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Fibrous Material Microstructure Design for Optimal Damping Performance

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5th Symposium on the Acoustics of Poro-Elastic Materials (SAPEM)

December 6th – 8th, 2017

Le Mans, France

FIBROUS MATERIAL MICROSTRUCTURE DESIGN FOR OPTIMAL DAMPING PERFORMANCE

Yutong (Tony) Xue, J. Stuart Bolton

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Xue

eux

Je suis heureux d'être ici!

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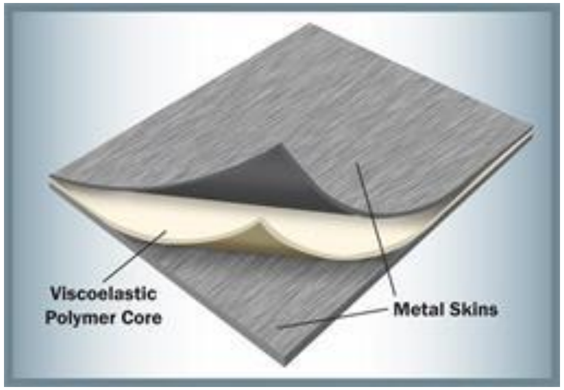
INTRODUCTION

- **Traditional Damping Treatments – Visco-elastic Core with Metal Skins**

Traditional Damping Material[1]



Structure of a Traditional Damper[2]

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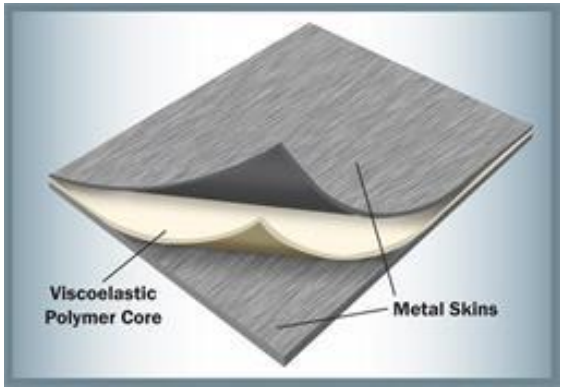

Structure of a Traditional Damper[2]

- **Fibrous Damping Treatments – Target Material of this Study**

Fibrous Damping Material[3]

Test on Fibrous Dampers[4]

INTRODUCTION

- **Literature Review**

- **Bruer & Bolton, *AIAA 1987*[5]** – Analysis of different waves propagating in the layered damping structures
- **Wahl & Bolton, *JASA 1992*[6]** – Analysis by Inverse Discrete Fourier Transform (IDFT) on the spatial / temporal response of the layered damping system under line-driven force
- **Lai & Bolton, *Noise-Con 1998*[7]** – Modeling to prove reasonable structural damping effect from the light fibrous materials through dissipating nearfield energy
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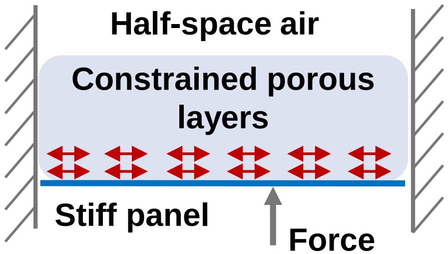
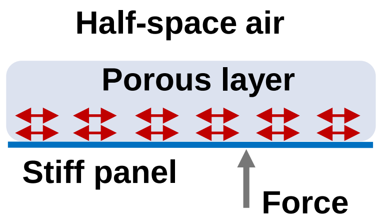
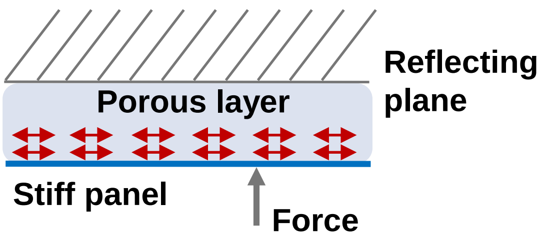
- **Layered Structures Shown in the Literature**







- The panel damping mostly arises because of the viscous interaction of the fibrous medium and the evanescent near-field of the panel associate with subsonic panel motion

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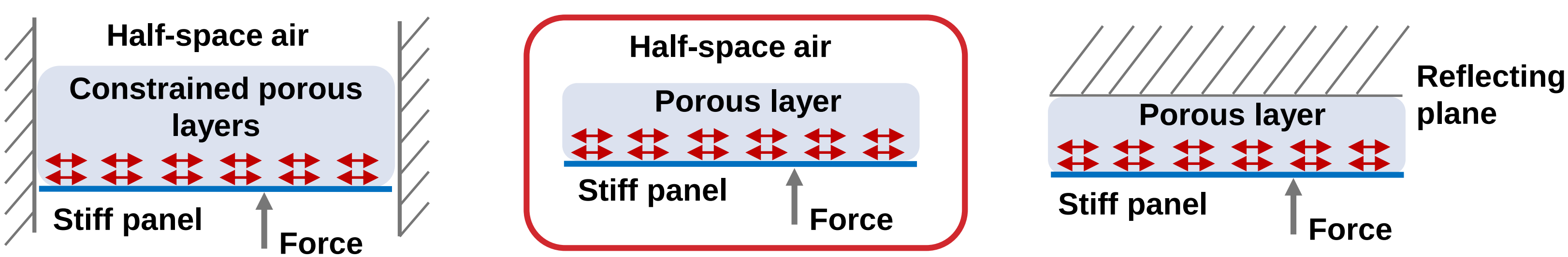

Target Structure of this Study

INTRODUCTION

PURDUE MECHANICAL ENGINEERING

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- **Acoustical / Damping Performance Prediction Process**

Microstructure Inputs → Airflow resistivity → Acoustical properties → Damping properties

Airflow Resistivity Model **(AFR)**

Biot's Theory & B.C.s **(TMM)**

Beam Theory & IDFT **(NFD)**

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- **Noise Control Materials Microstructure Design Process**

Input a range of airflow resistivities → **TMM** + **NFD** → Acoustical & damping properties → **AFR** + Optimizing → Optimal airflow resistivity & microstructure

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- **Objectives of this Study**
  - **Identify the airflow resistivity providing optimal damping performance given panel structure and frequency range of interest**
  - **Translate the optimal airflow resistivity into optimal fiber sizes for fibrous material microstructure design**

MODELING

- **Modeling Process[6], [10]**

Limp porous medium bulk properties: thickness ($\boldsymbol{d}$), AFR ($\boldsymbol{\sigma}$), bulk density ($\boldsymbol{\rho_b}$), porosity ($\boldsymbol{\phi}$), tortuosity ($\boldsymbol{\alpha_\infty}$)

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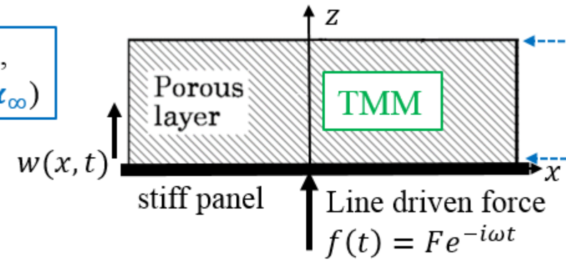

