





# **Financial Frictions, Monetary Policy and Business Cycles**

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## **Abstract**

This dissertation consists of three essays, which study the implication of financial frictions in business cycles and monetary policy making. The first essay develops a Dynamic Stochastic General Equilibrium (DSGE) model to study how the instability of the banking sector can amplify and propagate business cycles. Model simulations show that in an economic downturn, in addition to credit demand contraction induced by low firm net worth, low bank capital position can create strong credit supply contraction, and have a quantitatively significant effect on business cycle dynamics. The second essay studies the optimal Taylor-type monetary policy rules based on the model developed in the first chapter and find that with interest rate smoothing, 'leaning against the wind' can significantly dampen the procyclicality of financial distortions, and increase the welfare of the economy. The third chapter examines the role of households frugality in a financial crisis and finds that higher savings by more frugal households provide an important cushion for the fall in private investment funding.

*Keywords:* asset prices, financial frictions, monetary policy, banking instability, savings, investment, business cycles



# Zusammenfassung

Diese Dissertation besteht aus drei Aufsätzen, die die Auswirkungen von Kapitalmarktfriktionen auf Geldpolitik und Konjunkturverlauf untersuchen. Ich entwickle ein dynamisch-stochastisches Gleichgewichtsmodell (DSGE) um zu prüfen, wie die Instabilität im Bankensektor Konjunkturzyklen verstärken und verbreiten kann (Kapitel 1); um ferner zu prüfen, ob optimale Geldpolitik in einer durch Kreditfriktionen geprägten Wirtschaft gegen den Strom schwimmen sollte (Kapitel 2); und um die Rolle privaten Sparens auf die Finanzkrise zu untersuchen (Kapitel 3).

Das Thema dieser Arbeit wurde zu einer Zeit gewählt, in der die Weltwirtschaft ihre schlimmste Finanzkrise seit Jahrzehnten erlebte. Die Krise griff schnell auf die Realwirtschaft über und resultierte in einer weltweiten Rezession. Es wurde vielfach argumentiert, dass die Zentralbanken zu sehr um die Inflation von Güterpreisen besorgt waren und beim Setzen der Leitzinssätze zu wenig Augenmerk auf Kredit- und Kapitalkosten richteten. Dies spielte eine beträchtliche Rolle in der Entstehung der Krise. Zudem spielten in der existierenden makroökonomischen Literatur über Konjunkturzyklen der Bankensektor und mit ihm zusammenhängende Kapitalmarktfriktionen eine untergeordnete Rolle. Für lange Zeit glaubte man an das Modigliani-Miller Theorem, welches besagt, dass die Finanzstruktur für realwirtschaftliche Ergebnisse irrelevant sei und daher bei der Betrachtung von Konjunkturzyklen vernachlässigt werden könne. Dies führte zur Entstehung einer großen Bandbreite von DSGE-Literatur, welche sich lediglich mit realen Friktionen und nominalen Rigiditäten und den entsprechenden Implikationen auf Geldpolitik beschäftigten. Die Finanzkrise der letzten Jahre zeigte, dass Kapitalmarktfriktionen eine zentrale Rolle bei der Bestimmung der Länge und des Ausmaßes einer Rezession spielen und Finanzschocks selbst Auslöser von Konjunkturzyklen sein können. Zudem kann die Analyse optimaler Geldpolitik in einem theoretischen wirtschaftlichen Umfeld ohne Kapitalmarktfriktionen zu verzerrten Ergebnissen und mit der Anwendung dieser Ergebnisse zu ernsthaften Konsequenzen für die reale Welt führen. Vor diesem Hintergrund erweitere ich ein kanonisches DSGE-Modell um den Bankensektor und dazugehörige Kapitalmarktfriktionen, um die Interaktion von Realwirtschaft und dem Finanzsektor zu untersuchen (Kapitel 1). Aufbauend auf dem in Kapitel 1 entwickelten Modell untersuche ich dann, welche Implikationen die Berücksichtigung von Kapitalmarktfriktionen für die Geldpolitik hat. Dabei interessiert besonders, ob Geldpolitik Wertpapierpreise über die Zyklen stabilisieren sollte um damit die Prozyklizität finanzieller Verwerfungen zu dämpfen.

Allgemein kann die Instabilität des Bankensektors von beiden Seiten der Bilanz herrühren. Eine repräsentative Arbeit von Diamond und Dybvig (1983) zeigt die natürliche Instabilität auf Seiten der Verbindlichkeiten. Aufgrund impliziter oder expliziter Garantien seitens des Staats für Einlagen bei Banken werden Bank Runs weitgehend vermieden. Kapitel 1 konzentriert sich daher auf Risiken auf Seiten der Aktiva der Banken. Diese Risiken beziehen sich dabei nicht auf das Eingehen exzessiver und irrationaler Risiken seitens der Banker, sie spiegeln vielmehr institutionelle Schwächen der Banken wider. Banken können bspw. zwar spezifische Schocks durch breite Anleihenportfolios minimieren, bleiben aber dennoch gegenüber systemischen Risiken empfindlich. Grundlegende Ursache dafür ist, dass Banken mit Kreditnehmern kein Set zustandsabhängiger Verträge abschließen können, um sich damit gegen alle makroökonomischen Umstände abzusichern.

Als Startpunkt habe ich entschieden, das Bernanke et al. (1999) Modell (im Folgenden BGG) zu erweitern, welches die Rolle von Kreditnachfragefraktionen als Folge von asymmetrischen Informationen zwischen Kreditnehmer und geber untersucht. Das Modell stellt einen Zusammenhang zwischen den Fremdkapitalkosten eines Unternehmens und seinem Nettowert her. Im Zuge eines Abschwungs erhöht sich die Verschuldungsquote des Unternehmens, was aufgrund sich verschärfender Informationsasymmetrien wiederum seine Fremdkapitalkosten erhöht. Die höheren Kosten führen folglich zu geringerer Kapitalnachfrage. Der Einbruch der Kapitalnachfrage verstärkt den anfänglichen Rückgang des Netto-Unternehmenswerts und verstärkt somit konjunkturellen Abschwung. Dieser Mechanismus ist in der Literatur als Financial Accelerator bekannt. Allerdings können durch die Verwendung von Verträgen, welche die Banken vor aggregierten Schocks schützen, das Kernproblem der Verknüpfung von systemischen Risiken mit den Bilanzen der Banken sowie die damit verbundene Instabilität im Bankensektor, welche über den Kreditmarkt auf die Volkswirtschaft übertragen wird, vermieden werden. Dieser Zusammenhang zeigte in der Finanzkrise seine große Bedeutung.

Dieses erste Kapitel fokussiert sich auf die Finanzstruktur von Banken und die damit zusammenhängenden Friktionen des Kreditangebots. Das grundlegende Modell ähnelt dem BGG mit dem zentralen Unterschied, dass finanzielle Verträge mit einbezogen werden, auf deren Grundlage sich Kreditnehmer und geber systemische Risiken teilen. Am Ende jeder Periode wird ein Kreditvertrag basierend auf den Erwartungen der beiden Parteien im Hinblick auf zukünftige wirtschaftliche Bedingungen unterzeichnet. Aggregierte Schocks in der nächsten Periode werden zu einer höher als erwarteten Kreditausfallrate führen und



daher nicht nur die Bilanz der Firma, also ihren Nettowert, sondern auch die Bilanz der Bank bzw. ihre Kapitalposition beeinflussen, weil die Bank mit hohen Abschreibungen infolge der unerwarteten Kreditausfälle konfrontiert wird. Dies unterscheidet sich vom BGG Modell insofern, als dass dort zustandsabhängige Verträge mit Unternehmern abgeschlossen werden, die dazu führen, dass das Kreditportfolio der Bank von aggregierten Schocks unabhängig ist. Die Bank muss nun zwischen einer Ausweitung seiner Aktiva und der Erhöhung der Finanzierungskosten abwägen, da Haushalte ein geringeres Verhältnis von Kapital und Aktiva als instabilere Finanzstruktur auffassen und eine höhere Prämie für das Halten von Bankanteilen verlangen. Zusätzlich zu den vom Financial Accelerator erfassten Kreditnachfragefraktionen aufgrund des geringen Nettowerts der Firmen, führt eine ungünstige Kapitalposition von Banken zu starken Kreditangebotsfraktionen. Diese beidseitigen Kreditfraktionen interagieren und verstärken einander, was zu einem weiter verstärkten Abschwung der Wirtschaft führt.

Modellsimulationen zeigen, dass die Instabilität im Bankensektor alleine zu starken Kreditangebotsfraktionen führen kann, welche kurzfristige Abschwünge signifikant verstärken können. Aus dem Bankensektor entstehende Schocks, wie etwa eine plötzliche Schrumpfung des Bankenkapitals, können zu einem starken Abschwung der Realwirtschaft führen. Langfristig impliziert die Instabilität im Bankensektor einen niedrigeren Kapitalstock in der Wirtschaft, dessen Folgen geringeren Investitionsniveaus und geringerer Wirtschaftsleistung sind. Ich vergleiche zudem die relativen Beiträge der verschiedenen Friktionen bei der Übermittlung von Schocks und komme zu dem Schluss, dass das Bankkapital als Übermittler wichtiger bei der Verstärkung von Geldpolitikschocks ist, als der Financial Accelerator. Dies stimmt mit früheren Befunden aus der Literatur überein welche besagen, dass der Financial Accelerator nur marginal zur Übertragung von Geldpolitik beiträgt. Die relative Bedeutung der beiden Übertragungswege ist jedoch umgekehrt, wenn die Wirtschaft durch einen positiven Technologieschock getroffen wird. In diesem Fall spielen starke Unternehmensbilanzen eine wichtige Rolle bei der Erhöhung von Vermögenswerten und Investitionsvolumina.

Das Modell kann auch das seit langem bestehende Rätsel lösen, wieso aggregiertes Leihen nicht sofort nach einem restriktiven Geldpolitikschock sinkt, sondern erst für vier bis sechs Quartale steigt und dann erst sinkt. Der dahinter stehende Mechanismus ist, dass der Nettowert von Firmen in der ersten Periode nach dem Schock stärker als die Preise der Aktiva sinken und Firmen daher mehr auf externe Finanzierung angewiesen sind. In der folgenden Periode sinken der Nettofirmenwert langsamer und die Preise der Aktiva schneller, so dass externes Ausleihen der Firma reduziert wird.

Kapitel 2 verwendet eine Approximation zweiter Ordnung des Modells aus Kapitel 1, um die wohlfahrtsmaximierende Geldpolitik zu untersuchen. Es gibt drei zentrale Ergebnisse: Erstens ist, ungeachtet dessen, ob die Zentralbank eine starke oder schwache Haltung gegenüber Inflation hat, eine Glättung des Zinssatzes als Reaktion auf Vermögenspreise strikt wohlfahrtsverbessernd. Zweitens ist der Wohlfahrtsgewinn kleiner aber immer noch signifikant, wenn eine starke Reaktion auf Output Teil der Geldpolitik ist. Drittens kann ohne Zinssatzglättung eine zu starke Reaktion auf Vermögenspreise zu Unbestimmtheit des Modells führen. Diese Ergebnisse stehen in Kontrast zu früheren Befunden welche unter der Annahme einer starken Anti-Inflations-Haltung einen marginalen Wohlfahrtsgewinn der Reaktion auf Vermögenspreise ermitteln. Der Grund hierfür ist, dass die Einbeziehung des Zusammenhangs zwischen Vermögenspreisen und Instabilität des Bankensektors in das Modell die Volatilität von Vermögenspreisen zu Instabilität auf dem Bankensektor führt, welche auf die Realwirtschaft übergreift und dadurch Konjunkturzyklen verstärkt und verbreitet. Es ist daher für Zentralbanken entscheidend, gegen den Strom zu schwimmen und die Prozyklität finanzieller Verwerfungen und damit ihren Einfluss auf die Realwirtschaft zu mindern. Eine weitere Beobachtung in der Krise, welche große Aufmerksamkeit erhielt, war die plötzlich steigende Sparquote der Haushalte in vielen Ländern. In den USA brachten sowohl Politiker als auch Wissenschaftler ihre Besorgnis zum Ausdruck, dass erhöhte Sparsamkeit der Haushalte aufgrund des aus ihr resultierenden Rückgangs der aggregierten Nachfrage zu einer tieferen und längeren Rezession führen wird. In Kapitel 3 lege ich dar, dass die Sparsamkeit der Haushalte eine zentrale Rolle bei der letzten Rezession gespielt haben könnte, sie allerdings nicht destabilisierend wirkte.

Die kürzlich erfolgte Rezession war größtenteils eine Bilanz-Rezession. Sie wurde durch sinkendes Vertrauen in das Finanzsystem und einen widrigen Schock auf den Nettowert von Unternehmen verursacht. Ein derartiger Schock wird sich in fallenden Unternehmensgewinnen und daher auch sinkenden einbehaltenen Gewinnen dem unternehmerischen Sparen niederschlagen. Bei derartigen finanziellen Spannungen können sparsamere Haushalte zur Stabilisierung der Wirtschaft beitragen, statt sie zu destabilisieren. Wie die Spar-Investitions-Identität zeigt, muss das private Kapital einer Wirtschaft mit der Summe aus unternehmerischer Binnenersparnis, der Binnenhaushaltersparnis, dem Haushaltsüberschuss der Regierung und externer Ersparnis übereinstimmen. Daten der USA und einigen anderen industrialisierten Volkswirtschaften legen nahe, dass sowohl Unternehmensgewinne, als auch Haushaltsüber-

schüsse der Regierungen während der Krise um durchschnittlich 5 % des BIP gesunken sind, während gesamte Investitionsaufwendungen um 3,2 % des BIP sanken (der nicht-Binnenanteil der Investitionen fiel sogar nur um 1 %). Dies legt nahe, dass die zusätzliche Haushaltsersparnis den freien Fall der Investitionen aufgefangen hat. Aufgrund des zurückgegangenen Konsums ist die Haushaltsersparnis um ca. 1,3 % des BIP angestiegen.

Ich untersuche das Zusammenspiel zwischen unternehmerischen Ersparnissen, Haushaltsersparnissen und privaten Kapitalinvestitionen in Zeitperioden finanzieller Spannungen in einem allgemeinen Gleichgewichtsmodell. Das Modell folgt größtenteils dem BGG-Ansatz, wurde aber modifiziert, um eine Geldstromanalyse zu vereinfachen. Ich modelliere eine Bilanzrezession als widrigen Schock für netto-Unternehmenswerte. Modellsimulationen sind konsistent mit empirischen Beobachtungen und zeigen, dass während Perioden finanzieller Spannungen, wenn unternehmerische Ersparnisse sinken, zusätzliche Haushaltsersparnisse eine wichtige alternative Finanzierungsquelle zur Dämpfung des Falls der Investitionen sind.

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# Introduction

This dissertation consists of three essays, which study the implication of financial frictions on monetary policy making and business cycles. In particular, I develop a Dynamic Stochastic General Equilibrium (DSGE) model to study how the instability of the banking sector can amplify and propagate business cycles (Chapter 1); study whether the optimal monetary policy should "lean against the wind" in an economy that is characterized by credit frictions (Chapter 2); and investigate the role of household frugality in a financial crisis (Chapter 3).

The topic of the thesis was chosen at a time when the world was experiencing the worst financial crisis in decades, which then rapidly spilled over to the real economy and resulted in a worldwide recession. Many have argued that the focus on goods price inflation has made central banks place insufficient weight on credit and asset prices in interest rate setting, which played a non-negligible role in the buildup to the crisis. However, the existing macro literature on business cycles paid very little attention to the banking sector and related financial frictions. As for a long time, people have believed in the Modigliani-Miller proposition, which states that financial structure is irrelevant for real economic outcomes and therefore can be omitted in business cycles analysis. This has given rise to an exuberance of DSGE literature focusing only on real frictions and nominal rigidities and studying the corresponding implications for monetary policy making.

The recent crisis has demonstrated that financial frictions can play a central role in determining the length and depth of a recession and financial shocks themselves can be a trigger of business cycles. Also, studying optimal monetary policy based on an economy structure where financial frictions are assumed away can lead to biased results and such policy making may have serious consequences in the real world. Against this backdrop, I extend a canonical DSGE model with banking sector and related credit frictions to study the interaction between the real economy and the financial sector (Chapter 1). Based on the model developed in Chapter 1, I study what are the implications for monetary policy after taking financial frictions into consideration, especially I am interested in whether monetary policy should stabilize asset prices over the cycles and consequently dampen the procyclicality of financial distortions (Chapter 2).

Generally speaking, the instability of the banking sector can arise on both sides of the balance sheet. Representative work from Diamond and Dybvig (1983) shows the inherent

instability coming from the bank's liability side. However, given the explicit or implicit government guarantee on bank deposits, bank runs have been mitigated to a large extent. Chapter 1 therefore focuses on risks from the asset side. To be more specific, this risk does not refer to the irrational excessive risk taking of bankers, rather, it reflects the institutional weakness of banks, i.e., although banks can diversify away idiosyncratic shocks by holding a large loan portfolio, they are still vulnerable to any systemic risk. A more fundamental reason for this is that banks cannot sign a complete set of state-contingent contracts with borrowers to insure against each macroeconomic state.

As a starting point, I choose to extend the Bernanke et al. (1999) (hereinafter BGG) model, which investigated the role of credit demand friction, as a result of asymmetric information between the borrower and the lender. Their model established a link between a firm's borrowing cost and its net worth. In an economic downturn, firms' leverage ratios increase, causing them to face a higher external finance premium because information asymmetry is exacerbated. This higher premium in turn dampens capital demand. The drop in capital demand thus reinforces the initial decline of firms' net worth and the business cycle is propagated. This mechanism is known in the literature as the "financial accelerator". However, by using financial contracts that insulate banks from aggregate shocks, they have avoided the key issue of linking systemic risk to banks' balance sheets and the related banking instability, which is then passed on to the macro economy through the credit market. This linkage is shown to be of critical importance in the recent crisis.

This first chapter focuses on the financial structure of banks and related credit supply frictions. The basic model is similar to BGG, while the key difference is the integration of a financial contract where borrowers and lenders share systemic risk. At the end of each period, a loan contract is signed based on the two parties' expectation of future economic conditions. Contractionary aggregate shocks in the next period will lead to a higher- than-expected loan default rate and will therefore not only influence the firm's balance sheet, i.e., net worth, but also the bank's balance sheet, or capital position, as the bank faces large write-offs from the unexpected loan losses. This contrasts with the BGG model, in which banks issue state-contingent contracts with entrepreneurs; therefore, returns on their loan portfolio are independent of any aggregate shocks. Given their initial capital position, banks now face the trade-off between increasing asset size and raising funding costs, as households perceive lower capital-asset ratio as a more unstable financial structure and charge a higher premium for holding banks' equity.

Therefore, in an economic downturn, in addition to the credit demand friction induced by low firm net worth, as captured in a financial accelerator, a low bank capital position also gives rise to strong credit supply friction. Credit frictions from both sides will interact, and reinforce each other, and drive the economy down further.

Model simulations show that the instability of the banking sector alone can create strong credit supply frictions and can amplify and propagate short-run cycles significantly. Shocks that originate from the banking sector, e.g., a sudden decline in the bank capital, can lead to strong contraction in the real economy. In the long run, the instability of the banking sector implies a lower capital stock in the economy and therefore a lower level of investment and output.

I also compare the relative contribution of various frictions in shock transmission and find that the bank capital channel is more important than the financial accelerator in amplifying monetary policy shocks. This is consistent with previous findings in the literature that the financial accelerator contributes only marginally to monetary policy transmission. However, the relative importance of the two channels is reversed when a positive technology shock hits the economy. In this case, strong corporate balance sheets play an important role in driving up asset prices and investment.

The model can also explain the long-established puzzle that aggregate lending does not decline immediately following a contractionary monetary policy shock but increases for four to six quarters and then falls. The mechanism behind this phenomenon is that firm's net worth contracts faster than asset prices in the initial period following a negative policy shock, and that therefore firms have to rely more on external financing. In the following period, contraction of firm's net worth slows down, while asset price is declining faster, so that firm's external borrowing declines.

Chapter 2 uses a second-order approximation of the model developed in Chapter 1 to study the welfare-optimizing monetary policy. The main findings are threefold: First, with interest rate smoothing, responding to asset price is strictly welfare improving regardless whether the central bank has a strong or weak stance on inflation; second, the additional welfare gain is smaller but still significant if strong reaction to output is included in the policy rule; Third, without interest rate smoothing, too strong reaction to asset prices may lead to model

indeterminacy. These results are in contrast with previous findings that conditional on strong anti-inflation stance, responding to asset price has marginal welfare gain. The reason is that once the link between asset prices and banking instability is incorporated into the model, high volatility of asset prices leads to banking instability, which spills over to the real economy, therefore amplifying and propagating business cycles. It is therefore crucial for central banks to 'lean against the wind' to dampen the procyclicality of financial distortions and their impact on real economy.

Another observation from the crisis that received great attention is the sudden increase of household savings in many countries. In the U.S., both policy makers and scholars have expressed their concerns that the emergence of household frugality will lead to a deeper and prolonged recession by reducing aggregate demand. In Chapter 3, I argue that while household frugality may have played a key role in the recent global downturn, that role may not have been a destabilizing one.

The recent economic downturn was in large part a "balance sheet" recession. It was brought on by a fall of confidence in the financial system and an adverse shock to corporate net worth. Such a shock will be reflected in a fall in corporate profits, including the retained component — corporate savings. Under such financial stress, more thrifty households can help stabilize the economy - rather than destabilize it. As the saving — investment identity shows, an economy's private capital investment must equal to the sum of domestic corporate savings, domestic household savings, the government budget surplus, and external savings. Data from US and several other industrialized economies suggest that both corporate earnings and government budget surpluses fell substantially by an average of around 5% of GDP during the crisis, while total investment expenditures fell by about 3.2% of GDP (the non-residential component of investment fell only by 1%). This suggests that extra household savings helped cushion the free fall of investment. Owing to a fall in their consumption, savings by household rose by about 1.3% of GDP.

I examine the interplay between corporate saving, household saving, and private capital investment during periods of financial stress in a general equilibrium framework. The model largely follows the BGG model, but are reinterpreted to facilitate a flow-of-funds analysis. I model a "balance sheet recession" as an adverse shock to corporate net worth. Model simulations are broadly consistent with empirical observations and show that: during periods

of financial stress, when corporate earnings fall, extra household savings are an important alternative financing source to buffer the fall of investment.

# 1 Bank Capital Regulation, the Lending Channel and Business Cycles

*This chapter develops a Dynamic Stochastic General Equilibrium (DSGE) model to study how the instability of the banking sector can amplify and propagate business cycles. The model builds on Bernanke, Gertler and Gilchrist (BGG) (1999), who consider credit demand frictions due to agency cost, but it deviates from BGG in that financial intermediaries have to share aggregate risk with entrepreneurs, and therefore bear uncertainty in their loan portfolios. Unexpected aggregate shocks will drive loan default rate away from expected, and have an impact on both firm and bank's balance sheets via the financial contract. In an economic down turn, in addition to credit demand contraction induced by low firm net worth, low bank capital position can create strong credit supply contraction, and have a quantitatively significant effect on business cycle dynamics.*

## 1.1 Introduction

Financial frictions have long been ignored in the literature on business cycles. The main theoretical justification for this omission is the Modigliani-Miller proposition, which implies that financial structure is irrelevant for real economic outcomes. However, the 2008 financial crisis has demonstrated that financial conditions play a central role in determining how real shocks are transmitted through the economy. It has also shown that financial disturbance itself can be a source of economic fluctuations. Moreover, many historical episodes illustrate that distressed banking systems and adverse credit market conditions have either triggered or contributed to serious macroeconomic contractions.<sup>1</sup> Yet in the canonical Dynamic Stochastic General Equilibrium (DSGE) models, there is no financial sector and consequently no financial shocks. Recently, a number of authors have sought to incorporate the banking sector and related credit frictions into DSGE models to study the interaction between the real economy and the financial sector. This paper is part of that effort.

