12-2016

How training set and prior knowledge affect preschoolers' perception of quantity and early number learning

Arum Han
Purdue University

Graduate School Form 30
Updated 1/15/2015

PURDUE UNIVERSITY
GRADUATE SCHOOL
Thesis/Dissertation Acceptance

This is to certify that the thesis/dissertation prepared

By Arum Han

Entitled
How Training Set and Prior Knowledge Affect Preschoolers' Perception of Quantity and Early Number Learning

For the degree of Doctor of Philosophy

Is approved by the final examining committee:

George Hollich
Chair

Sara A. Schmitt

Barbara A. Younger-Rossmann

David J. Purpura

To the best of my knowledge and as understood by the student in the Thesis/Dissertation Agreement, Publication Delay, and Certification Disclaimer (Graduate School Form 32), this thesis/dissertation adheres to the provisions of Purdue University's "Policy of Integrity in Research" and the use of copyright material.

Approved by Major Professor(s): George Hollich

Approved by: Christopher R. Agnew 11/21/2016
Head of the Departmental Graduate Program Date

HOW TRAINING SET AND PRIOR KNOWLEDGE AFFECT PRESCHOOLERS’

PERCEPTION OF QUANTITY AND EARLY NUMBER LEARNING

A Dissertation

Submitted to the Faculty

of

Purdue University

by

Arum Han

In Partial Fulfillment of the

Requirements for the Degree

of

Doctor of Philosophy

December 2016

Purdue University

West Lafayette, Indiana

TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Han, Arum. Ph.D., Purdue University, December 2016. How Training Set and Prior Knowledge Affect Preschoolers' Perception of Quantity and Early Number Learning. Major Professor: George Hollich.

This dissertation examines how training on the iPad can improve children's quantity recognition, and whether different types of training might be warranted for children with different levels of experience. Study 1 tested the effects of multiple exemplar training (3 cars / 3 apples / 3 ducks, etc.) versus single exemplar training (3 cars / 3 cars / 3 cars, etc.) in recognizing quantities. For children just learning to recognize quantities (0-2 knowers), training with multiple exemplars was most effective for quantities three and four. For 3-6 knower children, single exemplar training was most effective for learning quantities five and six. Study 2 tested the effects of using a training set with perceptually distinct dice-like arrangements versus linear arrangements of objects in the quantity recognition task. 0-2 knower children tended to choose the familiar arrangements which were shown in the training session (regardless of quantity), while 3-6 knowers could pick out the correct quantity regardless of arrangement. This result suggests that selecting the right type of training is important for facilitating children's early number learning.

INTRODUCTION

Psychologists' understanding of the nature of young children's informal mathematical knowledge has been changing (Baroody, Lai, & Mix, 2006). For most of the twentieth century, psychologists believed that young children's mathematical ability was limited (Piaget, 1965; Thorndike, 1922). Due to this pessimistic view, there was very little emphasis on early mathematics education for preschool children. This pessimistic view was replaced with a highly optimistic view in the last quarter of the twentieth century. In this period, psychologists, including Wynn (1998), discovered that infants and young children possess innate mathematical competencies (such as ability to discriminate between different small numbers of entities). While some of the most optimistic claims have been tempered over the last 15 years (Huttenlocher, Jordan, & Levine, 1994), it is still widely accepted that even young infants develop an informal understanding of mathematical concepts and that their abilities to discriminate different quantities of discrete objects is linked to later mathematical skill. Thus, children's early number competence (e.g., counting, number recognition, number comparisons) predicts their later mathematics achievement and this correlation is strong and persistent (Jordan, Kaplan, Ramineni, & Locuniak, 2009). In addition, targeted programs such as number board games seem to improve young children's number competencies (Booth & Siegler, 2008; Ramani & Siegler, 2008), and a significant relation between the mathematical input in the speech of preschool teachers and growth of children's mathematical knowledge over the school year was found (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). Such work suggests a clear value for training children on quantity recognition and other early numeracy concepts.

Unfortunately, in the United States, historically, the focus of early childhood programs has been mainly on language and literacy (National Research Council, 2009). There has been very little emphasis on mathematical experiences at the child daycare and preschool level (although progress has been made to improve elementary and middle school students' mathematics performance). Nor has there been much study of how to best achieve early mathematical education. Can children learn to better recognize quantities? What sorts of experiences might be most beneficial? How does early mathematical learning relate to other types of early learning, including language development? Therefore, to improve early mathematics education for young children, it may be important to understand how the early mathematical learning system develops and how we might better train children in early quantity recognition as a result of that understanding.

One potentially fruitful approach to understanding the development of the early math system lies in its apparent connection to other systems, the language system in particular. Research shows a link between early mathematics development and language development. For example, early literacy skills predict early numeracy development (Purpura, Hume, Sims, & Lonigan, 2011) and the relationship between general oral language and early numeracy is mediated by mathematics language (e.g., individual number names, *more*, *how many*, *triangle*) (Toll & Van Luit, 2014). Understanding the true nature of this connection could help explain why some types of training could be more beneficial than others.

There are three possible explanations for the relationship between language and mathematics learning in young children. First, there is the possibility that part of language learning may depend on early number skills. According to cross-species research on number detection, it seems that a system for detecting quantity or frequency is an evolutionarily older system in comparison with language system. Human infants, even at birth, show non-verbal representation of number (Izard, Sann, Spelke, & Steri, 2009) and even primitive arithmetic. For example, a classic experiment by Wynn (1992) showed that 5-month-old infants understand simple arithmetic calculations for small numbers, such as ‘1+1’ and ‘2-1’. In the experiment using a looking-time procedure, infants in the ‘1+1’ group, for example, saw a single object in a display area then a small screen came up and hid the object from view. The experimenter showed one more object and placed it behind the screen. By doing so, infants could see the arithmetical operation being performed but could not see the result because objects were hidden by the screen. After a sequence of events, infants were shown either a possible outcome or an impossible outcome. Infants looked longer at the incorrect outcomes (unexpected events) than the correct outcomes (expected events) showing their understanding of numerical computation for small numbers (Wynn, 1992).

In addition, research over the last decades has provided evidence for the representation of small number across species. For example, non-verbal animals, such as monkeys, represent the exact number of objects, up to four objects, in a scene even without training (Hauser, Carey, & Hauser, 2000). When the number is small, even monkeys are able to pay attention to the effects of addition and subtraction. When it comes to tracking a large number of individual objects, infants appear to have a primitive mental system of nonverbal representations that produces an intuitive 'number sense', which is known as the Approximate Number System (ANS) (Mazzocco, Feigenson, & Halberda, 2011). The ANS provides "a sense of approximate numerical values and relationships" (Spelke, 2003, p. 284) and follows Weber's law, which describes the smallest perceptual differences that can be reliably detected (Gallistel & Gelman, 1992; Spelke, 2003). Researchers found that this evolutionarily old ANS is shared by humans and non-verbal animals. Monkeys, trained and untrained birds, rats, and chimpanzees all appear to represent approximate numerosity (Dehaene, Dehaene-Lambertz & Cohen, 1998; Gallistel & Gelman, 2000; Gelman & Gallistel, 2004). Also, trained dolphins can discriminate simultaneously presented two visual stimuli on the basis of numerosity feature and can accomplish a transfer to novel numerosities as well (Kilian, Yaman, Fersen, & Güntürkün, 2003).

In sum, researchers have found evidence for the presence of an evolutionarily ancient system for early number processing which is independent of language and symbolic representations. If the early number system develops prior to language and symbolic counting, it is possible that this old number system may play a role in language learning, and this could explain some of the apparent connection between the two systems. If so, than improving quantity recognition could conceivably lead to improvements in language learning. Unfortunately, because the direction of the connection is one way, we wouldn't expect that improvements in the early language system would help the early quantity recognition system, and we would find little inspiration for how modifying training might help quantity recognition. Fortunately, there are other possibilities for the connection between language and early math.

A second possibility for the connection between language and early mathematics learning is that aspects of early mathematics learning may depend on language. As shown above, approximate number representations appear in non-verbal animals as well as humans. However, exact representation of number is necessary for successful numerical learning. Spelke (2000; 2003) suggested that the language of number words provides a source of mathematical thinking. Counting, in particular, seems to be linked to early language abilities. Counting is, in many ways a kind of language with rules and a grammar that helps children learn to associate number words with certain quantities. In the emergence of counting, children show systematic growth in understand of what Gelman and Gallistel (1978) claimed are number-specific five principles that underlie children’s counting abilities: the one-one principle states that each of the items to be counted should be assigned one, and only one, distinct number name; the stable-order principle states that the list of number tags must be in a fixed order; the cardinality principle states that the counting tag allocated to the final object in a collection represents the cardinality of the collection of items; the abstraction principle states that any collection of objects, whether physical or not, can be grouped together for a count; the order-irrelevance principle states that the order in which a set of items are counted is irrelevant. Similar heuristics have been identified for children's learning the meaning of new words (like novel-name-nameless category principle, in which children recognize that there is one and only one word per object) and that there order of words matters for language (Hirsh-Pasek & Golinkoff, 1999). Thus the exact number system appears dependent on (or is even a form of) early language.

Further evidence for a language-dependent system for mathematical thinking, especially the representation of large, exact number, comes from brain research, research with bilingual subjects, and research with speakers with a small lexicon of number words. Brain research suggests that language contributes to exact number representations and arithmetics. For example, fMRI and ERP data showed evidence that exact calculation depends on language, whereas approximate calculation does not depend on language. Dehaene and colleagues (1999) gave adult subjects an addition problem (e.g., 4+5?), and asked them to select one answer as quickly as possible after two candidate answers were flashed. In the approximate addition task, where subjects were asked to choose the most plausible answer (e.g., candidate answers: 8 or 3), the bilateral intraparietal area, which is involved in visuo-spatial processing, was activated. In contrast, in the exact addition task where subjects were asked to choose the correct answer (e.g., candidate answers: 9 or 7) for the same problem used in the approximate addition task, the left inferior frontal area, which is involved in word-association processes, was activated (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). Baldo and Dronkers (2007) also found a common neural substrate for language and exact calculation, including the inferior frontal gyrus and the middle and superior temporal gyri, suggesting that language comprehension and arithmetic process are mediated by overlapping neural networks. Research with bilingual training methods also suggests that the human ability for representing exact numbers is dependent on the language faculty. For example, Russian-English bilingual college students were taught different sets of number operations (e.g., new numerical operations, new arithmetic equations) and some geographical or historical facts involving numerical or non-numerical information (Spelke & Tsivkin, 2001). They learned a set of items in each of their two languages and were tested in both languages. For the information about approximate numbers and non-numerical facts, performance was independent of language. Subjects responded equally well when queried in the two languages. For the information about large, exact numbers, performance was dependent on language; subjects retrieved more quickly and more accurately when queried in the language of training. These findings suggest that language plays a role in learning large, exact numbers but not approximate numbers. Furthermore, according to Spelke and Tsivkin (2001), people who speak more than one language tend to count and perform arithmetic calculation in the language in which they initially learned elementary arithmetic. Further evidence comes from research with speakers of Mundurukú, an Amazonian language that has number words only for the numbers 1 through 5 (Pica, Lemer, Izard, & Dehaene, 2004). For an approximation task with numbers greater than 4 or 5, speakers of Mundurukú were able to add and compare approximate numbers. For an exact arithmetic task with numbers greater than 4 or 5, subjects failed in the task suggesting that there may be a distinction between a system for number approximation and a system for exact number and representations for exact number may emerge only when number words are available (Pica et al., 2004).

If early language is necessary for exact early math learning, we would expect strong connections between math language skills and subsequent math abilities.

Indeed, work by Purpura and Logan (2015) found that math language (e.g., more/less, near/far) predicts mathematical performance across the preschool year. Under this possibility, then, labeling of quantities early might also help in early math competencies. So as children learn to better connect the number with specific quantities, they would also improve their math outcomes. This account would give special importance to training quantity recognition, since it represents the combination of language learning and the approximate number system.

Finally, there is a third possibility that language learning and mathematics learning may be connected because two systems are fundamentally similar in that both are abstract, symbolic, and rule-based. Mathematics is an abstract subject (Gallistel & Gelman, 1992), therefore young children often have problems learning mathematical concepts. For example, number words (such as *one, two, and three*) do not refer to any object in the external world (Bloom & Wynn, 1997), rather they refer to properties of sets. Children tend to take novel words as referring to objects (individuals), and this tendency makes it much harder to learn number words. However, for children, number words are not the only difficult words to learn, but researchers have noticed that verbs and other abstract relational words are hard to learn as well. For example, Gentner (1982) noted that generally nouns are learned first and verbs are learned later. For example, it seems that verb learning is more challenging for young children, therefore they tend to produce nouns like *cup* and *apple* early and produce verbs like *fly* and *think* later. However, this traditional account of verb and noun learning does not explain why some nouns, such as *peace* and *uncle*, are also learned relatively late. Such noun exceptions suggest that it may not be just about syntactic class (nouns versus verbs), rather it may be about word learning of abstract concepts in general (Maguire et al., 2006). Specifically, Maguire and colleagues (2006) proposed the SICI continuum. The SICI continuum posits a single continuum of “abstractness” to explain the developmental path of vocabulary acquisition across linguistic word classes. “SICI” is “an acronym for the factors that scale the difficulty of learning a particular word: shape, individuation, concreteness, and imageability” (Maguire et al., 2006, p. 17). According to the SICI continuum, children are likely to learn words that are shape-based (S), easy to individuate (I), more concrete (C), and easy to yield a mental image faster (I). Most verbs lie on the more abstract end of the SICI continuum and most nouns lie on the more concrete end of the SICI continuum. For this reason, in general, children learn nouns first and learn verbs later.