Acoustic field pressure $\boldsymbol{p_2}$ and normal velocity $\boldsymbol{v_{z2}}$ at $\boldsymbol{x = d}$

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MODELING

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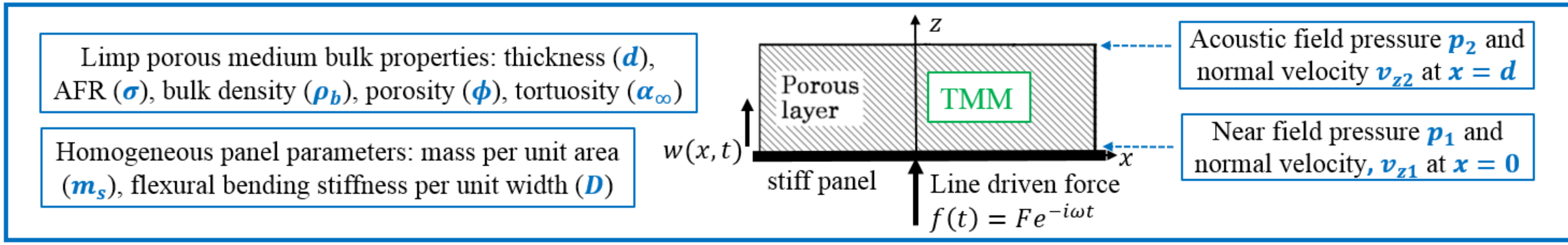

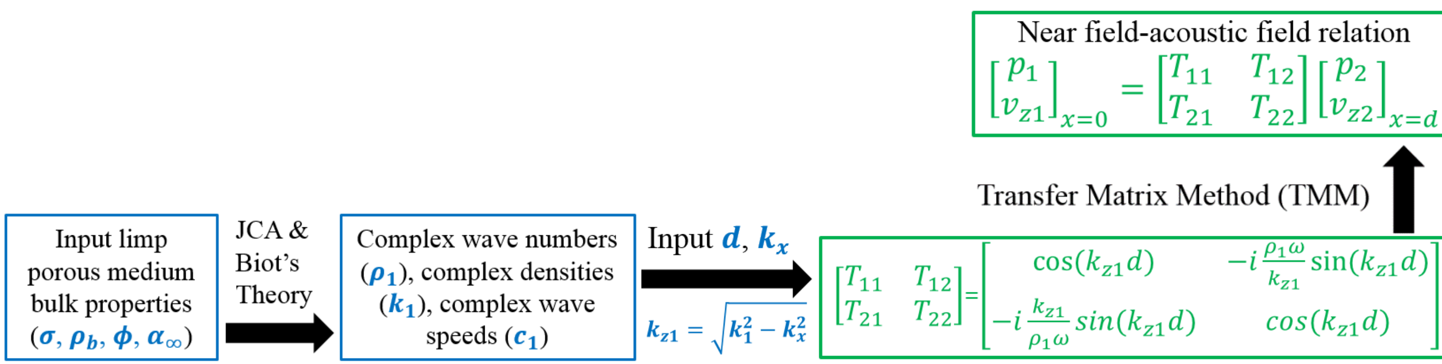

MODELING

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Limp porous medium bulk properties: thickness (d), AFR (σ), bulk density (ρ_b), porosity (ϕ), tortuosity (α_∞)

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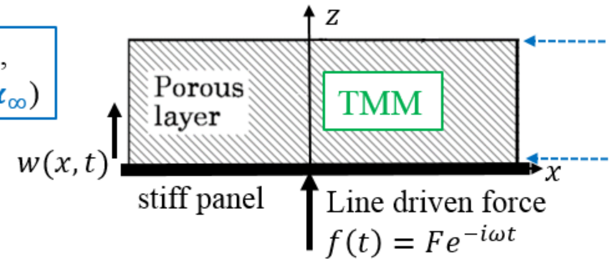

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Euler-Bernoulli beam theory (neglect rotary inertia & shear deformation)

Governing Equation (GE)

$$D\frac{\partial^4 w(x,t)}{\partial x^4} + m_s\frac{\partial^2 w(x,t)}{\partial t^2} = -p_1(x,t) + f(t)\delta(x)$$

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$$(Dk_x^4 - \omega^2 m_s)W(k_x,\omega) = -P_1(k_x,t) + F$$

Input N, γ_s for IDFT

Panel normal velocity response

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Transfer Matrix Method (TMM)

Input limp porous medium bulk properties (σ, ρ_b, ϕ, α_∞)

JCA & Biot's Theory

Complex wave numbers (ρ_1), complex densities (k_1), complex wave speeds (c_1)

Input d, k_x

$$k_{z1} = \sqrt{k_1^2 - k_x^2}$$

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MODELING

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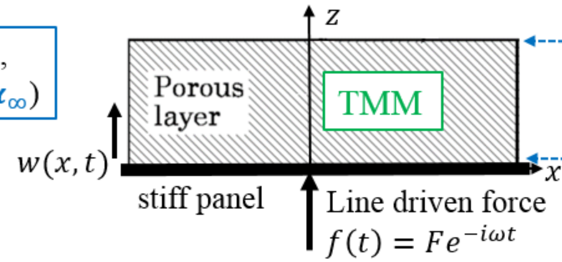

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Power radiation into the porous layer

$$\bar{P}_1 = \frac{1}{2}\mathrm{Re}\left\{\int_{-\infty}^{\infty} p_1 v_{z1}^* \,dx\right\} = \frac{1}{4\pi}\mathrm{Re}\left\{\int_{-\frac{\gamma_s}{2}}^{\frac{\gamma_s}{2}} Z_{a1}|v_{z1}(k_x,\omega)|^2\,dk_x\right\}$$

Power radiation into the air

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AFR Model combined with least square optimization returns optimal porous material micro-structure details (e.g. fiber sizes)

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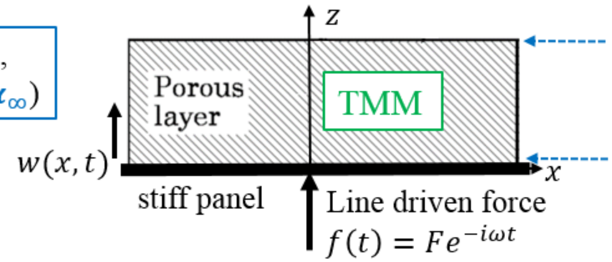

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PURDUE MECHANICAL ENGINEERING

RAY W. HERRICK LABORATORIES

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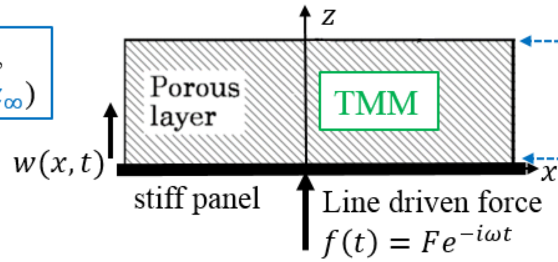

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$$\begin{bmatrix} p_1 \\ v_{z1} \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{bmatrix} p_2 \\ v_{z2} \end{bmatrix}_{x=d}$$

Transfer Matrix Method (TMM)

