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Exploratory study of students' representational fluency and competence of electric circuits

William Fernando Sandchez Cossio
Purdue University

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Alejandra J Magana
Chair

Dawn Laux
Co-chair

Baijian Yang
Co-chair

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Approved by Major Professor(s): Alejandra J Magana

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EXPLORATORY STUDY OF STUDENTS' REPRESENTATIONAL FLUENCY AND COMPETENCE OF ELECTRIC CIRCUITS

A Thesis

Submitted to the Faculty

of

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by

William Fernando Sanchez Cossio

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of

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To my parents, advisor and close friends who have helped me out through this process.

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LIST OF ABBREVIATIONS

AC: Alternating Current

CCI: Circuits Concept Inventories

DC: Direct Current

DIRECT: Determining and Interpreting Resistive Electric Circuits Concepts Test

ECCE: Electric Circuits Concept Evaluation

MBR: Models-Based Reasoning

MEA: Model-Eliciting Activities

MMP: Models and Modeling Perspective

GLOSSARY

Misconception: “An idea for which the student's interpretation is in conflict with the formal concept as understood by a physicist” (McDermott & Shaffer, 1992a, p. 1002). In this document, the term will be used with a more general meaning, as “… the formal concept as understood by an *expert in the field*”.

Model: Bridge between a phenomenon and its representation in a particular language (e.g., mathematical representation, spoken language), along with the mental representations used to carry out reasoning and the means to think and understand the concepts (Nersessian, 1999).

Modeling: Process of developing representational descriptions or models for specific purposes in specific situations (Lesh & Lehrer, 2003).

Model-based reasoning: Form of scientific cognition that investigates how novel scientific representations are created from existing representations (Nersessian, 2002).

Representational competence: Set of skills that support the ability to use, think about and communicate with representations, but also to act on phenomena based on those representations (Kozma & Russell, 2005).

Representational fluency: Ability of the learner to represent a concept in multiple ways and to be able to seamlessly translate within and among those representations (Moore, Miller, Lesh, Stohlmann, & Kim, 2013).

ABSTRACT

Sanchez Cossio, William Fernando. M.S., Purdue University, August 2016. Exploratory Study of Students' Representational Fluency and Competence of Electric Circuits. Major Professor: Alejandra J. Magana, Ph.D.

Electric circuits are extensively used in today's devices as computers, phones, cameras and others. This makes them a crucial topic in engineering because almost every engineering branch could be related of used them at different levels. Even though their importance, students often struggle during the learning process of circuit analysis topics. Additionally, other very important abilities for engineering students are the capacities to create, use, express and think about models and representations of technical concepts; and the capacities to translate and map from one representation to another. These abilities are known as representational competence and representational fluency respectively.

The purpose of this exploratory study was to analyze how the use of multiple representations of technical concepts is related to the conceptual understanding of those concepts. The methodological approach employed was case study, which was implemented through two cases and focuses on electric circuit analysis at the college level. An activity based on model-eliciting activities was used to assess representational competence, fluency and conceptual understanding of the students in order to explore the relationship between (1) using multiple representations and conceptual understanding and (2) the ability to map between representations and conceptual understanding.

The results of this exploratory study indicate that a multi-representational approach can support and foster the learning process and conceptual understanding of electric circuits. Furthermore, the results also suggest a positive relationship between representational competence and fluency and conceptual understanding. Which suggest that students with high representational competence and fluency may interpret concepts more deeply. Results also indicate that students with a deep conceptual understanding are able to create more accurate representations and to map between representations accurately as well. Finally, the contribution of this exploratory study relies on (1) the application of a multi-representational analysis of conceptual understanding of electric circuit and (2) the probe of using multiple and additional representations during the learning process of electric circuits.

CHAPTER 1. INTRODUCTION

This chapter presents the introduction to the research study. The chapter describes the significance and the purpose that led to the research questions. The boundaries of the study are defined as well in the assumptions, limitations and delimitations sections.

1.1 Statement of purpose

This research employed a case study approach and focused on analyzing how the use of multiple representations of technical concepts relates to the conceptual understanding of such concepts. The case studied was electric circuit analysis in a college level course. The course has the purpose of applying different models and representations, as well as modeling practices to introduce students to electric circuit theory. The course also focuses on developing abilities for analyzing and understanding electric components and their characteristics, behavior, and performance.

Electric circuit analysis has become an important ability for almost every branch of engineering. This is due to technology development, instrumentation and digitization of systems in both, household and technical environments. Although the topic is most relevant in engineering fields, students often struggle through the learning process, mainly because they have misconceptions related to electrical circuits.

The aim of this research study was to identify the effects of using multiple representations in students' misconceptions and conceptual understanding of electric circuits. This goal was pursued by evaluating students' performance in each representation (i.e., representational competence) and their ability to transfer knowledge and map between the representations (i.e., representational fluency). The outcome of this research can influence and contribute to the improvement and the design of micro and macro curricula related to electric circuits. Additionally, this research has given insights for applying multiple representations perspective to other models commonly used in science fields and other engineering programs.

1.2 Significance

The technology development, instrumentation, and digitization of systems in both household and technical environments have made electric circuit analysis an important ability for almost every branch of engineering. During school years, future engineers are expected to develop abilities for analyzing and understanding electric components and their characteristics, behavior, and performance. Although the relevance of electric circuit analysis within engineering fields has been widely emphasized, students often struggle through the learning process, mainly because they have misconceptions related to electrical circuits. The purpose of this study was to investigate how the use of multiple representations can affect the conceptual understanding of electric circuits.

The significance and impacts of this project have involved the improvement of the design of micro-and macro-curricula related to electric circuits in several engineering disciplines. Besides, this case study has provided insights for applying models and modeling perspective to other STEM topics where multiple representations and models are worthy.

1.3 Scope

The scope of this case study included developing learning assessment tools and using them to investigate the influence of multiple representations in conceptual understanding of electric circuits. The context was a college level circuit analysis course offered by Purdue University. The goal of this study was to analyze how the use of multiple representations of the same concepts relates to students' conceptual understanding.

1.4 Research questions

1. How does the use of multiple representations relate to student conceptual understanding of electric circuits?
2. What is the relationship between student conceptual understanding of electric circuits and their ability to map between multiple representations?

1.5 Assumptions

The study had the following assumptions:

1. Participants of this study have been honest and will put their best effort in answering the assessment materials.
2. Participants had previous knowledge of differential and integral calculus, complex numbers, vector and matrices, and experience with MATLAB® or an equivalent programming language.
3. Participants have completed the assessments materials based on the knowledge they possess.

1.6 Limitations

The study had the following limitations:

1. Participants in this study were enrolled in a level one Electric Circuits Analysis course at Purdue University. Thus, findings from this study will be applicable to students with similar characteristics; however, in order to generalize them, further research should be done.
2. The study considered students enrolled in Electric Circuit Analysis course and their participation was voluntary.

1.7 Delimitations

The study had the following delimitations:

1. This study analyzed representational competence and representational fluency in the specific context of electric circuits.

2. The analysis was performed on the data collected from the assessments materials used in a homework assignment emphasizing the use of multiple representations.

1.8 Chapter summary

This chapter explained the motivation for conducting this study. It described the scope of the study and the contribution to the field this research could make. It also addressed the boundaries that limited the study.

The following chapter contains background information about conceptual understanding and common misconceptions in electric circuit analysis. A review of previous research work related to misconceptions in this domain is also developed.

CHAPTER 2. LITERATURE REVIEW

The literature review presents previous research related to the difficulties students face with understanding electric engineering concepts, specifically targeted to conceptual understanding of electric circuits. Then, a general introduction to conceptual knowledge and misconceptions in science and engineering is made before going to the remaining sections. In the first section, a description of the most relevant literature about conceptual understanding of electric circuits is presented. Secondly, concept inventories are discussed as the instruments most frequently used to measure conceptual understanding and to also identify misconceptions in sciences and engineering. The third section presents the most relevant misconceptions identified in the circuit theory domain. The fourth section describes the responses of curriculum designers and educators to address these misconceptions. At the end of this chapter, a chapter summary is presented.

2.1 Overview of conceptual understanding in science and engineering

Conceptual understanding or conceptual knowledge could be defined as the "understanding of principles governing a domain and the interrelations between units of knowledge in a domain" (Rittle-Johnson, 2006, p. 2). This definition makes clear that it is not enough to understand the concepts of a particular topic, but to achieve conceptual understanding the relationship among the different concepts is crucial. For instance, conceptual knowledge in the engineering domain contains quantities such as force, heat or electric current; but also relationships as Newton’s law, the laws of thermodynamics or Ohm’s law (Streveler, Litzinger, Miller, & Steif, 2008).

Conceptual knowledge is helpful for students in the process of: (a) identifying details of a problem based on deeper understanding of the topic, (b) detecting mistakes in problem solving procedures, and (c) selecting which could be the best procedure or, even, generating new procedures to solve problems (Rittle-Johnson, Siegler, & Alibali, 2001). More particularly, conceptual understanding is one of the three core components of the engineering practice: (a) engineering as problem solving, systematic method used to describe, delineate and solve problems; (b) engineering as knowledge, spatialized knowledge required for problem solving; (c) engineering as a merge of knowledge and problem solving (Sheppard, Colby, Macatangay, & Sullivan, 2006).

Therefore, conceptual understanding is a key element to developing the competence and the skills needed for engineering students and professionals. A clear example is engineering judgment. Engineers rely on conceptual knowledge to make intuitive and educated suppositions about the expectations of how a system is going to behave under specific circumstances without appeal to prototypes or complicated models, often due to time constraints (Streveler et al., 2008). Despite the importance of conceptual understanding, students often struggle understanding or they have wrong ideas about key concepts and this generates *misunderstandings* or *misconceptions*.

The term *misconception* refers to (a) “an idea for which the student's interpretation is in conflict with the formal concept as understood by a physicist” (McDermott & Shaffer, 1992a, p. 1002), (b) student non-scientific conceptions (Taslidere, 2013), and (c) incorrect patterns of response by students. A pattern “could be part of a coherent naive theory of some physical phenomena or a more fragmented and primitive response produced on the spot as a result of the questions posed” (Engelhardt & Beichner, 2004, p. 98). This study assumes the definition of McDermott and Shaffer (1992a) applied to science fields in general.

Other terms to identify misconceptions are: (a) *misunderstandings* (Picciarelli, Di Gennaro, Stella, & Conte, 1991a, 1991b), (b) *preconceptions* (Clement, 1982), (c) *alternative frameworks* (Driver & Erickson, 1983) and (d) *alternative conceptions* (Gilbert & Watts, 1983). Although the misconceptions have been labeled using different terms, it is very important to address them in order to improve conceptual understanding among students.

2.2 Conceptual understanding of electric circuits

A clear understanding of concepts related to electricity is important for students because many of the current and novel computer technologies are based on them (Chabay & Sherwood, 2006).

Students’ conceptual understanding of science concepts about electricity has been a proficient and well-documented research area (Mulhall, McKittrick, & Gunstone, 2001). Since the 1970s, there has been a widely cited bibliography of studies about student and teacher conceptions of sciences and other topics, where electricity is among them (Duit, 2009). The long-standing interest and strong concentration of research in electricity related conceptions are attributed to two essential reasons: (a) electricity is one of the central areas of science curricula at all levels of education, and (b) its concepts are particularly difficult to teach and learn because they are abstract and complex (Gott, 1985b). Therefore, both teachers and students face several challenges throughout the learning process (Gott, 1985a).

The abstract and complex nature of electricity-related concepts make many students generate conceptions and ideas that can be in conflict with the formal science perspectives. These are called misconceptions (Treagust & Duit, 2008). They are difficult to solve inside the classroom because these fragmented or incorrect deep-rooted ideas are developed prior to instruction (Chi, Slotta, & De Leeuw, 1994). Thus, they are often held after formal instruction (Engelhardt & Beichner, 2004; McDermott & Shaffer, 1992a; Mulhall et al., 2001).

2.3 Learning measures for conceptual understanding of electric circuits

The most common method used to measure academic mastery and knowledge in the electric circuit domain has been student ability to solve quantitative problems (McDermott & Shaffer, 1992a). However, this assessment mechanism has some limitations because success in calculating mathematical answers does not necessarily mean an equivalent ability in conceptual understanding (Kim & Pak, 2002). Although students may find the right solution for a specific formula or equation, they often struggle to interpret or explain the physical meaning of these problems (McDermott, 1991).

As a result, a particular assessment tool called concept inventory is commonly employed to measure conceptual understanding and not just problem solving skills. Sangam and Jesiek (2010) presented concept inventories as assessment instruments for fundamental concepts for a particular domain, with direct application in assessing curricula, instruction and students' conceptual understanding. Concepts inventories address these concepts through multiple-choice questions where the common misconceptions are used as distracters (Sangam & Jesiek, 2010; Sangam, 2012). Thus, concept inventories are often used to measure the effectiveness of an intervention, commonly applied in a pre- and post-test fashion.

Sangam and Jesiek (2010) presented a review of the different concept inventories used to assess basic electric circuits concepts, including direct current (DC) and alternating current (AC) circuits. There is a more detailed version of this review by Sangam (2012). The concept inventories reviewed were: Determining and Interpreting Resistive Electric Circuits Concepts Test (DIRECT) (Engelhardt & Beichner, 2004), Circuits Concept Inventories (CCI) (Rancour & Helgeland, 2003), AC/DC Concepts Test (Holton, Verma, & Biswas, 2008), and Electric Circuits Concept Evaluation (ECCE) (Sokoloff, 1996). In their comparison, the author remarks the characteristics and differences between these tests in terms of their scope and statistical quality. DIRECT and AC/DC Concepts Test have many reported results. In contrast, CCI and ECCE do not have the same among of information reported. DIRECT and AC/DC concepts test were developed based on student interaction and experts' views, with a focus on qualitative assessment of misconceptions. Their developers reported the statistical quality and their target population was high school and early college students; although, DIRECT only focuses on DC circuits. CCI has a quantitative approach and sophomore students are its target population. More details of the comparison are shown in Table 2.1, which has been taken from Sangam and Jesiek (2010) and Sangam (2012), the table was modified to meet APA standards.

