

A Highway-Driving System Design Viewpoint using an Agent-based Modeling of an Affordance-based Finite State Automata

Kasin Ransikarbum, Namhun Kim*, Sangho Ha, Richard A. Wysk, and Ling Rothrock

Abstract—This paper presents an agent-based modeling framework for affordance-based driving behaviors during the exit maneuver of driver agents in human-integrated transportation problems. We start our discussion from one novel modeling framework based on the concept of affordance called the Affordance-based Finite State Automata (AFSA) model, which incorporates the human perception of resource availability and action capability. Then, the agent-based simulation illustrates the validity of the AFSA framework for the Highway-Lane-Driver System. Next, the comparative study between real driving data and agent-based simulation outputs is provided using the transition diagram. Finally, we perform a statistical analysis and a correlation study to analyze affordance-based driving behavior of driver agents. The simulation results show that the AFSA model well represents the perception-based human actions and drivers' characteristics, which are essential for the design viewpoint of control framework of human driver modeling. This study is also expected to benefit a designed control for autonomous/self-driving car in the future.

Index Terms—agent-based modeling, affordance, finite state automata, driving behavior, human-machine interactions.

I. INTRODUCTION

A Human-machine system is often regarded as a complex one, in which the integration between the functionalities of a human and the opportunities that the machine or the environment presents to the human should be considered simultaneously [1]. One challenging, popular application area of such system is a control framework of the human-involved manufacturing system. Part of the problem of considering humans performing critical roles is that the human behaviors are nondeterministic and the human can play several roles in

terms of beneficial and detrimental actions. One way to explain these human behaviors is based on the concept of affordance [3] and prospective controls [4]. The Affordance Theory has later been adopted in various domains including human-computer interaction, interaction design, and user interface designs [e.g., 5-8, 52-62]. Using the Affordance Theory, an Affordance-based Finite State Automata (AFSA) modeling formalism is developed for the manufacturing control by directly relating the transition rules with the juxtaposition process [9]. Since then, various researchers have applied the AFSA model in various domains due to its ability to describe the control ability and the interaction between human and the system environments [10-15, 55-56].

In the context of road traffic analysis, driver's behavior simulations are one of the most important challenges in the context of building autonomous vehicles using public roads, where there is a need of exact mapping and prediction of the human behavior. Although the AFSA model was developed, the model has not been extended through agent-based simulation and real experiments in the context of highway-driving system [10-11]. On the contrary, existing agent-based simulation models for driving applications lack a perspective of AFSA in terms of sensing its environment [63-69]. Thus, we propose the agent-based AFSA model, which well reflects the characteristics of people's behavior on the roadway on the basis for modeling human-machine behavior. Using the Highway-Lane-Driver System (HLDS) previously studied by other researchers [16-17, 44], the mathematical definition of the HLDS problem is modeled with the AFSA model and the transition diagrams are created on the basis of real tests in our study. The main objective is to study the obtained experiments' statistical data of a given runway segment using the AFSA model to validate the model correctness against the real driving. We note that this study is expected to offer insights toward a design of control framework for not only human-driver behavior modeling, but also a control of autonomous/self-driving car, in which control systems need to detect surroundings, interpret sensory information, distinguish between different cars, and plan a path on the road to the desired location [59, 62, 64, 66-67].

The remaining sections of this paper are organized as follows. We overview the pertinent literature in Section 2 and discuss the AFSA framework in Section 3. Next, Sections 4 and 5 provide the highway driving problem formulated using

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the AFSA model and an agent-based modeling, respectively. Then, we provide results and discussion in Section 6. Finally, Section 7 presents our research conclusions.

II. LITERATURE REVIEW

The study of Finite State Automata (FSA) has been widespread as a tool for modeling control of complex systems [9, 23]. For example, Smith *et al.* suggested a formal model of a control scheme for manufacturing systems by using communicating FSA, called Message-based Part State Graph (MPSG) [24]. The authors implemented the model in a shop floor manufacturing problem without any human involvement. Shin *et al.* [2] further considered human activities in the FSA. Even though their work describes human activities in manufacturing systems, the study considers the human as a system component without any physical/environmental constraints. With regard to the Affordance Theory, Gibson [3] initially defined affordances as action possibilities in the environment, objectively measurable and independent of the individual's ability to recognize them. The affordance is thus a relation between an environment and a subject that, through collection stimuli, affords the opportunity for that subject to perform an action [3-4, 36-37, 42]. Since its development, affordance plays a key role in several studies in not only an engineering design context [52-54], but also in a highway driving [16-17, 59-62]. Recent studies in the control design of autonomous driving also implement affordance as a key element [58, 61].

One of an extended concept from FSA is the Discrete Event System Specification (DEVS), which is a hierarchical formalism for modeling control of general systems with discrete events. The extension in DEVS provides a hierarchical concept to define both system behavior and system structure [20, 43]. The DEVS formalism thus is a basis for the AFSA model, in which the FSA and the affordance capture an outer system level and the inner behavior model, respectively. Kim *et al.* [1, 9] provided in their study a link between System Theory of FSA and Affordance Theory suggesting that the FSA corresponds to the ecological sense of affordances. In particular, the authors developed a formal modeling framework called the AFSA model and illustrated a human-machine cooperative manufacturing system in their study. Recent researchers have applied the AFSA model in various problems [12-15]. Ko *et al.* [12] proposed a formal representation of design knowledge for customized design for additive manufacturing (e.g., [45]) using FSA and the concept of affordance to identify the interrelations between AM constraints, user's desire and capabilities, and product's customized features. Oh *et al.* [13] presented a hybrid discrete event system and agent-based model to simulate the performance of a human operator in a human-machine cooperative environment. The authors integrated an affordance-based MPSG control model into a simulation model of human and machine behaviors to aid a manufacturing process plan and control under dynamic situations. Ryu *et al.* [15] presented the modified AFSA model with considering memory decay function of human operators for training and

control of safety-critical human-machine systems. Ko *et al.* [55-56] proposed a design method and architecture for product-service system based on affordance and FSA. The authors illustrated an automotive system and additive manufacturing in their studies.

Another line of research involves agent-based simulation modeling for a highway driving [63-67] and an integrated affordance and agent-based simulation modeling [14, 57-58, 68-69]. Joo *et al.* [14] proposed a simulation model of affordance-based human behaviors for emergency evacuation to mimic perception-based dynamic human actions interacting with emergent environmental changes, such as fire in a warehouse-fire-evacuation case study. The authors argue that existing studies lack a perspective on both the ecological concept of affordance and a formal system that enables human perceptions of dynamic environmental elements. Busogi *et al.* [57] also integrated affordance in the agent-based simulation for evacuation problem. The authors used a cost-based affordance in an agent model to trigger an evacuee movement from a building. Klügl [58] developed an approach to capture agent-environment interactions based on the affordance concept and illustrated their method in a post-earthquake event. Recently, researchers have proposed an integrated affordance and agent-based modeling for autonomous driving context, in which an autonomous or driverless car is capable of sensing its environment and detecting surrounding information [68-69].

