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CFS Working Paper No. 2011/19

Default Risk in an Interconnected Banking System with Endogeneous Asset Markets*

Marcel Bluhm[1] and Jan Pieter Krahnen[2]

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Abstract:
This paper analyzes the emergence of systemic risk in a network model of interconnected bank balance sheets. Given a shock to asset values of one or several banks, systemic risk in the form of multiple bank defaults depends on the strength of balance sheets and asset market liquidity. The price of bank assets on the secondary market is endogenous in the model, thereby relating funding liquidity to expected solvency - an important stylized fact of banking crises. Based on the concept of a system value at risk, Shapley values are used to define the systemic risk charge levied upon individual banks. Using a parallelized simulated annealing algorithm the properties of an optimal charge are derived. Among other things we find that there is not necessarily a correspondence between a bank's contribution to systemic risk - which determines its risk charge - and the capital that is optimally injected into it to make the financial system more resilient to systemic risk. The analysis has policy implications for the design of optimal bank levies.

JEL Classification: G01, G18, G33

Keywords: Systemic Risk, Systemic Risk Charge, Systemic Risk Fund, Macroprudential Supervision, Shapley Value, Financial Network

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1 Introduction

In a manner unexpected only a few years ago, the global financial crisis which started in 2007 has demonstrated that a system of interconnected financial institutions may be subject to a systemic breakdown, with large effects on the real economy. In this paper a numerical model is used to analyze a network of financial institutions subject to capital requirements. The model allows to replicate important stylized facts of systemic risk which emerged during the recent financial crisis. We then introduce the concept of a Systemic Value at Risk (SVaR) which allows to simultaneously determine both, a fair risk charge as well as the optimal macro-prudential capital endowment, for financial institutions in the system. Among other things we find that there is not necessarily a correspondence between a bank's[1] contribution to systemic risk – which determines its risk charge – and the optimal capital injection which would render the financial system more resilient with respect to systemic risk.

As there are many different sources of systemic risk, and also different potential consequences for the real economy, there is not a single definition of systemic risk.[2] An early definition of systemic risk was given in Group of Ten: "Systemic financial risk is the risk that an event will trigger a loss of economic value or confidence in, and attendant increases is uncertainly about, a substantial portion of the financial system that is serious enough to quite probably have significant adverse effects on the real economy. Systemic risk events can be sudden and unexpected, or the likelihood of their occurrence can build up through time in the absence of appropriate policy responses. The adverse real economic effects from systemic problems are generally seen arising from disruptions to the payment system, to credit flows, and from the destruction of asset values."[3] Lo (2009) proposes analyzing a set of risk measures to capture systemic risk in the entire financial system. These risk measures capture the six dimensions 'leverage', 'liquidity', 'correlation', 'concentration', 'sensitivities', and 'connectedness'. The IMF defines systemic risk as "large losses to other financial institutions induced by the failure of a particular institution due to its interconnectedness"[4] and the Financial Stability Board, International Monetary Fund, and Bank for International Settlements describe systemic risk in a report to the G-20 as "a

[1] In the following the terms 'banks' and 'financial institutions' will be used interchangeably.

[2] See paper 2 of International Monetary Fund (2009) for a comprehensive discussion of different definitions of systemic risk.

[3] Group of Ten (2001), p. 126.

[4] Chapter 2 of International Monetary Fund (2010), p. 2.

risk of disruption to financial services that is (i) caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy".[5] Following closely the latter definition, in this paper we define systemic risk as the danger that failures within the financial system will mean that an adequate supply of credit and financial services to the economy is no longer guaranteed, so that negative real effects will follow.

A main driver of the recent financial crisis was the state of the financial system.[6] Large financial institutions tended to be highly leveraged, while their portfolio structures were relatively homogeneous, and returns were highly correlated.[7] There were also close ties to the so-called shadow banking sector, obscuring balance sheets and rendering the financial system fragile. In the course of the crisis numerous institutions had to be bailed out because their insolvency would have put the financial system at risk via triggering a cascade of other financial institutions' defaults. The increase in systemic risk was essentially driven by three factors, the size of the financial institutions, the direct links among these institutions, as well as the indirect, asset market-driven links.

First of all, the default of a financial institution which is relatively large can put the financial system at risk. For example, in line with our definition of systemic risk, one can expect that the insolvency of even a single large bank constitutes a serious threat to the financial system and the real economy of the entire country. Switzerland is a good example, as its two global banks, UBS and Credit Suisse, pose a significant risk for the country's financial system and the wider economy because of their mere size. This is why banks like UBS or CS were called 'too-big-to-fail' in the recent financial crisis.

Second, banks that are highly interlinked with other financial institutions can also threaten the financial system through their network of exposures to other banks, domestically and abroad. If such a bank defaults on its liabilities it can directly induce losses on its creditor banks which on their part might spread the shock further in case they also default. For example, during the recent financial crisis, the insurance company American International group (AIG) was bailed out because of its interlinkages, via CDS (Credit Default

[5]Financial Stability Board, International Monetary Fund, and Bank for International Settlements (2009), p. 2.

[6]For a general overview on the causes and consequences of the recent financial crisis see, *inter alia*, Issing, Asmussen, Krahnen, Regling, Weidmann, and White (2009), Borio (2008), Brunnermeier (2009), and Gorton (2010a).

[7]For an analysis of the role of the shadow banking system in the recent financial crisis see Gorton (2010b) who compares the breakdown of the shadow banking system to historical bank runs.

Swap) counterpart exposures, with several other large financial institutions. A default of AIG would thus have exposed a large part of the financial system to significant expected losses.

Third, indirect connections between financial institutions, too, may render the financial system vulnerable. If banks invest in identical or correlated financial products, their balance sheets become more highly correlated. Furthermore, losses may induce banks to deleverage via the liquidation of assets on the market, eventually resulting in a decline of prices for these assets. Other banks that have invested into the same or into correlated assets will thus also face losses when marking their assets to market. Accordingly, these banks are induced to sell assets on the market which will likely further depress prices, eventually forcing other banks to engage in deleveraging tmemnselves. Ultimately this cascade creates firesales[8] and indirectly transmits shocks across financial institutions with correlated balance sheets. Shocks can thus spread directly and indirectly through the financial system. Institutions that threaten the financial system through a contagious casacade of defaults because of their interconnectedness with the financial system were labelled 'too-interconnected-to-fail' during the recent financial crisis.

Figure 1 gives an outline of how balance sheets of financial institutions are interconnected. Solid lines depict direct interconnections while dashed lines depict indirect interconnections. The direction of the arrows indicates exposure towards another bank. For example, the arrow from the interbank lendings of bank 2 to the interbank borrowings of bank 1 represents counterparty exposure of bank 2 towards bank 1.

On the stylized balance sheet from Figure 1 banks' assets consist of liquid and non-liquid assets as well as interbank lendings. Liquid assets are, for example, cash and cash equivalents. Non-liquid assets are, for example, Collateralized Debt Obligations (CDO) and need to be marked to market if they are held in a bank's trading book. Interbank lendings are, for example, credits given to other financial institutions. Distinguishing between liquid and illiquid assets is important because one of the main drivers of systemic risk during the recent financial crisis consisted of banks which were cut off from liquidity on the interbank markets and thus had to sell illiquid assets, resulting in self-energizing firesales. Banks' liabilities consist of deposits, interbank borrowings, and equity. Below the stylized balance sheets on Figure 1 in dashed lines are conditional assets and liabilities, for example CDS.

To mitigate the risk of future financial meltdowns it has become consensus that, in addition to microprudential supervision, supervisors need

[8]See Gorton and Metrick (2009) and Gorton (2009) for a detailed analysis of how firesales have affected secondary asset markets during the current financial crisis.

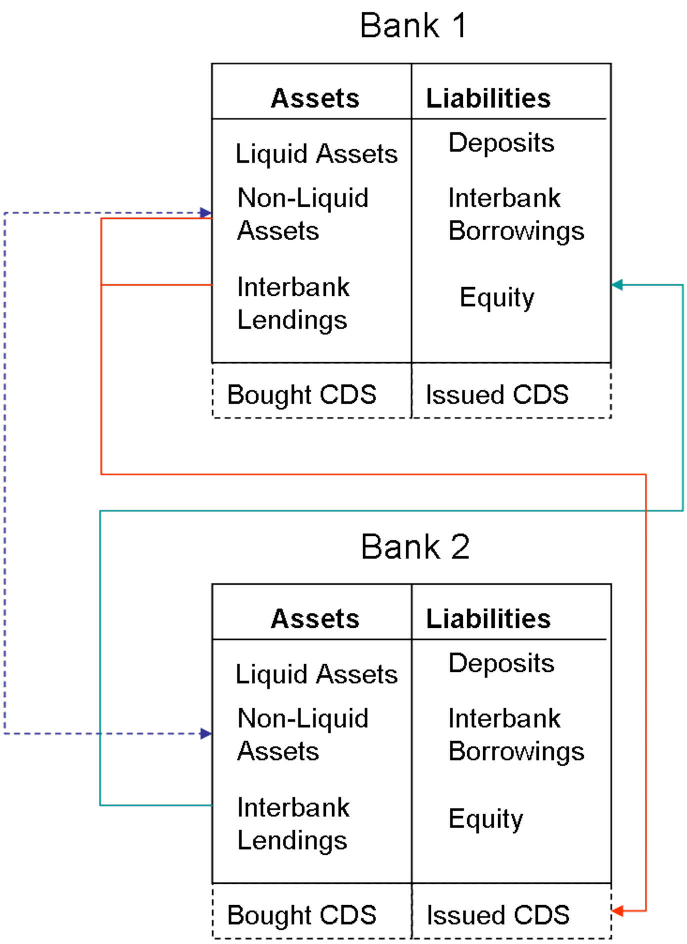

Figure 1: Interconnections Between Financial Institutions

to set up an additional layer of macroprudential regulation and supervision which shall allow to identify system-wide risk drivers, monitor systemic risk, and react adequately to it. Systemic risk is a negative externality of financial institutions on the financial system. Without charging them for this negative externality, financial institutions are perversely incentivized to increase their contribution to systemic risk via becoming too-big-to-fail or too-interconnected-to-fail because it allows them to take advantage from resulting cheap refinancing opportunities.

To analyze systemic risk and banks' contributions to it, we develop a network of interrelated bank balance sheets with endogeneous asset markets. Our model reproduces the main stylized facts with regards to systemic risk that emerged during the recent financial crisis. We then introduce the concept of SVaR in which a Pigouvian tax is used to capitalize a systemic risk fund. The capital from the systemic risk fund is re-injected into the financial system to make it more resilient to systemic risk. The optimal amount of capital for the systemic risk fund as well as the necessary proportions of capital injected into financial institutions are determined with a parallelized simulated annealing approach.

Our analysis provides evidence that there is not neccessarily a correspondence between a bank's contribution to systemic risk – which determines its risk charge – and the capital that is injected into it to make the financial system more resilient to systemic risk. In addition, the analysis provides evidence that a systemic risk fund which is immediately re-injected into the financial system requires less capital than a systemic risk fund which stores the capital in a central depository and is used to bail out banks ex-post.

The remainder of the paper is organized as follows: Section 2 gives an overview on the previous literature. Section 3 outlines our model, and Section 4 shows how it can be used to analyze systemic risk as well as individual institutions' contribution to systemic risk along various parameters. Using the outlined model, Section 5 develops and analyzes a proposed systemic risk charge and fund subject to our SVaR concept within a systemic risk management approach. Section 6 concludes. Further details regarding different model structures analyzed as well as the parallelized simulated annealing algorithm employed for analysis are described in several appendices at the end of the paper.

2 Review of Previous Literature

To get a general overview on systemic risk, Haldane (2009) considers the financial network as a complex and adaptive system and applies several lessons from other disciplines such as ecology, epidemiology, biology, and engineering to gain insights to systemic risk in the financial system. More specifically and regarding the various approaches to assessing systemic risk it is sensible to distinguish between (i) 'market-based' and (ii) 'network-based' approaches.[9] While the former use correlations and default probabilities that can be extracted from market prices of financial instruments, the latter explicitely model linkages between financial institutions, mostly using balance sheet information.

As regards the market-based literature, Lehar (2005) uses standard tools which regulators require banks to use for their internal risk management – however at the level of the entire bank system – and shows that in a sample of international banks over the period from 1988 to 2002 the North American banking system increased its stability while the Japanese banking sector has become more fragile. Bartram, Brown, and Hund (2007) develop three distinct methods to quantify the risk of systemic failures in the global banking system. Using a sample of 334 international banks during 6 financial crises the authors come to the conclusion that the existing institutional framework could be regarded as adequate to handle major macroeconomic events. Bårdsen, Lindquist, and Tsomocos (2006) evaluate the usefulness of macroeconomic models for policy analysis from a financial stability perspective. They find that a suite of models is needed to evaluate risk factors because financial stability depends on a wide range of factors.

To measure systemic risk, more recent research from the market-based literature focuses mainly on detecting systemic risk in groups of financial institutions, in particular using multivariate measures such as tail risk indicators or multivariate distress dependences.[10] For example, Gray and Jobst (2010) find that using equity option information to calculate (joint) tail risk indicators between institutions yields timely information about the extent of systemic risk. Segoviano and Goodhart (2009) compute the multivariate density of a portfolio of banks to capture linear and non-linear distress dependences and apply their methodology to a number of country and regional examples. Among other findings they show that U.S. banks are highly interconnected, and that distress dependence rises in times of crises. Finally, Adrian and Brunnermeier (2009) propose CoVaR, defined as the value at risk

[9] See the background paper of Financial Stability Board, International Monetary Fund, and Bank for International Settlements (2009) for a similar distinction.

[10] See Chapter Three of International Monetary Fund (2009) for a similar subsumption.

of financial institutions conditional on other institutions being in distress to assess systemic risk in the financial system. Using this measure, the authors quantify the extent to which financial key figures such as the leverage ratio and maturity mismatch can predict systemic risk.

As regards the network-based literature, Upper and Worms (2004) use balance sheet information to analyze whether there is the risk of contagion in the German interbank market and find that the failure of a single bank can lead to a loss of up to 15% of the banking system's assets. Cifuentes, Ferrucci, and Shin (2005) integrate a mechanism of marking to market assets in a network model and show that liquidity requirements can serve as an effective means to forestall contagious defaults in the financial system. Elsinger, Lehar, and Summer (2006) use standard tools from risk management in combination with a network model of interbank loans. Applying their methodology to a dataset of all Austrian banks they provide evidence that correlations in banks' asset portfolios are a main source of systemic risk. Mueller (2006) employs a data set of bilateral bank exposures and credit lines in a network model and finds a substantial potential for contagion in the Swiss interbank market. Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne, and Willison (2009) combine a network model of the financial system with funding liquidity risk and incorporate this to a suite of models that allow to model various aspects of systemic risk. The authors provide evidence that large losses at some banks can be exacerbated by liquidity feedbacks and thus can lead to system-wide instability.

Castaglionesi and Navarro (2007) study the endogeneous formation of financial networks and show that an efficient financial network and a decentralized financial network both display a core-periphery structure in which core banks are all connected among themselves and choose to hold a safe asset while periphery banks can eventually be connected to other banks and choose to hold a risky asset. Gai and Kapadia (2010) develop a network framework where asset prices are allowed to interact with balance sheets. The authors find that greater connectivity in financial systems reduces the likelihood of widespread default in case of relatively small shocks, while the impact on the financial system in case of large shocks increases this likelihood. Espinosa-Vega and Solé (2010) show how a cross-border network analysis can be used to efficiently monitor direct and indirect systemic linkages between countries, in particular in the face of different credit and funding shocks. The authors provide evidence that the inclusion of risk transfers can modify the risk profile of entire financial systems.

