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Screened perturbation theory for 3d Yang-Mills theory and the magnetic modes of hot QCD

Owe Philipsen*
Institut für Theoretische Physik, Westfälische Wilhelms-Universität Münster, 48149 Münster, Germany
E-mail: ophil@uni-muenster.de

Daniel Bieletzki
Institut für Theoretische Physik, Westfälische Wilhelms-Universität Münster, D-48149 Münster, Germany
E-mail: Daniel.Bieletzki@uni-muenster.de

York Schröder
Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany
*E-mail:*yorks@physik.uni-bielefeld.de

Perturbation theory for non-abelian gauge theories at finite temperature is plagued by infrared divergences which are caused by magnetic soft modes $\sim g^2T$, corresponding to gluon fields of a 3d Yang-Mills theory. While the divergences can be regulated by a dynamically generated magnetic mass on that scale, the gauge coupling drops out of the effective expansion parameter requiring summation of all loop orders for the calculation of observables. Some gauge invariant possibilities to implement such infrared-safe resummations are reviewed. We use a scheme based on the non-linear sigma model to estimate some of the contributions $\sim g^6$ of the soft magnetic modes to the QCD pressure through two loops. The NLO contribution amounts to $\sim 10\%$ of the LO, suggestive of a reasonable convergence of the series.

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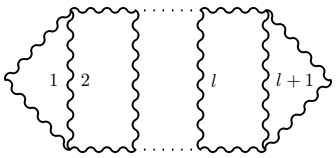

Figure 1: Feynman diagram contributing to the pressure.

1. Introduction

Perturbation theory for static quantities of non-abelian gauge theories at finite temperature (T) features three scales: πT associated with the non-zero Matsubara modes that arise due to the compactness of the Euclidean time direction, and gT, g^2T, where g is the gauge coupling, which are associated with the screening of colour-electric and magnetic fields, respectively. A prohibitive obstacle for the evaluation of perturbative series are the well-known infrared divergences due to the magnetic modes. For example, the thermodynamic pressure corresponds to all vacuum loop diagrams evaluated with finite temperature Feynman rules, and one $(l+1)$-loop example is shown in Fig. 1. The Matsubara sum also contains a bosonic zero mode, and its contribution to the pressure is, parametrically,

$$\sim g^{2l}\left(T\int d^3p\right)^{l+1} p^{2l}(p^2+m^2)^{-3l} \sim g^6T^4(g^2T/m)^{l-3}\text{for} \quad l>3, \tag{1.1}$$

where we have introduced a mass as infrared regulator in the propagators. Clearly, for $m\to 0$ the contribution diverges. One may argue that the full theory dynamically generates a magnetic mass scale, $m = const.g^2T$, to regulate this divergence. After all, we know that non-perturbatively the theory is finite. However, in that case we see that the coupling constant drops out of the expansion parameter and *all* higher loop orders contribute to the order g^6 in the pressure. This is the Linde-problem [1], which occurs sooner or later in any perturbatively computed observable, e.g. in the Debye mass already at NLO, no matter how weak the coupling. Thus, at finite temperature, perturbation theory is only well-defined to some observable-dependent low order. Note that this IR-problem in the zero-mode sector is equivalent to the one in 3d Yang-Mills theory.

In this contribution we reconsider a resummation method for 3d Yang-Mills theory, viz. the finite T zero mode sector, which has been developed a while ago [2,3]. In view of current and future heavy ion collision experiments there is an urgent need for non-perturbative theoretical predictions. On the other hand, straightforward Monte Carlo simulations of lattice QCD do not work for finite baryon densities or dynamical problems involving real time (for a recent review, see e.g. [4]), motivating also analytical attempts. As the simplest observable to test our resummation scheme, we therefore consider the contribution of the infrared sector represented by the 3d Yang-Mills theory to the thermodynamic pressure of the QCD plasma. Of course, the equation of state *can* be computed without major problems on the lattice, at least for a temperature range $T\lesssim 5T_c$. The idea here is to try and develop resummation schemes, test them against the pressure and in case of satisfactory results apply them to other quantities. In the thermal context, resummation schemes

have already been successfully employed in scalar theories through four-loop level [5], but it is not straightforward to generalise these methods to gauge theories. Recently a scheme based on Hard Thermal Loop terms was attempted for non-abelian gauge theory, though not including the magnetic modes [6]. We therefore briefly review the contribution of the 3d gauge theory to 4d thermodynamics, before presenting our resummation scheme and evaluating it through two-loop order.