Specifically, in the context of driver models, several other researchers have proposed different methods in terms of theoretical and modeling framework not directly related to an integrated agent-based AFSA model to understand human driver behavior in the literature [27-35, 46-51, 59-69]. Macadam [49] provided a systematic review for issues related to human driver modeling. The author suggested that as the vehicle and driver constitute a complex feedback system, the idea of treating the driver and vehicle together as a combined 'man-machine' system is an important aspect. Yang and Koutsopoulos [27] initially classified between the Mandatory Lane Change (MLC) and the Discretionary Lane Change (DLC) concepts. Later, Ahmed [28] proposed the acceleration and lane-changing models in the study. According to the author, the MLC is performed when the driver must leave the current lane, while the DLC is performed to improve driving conditions. Salvucci [29] proposed the mind tracking system with a case study in lane changing detection. Later, Salvucci [30] and Salvucci *et al.* [31] developed different models to understand driver behavior in changing lane and acceleration. In the same year, Toledo *et al.* [32] suggested the model that integrates acceleration, lane changing, and gap acceptance models based on the concepts of short-term goal and short-term plan. A microscopic traffic simulator was used to validate and compare their model against an independently developed model [33-35]. Sun and Elefteriadou [46] studied the behavior of drivers using focus group and use results in micro simulators. Four types of drivers were identified in their study; drivers who always want to keep their current lane and are risk averse; drivers who prefer a better position under low risk; drivers who aim to get a better position with increasing risk; and drivers who always try to get

a better position. Sadigh *et al.* [47] used different approach to model stochastic nature of driver behavior by using convex Markov chains and showed that their model suits well the driving pattern with the presence of threats. In the same year, Mars *et al.* [48] analyzed the driver-vehicle system by varying degrees of haptic shared control. According to the authors, the shared control is more beneficial to the drivers in low visibility conditions. Markkula [50] has further proposed that current driver models in the literature need to be validated on relevant critical situations, such as the near-crash situation.

Given that we intend to fill the void of research gaps for highway-driving studies that implement the AFSA as well as the agent-based modeling, we summarize research gaps in Table I. We note that this literature review is not meant to cover all ranges of countless models for driving behaviors, but to represent existing research gaps related to modeling aspects and how our study contributes to researchers interested in modeling and theory of AFSA as well as practitioners desiring to design a controlling scheme of driving model. This study is also expected to benefit a designed control for autonomous/self-driving car, in which a control system needs to be capable of sensing and navigating its environment, to distinguish between different cars on the road, to detect surrounding information, and to plan a path on the road to the desired location, which are essential elements for ‘affordance-effectivity’ pair of AFSA [59, 62].

In particular, we highlight gaps in the existing research and discuss our contributions as follows:

- A control framework that integrates an agent based model and AFSA in a highway driving system has not been studied and developed in the literature. Thus, we offer a

combined theoretical-practical model in this research.

- Although existing models consider a perspective on the affordance and/or FSA, they are mainly used in an engineering design context and have not been investigated with real data especially for highway driving applications.
- An integrated agent-based simulation with AFSA proposed in this study is intended to provide an understanding of a control framework for driver behavior and for autonomous/self-driving car applications.
- The statistical and correlation analysis in our study suggests an improvement toward the theoretical aspect of AFSA model.

III. AFFORDANCE-BASED FSA MODELING

A. FSA-based Model

The FSA is a mathematical model of computation conceived as an abstract machine that can be one of a finite number of states. The machine will be only one state at a time. Then, it can change from one state to another when initiated by a triggering event or condition called a transition. Finally, the machine will go to accepting or final states represented by double circles [23-24]. A commonly used FSA can be defined in a mathematical form using a quintuple (1) as follows [23, 24].

$$M^{DFA} = \langle \Sigma, Q, q_o, \delta, F \rangle \quad (1)$$

- Σ : a set of input alphabets (a finite non-empty set of symbols);
- Q : a set of finite and non-empty states;
- q_o : an initial state such that $q_o \in Q$;
- δ : a state transition function, such that $\delta: Q \times \Sigma \rightarrow Q$; and
- F : a set of final states, such that $F \subseteq Q$.

Considering an example of a ‘person-climbing-stairs’ system, a transition from a lower level (i.e., an initial state) to an upper

TABLE I
LITERATURE BASED ON MODELING CONTEXT AND HIGHWAY DRIVING APPLICATIONS

Authors	Year	Modeling context			Application	
		Affordance	Finite State Automata	Agent-based model	Highway driving	Others
Shin et al. [2]	2006		×			Shop floor mfg.
Ko et al. [12]	2015	×	×			Additive mfg.
Joo et al. [14]	2013	×		×		Evacuation
Thiruvengada and Rothrock [16]	2007	×			×	
Thiruvengada et al. [17]	2007	×			×	
Smith et al. [23]	2003		×			Shop floor mfg.
Thiruvengada et al. [26]	2007	×	×			Eng. design
Maier and Fadel [52]	2009	×				Eng. design
Ciavola et al. [53]	2015	×				Eng. design
Ciavola and Gershenson [54]	2016	×				Eng. design
Ko et al. [55]	2016	×	×			Eng. design
Ko et al. [56]	2015	×	×			Additive mfg.
Busogi et al. [57]	2017	×		×		Evacuation
Klügl [58]	2014	×		×		Evacuation
Chen et al. [59]	2015	×			×	(Self driving)
Morice et al. [60]	2015	×			×	
Vanderhaegen [61]	2016	×			×	
Krome et al. [62]	2017	×			×	(Self driving)
Nguyen et al. [63]	2014			×	×	
Fagnant and Kockelman [64]	2014			×	×	(Self driving)
Bazzan and Klügl [65]	2014			×	×	
Mladenovic and Abbas [66]	2014			×	×	(Self driving)
Mladenovic and Abbas [67]	2013			×	×	(Self driving)
Ksontini et al. [68]	2013	×		×	×	
Ksontini et al. [69]	2015	×		×	×	
This study		×	×	×	×	

level (i.e., a final state) can occur immediately following the action ‘climb stairs’, which is an input symbol to a current state ‘lower level’. In spite of the FSA success in automated systems design, the model falls short of adequately addressing human aspects. In particular, it only represents the physical aspects of systems behavior without considering the resource availability, a person’s attention, and capability to accomplish a specific action [9].

B. Theory of Affordance

The terms affordance and effectivity represent an environmental property that guides an action opportunity to a human being and the action capability of humans in a certain environment [3]. This notion of affordance is conjectured for a prospective control, in which formal definition of affordance using a juxtaposition function can be defined [4]. Let $W_{pq} = j(X_p, Z_q)$ be a function that is composed of an animal (Z) and an environmental object (X); furthermore, let p and q be properties of X and Z , respectively. Then, p refers to an affordance of X and q is the effectivity of Z if and only if there exists a third property r such that

- $W_{pq} = j(X_p, Z_q)$ possesses r ;
- $W_{pq} = j(X_p, Z_q)$ possesses neither p nor q ; and
- Neither X nor Z possesses r , where r is the third property.