Like most verbs, number words (with ill-defined shape, hard individuation, low concreteness, low imageability) lie on the more abstract end of the SICI continuum. This may explain why children find learning number words difficult. If number word learning is difficult because of the abstract nature of number words, we may help children master them faster by reducing abstractness, or by using a training set which encourages children to notice broader abstractions. Thus, language and mathematics learning are connected because they occupy an overlapping problem space. Just as words that are abstract are more difficult to learn (Gentner, 1982), number words can be difficult to learn for the same reasons. Thus, for successful word and numerical learning, it is important to reduce abstractness and uncertainty of concepts.

How might one do this? The Emergentist Coalition Model (ECM) by Hollich and colleagues (2000) suggests that the youngest language learners rely on perceptual information in early learning and only after having learned a few words move to more social and cognitive sources of information. This has been conclusively demonstrated for word learning: Words like *peace* or *believe* with weak perceptual links are learned late because those words, regardless of syntactic class, that are less perceptually accessible and more abstract require additional coordination of perceptual, social, and linguistic inputs. In contrast, words like *ball* or *jump* that are perceptually more salient are learned early (Maguire, Hirsh-Pasek, & Golinkoff, 2006). This same development from perceptually dominant mindset to understanding of cognitive and social intention is likely to show up across a range of learning phenomenon, including quantity learning, one of the earliest number skills children develop. The emergentist model's focus on early perceptual knowledge suggests two possible avenues for aiding word and quantity learning: cross-situational learning, and increasing perceptual distinctiveness.

The first possible solution to help children discover abstract properties involves cross-situational learning: learning "a new word by paying attention to the element that remains constant [and those elements that change] across multiple uses of that word" (Akhtar & Montague, 1999, p. 347). The abstract nature of word learning makes it hard to resolve referential ambiguity when children first encounter a novel word. That is, when children learn new words in everyday contexts, there are many words and many potential referents. Because abstract words are not concrete and not obvious, to reduce this ambiguity and uncertainty, it is important to be exposed to multiple exemplars. This cross-situational learning is known as particularly useful in the case of adjectives. For example, when a child sees a red truck and a red apple and hears 'red' to describe both, the child may be confused what ‘red’ exactly means and even search for commonalities in shape between truck and apple. The child may comprehend the term ‘red’ only after multiple exposures to the same word ‘red’ used to describe many red-colored objects if those objects do not share any characteristics other than their red color. In Akhtar and Montague’s (1999) study, two-year-olds were shown novel objects that varied in shape and texture without labeling. In the training trials, an experimenter showed one target object and labeled, ‘This is a *modi* one’. The experimenter then showed the two other training objects and labeled, ‘This is a *modi* one, too’, and ‘There is another *modi* one’. In the Shape condition, the objects were grouped by shape, so two training objects matched the target objects in shape but differed in texture. In the Texture condition, two training objects matched the target in texture but differed in shape. The results revealed that they were able to pay attention to the characteristic, either shape or texture, which was constant across trials. The results show that 2-year-old children were able to reduce uncertainty through cross-situational learning.

Similarly, cross-situational learning may be particularly useful for number word learning. Before a child can count well (before they even know what 1 or 2 means), when he/she sees three firetrucks and three apples and hears ‘three’ to describe both, the child may try to guess what ‘three’ exactly means. The child may think the word ‘three’ is connected to the object itself, such as its color or texture. The problem that the child encounters is the same as the one in adjective learning reported above. Number words (such as one, two, and three) do not refer to any concrete object in the external world (Bloom & Wynn, 1997), rather they refer to properties of sets. Thus, after lots of exposure to the same word ‘three’ used to describe the numerosity ‘3’, such as ‘three balls’ and ‘three cars’, the child may be able to match the word ‘three’ to the numerosity ‘3’.

Study 1 was designed to examine the importance of multiple exemplars versus single exemplar in learning numbers. A study by Twomey and colleagues (2014) showed exposure to multiple exemplars aided 30-month-old children’s word learning. In their study, children either saw the same exemplar repeatedly or saw multiple exemplars across word learning trials. Results showed that children who were exposed to multiple exemplars retained name-object mappings better. As in word learning, perhaps exposure to multiple exemplars may aid young children’s early number learning. In this work, children were exposed to either three identical sets of objects (cars / cars / cars) or three different sets of objects (cars / apples / ducks), and it was expected that children in the multiple exemplars training condition would show better performance as in word learning.

Another related factor in number learning concerns issues of ease of perception. As described above, early word learning depends heavily on early perceptual abilities. As in early word learning, young children’s direct perceptual judgment of the numerosity, which is called subitizing (Kaufman, Lord, Reese, & Volkmann, 1949), is important in development of mathematical concepts. According to Clements (1999), there are two types of subitizing. First, perceptual subitizing is recognizing a number without using any mathematical knowledge or processes. This perceptual subitizing skill is related to innate abilities of infants to discriminate between different small numbers (Wynn, 1998). Second, conceptual subitizing is recognizing a number pattern.

For example, people just know the domino's number by focusing on both the whole and the unit (or subset). Clements (1999) noted that using complex objects which are not simple in design and using irregular arrangements might increase children's conceptual subitizing errors.

From word learning research, we learned that words that are perceptually more salient or distinct are learned easily. Then, does creating a more perceptually salient array aid children's number recognition by making conceptual subitizing easier? Many researchers reported how spatial arrangement of objects affects young children's performance. Beckwith and Restle (1966) found that children's counting speed was the fastest when they saw rectangular arrays, followed by line arrays, circle arrays, and scrambled arrangements. Furthermore, children's error rates were the lowest for rectangular arrangements, followed by line, circle, and scrambled arrangements. Researchers also examined whether one representation is easier to form than another. Siegler and Ramani (2009) found that children who had played the linear number board game showed better performance on numerical magnitude comparison task and number line estimation task than children who had played a circular number board game. Surprisingly, the linear number board game was effective not only on tasks that directly measures understanding of numerical magnitudes but also on arithmetic problems. They suggested that the linear board game enabled more direct mapping to the desired mental representation, and it increased preschoolers' numerical knowledge. It is possible that creating perceptually distinctive arrays might not be as useful, especially for more experienced preschoolers.

Researchers also reported how children's spatial structuring abilities affect early mathematics. Van Nes and de Lange (2007) defined a spatial structure in terms of a pattern which is "a numerical or spatial regularity and the relationship between the elements of a pattern" (p.217, van Nes & de Lange, 2007). Spatial structures are related to the development of number sense (Bobis, 2008; van Nes & de Lange, 2007), and patterning skills are important in the development of mathematical representation (Mulligan, Prescott, & Mitchelmore, 2004; Papic & Mulligan, 2005). Children who showed poor performance on patterning tasks in preschool did poorly on other numeracy assessments a year later (Papic & Mulligan, 2005).

Study 2 was designed to look at whether a dice-like arrangement is better or worse than a linear arrangement for connecting quantities to numbers. For small numbers (1 to 3), the number of possible displays for each quantity is limited. For example, two dots make perceptually straight line and three dots make a triplet triangular pattern (Mandler & Shebo, 1982), therefore, random, dice, and linear configurations differ minimally for small numbers (Jansen et al., 2014). However, for large numbers greater than 3, there are so many possible displays and most do not produce patterns. Therefore, it is possible that one presentation is more effective than another. For example, Benoit and colleagues (2004) found that for small numbers (1 to 3), 3-years-old children performed better at verbal naming of the exam number of items that they saw when the items were presented simultaneously than when the item were presented in succession, suggesting the importance of subitizing for acquiring the first few number words. Also, with small numbers, there was no difference between performance for the dice arrangement and for the linear arrangement. With large numbers, children's performance was configuration-sensitive: children performed better with a dice-type arrangement. Likewise, in this work, it was expected that dice configuration would yield better performance than linear configuration, because a line pattern did not relate to a specific quantity whereas a dice pattern was closely connected to a specific quantity (e.g., three-triangle, four-square).

Thus while Study 1 looks at how multiple exemplars affects quantity recognition, Study 2 looks at how perceptual distinctiveness might also help with training quantity recognition. These two studies are significant because no prior studies have looked at quantity recognition independent of counting and labeling of quantities. This is important because many factors that have been associated with math development could be only associated with the counting portion, it is for this reason that many previous studies break subjects into those who can recognize small numbers versus those that can count. (Slusser & Sarnecka, 2011).

Another factor that could be related to counting is executive functioning. Research shows that executive functioning, like language, is related to young children's mathematical abilities. For example, there is a link between the executive function, particularly the inhibitory control aspect, and early mathematics in kindergarten (Blair & Razza, 2007); children's executive function is important in development of counting skills (Kroesbergen, van Luit, van Lieshout, van Loosbroek, & van de Rijt, 2009); inhibitory control contributes to mathematical performance in preschool children (Espy et al., 2007); low-achieving children show difficulties on measures of executive functioning (e.g., Stroop task, the Wisconsin Card Sorting Task) (Bull & Scerif, 2001); and children's developing executive function prior to school entry predicts children’s early mathematics achievement at early school age (Clark, Pritchard, & Woodward, 2010; Viterbori, Usai, Traverso, & de Franchis, 2015). Therefore, this work measured children’s executive functioning to see if that would make a difference in quantity learning. The Day-night task was used because this task was expected to capture the greatest variability of individual differences at a given age group in this work (Carlson, 2005).

STUDY 1: CHILDREN'S NUMBER LEARNING FROM EXEMPLARS

Study 1 looked at how children's exposure to object exemplars influences their number learning. Specifically, this study tested children's learning from single / multiple exemplars. We were also interested in whether there might be any differences between those children who already knew a few words and those who did not. We expected that, as in word learning, multiple exemplars might be particularly beneficial to novice learners. As an additional control, we also measured children's executive functioning to see if that would make a difference in number learning, or if that might interact with the type of training set.

Method

Participants

A total of forty typically developing children (*M* = 42.08 months, *SD* = 5.96 months, age range: 30.6 – 54.4 months, 20 girls) participated in the study. They were recruited via mass distribution in daycare centers and preschools of consent forms and letters explaining the study. Only children of parents who gave consent by returning those signed consent forms participated in the study. Data from an additional 12 children were not included because of fussiness (6), unwillingness (5), and which the proportion of English use was less than 50% (1). Children (*n* = 40) were randomly assigned to one of three conditions: the single exemplar condition (*n* = 13), the multiple exemplars condition (*n* = 14), and the control condition (*n* = 13).

Procedure

As Table 1 shows, before training, children completed a standard Give-N task and a quantity recognition task using an iPad. These tasks were included to get a baseline for the numbers that these children could recognize. During training session, children in the single exemplar condition and children in the multiple exemplar condition were trained with linear arrangements of objects. Both children in the single and multiple exemplar condition were given testing trials on quantities 3~6. Children in the control condition had no training session, instead they had free play time, then were given testing trials on quantities 3~6. After the training session, the Give-N task and the quantity recognition task were given one more time to see if children's counting skills were improved after training. After that, 16 testing trials used in the training session (four trials for each quantity) were given to test children's retention even after a short delay. Then an iPad version of the day-night task (created specifically for this project) was given to tap children's executive functioning. The creation of the iPad version of the day-night task enabled easier data collection than the traditional version and allowed examination of the possible connections between learning in the quantity recognition task and executive function.

Give-N task. The procedure for the Give-N task was adopted from Slusser and Sarnecka (2011). Children were asked for one block for the first trial and three blocks for the second trial. If a child succeeded on both requests, five blocks were requested for the third trial. If the child failed to give either one or three blocks (or both), two blocks were requested for the third trial. Depending on the child's responses, the experimenter requested differently. If the child succeeded at giving a number n, the next request was n+1. The experimenter went on until the experimenter's request was number six. If the child failed at giving a number n, the child was asked for n-1. The child did not get any feedback. The task continued until the child had at least two success at a given number n and at least two failures at n+1. If the child was asked for two blocks but gave four blocks instead, this was counted as two errors: one error for number two and one error for number four (Slusser & Sarnecka, 2011).

Quantity recognition task. After the standard Give-N task using blocks, children were given the quantity recognition task on the iPad. The child was shown four boxes with different numbers of objects, and was asked to touch one of boxes (e.g., "Which box has one?"). The procedure was similar to the standard Give-N task except that children were presented with four options to choose and this task was done on the iPad.

Training. As shown in Figure 1, there were two training conditions, the single exemplar condition and the multiple exemplars condition. During training, children in the single exemplar condition saw three identical sets of objects for the quantities three to six. For example, children were presented with three cars for in the first training trial, and those three cars were presented repeatedly for the next two training trials : cars / cars / cars. The audio provided the label of the set's quantity first (e.g., "Look, there are three cars"), then counted the same set of objects right after the labeling (e.g., "Let's count them, one, two, three! Three cars!"). The procedure for the multiple exemplars condition was similar to the single exemplar condition, except that children were trained with three different sets of objects: cars/ apples/ ducks. For example, for number three learning, children in the multiple exemplars condition were presented with three cars in the first training trial. Then children saw three apples and three ducks for the next two training trials. For children who were assigned to the control condition, there was no training session for them, instead, they had free playtime.

Testing. During the testing phase, two sets of objects with different quantities appeared on the iPad, and the audio requested to find the target number. For example, right after the training trials on number three and four, children saw three objects on the top and four objects at the bottom or vice versa and were asked to choose either three or four (e.g., “Which box has three?” or “Which box has four?”). Children were shown 16 testing trials (four trials for each number). For the multiple exemplars condition, the testing trials were exactly the same as the ones in the single exemplar condition. As shown in Figure 3, there were four types of testing trials: ‘Extension’, ‘Original’, ‘Target is Original’, and ‘Target is New’. For ‘Extension’ trials, both target and non-target quantities were shown as a linear arrangements of novel objects (balls). For ‘Original’ trials, both target and non-target quantities were shown as a linear arrangements of familiar objects (cars). For ‘Target is Original (or New)’ trials, the target quantity was shown as a set of cars (or balls). Children were shown a total of sixteen testing trials.

