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# Multi objective $H_\infty$ active anti-roll bar control for heavy vehicles

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**Abstract:** In the active anti-roll bar control on heavy vehicles, roll stability and energy consumption of actuators are two essential but conflicting performance objectives. In a previous work, the authors proposed an integrated model, including four electronic servo-valve hydraulic actuators in a linear yaw-roll model on a single unit heavy vehicle. This paper aims to design an active anti-roll bar control and solves a Multi-Criteria Optimization (MCO) problem formulated as an $H_\infty$ control problem where the weighting functions are optimally selected through the use of Genetic Algorithms (GAs). Thanks to GAs, the roll stability and the energy consumption are handled using a single high level parameter and illustrated via the Pareto optimality. Simulation results emphasize the simplicity and efficiency of the use of the GAs method for a MCO problem in $H_\infty$ active anti-roll bar control on heavy vehicles.

*Keywords:* Active anti-roll bar, $H_\infty$ control, Multi-criteria optimization, Genetic algorithms.

## 1. INTRODUCTION

The aim of rollover prevention is to provide the vehicle with the ability to resist overturning moments generated during vehicle maneuvers. Roll stability is determined by the height of the center of mass, the track width and the kinematic properties of the suspension. The primary overturning moment arises from the lateral acceleration acting on the center of gravity of the vehicle. More destabilizing moment arises during the cornering manoeuver when the center of gravity of the vehicle shifts laterally. The roll stability of the vehicle can be guaranteed if the sum of the destabilizing moment is compensated during a lateral manoeuver.

Several schemes concerned with the possible active intervention into vehicle dynamics have been proposed. These approaches employ active anti-roll bars, active steering, active braking, active suspensions, or a combination of them (Gaspar et al., 2004). The active anti-roll bar system is the most common method used to improve the roll stability of heavy vehicles. Several control design problems for active anti-roll bar systems have been investigated with many different approaches during the last decades. In (Gaspar et al., 2005a) the authors present Linear Parameter Varying *(LPV)* techniques to control the active anti-roll bars, combined with an active brake control on single unit heavy vehicles. The forward velocity is considered as the varying parameter. Other works concerning the yaw-roll model on single unit heavy vehicles have dealt with optimal control (Sampson and Cebon, 2003a), robust control (Vu et al., 2016b), and neural network control (Boada et al., 2007).

The $H_\infty$ control design approach is an efficient tool for improving the performance of a closed-loop system in pre-defined frequency ranges. The key step of the $H_\infty$ control design is the selection of weighting functions which depends on the engineer skill and experience (Skogestad and Postlethwaite, 2003). In many real applications, the difficulty in choosing the weighting functions still increases significantly because the performance specification is not accurately defined i.e., it is simply to achieve the **best possible** performance (optimal design) or to achieve an optimally joint improvement of more than one objective (multi-objectives design). So the optimization of weighting functions to satisfy all the desired performances is still an interesting problem. In (Hu et al., 2000) it is proposed to consider each system, no matter how complex it is, as a combination of sub-systems of the first and second order, for which it is easy to find the good weighting functions to be used in the $H_\infty$ control methodology. However, there is no explicit method to find these functions in the general case. The usual way is to proceed by trials-and-errors. Recently, the idea to use an optimization tool was proposed in (Alfaro-Cid et al., 2008). The choice of GAs seems natural because their formulation is well suited for this type of problems (Do et al., 2011).

Based on the integrated model presented in (Vu et al., 2016a), this paper proposes an $H_\infty$ control for active anti-roll bar, and the GAs method is used to solve the Multi-Criteria Optimization (MCO) problem for the $H_\infty$ synthesis. The latter work is here extended and provides two new main contributions:

- We design here an $H_\infty$ controller for active anti-roll bar system on the integrated model for the single unit heavy vehicle. The aim is to improve the roll stability of the heavy vehicle. The normalized load transfers and the limitation of the input currents generated by the controllers are considered in the MCO problem.
- The GAs method is applied to find the optimal weighting functions solving the MCO $H_\infty$ control problem. Thanks to GAs, the conflicting objectives between the normalized load transfers and the input currents are handled using only one single high level parameter.

This paper is organised as follows: Section 2 gives the integrated model for a single unit heavy vehicle. Section 3 presents the MCO problem of active anti-roll bar control. Section 4 illustrates the $H_\infty$ robust control synthesis to improve roll stability of heavy vehicles. In section 5, the GAs method is used for MCO in the $H_\infty$ anti-roll bar control. Section 6 shows some simulation results in the frequency domain. Finally, some conclusions are drawn in section 7.