In the case of a ‘person-climbing-stairs’ system (W) discussed earlier, a person (Z) can walk (q), stair (X) can support something (p), and this combination yields climbing property (r). These definitions of affordance, effectivity, and the juxtaposition function are mapped to the state transitions in the FSA and provide a foundation to incorporate the concept of affordance into system modeling and control.

C. Affordance-based FSA in Human-Machine Interaction

The AFSA model incorporates dynamic and perceivable properties of affordance into a formal control model in such a manner that a human operator has a set of possible actions and can take an action based on perceived system conditions (affordances) and his or her capabilities (effectivities) [9]. Mathematically, the AFSA is defined with a six-tuple FSA called M^{comb} (combined model), which describes the rules of state transitions, and a 12-tuple FSA called M^{atom} (atomic model), which contains both human and environmental components as follows ((2) and (3)).

$$M^{comb} = \langle \sum, S, s_o, M^{atom}, \delta_{ext}, F \rangle \quad (2)$$

$$M^{atom} = \langle \{X, Z, W\}, \{P, Q, PA\}, Pr, j, \pi, ta, \delta_{int}, t_{int} \rangle \quad (3)$$

- \sum : a set of transitions among system states;
- S : a set of system states;
- s_o : an initial (starting) state in the system;
- δ_{ext} : a system state (external) transition function, $\delta_{ext}: S \times \sum \rightarrow S$;
- F : a set of final (halting) states;
- M_{atom} : a sub system (atomic model) containing both human and environmental perception states;

- X : an environment system;
- Z : a human (animal) in the environment system;
- W : an Animal-Environment System (AES);
- P : a set of affordances, $P = \{p_1, p_2, \dots, p_m\}$, m is a positive integer;
- Q : a set of effectivities, $Q = \{q_1, q_2, \dots, q_n\}$, n is a positive integer;
- PA : a set of possible actions, $PA = \{pa_1, pa_2, \dots, pa_r\}$, r is a positive integer;
- Pr : a perceptual predicate function for higher order properties, $Pr: X \rightarrow P$, $Pr: Z \rightarrow Q$, $Pr: W \rightarrow PA$, p , q , and pa is a property of X , Z , and PA , respectively;
- j : a juxtaposition function $J: X \times Z \rightarrow W$;
- π : a possible action generation function, $\pi: P \times Q \times C \rightarrow PA$; C is a set of physical preconditions for realization of an action in AES;
- ta : a target action; $ta \in PA$ and $ta \in \sum$;
- δ_{int} : a time advance (internal) transition function, $\delta_{int}: \{P, Q\} \times t_{int} \rightarrow \{P, Q\}$;
- t_{int} : a time advance function.

The internal transition (δ_{int}) connects two sub-states that contain a specific duality of affordance (p_m) and effectivity (q_n) properties. These properties change over time (t_{int}) and the juxtaposition function (j) generates a set of possible human actions (PA). Then, the system transition (δ_{ext}) is made available by the human taking the action (ta) if and only if a physical condition (C) is met within the same time and space.

IV. HIGHWAY-LANE-DRIVER SYSTEM (HLDS)

A. HLDS Problem Description

The HLDS problem and an experiment with real test by Thiruvengada and Rothrock [16] were adapted in this study. This problem contains three highway lanes, two drivers, and an exit as illustrated in Fig. 1(a). Two drivers share the highway lanes and take actions to exit the HLDS. One of the critical considerations for a driving experiment is safety of drivers. Thus, key assumptions for the HLDS problem were defined following the previous study as follows [1, 9, 44].

- Multiple drivers can share the HLDS.
- A lane (L_i) provides the affordance “ L_i is drivable” to a driver (d_j) if and only if the lane is empty for at least three-car length (i.e., drivers are instructed to use this decision criteria for moving into a lane) at any given time accounting for safety factor for moving into lane without a crash.
- The drivers possess the capability to perceive the affordances offered by the environment (other cars and highway lanes) based on their visual information and view angle through a front, a rear view mirror, and side mirrors.
- The drivers drive with speed instructed in each scenario and maintain their velocity throughout their driving.

B. AFSA Representation for the HLDS

Kim *et al.* [1] illustrate the AFSA model using a set of nodes (discrete states of the system) and arcs (the transitions between states), where a set of potential properties (affordances and effectivities) are defined by a set of transitions in each state for the HLDS problem. Whereas the set of nodes or states (S) is lane 1, lane 2, lane 3, exit (i.e., goal state), and error state (i.e., absorbing state); the set of final (halting) states (F) includes exit and error state. The set of final states in the model implies that the model will be terminated if either ‘exit’ or ‘error state’ is reached (Fig. 1(b)). To make a transition to the next state, the human driver considers appropriate perceptual conditions of affordance and effectivity to take a possible action. The perceptual information is represented by functions of visually perceivable elements, such as dimension and location within a specific time and space range. In particular, the sub-state is defined with system affordances (i.e., perceived drive-ability of the lane 1, 2, 3, and exit) and driver effectivities (i.e., driver’s perceived capability to make a lane change to or keep going on the lane 1, 2, 3, and exit). Next, the physical pre-conditions can be defined as c_i , where $i = \{1,2,3,4\}$ to represent the physical requirements for realization of a specific action that the L_i is empty for at least three times the car length and a driver does not pass by the exit. Finally, the set of possible actions (PA) can be included (Fig. 1(c)). Mathematically, the HLDS problem can be modeled with the AFSA model as follows ((4) and (5)).

$$\mathbf{M}^{comb} = \langle \Sigma, S, s_o, \mathbf{M}^{atom}, \delta_{ext}, F \rangle \quad (4)$$

$$\mathbf{M}^{atom} = \langle \{X, Z, W\}, \{P, Q, PA\}, Pr, j, \pi, ta, \delta_{int}, t_{int} \rangle \quad (5)$$

- Σ : a set of transitions among system states, $\Sigma = PA$;
- $S = \{s_0 = \text{lane 1}, s_1 = \text{lane 2}, s_2 = \text{lane 3}, s_3 = \text{exit lane}, s_4 = \text{absorbing state (error state)}\}$;
- $\delta_{ext}: S \times \Sigma \rightarrow S$;
- $F = \{s_3, s_4\}$;
- X : confederate driver and highway lanes;
- Z : subject driver;
- W : HLDS;
- $P = \{p_1 = \text{drive-on/change-to-lane-1-able}, p_2 = \text{drive-on/change-to-lane-2-able}, p_3 = \text{drive-on/change-to-lane-3-able}, p_4 = \text{exit-the-highway-able}\}$;
- $Q = \{q_1 = \text{drive on/change to lane 1}, q_2 = \text{drive on/change to lane 2}, q_3 = \text{drive on/change to lane 3}, q_4 = \text{exit the highway}\}$;
- $C = \{c_1 = L1 \text{ is empty for at least three times the car length and a driver does not pass by the exit}, c_2 = L2 \text{ is empty for at least three times the car length and a driver does not pass by the exit}, c_3 = L3 \text{ is empty for at least three times the car length and a driver does not pass by the exit}, c_4 = \text{the exit is empty and a driver does not pass by the exit}\}$;
- $J: X \times Z \rightarrow W$;
- $Pr: X \rightarrow P, Pr: Z \rightarrow Q, Pr: W \rightarrow PA$;
- $\pi: P \times Q \times C \rightarrow PA$;
- ta : a target action; $ta \in PA$ and $ta \in \Sigma$;
- $\delta_{int}: \delta_{int}: \{P, Q\} \times t_{int} \rightarrow \{P, Q\}$;
- t_{int} : a time advance function;
- $PA = \{\text{drive to/change to lane 1 iff } c_1, \text{ drive to/change to lane 1 iff } c_2, \text{ drive to/change to lane 3 iff } c_3, \text{ exit the highway iff } c_4\}$.