Post-training. After training session, the Give-N task and the quantity recognition task were given one more time to see if children’s skills were improved after training. After that, 16 testing trials used in the training block (four trials for each quantity) were given to test children’s retention even after a short delay.

Day-night task. An iPad version of the day-night task was given as a final task. A child was introduced to a “day” card, a white card with a yellow sun, and a “night” card, a black card with yellow stars and a white moon. Then the child was asked to play a new game. Two cards (“day” and “night”) appeared on the screen simultaneously (left/right). The child was asked to touch the “day” card when the audio requests “night”. Also, the child was asked to touch the “night” card when the audio requests “day”. After the instructions, the child had sixteen testing trials. If the child responded correctly to two trials in a row, testing continued without any repetition of the rules. If the child failed to respond correctly to either of the first two trials, the rules were repeated (e.g., “Remember, when he says day card, you touch this card”). The child had to repeat the first two trials until the child was correct on both first and second trials. If the child had not passed the first two trials in the third attempt, a final explanation of rules was given. Then the third card pair was presented and testing continued without any feedback and correction.

Coding

Children’s behaviors were recorded via digital video camera (GoPro) subsequently coded off-line. In this study, children’s touching behavior was coded and used as a measure of children’s performance in the testing trials.

Results

Pre-Training Results

Give-N results. The Give-N task yielded 2 zero-knowers, 7 one-knowers, 6 two-knowers, 6 three-knowers, 5 four-knowers, 2 five-knowers, and 6 six-knowers. Children’s pre-training performance on the Give-N task was correlated with age, r =

.33, $p = .040$, reflecting the fact that generally older children knew number words better than younger children.

Quantity recognition results. The quantity recognition task before training yielded 3 zero-knowers, 6 one-knowers, 11 two-knowers, 10 three-knowers, 3 four-knowers, 1 five-knower, and 6 six-knowers. Children's pre-training performance on the quantity recognition iPad task was not significantly correlated with age, $r = .25$, $p = .12$.

Give-N task and quantity recognition task. As shown in Table 3, for all 40 children, pre-training performances on the Give-N task and the quantity recognition iPad task were positively correlated, $r = 0.63$, $p < .0001$, suggesting that these tasks tested some common underlying skills. Interestingly, children's performance on the pre-training Give-N task ($M = 3.48$) was consistently better than their performance on the pre-training quantity recognition task ($M = 2.78$), $t(39) = 2.66$, $p < .05$. This suggests that these tasks measured slightly different numerical abilities in young children; it is possible that the Give-N task measured children's ability to count up to a given number while the quantity recognition task captured children's direct perception of quantity. In fact, most children did not show any counting behavior (e.g., verbal counting or pointing) when they were given the quantity recognition task even though they were able to count objects in the Give-N task.

Training Session

Preliminary analysis looking at performance on the testing trials during training session, with age as a covariate, showed no significant effect of gender, $F(1, 37) = 2.28$, $p = .14$. Therefore, data were collapsed over gender in further analysis.

How much of a difference does children's prior knowledge of numbers make in children's learning? A univariate analysis of variance (ANOVA) looking at performance on the testing trials with age as a covariate showed no significant effect of pre-training knower level determined by the Give-N task, $F(6, 32) = 1.82$, $p = .13$, but showed a significant effect of pre-training knower level determined by the quantity recognition task on the iPad, $F(6, 32) = 4.19$, $p < .01$. Therefore, for further analysis, children were grouped by their pre-training quantity recognition task performance. First, 0-2 knowers ($N = 20$) included children who were able to recognize quantity up to either two or one and children who did not complete the task. Second, 3-6 knowers ($N = 20$) included children who were able to recognize quantity up to three or beyond three. The reason why children were grouped this way was that the testing trials requested quantities from three to six. 0-2 knowers were expected to have no prior knowledge on the requested quantities in this study while 3-6 knowers were expected to have prior knowledge.

Table 4 shows children's performance on the testing trials during the training session by condition. For 0-2 knowers, only children in the multiple exemplars training condition responded above chance for the testing trials for quantities three and four, $t(6) = 3.65$, $p = .005$ (one-tailed). As expected, 3-6 knowers, who already had mastered numbers three and four, easily responded well on the testing trials for three and four regardless of the condition they were in. For quantities five and six, only 3-6 knowers in the single exemplar training condition responded above chance, $t(5) = 3.46$, $p < .01$ (one-tailed). As expected, 0-2 knowers showed poor performance on the testing trials for five and six regardless of the condition they were in.

0-2 knowers' performance by condition and testing trial type. While 0-2 knowers showed poor performance overall, we wanted to examine whether this was consistent across the testing trial types. A two-way repeated measures ANOVA [3 (Condition; Single, Multiple, Control) x 8 (Trial type)] revealed no main effect of condition ($F = 1.83$, $p = .19$) or trial type ($F = 1.86$, $p = .08$). There was no significant interaction between condition and trial type, $F = .69$, $p = .78$.

Figure 4 shows 0-2 knower children's performance on quantities three and four trials by testing trial type. 0-2 knower children in the multiple exemplars condition showed better performance than children in the control or the single exemplar conditions for 'extension', 'target is original', and 'target is new' trials. This suggests that exposure to different objects with the same quantity was particularly helpful when the array for testing trials included new objects which were not shown in the training session. For quantities five and six, as expected, 0-2 knower children did not respond above chance for any testing trial type.

3-6 knowers' performance by condition and testing trial type. For 3-6 knowers, children in the single exemplar condition showed better performance, and we wanted to look at whether this performance was consistent across the testing trial types. A two-way repeated measures ANOVA [3 (Condition; Single, Multiple, Control) x 8 (Trial type)] revealed no main effect of condition ($F = .77$, $p = .47$) but a significant main effect of trial type ($F = 9.70$, $p < .0001$). Also there was no significant interaction between condition and trial type, $F = 1.40$, $p = .16$.

As shown in Figure 5, as expected, 3-6 knowers, who already had mastered numbers three and four, easily responded well on the testing trials for quantities three and four regardless of the testing trial type. For quantities five and six trials, 3-6 knower children in the single exemplar condition responded above chance numerically did better than children in the multiple exemplars condition or the control condition regardless of the testing trial type, suggesting that training with only one type of exemplar was most effective for experienced children, 3-6 knowers.

Post-Training Results

Twenty-five children in the training conditions (twelve 0-2 knowers and thirteen 3-6 knowers) were given the same testing trials after a short delay to check if they could remember even after a delay. Two additional children were excluded because of fussiness, and children in the control group were not given the post-training testing trials. As shown in Figure 6, for 0-2 knowers, again, children in the multiple exemplars condition responded above chance (.5) in the testing trials for quantities three and four after the training session, $M = .73$, $t(4) = 2.25$, $p < .05$. For 3-6 knowers, both the single ($M = .94$) and the multiple ($M = .79$) training groups responded above chance in the testing trials for quantities three and four, $t(5) = 15.65$, $p < .0001$; $t(6) = 2.03$, $p < .05$. Also, in the testing trials for quantities five and six, again, only 3-6 knowers in the single exemplar condition responded above chance, $M = .77$, $t(5) = 3.61$, $p < .01$.

Similar to the training session, 0-2 knowers in the multiple exemplars condition ($M = .73$) showed numerically better performance than the single exemplar condition ($M = .52$) for the testing trials for quantities three and four, $t(10) = 2.13$, $p = .18$. Also, for 3-6 knowers in the testing trials for quantities five and six, the single exemplar group ($M = .77$) did better than the multiple exemplars group ($M = .48$) as in during training session, $t(10) = 2.08$, $p = .03$. Thus, there were no significant changes in children's performance during training and after a short delay for the multiple exemplars condition and the single exemplar condition.

Give-N and quantity recognition task performance before and after training. Paired-samples two-tailed t-tests indicated no significant difference in the Give-N task performance before and after training for the single exemplar condition ($M_{\text{pre}} = 3.62$, $M_{\text{post}} = 3.46$), $t(12) = .43$, $p = .67$, for the multiple exemplars condition ($M_{\text{pre}} = 3.14$, $M_{\text{post}} = 3.43$), $t(13) = .81$, $p = .78$, or for the control condition ($M_{\text{pre}} = 3.69$, $M_{\text{post}} = 3.62$), $t(12) = .56$, $p = .58$.

Even though there was no significant difference between pre-training and post-training Give-N task performance, children in the training group showed improved performance in the quantity recognition task. A paired-samples one-tailed t-test indicated that post-training quantity recognition performance was significantly higher than pre-training quantity recognition performance for the single exemplar condition ($M_{pre} = 2.15$, $M_{\text{post}} = 3.08$), $t(12) = 2.52$, $p = .01$. The same test for the multiple exemplars condition also showed that post-training quantity recognition performance was significantly better than pre-training quantity recognition performance ($M_{pre} = 2.79$, $M_{\text{post}} = 3.29$), $t(13) = 1.84$, $p = .04$. As expected, in the control condition, when no training was given, there was no significant difference in the quantity recognition performance ($M_{pre} = 3.38$, $M_{\text{post}} = 3.38$). This suggests that the training influenced children's performance on the quantity recognition task.

Day-Night Task Results

There was a significant linear relationship between age and children's performance on the day-night task, $r = .35$, $p < .05$. However, the day-night task did not predict performance on any of the quantity tasks including the Give-N task. The best prediction of performance across conditions was how much they knew about number words, rather than executive function.

Discussion

Results from Study 1 revealed that children's prior knowledge and experience make a difference in children's learning: Children with limited number knowledge benefited from the multiple exemplars training, especially when the array for testing trials included novel objects which were not shown in the training session, while children with extended number knowledge benefited from the single exemplar training.

Why did not 0-2 knowers benefit from the single exemplar training? One possible explanation is that it was due to young children's tendency to pay more attention to objects or agents (Hollich et al., 2000; Kersten & Smith, 2002), not the relations. Gentner (2003) claimed that children need to be exposed to multiple exemplars to learn abstract and relational terms, such as action verbs. Likewise, the results from this work suggest that multiple exemplars are necessary for inexperienced children to relate the number word to the quantity of a set by finding the relational commonality. This finding is consistent with prior research that emphasizes the importance of cross-situational learning for word learning. For example, an experiment by Smith and Yu (2008) showed infants could use cross-situational observation to learn novel noun words by accumulating the statistical evidence across many ambiguous word-referent pairs. Scott and Fisher (2012) also found that 2.5-year-old children could use statistical information in action verb learning. In addition to noun and verb learning, Akhtar and Montague (1999) reported that 2-year-old children learned new adjectives after encountering multiple exemplars.

We expected that 3-6 knower children also would benefit from the exposure to multiple exemplars. However, the results showed that children with extended number knowledge did not benefit from the training with multiple exemplars as much as we expected. Interestingly, the training with single exemplar seemed to help 3-6 knowers do better on the task. Researchers reported similar findings in word learning research. For example, Maguire and colleagues (2008) found that both 2.5- and 3-year-olds learned new verb labels better when they were trained with one actor than with four actors suggesting that training with fewer exemplars may help early verb learning. They suggested that repeated exposure to the same exemplar allowed children to focus more attention on the action relation, therefore the single exemplar training was more helpful (Maguire et al., 2008). Likewise, in the current work, 3-6 knowers who already understand the number-word meanings and are already able to match the number word to specific numerosity or quantity may benefit from the single exemplar training because they are be able to focus more on the task itself when there is no extraneous or distracting information (such as changing objects). This is consistent with the “less is more” hypothesis proposed by Newport (1990) that less information is useful for learning language. According to Newport (1990), the ability to learn a new language declines as nonlinguistic cognitive abilities increase. In other words, young children’s less well-developed cognition, such as their limited perception and memory, actually allows children to focus on smaller linguistic units without over-analyzing. It is possible that once children get the idea of abstract numerical relation, they are able to connect a number word to a specific quantity of elements. Then, showing multiple exemplars might not benefit to those children with understanding of numerical relations.

Our result showing the significant relationship between age and the children's performance on the day-night task replicates previous studies that show age-related changes in executive functioning in preschool children (Carlson, 2005). However, there was no significant relationship between children's performance on the day-night task and the quantity task. It is possible that executive functioning did not appear to make any differences because our training was brief, we might expect to see effects of executive function show up over a longer trading period.

Similarly, it would appear that small differences in the training set can make a big difference in effectiveness of the training. Twomey and colleagues (2014) examined how the within-category variability influences 30-month-old children's word retention. In the narrow multiple exemplars condition with low within-category variability, children were exposed to novel objects that varied along one dimension, which was color. In contrast, in the broad multiple exemplars condition with high within-category variability, children were exposed to novel objects that varied along multiple dimensions (color, texture, size and slightly in overall shape). The results showed that children who saw objects that only varied in color could retain names for objects categories better after a short delay. One possible explanation for poor retention in the high within-category variability condition is that broad exemplars may have required more attentional demands, therefore children with limited cognitive capacity may have not used resources to memorize name-object mappings. If we apply this to teaching number words, the categories would be number words, thus it would be better to give exemplars that only varied in quantity. If a child sees a series of examples from a board book, such as ‘one car, two apples, and three ducks’, the uncertainty as to the number word’s meaning would increases because those exemplars varied along multiple dimensions: quantity and kind of objects. This work tested the effectiveness of multiple exemplars with different objects (e.g., 3 cars / 3 apples / 3 ducks). Future studies may include exemplars that only vary in color (e.g., 3 red cars / 3 blue cars / 3 yellow cars) to investigate if lower within-category would make any difference in children’s learning.