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos(k_{z1}d) & -i\frac{\rho_1\omega}{k_{z1}}\sin(k_{z1}d) \\ -i\frac{k_{z1}}{\rho_1\omega}\sin(k_{z1}d) & \cos(k_{z1}d) \end{bmatrix}$$

AFR Model combined with least square optimization returns optimal porous material micro-structure details (e.g. fiber sizes)

Input limp porous medium bulk properties ($\boldsymbol{\sigma}$, $\boldsymbol{\rho_b}$, $\boldsymbol{\phi}$, $\boldsymbol{\alpha_\infty}$)

JCA & Biot's Theory

Complex wave numbers ($\boldsymbol{\rho_1}$), complex densities ($\boldsymbol{k_1}$), complex wave speeds ($\boldsymbol{c_1}$)

Input $\boldsymbol{d}$, $\boldsymbol{k_x}$

$$k_{z1} = \sqrt{k_1^2 - k_x^2}$$

- **Results Observations**

MODELING – KEY POINT 1

- **Choice of IDFT sampling rate γ_s and sampling points number N**

➢ **Target of the NFD model**: calculate spatial responses for wide frequency range

➢ **Key point**: for each frequency input, choosing proper γ_s and N to ensure **accurate IDFT results over a large enough spatial span** for observation

MODELING – KEY POINT 1

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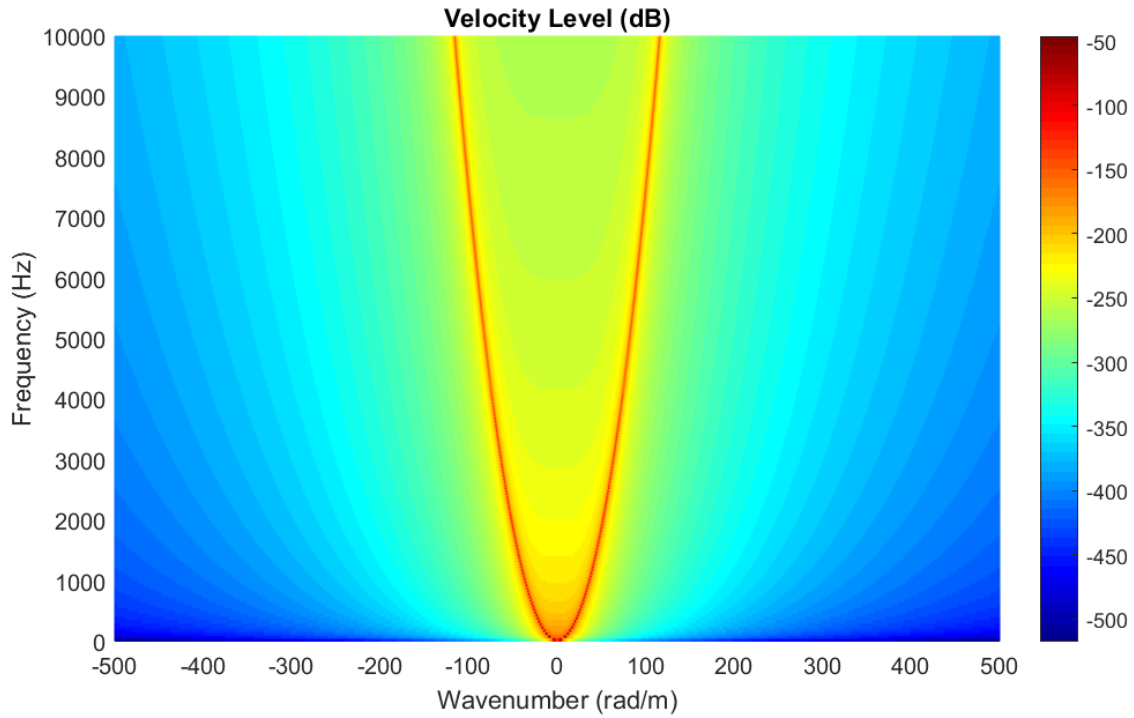

- Step 1: evaluate the wave number domain response of the panel

MODELING – KEY POINT 1

- **Choice of IDFT sampling rate γ_s and sampling points number N**

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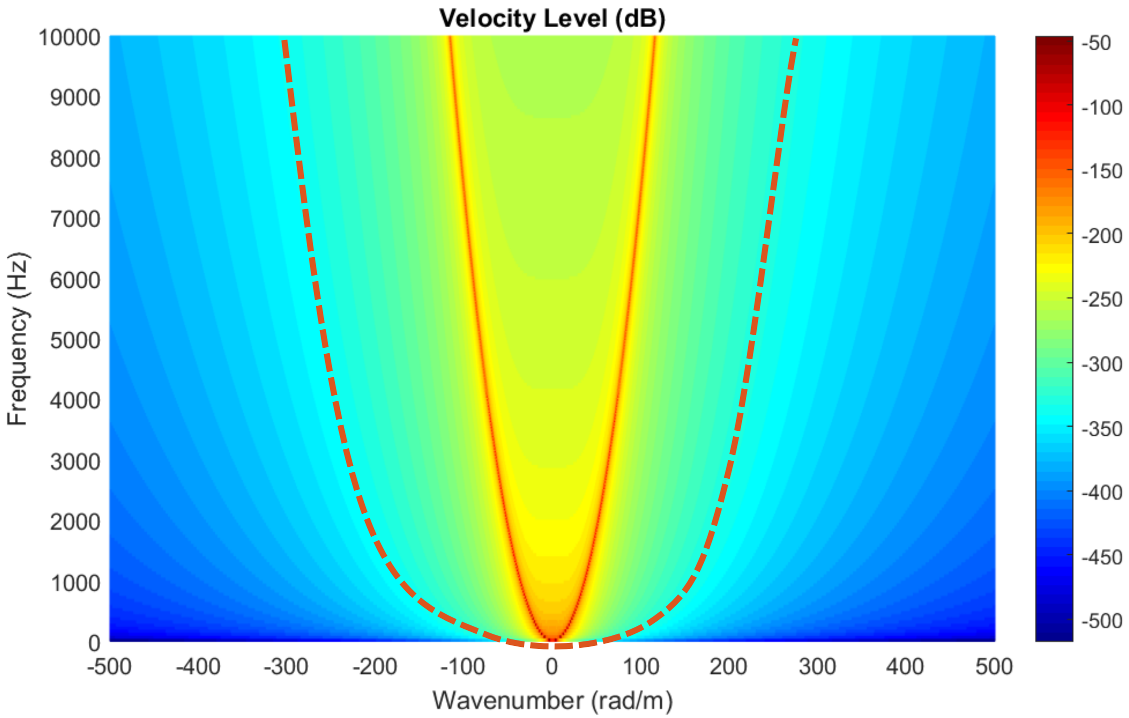
➢ **Key point**: for each frequency input, choosing proper γ_s and N to ensure **accurate IDFT results over a large enough spatial span** for observation