C. Designed Experiment

The experiment conducted by Thiruvengada and Rothrock with two drivers, three lanes, and a length of 80 blocks with a

block of 4.5 meters is adapted in this paper [16, 17, 44]. We use the real test results obtained from the authors to test the AFSA modeled in the agent-based simulation environment. A designed experiment is briefly discussed in this section due to space limit and we encourage interested readers to check ref. [16, 44]. Fig. 2 adapted from Ref. 44 shows a layout of the PTI driving track with real test setup. We note that an exit (lane 1) in Ref. [44] is replaced with an exit (lane 3) in our study to aid comprehension without loss of generality. In particular, four test drivers were randomly grouped into two pairs, in which one driver was randomly assigned the role of driver 1 (i.e., subject driver (SD)) and the other driver was assigned the role of driver 2 (i.e., confederate driver (CD)). Each of the drivers was male aging between 40-65 years and possessed a valid commercial driver’s license at the time of the experiment with at least 15 years of driving experience. The experiment was also conducted during daytime between 2- 4:30 pm eastern standard time to ensure ample daylight while driving. During the experiment, the CD was instructed to follow a pre-scripted path, whereas the SD’s behavior was studied. The CD was instructed that the lane can be changed after passing a visual cue (orange cone) on the driving track. Both drivers received specific instructions prior to beginning each trial about their starting location and the target velocity to maintain [44]. We summarize the designed experiment in Table II with three factors based on relative velocities (e.g., whether SD was driving faster than CD), starting lane positions (e.g., whether CD was vertically closer to exit lane), and starting block positions (e.g., whether CD was horizontally closer to exit lane) to observe and analyze driver behavior. Then, based on all

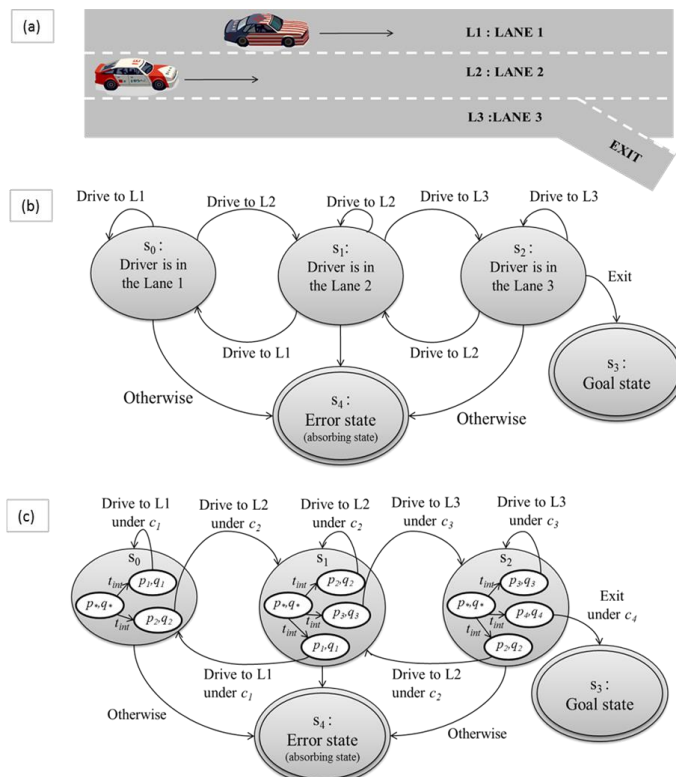


Fig. 1. (a) HLDS problem, (b) FSA model for HLDS, (c) AFSA model for HLDS (adapted from [1], [16])

possible scenarios, a subset of 12 scenarios were chosen, in which two levels of relative velocity ($V_{SD} > V_{CD}$ or $V_{SD} = V_{CD}$) were used [44] to properly observe the SD's interaction with CD as shown in Table III. In the first pair (i.e., experiment 1), the driver who was assigned the role of SD committed driving from scenarios 1 to 12 with the 1st and 2nd round. Next, in the second pair (i.e., experiment 2), data were collected from the other SD, who committed the same driving scenarios with the 1st and 2nd rounds. Thus, there are a total of 48 experimental trials (i.e., with 12 different scenarios, two replications between drivers, and two replications within drivers). Experiments 1 and 2 contain trials 1-24 and trials 25-48, respectively. To track the positions of drivers, a vehicle was equipped with the Differential Global Positioning System (DGPS) unit, in which the primary reference point is the location of the DGPS's base station unit and the secondary reference point is the point at the beginning of the starting lane's first block (Fig. 2). Both of these DGPS units provide positional information about the respective test vehicles in terms of latitude, longitude and altitude, which is then transformed into x-y Cartesian coordinate system (with reference to the secondary reference point) [See ref. 44]. Thus, the experimental lane position data for each driver at a particular time and space can be obtained. Given the real test outputs from the 48 trials, we then compare with outputs from the agent-based AFSA modeling framework (Fig. 3). The hypothesis testing is conducted to see the impact of agent preference and affordance based model (H_0 : *There exists no difference in driving behavior between using the proposed agent-based AFSA simulation framework and the actual driving experiment*), which is further analyzed using comparative and correlation study. The horizontal arrows in Fig. 3 suggest that the agent-based model uses experimental settings (e.g., number of drivers, lane positions, etc.) from a previous study as an input set for a modeling basis. In addition, the results from the simulated data are compared with the actual driving as a proof of modeling concept. This recursive process ensures the verification and validation of the simulation model.

V. AGENT-BASED SIMULATION OF AFSA-BASED HLDS PROBLEM

A. HLDS Simulation Model Description

Simulation model and verification/validation process are essential [18-22]. Sargent [22] suggested that validation techniques can be used either subjectively (e.g., exploring model behavior) or objectively (e.g., comparing using statistical tests and procedures). In this study, we simulate the AFSA representation for the HLDS problem using the agent-based simulation approach. The transition diagrams with the lane position data are obtained from the simulation model and are compared with the real data obtained from the actual driving experimental trials. Fig. 4 illustrates the agent-based simulation modeling for the HLDS using a software package called AnyLogic, which is capable of modeling agent-based, system-dynamics, and discrete-event simulation [38]. Fig. 4(a) presents the rule-based state charts for the SD and the CD.