Table 2.1: Comparison of electric circuit concept inventories

Criteria	DIRECT	CCI	AC/DC Concepts test	ECCE
Scope	DC resistive circuits	AC and DC circuits	AC and DC circuits	AC and DC circuits
Instrument design methodology	Experts and field testing	Not known*	Student interviews and expert views	Not known*
DC concept question ratio	29/29	18/25	13/20	38/45
Number of options per question	3 to 5	4	3 to 5	3 to 6
Basis for choice of distracters	Misconceptions (literature)	Not known*	Invariants and misconceptions	Not known*
Type of assessment used for	Qualitative	Quantitative	Qualitative	Qualitative
Application	Pre/post-test results for assessment of curriculum, or instruction, and students' conceptual understanding	Pre/post-test to gauge knowledge prior to instructions, and assess instructions or curriculum	Pre/post-test results for assessment of curriculum, or instruction, and students' conceptual understanding	Not known*

Table 2.1: Comparison of electric circuit concept inventories (Continued)

Statistical reports of quality	• KR-20 = 0.71 (reliability) • Content validity • Construct validity • Point biserial correlation • Discrimination index • Difficulty index	Not known*	• KR-20 = 0.687 (reliability) • Point biserial correlation • Item difficulty • Ferguson's Delta • Item discrimination index	Not known*
Target population	High school and college / university students	Sophomore students	High school and early undergraduate	Not known*
Other characteristics	Uses multiple representations of problems (symbols, words, pictures)	Uses only symbols	Uses symbols, words, and pictures; Questions are presented in context (situated in everyday language); Revised for test wiseness+	Uses symbols

* Information not found in published sources or personal communication.

+ Ability of students to do well in test due to cues found in question wording.

2.4 Common misconceptions in electric circuit theory

There is an extensive literature related to student misconceptions and difficulties in electric circuits, even though, the most relevant and cited work has been the one done by McDermott and Shaffer (1992a). These authors present their identified common difficulties and misconceptions students experience in understanding electric circuits. Their conclusion is based on data gathered from interviews based on interaction with real equipment and simulations, laboratory activities, class discussions, homework assignments and examinations; and with participants with difference levels of expertise in the field. In their work, these authors grouped student difficulties into three general, non-exclusive categories. Students struggle to (a) implement formal concepts into electric circuits, this is the ability to understand and associate the correct meaning to a formal concept, conceptual knowledge; (b) relate formal and graphic representations to technical concepts, this is the ability to manipulate mathematics equations and diagrams relating them to the conceptual meaning; and (c) apply qualitative reasoning in electric circuits, this represents the lack of a conceptual framework that allows them to reason holistically about a particular problem in a qualitative manner (McDermott & Shaffer, 1992a).

Student misconceptions represent the inability to employ formal concepts in electric circuits, which is conceptual understanding by itself. This is particular interesting for this study because, the other misconceptions are surely related to a lack of conceptual knowledge due to how students with conceptual understanding could use it to deal with the other misunderstandings, by reasoning qualitative or identifying details of the problem to solve, detecting possible mistakes or selecting the best procedure to solve it. Moreover, the misconceptions related to the relationship among different types of representations and conceptual knowledge are also particularly interesting for this study because they demonstrate in context the importance of representational competence and fluency; this is going to be expanded later in this document.

The use of formal concepts in electric circuits is compounded by specific misconceptions about electric current, voltage and resistance as an element and a current-voltage relationship (McDermott & Shaffer, 1992a). Sangam and Jesiek (2012) take and probe these findings in a study over approximately 150 first year engineering students where they found students difficulties and misconceptions can still remain after formal instruction. Based on McDermott & Shaffer (1992a), Sangam and Jesiek (2012) presented a summary of the common student difficulties in electric circuits. Students often mistakenly think the current in a circuit (a) depends on how the elements are arranged, (b) is consumed by the elements, and (c) a battery is a source of current; and about the voltage students misunderstand the concepts of potential and potential difference (Sangam & Jesiek, 2012). Table 2.2 shows a summary of the common misconceptions reported by Sangam and Jesiek (2012).

Table 2.2: Common misconceptions in electric circuits

Concept	Difficulty/Misconception
Current	Current in a circuit depends on the order and direction of elements
	Current is used up
	Battery is a source of current
Voltage	Ideal battery maintains a constant potential difference across its terminals
	Difference between potential and potential difference

In another study, Taslidere (2013) compiled the extensive literature on the topic in 11 misconceptions or wrong models student may have.

These are: (1) sink model is the misconception in which students think just one wire connection from a battery to an electric device would be enough power up the device. (2) In the attenuation model misconception students think the current is consumed by the elements along the circuit, or students can also think (3) the current is shared equally by the elements in the circuits regardless its characteristics in the shared current model. (4) In the sequential model, the misconception is to think a change in the circuit affects only the following elements in the circuit and not every one of them. (5) In the clashing current model, students think that what made a circuit element works is the clash of the positive and negative electricity coming from the battery's terminals. (6) Students follow an empirical rule model that says a bulb farther from the source is dimmer than a closer bulb. (7) The short circuit misconception is when wires with no electrical elements are ignored for the analysis. (8) The understanding of the power supply as a constant current source and not as an electric energy source. (9) In the parallel circuit misconception, students think any increase in the number of resistors would increase the overall resistance. (10) Another difficulty students often have is local reasoning ignoring the other part of the circuits or thinking of the circuit as a whole. (11) Understanding current flow as water flows in the nodes, therefore, most of the current goes through the straight wire regardless the elements in each branch.

Although the misconceptions Taslidere (2013) summarized are very common among students, these are very dependent on the problem context and are not oriented from a conceptual understanding perspective in contrast to the ones identified by McDermott and Shaffer (1992a). These two identified groups of misconceptions are highly related and overlap as follows. When Taslidere's misconceptions are thought form a conceptual understanding perspective they fall into the categories given by McDermott and Shaffer of difficulties with formal concepts (i.e., current, voltage and resistance) or multiple representations.

2.5 Instructional strategies to address misconceptions in electric circuits

McDermott and Shaffer (1992a) recognized the need for instructional materials that will enhance the learning process and promote conceptual understanding inside the classroom. This need has led to developing different strategies to address misconceptions in the domain of electric circuits. For instance, researchers have developed modifications to the curricula such as the application of deeper educational interventions with computational and laboratory activities and components of cyberlearning tools as opposed to traditional materials and pedagogies. The learning materials employed are intended to be used along with course lectures and textbooks. Some examples of this are tutorial modules (McDermott & Shaffer, 1992b) or concepts cartoons worksheets (Taslidere, 2013).

Besides, Baltzis and Koukias (2009) researched the efficacy of traditional teaching methods along with computer tools. Findings of their implementation in a circuit analysis course suggested an overall improvement in student performance and engagement. The cyberlearning implementation that the authors used allowed students to see the electric diagram, and to study and analyze the circuit's characteristics (Baltzis & Koukias, 2009).

In addition, Kezunovic, Abur, Huang, Bose and Tomsovic (2004) proposed enhancing traditional teaching methods with computer modeling and simulations. In their study, Kezunovic and colleagues found that students appreciated applying cyberlearning environments in electrical engineering courses.

Similarly, Li and Khan (2005) argued that the application of a cyberlearning environment in measurement, analysis, design and simulation of circuits improved student understanding of theories. The authors attributed student improvement to the schematic model and the simulated characteristics of the circuit, and with this, students were able to do comparisons among measured, simulated, and theoretically calculated results (Li & Khan, 2005).

2.6 Chapter summary

This chapter explored the literature related to misconceptions in science and engineering in general and specifically with misconceptions associated with electric circuit theory. It began with and introduction to the idea of conceptual understanding moving across common misconceptions students encounter, and ways to identify and measure them. Finally, the chapter explored the solutions and strategies educators have used to address the common misconceptions in the domain. The last study explored has particular importance for this research. This study suggests that the use of different schematic models, along with the use of simulations of circuits, significantly improved students' understanding of circuit theory. The authors of this study will explore this idea deeper in the following sections.

CHAPTER 3. THEORETICAL FRAMEWORK

Model-based reasoning (MBR) was the theoretical framework that guided this investigation. This type of reasoning studies how existing representations can be used to create novel scientific representations (Nersessian, 2002). MBR assumes that constructing new representations in science often starts with modeling and then is followed by a quantitative formulation (Nersessian, 2002). MBR has a strong focus on the processes or practices that lead to concept formation and concept understanding, more than on the product resulting from the modeling process. Thus, the emphasis is placed on the modeling process instead of the concept being modeled.

Nersessian (1999, 2002) has claimed three important forms of reasoning that take place during MBR episodes: analogical modeling, visual modeling, and thought experimentation. These forms of reasoning can lead to concept formation, conceptual understanding, or conceptual change. Analogical modeling refers to the process of borrowing mental models from another domain to exemplify the principles and constraints of the studied domain (Nersessian, 2002). Visual modeling uses images to provide support and to reason with a mental model (Nersessian, 2002). Finally, thought experimentation, also called simulative modeling, is a mental simulation of the phenomenon and normally employs hypothetic situations to represent the concept.

Although these forms of reasoning are considered separately, they are often used together in reasoning and problem solving during MBR (Nersessian, 2002).

Under an MBR perspective, models are considered a bridge between a phenomenon and its representation in a particular language (e.g., mathematical representation, spoken language), along with the mental representations used to carry out reasoning and the means to think and understand the concepts (Nersessian, 1999). In practice, MBR allows students to express their understanding by generating models of the concepts studied in class, which is often achieved by making students create external representations of these same concepts (Lesh & Doerr, 2003).

One form of MBR is models and modeling perspective (MMP) where the development of mental models used to represent particular concepts is studied (Lesh & Doerr, 2003). Models are conceptual systems used to generate and describe other systems by representing them with external notation systems (Lesh & Doerr, 2003). This definition is close to the one given by Treagust and Duit (2008) where models are ways to represent concepts externally or internally during reasoning processes. Along this work, the ability to express, use and think about models in each media has been labeled as representational competence (Kozma & Russell, 2005).

Modeling is the process of generating representational descriptions or models with a particular aim in specific circumstances (Lesh & Lehrer, 2003). This process is based on the ability to map between different representations, which is called representational fluency (Moore et al., 2013). Moreover, this ability has recognized as a mechanism to develop and assess conceptual understanding (Johri & Lohani, 2011;

McCracken & Newstetter, 2001; Moore et al., 2013). Representational fluency is also known as representational transformation (McCracken & Newstetter, 2001), or representational literacy, which is a broader term that includes representational competence (Johri & Lohani, 2011).

Specifically, MMP focuses on the development and creation of mathematical, computational, and other forms of representations where individuals produce conceptual tools. Such conceptual tools may include models designed to identify aspects that may be related to how students interpret particular problem-solving situations. Based on this perspective, MMP assumes that humans interpret their experiences using internal conceptual systems in order to select, filter, organize, transform or infer patterns from information (Lesh & Lehrer, 2003).

However, MMP also recognizes that in order to have sufficient tools for dealing with real-life problems, relevant conceptual systems must be expressed in a variety of interacting media, including spoken language, written symbols, diagrams, metaphors and computational simulations (Cramer, 2003; Johnson & Lesh, 2003; Lesh & Harel, 2003). To accomplish this, students are exposed to model-eliciting activities (MEAs) in which they develop solutions that indicate their representational competence and fluency (Moore et al., 2013). Thus, MEAs are problem solving assignments based on real-world situations, where students develop, construct, describe or explain different representations in order to find an appropriate solution (Hamilton, Lesh, Lester, & Brilleslyper, 2008; Lesh, 2010; Moore et al., 2013).

MBR as the theoretical framework of this study supports the use of MMP as (a) a guide for creating the learning design highlighting representational competence and fluency, and (b) a method for identification and assessment of student representational competence and fluency through the representations generated by the students.

3.1 Chapter summary

This chapter explored the theoretical framework that guided this research. It began by explaining Model-based reasoning (MBR) as on form of scientific cognition that investigates how novel scientific representations are created with a strong focus on the modeling process. After that, the authors summed up the forms of reasoning during MBR. Then, we introduced Models and modeling perspective (MMP) as a form of MBR focused on the process of creating representations where individuals produce conceptual tools. This chapter also defined the terms representational competence and fluency. Lastly, the implications of the use of MBR as the theoretical framework for this study were shown.

CHAPTER 4. LEARNING DESIGN

This research study was developed in the context of a circuit analysis course. These college level courses are usually taught not only for second year engineering students in fields such as electrical, electronic, and computer engineering, but also students from other disciplines such as mechanical engineering or sciences. The course attempts to introduce students to circuit analysis and diverse topics as general circuit elements, Kirchhoff's laws and circuit equations, Thevenin's and Norton's theorems, and step response. Common learning objectives these courses have are:

- Learning the vocabulary, principles, and analysis methods of circuit theory.
- Learning to write and solve equilibrium equations for electric circuits.
- Learning to calculate the step response for several types of circuits.
- Learning to apply and use Thevenin's and Norton's theorems.
- Foster skill development of rational and critical thinking, problem solving, and self-evaluation.

Circuit analysis courses have some previous requirements student must achieve in order to be able to understand and work with the mathematical support of the concepts. The requirements normally include some calculus knowledge such as differential and integral calculus; capacity to work with complex numbers and variables;

basic knowledge of linear algebra, vectors, and matrices; and some computer literacy and experience with programming in engineering.

4.1 Pedagogical approach

Model-eliciting activities (MEAs) were chosen from models and modeling perspective to guide the design of the learning experience. MEAs are thought revealing activities that can be used to demonstrate student modeling abilities, representational competence, and representational fluency (Lesh, 2010). Thus, MEAs are appropriate mechanisms because the solution process requires to constantly shift among several models or representations.

