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Benjamin Bental / Benjamin Eden

Institut für Kapitalmarktforschung ■ Center for Financial Studies
an der Johann Wolfgang Goethe-Universität, Frankfurt am Main

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Institut für Kapitalmarktforschung - Center for Financial Studies
an der Johann Wolfgang Goethe-Universität ▪ Taunusanlage 6 ▪ D-60329 Frankfurt am Main
Tel: (+49)069/242941-0 ▪ Fax: (+49)069/242941-77 ▪ E-Mail: ifk@ifk-cfs.de ▪ Internet: http://www.ifk-cfs.de

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The Real Effects of Reserve Requirements*

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Keywords: Reserve requirements, money creation, intermediation, monetary shocks

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** Department of Economics, University of Haifa, Haifa 31905, Israel, Tel.: ++972 (0)4 824 9586, Fax: ++972 (0)4 824 0059, E-mail: bbental@econ.haifa.ac.il

*** Department for Economics, University of Haifa, Haifa 31905, Israel

THE REAL EFFECTS OF RESERVE REQUIREMENTS

Benjamin Bental

Benjamin Eden*

February 1998

ABSTRACT

We review arguments for and against reserve requirements and conclude that the main question is whether a distinction between money creation and intermediation can be made. We argue that such a distinction can be made in a money-in-advance economy and show that if the the money-in-advance constraint is universally binding then reserve requirements on checkable accounts have no effect on intermediaiton. We then proceed to show that in a model in which trade is uncertain and sequential, a fractional reserve banking system gives rise to endogenous monetary shocks. These endogenous monetary shocks lead to fluctuations in capacity utilization and waste. When the money-in-advance constraint is universally binding, a 100% reserve requirement on checkable accounts can eliminate this waste.

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1. INTRODUCTION

There seems to be an ongoing trend towards decreasing reserve requirements. For example, in the US the ratio of required reserves to checkable deposits decreased from about 20% in the early fifties to less than 10% in the early ninties. (See Barro [1993] page 462). Surprisingly enough there is very little discussion of this policy trend. Text-books try to provide some explanation. For example, Barro (1993, page 478) argues that high reserve requirements are associated with a large spread between borrowing and lending rates and less intermediation between borrowers and lenders. As a result resource allocation becomes less efficient.

On the other hand, Friedman (1959) argues that reserve requirements do not affect intermediation. He envisaged that under 100% reserve requirement there will be two institutions. One that stores deposits and provides checking services for a fee and one that does the intermediation between lenders and borrowers.[1] Friedman recommends 100% reserve requirement to improve the control of the money supply and reduce fluctuations in real output.

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[1] In describing "how a 100% reserves would work", Friedmans says (page 69-70) "The effect of this proposal would be to require our present commercial banks to divide themselves into two separate institutions. One would be a pure depository institution, a literal warehouse for money. It would accept deposits payable on demand or transferable by check. ...The other institution that would be formed would be an investment trust or brokarage firm. It would acquire capital by selling shares or debentures and would use the capital to make loans or acquire investments."

These seemingly conflicting views can be reconciled if we distinguish between intermediation and money creation. This distinction can be clearly made in money-in-advance economies in which demand (checkable) deposits may be used to satisfy the money-in-advance constraint while time deposits do not satisfy the money-in-advance constraint. In such economies it matters whether reserve requirements are imposed on demand or time deposits. In the first part of the paper we argue that reserve requirements on time deposits act like a tax on intermediation and distort the allocation of resources. Reserve requirements on demand deposits act like a tax on the creation of inside money and in the absence of uncertainty, have no effect on the allocation of resources.

In the second part we consider a money-in-advance economy with uncertain and sequential trade to study the effects of reserve requirements on output fluctuations. In this model a fractional reserve banking system leads to uncertainty about the currency to deposit ratio and to fluctuations in the money supply and real output.

The money-in-advance model can thus be used to support Friedman's proposal for 100% reserve requirements on demand deposits. However, a money-in-advance model in which some buyers can circumvent the money-in-advance constraint (by using credit cards, for example) may change this conclusion. We discuss this possibility as well as other models which have been used to argue against Friedman's proposal in the last sections.

2. THE EFFECT OF RESERVE REQUIREMENTS ON INTERMEDIATION

We assume that demand deposits can be used to satisfy the money-in-advance constraint on consumption while time deposits cannot. Using this definition we argue that reserve requirements on time deposits may indeed affect intermediation but reserve requirements on demand deposits are neutral. The first point was demonstrated by Chari, Jones and Manuelli (1995).[2] We now demonstrate the second point by an example.

There are two representative agents in the economy. Each agent is endowed with one unit of labor every other period. The agents are infinitely lived with preferences given by: $\Sigma_t \beta^t u(c_t)$, where $0 < \beta < 1$ is the discount factor, c_t is consumption at time t and u( ) is a strictly concave single period utility function.

There is a single firm which converts labor into consumption good at a rate of one for one. It hires labor and pays money wages at the end of the period, after selling its output at the price P_t for money-in-advance. It pays the entire revenues of M1 dollars to the worker.

For simplicity, it is assumed that there are only two assets: demand deposits (DD) and time deposits (TD). The interest rates on loans, i_L, on time deposits, i_T, and demand deposits, i_D, are constant over time. The evolution of assets is given by:

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[2] Chari, Jones and Manuelli (1995) use a cash-in-advance model in which there is a special kind of capital that can be bought only with bank loans. In their model bank loans are financed by deposits which are subject to reserve requirements but these deposits cannot be used to satisfy the cash-in-advance constraint on consumption (their equation [25]). According to our definition, these are time deposits.

$$(1) \qquad DD_t^h + TD_t^h$$

$$= W_t^h - (1 + i_L)L_{t-1}^h + L_t^h + (1 + i_T)TD_{t-1}^h + (1 + i_D)DD_{t-1}^h - P_{t-1}c_{t-1}^h ,$$

where h indexes the individual, t indexes time, L is the amount of loans, W is the wage payment (= M1 for the agent who worked last period and zero otherwise) P is the dollar price of consumption and c is the quanitity of consumption. All the magnitudes in (1) are non-negative.

In addition to (1), agents face a money-in-advance constraint. They use checks only (no cash) so that:

$$(2) \qquad P_t c_t^h \leq DD_t^h .$$

Agent h chooses (L_t, TD_t, DD_t, c_t) to solve:

$$(3) \qquad \max \Sigma_{t=1}^{\infty} \quad \beta^t u(c_t^h)$$

s.t. (1), (2), initial values of: L_0^h , TD_0^h , DD_0^h and non-negativity constraints.

There is a price taking bank which chooses the amounts of loans (L), reserves (R), time deposits (TD) and demand deposits (DD) subject to the balance sheet identity:

$$(4) \qquad L + R = TD + DD$$

and the reserve requirement:

$$R \geq rrDD + \varepsilon TD, \tag{5}$$

where rr is the reserve ratio for demand deposits and ε is the reserve ratio for time deposits. We assume $\varepsilon \leq rr$. The bank chooses (L, TD, DD, R) to solve the following problem:

$$\max\ (1 + i_L)L + R - (1 + i_T)TD - (1 + i_D)DD \quad \text{s.t (4) and (5).} \tag{6}$$

Interior solution to the bank's problem requires[3]:

$$i_D = i_L(1 - rr) \text{ and } i_T = i_L(1 - \varepsilon). \tag{7}$$

Since $\varepsilon \leq rr$, at the solution to the consumer's problem (3) the money-in-advance constraint (2) holds with equality. This allows for an easy comparison of the cost of current consumption which the borrower faces to the cost which the lender faces.