The recent financial crisis has revealed that individual financial institutions impact differently on systemic risk. There are particularly two reasons why it is important to assess financial institutions' individual contribution to systemic risk. First of all, to prevent the insecurity surrounding potential defaults such as the Lehmann bankruptcy in 2008, a supervisor should be able to assess the impact of individual institutions' defaults on the stability of the financial system. Second, as already outlined in the previous section, individual financial institutions should be charged to incentivize them to internalize the cost of their negative externality on the financial system. Tarashev, Borio, and Tsatsaronis (2009) use the Shapley value methodology to identify the contribution of individual financial institutions to systemic risk. The authors show that none of the drivers of contribution to systemic risk, such as the institution's size or its probability of default, in isolation provide a fully satisfactory proxy for systemic importance. Following the authors, it is thus important to carefully take into consideration the interactions between the various risk factors when analyzing systemic risk and the individual institutions' contribution to it. Gauthier, Lehar, and Souissi (2010) compare alternative mechanisms for allocating the overall risk of a banking system to its member banks. Using a data set of the Canadian banking system the authors find that capital allocations that are optimal with respect to systemic risk can differ by up to 50% from actually observed capital levels. Similarly to Tarashev, Borio, and Tsatsaronis (2009) these allocations are not trivially related to different risk factors.

The following section outlines the network model that will be used for our analysis.

3 Model of an Interrelated Financial Network

The model which is set up in this section captures important features of the financial system and replicates several stylized facts encountered during the recent financial crisis. It consists of (i) a system of three interconnected financial institutions that adjust their portfolio to fulfill a capital requirement and (ii) the Rest of the World (ROW). Banks have deposits, lend to each other, and hold liquid assets (LA) and non-liquid assets (NLA) on their balance sheet. Non-liquid assets are marked to market[11] while liquid assets do not change their value on banks' balance sheets. The financial system is mapped into a matrix of assets and liabilities as displayed on Figure 2.

Assets

Liabilities

	Bank 1	Bank 2	Bank 3	ROW	
				NLA	LA
Bank 1				X	Y
Bank 2	W				
Bank 3					
ROW	Z				

Figure 2: Matrix of the Financial System Model

Figure 2 summarizes bank balance sheets and their interconnections in a matrix from. The second row, for example, displays bank 1's assets, while its liabilities are captured in the second column. Item 'W' in matrix entry 3/3 represents bank 2's interbank lending to bank 1. W is an asset for bank 2, and a liability for bank 1. Item 'X' represents bank 1's holdings of non-liquid claims on the set of the world like, for example, collateralized debt obligations or corporate loans. Similarly, item 'Y' in matrix entry 2/6 refers to bank1's holdings of liquid assets, i.e. cash and trading book assets. Item 'Z', finally, entails bank 1's deposits and outstanding bonds, held by the rest of the world.

[11] Note that there is no distinction between banking and trading book in the model, all non-liquid assets are marked to market in the model.

Banks have to fulfill a minimum capital requirement, γ, which is defined for bank i according to equation 1,

$$\gamma = \frac{\sum_j a_j + p \cdot b_i + c_i - \sum_j l_j - d_i}{\sum_j a_j + p \cdot b_i}, \tag{1}$$

where $i, j \in (1, 2, 3), i \neq j$, are indices for the three banks in the system, b_i are non-liquid assets, c_i are liquid assets, a_j are interbank lendings, l_j are interbank borrowings, p is the market price of the non-liquid asset, and d_i are deposits. Note that the liquid asset does not show up in the denominator of Equation 1 because banks do not have to hold capital for their liquid asset holdings.[12] If a bank's equity ratio is lower than the capital requirement, γ, it tries to net its interbank exposure and, if that is not sufficient to adequately recapitalize, sells non-liquid assets on the market. In both cases the denominator in Equation 1 decreases relative to the numerator. If a bank cannot meet the capital requirement, it defaults.

Equation 2 shows the capital ratio after netting its exposures against other banks by θ units.

$$\gamma^* = \frac{(\sum_j a_j - \theta) + p \cdot b_i + c_i - (\sum_j l_j - \theta) - d_i}{(\sum_j a_j - \theta) + p \cdot b_i} \tag{2}$$

Netting reduces the denominator by θ units while the numerator remains unchanged. Note that in the model, banks may net any cross-exposure as long as their balance sheet equity value remains non-negative, that is $\sum_j a_j + p \cdot b_i + c_i - \sum_j l_j - d_i \geq 0$.[13] The term cross-exposure means that two banks have borrowed from and lent to each other. Note that a bank which has cross-exposure with another bank can have net-exposure with the same bank. Here and in the following net-exposure is defined as one bank having lent more to another bank than borrowed from the same bank.

Solving Equation 2 for the amount of bank i's desired netting yields Equation 3

$$\theta_i^d = -\mathbf{1}_{[nv_i \geq 0]} \frac{(1-\gamma)(\sum_j a_j + p \cdot b_i + c_i - \sum_j l_j - d_i)}{\gamma}, \tag{3}$$

where $\mathbf{1}$ is an indicator function and nv_i is bank i's net-value defined as $\sum_j a_j + p \cdot b_i + c_i - \sum_j l_j - d_i$. The amount of netting the j'th bank is willing to accept with bank i is given by Equation 4

$$\theta_j^s = \mathbf{1}_{[nv_j \geq 0]} min(a_i, l_i). \tag{4}$$

[12] See Cifuentes, Ferrucci, and Shin (2005) for a similar set up.

[13] If a bank's liabilities exceed its assets, it is taken into custody by the supervisor to protect creditors. In this case no netting is possible.

Note that the minimum operator is used since only cross-exposures can be netted. The resulting amount netted between bank i and bank j is given by Equation 5

$$\theta_{ji} = min(\theta_j^s, \theta_i^d). \tag{5}$$

Note that in the model banks never increase their lending to each other.

Furthermore, in order to meet the minimum capital requirement, the bank may engage in asset sales. Equation 6 shows the capital ratio bank i expects to obtain if it engages in selling s_i units of its non-liquid assets.

$$\gamma^* = \frac{\sum_j a_j + p(b_i - s_i) + c_i + p \cdot s_i - \sum_j l_j - d_i}{\sum_j a_j + p(b_i - s_i)} \tag{6}$$

Consider the indirect effects of the above responses to violations of Equation 1. Netting by bank i increases γ for banks i and j, where bank j is holding the cross exposure. Asset sales by bank i, in contrast, have further repercussions on all banks with positive exposure[14] in that very asset, because asset sales have an impact on its secondary market price. In the model it is assumed that market prices of non-liquid assets, p, are a function of supply and demand on the market. If banks engage in liquidating (part of) their non-liquid assets, several effects on banks' balance sheets have to be considered: the seller obtains cash, a liquid asset, and hence improves her capital ratio. However, at the same time an increased supply of non-liquid assets to the market decreases the market price of the asset, lowering the market value of the bank's remaining portfolio holdings of the same asset. Furthermore, the price effect also influences other banks' balance sheets since the market value of their non-liquid assets is reduced as well.

In the model, the market price of the non-liquid asset is found via a tâtonnement process between supply and demand. Following Cifuentes, Ferrucci, and Shin (2005), the inverse demand function is assumed to follow Equation 7

$$p = exp(-\xi \sum_i s_i), \tag{7}$$

where ξ is a positive constant to scale the price responsiveness with respect to non-liquid assets sold, and s_i is the amount of bank i's non-liquid assets sold in the market.

Solving Equation 6 for the amount of non-liquid assets sold by bank i to fulfill the capital requirement, and noting that a bank can only sell non-liquid

[14] We restrict b_i to be non-negative, assuming that bank asset holdings refer to cash flow streams outside the financial sector. Put differently, bonds issued by banks are included in l_j.

assets it holds in positive quantities[15], leads to Equation 8. It shows bank i's supply of non-liquid assets on the market as a function of the market price.

$$s_i = min\left(b_i, \frac{-(1-\gamma)(p \cdot b_i + \sum a_i) - c_i + \sum l_i + d_i}{\gamma p}\right) \tag{8}$$

Since each s_i is decreasing in p, the aggregate sales function, $S(p)$, is also decreasing in p. The tâtonnement-process leading to the equilibrium market price is depicted in Figure 3.

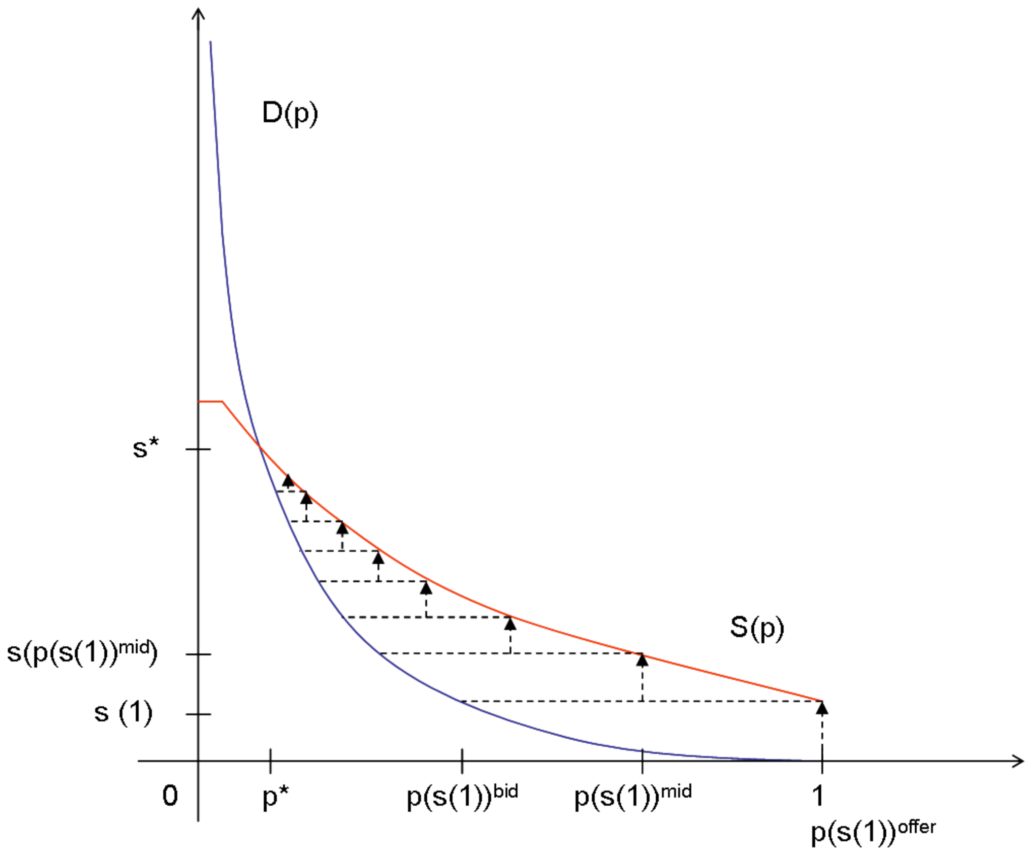

Figure 3: Tâtonnement Process in the Model

Prior to any shock, the market price equals 1, which is the initial price when all banks fulfill their respective capital requirements, and sales of the non-liquid asset are zero. A shock to bank i, say a certain loss of cash, c_i, shifts the supply curve upwards, resulting in $S(1) = s_i \succ 0$ because

[15]Note that banks do not engage in buying or short-selling non-liquid assets in the model.

bank i starts selling non-liquid assets to fulfill its capital ratio. However, for $S(1)$ the bid price equals only $p(S(1))^{bid}$, while the offer price is one. The resulting market price is $p(S(1))^{mid}$, the midprice between bid and offer prices. Since the market price thus decreases and banks have to mark their non-liquid assets to market, additional non-liquid asset sales may be needed to fulfill the capital requirement. The stepwise adjustment process continues the demand and the supply curves intersect at p^*. Note that the supply curve may become horizontal from some value of non-liquid assets sold onwards, as the total amount of non-liquid assets on the banks' balance sheets is limited. Since a shock to a bank will always result in an upward shift of the supply curve, and the maximum price of the non-liquid asset being equal to 1, while the initial equilibrium prior to the shock equals zero, a market price $p \in (0, 1)$ always exists.

In the framework just outlined, there are two main shock transmission channels, the direct connection between banks via interbank holdings (credit risk), and indirect connections via marking to market of non-liquid assets on the balance sheet (market risk).

The following sub-section explains how different configurations of a financial network can be captured in the model.

3.1 Generating Specific Realizations of the Financial System Matrix

Any specific set up of a financial system is described by a consistent matrix, that is, when all banks fulfill their capital requirement ratio, with concrete values for all assets and liabilities, as depicted in figure 2. Accordingly, a setup is defined by (i) the structure of the system, that is, the network of exposures and cross-exposures among banks and the rest of the world; (ii) the banks' individual ratio of interbank lending to other assets (that is, her non-liquid and liquid asset holdings), α, with α the overall amount lent to other banks, and $1 - \alpha$ the amount invested in other assets; (iii) the ratio of investment in non-liquid to liquid assets, β, where β is the fraction invested in non-liquid assets and $1 - \beta$ is the fraction invested in liquid assets; (iv) the capital requirement, γ; and (v) an initial endowment of capital, A, that is allocated to banks' assets according to α and β. Note that $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$.

To determine all except the last row of the financial system matrix in Figure 2, the structure of interlinkages, i.e. the net of exposures has to be defined, and concrete values for α, β, A, and γ have to be assigned. In the model, it is assumed that banks invest all their borrowed funds into liquid and non-liquid assets. The overall amounts bank i holds in non-liquid and liquid assets are $((1-\alpha)\cdot A + \sum_j l_j)\beta$ and $((1-\alpha)\cdot A + \sum_j l_j)(1-\beta)$, respectively. The entry for the i'th bank in the last row of the financial system matrix, that is, its deposits, is residual in the sense that the capital requirement is just met, using Equation 9

$$\begin{aligned} d_i =& A\cdot\alpha + \left((1-\alpha)\cdot A + \sum_j l_j\right)[\beta p + 1 - \beta] - \sum_j l_j \\ & -\gamma\left[A\cdot\alpha + (1-\alpha)A\cdot\beta\cdot p + \sum_j l_j\cdot\beta\cdot p\right]. \end{aligned} \tag{9}$$

As an example, Figure 4 illustrates the symmetric case. All banks have identical initial capital, A, they borrow from and lend to each other, and they have identical portfolio allocations, α and β.

	Bank 1	Bank 2	Bank 3	ROW	
				NLA	LA
Bank 1		Aα/2	Aα/2	Aβp	A(1-β)
Bank 2	Aα/2		Aα/2	Aβp	A(1-β)
Bank 3	Aα/2	Aα/2		Aβp	A(1-β)
ROW	d	d	d		

Figure 4: Symmetric Case of the Financial System Matrix

In the example on Figure 4 each bank's balance sheet then looks as displayed on Table 1.

Note that with different interlinkage structures the relative size of banks vis-à-vis each other, measured by the sum of their assets, changes because banks choose their leverage independently, and are free to select their desired balance sheets.