From the current study, it would appear that training set makes a difference and prior knowledge of subjects matters. Specifically, multiple exemplars help novices (0-2 knowers) recognize quantity, as would be predicted by theories that tie early math learning to perceptual learning. What else might increase the perceptual strength of the training set? Study 2 examines whether increasing the perceptual distinctiveness of the displays might help as well.

STUDY 2: CHILDREN'S NUMBER LEARNING AND SPATIAL ARRANGEMENT OF OBJECTS

How does spatial arrangement of objects influence young children's number learning? Study 2 investigated whether children's exposure to different arrangements of arrays might have differential effects on children's numerical learning. In the previous study, the objects were always presented in a line. The difference between three and four or five and six was thus mostly one of length. Would having a more distinct arrangement of object allow children to more quickly and efficiently recognize the different quantities, just as perceptual salience provides a power cue in language learning? Specifically, this study used two kinds of object arrays: a linear arrangement (as used in study 1) and a "dice" style arrangement. It was expected that children might be helped or swayed by the surface perceptual features in the dice array that made the quantities more distinct.

Method

Participants

A total of forty-three typically developing children (*M* = 47.5 months, *S*D = 6.96, age range: 33 —59.6 months, 20 girls) participated in this study. They were recruited via mass distribution (across daycare centers and preschools) of consent forms and letters explaining the study. Only children of parents who gave consent by retiring those signed consent forms participated in the study. Data from an additional three children were not included because of fussiness (1) and unwillingness (2). Children (*n* = 43) were randomly assigned to one of three conditions: the dice arrangement condition (*n* = 15), the linear arrangement condition (*n* = 14), and the control condition (*n* = 14).

Procedure

As in Study 1, children completed a standard Give-N task and a quantity recognition task using an iPad before training. During training session, as shown in Figure 7, children in the dice arrangement condition were trained with dice arrangements of objects (cars), and children in the linear arrangement condition were trained with linear arrangements of objects. Both children in the dice and the linear condition were given testing trials on quantities 3~6. Children in the control condition had no training session, instead they had free play time, then were given testing trials on quantities 3~6. After the training session, children completed a day-night touch game using the iPad. Then children completed the Give-N task and the quantity recognition task on the iPad one more time. As a final test, the same testing trials used in the training session were given.

Give-N task. As in Study 1, the Give-N task was given to children before and after the training session.

Quantity recognition task. As in Study 1, after the standard Give-N task using blocks, children were given the quantity recognition task on the iPad.

Training. As shown in Figure 7, there were two training conditions, the dice arrangement condition and the linear arrangement condition. Children in the dice condition saw dice arrangements of cars for quantities 3, 4, 5, and 6. Children in the linear condition saw linear arrangement of cars for quantities 3, 4, 5, and 6. Children in the control condition had no training session.

Testing. During the testing phase, two sets of balls with different quantities appeared on the iPad, and the audio requested children to find the target number. For example, right after the training trials on number three and four, children saw three balls on the top and four objects at the bottom or vice versa and were asked to choose either three or four (e.g., “Which box has three?” or “Which box has four?”). As shown in Figure 8, there were four types of testing trials: ‘Dice vs. Dice’, ‘Linear vs. Linear’, ‘Target is Dice’, and ‘Target is Linear’. For ‘Dice vs. Dice’ trials, both target and non-target quantities were shown as dice arrangements of balls. For ‘Linear vs. Linear’ trials, both target and non-target quantities were shown as linear arrangements of balls. For ‘Target is Dice (or Linear)’ trials, the target quantity set of balls was shown in the dice (or linear) arrangement. Children were shown a total of sixteen testing trials.

Post-training. After training, an iPad version of the day-night task used in Study 1 was given. After the day-night task, the Give-N task and the quantity recognition task were given one more time to see if children learned something from training. After that, the 16 testing trials used in the training block (four trials for each quantity) were given as a final test.

Coding

As in Study 1, children’s behaviors were recorded via digital video camera (GoPro). Children’s touching behavior was coded and used as a measure of children’s performance in the testing trials.

Results

Pre-Training Results

Give-N results. The Give-N task yielded 6 zero-knowers, 8 one-knowers, 4 two-knowers, 8 three-knowers, 2 four-knowers, 1 five-knowers, and 14 six-knowers. As shown in Table 6, children's pre-training performance on the Give-N task was again correlated with age, $r = .49$, $p < .0001$, reflecting the fact that generally older children knew number words better than younger children.

Quantity recognition results. The quantity recognition task before training yielded 11 zero-knowers, 5 one-knowers, 4 two-knowers, 7 three-knowers, 6 four-knowers, and 10 six-knowers. Unlike in Study 1, children's pre-training performance on the quantity recognition iPad task was correlated with age, $r = .49$, $p < .001$, reflecting the fact that generally older children were better able to recognize quantities than younger children.

Give-N task and quantity recognition task. For all 43 children, children's pre-training performances on the Give-N task and on the quantity recognition iPad task were strongly positively correlated, $r = .88$, $p < .0001$, suggesting that these tasks tested some common underlying skills.

Training Session

Preliminary analysis looking at performance on the testing trials during training session, with age as a covariate, showed no significant effect of gender, $F(1, 40) = .56$, $p = .46$. Therefore, data were collapsed over gender in further analysis.

As in Study 1, does children's prior knowledge of numbers influence children's quantity perception on the iPad? A univariate analysis of variance (ANOVA) looking at performance on the testing trials with age as a covariate showed a significant effect of pre-training knower level determined by the Give-N task, $F(6, 35) = 7.94$, $p < .0001$. Also, the same analysis showed a significant effect of pre-training knower level determined by the quantity recognition task, $F(5, 35) = 20.01$, $p < .0001$. As in Study 1, for further analysis, children were grouped into two groups by their pre-training quantity recognition performance. First, 0-2 knowers ($N = 20$) included children who had mastered number words up to either two or one and children who had no prior knowledge of numbers. Second, 3-6 knowers ($N = 23$) included children who had mastered number words up to three or beyond three. This 3-6 knower group was expected to complete the number three and four testing trials relatively easily because of their prior knowledge on numbers.

Table 7 shows children's performance on the testing trials during the training session by condition. For 0-2 knowers, only children in the linear arrangement condition responded above chance for the testing trials for quantities three and four ($M = .66$) at the margin of statistical significance, $t(6) = 1.89$, $p = .05$ (one-tailed). For quantities five and six, 0-2 knowers showed poor performance regardless of the condition they were in. As expected, 3-6 knowers, who already had mastered numbers three and four, easily responded well on the testing trials for quantities three and four regardless of the condition they were in. For quantities five and six, only 3-6 knowers in the control condition responded above chance ($M = .75$), $t(6) = 3.06$, $p < .05$ (one-tailed). While this result was unexpected, it could be that too much variation in the arrangements of the training set presented difficulties for the more experienced learners, just as multiple exemplars presented a problem for them in Study 1.

0-2 knowers' performance by condition and testing trial type. While 0-2 knowers showed poor performance overall, we wanted to examine whether this was consistent across the testing trial types. A two-way repeated measures ANOVA [3 (Condition; Dice, Linear, Control) x 8 (Trial type)] revealed no main effect of condition ($F = 1.99$, $p = .17$) or trial type ($F = .90$, $p = .51$), but showed a significant interaction between training condition and trial type, $F = 1.98$, $p = .03$. This suggests that children's performance during training session was dependent on both the training type (condition) and the testing trial type. For quantities three and four trials, there was a significant main effect of condition when both target array and non-target array were linear object arrangements, $F(2, 17) = 5.85$, $p = .01$, in particular, as shown in Figure 9, only the children who were exposed to the linear object arrangements ($M = .93$) responded above chance, $t(6) = 6.0$, $p < .001$. This suggests that training sessions with linear arrangements were most effective for recognizing and comparing two different numbers of objects when 0-2 knowers were tested with the linear arrangements. For other testing trial types, this analysis did not reveal a significant main effect of condition. However, it is notable that when the target array was dice arrangement and non-target was linear arrangement, children in the dice training condition numerically performed better ($M = .75$) than children in the control training condition ($M = .50$) or in the linear training condition ($M = .64$), even though the analysis did not reach statistical significance.

For quantities five and six trials, there was a main effect of condition when the target array was a dice arrangement and the non-target array was a linear one, $F(2, 20)$

$= 4.28$, $p = .03$. In these ‘Target = Dice’ trials, only children in the dice training condition ($M = .83$) performed above chance, $t(5) = 3.16$, $p = .01$.

To see if children just chose the familiar arrangement as an answer regardless of requested quantities, the proportions of responses to familiar patterns which were already exposed in the training trials were calculated for ‘Target=Linear’ and ‘Target=Dice’ trials. For quantities three and four, children in the dice training condition (.63) were more likely to follow the familiar patterns than children in the linear training condition (.46). For quantities five and six, as in the quantities three and four, children in the dice training condition (.79) were more likely to follow the familiar arrangements children in the linear training condition (.64). The results show that when the testing trials had familiar vs. unfamiliar arrangements, inexperienced children tend to choose familiar arrangement (regardless of quantity) as an answer when they are unsure of answer.

3-6 knowers’ performance by condition and testing trial type. A two-way repeated measures ANOVA [2 (Condition) x 8 (Trial type)] showed no main effect of condition, $F = .05$, $p = .95$. This analysis showed main effect of trial type, $F = 6.56$, $p < .0001$, and a significant interaction between training condition and trial type, $F = 1.89$, $p = .03$. Tukey’s post hoc comparisons indicated that 3-6 knowers’ performance on testing trials for quantities three and four were different from their performance on testing trials for quantities five and six. Perhaps, this is because 3-6 knowers who already had mastered numbers three and four easily responded to these quantities. For quantities five and six trials, when the target array was a dice arrangement and the non-target array was a linear arrangement, no 3-6 knower children in any condition responded above chance. When the target array was a linear arrangement and the non-target array was a dice arrangement, only 3-6 knowers in the dice training condition responded above chance (M =.81), $t(7) = 3.42$, $p < .01$.

Post-Training Results

Thirty-three children (15 0-2 knowers and 18 3-6 knowers) were given the same testing trials after a short delay to check if they could remember even after a delay. Ten additional children were excluded because of children's unwillingness. As in the training session, 0-2 knowers showed poor performance for both quantities three and four trials and quantities five and six trials regardless of the condition they were in. As expected, all 3-6 knowers performed well for quantities three and four. For quantities five and six, only 3-6 knowers in the dice training condition (M = .77) only responded above chance, $t(6) = 3.60$, $p < .01$.

Give-N and quantity recognition task performance before and after training. Paired-samples t-tests indicated no significant difference in the Give-N task performance before and after training for the dice condition (M_{pre} = 3.36, M_{post} = 3.36), for the linear condition (M_{pre} = 3.4, M_{post} = 3.3), or for the control condition (M_{pre} = 2.79, M_{post} = 2.57).

For the quantity recognition task, paired-samples t-tests indicated no significant difference in the performance before and after training for the dice condition (M_{pre} = 2.93, M_{post} = 3.00) or for the control condition (M_{pre} = 2.5, M_{post} = 2.43). For the linear condition, even though the analysis did not reach statistical significance, children did numerically better after training (M_{post} = 3.13) than before training (M_{pre} = 2.8).

Day-Night Task Results

A total of forty children completed the day-night task. Three children were excluded from the analysis because of unwillingness and fussiness. The results from the day-night task data demonstrated that there was a significant correlation between age and performance on the day-night task, r = .42, p = .0076 . However, as in Study 1, the day-night task did not predict performance on any of the quantity tasks used in the current work.

Discussion

Study 2 results reveal that 0-2 knower children tend to choose the familiar arrangements which were presented in the training session. There was no significant main effect of training condition for more experienced 3-6 knowers. Overall, 0-2 knower children showed poor performance regardless of the condition they were in. This work did not find evidence showing that certain arrangements were easier for 0-2 knower children. Instead, we found that they tended to choose just the familiar arrangements suggesting that they focused more on the perceptual features (such as whole configuration) of sets, not on the quantities of sets. For example, when the target array was unfamiliar arrangement and the non-target array was familiar arrangement, 0-2 knower simply tended choose the familiar arrangement as an answer without counting. It is possible that those 0-2 knower children did not develop their representations of both spatial and mathematical structure yet. Mulligan and colleagues (2004) explained children's structural development with regard to early mathematics. In their study, children aged from 5 years 6 months to 6 years 8 months completed thirty tasks designed to examine children's mathematical and spatial structures within number, measurement, space and data. For example, in a triangular pattern task which was one of the space and data tasks, children were shown a flash card with triangular pattern of six dots and were asked to draw exactly what they saw using their memory. In this task, children were required to integrate the spatial pattern (triangle-shape) and the numerical pattern (six dots) to succeed on this task. In their results, children whose representations lacked both spatial and mathematical structure drew too many dots in a linear arrangement; children with little awareness of the structure drew a christmas tree as an attempt to show the triangular pattern, or drew the correct quantity of circles in a random arrangement; children with partial structure representations drew a triangle; and children with well-developed mathematical and spatial structure were able to draw the exact same pattern from memory. In this work, perhaps, those 0-2 knower children may be at the partial structure representation stage, therefore they were sensitive to the perceptual information but not sensitive to the numerical information. In any case, we saw no evidence of a benefit for novice learners by having more perceptually distinct arrays.

