

Mixed Ice Accretion on Aircraft Wings

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Abstract

Ice accretion is a problematic natural phenomenon that affects a wide range of engineering applications including power cables, radio masts and wind turbines. Accretion on aircraft wings occurs when supercooled water droplets freeze instantaneously on impact to form rime ice or runback as water along the wing to form glaze ice. Most models to date have ignored the accretion of mixed ice, which is a combination of rime and glaze. A parameter we term the ‘freezing fraction’, is defined as the fraction of a supercooled droplet that freezes on impact with the top surface of the accretion ice to explore the concept of mixed ice accretion. Additionally we consider different ‘packing densities’ of rime ice, mimicking the different bulk rime densities observed in nature. Ice accretion is considered in four stages: rime, primary mixed, secondary mixed and glaze ice. Predictions match with existing models and experimental data in the limiting rime and glaze cases. The mixed ice formulation consequently however provides additional insight into the composition of the overall ice structure, which ultimately influences adhesion and ice thickness; and shows that for similar atmospheric parameter ranges,

this simple mixed ice description leads to very different accretion rates. A simple one-dimensional energy balance was solved to show how this freezing fraction parameter increases with decrease in atmospheric temperature, with lower freezing fraction promoting glaze ice accretion.

6 *Keywords:*

7 mixed ice, aircraft wings, freezing fraction, glaze, rime

8 **Nomenclature**

9	a	Aerodynamic heating constant
10	c_a	Specific heat capacity of air
11	H_{aw}	Air-water heat transfer coefficient
12	L_f	Latent heat of fusion
13	c_w	Specific heat capacity of water
14	x_e	Evaporative coefficient
15	e_0	Evaporative function derivative
16	U_∞	Air speed
17	d	Droplet diameter
18	T_f	Freezing temperature
19	k_i	Thermal conductivity of ice
20	k_w	Thermal conductivity of water
21	\dot{M}	Mass flux of supercooled droplets on the wing
22	x_s	Sublimation coefficient
23	r	Rime height

24	b	Glaze height
25	h	Water height
26	w	Limit of unfrozen water droplets for secondary mixed ice
27	t_w	Rime transition time
28	t_b	Primary mixed transition time
29	α	Angle of attack
30	β	Collection efficiency
31	ρ_l	Liquid water content
32	ρ_w	Density of water
33	ρ_i	Density of glaze ice
34	ρ_r	Density of rime ice

35 **1. Introduction**

36 Ice accretion is a natural phenomenon that affects a wide range of exter-
 37 nal engineering structures, such as aircraft, power cables, radio masts and
 38 wind turbines. A motivation for the modelling described in this paper, is
 39 to develop an understanding of the basic physical processes leading to ice
 40 accretion to inform strategies for improving anti-icing techniques.

41 The ice accretion process is not straightforward with different types of ice
 42 growing under different environmental conditions. Rime ice results from a
 43 relatively simple heat transfer process, whereby supercooled water droplets
 44 freeze instantaneously on impact with any very cold wing surface [1, 2, 3, 4, 5].
 45 This process creates a relatively low density, porous ice with a distinctive

46 white appearance. In contrast, glaze ice typically grows at temperatures
47 closer to the melting point via a Stefan-type boundary condition, to form
48 a dense and translucent ice layer. Glaze formation requires the presence of
49 water, either due to partial freezing of the supercooled water droplets, or as
50 *runback* water as the electrothermal systems in the leading edge melt any
51 ice structures accreted there. In practice, the ice that forms on an aircraft
52 wing tends to be a combination of rime and glaze ice; potentially with some
53 retained pockets of air and described as *mixed ice*. The inclusion of porosity
54 of rime ice is similar to the approach of Rios (1991) [6] who developed a den-
55 sity formula for accreted ice but did not define different stages of ice accretion.
56 Commercial icing codes such as LEWICE, TRAJICE, ONERA and ICECREMO
57 code [7, 8, 9, 10, 11] have been developed over a number of years. The first
58 three icing codes employ a Messinger [1] approach which includes limitations
59 such as non inclusion of the conduction term in the energy balance, freezing
60 fraction is assumed constant through out accretion and inaccurate descrip-
61 tion of water movement [12]. The ICECREMO code considers the dynamic
62 behaviour of the runback water film on the accretion rate and uses a Stefan
63 [13] condition at the ice-water interface to overcome these limitations. Thus
64 this will form a basis for our analysis. The ICECREMO code modelled so-
65 lidification of the runback water film, which resulted in a wider applicability
66 of the model to different icing conditions. However, all of these models how-
67 ever tend to over-predict ice accretion, leading to potential energy wastage
68 through over-use of the electro-thermal de-icing system. A limitation is these

69 do not differentiate between different ice types - an important consideration
70 when considering adhesion to possible wing coatings. More recently, Zhang
71 et al. (2017) [14] included the effects of runback water and porosity of rime
72 ice in the accretion process.

73 This paper considers the formation of mixed ice on an aircraft wing from
74 the partial freezing of impinging supercooled droplets, which deposit onto
75 the growing interface as a combination of solid rime ice particles and wa-
76 ter. We characterise this as a multiphase layer comprising a porous medium
77 created by the solid rime ice particles and water at longer accretion times
78 and different atmospheric conditions. This is in contrast to more traditional
79 'spongy ice' models that envisage unfrozen water from supercooled droplets
80 entrapped within the growing ice dendrites [15]. We note mushy layers as
81 seen in sea ice for example, form due to a secondary diffusive process at
82 the solidification front with a presence of multiphase microstructures [16].
83 Although not explicitly modelled, a multiphase porous medium can provide
84 additional insight towards the inclusion of a mushy structure in future ic-
85 ing models. Generally the water within a rime ice matrix can solidify as
86 glaze ice within the pore space. A solidifying glaze ice grows through the
87 rime particles to create a layer of co-existing rime and glaze, i.e. mixed ice.
88 Anderson and Feo (2002) [17] also included varied freezing fraction values
89 to determine water film thickness and ice shape without defining different
90 stages of ice accretion. In this work we determine a model for the growth
91 rate and accretion based on prevailing atmospheric conditions. Including the

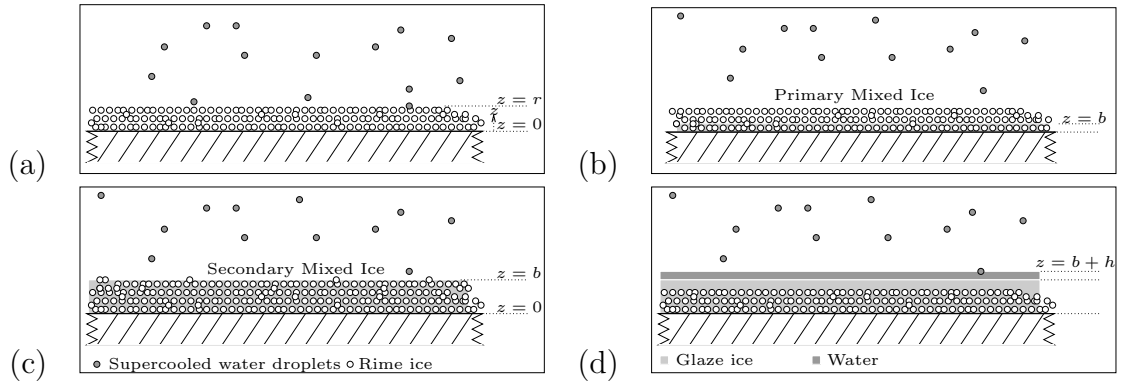


Figure 1: Schematic representation of the modes of accretion in mixed ice formation. (a) At times $t < t_w$, supercooled droplets freeze on impact with the wing, forming rime ice (with air in the pore space). (b) When the layer of rime ice is thick enough that the latent heat released on freezing can no longer be conducted through it, a proportion of the incoming supercooled droplets remain as water. This water quickly freezes as glaze ice within the pore spaces of the original rime layer ($t_w \geq t < t_b$). (c) Once the glaze ice fully occupies the pore space between the rime crystals, rime and glaze can accrete simultaneously in a mixed ice layer, with better thermal conductivity than the rime ice alone. (d) Once this *mixed ice* layer is again too thick to transport released latent heat on the droplets freezing, a water film forms on its surface. This may be supplemented by runback water formed by melting ice formations at the wing leading edge.