A borrower who wants to consume an additional unit today, will borrow P_t dollars and deposit it in demand deposit. At the beginning of next period he will have $P_t(1 + i_L - i_D)$ dollars less. A lender who wants to consume an additional unit today, will transfer P_t dollars from time to demand deposit and will therefore have, at the beginning of next

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[3] A fraction $(1 - \varepsilon)$ of a dollar in time deposits will earn a gross interest of $(1 + i_L)$ and a fraction ε will earn a gross interest of 1. Therefore an interior finite solution requires: $(1 + i_T) = (1 + i_L)(1 - \varepsilon) + \varepsilon$. For the same reason: $(1 + i_D) = (1 + i_L)(1 - rr) + rr$.

period, $P_t(1 + i_T - i_D)$ dollars less. When $\varepsilon = 0$ the change in the asset position of both is the same, and therefore both face the same cost of current consumption in terms of future consumption. Furthermore, since changes in rr do not affect the difference between i_L and i_T, such changes do not introduce a wedge between the relative price of current consumption which is faced by the borrower and the relative price which is faced by the lender.

To illustrate the working of the system and to distinguish between loans which create inside money and loans which take part in the intermediation process, we now discuss a steady state equilibrium.

In the steady state each agent chooses (L, TD, DD, c) when receiving salary and (L^*, TD^*, DD^*, c^*) when not receiving salary. The bank holds the entire stock of outside money (H dollars) as reserves. This stock does not change over time. Agents owe the bank the M1 - H dollars necessary to create inside money. In addition an agent may take a loan financed by the time deposit of the other agent for consumption smoothing purposes. We now define a steady state equilibrium and solve an example.

A steady state equilibrium is a vector
(P, R, i_L, i_T, i_D, L, TD, DD, c, L^*, TD^*, DD^*, c^*) such that:
(a) Given the interest rates (i_L, i_T, i_D), the strategy of choosing the vector (L, TD, DD, c) when receiving salary and the vector (L^*, TD^*, DD^*, c^*) when not receiving salary maximizes the consumer problem (3) for the initial conditions:
$DD_0^h = DD^*$, $L_0^h = L^*$ and $TD_0^h = TD^*$ for the agent who receives salary in the first period (t = 1) and $DD_0^h = DD$, $L_0^h = L$ and $TD_0^h = TD$ for the agent who does not receive salary at t = 1;

(b) Given the interest rates (i_L, i_T, i_D), the vector ($L + L^*$, $TD + TD^*$, $DD + DD^*$, R) solves the bank's problem (6);

(c) R = H (outside money), $P = DD + DD^*$ and $c + c^* = 1$.

We now show that in the absence of reserve requirements on time deposits, there exists a steady state equilibrium in which consumption is perfectly smooth.

Claim: Assume that there is no reserve requirement on time deposits ($\varepsilon = 0$). Then there exists a steady state equilibrium in which:

$P = H/rr = M1;$

$i_D = (1 - rr)(1/\beta - 1)$

$i_L = i_T = (1/\beta - 1)$

$c = c^* = 1/2$

$DD = DD^* = (1/2)H/rr$

$L = (1/2)(1 - rr)(H/rr) = (1/2)(M1 - H)$

$TD = (1/2)[\beta/(1+\beta)](H/rr)$

$L^* = (1/2)(1 - rr)(H/rr) + (1/2)[\beta/(1+\beta)](H/rr) = (1/2)(M1 - H) + TD$

$TD^* = 0.$

Note that in the steady state both agents owe the bank a loan of $(1/2)(M1 - H)$ which is necessary to create inside money. The agent who does not receive salary owes the bank an additional amount which is equal to the amount that the other agent has in time deposit. When rr = 1, the inside money component of the loan to the bank disappears and we are left with the intermediaiton component which does the job of smoothing consumption.

To show the Claim let $td = TD - L$ and $td^* = TD^* - L^*$ denote the net position in non-checkable accounts. According to the steady state strategy, the evolution of td and td^* is given by:

$$(8) \qquad (^1/_2)H/rr + td = H/rr + td^*/\beta + (^1/_2)(1 - rr)(1/\beta - 1)H/rr,$$

$$(9) \qquad (^1/_2)H/rr + td^* = td/\beta + (^1/_2)(1 - rr)(1/\beta - 1)H/rr.$$

Where these equations are derived from (1) after substituting $P_t c_t^h = DD_t^h = (^1/_2)H/rr$ and $W_t = H/rr$ when receiving salary.

The solution to these equations is given by:

$$(10) \qquad td = (^1/_2)[\beta/(1+\beta) - (1 - rr)](H/rr)$$

$$(11) \qquad td^* = (^1/_2)[-\beta/(1+\beta) - (1 - rr)](H/rr).$$

The existence of a solution to equations (8) and (9) implies that the present value of consumption at each point in time is equal to the wealth at that point. Since the consumption is the same for both agents it follows that the beginning of period wealth is the same for both agents and in particular, it does not depend on whether the agent receives salary this period. This occurs because an agent who receives salary is in debt.

Since the smooth consumption path $c = c^* = {}^1/_2$ is feasible and its present value is equal to wealth, it is also optimal in the sense of maximizing (3). We have thus shown that there exists a steady state equilibrium allocation which is independent of reserve requirements on demand deposits.

3. THE EFFECT OF RESERVE REQUIREMENTS ON OUTPUT FLUCTUATIONS

Friedman (1959) argues that high reserve requirements allow for better control of the money supply and therefore reduce output fluctuations. We examine this hypothesis within the framework of uncertain and sequential trade (UST) models.

It has been shown (Eden [1994], Lucas and Woodford [1994] and Bental and Eden [1996]) that exogenous fluctuations in the money supply lead to waste. Here we demonstrate a similar proposition for the case in which the fluctuations in the money supply (M1) arise endogenously as a result of fluctuations in the demand for cash.

In the spirit of Lucas and Stokey (1987), we assume that goods have to be purchased with money. However, the definition of money differs across buyers. Buyers who stay in their own neighborhood can pay with either cash or checks. Buyers who travel to other neighborhoods must use cash. Checkable deposits can thus be used to satisfy the money-in-advance constraint for non-travelers and therefore checkable deposits will be demanded by them.

In the model, the fraction of buyers who stay in their own neighborhood is random. Therefore, in a fractional reserve system, there is uncertainty about the currency/deposit ratio and about M1. This uncertainty about the money supply has real effects because of the sequential nature of trade in the goods market and the fact that prices at each stage of the trading process cannot depend on information which will be revealed at the end of the process. From the point of view of a typical seller, dollars arrive in batches. The seller, who does not know how many batches will arrive, makes a contingent plan which specifies the amount that will be sold in exchange for each batch of dollars. The amount actually sold depends on the number of batches that arrive. Goods which are not sold are lost. Since the total amount produced will be sold only if M1 attains its maximal value, uncertainty about M1 causes waste.

3.1 The model

We consider a discrete time economy with infinitely lived households. Each household consists of two members: a worker and a buyer. The households evenly populate two identical islands. Households turn out to be one of three types. Some households will consume at the current period and some will not. Out of the households who will consume in the current period some will shop in their home island and some will travel to the other island.

To simplify, we assume that a constant fraction, α, of the households are non-consumers.

A random fraction $\tilde{\phi}$ of the households which do consume, shop in their home island. This fraction is an identically and independently distributed random variable, which takes S possible realizations: $0 < \phi_1 < \phi_2 < \ldots < \phi_S$. The probability that $\tilde{\phi} = \phi_s$, is denoted by Π_s and the probability that $\tilde{\phi} \geq \phi_s$ is denoted by q_s. The identity of the households who belong to each type is determined every period by an i.i.d. lottery.

All agents first trade in a securities market. Then they go to a bank and learn their type. Finally, they go to the goods market and learn the price (market) at which they can buy.

Travelers can use only cash to buy goods. Non-travelers can use checks and cash to buy goods. In equilibrium, with a fractional reserve banking system, only non-travelers will use checks and therefore the amount of inside money depends on the number of non-travelers. Accordingly, total purchasing power (M1) depends on the realization of $\tilde{\phi}$.

We start from describing the arrival of purchasing power from the sellers' point of view.

3.2 Firms

From the point of view of the representative firm, demand arrives sequentially in batches. The number of batches that will arrive is denoted by the random variable $\tilde{s}$, where $\tilde{s}$ takes values from 1 to S. The amount of dollars in each batch is determined endogenously. The number of batches that will arrive depends on the realization of $\tilde{\phi}$. In particular, the probability that $\tilde{s} = s$ is: $\Pi_s = \text{prob}(\tilde{\phi} = \phi_s)$. The only information that is revealed by the arrival of batch j is that $\tilde{s} \geq j$.

