From Summit to Seafloor - Lifted Weight as a Function of Altitude and Depth

A comparison of lifted weight as a function of altitude and depth reveals big differences by a factor of about 3'000 in total from 11 km above sea level to 10 km below sea level, divided into factors of:

- **~ 4 between summit and sea level,**
- **~ 400 between flying close to the ground and planing on water,**
- **~ 2 between planing on water and in a fully submerged state.**

The most dramatic changes are due to different fluids and levels of altitude. The most interesting sector to discuss lift is close to sea level: aircraft approaching the ground, plates planing on water and hydrofoils only barely submerged in water. There is one basic similarity across of these: *Almost any shape, as long as it is not too thick, will work as an (air)foil and produce lift when the angle of attack is in the right range.* [1]

1. Subject of discussion

One may consider a rectangular, flat plate as prototype for any foiling shape (fixed or rotary wings) to discuss similarities in different fluids (air or water): (wing)span b = 10 m, chord c = 1 m, thickness t = 0, neglecting buoyancy according to Archimedes' principle (e.g. for a metal plate) and at speeds as follows: subsonic in air, planing on water and subcavitating in water.

2. Pressure fields, large and small scale

Any object in any fluid on earth is supported by the ground, no matter whether it is up in the air or submerged underwater (see fig. 1).

Figure 8.5.3 The airplane's pressure footprint on the ground in steady, level flight. From Prandtl and Tietjens, (1934). Used with permission of Dover Publications, Inc.

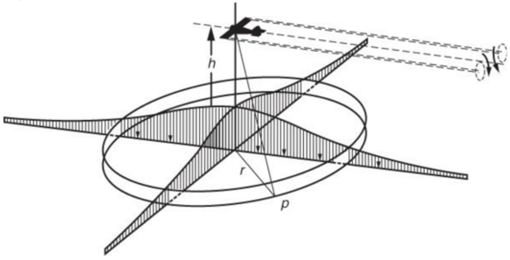


fig. 1: Pressure field at large scale (from [1]).

Objects of discussion in fluids can be moved by applying pressure - but under no circumstances by "suction". To avoid a basic misconception, the pressure field around a foil in fig. 2 is shown with only positive pressure differences. This represents how lift could be felt on the camber mean line.

Figure 7.3.3 Airfoil pressure distributions represented graphically as vectors. (a) Arrows proportional to the pressure difference $p - p_\infty$. (b) Arrows proportional to the absolute pressure

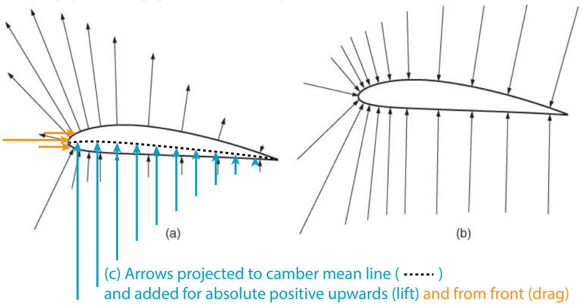
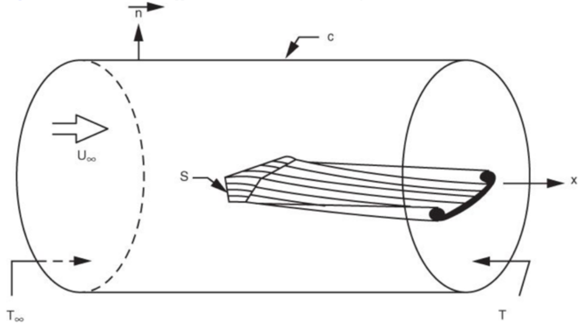


fig. 2: Pressure field at small scale (from [1]), represented by positive pressure differences on foil only (↑→).

3. Flow field: downwash/upwash and vortices

Lifting foils produce downwash and upwash (see fig. 3). The equalisation of pressure differences appears as vortices. Outside a (large enough) control volume, all pressure is equal (to the state before the object moved through it). Therefore all mass of fluid moved downwards (by the foil) must equally come up again.

Figure 6.1.1 Control volume for application of conservation laws in body-centered reference frame



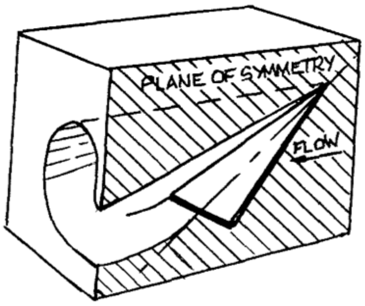


fig. 3: Downwash/upwash and vortices behind an airfoil (from [1]) and hydrofoil at cavitational state (from [5]).

See par. 9, fig. 14-16 for illustrations of phenomena.

4. Calculation of lift

Lifted Weight $W = \frac{C_L \cdot \rho \cdot A \cdot v^2}{2 \cdot g} \cdot C$

Lift Coefficient C_L = 0.11 per °deg a.o.a. for high aspect ratio

Area A [m^2] true speed v [m/s] g = 9.8 m/s^2

ρ : function of altitude and depth (see paragraph 5).

C : correction factor for ground / submerge effect.

Angle of attack a.o.a.

All flying craft use lift produced by a.o.a. for most or all of the time that they are airborne. A.o.a. needs to stay within certain limits (see fig. 5).

fig. 5a-b: Instruments showing angle of attack: strings used as tell-tales on gliders and sailboats.

Additional lift produced by the cambered shape of foils increases economy, but is neither necessary nor sufficient to support an aircraft – except in extraordinary configuration, e.g. very low weight or very large wing area, or in extreme operation status such as high speed cruise. See fig. 6 for modern general aviation aircraft and military trainer aircraft.