92 formation of a porous ice matrix layer within this model enables more general
 93 possibilities to explore in the future, such as forced convective transfer modes.

94

95 2. Model Formulation

96 In this section we introduce four, possibly consecutive, modes of ice accre-
 97 tion within the context of an application to icing on aircraft wings; including
 98 any aircraft passing through cloud cover, where the local temperatures can be
 99 very cold and available water droplets are supercooled. As the aircraft pro-
 100 gresses, these water droplets may impinge on the wing surface. For simplicity,

101 we restrict our initial interest to one-dimensional accretion with calculation
 102 of the time evolution of ice growth normal to the wing surface; assuming that
 103 the ambient conditions and influx of droplets are steady. A first aim is to
 104 understand the influence of rime versus glaze versus mixed accretion modes
 105 on the resulting accretion profiles.

106 *2.1. Rime Ice*

107 In the earliest stage of accretion, the wing is approximately at the temper-
 108 ature of the ambient air and the supercooled droplets collected will freeze on
 109 impact to form rime ice [7], as illustrated in Fig. 1(a). The wing is generally
 110 an effective thermal conductor, so the latent heat released as the supercooled
 111 droplets solidify is easily transported away.

112 A key feature of rime r is that air is generally trapped within the pore
 113 spaces between tiny crystals of ice. Thus we define a solid volume fraction
 114 of ice in this rime layer

$$\phi = \frac{\rho_r}{\rho_i}, \quad (1)$$

115 where ρ_r is the bulk density of the rime ice layer and ρ_i is the density of pure
 116 ice (we assume that all ice, rime particles or glaze, has density ρ_i , which is
 117 independent of temperature). For typical values of opaque rime bulk density,
 118 $\rho_r \geq 610 \text{ kg m}^{-3}$ [18], the equivalent solid volume fraction follows, $\phi \geq 0.67$.
 119 This is consistent with the maximum packing fraction of spheres [19].

120 For simplicity, we consider one-dimensional accretion on a flat plate, vary-
 121 ing with time. The rate of growth of the rime front at $z = r$ (Fig. 1(a)) is

122 given by a mass balance

$$\frac{\partial r}{\partial t} = \frac{1}{\rho_i \phi} \dot{M}, \quad (2)$$

123 where \dot{M} is the mass flux of incident water droplets. For constant values of
124 \dot{M} and ϕ , the ice height thickness follows,

$$r = \frac{\dot{M}t}{\rho_i \phi}. \quad (3)$$

125 where t is the elapsed time and $r(t = 0) = 0$.

126 2.2. Mixed Ice

127 Rime ice will continue to accrete as in Sec. 2.1 (Fig. 1(a)) until the layer
128 becomes sufficiently thick that the latent heat released on solidification of the
129 droplets can no longer be conducted away through the wing; the top surface
130 of the rime is now at the freezing temperature T_f [20]. This transition time,
131 t_w , can be evaluated from an energy balance (Sec. 2.3).

132 At the surface interface, only a proportion λ of the incoming supercooled
133 droplets freeze. Subsequently water forms alongside the rime ice and the ice
134 becomes ‘*mixed*’. This water can percolate through the pore space in the rime
135 layer and freeze as glaze ice inside the pore space (Fig. 1(b)). For simplicity,
136 we assume that there is no air in the ice accretion, but in our model it would
137 be straightforward to include air (as an essentially free parameter). This may
138 be important since observations indicate air occupying up to 35% of the total
139 interstitial space for rime accretion [21]. Such a glaze freezing process inside

140 the pore space occurs over a short timescale and is not explicitly modelled.

141 Once the glaze ice front reaches the top of the rime (Fig. 1(c)) mixed
142 ice will continue to grow. Rime and glaze form alongside each other in the
143 form of secondary mixed ice because the conduction through the glaze ice
144 freezes all of the unfrozen water at the air-rime interface initially. As the rate
145 of conduction reduces with the increase in ice thickness; eventually, a water
146 film begins to appear within the interstitial rime matrix, above the growing
147 glaze. This can rapidly lead to the film occupying the porous spaces and
148 flowing above the rime matrix as well when the packing fraction is high and
149 freezing fraction is low.

150 At longer accretion times (Fig. 1(d)), a water film may grow above the
151 rime boundary. Subsequent supercooled droplets will impact on a water film
152 directly instead of the rime matrix and both rime and glaze will accrete
153 simultaneously.

154 Thus, the freezing fraction λ provides a dimensionless quantity for the
155 fraction of a supercooled droplet that solidifies on impact with the aircraft
156 wing or ice layer. λ depends on the energetics of droplet impact. This is
157 different to the freezing fraction as the ratio of the amount of ice formed to
158 the mass flux of incoming supercooled droplets, describing the solidification
159 process of glaze ice [10]. λ in contrast determines both the rate and type of
160 ice formation which will be apparent in our subsequent discussion.

161 *2.3. Energy Balance*

162 While the water and ice (both glaze and rime) layers remain thin, we
 163 can assume that conduction across the layer is the primary mode of heat
 164 transfer. The reduced pseudo-steady state conduction equations for heat
 165 transfer through the ice and water layers are

$$\frac{\partial^2 T}{\partial z^2} = 0, \quad (4)$$

166 and

$$\frac{\partial^2 \theta}{\partial z^2} = 0, \quad (5)$$

167 where T and θ are the temperatures in the ice and water layers respectively
 168 [10]. Considering first the early stage of rime accretion (Fig. 1(a)), the tem-
 169 perature distribution through the rime layer is determined by fixing the rime
 170 temperature at the wing to be the wing surface temperature (i.e. the ambient
 171 air temperature), $T(z = 0) = T_a$. Further, the heat flux through the air-rime
 172 interface is determined by an energy balance there [10, 20],

$$\left. \frac{\partial T}{\partial z} \right|_{z=r} = \frac{1}{k_i} [Q_l + Q_k + Q_a - (Q_r + Q_h + Q_s)], \quad (6)$$

173 where

$$\begin{aligned}
Q_k &= \frac{1}{2}\dot{M}U_\infty^2 && \text{is the droplet kinetic energy,} \\
Q_a &= \frac{1}{2c_a}aH_{aw}U_\infty^2 && \text{is the aerodynamic heating,} \\
Q_l &= \lambda\dot{M}L_f && \text{is the release of latent heat,} \\
Q_d &= c_w(1-\lambda)\dot{M}(T-T_d) && \text{is the droplet thermal energy after stage} \\
&= q_d(1-\lambda)(T-T_a) && \text{one of rime accretion } (q_d = c_w\dot{M}),
\end{aligned}$$

$$\begin{aligned}
Q_r &= c_w\lambda\dot{M}(T-T_d) && \text{is the droplet thermal energy during} \\
&= q_r\lambda(T-T_a) && \text{stage one of rime accretion } (q_r = c_w\dot{M}),
\end{aligned}$$