The representative firm hires labor, l, and produces according to a linear production function k = l, where k denotes total capacity. Units of capacity can be costlessly converted to units of output at the rate of one to one.

The firm knows that it can sell to batch s at the price P(s), if batch s arrives. It makes a contingent plan: k(s) units of output will be sold to batch s if it arrives. Unsold units are wasted.

We say that the arrival of the first batch opens the first market. The arrival of each additional batch opens an additional market. Using this language, the firm allocates total supply among the S potential markets. Thus,

$$1 = \Sigma_s \; k(s). \tag{12}$$

Units allocated to market j bring P(j) dollars if market j opens and zero if it does not. The nominal revenue if exactly s markets open is:

$$y(s) = \Sigma_{j \le s} \; P(j)k(j). \tag{13}$$

At the beginning of the period there are complete markets for contingent claims, to be described below. The price at the beginning of the period of a dollar that will be delivered at the end of the period if exactly s markets open is n_s. The nominal wage is given by W and is paid at the beginning of the period.

The firm chooses k(s) to maximize the present value of profits. It solves:

(14) $$\max Y = \Sigma_s n_s [y(s)] - Wl$$

s.t.

$$y(s) = \Sigma_{j \leq s} P(j)k(j) ;$$

$$1 = \Sigma_s k(s) ;$$

$$k(s) \geq 0.$$

3.3 Banks

The representative bank faces three interest rates: i_D for checkable deposits, i_T for time deposits and i_L for loans.

Let DD(s), TD(s) and LL(s) denote the amounts of checkable (demand) deposits, time deposits and loans that the bank has in state s. The bank's profits in state s are given by:

(15) $$z(s) = i_L LL(s) - i_D DD(s) - i_T TD(s).$$

Let,

(16) $$Z = \Sigma_s n_s z(s);\ D = \Sigma_s n_s DD(s);\ T = \Sigma_s n_s TD(s);\ L = \Sigma_s n_s LL(s),$$

denote the expected value of the corresponding quantities when using "risk neutral probabilities". (We later show that in equilibrium $n_s = \Pi_s$ and these are standard mathematical expectations).

While banks cannot observe the state s at the time they operate, they can still control the expected values D, T and L. This assumption is motivated by the following Bertrand type argument. We assume that if the bank sets the market interest rates it will get the market average quantities. By deviating slightly from the market rates and setting appropriate quantity limits, a bank can attract any amount it wants.

There is a regulator who can infer from the operating procedures of the bank the expected values (16). It is assumed that the regulator imposes an "average" reserve requirement on demand deposits[4]:

$$(D + T - L)/D \geq rr, \tag{17}$$

where $0 < rr \leq 1$ is the average reserve requirement. There is no reserve requirement on time deposits. The bank chooses D, T and L to solve:

(18) $\max Z = i_L L - i_D D - i_T T$; s.t. (17) and non-negativity constraints.

In equilibrium, $i_L \geq i_D$ and therefore (17) will hold with equality. Substituting (17) into (18) allows us to write the profit of the bank as:

$$[(1 - rr)i_L - i_D]D + (i_L - i_T)T. \tag{19}$$

In equilibrium D and T must be finite and positive and therefore:

$$(1 - rr)i_L = i_D \text{ and } i_L = i_T. \tag{20}$$

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[4] In practice, central banks control reserve requirement by computing periodic averages.

3.4 Households

The objective function of the household is given by

(21) $$\Sigma_t \beta^t \theta_t u(c_t),$$

where c_t is consumption at time t, θ_t is an i.i.d. random variable that may take the value of 1 (if the household wants to consume) and 0 (otherwise) and $0 < \beta < 1$ is a discount factor. The single period utility function u( ) is differentiable and strictly concave with $u'(0) = \infty$. The amount of consumption depends on the realizations of three shocks: the aggregate shock, $(\tilde{s}_t)$, the (idiosyncratic) market at which the buyer participates $(\tilde{j}_t \leq \tilde{s}_t)$ and the (idiosyncratic) type of the household $(\tilde{\tau}_t)$. The type of the household is determined both by its desire to consume and the traveling status of the buyer. The buyer is of type 0 if he wants to consume $(\theta = 1)$ and he is non-traveler, he is of type 1 if he wants to consume and he travels and of type 2 if he does not want to consume $(\theta = 0)$.

The household starts the period with A_t dollars. It owns a firm and a bank which are valued at Y_t and Z_t dollars, respectively. It sells (inellastically) a unit of labor for W_t dollars. It first goes to the securities market and buys or sells (from and to the "market") contingent dollars that will be delivered at the end of the period. The contingencies are on the realizations of the aggregate shock, $(\tilde{s}_t)$, the (idiosyncratic) market at which the buyer participates $(\tilde{j}_t)$ and the (idiosyncratic) type of the buyer $(\tilde{\tau}_t)$.

The price of a dollar that will be delivered if the realization of $(\tilde{s}_t, \tilde{j}_t, \tilde{\tau}_t)$ is (s,j,τ) is denoted by $n_t(s,j,\tau)$ and the number of dollars that will be delivered in this case is $\zeta_t(s,j,\tau)$. Note that $\zeta_t(s,j,\tau)$ is defined only for $s \geq j$. For notational convenience we set $\zeta_t(s,j,\tau) = 0$ for $s < j$. The total cost of these contingent claims is thus, $\Sigma_s\Sigma_j\Sigma_\tau\, n_t(s,j,\tau)\zeta_t(s,j,\tau)$. The amount of money that the household carries after the end of transactions at the securities market is:

$$BD_t = A_t + Y_t + Z_t + W_t - \Sigma_s\Sigma_j\Sigma_\tau\, n_t(s,j,\tau)\zeta_t(s,j,\tau). \tag{22}$$

After the end of trade in the securities market, one member of the household goes to work (the worker) and the other member goes to the bank (the buyer). At the bank, the buyer learns his type, τ, and chooses the amount of spendable dollars, $SD_t(\tau)$.

After the end of bank transactions buyers go to their shopping island (non-travelers stay in their island of origin and travelers go to the other island). In each island, buyers form a line. The place of an individual buyer in the line is exogenously determined by an i.i.d. lottery. Buyers arrive at the goods market sequentially according to their place in line. Buyers cannot resell goods.

Upon arrival at the market-place, buyers find out the lowest price, $P_t(j)$, at which goods are still available. They thus learn that they participate in market j. A buyer of type τ who participates in market j, chooses to spend $E_t(j,\tau)$ dollars which buy:

$$c_t(j,\tau) = E_t(j,\tau)/P_t(j), \tag{23}$$

units of consumption. The money in advance constraint is:

(24) $$E_t(j,\tau) \le SD_t(\tau).$$

The asset transition equation for the household is:

(25) $$A_{t+1}(s,j,\tau) = (1+i_{BDt})BD_t - i_{SDt}(\tau)SD_t(\tau_t) - E_t(j,\tau) + \zeta_t(s,j,\tau),$$

where i_{BDt} and i_{SDt} are shadow interest rates: i_{BDt} is the interest applicable to BD_t and $i_{SDt}(\tau_t)$ is the interest cost of a spendable dollar, which is type dependent.

The household chooses $\zeta_t(s,j,\tau)$, $SD_t(\tau)$ and $E_t(j,\tau)$ to maximize the expected value of (21) with respect to (22) - (25). A dynamic programming formulation of the household's maximization problem is in Appendix A. We now turn to specify the shadow interest rates as a function of the bank's rates: i_L, i_T and i_D.

The shadow interest rates:

The shadow interest rate on BD can be computed by holding SD constant and adding a dollar to BD. If the buyer borrows from the bank (SD > BD), a dollar added to BD will reduce the amount of loans by one dollar and will cut the interest cost by i_L. If he does not borrow, a dollar added to BD will simply be deposited at the interest $i_T = i_L$. Thus, the shadow interest rate applicable to BD is: $i_{BD} = i_L$.