For experienced 3-6 knower children, we expected that a perceptually distinct array, which was the dice arrangement in Study 2, might aid children's learning. However, our analysis also did not detect a significant difference between the dice training condition and the linear training condition. It could be, that as in Study 1, more experienced learners do not need, and in fact are distracted by, perceptually distinct arrays. Thus, it is possible that, unexpectedly, dice arrangements are not simple enough in design for young children, so that they may be distracted by the dice configuration and have trouble paying attention to one-by-one counting. For example, for the dice training condition, there was more than one possible path from top left to bottom right when counting objects. For the linear arrangements, young children might have no problem with keeping track of items, because only moving across horizontally to the right does not require lots of load on memory (Potter & Levy, 1968). However, for the dice arrays, children had to remember what they had touched and what they had not. Even though the hand on the screen moved to point to the corresponding objects while counting to help children count correctly, having more than one possible path could simply be too complex to young children. Due to this difficulty that less orderly young children had with arrays with columns and rows, it is possible that the dice training did not show significantly better performance than the linear training. If we conduct research with older children or adult subjects who are able to use spatial strategies well, subjects may find the dice configurations easier (Beckwith & Restle, 1966).

As in Study 1, the significant relationship between age and the children's performance on the day-night task was found in Study 2. However, there was no significant relationship between children's performance on the day-night task and the quantity task. It is possible that executive functioning did not to make any differences in our quantity recognition task because this task was dependent on children's perceptual abilities. Again, it was possible that for effects of executive function to show up, the training would need to be over a longer period of time. That is, more transitory attentional effects made more of a difference, than executive function.

GENERAL DISCUSSION

To date, the empirical research on young children's mathematics learning from differing types of exemplars or arrangements is limited. This study provides some evidence that perception of quantity and number learning can change with children's knowledge and type of training set.

From early language learning research, we learned the importance of exposure to multiple exemplars for early word learning. To see if this was applicable to children's early number learning, Study 1 compared children's performance on the quantity recognition task on the iPad after training with multiple exemplars (e.g., cars / apples / ducks) or a single exemplar (e.g., cars / cars / cars). Results suggested that children's prior knowledge of numbers determines which training condition will work best. Inexperienced 0-2 knowers needed multiple exemplars to get their start in recognizing quantity, while experienced 3-6 knowers benefited from single exemplars to help them focus on learning new quantities.

Study 2 compared children's performance on the quantity recognition task on the iPad after training with dice arrangement of objects or a linear arrangement of objects to examine whether a perceptually more distinct array, a dice arrangement, might be more helpful for young children's quantity recognition performance. Results suggested that inexperienced 0-2 knowers tended to choose the familiar arrangements which were shown in the training session, while more experienced 3-6 knowers picked out the correct quantity regardless of arrangement. Also, unexpectedly, 3-6 knower children's performances in the linear training condition and in the dice training condition were not significantly different. This suggests that having perceptually more distinct array does not actually help young children's learning to recognize quantity.

Thus, this dissertation suggests that selecting the right type of training is important for facilitating children's early number learning, specifically quantity recognition. For 0-2 knowers, training with multiple exemplars may be helpful, while for 3-6 knowers, training with a single exemplar set may be helpful, as multiple exemplars or unusual arrangements of items seems to impair performance. Also, 0-2 knowers appear to be influenced by surface perceptual features, therefore choosing an array with the right type of spatial arrangement of objects may be critical.

One limitation of this work is small sample size. If we conduct research with larger sample size, we may get greater power to detect differences in children's performance for each training condition. In addition, children in the current work had only a limited training session. Certainly, learning the complexities of counting out arrays may take longer than a single training session. Children may need more time to process new information. To examine the long-term effectiveness of training, it may be worth having multiple training sessions. Such a long-term training would also allow us to detect effects of executive functioning, which we also did not see in the current short-term training periods. Similarly, the 5-10 minute long delay may have been too short to examine children's long-term retention.

Finally, children in this work had no social interaction in the training. Work in language suggests a central role for social interaction as children become more experienced (Hollich, Hirsh-Pasek, & Golinkoff, 2000). Even though our training video tried to attract children's attention by asking questions (e.g., Audio: "Can you say it with me? Three cars!") or by having visual effects (e.g., The hand moved to the corresponding objects while counting), we do not know if those manipulations in interaction with the iPad were more or less successful than other types of social interaction, such as the guidance of an adult. This could be explicitly examined in subsequent studies by using the same training sets but instead training with a live person.

Significance and Broader Impacts of Current Studies

Despite the limitations above, this is among the first studies to demonstrate that quantity recognition can be trained in a short amount of time and can be improved by careful selection of the training sets. If early quantity recognition is causally linked to later language, such training could have long-term beneficial effects on later language development. Similarly, by using methods inspired by language learning, these studies allow for new avenues to improve math education by improving children's early labeling of quantities (whether by using multiple exemplars for novices or single exemplars for more experienced learners). Finally, this work also helps highlight the numerous similarities between early quantity recognition and language learning, potentially outlining an approach that unifies both types of learning under a common mechanism.

With regard to broader impacts, this work may have implications for educators and parents. This dissertation suggests that at the early stage of number learning, similar to word learning, it is important to provide many examples to help children connect number word meaning to specific quantity (e.g., point to three chairs and say “three”, then point to three spoons and say “three”). As children acquire some number concepts and accumulate experience with number words, children do not require multiple exemplars. Once they understand that number words (‘one’, ‘two’, ‘three’, …) are connected to quantity of a set of objects, not to the name or color of objects, they may no longer need to experience multiple exemplars to learn higher numbers. At this stage, teaching with a few exemplars may be as effective as teaching with many exemplars. Furthermore, as Clements (1999) has stated, this work also suggests that inefficient presentation of objects might hinder children’s learning. Given our findings from Study 2, dice-like arrangements may not always guarantee better learning than linear arrangements even with the perceptual distinctness of dice-like arrangements with regard to recognizing quantities. This work indicates that the spatial arrangement of sets influences how difficult they are to recognize, and finding arrangements yielding a better fit for certain group of children (e.g., 0-2 knower vs. 3-6 knower) is important. Again, perception of quantity and number learning can change with children’s experience and knowledge level and type of training set, thus educators and teachers need to be careful when they develop the educational materials for children’s early number learning.

Future Directions

Some open questions still remain. First, it remains an open question as to whether training with tangible objects lead to the same or different patterns of results from the current work. Many educators believe that young children only learn mathematics with physical manipulatives which are concrete and tangible (Lee & Ginsburg, 2009). Even though children in this work were able to touch the iPad screen and point to objects while counting, again, we do not know if this touching behavior was more or less successful than typical interaction with real objects. Again, subsequent studies looking at training with a live person could also use tangible objects to see how much of a difference concrete objects make. Even so, the results from this dissertation suggest that the medium for mathematics instruction could be anything, tangible object are not totally necessary, as long as the training set can be used to encourage children to think about the abstract mathematical idea (e.g., abstracting the idea of the number by generalizing from many experiences; understanding addition/subtraction) (Lee & Ginsburg, 2009). Further research should investigate whether training with tangible objects is better teaching method than training with the touch-screen device.

Second, Study 1 did not include combined condition where children were shown both the single exemplar and the multiple exemplars. Goldenberg and Sandhofer (2013) found that 2-year-old children showed better performance in generalizing a new label in a new context when training was in both same and varied contexts. They included three conditions: the same context condition where all category instances were presented in the same context, the varied condition where all category instances were presented in varied contexts, and the interleaved condition where some instances were presented in the same context and some were presented in varied contexts. The results showed that children in the interleaved condition showed better performance because this combined condition could support both aggregation (detecting the covarying features) and decontextualization (successfully generalizing in a new context) processes. Thus, it is possible that young children's learning would improve more when they were trained with both the single exemplar and the multiple exemplars in the current work. Further research should investigate whether combining the single and multiple exemplars is better or worse than the single or the multiple exemplars condition.

A third open question is whether children in Study 2 may benefit from viewing multiple different patterns (e.g., dice / linear / random arrangement). From Study 1 results, we learned that inexperienced learners may benefit from the exposure to multiple exemplars. However, children in Study 2 in each condition were only exposed to one type of arrangement (dice training-dice/dice/dice; linear training-linear/linear/linear). Solnick and Baer (1984) found that training with multiple formats of workbooks was effective for improving young children's skills in number-numeral correspondence. Further research should include training condition with multiple arrangements to examine whether this yields better performance than training with single pattern.

LIST OF REFERENCES

LIST OF REFERENCES

Akhtar, N., & Montague, L. (1999). Early lexical acquisition: The role of cross-situational learning. *First Language*, *19*(57), 347-358.

Baldo, J. V., & Dronkers, N. F. (2007). Neural correlates of arithmetic and language comprehension: A common substrate? *Neuropsychologia*, *45*(2), 229–235.

Baroody, A. J., Lai, M., & Mix, K. S. (2006). The development of young children's early number and operation sense and its implications for early childhood education. In B. Spodek & O. Saracho (Eds.), *Handbook of research on the education of young children* (pp. 187-221). Mahwah, NJ: Lawrence Erlbaum Associates.

Beckwith, M., & Restle, F. (1966). Process of enumeration. *Psychological Review*, *73*(5), 437–444.

Benoit, L., Lehalle, H., & Jouen, F. (2004). Do young children acquire number words through subitizing or counting? *Cognitive Development*, *19*(3), 291–307.

Blair, C., & Razza, R. P. (2007). Relating effortful control, executive function, and false belief understanding to emerging math and literacy ability in kindergarten. *Child Development*, *78*(2), 647–663.

Bloom, P., & Wynn, K. (1997). Linguistic cues in the acquisition of number words. *Journal of Child Language*, *24*(03), 511-533.

Bobis, J. (2008). Early spatial thinking and the development of number sense. *Australian Primary Mathematics Classroom*, *13*(3), 4–9.

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, *79*(4), 1016–1031.

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: inhibition, switching, and working memory. *Developmental Neuropsychology*, *19*(3), 273–293.

Carlson, S. M. (2005). Developmentally sensitive measures of executive function in preschool children. *Developmental Neuropsychology*, *28*(2), 595–616.

Clark, C. A. C., Pritchard, V. E., & Woodward, L. J. (2010). Preschool executive functioning abilities predict early mathematics achievement. *Developmental Psychology*, *46*(5), 1176–1191.

Clements, D. H. (1999). Subitizing: What is it? Why teach it? *Teaching Children Mathematics*, *5*(7), 400-405.

Dehaene, S., Dehaene-Lambertz, G., & Cohen, L. (1998). Abstract representations of numbers in the animal and human brain. *Trends in Neurosciences*, *21*(8), 355–361.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of Mathematical Thinking: Behavioral and Brain-Imaging Evidence. *Science*, *284*(5416), 970–974.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., . . . Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428-1446.

Espy, K. A., McDiarmid, M. M., Cwik, M. F., Stalets, M. M., Hamby, A., & Senn, T. E. (2004). The contribution of executive functions to emergent mathematic skills in preschool children. *Developmental Neuropsychology*, *26*(1), 465–486.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, *44*(1–2), 43–74.

Gallistel, C. R., & Gelman, R. (2000). Non-verbal numerical cognition: from reals to integers. *Trends in Cognitive Sciences*, *4*(2), 59–65.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Gelman, R., & Gallistel, C. R. (2004). Language and the Origin of Numerical Concepts. *Science*, *306*(5695), 441–443.

Gentner, D. (1982). Why nouns are learned before verbs: Linguistic relativity versus natural partitioning. In S. A. Kuczaj (Ed.), *Language development: Vol. 2. Language, thought and culture* (pp. 301 - 334). Hillsdale, NJ: Erlbaum.

Gentner, D. (2003). Why we're so smart. In D. Gentner & S. Goldin-Meadow (Eds.), *Language in mind: Advances in the study of language and thought* (pp. 195-236). Cambridge, MA: MIT Press.

Goldenberg, E. R., & Sandhofer, C. M. (2013). Same, varied, or both? Contextual support aids young children in generalizing category labels. *Journal of Experimental Child Psychology*, *115*(1), 150–162.

Hauser, M. D., Carey, S., & Hauser, L. B. (2000). Spontaneous number representation in semi–free–ranging rhesus monkeys. *Proceedings of the Royal Society of London B: Biological Sciences*, *267*(1445), 829-833.

Hirsh-Pasek, K., & Golinkoff, R. M. (1999). *The origins of grammar: Evidence from early language comprehension*. Cambridge, MA: MIT Press.

Hollich, G., Hirsh-Pasek, K., & Golinkoff, R. (2000). Breaking the language barrier: An emergentist coalition model of the origins of word learning. *Monographs of the Society for Research in Child Development*, 65(3, Serial No. 262).

Huttenlocher, J., Jordan, N. C., & Levine, S. C. (1994). A mental model for early arithmetic. *Journal of Experimental Psychology: General*, *123*(3), 284-296.

Izard, V., Sann, C., Spelke, E. S., & Streri, A. (2009). Newborn infants perceive abstract numbers. *Proceedings of the National Academy of Sciences*, *106*(25), 10382–10385.

Jansen, B. R. J., Hofman, A. D., Straatemeier, M., van Bers, B. M. C. W., Raijmakers, M. E. J., & van der Maas, H. L. J. (2014). The role of pattern recognition in children's exact enumeration of small numbers. *British Journal of Developmental Psychology*, *32*(2), 178–194.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early Math Matters: Kindergarten Number Competence and Later Mathematics Outcomes. *Developmental Psychology*, *45*(3), 850– 867.

Kaufmann, E. L., Lord, M.W., Reese, T.W., & Volkmann, J. (1949). The discrimination of visual number. *American Journal of Psychology, 62*, 498–525.

Kersten, A. W., & Smith, L. B. (2002). Attention to novel objects during verb learning. *Child Development*, *73*(1), 93–109.