174

$$\begin{aligned}
Q_h &= H_{aw}(T-T_a) && \text{is the convective heat transfer from rime} \\
&= q_h(T-T_a) && \text{to air,}
\end{aligned}$$

$$\begin{aligned}
Q_s &= x_s e_0(T-T_a) && \text{is the heat of sublimation } (q_s = x_s e_0), \\
&= q_s(T-T_a)
\end{aligned}$$

$$\begin{aligned}
Q_i &= c_w\lambda\dot{M}(T_f-T_d) && \text{is the energy to raise the temperature of} \\
&= \lambda q_d(T_f-T_a) && \text{solidifying fraction of the droplet to the} \\
&&& \text{freezing point after stage one of rime} \\
&&& \text{accretion.}
\end{aligned}$$

175

176 For pure rime accretion $\lambda = 1$ in the above. Myers (2001) [10] and Myers
177 & Charpin (2004) [20] show that Eqn. 6 can be simplified in the form

$$\frac{\partial T}{\partial z} = E_{rz} - F_{rz}T, \tag{7}$$

178 where

$$E_{rz} = \frac{1}{k_i} [Q_k + Q_a + Q_l + q_r T_d + (q_h + q_s) T_a], \quad (8)$$

179 and

$$F_{rz} = \frac{1}{k_i} (q_r + q_h + q_s). \quad (9)$$

180 Solving the conduction equation (Eqn. 4) for T subject to the fixed tem-
 181 perature at the wing surface and flux determined by the simplified energy
 182 balance (Eqn. 7),

$$T = T_a + \frac{E_{rz} - F_{rz} T_a}{1 + F_{rz} r} z, \quad (10)$$

183 within the pure rime layer at $0 \leq z \leq r$.

184

185 After a period of rime accretion, water will first appear when the air-rime
 186 interface reaches the freezing temperature, i.e. $T(z = r_w) = T_f$ [20]. Thus

$$r_w = k_i \frac{T_f - T_a}{Q_l - Q_r + Q_a + Q_k - (q_h + q_s)(T_f + T_s)}, \quad (11)$$

187 with a corresponding transition time from the rime mass balance (Eqn. 3)

$$t_w = \frac{\phi \rho_i r_w}{\dot{M}}. \quad (12)$$

188 At times $t > t_w$, some of the supercooled water droplets must remain as
 189 water and $0 \leq \lambda < 1$. This is now the regime for the mixed ice accretion.
 190 However, under a range of ambient conditions, water could form within the

191 rime layer before this transition time. The atmospheric conditions required
 192 for complete, partial or no freezing of the supercooled droplets, and the
 193 resulting ice type that forms is described in Sec. 2.4 and summarised in
 194 Table 1.

195 Assuming that glaze ice freezes quickly within the pore space of deposited
 196 rime ice, the propagation of the glaze ice through the rime is determined by
 197 a mass balance (Fig. 1(b)). Under steady conditions, glaze ice (b_b) will thus
 198 reach the top surface of the accretion at a time

$$t_b = t_w + \frac{(1 - \phi) \rho_i b_b}{(1 - \lambda) \dot{M}}, \quad (13)$$

199 where at this time

$$r_b = r_w + \frac{\lambda \dot{M} (t_b - t_w)}{\rho_i \phi}. \quad (14)$$

200 To complete the specification to determine r_b , b_b and λ , we modify the
 201 surface conditions of the energy balance (Eqns. 8 and 9) by including terms
 202 Q_i and $q_d T_d$ so as to account for the energy of partially frozen droplets at
 203 the air-ice interface,

$$E_{rm} = \frac{1}{k_i} [Q_k + Q_a + Q_l - Q_i + q_d T_d + (q_h + q_s) T_a], \quad (15)$$

204 and

$$F_{rm} = \frac{1}{k_i} [q_d + q_h + q_s]. \quad (16)$$

205 A modification of Eqn. 7 gives the flux condition

$$\frac{\partial T}{\partial z} = E_{rm} - F_{rm}T. \quad (17)$$

206 Solving Eqn. 4 with the flux condition as in Eqn. 17 and a fixed temperature
 207 at the wing surface gives

$$T = T_a + \frac{E_{rm} - F_{rm}T_a}{1 + F_{rm}r}z \quad (18)$$

208 Substituting for $T = T_f$ at $z = r$, the explicit formula for λ during Stage 2
 209 of accretion is given by rearranging Eqn. 18

$$\lambda = \frac{1}{Q_l} \left\{ \left[\frac{1}{r} + \frac{1}{k}(q_d + q_h + q_s) \right] \left[k_i(T_f - T_a) \right] - Q_a - Q_k \right\}, \quad (19)$$

210 Eqn. 19 together with the mass balance Eqn. 13 and 14 allows us to solve
 211 for λ , r_b and b_b during the second stage of accretion.

212 Any water layer formed over the ice surface after the secondary mixed
 213 ice accretion stage, will interact with the air flow over the wing. The water
 214 film does not affect the wing surface boundary condition, which remains T_s
 215 = T_a , as for the rime accretion case. At the growing ice front $z = b$, the
 216 temperature in both the ice and water phases is the freezing temperature,
 217 $T(z = b) = \theta(z = b) = T_f$. Integrating Eqn. 4 subject to these boundary

218 conditions, the temperature in the ice layer becomes

$$T = \left(\frac{T_f - T_s}{b} \right) z + T_s, \quad (20)$$

219 for $0 \leq z \leq b$.

220 The water film on top of this glaze ice is at the freezing temperature at
 221 the ice front and T at its upper surface, $z = b + w$, the heat flux is determined
 222 by an energy balance

$$\left. \frac{\partial \theta}{\partial z} \right|_{z=b+h} = \frac{1}{k_w} [Q_l + Q_k + Q_a - (Q_i + Q_d + Q_h + Q_e)], \quad (21)$$

223 where $Q_e = x_e e_0 (\theta - T_a) = q_e (\theta - T_a)$ is the heat of evaporation. The
 224 difference between this heat flux boundary condition and that for the rime
 225 growth at $z = r$, is that evaporation replaces sublimation due to the water
 226 film replacing ice at the exposed surface. Eqn. 21 can be simplified to give

$$\frac{\partial \theta}{\partial z} = E_{gz} - F_{gz} \theta, \quad (22)$$

227 where

$$E_{gz} = \frac{1}{k_w} [Q_k + Q_a + Q_l - Q_i + q_d T_d + (q_h + q_s) T_a], \quad (23)$$

228 and

$$F_{gz} = \frac{1}{k_w} (q_d + q_h + q_e). \quad (24)$$

229 Taking conduction is the leading heat transfer mechanism (Eqn. 5), the

230 boundary conditions determine the temperature in the water layer as

$$\theta = T_f + \frac{E_{gz} - F_{gz}T_f}{1 + F_{gz}h} (z - b), \quad (25)$$

231 for $b \leq z \leq b + w$.

232 2.4. Freezing Fraction

233 Ultimately, the freezing fraction λ determines the type of ice accretion
234 in this model, but its value is determined by the prevailing conditions. For
235 example, the water temperature Eqn. 25, is physically constrained. If theo-
236 retically a water film exists above the freezing temperature, then both $h > 0$
237 and $\theta \geq T_f$, implying that $F_{gz}T_f \leq E_{gz}$. From their definitions, Eqn. 25 can
238 be re-written

$$T_f - T_a \leq \frac{Q_k + Q_a + Q_l}{q_d + q_h + q_e}. \quad (26)$$

239 Since Q_l and q_d are dependent on λ , we thus have ambient constraint on the
240 value λ can take given typical ambient conditions for icing on an aircraft wing
241 [20]. Pure rime forms at very cold temperatures, until a water layer would
242 be thermodynamically possible, as dictated by this inequality. Thus $\lambda = 1$
243 is possible if $T_a \leq -15.3^\circ\text{C}$. At temperatures greater than this, some glaze
244 ice will be present and the overall accretion will be mixed in appearance.
245 Conversely, pure glaze ice corresponding to $\lambda = 0$ will form if $-1.8 \leq T_a \leq$
246 0.0°C . -1.8°C is thus the highest ambient temperature at which mixed ice
247 can form. These constraints are summarised in Table 1.