2. High T effective theory from dimensional reduction

At sufficiently high temperatures, the previously mentioned scales get separated hierarchically, $g^2T \ll gT \ll \pi T$. This suggests to successively integrate out those scales, which has been done systematically at the two-loop level. Here we follow [7], to which we refer for details and references.

In the first step, the hard non-zero Matsubara modes are integrated out, leading to so-called EQCD as an effective theory for modes $\sim gT$ and softer,

$$\begin{aligned} p_{QCD}(T) &= p_E(T) + \frac{T}{V}\ln\int DA_k^a DA_0^a \exp\left(-\int d^dx \mathscr{L}_E\right) \\ \mathscr{L}_E &= \frac{1}{2}TrF_{kl}^2 + Tr\left[D_k, A_0\right]^2 + m_e^2 TrA_0^2 + \lambda_E^{(1)}(TrA_0^2)^2 + \lambda_E^{(2)} TrA_0^4 + \dots , \end{aligned}$$

where to leading order the parameters of the effective theory are $p_E \sim T^4$; $\quad m_E^2 \sim g^2T^2$; $\quad g_E^2 \sim g^2T$; $\quad \lambda_E^{(1)} \sim g^4T$; $\quad \lambda_E^{(2)} \sim g^4T$. Note that by performing the Matsubara sum the theory has become effectively three-dimensional, and the electric gauge field component now represents an adjoint scalar field. The dots represent higher dimension operators that are suppressed by inverse powers of T. This effective theory still contains the two dynamically generated scales gT and g^2T. In a second step, the A_0 field and all modes living on the scale gT can be integrated out as well. This step already requires some resummation, but can still be performed as a series in the coupling,

$$\begin{aligned} \frac{T}{V}\ln\int DA_k^a DA_0^a \exp\left(-\int d^dx \mathscr{L}_E\right) &= p_M(T) + \frac{T}{V}\ln\int DA_k^a \exp\left(-\int d^dx \mathscr{L}_M\right) \\ \mathscr{L}_M &= \frac{1}{2}TrF_{kl}^2 + \dots , \end{aligned} \tag{2.1}$$

with matching coefficients $p_M \sim m_E^3 T$ and $g_M^2 \sim g_E^2$. This is an effective theory for the ultra-soft modes $\sim g^2T$, its leading term being equivalent to 3d Yang-Mills theory.

At this stage we can evaluate the contributions to the pressure from the different momentum scales separately and with appropriate methods. The contributions $p_E(T) + p_M(T)$ can be computed as a series in the coupling constant, whereas the contribution of $p_M(T)$ is completely non-perturbative. This is the part of the theory giving the g^6 contribution to the pressure where the Linde-problem surfaces, and for dimensional reasons one can give its dependence on the coupling constant,

$$\begin{aligned} p_G(T) &= \frac{T}{V}\ln\int DA_k^a \exp\left(-S_M\right) \sim Tg^6 \\ \frac{p_G(T)}{T\mu^{-2\varepsilon}} &= d_A C_A^3 \frac{g_M^6}{(4\pi)^4}\left[\alpha_G\left(\frac{1}{\varepsilon} + 8\ln\frac{\bar{\mu}}{2m_G}\right) + \beta_G + O(\varepsilon)\right] . \end{aligned} \tag{2.2}$$

The coefficients α_G, β_G first receive contributions at the four-loop level. There are similar (and more lengthy) expressions for $p_E(T) + p_M(T)$, for the full result see [7]. It is well-known that whatever renormalisation is necessary and sufficient in the vacuum will also be sufficient at finite temperature. The epsilon poles appearing in the contributions of the different scales are due to the break up of the momentum integration range, thus the sum of all divergences has to cancel in the full pressure.