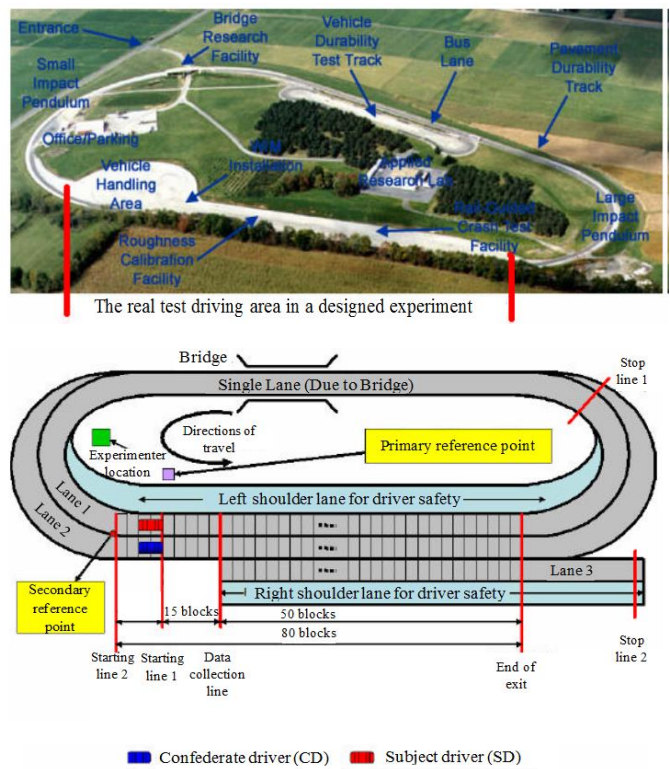


Fig. 2. PTI's real driving track and experimental setup (adapted from [44])

TABLE II
A DESIGNED EXPERIMENT WITH THREE FACTORS

Factors	Levels
Starting lane position	Lane 1; Lane 2
Starting block position	Block 1; Block 15
Relative velocity (V)	$V_{SD} (40 \text{ mph}) > V_{CD} (20 \text{ mph})$; $V_{SD} (20 \text{ mph}) = V_{CD} (20 \text{ mph})$; $V_{SD} (20 \text{ mph}) < V_{CD} (40 \text{ mph})$;

TABLE III
A SELECTED 12 SCENARIOS USED IN THE EXPERIMENT

#	Relative starting position of drivers				Relative velocity
	Subject driver		Confederate driver		
	Lane position	Block position	Lane position	Block position	
1	Lane 2	Block 1	Lane 2	Block 15	$V_{SD} > V_{CD}$
2	Lane 2	Block 1	Lane 1	Block 15	$V_{SD} > V_{CD}$
3	Lane 2	Block 15	Lane 1	Block 15	$V_{SD} > V_{CD}$
4	Lane 1	Block 1	Lane 2	Block 15	$V_{SD} > V_{CD}$
5	Lane 1	Block 1	Lane 1	Block 15	$V_{SD} > V_{CD}$
6	Lane 1	Block 15	Lane 2	Block 15	$V_{SD} > V_{CD}$
7	Lane 2	Block 1	Lane 2	Block 15	$V_{SD} = V_{CD}$
8	Lane 2	Block 1	Lane 1	Block 1	$V_{SD} = V_{CD}$
9	Lane 2	Block 1	Lane 1	Block 15	$V_{SD} = V_{CD}$
10	Lane 1	Block 1	Lane 2	Block 1	$V_{SD} = V_{CD}$
11	Lane 1	Block 1	Lane 2	Block 15	$V_{SD} = V_{CD}$
12	Lane 1	Block 1	Lane 1	Block 15	$V_{SD} = V_{CD}$

While the CD considered as a part of an environment was instructed to follow a pre-scripted path with a deterministic route, the SD makes a decision to drive to different lanes or to go straight in the same lane based on the SD's preconditions (C) representing driver's decision criteria until the SD in the

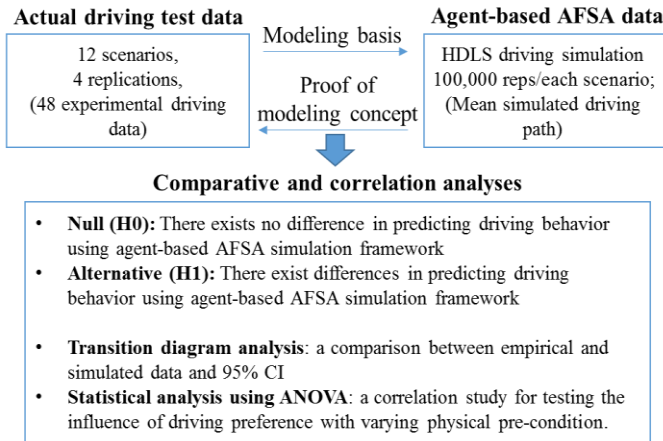


Fig. 3. Structure of experimental design

simulation reaches the exit lane. This allows us to observe and compare the simulation model’s behavior against the real driving experiment.

B. Model Verification

One of the most important, difficult tasks facing a model developer is the verification and validation process of the model. Researchers suggest that this process should be performed during a model development and typically requires an experiment with the real system [19]. In particular, a simplified version of the modeling process includes three key components: the problem entity, the conceptual model, and the computerized model [22]. The problem entity represents the proposed system of interest (i.e., HLDS), the conceptual model is a model representing the problem entity (i.e., the AFSA mathematical model), and the computerized model is a computer representation of the conceptual model (i.e., the agent-based simulation model). This modeling process is iterative and continues until a consensus among model developers, stakeholders, and decision makers is reached [10].

Fig. 4(b) and Fig. 4(c) illustrate a screenshot of the simulated HLDS problem after the model is run reasonably long until the agent driver reaches a visual cue. Whereas the SD decides to continue driving in lane 2 without any lane changing, the CD is instructed to change from lane 2 to lane 3 after passing a visual cue. In order to verify the model, experts’ comments for the AFSA and simulation model representing the HLDS problem were used, which helped us to improve the model. Next, the operational validation is performed with a comparative study with actual test driving output. Given 48 experimental outputs based on actual lane-position data and observations of lane changing for each driver at a particular time and space from the driving scenarios, the agent-based AFSA simulation outputs are similarly reported in terms of a transition diagram representing the average lane position data in time-space dimension to aid a comparative study. We also perform a model correlation analysis to see the relationship between varied physical preconditions and the AFSA-based simulation model.