248 Myers & Charpin (2004) [20], Myers (2001) [10] and Myers & Hammond
 249 (1999) [9] evaluate from their respective models the highest ambient tempera-
 250 ture for pure rime accretion as -16.6°C , -18.35°C and -15.98°C , fitting with
 251 our analysis. Data from observations of actual inflight icing also supports
 252 our predictions. For example, Lynch & Khodadoust (2001) [22] reported
 253 pure glaze accretion in the temperature range $-3 < T < 0^\circ\text{C}$ where Mirzaei
 254 et al (2009) [23] suggest temperatures close to 0°C .

255 Now that we know the approximate ranges for mixed ice accretion, we
 256 can analyse the composition of mixed ice at ambient temperatures in the
 257 range $-15.3 < T_a < -1.8^\circ\text{C}$, if and when it grows above r_w , the originally
 258 deposited rime layer. If we assume the water film is isothermal at T_f and
 259 exists only when $t > t_w$, such that water is well mixed and thereby isothermal
 260 [10], the proportion of the incoming droplets that will freeze

$$\lambda_m = \frac{(T_f - T_a)(q_d + q_h + q_e) - Q_k - Q_a}{Q_t}. \quad (27)$$

261 Thus, for the typical environmental conditions in Table A, we can thus predict
 262 that mixed ice accreting at -5 , -10 and -15°C has $\lambda_m = 0.24, 0.61$ and 0.98
 263 respectively.

264 The physical implications of this are open to debate, but adhesion tests
 265 indicate that rime ice and mixed ice or glaze ice have very different properties
 266 and would require different approaches to anti- or de-icing. The nature of
 267 the bond between the ice and the wing surface, in particular the proportion

Table 1: Ice type variation with droplet freezing fraction, λ .

Droplet Freezing Fraction	Type of Ice	Ambient Temperature Range, °C
$\lambda = 1$	Ice Crystals	$T_a \leq -40.0$
$\lambda = 1$	Rime	$-40.0 \leq T_a \leq -15.3$
$0 \leq \lambda \leq 1$	Mixed	$-15.3 \leq T_a \leq -1.8$
$\lambda = 0$	Glaze	$-1.8 \leq T_a \leq 0.0$
$\lambda = 0$	No Freezing	$0.0 \leq T_a$

268 of glaze ice present to increase the strength of that bond, could be a signifi-
 269 cant consideration in optimising strategies. In reality, icing encounters below
 270 -20°C are extremely rare [24], the temperature range we consider applying
 271 to the overwhelming majority of commercial aircraft applications falls within
 272 this mixed accretion band.

273 2.5. Secondary Mixed Ice

274 It is useful to differentiate the early-deposited mixed ice that grows through
 275 the rime layer with a ‘secondary’ mixed ice that accretes after t_b due to par-
 276 tially freezing supercooled droplets. This consists of the simultaneous depo-
 277 sition of a matrix of rime ice, with water freezing in its pore space as glaze
 278 ice.

279 At times $t > t_b$, the growth rate of the rime matrix follows the rime ac-
 280 cretion rate of primary mixed described in Eqn. 14, with $\lambda = \lambda_m$ as discussed
 281 in Sec. 2.4, i.e.

$$r = \frac{\lambda_m \dot{M} (t - t_b)}{\rho_i \phi} + r_b. \quad (28)$$

282 The next stage is to determine the level of the unfrozen portion of the
 283 supercooled droplets that impact the air-rime interface before incorporating
 284 conduction via the Stefan condition [20]. The condition implies that the
 285 velocity of the boundary of phase change is proportional to the temperature
 286 gradients across it. This is because either the unfrozen portion will occupy
 287 a space within the rime matrix or will fill this space up and engulf the rime
 288 matrix. The limit of the unfrozen supercooled droplets is given by w ,

$$w = \frac{(1 - \lambda_m)\dot{M}(t - t_b)}{\rho_w(1 - \phi)} + r_b. \quad (29)$$

289 If $w \geq r$, we define the new limit as $u = r + h_w$ where h_w is given by,

$$h_w = (1 - \phi)(w - r). \quad (30)$$

290 The glaze ice growth rate is determined by a Stefan condition [25] match-
 291 ing the rate of release of latent heat as the liquid solidifies to the rate at
 292 which that heat can be transported from the glaze solidification front. After
 293 fixing the limit of w , we incorporate a single time step of glaze ice growth b
 294 due to Stefan conduction. While w is within r , b is given by

$$(1 - \phi)\rho_i L_f \frac{\partial b}{\partial t} = k_i \frac{\partial T}{\partial z}, \quad (31)$$

295 which can be integrated with conditions $b = r_b$ at $t = t_b$ to give the

296 location of this ‘glaze front’ here b_m as thus.

$$b_m = r_b + \sqrt{\frac{2k_i(T_f - T_s)(t - t_b)}{(1 - \phi)\rho_i L_f}}. \quad (32)$$

297 and if w is outside r , b is given by

$$\rho_i L_f \frac{\partial b}{\partial t} = k_i \frac{\partial T}{\partial z}, \quad (33)$$

298 where the term on the right hand side corresponds to the transport of heat
 299 through the already-formed ice behind the front. Conduction through the
 300 water film is neglected since it is typically small, and since the water film is
 301 approximately isothermal, [10],

$$b_m = r_b + \sqrt{\frac{2k_i(T_f - T_s)(t - t_b)}{\rho_i L_f}}. \quad (34)$$

302 A water film appears when the limit of b_m is less than w . This implies that
 303 there is now some unfrozen water within the rime matrix or overlying; ice
 304 will grow underneath and the amount of heat conducted away through the
 305 ice to the wing is insufficient to freeze all of the unfrozen supercooled droplet
 306 fraction. When w is less than b_m , we do not differentiate between the rime
 307 and glaze fronts since the process is so rapid that everything freezes almost
 308 instantaneously. The mass balance for glaze ice accretion (Stage 4 of ice
 309 accretion) when the glaze ice and water remain within the rime matrix can

310 be given by

$$\dot{M} = (1 - \phi)\rho_i \frac{\partial b}{\partial t} + (1 - \phi)\rho_w \frac{\partial w}{\partial t} + \phi\rho_i \frac{\partial r}{\partial t}. \quad (35)$$

311 2.6. Accretion Types

312 The atmospheric conditions given in Table 1 show three possible accretion
313 regimes. At cold atmospheric temperatures, $T_a \leq -15.3^\circ\text{C}$, only rime ice can
314 form. Thus, accretion follows Eqn. 3 under steady conditions.

315 Conversely, if the atmospheric temperature is close to the freezing point,
316 $-1.8 \leq T_a \leq 0^\circ\text{C}$, incoming droplets deposit as water which then forms glaze
317 ice. The mass balance described in Eqn. 35, where the solid volume fraction
318 $\phi \approx 0$, describes the total ice and water layer with the Stefan solidification
319 condition, Eqn. 35, determining the extent of the glaze ice within this. This
320 model is the one employed in most current major icing codes, such as glaze
321 ice model [10] and includes conduction through the water film unlike our
322 model.

323 In intermediate conditions, $-15.3 \leq T_a \leq -1.8^\circ\text{C}$, mixed ice forms, de-
324 scribed by the model we have introduced here. Thus initially, for $t \leq t_w$, rime
325 ice forms following Eqn. 3 until $r = r_w$. At this point, λ decreases as glaze
326 ice forms in the previously deposited rime pore space, until $t = t_b$ and the
327 (primary) mixed ice thickness $b = b_b = r_b$ (Eqns. 13 and 14). Now, glaze and
328 rime continue to grow simultaneously as secondary mixed ice, with freezing
329 fraction λ_m . Rime and glaze ice heights are described by Eqns. 28 & 35 &
330 34 respectively where applicable.