The next sub-section outlines how shocks to the financial system matrix are modeled.

Assets	Liabilities
LA: $A(1-\beta)$ NLA: $A\beta p$ Interbank lendings: $A\alpha$	Deposits: $A(\beta(p-1)-\gamma(\alpha+\beta p)+1)$ Interbank borrowings: $A\alpha$ Equity: $A(\gamma(\alpha+\beta p))$
$\sum = A(\alpha+\beta(p-1)+1)$	$\sum = A(\alpha+\beta(p-1)+1)$

Table 1: Banks' Balance Sheets in the Symmetric Case

3.2 Shocks in the Financial System Matrix and the Measure for Systemic Risk

As explained in the beginning, systemic risk is defined as the hazard of bank failures causing a decrease in the supply of credit and financial services to the economy which, resulting in negative real effects. Accordingly, we define systemic risk conditional on a shock as the relative size of the financial system that breaks down. It is measured by the banks' balance sheet size, that is, the sum of their assets. Intuitively, when banks default, the resulting liquidation costs as well as the the banks' overall importance to the real economy will be closely related to the size of its balance sheet.

Shocks in the model come in the form of percentage loss in asset values. The resulting systemic risk is calculated as the ratio of assets from defaulting banks to system-wide asset total, both measured prior to the shock. For example, if subsequent to a shock only bank 1 defaults, while all other banks in the financial system remain solvent, then systemic risk is $\frac{\text{Sum of Bank 1's Assets Prior to the Shock}}{\text{Sum of all Banks' Assets Prior to the Shock}}$.

A wide range of possible shock events, from mild to severe, are considered in the simulations. Strongly adverse scenarios with high unexpected losses will be included among these scenarios, as such shocks are likely candidates to trigger systemic risk events, involving defaults of parts of the financial system. The expected systemic risk in a particular point in time is calculated as the weighted sum of systemic risk events caused by a distribution of shock realizations. The weights are derived from the probability distribution of shock realizations. Equation (10) defines this measure of expected systemic risk.

$$\Phi^E = \sum_j \frac{\text{Sum of Insolvent Bank's Assets Prior to Shock}_j}{\text{Sum of all Banks' Assets Prior to Shock}_j} \cdot prob_j \qquad (10)$$

where Φ^E is expected systemic risk and $prob_j$ is the probability assigned to shock scenario j. Defining systemic risk this way, i.e. assuming a distribution of shock realizations for each bank, allows to identify the contribution of individual banks to overall systemic risk. At the same time, it allows to handle the three main risk-channels of interbank risk in a unified framework, namely bank size, bank interconnectedness, and bank asset fire sales. As it turns out, given the parametrization chosen in our model, the fire sale channel is particularly sensitive to systemic risk, since even small shocks have a significant effect on financial system default rates. The interlinkage channel, on the other hand, requires a relatively large shock to generate a system-wide default. Our modeling strategy allows to adopt the Value at Risk (VaR) metros,[16] the standard risk management tool used in microprudential supervision, for a macroprudential problem. The resulting metric, the System Value-at-Risk (SVaR), is effectively a set of stress tests for an entire banking system. This metric, expected systemic risk, will be used subsequently to analyze systemic risk in the financial system.

Each possible shock to the banking system is modeled as a vector of percentage losses to a bank's (non-weighted) sum of assets over a discrete grid, ι, ranging from 1% to ς%, with ς being the highest conceivable shock. Considering all combinations of shocks for the three banks yields a total number of ι^3 shock vectors. Each shock vector consists of n elements, i.e. the loss associated with the shock for each institution our model with n banks. In this paper,n=3. The probability of a shock realization is captured by a multivariate normal distribution centered at a value between 1 and ς. The extent of correlation between the shocks is modeled with the variance-covariance matrix of the multivariate normal density function. The correlation between shocks in a given scenario, say a shock to banks 1 and 2 in scenario 1, is then calculated as $\frac{cov_{1,2}}{\sigma_1\sigma_2}$, where $cov_{1,2}$ designates the covariance between shocks 1 and 2 and σ_1 and σ_2 are the standard deviations of shocks to banks 1 and 2, respectively.[17] Since shocks only range from 1 to ς, the multivariate normal density is rescaled such that the integral of the volume described by the discrete grid of shocks, ranging from 1 to ς in all three dimensions equals 1.

As previously outlined, if subsequent to a shock realization, the bank cannot fulfill its capital requirement, it will net its counterparty exposures first. Next, if netting is not enough to meet the capital constraint, the bank will sell non-liquid assets, thereby indirectly transmitting the shock to other banks, via a downward pressure on the market prices of non-liquid assets. If it still cannot fulfill the capital requirement,the bank will go into defaults. In default, seniority of deposits over other liabilities is respected.

The clearing algorithm for shock transmission is an iterative process in

[16] See Jorion (2006) for an outline of the VaR methodology.

[17] Apart from the firesale channel for non-liquid assets, the correlation between direct shocks to different banks captures an additional element of common bank exposure within the financial system.

which banks sequentially absorb the shock. Banks initially try to fulfill their capital requirement via netting counterparty exposures, and, after that stage, via selling non-liquid assets into the market. Banks with negative net-value, i.e. negative equity then transmit a shock to their creditors, and the iterative process restarts. The process ends when shocks to solvent banks are fully absorbed. Figure 5 depicts the procedure of modeling the shock transmission.

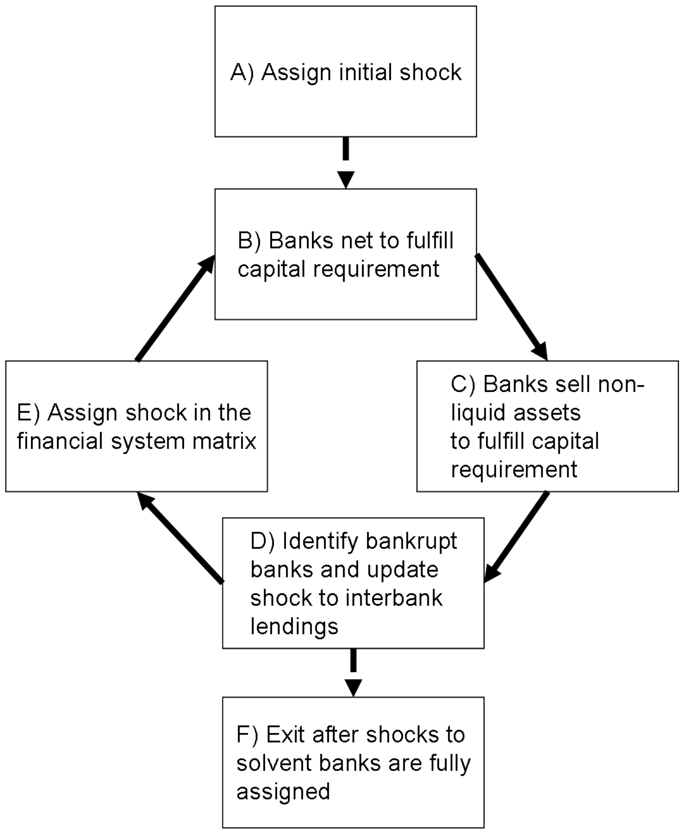

Figure 5: Shock Transmission in the Financial System Model

Banks' assets are contracted by the initial shock (step A on Figure 5). Banks that do not fulfill the capital requirement first try to improve their capital ratio through netting interbank liabilities with other banks, since netting has no negative repercussions on the balance sheet (step B on Figure 5). Next, banks that still do not fulfill the capital requirement start selling non-liquid assets in the market (step C on Figure 5).

Banks that are not able to fulfill the capital requirement even after selling all their non-liquid assets, will enter into default. If insolvent banks have negative net-value they will transmit shocks to their creditors, that is, banks that have exposure with them, or ultimately to the bank's depositors. A bank with negative net-value transmits shocks to its creditors, respecting seniority, until it has a net-value of zero. The overall shock prepared for transmission to the insolvent banks' creditors equals the absolute value of their negative net-value and is assigned proportionally to a bankrupt's bank interbank liabilities as long as they are positive (step D on Figure 5).

In case the interbank liability shock matrix contains nonzero entries it is assigned (step E on Figure 5), and the iteration restarts (step A on Figure 5). If the interbank liability shock matrix is empty the shock has been assigned, and the resulting systemic risk is computed (step F on Figure 5).

The following sub-section outlines how the model can be used to analyze individual financial institutions' contribution to expected systemic risk.

3.3 Analyzing Banks' Contribution to Expected Systemic Risk

To identify the contribution of an individual bank to expected systemic risk, the Shapley value methodology can be employed.[18] In game theory this value is used to find the fair allocation of gains obtained by cooperation among players. For a game consisting of three players the Shapley value is defined as

$$\phi_i(v) = \sum_{K \ni i; K \subset N} \frac{(k-1)!(n-k)!}{n!} \left[v(K) - v(K - \{i\}) \right], \tag{11}$$

where k is the number of players in coalition K, N is the set of all players, v(K) is the value obtained by coalition K including player i and $v(K - \{i\})$ is the value of coalition K without player i. The Shapley value for player i is the average contribution to the gain of the coalition over all permutations in which players can form a coalition.

The analogy between gains allocation in game theory and systemic risk contribution in financial economics is evident, as individual banks through their portolio structures and their interconnections to other banks and to the rest of the world may increase or decrease the likelihood of a given financial

[18]See Shapley (1953). Tarashev, Borio, and Tsatsaronis (2009) also rely on the Shapley value to compute individual financial institutions' contribution to systemic risk. Note that in general also other measures for financial institutions' contribution to systemic risk could be employed, for example the CoVaR methodology developed by Adrian and Brunnermeier (2009). However, for a simulation based approach to systemic bank risk, the Shapley value methodology is suited particularly well, as different patterns of interbank dependencies, i.e. via portfolio structures and via interbank lending and borrowing, can be accounted for. The CoVaR methodology, in comparison, relies on reduced form representation instead.

system experience multiple bank defaults. In this sense, a bank's contribution to overall system risk can a priori be positive or negative. Furthermore, the marginal effect of a bank on overall systemic risk cannot be estimated from bank-individual data alone; the interplay with other banks' balance sheets and their portfolio compositions is needed to assess the bank's impact on system stability.

The Shapley value has a number of well-known properties:

- Pareto efficiency: The total gain of a coalition is distributed;
- Symmetry: Players with equivalent marginal contributions obtain the same Shapley value;
- Additivity: If one coalition can be split into two sub-coalitions then the pay-off of each player in the composite game is equal to the sum of the sub-coalition games;
- Zero player: A player that has no marginal contribution to any coalition has a Shapley value of zero.

Of course, expected systemic risk is a cost to the financial network. Therefore, the Shapley value can be employed to to compute the marginal contribution of any single bank to the overall cost of systemic risk.

From the financial network matrix, the contribution of each single bank to systemic risk is determined in Equation 11, given a shock of a particular magnitude. As outlined before, systemic risk conditional on the realization of a shock is defined as the proportion of the assets of all banks that enter default because of a system wide asset shock, where pre-shock asset values are used to define the proportions. $v(K)$ is the coalition K of 'all banks that can default and transmit shocks' and hence contribute to the measure for expected systemic risk, and $v(K - \{i\})$ is the coalition K without the i'th bank. Intuitively, the latter can be imagined as the situation in which bank i cannot default and thus not transmit shocks to the financial system. In the model this is done via temporarily adding a large amount of liquid assets to a bank that shall not transmit shocks. Such a 'safe' bank does not try to net counterparty exposure[19] or sell non-liquid assets on the markets because it always fulfills the capital requirement. Following this approach, one calculates for each permutation of banks the systemic risk if only the first bank in the order can default, next the marginal contribution to systemic risk if the following bank can also default, and finally the marginal contribution to

[19] Though it accepts netting requests from other banks.

systemic risk if all three banks in the actual order can default. The Shapley value for a bank is then the average of its marginal contributions over all possible permutations. Since systemic risk is defined as a proportion here, its value and the Shapley values are restricted to lie between 0 and 1.

Similar to calculating expected systemic risk as a weighted sum of systemic risk from a set of scenarios, Equation (12) outlines bank i's contribution to expected systemic risk from a weighted sum of its Shapley values.

$$\phi_i^E = \sum_j \phi_{ij} \cdot prob_j \tag{12}$$

where ϕ_{ij} is bank i's contribution to systemic risk in scenario j and $prob_j$ is the probability that scenario j realizes. Note that $\Phi^E = \sum_i^3 \phi_i^E$.

Using the model just developed, the nest section analyzes the main determinants of systemic risk.

4 Applying the model: Systemic risk and its determinants

In the model banks' contribution to expected systemic risk is driven in particular by three characteristics, namely a bank's size within the financial system, as well as the extent of direct and indirect links among the banks.[20] First of all, the size of an individual bank matters for its contribution to expected systemic risk because our measure of systemic risk, total assets of defaulting banks relative to system-wide assets, both measured prior to the shock realization, increases with the size of the 'shocked' bank's balance sheets. Second, banks that have borrowed from other banks are likely to contribute more to expected systemic risk than banks inactive in interbank borrowing. Furthermore, a defaulting bank with outstanding interbank liabilities transmits a shock to its creditor banks. Third, with significant amounts of non-liquid assets on banks' balance sheets, the financial system becomes vulnerable to fire sales. Non-liquid assets on a bank's balance sheet creates vulnerabilities with respect to movements of asset prices. Furthermore, subsequent to a loss, banks may be forced to sell non-liquid assets, thereby furthering the downward spiral of asset prices and transmitting the shock to other banks in the financial system. The following analyses will consider these three main risk-channels in turn.

Expected systemic risk will first be explored in a baseline specification of the model. Subsequent analyses will then investigate the impact of the above risk-channels. To shed some light on the role of banks' capitalization and its role as a shock buffer, the effect of different capital requirement ratios on expected systemic risk will also be investigated.

In the baseline specification parameters are set such that banks' resulting balance sheets roughly corresponds to the proportions actually found in a real-word financial system. Concerning the relative importance of interbank lending, Upper and Worms (2004) in their study on the German interbank market report an average level of interbank lending of 2.96 times the amount of their own capital. Scaling the parameter α to 0.3 in our model generates approximately the aforementioned relative amount of interbank lending, assuming the bank engages in such lending at all. Furthermore, the proportion of non-liquid assets to cash and cash equivalents at an international universal bank, as for example Deutsche Bank in 2009 was roughly 0.8.[21] In the model, we set β to 0.8, roughly mimicking the proportion found at an interna-

[20] Throughout the remainder of the paper we will refer to expected systemic risk caused by direct and indirect interconnections as 'interlinkage' and 'firesale' channels, respectively.

[21] See Deutsche Bank AG (2010).

tional bank. As regards banks' capitalization, following the Basel Commitee on Banking Supervision (2006), the capital requirement ratio, parameter γ, is set to 8% of risk weighted assets, where we assume risk weights to be uniformly one for non-liquid assets, and zero for liquid assets. The scaling parameter for the price responsiveness of non-liquid assets, parameter ξ, is set to 0.03 which results in a price decrease of approximately 7% of its market price if banks sell all their non-liquid assets on the market. Banks in the system are initially equipped with one unit of capital, parameter A. Since systemic risk is measured as a proportion throughout the following exercises, A is effectively a scaling parameter. It affects results only if banks were to obtain different amounts of initial capital because only then banks' relative sizes will be affected.