Here our interest is merely in the contribution from the 3d Yang-Mills part. Whereas α_G is known analytically [7], the infrared coefficient β_G is receiving contributions from all loop levels and needs to be evaluated non-perturbatively, e.g. by lattice simulations of the effective theory [8], leading to a numerical value (with a certain error bar). In the following we discuss possibilities for an analytical evaluation by resummation methods.

3. Resummation schemes: general idea

The general idea of a resummation is, at some given order in a perturbative scheme, to sum up higher order contributions (infinitely many in our case) into the current one. In order to avoid double counting, these contributions must then be left out at the order where they naturally occur. In other words, the perturbative scheme gets reorganised in some systematic way. (For a discussion of various schemes along those lines used in the context of thermal field theory, see [9]). This can be formalised by rewriting the Lagrangian serving for the perturbative expansion as [10]

$$\mathscr{L} = \frac{1}{\ell}\left[\mathscr{L}_M(\sqrt{\ell}X) + \mathscr{L}_\phi(\sqrt{\ell}X) - \ell\mathscr{L}_\phi(\sqrt{\ell}X)\right] , \tag{3.1}$$

where X stands for a generic field; $\mathscr{L}_M$ is the theory under study, here taken as the 3d Yang-Mills theory of Eq. (2.1); the modification $\mathscr{L}_\phi$ contains fields of the original Lagrangian and possibly auxiliary fields; and ℓ is a counting parameter introduced to systematise the resummed expansion, which is now an expansion in powers of ℓ. At the end of a calculation it is to be set to $\ell = 1$, for which the Lagrangian is identical to the original one and the theory is unchanged. However, during a perturbative evaluation the contributions of $\mathscr{L}_\phi$ are subtracted out one order higher than they are added in, hence at any finite order of the expansion the result will differ from the unresummed one. For example, the one-loop vacuum diagrams contributing to the pressure will be of order ℓ^0 while two-loop contributions count as ℓ^1. In particular, if $\mathscr{L}_\phi$ is chosen to represent a mass term for the gluon, this will regulate the infrared divergences.

Simply adding and subtracting a mass term for the gluon will not be sufficient, however, as we would obtain gauge-dependent and hence arbitrary results at every finite order in our resummed scheme. The question then is how to resum in a gauge-invariant way. The answer is that we need to resum such that we maintain the Slavnov-Taylor identities of our gauge theory. Clearly, this is not just to make things pretty, but a necessity in order not to change the original theory in the process of resumming.

4. The non-linear sigma model providing a gluon mass term

A well-known and time honoured way to introduce mass for gauge fields is by means of the Higgs effect, i.e. gauged sigma models [11]. For example, in the $SU(2)$-Higgs model used in the

electroweak theory, we have

$$\begin{aligned} \mathscr{L}_\sigma &= \mathrm{Tr}\,(D_i\Phi)^\dagger(D_i\Phi) + \mu^2\mathrm{Tr}\,(\Phi^\dagger\Phi) + 2\lambda\,\mathrm{Tr}\,(\Phi^\dagger\Phi)^2 \\ \phi &= \frac{1}{2}(\sigma + i\tau^a\pi^a) \quad \text{with} \quad T^a = \frac{\tau^a}{2} \\ \sigma^2 &= v^2 + (\pi^a)^2 \quad \text{with} \quad <\phi>= v\,. \end{aligned} \tag{4.1}$$

The gauge field gets a mass $m = g_M v/2$ through the expectation value of the scalar field.