331 3. Results and Analysis

332 In the following, we describe the variation in accretion profile of mixed
333 ice with variation in ambient conditions. We describe A) the initial rime
334 accretion phase at $0 \leq t \leq t_w$, B) a primary mixed accretion phase at
335 $t_w \leq t \leq t_b$, C) a secondary mixed accretion phase at $t_b \leq t \leq t_m$ and finally
336 (D), a glaze accretion process when $t \geq t_m$.

337 Figure 2(a) shows the ice accretion profile at an ambient temperature of
338 -5°C and rime packing fraction $\phi = 0.65$. The accretion time of 100 seconds
339 shows a clear transition between the various stages such as the rime stage
340 (0–13 s), primary mixed stage (13–35 s), secondary mixed stage (35–68 s) and
341 glaze stage (68–100 s). Figure 2(b) shows accretion for the same conditions
342 as Figure 2(a) but assumes air bubbles of $\phi_a = 0.3$ in Stage B respectively.
343 We can see the inclusion of air in ice freezing from the substrate upwards
344 results in the reduction of primary mixed ice stage time from 22 s to 7 s for
345 $\phi_a = 0.3$. Figure 3(a) shows how the freezing fraction λ changes with time
346 for the conditions in Figure 2(a). After the initial rime stage, λ steadily
347 decreases during primary mixed stage as the ability to conduct latent heat
348 away from the air-rime interface through the rime decreases. The decrease
349 will be more dramatic with higher values of ϕ since the porous space will be
350 filled up quicker by the unfrozen supercooled droplets.

351 For the secondary mixed and glaze stages, our model assumptions lead to a
352 constant value of λ which is dependent only on the atmospheric temperature
353 since the air-ice or air-water interface is fixed at the freezing temperature

354 T_f . Figure 3(b) describes the temperature profile during the different stages
355 of accretion for conditions described in Figure 2(a). We can observe that
356 the transition from rime to primary mixed occurs when the temperature
357 of the air-rime interface reaches T_f , according to Myers (2001) [10]. The
358 temperature profile through the glaze growing from the wing upwards during
359 primary mixed follows a linear profile dependent on the location of the top
360 interface. A linear temperature profile is also apparent during the secondary
361 mixed and glaze stages with water film assumed isothermal at T_f .

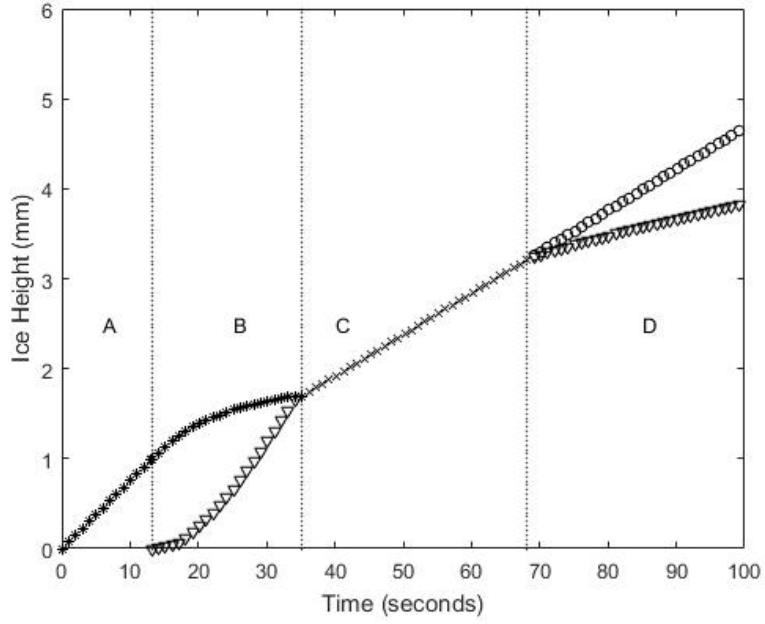
362 It has been reported that just 0.13 mm and 0.77 mm of ice accretion can
363 reduce lift characteristics of an aircraft in flight by 20% and 40% respectively
364 [26]. Several devices currently exist to detect ice near the leading of the wing
365 to aid in the visual capacity of crew members including cylinders, sensors
366 using stiffness, hot rods etc. Modern optical sensors provide a consistent
367 signal at 1.27 mm of ice thickness [27]; while ultrasonic sensors under devel-
368 opment in a high frequency mode of 2 MHz are sensitive to even 0.2 mm of
369 ice accretion [28]. It is for this reason that the key comparative portion of
370 this analysis focuses on what we term as the ‘rapid accretion’ phase of 30 sec-
371 onds when the aircraft is descending or ascending through a cloud structure
372 containing supercooled droplets. After this time period, the aerodynamic
373 characteristics are negatively affected and the ice detection and protection
374 systems begin functioning to mitigate the problem.

375 Figure 4(a) and figure 4(b) show ice growth under different conditions of
376 ambient temperature and packing fraction with accretion time of 30 seconds.

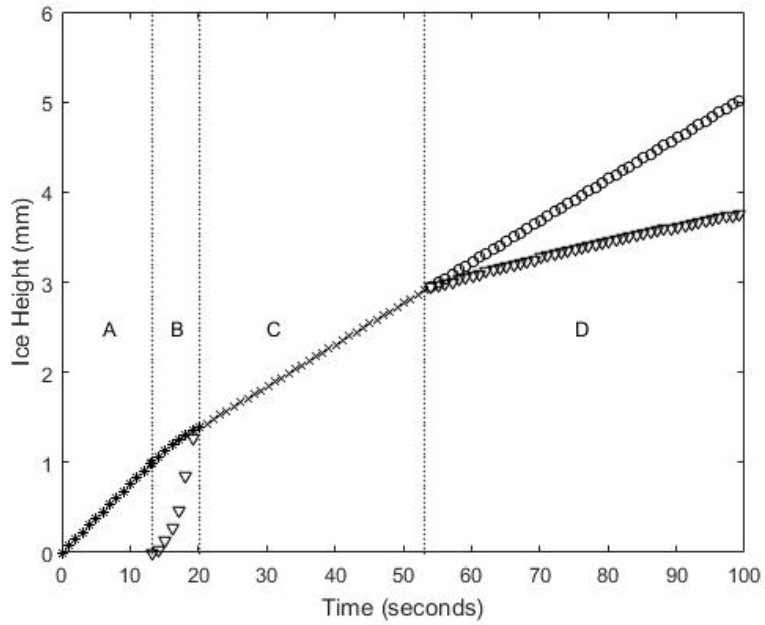
377 Figure 5(a) and Figure 5(b) provide a comparison between the ice growth
378 in the author's model and the traditional glaze ice model [10] with $\phi = 0.96$
379 and ambient temperature of -3°C and -6°C respectively.

380 We can determine the influence of the two key parameters: packing frac-
381 tion and ambient temperature on the ice accretion rate and time and tran-
382 sition between different ice regimes. When comparing Figure 4(a) and Fig-
383 ure 5(a), we can see that an increase in the packing fraction from 0.65 to
384 0.96 while the temperature is fixed at -3°C causes no reduction in overall ice
385 height. The former however sees a prolonged primary mixed stage owing to
386 the lower value of ϕ as compared to the latter. Similarly, comparing Fig-
387 ure 4(b) and Figure 5(b), we can see that an increase in the packing fraction
388 from 0.65 to 0.96 while the temperature is fixed at -6°C causes a reduction
389 in rime and glaze ice height by 21% and 33% respectively. For both cases,
390 it is apparent that higher values of ϕ result in longer rime ice stage, smaller
391 primary mixed stage and drastically affects the ice height; thereby affecting
392 the adhesion characteristics of the accreted ice immensely.