Essentially, MEAs are open-ended problems that simulate interdisciplinary real-world scenarios and employ teamwork to work to solve the problem over brief periods of time (Hamilton et al., 2008; Lesh, 2010; Moore et al., 2013).

During the solving process, the students clearly recognize the need to use, develop, describe, or explain different models, even from diverse engineering concepts and disciplines (Moore et al., 2013). Real-world scenarios are typically achieved by using client-driven situations. Teamwork approach is used mainly because the short time design for the activities. The teams could have three to five students who work on the solution usually over a period of one or two classes, which means 60-90 minutes work (Hamilton et al., 2008). Even though the time period is related to the MEA complexity and whether or not it was meant to be solved in the classroom or at home. Yildirim, Shuman, and Besterfield-Sacre (2010) have found the MEA solutions developed as out of class activities have usually more depth due to students having more information and time available to the problem solving process.

MEAs were originally developed in mathematics education, but they have been adapted and used for engineering education (Diefes-dux, Hjalmarson, Miller, & Lesh, 2008; Diefes-Dux & Imbrie, 2008; Zawojewski, Hjalmarson, Bowman, & Lesh, 2008). Four specific instructional benefits have been found in implementing MEAs. They improve conceptual understanding, problem solving, teamwork skills, and ethical reasoning (Yildirim et al., 2010). Yildirim et al. (2010) also postulated MEAs can have three 3 roles in the learning process of engineering concepts:

- Reinforcement: Reinforce concepts in the topic studied currently;
- Integration: Integrate pre-acquired knowledge with new information; and
- Discovery: Discover concepts before they have been formally introduced.

Besides using MEAs to improve conceptual understanding, they have also been used to address student misconceptions. For instance, the study developed by Self et al. (2008) addressed student misconceptions in mechanics and thermal science using MEAs. Other impacts made with MEAs implementations include decreasing educational gaps between genders in engineering, improving student creativity, and encouraging students to use more advanced engineering knowledge (Yildirim et al., 2010).

4.1.1 Design of an MEA

MEA design requires the application of the six guiding principles: (1) reality principle, (2) model construction, (3) model documentation, (4) self-evaluation, (5) model generalization, and (6) effective prototype (Hamilton et al., 2008; Lesh, Hoover, Hole, Kelly, & Post, 2000):

- Reality principle: The MEA presents a real life engineering context and allows students to consider the constraints and needs of the particular situation with different levels of knowledge and ability (Diefes-dux et al., 2008; Moore, 2008).
- Model construction: The MEA requires the construct of a model as an explicit construction or description. Students have just enough information to make an educated decision about how to reach the problem requirements (Moore & Diefes-Dux, 2004).
- Model documentation: Students are required to provide documentation that aims to reveal the thinking process employed to solve the problem (Moore, 2008).
- Self-evaluation: The MEA should provide opportunities that allow students to judge if the proposed solution or the thinking process need to be improved or extended for a given requirement (Lesh et al., 2000; Moore, 2008).
- Model generalization: Also called model share-ability or re-usability. The models developed during the activity and the way of thinking must be applicable in similar engineering problems (Lesh et al., 2000; Moore, 2008).
- Effective prototype: The solution and the models developed should be a product generalizable or modifiable and facilitate the design of other models in similar situations (Diefes-dux et al., 2008).

4.1.2 Fostering representational fluency and competence through MEA

MEAs have been used to assess and prove student modeling abilities, and representational competence and fluency (Lesh, 2010). Throughout the solution process of an MEA problem, students go through development cycles to create models for the final solution (Moore & Diefes-Dux, 2004). The modeling cycles contain four steps that involve representational competence and fluency in its usage. The four steps include: (1) description, mapping process from the real world to the model world; (2) manipulation of the model to think about the original problem to solve, in order to generate predictions or actions for the problem; (3) translation (or prediction) of relevant results to the real world again; (4) verification of the actions and predictions generated in the manipulation stage (Lesh & Doerr, 2003).

Thus, in the solution process of an MEA students use, visualize, and think about models in different representations, which is representational competence (Moore et al., 2013). Additionally, students go through the modeling process by creating several types of representations mainly while mapping from one representation to another one, which is representational fluency (Moore et al., 2013). These abilities have been identified as fostering and assessment instruments for conceptual knowledge understanding (Johri & Lohani, 2011; McCracken & Newstetter, 2001; Moore et al., 2013). As a result, representational competence and fluency could be measured by analysis student responses to an MEA, one example of this is when Moore et al. (2013) developed a qualitative analysis of MEA solutions to measure these abilities.

4.1.3 Assessment and rubric of an MEA

Usually, the assessment of student performance in MEA solutions is done through a rubric based on the design principles (Clark, Shuman, & Besterfield-Sacre, 2010; Yildirim et al., 2010). Each element of the rubric is graded on a one-to-five scale, indicating the level in which the solutions achieved the principles. The element used by Yildirim et al. (2010) were:

- Generalizability: Accuracy of the solution for the problem and future similar cases.
- Self-Assessment or testing: Level in which the solution has been tested and the thinking process and procedures have been revised.
- Model documentation: Level of detail and quality of the writing, taking into account the clarity, grammar, and readability.
- Effective prototype: Level of quality of the solution in terms of refinement and elegance.

4.2 Chapter summary

This chapter has described the learning design used during this study to promote conceptual understanding, representational fluency, and representational competence in circuit design. As implication on the using model-based reasoning and models and modeling perspective, model-eliciting activities were implemented. First, the context of the activity was described. This is the description of the class in where the MEA took place.

Secondly, there was presented a detail introduction about MEAs, their definition, their possible roles in the learning process, their use and benefits in engineering education, specifically in conceptual understanding and addressing student misconceptions. In this section, there were also exposed the design principles, solution process and assessment of MEAs.

CHAPTER 5. METHODS

This research followed a case study approach to explore the effect of employing multiple representations and students’ ability to transfer knowledge between representations upon conceptual understanding of electric circuits in an undergraduate level course. The aim of this study was not to generalize findings over conceptual understanding in electric circuits or students’ representational competence and fluency, but rather to explore how conceptual understanding in electric circuits is related to students’ representational competence and fluency ability.

A case study approach usually consists of a detailed investigation of a phenomenon within its context, because the main purpose is to understand how the phenomenon is influenced by, and influence a concrete situation (Hartley, 2004). The strength of the case study methodology lays down over the concrete and context dependent nature of the targeted phenomena (Case & Light, 2011). Therefore, this approach was appropriate for the present study because the researchers were exploring specific instances of representational competence and fluency in a particular situation, which is conceptual understanding of electric circuits. Additionally, the findings of this research are relative to the phenomenon and context dependent, which is useful to “address research questions concerned with the specific application of initiatives or innovations to improve or enhance learning and teaching” (Case & Light, 2011, p. 191). Moreover, case studies take the advantage of using diverse sources and forms of data like quantitative, qualitative or both (Hartley, 2004; Kohlbacher, 2006), in order to provide a deeper definition of the phenomenon. This research study has employed a qualitative categorical analysis followed by a quantitative statistical analysis.

Two studies were developed during this research. The case study one was performed in the fall of 2014 and case study two was executed in spring of 2016. Both data collections were voluntary extra-credit homework assignment for a circuit analysis course, although during the case one, students received credit for partial answers which allowed them to choose not to develop every representation requested generating a “no-response” scenario. In order to avoid the latter phenomenon, during the case two, participants only received extra credit if they completely solved the assignment.

Until now, this chapter has described the methodological approach employed. The next subsections present the participants, materials, procedures and data analysis of the two case studies.

5.1 Participants

The participants for case study one were 25 sophomore engineering students enrolled in a linear circuits analysis course (i.e., ECE 201 Linear Circuit Analysis I) in the fall semester of 2014. The participants for case study two were 26 sophomore students enrolled in the same course but during the spring semester of 2016. The two courses were offered by different instructors. The course is regularly offered by the Electrical and Computer Engineering Department (ECE) at Purdue University. Throughout the first year, students are required to take courses that provide the foundation needed for the course. Previous foundation courses provide background knowledge about differential and integral calculus, complex numbers, vectors and matrices, computer literacy and experience with MATLAB® or an equivalent programming language. Therefore, when students get to the electric circuits course, they are expected to understand and successfully employ multiple representations (e.g., mathematical representations), as well as the basic programming structures used in computational representations. Throughout the course, students learn how to employ those representations in the circuit analysis context.

5.2 Materials

For this project, the assignment designed with the principles of model-eliciting activities (MEA) was employed to measure the students' representational competence and fluency, as well as conceptual understanding. It was given to the students as a homework assignment as part of the above mentioned electric circuit course. The assignment was chosen to be based on MEAs due to their focus on conceptual understanding, modeling abilities, representational competence, representational fluency.

Our MEA was designed with the goal of reinforcing student acquired knowledge in the class. In addition, its goal was also to expose student misconceptions about current, voltage and resistance by probing student representational competence and fluency and their relationship with student conceptual understanding of the topic.

The assignment was designed by applying design principles of MEAs mentioned above in the learning design chapter (CHAPTER 4 LEARNING DESIGN). This was done by using a problem from a real situation from the student perspective where students had to construct different models and representations to solve the tasks and provide documentation that shows their thinking process. The assignment used an adapted textbook problem from DeCarlo and Lin (2009, p. 142, Problem 3.2) about starting a car, which battery has died, with the help of a friend's battery. Moreover, along the questions in the MEA, the students had different opportunities for self-evaluation and judgment of their own work with the representations and models. Although the assignment was context-based, the models generated during the solving process could be applicable to other engineering problems on a related topic. Finally, the learning outcomes and the models generated during the solution of the MEA could be used for other contexts and similar situations as well.

Despite the teamwork approach of MEAs, this characteristic was not intended to be part of our MEA design for this project due to the course context; individual work was preferred in order to gather data from each student and uncover individual misconceptions and representational abilities.

The MEA analyzed 3 forms of representations: diagrammatic, mathematical and computational. Results' interpretation and conceptual inferential questions were placed to measure the conceptual understanding students have about the concepts in the topic.

The challenge consisted of two different tasks, each one with 3 section: section (1) is to develop the diagram (only for the second task), and the equations of the circuits; section (2) is to create the computational representation, and section (3) is to explain the solutions and answer inferential questions for conceptual understanding. The context of the challenge was a situation in which the battery of student's car was dead and they would have to start the vehicle with the help of the car's battery of a friend.

In the first task, students were provided with a basic circuit connection for the batteries in a form of a diagram. From there, students should have analyzed the electric circuit and created and solved the mathematical model and equations of the problem, this is the mathematical representation. Then, students used a computational tool (i.e., MATLAB® or an equivalent software) to create the computational representation and generate graphs of the system output. With the this in hand, students should have found the minimum voltage the dead battery needed to crank the engine in the basic circuit configuration. By comparing the minimum voltage required and the voltage of the dead battery, students should have found out that in the basic configuration the car would not start.

In the task number two, students optimized the basic configuration in order to design a circuit able to start the car. Here students expressed the optimized circuit through its diagrammatic representation, and also throughout its mathematical and computational representation as in the task number one. At the end of each task, students were asked to explain their solution for each circuit and answer inferential questions to measure their conceptual understanding of the covered concepts.

The assignment was built by a graduate student with a circuit analysis and teaching background and reviewed by other four circuit analysis experts (i.e., both instructors and two additional graduate students with a circuit analysis and teaching background) and one engineering education expert. The instructors established the assignment had questions and a level of difficulty appropriate for the course and student' previous knowledge. The engineering education expert revised the questions to meet the specific educational objective of interest. The assignment developed was revised after the case study one with no significant changes (i.e., the changes were made for clarity in the document). The assignment can be found in Appendix A.

5.3 Procedures

In both case studies, the assignment was an optional homework (out of class work) for the electric circuit course. For the case study one, the lecturer gave the assignment in a document with PDF format at the end of the class, students had one week to solve it individually. The ones who did it got extra credit based on their scores, which, as the authors later noticed, allowed participants to choose not to develop all representations asked. For the case study two, only the students who completely solve the assignment got extra credit, thus, during the data collection students were asked to respond the assignment completely to earn the extra credit, with this, the authors sought that all the participants generate every representation requested. For this case two, the students had two weeks to solve the assignment individually.

5.4 Data scoring and data analysis

The analysis for the MEA assignment began with a qualitative approach followed by a quantitative one. Thus, a categorical analysis was performed first, with this the initial rubric was revised during the case one; secondly, the student responses were assessed with the rubric and analyzed through descriptive statistics and correlational analysis.

The qualitative analysis of the representations consisted of comparing and contrasting students' individual responses to each question in order to identify categories of competence in each response. The answers to the conceptual understanding questions were analyzed with categorical analysis to characterize student conceptual understanding of the concepts. With this categorization, the initial rubric was revised during the case one in order to assess the student responses. The rubric had four sections; one for each representation (three representations at total) and one for conceptual understanding. The scoring level ranged from one to four and included a grading criterion of "no response".

Thus, the level of competence for each section of the rubric scored as (1) below basic, for responses without a clear representation or the representation and conceptual understanding showed by the student has a clear conceptual misconception; (2) basic, when students' solutions had an appropriate representation or conceptual understanding, but there might be some conceptual errors affecting the responses; (3) proficient, when the solutions had an appropriate representation or conceptual understanding but, there might be some errors due to issues outside the focus of the study (e.g., calculation, programming or mathematical errors); and (4) advanced, when students demonstrated an appropriate representation or conceptual understanding without any conceptual misunderstanding or mathematical errors. Examples of these categories are presented in the categorical analysis subsection (5.5 Categorical analysis). The rubric developed can be found in the Appendix B. Furthermore, the qualitative analysis was developed with a further interest in other trends that might occur, such as underlining misconceptions.