However, in the current context we don't want an additional physical scalar particle, so we decouple the σ by sending $\mu, \lambda \to \infty$ while keeping v constant, which takes us to the non-linear sigma model. Choosing a parametrisation by unitary matrices generalises to $SU(N)$ and we have

$$\mathscr{L}_\phi = \mathrm{Tr}\,[(D_i\phi)^\dagger D_i\phi], \quad \phi(x) = \frac{m}{g_M}\exp(i\frac{g_M}{m}\pi'^a T^a), \quad \pi'^a \to \pi^a + O(\pi^3, \pi^5, ...), \tag{4.2}$$

with the usual covariant derivative $D_i = \partial_i - ig_M A_i$. What is the meaning of the remaining scalar degrees of freedom? This can be understood by considering the resummed partition function [10]

$$Z = \int DAD\phi\ \Delta_{FP}\exp\left[-\frac{1}{\ell}(S_{YM} + S_\phi - \ell S_\phi + S_{gf})\right]\,, \tag{4.3}$$

with some gauge fixing that only depends on the gauge field, and the corresponding Faddeev-Popov determinant Δ_{FP}. We can now transform to unitary gauge by $A_i \to A_i^U$ with $U = \exp(i\pi^a T^a)$ and integrate over the auxiliary field. Up to a delta-function this gives

$$\int D\phi\ \Delta_{FP}\, e^{-S_{gf}} = 1\,, \tag{4.4}$$

such that we end up with

$$Z = \int DA\ \exp\left[-\frac{1}{\ell}(S_{YM} - m^2\int \mathrm{Tr}A^2 + \ell m^2\int \mathrm{Tr}A^2)\right]\,, \tag{4.5}$$

which is just pure gauge theory with a gauge invariantly resummed mass term. Unitary gauge is not well suited for perturbative higher order calculations because of the bad UV-behaviour of gauge field propagators. Hence we shall proceed in a slightly different way later on, but these manipulations illustrate that the auxiliary field is merely a gauge degree of freedom and does not add anything undesired to the theory.

Obviously, the choice of the resummation term $\mathscr{L}_\phi$ above is not unique. There are other choices compatible with gauge invariance that have been considered in the past, like $\mathscr{L}_\phi = F_\mu \frac{1}{D^2} F_\mu$, with $F_\mu = \varepsilon_{\mu\alpha\beta}F_{\alpha\beta}$ [10]. This term does not require an auxiliary field, but at the price of being non-local. Another non-local choice is based on the Chern-Simons eikonal and appears naturally in the framework of Hard Thermal Loop resummations [12]. Yet another possibility to give mass to the gauge field in a gauge invariant way is by the pinch technique [13]. Different gauge invariant additions/subtractions correspond to different ways of resumming the theory, and it is difficult to say a priori which ones are better or worse. Moreover, there is no small expansion parameter here. Writing $m = Cg_M^2 = Cg^2T$, the dimensionless expansion parameter (modulo factors of $1/(4\pi)$) is

$$\frac{g_M^2}{m} = \frac{g_M^2}{Cg_M^2}, \tag{4.6}$$

	ref.	m/g_M^2
1-loop gap eq.	[12]	0.38
	[2, 10]	0.28
	[13]	0.25
2-loop gap eq.	[14]	0.34
lattice Landau	[15]	0.456(6)

Table 1: Comparison of magnetic mass values for the pole mass of transverse gluons in SU(2), $m = Cg_M^2$, as obtained from gap equations and from gauge fixed lattice simulations.

where C is a numerical coefficient that will be discussed in the next section (see also Table 1). The coupling constant drops out and we are effectively expanding in a dynamically generated number. Thus, there is no limit of parameters (not even $g_M \to 0$) in which the scheme is guaranteed to work. The convergence properties are due to the dynamics of the theory and can only be seen empirically by computing several orders of a given quantity.

5. Self-consistency and the magnetic mass

A priori the coefficient C for the infrared cut-off is not determined. Since it gets subtracted out again at higher orders in the resummation scheme it could, e.g., be treated as a variational parameter with some freedom to optimise the expansion. On the other hand, the resummation scheme of choice can be used to compute any observable, in particular one might consider the self-energy of the gluon. Defining the "magnetic mass" to be the pole mass of the transverse part of the full gluon propagator, we obtain a gap equation for m,

$$D_{\rm trans}(p^2) = \frac{1}{p^2+m^2-\Pi_{\rm trans}(p^2)} \sim \frac{1}{p^2+m^2} \quad \text{for} \quad p^2=-m^2$$
$$\Pi_{\rm trans}(p^2=-m^2)\left(1+\frac{\partial \Pi_{\rm trans}}{\partial p^2}(p^2=-m^2)\right)=0$$