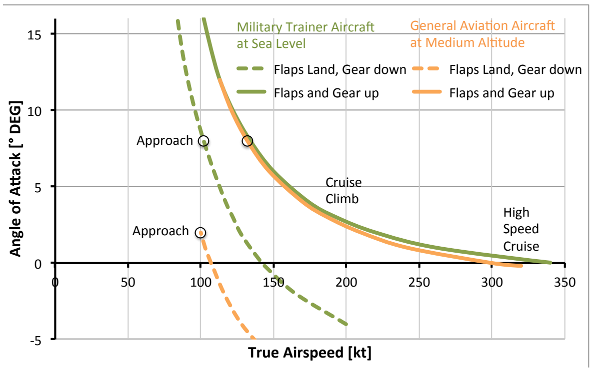


fig. 6: A.o.a. vs. true air speed: Actual data of modern aircraft [6].

Many aircraft have symmetrical foils and therefore cannot produce lift other than by a.o.a., for instance

- Extra Aircraft Aerobatic Sportster 300L (MA15S to MA12S);
- Lockheed Martin F-16 Fighting Falcon (NACA 64A204);
- Sikorsky helicopter S-300 (Hughes/Schweizer), (NACA 0015), S-300 hovers at about 2° deg;

and many more.

Lift coefficient C_L vs. angle of attack a.o.a.

Lift coefficient per deg. a.o.a. (angle of attack) depends on the ratio of wingspan to chord and is not bigger than 0.11. This function can be described for airfoils, hydrofoils and for plates planing on water. See par. 9, fig. 17-18 for illustration and reference.

airfoils: wings of different aspect ratio; foil shape cambered vs. symmetrical

hydrofoils from sub- to supercavitational state for short wings (low aspect ratio)

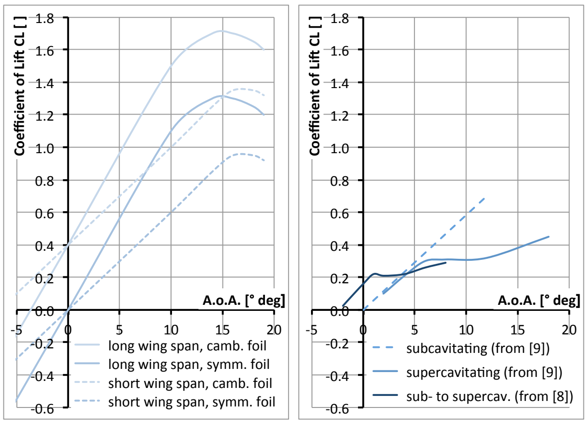


fig. 7a-b: Lift coefficient C_L vs. a.o.a.: Typical values from different references.

Maximum possible C_L (airborne at stall) is no more than ~2.0, while C_L at a.o.a = 0° for cambered foils is not bigger than ~0.4. This means that a.o.a. under most circumstances contributes much more to maximal lift than the non-symmetrical shape of a foil (in any case for a.o.a. > 4 deg. and ratio >10).

Shape of airwings / hydrofoils

Airwings of high aspect ratio are designed to generate as much lift as possible at low to moderate speeds. The higher the max. speed the more:

- the wings are swept back ;
- the aspect ratio is low (wingspan vs. chord);
- the profiles are symmetrical (not cambered).

Hydrofoils resemble airfoils for high speed aircraft that the aspect ratio is low and in some configurations foils are swept back (par. 9, fig. 22).

Ground approach and submergence in water

Lifted weight by a wing or foil increases when approaching the ground or submerging in water. These effects have significant influence only within a small range of distance from ground / sea level of about 10 to 25 % of wingspan while they can already be felt when the height above ground or depth below sea level is of the order of the wing span.

Due to **ground effect**, the induced lift increases significantly by a factor of up to maximum of about 1.25 when approaching or flying near the ground, while maximum a.o.a. (at stall) decreases. Therefore the maximum possible lift near the ground is smaller than further up in the air.

In practice when an aircraft is flying, lift should not increase because in any case it needs to be equal to the aircraft weight. The effect is that a lower a.o.a. and or lower speed is required to produce the same lift, and that induced drag is greatly reduced.

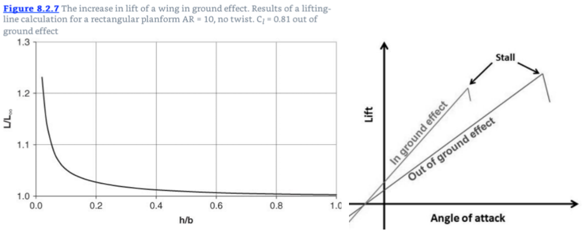


fig. 8: Increase of lift in ground effect (from [1], left); Ground effect on lift and max. a.o.a (from skybrary.aero) (right).

The basic reduction factor for induced lift for plates **planing on water** is 0.45 compared to plates at submergence greater than ~25 % of span. [7|

Due to **submerge effect** induced lift rises up to factor 1.0. as a plate or foil is going deeper underwater. Induced lift respectively decreases from 1.0 to 0.50 when approaching the water surface from below (submerge effect). [9]

5. Lift as a function of altitude and depth

Lifted weight is proportional to plate area, C_L and density [rho] of fluid, thus air or water – and to true air speed in the square. See fig. 9 for standard atmosphere and for salt water.