## 2. INTEGRATED MODEL FOR HEAVY VEHICLES

The proposed integrated model includes four Electronic Servo-Valve Hydraulic (ESHV) actuators (two at the front axle and two at the rear axle) in a linear single unit heavy vehicle yaw-roll model (Gaspar et al., 2005b). The control signal is the electrical current $u$ opening the electronic servo-valve, the output is the force $F_{act}$ generated by the hydraulic actuator. The symbols and parameters of the integrated model are detailed in (Gaspar et al., 2005a), (Vu et al., 2016a).
In the linear single unit heavy vehicle yaw-roll model, the differential equations of motion, i.e., the lateral dynamics, the yaw moment, the roll moment of the sprung mass, the roll moment of the front and the rear unsprung masses, are formalized in the equations (1):

$$\begin{cases} mv(\dot{\beta}+\dot{\psi}) - m_s h\ddot{\phi} = F_{yf} + F_{yr} \\ -I_{xz}\ddot{\phi} + I_{zz}\ddot{\psi} = F_{yf}l_f - F_{yr}l_r \\ (I_{xx}+m_s h^2)\ddot{\phi} - I_{xz}\ddot{\psi} = m_s gh\phi + m_s vh(\dot{\beta}+\dot{\psi}) \\ \quad -k_f(\phi-\phi_{tf}) - b_f(\dot{\phi}-\dot{\phi}_{tf}) + M_{ARf} + T_f \\ \quad -k_r(\phi-\phi_{tr}) - b_r(\dot{\phi}-\dot{\phi}_{tr}) + M_{ARr} + T_r \\ -rF_{yf} = m_{uf}v(r-h_{uf})(\dot{\beta}+\dot{\psi}) + m_{uf}gh_{uf}.\phi_{tf} - k_{tf}\phi_{tf} \\ \quad +k_f(\phi-\phi_{tf}) + b_f(\dot{\phi}-\dot{\phi}_{tf}) + M_{ARf} + T_f \\ -rF_{yr} = m_{ur}v(r-h_{ur})(\dot{\beta}+\dot{\psi}) - m_{ur}gh_{ur}\phi_{tr} - k_{tr}\phi_{tr} \\ \quad +k_r(\phi-\phi_{tr}) + b_r(\dot{\phi}-\dot{\phi}_{tr}) + M_{ARr} + T_r \end{cases} \quad (1)$$

In (1) the lateral tyre forces $F_{y;i}$ in the direction of velocity at the wheel ground contact points are modelled by a linear stiffness as:

$$\begin{cases} F_{yf} = \mu C_f \alpha_f \\ F_{yr} = \mu C_r \alpha_r \end{cases} \quad (2)$$

with tyre side slip angles:

$$\begin{cases} \alpha_f = -\beta + \delta_f - \dfrac{l_f\dot{\psi}}{v} \\ \alpha_r = -\beta + \dfrac{l_r\dot{\psi}}{v} \end{cases} \quad (3)$$

$M_{ARf}$ and $M_{ARr}$ are the moments of the passive anti-roll bar acting on the unsprung and sprung masses at the front and rear axles (Vu et al., 2016a).
The torque generated by the active anti-roll bar system at the front axle is now determined by:

$$T_f = 2l_{act}F_{actf} = 2l_{act}A_p\Delta_{Pf} \quad (4)$$

and the torque generated by the active anti-roll bar system at the rear axle is:

$$T_r = 2l_{act}F_{actr} = 2l_{act}A_p\Delta_{Pr} \quad (5)$$

where $\Delta_{Pf}$ and $\Delta_{Pr}$ are respectively the difference of pressure of the hydraulic actuator at the front and rear axles.
The equations of these electronic servo-valve actuators are given by (6):

$$\begin{cases} \dfrac{V_t}{4\beta_e}\dot{\Delta}_{Pf} + (K_P + C_{tp})\Delta_{Pf} - K_x X_{vf} \\ \quad +A_p l_{act}\dot{\phi} - A_p l_{act}\dot{\phi}_{uf} = 0 \\ \dot{X}_{vf} + \dfrac{1}{\tau}X_{vf} - \dfrac{K_v}{\tau}u_f = 0 \\ \dfrac{V_t}{4\beta_e}\dot{\Delta}_{Pr} + (K_P + C_{tp})\Delta_{Pr} - K_x X_{vf} \\ \quad +A_p l_{act}\dot{\phi} - A_p l_{act}\dot{\phi}_{ur} = 0 \\ \dot{X}_{vr} + \dfrac{1}{\tau}X_{vr} - \dfrac{K_v}{\tau}u_r = 0 \end{cases} \quad (6)$$

Defining the state vector:

$$x = \begin{bmatrix} \beta & \dot{\psi} & \phi & \dot{\phi} & \phi_{uf} & \phi_{ur} & \Delta_{Pf} & X_{vf} & \Delta_{Pr} & X_{vr} \end{bmatrix}^T$$

The motion differential equations (1)-(6) can be rewritten in the LTI state-space representation as:

$$\dot{x} = A.x + B_1.w + B_2.u \quad (7)$$

where $A$, $B_1$, $B_2$ are the model matrices of appropriate dimensions, $w = \begin{bmatrix} \delta_f \end{bmatrix}^T$ the exogenous disturbance (steering angle), $u = \begin{bmatrix} u_f & u_r \end{bmatrix}^T$ the control inputs (input currents).