We now turn to specify the interest cost of a spendable dollar. For type 2 consumers, SD = 0 and the specification of the interest cost is superfluous. For types 1 and 0 the interest cost depends on the exact specification of the money in advance constraint for travelers and non-travelers.

The (generic) buyer chooses the amount of loans, ll, and allocates the total of BD + ll between cash, cu, demand deposits, dd, and time deposits, td. Thus,

$$cu + dd + td = BD + ll. \tag{26}$$

A buyer who travels must satisfy the cash-in-advance constraint:

$$E \leq cu\ , \tag{27}$$

where E is the nominal expenditures on goods. A buyer who does not travel, must satisfy the less stringent constraint:

$$E \leq cu + dd. \tag{28}$$

The money in advance constraint (24) takes the form in (27) for a traveler and (28) for a non-traveler. Accordingly, the amount of spendable dollars, SD, is the right hand side of (27) for a traveler and of (28) for a non-traveler.

A traveler who wants to add a dollar to the spendable amount and has no time deposits ($SD \geq BD$) will have to borrow the additional dollar and add it to cash. The interest cost is i_L. If he has time deposits ($SD < BD$), he will withdraw the dollar from time deposits and loose i_L. Thus, $i_{SD}(1) = i_L$. A non-traveler who wants to add a dollar to his spendable amount, will take a loan and deposit the dollar in a checkable account if SD ≥ BD. The net interest cost for doing that is $i_L - i_D$. If SD < BD he will transfer the dollar from time to demand deposits and the interest cost is also $i_L - i_D$. Thus,

$$(29) \qquad i_{SD}(0) = i_L - i_D; \; i_{SD}(1) = i_L.$$

This difference in the shadow price of a spendable dollar turns out to be crucial for generating endogenous fluctuations in M1.

3.5 Equilibrium

We assume that when $\tilde{\phi} = \phi_s$ the first s markets open. The amount of money that will be spent in each market is determined endogenously in the following way.

Assuming that the fraction of travelers and non-travelers is the same in all markets, total amount of expenditures per seller if $\tilde{\phi} = \phi_s$ is given by:

$$(30) \qquad TE(s) = (1 - \alpha)[\phi_s \Sigma_{j\leq s} E(j,0) + (1 - \phi_s)\Sigma_{j\leq s} E(j,1)].$$

Without loss of generality, we assume E(j,0) ≥ E(j,1). (Otherwise we redefine indices). This implies: TE(s) ≤ TE(s+1).

We say that the minimum amount of dollars that will arrive, TE(1), are spent in market 1. If more than TE(1) dollars arrive, then market 2 opens. If more than TE(2) dollars arrive, then market 3 opens and so on. The nominal demand per seller in market s is:

$$\Delta(s) = TE(s) - TE(s-1), \tag{31}$$

where we set: $TE(0) = 0$.

Note that the amount of dollars that will be spent in each market is endogenous and depends on the choices of $E(s,\tau)$. In the special case in which $E(j,0) = E(j,1)$ for all j, $TE(s) = TE(1)$ for all s and $\Delta(s) = 0$ for all $s > 1$. This case of full capacity utilization occurs when the average reserve ratio, rr, is unity, because in this case (20) implies that $i_D = 0$ and (29) implies $i_{SD}(0) = i_{SD}(1) = i_L$. However, if $E(j,0) > E(j,1)$ for all j, then $\Delta(s) > 0$ for all s.

The probability that a dollar will be spent at market j when exactly s markets open is:

$$\upsilon_j^s = \Delta(j)/TE(s). \tag{32}$$

Market clearing requires:

$$\Delta(s) = P(s)k(s), \quad \text{for all s.} \tag{33}$$

We assume that the representative household starts with a nominal wealth $A = H$, where H is outside money and define equilibrium as follows.

A <u>stationary symmetric equilibrium</u> for the reserve requirement rr ($0 \le rr \le 1$) and $A = H$, is a vector [W, n_s, P(s), k(s), y(s), i_L, i_D, i_T, $i_{SD}(\tau)$, LL(s), TD(s), DD(s), z(s), L, T, D, Y, Z, $n(s,j,\tau)$, $\zeta(s,j,\tau)$,

BD, SD(τ), E(j,τ), Δ(s), υ_j^s, A'(s,j,τ); s,j = 1,...,S; and τ = 0,1,2]
such that:

(a) Δ(s) and υ_j^s satisfy (30) - (32), y(s), z(s) are defined by (13) and (15); Y is defined by (10); L, T, D, Z are defined by (16), $i_{SD}(\tau)$ satisfy (29) and $A'(s,j,\tau) = (1+i_L)BD - i_{SD}(\tau)SD(\tau) - E(j,\tau) + \zeta(s,j,\tau)$.

(b) Maximizing behavior

Given (W, n(s,j,τ), i_L, i_D, P(s), Y, Z) the quantities $\zeta(s,j,\tau)$, BD, SD(τ), E(j,τ) solve the household's maximization problem (maximizing the expected value of (21) subject to (18)-(25): see Appendix A for a complete dynamic programming formulation);

Given (W, n_s, P(s)), the quantities l and k(s) solve the firm's problem (14);

Given (i_L, i_D, i_T) the expected quantities (L, D, T) solve the bank's problem (18).

(c) Market clearing

Securities:

$$\Sigma_{j\leq s}\, \upsilon_j^s\, [(1-\alpha)\phi_s\zeta(s,j,0) + (1-\alpha)(1-\phi_s)\zeta(s,j,1) + \alpha\zeta(s,j,2)]$$
$$= z(s) + y(s), \text{ for all } s;$$

The left hand side is the total amount of dollars claimed when s markets open and the right hand side is the supply of dollars in this case.

Money:

$BD = H;$

$\Sigma_{j \leq s} \upsilon_j^s [(1-\alpha)\phi_s A'(s,j,0) + (1-\alpha)(1-\phi_s)A'(s,j,1) + \alpha A'(s,j,2)] = H,$

for all s;

This says that H will always be willingly held. The first requirement insures that H is willingly held after the end of transactions in the securities market. The second requirement insures that outside money is willingly held by the household at the end of the period. (The first order conditions for the banks and the travelers insure that money is willingly held during the period).

Banks:

$LL(s) = (1 - \alpha)[\phi_s \max\{0, SD(0) - H\} + (1 - \phi_s)\max\{0, SD(1) - H\}];$

$DD(s) = \phi_s(1 - \alpha)SD(0);$

$TD(s) = \alpha H + (1 - \alpha)\{\phi_s \max(0, H - SD(0)) + (1 - \phi_s)\max(0, H - SD(1))\}$

On the left hand side are banks' supplies. On the right hand side are aggregate demands. Since non-consumers do not take loans we aggregate the demand for loans of types 0 and 1 only (first condition). Since $i_T \geq i_D$, travelers use time deposits rather than demand deposits, in case they choose SD < BD. Therefore, only non-travelers use checkable deposits (second condition). The last condition aggregates demand for time deposits over all types.

Goods: $\Delta(s) = P(s)k(s)$, for all s;

Markets which are opened are cleared.

Labor: $l = 1$.

Stationarity of wealth distribution:

$A'(s,j,\tau) = H$ for all s, j, τ.

In Appendix B we show the following main results.

Proposition 1: There exists a unique stationary symmetric equilibrium.

Proposition 2: The allocation obtained when rr = 1 is Pareto efficient.[5]

The intuition for the second result is that setting rr = 1 eliminates the endogenous fluctuations in M1 and leads to full capacity utilization. In detail, when rr = 1, $i_D = 0$ and $i_{SD}(0) = i_{SD}(1) = i_L$. Therefore, $E(j,0) = E(j,1)$ for all j, $TE(s) = TE(1)$ for all s and $\Delta(s) = 0$ for all $s > 1$. Market clearing implies that all the capacity is supplied to the first market and since this market always open, capacity is fully utilized. When $rr < 1$, $i_D > 0$ and $i_{SD}(0) = i_L - i_D \neq i_{SD}(1) = i_L$. In this case, $E(j,0) > E(j,1)$ for all j, and $\Delta(s) > 0$ for all s. Strictly positive capacity will be supplied to all markets and capacity in markets which do not open is wasted.