Kilian, A., Yaman, S., Fersen, L., & Güntürkün, O. (2003). A bottlenose dolphin discriminates visual stimuli differing in numerosity. *Animal Learning & Behavior*, *31*(2), 133–142.

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk." *Developmental Psychology*, *42*(1), 59–69.

Kroesbergen, E. H., Luit, J. E. H. V., Lieshout, E. C. D. M. V., Loosbroek, E. V., & Rijt, B. A. M. V. de. (2009). Individual differences in early numeracy: The role of executive functions and subitizing. *Journal of Psychoeducational Assessment*, *27*(3), 226–236.

Krajewski, K., & Schneider, W. (2009). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, *19*(6), 513-526.

Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood*, *34*(4), 37-45.

Maguire, M., Hirsh-Pasek, K., & Golinkoff, R. M. (2006) A unified theory of word learning: Putting verb acquisition in context. In K. Hirsh-Pasek & R.M. Golinkoff (Eds.), *Action meets word: How children learn verbs*. (pp. 364 - 391). New York, NY: Oxford University Press.

Maguire, M. J., Hirsh-Pasek, K., Golinkoff, R. M., & Brandone, A. C. (2008). Focusing on the relation: Fewer exemplars facilitate children's initial verb learning and extension. *Developmental Science, 11*(4), 628-634.

Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General*, *111*(1), 1–22.

Mulligan, J., Prescott, A., & Mitchelmore, M. (2004). Children's development of structure in early mathematics. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 3* (26), 393-400.

NAEYC/NCTM. (2002). *Early childhood mathematics: Promoting good beginnings*. Washington, DC: National Association for the Education of Young Children.

National Research Council (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington, DC: The National Academies Press.

Newport, E. L. (1990). Maturational constraints on language learning. *Cognitive Science*, *14*(1), 11–28.

Papic, M. & Mulligan, J. (2005). Preschoolers' mathematical patterning. *The Proceedings of the 28th Mathematical Education Research Group of Australasia Conference* (pp. 609-616).

Papic, M., & Mulligan, J. (2005). Preschoolers' mathematical patterning. In P. Clarkson, A, Downton, D. Gronn, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building Connections: Theory, research and practice* (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, Melbourne, Vol. 2, pp. 609-616). Sydney, Australia: MERGA.

Piaget, J (1965). *The child's conception of number*. New York, NY: Norton.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and Approximate Arithmetic in an Amazonian Indigene Group. *Science*, *306*(5695), 499–503.

Potter, M. C., & Levy, E. I. (1968). Spatial enumeration without counting. *Child Development*, *39*(1), 265-272.

Purpura, D. J., Hume, L. E., Sims, D. M., & Lonigan, C. J. (2011). Early literacy and early numeracy: The value of including early literacy skills in the prediction of numeracy development. *Journal of Experimental Child Psychology, 110*(4), 647-658.

Purpura, D. J., & Logan, J. A. R. (2015). The nonlinear relations of the approximate number system and mathematical language to early mathematics development. *Developmental Psychology*, *51*(12), 1717–1724.

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development*, *79*(2), 375–394.

Scott, R. M., & Fisher, C. (2012). 2.5-Year-olds use cross-situational consistency to learn verbs under referential uncertainty. *Cognition, 122*(2), 163–180.

Slusser, E. B., & Sarnecka, B. W. (2011). Find the picture of eight turtles: A link between children's counting and their knowledge of number word semantics. *Journal of Experimental Child Psychology, 110*(1), 38-51.

Smith, L., & Yu, C. (2008). Infants rapidly learn word-referent mappings via cross-situational statistics. *Cognition, 106*(3), 1558–1568.

Solnick, J. V., & Baer, D. M. (1984). Using multiple exemplars for teaching number-numeral correspondence: Some structural aspects. *Analysis and Intervention in Developmental Disabilities*, *4*(1), 47–63.

Spelke, E. S. (2003). What makes us smart? Core knowledge and natural language. In A. A. Gentner & S. Goldin-Meadow (Eds.), *Language in mind: Advances in the study of language and thought*. (pp. 277-311). Cambridge, Mass: MIT Press.

Spelke, E. S., & Tsivkin, S. (2001). Language and number: a bilingual training study. *Cognition, 78*(1), 45–88.

Toll, S. W. M., & Luit, J. E. H. V. (2014). The developmental relationship between language and low early numeracy skills throughout kindergarten. *Exceptional Children, 81*(1), 64-78.

Thorndike, E. L. (1922). *The psychology of arithmetic*. New York, NY: Macmillan

Twomey, K. E., Ranson, S. L., & Horst, J. S. (2014). That's more like it: Multiple exemplars facilitate word learning. *Infant and Child Development*, *23*(2), 105-122.

Van Nes, F., & de Lange, J. (2007). Mathematics Education and Neurosciences: Relating Spatial Structures to the Development of Spatial Sense and Number Sense. *Montana Mathematics Enthusiast*, *4*(2), 210–229.

Viterbori, P., Usai, M. C., Traverso, L., & De Franchis, V. (2015). How preschool executive functioning predicts several aspects of math achievement in Grades 1 and 3: A longitudinal study. *Journal of Experimental Child Psychology*, *140*, 38–55.

Wynn, K. (1992). Addition and subtraction by human infants. *Nature, 358*(6389), 749 - 750.

Wynn, K. (1998). Psychological foundations of number: Numerical competence in human infants. *Trends in Cognitive Sciences*, *2*(8), 296-303.

APPENDICES

Appendix A

Table 1

Study 1 Procedure

Condition	Pre-Training		Training		Post-Training			
Single	Give-N Task	Quantity Recognition Task on iPad	Number 3&4 <u>Single exemplar</u> training and testing	Number 5&6 <u>Single exemplar</u> training and testing	Give-N Task	Quantity Recognition Task on iPad	Same testing trials used in the training session	Day-Night touch game
Multiple			Number 3&4 <u>Multiple exemplar</u> training and testing	Number 5&6 <u>Multiple exemplar</u> training and testing				
Control			No training, instead free play time, then testing				No testing	

Table 2

Give-N Task

Give-N task Script	Coding
If the child succeeds at giving a number n, the next request will be n+1.	Coding instruction
If the child fails at giving a number n, the child will be asked for n-1.	- “S(a)”: if correct
	- “F(a,b)”: if wrong
	- a: requested number
The task continues until the child has at least two success at a given number n and at least two failures at n+1.	- b: child’s wrong answer
Look, we have blocks. Can you give me 1 block?	*Coding Example*
Thank you. Can you give me 3 blocks?	*S(1)*
	S(3)
If the child succeeds on both requests, go to ***e****.*	
Otherwise, go to ***b****.*	
a. Can you give me 1 block?	
b. Can you give me 2 blocks?	
c. Can you give me 3 blocks?	*S(3)*
d. Can you give me 4 blocks?	*F(4,5), F(4,6)*
e. Can you give me 5 blocks?	*F(5,6)*
f. Can you give me 6 blocks?	

Table 3

Correlations Between Performance and Age for Study 1

	Pre-training Give-N	Pre-training Quantity	Post-training Give-N	Post-training Quantity	Day-Night	Age
Pre-training-Give-N (*N* = 40)	–					
Pre-training Quantity (*N* = 40)	.63****	–				
Post-training Give-N (*N* = 40)	.86****	.67****	–			
Post-training Quantity (*N* = 40)	.66****	.80****	.70****	–		
Day-Night (*N* = 32)	.30	.17	.13	.33	–	
Age (*N* = 40)	.33*	.25	.21	.42**	.35*	–

Note. **** $p < .0001$; ** $p < .01$; * $p <. 05$.

Table 4

Study 1 Results From Training Session by Pre-Training Quantity Recognition Performance and Condition

Pre-Training Quantity Recognition	Number of Children	Mean Age (months)	Proportions of Correct Responses for Quantities Three and Four	Proportions of Correct Responses for Quantities Five and Six
0-2 knower				
Single	7	38.71	0.59	0.41
Multiple	7	39.71	0.73 **	0.54
Control	6	44.45	0.58	0.44
3-6 knower				
Single	6	42.80	0.90***	0.75**
Multiple	7	42.81	0.93****	0.43
Control	7	44.43	0.91***	0.59

Note. A one-tailed t-test was conducted to see if the mean correct response was significantly above chance (.5), **** $p < .0001$, *** $p < .001$, ** $p < .01$.

Table 5

Study 2 Procedure

Condition	Pre-Training		Training		Post-Training			
Linear			Numbers 3&4 <u>Linear</u> training and testing	Numbers 5&6 <u>Linear</u> training and testing				
Dice	Give-N Task	Quantity Recognition Task on iPad	Number 3&$ <u>Dice</u> training and testing	Number 5&6 <u>Dice</u> training and testing	Day-Night Touch game	Give-N Task	Quantity Recognition	Same testing trials used in the Task on iPad training
Control			No training, instead free play time, then testing					

Table 6

Correlations Between Performance and Age for Study 2

	Pre-training Give-N	Pre-training Quantity	Post-training Give-N	Post-training Quantity	Day-Night	Age
Pre-training-Give-N (*N* = 40)	–					
Pre-training Quantity (*N* = 40)	.88****	–				
Post-training Give-N (*N* = 40)	.97****	.88****	–			
Post-training Quantity (*N* = 40)	.90****	.95****	.91****	–		
Day-Night (*N* = 32)	.17	.04	.13	.14	–	
Age (*N* = 40)	.49***	.49***	.45**	.49***	.42**	–

Note. **** $p < .0001$; *** $p < .001$; ** $p < .01$.

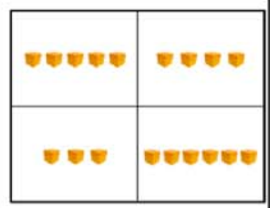
Table 7

Study 2 Results From Training Session by Pre-Training Quantity Recognition Performance and Condition

Pre-Training Quantity Recognition	Number of Children	Mean Age (months)	Proportions of Correct Responses for Quantities Three and Four	Proportions of Correct Responses for Quantities Five and Six
0-2 knower				
Linear	7	45.49	.66+	.48
Dice	6	47.35	.56	.54
Control	7	42.09	.46	.46
3-6 knower				
Linear	8	48.96	.98****	.64
Dice	8	49.01	.97****	.66
Control	7	51.83	.82**	.75*

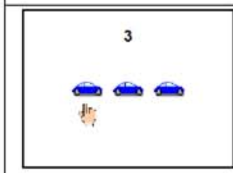
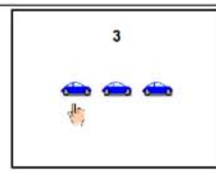
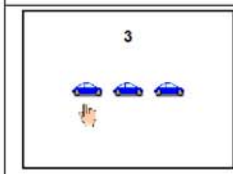
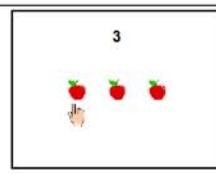
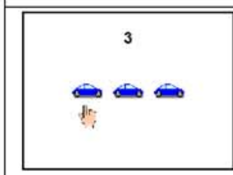
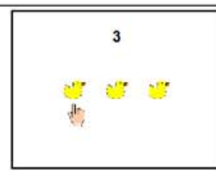
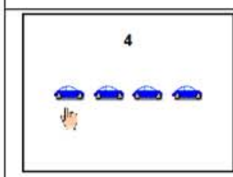
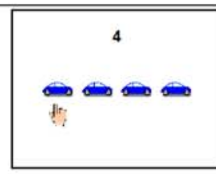
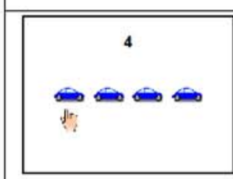
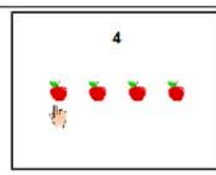
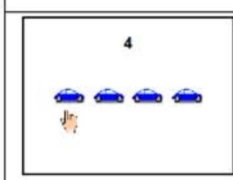
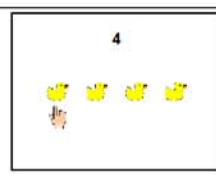
Note. A one-tailed t-test was conducted to see if the mean correct response was significantly above chance (.5), **** $p < .0001$, ** $p < .01$, * $p < .05$, + $p = .05$.

Appendix B

Screen	Audio	Screen	Audio
	Which box has 1? Which box has 2? Which box has 3? Which box has 4?		Which box has 5? Which box has 6?

Note. The order of presentation was determined by a child's response.

Figure 1. Quantity recognition task on iPad.

Single Exemplar Condition		Multiple Exemplars Condition	
Screen	Audio	Screen	Audio
	Look, there are *3* cars. Can you say it with me? *3* cars. *Let's count them, 1, 2, 3. 3 cars!		Look, there are *3* cars. Can you say it with me? *3* cars. *Let's count them, 1, 2, 3. 3 cars!
	Look, there are *3* cars. Can you say it with me? *3* cars. *Let's count them, 1, 2, 3. 3 cars!		Look, there are *3* apples. Can you say it with me? 3 *apples*. *Let's count them, 1, 2, 3! 3 apples!
	Look, there are *3* cars. Can you say it with me? *3* cars. *Let's count them, 1, 2, 3. 3 cars!		Look, there are *3* ducks. Can you say it with me? 3 *ducks*. *Let's count them, 1, 2, 3! 3 ducks!
	Look, there are *4* cars. Can you say it with me? *4* cars. *Let's count them, 1, 2, 3, 4. 4 cars!		Look, there are *4* cars. Can you say it with me? *4* cars. *Let's count them, 1, 2, 3, 4. 4 cars!
	Look, there are *4* cars. Can you say it with me? *4* cars. *Let's count them, 1, 2, 3, 4. 4 cars!		Look, there are *4* apples. Can you say it with me? *4 apples*. *Let's count them, 1, 2, 3, 4! 4 apples!
	Look, there are *4* cars. Can you say it with me? *4* cars. *Let's count them, 1, 2, 3, 4. 4 cars!		Look, there are *4* ducks. Can you say it with me? *4 ducks*. *Let's count them, 1, 2, 3, 4! 4 ducks!