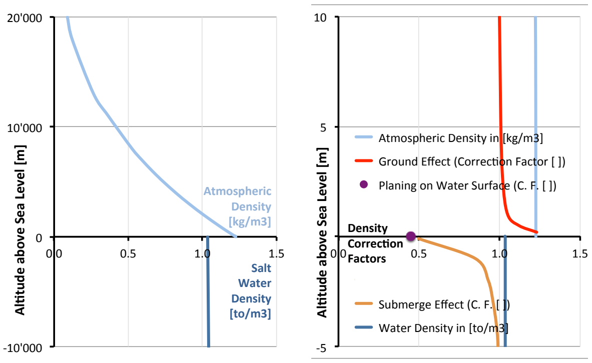


fig. 9: Density of air [kg/m^3] and salt water [to/m^3] (left, from 20 km above to 10 km below sea level) and correction factors for ground effect [-], planing on water surface [•] and submerge effect [-] (right, from 10 m above to 5 m below sea level).

Combining discussed parameters one can deduce:

- Lift per °deg a.o.a.,
- Max. lift due to cambered profile,
- Max. lift due to max. a.o.a. + cambered profile

from summit to seafloor, assuming that maximum a.o.a. underwater is similar to air at ~15° (see fig. 10).

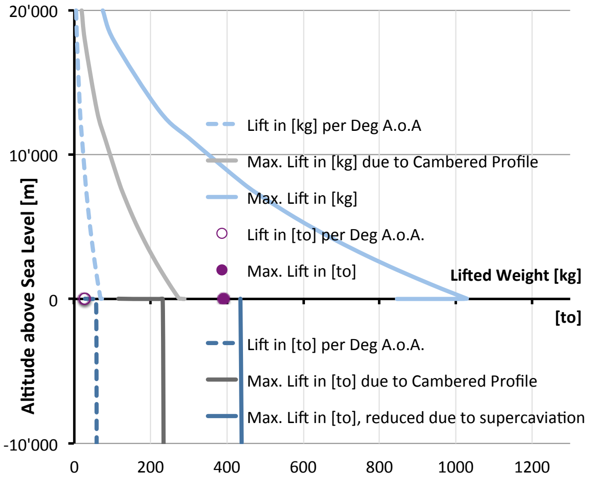


fig. 10: Lifted weight from 20 km above to 10 km below sea level: [kg] airborne, [to] waterborne.

Probably the most interesting sector is close to sea (ground) level, e.g. from 10 m above to 5 m below, still considering a rectangular, flat plate with span b = 10 m and chord c = 1 m as prototype. See fig. 12 and illustration in par. 9, fig. 19-21.

As one can see, the most dramatic changes are due to different fluids (air or water). Further reasons for changes are high altitude, ground and submerge effect and – with almost no effect – great depth. Calculated lift will (at the same speed) increase from air- to hydrofoiling for a flat plate by a factor of:

summit (+ 11 km)	**1.0**	0.30	$7.8 \cdot 10^{-4}$
5 m above sea level	3.4	**1.0**	$2.7 \cdot 10^{-3}$
next to water surface °	4.1	1.25	$3.2 \cdot 10^{-3}$
water surface (planing)	**1’280**	**380**	**0.9**
next to water surface °°	1’420	423	**1.0**
5 m below sea level	2’840	845	2.0
sea floor (-10 km)	2’860	850	2.0

° airborne °° submerged underwater

fig. 11: From summit to sea floor: Factors for calculated lift at different altitudes and depths for low a.o.a. at subcavitational state.

Further insights could be gained by discussing the thrust needed at every altitude/depth, induced drag in different fluids, effects of cavitation, structural design principles and practical aspects etc.