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[5] This result uses the assumption that labor supply is inelastic. Otherwise, the Friedman zero nominal interest rate rule is required to achieve efficiency.

4. THE EFFECT OF CREDIT CARDS

We view credit cards as a way of circumventing the money-in-advance constraint: A buyer with a credit card (a credit buyer) does all payments using the card and then, at the end of the period, he uses time deposits to cover the debt.[6]

Shocks to the number of credit card users cause "velocity shocks" which are analogous to the shocks generated by changes in the currency to deposits ratio. To illustrate, we adapt the above UST model to allow for credit buyers. There are three types of households characterized by their desire to consume and the credit status of the buyer. The buyer is of type 0 if he wants to consume and is creditworthy. The buyer is of type 1 if he wants to consume and is not creditworthy and the buyer is of type 2 if he does not want to consume. As before, we assume that a constant fraction, α, of the households are non-consumers.

To simplify we assume that all buyers can use checks so that cash is not used. We also simplify by assuming that creditworthiness is assigned arbitrarily at the bank: A random fraction $\tilde{\phi}$ of the households which consume, are creditworthy. The identity of the households who belong to each type is determined every period, at the bank, by an i.i.d. lottery.[7] The bank can observe the type of each buyer and supplies credit cards only to buyers who are creditworthy.

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[6] Note that unlike Lucas and Stokey (1987), here the ability to use credit is a characteristic of a buyer, rather than of a good.

[7] Creditworthiness is actually determined on the basis of past behavior. For our purpose, it is enough that there be some random element in the process of determining creditworthiness.

Since a credit buyer pays at the end of the period, the interest cost of a spendable dollar is zero for this type of buyer ($i_{SD}(0) = 0$)[8]. The interest cost of the non-creditworthy buyer is the same as for the a check user in the previous section ($i_{SD}(1) = i_L - i_D$).

As before the interest cost differential will lead to different expenditure functions $E(j, \tau)$ and therefore to uncertainty about total demand. This will lead to strictly positive demand (Δ in equation [31]) in markets with $s > 1$ and to less than full capacity utilization.

Thus random number of credit buyers may generate velocity shocks which may lead to waste. When cash is not used, we can increase capacity utilization by choosing low rr. This will reduce the interest cost to the non-creditworthy buyers. At the limit, with zero reserve requirement, the interest cost is $i_{SD}(1) = i_L - i_D = 0$ and all the demand is in the first market. However in this case the price level cannot be determined.

When cash is used, the only way to achieve full capacity utilization is by prohibiting the use of credit cards (and allow only the use of debit cards which do not circumvent the money-in-advance constraint). Otherwise, there is a tradeoff between random fluctuations in nominal demand which stem from changes in the number of credit users and fluctuations which stem from changes in the number of cash users. When rr is reduced, the first source of fluctuations becomes less important but the second source gains importance. To minimize random

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[8] The calculation of this interest cost is analogous to the caluclation of the interest cost of a spendable dollar to the non-traveler in the previous section, where i_T replaces i_D in (29).

fluctuations in nominal demand, it is likely that an interior reserve requirement ($0 < rr < 1$) should be chosen.

5. DISCUSSION

We have shown that if the money-in-advance constraint is universally applicable then the Friedman case for a 100% reserve requirement on checkable accounts is justifiable.

To understand Friedman's position it is useful to distinguish between the individual and the social points of view, regarding the creation of real balances.[9] While from the individual point of view banks alleviate the money-in-advance constraint, from the social point of view they do not: The increase in inside money simply increases the price level. Moreover, we have shown that when the money-in-advance constraint is universally applicable, reserve requirements on checkable accounts have no effect on intermediation which is done by the use of time deposits.

We have also shown that endogenous fluctuations in M1 lead to fluctuations in output, as argued by Friedman. In our UST model fluctuations in the currency/deposit ratio create endogenous monetary shocks. These fluctuations are non-neutral here for the same reason that fluctuations in the money supply are non-neutral in other UST models: actual trade occurs before all the information about the current money supply and demand is revealed. To insure full capacity utilization, sellers must know the current demand. In our model, this is achieved by

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[9] This distinction is present in Friedman (1959) and Friedman (1969).

imposing a 100% reserve requirement which eliminates the endogenous fluctuations in the money supply.

However, when the nominal interest rate is positive, there are incentives to circumvent the money-in-advance constraint. The use of credit cards is a good example. We view credit cards as allowing buyers to use time deposits to buy goods. In general, there will be three types of buyers: cash users, check users and credit users. And there will be a difference in the interest cost of consumption which cannot be entirely eliminated. If we adopt the 100% reserve requirement we eliminate the cost difference between cash users and check users but maximize the cost difference between these two types and credit users. A 0% reserve requirement (if possible) will eliminate the difference in cost between credit users and check users but cash users will pay more.[10]

In our UST model, prohibiting the use of credit cards combined with the 100% reserve requirement, will ensure full capacity utilization. But it is not clear whether such regulations can be enforced.

This is not a problem at the Friedman zero nominal interest rate rule because at zero nominal interest rate there are no incentives to circumvent the money-in-advance constraint. However, other problems may arise. If the fraction of non-consumers (α) is random, then at the Friedman rule there will be uncertainty about nominal demand because non-consumers will have no incentive to lend money which they do not

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[10] It is possible that the observed recent reductions in reserve requirements can be explained by the growing importance of credit card transactions.

plan to spend. In a UST environment, this uncertainty leads to waste. It may thus be desirable to have a small positive nominal interest rate. In this case, the non-consumers will lend their money and the amount of money which arrives at the goods market (under 100% reserve requirement) is non random. (See related arguments in Eden [1986] and Williamson [1996]). But as was mentioned before, there will be incentives to use credit cards for circumventing the money-in-advance constraint.

We may therefore say that UST models which incorporate the money-in-advance constraint do not give unambiguous support for the Friedman zero nominal interest rate rule nor to the 100% reserve requirement. Still these models provide a framework for analyzing the relevant tradeoffs associated with the choice of reserve requirements and nominal interest rate.

Other models have been used to discuss reserve requirements. Sargent and Wallace (1982) argue against the imposition of any legal restrictions on the operation of banks. They argue for the elimination of all interest rate differentials. However, Sargent and Wallace have only one type of deposits. Therefore they do not make the distinction between time and demand deposits which we argue is crucial.

The Diamond and Dybvig (1983) model has also been used to make a case against imposing (100%) reserve requirements. In their model demand deposits serve agents who are not sure about the timing of their consumption. Accordingly, the Diamond-Dybvig definition of demand deposits is different from ours. They emphasize the flexible maturity (the ability to withdraw a known quantity of cash upon demand) aspect of these accounts while we emphasize the circulating debt (the ability to use these accounts for writing checks and satisfy the money in advance constraint) aspect. The risk pooling role of the Diamond-Dybvig banks can be performed by other financial institutions as argued by Jacklin (1987). We therefore think that the main distinguishing feature of banks is in the creation of circulating debt and not the creation of flexible maturity debt.

APPENDIX A

Dynamic programming formulation

Here we specify the dynamic programming problem faced by the household.

There are three sessions of trade. At the first session there is trade in securities and labor. The household brings from the previous period a nominal wealth A, and after completion of transactions (choosing ζ, receiving profits from the firm and the bank, and selling labor) its wealth at the end of the period is: A_1 dollars. The buyer then goes to the bank and chooses SD, which changes his wealth to A_2. Finally, the buyer goes to the goods market and chooses E, changing the wealth to A_3 which is carried over to the next period as A'. Thus, A_{i-1} denotes the (random, end of period) wealth at the beginning of session i. We use v_i to denote the maximum expected utility in session i, which depends on A_{i-1}.

Using the logic of dynamic programming we start from the last session.