To repeat, shocks that affect individual banks are modeled as a loss of a bank's assets ranging from 1% to 9% of its balance sheet sum in discrete steps of 2%. Note that a shock always manifests itself via a loss in liquid asset value.[22] The multivariate normal shock distribution which determines the shock scenario realizations is centered at a loss of 6% of banks' assets. The main diagonal of the variance-covariance matrix is set to 3, and the covariances are set to yield a pairwise correlation coefficient of $\frac{1}{6}$ between shocks to all banks.[23]

Note that the distribution of shock scenarios will likely influence the outcomes of the following analyses. For example, choosing the parameters of the distribution such that small shocks receive a relatively high likelihood will generally reduce the expected risk contribution of the interlinkage channel. This property of the mechanism is due to the fact that banks only transmit shocks via the interlinkage channel if a shock is large enough to reduce the sum of a bank's assets below the sum of its liabilities, that is, its equity is exhausted. Similarly, if very large shocks have a high probability, the size channel dominates the outcome as regards banks' contribution to expected systemic risk. In the extreme case when all banks lose all equity from an initial shock and cannot recapitalize, the whole banking system defaults. In

[22]A direct loss assigned to non-liquid assets might affect the firesales channel in the model. A larger shock to an institution's non-liquid assets can theoretically cause lower risk in the financial system through a reduced volume of firesales. In the extreme case of a bank losing all its non-liquid assets subsequent to a shock, its potential to transmit the shock via the firesales channel has vanished.

[23]Concerning mean and variance of the shock distribution, there is little empirical guidance as to how these parameters can be chosen. Moody's Investor Service (2005) estimates the asset correlations for major structural finance sectors to range between 2% and 18%. Given that the recent financial crisis has demonstrated that correlations in the financial sector can be even higher than was previously assumed, a value slightly above the upper range of the interval has been chosen.

this case, there is no room for contagion via firesales, or interlinkages. In this respect, the variance and covariance of shocks matter as well. For example, to identify banks which contribute to expected systemic risk via the interlinkage channel it is necessary to model shock scenarios in which creditor banks are subject to a relatively small shock. 'Small' implies it does not cause the bank to default, even if, at the same time, its debtor banks are subject to a relatively large shock which makes them default on their liabilities. thus ultimately imposing some default risk on creditor banks. The distribution parameters thus influence expected systemic risk directly as well as indirectly, through banks' systemic risk contributions, via different channels.

Our choice of parameters governing the distribution of shock scenarios has been taken mainly with a view on generating shock scenarios which, on the one hand, allow for the emergence of systemic risk through all risk-channels, and, on the other hand, to identify through which of the channels banks primarily contribute to expected systemic risk. It is important to note that while the analyses are in some cases affected by distributional assumptions and interactions between the risk-channels themselves, the *insights* obtained from the *outcomes* of the experiments are qualitatively robust to changes in these underlying parameters because they are always corroborated with a view on the model's underlying mechanics. Furthermore, in case the distributional assumptions particularly matter, we will discuss the robustness of the results in question.

Given that a bank can engage in borrowing and lending to and from other banks simultaneously, there exist 2^6 possible banking structures, i.e. patterns of interbank exposures. Appendix A, at the end of the paper,describes all possible structures of the financial network matrix.

In the next section, the properties of a model of interconnected banking with endogenous asset markets is explored using a baseline specification. The role of different channels of risk contagion in the emergence of systemic risk is analyzed. The discussion will be in terms of bank 1's contribution to systemic risk. This is without loss of generality since the interlinkage structures as seen from banks 2 and 3 are symmetric, and it therefore suffices to report results from the view of one bank only. For example, as can be seen in Appendix A, structure 19 from the perspective of bank 1 is the same as structure 25 from the perspective of bank 3.

Finally note that all following analyses will frequently refer to specific structures of the financial system as well as to banks' size, counterparty exposure, and amount of non-liquid asset investment. Besides the general structural overview given in Appendix A, a presentation of specific banking network structures as well as their size distribution can be found in Appendix B, along with banks' relative size.

The following sub-section analyzes expected systemic risk in the baseline specification.

4.1 Expected Systemic Risk in the Baseline Specification

Figure 6 displays expected systemic risk in the baseline specification of the model. The upper panel shows the contribution of bank 1 to expected systemic risk (y-axis). The possible interlinkage structures outlined in Appendix A have been ordered from lowest to highest contribution to expected systemic risk (x-axis).

In the baseline model, it is interlinkage structure 31 in which bank 1 contributes least to expected systemic risk (Table 13 in Appendix B). Looking at the three main risk-channels, i.e. size, interlinkages, and firesales, suggests why this is the case. First of all, in this network structure, bank 1 is relatively small, holding merely 28% of total assets in the financial system. Second, it has no direct connections to other banks. This prevents it from being involved in shock transmissions via interbank lending. Third, in this network structure, bank 1 holds the same amount of non-liquid assets as the other two banks and thus is not particularly involved in the firesales channel either. At the other end of the spectrum are network structure 12 and 64, in which Bank 1 is the major contributor to expected systemic risk (Tables 7 and 16, respectively). Here, bank 1 holds 36% of financial system total assets. It thus contributes more prominently to expected systemic risk via the size channel. Furthermore, due to its interlinkages with other banks in both network structures, it can directly issue or indirectly transmit a shock to its creditor banks. Finally, in this network structure, bank1 has a major amount of non-liquid assets on its balance sheet, rendering participation in firesales more likely.

As outlined at the beginning of this section, expected systemic risk and bank 1's contribution to it may depend on the distributional assumptions of the shock scenarios. Note, for example, that in structure 16 (Table 8), though bank 1 is the largest bank in the financial system (44%), two banks have net-exposure to it, and it has the largest holdings of non-liquid assets, it contributes slightly less to expected systemic risk than in structures 12 or 64. This result is due to the fact that shocks large enough to throw several banks into default, as in network structure 16, are at the extreme end of the shock distribution and thus receive relatively little weight in the calculation of expected systemic risk (Equation (10)).In this particular scenario, Similarly, bank 1's contribution to overall systemic risk is limited (Equation (12)).

In contrast, in network structures 12 or 64, an eventual loss from bank 1 is transferred forcefully to its single creditor, rendering a siginficant shock transfer more likely, particularly for relatively smaller shocks with a higher probability weight in the shock distribution.

The lower panel in Figure 6 displays expected systemic risk (y-axis) in the financial system over the different possible interlinkage structures (x-axis). The structures have been ordered by expected value of systemic risk. In the baseline specification expected systemic risk is lowest in interlinkage structure 32 (Table 14), where banks are not connected at all by interbank lending, and are otherwise equal with respect to size, and non-liquid asset holdings. Expected systemic risk peaks when network structures display unidirectional links, as for example in structures 10 and 61 (Tables 6 and 15, respectively). Note that in these structures, the arrows are 'pointing' into the same direction, that is, from bank 1 via bank 2 to bank 3, and from bank 3 back to bank 1, or vice versa, such that each bank can send shocks via interbank linkages to all other banks in the financial system. In these network structures with maximal risk, banks are akin with respect to size and non-liquid asset holdings.

We now turn to the main risk-channels for the emergence of expected systemic risk. The systemic risk contribution of individual banks will be analyzed in the next sub-section. To isolate the effect of any particular channel, we will modify the simulations such that other channels are temporarily (partially) shut down. Note that the size variations in our model do hinge upon initial amounts of capital, A, as well as upon bank interconnectedness, since bank borrowings increase the scale of their operations.

The next sub-section analyzes the effect of firesales on expected systemic risk.

4.2 The Effect of Firesales on Expected Systemic Risk

The effect of the 'firesales' channel on expected systemic risk can be analyzed if the 'interlinkage' channel is shut down and all banks start with the same amount of initial assets. This can be done using structure 32 (Table 14), where all banks have the same size with respect to the financial system and do not lend to each other. The price responsiveness of the non-liquid asset, parameter ξ, is increased from 0 to 0.05. If all non-liquid assets are sold on the market, the percentage loss of the price of the non-liquid asset then ranges from 0% to 11%, respectively. Figure 7 displays the effect of an increase in the price responsiveness of the non-liquid asset (x-axis) on expected systemic risk (y-axis) on the lower panel and bank 1's contribution to it (y-axis) on the upper panel, both in structure 32. Not surprisingly, the impact of the firesale

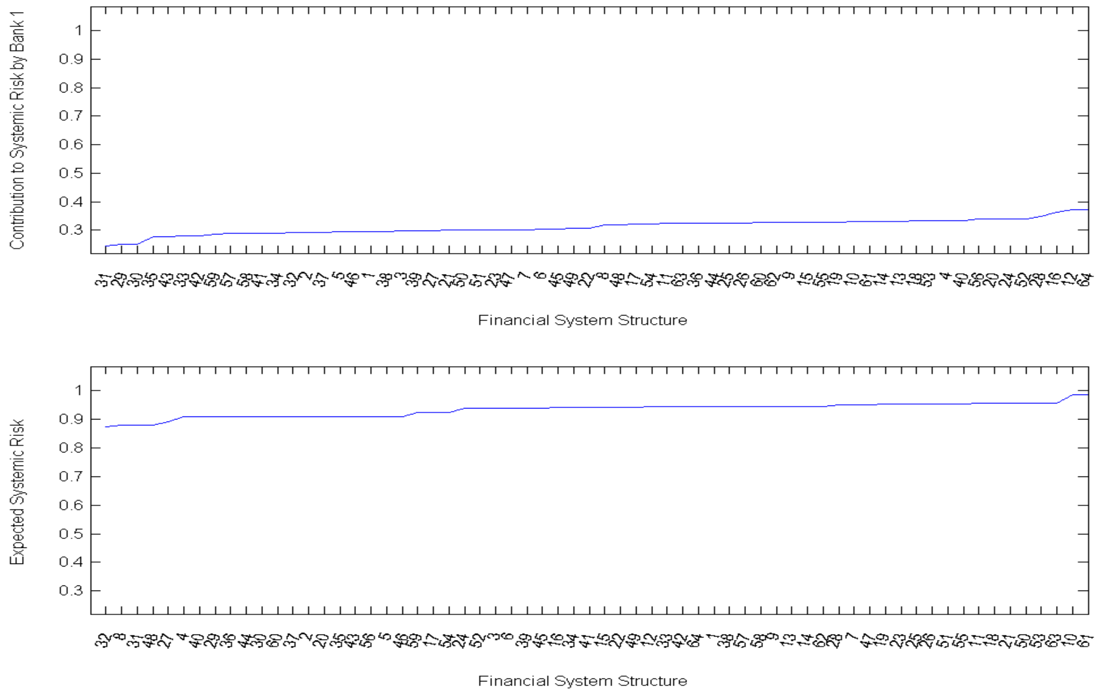

Figure 6: Expected Systemic Risk in the Financial System Model's Baseline Specification

channel depends upon the price sensitivity of secondary asset markets to an increase of sales. High price sensitivities translate into increased expected systemic, while bank 1's contribution rises as well. For parameter value of 0.05 and above, even small shocks to asset value may translate into the default of the entire financial system.Because relatively small amounts of non-liquid assets sold in order to recapitalize the balance sheet may lead to significant price effects, triggering a firesale spiral.

Note that the functions displayed on Figure 7 do not follow a smooth pattern because of the coarseness of the grid of shocks, featureing a stepsize of 2% over a range of losses. For example, assume that given price responsiveness, a bank losing 5% of its assets is not able to recapitalize successfully, and thus is forced to sell all its non-liquid assets on the market before defaulting. If the price responsiveness is then *ceteris paribus* slightly increased, this bank would start liquidating its assets earlier, say at a loss rate of 4% before defaulting. However, since the next smaller shock considered is 3%, the price responsiveness needs to be raised sizeably to increase expected systemic risk and banks' contribution to it over some ranges of the grid. The upshot is

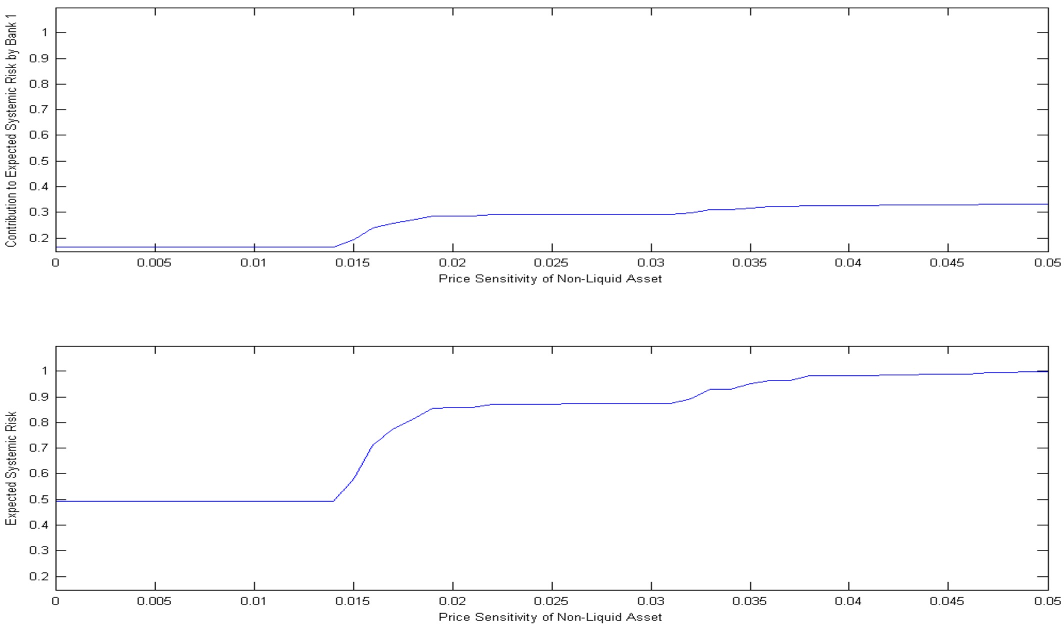

Figure 7: Effect of Firesales on Expected Systemic Risk in Financial System Structure 32

that over some regions of the parameter space of ξ, a significant increase in price elasticity is required to cause an offsetting increase in expected systemic risk.

The simulation results presented in this section suggest the importance to understand the price elasticity of non-liquid assets in order to estimate expected systemic risk properly. The same holds true for a bank's contribution to systemic risk.

The next section turns to the role of interbank lending in the emergence of systemic risk.