After the categorical analysis, each participant' solution was assessed with the rubric. During the case study one only whole points (i.e., four for advance, three for proficient, two for basic and one for below basic) were given to student representation, while during the case study two partial scores were given to appropriate but incomplete answers that fell between the categories developed (e.g., 3.5 for a representation between the advance and proficient level).

The scores were then analyzed through descriptive statistics for each representation and conceptual understanding section in order to measure representational competence. Three levels of achievement were considered, low for scores below 1.5, moderate for scores between 1.6 and 3.5, and high for scores over 3.6 (Sanchez, Magana, & Bermel, 2016).

Finally, a correlational analysis (i.e., Pearson product-moment correlation coefficient also known as Pearson's r) was performed to assess the relationship between and within representations and conceptual understanding, which is representational fluency. Five levels of correlation were considered, strong for correlations of 0.5 or higher, moderate-to-strong for correlation of 0.4 or higher, moderate for correlations of 0.25 or higher, weak-to-moderate for correlations of 0.2 or higher and weak for correlations lower than 0.2 (Rubin, 2012).

5.5 Categorical analysis

Specific trends were found after going through the students' responses. These trends were used to revise the scoring levels of the rubric during the case one. The categories were (1) below basic, (2) basic, (3) proficient, and (4) advanced depending on the quality of the answers, an additional scoring level of "no response" was also included because in the study one not all participants chose to generate all the representations asked. In the rubric, more weight was given to conceptual understanding errors than to other types of error, such as calculation error. This is due to the fact that the study is focused on conceptual understanding and students are assumed to have sophomore-level engineering competencies such as calculus and other areas required for the course. Descriptions and examples of the categories for each section are presented next.

5.5.1 Diagrammatic representation

Participants only generated this representation for the optimized circuit because it was the input representation for the basic circuit. Participants in the advanced category should have created a clear and fully accurate representation, taking into account all the circuit elements and the components in the problem context (e.g., the batteries and their internal resistor are not separable in the diagram because they are not separable physically in the real battery). Figure 5.1 shows two examples of this category.

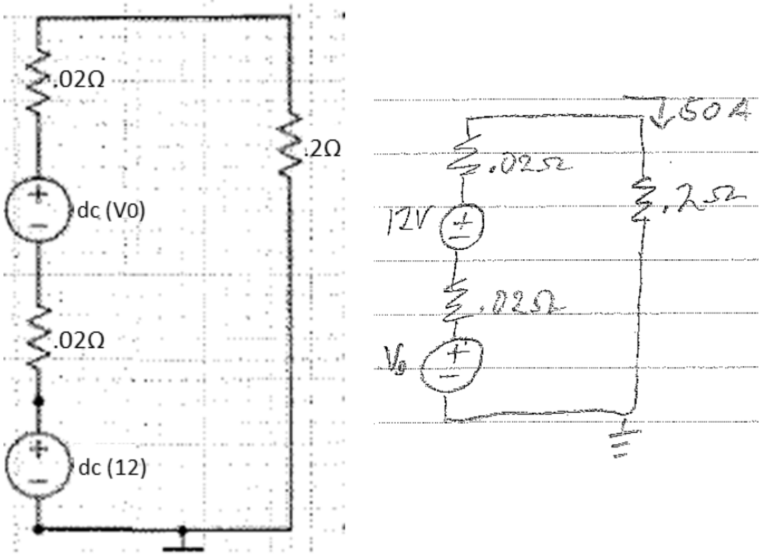


Figure 5.1: Diagrammatic representation, advanced category examples

Participants in the proficient category should have created a correct representation for a particular subcase of the problem but they did not take into account all the components in the circuits. Figure 5.2 displays one instance of this category, the participant did not represent the "dead battery" presumably because it was not adding voltage to the circuit. In this case, the circuit response would not change, but in order to have a fully accurate diagram the battery should have been included to take account of other cases.

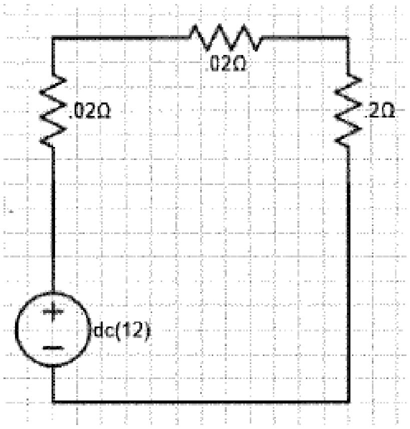


Figure 5.2: Diagrammatic representation, proficient category example

In the basic category, participants had the basic idea of the representation but they created a representation with some errors that affect the outcome of the circuit, Figure 5.3 illustrates one example of this category.

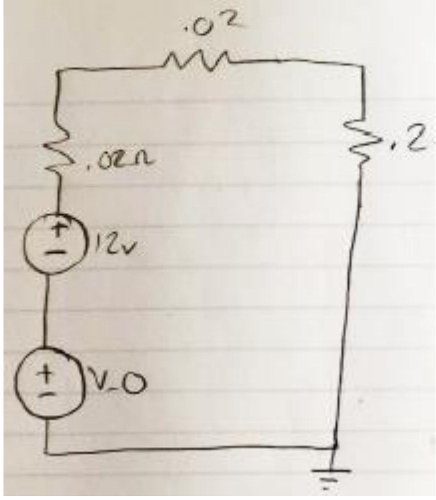


Figure 5.3: Diagrammatic representation, basic category example

Participants at the below basic level created a fully incorrect diagram for the optimized circuit, for instance, the representation in Figure 5.4.

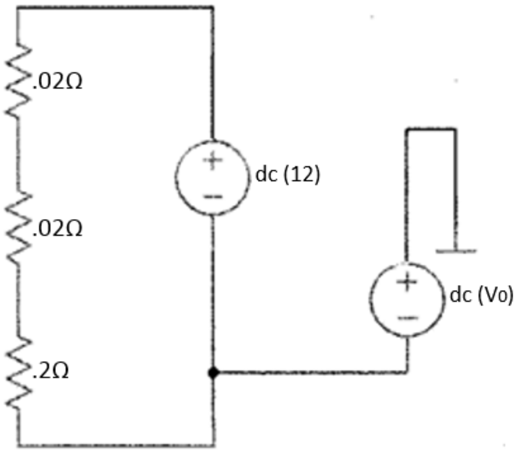
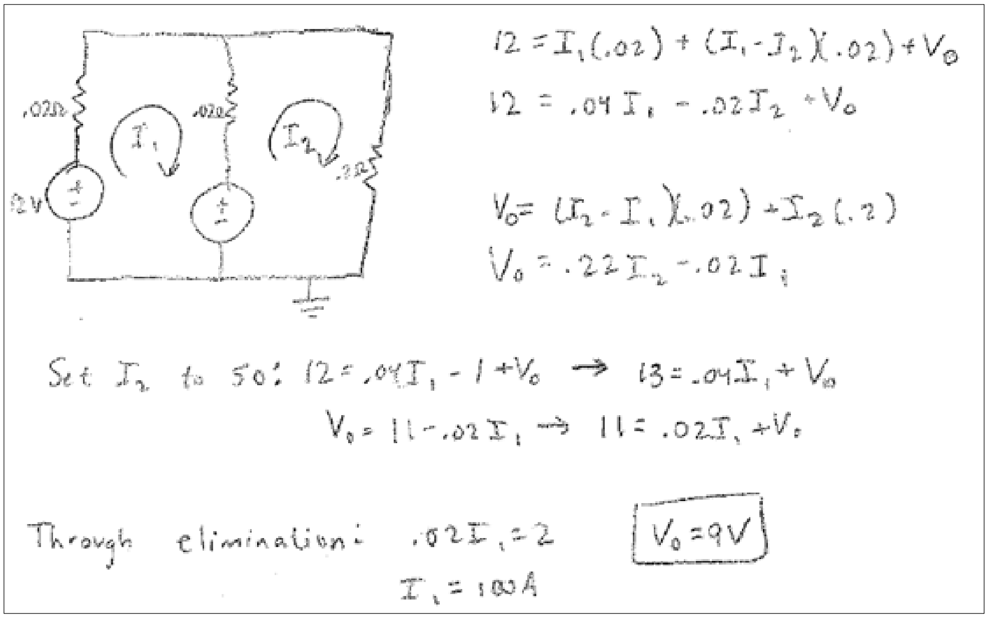


Figure 5.4: Diagrammatic representation, below basic category example

5.5.2 Mathematical representation

This representation was developed for both circuits, then, participants developed a total of two mathematical representations in the MEA. Additionally, for this representation, the circuit analysis had several solution methods for both circuits, methods such as nodal analysis using Kirchhoff's current law, mesh analysis using Kirchhoff's voltage law, superposition theorem, and source equivalences as Thévenin's theorem and Norton's theorem. However, the method employed by the participant to create the mathematical representation was not discriminatory and only the correctness of the answer was considered for scoring. Hence, participants in the advanced category developed a fully accurate representation free of conceptual understanding issues or calculation errors. In the example presented in Figure 5.5, the participant developed the mathematical representation of the basic circuit from the diagrammatic representation given with a mesh analysis. It can be seen how the students mapped from the diagrammatic representation to the mathematical representation in the mesh analysis, which in turns, make observable the importance of the representational fluency.



$12 = I_1(.02) + (I_1 - I_2)(.02) + V_0$

$12 = .04 I_1 - .02 I_2 + V_0$

$V_0 = (I_2 - I_1)(.02) + I_2(.2)$

$V_0 = .22 I_2 - .02 I_1$

Set I_2 to 50: $12 = .04 I_1 - 1 + V_0 \rightarrow 13 = .04 I_1 + V_0$

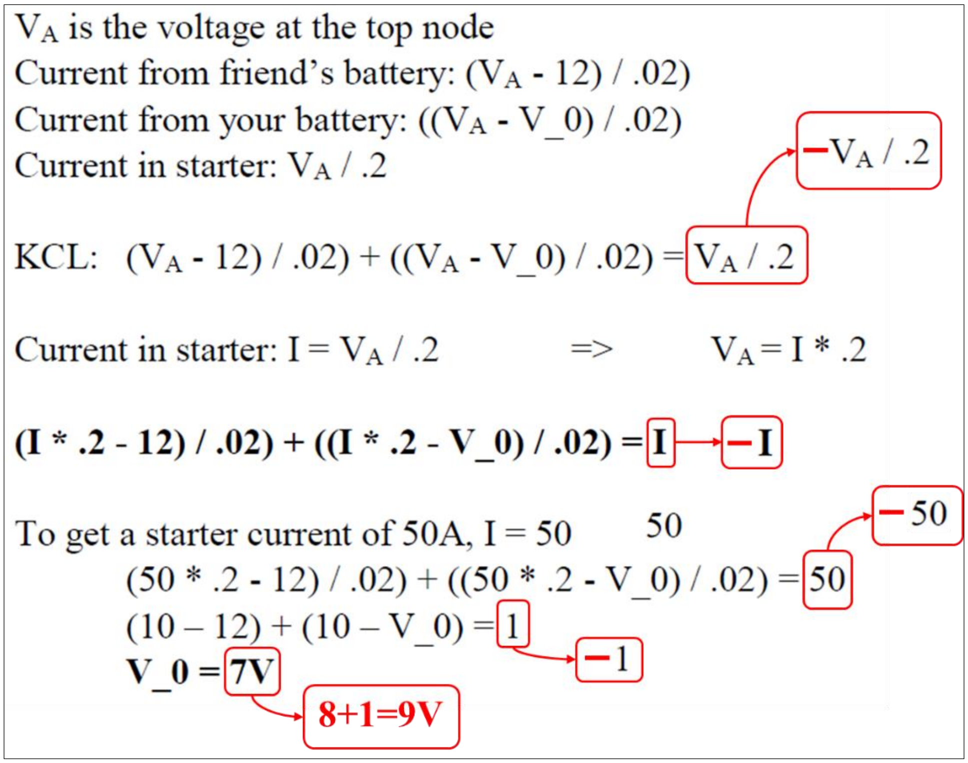
$V_0 = 11 - .02 I_1 \rightarrow 11 = .02 I_1 + V_0$

Through elimination: $.02 I_1 = 2$ $\boxed{V_0 = 9V}$

$I_1 = 100A$

Figure 5.5: Mathematical representation, advanced category example

Participants in the proficient category created the accurate mathematical model but the mathematical process possessed errors due to calculation issues or errors not related to conceptual understanding. One instance of this category is displayed in Figure 5.6, the errors are highlighted with red boxes. The mathematical model is correct, although this participant had an error with the sign of the current in the starter. The mathematical error is propagated until the final answer, where it is subtracting one from eight instead of summing up as it is supposed to do in the algebraic procedure. This gives seven as a result instead of the correct answer which is nine volts.



V_A is the voltage at the top node

Current from friend's battery: (V_A - 12) / .02)

Current from your battery: ((V_A - V_0) / .02)

Current in starter: V_A / .2

KCL: (V_A - 12) / .02) + ((V_A - V_0) / .02) = $\boxed{V_A / .2} \rightarrow \boxed{-V_A / .2}$

Current in starter: I = V_A / .2 => V_A = I * .2

(I * .2 - 12) / .02) + ((I * .2 - V_0) / .02) = $\boxed{\mathbf{I}} \rightarrow \boxed{\mathbf{-I}}$

To get a starter current of 50A, I = 50 50

(50 * .2 - 12) / .02) + ((50 * .2 - V_0) / .02) = $\boxed{50} \rightarrow \boxed{-50}$

(10 – 12) + (10 – V_0) = $\boxed{1} \rightarrow \boxed{-1}$

V_0 = $\boxed{\mathbf{7V}} \rightarrow \boxed{\mathbf{8+1=9V}}$

Figure 5.6: Mathematical representation, proficient category example

In the basic level, there were participants who realized a correct circuit analysis but had and modeling error due to conceptual understanding issues. In Figure 5.7, an example of this category is depicted. The errors are highlighted with red boxes. The red V_A represent the voltage in the homonym point of the circuit diagram. In this participant's model, the resistor's voltage is only depending on the voltage of the closest battery and not on the circuit itself. The voltage should depend on the voltage difference between both terminals of the resistor as is marked in red. This conceptual error impacted the student's mathematical representation and the correctness of the answer was compromised.