At the goods market:

We use $A_2(\tilde{s},j,\tau)$ to denote the end of period wealth of the household at the beginning of trade in the goods market j. This value depends on the yet unknown realization of $\tilde{s}$ and the indices j and τ (which are known at this stage) because contracts signed at previous stages are contingent on these variables. Note that $A_2(\tilde{s},j,\tau)$ is defined only for realizations $s \geq j$. For notational convenience we set

$A_2(s,j,\tau) = 0$ for $s < j$. The same convention is adopted below for similar cases.

We use the vector:

$A_2(\bullet,j,\tau) = \{A_2(1,j,\tau), A_2(2,j,\tau), \ldots, A_2(S,j,\tau)\}$, to denote all possible realizations of $A_2(\tilde{s},j,\tau)$. The buyer faces the price $P(j)$ and chooses to spend $E(j,\tau)$ dollars subject to the constraint: $E(j,\tau) \leq SD(\tau)$. The end of period nominal wealth after spending is given by $A'(\tilde{s},j,\tau) = A_2(\tilde{s},j,\tau) - E(j,\tau)$. This amount yields next period the expected utility $EV(A'(\tilde{s},j,\tau))$. We require that bankcruptcies do not occur so that $A'(\tilde{s},j,\tau)$ is positive.

The buyer who found out his type in the previous stage, has used Bayes law to update the probability of state s in a way which will be described below. As a result, buyer of type τ assigns the probability $\pi_s(\tau)$ to the event: $\tilde{\phi} = \phi_s$. When the buyer finds that he participates in market j and that $\tilde{s} \geq j$, he updates the probability again: $Prob(\tilde{s} = s | \tilde{s} \geq j, \tau) = (\pi_s(\tau)/q_j)$. Taking $SD(\tau)$ and $A_2(\bullet,j,\tau)$ as given the buyer chooses $E(j,\tau)$ to solve:

(A1) $$v_3(A_2(\bullet,j,\tau), SD(\tau), j) =$$

$$\max \{\theta u(E(j,\tau)/P(j)) + \beta \Sigma_{s\geq j} (\pi_s(\tau)/q_j) V(A'(s,j,\tau))\}$$

s.t

$$0 \leq E(j,\tau) \leq SD(\tau) ;$$

$$A'(s,j,\tau) = A_2(s,j,\tau) - E(j,\tau) \geq 0.$$

At the bank:

When the buyer learns his type, he uses Bayes rule to update the probability that $\tilde{\phi} = \phi_s$. This probability conditional on τ, is:

$$\text{(A2)} \qquad \pi_s(0) = \{[(\Pi_s\phi_s)/\psi],$$

$$\pi_s(1) = [(\Pi_s(1-\phi_s))/(1-\psi)] \text{ and } \pi_s(2) = \Pi_s,$$

where $\psi = \Sigma_s\Pi_s\phi_s$ is the probability of being a non-traveler given $\theta = 1$.[11]

Before transacting at the bank, the end of period wealth is $A_1(\tilde{s},\tilde{j},\tau)$. After the completion of transactions at the bank, the end of period wealth is:

$$\text{(A3)} \qquad A_2(\tilde{s},\tilde{j},\tau) = A_1(\tilde{s},\tilde{j},\tau) - i_{SD}(\tau)SD(\tau).$$

At the goods market, the expected utility of the household which participates in market j is: $v_3(A_2(\bullet,j,\tau),SD(\tau),j)$. However, at the banking stage, $\tilde{j}$ is still a random variable. To compute expectations, we use υ_j^s to denote the probability that the buyer will participate in market j given that $s \geq j$ markets open. Using this notation the probability that a buyer of type τ assigns to the event that he will participate in market j is given by $f_j(\tau) = [\Sigma_{s\geq j}\ \upsilon_j^s\pi_s(\tau)]$. (Note that the index j is not relevant for type 2 but we include it for notational convenience). Therefore the maximum expected utility at the beginning of the third session is:

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[11] For example,
$\text{prob}(\tilde{\phi} = \phi_s|\ \tau = 0) = \text{prob}(\{\tilde{\phi} = \phi_s\}\cap\{\tau = 0\})/\text{prob}(\tau = 0) = (\Pi_s\phi_s)/\psi$.

$Ev_3(A_2(\bullet,\tilde{j},\tau),SD(\tau),\tilde{j}) = \Sigma_j f_j(\tau)v_3(A_2(\bullet,j,\tau),SD(\tau),j)$.

At the bank, the buyer chooses SD ≥ 0 to solve:

$$\text{(A4)} \qquad v_2(A_1(\bullet,\bullet,\tau),\tau) = \max_{SD} \Sigma_j f_j(\tau)v_3(A_2(\bullet,j,\tau),SD(\tau),j) \quad \text{s.t. (A3)},$$

where $A_1(\bullet,\bullet,\tau)$ is the matrix of all possible realizations of $A_1(\tilde{s},\tilde{j},\tau)$.

At the securities market:

The household starts with A dollars and after receiving the profits from the firm and the bank and selling labor it has A + Y + Z + W dollars. It then chooses BD and ζ out of the budget constraint (22) in the text to maximize the expected value of $v_2(A_1(\bullet,\bullet,\tilde{\tau}), \tilde{\tau})$. The shadow interest rate for BD is i_L and therefore the asset transition equation is:

$$\text{(A5)} \qquad A_1(\tilde{s},\tilde{j},\tilde{\tau}) = (1 + i_L)BD + \zeta(\tilde{s},\tilde{j},\tilde{\tau}).$$

Before learning its type, the household chooses $\zeta(\tilde{s},\tilde{j},\tilde{\tau})$ to maximize $Ev_2(A_1(\bullet,\bullet,\tilde{\tau}), \tilde{\tau})]$. The household thus solves:

$$\text{(A6)} \quad V(A) = \max\ (1 - \alpha)[\psi v_2(A_1(\bullet,\bullet,0), 0) + (1-\psi)v_2(A_1(\bullet,\bullet,1), 1)] + \alpha v_2(A_1(\bullet,\bullet,2), 2)$$

s.t. (22) in the text and (A5),

where as before ψ denotes the probability that a buyer will not travel given that he wants to consume.

Existence and Characterization of equilibrium

We start by valuing an additional dollar at the beginning of the period under the assumption that an equilibrium exists. This is:

(A7)
$$V'(A) = \Sigma_s \Pi_s \Sigma_{j \le s} \upsilon_j^s \{(1-\alpha)\phi_s\{\beta i_D V'(A'(s,j,0)) + u'(E(j,0)/P(j))/P(j)\} + (1-\alpha)(1-\phi_s)u'(E(j,1)/P(j))/P(j) + \alpha\beta(1+i_L)V'(A'(s,j,2))\}$$

This follows from the envelope argument applied to (A6). The intuition is as follows.

A type 0 buyer cannot do better than deposit the dollar in a checkable account and spend it. This follows from the fact that we have an interior solution. In detail, spending the dollar on consumption will be the strictly preferred option in case the money-in-advance constraint is binding. Since the buyer always buys a strictly positive amount of consumption, when the money-in-advance constraint is not binding the buyer is indifferent between spending the dollar on consumption and carrying it over to the next period. So in either case we may assume that the dollar is spent on consumption. Since the dollar is deposited, the end of period wealth increases by i_D dollars due to the interest on demand deposits and this is valued by: $\beta i_D V'(A'(s,j,0))$. Since the event

$\{\theta = 1$, s markets open, and the buyer is a non-traveler who participates in market $j \leq s\}$ occurs with probability $\Pi_s \upsilon_j^s (1 - \alpha)\phi_s$, the second line in (A7) is the total value of the additional dollar to a non-traveler.

A type 1 buyer will take the dollar as cash and spend it. Since the event $\{\theta = 1$, s markets open and the buyer is a traveler who participates in market $j \leq s\}$ occurs with probability $\Pi_s \upsilon_j^s (1 - \alpha)(1 - \phi_s)$, the value from doing it is the third line of (A7).