4.3 The Effect of Interlinkages on Expected Systemic Risk

As a first inspection of the effect of interlinkages on expected systemic risk, Figure 8 displays a boxplot of expected systemic risk (lower panel) as well as bank 1's contribution to it (upper panel), for different number of interbank links, according to the 64 possible financial network structures in the model. Note that two banks are considered as being connected if there is a single link between them. To focus on the pure effect of interbank connectedness, we have to abstract from other risk determinants, like asset firesales and bank size. Therefore, the parameter of price responsiveness has been set to zero, and initial assets of bank j is equal for all banks. Results are presented in a

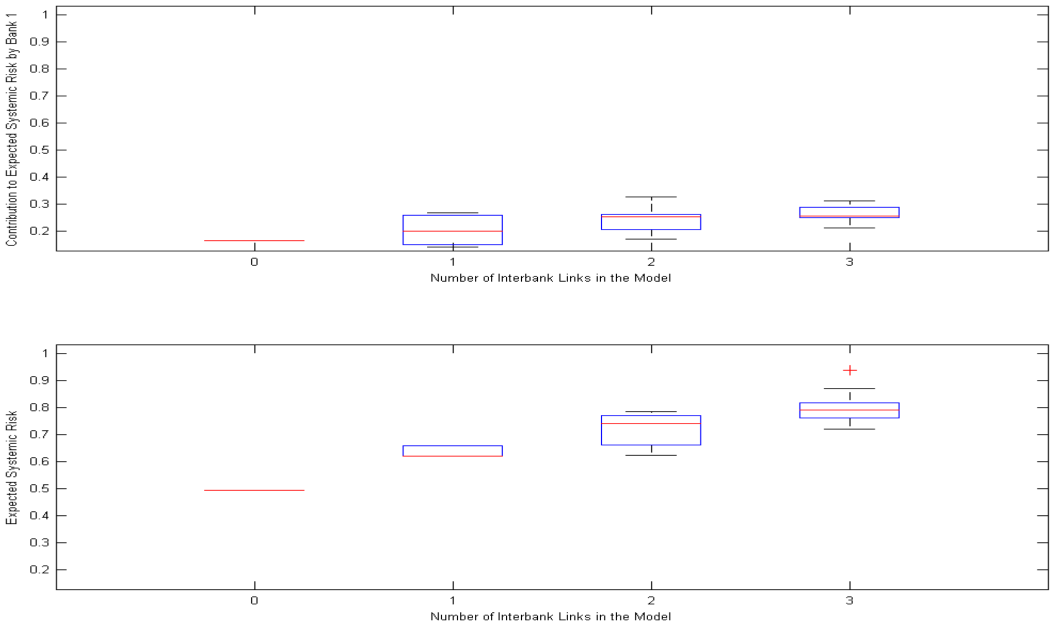

Figure 8: Effect of Number of Interlinks on Expected Systemic Risk

box plot diagram. When investigating the medians (red lines), the plots suggest that expected systemic risk, as well as bank 1's contribution to it, tend to increase with the number of active links across banks. However, focusing on the upper and lower quartiles (designated by the blue boxes), the whiskers which extend to the extreme data points (black lines), and to outliers (red plus symbol), demonstrate that there is no monotonous relationship between number of interbank links and expected systemic risk, or contribution to it.

In the network literature this property is sometimes labeled 'robust, yet fragile', meaning that a growing number of interbank linages will render the network more robust vis-à-vis small shocks, and at the same time more vulnerable to large shocks. Since in this case the shock vectors are the same, the 'robust yet fragile' property follows from a specific network structure, namely cross-exposures between two banks, akin to a mutual insurance.

The box-plots in Figure 8 suggest that expected systemic risk as well as a bank's contribution to it increase with the number and the intensity of

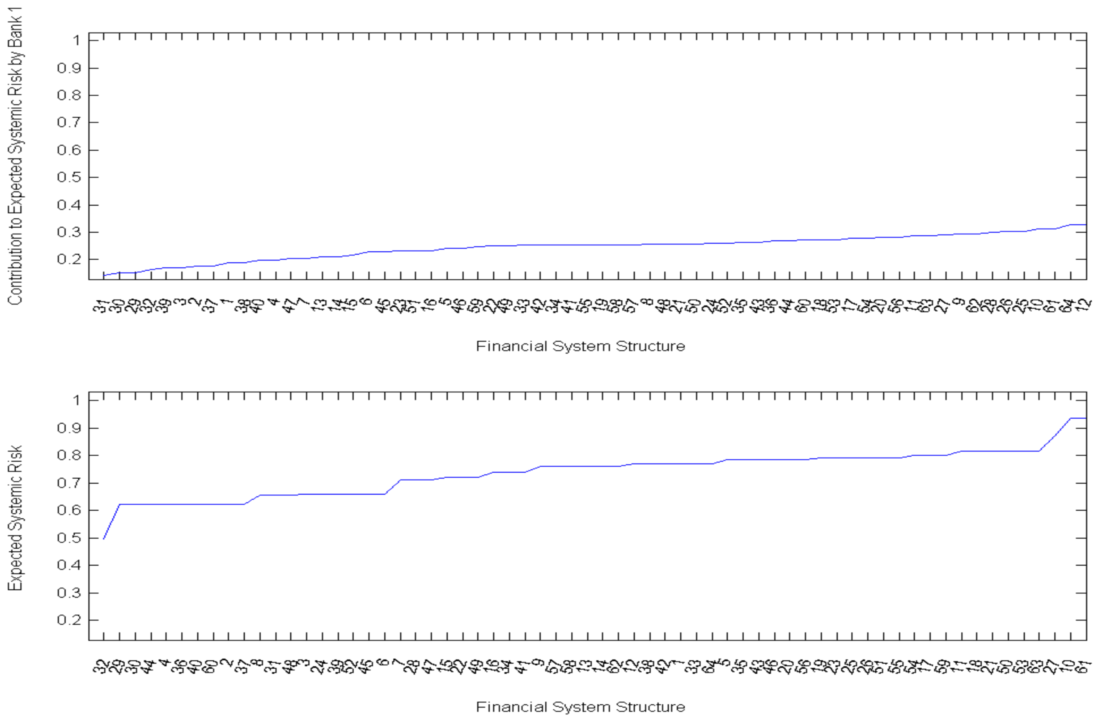

Figure 9: Effect of Financial System Structures on Expected Systemic Risk

interlinkages in the financial system.

As an alternative representation, the effect of the interlinkage-channel on expected systemic risk is presented in Figure 9 analogously to Figure 6. It again relies on the baseline specification, but this time the firesales channel is shut down. In other words, the parameter for price responsiveness, ξ, is set to zero, while all banks are starting with the same amount of initial assets, parameter A.

Qualitatively the results remain broadly the same. However, two points deserve mentioning. First, in Figure 9 expected systemic risk (lower panel) as well as bank 1's contribution to it (upper panel) turn out to be lower than before. For some structures with a low level of expected systemic risk, such as structure 32, the decrease of expected systemic risk (from 0.87 on Figure 6 to 0.49 on Figure 9) and bank 1's contribution to it (from 0.29 on Figure 6 to 0.17 on Figure 9) are significant. For other structures, such as for example structure 61 which is at the high end of expected systemic risk, the effect is relatively small (expected systemic risk decreases from 0.99 on Figure 6 to 0.94 on Figure 9 and bank 1's contribution to it from 0.33 on Figure 6 to 0.31 on Figure 9.).

Second, the ordering of structures along the x-axis can be affected, providing further evidence that the firesales channel impacts expected systemic risk arising through different interlinkage structures to different extents. Shock transmission via direct interlinkages takes only place if a debtor bank is hit by a shock which is strong enough to turn the bank's net-value negative because the direct interlinkage channel only gets contagious once the debtor bank's equity has been completely extinguished. The analysis of Sub-Section 4.2 has already provided evidence that the firesales channel increases the impact of shocks, as, for example, a high value for the parameter for price responsiveness, ξ, causes the whole financial system to default at even tiny shocks. This feature indirectly also impacts the effect of interlinkages and can thus affect expected systemic risk as well as banks' contribution to it in some structures.

Consider, for example, expected systemic risk and bank 1's contribution to it in structures 19 and 25 (Tables 9 and 10, respectively) on Figures 6 and 9. Both structures yield the same expected systemic risk on the same figure (0.96 on Figure 6 and 0.79 on Figure 9.). However, comparing bank 1's contributions to expected systemic risk (upper panel) on Figure 6 with the firesales channel being active, bank 1 contributes less to expected systemic risk in structure 25 (0.32) than in structure 19 (0.33). By contrast, with the firesales channel shut down, on Figure 9 bank 1 contributes relatively more to expected systemic risk in structure 25 (0.30) than in structure 19 (0.25).

This change in the relative magnitudes of systemic risk attached to particular network structures are a consequence of the interaction between lending (cross-) exposures and asset firesales, as well as the mean shock size. The underlying mechanism can be investigated by quantifying the risk-channels through which bank 1 contributes to expected systemic risk. Considering the interlinkage channel in isolation, bank 1's contributions to expected systemic risk are considerably larger in structure 25 than in structure 19.[24] Furthermore, in structure 25 bank 1 constitutes a larger proportion of the financial system (0.37) and has more non-liquid assets (0.92) than in structure 19 (0.33 and 0.8, respectively). Depending on the shock scenario, bank 1 can

[24] As can be seen on Tables 9 and 10, in structure 25 bank 3 has net-exposure to bank 1 and bank 2 has net exposure to bank 3, while in structure 19 bank 2 has net-exposure to bank 1 and bank 1 has net exposure to bank 3. This means that in structure 19 bank 1 can directly send a shock to bank 2 and/or transmit a shock from bank 3 to bank 2. In structure 25, however, bank 1 can directly send a shock to bank 3 which will transmit the shock even further to bank 2. *Ceteris paribus*, in the model a bank X that transmits a shock to another bank Y, which inn turn forwards the shock to a third bank Z contributes more to systemic risk than a second bank X which sends a shock to another bank Y but also forwards a shock from one bank Z to another bank Y.

contribute more to systemic risk in structure 25 than in structure 19, across all three channels.

This is reflected on Figure 9 where bank 1 contributes more to expected systemic risk in structure 25 than in structure 19. Note that when the firesales channel is shut down, the interconnection channel is generally weak in the baseline specification. It therefore merely plays a minor role in this bank's contribution to expected systemic risk.[25] We summarize these observations by stating that bank 1's larger systemic risk contribution in structure 25 as opposed to structure 19 in Figure 9 is apparently driven by the larger size of bank 1 in scenario 25.

Furthermore, according to Figure 6, bank 1 contributes slightly more to expected systemic risk in structure 19 than in structure 25. The change of order between the two structures when the firesales channel is active – rendering shocks more severe – can be traced to the properties of the shock distribution.[26] Since shocks close to the mean receive a higher probability weight in the computation of the contribution to expected systemic risk than shocks on the upper range of the interval of shocks analyzed, bank 1 contributes more to expected systemic risk via the interlinkage channel – which in this case outweighs its relatively smaller contribution from the other two channels – in structure 19 than in structure 25 on Figure 6.[27]

Shutting down the firesales channel also has mixed effects on expected systemic risk, depending on the actual network structure of the financial system (lower panels on Figures 6 and 9). For example, the second lowest expected systemic risk is found in structure 8 (0.88; Table 5) on Figure 6. This structure is relatively safe because only banks 1 and 3 which have cross-

[25]Banks that have borrowed from other banks to invest into non-liquid assets are relatively safe, given the firesales channel shut down, because non-liquid assets are similar to liquid assets in these circumstances.

[26]With the firesales channel open, shocks to banks in the financial system have more impact, increasing also the influence of the interconnection channel. Taking into account the mean size of shocks to the system, a further needs to be considered: a large shock to bank 1, that is, a shock on the upper range of the shocks considered, to bank 1, quickly erases equity such that the bank cannot use netting anymore to reduce its counterparty exposures. In case of a medium shock to bank 1, i.e. a shock close to the mean of the shock distribution, equity will not be wiped out, so it may improve its capital ratio via netting. Since bank 1 can net more counterparty exposure in structure 25 (0.3) than in structure 19 (0.15), it has less chances to recover via netting in the latter structure and is thus more likely to forward a shock to a bank that has exposure to it.

[27]Note that this interpretation is corroborated by the fact that summing up all contributions to expected systemic risk by bank 1 with equal weights, that is, relaxing the assumption that shocks near the mean have a higher probability and all other parameters set as in the baseline specification, results, as expected, in bank 1 contributing more to expected systemic risk in structure 25 than in structure 19.

exposure but no net-exposure to each other are interlinked. This pattern allows the banks to engage in self-insure against shocks, via netting. However, with the firesales channel shut down (Figure 9), the second lowest level of expected systemic risk can be found in structure 29 (0.62; Table 12). Again, this change of ranks results from the particular role of interlinkages when firesales are disallowed: In structure 29, bank 2 has a net exposure vis-à-vis bank 3. The latter is leveraged and holds more non-liquid assets than the other banks. However, with the firesales channel shut down, bank 3 appears safe because the non-liquid assets are now similar to liquid assets, so shock are transferred to bank 2 via the interlinkage channel only infrequently. The non-liquid asset-based shock buffer lowers systemic risk by more than the quasi mutual insurance provided by the cross-exposure between banks 1 and 3 in structure 8. In addition, in the latter structure all banks hold the same amount of non-liquid assets, so banks that theoretically may send shocks via interbank lendings have no particularly large shock buffer from non-liquid asset holdings, given the firesales channel is shut down.

Summing up, the baseline specification with no firesales and no size differences, we find four insights relating to the role of the the interlinkage channel. First, expected systemic risk as well as the bank's own contribution to systemic risk tend to increase with the amount of interlinkages in the financial system. Second, cross-exposure is a form of mutual insurance (since netting on the interbank market tends to increase the capital ratio) and thus can lower expected systemic risk, and also banks' contribution to it. Third, a positive net-exposure increases expected systemic risk as well as the contribution to it provided the banks remain net borrowers. Fourth, the effect of the interlinkage channel on expected systemic risk and bank 1 's contribution to it depends on the magnitude of the shock to the financial system which, in turn, is also impacted by the firesales channel. Since the interlinkage channel only becomes contagious at relatively large shock levels, that is, those shocks which turn the net-value of banks negative, and the firesales channel amplifies the effect of shocks to the financial system, the effect of the interlinkage channel on expected systemic risk as well as banks' contribution to it increase with the extent of firesales in the financial system.

The next sub-section analyzes the effect of a bank's size on expected systemic risk.

4.4 The Effect of Banks' Size on Expected Systemic Risk

The effect of banks' size on expected systemic risk is isolated via shutting down the interlinkage and firesales channels. Using structure 32 (Table 14) in which no banks borrow from or lend to each other and the price responsiveness of the liquid asset, parameter ξ, set to zero, the amount of initial assets of bank 1 is increased over a range from 1 to 3, while the amount of initial assets of banks 2 and 3 remains set to 1 as in the baseline specification.