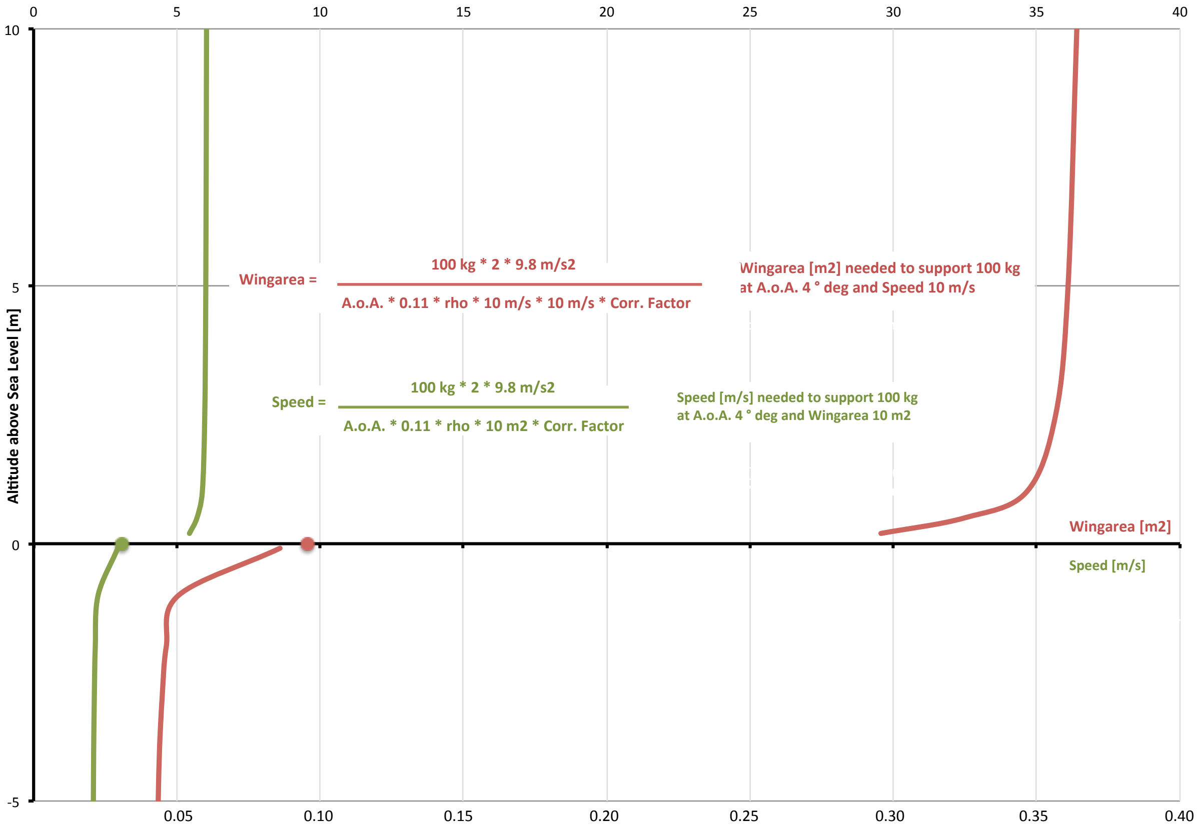


fig. 12a: Wing area [m^2] and speed [m/s] needed to support 100 kg, (+10 m to -5 m from sea level).

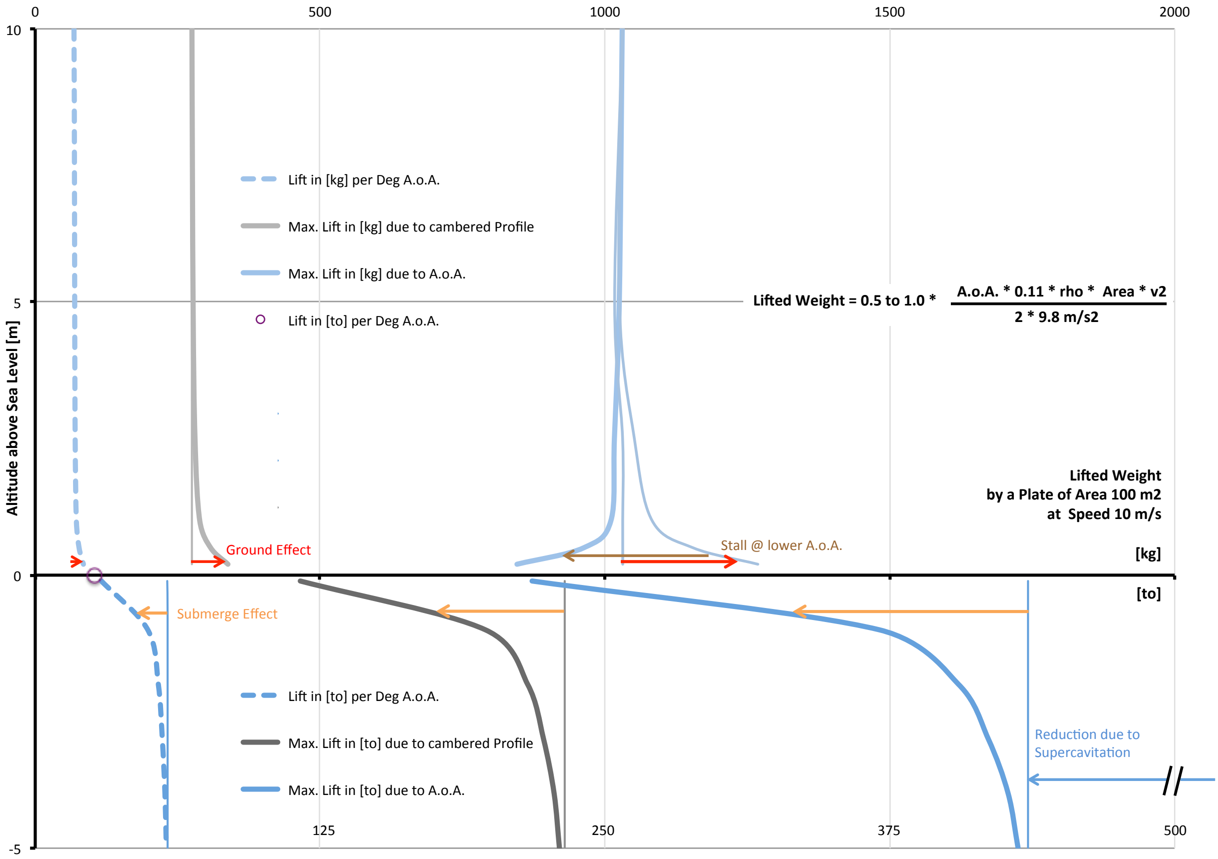


fig. 12b: Supported weight lifted in air [kg], planing on water surface [to] and submerged [to] with impact of relevant correction factors (from +10 m to -5 m).

6. Physical description of flight / lift in general

Competition for best "explanation"

McLean [1] thoroughly discusses theoretical idealizations, popular misconceptions and physical aspects. *When a relatively thin, flat lifting surface such as a wing, a sailboat sail, or a shark's fin moves through air or water, it can produce a force perpendicular to its direction of motion, [ ] called lift [ ]. Mathematical theories of lift have been agreed on by the experts since the early twentieth century, but there has been a long history of disagreement on how to explain lift in [ ] physical terms [ ]*, that *is more difficult than most people realize. The difficulty is inherent in the basic nature of fluid mechanics,* i.e. *it is compounded by the circular nature of the cause-and-effect relationship between pressure and velocity.* He makes reference to concepts from modern to classic: Kutta-Joukowski theorem, potential-flow theory, Navier-Stokes-equations and Newton's laws - and to more than 250 books and academic and scientific papers dealing with a whole range of questions from the basic to the sophisticated, e.g.:

- Anderson, David; Eberhardt, Scott (2009): A Physical Description of Flight; Revisited.
- Yu, N.J., Kao, T.J., and Bogue, D.R. (2000) Computational simulations of commercial airplane configuration with vortex generators.