A type 2 buyer will deposit the dollar in a time deposit at an interest rate i_L. His end of period wealth increases by $(1 + i_L)$ dollars and his expected utility by $\beta(1 + i_L)V'(A'(s,j,2)$. Since the event $\{\theta = 0$, s markets open, and the buyer (fictitiously) participates in market $j \leq s\}$ occurs with probability $\alpha\Pi_s \upsilon_j^s$, the last expression under the summation on the right hand side of (A7) is the value of an additional dollar to a type 2 buyer.

Since u is concave, it can be shown (following Stokey and Lucas [1989]) that V(A) is concave. Concavity and the market clearing condition:

$\Sigma_j \upsilon_j^s [(1-\alpha)\phi_s A'(s,j,0) + (1-\alpha)(1-\phi_s)A'(s,j,1) + \alpha A'(s,j,2)] = H$ for all s leads to stationarity: $A'(s,j,\tau) = H$ for all s, j, τ. This follows from the fact that the sum of the weights in the above market clearing condition is unity: $\Sigma_j \upsilon_j^s [(1-\alpha)\phi_s + (1-\alpha)(1-\phi_s) + \alpha] = 1$, for all s.

Stationarity and (A7) imply:

$$(A8)\quad V'(H) = \Sigma_s \Pi_s \Sigma_{j\leq s} \upsilon_j^s \{\phi_s u'(E(j,0)/P(j))[1/P(j)] + (1-\phi_s)u'(E(j,1)/P(j))[1/P(j)]\}/\Gamma,$$

where $\Gamma = [1 - \alpha\beta(1+i_L) - (1 - \alpha)i_D\beta\psi]/(1 - \alpha)$.

Next we use the first order conditions that govern the choice of $SD(\tau)$ to show that in equilibrium:

$$\text{(A9)} \quad \Sigma_s \pi_s(\tau) \Sigma_{j \le s} \upsilon_j^s \, u'(E(j,\tau)/P(j))[1/P(j)] = \beta[1 + i_{SD}(\tau)]V'(H).$$

Condition (A9) uses the following reasoning. At the optimum the buyer is indifferent between taking an additional spendable dollar and actually spending it to not doing so (this is true even when he is not constrained by the money in advance constraint, see the reasoning for (A7)). If he spends an additional dollar he will get the additional expected utility from consumption calculated by the left hand side of (A9). The cost of doing so, which is on the right hand side of (A9), arises because he will have $1 + i_{SD}(\tau)$ dollars less at the end of the period. Consistency of conditions (A9) and (A8) requires $(1 + i_L) = 1/\beta$. We will argue soon that this must hold in equilibrium.

The first order conditions that govern the choice of E in the goods market (invoking stationarity) imply:

$$\text{(A10)} \quad u'(E(j,\tau)/P(j))/P(j) \ge \beta V'(H); \quad \text{with equality if } E(j,\tau) < SD(\tau).$$

Stationarity and the concavity of V( ) imply nominal prices which are actuarially fair:

$$\text{(A11)} \quad n(s,j,0) = \beta(1 - \alpha)\phi_s \Pi_s \upsilon_j^s; \qquad n(s,j,1) = \beta(1 - \alpha)(1-\phi_s)\Pi_s \upsilon_j^s;$$

$$n(s,j,2) = \beta\alpha\Pi_s \upsilon_j^s; \qquad n_s = \Sigma_j \Sigma_\tau \, n(s,j,\tau) = \beta\Pi_s; \qquad \Sigma_s \, n_s = \beta.$$

The absence of arbitrage opportunities implies, (A12) $$(1 + i_L) = 1/\beta.$$

To see why $(1 + i_L) = 1/\beta$, note that the price of a dollar in the next period is $\Sigma_s n_s = \beta$ and the implied gross interest rate in the securities market is $1/\beta$. Suppose now that $(1 + i_L) > 1/\beta$. Then a household can choose large BD by selling claims on dollars in the securities market and deposit (BD - SD) at the bank as time deposits, making an unbounded amount of money with certainty. If $(1 + i_L) < 1/\beta$, it will choose large negative BD by buying claims on dollars and take loans from the bank to get the desired level of SD.

From (13) in the text and (A12) it follows immediately that:

(A13) $$i_L - i_D = rr(\beta^{-1} - 1).$$

Thus, the interest spread is increasing in the reserve requirement, rr.

The first order condition for an interior solution to the firm's problem (14), implies:

(A14) $$q_s P(s) = P(1) = W/\beta.$$

In Appendix B we use the above result to show that:

<u>Proposition 1:</u> There exists a unique stationary symmetric equilibrium.

We now turn to discuss the optimal choice of reserve requirements. We first show, Claim 1: If $SD(\tau) \geq SD(\tau')$, then $E(j,\tau) \geq E(j,\tau')$ and vice versa.

This follows from (A10).

Claim 2: When rr < 1, SD(1) < SD(0).

To show this Claim, suppose SD(1) ≥ SD(0). Then E(j,1) ≥ E(j,0) by Claim 1. It follows that travelers spend more both on consumption and on interest than non-travelers. Since prices are actuarially fair (see, [A11]) the strategy of consuming more when traveling is worse than a strategy of consuming an amount that does not depend on the traveling status. To see this point, note that if the mean is the same, concavity of u( ) works in favor of the alternative. Moreover, average consumption is higher under the alternative because interest costs, i_{SD}, are lower. Thus, by contradiction, SD(1) < SD(0).

When rr = 1, (20) implies $i_D = 0$ and (29) implies that both types face the same shadow interest rate for SD: $i_{SD}(0) = i_{SD}(1) = i_L$. Therefore,

Claim 3: When rr = 1, SD(1) = SD(0) and E(s,0) = E(s,1) for all s.

This leads to:

Proposition 2: The allocation obtained when rr = 1 is Pareto efficient.

When rr = 1, Claim 3 and the definitions of Δ in (31) imply that only the first market is active. Therefore, in equilibrium k(1) = 1, k(s) = 0 for all s > 1. Consumption per household is unity and does not depend on the traveling status.

APPENDIX B

We compute a stationary and symmetric equilibrium in the following way. We first arbitrarily choose a vector SD = [SD(0), SD(1)] and compute prices, consumption and the marginal utility of a dollar as functions of SD. We then use these functions to solve for a vector SD that satisfies the first order conditions at the banking session. It turns out that if SD is a solution then λSD is also a solution for all $\lambda > 0$. We use the reserve requirement and BD = H, to scale the SD vector and to show the existence of a unique stationary and symmetric equilibrium.

Proof of Proposition 1

We first define equilibrium in the goods market for a given vector SD = [SD(0), SD(1)].

The vector [(P(1),...,P(S), k(1),...,k(S), E(0,1),...,E(0,S), E(1,1),...,E(1,S), υ_j^s, V'] is an equilibrium in the goods market if:

(B1) $$q_s P(s) = P(1)$$

(B2) $$\Sigma_s\, k(s) = 1$$

(B3) $\Delta(s)/P(s) = k(s)$, where $\Delta(s)$ is from (31) in the text;

(B4) $$E(j,\tau) \leq SD(\tau)$$

(B5) $u'(E(j,\tau)/P(j))/P(j) \geq \beta V'$ with equality if $E(j,\tau) < SD(\tau)$.

(B6) $$V' = \Sigma_s \Pi_s \Sigma_{j \leq s}\, \upsilon_j^s\{(1-\phi_s)u'(E(j,1)/P(j))/P(j) + \phi_s u'(E(j,0)/P(j))/P(j)\}/\Gamma;$$

where (after substituting [A12]) $\Gamma = 1 - i_D\beta\psi$, and υ_j^s is given by (32) in the text.

Claim B1: For any SD > 0, there exists a unique equilibrium in the goods market: [(P(1;SD),...,P(S;SD), k(1;SD),...,k(S;SD), E(0,1;SD),...,E(0,S;SD), E(1,1;SD),...,E(1,S;SD), υ_j^s(SD), V'(SD)].