Figure 10 displays the effect of varying bank 1's initial assets on expected systemic risk (lower panel) as well as its contribution to it (upper panel) in structure 32. Controlling for the effect of the firesales and interlinkage

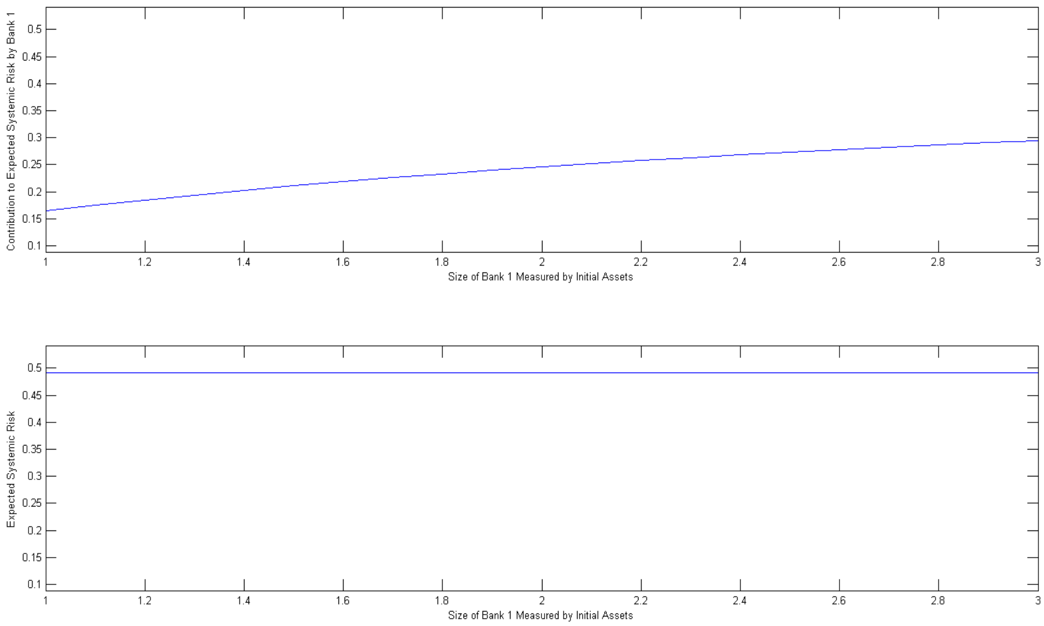

Figure 10: Effect of an Increase of Size on Expected Systemic Risk in Financial System Structure 32

channels and increasing bank 1's size results in increasing its contribution to expected systemic risk (from 0.16 to 0.29). However, given the definition of systemic risk as well as the symmetry of the shock vectors and assigned probabilities which are used in the computation of expected systemic risk, the expected amount of systemic risk does not change (constantly at 0.49). This outcome is driven by the fact that in the weighted sum of systemic risk over all shock scenarios the changes in systemic risk resulting from increasing bank 1's size relatively to the other banks in the financial system exactly offset each other.

Increasing bank 1's size does not change its probability of default in any shock scenario but only increases its proportion in the financial system as measured by the sum of its assets and reduces the proportion of the remainder two banks by the same amount. When increasing bank 1's size, systemic risk thus increases in scenarios in which only bank 1 or bank 1 and one other bank default, decreases in scenarios in which only bank 2 or 3 or both default, and remains unchanged in scenarios where all banks or none of the banks default. For example, say in scenario A only bank 1 defaults while in scenario B banks 2 and 3 default with both scenarios having the same probabilities. Increasing the relative size of bank 1 in the financial system results in increasing systemic risk in scenario 1 and lowering it by the same amount in scenario 2. Expected systemic risk computed according to Equation (10) including both scenarios does not change. Note that the level of expected systemic risk can be affected of course if the distribution of shocks is not symmetric.

In summary, controlling for the effect of the interlinkage and firesales channels, increasing a bank's size with respect to the financial system increases the contribution to expected systemic risk from that bank and lowers the contribution of the remainder two banks by the same amount such that expected systemic risk remains unaffected.

The next sub-section investigates the effect of the capital requirement ratio on expected systemic risk.

4.5 The Effect of Capital Requirements on Expected Systemic Risk

In order to lower systemic risk in the financial system, several calls have been voiced to increase banks' capitalization. Since capital held in excess of liabilities is the main shock buffer before a bank starts emitting shocks via its interbank liabilities it is regarded to be one of the most effective tools in macro- and micro-prudential regulation. For example, under the proposed Basel III framework an essential strengthening of banks' capitalization is envisaged to make the financial system more resilient.[28]

In the following analysis the implications of different levels of banks' capital ratios on expected systemic risk will be analyzed with all remainder parameters set as in the baseline specification. Figure 11 displays expected systemic risk (lower panel) as well as bank 1's contribution to expected sys-

[28] Bank for International Settlements (2010).

temic risk (upper panel) when the required equity ratio in the financial system is varied over a range from 1% to 25%.

Expected systemic risk and bank 1 's contribution to it are displayed along the y-axis, the varying levels of required capital are displayed along the x-axis, and the interlinkage structures have been ordered along the z-axis according to their highest sum of expected systemic risk or contribution to it, that is, the integral over the x-axis for a given structure. For example, adding up all contributions to expected systemic risk from bank 1 over the range of analyzed required capital ratios, structures 12 and 64 (Tables 7 and 16, respectively) yield the highest values, which is the reason for these structures being farthest right on the upper panel.

As regards the ordering of financial system structures along the z-axis, results remain broadly the same with respect to figure 6. In the model increasing the parameter for the required capital ratio results in lowering expected systemic risk as well as bank 1's contribution to it. The lowest sum of contribution to expected systemic risk from bank 1 is achieved in structure 31 (Table 13). The highest expected systemic risk over all capital requirements analyzed is achieved in structures 10 and 61 (Tables 6 and 15, respectively). The lowest sum of expected systemic risk is obtained in structure 27 (Table 11), where at high levels of bank capitalization the self-insurance mechanism via cross-exposures becomes very effective, making it thus less risky than structure 32 (Table 14) which yields the lowest level of expected systemic risk on Figure 6.

The analysis in this sub-section provides evidence that increasing the capital requirement is an effective means to lower expected systemic risk and banks' contribution to it.

Overall, the results in this section show that our model reproduces the stylized facts which could be observed during the recent financial crisis. The main risk-channels which cause the emergence of systemic risk are interlinkages, firesales and the size of a bank with respect to the financial system. It has been shown that banks' capital requirements are an effective shock buffer and can make the financial system more resilient to expected systemic risk as well as banks' contribution to it.

In the following section the model will be used to explore a systemic risk charge and fund which address systemic risk from a macroprudential perspective.

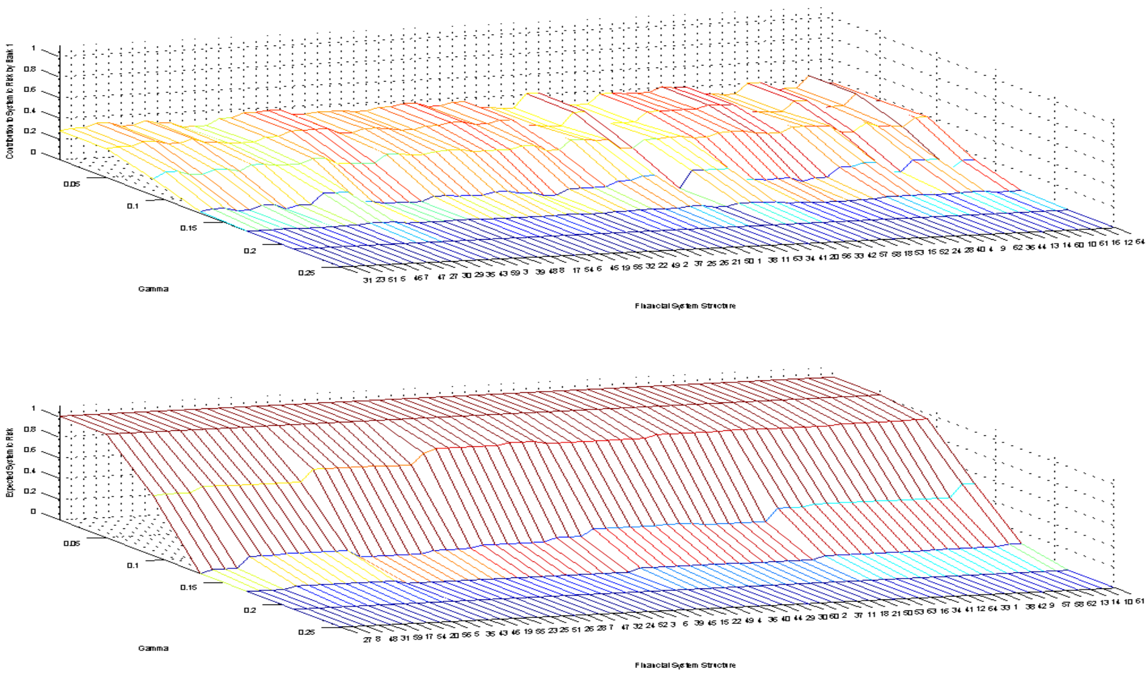

Figure 11: Effect of the Capital Requirement on Expected Systemic Risk

5 Developing a Systemic Risk Charge and Fund

A supervisor's approach to manage systemic risk should feature in particular three characteristics. First of all, it should address extreme shock scenarios, that is, shock events with an unusually high impact on the financial system. Systemic risk arises primarily through unexpectedly high losses which generally lead to firesales, contagion, and the default of individual institutions. To properly identify risk-channels and banks' contribution to expected systemic risk, these scenarios should cover a sufficient range of different shocks. Second, addressing systemic risk should not give wrong incentives, that is, it should not cause moral hazard[29] but, akin to a Pigouvian tax, incentivize financial institutions to lower their negative externality on the financial system which arises through their contribution to systemic risk. Third, the approach should envisage to preserve with a high probability even in strongly adverse scenarios a fraction of the financial system which is deemed necessary to prevent a financial shock from severely affecting the real economy.

It has been shown in the previous section that banks' contribution to systemic risk is driven by three risk-channels, (i) the extent of direct shock transmission through interbank liabilities which itself depends on the interlinkage structure and the amounts lent and borrowed, (ii) the extent of firesales which themselves depend on the amount of non-liquid asset holdings and the price responsiveness of the non-liquid asset, and (iii) banks' size relative to the financial system. If the supervisor wants to lower systemic risk, it is unlikely that he starts regulating all these dimensions involved in banks' contribution to expected systemic risk. However, it makes sense to use in particular one instrument, additional capital, to make the financial system more resilient to expected systemic risk. On the one hand, as has become clear in the previous section, this instrument has a high impact as shock buffer to lower expected systemic risk and, on the other hand, remaining administrative regulatory approaches such as, for example, forcing a bank to change its portfolio composition or counterparties, would be unfeasible in reality.

Our model will be used to investigate a systemic risk management approach in which a systemic risk charge and systemic risk fund are determined within an SVaR concept.[30] As will become clear, the SVaR concept combines

[29] See Poole (2008) for a discussion of financial institutions, financial stability, and moral hazard.

[30] The following SVaR approach features some of the characteristics of the VaR concept which is a well established measure in risk management used on the level of individual banks. The VaR indicates for a given portfolio the loss it will not exceed in a specified time horizon with a given probability. See, for example, Jorion (2006).

the previously outlined characteristics in a unified framework. The general idea is to charge banks according to their contribution to expected systemic risk. Banks which contribute more to expected systemic risk have to pay a higher risk charge than banks which contribute less. These payments are used to capitalize a systemic risk fund which is re-injected into the financial system in an optimal way to make it more resilient to systemic risk.

In the following, the approach to determine (i) the optimal amount of capital for the risk fund and (ii) the individual financial institution's contribution to it, as well as (iii) the optimal (macroprudential) capital amounts individual financial institutions are injected from the systemic risk fund to make the financial system more resilient to systemic risk will be outlined.

To set up the systemic risk charge and fund, the supervisor first of all defines a distribution of extreme shock scenarios deemed possible. Given our model, the supervisor will be able to compute the expected systemic risk as well as individual institutions' contribution to it associated with the stress scenarios. Next, the supervisor chooses an SVaR. The SVaR is defined as the proportion of the financial system which the supervisor is willing to accept to become insolvent in a given quantile of the shock distribution. For example, an SVaR could be defined as 'In 95% of all shock-scenarios systemic risk shall not exceed 0.37'. Given all shock scenarios the supervisor then computes the minimum (macroprudential) capital amounts which banks in the financial system need to be injected to fulfill this SVaR. The sum of these additional capital injections (which need not be the same across banks) constitutes the overall necessary amount of capital in the systemic risk fund.

The fund is capitalized via charging financial institutions according to their contribution to systemic risk. Equation (13) displays the systemic risk charge, H, for the i'th bank.

$$H_i = \Psi \cdot \frac{\phi_i^E}{\sum_j^3 \phi_j^E}. \tag{13}$$

where $i \in j$, Ψ is the amount of capital to be collected for the entire systemic risk fund, and ϕ_i^E designates the contribution to expected systemic risk by bank i as measured by the Shapley value. After collecting all individual charges in the fund,[31] the money is re-injected into 'neuralgic' financial institutions, that is, those institutions which increase the financial system's resilience most, as additional capital which they are required to hold as liquid assets. The additional capital can be injected on top of the required capital from microprudential regulation in the form of preferred stock such that its

[31] Note that at this point it is assumed that banks can pay these charges from profits, for example, by deferring dividend payments.

function as an additional shock-buffer only emerges after other shareholders' equity has been extinguished.

As will become clear in the following, requiring banks to hold this macroprudential capital in addition to the microprudential capital requirement, the risk fund primarily addresses systemic risk arising through the interlinkage channel. The other two risk channels are only indirectly affected. The size channel is not directly affected because the additional capital is not included in computing banks' proportion of the financial system. Furthermore, banks default if they cannot fulfill both the macro- and microprudential capital requirement, that is, a default does not get more unlikely through additional capital. The firesales channel is also not directly affected because of a same argument. Since banks have to maintain the higher capitalization their market behaviour as regards sales of non-liquid assets does not change. However, both channels are indirectly affected as, for example, a reduced impact from the interlinkage channel because of a higher capitalization can prevent a shock from being spread further via that channel to a bank with a sizeable amount of non-liquid assets on the balance sheet. The firesales channel is thus indirectly dampened via less shock transmission through the interlinkage channel.[32]

An important feature of the SVaR concept is that individually all banks can default. If none of the banks' sizes exceeds the proportion of the financial system that is accepted to break down under the SVaR, then, depending on which scenario realizes, all three banks are threatened with insolvency. This reduces the risk of moral hazard from the systemic risk fund.

In the following, the outlined systemic risk charge and fund will be computed for the baseline specification of the financial system developed in the previous section. The structure of the financial system analyzed is structure 19 (see Appendix A as well as Table 9 in Appendix B at the end of this paper). The shocks are modeled as outlined in Section 2.

The supervisor's SVaR is defined as 'In at least 95% of possible shock scenarios, not more than 37% of the financial system shall default'. The following exercise consists of finding the minimum additional capital amounts

[32]The firesales and size channels could be addressed via 'triggering' in case a systemic shock emerges either (i) reduced required capital ratios, if the additional (macroprudential) capital had been injected as preferred stock, or, (ii) the conversion of debt into equity, in case the additional (macroprudential) capital had been injected in the form of Conditional Convertible Bonds (CoCos). (For a review on the advantages and disadvantages of contingent capital see Pazarbasioglu, Zhou, Leslé, and Moore (2011).) However, any threshold value, be it triggered by a market based measure or by a supervisory authority, can lead to perverse incentives and cause moral hazard. Investigating the effect of such triggers in our model would be interesting to pursue but is beyond the scope of this paper and thus not addressed.

that need to be injected into financial institutions to fulfill this SVaR. Since in structure 19 the biggest bank constitutes 37% of the financial system (measured by the size of their balance sheets, banks 1 to 3 constitute, in rounded values, 33%, 29%, and 37% of the financial system, respectively), theoretically each bank can default with the SVaR still being fulfilled if the other two banks remain solvent in a given shock scenario.