First of all, one should think of lift as a result of an object (e.g. a flat plate or symmetrical foil) moving through a fluid with a relative a.o.a. to it.

An intuitive way to imagine [ ] lift [ ] is to think [ ] that the airfoil pushes downward on the fluid as it flows past. The fluid must then push back with an equal and opposite (upward) force, which is the lift [Newton's third law: actio = reactio]. *Thus [ ] the airfoil and the fluid exchange equal and opposite forces.* [Further explanation] *will have to do with how the moving fluid actually pushes back.*

Explaining how the flow maintains the pressure difference [above and below a foil] *requires looking at the forces exerted on the air and the resulting accelerations of the air, [ ] in an extended region around the airfoil* [to see how it] *satisfies Newton's second law* [F = m * a]*. [1]*

Momentum-based description

In momentum-based explanations [descriptions]*, it is generally argued that the airfoil produces a flowfield in which some of the air is "deflected" downward and thus has downward (continuous) momentum imparted to it. To acquire downward momentum, the air must have a downward force exerted on it by the airfoil, and thus, by Newton's third law, the airfoil must have an upward force exerted on it by the air. [ ] The mutuality of the force exchange between the airfoil and the air is explicitly acknowledged. [ ] Some momentum-based explanations emphasize that it is not just the lower surface of the airfoil that deflects the flow, and that the flow pattern over the upper surface also contributes strongly to the overall downward deflection. [1]*

How the upper part of the stream is deflected downwards is subject to discussion. However, any explanation of pressure fields applying a force on the foil in an upward direction (from the air beneath) also implies a force in a downward direction from the air further above on the (deflected) volume of air with lower pressure above the foil.

From research on hydrodynamic foils one can learn that both sides – upper and lower – contribute equally to lift of a flat plate [7]. Only for wings or foils of cambered shape can the upper side be seen as contributing more to lift, due to the lower pressure on the upper side according to Bernoulli's law.

The Bernoulli equation says that, in incompressible flow, an increase in flow velocity is accompanied by a decrease in pressure and vice versa. One may consider this as a useful mathematical model for reality rather than an "explanation". To say that one causes the other is not correct: Is it the lower pressure causing an increase in velocity or is it the other way around? [6]

In any case, the curved shape of the upper side of a foil leads to lower pressure on that same side and therefore generates lift independently from a.o.a. or additionally to a.o.a respectively - keeping in mind that objects in fluids can be moved by positive pressure only (from below) and not by "suction" (from above).

If one follows the momentum-based description, there is also a backwards momentum needed to make an aircraft fly (a surfboard plane, a sailboat foil i.a.), often produced by engine(s). For helicopters in forward motion rotary foils impart a momentum downward and backwards by the same motion.

Nature of momentum: continous or pulsing

One can always observe or easily imagine flowfields in which some fluid is pushed downwards - (and backwards) - and thus has downward (pulsing) momentum imparted to it, for example.:

- strokes of wings of birds (airborne);
- bounces of skipping stones (waterborne);
- strokes of swimmers or fish (submerged).

A principal difference between these examples and an aircraft lies in continuous vs. pulsing impartation of momentum.

7. Conclusion

All explanations for lift in air and water, and on water surfaces, are found to rely on the same theoretical and experimental concepts as well as physical laws.

Key to understanding: angle of attack a.o.a.

Lifted weight for a flat, planing plate on water surface is half the lifted weight in submerged state (for more than about 0.5 of the span of the foil). Thus, lift for a flat plate can obviously be equally explained by flow, pressure or force on both sides of the plate or foil - keeping in mind that objects in fluids can only be moved by pressure (from below) and not by "suction" (from above).

On the water surface, the pressure or flow field above the foil contributes only 0.5 ‰ to lifted weight, due to the difference in density between air and water by a factor of about 1'000.

Experiencing lift on a different scale

Objects on water or only barely submerged by water are a fairly simple way to experience and produce lift. To experience lift forces one might, for example:

- produce lift by pushing one's hand through or on the water like a foil with different a.o.a. and feel the different levels of (momentum) force and positive pressure field on the underside;
- use a flat plate made of wood (without enough boyancy to carry the surfer's weight) to experience the need for different a.o.a. at different velocities of the water from midstream to the riverbank (where the plate is fixed with a rope).

To experience the same forces on flying objects (e.g. kites), one needs to multiply the lifting area by a factor of 375 to 835 – setting aside the influence of span to chord ratio and practical challenges for now. Or, at the speed of a car driving down a motorway (thus at ten times the speed) the multiplication factor for required wing area is about 4 to 8 to experience the same lift force.

size of plate in/on water	airborne (e.g. for kites) basic speed	10 times faster
hand	bedroom floor	dish
waterboard	penalty area	king size bed

fig. 13: Area needed to produce same lift force.

Illustration of large to giant kites: par. 9, fig. 23-24.

From this point, one might describe how to stabilise and trim these plates using commonly known aircraft design principles or go into more detailed discussion about lift and drag (and their integration) as well as flow attachement and separation, boundary-layer flows etc. etc.

Additional lift due to shape of foil

Additional lift produced by the cambered shape of foils increases economy, but is neither necessary nor sufficient to support an aircraft – except in extraordinary configuration, e.g. very low weight or very large wing area, or in extreme operation status such as high speed cruise. Only for wings or foils of cambered or curved shape can the contribution to lift of the upper side be seen as superior to that of the lower side, due to the lower pressure on the upper side according to Bernoulli's law– while the foil will still be moved only by positive pressure from below.