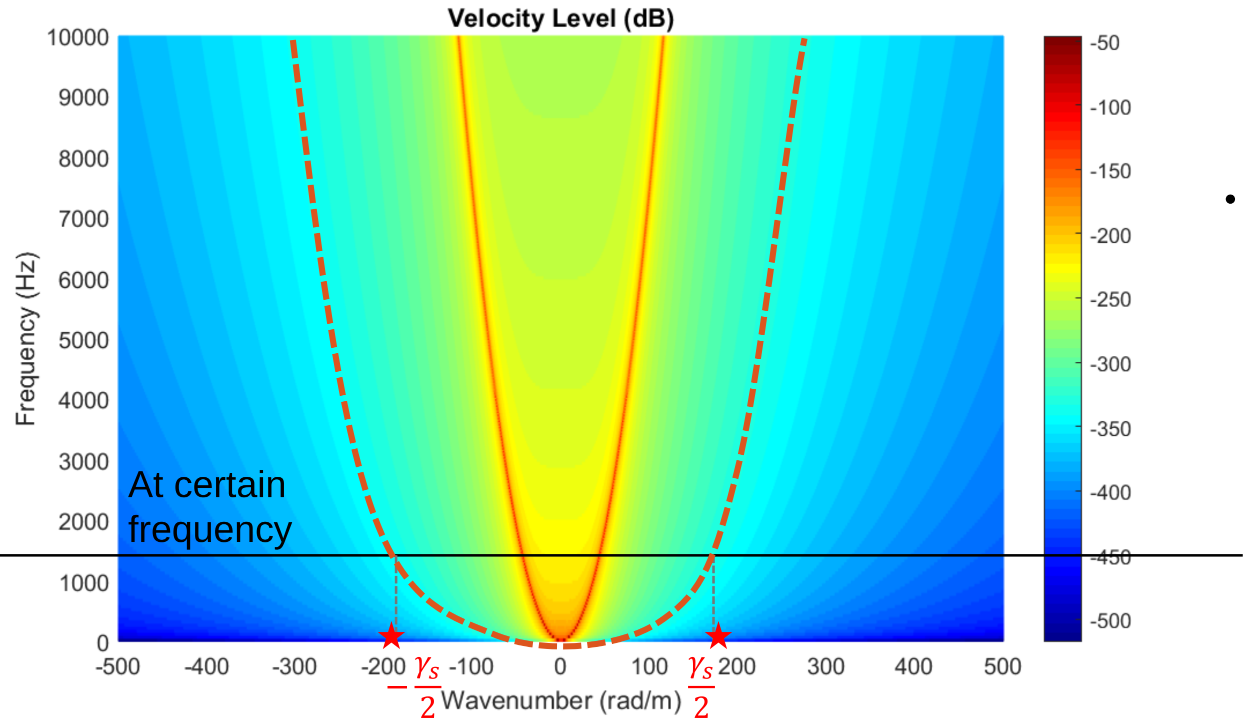
- Step 2: decide a proper cutoff level to avoid windowing/truncation effect

MODELING – KEY POINT 1

- **Choice of IDFT sampling rate γ_s and sampling points number N**

➢ **Target of the NFD model**: calculate spatial responses for wide frequency range

➢ **Key point**: for each frequency input, choosing proper γ_s and N to ensure **accurate IDFT results over a large enough spatial span** for observation



- N should be large enough to avoid bias

- Step 3: find the proper sampling rate γ_s for each input frequency

MODELING – KEY POINT 1

- **Choice of IDFT sampling rate γ_s and sampling points number N**

➢ **Target of the NFD model**: calculate spatial responses for wide frequency range

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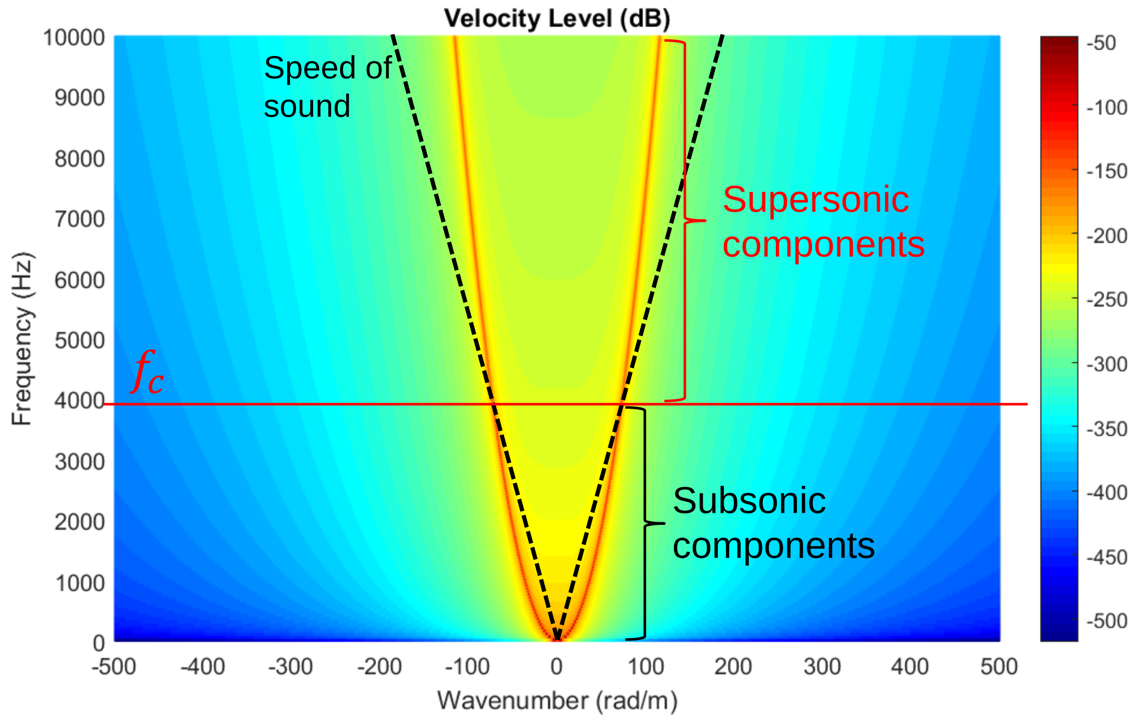
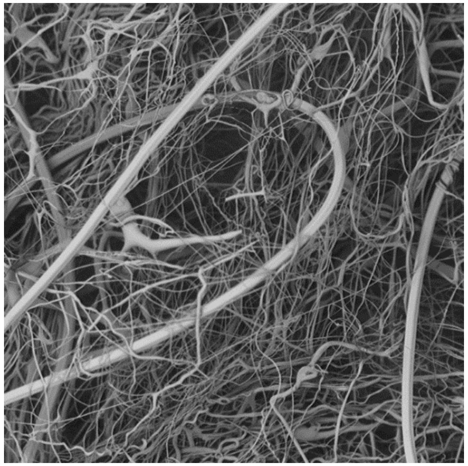


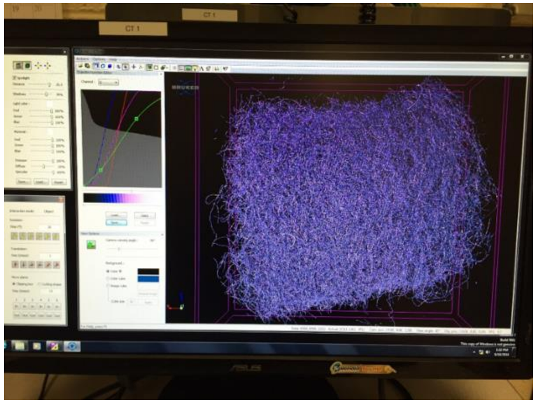
- Step 4: identify the critical frequency f_c

