_s is not specified at this level of perturbation theory. Expressions of $C^{P(S)}$ are complicated and not illuminating, essentially due to the HYP-smeared static action and the improved part of the gluon propagator [10]. Thus we have simply collected the numerical values of Z_p^{stat} and Z_s^{stat} in Table 1 for a boosted coupling $g_p^2 = g_0^2 / \langle U_P \rangle$ (where $g_0^2 = 6/\beta$ and $\langle U_P \rangle$ is the average plaquette value). It turns out that the systematic error introduced by a poor determination of the ratio $z_r = Z_s^{\text{stat}} / Z_p^{\text{stat}}$ is minimal, especially on the

β	Z_p^{stat}	Z_s^{stat}
3.9	0.849	0.933
4.05	0.859	0.938

Table 1: First order perturbation theory renormalisation factors of the pseudoscalar and scalar static-light dimension 3 operators in the \overline{MS} scheme at the scale $\mu = 1/a$.

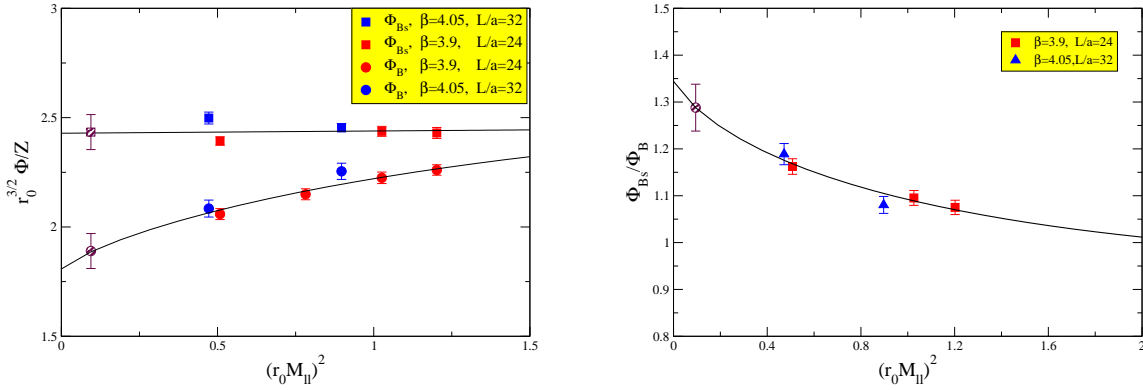


Figure 1: Left plot: unrenormalised heavy-light decay constant combination $r_0^{3/2} \Phi/Z(4.05)$ (with $Z \equiv (Z_P^{\text{stat}} + Z_S^{\text{stat}})/2$) versus the squared mass of the pion built of the light sea quarks. The circles represent the B meson case, where the valence light quark is equal to the sea quarks. The squares represent the B_s meson case, where the light valence quark is the strange quark. The data at $\beta = 3.9$ (red symbols) have been multiplied by the appropriate factor to match the same scale for the data at $\beta = 4.05$. The curves represent the NLO HMChPT theory expressions. Right plot: the ratio $\frac{\Phi_{B_s}}{\Phi_B}$ versus the squared mass of the pion built of the light sea quarks. The curve represents the NLO heavy quark chiral perturbation theory.

ratio of the B and B_s decay constants. We thus present in fig. 1 the bare matrix element, which depends on the ratio z_r only.

Once the matrix element $\Phi^{\overline{MS}}(\mu = 1/a)$ has been renormalised in the \overline{MS} scheme at the scale $\mu = 1/a$ a NLO running of perturbation theory [11] has been applied to evolve it to a scale $\mu = M_B^{\text{exp}}$. This is what is needed to perform a fit together with the relativistic data matched to HQET at the same scale (see next section).

The extrapolation of Φ_B down to the physical pion has been performed with Heavy Meson Chiral Perturbation Theory (HMChPT) at NLO by using the formula [12 – 14]

$$\begin{aligned} \frac{\Phi_B}{\Phi_0} &= 1 - \frac{3(1 + 3\hat{g}^2)}{4} \frac{M_{ll}^2}{(4\pi f)^2} \log\left(\frac{M_{ll}^2}{(4\pi f)^2}\right) + \alpha_1 M_{ll}^2, \\ \frac{\Phi_{B_s}}{\Phi_{0s}} &= 1 + \alpha_{1s} M_{ll}^2, \end{aligned} \quad (2.4)$$

where M_{ll} denotes the simulated pion masses, f stands for the light decay constant in the chiral limit, while $\Phi_{0(s)}$ and $\alpha_{1(s)}$ are free fit parameters. The \hat{g}^2 coupling has been fixed to 0.2 [15, 16], and we have checked that a change of 50% in the value of \hat{g}^2 results in a shift in Φ_B which is well below the statistical error. The chiral extrapolation of Φ_B , Φ_{B_s} and the ratio Φ_{B_s}/Φ_B is shown in fig. 1. This figure also illustrates that we find consistent results at our two available lattice spacings within the relatively large errors. We do not have enough data to include explicit discretisation error terms in the fit formula. However it seems that cut-off effects are quite small. This is more evident for the ratio $\frac{\Phi_{B_s}}{\Phi_B}$ which is consistent with having no cutoff effects (see right plot of fig. 1).