Proof: Let p denote the expected revenue per unit in the goods market. We choose p > 0 arbitrarily and set:

$$P(s;p) = p/q_s. \tag{B7}$$

Lemma B1: Given SD and p, there exists a solution, E(j, τ; SD, p) to:

$$\begin{aligned} &u'(E(j,\tau)/P(j;p))/P(j;p) \geq \\ &\beta\Sigma_s\Pi_s\Sigma_{i\leq s}\ \upsilon_i^s\{(1-\phi_s)u'(E(i,1)/P(i;p))/P(i;p) \\ &+\phi_s u'(E(i,0)/P(i;p))/P(i;p)\}/\Gamma \end{aligned} \tag{B8}$$

with equality if $E(j,\tau) < SD(\tau)$.

Note that to get (B8) we substitute (B6) into (B5) and therefore (B8) insures that both conditions are satisfied. To show existence of a solution to (B8), we choose κ as our guess for V' and define:

$$\begin{aligned}\kappa' = \Sigma_s\Pi_s\Sigma_{j\leq s}\ \upsilon_j^s\{&(1-\phi_s)\min[u'(SD(1)/P(j;p))/P(j;p),\ \beta\kappa] + \\ &\phi_s\min[u'(SD(0)/P(j;p))/P(j;p),\ \beta\kappa]\}/(\beta\Gamma).\end{aligned} \tag{B9}$$

If κ is small then $\kappa' = \kappa/\Gamma > \kappa$ since $\Gamma < 1$. If κ is sufficiently large, then $\kappa' = \Sigma_s \Pi_s \Sigma_{j \leq s} \upsilon_j^s \{(1-\phi_s)u'(SD(1)/P(j;p))/P(j;p) + \phi_s u'(SD(0)/P(j;p))/P(j;p)\}/(\beta\Gamma) < \kappa$.

By continuity, there exists a fixed point $\kappa(SD, p)$ of (B9). Since the mapping is monotone, $\kappa(SD, p)$ is unique.

We now set $E(j, \tau; SD, p) = SD(\tau)$ for all j and τ such that: $u'(SD(\tau)/P(j;p))/P(j;p) \geq \beta\kappa(SD, p)$. Otherwise, $E(j, \tau; SD, p)$ is given by the solution to: $u'(E(j,\tau)/P(j;p))/P(j;p) = \beta\kappa(SD, p)$. Thus we have shown Lemma B1.

Let $TE(s;SD,p)$ and $\Delta(s;SD,p)$ be defined by (30) and (31) when using $E(j, \tau) = E(j, \tau; SD, p)$. Then,

<u>Lemma B2:</u> $\Sigma_s\, q_s\Delta(s;SD,p)/p$ is decreasing in p.

To show this claim note that:

(a) $E(s, \tau; \lambda SD, \lambda p) = \lambda E(s, \tau; SD, p)$;

(b) $E(s, \tau; SD, p)$ is increasing in SD.

From the definition of Δ in the text (31) and (a) and (b) it follows that:

(a') $\Delta(s;\lambda SD,\lambda p) = \lambda\Delta(s;SD,p)$;

(b') $\Sigma_s\, q_s\Delta(s;SD,p)$ is increasing in SD.

From (a') and (b') we get for $\lambda > 1$:

$\Sigma_s\, q_s\Delta(s;SD,\lambda p)/\lambda p < \Sigma_s\, q_s\Delta(s;\lambda SD,\lambda p)/\lambda p = \Sigma_s\, q_s\Delta(s;SD,p)/p$.

This completes the proof of Lemma B2.

To continue with the construction of equilibrium in the goods market, we note that the real demand in market s at the prices P(s;p), is: $k^d(s;p) = \Delta(s;SD,p)/P(s;p) = q_s\Delta(s;SD,p)/p$. The total real demand is: $K^d(p) = \Sigma_s k^d(s;p) = \Sigma_s q_s\Delta(s;SD,p)/p$. Since total supply is unity, market clearing requires:

(B10) $$K^d(p) = 1.$$

By Lemma B2, $K^d(p)$ is continuously decreasing. When p is arbitrarily large, K^d is arbitrarily small and vice versa. This leads to a unique solution of the expected revenue per unit: p(SD). We can now compute equilibrium magnitudes. This completes the proof of Claim B1.

To compute a stationary symmetric equilibrium we use the following goods market equilibrium magnitudes:

$P(s;SD) = P(s;p(SD))$;

$E(j, \tau;SD) = E(j, \tau;SD,p(SD))$;

$V'(SD) = \kappa(SD, p(SD))$;

$\upsilon_j^s(SD) = \Delta(j;SD,p(SD))/TE(s;SD,p(SD))$

We now look for a vector SD that will satisfy the first order condition at the banking session, given P(s;SD), E(j, τ;SD) and V'(SD). We denote the expected marginal utility of a dollar to a type τ buyer, given that s markets open and the dollar is actually spent, by:

(B11) $$X(s,\tau,SD) = \Sigma_{j\leq s}\, \upsilon_j^s(SD)\; u'(E(j,\tau;SD)/P(j;SD))/P(j;SD).$$

The first order conditions at the banking session (A6) are:

(B12a) $\Sigma_s[(\Pi_s\phi_s)/\psi]X(s,0,SD) = \beta(1 + i_{SD}(0))V'(SD)$

(B12b) $\Sigma_s[(\Pi_s(1-\phi_s))/(1-\psi)]X(s,1,SD) = \beta(1 + i_{SD}(1))V'(SD)$

Lemma B3: There exists a vector SD that solves (B12).

Note that if SD solves (B12) then λSD is also a solution for any $\lambda > 0$. Note also that (B6) is a linear combination of (B12a) and (B12b). To see this multiply (B12a) by ψ and (B12b) by $(1-\psi)$ and add the two while using $(1 + i_L) = 1/\beta$, to get (B6), which holds by construction. We can therefore look at a single equation, say (B12a), normalize SD(1) = 1 and solve for SD(0). Let us rewrite (B12a) as:

(B13) $\Sigma_s[(\Pi_s\phi_s)/\psi]X(s,0,[SD(0), 1]) = \beta(1 + i_{SD}(0))V'([SD(0), 1])$.

Note that when SD(0) is large, the consumption of type 1 goes to zero and X(s,0,[SD(0), 1]) is large because we assume: $u'(0) = \infty$. Since V' is a linear combination of X(s,0,[SD(0), 1]) and X(s,1,[SD(0), 1]), it follows that V' is large. In particular it is larger than the LHS of (B13). The opposite holds when SD(0) is small. Thus there exists a solution: $\hat{SD} = [\hat{SD}(0), 1]$. This completes the proof of Lemma A3.

We now scale $\hat{SD}$ to satisfy the reserve requirements. For this purpose, we characterize all combinations of SD = [SD(0), SD(1)] that satisfy the reserve requirement. In equilibrium when BD = H, the non-consumers (a fraction α of the population) deposit BD in time deposits. In addition, consumers deposit any amount beyond SD in time deposits. Thus,

(B14) $T = \alpha H + (1 - \alpha)\{\psi \max(0, H - SD(0)) + (1 - \psi)\max(0, H - SD(1))\}$.

Consumers who choose $SD(\tau) > H$, take loans and therefore:

(B15) $L = (1 - \alpha)\psi \max(0, SD(0) - H) + (1 - \alpha)(1 - \psi)\max(0, SD(1) - H)$.

Only non-travelers use demand deposits and therefore:

(B16) $$D = (1 - \alpha)\psi SD(0).$$

In equilibrium there will be no excess reserves (on average) and therefore (using [17] in the text):

(B17) $$(1 - rr)D + T = L.$$

Substituting (B14)-(B16) into (B17) yields:

(B18) $$\Psi_0 SD(0) + \Psi_1 SD(1) = 1.$$

where, $\Psi_0 = (1 - \alpha)rr\psi/H$; $\Psi_1 = (1 - \alpha)(1 - \psi)/H$. Thus we can scale the solution $\hat{SD}$ by $1/(\Psi_0 \hat{SD}(0) + \Psi_1)$ to get a stationary symmetric equilibrium. With this we have shown, existence and uniqueness of a stationary symmetric equilibrium.

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