Note. When the audio starts counting, the hand moves to the right to point to the corresponding objects.

Figure 2. Single exemplar and multiple exemplars training conditions for the quantities three and four.

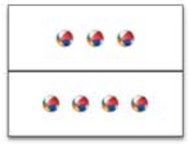
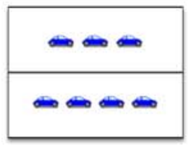
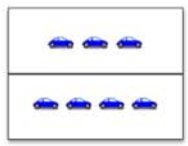
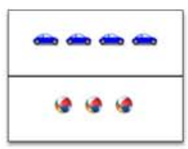
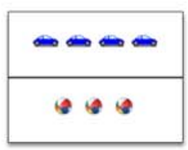
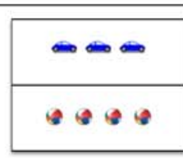
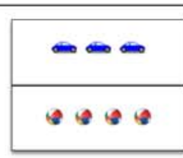
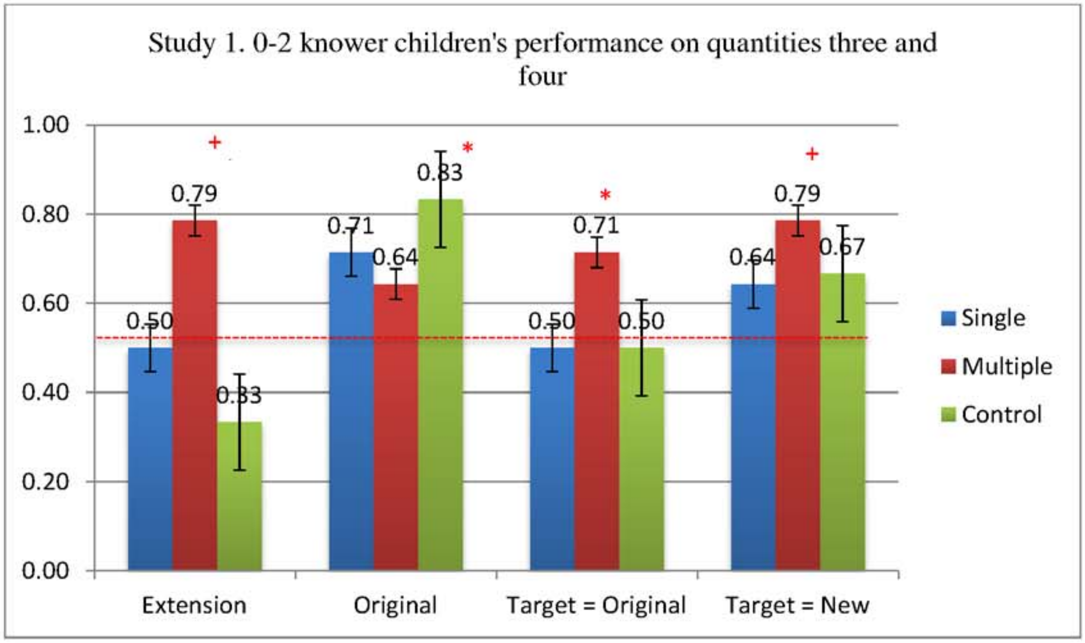
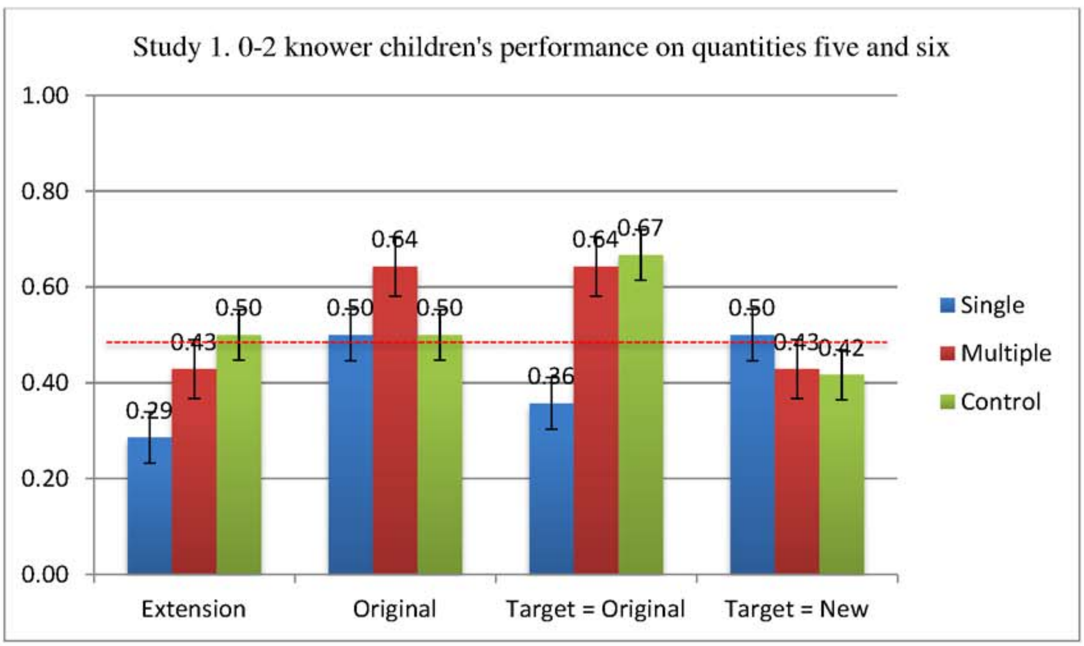
Testing Trial Type	Video	Audio
Extension Novel target vs. Novel distractor *Ball: Novel object*		Which box has 3?
Extension Novel target vs. Novel distractor		Which box has 4?
Original Familiar target vs. Familiar distractor *Car: Familiar object*		Which box has 4?
Original Familiar target - Familiar distractor		Which box has 3?
Crossed: Target = New Novel target - Familiar distractor		Which box has 3?
Crossed: Target = Original Familiar target - Novel distractor		Which box has 4?
Crossed: Target = New Novel target - Familiar distractor		Which box has 4?
Crossed: Target = Original Familiar target - Novel distractor		Which box has 3?

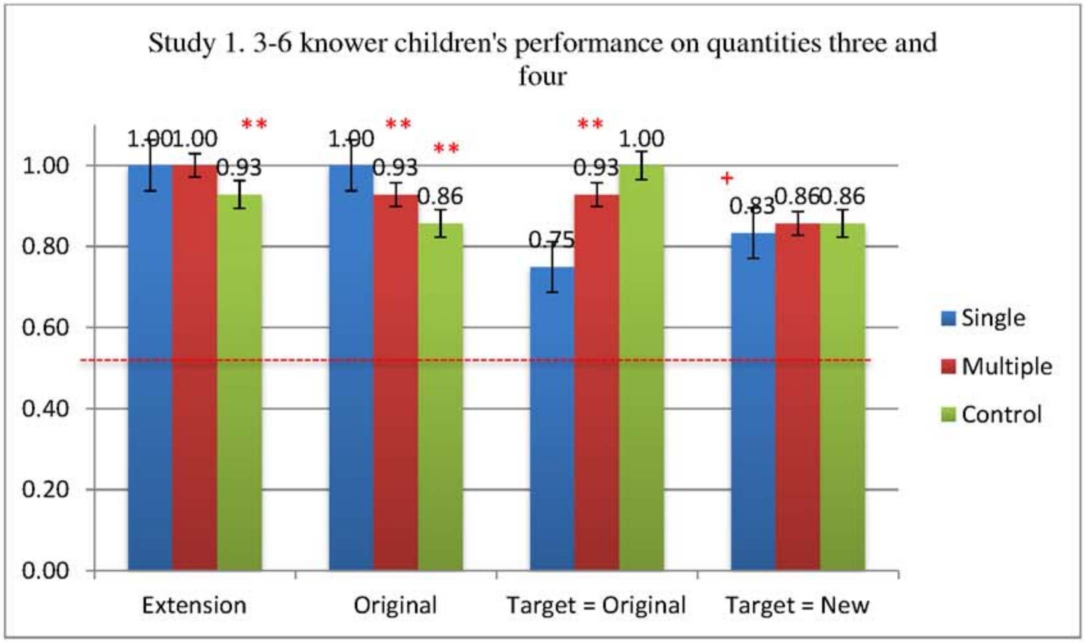
Figure 3. Testing trials for Study 1.

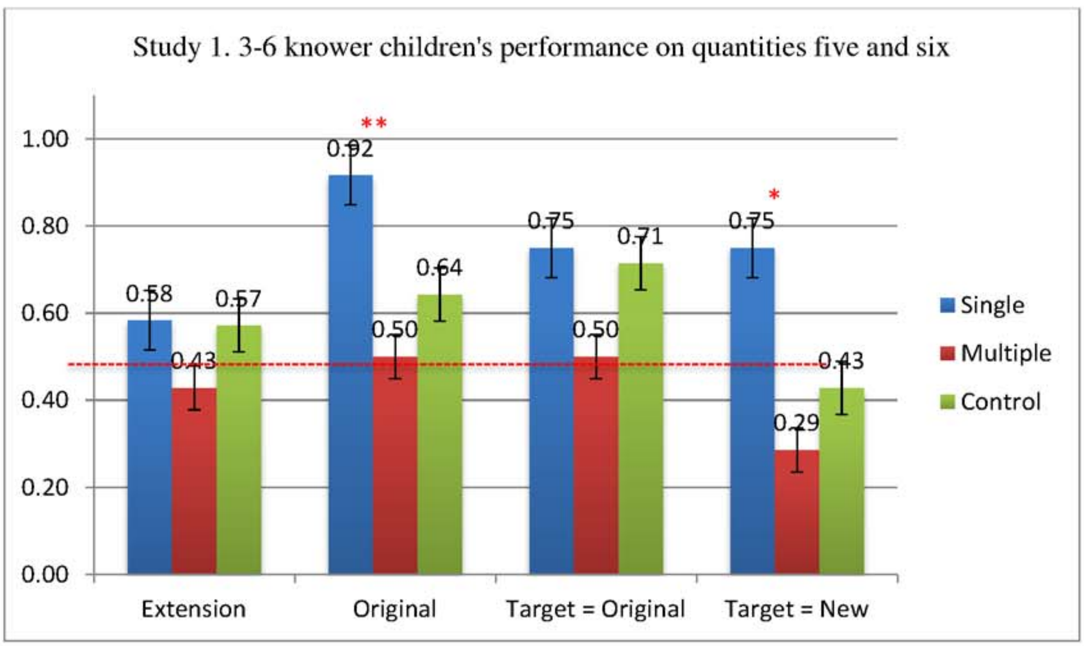




Note. One-tailed t-tests were conducted to see if the response rates were above chance (.5), * $p < .05$, + $p = .05$.

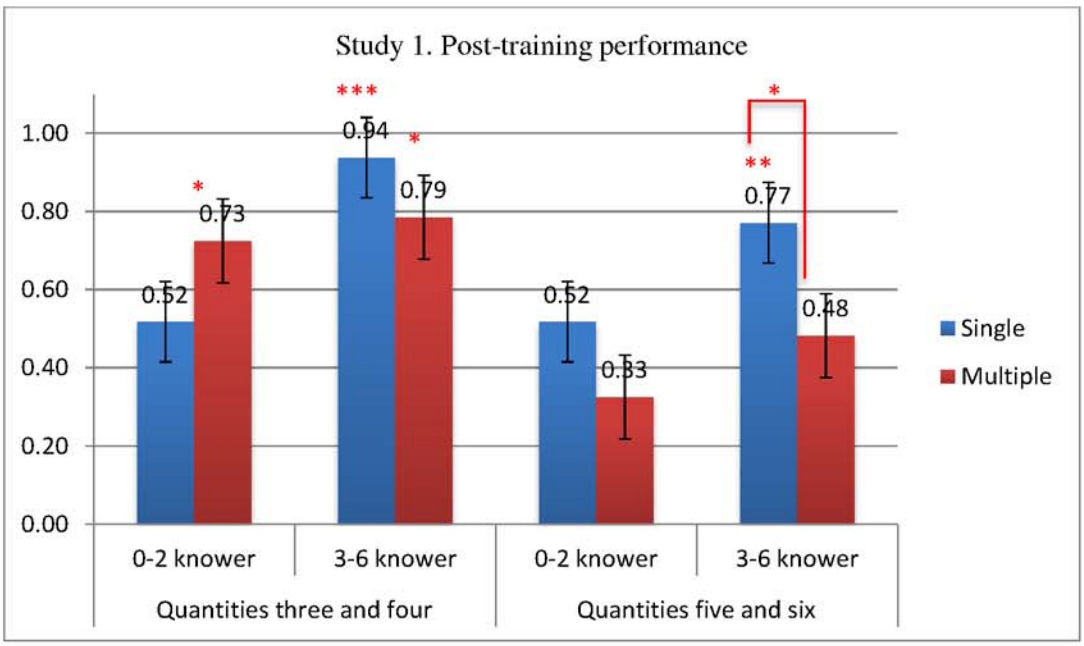
Figure 4. 0-2 knower children's performance on quantities three and four trials (*top*) and five and six trials (*bottom*) by testing trial type.