Generally speaking, there are two aspects in integrating credit market frictions: one is the credit friction from the demand side, and the other is from the supply side. Bernanke and Gertler (1989) and Bernanke et al. (1999) (hereinafter BGG) investigated the role of credit demand friction, as a result of asymmetric information between the borrower and the lender. Their model established a link between a firm's borrowing cost and its net worth. In an

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<sup>1</sup>These include past crises in Scandinavia, Latin America, Japan, and other East Asian countries.

economic downturn, firms' leverage ratios increase, causing them to face a higher external finance premium because information asymmetry is exacerbated. This higher premium in turn reduces capital demand. The drop in capital demand thus reinforces the initial decline of firms' net worth and the business cycle is propagated. This mechanism is known in the literature as the "financial accelerator".

In terms of supply side friction, however, the vulnerability of the financial intermediary itself has not been incorporated into DSGE models. Recent models have tried to link the financial structure of banks to their lending rate to motivate the role of bank capital (e.g. Markovic (2006), Aguiar and Drumond (2007)) or have explained the function of banks in a detailed manner (Gerali et al. (2009), Christiano et al. (2007) ). However, by using financial contracts that insulate banks from aggregate shocks, they have avoided the key issue of linking systemic risk to banks' balance sheets and the related banking instability, which is then passed on to the macro economy through the credit market. This linkage is shown to be of critical importance in the recent crisis.

This paper focuses on the financial structure of banks and related credit supply frictions. The basic model is a closed economy DSGE model similar to BGG. The key deviation from the basic model is the integration of a financial contract where borrowers and lenders share systemic risk. At the end of each period, a loan contract is signed based on the two parties' expectation of future economic conditions. Contractionary aggregate shocks in the next period will lead to a higher- than-expected loan default rate and will therefore not only influence the firm's balance sheet, i.e., net worth, but also the bank's balance sheet, or capital position, as the bank faces large write-offs from the unexpected loan losses. This contrasts with the BGG model, in which banks issue state-contingent contracts with entrepreneurs; therefore, returns on their loan portfolio are independent of any aggregate shocks. Given their initial capital position, banks face the trade-off between increasing asset size and raising funding costs, as households perceive lower capital-asset ratio as a more unstable financial structure and charge a higher premium for holding banks' equity. Therefore, in an economic downturn, in addition to the credit demand friction induced by low firm net worth, as captured in a financial accelerator, a low bank capital position also gives rise to strong credit supply friction. Credit frictions from both sides will interact, and reinforce each other, and drive the economy down further.

Model simulations show that the instability of the banking sector alone can create strong



credit supply frictions and can amplify and propagate short-run cycles significantly. Shocks that originate from the banking sector, e.g., a sudden decline in the bank capital, can lead to strong contraction in the real economy. In the long run, the instability of the banking sector implies a lower capital stock in the economy and therefore a lower level of investment and output.

This paper also compares the relative contribution of various frictions in shock transmission. Three cases are considered. In the first case, only nominal rigidities and capital adjustment costs are considered; in the second, a financial accelerator effect is added; in the third case, the bank balance sheet channel is incorporated. Model simulations show that the bank capital channel is more important than the financial accelerator in amplifying policy shocks. This is consistent with previous findings in the literature that the financial accelerator contributes only marginally to monetary policy transmission.<sup>2</sup> However, the relative importance of the two channels is reversed when a positive technology shock hits the economy. In this situation, strong corporate balance sheets play an important role in driving up asset prices and increasing aggregate investment.

The model can also explain the long-established puzzle that aggregate lending does not decline immediately following a contractionary monetary policy shock but increases for four to six quarters and then falls. (Christiano et al. (1996) ). The mechanism behind this phenomenon is that firm's net worth contracts faster than asset prices in the initial period following a negative policy shock, and that therefore firms have to rely more on external financing. In the following period, contraction of firm's net worth slows down, while asset price is declining faster, so that firm's external borrowing declines.

This paper is also related to the banking literature which focuses on the fragility of financial intermediaries. Representative work from Diamond and Dybvig (1983) shows the inherent instability coming from the bank's liability side. Given the explicit or implicit government guarantee on bank deposit, this problem has been mitigated to a large extent. This paper shows the banking instability arising from the asset side. i.e., although banks can diversify away idiosyncratic shocks by holding a large loan portfolio, they are still vulnerable to any systemic risk. Another key difference is that in this model, financial instability is driven by fundamentals rather than pure self-fulfilling expectations.

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<sup>2</sup>See Meier and Mueller (2006), Christensen and Dib (2008).

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 describes the calibration strategy. Section 4 discusses the effect of the bank capital channel on long-run steady states and short-run dynamics. Section 5 concludes.

## 1.2 Model

The economy is inhabited by four types of agents: households, entrepreneurs, retailers and bankers. The structure of the basic model is the following: Bankers raise equity and deposit from the households, and then intermediate these funds to the entrepreneurs. Entrepreneurs combine their own net worth and the money they borrowed from banks to purchase physical capital, which will be used in aggregate production together with labor supplied by households. The product will then be differentiated in the retail sector to become final goods, which is either invested or consumed by the agents. Nominal Return on risk-free assets (i.e., deposit) is set by the central bank, who conducts monetary policy following a Taylor rule. Banks are subject to regulatory requirement on minimum capital ratio.

Next, we will present a financial contract where borrowers and lenders share aggregate risk, and then integrate it into a general equilibrium.

### 1.2.1 The Financial Contract

In this part, we discuss the design of an optimal financial contract between entrepreneurs and banks, which is the key deviation of our model to the original BGG model. The contract is derived in a partial equilibrium setting, taking the price of capital goods, entrepreneurs' net worth, the cost of deposits and bank capital as given. We then imbed the optimal contract in the general equilibrium setting.

There are two parties to the contract: an entrepreneur with net worth and a financial intermediary, which we call "bank". Bank takes deposit from and issue equity to households to finance the loan demanded by entrepreneurs. We will discuss the detail of the banking sector later. Both parties are assumed to be risk-neutral. At the end of period  $t$ , a continuum of entrepreneurs (indexed by  $i \in (0, 1)$ ) need to purchase capital for production at  $t + 1$ . The quantity of capital purchased by entrepreneur  $i$  is denoted  $K_{t+1}^i$ . The price of capital in period  $t$  is  $q_t$  (in real term). The return on capital is subject to both idiosyncratic and aggregate risk. The *ex-post* gross return to entrepreneur  $i$  is  $\omega_{t+1}^i R_{t+1}^k$ , where  $\omega_{t+1}^i$  is an idiosyncratic productivity

shock to entrepreneur  $i$ , and  $R_{t+1}^k$  is the *ex-post* aggregate rate of return on capital.  $\omega_{t+1}^i$  is identically and independently distributed (across time and entrepreneurs) with log-normal distribution and unit mean.

To finance the purchase of capital, entrepreneurs use internal funds (net worth) and borrow the rest from a bank. Let  $N_{t+1}^i$  denote the net worth of entrepreneur  $i$  at the end of period of  $t$ ; then borrows from the bank is the following:

$$L_{t+1}^i = q_t K_{t+1}^i - N_{t+1}^i \quad (1.1)$$

$\omega_{t+1}^i$  is private information to entrepreneur  $i$  and the bank has to pay a monitoring cost to observe it. Entrepreneurs observe the realization of  $\omega_{t+1}^i$  and decide whether to repay the debt or default. If they repay the debt, they pay  $R_{t+1}^L L_{t+1}^i$ .  $R_{t+1}^L$  is the gross loan rate specified in the contract that the entrepreneur need to pay to the bank. It can be fixed or state-contingent. If they default, the bank seizes the entrepreneur's remaining assets after paying the monitoring cost.<sup>3</sup> For a particular value of  $R_{t+1}^k$ , there is a corresponding cut-off value of idiosyncratic productivity  $\bar{\omega}_{t+1}^i$ , such that, if the realization of the idiosyncratic productivity falls below it, the entrepreneur defaults. That is:

$$\bar{\omega}_{t+1}^i R_{t+1}^k q_t K_{t+1}^i = R_{t+1}^L L_{t+1}^i \quad (1.2)$$

The monitoring cost is assumed to equal a proportion  $\mu$  of the realized gross capital return  $\omega_{t+1}^i R_{t+1}^k q_t K_{t+1}^i$ . Parameter  $\mu$  captures the degree of monitoring cost or information asymmetry.

<sup>4</sup>

In BGG, entrepreneurs are assumed to bear all the aggregate risk. By issuing state-contingent loan contract, banks are insulated from aggregate shocks and always obtain risk-free rate of return on loan portfolios. The optimal contract, as a result, maximizes the expected return to entrepreneurs as following:

$$\max E_t \left\{ \int_{\bar{\omega}_{t+1}^i}^{\infty} \omega_{t+1}^i R_{t+1}^k q_t K_{t+1}^i f(\omega_{t+1}^i) d\omega - (1 - F(\bar{\omega}_{t+1}^i)) \bar{\omega}_{t+1}^i R_{t+1}^k q_t K_{t+1}^i \right\} \quad (1.3)$$

where expectations are taken with respect to the random variable  $R_{t+1}^k$ , and  $\bar{\omega}_{t+1}^i$  is a function of realization of  $R_{t+1}^k$  (and therefore, function of the states).  $f(\cdot)$  and  $F(\cdot)$  are respectively the

<sup>3</sup>see Townsend (1979) and Gale and Hellwig (1985).

<sup>4</sup>The existence of the banking sector in this paper is taken as given. It could also be motivated by assuming that banks have information advantage compared to households in monitoring the project outcome, i.e.  $\mu_{bank} < \mu_{households}$ .

density function and the cumulative distribution function of the random variable  $\omega$ . The optimal contract must observe the participation constraints of the bank as well, such that, for each possible realization of states of nature (and therefore,  $R_{t+1}^k$  and  $\bar{\omega}_{t+1}^i$ ) the contract satisfies:

$$(1 - F(\bar{\omega}_{t+1}^i))R_{t+1}^L L_{t+1}^i + (1 - \mu) \int_0^{\bar{\omega}_{t+1}^i} \omega E_t R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega = R_{t+1}^f L_{t+1}^i \quad (1.4)$$

In equation (4), the left hand side shows that banks' return on the loan portfolio has two components: the loan amount that is paid back by the entrepreneurs, and, in the default case, the acquisition of the firm' remaining assets after paying off the monitoring cost.  $R_{t+1}^f$  is the funding cost of the bank, which will be determined in the general equilibrium. Since the participation constraints hold for each realization of  $R_{t+1}^k$ , banks face no uncertainty in the return on loan portfolio, which equals to the risk-free rate.

The risk-sharing rule among entrepreneurs and banks is a bit stylized, nonetheless.<sup>5</sup> In reality, banks face great uncertainty in their loan portfolio. The major source of uncertainty is shocks to default risk. To account for this, *we assume that aggregate risk is shared between banks and entrepreneurs. The financial contract cannot be therefore contingent on the realized capital return but has to be written based on the two parties' expectation of capital return in the next period.*<sup>6</sup> Under this risk sharing rule, we have to make a distinction between the ex-post loan default threshold  $\bar{\omega}_{t+1}^{i,b}$  and the ex-ante  $\bar{\omega}_{t+1}^{i,a}$ .

Let  $E_t R_{t+1}^k$  denote the expected capital return at the end of period t. *We assume that the entrepreneur can only offer the contract based on  $E_t R_{t+1}^k$  instead of all possible realizations of  $R_{t+1}^k$ .*<sup>7</sup>The contract maximizes the expected return of the entrepreneur as following:

$$\int_{\bar{\omega}_{t+1}^{i,a}}^{\infty} \omega_{t+1}^i E_t R_{t+1}^k q_t K_{t+1}^i f(\omega_{t+1}^i) d\omega - (1 - F(\bar{\omega}_{t+1}^{i,a})) \bar{\omega}_{t+1}^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i \quad (1.5)$$

where  $\bar{\omega}_{t+1}^{i,a}$  is the cut-off idiosyncratic productivity that the entrepreneur is *expected* to default in period t+1 based on information up to period t. Correspondingly, the participation constraint of banks is also based on  $E_t R_{t+1}^k$  :

$$(1 - F(\bar{\omega}_{t+1}^{i,a}))R_{t+1}^L L_{t+1}^i + (1 - \mu) \int_0^{\bar{\omega}_{t+1}^{i,a}} \omega_{t+1}^i E_t R_{t+1}^k q_t K_{t+1}^i f(\omega_{t+1}^i) d\omega = R_{t+1}^f L_{t+1}^i \quad (1.6)$$

<sup>5</sup>See footnote 10 in their paper.

<sup>6</sup>A state-contingent contract could be prevented by assuming that the state of the economy is not observed by the enforcement of the contract, but only observed at the very end of the period when people form expectations for the next period.

<sup>7</sup>Our assumption actually simplifies the characterization of the financial contract, as it corresponds to the problem of solving one case of no aggregate risk in the original BGG.

By solving the contract we obtain the credit demand equation (see Appendix A ):

$$E_t R_{t+1}^k = S\left(\frac{q_t K_{t+1}^i}{N_{t+1}^i}\right) R_{t+1}^f \quad (1.7)$$

The property and interpretation of  $S(\cdot)$  is identical to BGG, where  $S$  denotes the external finance premium, which captures the wedge (driven by the existence of monitoring cost) between the cost of finance from the firm's side and the cost of funds from the bank's side.  $S' > 0$ , implying that the higher is the leverage ratio of firms, the higher is the external finance premium.

After solving the optimal contract, the contractual lending rate could be derived as

$$R_{t+1}^L = \frac{\bar{\omega}_{t+1}^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i}{L_{t+1}^i} \quad (1.8)$$

Note that in this model the contractual lending rate is *fixed* and independent to the realizations of the return on capital in  $t+1$ , whereas in BGG the lending rate is *state-contingent*:

$$R_{t+1}^L = \frac{\bar{\omega}_{t+1}^i R_{t+1}^k q_t K_{t+1}^i}{L_{t+1}^i} \quad (1.9)$$

In period  $t+1$ , given the specified loan rate  $R_{t+1}^L$  and the realized return on capital, the ex-post default threshold  $\bar{\omega}_{t+1}^{i,b}$  is now determined by:

$$\bar{\omega}_{t+1}^{i,b} = \frac{R_{t+1}^L L_{t+1}^i}{R_{t+1}^k q_t K_{t+1}^i} \quad (1.10)$$

Recall that the expected default threshold is defined by:

$$\bar{\omega}_{t+1}^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i = R_{t+1}^L L_{t+1}^i \quad (1.11)$$

This implies:

$$\bar{\omega}_{t+1}^{i,b} = \frac{\bar{\omega}_{t+1}^{i,a} E_t R_{t+1}^k}{R_{t+1}^k} \quad (1.12)$$

From this expression, we see that any deviation of the realized capital return from expected one will drive a wedge between ex-post loan default rate and ex-ante. We will discuss its impact on banking sector and aggregate economy later.

## 1.2.2 General Equilibrium

In this section, we analyze how aggregate shocks can influence firm and bank's balance sheet via the financial contract in a general equilibrium. In addition to a firm's credit demand curve which is contingent on its net worth ( capturing the traditional financial accelerator effect), this model also derives an implicit credit supply curve, which is contingent on bank's capital position.

**Households** There is a continuum of households in the economy, each indexed by  $i \in (0, 1)$ . They consume the final good,  $c_t$ , invest in risk free bank deposits,  $d_{t+1}$ , and bank equity,  $e_{t+1}$ , supply labor  $h_t$  and own shares in a monopolistically competitive sector that produces differentiated varieties of goods. The households maximize the utility function:<sup>8</sup>

$$\max E_t \sum_{k=0}^{\infty} \beta^k [\ln(c_{t+k}) + \frac{d_{t+k+1}^{1+\varphi}}{1+\varphi} + \rho \ln(1 - h_{t+k})] \quad (1.13)$$

subject to the sequence of budget constraints:

$$d_{t+1} + e_{t+1} + c_t = w_t h_t + R_t^d d_t + R_t^e (1 - \phi_t) e_t + \Pi_t \quad (1.14)$$

$d_{t+1}$  and  $e_{t+1}$  are deposits and bank equity (in real terms) held by the household from  $t$  to  $t+1$ .  $R_t^d$  and  $R_t^e$  reflect the gross real return on holding deposit and bank equity, and  $\phi_t$  is the default rate on bank capital.  $h_t$  is household labor supply,  $w_t$  is the real wage for household labor,  $\Pi_t$  is dividends received from ownership of retail firms. Following Van den Heuvel (2008), the liquidity services of bank deposits are modeled by assuming that the household has a derived utility function that is increasing in the amount of deposits. The households' optimization problem yields following first-order conditions:

$$U_c(c_t) = \beta E_t R_{t+1}^e (1 - \phi_{t+1}) U_c(c_{t+1}) \quad (1.15)$$

$$U_c(c_t) - U_d(d_{t+1}) = \beta E_t R_{t+1}^d U_c(c_{t+1}) \quad (1.16)$$

$$-U_{c,t} / U_{h,t} = w_t \quad (1.17)$$

Equation (15) shows that households' intertemporal consumption decisions are determined by the default-adjusted return on holding bank equity. Equation (16) shows the optimality condition on bank deposit. Equation (17) describes the usual trade-off between consumption and

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<sup>8</sup>Inserting deposits into the utility function is just a modeling device to capture the bank' liquidity creation function. Model dynamics are robust if we consider a standard utility function with only consumption and leisure.

leisure. In the model set up, bank equity has to offer higher return than deposit for two reasons: the first is the liquidity premium, since deposits can provide households extra utility in addition to carry a monetary reward; the second is to compensate for the default risk. As will be discussed later, banks will be shut down and default on capital return when their capital ratios fall below the regulatory threshold.<sup>9</sup>

**Entrepreneurs** Other than difference in the financial contract, the entrepreneur sector at the aggregate level is identical to the BGG. We describe the entrepreneur sector for completeness purpose below. After signing the financial contract, entrepreneurs combine loans acquired from the bank and their own net worth to purchase capital. They use capital and labor to produce wholesale goods and sell them on a perfect competitive market at a price equal to their nominal marginal cost. The aggregate production function is given by :

$$Y_t = A_t K_t^{\alpha_k} (h_t)^{\alpha_h} (h_t^e)^{\alpha_e} (h_t^b)^{\alpha_b} \quad (1.18)$$

Following BGG, We assumed entrepreneurs and bankers supply one unit of labor services inelastically to the general labor market:  $h_t^e = h_t^b = 1$ . As will be see later,  $\alpha_e$  and  $\alpha_b$  are calibrated so that these two additional labor forces have a negligible effect on the output level and model dynamics.<sup>10</sup>

The optimization problem of production remains standard:

$$z_t = \alpha_k m c_t \frac{Y_t}{K_t} \quad (1.19)$$

$$w_t = \alpha_h m c_t \frac{Y_t}{h_t} \quad (1.20)$$

$$w_t^e = \alpha_e m c_t \frac{Y_t}{h_t^e} \quad (1.21)$$

$$w_t^b = \alpha_b m c_t \frac{Y_t}{h_t^b} \quad (1.22)$$

where  $z_t$  is the real rental rate of capital and  $w_t, w_t^e$  and  $w_t^b$  are, respectively, the real wage of households, entrepreneurs and bankers.  $m c_t$  denotes real marginal cost. The expected return on capital is then:

$$E_t R_{t+1}^k = E_t \left( \frac{z_{t+1} + (1 - \delta) q_{t+1}}{q_t} \right) \quad (1.23)$$

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<sup>9</sup>This paper assumes a relationship between households and bankers as delegated monitoring. Therefore, households do not care about the capital structure of banks in their decision.

<sup>10</sup>The salary that bankers earn from labor supply could be understood as fee income collected from transaction services, a function of financial intermediaries that is not modeled in the paper.

The accumulation of entrepreneurs' net worth consists of two parts: profits from operating the firms and labor income. It is assumed that, in every period, entrepreneur will die with the probability  $1 - \gamma$ . This assumption ensures that entrepreneurs never accumulate enough net worth to finance a project without external financing. Those entrepreneurs who die at time  $t$  will consume  $(1 - \gamma)V_t$ . The evolution of aggregate net worth is therefore given by:

$$N_{t+1} = \gamma V_t + w_t^e \quad (1.24)$$

where  $V_t$  represents gross return on operating business. It is the difference between gross capital return and loan payment.

$$V_t = \int_{\bar{\omega}^b}^{\infty} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega - (1 - F(\bar{\omega}^b)) R_{t+1}^L L_{t+1}^i \quad (1.25)$$

**Capital Producers** Capital producers purchase a fraction of final goods from the retailer as investment goods  $i_t$  and combine this with the existing capital stock to obtain capital stock in the next period. A quadratic capital adjustment cost is included to motivate a variable price of capital, which contributes to the volatility of firm net worth and bank capital. Capital producers will choose the quantity of investment goods to maximize profit subject to the adjustment cost:

$$\max E_t \left[ q_t i_t - i_t - \frac{\chi}{2} \left( \frac{i_t}{k_t} - \delta \right)^2 k_t \right] \quad (1.26)$$

where  $q_t$  is the real price of capital. The optimization problem yields the following capital supply curve:

$$q_t = 1 + \chi \left( \frac{i_t}{k_t} - \delta \right) \quad (1.27)$$

where  $\chi$  captures the sensitivity of capital price to investment fluctuation. The higher  $\chi$  is, the more volatile the price of capital. The aggregate capital stock evolves according to:

$$k_{t+1} = i_t + (1 - \delta)k_t \quad (1.28)$$

where  $\delta$  is the depreciation rate.

**Banking Sector** The banks' equity value is accumulated through retained earnings:

$$e_{t+1} = (1 - \phi_t)e_t + [R_{t+1}^L L_{t+1} (1 - F(\bar{\omega}^b)) + (1 - \mu) \int_0^{\bar{\omega}^b} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega - R_{t+1}^f L_{t+1}] + w_t^b$$



where  $\phi_t$  is the bank default rate, which will be explained in the bank regulation section. Aggregate bank equity at time  $t+1$  consists of three parts:  $(1 - \phi_t)e_t$  is equity from those banks who did not default at time  $t$ ; the term inside the square bracket is unexpected gains or losses in the loan portfolio;  $w_t^b$  is bankers' wages.

Substituting equation (6) into the above equation, we get:

$$\begin{aligned} e_{t+1} = & (1 - \phi_t)e_t + R_{t+1}^L L_{t+1} (F(\bar{\omega}^a) - F(\bar{\omega}^b)) \\ & + (1 - \mu) \int_0^{\bar{\omega}^b} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega \\ & - (1 - \mu) \int_0^{\bar{\omega}^a} \omega E_t R_{t+1}^k q_t K_{t+1} f(\omega) d\omega + w_t^b \end{aligned}$$

Notice from the financial contract, we have derived following relationship between loan default threshold and aggregate capital return:

$$\bar{\omega}_{t+1}^b = \frac{\bar{\omega}_{t+1}^a E_t R_{t+1}^k}{R_{t+1}^k} \quad (1.29)$$

Consider the case when a contractionary shock hits the economy, which reduces realized capital return  $R_{t+1}^k$  below the expected value  $E_t R_{t+1}^k$ . This will lead to higher ex-post loan default threshold  $\bar{\omega}_{t+1}^b$  than expected  $\bar{\omega}_{t+1}^a$ , correspondingly higher loan default rate  $F(\bar{\omega}^b)$  than anticipated  $F(\bar{\omega}^a)$ , and create unexpected losses  $R_{t+1}^L L_{t+1} (F(\bar{\omega}^a) - F(\bar{\omega}^b))$  that write down bank's capital position. This is the key difference of our model from the original BGG model in terms of shocks transmission. In the BGG setting, all aggregate shocks are absorbed by firms' balance sheets; while in our model, aggregate shocks are absorbed partly by firms' balance sheets and partly by banks' balance sheets via the financial contract.