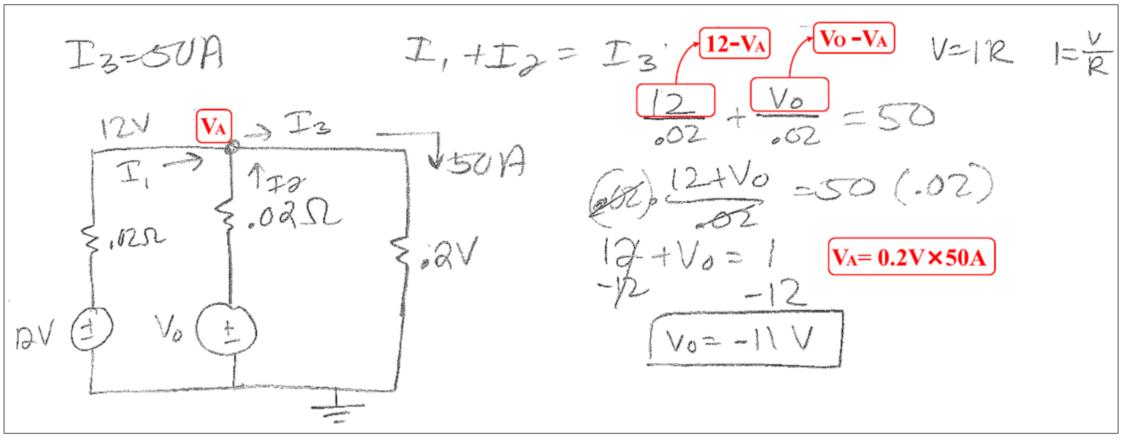
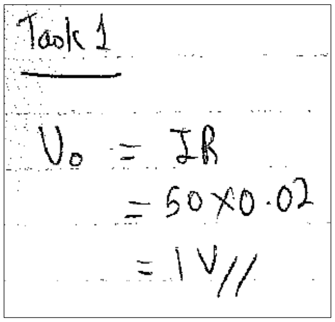


Figure 5.7: Mathematical representation, basic category example

Lastly, some responses were categorized as below basic because they did not present any circuit analysis or a clear misconception was evident. Figure 5.8 shows one instance of this category. The participant presented the Ohm's law equation but there is no circuit analysis using this equation. Additionally, the values employed are wrong without any further analysis. The mathematical representation was clearly uncompleted and without any circuit analysis or mathematical modeling.



Task 1

$V_o = IR$

$= 50 \times 0.02$

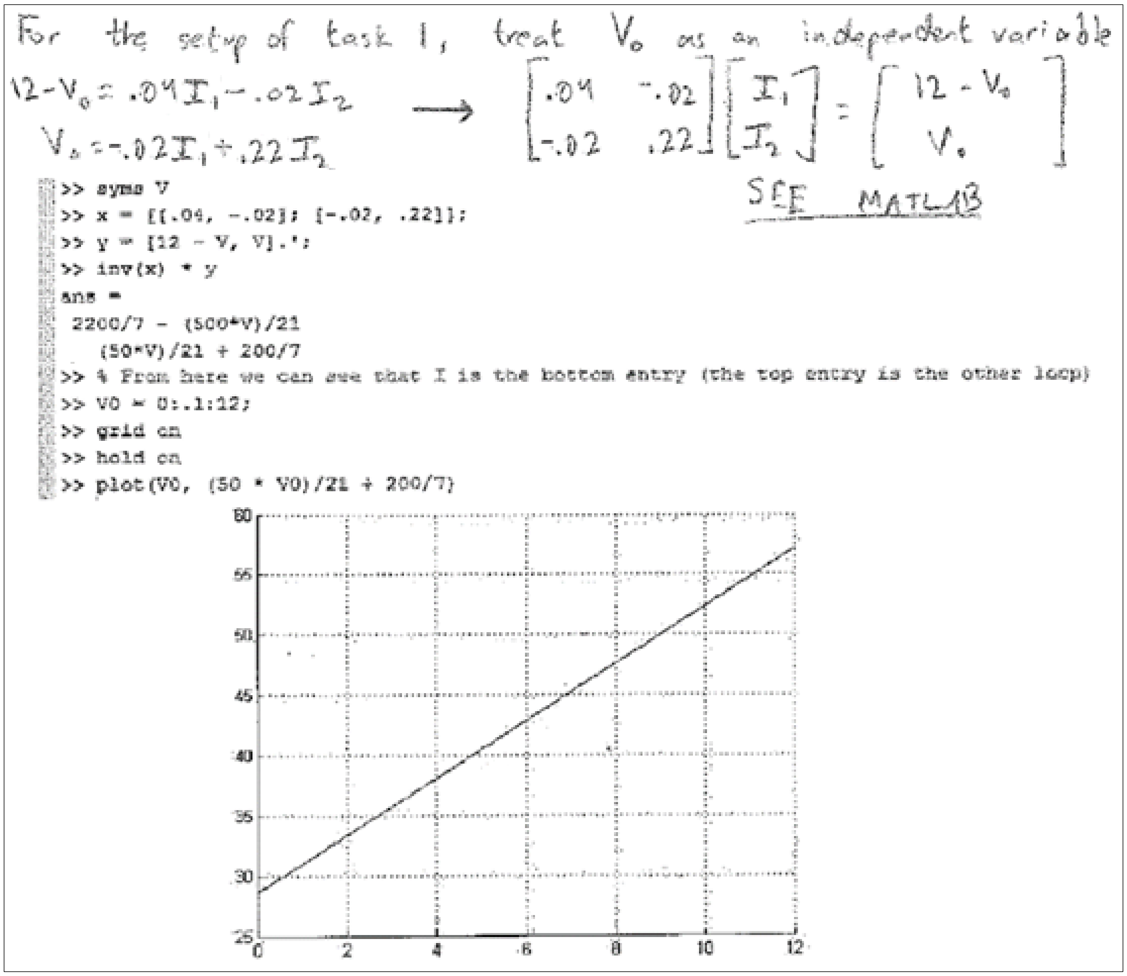
$= 1V$ //

Figure 5.8: Mathematical representation, below basic category example

5.5.3 Computational representation

This representation was also developed for both circuits. Based on the instructor's advice and the background students should have had, participants were asked to generate the computational representation using MATLAB®. However, as the research study is focused on the representation and not on the software, participants had the option to use a different computational program. Furthermore, there were different approaches to solving this task, students could have chosen to use an analytical or numerical approach, and, in turn, to use systems of equations or equivalent matrixes. The answers were categorized depending on their correctness and not on the method employed.

Participants in the advanced category generated fully accurate computational representations in which they solved the problem and generated the graphs requested. As the graphs are an important part of the computational representation, they were also taken into account for the categorization. For instance, it can be seen in Figure 5.9 how the student generated and solved the matrixes of the system to, then, generate the graph of the system output. In the example it can be also noticed how the student used the mathematical representation to map towards the computational representation of the problem.



For the setup of task 1, treat V_o as an independent variable

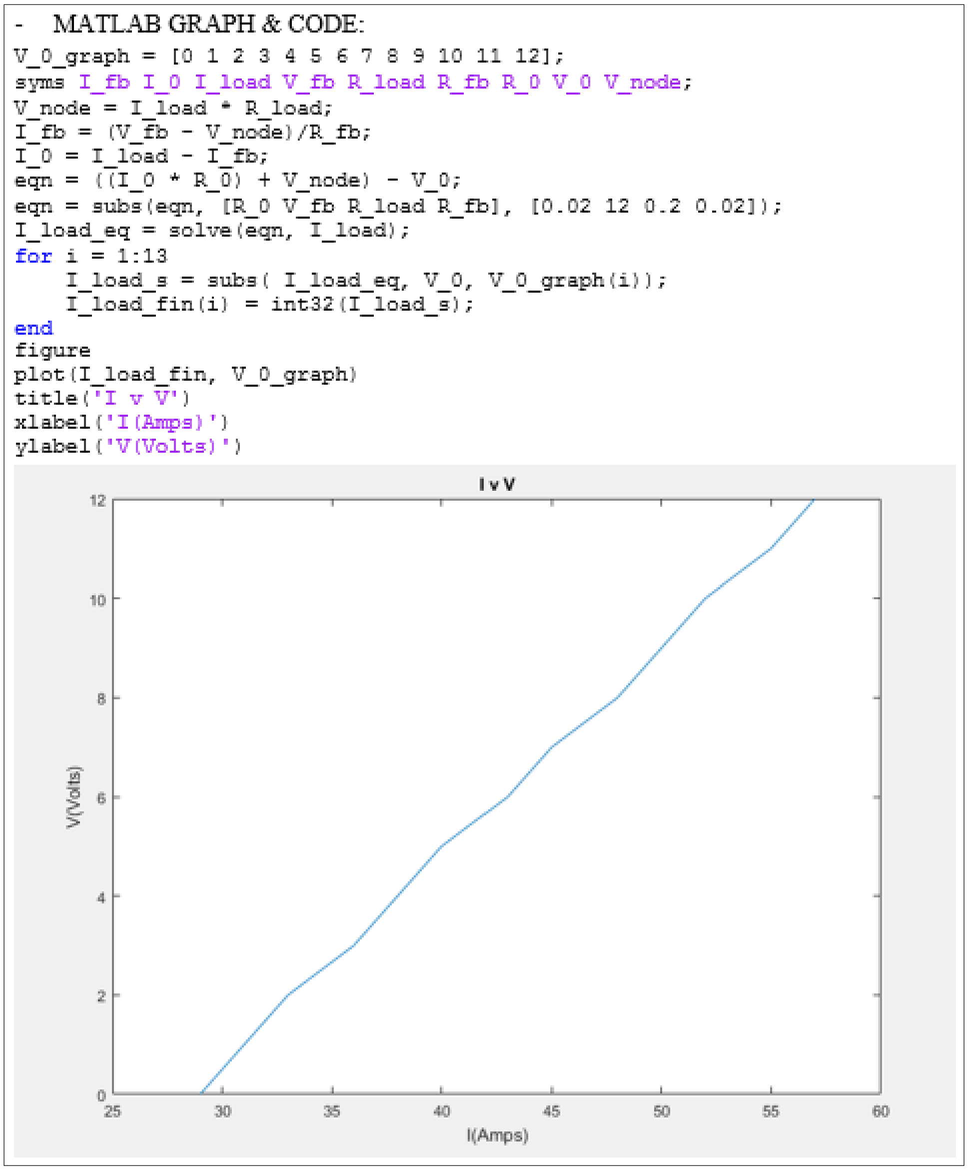
$$12 - V_o = .04 I_1 - .02 I_2$$
$$V_o = -.02 I_1 + .22 I_2 \longrightarrow \begin{bmatrix} .04 & -.02 \\ -.02 & .22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 - V_o \\ V_o \end{bmatrix}$$

SEE MATLAB

```
>> syms V
>> x = [[.04, -.02]; [-.02, .22]];
>> y = [12 - V, V].';
>> inv(x) * y
ans =
 2200/7 - (500*V)/21
   (50*V)/21 + 200/7
>> % From here we can see that I is the bottom entry (the top entry is the other loop)
>> V0 = 0:.1:12;
>> grid on
>> hold on
>> plot(V0, (50 * V0)/21 + 200/7)
```

Figure 5.9: Computational representation, advanced category example

The proficient category contained responses with correct reasoning but with programming or mathematical errors that affected the outcome on a small scale. The Figure 5.10 presents an example of this category where the participant solved the systems of equations and then generated the graph, which is slightly different than the expected. The student used a correct symbolic approach to solve the task and create the computational representation, although, the participant had two errors. The first error was to approximation the current response to integers (i.e., int32 function), with this the student lost the continuous characteristic of the current. The second error was that the participant confused the axes of the graph. The graph should be current against voltage (usually represented as “I vs V”), then the volts should be in the horizontal axis and the amperes should be in the vertical axis.