MODELING – KEY POINT 2

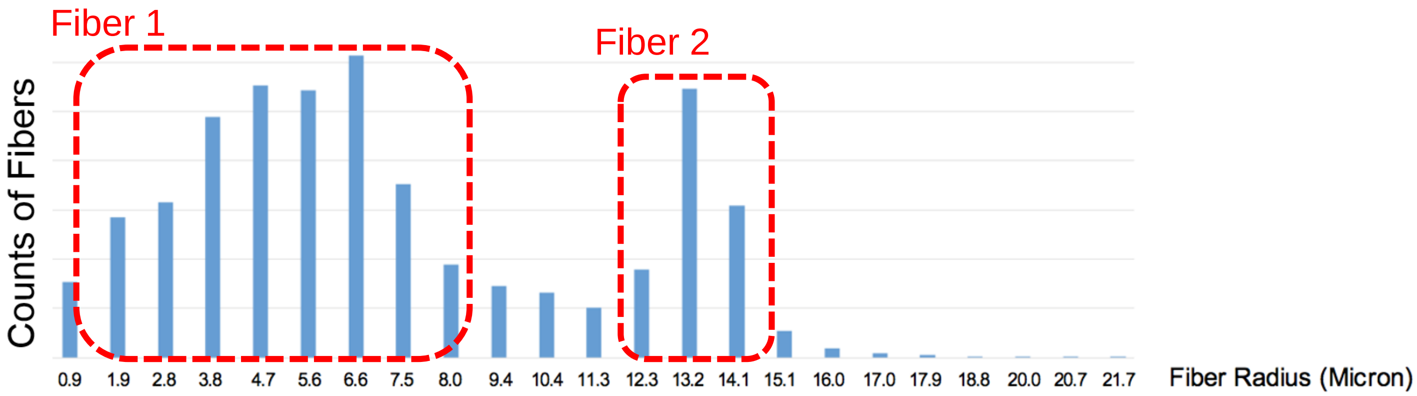
- **Fibrous Medium Airflow Resistivity Prediction[7]**



SEM of the target fibrous medium



Fibrous medium micro-CT scanning



Micro-CT scanned fiber radii distribution of the fibrous medium

MODELING – KEY POINT 2

PURDUE MECHANICAL ENGINEERING

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- **Fibrous Medium Airflow Resistivity Prediction[7]**

Inputs

Fiber mean radii: r_1, r_2, distribution parameters

Fiber bulk density: ρ_b

Component weight fractions: X_1, X_2

Solid material densities: ρ_1, ρ_2

1

Solidity:

$$C = X_1 \frac{\rho_b}{\rho_1} + X_2 \frac{\rho_b}{\rho_2}$$

2

Fiber mean spacing:

$$b^2 = \frac{\pi}{C} \frac{\left(\sum_{p=1}^{j} n_{1,p} r_{1,p}^2 + \sum_{q=1}^{k} n_{2,q} r_{2,q}^2\right)}{\left(\sum_{p=1}^{j} n_{1,p} + \sum_{q=1}^{k} n_{2,q}\right)}$$

3

Output

Airflow Resistivity: $\sigma = \dfrac{4\pi\eta}{b^2\left[0.640\ln\left(\frac{1}{C}\right) + C - 0.737\right]}$

- **Step 1**: C calculation based on $\rho_b, X_1, X_2, \rho_1, \rho_2$
- **Step 2**: b^2 calculation based on r_1, r_2, distribution parameters and C
- **Step 3**: σ calculation base on C and b^2

MODELING – KEY POINT 2

- **Fibrous Medium Airflow Resistivity Prediction[7]**

Inputs

Fibrous medium verified microstructure inputs

AFR →

Output

Airflow Resistivity Prediction

MODELING – KEY POINT 2

- **Fibrous Medium Airflow Resistivity Prediction[7]**

Inputs

Fibrous medium verified microstructure inputs

AFR →

Output

Airflow Resistivity Prediction

- **Fiber Microstructure Design for Optimal Damping Performance**

Inputs

Range of Airflow Resistivity

TMM & NFD →

Range of damping properties (power dissipation from the limp fibrous layer)

Select peak value →

Optimal damping and corresponding optimal airflow resistivity

↓ **AFR**

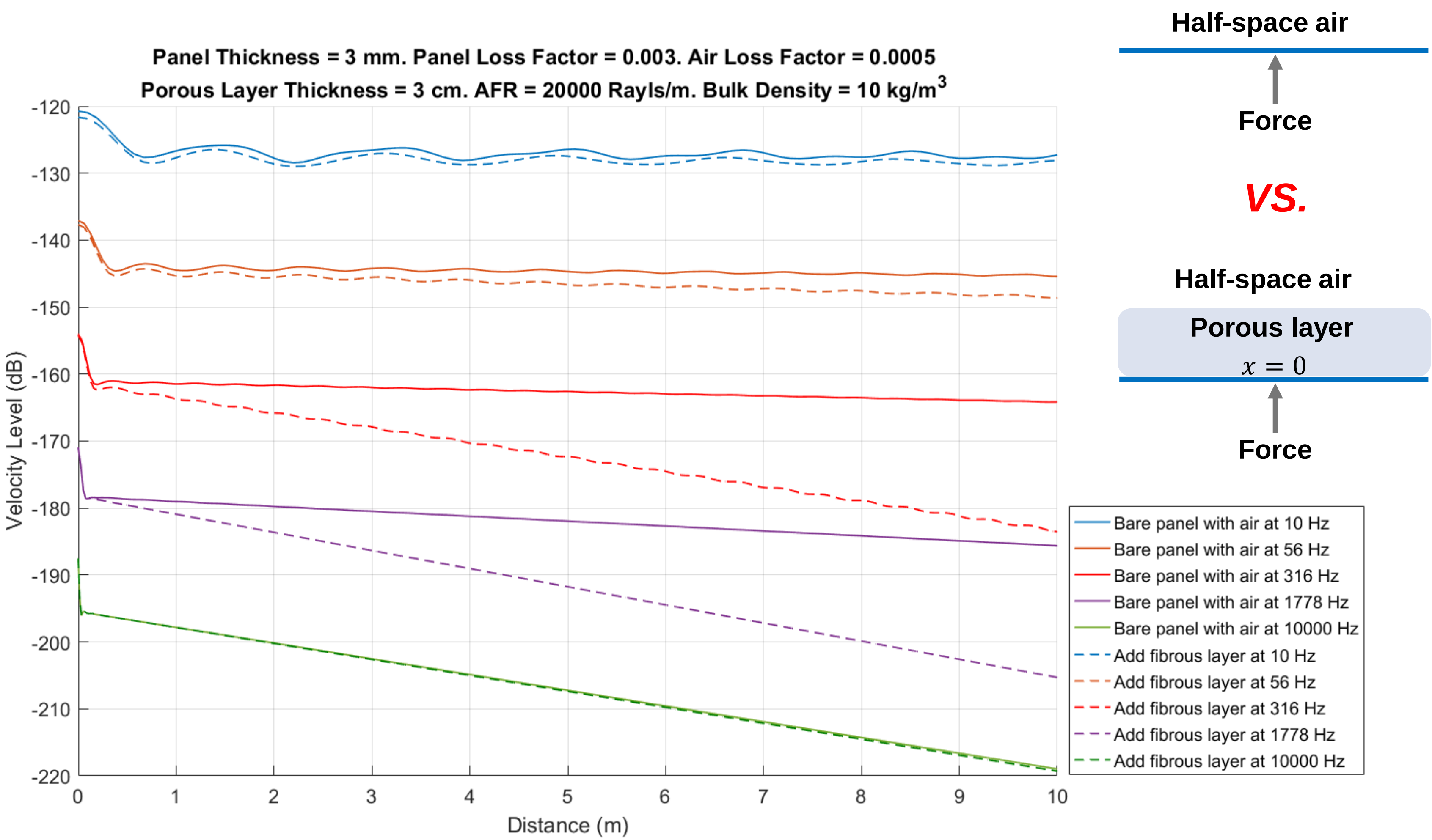
Numerical optimization by least square fitting