Given the aggregate loan size,  $L_t$ , and bank equity, we obtain the aggregate capital ratio:

$$\Delta_t = \frac{e_t}{L_t} \quad (1.30)$$

The rest of bank funding

$$d_t = L_t - e_t \quad (1.31)$$

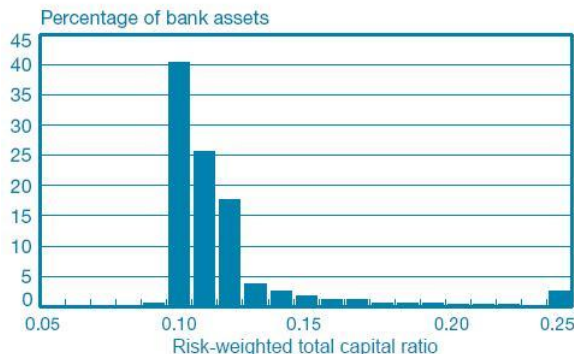
will be collected from the households in the form of deposits. Therefore, from an aggregate level, the opportunity cost of bank funding is a linear combination of cost of bank equity and cost of deposits, where the proportion of each type of funding varies according to the bank capital ratio.

$$R_{t+1}^f = \Delta_t R_{t+1}^e + (1 - \Delta_t) R_{t+1}^d \quad (1.32)$$

The respective costs of deposits  $R_{t+1}^d$  and equity  $R_{t+1}^e$  are derived endogenously from households' optimization problem.

**Bank regulation** In modern banking regulation, capital requirement has become the focal point.<sup>11</sup> Given the implicit or explicit government guarantee on bank deposit, bank capital regulation is imposed to curb banks' excessive risk-taking. In 1987, the Basel Committee of Banking Supervision established the Basel I Accord, which provided a uniform capital standard for all banks in the member countries. Basel I required the ratio of banks' capital to risk-weighted assets to amount to a minimum of 8 percent, with at least 50 percent of it being tier 1 capital. By 1993, nearly all of the world's big banks satisfied the Basel capital requirement. Many of them have been increasing their capital ratio. Figure 1.1 presents a histogram of the risk-based total capital ratios of U.S. commercial banks in the fourth quarter of 2000. As we can see from the figure, capital ratios vary across banks, with most of them between 10 and 11 percent, and very few below 10 percent.

Figure 1.1: Distribution of Bank Capital Ratio of U.S. Banks in 2000:4



Source: Federal Reserve Bank of Chicago

Motivated by this empirical observation, the capital ratio across banks in the model is assumed to have *log-normal* distribution. The mode of the distribution is given by the aggregate capital ratio derived above.  $\Delta_{i,t} \sim \log\text{-normal}(\Delta_t, \sigma)$ .<sup>12</sup> The health of the banking sector as a

<sup>11</sup>In this paper, bank capital regulation is taken as given, instead of being motivated from a micro perspective. It could be understood to mean that the threshold requirement is set to keep the government or the central bank from having to shoulder the burden of massive bank failures.

<sup>12</sup>The conditional distribution of bank capital ratio could be derived endogenously from the bank equity accumulation equation. For simplicity, in the simulation only the mean of the distribution is used, while the variance is assumed constant. As Krusell and Smith (1998) has shown, the behavior of the

whole will depend largely on the variation of aggregate capital ratio. With a higher aggregate ratio, the distribution moves to the right, and fewer banks will fall short of the 8 percent threshold and thus default, and vice versa. The default probability is given by the cumulative distribution function up to the regulatory threshold:<sup>13</sup>

$$\phi_t = cdf(\Delta_t, \sigma) \quad (1.33)$$

The higher the default probability, the more it costs banks to raise equity. Therefore, a low capital position today will lead to higher equity costs in the next period. This increase in funding costs will dampen banks' incentive to supply credit, and reduce aggregate investment.

By contrast, in the BGG model, banks' funding costs are independent of banks' capital structure, and always equal to the risk free rate. In economic downturns, even though large loan losses lead to a weak capital position, funding costs remain the same, as households do not charge a risk premium for the increased banking instability; therefore, there is no amplification effect of business cycles from banks.

**Retail Sector** The retail sector is introduced into the model to motivate sticky prices. We assume monopolistic competition and Calvo pricing. Retailers purchase the wholesale good from entrepreneurs at a price equal to its nominal marginal cost and differentiate them at no cost. They then sell these differentiated retail goods in a monopolistically competitive market. Let  $Y_t(i)$  be the quantity of output sold by retailer  $i$ , measured in units of wholesale goods, and let  $P_t(i)$  be the nominal price. Total final usable goods  $Y_t$  are the following composite of retail goods:

$$Y_t = \left[ \int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)} \quad (1.34)$$

with  $\epsilon \geq 1$  representing the degree of monopolistic competition. The corresponding price index is given by

$$P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} di \right]^{1/(1-\epsilon)} \quad (1.35)$$

Following Calvo (1983), in a given period the retailer receives the signal to adjust the price with probability  $1 - \theta$  and otherwise has to maintain the previous price. Let  $P_t^*(i)$  denote the price set by retailers who are able to change price at  $t$ , and  $Y_t^*(i)$  the demand given this price. The

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macroeconomic aggregates can be described almost perfectly using only the mean of the wealth distribution.

<sup>13</sup>Since banks that fall below the regulatory threshold cannot make new loans, they exit from the industry. Note that the default case in this model is benign, i.e. banks default because of bad fundamentals. Irrational bank runs caused purely by shifts in people's expectations are not considered here.

retailer will thus choose this price to maximize future expected discounted real profits, given by:

$$\max E_t \sum_{k=0}^{\infty} \left[ \theta^k \Lambda_{t,k} \Omega_{t+k}(i) / P_{t+k} \right] \quad (1.36)$$

subject to the demand function

$$Y_{t+k}^*(i) = \left( \frac{P_t^*(i)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \quad (1.37)$$

where the discount rate  $\Lambda_{t,k} = \beta^k C_t / C_{t+k}$  (given assumed log utility in consumption) is the household intertemporal marginal rate of substitution, which the retailer takes as given.  $\Omega_{t+k}$  is nominal profits given by  $(P_t^*(i) - MC_{t+k}) Y_{t+k}^*(i)$ . The optimization problem yields the following condition:

$$P_t^*(i) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} MC_{t+k}(i) Y_{t+k}^*(i) / P_{t+k}}{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} Y_{t+k}(i) / P_{t+k}} \quad (1.38)$$

Given that the share  $\theta$  of retailers do not change their price in period  $t$ , the aggregate price evolves according to:

$$P_t = [\theta P_{t-1}^{1-\epsilon} + (1 - \theta)(P_t^*)^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \quad (1.39)$$

Combining the optimal pricing and the evolution of aggregate price and then log-linearizing, we obtain a standard Phillips curve where  $\hat{m}c_t$  represents the real marginal cost gap.

$$\beta E_t \pi_{t+1} = \pi_t - (1 - \beta\theta) \frac{1 - \theta}{\theta} \hat{m}c_t \quad (1.40)$$

**Monetary Policy** To facilitate comparison with previous models, we assume a simple rule according to which the central bank adjusts the current nominal interest rate in response to the lagged inflation rate and the lagged interest rate.

$$r_t^n = \rho_r r_{t-1}^n + \rho_\pi \pi_{t-1} + \epsilon_t \quad (1.41)$$

### 1.3 Calibration

In the household utility function,  $\rho$  is chosen so that steady-state labor is 0.3.  $\varphi$  is calibrated so that the steady-state liquidity premium is 380 bp on an annual basis.  $\beta$  is calibrated at 0.983..

In the aggregate production function, the capital share is 0.33, the share of household labor is 0.66, the share of entrepreneur labor is 0.00956 and the share of banking labor is 0.00044. Capital depreciates at 2.5 percent quarterly. Capital adjustment parameter  $\chi$  is calibrated at 2

based on the estimates in Chirinko (1993).

In the retail sector, the degree of monopolistic competition  $\epsilon$  is calibrated at 6, which implies a steady-state mark-up of 20 percent. The Calvo probability that a firm does not change price in a given period  $\theta$  is set to 0.75, which implies that prices in the economy are adjusted every four quarters on average. In monetary policy, the autoregressive coefficient is set to 0.65 and the coefficient of lagged inflation 1.2. These calibrations are standard in the literature.

In the financial contract, the monitoring cost parameter  $\mu$  is set to 0.12, following BGG 1999. The probability that entrepreneurs die in a given period  $1 - \gamma$  is set to 0.019. The variance of idiosyncratic productivity is set to 0.265. These parameterizations lead to a capital-to-net worth ratio of 2 (leverage ratio of 0.5), an annual loan default rate of 2.56 percent and an annual external finance premium of 180 bp. In the distribution of the bank capital ratio, the steady-state ratio is calibrated at 10 percent and the variance of the distribution is set to match a steady-state annual bank default rate of 1 percent. Based on Dimson et al. (2002), Annualized return on equity is calibrated at 5.8 percent and return on deposit 1 percent.

Based on King and Rebelo (1999), the aggregate productivity shock follows an AR (1) process, with a coefficient of 0.9 and a standard deviation of 0.0056.

## 1.4 Simulation

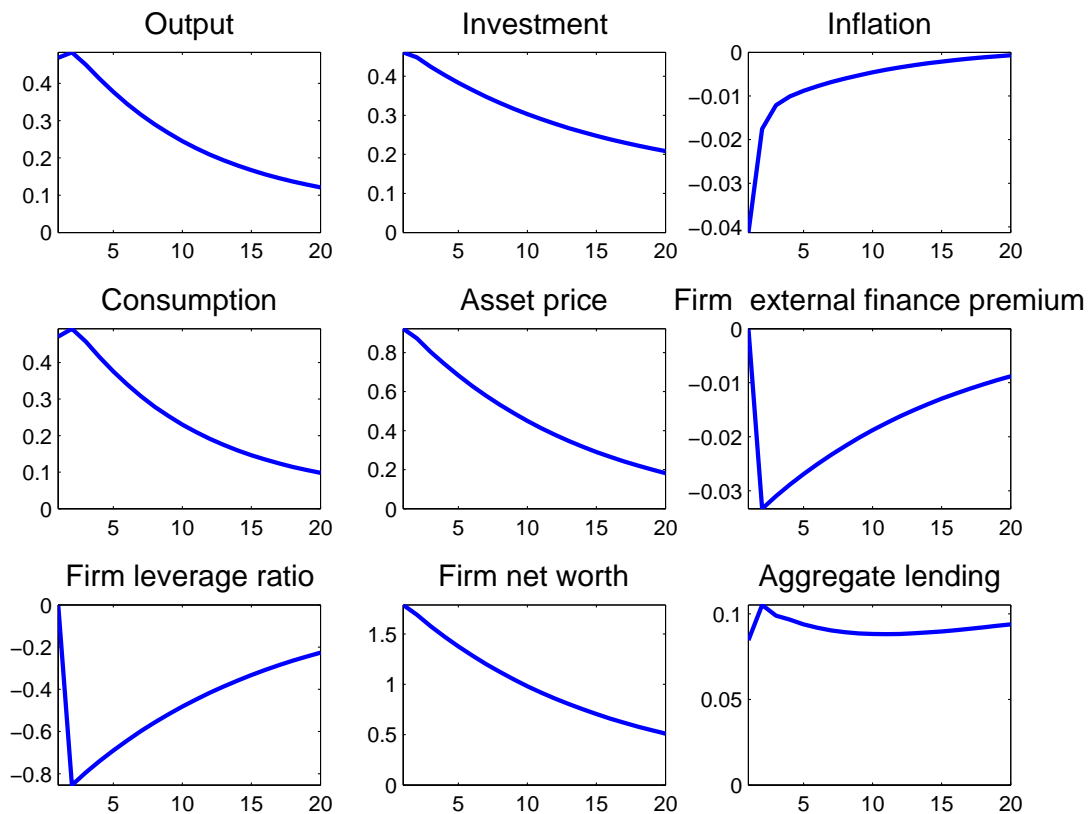
Technology shocks, monetary policy shocks and financial shocks are considered in the simulation. First, the impulse responses to shocks are analyzed; then the model is compared with a model where the only financial friction comes from the credit demand side and with a baseline model with no financial friction. The marginal contributions of the bank capital channel to the long-run steady state and short-run dynamics are studied.

### 1.4.1 Technology Shocks

Figure 1.2 and Figure 1.3 display impulse responses to a positive technology shock with the size of one standard deviation. After a positive technology shock, the realized capital return is higher than expected, leading to lower than expected loan default rate. This generates unexpected gain on the loan portfolio, which strengthens banks' capital position. Given the improvement in banks' balance sheets, households expect a lower bank default rate in the next period and are therefore willing to hold bank capital at lower rates of return. The reduction

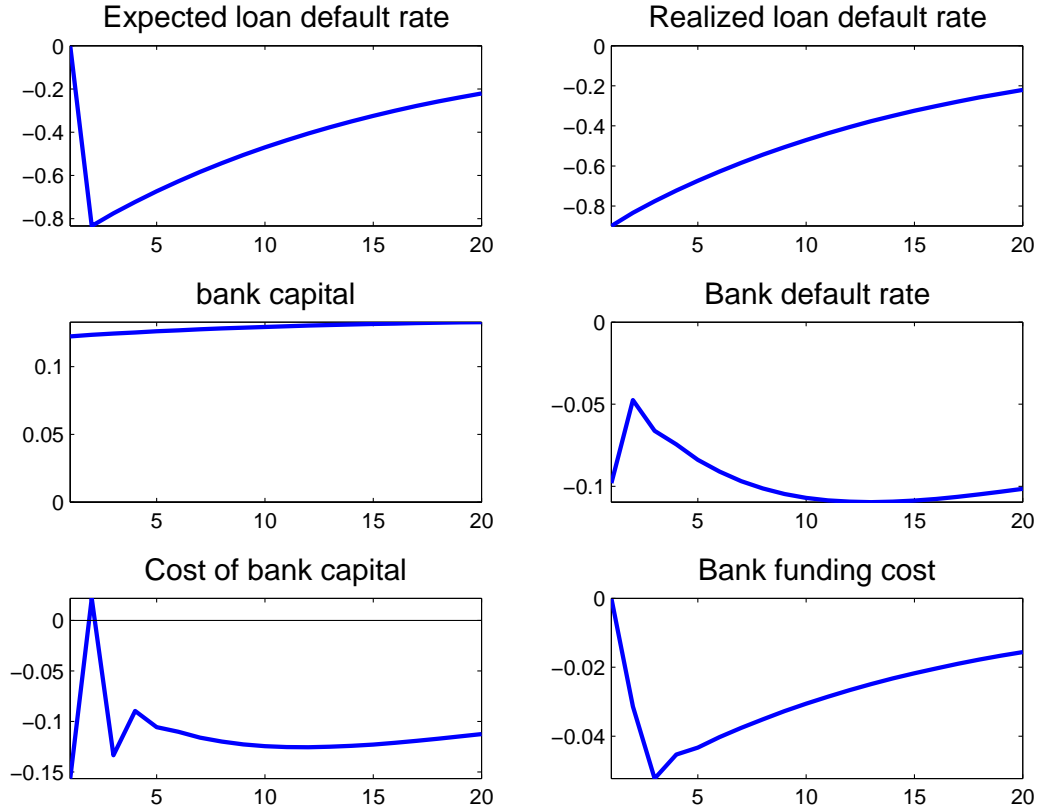
in the cost of funding from the banks' side expands credit supply and drive up investment in equilibrium. On the other hand, after a positive technology shock, firms' net worth increases and leverage ratios decline, causing them to face lower agency costs in the credit market and enabling them to obtain loans at lower external finance premiums. The positive reaction from both the credit supply and credit demand side drive up aggregate lending to a large extent, which implies an investment boom. This raises output, consumption, and asset prices. The marginal cost of production falls after productivity increases; therefore, inflation falls.<sup>14</sup>

Figure 1.2: Impulse responses to a productivity shock



<sup>14</sup>In all the graphs in the simulation part, the X-axis represents the number of quarters after shocks hit the economy, the Y-axis represents the percentage point deviation from the steady state value.

Figure 1.3: Impulse responses to a productivity shock



## 1.4.2 Monetary Policy Shocks

Figure 1.4 and Figure 1.5 show impulse responses to an unanticipated twenty-five basis point increase in the policy rate. After a monetary policy tightening, the cost of deposits rises, bank credit supply declines, ex-post loan default rate goes up. The unexpected loss in loan portfolio will write off bank's capital. The deterioration in banks' balance sheets will lead households to demand higher returns for holding bank capital in the next period. The difficulty in raising capital will further depress banks' credit supply and propagate the monetary policy shock. On the other hand, the net worth of entrepreneurs falls, the leverage ratio rises. This makes them look less attractive in the credit market and forces them to pay a higher external finance premium. *Note that, despite the contraction in both credit supply and credit demand, the aggregate lending rises for about four to six quarters and then falls.* This behavior has been well documented in empirical

studies. Christiano et al. (1996) argue that "following a contractionary shock to monetary policy, net funds raised by the business sector increases for roughly a year, and then fall". Recall aggregate lending is determined by:  $L_{t+1} = q_t K_{t+1} - N_{t+1}$ . The reason for the temporary increase in the loan amount is that, after a monetary policy tightening, there is contraction in firm net worth, capital stock and asset prices. The adjustment speed of capital is low; therefore, the change in aggregate lending depends on the adjustment speed of net worth and asset prices. Since at the beginning net worth decreases much faster than the asset price, the firm has to borrow more external funds to finance a reduced amount of investment. In the following period, contraction of firm net worth slows down, while asset price is declining faster, firm's external borrowing therefore goes down. The rest of dynamics are standard: after interest rates are increased, inflation and consumption fall. Contraction of investment and consumption reduces the output level.

Figure 1.4: Impulse responses to a monetary policy shock

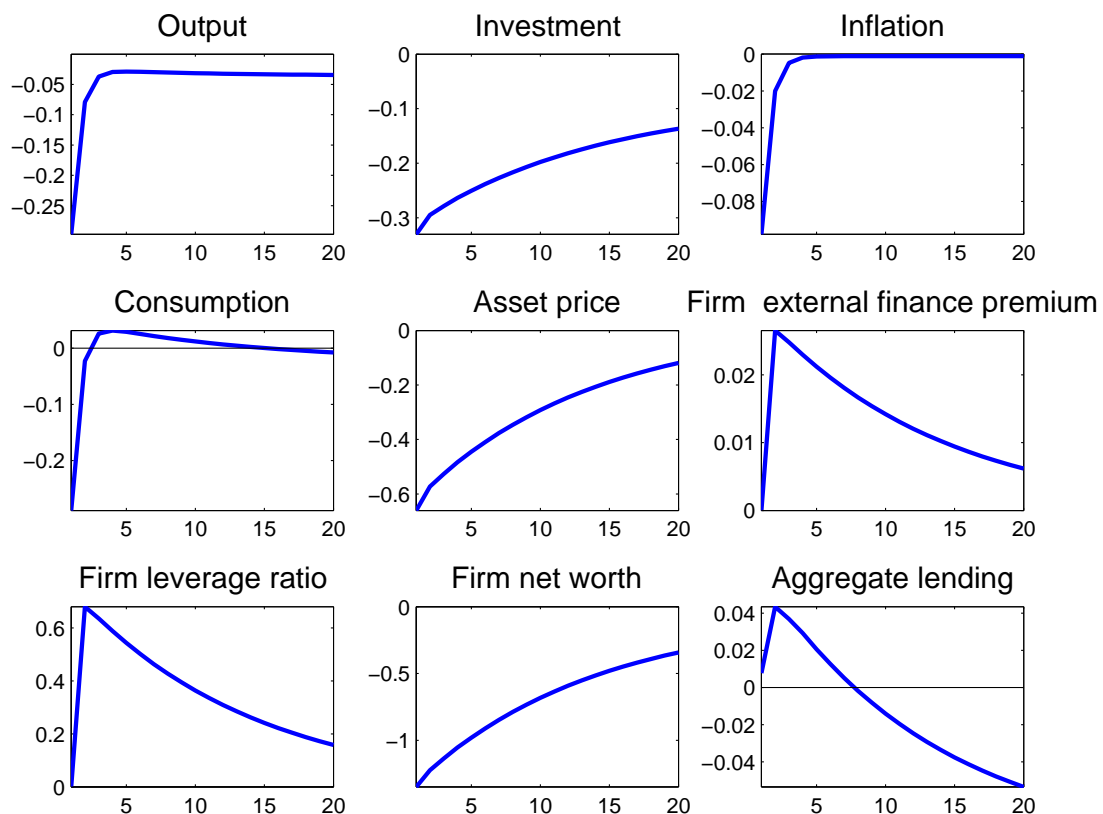
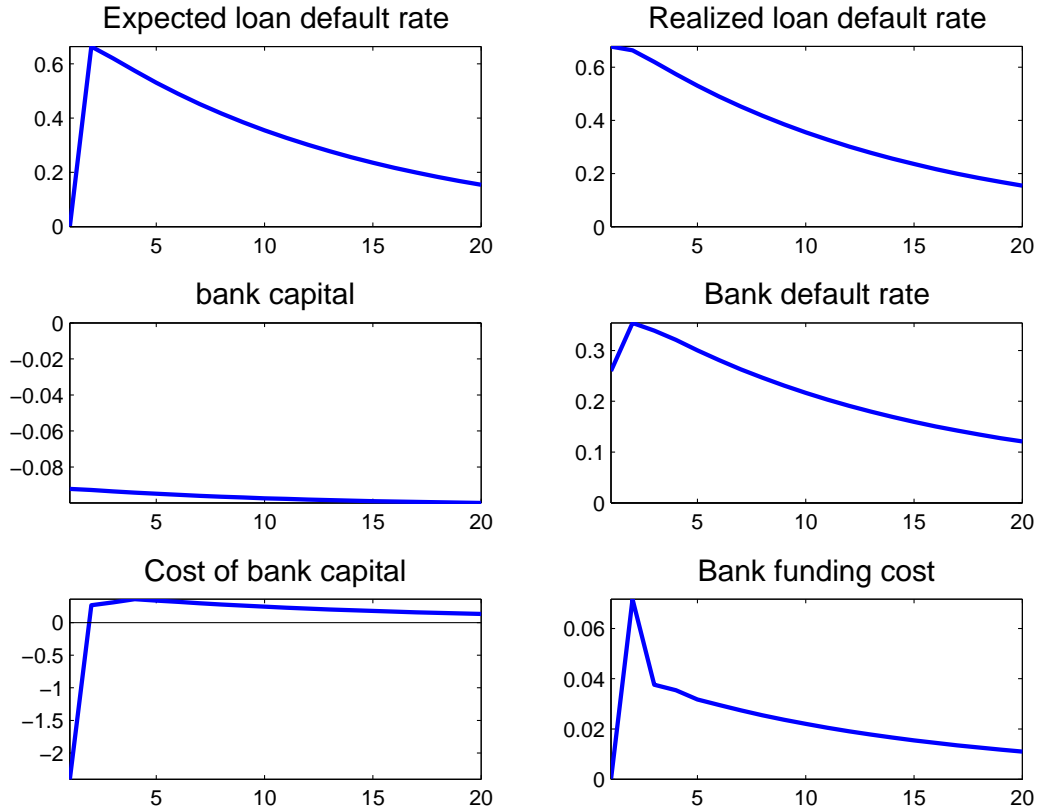




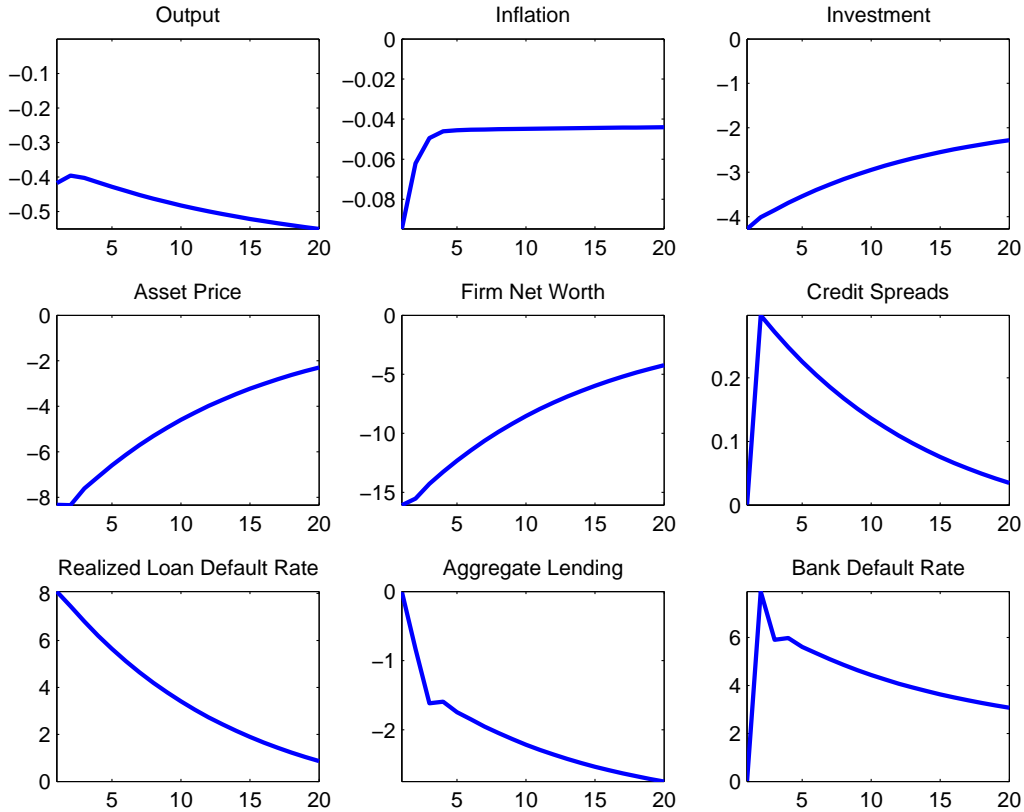
Figure 1.5: Impulse responses to a monetary policy shock



### 1.4.3 Financial Shocks

Figure 1.6 depicts the model dynamics after a negative shock to bank capital. Assume that there is an exogenous deterioration of bank's balance sheet and therefore a sudden drop of bank capital, possibly due to the burst of an asset price bubble, which leads to larger write-offs of bank equity compared to the case where asset swing is only driven by fundamental as modeled in this paper. From the simulation we can see that, a sudden drop of bank's capital position leads to strong contraction in bank's credit supply. We observe a decrease in aggregate lending and an increase in credit spreads. Tightening of credit market leads to dampened aggregate investment, which further deteriorates firm's balance sheet, loan default rate goes up. Weak aggregate demand leads to both low output and inflation.