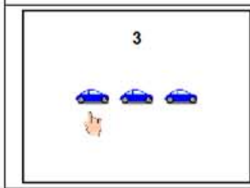
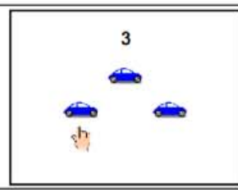
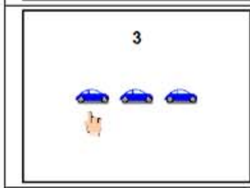
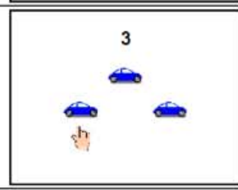
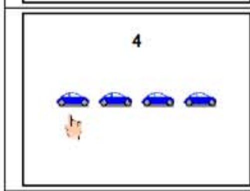
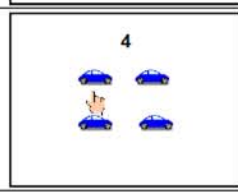
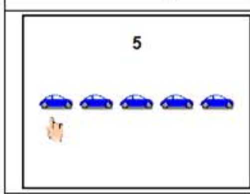
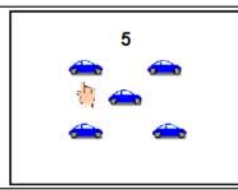
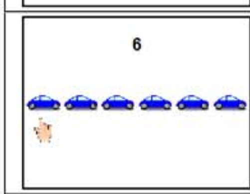
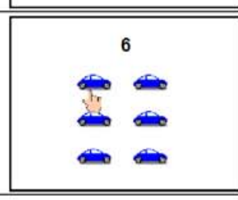
Note. One-tailed t-tests were conducted to see if the response rates were above chance (.5), ** $p < .01$, * $p < .05$, + $p = .05$.

Figure 5. 3-6 knower children's performance on quantities three and four trials (*top*) and five and six trials (*bottom*) by testing trial type.



Note. One-tailed t-tests were conducted to see if the response rates were above chance (.5), **** $p <$ .0001, *** $p < .001$, ** $p < .01$, * $p < .05$.

Figure 6. Twenty-five children's (twelve 0-2 knowers) performance on the testing trials after the training session.

Linear	Dice	Audio	Training Trial Type
		Look, there are *3* cars. Can you say it with me? *3* cars. *Let's count them, 1, 2, 3. 3 cars!	Training trial for number **three**
		Look, there are *3* cars. Can you say it with me? *3* cars. *Let's count them, 1, 2, 3. 3 cars!	Training trial for number **three**
		Look, there are *3* cars. Can you say it with me? *3* cars. *Let's count them, 1, 2, 3. 3 cars!	Training trial for number **three**
		Look, there are *4* cars. Can you say it with me? *4* cars. *Let's count them, 1, 2, 3, 4. 4 cars!	Training trial for number **four** × ***3***
Testing trials for quantities three and four. Four trials for each quantity were given.			
		Look, there are *5* cars. Can you say it with me? *5* cars. *Let's count them, 1, 2, 3, 4, 5. 5 cars!	Training trial for number **five** × ***3***
		Look, there are *6* cars. Can you say it with me? 6 cars. *Let's count them, 1, 2, 3, 4, 5, 6. 6 cars!	Training trial for number **six** × ***3***
Testing trials for quantities five and six. Four trials for each quantity were given.			

Note. When the audio starts counting, the hand moves to point to the corresponding objects.

Figure 7. Training trials for the linear and dice arrangement conditions.

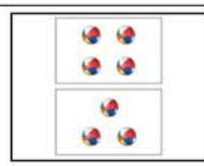
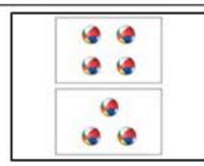
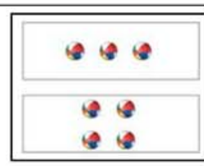
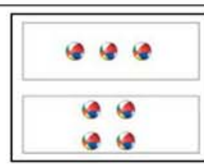
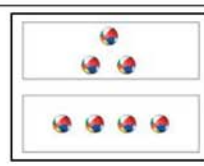
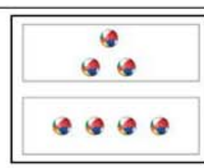
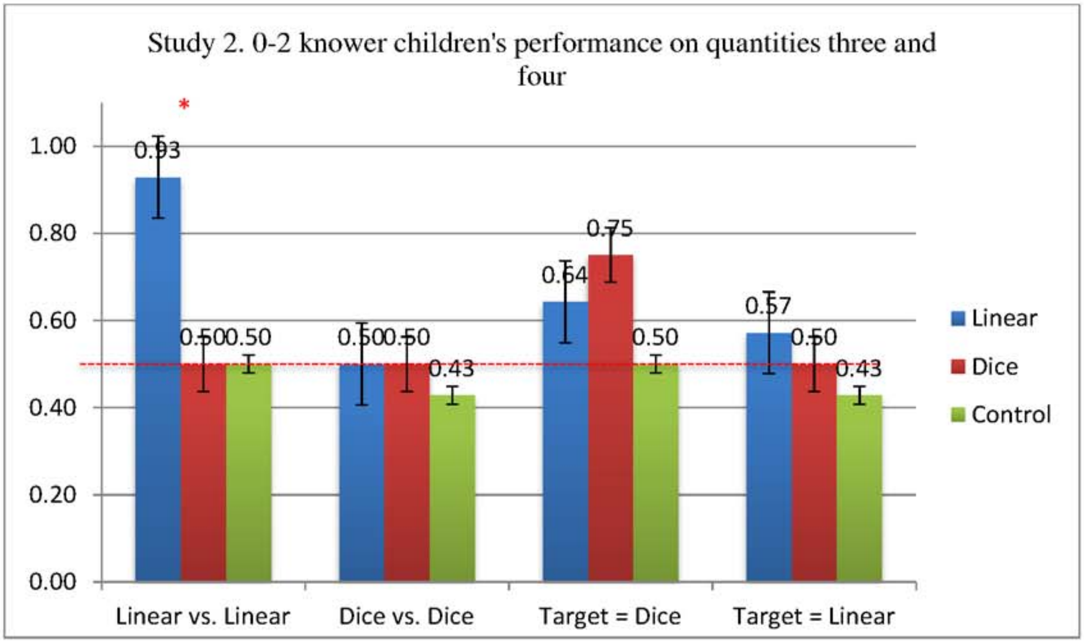
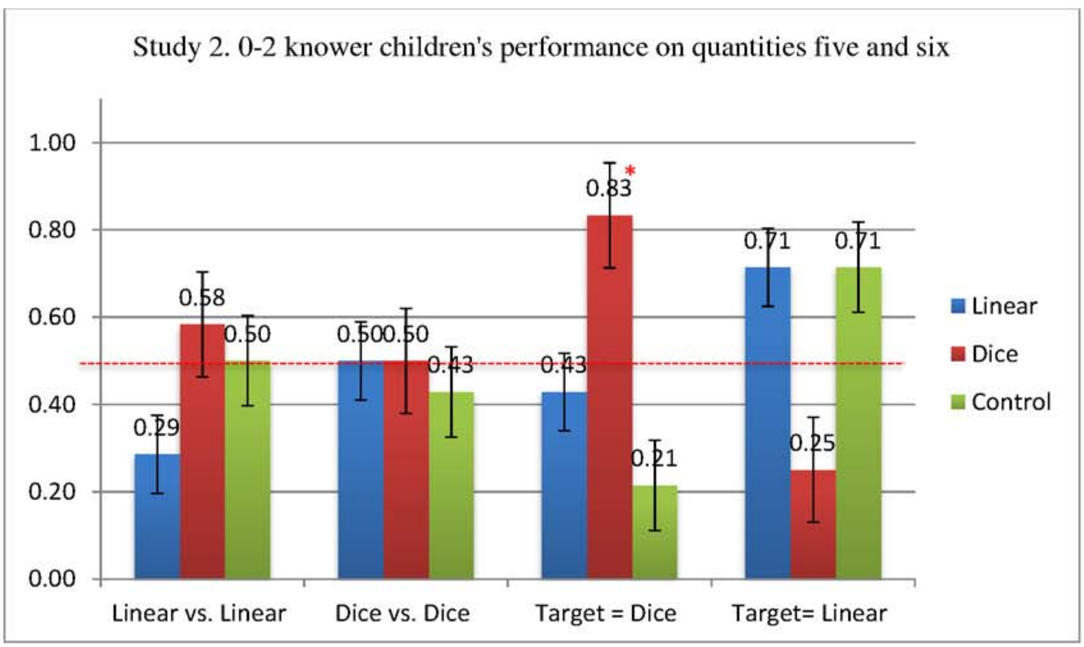
Testing Trial Type	Video	Audio
Linear vs. Linear		Which box has 4?
Linear vs. Linear		Which box has 3?
Dice vs. Dice		Which box has 3?
Dice vs. Dice		Which box has 4?
Target is *Linear*		Which box has 3?
Target is *Dice*		Which box has 4?
Target is *Dice*		Which box has 3?
Target is *Linear*		Which box has 4?

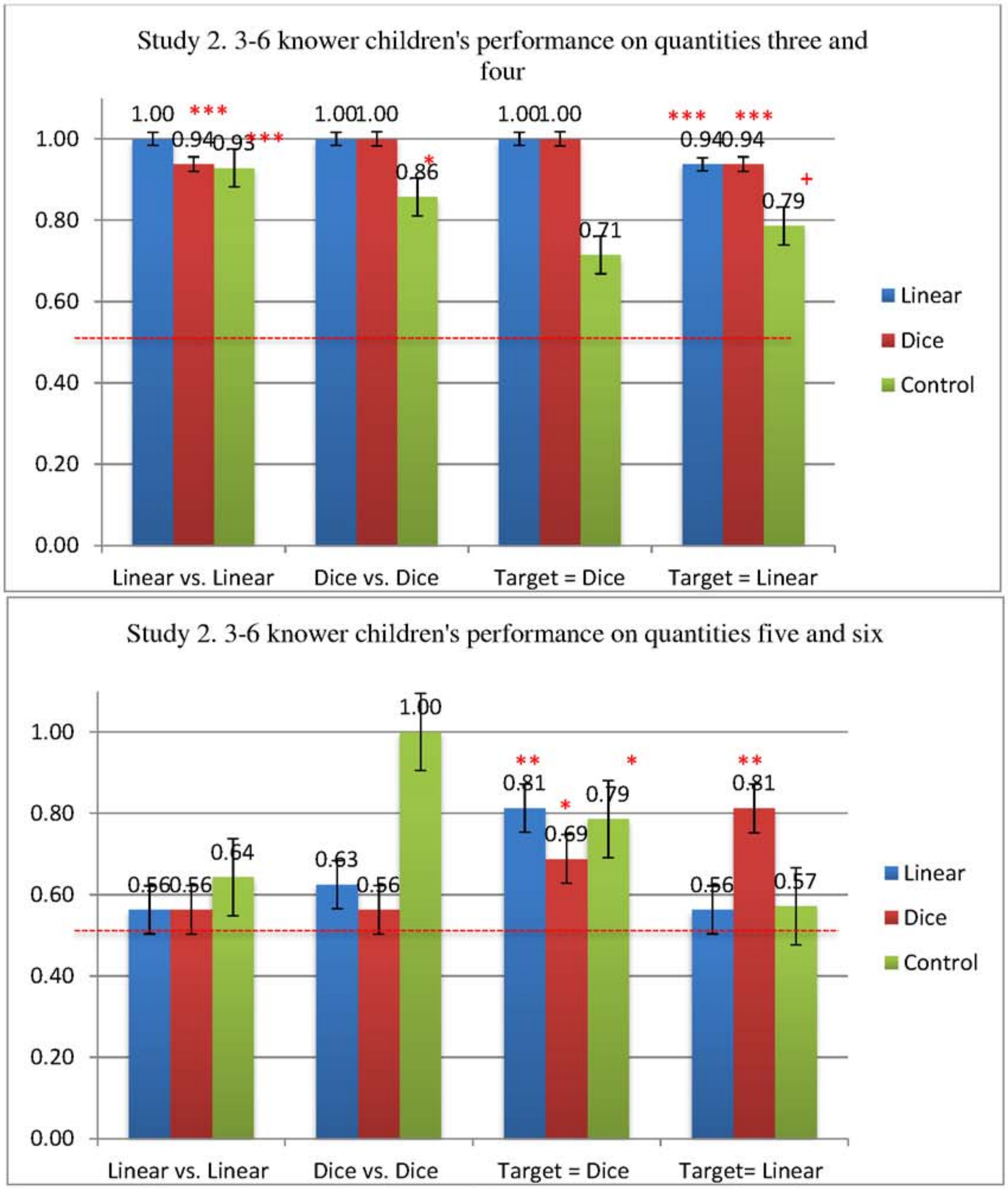
Figure 8. Testing trials for Study 2.





Note. One-tailed t-tests were conducted to see if the response rates were above chance (.5), * $p < .05$, + $p = .05$.

Figure 9. 0-2 knower children's performance on quantities three and four trials (*top*) and five and six trials (*bottom*) by testing trial type.



Note. One-tailed t-tests were conducted to see if the response rates were above chance (.5), *** p < .001; ** p < .01; * *p* < .05; + *p* = .05.

Figure 10. 3-6 knower children's performance on quantities three and four trials (*top*) and five and six trials (*bottom*) by testing trial type.

VITA

VITA

Arum Han was born in Seoul, South Korea in 1986. She attended Seoul Science High School and graduated in 2004. She attended Seoul National University and earned a Bachelor of Science in Chemistry and a Bachelor of Arts in Psychology in 2008. In 2010, she joined the graduate program of Department of Psychological Sciences at Purdue University. She obtained a Master of Science in Psychology in 2012, and her doctoral degree in 2016.