## 3. MULTI-CRITERIA OPTIMIZATION OF ACTIVE ANTI-ROLL BAR CONTROL

### *3.1 Multi-criteria optimization and Pareto-optimal solutions*

A multi-Criteria Optimization (MCO) problem can be described in mathematical terms as follows (Ehrgott, 2005):

$$\min_{x\in S} F(x) = [f_1(x), f_2(x), ..., f_n(x)] \quad (8)$$

where $n > 1$ and $S$ is the set of constraints defined above. The space in which the objective vector belongs is called the objective space, and the image of the feasible set under $F$ is called the attained set. In the following, such a set will be denoted by $C = \{y \in R^n : y = f(x), x \in S\}$. The scalar concept of optimality does not apply directly in the multi-criteria setting. Here the notion of Pareto optimality is introduced. Essentially, a vector $x^* \in S$ is said to be Pareto optimal for a multi-criteria problem if all other vectors $x \in S$ do have a higher value for at least one of the objective functions $f_i$, with $i = 1, ..., n$, or have the same value for all the objective functions.
There are many formulations to solve the problem (8) such as weighted min-max method, weighted global criterion method, goal programming methods... (Marler and Arora, 2004) and references therein. Here, one uses a particular case of the weighted sum method, where the multi-criteria functions vector $F$ is replaced by the convex combination of objectives:

$$\min_{x\in S} J = \sum_{i=1}^{n} \alpha_i f_i(x), \quad \sum_{i=1}^{n} \alpha_i = 1 \quad (9)$$

The vector $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$ represents the gradient of function $J$. By using various sets of $\alpha$, one can generate several points in the Pareto set.

### *3.2 Control objective, and MCO for $H_\infty$ active anti-roll bar control*

The main objective of the active anti-roll bar control system is to maximize the roll stability of the vehicle to prevent a rollover phenomenon in an emergency. Two main criteria are commonly used to assess the roll stability of the heavy vehicle:

- The normalized load transfer $R_{f,r}$ at the two axles, defined as follows (Hsun-Hsuan et al., 2012):

$$R_f = \frac{\Delta F_{zf}}{F_{zf}}, \quad R_r = \frac{\Delta F_{zr}}{F_{zr}} \quad (10)$$

where $F_{zf}$ is the total axle load at the front axle and $F_{zr}$ at the rear axle. $\Delta F_{zf}$ and $\Delta F_{zr}$ are respectively the lateral load transfers at the front and rear axles, which can be given by:

$$\Delta F_{zf} = \frac{k_{uf}\phi_{uf}}{l_w}, \quad \Delta F_{zr} = \frac{k_{ur}\phi_{ur}}{l_w} \tag{11}$$

where $k_{uf}$ and $k_{ur}$ are the stiffness of the tyres, $\phi_{uf}$ and $\phi_{ur}$ are the roll angles of the unsprung masses at the front and rear axles, $l_w$ the half of the vehicle's width.
The normalized load transfer $R_{f,r} = \pm 1$ value corresponds to the largest possible load transfers. The roll stability is achieved by limiting the normalized load transfers within the levels corresponding to wheel lift-off.

- The roll angles between the sprung and unsprung masses $(\phi - \phi_u)$, give the maximum stabilizing moment of the active anti-roll bar system to be increased. They should stay within the limits of the suspension travel $7-8deg$ (Sampson and Cebon, 2003b).

As mentioned above, the objective of the active anti-roll bar control system is to improve the roll stability of heavy vehicles. However, such a performance objective must be balanced with the energy consumption of the anti-roll bar system due to the input current entering the electronic servo-valve of the actuators. Therefore the objective function is selected as follows:

$$f = \alpha f_{Normalized-load-transfer} + (1-\alpha) f_{Control-cost} \tag{12}$$