393 When comparing Figure 4(a) and Figure 4(b), we find that the decrease
394 in temperature from -3°C to -6°C , whilst the packing fraction is kept constant
395 at 0.65, results in an overall ice height of 1.4 mm (1 mm glaze and 0.4 mm
396 mixed) and 1.9 mm (0.6 mm glaze and 1.3 mm rime) respectively. There is
397 a change in the accretion height of glaze and the former case also makes an
398 earlier transition to secondary mixed which is understandable as this is a
399 pre-requisite to moving onto a glaze ice stage which is favoured at higher

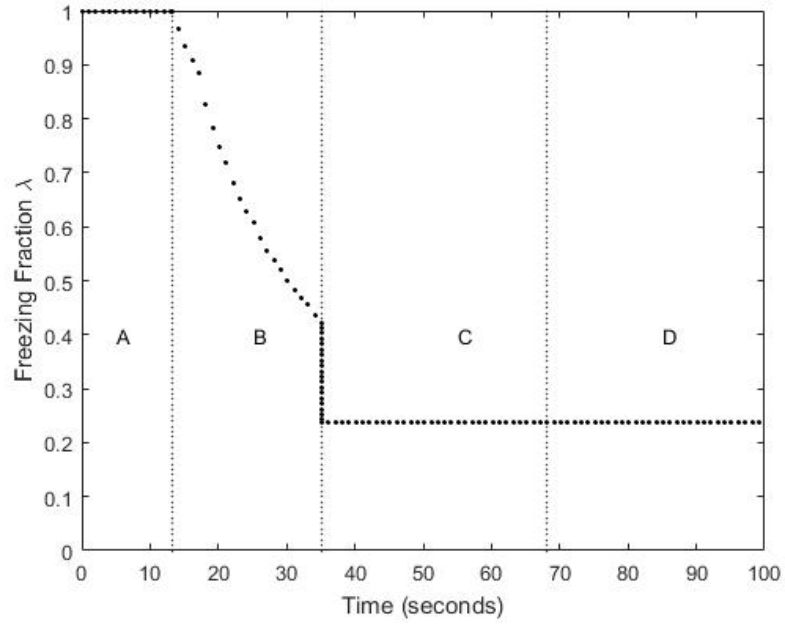


(a)

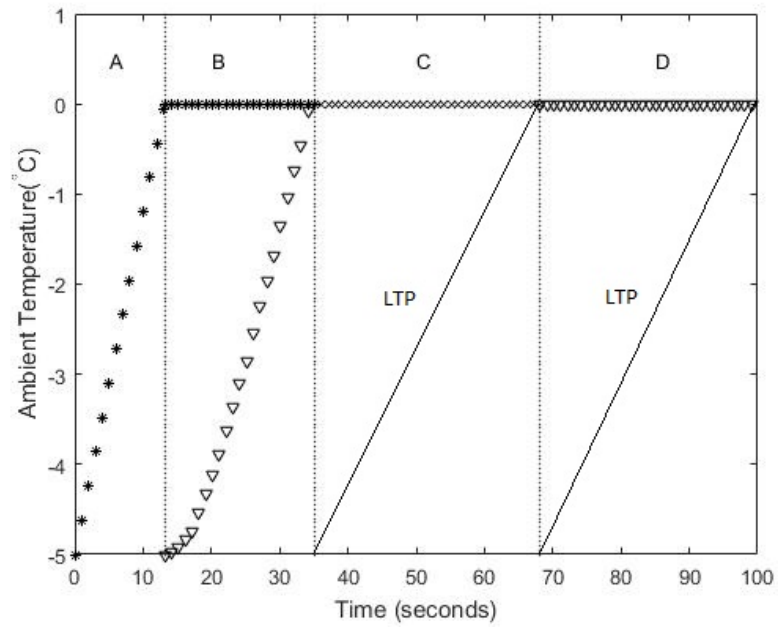


(b)

Figure 2: Mixed ice accretion at $T_a = -5^\circ\text{C}$ and $\phi = 0.65$ for 100 s of droplet impingement. The different ice and water limits are represented by the following symbols: rime (*), glaze (∇), mixed (x) and water film (\circ). (a) The ice profile in four different stages. (b) The ice profile with air bubbles occupying $\phi = 0.3$ in Stage B.

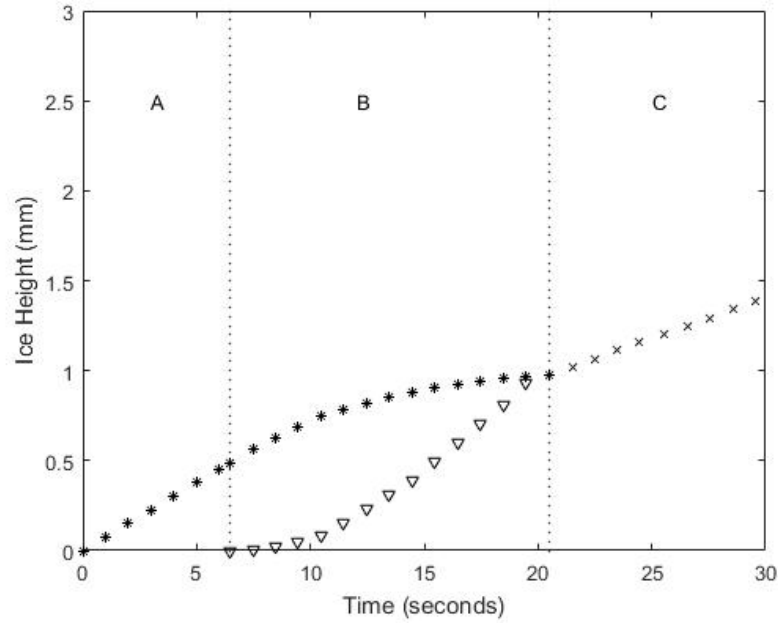


(a)

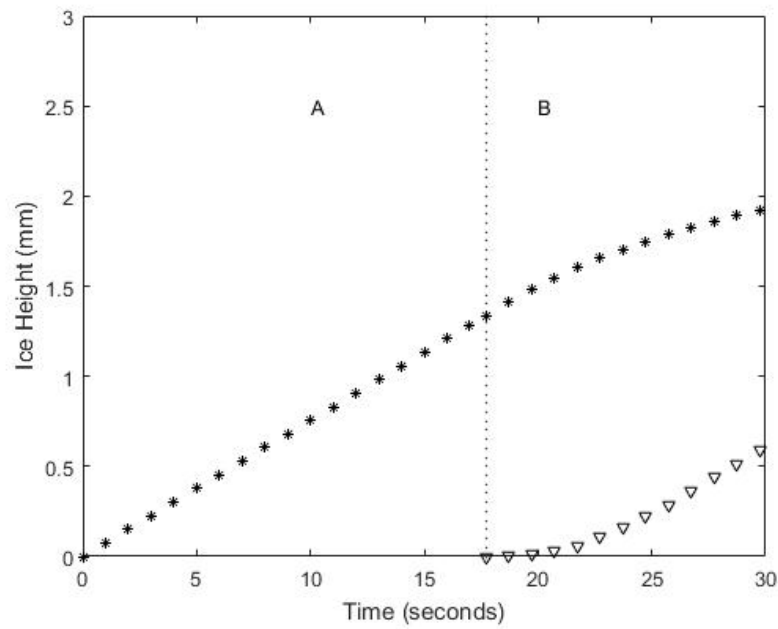


(b)

Figure 3: Mixed ice accretion at $T_a = -5^\circ\text{C}$ and $\phi = 0.65$ for 100 s of droplet impingement. The different ice and water limits are represented by the following symbols: rime (*), glaze (∇), mixed (x) and water film (\circ) in (b). (a) The freezing fraction for Figure 2(a). (b) Temperature profile through ice for Figure 2(a). LTP stands for linear temperature profile through the ice.