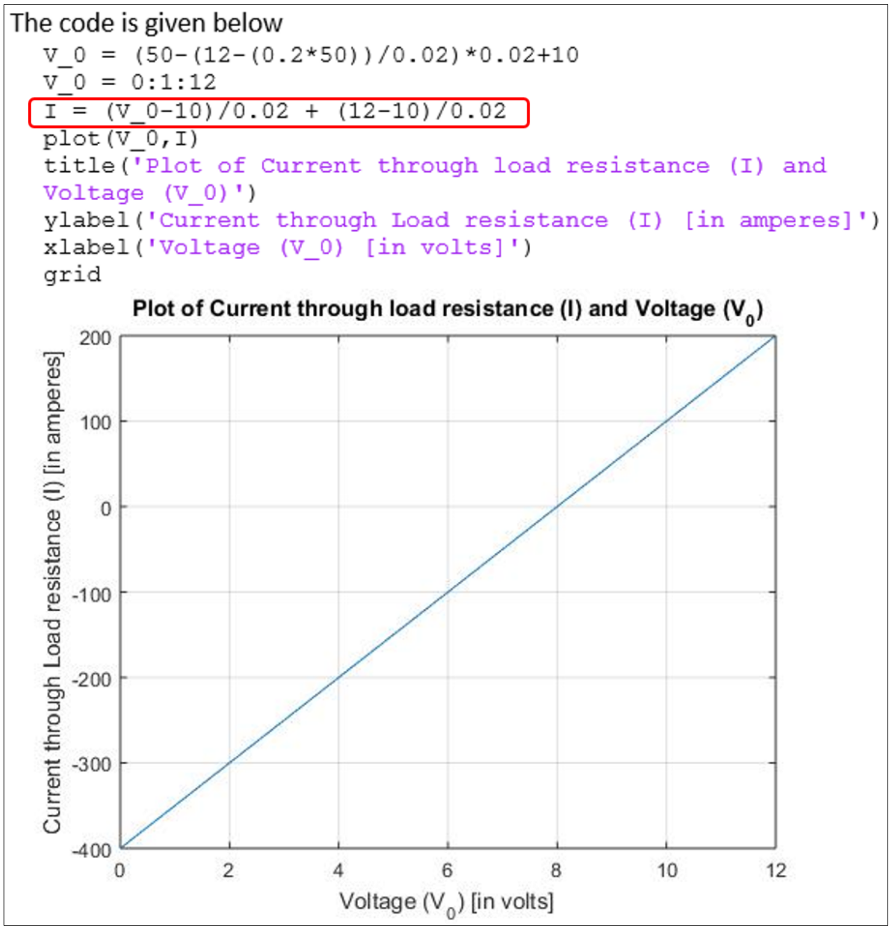
- MATLAB GRAPH & CODE:

```
V_0_graph = [0 1 2 3 4 5 6 7 8 9 10 11 12];
syms I_fb I_0 I_load V_fb R_load R_fb R_0 V_0 V_node;
V_node = I_load * R_load;
I_fb = (V_fb - V_node)/R_fb;
I_0 = I_load - I_fb;
eqn = ((I_0 * R_0) + V_node) - V_0;
eqn = subs(eqn, [R_0 V_fb R_load R_fb], [0.02 12 0.2 0.02]);
I_load_eq = solve(eqn, I_load);
for i = 1:13
    I_load_s = subs( I_load_eq, V_0, V_0_graph(i));
    I_load_fin(i) = int32(I_load_s);
end
figure
plot(I_load_fin, V_0_graph)
title('I v V')
xlabel('I(Amps)')
ylabel('V(Volts)')
```

Figure 5.10: Computational representation, proficient category example

Responses where the outcome was completely wrong due to other types of errors, were considered as basic. One instance of this category is shown in the Figure

5.11. The student’s error is in the four row and it is highlighted in a red box. In this error, the participant assumed the voltage in the common node of the batteries for the first configuration is constant. The voltage in this node is actually changing as the voltage of the batteries changes, then, it should be modeled through an equation instead of just a constant. Such error affected the graphic outcome making it go to extreme and not correct values.

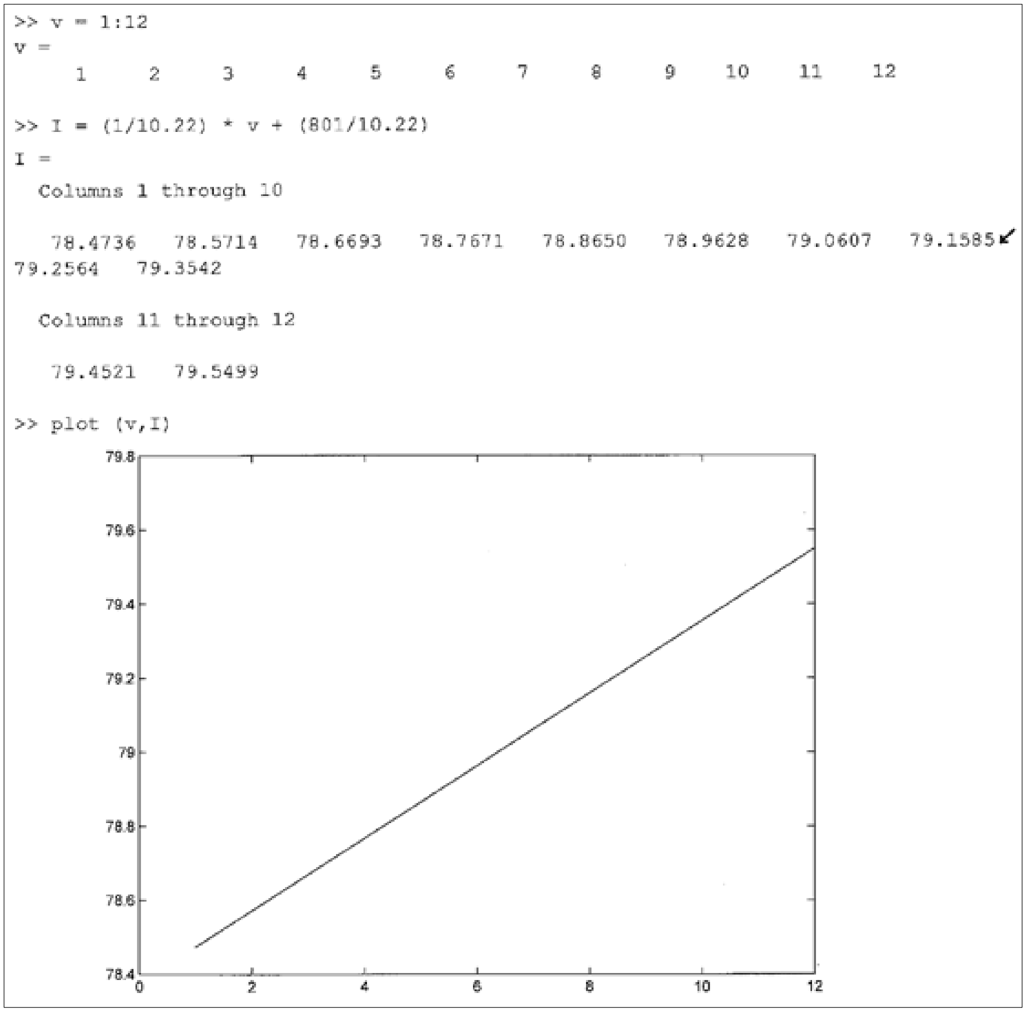


The code is given below

```
V_0 = (50-(12-(0.2*50))/0.02)*0.02+10
V_0 = 0:1:12
I = (V_0-10)/0.02 + (12-10)/0.02
plot(V_0,I)
title('Plot of Current through load resistance (I) and
Voltage (V_0)')
ylabel('Current through Load resistance (I) [in amperes]')
xlabel('Voltage (V_0) [in volts]')
grid
```

Figure 5.11: Computational representation, basic category example

Responses, when a conceptual error affected the computational representation or the outcome, were described under the below basic category. In this category were also categorized responses where a not clear computational response was presented. An example of this category is displayed in the Figure 5.12. The student provided an undeveloped computational representation and an accurate analysis was not evident in it. The participant used equations with wrong constant numbers and without any previous or further analysis.



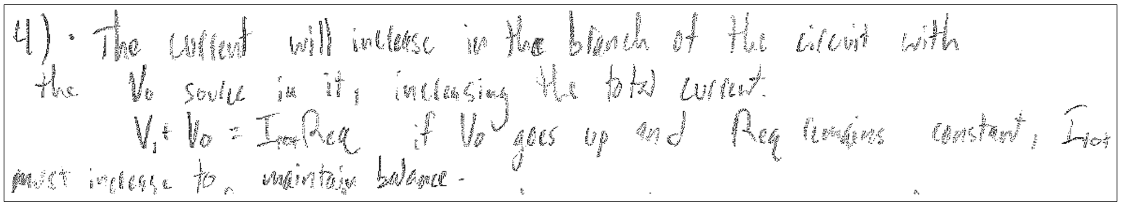
```
>> v = 1:12
v =
     1      2      3      4      5      6      7      8      9     10     11     12

>> I = (1/10.22) * v + (801/10.22)
I =
  Columns 1 through 10

   78.4736   78.5714   78.6693   78.7671   78.8650   78.9628   79.0607   79.1585↙
79.2564   79.3542

  Columns 11 through 12

   79.4521   79.5499

>> plot (v,I)
```

Figure 5.12: Computational representation, below basic category example

5.5.4 Conceptual understanding

The conceptual understanding questions were categorized based on the correctness of each answer. The total conceptual understanding score was given depending on the number of correct responses achieved and how complete they were.

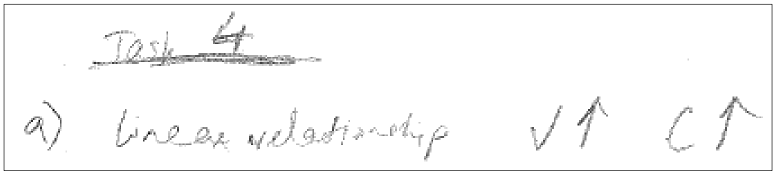
A fully accurate response had the correct answer and the respective reason or interpretation. For the questions regarding the system response to change the dead battery voltage, an accurate answer could be: *"the current will increase in the branch at the circuit with the* $\mathrm{V_0}$ *source* [the dead battery] *in it, increasing the total current.* $\mathrm{V_1} + \mathrm{V_0} = \mathrm{I_{eq}R_{eq}}$ *if* $\mathrm{V_0}$ *goes up and* $\mathrm{R_{eq}}$ *remains constant,* $\mathrm{I_{eq}}$ *must increase to maintain balance* [Ohm's law]*"* (Figure 5.13).



4) • The current will increase in the branch of the circuit with the V_0 source in it, increasing the total current.
$V_1 + V_0 = I_{eq}R_{eq}$ if V_0 goes up and R_{eq} remains constant, I_{tot} must increase to maintain balance.

Figure 5.13: Accurate answer example of the system response question

For the same question, a correct answer that needs elaborations could be: *"Linear relationship* V ↑ C ↑ [if the voltage increases the current increases]*"* (Figure 5.14).

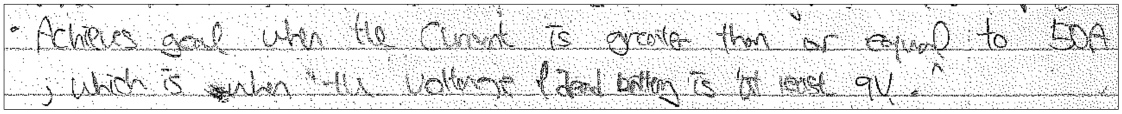


Task 4

a) Linear relationship V↑ C↑

Figure 5.14: Example of answer with improvement opportunities of the system response question

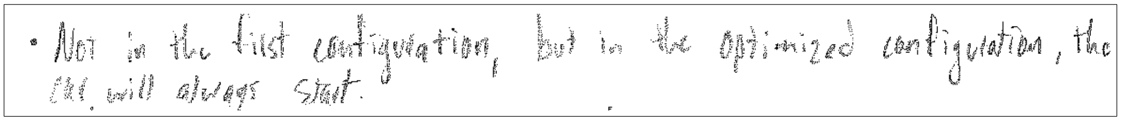
For the question regarding the understanding of the system response and the interpretations of the results based on the context other contexts, a fully accurate answer could be: “[The circuit] *Achieves* [the] *goal* [of starting the car’s engine] *when the current is greater than or equal to 50A, which is when the voltage of* [the] *dead battery is at lease 9V”* (Figure 5.15).



- Achieves goal when the current is greater than or equal to 50A ; which is when the voltage of dead battery is at least 9V.

Figure 5.15: Accurate answer example about understanding the system response

In the same question, a correct answer with room for improvement could be: “[The circuit does] *Not* [always achieve the goal of starting the engine] *in the first configuration, but in the optimized configuration, the car will always start”* (Figure 5.16).



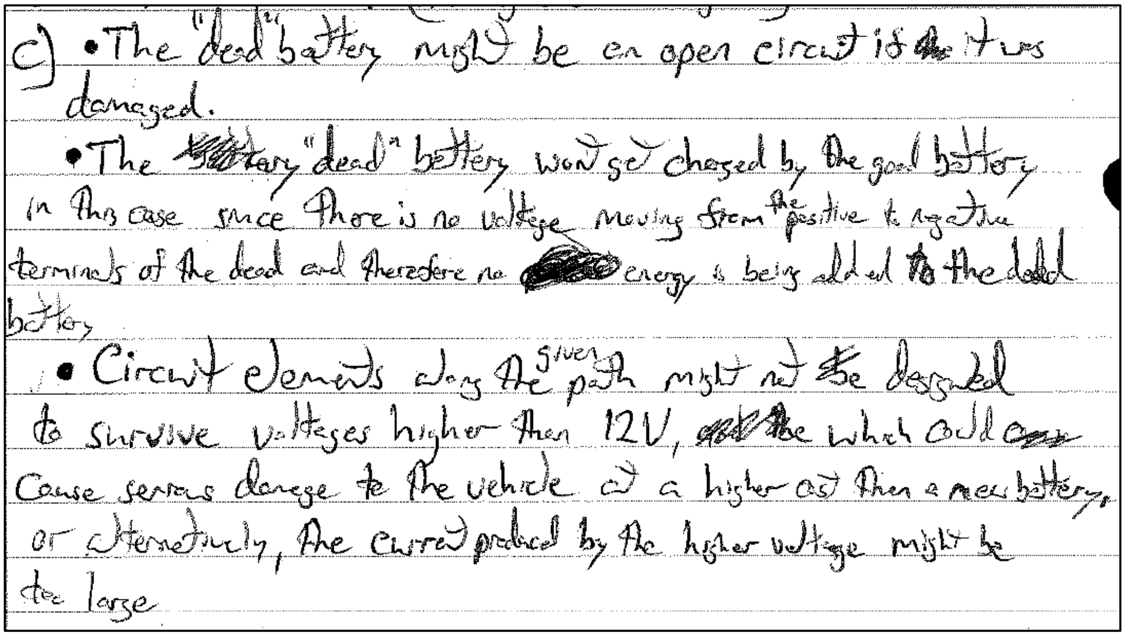
- Not in the first configuration, but in the optimized configuration, the car will always start.