Output

Optimal fiber size

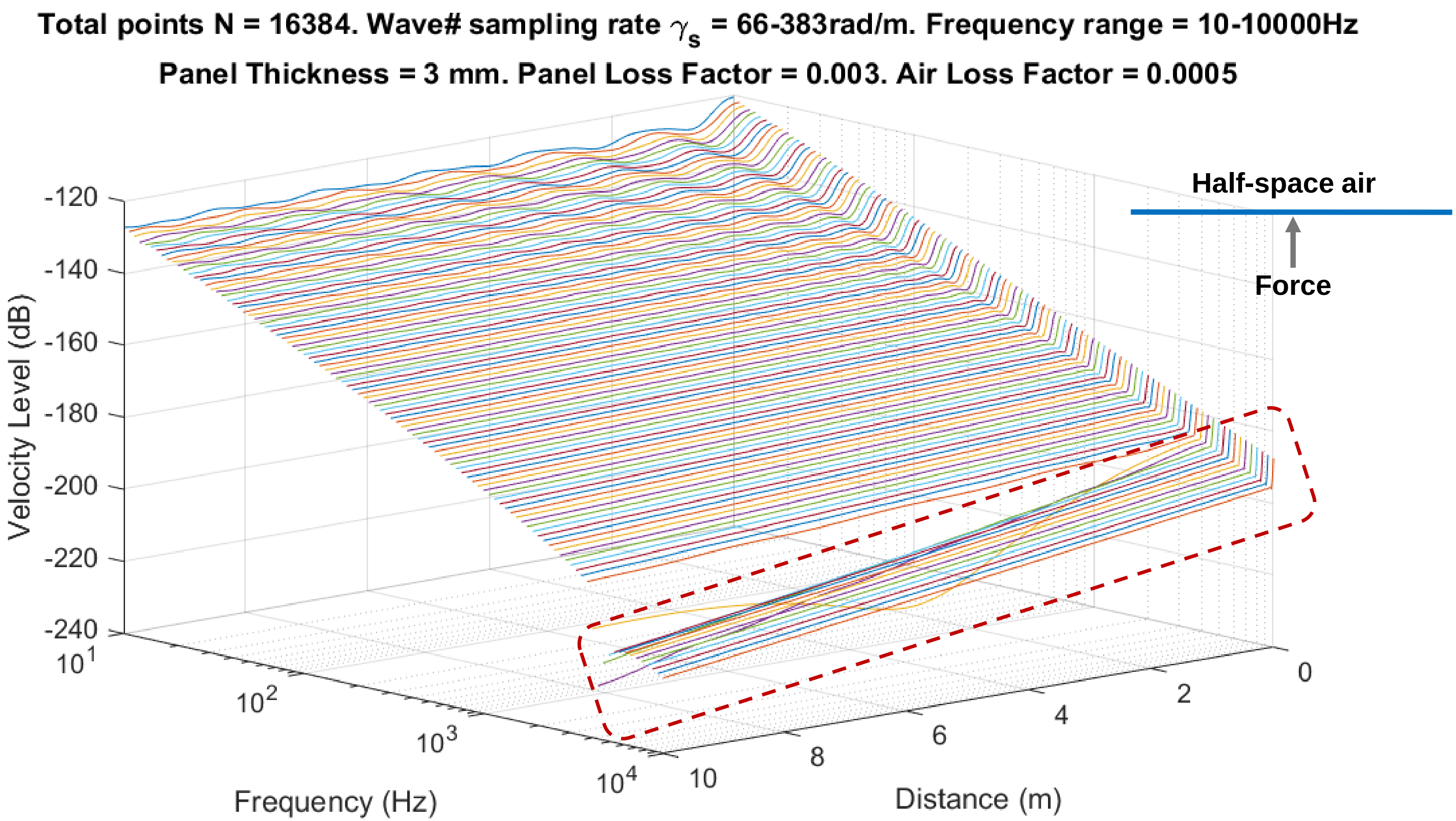
RESULTS – OBSERVATION 1

- **Spatial Velocity Level (dB)**



RESULTS – OBSERVATION 1

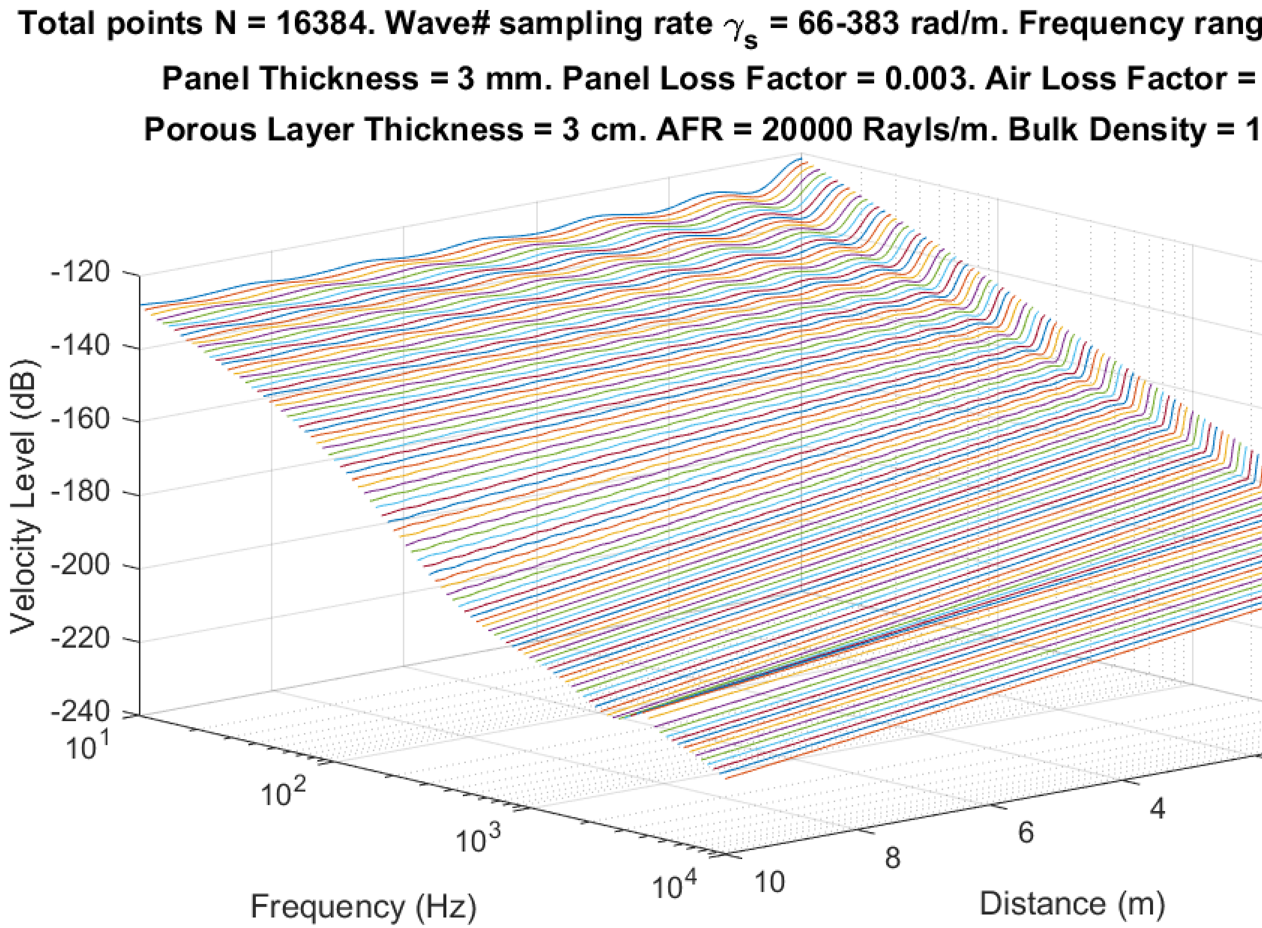
- **Spatial Velocity Level (dB)**

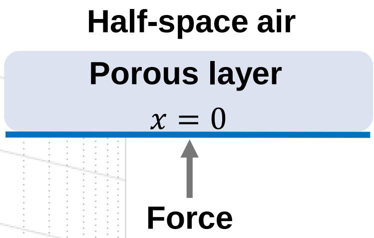


- **Spatial resonance in supersonic region above the critical frequency**