8. References and acknowledgement

[1] McLean, Doug J. (2013): Understanding Aerodynamics: Arguing from the Real Physics (Aerospace Series). John Wiley & Sons.

[2] Bocquet, Lydéric (2003): The physics of stone skipping. Université Lyon France. American Journal of Physics 71, 150.

[3] Cantwell, Brian J. (2000?): Wings of finite span. Stanford University.

[4] Clanet, Christophe; Hersen, Fabie; Bocquet, Lydéric (2004): Secrets of successful stone-skipping. Nature 427, 29. doi:10.1038/427029a.

[5] Conolly, A.C. (1975): Prospects for Very-High-Speed Hydrofoils. Marine Technology, Vol. 12, No. 4, Oct. 1975, pp. 367-377.

[6] Manfriani, Leonardo (2016): A.oA. vs. True Air Speed. ZHAW Zurich University of Applied Sciences, SoE School of Engineering, ZAV Centre for Aviation.

[7] Perry, Byrne (1952): The Effect of Aspect Ratio on the Lift of Flat Planing Surfaces. Hydrodynamics Laboratory, California Institute of Technology, Pasadena, California.

[8] Shen, Young T.; Wermter, Raymond (1979): Recent Studies of Struts and Foils for High-Speed Hydrofoils. Marine Technology, Vol 16, No. 1 , Jan 1979, pp 71-82.

[9] Wadlin, Kenneth L.; Christopher, Kenneth W. (1958): A Method for Calculation of Hydrodynamic Lift for submerged and planing rectangular lifting surfaces. Langley Aeronautical Laboratory Langley Field, Va.

Thanks for the reviewing of this document to Capone, Pierluigi Dr.; Manfriani, Leonardo Dr. Prof.; Maurer, Werner Prof. and Neininger, Bruno Dr. Prof. (all from ZHAW Zurich University of Applied Sciences, SoE School of Engineering).

9. Illustrations

fig. 14a-d: from [1, title page]: commercial airliner; Rivers Surfer Magazine; Wikipedia; [4]: skipping stone.

Vortices, Downwash and Upwash

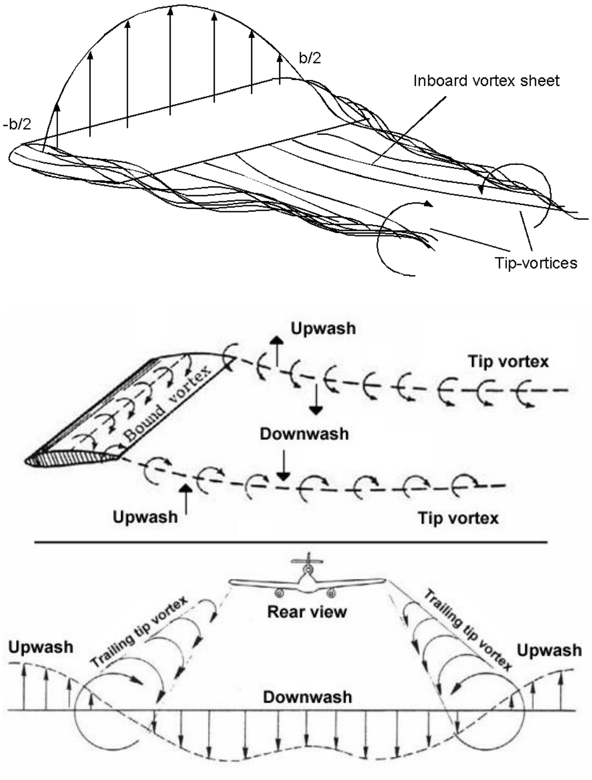


fig. 15a-b: from aerospaceweb.org.

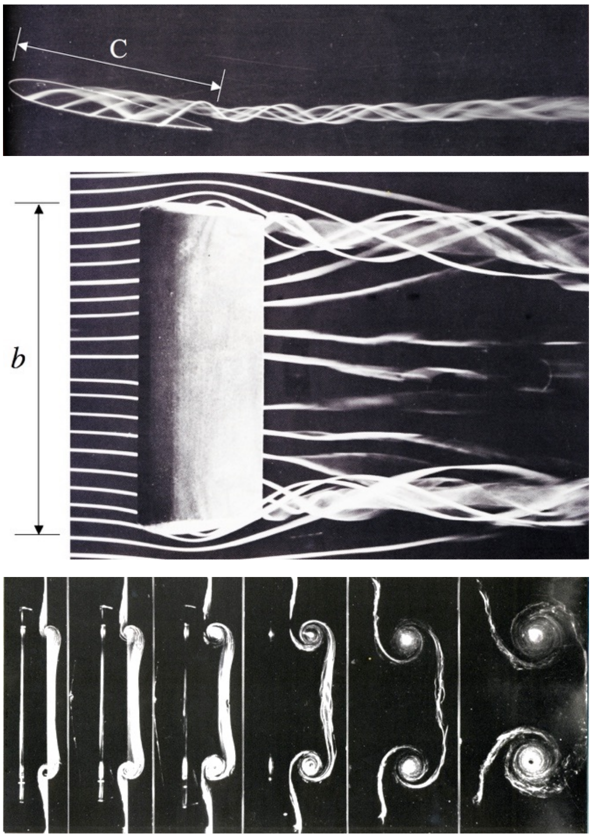
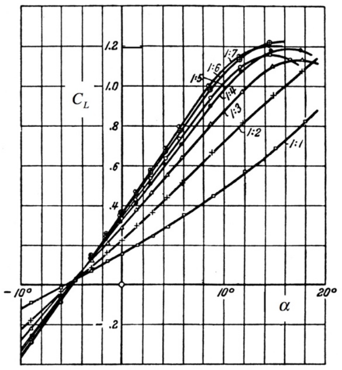


fig. 16a-c: from [3].