Figure 1.6: Impulse responses to a financial shock



#### 1.4.4 Model Comparison: Marginal Effect of Banking Instability

Next, we compare this model with a model where only the BGG type of financial friction exists as well as with a standard model with no financial friction. The results show that banking instability can lead to lower capital stock and investment in the long run and have an acceleration effect on the short-run dynamics of the model.

**Long-run effect** Table 1 displays steady states of model economy with different frictions. In the long run, instability of the banking sector implies higher bank funding cost, compared to the risk-free rate in BGG model. Given the increased funding cost, banks are only willing to finance project with higher return. Since the marginal return on capital is decreasing at the aggregate level, this implies a lower capital stock in the equilibrium, and therefore lower investment, output and consumption level.

Table 1.1: Steady States Comparison

Variable	Zhang	BGG
Capital	7.1621	7.4116
Investment	0.17905	0.1853
Output	0.86509	0.875
Consumption	0.68604	0.68964

**Short-run effect** Figure 1.7 and Figure 1.8 compare the relative importance of various frictions in shock transmission. The dashdot line describes impulse responses in a standard DSGE model, where only nominal rigidity and capital adjustment costs are considered. The dashed line incorporates the additional friction coming from the credit demand side, or the financial accelerator effect. The solid line captures the model dynamics where the bank capital channel is added to the previous frictions.

Figure 1.7: Impulse responses to a monetary policy shock

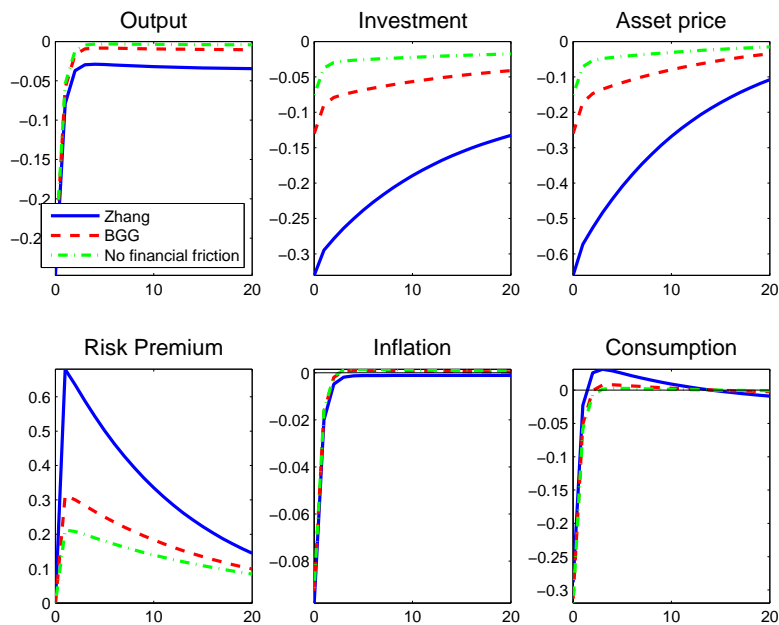
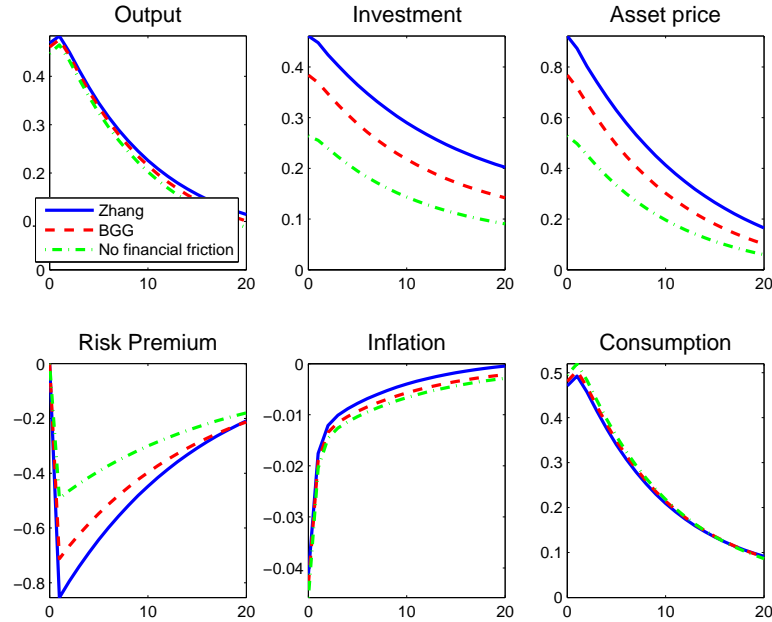


Figure 1.8: Impulse responses to a productivity shock



As we can see from the figures, the bank capital channel has a strong acceleration and propagation effect on both the impulse responses to the technology shock and the monetary policy shock. Compared to previous literature (e.g. Markovic (2006)), where the bank capital channel can generate the acceleration effect, but very little propagation effect, as the marginal contribution of credit supply friction vanishes after around 8 quarters following a policy shock. In this model, by introducing bank capital as a state variable, low capital position not only amplifies the cycle, but also creates more persistence of cycles. This corresponds to the real world scenario, where after a one-time deterioration of banks' balance sheets, it takes time to repair the balance sheets and restore credit supply.

The most significant effect of bank capital is on investment, asset prices, and credit spreads. The instability in the banking sector introduces extra volatility to these variables, while its impact on output is relatively minor. This is because consumption, which accounts for 80 percent of output in the model calibration, is not strongly subject to the influence of banking instability. If consumer loan is incorporated, bank capital channel will have a much larger effect on the consumption level, and therefore a more significant impact on output.

Another observation from Figure 1.7 is that the bank capital channel is more important than

the financial accelerator in amplifying policy shocks. This is consistent with previous findings in the literature that the financial accelerator contributes only marginally to monetary policy transmission. However, the relative importance of the two channels is reversed when a positive technology shock hits the economy, where strong corporate balance sheets play an important role in driving up asset prices and aggregate investment.

## 1.5 Conclusion

This paper extends a general equilibrium model with a BGG-type financial accelerator to a model in which financial friction coming from both the credit supply and credit demand sides are considered. By integrating a financial contract, in which entrepreneurs and banks share aggregate risk, aggregate shocks will have impact not only on firms' balance sheet, but also banks' balance sheet. In economic downturn, in addition to credit demand friction induced by low firm net worth, this paper shows that low bank capital position also creates strong credit supply friction, and leads the economy to contract further. This bank balance sheet to credit market linkage has been shown to be critical importance in the current crisis.

The extended model enables us to study how real shocks, e.g., technology shocks, monetary shocks, affect the financial sector and how shocks originate in the banking sector can influence the real economy. The model also facilitate us to understand the role of different frictions in shock transmissions.

In future research, this model could be extended to consumer loans. Since consumption is the major component of output, once the feedback from banking instability to consumption is incorporated, the effect on output will be much more significant compared to the corporate-loans-only case. The model could also be extended to an open economy and study how the instability of a financial intermediary in one country could influence the real sector in the other economy.

Table 1.2: Notation of key variables

Symbol	Variable
$K$	Capital
$I$	Investment
$Y$	Output
$C$	Consumption
$d$	Bank deposit
$e$	Bank equity
$N$	Firm net worth
$q$	Asset price
$L$	Loan
$R^k$	Gross return on capital
$R^d$	Gross return on bank deposit
$R^e$	Gross return on bank equity
$R^L$	Contractual loan rate
$\phi$	Bank default rate
$F(\bar{\omega}^a)$	Expected loan default rate
$F(\bar{\omega}^b)$	Realized loan default rate

## 2 Monetary Policy and Asset Prices: the Role of Banking Instability

*This chapter studies optimal Taylor-type monetary policy rules based on the model developed in the first chapter, where the role of bank balance sheets in business cycles is incorporated into a canonical DSGE framework. We find that with interest rate smoothing, 'leaning against the wind' can significantly dampen the procyclicality of financial distortion, and increase the welfare of the economy.*

### 2.1 Introduction

The financial crisis that started in 2007 has triggered a heated debate among academics and policy makers about the appropriate stance of monetary policy regarding asset price movement. Many have argued that the focus on goods price inflation has made central banks place insufficient weight on credit and asset prices in interest rate setting, which plays a non-negligible role in the buildup to the current crisis. In turn, this has led many to argue that concern for macro financial stability be explicitly included in central banks' mandates. In its World Economic Outlook 2008, the International Monetary Fund has argued that strong monetary reaction to an overheating of credit and asset prices can bring stabilization benefits. Frankel (2009) also argues that narrow inflation targeting has already seen its best days and fighting asset bubbles is the way ahead. On the other hand, there is a long and established view that inflation-targeting central banks need not respond to asset prices, except insofar as they affect the inflation forecast. The Federal Reserve Board has consistently shared this view and excluded asset price from monetary policy decision. Former Chairman Alan Greenspan has emphasized the immense difficulty of identifying asset price bubbles and pointed out it is enough for central banks to cut interest rate after the bubble bursts to protect the economy. Apparently, the painful adjustment caused by the current crisis, despite aggressive monetary stimulus, has challenged this view.

In the literature, the divergence of view regarding 'leaning against the wind' has been long-standing. Current Fed Chairman Ben Bernanke has argued (Bernanke and Gertler (1999, 2001)) that conditional on a strong policy response to inflation, gearing monetary policy to asset prices brings little additional gains in terms of stabilizing the economy. Their conclusion is drawn by maximizing output-inflation volatility frontier on a simulated economy that emphasizes credit friction. Faia and Monacelli (2007) conducts welfare-based policy analysis on

a similar structural model and also concludes that the marginal welfare gain of responding to asset prices vanishes given a strong anti-inflationary stance. On the other hand, Cecchetti et al. (2000, 2002) argue that monetary policy should attempt to identify and respond to asset price misalignments. Fukunnaga and Saito (2009) similarly argue that if asset prices movements are closely related to financial market imperfection, there maybe potential benefit in responding to asset price.

This paper seeks to add to this literature by exploring whether and how our answer would change if banking instability is incorporated into the structural model. Previous studies have conducted optimal policy in different Dynamic Stochastic General Equilibrium (DSGE) frameworks. Although some have built credit frictions into the model, e.g. 'financial accelerator' in Bernanke and Gertler (2001), none of previous studies have allowed for banking stability, and the role of bank balance sheet on business cycles, which have played an important role in the current crisis. The analysis in this paper is based on the structural model developed in Zhang (2009), where the credit market is characterized by both demand and supply frictions. While credit demand frictions capture how firms' ability to borrow relates to their net worth; credit supply frictions refer to how banks' willingness to lend depends on its capital position, or financial structure. Given the presence of credit frictions, fluctuation in asset price are amplified, and leads to larger fluctuation on firms' and banks' balance sheets, and reinforce the volatility of the degree of financial friction. In short, the new channel compared to previous study is the link between asset prices and bank balance sheets, as large swing in asset prices lead to a higher degree of banking instability and turbulence in financial markets, which is detrimental to welfare.

Another highlight of the paper is methodological. In most of previous studies of asset prices and monetary policy, policy rules are compared based on an output-inflation volatility frontier. This type of unconditional welfare criteria ignores the welfare effects of transitioning from the initial state to the stochastic steady state induced by the policy. Kim and Kim (2003) and Schmitt-Grohe and Uribe (2004) have shown that absence of transitional dynamics may distort the ranking of alternative policy rules. This paper therefore chooses the conditional life time utility of households as the welfare measure. Secondly, most of previous analysis are based on a first-order approximation of the economic system and require the assumption that steady state of the economy is efficient. However, for an economy characterized by high degree



of financial distortion, the steady state is significantly lower than the friction-free economy<sup>15</sup>. Follow the method developed in Schmitt-Grohe and Uribe (2007), this paper approximates the original model to second order, which allows us to study policy rules in a dynamic economy that evolves around an inefficient steady state. The main findings of the paper are threefold: First, with interest rate smoothing, responding to asset price is strictly welfare improving regardless whether the central bank has a strong or weak stance on inflation; second, the additional welfare gain is smaller but still significant if strong reaction to output is included in the policy rule; Third, without interest rate smoothing, too strong reaction to asset prices may lead to model indeterminacy. These results are in contrast with previous findings that conditional on strong anti-inflation stance, responding to asset price has marginal welfare gain. The reason is that once the link between asset prices and banking instability is incorporated into the model, high volatility of asset prices leads to banking instability, which spills over to the real economy, therefore amplifying and propagating business cycles. It is therefore crucial for central banks to 'lean against the wind' to dampen the procyclicality of financial distortions and their impact on real economy.

The rest of the paper is as follows, Section 2 presents the model. Section 3 describes the calibration and solution strategy. Section 4 discusses the implication of bank capital channel on model dynamics. Section 5 analyzes the welfare effect of alternative interest rate rules. Section 6 concludes.

## 2.2 Model

The model is discussed here briefly; for a more detailed discussion, please see Zhang (2009). The economy is inhabited by four types of agents: households, entrepreneurs, retailers and bankers. The structure of the basic model is the following: Bankers raise equity and deposits from households, and intermediate these funds to the entrepreneurs. Entrepreneurs combine their own net worth and the money they borrowed from banks to purchase physical capital, which is used in aggregate production together with labor supplied by households. The product will then be differentiated in the retail sector to become final goods, which are either invested or consumed by the agents. Nominal return on risk-free assets (i.e., deposit) is set by the central bank, who conducts monetary policy following a Taylor rule. Banks are subject to a minimum capital ratio.

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<sup>15</sup>See Zhang (2009) for a comparison of steady states with different frictions.

Next, we will present a financial contract where borrowers and lenders share aggregate risk, and then integrate it into a general equilibrium.

### 2.2.1 The Financial Contract

In this part, we discuss the design of an optimal financial contract between entrepreneurs and banks, which is the key deviation of our model from the original BGG model. The contract is derived in a partial equilibrium setting, taking the price of capital goods, entrepreneurs' net worth, the cost of deposits and bank capital as given. We then imbed the optimal contract in general equilibrium.

There are two parties to the contract: an entrepreneur with net worth and a financial intermediary, which we call "bank". Banks take deposits from and issue equity to households to finance the loan demanded by entrepreneurs. We will discuss the detail of the banking sector later. Both parties are assumed to be risk-neutral. At the end of period  $t$ , a continuum of entrepreneurs (indexed by  $i \in (0, 1)$ ) need to purchase capital for production at  $t + 1$ . The quantity of capital purchased by entrepreneur  $i$  is denoted  $K_{t+1}^i$ . The price of capital in period  $t$  is  $q_t$  (in real term). The return on capital is subject to both idiosyncratic and aggregate risk. The *ex-post* gross return to entrepreneur  $i$  is  $\omega_{t+1}^i R_{t+1}^k$ , where  $\omega_{t+1}^i$  is an idiosyncratic productivity shock to entrepreneur  $i$ , and  $R_{t+1}^k$  is the *ex-post* aggregate rate of return on capital.  $\omega_{t+1}^i$  is identically and independently distributed (across time and entrepreneurs) with log-normal distribution and unit mean.

To finance the purchase of capital, entrepreneurs use internal funds (net worth) and borrow the rest from a bank. Let  $N_{t+1}^i$  denote the net worth of entrepreneur  $i$  at the end of period of  $t$ ; then borrows from the bank is the following:

$$L_{t+1}^i = q_t K_{t+1}^i - N_{t+1}^i \quad (2.1)$$

$\omega_{t+1}^i$  is private information to entrepreneur  $i$  and the bank has to pay a monitoring cost to observe it. Entrepreneurs observe the realization of  $\omega_{t+1}^i$  and decide whether to repay the debt or default. If they repay the debt, they pay  $R_{t+1}^L L_{t+1}^i$ .  $R_{t+1}^L$  is the gross loan rate specified in the contract that the entrepreneur needs to pay to the bank. It can be fixed or state-contingent. If they default, the bank seizes the entrepreneur's remaining assets after paying the monitoring cost.<sup>16</sup> For a particular value of  $R_{t+1}^K$ , there is a corresponding cut-off value of idiosyncratic

<sup>16</sup>see Townsend (1979) and Gale and Hellwig (1985).

productivity  $\bar{\omega}_{t+1}^i$ , such that, if the realization of the idiosyncratic productivity falls below it, the entrepreneur defaults. That is:

$$\bar{\omega}_{t+1}^i R_{t+1}^k q_t K_{t+1}^i = R_{t+1}^L L_{t+1}^i \quad (2.2)$$

The monitoring cost is assumed to equal a proportion  $\mu$  of the realized gross capital return  $\omega_{t+1}^i R_{t+1}^k q_t K_{t+1}^i$ . Parameter  $\mu$  captures the degree of monitoring cost or information asymmetry.

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In BGG, entrepreneurs are assumed to bear all the aggregate risk. By issuing state-contingent loan contract, banks are insulated from aggregate shocks and always obtain risk-free rate of return on loan portfolios. The optimal contract, as a result, maximizes the expected return to entrepreneurs as following:

$$\max E_t \left\{ \int_{\bar{\omega}_{t+1}^i}^{\infty} \omega_{t+1}^i R_{t+1}^k q_t K_{t+1}^i f(\omega_{t+1}^i) d\omega - (1 - F(\bar{\omega}_{t+1}^i)) \bar{\omega}_{t+1}^i R_{t+1}^k q_t K_{t+1}^i \right\} \quad (2.3)$$

where expectations are taken with respect to the random variable  $R_{t+1}^k$ , and  $\bar{\omega}_{t+1}^i$  is a function of realization of  $R_{t+1}^k$  (and therefore, function of the states).  $f(\cdot)$  and  $F(\cdot)$  are respectively the density function and the cumulative distribution function of the random variable  $\omega$ . The optimal contract must observe the participation constraints of the bank as well, such that, for each possible realization of states of nature (and therefore,  $R_{t+1}^k$  and  $\bar{\omega}_{t+1}^i$ ) the contract satisfies:

$$(1 - F(\bar{\omega}_{t+1}^i)) R_{t+1}^L L_{t+1}^i + (1 - \mu) \int_0^{\bar{\omega}_{t+1}^i} \omega E_t R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega = R_{t+1}^f L_{t+1}^i \quad (2.4)$$

In equation (4), the left hand side shows that banks' return on the loan portfolio has two components: the loan amount that is paid back by the entrepreneurs, and, in the default case, the acquisition of the firm' remaining assets after paying off the monitoring cost.  $R_{t+1}^f$  is the funding cost of the bank, which will be determined in the general equilibrium. Since the participation constraints hold for each realization of  $R_{t+1}^k$ , banks face no uncertainty in the return on loan portfolio, which equals to the risk-free rate.

The risk-sharing rule among entrepreneurs and banks is a bit stylized, nonetheless.<sup>18</sup> In reality, banks face great uncertainty in their loan portfolio. The major source of uncertainty is

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<sup>17</sup>The existence of the banking sector in this paper is taken as given. It could also be motivated by assuming that banks have information advantage compared to households in monitoring the project outcome, i.e.  $\mu_{bank} < \mu_{households}$ .

<sup>18</sup>See footnote 10 in their paper.

shocks to default risk. To account for this, we assume that aggregate risk is shared between banks and entrepreneurs. The financial contract cannot be therefore contingent on the realized capital return but has to be written based on the two parties' expectation of capital return in the next period.<sup>19</sup> Under this risk sharing rule, we have to make a distinction between the ex-post loan default threshold  $\bar{\omega}_{t+1}^{i,b}$  and the ex-ante  $\bar{\omega}_{t+1}^{i,a}$ .

Let  $E_t R_{t+1}^k$  denote the expected capital return at the end of period t. We assume that the entrepreneur can only offer the contract based on  $E_t R_{t+1}^k$  instead of all possible realizations of  $R_{t+1}^k$ .<sup>20</sup> The contract maximizes the expected return of the entrepreneur as following:

$$\int_{\bar{\omega}_{t+1}^{i,a}}^{\infty} \omega_{t+1}^i E_t R_{t+1}^k q_t K_{t+1}^i f(\omega_{t+1}^i) d\omega - (1 - F(\bar{\omega}_{t+1}^{i,a})) \bar{\omega}_{t+1}^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i \quad (2.5)$$

where  $\bar{\omega}_{t+1}^{i,a}$  is the cut-off idiosyncratic productivity that the entrepreneur is *expected* to default in period t+1 based on information up to period t. Correspondingly, the participation constraint of banks is also based on  $E_t R_{t+1}^k$  :

$$(1 - F(\bar{\omega}_{t+1}^{i,a})) R_{t+1}^L L_{t+1}^i + (1 - \mu) \int_0^{\bar{\omega}_{t+1}^{i,a}} \omega_{t+1}^i E_t R_{t+1}^k q_t K_{t+1}^i f(\omega_{t+1}^i) d\omega = R_{t+1}^f L_{t+1}^i \quad (2.6)$$

By solving the contract we obtain the credit demand equation (see Appendix A):

$$E_t R_{t+1}^k = S\left(\frac{q_t K_{t+1}^i}{N_{t+1}^i}\right) R_{t+1}^f \quad (2.7)$$

The property and interpretation of  $S(\cdot)$  is identical to BGG, where  $S$  denotes the external finance premium, which captures the wedge (driven by the existence of monitoring cost) between the cost of finance from the firm's side and the cost of funds from the bank's side.  $S' > 0$ , implying that the higher is the leverage ratio of firms, the higher is the external finance premium.