The vector $\alpha = [0 \div 1]$ is the gradient of function $f$. When $\alpha$ moves to 0, the optimal problem focusses on minimizing input currents. And conversely, when $\alpha$ moves to 1, the optimal problem focusses on minimizing the normalized load transfers. In the objective function (12), $f_{Normalized-load-transfer}$ and $f_{Control-cost}$ are performance indices corresponding to the normalized load transfers and input currents at the two axles, which are defined as follows:

$$\begin{cases} f_{Normalized-load-transfer} = \frac{1}{2}\left(\sqrt{\frac{1}{T}\int_0^T R_f^2(t)dt} + \sqrt{\frac{1}{T}\int_0^T R_r^2(t)dt}\right) \\ f_{Control-cost} = \frac{1}{2}\left(\frac{\sqrt{\frac{1}{T}\int_0^T u_f^2(t)dt}}{\sqrt{\frac{1}{T}\int_0^T u_f^2(t)_{max}dt}} + \frac{\sqrt{\frac{1}{T}\int_0^T u_r^2(t)dt}}{\sqrt{\frac{1}{T}\int_0^T u_r^2(t)_{max}dt}}\right) \end{cases} \tag{13}$$

where $u_{f,rmax}$ are defined when the optimal problem focusses only on the normalized load transfers (i.e., the input currents are then not considered in the optimisation problem). In that case, $\alpha = 1$ and $f = f_{Normalized-load-transfer}$.
The MCO problem represented by the equation (12) can not be resolved directly in the synthesis of $H_\infty$ controller. Thus, summarizing the implementation is done in this paper as described in Fig 1. The generalized plant includes the integrated model (section 2) and the weighting functions. The controller is synthesised by using the $H_\infty$ method (section 4). The conflicting objective between roll stability and energy consumption is the computation of the closed-loop performance (MCO problem in section 3). Depending on the purpose of the MCO problem, the weighting functions are appropriately selected by GAs (section 5). The optimal parameters obtained from GAs are sent to the weighting functions to calculate the controller.

## 4. $H_\infty$ ACTIVE ANTI-ROLL BAR CONTROL TO IMPROVE ROLL STABILITY OF HEAVY VEHICLES

### 4.1 Background on $H_\infty$ control

The interested reader may refer to (Skogestad and Postlethwaite, 2003), (Scherer and Weiland, 2005) for detailed explanations on $H_\infty$ control design.

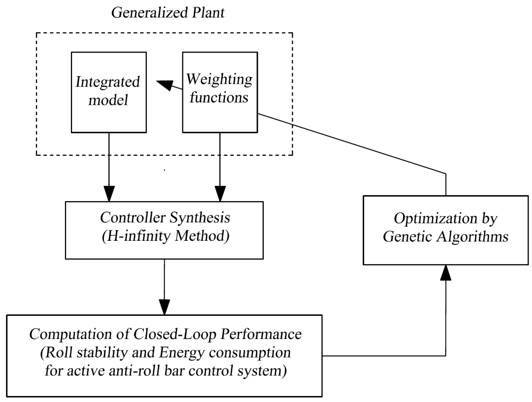


Fig. 1. Controller optimization for $H_\infty$ active anti-roll bar using Genetic Algorithms.

The $H_\infty$ control problem is formulated according to the generalized control structure shown in Fig 2.

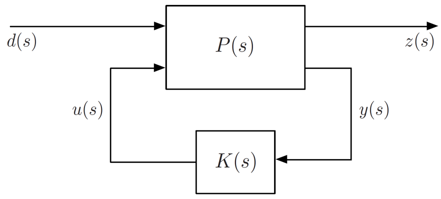


Fig. 2. Generalized control structure.

with $P$ partitioned as

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} d \\ u \end{bmatrix} \tag{14}$$

and

$$u = K(s).y \tag{15}$$

which yields

$$\frac{z}{d} = \mathcal{F}_l(P,K) := [P_{11} + P_{12}K[I - P_{22}K]^{-1}P_{21}] \tag{16}$$

The aim is to design a controller $K(s)$ that reduces the signal transmission path from disturbances $d$ to performance outputs $z$ and also stabilizes the closed-loop system. The $H_\infty$ problem is to find $K$ which minimizes $\gamma$ such that

$$\|\mathcal{F}_l(P,K)\|_\infty < \gamma \tag{17}$$

By minimizing a suitably weighted version of (17), the control aim is achieved, as presented below.

### 4.2 $H_\infty$ control design for active anti-roll bar system

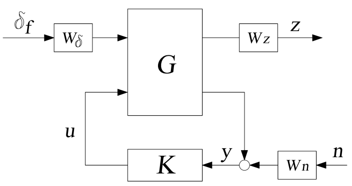


Fig. 3. Closed-loop structure of an $H_\infty$ active anti-roll bar control.