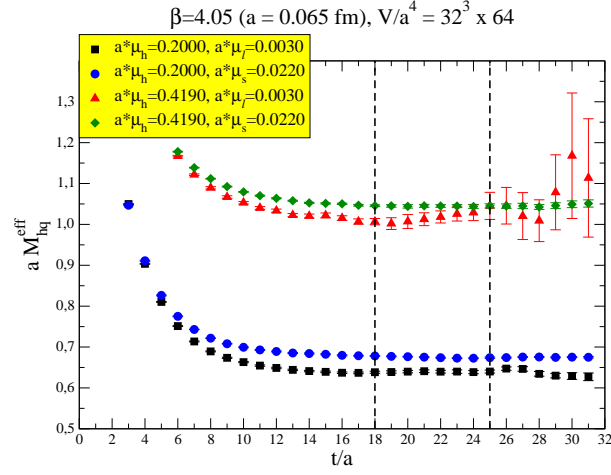


Figure 2: Effective masses at $\beta = 4.05$ for two heavy-light (hl) and two heavy-strange (hs) quark mass combinations. The two heavy quark masses correspond approximately to the physical charm quark mass and to $\sim 2/3$ of the value of the physical b quark mass.

3. Relativistic results and interpolation to the physical b quark mass

We perform an interpolation of the heavy-light (hl) decay constants from the charm region up to the bottom mass, by including data in the static limit calculated in the HQET as explained in the previous section. The lattice QCD data used in this analysis are at four values of the lattice spacing $a \approx 0.100, 0.085, 0.065, 0.050$ fm (corresponding to $\beta = 3.8, 3.9, 4.05, 4.2$), that is we have used the configuration ensembles denoted in [6, 7] as $A_{2,3}$, $B_{1,2,3,4,6,7}$, $C_{1,2,3}$ and D_2 , respectively. We have simulated for each ensemble 16 heavy quark masses in the range $m_c^{phys} \lesssim m_h \lesssim 0.8m_b^{phys}$. Quark propagators with different valence masses are obtained using the so called multiple mass solver method [17]. In fig. 2 we show for illustrative purpose the effective masses at $\beta = 4.05$ and for few quark mass combinations.

The analysis is performed by studying the dependence of the decay constants, more precisely of the quantity $\Phi_{hq} = f_{hq}\sqrt{M_{hq}}$, as a function of the meson masses, as in our recent analysis of the f_D and f_{D_s} decay constants [9].

In order to make use of the HQET scaling law we introduce for each simulated hq meson mass M_{hq} the HQET quantity that is finite in the static limit [11]:

$$\Phi_{hq} = \left(\frac{\alpha^{\overline{\text{MS}}}(M_{hq})}{\alpha^{\overline{\text{MS}}}(M_B^{\text{exp}})} \right)^{-\gamma_0/(2\beta_0)} \cdot \left[1 - \left(\frac{439}{1089} - \frac{28\pi^2}{297} \right) \frac{\alpha^{\overline{\text{MS}}}(M_{hq}) - \alpha^{\overline{\text{MS}}}(M_B^{\text{exp}})}{4\pi} \right] \cdot \left[1 + \frac{8}{3} \frac{\alpha^{\overline{\text{MS}}}(M_{hq})}{4\pi} \right] \cdot (\Phi_{hq})_{QCD}, \quad (3.1)$$

which has been obtained through the NLO matching from QCD to HQET and evolving at NLO to the renormalisation scale given by the experimental value of the B meson mass. For Φ_{hq} ($q = l, s$) we first study the dependence on the light/strange quark mass at fixed heavy mass through the

following functional forms

$$\begin{aligned}\Phi_{hl} &= A(a, m_h) \cdot \left(1 - \frac{3(1+3\hat{g}^2)}{4} \cdot \frac{M_{ll}^2}{(4\pi f)^2} \cdot \log \left(\frac{M_{ll}^2}{(4\pi f)^2} \right) + B \cdot M_{ll}^2 \right), \\ \Phi_{hs} &= A'(a, m_h) \cdot (1 + B' \cdot M_{ll}^2 + C'(a) \cdot M_{ss}^2). \end{aligned} \quad (3.2)$$

We note that the fit forms above follow from the HMChPT formulae [12–14], which we have already used in the static sector (see eq. (2.4)). A dependence of the coefficients A, A', C' on the lattice spacings is allowed, in order to account for discretisation effects. The extrapolation/interpolation to the physical light/strange quark mass is performed by replacing in eq. (3.2) $M_{ll}^2 = (M_\pi^{exp})^2$, $M_{ss}^2 = 2(M_K^{exp})^2 - (M_\pi^{exp})^2$. This first step provides the values of the decay constants at the physical light/strange quark mass for every simulated lattice spacing and heavy quark mass, or equivalently the quantities $\Phi_{hq^{phys}}$.