The loss function, ϵ, which is minimized to compute the optimal amount of additional capital that needs to be injected to fullfill the SVaR is given by Equation 14.

$$\epsilon = \sum_{i}^{3} \tau_i + \sum_{w}^{L} o_w(\tau), \tag{14}$$

where τ_i is the additional amount of capital injected into financial institution i. o_w is the systemic risk in scenario w, with L the number of scenarios that exceed the accepted proportion of systemic risk after exclusion of the percentage amount of scenarios the supervisor allows to attain or exceed the maximum systemic risk. For example, consider the supervisor sets up 100 scenarios, with each scenario assigned a different probability. According to the SVaR, in 95% of all scenarios the proportion of insolvent banks with respect to the financial system shall not exceed 0.37. Say, in case the supervisor injects no additional capital at all, that is $\sum_i^3 \tau_i = 0$, the sum of probabilities of scenarios resulting in excess of a systemic risk of 0.37 is 25%. Inspecting Equation (14), ϵ then consists of the sum of systemic risk resulting in all scenarios exceeding the SVaR, excluding those scenarios in excess of the SVaR which add up to the highest expected systemic risk based on 5% of the shock scenarios.[33]

Note that minimizing Equation (14) to find out the necessary additional capital and the financial institutions in which additional capital needs to be injected requires a non-standard optimization technique because the objective function can have multiple local minima. The simulated annealing approach,[34] a probabilistic metaheuristic optimization procedure, is used to find the optimal solution for Equation (14). A parallelized variant of the optimization algorithm is outlined in Appendix C at the end of this paper.

[33]In Equation (14) probabilities are not used to weight the scenarios. However, excluding the 5% of scenarios in excess of the SVaR which result in the highest expected value yields the lowest value of the loss function. In any case, in the exercise, all shock scenarios included in the second term of equation (14) consist of at least the systemic risk value arising through the insolvency of two banks which exceeds the first term of Equation (14). The supervisor has thus a strong incentive to make sure the SVaR is not exceeded in any of the scenarios representing 95% of the shock distribution.

[34]See Kirkpatrick, Gelatt, and Vecchi (1983).

Contribution to Expected Systemic Risk of Bank 1	0.3289
Contribution to Expected Systemic Risk of Bank 2	0.3017
Contribution to Expected Systemic Risk of Bank 3	0.3246
Contribution of Bank 1 to Systemic Risk Fund	0.0472
Contribution of Bank 2 to Systemic Risk Fund	0.0433
Contribution of Bank 3 to Systemic Risk Fund	0.0465
Amount of Capital Injected to Bank 1 from Systemic Risk Fund	0.0494
Amount of Capital Injected to Bank 2 from Systemic Risk Fund	0.0350
Amount of Capital Injected to Bank 3 from Systemic Risk Fund	0.0526

Table 2: Results of the Systemic Risk Fund Exercise in Financial System Structure 19

Table 2 displays the optimal results from the systemic risk fund exercise. The first three rows display the banks' weighted Shapley values, that is, their contribution to expected systemic risk, resulting from the set of all shocks. Note that these Shapley values are calculated following Equation (12) on the basis of the financial system without any capital injections from the systemic risk fund. Rows four to six display the resulting optimal capital risk charge – which depends on the necessary size of the systemic risk fund as well as banks' individual contributions to expected systemic risk – for each bank. These values are computed following Equation (13) where Ψ is obtained by minimizing Equation (14) and summing up the optimal individual capital injections. Rows seven to nine which are also obtained from the minimization of Equation (14) display the optimal amount of capital injected from the systemic risk fund into the respective banks to fulfill the SVaR.

Three points are worth mentioning. First of all, banks' contribution to expected systemic risk is driven by the three risk-channels outlined before, size, firesales, and interlinkages. In particular note that the higher contribution to expected systemic risk of bank 1 with respect to bank 3 in this structure has already been analyzed in Sub-Section 4.3 in the context of investigating the effect of interlinkages on expected systemic risk.[35] Bank 2 contributes least to expected systemic risk because the other banks have no net-exposure to it, it holds the smallest amount of non-liquid assets, and constitutes the smallest proportion of the financial system. This is reflected

[35] Given the symmetry of all structures, the contribution of bank 1 to expected systemic risk in structure 25 is the same as the contribution of bank 3 to expected systemic risk in structure 19. Sub-Section 4.3 clarified why in the baseline specification displayed on Figure 6 the contribution to systemic risk by bank 1 (upper panel) is larger in structure 19 than in structure 25.

in the contributions to expected systemic risk and the banks' contribution to the systemic risk fund on Table 2. Bank 1 contributes slightly more to expected systemic risk and thus has to pay the highest charge, followed by banks 3 and 2, respectively.

Second, the optimal size of the systemic risk fund (0.14), obtained when summing up rows 7 to 9 on Table 2 represents 3.5% of system-wide assets. Calculating for each shock the difference between the net-value of the financial system, that is, the sum of all banks' net-values, with and without pre-injecting the capital from the systemic risk fund into the banks, and summing up these differences weighted with the shock probabilities shows that in expectation the financial system would have to be injected ex-post an additional capital of about 4.1% in relation to system wide assets if the same outcome as with pre-injecting the capital amounts was desired. This expected size of an ex-post bail-out exceeds the size of the fund that is immediately re-injected into the financial system to fulfill the supervisor's SVaR.[36] This second result is driven by pre-emptively nipping the contagious effects of financial shocks in the bud, in particular knock-on defaults via the interlinkage channel and resulting firesales of non-liquid assets when the systemic risk fund is immediately injected into the financial system.

Third, the optimal amounts of additional capital injected from the systemic risk fund do not fully reflect the ranking which emerges in banks contribution to expected systemic risk. Although bank 1 contributes more to expected systemic risk than bank 3, it is optimal to inject more capital into bank 3 to fulfill the SVaR.[37] Taking a systemic perspective, the optimal macroprudential capitalizations thus need not necesarily reflect banks' contribution to systemic risk in a proportional way. This result is mainly driven by using different probability weights when computing banks' contribution to expected systemic risk, however, following the definition of the SVaR, equally weighting 95% of the shock scenarios for computing banks' optimal additional capital injections.[38]

[36]Note that the size of an ex-post bail-out fund gets even larger if one does not take the expected difference over all scenarios, but the largest difference that results in the 95% of scenarios in which the SVaR must be fulfilled.

[37]Note that this result is robust to controlling for scenarios in which more than 37% of the financial system default, that is, the 5% of scenarios which are accepted under the outlined SVaR to exceed the highest proportion the supervisor is willing to accept as insolvent in the system. Calculating the contribution to expected systemic risk without additional capital injections only for the 95% of scenarios in which 62% (due to rounding, the three banks' proportions add up to 0.99) or more of the banking system remains solvent with the optimal injections from the systemic risk fund, results in the same order of contribution to expected systemic risk as displayed on Table 2.

[38]The probability weights play no role in the 95% of scenarios in which the supervisor

Note that qualitatively, the same result emerges, however more robust to distributional assumptions about the shock scenarios, when taking into consideration that additional capital injections affect the channels of contribution to expected systemic risk to different extents. As outlined before, increasing a bank's capitalization does not directly affect its contribution to expected systemic risk via the size and firesales channels. The main impact of additional capital is lowering expected systemic risk emerging via the interlinkage channel. To make this point clear consider, for example, a slight modification of the baseline specification which consists of strongly increasing the size of one of the financial institutions while making the remainder two financial institutions highly interlinked in the financial system. Increasing bank 1's initial assets, parameter A, to 2, leaving all remainder parameter values as in the baseline specification, and taking financial system structure 60 results in the desired setting. Table 3 displays the financial system as well as the banks' proportions in the outlined set up.

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0.30	0.30	1.12	0.28	0.44
Bank 2	0		0	1.04	0.26	0.28
Bank 3	0	0		1.04	0.26	0.28
ROW	1.86	0.92	0.92			

Table 3: Financial System Structure 60 with Parameter A increased to 2 for bank 1

As can be seen, bank 1 constitutes the largest proportion of the financial system (44%) while banks 2 and 3 both constitute a relatively little proportion (28%, each). Furthermore, bank 1 holds the largest amount of non-liquid assets (1.12) while banks 2 and 3 hold a relatively small amount (1.04, each). With regards to interlinkages, bank 1 has net-exposure both to banks 2 and 3. In this setting bank 1 contributes most to expected systemic risk via the size and firesales channels and banks 2 and 3 contribute most to expected systemic risk via the interlinkage channel.

Defining the SVaR as 'In 95% of all shock-scenarios systemic risk shall not exceed 0.44' and repeating the systemic risk fund exercise, Table 4 displays

insures that 62% or more of the financial system remain solvent. In these scenarios the supervisor only tries to find the minimum amount of capital which ensures that at most one bank defaults. As has been outlined in Sub-Section 4.3, without giving different weights to the shock scenarios, bank 1 contributes more to expected systemic risk in structure 25 than in structure 19. Given the symmetry of all structures, the contribution of bank 1 to expected systemic risk in structure 25 is the same as the contribution of bank 3 in structure 19.

Contribution to Expected Systemic Risk of Bank 1	0.4693
Contribution to Expected Systemic Risk of Bank 2	0.2610
Contribution to Expected Systemic Risk of Bank 3	0.2610
Contribution of Bank 1 to Systemic Risk Fund	0.0731
Contribution of Bank 2 to Systemic Risk Fund	0.0407
Contribution of Bank 3 to Systemic Risk Fund	0.0407
Amount of Capital Injected to Bank 1 from Systemic Risk Fund	0.0000
Amount of Capital Injected to Bank 2 from Systemic Risk Fund	0.0772
Amount of Capital Injected to Bank 3 from Systemic Risk Fund	0.0772

Table 4: Results of the Systemic Risk Fund Exercise in Financial System Structure 60

the optimal results for the financial system outlined on Table 3.

Again, there is no correspondence between a bank's systemic risk charge and the capital that is optimally injected into it. Though bank 1 contributes most to expected systemic risk and thus pays the highest charge for the systemic risk fund, from a financial stability perspective it is optimal to inject this capital into banks 2 and 3, only. As outlined before, this outcome results from the fact that the contribution to expected systemic risk is driven by three different risk-channels which are affected to a different extent by the supervisor's instrument to lower expected systemic risk, additional capital injections. Since the contribution of bank 1 is only driven by the firesales and size channels which are not directly addressed in the model by additional capital, the SVaR is optimally attained via injecting all additional capital into banks 2 and 3 which contribute most to expected systemic risk via the interlinkage channel.[39]

Overall, the SVaR analysis shows that linking a bank's macroprudential capital requirements directly to its contribution to systemic risk, as, for example, suggested in Acharya, Pedersen, Philippon, and Richardson (2009),[40]

[39]Note that the result is robust to relaxing the distributional assumptions such that all scenarios emerge with the same probabilities. Furthermore, it is robust to controlling for scenarios in which more than 44% of the financial system default, that is the 5% of scenarios which are accepted under the outlined SVaR to exceed the highest proportion the supervisor is willing to accept as insolvent in the system. Calculating the contribution to expected systemic risk without additional capital injections only for the 95% of scenarios in which 56% or more of the banking system remain solvent with the optimal injections from the systemic risk fund, results in the same order of contribution to expected systemic risk as displayed on Table 4.

[40]The authors propose, *inter alia*, that "[c]apital requirements could be set as a function of a financial firm's marginal expected shortfall" (p. 8) which is their measure for a bank's contribution to systemic risk. See also V. Acharya and M. Richardson (2009).

is not necessarily an optimal and consistent policy approach when taking a systemic risk management perspective. Following the results in our framework, linking banks' macroprudential capital requirements directly to their contribution to expected systemic risk can be inconsistent or inefficient if, as is likely the case, the drivers of expected systemic risk are differently affected by additional macroprudential capital requirements. This result becomes more intuitive when pointing out that a variant of the Tinbergen rule applies in our setting. The Tinbergen rule implies that consistent economic policy requires the number of policy instruments to at least equal the number of policy targets.[41] In our systemic risk management approach a consistent and *efficient* economic policy calls for the same requirement because there are two policy targets which the supervisor tries to achieve. First of all, a numerical value with respect to expected systemic risk, the SVaR, and, second, to incentivize banks to lower their contribution to expected systemic risk via an appropriate risk charge. Though ultimately related, both targets can become distinct when the risk-channels through which banks contribute to expected systemic risk are affected by the instrument to achieve systemic stability, additional capital, to a different extent.

The solution to the dilemma in the SVaR concept is to use two instruments, a levy to fulfill the incentive requirement[42] and a capital injection to guarantee systemic stability. Though a proper incentive requirement should foster the target of financial stability, it is possible that both targets cannot be achieved by only one instrument in an efficient or in a consistent way if the risk-channels are unequally affected by the single instrument. Merging the two instruments in case the risk-channels are indeed affected differently by additional capital injections can result in not properly incentivizing financial institutions to lower their contribution to expected systemic risk[43] or in

[41]See J. Tinbergen (1952).

[42]Note that the incentive requirement implied by the SVaR is only fulfilled if financial institutions are aware of how they can lower their contribution to systemic risk. This however potentially depends in part on the decisions taken by other banks. In the model and SVaR approach the incentive requirement is only fulfilled to the extent that banks which contribute more to systemic risk face a higher risk charge. It still needs to be investigated, desireably in richer framework where banks do not only try to fulfill a capital requirement but also maximize their profit, whether a trade-off between maximizing profit and paying an adequate risk charge for the resulting contribution to systemic risk is feasible.

[43]This is the case if each bank is only charged the optimal amount of capital it will be required to hold as additional (macroprudential) capital. In the example on Table 4 this would be achieved via setting the contributions of banks to the systemic risk fund, rows 4 to 6, to the respective values displayed in rows 7 to 9. The SVaR would be optimally fulfilled, however, the incentive requirement not. Hence the policy approach would be inconsistent.

requiring a systemic risk fund with a larger amount than the one implied by the optimal SVaR approach[44] which then results in a sub-optimal capital allocation.

The next section concludes.

[44]This is the case if the incentive requirement is fulfilled, that is, banks are charged according to their contribution to systemic risk while fulfilling the SVaR, however, not in an optimal way. With respect to the example on Table 4 this is achieved via including the additional restriction in the optimization procedure that the amount injected into a bank must be equal to the amount charged from that bank. In the example, this restriction leads to a higher sum of necessary capital injections, that is, in a sub-optimal capital allocation with respect to not including the restriction.

6 Conclusion

In this paper a model that allows to replicate the main stylized facts of systemic risk which came up during the recent financial crisis has been developed. In our model, the three main risk-channels through which systemic risk arises are banks' size, their interlinkages, and firesales of non-liquid assets. Furthermore, a proposed systemic risk charge and fund are designed within an SVaR approach which allows to make the financial system more resilient to systemic risk and charges banks according to their contribution to expected systemic risk. This systemic risk management concept allows to simultaneously determine the necessary capital of a systemic risk fund, banks optimal (macroprudential) capitalization, and risk charge in a unified framework which is consistent and efficient.

Among numerous insights into the complex processes arising in an interdependent financial network two key results are of particular importance. First of all, keeping additional (macroprudential) capital obtained from charging banks according to their contribution to expected systemic risk in the financial system to make it more resilient to extremely adverse shock scenarios is likely to come at a lower cost than bailing out banks ex-post. The reason for this outcome is that re-injecting capital into 'neuralgic' points of the financial system helps nipping crisis developments and contagion effects in the bud before they can unfold their mischief. Besides the argument that a systemic risk fund which is not injected into the financial system but kept centralized in a 'government chest' sparks political interest to divert its intended use after a longer period with no systemic events, the result of our systemic risk fund analysis provides further evidence as to why it is better to keep macroprudential capital which is levied via a risk charge in the financial system.