Figure 5.16: Example of answer with improvement opportunities about understanding the system response

For the question regarding the consequences of the optimized configuration and its downsides an accurate answer could be the one presented in Figure 5.17:

> • The “dead” battery might be an open circuit if it was damaged. • The “dead” battery won’t get charged by the good battery in this case since there is no voltage moving from the positive to [the] negative terminals of the dead [battery] and therefore no energy is being added to the dead battery. • Circuit elements along the given path might not be designed to survive voltages higher

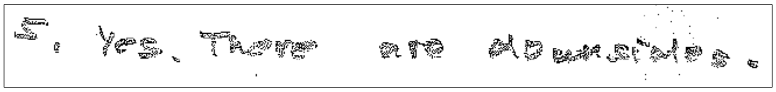
than 12V, which could cause serias [sic] damage to the vehicle at a higher ast [sic] than a car battery, or alternatively, the current produced by the higher voltage might be too large. (Figure 5.17)



c) • The "dead" battery might be an open circuit if ~~the~~ it was
damaged.
• The ~~battery~~ "dead" battery won't get charged by the good battery
in this case since there is no voltage moving from the positive to negative
terminals of the dead and therefore no [illegible] energy is being added to the dead
battery
• Circuit elements along the given path might not be designed
to survive voltages higher than 12V, ~~and the~~ which could ~~cause~~
Cause serias damage to the vehicle at a higher cost then a new battery,
or alternatively, the current produced by the higher voltage might be
too large

Figure 5.17: Accurate answer example for the consequences and downsides of the circuit

For the same question, an answer that is correct but required more elaboration is: *"Yes, there are downsides"* (Figure 5.18).



5. Yes, There are downsides.

Figure 5.18: Example of answer with improvement opportunities for the consequences and downsides of the circuit

The scoring procedure developed is the following: for the basic circuit, participants in the advanced level answered both of the 2 questions correct. In the proficient level, the answer to one question was correct and the other answer had the correct idea but it needed elaboration. Responses with one correct answer were categorized as basic. Below basic level was used when neither of the questions was answered correctly. For the optimized, the participants who answered three questions correctly were in the advanced level, the ones with two questions correct were at the proficient level. One correct answer meant basic level and below basic was reserved for no questions answered correctly.

5.6 Chapter summary

This chapter has described the methodology the two case studies have followed, it explained the participants and the context in which the research has been developed. Additionally, the chapter described the materials and procedures for the data collection methods and provided a description of the procedures for the data analysis methods. Lastly, the categorical analysis and its description were also presented in this chapter.

CHAPTER 6. RESULTS

This section presents the results presented as two different cases. Both research questions are address separately for each case. Thus, for each case, the results of the representational competence for each representation and conceptual understanding are presented in order to answer the first research question: how does the use of multiple representations relate to student conceptual understanding of electric circuits? Afterward, the representational fluency results are displayed to answer the second research question, what is the relationship between student conceptual understanding of electric circuits and their ability to map between multiple representations?

6.1 Results case one

6.1.1 How does the use of multiple representations relate to student conceptual understanding of electric circuits?

During the solution of the MEA activity for the case one, students were prompted to create five representations, two for the basic circuit and three for the optimized circuit. Because of differences in the scoring method, two analyses were developed; one for all responses and one for those students whose responses included all representations and answers for all the conceptual understanding questions. The 25 participants of the case study one developed on average three representations (M = 3.4, *SD* = 1.2) with an overall score rated as moderate (*M* = 2.9, *SD* = 0.9). The conceptual interpretation of the students was also rated as moderate (*M* = 2.0, *SD* = 1.6) (Sanchez et al., 2016).

A summary of student performance on each representation and conceptual understanding sections is presented through descriptive statistics in Table 6.1 (Sanchez et al., 2016). These results indicate that participants of the case study one demonstrated an overall good understanding of the mathematical representation and conceptual understanding of the basic circuit. Students were also able to represent the optimized circuit with the diagram, but failed to effectively create the mathematical representation or answer questions about its behavior. In both cases, basic and optimized, students unsuccessfully generated computational representations of the circuits (Sanchez et al., 2016).

Table 6.1: Descriptive statistics for each representation and conceptual understanding section (case one)

Task	Representation	N	Mean	Standard Deviation (SD)	Min	Max
Basic circuit	Diagrammatic	n/a	n/a	n/a	n/a	n/a
	Mathematical	25	3.53	1.07	1	4
	Computational	13	1.59	1.58	1	4
	Conceptual Understanding	17	3.29	0.98	2	4
Optimized circuit	Diagrammatic	25	3.08	1.04	1	4
	Mathematical	9	1.16	1.67	2	4
	Computational	13	1.08	1.47	1	4
	Conceptual Understanding	17	1.80	1.71	1	4

In the second analysis only the students who developed all the representations and answered the conceptual questions were included for each circuit. Participants who additionally completely solved the assignment were considered in a third non-exclusive group. The descriptive statistics of the second analysis is depicted in Table 6.2 (Sanchez et al., 2016).

Table 6.2: Descriptive statistics for each representation for students who completed all required representations (case one)

Task	Representation	N	Mean	Standard Deviation (SD)	Min	Max
Basic circuit	Diagrammatic	n/a	n/a	n/a	n/a	n/a
	Mathematical	12	3.75	0.87	1	4
	Computational	12	2.33	1.5	1	4
	Conceptual Understanding	12	3.5	0.9	2	4
Optimized circuit	Diagrammatic	5	3.08	0.45	3	4
	Mathematical	5	3.4	0.89	2	4
	Computational	5	3.4	1.41	1	4
	Conceptual Understanding	5	3.6	0.89	2	4
Basic circuit*	Diagrammatic	n/a	n/a	n/a	n/a	n/a
	Mathematical	5	4	0	4	4
	Computational	5	3	1.41	1	4
	Conceptual Understanding	5	4	0	4	4

* Participants who completely solved the assignment.

This analysis suggests that participants who completed all representations achieved moderate to high levels of competence, and this process was beneficial to interpret the circuits’ behavior (Sanchez et al., 2016).

6.1.2 What is the relationship between student conceptual understanding of electric circuits and their ability to map between multiple representations?

A Pearson product-moment correlation coefficient was calculated to assess the relationship between and within representations (i.e., diagrammatic, mathematical, and computational) and conceptual understanding. An alpha level of .05 has been used for these statistical tests. The overall results suggest a strong positive correlation between the total number of representations generated by students and their conceptual understanding achievement of the topic, $r(23) = .53$, $p = .006$. The correlation between the average score or quality of the representation created and conceptual understanding was also analyzed with no significant results, $r(23) = .37$, $p = .06$ (Sanchez et al., 2016).

The correlations between representations and conceptual understanding were limited to those participants who developed all the representations required for each circuit, this is 12 students for the basic circuit and five students for the optimized circuit. The Table 6.3 (Sanchez et al., 2016) presents the correlation coefficients for the basic circuit, this results suggests a strong positive correlation between the computational representation and conceptual understanding, although, the results were not significant.

Table 6.3: Correlation between representations and conceptual understanding for the base circuit n=12 (case one)

	Base circuit	1	2
1	Mathematical representation		
2	Computational representation	.28	
3	Conceptual understanding	-.17	.53

The correlation results for the optimized circuit suggest a strong positive relationship between and within students' representations and conceptual understanding, these results are shown in Table 6.4 (Sanchez et al., 2016).

Table 6.4: Correlation between representations and conceptual understanding for the optimized circuit n=5 (case one)

	Optimized circuit	1	2	3
1	Diagrammatic representation			
2	Mathematical representation	.87*		
3	Computational representation	1**	.87*	
4	Conceptual understanding	1**	.87*	1**

* p = .05. ** p < .0001

Finally, when the authors analyzed the responses of the participants who answered the conceptual understanding questions for each circuit (*n* = 14) a moderate-to-strong positive but not significant correlation was found between the number of representations generated and conceptual understanding, *r*(12) = .46, *p* = .102. For the mentioned subsample, a strong positive correlation between the quality of the representations created and conceptual understanding was found as well, $r(12) = .92$, $p < .001$ (Sanchez et al., 2016).

6.2 Results case two

6.2.1 How does the use of multiple representations relate to student conceptual understanding of electric circuits?

The students were prompted to generated five representations, two for the basic circuit and three for the optimized circuit. In the case two, students were asked to generate every representation to get the extra-credit, thus, there is a constant number of representations for each test ($n = 26$). The participants of this case had a moderate level of achievement for the average representations' scores ($M = 2.89$, $SD = 0.83$). The student conceptual interpretation was rated as moderate as well ($M = 2.93$, $SD = 0.82$).

Table 6.5 depicts a summary with the descriptive statistics of participants' performance on each representation and conceptual understanding sections. For case two, the participants' level of achievement for the representations and conceptual understanding sections were closely related. Even though, students showed a superior ability to generate the mathematical representation and answer the conceptual understanding questions of the basic circuit; while the ability to generate computational representations was the lowest of the group. For the optimized circuit, students generated all representations and demonstrated conceptual understanding with a similar ability, being conceptual understanding the lowest one.

Table 6.5: Descriptive statistics for each representation and conceptual understanding section n=26 (case two)

Task	Representation	Mean	Standard Deviation (SD)	Min	Max
Basic circuit	Diagrammatic	n/a	n/a	n/a	n/a
	Mathematical	3.15	0.98	1	4
	Computational	2.52	1.13	1	4
	Conceptual Understanding	3.04	1.13	1	4
Optimized circuit	Diagrammatic	2.98	1.20	1	4
	Mathematical	2.92	1.21	1	4
	Computational	2.92	1.17	1	4
	Conceptual Understanding	2.83	0.93	1	4

6.2.2 What is the relationship between student conceptual understanding of electric circuits and their ability to map between multiple representations?

Similar to case one, the Pearson product-moment correlation coefficient was calculated to evaluate the relationship between and within conceptual understanding and representations. An alpha level of .05 was used for the statistical tests. The overall results indicate a strong positive correlation between the average score for all representations generated per participant and their average on their conceptual interpretation, $r(24) = .70$, $p < .001$. More particularly, a strong positive correlation between average representational score per student and conceptual understanding was also found for each circuit, basic circuit: $r(24) = .67$, $p < .001$, optimized circuit: $r(24) = .56$, $p = .003$.

More specifically, Table 6.6 shows the correlation coefficients for the basic circuit. The results indicate a strong positive correlation between and within every representation and conceptual understanding, meaning positive correlations between mathematical with computational representations, mathematical representation and conceptual understanding, and computational representation and conceptual understanding.

Table 6.6: Correlation between representations and conceptual understanding for the base circuit n=26 (case two)

	Base circuit	1	2
1	Mathematical representation		
2	Computational representation	.71***	
3	Conceptual understanding	.67***	.58**

*p < .05. **p < .01. ***p < .001.

The results of the correlational analysis for the optimized circuit are depicted in Table 6.7. These results suggest a strong positive correlation between the mathematical and computational representations, and the computational representation and conceptual understanding. Additionally, the results also suggest a moderate-to-strong positive correlation between the diagrammatic and mathematical representations, and the mathematical representation and conceptual understanding. Such results are consistent with the results of the basic circuit.

Table 6.7: Correlation between representations and conceptual understanding for the optimized circuit n=26 (case two)

	Optimized circuit	1	2	3
1	Diagrammatic representation			
2	Mathematical representation	.47*		
3	Computational representation	.30	.82***	
4	Conceptual understanding	.24	.47*	.69***

*p < .05. **p < .01. ***p < .001.

Hence, these results suggest the relationship and importance of creating accurate representations and mapping between them for conceptual understanding. These results were also analyzed against the student scores of the exams given throughout the course (before and after the assignment) without significant results.

6.3 Chapter summary

This chapter presented the results of the two case studies for this study by focusing on each research question separately. For each case study, the representational competence results for multiple representations were shown through the research question: how does the use of multiple representations relate to student conceptual understanding of electric circuits? The representational fluency results of the research study were depicted through the research question: what is the relationship between student conceptual understanding of electric circuits and their ability to map between multiple representations?

CHAPTER 7. DISCUSSION

The purpose of this exploratory research study was to analyze how the use of multiple representations of technical concepts is related to the conceptual understanding of such concepts. The research followed a case study methodology implemented in two cases, the case studied was electric circuit analysis at the college level. The cases were particularly focused on (1) the student performance in each of the multiple representations employed, called representational competence, and (2) the ability to transfer or map from one representation to another, which was called representational fluency.

7.1 How does the use of multiple representations relate to student conceptual understanding of electric circuits?

Results from this exploratory study suggest that when students generate accurate representations, which means a high representational competence, they generally interpret the circuit behavior accurately as well (Sanchez et al., 2016). Only during the case study one the number of representations generated were variable. Hence, the results suggested that when students develop multiple representations usually they have a better understanding of the circuit behavior. This may suggest a relationship between the number of representations used in classrooms and a deeper conceptual understanding of formal concepts (Sanchez et al., 2016). Furthermore, the results from both case studies may indicate that when students generate accurate representations, they would generally understand the concepts involved accurately as well. This suggests a strong relationship between conceptual understanding and representational competence.

Findings of this research study are aligned with previous findings of the use of multiple representations. Kozma and Russell (2005) studied (1) how the use of multiple representations (i.e., computer-based molecular modeling, simulations and animations) can support conceptual understanding of difficult concepts in chemistry and (2) the role of multiple representations in the development of necessary skills for investigative practices. These authors have found that the multiple representations approach and representational competence can support the development of conceptual understanding and investigative practices of chemistry students (Kozma & Russell, 2005). Additionally, for ideal gases problems in chemistry, Madden, Jones, and Rahm (2011) examined students' conceptual understanding and learning gains from a multi-representational perspective. They found that when students use multiple representations heuristically with a high level of representational competence, they may benefit the development of conceptual understanding and problem solving skills (Madden et al., 2011).

Furthermore, the computational representation had the lowest scores in average when the results for each representation were analyzed and compared. The authors suspect it is because the circuit analysis course is strongly focused on the diagrammatic and mathematical representation, as well as conceptual understanding. Which could have led to a poor performance on the computational representations. The authors also suspect that this phenomenon could because students often have insufficient programming skills before taking the course. Those insufficiencies are typically not filled along the course because the computational representation is not the purpose of the class.

7.2 What is the relationship between student conceptual understanding of electric circuits and their ability to map between multiple representations?