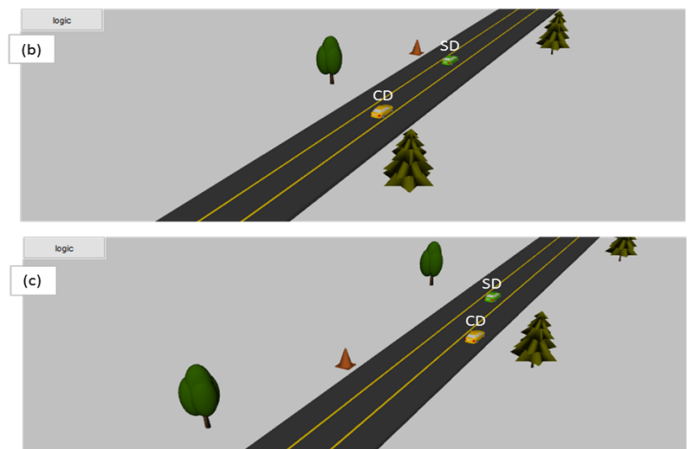
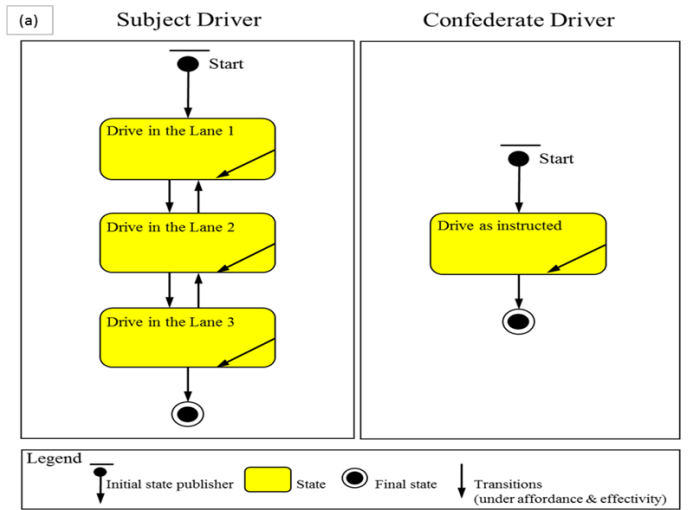


Fig. 4. Agent-based modeling framework for the HLDS: (a) state charts for a subject driver (SD) and a confederate driver (CD), (b) a screenshot before a visual cue, and (c) a screenshot after a visual cue

TABLE IV
NOTATION FOR THE TRANSITION DIAGRAM

Notation	Description
	Starting position (Block i , Lane j) of Driver 1 (Subject Driver) in each scenario, where $i = 1, \dots, 80$, and $j = 1, 2, 3$
	Starting position (Block i , Lane j) of Driver 2 (Confederate Driver) in each scenario, where $i = 1, \dots, 80$, and $j = 1, 2, 3$
	Current position from an empirical output of Driver 1 or Driver 2 at time t
	Current position on average from the agent-based simulation model of Driver 1 at time t
	Possible transitions from the Affordance-based FSA model (p^* , q^*) under initial pre-conditions (C) or adjusted pre-conditions (C^*) at time t
	Block-land diagram, where the x -axis denotes block numbers and the y -axis denotes lane numbers
	The exit lane positioned at lane 1 after passing 80 blocks of 360 meters

C. Transition Diagram Analysis

Given notations in Table IV, a transition diagram is generated for the outputs obtained from actual driving test and agent-based AFSA simulation for 48 experimental trials. For each trial, four sub-transition diagrams are next proposed to aid a comparative study (transition diagrams (a)-(d)). These sub-transition diagrams allow us to investigate observable behavior of the model and compare between simulated results and empirical outputs at the particular time-space dimension. Each sub-transition diagram (a)-(d) is discussed below.

1) *Sub-Transition Diagram (a): The plotted CD's actual driving path (Environment)* - This diagram (a) shows CD's positions at the particular time-space dimension. The CD acts as a part of environment following the pre-scripted path from the beginning to the end of the exit lane and is instructed to maintain the pre-specified speed. He or she is instructed that the lane can be changed after passing a visual cue.

2) *Sub-Transition Diagram (b): The shaded output of agent-based AFSA simulation model for SD alone* - This diagram represents the simulation output of one agent alone (SD) without any intervention of CD. That is, the shaded, grey area shows possible transitions (i.e., p^* and q^* from the AFSA model) generated from using the agent-based simulation approach under initial pre-conditions (C) for 100,000 replications.

3) *Sub-Transition Diagram (c): The shaded output of agent-based AFSA model for SD interacting with CD (SD's actual driving path vs. mean SD simulated path)* - This diagram shows the simulation outputs and actual lane position data of the SD (driver 1), given that there is an interaction between the two drivers and that CD acts as a part of the environment in the AES system. First, the simulation results of the SD under a set of physical pre-conditions (C) are shown using the shaded output area (possible transitions). Then, the simulated path of SD calculated as the mean path is plotted with a fixed interval in the diagram. The SD's positions from the actual driving data at the particular time-space dimension are also plotted from the beginning to the end of the exit lane. It is clear that the possible transitions from the model of SD without any intervention of CD (sub-transition diagram (b)) are affected by the existence of CD. In addition, a comparative study can be done between the SD's actual driving path and mean simulated path.

4) *Sub-Transition Diagram (d): The shaded output of agent-based AFSA model for SD interacting with CD (SD's actual driving path vs. mean SD simulated path under C^*)* -

This diagram shows a particular result from the correlation study of the AFSA model, where a set of physical preconditions called adjusted physical pre-conditions (C^*) are varied. These physical pre-conditions are treated as the driver's preference on the lane gap criterion of the driver's car with respect to the one in front of the driver, which can be varied in the AFSA model. Given an autonomous/self-driving environment, varying physical preconditions also implies setting parameters to detect different cars in a driving path. We illustrate the case of relaxing from the three-car length (C) to one-car length (C^*) to illustrate a case of conservative and aggressive driver, respectively. That is, the shaded area (possible transitions) shows the simulation results of the SD under a set of adjusted physical pre-conditions (C^*). The mean path of the simulation results at a fixed interval is compared with the path obtained from the actual driving path of the SD based on C^* .

VI. RESULTS AND DISCUSSION

The agent-based AFSA simulation model developed using AnyLogic software was run for 48 experimental trials on a PC with an Intel (R) Core (TM) i7 @3.50 GHz and 32.0 GB of RAM. Each run is terminated when the SD reaches the final state. Initially, the number of total run is set high enough to avoid any bias in the statistical inference that could affect the results. In particular, the number of 100,000 replications was run with reported computational time of approximately 10 seconds. The initial condition of each run for SD and CD follows a setup of relative position and velocity based on 12 scenarios. Next, we compute the mean simulated path from shaded output of simulation model at 95% confidence interval (CI) for all the 48 experimental trials. When the original physical pre-conditions (C) with three-car length are used, the mean simulated paths from the simulation model appear to coincide and fit well with the actual driving data of SD's moving paths in the space and time dimension for 94% of all the experimental trials (45 out of 48 trials). However, the less of the trials (scenarios 4 (experiment 1, trial 17) and 12 (experiment 1, trials 2 and 15)) show rejecting H_0 with statistically significant difference at 95% CI. Comparing between two pairs of drivers, while the experiment 2 shows coincide data, significant differences are found in experiment 1 (Recall that different pairs of drivers with different SD are

TABLE V
AN ILLUSTRATIVE CASE OF SCENARIO 12, EXP. 1 TRIAL 2 (SHADED AREAS SHOW AGGREGATED DRIVING PATHS AT 95% CI)