The second step consists in studying the dependence of $\Phi_{hq^{phys}}$, included the available static points, on the heavy quark mass and on the lattice spacing, in order to interpolate to the b quark mass and to extrapolate to the continuum limit. Several functional forms with different $\mathcal{O}(a^2)$ and $\mathcal{O}(a^4)$ discretisation terms have been tried, which can be written in a compact way as

$$\Phi_{hl^{phys}(hs^{phys})} = \sum_{n,k} P_{nk} a^{2n} M_{hq}^{2n-k}, \quad (n = 0, 1, 2; k = 0, 1, 2), \quad (3.3)$$

where M_{hq} is a reference meson mass with the same simulated heavy quark mass as in the fitted quantity Φ and the light quark mass is fixed to a similar value for all data. We have performed correlated fits by assuming the static results uncorrelated with the relativistic data.

The results for the decay constants f_B and f_{B_s} are finally obtained by replacing in eq. (3.3) $M_{hq} = M_{hs} = M_{B_s}^{exp}$, setting the lattice spacing equal to zero and performing the matching from HQET back to QCD at NLO.

The dependence of the decay constants on the hq meson mass is shown in fig. 3 where, for illustrative purpose, we also show curves corresponding to one of the various fits. The discretisation terms included in the shown fits are of $\mathcal{O}(a^2 M_{hq})$, $\mathcal{O}(a^2 M_{hq}^2)$ and $\mathcal{O}(a^4 M_{hq}^4)$ for both $\Phi_{hl^{phys}}$ and $\Phi_{hs^{phys}}$. We observe that with our data it is not possible to determine the coefficients of more than three discretisation terms for each fit and that, in some cases, only two out of three parameters turn out to be different from zero. About twenty of these fits have a chi square per degree of freedom of order one or smaller and are considered in deriving our final result for f_B and f_{B_s} . The spread among these fits is included in the systematic uncertainty.

Our preliminary results for f_B , f_{B_s} and the ratio read¹

$$\begin{aligned}f_B &= 191(6)(12)(3) \text{ MeV} = 191(14) \text{ MeV}, \\ f_{B_s} &= 243(6)(12)(3) \text{ MeV} = 243(14) \text{ MeV}, \\ f_{B_s}/f_B &= 1.27(3)(4) = 1.27(5), \end{aligned} \quad (3.4)$$

where: i) the first error is of statistical plus fitting origin, ii) the second error, estimated through the spread of the results obtained with functional forms containing different discretisation terms,

¹The results given in the present proceedings are based on a larger statistical sample w.r.t to the values presented at the Conference and cited in ref. [1].

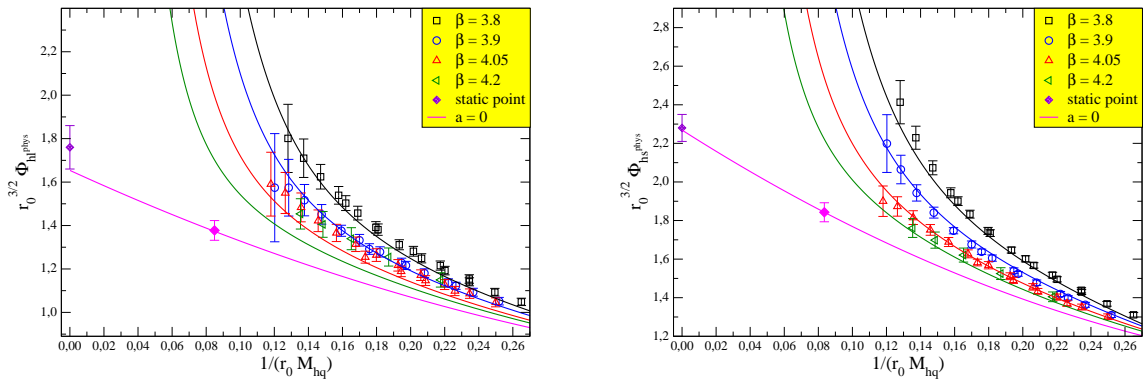


Figure 3: Interpolation to the b quark mass and continuum extrapolation of Φ_{hlphys} (left) and Φ_{hsphys} (right).

represents the residual uncertainty due to the continuum limit and to the b mass interpolation, iii) the third error takes into account the effect of the systematic uncertainty on the static point.

We conclude by comparing the results in eq. (3.4) with those obtained in ref. [2] using suitable ratios having an exactly known static limit. The latter values read

$$\begin{aligned} f_B &= 194(16) \text{ MeV}, \\ f_{B_s} &= 235(11) \text{ MeV}, \end{aligned} \quad (3.5)$$

where the uncertainty is the sum in quadrature of the statistical and systematic errors. The two sets of results are in very good agreement, thus providing further confidence on their robustness. We note that the results in eq. (3.5) are obtained from a subset of the data analysed in the present study. The inclusion of the full set of data is in program for a forthcoming publication.

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