RESULTS – OBSERVATION 1

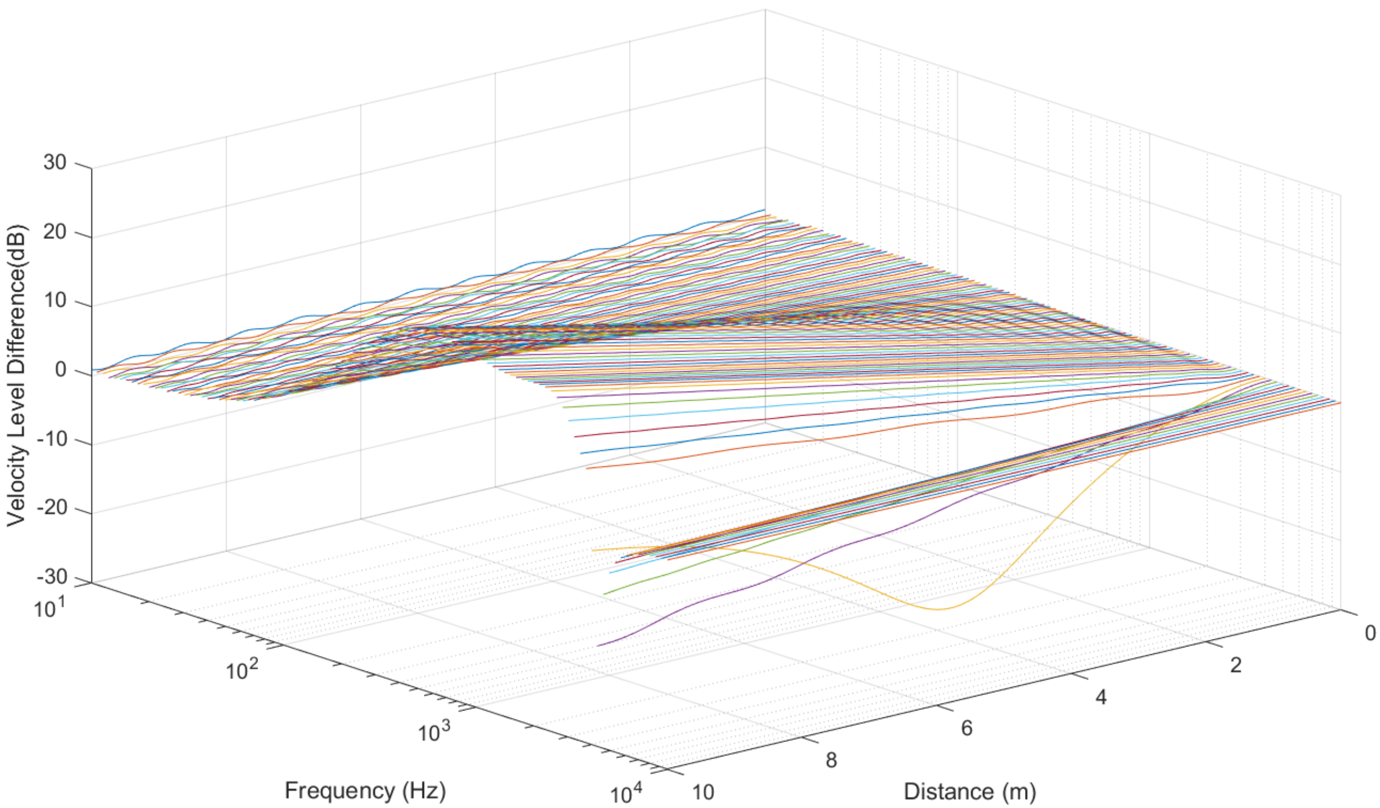
- **Spatial Velocity Level (dB)**





RESULTS – OBSERVATION 1

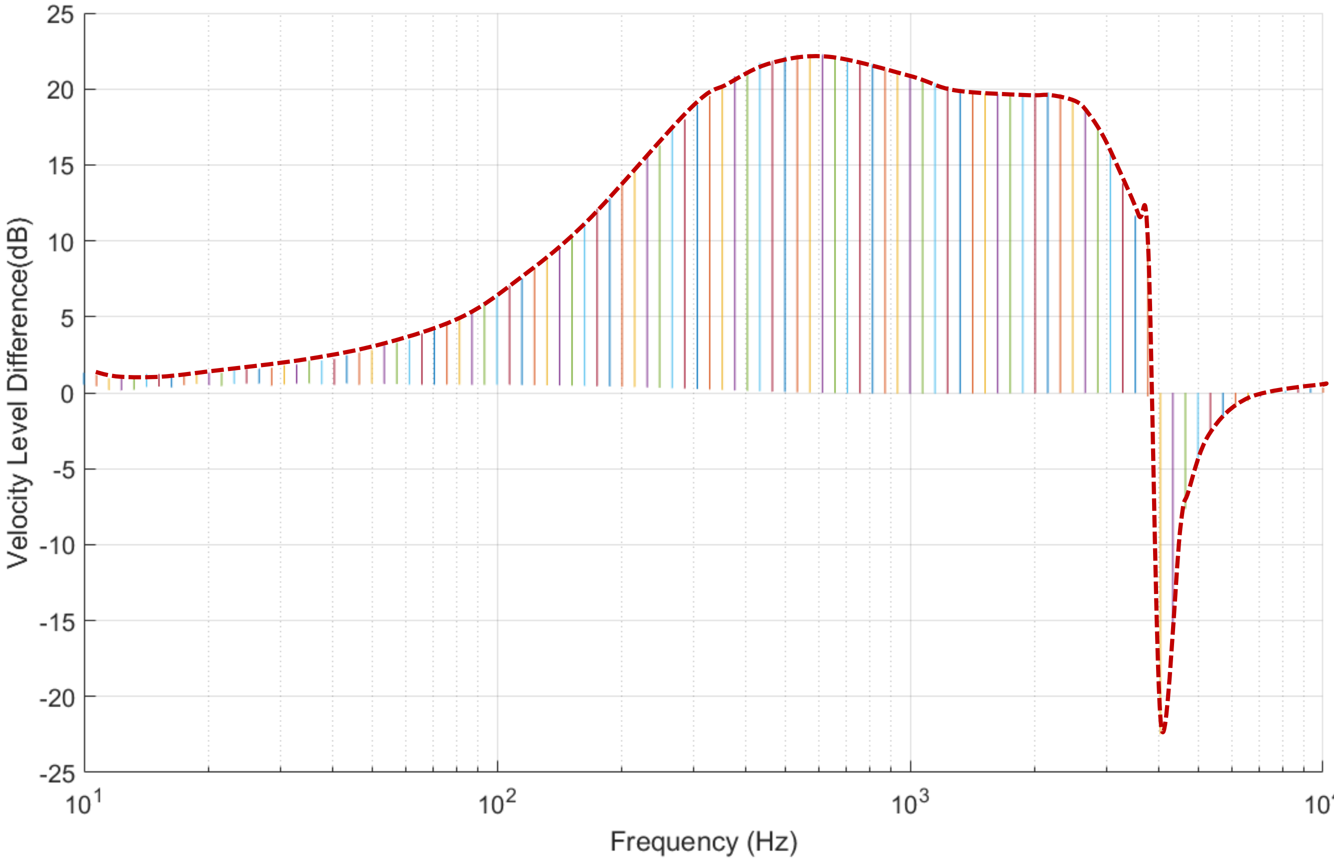
- **Spatial Velocity Level (dB)**
  - **Bare panel case** minus **Panel + fibers case**



- **Significant attenuation in subsonic region below critical frequency**