After solving the optimal contract, the contractual lending rate could be derived as

$$R_{t+1}^L = \frac{\bar{\omega}_{t+1}^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i}{L_{t+1}^i} \quad (2.8)$$

---

<sup>19</sup>A state-contingent contract could be prevented by assuming that the state of the economy is not observed by the enforcement of the contract, but only observed at the very end of the period when people form expectations for the next period.

<sup>20</sup>Our assumption actually simplifies the characterization of the financial contract, as it corresponds to the problem of solving one case of no aggregate risk in the original BGG.

Note that in this model the contractual lending rate is *fixed* and independent to the realizations of the return on capital in  $t+1$ , whereas in BGG the lending rate is *state-contingent*:

$$R_{t+1}^L = \frac{\bar{\omega}_{t+1}^i R_{t+1}^k q_t K_{t+1}^i}{L_{t+1}^i} \quad (2.9)$$

In period  $t+1$ , given the specified loan rate  $R_{t+1}^L$  and the realized return on capital, the ex-post default threshold  $\bar{\omega}_{t+1}^{i,b}$  is now determined by:

$$\bar{\omega}_{t+1}^{i,b} = \frac{R_{t+1}^L L_{t+1}^i}{R_{t+1}^k q_t K_{t+1}^i} \quad (2.10)$$

Recall that the expected default threshold is defined by:

$$\bar{\omega}_{t+1}^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i = R_{t+1}^L L_{t+1}^i \quad (2.11)$$

This implies:

$$\bar{\omega}_{t+1}^{i,b} = \frac{\bar{\omega}_{t+1}^{i,a} E_t R_{t+1}^k}{R_{t+1}^k} \quad (2.12)$$

From this expression, we see that any deviation of the realized capital return from expected one will drive a wedge between ex-post loan default rate and ex-ante. We will discuss its impact on banking sector and aggregate economy later.

## 2.2.2 General Equilibrium

In this section, we analyze how aggregate shocks can influence firm and bank's balance sheet via the financial contract in a general equilibrium. In addition to a firm's credit demand curve which is contingent on its net worth ( capturing the traditional financial accelerator effect), this model also derives an implicit credit supply curve, which is contingent on bank's capital position.

**Households** There is a continuum of households in the economy, each indexed by  $i \in (0, 1)$ . They consume the final good,  $c_t$ , invest in risk free bank deposits,  $d_{t+1}$ , and bank equity,  $e_{t+1}$ , supply labor  $h_t$  and own shares in a monopolistically competitive sector that produces differentiated varieties of goods. The households maximize the utility function:<sup>21</sup>

$$\max E_t \sum_{k=0}^{\infty} \beta^k [\ln(c_{t+k}) + \frac{d_{t+k+1}^{1+\varphi}}{1+\varphi} + \rho \ln(1 - h_{t+k})] \quad (2.13)$$

---

<sup>21</sup>Inserting deposits into the utility function is just a modeling device to capture the bank' liquidity creation function. Model dynamics are robust if we consider a standard utility function with only consumption and leisure.

subject to the sequence of budget constraints:

$$d_{t+1} + e_{t+1} + c_t = w_t h_t + R_t^d d_t + R_t^e (1 - \phi_t) e_t + \Pi_t \quad (2.14)$$

$d_{t+1}$  and  $e_{t+1}$  are deposits and bank equity (in real terms) held by the household from  $t$  to  $t+1$ .  $R_t^d$  and  $R_t^e$  reflect the gross real return on holding deposit and bank equity, and  $\phi_t$  is the default rate on bank capital.  $h_t$  is household labor supply,  $w_t$  is the real wage for household labor,  $\Pi_t$  is dividends received from ownership of retail firms. Following Van den Heuvel (2008), the liquidity services of bank deposits are modeled by assuming that the household has a derived utility function that is increasing in the amount of deposits. The households' optimization problem yields following first-order conditions:

$$U_c(c_t) = \beta E_t R_{t+1}^e (1 - \phi_{t+1}) U_c(c_{t+1}) \quad (2.15)$$

$$U_c(c_t) - U_d(d_{t+1}) = \beta E_t R_{t+1}^d U_c(c_{t+1}) \quad (2.16)$$

$$-U_{c,t}/U_{h,t} = w_t \quad (2.17)$$

Equation (15) shows that households' intertemporal consumption decisions are determined by the default-adjusted return on holding bank equity. Equation (16) shows the optimality condition on bank deposit. Equation (17) describes the usual trade-off between consumption and leisure. In the model set up, bank equity has to offer higher return than deposit for two reasons: the first is the liquidity premium, since deposits can provide households extra utility in addition to carry a monetary reward; the second is to compensate for the default risk. As will be discussed later, banks will be shut down and default on capital return when their capital ratios fall below the regulatory threshold.<sup>22</sup>

**Entrepreneurs** Other than difference in the financial contract, the entrepreneur sector at the aggregate level is identical to the BGG. We describe the entrepreneur sector for completeness purpose below. After signing the financial contract, entrepreneurs combine loans acquired from the bank and their own net worth to purchase capital. They use capital and labor to produce wholesale goods and sell them on a perfect competitive market at a price equal to their nominal marginal cost. The aggregate production function is given by :

$$Y_t = A_t K_t^{\alpha_k} (h_t)^{\alpha_h} (h_t^e)^{\alpha_e} (h_t^b)^{\alpha_b} \quad (2.18)$$

---

<sup>22</sup>This paper assumes a relationship between households and bankers as delegated monitoring. Therefore, households do not care about the capital structure of banks in their decision.

Following BGG, We assumed entrepreneurs and bankers supply one unit of labor services inelastically to the general labor market:  $h_t^e = h_t^b = 1$ . As will be shown later,  $\alpha_e$  and  $\alpha_b$  are calibrated so that these two additional labor forces have a negligible effect on the output level and model dynamics.<sup>23</sup>

The optimization problem of production remains standard:

$$z_t = \alpha_k m c_t \frac{Y_t}{K_t} \quad (2.19)$$

$$w_t = \alpha_h m c_t \frac{Y_t}{h_t} \quad (2.20)$$

$$w_t^e = \alpha_e m c_t \frac{Y_t}{h_t^e} \quad (2.21)$$

$$w_t^b = \alpha_b m c_t \frac{Y_t}{h_t^b} \quad (2.22)$$

where  $z_t$  is the real rental rate of capital and  $w_t, w_t^e$  and  $w_t^b$  are, respectively, the real wage of households, entrepreneurs and bankers.  $m c_t$  denotes real marginal cost. The expected return on capital is then:

$$E_t R_{t+1}^k = E_t \left( \frac{z_{t+1} + (1 - \delta) q_{t+1}}{q_t} \right) \quad (2.23)$$

The accumulation of entrepreneurs' net worth consists of two parts: profits from operating the firms and labor income. It is assumed that, in every period, entrepreneur will die with the probability  $1 - \gamma$ . This assumption ensures that entrepreneurs never accumulate enough net worth to finance a project without external financing. Those entrepreneurs who die at time  $t$  will consume  $(1 - \gamma)V_t$ . The evolution of aggregate net worth is therefore given by:

$$N_{t+1} = \gamma V_t + w_t^e \quad (2.24)$$

where  $V_t$  represents gross return on operating business. It is the difference between gross capital return and loan payment.

$$V_t = \int_{\bar{\omega}^b}^{\infty} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega - (1 - F(\bar{\omega}^b)) R_{t+1}^L L_{t+1}^i \quad (2.25)$$

---

<sup>23</sup>The salary that bankers earn from labor supply could be understood as fee income collected from transaction services, a function of financial intermediaries that is not modeled in the paper.

**Capital Producers** Capital producers purchase a fraction of final goods from the retailer as investment goods  $i_t$  and combine this with the existing capital stock to obtain capital stock in the next period. A quadratic capital adjustment cost is included to motivate a variable price of capital, which contributes to the volatility of firm net worth and bank capital. Capital producers will choose the quantity of investment goods to maximize profit subject to the adjustment cost:

$$\max E_t \left[ q_t i_t - i_t - \frac{\chi}{2} \left( \frac{i_t}{k_t} - \delta \right)^2 k_t \right] \quad (2.26)$$

where  $q_t$  is the real price of capital. The optimization problem yields the following capital supply curve:

$$q_t = 1 + \chi \left( \frac{i_t}{k_t} - \delta \right) \quad (2.27)$$

where  $\chi$  captures the sensitivity of capital price to investment fluctuation. The higher  $\chi$  is, the more volatile the price of capital. The aggregate capital stock evolves according to:

$$k_{t+1} = i_t + (1 - \delta)k_t \quad (2.28)$$

where  $\delta$  is the depreciation rate.

**Banking Sector** The banks' equity value is accumulated through retained earnings:

$$\begin{aligned} e_{t+1} = & (1 - \phi_t)e_t + [R_{t+1}^L L_{t+1}(1 - F(\bar{\omega}^b)) \\ & + (1 - \mu) \int_0^{\bar{\omega}^b} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega - R_{t+1}^f L_{t+1}] + w_t^b \end{aligned} \quad (2.29)$$

where  $\phi_t$  is the bank default rate, which will be explained in the bank regulation section. Aggregate bank equity at time t+1 consists of three parts:  $(1 - \phi_t)e_t$  is equity from those banks who did not default at time t; the term inside the square bracket is unexpected gains or losses in the loan portfolio;  $w_t^b$  is bankers' wages.

Substituting equation (6) into the above equation, we get:

$$\begin{aligned} e_{t+1} = & (1 - \phi_t)e_t + R_{t+1}^L L_{t+1}(F(\bar{\omega}^a) - F(\bar{\omega}^b)) + (1 - \mu) \int_0^{\bar{\omega}^b} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega \\ & - (1 - \mu) \int_0^{\bar{\omega}^a} \omega E_t R_{t+1}^k q_t K_{t+1} f(\omega) d\omega + w_t^b \end{aligned} \quad (2.30)$$

Notice from the financial contract, we have derived following relationship between loan default threshold and aggregate capital return:

$$\bar{\omega}_{t+1}^b = \frac{\bar{\omega}_{t+1}^a E_t R_{t+1}^k}{R_{t+1}^k} \quad (2.31)$$



Consider the case when a contractionary shock hits the economy, which reduces realized capital return  $R_{t+1}^k$  below the expected value  $E_t R_{t+1}^k$ . This will lead to higher ex-post loan default threshold  $\bar{\omega}_{t+1}^b$  than expected  $\bar{\omega}_{t+1}^a$ , correspondingly higher loan default rate  $F(\bar{\omega}^b)$  than anticipated  $F(\bar{\omega}^a)$ , and create unexpected losses  $R_{t+1}^L L_{t+1} (F(\bar{\omega}^a) - F(\bar{\omega}^b))$  that write down bank's capital position. This is the key difference of our model from the original BGG model in terms of shocks transmission. In the BGG setting, all aggregate shocks are absorbed by firms' balance sheets; while in our model, aggregate shocks are absorbed partly by firms' balance sheets and partly by banks' balance sheets via the financial contract.

Given the aggregate loan size,  $L_t$ , and bank equity, we obtain the aggregate capital ratio:

$$\Delta_t = \frac{e_t}{L_t} \quad (2.32)$$

The rest of bank funding

$$d_t = L_t - e_t \quad (2.33)$$

will be collected from the households in the form of deposits. Therefore, from an aggregate level, the opportunity cost of bank funding is a linear combination of cost of bank equity and cost of deposits, where the proportion of each type of funding varies according to the bank capital ratio.

$$R_{t+1}^f = \Delta_t R_{t+1}^e + (1 - \Delta_t) R_{t+1}^d \quad (2.34)$$

The respective costs of deposits  $R_{t+1}^d$  and equity  $R_{t+1}^e$  are derived endogenously from households' optimization problem.

**Bank regulation** In modern banking regulation, capital requirement has become the focal point.<sup>24</sup> Given the implicit or explicit government guarantee on bank deposit, bank capital regulation is imposed to curb banks' excessive risk-taking. In 1987, the Basel Committee of Banking Supervision established the Basel I Accord, which provided a uniform capital standard for all banks in the member countries. Basel I required the ratio of banks' capital to risk-weighted assets to amount to a minimum of 8 percent, with at least 50 percent of it being tier 1 capital.

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<sup>24</sup>In this paper, bank capital regulation is taken as given, instead of being motivated from a micro perspective. It could be understood to mean that the threshold requirement is set to keep the government or the central bank from having to shoulder the burden of massive bank failures.

In the model, the capital ratio across banks is assumed to have *log-normal* distribution. The mode of the distribution is given by the aggregate capital ratio derived above.  $\Delta_{i,t}$  log-normal  $(\Delta_t, \sigma)$ .<sup>25</sup> The health of the banking sector as a whole will depend largely on the variation of aggregate capital ratio. With a higher aggregate ratio, the distribution moves to the right, and fewer banks will fall short of the 8 percent threshold and thus default, and vice versa. The default probability is given by the cumulative distribution function up to the regulatory threshold:<sup>26</sup>

$$\phi_t = cdf(\Delta_t, \sigma) \quad (2.35)$$

The higher the default probability, the more it costs banks to raise equity. Therefore, a low capital position today will lead to higher equity costs in the next period. This increase in funding costs will dampen banks' incentive to supply credit, and reduce aggregate investment.

By contrast, in the BGG model, banks' funding costs are independent of banks' capital structure, and always equal to the risk free rate. In economic downturns, even though large loan losses lead to a weak capital position, funding costs remain the same, as households do not charge a risk premium for the increased banking instability; therefore, there is no amplification effect of business cycles from banks.

**Retail Sector** The retail sector is introduced into the model to motivate nominal rigidity. Monopolistic competition and quadratic price adjustment costs are assumed. Let  $Y_t(i)$  be the quantity of output sold by retailer  $i$ , measured in units of wholesale goods, and let  $P_t(i)$  be the nominal price. Total final usable goods  $Y_t$  are the following composite of retail goods:

$$Y_t = \left[ \int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)} \quad (2.36)$$

with  $\epsilon \geq 1$  representing the degree of monopolistic competition. The corresponding price index is given by

$$P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} di \right]^{1/(1-\epsilon)} \quad (2.37)$$

---

<sup>25</sup>The conditional distribution of bank capital ratio could be derived endogenously from the bank equity accumulation equation. For simplicity, in the simulation only the mean of the distribution is used, while the variance is assumed constant. As Krusell and Smith (1998) has shown, the behavior of the macroeconomic aggregates can be described almost perfectly using only the mean of the wealth distribution.

<sup>26</sup>Since banks that fall below the regulatory threshold cannot make new loans, they exit from the industry. Note that the default case in this model is benign, i.e. banks default because of bad fundamentals. Irrational bank runs caused purely by shifts in people's expectations are not considered here.

Assume retailers face a quadratic adjustment cost when setting price:

$$\frac{\vartheta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 P_t \quad (2.38)$$

The retailer will maximize future expected discounted real profits, given by:

$$\max E_t \sum_{t=0}^{\infty} [\Lambda_{0,t} \Omega_t(i) / P_t] \quad (2.39)$$

subject to the demand function

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \quad (2.40)$$

where the discount rate  $\Lambda_{0,t} = \beta^t C_t / C_0$  (given assumed log utility in consumption) is the household intertemporal marginal rate of substitution, which the retailer takes as given.  $\Omega_t$  is nominal profits given by  $(P_t(i)Y_t(i) - P_t m_{c,t} Y_t(i)) - \frac{\vartheta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 P_t$ . let  $\tilde{p}_t \equiv \frac{P_t(i)}{P_t}$  represents the relative price.

The optimization problem yields the following condition:

$$\vartheta \Lambda_t (\pi_t \frac{\tilde{p}_t}{\tilde{p}_{t-1}} - \pi) / p_{t-1}(i) = \Lambda_t \frac{Y_t}{P_t} \tilde{p}_t^{-\epsilon} ((1 - \epsilon) + \epsilon m_{c,t}) + \vartheta E_0 \Lambda_{t+1} (\pi_{t+1} \frac{\tilde{p}_{t+1}}{\tilde{p}_t} - \pi) \pi_{t+1} / p_t(i) \quad (2.41)$$

In a symmetric equilibrium,  $\tilde{p}_t = 1$ , we can therefore simplify the above equation to the following:

$$U_{c,t} (\pi_t - \pi) \pi_t = \epsilon U_{c,t} Y_t (m_{c,t} - \frac{\epsilon - 1}{\epsilon}) / \vartheta + \beta E_t \{ U_{c,t+1} (\pi_{t+1} - \pi) \pi_{t+1} \} \quad (2.42)$$

This is non-linear form of the New-Keynesian Phillips curve where deviation of real marginal cost from its steady state is driving inflation dynamics.

**Monetary Policy** The model is closed by assuming that interest rate is set according to a Taylor-type simple rule. The central bank may adjust the current nominal interest rate in response to lagged interest rate, current inflation rate, output and asset price. Alternative rules are compared in the welfare evaluation section.

$$\ln\left(\frac{R_t^n}{R_{ss}^n}\right) = \rho_r \ln\left(\frac{R_{t-1}^n}{R_{ss}^n}\right) + \rho_\pi \ln\left(\frac{\pi_t}{\pi_{ss}}\right) + \rho_y \ln\left(\frac{Y_t}{Y_{ss}}\right) + \rho_q \ln(q_t) \quad (2.43)$$

**Fiscal Policy** Government expenditure is assumed as following:

$$\ln\left(\frac{G_t}{G_{ss}}\right) = \rho_g \ln\left(\frac{G_{t-1}}{G_{ss}}\right) + \epsilon_t^g \quad (2.44)$$

Market clearing condition implies:

$$Y_t = C_t + I_t + G_t + C_t^e + \frac{\vartheta}{2}(\pi_t - \pi)^2 + \dots \quad (2.45)$$

The dotted part in the equation captures the bankruptcy cost of firms and banks. Since they are quantitatively very small, therefore dropped in the later simulation.

## 2.3 Calibration

In the household utility function,  $\rho$  is chosen so that steady-state labor is 0.3.  $\varphi$  is calibrated so that the steady-state liquidity premium is 380 bp on an annual basis.  $\beta$  is calibrated so that annual real return on deposit is 2 percent. In the aggregate production function, the capital share is 0.33, the share of household labor is 0.66, the share of entrepreneur labor is 0.00956 and the share of banking labor is 0.00044. Capital depreciates at 2.5 percent quarterly. Capital adjustment parameter  $\chi$  is calibrated at 2 based on the estimates in Chirinko (1993). In the retail sector, the degree of monopolistic competition  $\epsilon$  is calibrated at 6, which implies a steady-state mark-up of 20 percent. The price rigidity parameter  $\vartheta$  is calibrated so that the linearized version of the phillips curve corresponds to calvo probability of 0.75.

In the financial contract, the monitoring cost parameter  $\mu$  is set to 0.12, following Bernanke et al. (1999). The probability that entrepreneurs die in a given period  $1 - \gamma$  is set to 0.019. The variance of idiosyncratic productivity is set to 0.265. These parameterizations lead to the following steady-state values: a capital-to-net worth ratio of 2 (leverage ratio of 0.5), an annual loan default rate of 2.56 percent and an annual external finance premium of 180 bp. In the distribution of the bank capital ratio, the steady-state ratio is calibrated at 10 percent and the variance of the distribution is set to match a steady-state bank default rate of 1 percent.

Based on King and Rebelo (1999), the aggregate productivity shock follows an AR (1) process, with a coefficient of 0.9 and a standard deviation of 0.0056. The AR(1) coefficient in government expenditure is set to 0.9, standard deviation of  $\epsilon^g$  0.008. These are based on estimates in Perotti (2004).

The model is solved by computing a second-order approximation of the policy function around the deterministic steady state. In contrast to first-order approximation, where certainty equivalence holds, the expected value of a variable always equals to its non-stochastic steady

state; with second-order approximation, stochastic volatility will have an impact on both the first and second moments of the endogenous variables. This is critical for the welfare analysis later. For more details on the solution strategy, please see the appendix.

## 2.4 Welfare Evaluation

### 2.4.1 Model Dynamics

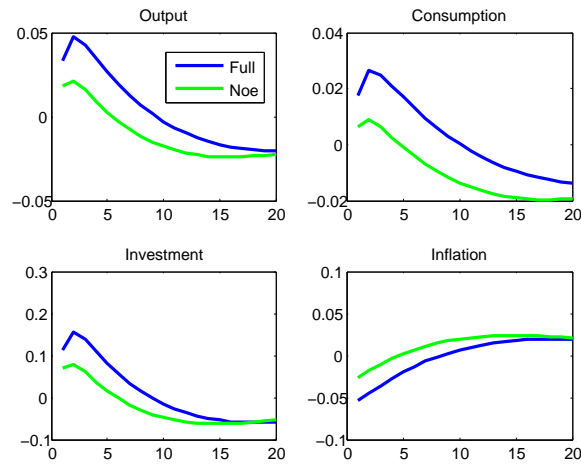
In this section, we study the dynamic behavior of the model. Especially, we highlight the role of bank balance sheet in shaping the equilibrium response in comparison to previous literature like BGG, where uncertainty in the banking sector is absent.

Figure 2.1 shows impulse responses of the model to a one standard deviation positive technology shock. The blue line shows dynamics of the model with both firms' and banks' balance sheet channel, while the green line shows model dynamics where bank balance sheet channel is absent. After a positive technology shock, the realized capital return is higher than expected, leading to lower than expected loan default rate. This generates unexpected gain on the loan portfolio, which strengthens banks' capital position. Given the improvement in banks' balance sheets, households expect a lower bank default rate in the next period and are therefore willing to hold bank capital at lower rates of return. The reduction in the cost of funding from the banks' side expands credit supply and drive up investment in equilibrium. On the other hand, after a positive technology shock, firms' net worth increases and leverage ratios decline, causing them to face lower agency costs in the credit market and enabling them to obtain loans at lower external finance premiums. The positive reaction from both the credit supply and credit demand side drive up aggregate lending to a large extent, which implies an investment boom. This raises output, consumption, and asset prices. The marginal cost of production falls after productivity increases; therefore, inflation falls.

As we can see from the graph, the bank capital channel has a strong acceleration and propagation effect on business cycles. This is because in previous models, technology shocks only reduce the degree of credit friction from the demand side because of higher firm net worth; while in this model those shocks also dampen credit supply friction by strengthening bank capital position.

For details on what are the roles of different frictions in the model dynamics to different shocks, see Zhang (2009).

Figure 2.1: Impulse responses to a productivity shock



Note: The X-axis represents the number of quarters after shocks hit the economy, the Y-axis represents the percentage point deviation from the steady state value.

## 2.4.2 Welfare Definition

The object that monetary policy aims to maximize in this paper is the expectation of household's life time utility conditional on the initial state of the economy. This is fundamentally different from previous studies that rank policy based on unconditional expectations of utility. As the previous method ignores the welfare effect of transitioning from deterministic steady states (or any other initial states) to the different stochastic steady states implied by the policy rule under consideration. Schmitt-Grohe and Uribe (2004) show that ignoring the transition effect may result in different ranking of policies than the case with unconditional utility. It is therefore crucial to choose the right welfare metric.

The structural model in this paper is characterized by heterogeneous agents: households, entrepreneurs and bankers. Entrepreneurs and banker don't have utility functions explicitly, rather they are assumed to rule-of-thumb consumers. This does not affect the welfare analysis. As both entrepreneurs and bankers are both risk neutral, their utility (whatever form that would be) will be determined by the mean level of their consumption and unaffected by the volatility of consumptions. Notice that the mean level of their consumption always equal to the deterministic steady state and do not vary across different policy rules. This simplifies our analysis to a great extent. We can therefore do the policy analysis based on the conditional utility of the household only.