Lift coefficient C_L, angle of attack a.o.a. and aspect ratio span b / chord c



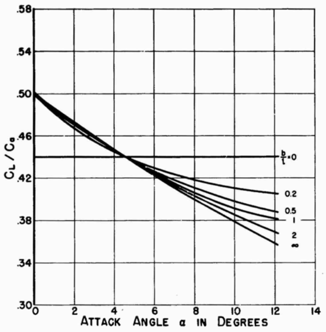


Fig. 9 - The ratio of planing to airfoil lift as assumed by Eq. (19)

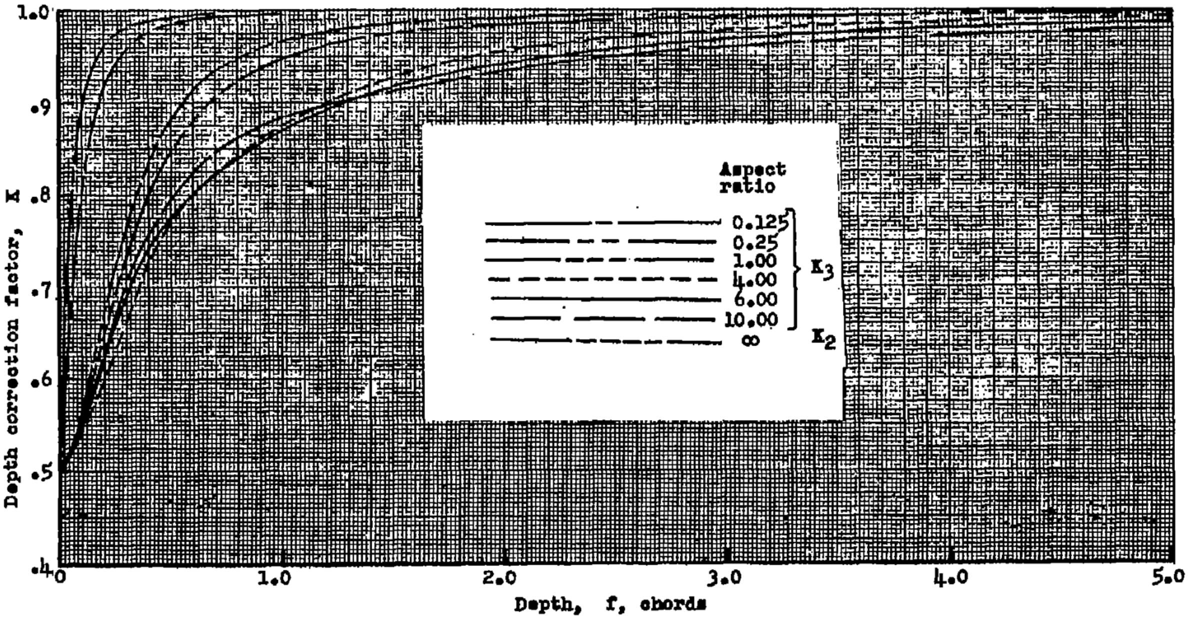
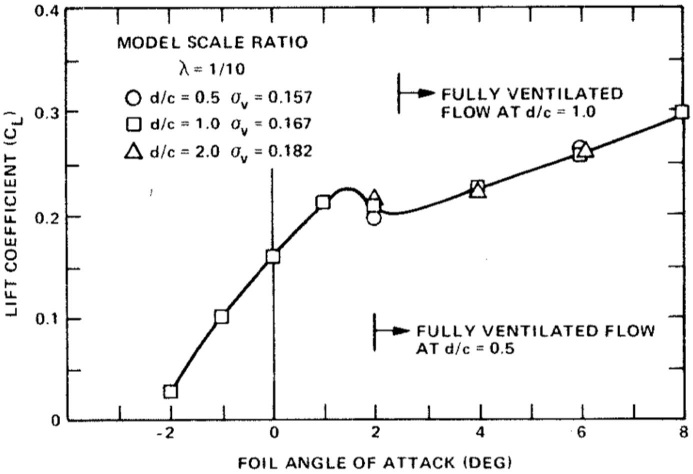


Figure 3.- Variation of depth correction factors with depth for typical aspect ratios at constant angle of attack. $\alpha = 8^0$.

fig. 17a-c: C_L, a.o.a. and ratio span b / chord c; from [3]: airborne, [7]: planing on water and [9]: submerged.

Lift coefficient C_L vs. angle of attack a.o.a. at submergence: from sub- to supercavitating state



(*a*) Lift-coefficient at one-chord submergence

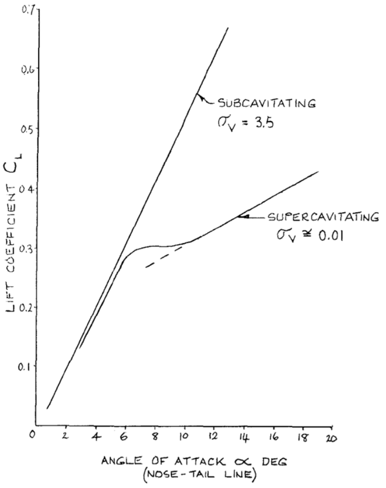


Fig. 10 Variation of lift coefficient with angle of attack for near-zero cavitation number (obtained from reference [2])

fig. 18a-b: C_L vs. a.o.a. for short wings: Reduced lift for a.o.a. > 2 -6° deg; from [8] (left) and [9] (right).