Figure 3 shows the closed-loop structure of an $H_\infty$ control designed for the active anti-roll bar system on a single unit heavy vehicle, using ESVH actuators. In the diagram, the feedback structure includes the nominal model $G$, the controller $K$, the performance output $z$, the control input $u$, the measured

output $y$, the measurement noise $n$. The steering angle $\delta_f$ is the disturbance signal, which is set by the driver. The weighting functions $W_\delta, W_z, W_n$ are presented below.
According to Figure 3, the concatenation of the linear model (7) with the performance weighting functions lead to the state space representation of $P(s)$:

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix} \tag{18}$$

where $w = \begin{bmatrix} \delta_f & n \end{bmatrix}$ is the exogenous input vector, $u = \begin{bmatrix} u_f & u_r \end{bmatrix}^T$ the control input vector, $z = \begin{bmatrix} u_f & u_r & R_f & R_r & a_y \end{bmatrix}^T$ the performance output vector, $y = \begin{bmatrix} a_y & \dot{\phi} \end{bmatrix}^T$ the measured output vector.
The input scaling weight $W_\delta$ normalizes the steering angle to the maximum expected command. It is selected as $W_\delta = \pi/180$, which corresponds to a $1^0$ steering angle command.
The weighting function $W_n$ is selected as a diagonal matrix, which accounts for sensor noise models in the control design. The noise weights are chosen here as $0.01(m/s^2)$ for the lateral acceleration and $0.01(^0/sec)$ for the derivative of roll angle $\dot{\phi}$ (Gaspar et al., 2004).
The weighting functions matrix $W_z$ represents the performance output, $W_z = diag[W_{zu}, W_{zR}, W_{za}]$. The purpose of the weighting functions is to keep the control inputs, normalized load transfers and lateral acceleration as small as possible over the desired frequency range. These weighting functions can be considered as penalty functions, that is, weights should be large in the frequency range where small signals are desired and small where larger performance outputs can be tolerated.
The weighting function $W_{zu}$ is chosen as $W_{zu} = diag[W_{zuf}, W_{zur}]$, corresponding to the input currents at the front and rear axles, and are chosen as:

$$W_{zuf} = \frac{1}{Z_1}; \quad W_{zur} = \frac{1}{Z_2} \tag{19}$$

The weighting function $W_{zR}$ is chosen as $W_{zR} = diag[W_{zRf}, W_{zRr}]$, corresponding to the normalized load transfers at front and rear axles, and are selected as:

$$W_{zRf} = \frac{1}{Z_3}; \quad W_{zRr} = \frac{1}{Z_4} \tag{20}$$

The weighting function $W_{za}$ is selected as:

$$W_{za} = Z_{51} \frac{Z_{52}s + Z_{53}}{Z_{54}s + Z_{55}} \tag{21}$$

From equations (19) - (21), $Z_i$ and $Z_{5,j}$ are constant parameters. Here, the weighting function $W_{za}$ corresponds to a design that avoids the rollover situation with the bandwidth of the driver in the frequency range up to more than $4rad/s$. This weighting function will directly minimize the lateral acceleration when it reaches the critical value, to avoid the rollover.
As said before, the key step of the $H_\infty$ control design is how to select the weighting function. The following variables are to be selected: $Z_1$, $Z_2$, $Z_3$, $Z_4$, $Z_{51}$, $Z_{52}$, $Z_{53}$, $Z_{54}$, $Z_{55}$. In the next section, the GAs method will be used to find these variables, suited for the MCO problem.

## 5. USING GENETIC ALGORITHMS FOR MULTI-CRITERIA OPTIMIZATION IN $H_\infty$ ANTI-ROLL BAR CONTROL

This section introduces the MCO problem for the $H_\infty$ active anti-roll bar control on heavy vehicles, which is solved by using the GAs method.

### *5.1 Genetic Algorithms*

A Genetic Algorithm, as presented by J.H. Holland (Holland, 1975) is a model of machine learning, which derives its behavior from a metaphor of the process of evolution in nature. GAs are executed iteratively on a set of coded chromosomes, called a population, with three basic genetic operations: selection, crossover and mutation. Each member of the population, called a chromosome (or individual) is represented by a string. GAs use only the objective function information and probabilistic transition rules for genetic operations. The primary operation of GAs is the crossover. The crossover happens with a probability of 0.9 and the mutation happens with a small probability 0.095.

### *5.2 Solving multi-criteria optimization by genetic algorithms*

From the objective function in (12), the MCO problem for the $H_\infty$ active anti-roll bar control can be defined as:

$$\begin{aligned} &\min_{p \in P} F(p) s.t. F(p) := \left[ f_{Normalized-load-transfer} \quad , \quad f_{Control-cost} \right]^T \\ &P := \left\{ p = [Z_1, Z_2, Z_3, Z_4, Z_{51}, Z_{52}, Z_{53}, Z_{54}, Z_{55}]^T \in R \mid p^l \leq p \leq p^u \right\} \end{aligned} \tag{22}$$

where $F(p)$ is the vector of objectives, $p$ denotes the vector of the weighting function parameters, $p^l$ and $p^u$ represent the lower and upper bounds of the parameters, as given in Table 1. Besides the minimization of the objective function from equations (12) and (22), we also have to account for the limitations of the normalized load transfers, roll angle of suspensions, as well as the input currents at each axle. These limitations are considered as the optimal conditions (binding conditions) shown in the Table 2.