The conditional welfare is derived as following: first, we write the utility of the household in recursive form:

$$\Pi_t = U(c_t, d_t, l_t^h) + \beta * E_t \Pi_{t+1} \quad (2.46)$$

We then add this equation to the optimality conditions of the model, and take a second-order approximation of the whole system, then solve the endogenous variables (including the conditional welfare  $\Pi_t$ ) as function of predetermined variables and shocks to the system. See appendix for detail.

### 2.4.3 Policy Optimization

In this section, we study the welfare implication of alternative policy rules.

**Case 1: With interest rate smoothing and no response to output** We first consider the case, where monetary policy does not systematically responds to output gap, i.e.,  $\rho_y = 0$ .  $\rho_r$  is set to 0.65 to capture interest rate smoothing. We then search numerically for the value of  $\rho_\pi$  and  $\rho_q$  that optimize the conditional welfare.

Figure 2.2: Conditional welfare varying response to inflation and to the asset price, with interest rate smoothing and no response to output

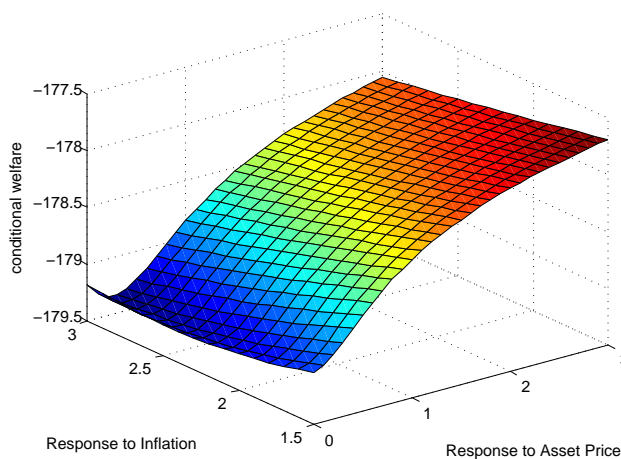
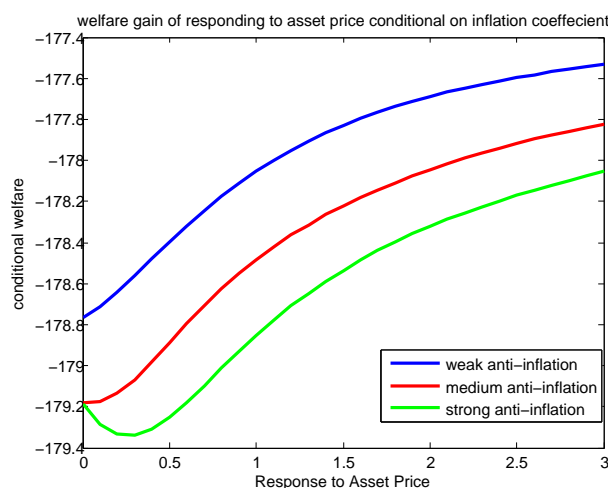


Figure 2.2 shows the result. Note that for any given value of  $\rho_\pi$  in the search grid, welfare is strictly increasing in  $\rho_q$ . This results can be seen more clearly in Figure 2.3, where the marginal welfare gain of responding to asset price is plot against three different monetary policy stances against inflation: the blue line describes the case with weak anti-inflation,  $\rho_\pi = 1$ ; the green line is the medium case with  $\rho_\pi = 2$ ; while the red line represents strong anti-inflation policy with  $\rho_\pi = 3$ . We can see that the welfare gain of 'leaning against the wind' does not vanish when central bank becomes tougher on anti-inflationary stance. This is in contrast with previous literatures that conclude that responding to asset price does not bring significant welfare improvement conditional on strong response to inflation.

Figure 2.3: Conditional welfare varying response to asset price conditional on inflation coefficient, with interest rate smoothing and no response to output



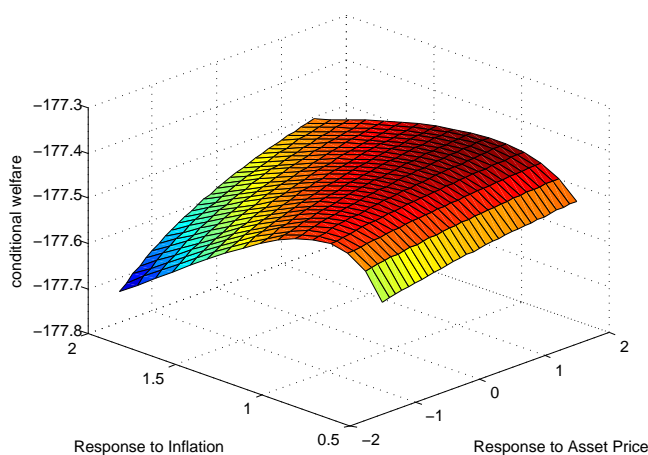
The difference in conclusions is mainly driven by the underlying financial distortions in the economy. In previous papers that emphasize credit demand friction coming from the firm side, inflation contains a lot of information regarding the degree of financial friction, therefore strong anti-inflation policy also dampens the procyclicality of financial friction; while in this paper, the newly incorporated credit supply friction seems to have less pronounced effect on inflation, which means inflation rate could be muted while financial distortion is already very high. In such case, responding to inflation rate is not sufficient to dampen financial frictions, and central banks have to turn to more informative indicator: the asset price.

**Case 2: With interest rate smoothing and response to output** Next we study the case where output (level) is taken into account in the policy rule. Figure 2.4 shows the simulation



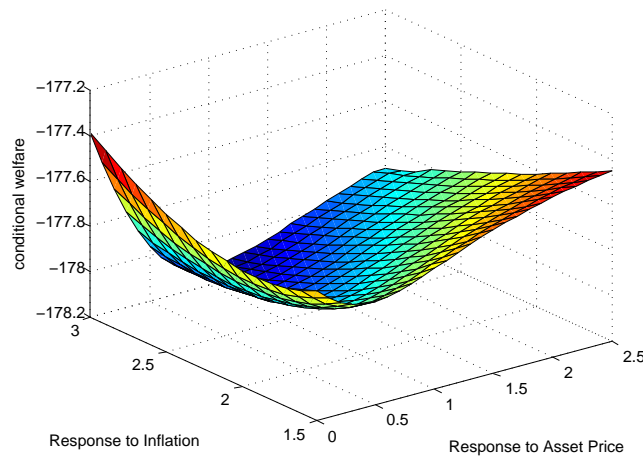
result with the interest rate smoothing parameter  $\rho_r = 0.65$  and response to output  $\rho_y = 0.8$ . We can see that responding to asset price is still strictly welfare improving, despite the quantitatively gain is not as significant as the case with  $\rho_y = 0$ . The reason is that output contains more information than inflation about the degree of financial distortion, therefore responding to output has dampened the cyclicalty of credit friction to some extent. Another observation is that including output in the policy rule is welfare improving rather than welfare deteriorating as concluded in Faia and Monacelli (2007), Schmitt-Grohe and Uribe (2007).

Figure 2.4: Conditional welfare varying response to inflation and to the asset price, with interest rate smoothing and response to output



**Case 3: With no interest rate smoothing and no response to output** In this section, we consider the case of neither interest rate smoothing nor a response to output in the policy rule. The degree of response to asset price now matters. As shown in Figure 2.5, in the lower range of  $\rho_q < 1$ , welfare is actually decreasing; for  $1 < \rho_q < 2.5$ , welfare is strictly increasing in  $\rho_q$ . At lower value of  $\rho_\pi$ , the optimal welfare is better than the case with no response to asset price, while for strong anti-inflation policy, the result is reversed; for  $\rho_q > 2.5$ , the model does not have unique local equilibrium. These observations have the following policy implications: First, over-reacting to asset price movement in setting interest rates may destabilize the economy; Second, the optimal degree of response to asset price depends on the inflation coefficient. For highly anti-inflation central banks, it could be desirable not to respond to asset prices; for central banks with relatively weak response to inflation, it is important to respond to asset prices sufficiently.

Figure 2.5: Conditional welfare varying response to inflation and to the asset price, with no interest rate smoothing and no response to output



## 2.5 Conclusion

This paper studies whether monetary policy should respond to asset prices if the linkage between swings of asset price and banking instability is incorporated into structural model. Taylor-type policy rules are compared based on welfare criteria. The study shows that with interest rate smoothing, responding to asset price is strictly welfare improving regardless whether the central bank has a strong or weak stance on inflation, although the additional welfare gain is less pronounced if strong reaction to output is included in the policy rule. On the other hand, without interest rate smoothing, too strong reaction to asset price may lead to model indeterminacy.

These results have important policy implications. As the conduct of monetary policy in most central banks explicitly or implicitly avoids too frequent change of policy rate, or preserve a relative smooth interest rate path, it seems desirable for central banks to take asset price into consideration when setting interest rates. With the increasing development of the financial sector, its interconnection with the real economy is also getting stronger. Banking instability is having more and more influence on business cycles. It is therefore critical to have systematic policy that minimize the cyclical of financial distortion and its impact on real economy.

In this paper, asset price is modeled as driven by fundamentals. For future research, it would

be interesting to build bubbles into the model and study its policy implications.

### 3 Household Savings: An Automatic Stabilizer in a Financial Crisis?

*Following the 2008 financial crisis, households dramatically and abruptly reduced their consumption and increased their saving. Many argued that such a shock to household savings would contribute to a deep and prolonged recession. Corporate savings flows — the undistributed profits that are critical for funding new capital expenditures — received less attention. As the recession began and balance sheets deteriorated, corporate savings fell dramatically. Investment expenditures also fell, but by less. This implies that savings from other sources must have risen; an important such source was households. Our general equilibrium framework broadly mimics such stylized facts. By cushioning the fall of investment, household frugality was thus a stabilizing response by households — not a destabilizing shock.*

#### 3.1 Introduction

During the 2008-09 crisis aggregate demand fell dramatically in the US and elsewhere. Figure 3.1 shows output growth and the contribution of household consumption in several industrialized countries. Households reduced their consumption and increased their savings. Policy makers became concerned about household behavior, insofar as lower consumption would also mean lower economic growth — at least in the very near term. Some, including Blanchard (2009), noted that lower consumption expenditures would not be replaced by higher private investment expenditures.

Such events have brought about renewed interest in the Keynesian "Paradox of Thrift" (POT). According to the POT, an abrupt increase in household saving will have a destabilizing effect on an economy by reducing aggregate demand. According to some, such a destabilizing impact will be even more severe if interest rates are downwardly rigid in nominal terms. For example, Christiano (2004) linked the POT to the liquidity trap, the case wherein the nominal interest rate is bounded on the downside at zero.

While household frugality may have played a key role in the recent global downturn, that role may not have been a destabilizing one. The recent economic downturn was in large part a "balance sheet" recession. It was brought on by a fall of confidence in the financial system and an adverse shock to corporate net worth. Such a shock will be reflected in a fall in corporate profits, including the retained component — corporate savings. Such savings flows

may be critical for funding for additional capital expenditures. Figure 3.2 shows the changes in corporate profits and capital expenditures in the same group of industrialized countries. In most of the countries show, these two variables move closely together - and especially so over the 2007-09 period.<sup>27</sup>

Under such financial stress, more thrifty households can help stabilize the economy - rather than destabilize it (as the POT suggests). As the saving — investment identity shows, an economy's private capital investment must equal the sum of domestic corporate savings, domestic household savings, the government budget surplus, and external savings (the current account deficit). Data from the US and several other industrialized economies (discussed in detail later in the paper) suggest that corporate earnings fell substantially – by an average of 5% of GDP.<sup>28</sup> Likewise, government budget surpluses also fell substantially - an average of about 6% of GDP. Capital formation also fell over the cycle – but by substantially less. Total investment expenditures fell about by about 3.2% of GDP, while the non-residential component of investment fell only by 1%.

Given the huge drop in corporate earnings, and large government deficit following the crisis, The evidence suggests that extra households' savings cushion the free fall of investment. Owing to a fall in their consumption, savings by household rose by about 1.3% of GDP. The savings — investment identity suggests a counterfactual: if households had not increased their savings by as much as they did, and all else had remained constant, investment spending would have fallen by even more.<sup>29</sup>

In this paper we examine the interplay between corporate saving, household saving, and private capital investment during periods of financial stress. To do so, we use a general equilibrium macroeconomic framework, for our analysis, namely Bernanke et al. (1999)

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<sup>27</sup>The charts show the changes in corporate earnings CORPEARN and fixed capital formation I, each in percent of base year gross domestic product (GDP).

<sup>28</sup>Our set of countries includes Australia, Canada, Finland, France, Germany, Japan, the Netherlands, Sweden, the U.K. and the U.S. The cycle is defined as peak-to-trough for corporate earnings. While the precise date differs by country, in most countries corporate earnings peak in mid-to-late 2007 and find their trough in 2009. Our data are calculated as the change in investment, savings, and consumption as a fraction of base period GDP. Since the change in retained corporate earnings is not reported for most countries, our proxy is the change in either gross corporate earnings or the gross operating corporate surplus.

<sup>29</sup>The data on the remaining source of savings from the external sector is mixed: it increased in some countries while decreased in others.

"financial accelerator" model (hereinafter BGG). Because the model includes financial frictions, it is well-suited for our analysis. In the model, external corporate financing costs fall as the net worth of the firm rises. These are deadweight losses: they reflect financial frictions that arise from informational asymmetries between borrowers and lenders. Such costs reflect "due diligence" by financial intermediaries.

We model a "balance sheet recession" as an adverse shock to corporate net worth. Such a shock may reflect a fall in financial confidence. Such shocks, which may be thought of as asset 'fire sales,' are not new in macroeconomics or finance literature, as Shleifer and Vishny (2011) point out. Seminal papers that discuss such shocks include Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997).<sup>30</sup>

In our model, we follow most of the assumptions in the original BGG paper. The economy is closed. Entrepreneurs own corporations whose external finance premium (interest spreads) falls as corporate net worth rises. When corporate earnings increase, firms can increase their net worth and invest more.

Corporate earnings are one key source of domestic savings. Another source is households, where saving is the difference between household income and total private consumption (entrepreneurs plus salaried agents). The public sector surplus is a residual that passively adjusts to other variables in the economy so as to close the model.<sup>31</sup> Monetary policy, summarized by a Taylor rule, is countercyclical.

Under financial stress (an adverse shock to corporate net worth), corporate earnings fall. Both consumer and investment demand also fall. This reduces inflation, but easing by the central bank (a lower interest rate) limits that fall. For this reason, capital investment falls by less than corporate earnings (savings). This means that other savings (household plus public) must rise. Importantly, total household consumption falls; this is an important source of extra saving.

In our baseline calibration, extra household saving cannot entirely fill this gap. Instead, the

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<sup>30</sup>Other recent papers by Curdia and Woodford (2009) and Gertler and Kiyotaki (2010) consider shocks to financial efficiency or 'capital quality.' We argue that our shock might reflect a fall in firms' ability to resell their capital, in the spirit of Kiyotaki and Moore (1997).

<sup>31</sup>See Leeper (1991) for the distinction between 'active' and 'passive' fiscal policy.

(passive) fiscal policy must accommodate by increasing its savings. Of course, we cannot rule out hypothetical alternative calibrations wherein household savings might rise "too much" so as to require a decrease in public savings. For this reason, we cannot rule out conditions under which a Keynesian countercyclical policy might be warranted.

We also present empirical evidence from nine industrialized countries for the 2007-09 period. Investment falls, but by less than corporate savings. Typically, household savings rise - but modestly so. Household consumption either falls or rises less than income. This confirms a key implication of the model: during periods of financial stress, when corporate earnings fall, extra household savings help buffer the fall of investment. Also, among industrial countries, increases in foreign savings (not explicitly considered in our closed economy model) also helped to cushion the fall of investment in several industrial countries.

Much recent research in macroeconomics has stressed the role of shocks to the financial system. Our work emphasizes that such shocks will have implications for a fundamental macroeconomic constraint, namely the investment - savings equality. An adverse financial shock will mean that an economy will have fewer resources to invest. Reductions in net worth imply deadweight costs to the economy - not simply losses on paper. The implications of household frugality associated with a recession depend critically on whether the shock is rooted in the financial sector or in households themselves (as some traditional frameworks emphasize). Our work suggests that if the shock is based in the financial sector, additional household frugality may be a stabilizing response that cushions the fall of investment - not a destabilizing shock. This has implications for fiscal responses. While countercyclical fiscal responses might be targeted to helping the most vulnerable households smooth their consumption streams, the aim of fiscal policy should not be simply replace spending and reduce overall saving. This is always true, but especially so when an economy has suffered a loss of resources.

The paper is organized as follows. In Part II, we present the model (calibration provided in an appendix). In Part III we present key impulse responses. In Part IV, we present some cross country evidence comparing the recent (2007-09) crisis to our model. Part V concludes.

## 3.2 Model

Our model largely follows BGG (1999). Our contribution lies in reinterpreting that model. Since the model is well-known we relegate several elements to an appendix.

### 3.2.1 Production Technology and Aggregate Demand

The economy's production function is assumed to be Cobb-Douglas:

$$Y_t = A_t K_{t-1}^\alpha (L_t)^{1-\alpha} \quad (3.1)$$

where  $Y_t$  is current output,  $K_{t-1}$  is capital installed in the previous period (following the "time-to-build" assumption), and  $L_t$  is the amount of labor currently employed. Labor is supplied by two types of agents, namely salaried (S) and entrepreneurial (E). Total labor supply is:

$$L_t = L_{S,t}^\lambda (L_{E,t})^{1-\lambda} \quad (3.2)$$

On the demand side, the national income accounting identity is:

$$Y_t = C_{S,t} + C_{E,t} + I_t + G_t \quad (3.3)$$

$C_{S,t}$  and  $C_{E,t}$  represent consumption by S and E households respectively,  $I_t$  is investment expenditures, and  $G_t$  is government purchases.

### 3.2.2 The Non-entrepreneurial Sector (salaried households plus government)

Salaried households have two sources of income. They earn a wage  $W_{S,t}$  from labor supplied to entrepreneurs  $L_{S,t}$  and they receive asset income (interest paid by banks)  $(R_t - 1)B_{t-1}$  where  $R_t$  is the gross real (risk-free) interest rate. The decision to consume  $C_{S,t}$  is the outcome of the maximization of the utility function:

$$\max E_t \sum_{k=0}^{\infty} \beta^k [\ln(C_{S,t+k}) + \eta \ln(1 - L_{S,t+k})]$$

subject to the following budget constraint:

$$\tilde{B}_t + C_{S,t} = W_{S,t} L_{S,t} + Z_t + R_t \tilde{B}_{t-1} \quad (3.4)$$

where  $W_{S,t}$  is the real wage paid to salaried agents,  $L_{S,t}$  is the number of hours they supply,  $R_t$  is the (gross real rate of return) on a risk free asset held by households (discussed later).  $Z_t$



represents implicit profits to a retail sector run by salaried agents net of a lump-sum tax. The corresponding intertemporal first-order (Euler) condition is:

$$U_C(C_{S,t}) = \beta E_t R_t U_C(C_{S,t+1}) \quad (3.5)$$

Likewise, the first order condition for the labor/leisure tradeoff yields a labor supply curve:

$$-U_C(C_{S,t})/U_L(L_{S,t}) = W_{S,t} \quad (3.6)$$

The government is treated as if it were simply a third class of consumer. Government purchases yield no utility for households. They have no effect on the household decision to consume or supply labor, and they are not productive. Following BGG, fiscal policy is passive: the government budget surplus (taxes minus expenditures) passively adjusts so as to accommodate monetary policy (see below).<sup>32</sup>

There is one risk-free financial instrument:  $B_t$  is a risk-free deposit that is lent forward to entrepreneurs. This occurs implicitly through a banking system. Such deposits are available to both households and the government. For this reason, it is useful to write a budget constraint that consolidates salaried households and the public sector:

$$B_t = R_t B_{t-1} + W_{S,t} L_{S,t} - C_{S,t} + GBS_t \quad (3.7)$$

where  $GBS_t$  is the (residual) government budget surplus. All saving by non-entrepreneurial agents (salaried households plus the public sector) is reflected one-to-one in the accumulation of such deposits.

$$SAV_{N,t} = B_t - B_{t-1} = (R_t - 1)B_{t-1} + W_{S,t} L_{S,t} - C_{S,t} + GBS_t \quad (3.8)$$

### 3.2.3 The Entrepreneurial Sector

Following BGG, we assume a continuum of entrepreneurial (E) households, indexed by  $i$ . In the aggregate, E households jointly own the economy's capital stock  $qK$ , where  $q$  is the familiar (Tobin) sale price of capital goods and  $K$  is the physical stock of capital. Optimal capital investment  $I^*$  and the relative price of capital goods  $q$  are jointly determined in the model by supply

<sup>32</sup>This setup resembles Leeper (1991) definition of a 'passive' fiscal policy that seems "irrelevant" but is quite critical insofar as "an equilibrium exists only because fiscal behavior supports the prevailing monetary policy."

and demand. Capital producing firms (owned by entrepreneurs) are assumed to maximize their expected profits:

$$\max E_t \left[ q_t I_t - I_t - \frac{\chi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \right] \quad (3.9)$$

where  $\delta$  is the capital depreciation rate,  $\frac{\chi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2$  reflects capital adjustment costs.

The optimization problem yields the following capital supply curve that links Tobin's  $q$  to the flow of capital expenditures:

$$q_t = 1 + \chi \left( \frac{I_t}{K_t} - \delta \right) \quad (3.10)$$

The capital stock evolves according to:

$$K_{t+1} = I_t^* + (1 - \delta)K_t \quad (3.11)$$

To finance capital accumulation, entrepreneurs rely to some extent on leverage. Their debt to households (via banks) is  $B_t$ . Thus aggregate net worth is  $N_t = q_t K_t - B_t$ .

A key variable for entrepreneurs is the real return to capital:

$$E_t R_{i,t+1}^k = E_t \left( \frac{MPK_{t+1} * MC_{t+1} + (1 - \delta)q_{t+1}}{q_t} \right) \omega_{t+1}^i \quad (3.12)$$

where  $MPK_{t+1}$  is the marginal product of capital,  $MC_{t+1}$  is the real marginal cost (the relative price of wholesale goods to retail goods). Equation (3.13) thus summarizes two components of  $R_{i,t+1}^k$ : the value of the marginal product of capital and the resale value of capital stock (net of depreciation).

We follow Gale and Hellwig (1985) discussion of an optimal contract between a bank (lender) and a firm (borrower). Each firm  $i$  will experience a random idiosyncratic shock  $\omega_{t+1}^i$  to its profitability. If the shock equals or exceeds a threshold value ( $\omega_{t+1}^i > \bar{\omega}_{t+1}$ ), that firm is a successful one (SF). Such a firm will use its realized capital return to pay back the loan while keeping the rest as firm equity. The aggregate value of successful firms is:

$$V_{t+1}^{SF} = \int_{\bar{\omega}_t}^{\infty} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega - (1 - F(\bar{\omega}_t)) R_t^L (q_t K_t - N_t) \quad (3.13)$$

where  $F(\bar{\omega}_t)$  is the cumulative distribution function of  $\omega_t$ , and  $R_t^L$  is the loan rate.