Note that the pole of the transverse self energy is gauge-invariant order by order, i.e. this definition of m is gauge invariant. Results for the solution of the gap equation using various resummation schemes and gauge fixed lattice propagators are summarised in Table 1. Expectedly there is some scatter in the values, but it is not much more than the two-loop correction computed with the scheme based on the non-linear sigma model [14], which amounts to about 15%. This suggests that the schemes might possibly lead to a reasonable convergence.

There is also an evaluation of the magnetic mass in a gauge invariant lattice calculation, based on the following observable [16]

$$\frac{\langle (D_i F_{ij})^a(x) U^{Ad}_{ab}(x,y) (D_k F_{lm})^b(y)\rangle}{\langle (F_{ij})^a(x) U^{Ad}_{ab}(x,y) (F_{lm})^b(y)\rangle} \sim \exp[-(3m-2m)|x-y|]\,, \tag{5.1}$$

where $U^{Ad}_{ab}(x,y)$ is a Wilson line in the adjoint representation. In perturbation theory, the correlator in the numerator is at large distances dominated by three-gluon exchange, whereas the denominator

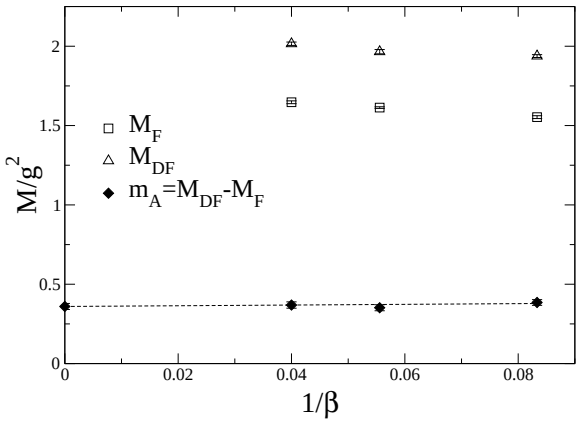

Figure 2: Lowest masses from the exponential fall-off of the field strength correlators in Eq. (5.1). Both masses are diverging at the same rate, but the difference can be extrapolated to the continuum.

is dominated by two-gluon exchange, hence the ratio should fall off with one gluonic pole mass. In a non-perturbative reasoning these correspond to gluelump correlators, i.e. glue with certain quantum numbers bound to a static adjoint source. The ratio of these correlators measures the smallest excitation energy with the quantum numbers of a gluon. Note that, due to the diverging self energy of the adjoint Wilson lines, neither gluelump correlator has a continuum limit, but the ratio does and the energy difference is finite. The resulting continuum extrapolation is shown in Fig. 2, and gives $m = 0.36(2)g_M^2$.

Finally, the mass parameter in the gluon propagator is not a physical observable and not the goal of our investigation, but merely a quantity that appears as a regulator for our resummation scheme. The test of the latter is in computing a physical observable. In the context of thermal field theory, this has been done by using the linear sigma model as a dimensionally reduced version of the electroweak theory in order to calculate the electroweak phase transition as a function of the Higgs mass. Standard perturbation theory to leading order predicts a first order phase transition for all Higgs masses, and cannot be extended beyond LO because of the massless W-bosons in the symmetric phase. Using the resummation similar to the one described here, the critical Higgs mass for which the electroweak phase transition disappears and turns into a smooth crossover was predicted [2, 17] and is within 10% of the corresponding lattice results [18, 19].