Table 1. Lower and upper bounds of the weighting functions.

| Bounds | $W_{zuf}$ | $W_{zur}$ | $W_{zRf}$ | $W_{zRr}$ | $W_{za}$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ | $Z_{51}$ | $Z_{52}$ | $Z_{53}$ | $Z_{54}$ | $Z_{55}$ |
| Lower bound | 0.001 | 0.001 | 0.1 | 0.1 | 0.5 | $\frac{1}{3000}$ | 1 | 10 | 0.001 |
| Upper bound | 10 | 10 | 100 | 100 | 100 | 10 | 900 | 1000 | 20 |

Table 2. Binding conditions.

| No | Note | Maximum value | Unit |
|---|---|---|---|
| 1 | $\mid \phi - \phi_{uf} \mid$ | $< 7$ | deg |
| 2 | $\mid \phi - \phi_{ur} \mid$ | $< 7$ | deg |
| 3 | $\mid R_f \mid$ | $< 1$ | - |
| 4 | $\mid R_r \mid$ | $< 1$ | - |
| 5 | $\mid u_f \mid$ | $< 20$ | mA |
| 6 | $\mid u_r \mid$ | $< 20$ | mA |

The proposed weighting function optimization procedure for the $H_\infty$ active anti-roll bar control synthesis is as follows:

- **Step 1:** Initialize with the weighting functions (it depends on the engineer skill and experience), the vector of weighting functions selected as $p = p_0$.
- **Step 2:** Select lower bound, upper bound, scaling factor, offset and start point.
- **Step 3:** Select the objective function (12) with the variation of the gradient from 0 to 1 and then solve the minimization problem.
- **Step 4:** Select the individuals, apply crossover and mutation to generate a new generation: $p = p_{new}$.
- **Step 5:** Evaluate the new generation by comparing with the binding conditions. If the criteria of interest are not satisfied, go to step 3 with $p = p_{new}$; else, stop and save the best individual: $p_{opt} = p_{new}$.

Table 3. Optimization results for the weighting functions of $H_\infty$ active anti-roll bar.

| Controllers | $W_{zuf}$ | $W_{zur}$ | $W_{zRf}$ | $W_{zRr}$ | $W_{za}$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ | $Z_{51}$ | $Z_{52}$ | $Z_{53}$ | $Z_{54}$ | $Z_{55}$ |
| $\alpha = 0$ | 0.060 | 0.020 | 0.100 | 0.965 | 0.673 | 0.948 | 1.063 | 972.212 | 0.855 |
| $\alpha = 0.25$ | 0.057 | 0.052 | 0.51 | 0.863 | 0.863 | 0.664 | 155.627 | 651.707 | 0.573 |
| $\alpha = 0.5$ | 0.099 | 0.0773 | 1.403 | 0.217 | 0.812 | 0.813 | 88.666 | 407.658 | 1.001 |
| $\alpha = 0.7$ | 0.057 | 0.066 | 0.412 | 0.221 | 0.832 | 0.514 | 139.609 | 357.401 | 1.901 |
| $\alpha = 0.9$ | 0.066 | 0.072 | 0.616 | 0.482 | 0.724 | 0.492 | 202.316 | 455.747 | 0.544 |
| $\alpha = 1$ | 0.07 | 0.090 | 0.655 | 0.305 | 0.545 | 0.245 | 444.397 | 839.299 | 0.163 |

## 6. SIMULATION RESULTS

### 6.1 Optimization results

Thanks to the GAs method, Table 3 gives a synthesis of the values of the variables $Z_i$, $Z_{5j}$ in six cases for $\alpha = [0; 0.25; 0.5; 0.7; 0.9; 1]$, as explained in (12). When $\alpha = 0$, $f = f_{Control-cost}$, the optimal problem focusses only on the input currents and when $\alpha = 1$, $f = f_{Normalized-load-transfer}$, the optimal problem focusses only on the normalized load transfers.
Figure 4 shows the conflicting relation between the normalized load transfers and control costs with some Pareto-optimal points, computed for the $H_\infty$ active anti-roll bar on heavy vehicles. They are generated for 6 values of $\alpha$ in the range [0; 1].

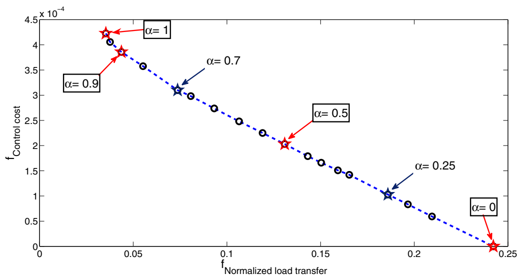


Fig. 4. The Pareto frontier for the active anti-roll bar on heavy vehicles using ESVH actuators.