By contrast, unsuccessful firms (UF) are those whose idiosyncratic shocks fall below the threshold value ( $\omega_{t+1}^i < \bar{\omega}_{t+1}$ ). These firms default. Their post-default value is zero:  $V_{t+1}^{UF} = 0$ . The bank will seize such a firm's remaining equity after paying the monitoring cost, which is

a fixed proportion  $\mu$  of the realized return. In this case, the bank receives the residual from the bankrupted firms

$$(1 - \mu) \int_0^{\bar{\omega}_t} \omega R_{t+1}^k q_t K_t f(\omega) d\omega \quad (3.14)$$

That is, banks pay a fixed portion  $\mu$  of their gross seizure of the liquidated firms as a monitoring cost. The parameter  $\mu$  summarizes monitoring costs inherent in a financial system, including the regime of supervision, regulation, and disclosure (see Gale and Hellwig (1985)), Carlstrom and Fuerst (1997) and Bernanke et al. (1999) for more discussion); Note also that  $\mu$  implies a deadweight loss associated with financial intermediation. Total deadweight loss  $DL_t$  for the economy is:

$$DL_t = \mu \int_0^{\bar{\omega}_t} \omega R_{t+1}^k q_t K_t f(\omega) d\omega \quad (3.15)$$

Also, under the optimal contract, the expected rate of return on capital must satisfy:

$$R_{t+1}(q_t K_t - N_t) + DL_t = E_t R_{t+1}^k (q_t K_t - N_t) \quad (3.16)$$

That is, loan repayment plus the deadweight loss term must equal the expected net return on capital.

Following BGG, the law of motion for aggregate net worth (for all E households) is:

$$N_{t+1} = \gamma(R_{t+1}^k q_t K_t - DL_t - R_t B_t) + W_{E,t} L_{E,t} \quad (3.17)$$

where  $\gamma$  is the fraction of entrepreneurial resources that is reinvested rather than consumed,  $W_{E,t} L_{E,t}$  is the wage bill paid to entrepreneurs, and  $R_t B_t$  is the amount owed to households (through banks). Note that the variable  $\gamma$  summarizes a "rule of thumb" that consumption by E households is a fixed portion of corporate net worth in any period:

$$C_{E,t+1} = (1 - \gamma)(R_{t+1}^k q_t K_t - DL_t - R_t B_t) \quad (3.18)$$

### 3.2.4 The Financial Accelerator

Equations (3.9) through (3.18) yield an investment decision (equilibrium value of  $I_t^*$ ) that takes into account both the real return to capital and the marginal cost of funds.

The cost of funds largely determined by a central feature of the BGG model, the "financial accelerator" that links the borrowing costs of firms to their balance sheets. The risk premium (spread)  $E_t R_{t+1}^k / R_{t+1}$  is:

$$E_t R_{t+1}^k / R_{t+1} = s \left( \frac{q_t K_t}{N_t} \right) \quad (3.19)$$

where  $s' > 0$ . This implies the higher is a firm's leverage ratio, the higher is its external finance premium.

### 3.2.5 A Flow of Funds Interpretation

Like any other macroeconomic model, this one respects the savings - investment identity:

$$I_t = SAV_{E,t} + SAV_{N,t} \quad (3.20)$$

where private investment must be either funded by internal sources  $SAV_{E,t}$ , or external sources  $SAV_{N,t}$  that are provided by households and government. Equation (3.20) may be rewritten in a way that is more compatible with actual macroeconomic data. First we consolidate the consumption of entrepreneurial and salaried households:

$$C_t = C_{E,t} + C_{N,t} \quad (3.21)$$

Second, we note that the model includes a measure of retained corporate earnings:

$$CORPEARN_t = (R_{t+1}^k - 1)q_t K_t - DL_t - (R_t - 1)B_t \quad (3.22)$$

Equation (3.22) summarizes the flow of resources available to finance capital investment that are internal to the firm: gross returns on capital minus interest payments and the deadweight loss. An expanded definition of private household savings is:

$$SAV_{HH,t} = Y_{HH,t} - C_t = (R_t - 1)B_t + W_{E,t}L_{E,t} + W_{S,t}L_{S,t} - C_t \quad (3.23)$$

Note, in equation (3.23) we have moved two key elements from the corporations to households: entrepreneurial saving and entrepreneurial consumption. Hence, savings-investment identity (3.20) may be rewritten as:

$$I_t = CORPEARN_t + SAV_{HH,t} + GBS_t \quad (3.24)$$

In simulations presented below, an adverse net worth shock will reduce  $CORPEARN_t$ . At the same time, since the rate of return on capital will fall, investment must also fall. A key question is whether that fall of investment is greater than or less than the fall in corporate earnings  $CORPEARN_t$ . The equation shows that if investment falls by less than corporate earnings, the reason must be that either household savings  $SAV_{HH,t}$ , the government budget surplus  $GBS_t$ , or both variables, must rise. Moreover, since a fall in household consumption relative to income will raise  $SAV_{HH,t}$ , that fall in household consumption is a stabilizing buffer for investment.

### 3.2.6 Inflation, Monetary Policy, and Fiscal Policy

As in BGG, we assume monopolistic competition and Calvo pricing in the retail sector. The loglinearize form of the Phillips curve is:( for details, please see chapter 1)

$$\beta E_t \pi_{t+1} = \pi_t - (1 - \beta\theta) \frac{1 - \theta}{\theta} \hat{m}c_t \quad (3.25)$$

where  $\pi_t$  is inflation,  $\theta$  is the Calvo price adjustment parameter (the probability that prices will remain sticky in any period), and  $\hat{m}c_t$  represents the real gap in marginal cost as a percent deviation from steady state.

A Taylor rule summarizes monetary policy: the central bank adjusts the current nominal interest rate in response to the lagged inflation rate and the lagged interest rate. In log linear form, that rule is:

$$r_t^n = \rho_r r_{t-1}^n + \rho_\pi \pi_{t-1} + \epsilon_t \quad (3.26)$$

The nominal interest rate, the real interest rate, and inflation are linked according to a Fisher relationship:

$$r_{t+1}^n = r_{t+1} + E_t \pi_{t+1} \quad (3.27)$$

Finally, as in BGG, government expenditure follows a simple AR(1) process in the log-linearized form.

$$g_t = \rho_g g_{t-1} + \eta_t \quad (3.28)$$

a passive fiscal policy (choice of  $GBS_t$ ) closes the model. That is, accommodates the interest rate — and, implicitly, the equilibrium level of capital investment.

### 3.3 Impulse Responses to Net Worth Shocks

Our simulations focus on net worth equation (3.17) whose logarithmic deviation from steady state is:

$$n_{t+1} = \gamma \frac{RK}{N} (r_t^k - r_t) + n_t + e_{n,t} \quad (3.29)$$

The shock term  $e_{n,t}$  may be interpreted as a disturbance to the ability to resell a capital good. It reflects the pecuniary value associated with the disruption of a market. This is similar to Kiyotaki and Moore (1997) 'resaleability constraint'.<sup>33</sup> We consider cases where there is some

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<sup>33</sup>There are alternative interpretations to this term. For example, the shock term may represent new information about future productivity or revenues that is initially reflected in equity values; such a relationship was recently uncovered by Beaudry and Portier (2006).

persistence to this shock:  $e_{n,t} = 0.65 * e_{n,t-1} + \xi_t$ . Such persistence helps the hump-shaped behavior of several key variables (i.e. positive autocorrelation at short horizons) that is noted in the data (see Carlstrom and Fuerst (1997)).

We now trace through several aspects of a negative shock to net worth. As shown in Figure 3.3, net worth falls sharply, which leads to a hike in credit spreads. This greatly dampens the credit demand, investment thus falls dramatically, so does asset price. Lower aggregate demand results in lower output and inflation. The central bank will thus lower nominal interest rate to accommodate the recovery of the economy. Note that on the consumption side, there are two opposite channels: for the salaried workers, consumption is determined by the interest rate according to the Euler equation; therefore a easing monetary policy buffers the fall of their consumption (substitution effect). While for the entrepreneurs, consumption follows a rule of thumb — a fixed proportion of net worth. After the fall of net worth, entrepreneurial consumption will fall correspondingly (wealth effect). The figure shows the total effect on consumption. Since the wealth effect dominates, total consumption initially falls before returning to steady state.

Figure 3.3 also summarizes the 'flow of funds' interpretation of the model. It shows deviations from steady state of corporate earnings  $CORPERAN$ , household savings  $SAV_{HH}$  and investment  $I$  in percent of steady state GDP. Retained corporate earnings falls dramatically reflecting both a lower level of investment and a lower rate of return on capital. However, investment, on the other hand, does not fall as much as corporate earnings, partly due to the stimulating effect of the expansionary monetary policy. Since Corporate earning is a key source of funding for investment, after its fall, alternative sources of saving must be available to fund investment. One source would be household saving. As Figure 3.3 illustrates, total household consumption does fall. This boosts household saving. However, that drop is initially less than the drop in household income. For this reason, household savings initially falls below its steady state value; only after several periods does it rise, before ultimately returning to steady state. Even so, household austerity, at the margin, helps cushion investment. Under our calibration, the remaining funding will come from the residual source - the public sector. The figure also shows the deadweight loss  $DW$  – deviation from its steady state in percent of steady state GDP. This reflects the loss of financing resources due to the liquidation of bankrupted firms.

### 3.4 Cross-country Evidence from the 2007-09 Crisis

The model's results are compared to actual data from the recent financial crisis. Table 3.1 presents data from several industrialized countries. (The principal data source is Eurostat; details are provided in an appendix.) A cycle is defined as peak to trough of corporate earnings (or its proxy, the gross operating surplus), which is typically from mid - 2008 to mid - 2009.

The table shows the change in real investment  $\Delta I$  over that cycle in percent of initial period GDP. (Unless otherwise noted, the investment number is private investment; where available, non-residential investment  $\Delta I(\text{nonres})$  is also shown. Likewise, the changes in corporate earnings and household savings,  $\Delta \text{CORPEARN}$  and  $\Delta \text{SAV}_{HH}$ , and private household consumption  $\Delta C_{TOT}$  are also shown; as before, the changes are for each country's cycle and are in percent of initial period GDP.<sup>34</sup>

Over the selected cycles, corporate earnings fell on average 5.6% of GDP; in the model, it fell 5.4% of GDP. In all countries except Australia, investment fell; the average fall of total investment was 2.8% of GDP; non-residential investment fell by 1.5%; the model yielded a fall of about 1%. These cross-country data confirm one of the model's predictions: capital investment will fall by less than corporate savings.

The data also show that household austerity helps compensate for the fall in corporate investment. In all countries except Germany and the Netherlands, household savings rose. Over the cycle, rose on average by 1.4%; the model posted a small loss of -.4% of GDP in the first period, but showed increase in subsequent periods. In six out of nine countries, consumption falls; on average household consumption falls by 0.4% of GDP; in the model, consumption falls by 1.1% of GDP.<sup>35</sup> These data illustrate another insight from the model: during periods of financial stress, extra household frugality can help buffer the fall of capital investment by compensating for financing that otherwise fell.

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<sup>34</sup>Additional data on the government budget surplus and the current account deficit are not shown but are available from the authors. A frequently-used proxy for national savings is investment plus the current account deficit. Because there were measurement errors, this measure differed substantially from the sum of corporate, household, and government saving.

<sup>35</sup>Note that both savings and consumption can rise; this happened in both Australia and the US, and it implies that while consumption did grow over the period, that growth was modest relative to income growth.

### 3.5 Conclusion

This paper studies the role of increased household savings in a financial crisis. Following the 2008 financial crisis, household savings increased significantly in several industrialized countries, including in the United States. This raised concern among many that higher saving and lower consumption will lead to stagnation in economic growth.

We examined the role of household frugality in a general equilibrium framework with financial frictions. When households reduce their consumption, output must fall; this is a standard demand-side effect. But, our analysis suggests that higher saving by more frugal households may also provide an important cushion for the fall in private investment funding. In a balance sheet recession, as firm net worth deteriorates, corporate savings must fall. Such savings are an important source for private investment funding. Private investment also falls - but by less. As in a closed economy, investment equals to savings from corporate, households and government. Given the huge drop in corporate earnings, and large government deficit following the crisis, extra households' savings cushion the free fall of investment.

More broadly, our paper has linked together two approaches to macroeconomic analysis. The more recent emphasis on financial sector shocks was shown to have implications for a more traditional approach, namely investment — savings balance. An adverse financial shock can mean that an economy has lost resources to invest. This is dissaving - not simply paper losses.

Many conventional analyses of a recession assume that household frugality is the result of a shock to their own consumption function. When seen this way, it is natural to conclude that the government should respond in an offsetting way - spending more when the private sector spends less. Our view differs. We have seen that an adverse financial shock will reduce consumption - mainly through a wealth channel. Since frugal households help offset the dissaving associated with a financial crisis, their behavior also helps cushion the fall of investment. This is a stabilizing response – not a destabilizing shock.

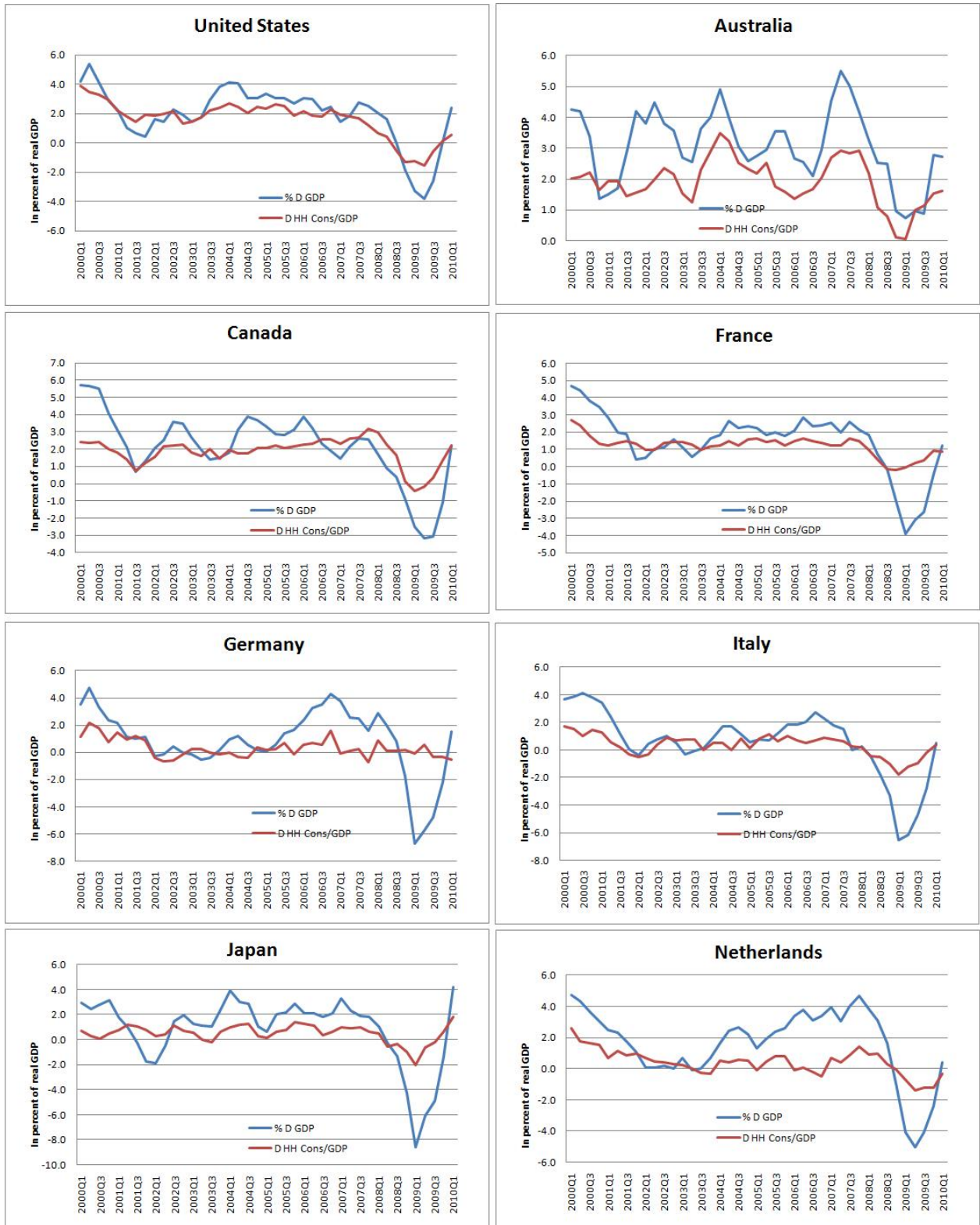
In the future, this model can be extended to open economy to study the role of foreign saving or capital flows during a financial crisis. A more full-fledged model on the fiscal side may also have implications for fiscal policy making.



Table 3.1: Saving and Investment during the 2007-2009 Crisis

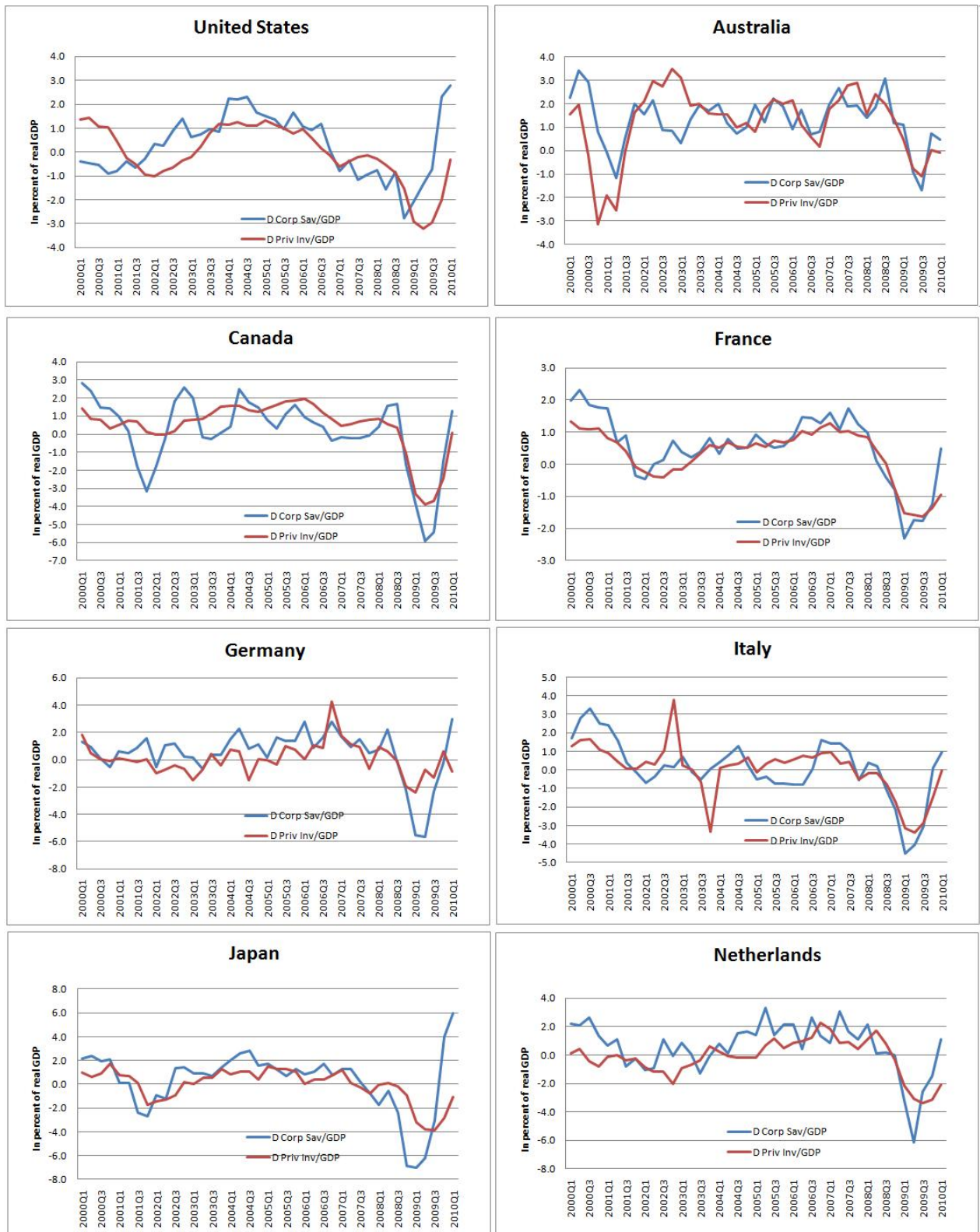
Country	Period	$\Delta I(priv)$	$\Delta I(nonres)$	$\Delta CORPEARN$	$\Delta SAV_{HH}$	$\Delta C_{HH}$
Australia	08/3–09/4	-0.9	-1.0	-0.2	2.6	1.5
France	08/1–9/4	-2.3	-1.0	-2.5	0.2	0.8
Germany	08/2–09/2	-1.1	-2.0	-5.7	-0.6	0.6
Netherlands	08/1–09/2	-3.0	-0.7	-6.3	0.8	-1.2
Sweden	08/4–09/3	-3.0	...	-5.1	1.6	-0.4
United Kindom	08/4–09/2	-7.4	-3.2	-7.7	4.2	-3.6
Canada	08/3–09/2	-3.9	-2.7	-6.2	2.0	-0.3
Japan	08/1–09/1	-2.5	-1.9	-8.6	1.0	-1.3
United Stated	06/3–08/4	-2.4	0.9	-5.3	2.3	0.6
Sample Average		-2.9	1.5	-5.3	1.6	-0.4
Model	1 period	-1.01	...	-5.4	-0.38	-1.1
	2 period	-0.91	...	-3.6	-0.09	-1.3
	3 period	-0.85	...	-2.3	0.05	-1.4
	4 period	-0.79	...	-1.4	0.14	-1.4
	5 period	-0.74	...	-0.9	0.19	-1.5
	Period Average	-0.86	...	-2.7	-0.02	-1.3

Figure 3.1: GDP growth and contribution of household consumption



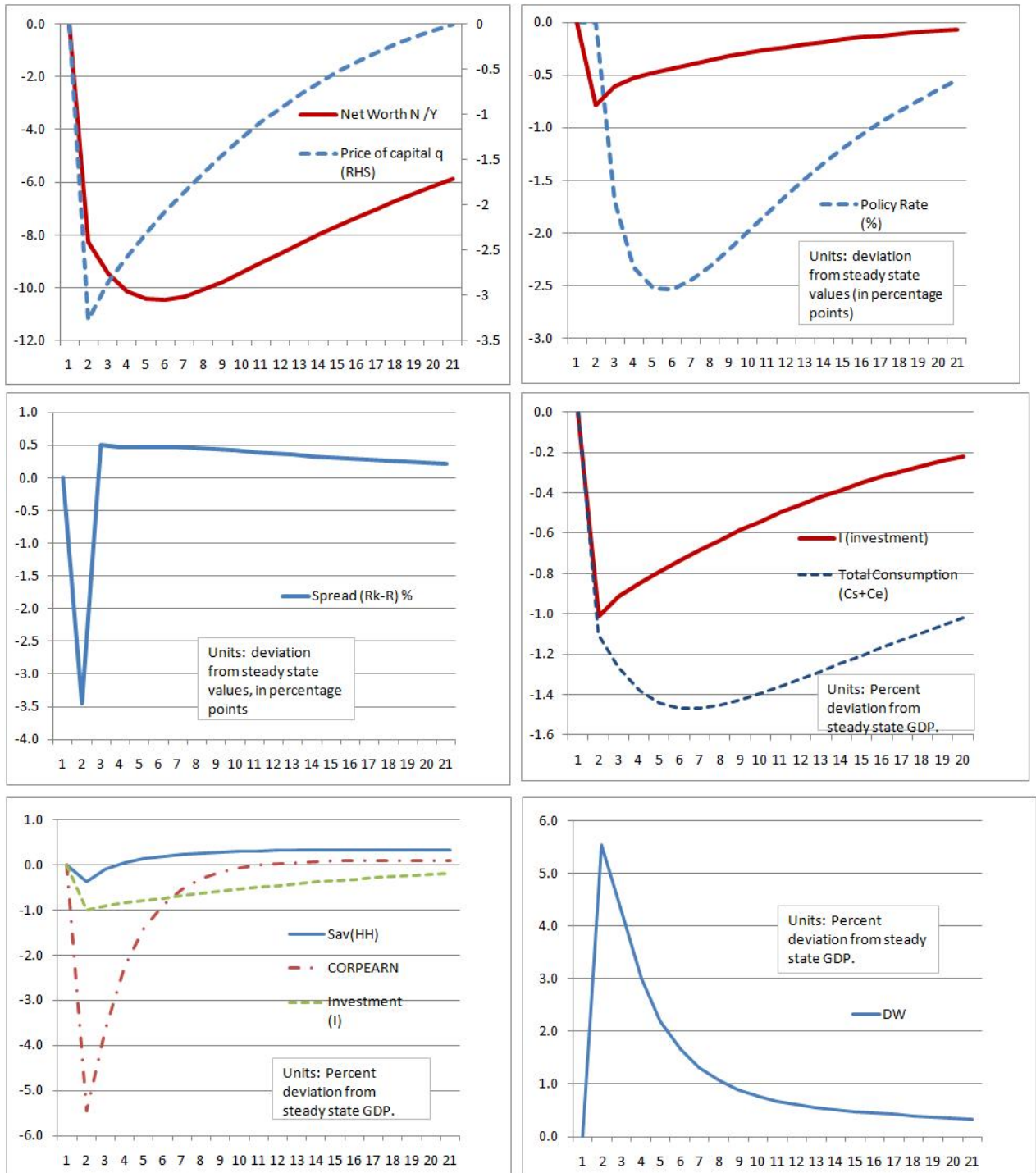
Source: National Statistical Agencies/ Eurostat/ Haver Analytics

Figure 3.2: Corporate savings and private investment



Source: National Statistical Agencies/ Eurostat/ Haver Analytics

Figure 3.3: Impulse responses to a net worth shock



## References

- Aguiar, A. and Drumond, I. (2007). Business cycle and bank capital: Monetary policy transmission under the basel accords. FEP Working Paper Series 242.
- Beaudry, P. and Portier, F. (2006). Stock prices, news and economic fluctuations. *American Economic Review*, 96(4):1293–1307.
- Bernanke, B. and Gertler, M. (1989). Agency costs, net worth, and business fluctuations. *American Economic Review*, 79:14–31.
- Bernanke, B. and Gertler, M. (1999). Monetary policy and asset price volatility. New Challenges for Monetary Policy: A Symposium Sponsored by the Federal Reserve Bank of Kansas City. Federal Reserve Bank of Kansas City, pp. 77-128.
- Bernanke, B. and Gertler, M. (2001). Should central banks respond to movements in asset prices? *American Economic Review Papers and Proceedings*, 91:253–257.
- Bernanke, B., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. and Woodford, M., editors, *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. North-Holland, Amsterdam.
- Blanchard, O. (2009). Sustaining a global recovery. *Finance and Development*.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12:383–398.
- Carlstrom, C. and Fuerst, T. (1997). Agency cost, net worth and business fluctuations: a computable general equilibrium analysis. *American Economic Review*, 87:893–910.
- Cecchetti, S., Genberg, H., Lipsky, J., and Wadhvani, S. (2000). *Asset Prices and Central Bank Policy*. CEPR and ICMB.
- Cecchetti, S., Genberg, H., and Wadhvani, S. (2002). Asset prices in a flexible inflation targeting framework. NBER Working Paper 8970.
- Chirinko, R. (1993). Business fixed investment spending: a critical survey of modeling strategies, empirical results, and policy implications. *Journal of Economic Literature*, 31:1875–1911.
- Christensen, I. and Dib, A. (2008). The financial accelerator in an estimated new keynesian model. *Review of Economic Dynamics*, 11:155–178.