Lift close to water surface

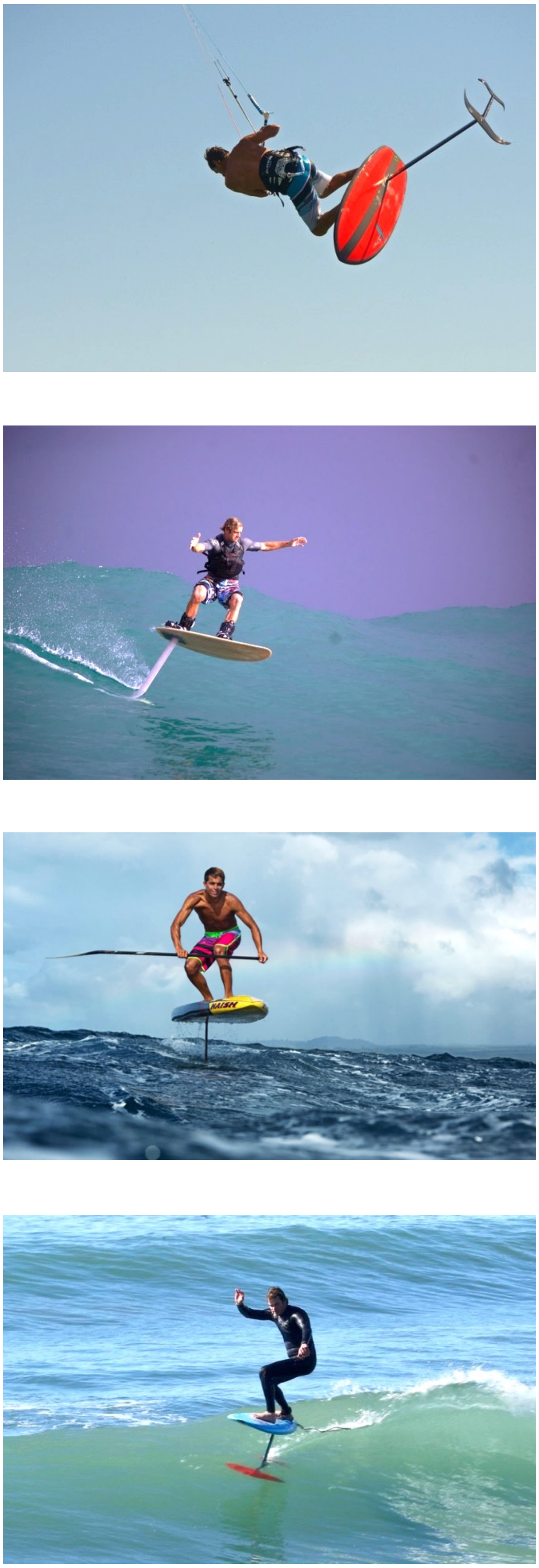


fig. 19a-d: Foilboards from lairdhamilton.com, sportsplanetmag.com, stretchboard.com.

fig. 20a-b: Riverbank-boards at high and low speed from unknown source.

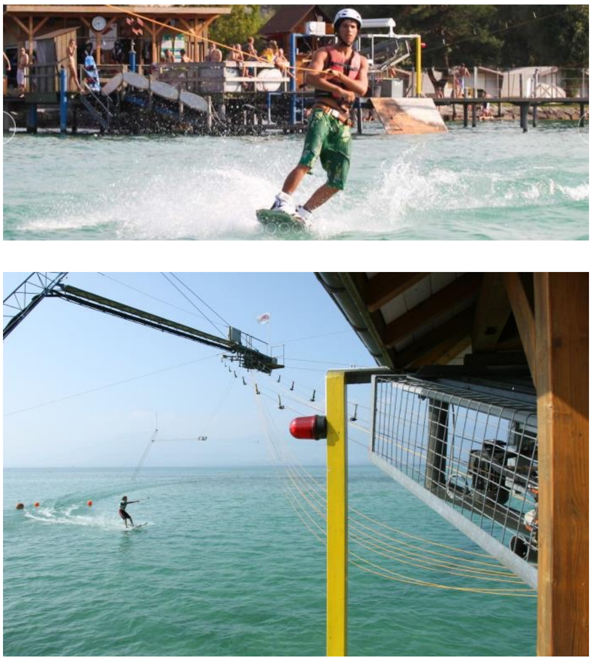


fig. 21c-d: Ski Nautique à Estavayer-Le-Lac from seilbahninventar.ch.

Swept back shape of hydrofoils

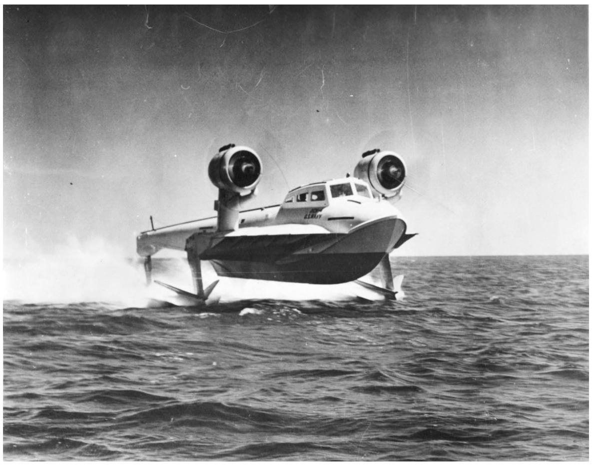


fig. 22: US Navy's XCH-4 from Wikipedia.org.

Large and giant kites

fig. 23: Large kite from kite.org.

fig. 24a-c: Sagami giant kites from Sagami-oodako.com.