To evaluate the optimization procedure, simulations in the frequency domain are done and compared for five cases: passive ARB (anti-roll bar) (Vu et al., 2016a) and $H_\infty$ AARB (active anti-roll bar) with $\alpha = [0; 0.5; 0.9; 1]$ .

### 6.2 Evaluation of optimization results in frequency domain

The frequency response of the heavy vehicle is shown in the nominal parameters case of the single unit heavy vehicle with the forward velocity V at $70Km/h$ and the road adhesion coefficient $\mu = 1$ (see, Gaspar et al. (2004) and Vu et al. (2016a)). Figures 5 and 6 show the transfer function magnitude of the normalized load transfers at the two axles $\frac{R_{f,r}}{\delta_f}$.
To assess the roll stability of the heavy vehicle using the four $H_\infty$ active anti-roll bar controllers, the reduction of the magnitude of transfer functions compared with the passive anti-roll bar case is considered at $10^{-2}rad/s$ and at $2rad/s$ as:

$$\lambda_{(X)} = \frac{X_{active}}{\delta_f} - \frac{X_{passive}}{\delta_f} \qquad (23)$$

where the variables of interest $X$ are the normalized load transfers $R_{f,r}$.
Figure 7 shows the reduction of the magnitude of transfer functions of the normalized load transfer compared with the passive anti-roll bar case at $10^{-2}rad/s$ and at $2rad/s$. We can see that at $10^{-2}rad/s$ the controller with $\alpha = 0$ decreases the roll stability, meanwhile, when $\alpha$ increases, the roll stability of the heavy vehicle increases. The curves are very different. From $2rad/s$ the transfer functions are not so different. This will be investigated in further studies.

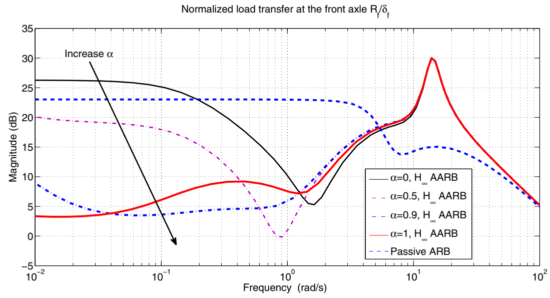


Fig. 5. Transfer function magnitude of the normalized load transfer at the front axle $\frac{R_f}{\delta_f}$.

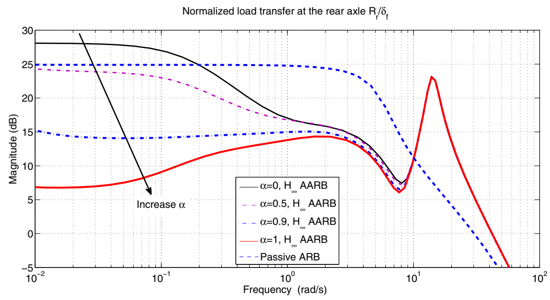


Fig. 6. Transfer function magnitude of the normalized load transfer at the rear axle $\frac{R_r}{\delta_f}$.

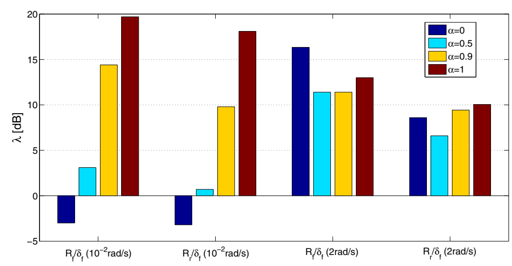


Fig. 7. Reduction of the magnitude of transfer functions of the normalized load transfers at the two axles compared with the passive anti-roll bar case (see (23)).

Figures 8 and 9 show the transfer function magnitude of the input currents at the two axles $\frac{u_{f,r}}{\delta_f}$: when $\alpha$ increases (the MCO problem focusses on minimizing the normalized load transfers), the total input currents also increase. It is proven for the normalized load transfer and the input current that they are two conflicting performance objectives.

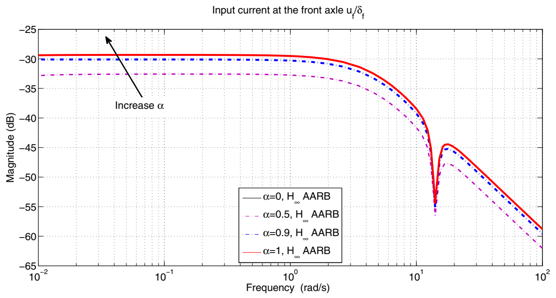


Fig. 8. Transfer function magnitude of the input current at the front axle $\frac{u_f}{\delta_f}$.