Results from this exploratory study indicate strong positive correlations between representational abilities and conceptual understanding. This may suggest that when students are able to accurately map between different representations, they are also usually able to accurately interpret and predict the circuit outcome (Sanchez et al., 2016).

Weaker and non-significant correlations were found between diagrammatic representations and (1) computational representation, and (2) conceptual understanding. This could be because of the importance of sequencing the use of representations. The standard procedure in circuit analysis is usually to map from the diagram to the mathematical representation where a moderate-to-strong positive correlation was found.

However, the data from this study suggest that when students are able to map between multiple representations accurately, students are also able to interpret formal concepts more deeply, which may indicate a strong relationship between conceptual understanding and representational fluency. This could also mean that students understand concepts deeply by generating multiple representations of them and mapping between these representations. Although, it is important to investigate the direction of this relationship, which is called causal effect (i.e., what is the cause of a particular effect or which one causes the other). This means to investigate if students with high representational abilities are able to understand technical conceptus more deeply or if students with a better conceptual understanding are able to develop and map between multiple representations more accurately. The causal analysis is important because it could give particular insights about what is the cause and what is the effect for the relationship between conceptual understanding and representational competence and fluency. Additionally, the analysis could also lead to a commensalism or mutual benefit relationship where students with a deep conceptual understanding have high representational abilities and vice-versa because one benefits the other. Thus, this could mean significant improvements to the engineering curricula.

The results of this research study are aligned with previous research for different areas. Moore et al. (2013) studied the effect of a multi-representational approach and representational fluency in a heat transfer modeling task. They found that expressing scientific concepts with multiple representations and then mapping between them fosters conceptual understanding of heat transfer (Moore et al., 2013). In another field, Stull, Hegarty, Dixon and Stieff (2012) found that representational fluency along with concrete models can be effective tools in the learning process of organic chemistry.

Thus, these other studies presented the relationship between conceptual understanding and representational competence and fluency.

7.3 Limitations of the study

Besides the limitations listed in the limitations subsection of the introduction (1.6 Limitations), the case study one was influenced by the "no response" scenarios because not all participants chose to completely solve the assignment. In these scenarios, participants did not provide development evidence for specific representations or answers to conceptual understanding questions. Therefore, it was not possible to measure representational competence or fluency for these scenarios. The "no response" scenarios could have occurred because some partial credit was given for the responses instead of only getting extra credit by solving the MAE completely.

Moreover, the data collected depended on the voluntary participation of students, as a result, the instructors and researchers control over the participation was limited. Thus, the sample size available for the statistical analysis was small. Additionally, this voluntary participation leads the authors to assume and rely on the honesty and commitment of the participants. Finally, further research that avoid the influence of these limitations is needed.

7.4 Implications for teaching and learning

These results about the relationship between representational competence and fluency with conceptual understanding could also suggest that students with a deeper conceptual understanding are capable of generating more accurate representations and map between them as well. Thus, conceptual knowledge can be also revealed through representational fluency and competence (Moss, Kotovsky, & Cagan, 2006). Hence, this research study suggested that a high level of representational competence and fluency may lead to a deeper interpretation of formal concepts.

It was particularly interesting that computational representations had the lowest score, this could be due to a poor student programming background because the latter is not the purpose of the course. For this reason, the computational representation scores suggest a possible improvement opportunity for circuit analysis courses. Discipline-based programming is one option to introduce computing and computational representations in the classroom while integrating technical knowledge with computational representations in learning experiences related to specific engineering disciplines (Magana, Falk, Vieira, & Reese, 2016). This approach can foster the learning process of concepts of computational representations and promote the development and application of this type of representations in the solution process of engineering problems (Magana, Falk, & Reese, 2013). Another option could be to implement computer simulations in the classrooms, Jaakkola, Nurmi, & Veermans have shown that computer simulations can improve conceptual understanding of electric circuits (2011).

Lastly, previous studies have also suggested that the use of simulations and computational representations in the learning process helped students to generate more accurate representations (Stieff, 2011) and to improve their representational fluency ability (Stieff & McCombs, 2006).

7.5 Conclusion and future work

The results of this exploratory research study suggest that representational competence and fluency have a positive relationship on the conceptual understanding of circuit analysis concepts. Besides, the results indicate the multiple representations approach may foster and support the learning process of circuit analysis concepts. These findings are aligned with the other findings from different engineering areas that suggest representational competence and fluency as a path to improve and measure conceptual understanding of technical knowledge, and the effect of using multiple representations approach to foster conceptual understanding.

Therefore, the main two contributions of this study are (1) the application of a multi-representational approach to circuit analysis by analyzing student conceptual understanding from this perspective, and (2) to explore the benefit of using multiple and additional representations during the learning process of the same topics, precisely, the consideration of computational representations in addition to diagrammatic and mathematical representations during a problem solving episode.

Although, this is an exploratory study that provides important insights, further research is needed in order to generalize the results. Thus, further analysis could include to study a causal effect for the relationship between representational abilities and conceptual understanding. The causal analysis could uncover the directional meaning of the correlational analysis developed during this research. Which could investigate if students with high representational abilities have a better conceptual understanding, or if students with a high conceptual understanding have higher representational abilities, or if conceptual understanding and representational abilities complement each other. Future work could also cover the application of multiple representational perspective to other engineering or science areas and the consideration of additional representations during investigation and application of this perspective.

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APPENDICES

Recharging a Car's Battery

Analysis and Optimization of an Electric Circuit

Figure 7.5.1: *Hybrid car*

Introduction

Electric circuits are very common in our diary life. Every appliance and electronic device have them, for instance, a fridge, oven, cell phone and cars. Generally, all cars have a battery that turns the engine on and powers other systems like the lights. Cars also have a power generation circuit to recharge the battery, thereby avoid running out of battery or having a dead battery. Even though batteries can do more than turning on some vehicles, they can also power them through electric engines, making an electric or a hybrid car.

Background:

A car battery charger circuit consists of an alternator, a regulator and the battery itself. When the engine is running, the alternator generates a current to feed the vehicle electric charge. But before using it, the regulator drives the voltage to working levels for the lights, radio, windows and to charge the battery. If any of the components does not work properly, the battery will not be charged and you will end up with a dead battery. In that case, you will have to use another way to turn on the car, such as a jump-start or push-start.

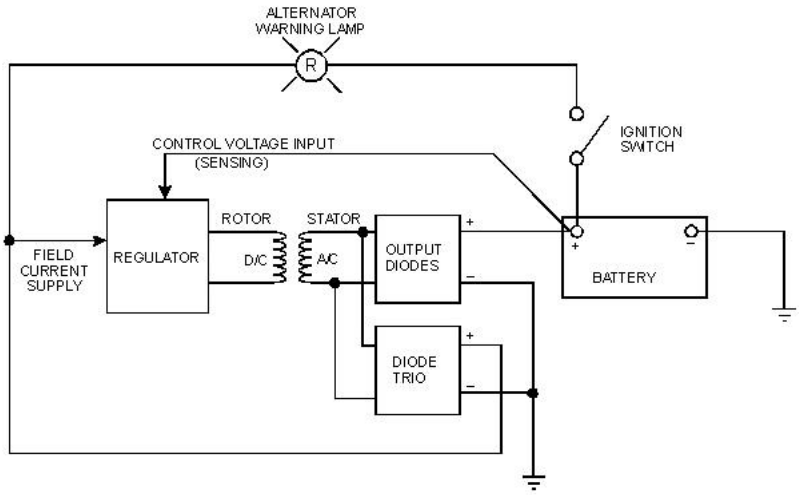

Figure 7.5.2: Electric circuit of a car

The Challenge:

The battery of your car suffered a sudden death by the sub-zero North wind and a faulty alternator. Unable to fight the elements, you wait for a few days hoping for a thaw, which eventually comes. You replace the alternator. Then, using your roommate's car, you attempt a jump-start. Nothing happens. You let it sit for a while with your roommate's car running juice into your battery for 20 minutes. Still, nothing happens. Why won't your car start?

Figure 7.5.3: Jump-start

Task 1:

a) Consider the circuit depicted in Figure 7.5.4. Notice that your "dead" battery is labeled as "V_0". Your roommate's battery is labeled 12V. Each battery has an internal resistance of 0.02 Ω. The starter, labeled "R_Load", has an internal resistance of 0.2 Ω. The starter motor requires 50A to crank the engine. Find the minimum value of voltage V_0 needed before the starter can draw 50A and work.

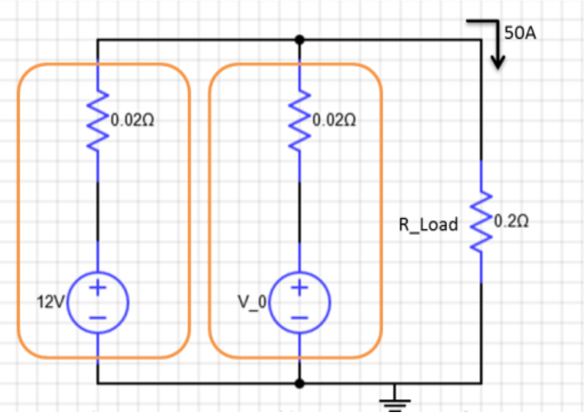

Figure 7.5.4: Jump-start circuit

b) Use MATLAB to solve the equations derived from (a) and substitute the constants for finding the voltage V_0, then find an equation for the current I on the load (R_Load) and graph I vs V, increasing the voltages from 0V to 12V. In your response, please provide the MATLAB code and the plots. HINT: This task could be accomplished by using the symbolic package of MATLAB or solving the system of equations in matrix form.

c) Based on the MATLAB implementation and the plots, answer the following questions:
   1. How does the current change as the voltage of the dead battery increases from 0V to 12V? Why?

2. Based on the last question and the graph generated in (b) for the circuit, does the current I always achieve the goal of starting the engine for each value of voltage from 0V to 12V? Why?

Task 2:

a) Optimize the circuit in Figure 7.5.4 so the minimum value of voltage V_0 needed is 0V (zero volts). You can find help for a starting point by running DC analysis in a simulation tool such as Circuit Sandbox that can be accessed at the link below. After having the optimized configuration, find the equations for this configuration and show that the voltage needed in V_0 is 0V. In your response, please provide the equations and its solution for this task and the circuit diagram found for the optimized configuration. Circuit Sandbox can be found following this link:
https://6002x.mitx.mit.edu/courseware/6.002_Spring_2012/Overview/Circuit_Sandbox/
A user guide on how to use Circuit Sandbox can be found here:
https://6002x.mitx.mit.edu/wiki/view/InteractiveLaboratoryUsage

b) Use MATLAB to solve the equations derived from (a) and substitute the constants for finding the voltage V_0, then find an equation for the current I on the load (R_Load) and graph I vs V, increasing the voltages from 0V to 12V. In your response, please provide the MATLAB code and the plots. HINT: This task could be accomplished by using the symbolic package of MATLAB or solving the system of equations in matrix form.

c) Based on the MATLAB implementation and the plots, answer the following questions:
   1. How does the current change as the voltage of the dead battery increases from 0V to 12V? Why?
   2. Based on the last question and the graph generated in (b) for this optimized circuit, does the current I always achieve the goal of starting the engine for each value of voltage from 0V to 12V? Why?
   3. Please discuss possible downsides to this optimized design and explain why.

Assessment Rubric:

Scoring	**Advanced (4)**	**Proficient (3)**	**Basic (2)**	**Below basic (1)**
Diagram	Correct diagram	Appropriate diagram not clear	Appropriate diagram but has some errors	No correct diagram
Mathematical representation	Good circuit analysis and good mathematical development	Good circuit analysis but with calculation error(s)	Good circuit analysis but with modeling error	No evidence of analysis or evident conceptual misunderstanding.
Computational representation	Good computational representation and good outcome	Good computational representation but the outcome has some errors (programming or mathematical)	Good computational representation but the outcome is incorrect	There is a computational representation but the outcome is incorrect due to a conceptual error
Conceptual understanding	Correct conceptual understanding (Three answers are correct)	Proficient conceptual understanding (2 answers are correct)	Basic conceptual understanding (1 answers are correct)	No correct conceptual understanding (Three answers are wrong)

Credits:

MATLAB is a copyright of MathWorks.
Main task: DeCarlo, R., and Lin, P. M. (2009). Linear Circuit Analysis: Time Domain, Phasor, and Laplace Transform Approaches (3rd ed.) (p.142, Problem 3.2). Dubuque, IA: Kendall Hunt Publishing.
Figure 7.5.1: http://auto.howstuffworks.com/hybrid-car-pictures.htm#page=8, (09/14/14)
Figure 7.5.2: http://alternatorparts.com/understanding-alternators.html, (09/14/14)
Figure 7.5.3: http://blog.cochran.com/wordpress/index.php/jump-start-car-battery/, (09/14/14)
Figure 7.5.4: Done with Circuit-Sandbox from MIT,
https://6002x.mitx.mit.edu/courseware/6.002_Spring_2012/Overview/Circuit_Sandbox/, (09/14/14)

Appendix B Rubric

Scoring	Advanced (4)	Proficient (3)	Basic (2)	Below basic (1)	No response (-)
Diagrammatic representation	Correct diagram	Appropriate diagram not clear	Appropriate diagram but has some errors	No correct diagram	No response
Mathematical representation	Good circuit analysis and good mathematical developmxent	Good circuit analysis but with calculation error(s)	Good circuit analysis but with modeling error	No evidence of analysis or evident conceptual misunderstanding.	No response
Computational representation	Good computational representation and good outcome	Good computational representation but the outcome has some errors (programming or mathematical)	Good computational representation but the outcome is incorrect	There is a computational representation but the outcome is incorrect due to a conceptual error	No response
Conceptual understanding	Correct conceptual understanding (Three answers are correct)	Some conceptual understanding (2 answers are correct)	Some conceptual understanding (1 answers are correct)	No correct conceptual understanding (Three answers are wrong)	No response