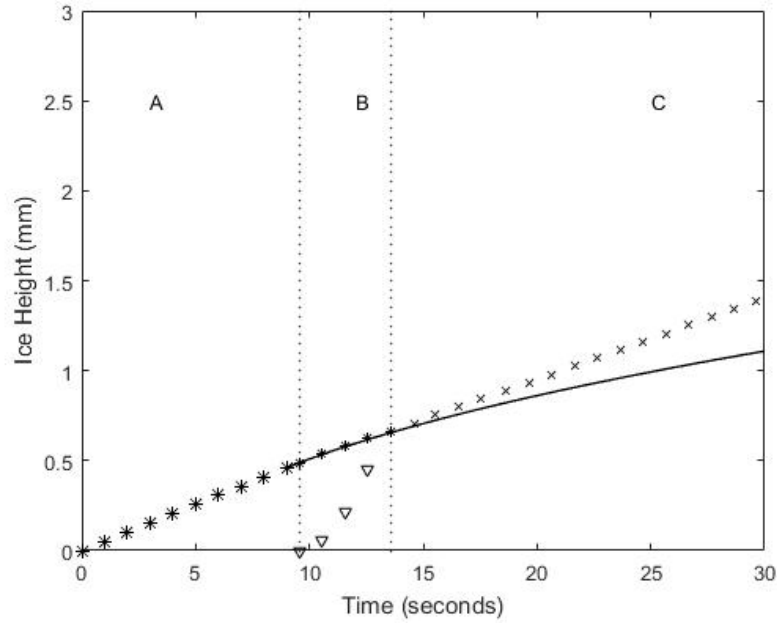


(a)

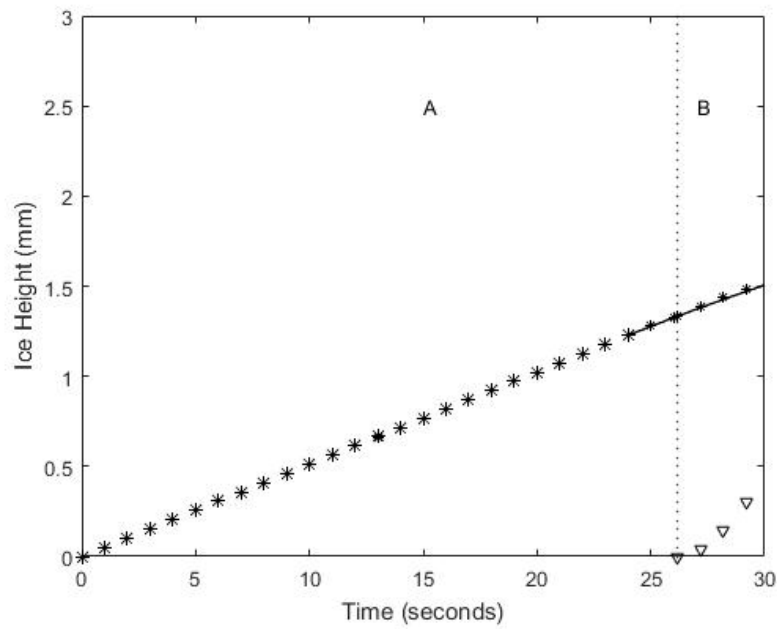


(b)

Figure 4: Mixed ice accretion at different ambient temperatures T_a and solid volume fractions ϕ respectively for 30 s of droplet impingement. The different ice and water limits are represented by the following symbols: rime (*), glaze (∇), mixed (x) and water film (\circ). (a) $T_a = -3^\circ\text{C}$ and $\phi = 0.65$. (b) $T_a = -6^\circ\text{C}$ and $\phi = 0.65$.



(a)



(b)

Figure 5: Mixed ice accretion model versus Myers [10] model. The different ice and water limits are represented by the following symbols: rime (*), glaze (∇), mixed (x) and water film (\circ). (a) $T_a = -3^\circ\text{C}$ and $\phi = 0.96$ versus Myers [10] model for glaze ice (-). (b) $T_a = -6^\circ\text{C}$ and $\phi = 0.96$ versus Myers [10] model for rime (*) and glaze ice (-).

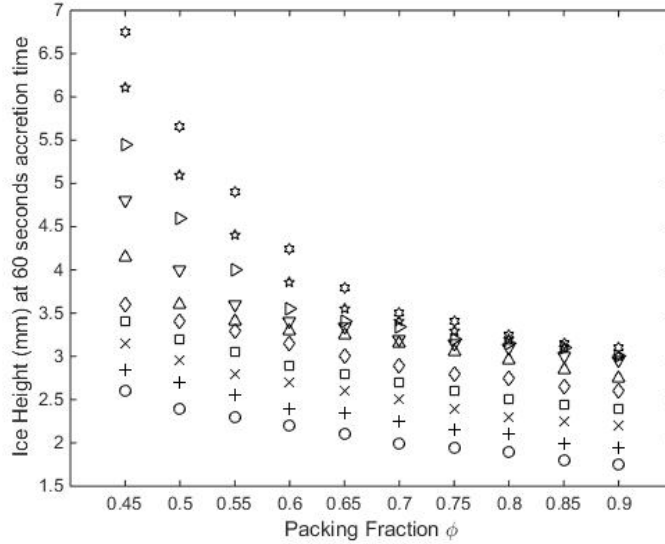


Figure 6: Overall ice height variation with changes in ϕ at -3°C (\circ), -6°C (\diamond) and -9°C (\square).

400 temperatures. Similarly, when comparing Figure 5(a) and Figure 5(b), we
 401 can see that the decrease in temperature from -3°C to -6°C whilst the packing
 402 fraction is kept constant at 0.96 results in an overall ice height of 1.4 mm
 403 (0.7 mm glaze and 0.7 mm mixed) and 1.5 mm (0.4 mm glaze and 1.1 mm
 404 rime) respectively.

405 Figure 5(a) shows that although the author's model prediction results in
 406 an overall ice height of 1.4 mm as compared to the glaze model [10] height of
 407 1.1 mm; only 0.7 mm of the former is glaze which has different adhesion char-
 408 acteristics than rime or mixed. Similarly Figure 5(b) shows that although the
 409 author's model prediction results in an overall ice height of 1.5 mm, exactly
 410 the same as the glaze model [10] height of 1.5 mm; only 0.4 mm of the former

411 is glaze which has different adhesion characteristics than rime or mixed. The
412 models match quite well even at smaller accretion times e.g. Figure 5(a) at
413 14s accretion time; and limiting cases. Table 2 exhibits the differences be-
414 tween the author's model which consistently predicts a lower glaze ice height
415 than the glaze ice model [10] at accretion time of 30 seconds and $\phi = 0.96$.
416 The difference in the height of the glaze varies with ambient temperature:
417 -2°C (33%), -3°C (30%), -4°C (31%), -5°C (7%) and -6°C (67%). The per-
418 centage fluctuations in the latter two values arise due to the completion of
419 primary mixed ice stage at -5°C and the presence of a prolonged rime ice
420 stage at lower temperatures of -6°C . Another key difference to note is the
421 absence of a water film at 30s for the author's model in all cases barring
422 accretion at -2°C . This is because secondary mixed ice occurs due to the
423 dual freezing actions exhibited by instantaneous freezing of a fraction of the
424 supercooled droplets at the air-rime interface and freezing at the Stefan in-
425 terface. This implies that the formation of the thin film may occur later than
426 initially expected on an aircraft wing. In the glaze ice model [10], the water
427 film appears immediately after the rime stage whereas in the author's model,
428 the water film appears much later after secondary mixed ice stage.