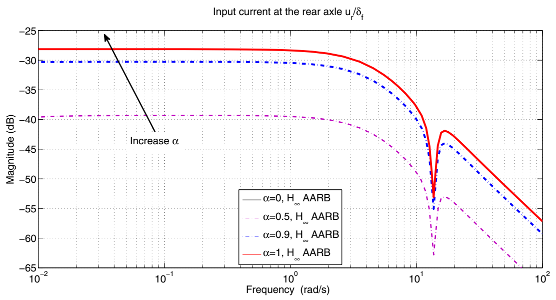


Fig. 9. Transfer function magnitude of the input current at the rear axle $\frac{u_r}{\delta_f}$.

Thus the MCO problem allows to get the weighting functions to enhance the roll stability of the heavy vehicle in the low frequency range as well as in the high frequency range up to over $4rad/s$, which is the limited bandwidth of the driver (Gaspar et al., 2004).

## 7. CONCLUSION

In this paper, the integrated model of a single unit heavy vehicle including four ESVH actuators is used to develop a linear $H_\infty$ control scheme maximizing its roll stability in order to prevent rollover. The normalized load transfers and the limitations of the input currents are considered in the design.

A weighting function optimization procedure using GAs for $H_\infty$ active anti-roll bar control on the integrated model has also been proposed. The conflicting objectives between the normalized load transfers and input currents are handled using only one high level parameter, which is a great advantage to solve the multi-objective control problem. The simulation results have shown the efficiency of the GAs to obtain a suitable controller to satisfy the MCO problem.

Even if a *LTI* controller seems to performs reasonably well here, the comparison with an *LPV* controller (scheduled by the vehicle velocity) will be of interest for future works.

## REFERENCES

Alfaro-Cid, E., McGookin, E., and Murray-Smith, D. (2008). Optimisation of the weighting functions of an $H_\infty$ controller using genetic algorithms and structured genetic algorithms. *International Journal of Systems Science*, 39(4), 335–347.

Boada, M., Boada, B., Quesada, A., Gaucha, A., and Daz, V. (2007). Active roll control using reinforcement learning for a single unit heavy vehicle. In *12th IFToMM World Congress*. Besancon, France.

Do, L.A., Sename, O., Dugard, L., and Boussaad, S. (2011). Multi-objective optimization by genetic algorithms in $H_\infty/LPV$ control of semi-active suspension. In *IFAC World Congress - 18$^{th}$ IFAC WC 2011*. Italy.

Ehrgott, M. (2005). *Multicriteria optimization*. Springer, 2$^{nd}$ edition.

Gaspar, P., Bokor, J., and Szaszi, I. (2004). The design of a combined control structure to prevent the rollover of heavy vehicles. *European Journal of Control*, 10(2), 148–162.

Gaspar, P., Bokor, J., and Szaszi, I. (2005a). Reconfigurable control structure to prevent the rollover of heavy vehicles. *Control Engineering Practice*, 13(6), 699–711.

Gaspar, P., Szabo, Z., and Bokor, J. (2005b). The design of an integrated control system in heavy vehicles based on an lpv method. In *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference*. Seville, Spain.

Holland, J.H. (1975). *Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence*. Ann Arbor: University of Michigan Press.

Hsun-Hsuan, H., Rama, K., and Dennis, A.G. (2012). Active roll control for rollover prevention of heavy articulated vehicles with multiple-rollover-index minimisation. *Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility*, 50(3), 471–493.

Hu, J., Bohn, C., and Wu, H. (2000). Systematic $H_\infty$ weighting function selection and its application to the real-time control of a vertical take-off aircraft. *Control Engineering Practice*, 8, 241–252.

Marler, R. and Arora, J. (2004). Survey of multi-objective optimization methods for engineering. *Structural and Multidisciplinary Optimization*, 26, 369–395.

Sampson, D. and Cebon, D. (2003a). Achievable roll stability of heavy road vehicles. In *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, volume 217, 269–287. United Kingdom.

Sampson, D. and Cebon, D. (2003b). Active roll control of single unit heavy road vehicles. *Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility*, 40(4), 229–270.

Scherer, C. and Weiland, S. (2005). Linear matrix inequalities in control. University Lecture.

Skogestad, S. and Postlethwaite, I. (2003). *Multivariable Feedback Control*. John Wiley & Sons, 2$^{nd}$ edition.

Vu, V.T., Sename, O., Dugard, L., and Gaspar, P. (2016a). Active anti-roll bar control using electronic servo-valve hydraulic damper on single unit heavy vehicle. In *IFAC Symposium on Advances in Automotive Control - 8$^{th}$ AAC 2016*. Norrkoping, Sweden.

Vu, V.T., Sename, O., Dugard, L., and Gaspar, P. (2016b). $H_\infty$ active anti-roll bar control to prevent rollover of heavy vehicles: a robustness analysis. In *IFAC Symposium on System Structure and Control - 6$^{th}$ SSSC 2016*. Istanbul, Turkey.