Block	B1	B6	B11	B17	B23	B29	B34	B40	B46	B51	B57	B63	B69	B74	B80
Lane 1	0.25	0.25	0.25	0.25	0.25	0.21	0.16	0.12	0.08	0.04	0	0	0	0	0
	0.38	0.38	0.38	0.38	0.38	0.31	0.24	0.18	0.12	0.06	0	0	0	0	0
	0.25	0.25	0.25	0.25	0.25	0.23	0.20	0.18	0.14	0.11	0.08	0.05	0.03	0	0
	0.13	0.13	0.13	0.13	0.13	0.14	0.16	0.18	0.17	0.16	0.16	0.11	0.06	0	0
Lane 2	0	0	0	0	0	0.06	0.12	0.18	0.20	0.21	0.23	0.16	0.08	0	0
	0	0	0	0	0	0.04	0.08	0.12	0.14	0.17	0.19	0.17	0.15	0.13	0.13
	0	0	0	0	0	0.02	0.04	0.06	0.09	0.12	0.15	0.18	0.22	0.25	0.25
Lane 3	0	0	0	0	0	0	0	0	0.04	0.08	0.11	0.20	0.28	0.38	0.38
	0	0	0	0	0	0	0	0	0.03	0.05	0.08	0.13	0.19	0.25	0.25
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

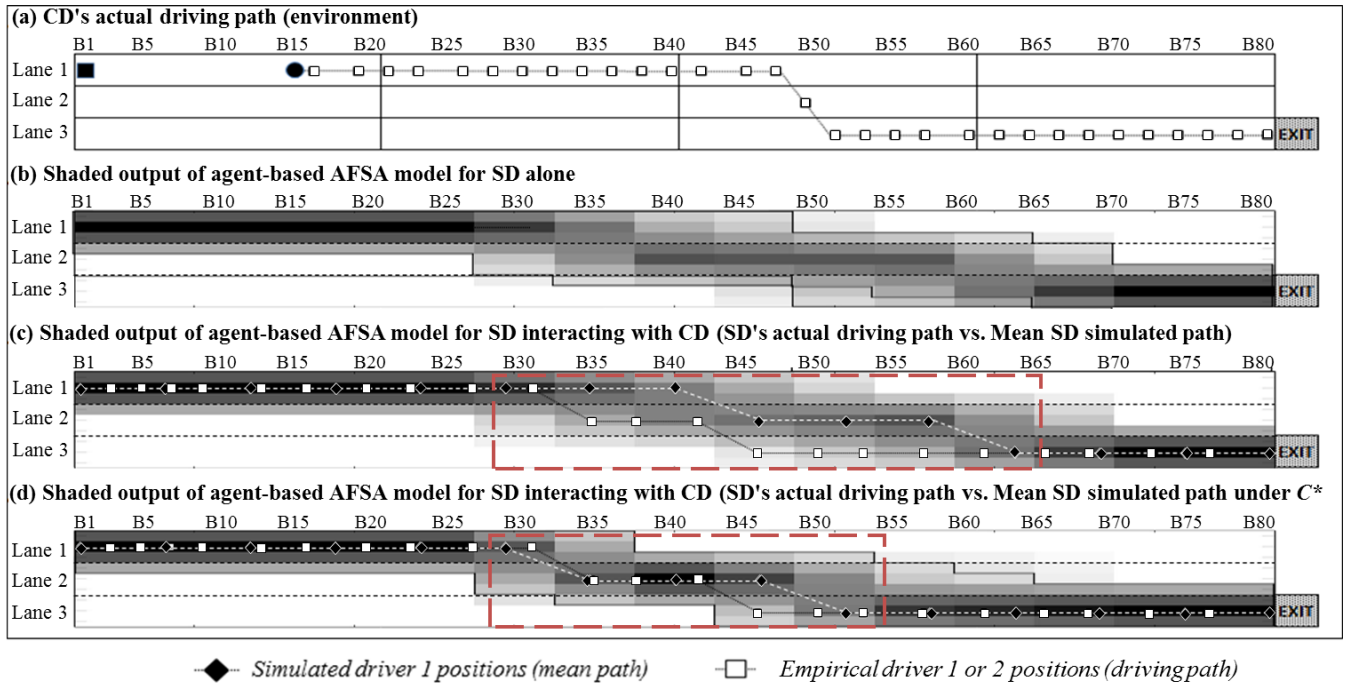


Fig. 5. Transition diagram for scenario 12, exp. 1, trial 2

termed experiments 1 and 2). This result implies that different drivers/agents may be more or less conservative when driving and a set of physical pre-conditions can be modeled to reflect such behavior. A design of control framework to simulate an autonomous driving car could also exploit such varying parameter of physical pre-conditions to reflect a driver (or a passenger in a driverless car) preference. Table V illustrates the simulation results for the scenario 12 (experiment 1, trial 2) visualized as shaded areas. The shaded output area is calculated based on the probabilities of driving paths of agent-based AFSA modeling of SD at 95% CI.

We illustrate the transition diagrams associated with scenario 12 (experiment 1, trial 2), which present significant different paths when the original physical pre-conditions (C) with three-car length are used (Fig. 5). When comparing the sub-transition diagrams (b) and (c), it is clear that the shaded area of SD alone is affected by the existence of CD. Further, while the mean simulated path of SD at the 30th block suggests that the driver should go straight, the actual driving data show that the driver decides to change from lane 3 to lane 2. However, when we relax the set of physical pre-conditions from the three-car length (C) to one-car length (C^*) in the sub-transition diagram (d), the mean simulated path of the SD at around the 30th block suggests that the SD should change from lane 3 to lane 2, which coincides with SD's decision in the actual path. As the most critical parts of a driver model can be validated by analyzing the most important observable data [31], we further discuss three observations related to the human error, the lane change decision, and the model's physical pre-conditions.

A. Human Errors of Commission and Omission

One possible reason can be attributed to the driver (human) error. By definition, human error implies that something has

been done that is not intended by the human and is deviated from the goal [39]. In this situation, the human errors called errors of commission and omission can be used to enlighten driver behavior [40, 41]. That is, SD may incorrectly perceive that the empty length of a lane between longitudinal positions of SD and CD is still more than three-car length (i.e., error of commission), or that SD may completely fail to pursue a lane change manoeuvre during a close-call situation (i.e., error of omission). Although it is not always the case, an accident may occur if either of these errors is present. Thus, human errors should be well incorporated in the computational model of human-involved complex systems and behavioral prediction.

B. MLC and DLC with Lane Change Decision

Significant differences in transition diagrams between simulated and actual driving are found during the lane change decision. Typically, the lane change behavior can be classified as either the MLC or the DLC [28]. Salvucci *et al.* [31] suggested that while the MLC is performed when the driver must leave the current lane, such as facing a lane drop, the DLC is performed to improve driving conditions. Further, when MLC conditions do not apply, the driver will decide whether to perform DLC by considering two conditions: whether current driving conditions in the same lane are satisfactory (e.g., based on desired speed) and, if not, whether any other lane is better than the current lane (e.g., based on the density of traffic).