6. Application to the pressure

Let us now apply the screened perturbation theory based on the non-linear sigma-model to the calculation of the pressure of the soft magnetic field modes, Eq. (2.2). In the framework of the resummation outlined above, the Lagrangian in the exponent of the first line is supplemented by the sigma-model term, the counter term as well as gauge fixing and ghost terms. How many powers of the auxiliary fields $\pi^a(x)$ are needed depends on the order of the calculation. At the two loop

level, or to order ℓ^1, a four-point vertex is the highest that is needed, and we have

$$\begin{aligned}
\mathscr{L}_{\rm eff} = &\frac{1}{4}\left(\partial_i A_j^a - \partial_j A_i^a\right)^2 + \frac{1}{2\xi}\left(\partial_i A_i^a\right)^2 + \frac{1}{2}m^2 A_i^a A_i^a \\
&+ \frac{1}{2}\left(\partial_i \pi^a\right)^2 + \frac{1}{2}\xi m^2 \pi^a \pi^a + (\partial_i (c^a)^*)\partial_i c^a + \xi m^2 c^{a*} c^a \\
&+ g_M \sqrt{\ell} f^{abc} A_i^b A_j^c \partial_i A_j^a + \frac{1}{4} g_M^2 \ell f^{abe} f^{cde} A_i^a A_j^b A_i^c A_j^d \\
&+ \frac{1}{2} g_M \sqrt{\ell} f^{abc} (\partial_i \pi^a) A_i^b \pi^c + g_M \sqrt{\ell} f^{abc} (\partial_i (c^a)^*) A_i^b c^c - \frac{1}{2} g_M \sqrt{\ell} \xi m f^{abc} \pi^a c^b (c^c)^* \\
&+ \frac{1}{8}\frac{g_M^2 \ell}{m^2}\left(\frac{2}{N}\delta^{ab}\delta^{cd} + d^{abe} d^{cde}\right) \pi^a \pi^c (\partial_i \pi^b) \partial_i \pi^d \\
&- \frac{1}{8} g_M^2 \ell \xi \left(\frac{2}{N}\delta^{ab}\delta^{cd} + d^{abe} d^{cde} - f^{abe} f^{cde}\right) (c^a)^* \pi^b \pi^c c^d \\
&- \frac{1}{2} m^2 \ell A_i^a A_i^a - \frac{1}{2}\xi m^2 \ell \pi^a \pi^a - \xi m^2 \ell c^{a*} c^a .
\end{aligned} \tag{6.1}$$

Note the additional Feynman rules that result from this effective Lagrangian. Besides the vertices for the auxiliary fields and their interactions with gluons, ghosts and themselves, there are also two-point vertices associated with the counter terms, cf. Fig. 3 (left), since they are treated as interactions. Through order ℓ^1, i.e. two loops, we then have to compute the diagrams in Fig. 3 (right).

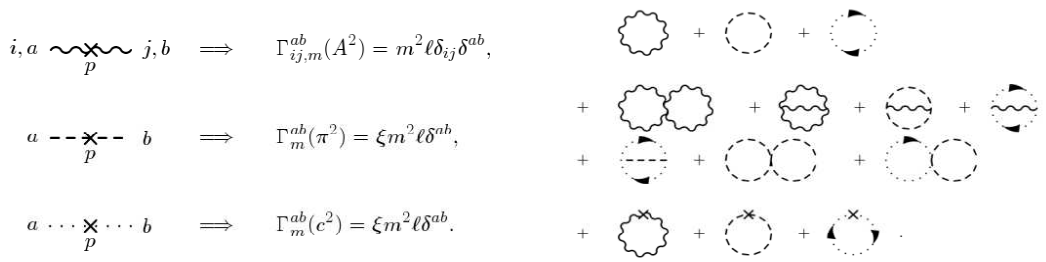

Figure 3: Left: New counterterms induced by the effective Lagrangian Eq. (6.1). Right: Feynman diagrams for the contribution of magnetic QCD to the pressure at order ℓ.

Our calculation is performed in a general R_ξ-gauge. This is an excellent way of checking the calculation – the pressure as a physical quantity has to be independent of the gauge parameter ξ. For intermediate checks, it is also useful to observe that the set of diagrams has three obviously gauge invariant subsets. The one-loop diagrams from the first line, and the two-loop diagrams just correspond to an unresummed non-linear sigma model which must be gauge invariant order by order. Correspondingly, the counter term diagrams in the last line are also a gauge invariant subset. According to the reshuffling of the expansion, they appear only at order ℓ although they technically are one loop diagrams.