Second, using the model to analyze the proposed systemic risk charge and fund provides evidence that there is not necessarily a correspondence between a bank's contribution to systemic risk – which determines its risk charge – and the capital that is optimally injected into it to make the financial system more resilient to systemic risk. If the drivers of systemic risk are affected by additional (macroprudential) capital to different extents one is well advised to carefully distinguish between a bank's contribution to systemic risk as a determinant of its risk charge and the amount of capital injected into it to make the financial system more resilient. Increasing a bank's capital is an efficient administrative instrument to lower systemic risk and banks' contribution to it. However, not distinguishing between a bank's risk charge and its macroprudential capitalization can result in inconsistent or inefficient economic policy.

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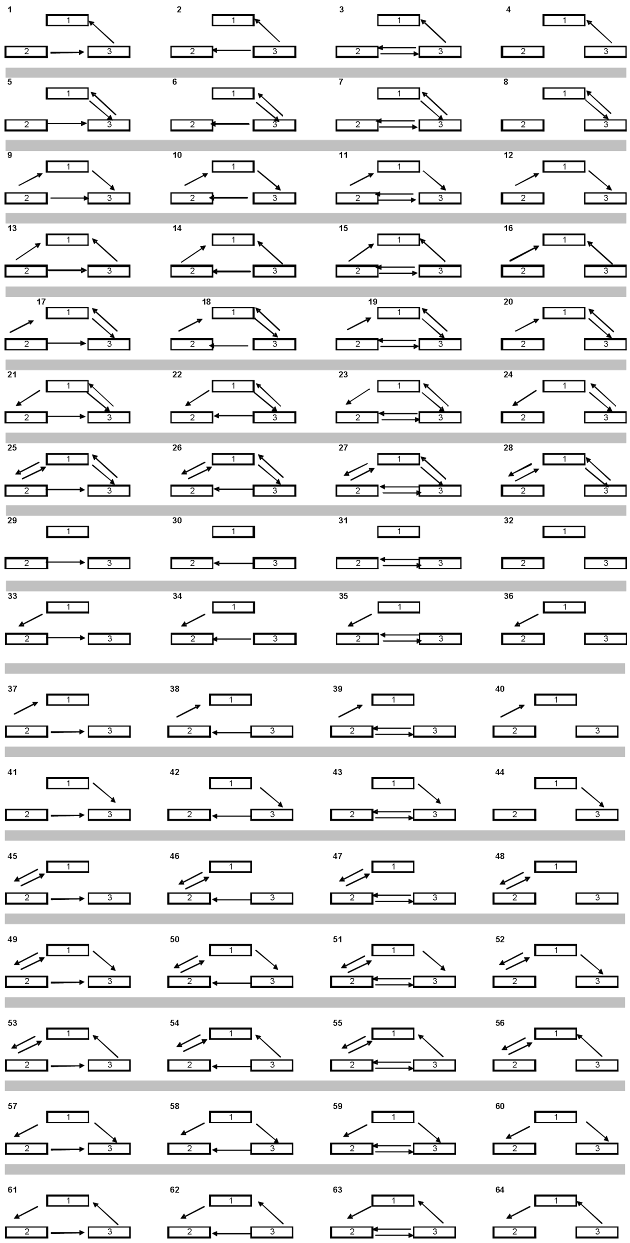
Appendix A: Structures of the Financial Network Matrix[45]



[45]An arrow from a bank to another bank symbolizes that this bank has exposure to the other bank through interbank lending.

Appendix B: Structures Referred to in the Analysis

The structures of the financial system outlined in the following tables have been referred to in the analysis of Sections 4 and 5. The entries in the tables are generated along the parameter settings in the baseline specification. The left part of each table is built up as outlined on Figure 2, and the right side outlines the respective bank's proportion in the financial system as measured by the amounts of its assets relative to system-wide assets.

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0	0.3	0.80	0.20	0.36
Bank 2	0		0	0.80	0.20	0.28
Bank 3	0.3	0		0.80	0.20	0.36
ROW	0.912	0.936	0.912			

Table 5: Financial System Structure 8

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0	0.3	0.80	0.20	0.33
Bank 2	0.3		0	0.80	0.20	0.33
Bank 3	0	0.3		0.80	0.20	0.33
ROW	0.912	0.912	0.912			

Table 6: Financial System Structure 10

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0	0.3	0.8	0.2	0.36
Bank 2	0.3		0	0.56	0.14	0.28
Bank 3	0	0		1.04	0.26	0.36
ROW	0.912	0.9312	0.9168			

Table 7: Financial System Structure 12

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0	0	1.28	0.32	0.44
Bank 2	0.3		0	0.56	0.14	0.28
Bank 3	0.3	0		0.56	0.14	0.28
ROW	0.8976	0.9312	0.9312			

Table 8: Financial System Structure 16

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0	0.3	0.8	0.2	0.33
Bank 2	0.15		0.15	0.68	0.17	0.29
Bank 3	0.15	0.15		0.92	0.23	0.37
ROW	0.912	0.9216	0.9024			

Table 9: Financial System Structure 19

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0.15	0.15	0.92	0.23	0.37
Bank 2	0.15		0.15	0.68	0.17	0.29
Bank 3	0.3	0		0.8	0.2	0.33
ROW	0.9024	0.9216	0.912			

Table 10: Financial System Structure 25

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0.15	0.15	0.80	0.20	0.33
Bank 2	0.15		0.15	0.80	0.20	0.33
Bank 3	0.15	0.15		0.80	0.20	0.33
ROW	0.912	0.912	0.912			

Table 11: Financial System Structure 27

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0	0	0.8	0.2	0.30
Bank 2	0		0.3	0.56	0.14	0.30
Bank 3	0	0		1.04	0.26	0.39
ROW	0.936	0.9312	0.9168			

Table 12: Financial System Structure 29

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0	0	0.80	0.20	0.28
Bank 2	0		0.3	0.80	0.20	0.36
Bank 3	0	0.3		0.80	0.20	0.36
ROW	0.936	0.912	0.912			

Table 13: Financial System Structure 31

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0	0	0.80	0.20	0.33
Bank 2	0		0	0.80	0.20	0.33
Bank 3	0	0		0.80	0.20	0.33
ROW	0.936	0.936	0.936			

Table 14: Financial System Structure 32

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0.3	0	0.80	0.20	0.33
Bank 2	0		0.3	0.80	0.20	0.33
Bank 3	0.3	0		0.80	0.20	0.33
ROW	0.912	0.912	0.912			

Table 15: Financial System Structure 61

	Bank 1	Bank 2	Bank 3	NLA	LA	Proportion
Bank 1		0.3	0	0.8	0.20	0.36
Bank 2	0		0	1.04	0.26	0.36
Bank 3	0.3	0		0.56	0.14	0.28
ROW	0.912	0.9168	0.9312			

Table 16: Financial System Structure 64

Appendix C: A Parallelized Simulating Annealing Algorithm

To minimize the loss-function outlined in Section 5 (Equation (14)) the simulated annealing algorithm is used. The algorithm has been developed by Kirkpatrick, Gelatt, and Vecchi (1983) and is a heuristic optimization procedure to approximate the global minimum of a complex function that has multiple local minima.[46] It has been inspired from the annealing process in metallurgy where a slow cooling down of metal insures that atoms have enough time to form stable crystals without defects.

To minimize a function with the simulated annealing algorithm, new function values are generated along random changes to the control parameters in a Markov chain. New solutions that lead to improvements, that is, decreasing values, in the function are always accepted as new element in the Markov chain, whereas new solutions that lead to an increase in the function value are only accepted with a certain probability. This acceptance probability is influenced by a temperature used in the algorithm. At high temperature values the acceptance probability is high, and at low temperatures this probability is small. The optimization procedure consists of numerous sub-optimizations along Markov chains. After each Markov chain the temperature is gradually lowered which decreases the initially high probability of 'uphill-moves' – thus preventing the optimization routine to get 'trapped' in local minima. The final solution is found when the system has 'frozen', that is, when for the length of one Markov chain no new solutions are accepted. Figure 12 displays the simulated annealing algorithm.

In the following, a variant of simulated annealing developed for our application is outlined. It uses parallel Markov chains as well as an automatic adjustment of the stepsize and temperature to increase accuracy and the chance that the global minimum is found.

Following Parks (1990) new solutions are generated following Equation 15

$$\boldsymbol{\rho}_{i+1} = \boldsymbol{\rho}_i + \boldsymbol{D} \cdot \boldsymbol{u}, \tag{15}$$

where $\boldsymbol{\rho}$ is the vector of control variables, $\boldsymbol{D}$ is a diagonal matrix scaling the stepsize of changes to the control variables, and $\boldsymbol{u}$ is a vector of uniformly distributed numbers on the interval (-1,1). $\boldsymbol{D}$ is updated after a successful draw as $\boldsymbol{D}^* = (1 - \pi)\boldsymbol{D} + \pi\omega\boldsymbol{R}$, where $0 < \pi < 1$ is a parameter that controls how fast $\boldsymbol{D}$ is updated, ω is a scaling parameter, and $\boldsymbol{R}$ is a diagonal matrix containing the absolute value of successfully implemented steps, that is $\boldsymbol{R} = |\boldsymbol{D}\boldsymbol{u}|$. Following Parks (2010), the values of π and ω are set to 0.1

[46] The following outline also draws strongly upon Parks (2010).

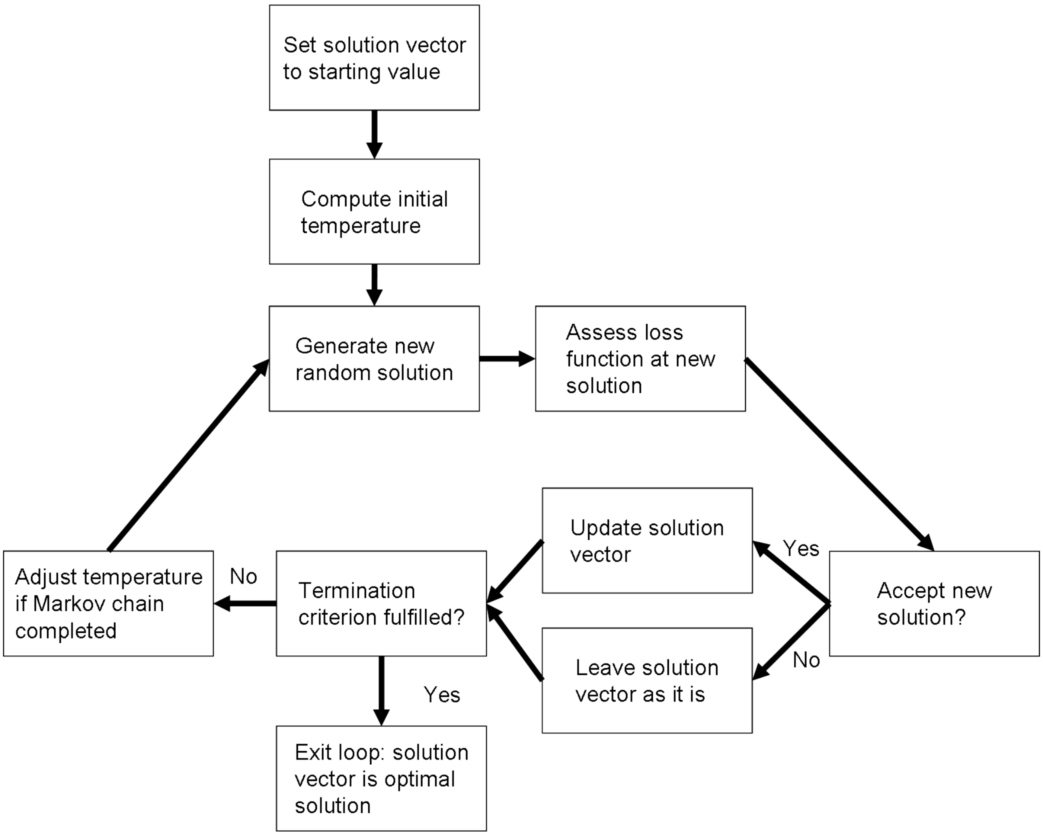


Figure 12: Simulated Annealing Algorithm

and 2.1, respectively.

Since the stepsize is flexibly adjusting to the functions' topography, the acceptance probability for uphill movements, that is increasing function values, needs to take this into account and is calculated following Equation 16

$$prob = exp\left(-\frac{\delta f^{+}}{T\bar{d}}\right), \tag{16}$$

where $\bar{d}$ is the average step size, that is, $\bar{d} = \sum_k |D_{kk}u_k|$, and δf^{+} is the increase in the loss function at the updated vector of control variables.

Following Kirkpatrick, Gelatt, and Vecchi (1983) the initial temperature is set such that the average probability of a function increase equals 0.8. The initial temperature, T_0, can be found via an initial search with the initial stepsize set to 1, with all function changes being accepted, and then applying

Equation 17

$$T_0 = -\frac{\delta \bar{f}^+}{ln(0.8)}, \tag{17}$$

where $\delta \bar{f}^+$ is the average positive change in the loss function during the initial search's Markov chain.

The maximum length of one Markov chain is set such that the search, given the initial step size theoretically can pace several times through the whole search space deemed realistical for the problem at hand, which in this application is set to be a cube with side length $2*A$, with A the inital assets of banks in the model.[47] In this application, with the initial maximum stepsize set to 1, the length of the Markov chain is set to fifty times the searchspace's volume divided by the initial maximum stepsize, that is $(2 \cdot A)^3 \cdot 50 = 400$. Clearly, the length of the Markov chain is a relatively arbitrary parameter. Setting its length too short can result in the system freezing prematurely, that is, getting stuck in a local optimum. Setting it too long can result in unnecessarily long computation time. In practice, the adequacy of the length of the Markov chain for the function to be minimized can be evaluated via taking out several optimizations with different starting values to cross-check whether they lead to the same optimal solution, also when taking random starting values.[48]

After a Markov chain of new random solutions has been completed the temperature is adjusted following an adaptive approach from Huang, Romeo, and Sangiovanni-Vincentelli (1986) where the temperature is decremented following Equation 18

$$T_{k+1} = \iota_k \cdot T_k, \tag{18}$$

and ι_k is given by Equation 19

$$\iota_k = max \left\{ 0.5, exp\left(-\frac{0.7 \cdot T_k}{\sigma_k} \right) \right\}, \tag{19}$$

where σ_k is the standard deviation of the loss function values that have been accepted during the Markov chain at temperature T_k. Note that the

[47] Note that the algorithm theoretically can explore far beyond this limit since the stepsize is adjusting freely to the necessary length. As robustness check totally unrealistic starting values of up to $1000 \cdot A$ have been chosen, always resulting in the same optimal solution, though eventually taking a long time to compute.

[48] Note that no matter which length the Markov chain is assigned, it is very unlikely to end up at exactly the same solution in each optimization given the heuristic nature of the algorithm. However, same solutions can be characterized as being in the same close neighborhood.

Markov chain is interrupted before its maximal length has been reached if the number of accepted random draws along the Markov chain equals 60% of the length of the Markov chain.

After the temperature has been decreased or at the beginning of the optimization procedure, the actual optimal value as well as stepsize and temperature are given to q parallel Markovian processes, where q is the number of CPUs used for parallel computing. Each process then optimizes the Markov chain along the lines outlined above until it is completed or interrupted because the number of accepted draws attained 60%. Next, the best solution as well as the according temperature and stepsize of these sub-optimizations from the parallel Markov chains are taken as new best value for the parallel optimization and given again as input to q parallel Markovian processes.

The algorithm terminates when the number of accepted changes in the entire optimal Markov chain is zero.

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