- Christiano, L. (2004). *The Zero-Bound, Zero-Inflation Targeting, and Output Collapse*. Northwestern University.
- Christiano, L., Motto, R., and Rostagno, M. (2007). Financial factors in business cycles. Manuscript.
- Christiano, L. J., Eichenbaum, M., and Evans, C. (1996). The effects of monetary policy shocks: Evidence from the flow of funds. *Review of Economics & Statistics.*, 78:16–34.
- Curdia, V. and Woodford, M. (2009). Credit spreads and monetary policy. National Bureau of Economic Reserach Working Paper No. 15289.
- Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91:401–19.
- Dimson, E., Marsh, P., and Staunton, M. (2002). Long-run global capital market returns and risk premia. available at <http://papers.ssrn.com>.
- Faia, e. and Monacelli, T. (2007). Optimal interest rules, asset prices and credit friction. *Journal of Economic Dynamics and Control*, 31:3228–3254.
- Frankel, J. (2009). What’s in and out in global money. *Finance and Development*, 3:13–17.
- Fukunnaga, I. and Saito, M. (2009). Asset prices and monetary policy. IMES Discussion Paper No. 2009-E-21.
- Gale, D. and Hellwig, M. (1985). Incentive-compatible debt contracts: the one-period problem. *Review of Economic Studies*, 52:647–664.
- Gerali, A., Neri, S., Sessa, L., and Signoretti, F. M. (2009). Credit and banking in a dsge model of the euro area. Bank of Italy Economic Working Papers 740.
- Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy. NYU Woking Paper.
- Kim, J. and Kim, S. (2003). Spurious welfare reversals in international business cycle models. *Journal of International Economics*, 60:471–500.
- King, R. G. and Rebelo, S. T. (1999). Resuscitating real business cycles. In Taylor, J. and Woodford, M., editors, *Handbook of Macroeconomics*, chapter 14, pages 927–1007. North-Holland, Amsterdam.

- Kiyotaki, N. and Moore, J. (1997). Credit cycles. *Journal of Political Economy*, 105(2):211–248.
- Krusell, P. and Smith, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106:867–896.
- Leeper, E. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics*, 27:129–147.
- Markovic, B. (2006). Bank capital channels in the monetary transmission mechanism. Bank of England Working Paper No. 313.
- Meier, A. and Mueller, G. (2006). Fleshing out the monetary transmission mechanism: Output composition and the role of financial frictions. *Journal of Money, Credit and Banking*, 38:1999–2133.
- Perotti, R. (2004). Estimating the effects of fiscal policy in oecd countries. Working Papers 276, IGER Bocconi University.
- Schmitt-Grohe, S. and Uribe, M. (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control*, 28:755–775.
- Schmitt-Grohe, S. and Uribe, M. (2007). Optimal, simple, and implementable monetary and fiscal rules. *Journal of Monetary Economics*, 54(6):1702–1725.
- Shleifer, A. and Vishny, R. (2011). Fire sales in finance and macroeconomics. *Journal of Economic Perspectives*, 25:29–48.
- Townsend, R. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21:265–293.
- Van den Heuvel, S. (2008). The welfare cost of bank capital requirements. *Journal of Monetary Economics*, 55:298–320.
- Zhang, L. (2009). Bank capital regulation, the lending channel and business cycles. Bundesbank Discussion Paper Series 1: Economic Studies, No 33.

# A Appendix to Chapter 1

## A.1 The Financial Contract

In the financial contract, the entrepreneurs maximize expected profit subject to the participation constraint of the bank,

$$\max \int_{\bar{\omega}^{i,a}}^{\infty} \omega E_t R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega - (1 - F(\bar{\omega}^{i,a})) \bar{\omega}^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i$$

subject to

$$(1 - F(\bar{\omega}^{i,a})) R_{t+1}^L L_{t+1}^i + (1 - \mu) \int_0^{\bar{\omega}^{i,a}} \omega E_t R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega = R_{t+1}^f L_{t+1}^i$$

Recall that

$$\bar{\omega}_t^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i = R_{t+1}^L L_{t+1}^i$$

The key difference in solving the contract compared to BGG is that, in BGG, the expectation operator is outside the brackets, since  $\omega$  itself is not fixed but instead contingent on  $R_{t+1}^k$ :

$$\max E_t \left\{ \int_{\bar{\omega}^{i,a}}^{\infty} \omega R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega - (1 - F(\bar{\omega}^{i,a})) \bar{\omega}^{i,a} R_{t+1}^k q_t K_{t+1}^i \right\}$$

subject to

$$(1 - F(\bar{\omega}^{i,a})) R_{t+1}^L L_{t+1}^i + (1 - \mu) \int_0^{\bar{\omega}^{i,a}} \omega R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega = R_{t+1}^f L_{t+1}^i$$

This participation constraint has to hold for each realization of  $R_{t+1}^k$ ; therefore,  $\bar{\omega}^a$  is a function of  $R_{t+1}^k$ . By contrast, in our model the participation constraint only holds for  $E_t R_{t+1}^k$  and will break down ex-post if the realization of  $R_{t+1}^k$  deviates from expectation.

Define

$$\Gamma(\bar{\omega}^{i,a}) = \int_{\bar{\omega}^{i,a}}^{\infty} \omega f(\omega) d\omega - (1 - F(\bar{\omega}^{i,a})) \bar{\omega}^{i,a} \tag{A.1}$$

$$G(\bar{\omega}^{i,a}) = \int_0^{\bar{\omega}^{i,a}} \omega f(\omega) d\omega \tag{A.2}$$

The financial contract can then be transformed into

$$\max_{K_{t+1}^i, \bar{\omega}^{i,a}} (1 - \Gamma(\bar{\omega}^{i,a})) E_t R_{t+1}^k q_t K_{t+1}^i$$

subject to

$$(\Gamma(\bar{\omega}^{i,a}) - \mu G(\bar{\omega}^{i,a})) E_t R_{t+1}^k q_t K_{t+1}^i = R_{t+1}^f (q_t K_{t+1}^i - N_{t+1}^i)$$



Define external finance premium  $s^i = \frac{E_t R_{t+1}^k}{R_{t+1}^f}$  and firm leverage ratio  $k^i = \frac{K_{t+1}^i}{N_{t+1}^i}$  and let  $\lambda$  be the Lagrange multiplier on the bank participation constraint. First-order conditions imply that:

$$\lambda = \frac{\Gamma'(\bar{\omega}^{i,a})}{\Gamma'(\bar{\omega}^{i,a}) - \mu G'(\bar{\omega}^{i,a})} \quad (\text{A.3})$$

$$s^i = \frac{\lambda}{1 - \Gamma(\bar{\omega}^{i,a}) + \lambda(\Gamma(\bar{\omega}^{i,a}) - \mu G(\bar{\omega}^{i,a}))} \quad (\text{A.4})$$

Combining first-order conditions with the participation constraint enables us to derive a one-to-one relationship between the external finance premium and the cut-off threshold value, as well as a one-to-one relationship between the leverage ratio and the cut-off threshold value:

$$s^i = s(\bar{\omega}^{i,a}) = \frac{\lambda(\bar{\omega}^{i,a})}{1 - \Gamma(\bar{\omega}^{i,a}) + \lambda(\bar{\omega}^{i,a})(\Gamma(\bar{\omega}^{i,a}) - \mu G(\bar{\omega}^{i,a}))} \quad (\text{A.5})$$

$$k^i = k(\bar{\omega}^{i,a}) = 1 + \frac{\lambda(\Gamma(\bar{\omega}^{i,a}) - \mu G(\bar{\omega}^{i,a}))}{1 - \Gamma(\bar{\omega}^{i,a})} \quad (\text{A.6})$$

Therefore there exists a one-one relationship between the firm leverage ratio and the external finance premium:

$$k^i = \varphi(s^i) \quad (\text{A.7})$$

or  $q_t K_{t+1}^i = \varphi(s^i) N_{t+1}^i$ . Since the leverage ratio is the same across firms, they pay the same external risk premium  $s$ . We can thus easily aggregate this equation, and derive the following:

$$q_t K_{t+1} = \varphi\left(\frac{E_t R_{t+1}^k}{R_{t+1}^f}\right) N_{t+1} \quad (\text{A.8})$$

where  $K_{t+1}$  and  $N_{t+1}$  represent aggregate capital and firm net worth. We can also rewrite this equation into equation (1.7) in the paper:

$$E_t R_{t+1}^k = s \left(\frac{q_t K_{t+1}}{N_{t+1}}\right) R_{t+1}^f \quad (\text{A.9})$$

## A.2 First-order Conditions

$$U_c(c_t) = \beta E_t R_{t+1}^e (1 - \phi_{t+1}) U_c(c_{t+1}) \quad (\text{A.10})$$

$$U_c(c_t) - U_d(d_{t+1}) = \beta E_t R_{t+1}^d U_c(c_{t+1}) \quad (\text{A.11})$$

$$-U_{c,t}/U_{h,t} = w_t \quad (\text{A.12})$$

$$z_t = \alpha_k m c_t \frac{Y_t}{K_t} \quad (\text{A.13})$$

$$w_t^h = \alpha_h m c_t \frac{Y_t}{h_t} \quad (\text{A.14})$$

$$w_t^e = \alpha_e m c_t \frac{Y_t}{h_t^e} \quad (\text{A.15})$$

$$w_t^b = \alpha_h m c_t \frac{Y_t}{h_t^b} \quad (\text{A.16})$$

$$q_t = 1 + \chi \left( \frac{i_t}{k_t} - \delta \right) \quad (\text{A.17})$$

$$k_{t+1} = i_t + (1 - \delta)k_t \quad (\text{A.18})$$

$$R_{t+1}^k = \frac{z_{t+1} + (1 - \delta)q_{t+1}}{q_t} \quad (\text{A.19})$$

$$E_t R_{t+1}^k = S \left( \frac{q_t K_{t+1}}{N_{t+1}} \right) R_{t+1}^f \quad (\text{A.20})$$

$$R_{t+1}^f = \Delta_t R_{t+1}^e + (1 - \Delta_t) R_{t+1}^d \quad (\text{A.21})$$

$$\bar{\omega}_{t+1}^a E_t R_{t+1}^k q_t K_{t+1} = R_{t+1}^L L_{t+1} \quad (\text{A.22})$$

$$\frac{q_t K_{t+1}}{N_{t+1}} = 1 - s(\bar{\omega}_{t+1}^a) (\Gamma(\bar{\omega}_{t+1}^a) - \mu G(\bar{\omega}_{t+1}^a)) \quad (\text{A.23})$$

$$\bar{\omega}_{t+1}^b = \frac{\bar{\omega}_{t+1}^a E_t R_{t+1}^k}{R_{t+1}^k} \quad (\text{A.24})$$

$$N_{t+1} = \gamma V_t + w_t^e \quad (\text{A.25})$$

$$V_t = \int_{\bar{\omega}^b}^{\infty} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega - (1 - F(\bar{\omega}^b)) R_{t+1}^L L_{t+1}^i \quad (\text{A.26})$$

$$\phi_t = \text{cdf}(\Delta_t, \sigma) \quad (\text{A.27})$$

$$P_t^*(i) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} M C_{t+k}(i) Y_{t+k}^*(i) / P_{t+k}}{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} Y_{t+k}(i) / P_{t+k}} \quad (\text{A.28})$$

$$y_t = c_t + i_t \quad (\text{A.29})$$

$$\begin{aligned} e_{t+1} &= (1 - \phi_t) e_t + R_{t+1}^L L_{t+1}^i (F(\bar{\omega}^a) - F(\bar{\omega}^b)) \\ &\quad + (1 - \mu) \int_0^{\bar{\omega}^b} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega \\ &\quad - (1 - \mu) \int_0^{\bar{\omega}^a} \omega E_t R_{t+1}^k q_t K_{t+1} f(\omega) d\omega + w_t^b \end{aligned} \quad (\text{A.30})$$

### A.3 Steady States

$$R_{ss}^e(1 - \phi_{ss}) = 1/\beta \quad (\text{A.30})$$

$$\frac{c_{ss}^\sigma}{d_{ss}^\phi} = R_{ss}^e(1 - \phi_{ss}) - R_{ss}^d \quad (\text{A.31})$$

$$\frac{c_{ss}^{-\sigma}}{l_{ss}^\phi} = w_{ss}^h \quad (\text{A.32})$$

$$mc_{ss} = \frac{\theta - 1}{\theta} \quad (\text{A.33})$$

$$z_{ss} = \alpha_k mc_{ss} \frac{y_{ss}}{k_{ss}} \quad (\text{A.34})$$

$$w_{ss}^h = \alpha_h mc_{ss} \frac{y_{ss}}{h_{ss}} \quad (\text{A.35})$$

$$w_{ss}^e = \alpha_e mc_{ss} \frac{y_{ss}}{h_{ss}^e} \quad (\text{A.36})$$

$$w_{ss}^b = \alpha_b mc_{ss} \frac{y_{ss}}{h_{ss}^b} \quad (\text{A.37})$$

$$q_{ss} = 1 \quad (\text{A.38})$$

$$i_{ss} = \delta k_{ss} \quad (\text{A.39})$$

$$R_{ss}^k = z_{ss} + 1 - \delta \quad (\text{A.40})$$

$$lev_{ss} = \frac{q_{ss} K_{ss}}{N_{ss}} \quad (\text{A.41})$$

$$R_{ss}^k = S(lev_{ss})R_{ss}^f \quad (\text{A.42})$$

$$R_{ss}^f = \Delta_{ss}R_{ss}^e + (1 - \Delta_{ss})R_{ss}^d \quad (\text{A.43})$$

$$\bar{w}_{ss}^a = \frac{R_{ss}^l L_{ss}}{R_{ss}^k q_{ss} K_{ss}} \quad (\text{A.44})$$

$$\bar{w}_{ss}^b = \bar{w}_{ss}^a \quad (\text{A.45})$$

$$lev_{ss} = 1 - s(\bar{w}_{ss}^a)(\Gamma(\bar{w}_{ss}^a) - \mu G(\bar{w}_{ss}^a)) \quad (\text{A.46})$$

$$N_{ss} = (1 - \gamma)w_{ss}^e \quad (\text{A.47})$$

$$\phi_{ss} = pdf(\Delta_{ss}, \sigma) \quad (\text{A.48})$$

$$e_{ss} = (1 - \phi_{ss})w_{ss}^b \quad (\text{A.49})$$

$$\pi_{ss} = 1 \quad (\text{A.50})$$

$$y_{ss} = c_{ss} + i_{ss} \quad (\text{A.51})$$

## B Appendix to Chapter 2

### B.1 Solution Strategy and Welfare

The optimality conditions of the model is summarized into the following non-linear system:

$$E_t f(Y_{t+1}, Y_t, X_{t+1}, X_t) = 0 \quad (\text{B.1})$$

where  $E_t$  denotes the mathematical expectations operator, conditional on information available at time  $t$ ,  $Y_t$  is the vector of non-predetermined variables, and  $X_t$  is the state vector, and can be partitioned as  $[X_{1,t}, X_{2,t}]$ , where  $X_{1,t}$  denotes the vector of endogenous predetermined variables, while  $X_{2,t}$  is the vector of exogenous variables which follows a stochastic process:

$$X_{2,t+1} = \Lambda X_{2,t} + \Sigma \sigma \epsilon_{t+1} \quad (\text{B.2})$$

where the scalars  $\sigma$  and matrix  $\Sigma$  are known parameters, innovation  $\epsilon_t \sim \text{i.i.d. N}(0, \mathbf{I})$ . The solution of the model is of the form (Schmitt-Grohe and Uribe (2004)):

$$Y_t = g(X_t, \sigma) \quad (\text{B.3})$$

$$X_{t+1} = h(X_t, \sigma) + \Sigma \sigma \epsilon_{t+1} \quad (\text{B.4})$$

Equation (3) and (4) describe the policy function and the transition function respectively.

The deterministic steady states of the model is defined as:

$$f(\bar{Y}, \bar{Y}, \bar{X}, \bar{X}) = 0 \quad (\text{B.5})$$

The solution method is to find a second-order approximation of the function  $g(X_t, \sigma)$  and  $h(X_t, \sigma)$  around the non-stochastic steady state, where  $X_t = \bar{X}$  and  $\sigma = 0$ .

Schmitt-Grohe and Uribe (2004) show that in contrast to a first-order approximation of the model, where the expected value of any variable equals to its value in the non-stochastic steady state, in a second-order approximation of the model, the expected value of any variable differs from its deterministic steady-state value by a constant term. This is critical to our welfare evaluation. As the conditional expected welfare  $\Pi_{0,t}$  is derived as following:

$$\Pi_{0,t} = \bar{\Pi}_0 + \frac{1}{2} \sigma^2 g_{\sigma\sigma}[\Pi_0] \quad (\text{B.6})$$

where  $\bar{\Pi}_0 = U(\bar{c}, \bar{d}, \bar{l}) / (1 - \beta)$ , the life time utility of the households evaluated at the deterministic steady state.  $g_{\sigma\sigma}$  is a vector that captures how non-predetermined variables  $Y_t$

reacts to stochastic volatility in the second-order approximation of policy function  $g$ .  $g_{\sigma\sigma}[\Pi_0]$  is the element in  $g_{\sigma\sigma}$  that corresponds to the variable  $\Pi_0$ .

The author has used Matlab codes provided by Schmitt-Grohe and Uribe (available at <http://www.econ.duke.edu/Grohe>) to analytically derive and numerically evaluate the first and second order derivatives of policy functions.

## C Appendix to Chapter 3

### C.1 Calibration

In the utility function for salaried households, the labor supply elasticity is chosen such that in steady state, agents spend 1/3 of their time working. The degree of risk aversion is calibrated at 1 in the baseline analysis; this corresponds to a logarithmic utility function for consumption. The quarterly discount rate is set to 0.99 to match a steady state riskless return of 1 percent.

On the production side, the capital share is calibrated to be 0.33, the share of household labor is 0.66; the share of entrepreneur labor 0.01. The quarterly depreciation rate of capital is 2.5. Capital adjustment parameter is set to 2, based on estimates by Chirinko (1993).

In the retail sector, the degree of monopolistic competition is calibrated at 6. This implies a steady state mark-up of 2 percent. The Calvo probability that a firm cannot adjust price in a given period is calibrated at 0.75. This implies that retail prices are adjusted on average every four quarters. For monetary policy, the coefficient on lagged inflation rate is 1.2 (satisfying the Taylor principle) and the autoregressive coefficient is set to 0.65; this places some interest rate smoothing in monetary policy.

## C.2 Data Description

All data were downloaded from the "G10+ Economic and Financial Indicators Section of Haver Analytics. The principal source variables, namely GDP, consumption, and investment, were taken from the national accounts. Unless otherwise noted, the proxy for corporate earnings was the "gross operating surplus."<sup>36</sup> The ultimate source of the variables is listed by country as follows:

Australia	Australian Bureau of Statistics
Canada	Statistics Canada
Finland	Statistics Finland
France	Institut National de la Statistique et des Etudes Economiques
Germany	Statistisches Bundesamt, Eurostat
Italy	Istituto Nazionale di Statistica;Eurostat
Japan	Economic and Social Research Institute, Cabinet Office
Netherlands	Centraal bureau voor de statistiek
Spain	Instituto Nacional de Estadística
Sweden	Statistics Sweden
United Kingdom	Office for National Statistics
United States	Bureau of Economic Analysis

<sup>36</sup>For UK and Canada, "corporate profit" were used. For Finland, "gross operating surplus plus mixed income" was used. For US, "corporate profits with inventory valuation and capital consumption adjustments" was used. For the Netherlands, "gross operating surplus plus mixed income" was used.

## Curriculum Vitae

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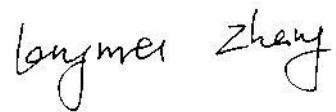
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## Ehrenwörtliche Erklärung

Ich habe die vorgelegte Dissertation selbst verfasst und dabei nur die von mir angegebenen Quellen und Hilfsmittel benutzt. Alle Textstellen, die wörtlich oder sinngemäß aus veröffentlichten oder nicht veröffentlichten Schriften entnommen sind sowie alle Angaben, die auf mündlichen Auskünften beruhen, sind als solche kenntlich gemacht.

Longmei Zhang

A handwritten signature in black ink that reads "Longmei Zhang". The letters are cursive and connected.

1 September 2011