The results for these three sets of diagrams for $SU(N)$ are (using $g_M^2 = g^2 T$, cf. Section 2)

$$\begin{aligned}
p_{G,\ell^0}(T) &= C^3(N^2-1)\frac{g^6}{6\pi}T^4 \\
p_{G,ct}(T) &= -\ell C^3(N^2-1)\frac{g^6}{4\pi}T^4 \\
\frac{p_{G,\ell^1}}{\mu^{-2\varepsilon}T^4} &= \ell C^2 N(N^2-1)\frac{g^6}{(4\pi)^2}\left[-\frac{21}{64}\left(\frac{1}{\varepsilon}+4\ln\frac{\bar{\mu}}{2Cg^2T}\right)+\frac{3}{16}-\frac{21}{32}-\frac{21}{16}\ln\frac{2}{3}+O(\varepsilon)\right] \quad (6.2)
\end{aligned}$$

Because of the resummation, we are now getting a contribution of order g^6 already to leading order in this scheme, whereas in unresummed perturbation theory this would occur at four loops. We then extract the following coefficients (with numerical values for $\ell = 1, N = 2$),

$$\begin{aligned}
\alpha_G &= -\frac{21}{4}\frac{C^2}{N^2}\pi^2\ell = -1.015582 \\
\beta_{G,\ell^0} &= \frac{128}{3}\frac{C^3}{N^3}\pi^3 = 3.630132 \\
\beta_{G,ct} &= -64\ell\frac{C^3}{N^3}\pi^3 = -5.445198 \\
\beta_{G,\ell^1} &= -\left(\frac{15}{2}+21\ln\frac{2}{3}\right)\ell\frac{C^2}{N^2}\pi^2 = 0.196301 \quad (6.3)
\end{aligned}$$

In order to evaluate these contributions, a number for C had to be specified. To be fully self-consistent, we have taken this to be the magnetic mass evaluated in the same resummation scheme for the case of $SU(2)$, i.e. $C \approx 0.28$ [2]. We now have two consecutive orders for β and may get a first glimpse of the convergence properties. While the resummed perturbation is organised in orders of ℓ, a convergence check by comparing different orders in ℓ would not seem to make much sense. By construction, the counter term diagrams get subtracted one order in ℓ higher than the other diagrams with the same number of loops and the same integral structure. Hence, when asking for the convergence properties, it would seem natural to proceed loopwise. Thus, in Eq. (6.3) the contribution of all one loop diagrams is $\beta_{G,\ell^0}+\beta_{G,ct} = -1.81$, while the genuine two-loop contribution is β_{G,ℓ^1}, which is about 10% correction. We would expect the counter terms with two loops, entering at l^2-level, to be of the same order of magnitude as the two-loop contributions evaluated here. Thus, comparing the one-loop and two-loop results appears to be promising in terms of convergence. We are presently performing the three-loop computation in order to check this behaviour.

Another interesting observation is the fact that if we take C to be the coefficient of the magnetic mass, evaluated in the same resummation scheme, then $C \sim N$. Thus, N drops out of the expressions in Eq. (6.3) and β_G becomes N-independent, as expected on general grounds [7].

7. Conclusions

We discussed the possibility to gauge-invariantly resum the 3d Yang-Mills theory such as to self-consistently include a dynamically generated pole mass for the gluons, which serves as a regulator for infrared divergences. In such a scheme a perturbative expansion to arbitrary orders

is possible. However, the dimensionless expansion parameter is a dynamically generated number, hence there is no parametric limit in which convergence of the series is guaranteed. Rather, the convergence properties can only be inspected after the calculation of several orders. We have applied a resummation scheme based on the non-linear sigma-model to evaluate the contribution of the soft magnetic gluons to the pressure of the QCD plasma. Comparison between one- and two-loop contributions suggests that there is hope for a reasonable convergence of the series.

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