429 Figure 6 exhibits the change in overall accretion height at 60 seconds with
430 variation in ϕ . It is apparent from the figure that the height consistently
431 increases with a reduction in ϕ . We see that the effect of packing fraction on
432 the accretion height increases as the ambient temperature reduces.

433 In terms of a qualitative comparison, Myers (2001) [10] grew ice in a wind

Table 2: Ice height comparison between mixed ice accretion predicted in the present work, and a pure glaze ice accretion [10].

T_a °C	Glaze mm, <i>glaze only</i> [10]	Water	Total	Rime mm, <i>present work</i>	Glaze	Water	Mixed	Total
-2	0.9	0.6	1.5	0	0.6	0.2	0.6	1.4
-3	1	0.5	1.5	0	0.7	0	0.7	1.4
-4	1.3	0.2	1.5	0	0.9	0	0.5	1.4
-5	1.4	0.1	1.5	0	1.3	0	0.2	1.5
-6	1.5	0.1	1.6	1	0.5	0	0	1.5

434 tunnel in Cranfield University under conditions matching those mentioned in
 435 Table A except for a change in collection efficiency from 0.5 to 0.55. The tem-
 436 perature of the wind tunnel was kept at -10°C and the overall accretion time
 437 was 12 minutes. He noticed that the height of the rime ice during transition
 438 to glaze was between 2–3 mm and occurred at approximately 46 seconds.
 439 Using our predictions, a packing fraction range of 0.87–0.98 gives us an ice
 440 height of 2.9–2.6 mm at the same accretion time which is an encouraging
 441 sign covering different density values of hard rime.

442 It is important at this stage of the model development to also test com-
 443 putational results with actual experimental results in an icing facility. For
 444 this purpose, we consider two studies. Palacios et al. (2010) [29] conducted
 445 icing experiments on an Adverse Environment Rotor Test Stand (AERTS).
 446 Essentially, the experiment consisted of 1 inch diameter-50 inch radius rotor
 447 connected to a 125 HP motor; inside a cold chamber with nozzle location on
 448 the ceiling. Similar tests were conducted by Ruff (1985) [30] in the Air Force
 449 Arnold Engineering Development Center. Since solid volume fraction ϕ is

Table 3: Ice height comparison between author’s model and experimental data

T_a (°C)	U_∞ m s^{-1}	ρ_a (g m^{-3})	t (s)	Current Model (mm)	Experimental Data (mm)	
-15	60.9	1.3	150	13.7	15.2	[30]
-13.75	60.9	1.2	300	25.3	25.4	[29]
-15	60.9	1.2	300	25.4	27.9	[30]
-12	60.9	0.8	225	12.8	13.5	[29]
-11.4	60.9	0.9	225	14.4	14	[30]
-5.5	60.9	1.3	150	4.8	5.7	[29]
-5	60.9	1.2	150	4.6	5.3	[30]

450 currently not linked to the freezing fraction λ in our code, we back calculate
 451 ice height from a single experimental reading of Palacios et al. (2010) [29] to
 452 give rime ice density as 743 kg m^{-3} and subsequently $\phi = 0.81$. Mean volume
 453 diameter (MVD) was fixed at $20 \mu\text{m}$ and air velocity at 60.9 m s^{-1} to reduce
 454 the amount of variables during comparison with the experimental results.

455 AoA was kept at 0° and β was calculated from Palacios et al. (2010) [29]
 456 as ~ 0.7 . Table 3 shows the comparison of results from the author’s model
 457 and experimental data. We see that for a wide array of icing conditions, dis-
 458 crepancies are between 0.4–15%; which is a promising sign for future work.
 459 The ice height from the experimental data was measured from the stagna-
 460 tion point. The errors accumulated are due to experimental calculation of
 461 collection efficiency and LWC; as well as being unable to account for changes
 462 in ϕ when LWC and ambient temperature change. This is an area of future
 463 research to build upon when λ and ϕ can be linked to compare to a wider

464 array of experimental data.

465 **4. Conclusion**

466 An initial one dimensional mixed ice accretion model has been developed
467 that incorporates both rime and glaze. A dimensionless parameter λ has been
468 introduced to account for the accretion of mixed ice on an aerofoil in nature.
469 Current icing models account for individual rime and glaze ice accretion on
470 an aircraft wing. The development of a mixed ice model is the first step in
471 quantifying the accretion of the third type of in-flight icing seen in nature and
472 shows that for similar atmospheric parameter ranges, this simple mixed ice
473 description leads to very different accretion rates. The boundary tempera-
474 tures provided for the atmosphere in relation to the type of icing experienced
475 correspond well with the literature. The authors' mixed ice model reduces
476 to the glaze ice model [10] when $\lambda = 0$ and states that the model [10] is
477 valid only for temperatures between -1.8° and 0°C . It predicts lower glaze ice
478 heights than the former at lower temperatures. Lower temperatures favour
479 higher ice growth and increasing the packing fraction corresponds to lower
480 ice height and glaze icing regime. The model shows a promising comparison
481 with both previously published computational data and experimental results.

482

483 Future work will include determining a transient value of freezing fraction to
484 provide results for longer accretion times, changing water film temperatures
485 and the effect of droplet size. The freezing fraction must also be linked to

486 the packing fraction to allow for variation in ϕ . Eventually, the model must
487 be expanded to account for two-dimensional accretion on an aircraft wing.

488 **Acknowledgements**

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Table A: Notation and values ascribed to parameters throughout this paper [20].

Parameter	Value
a	0.895
c_a	$1014 \text{ J kg}^{-1}\text{K}^{-1}$
H_{aw}	$500 \text{ W m}^{-2}\text{K}^{-1}$
L_f	334000 J kg^{-1}
c_w	$4220 \text{ J kg}^{-1}\text{K}^{-1}$
x_e	9.53 m s^{-1}
e_0	44.4 Pa K^{-1}
ρ_l	0.001 kg m^{-3}
β	0.5
U_∞	90 m s^{-1}
α	0°
ρ_w	1000 kg m^{-3}
ρ_i	917 kg m^{-3}
T_f	273.15 K
k_i	$2.18 \text{ W m}^{-1}\text{K}^{-1}$
k_w	$0.571 \text{ W m}^{-1}\text{K}^{-1}$
\dot{M}	$0.045 \text{ kg m}^{-2}\text{s}^{-1}$
x_s	11.65 m s^{-1}

491 Appendix

492 4.1. Mass Flux of Droplets

493 Fig. 1(a) exhibits the supercooled droplet mass flux prior to the accretion
494 stage. The first step for the icing code is to define the different terms in the
495 mass balance. The rate of impinging droplets incident on an aircraft wing \dot{N}
496 can be expressed as

$$\dot{N} = \frac{V_{air}\rho_l\beta}{V_d\rho_w}, \quad (36)$$

497 where $V_d = \frac{\pi d^3}{6}$ is the volume of a single (spherical) droplet and V_{air} is the
 498 volume of air incident on the aircraft wing per second. β i.e. the collection
 499 efficiency, can be defined as the distance between two droplets in the free
 500 stream and along the body surface when they impact the aerofoil respectively
 501 [12]. V_{air} is given by,

$$V_{air} = AU_\infty \cos \alpha, \quad (37)$$

502 where α is the angle of attack. Beyond $\alpha = 20^\circ$, the aircraft begins to stall.
 503 Hence α generally ranges between 5° and 15° . The incoming mass flux of
 504 supercooled droplets incident per unit area of the wing \dot{M} can be defined as

$$\dot{M} = \frac{V_d \dot{N} \rho_w}{A}, \quad (38)$$

505 which reduces to

$$\dot{M} = \beta U_\infty \rho_l \cos(\alpha). \quad (39)$$

506 For one-dimensional accretion on a flat plate, $\alpha = 0$. Using the values from
 507 Table A, we get $\dot{M} = 0.045 \text{ kg m}^{-2} \text{ s}^{-1}$.

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