Let's examine Fig. 5, for example. As the actual driving experiment was controlled in the case study, the MLC conditions are not relevant (e.g., the road quality was checked prior to an experiment). Thus, the DLC of SD in the 'time-space' dimension is investigated. At time ' t_9 ', the block-land positions of the SD and CD are at (Block 27, lane 1) and (Block 30, lane 1), respectively. Next, at time ' t_{10} ', the SD decides to continue driving in the same lane and his or her next

position is at (*Block 31, lane 1*), while the CD's position is at (*Block 32, lane 1*). At this time, the SD perceives that the current lane's conditions are not satisfactory due to the existence of CD's vehicle in front of him or her and prefers to improve the driving condition. The SD perceives that the adjacent lane (lane 2)'s conditions are better and decides to change to lane 2. Then, at time '*t11*', the SD makes the lane change and the new position is at (*Block 35, lane 2*) as shown in the figure.

C. Human Behavioral Propensities to Physical Pre-Conditions (C)

As shown in the AFSA framework, the SD will follow the set of physical pre-conditions, where $(C) = \{c_1, c_2, c_3, \text{ and } c_4\}$. However, driver behaviors in reality are nondeterministic and SD may be more or less conservative than what is estimated in the model. When the assumption related to the set of physical pre-conditions is adjusted, such that $(C^*) = \{c_1^*, c_2^*, c_3^*, \text{ and } c_4^*\}$, a different driving path can be simulated from the model. The sub-transition diagrams (d) in Fig. 5 illustrate different paths when the lane gap criterion is relaxed from the three-car length to the one-car length.

We can similarly examine SD's behavior in the time-space dimension using the adjusted set of physical pre-conditions $(C^*) = \{c_1^*, c_2^*, c_3^*, \text{ and } c_4^*\}$. That is, at time '*t9*', SD and CD are at the block-lane positions (*Block 27, lane 1*) and (*Block 30, lane 1*) respectively. The SD perceives that the empty gap length between two vehicles (i.e., his/her car and the CD's car) is two-car length and he or she can choose to continue driving in the same lane (lane 1) or make a lane change to the adjacent lane (lane 2), without breaking the set of physical pre-conditions (C^*) . Next, at time '*t10*', the SD decides to continue driving in the same lane and his or her next position is at (*Block 31, lane 1*), while the CD's position is at (*Block 32, lane 1*). At this time, the SD perceives that the physical pre-condition (c_i^* = the empty length for at least one car length) will be broken and makes a lane change to lane 2. Then, at time '*t11*', the SD's position is at (*Block 35, lane 2*). The above examination suggests that the set of physical pre-conditions (C) is a system property of the AFSA model that is dependent on characteristics of human participants. We note that understanding a physical pre-condition is important in controlling viewpoint for a number of applications. For example, driver driving on a passing lane (i.e. the leftmost lane in the U.S.) on a multi-lane highway may also differ from a driver driving on a regular lane (i.e. the right lane in the U.S.). Varying physical preconditions also implies a parameter setup of detecting different cars in an autonomous/self-driving environment. Mathematically, we propose that $\pi: P \times Q \times C(Z) \rightarrow PA; C(Z)$ is a proper set of physical pre-conditions for realization of an action, dependent on Z in the AFSA framework.

We further investigate the Analysis of Variance (ANOVA) of the positional errors between actual driving and simulation data. The statistical results show that the mean error rates of the driving paths are significantly dependent on the set of physical pre-conditions, $C(Z)$, of each driver agent at 95% CI (Table VI). The P-value for the F test statistic for both C and Z (i.e., driver)

TABLE VI
ANOVA ANALYSIS FOR THE SET OF PHYSICAL PRE-CONDITIONS (C)

Source	DF	Seq SS	Adj MS	F	P
<i>Scenario</i>	1	0.21	0.21	3.54	0.08
<i>C</i>	1	1.38	1.38	23.01	0.00
<i>Person</i>	1	0.99	0.99	16.52	0.00
<i>Error</i>	12	0.72	0.06		
<i>Total</i>	15	3.31			
$S = 0.24$ $R-Sq = 78.21\%$ $R-Sq(Adj) = 72.76\%$					

is less than 0.005 providing strong evidence against the null hypothesis. The squared multiple correlation (R^2) also indicates that 78.21 % of the variability in the mean simulated path can be explained by $C(Z)$. That is, the set of physical pre-conditions dependent on agents is the significant factor of determining the driving patterns. By adjusting the parameter $C(Z)$ for the driver's driving preference, the agent-based simulation approach for the AFSA model provides us with an appropriate prediction of driving patterns of drivers.

VII. CONCLUSIONS AND FUTURE RESEARCH

Researchers have studied the modeling and control framework of human-machine system for better prediction of the system behaviors, improved flexibility, and seamless integration in the system operations. The development of AFSA model is once such novel framework that incorporates stochastic human behaviors with environmental opportunities in a systematic way. Road traffic analysis and driver's behavior simulations are also one of the most important challenges in the context of building autonomous vehicles using public roads where there is a need of exact mapping and prediction of the human behavior. In this study, we proposed the first agent-based AFSA simulation model for the affordance-based highway driving and exit maneuver and analyzed the results using comparative and correlation study. We mapped each of real driving trajectory with agent-based AFSA simulation results for all the 48 experimental data sets. The statistical results show that the agent-based AFSA simulation fits well with driver's behavior in the designed experiment for 94% of all the trials. The less of the trials with significantly statistical difference in mapping driver's behavior with the model at 95% CI were then analyzed using the viewpoint of human errors, lane change decision, and human behavioral propensities. The ANOVA analysis was done to explore the influences of the physical preconditions on agents that constitute the existence of affordances.

The integrated affordance-based FSA with agent-based transportation simulation and experimental design provided in this paper are critical for practitioners and developers to enhance the understanding in control framework of highway driving system from the viewpoint of human-machine cooperative tools. The proposed research is also expected to benefit a design in smart transportation systems, in which both autonomous driving and manual driving coexist.

Understanding interconnection between affordance-based human driving behaviors and fully automated driving system is critical to avoid a possible tragic event in the transportation systems. The presented simulation framework can also support the planning of the appropriate positions of the highway exits, given human driving preferences. We expect that this research will provide the systematic approach for the design of efficient highway driving system.

Regardless, some limitations exist and future directions are discussed next. Given that data of an actual driving case study from a previous study is used for a comparative purpose, an extended experimentation in highway driving domain with more number of drivers for a larger-scale of simulation is one critical future direction. In a real environment, people also behave differently depending on different types of road ways and subject to their age, gender, and so on. Thus, further understanding of these elements is needed. In addition, it is interesting to integrate the modeling framework of this study at the control level with other driving models in transportation management. Finally, as the AFSA model is generic and can effectively represent human-system interactions, the AFSA model's validity can be increased by further applying to other problems that integrate humans and system operations, such as the driver-transportation system, operator-robot cooperative manufacturing system, etc.

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