RESULTS – OBSERVATION 1

- **Spatial Velocity Level (dB)**
  - **Bare panel case** minus **Panel + fibers case**



- **Significant attenuation in subsonic region below critical frequency**

RESULTS – OBSERVATION 2

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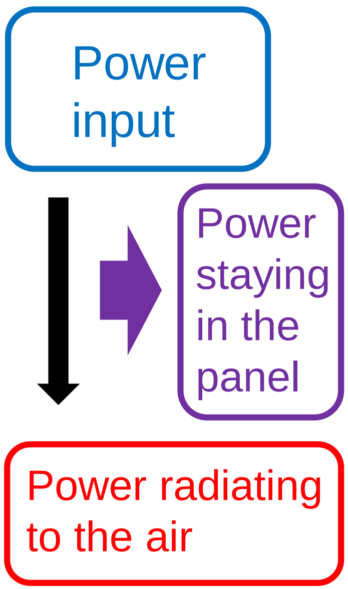
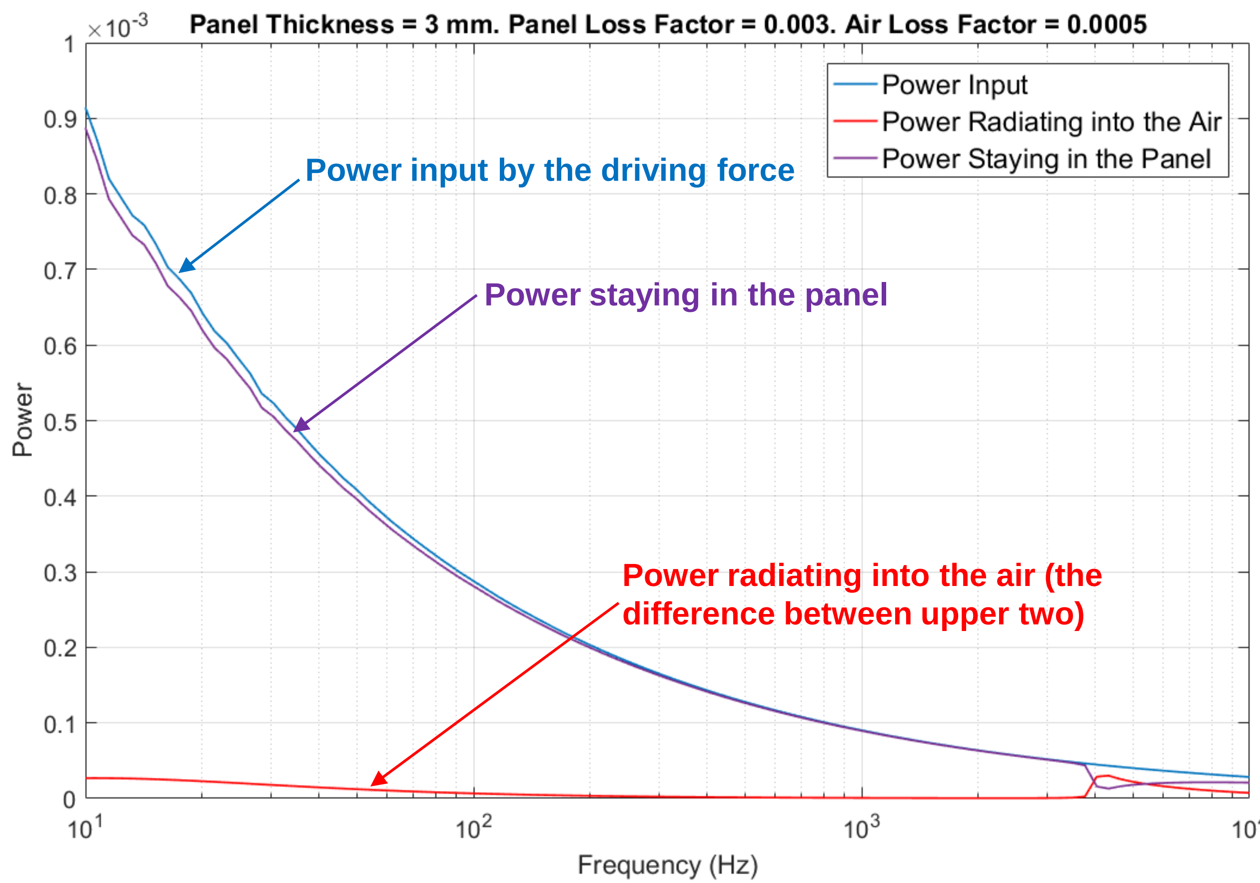
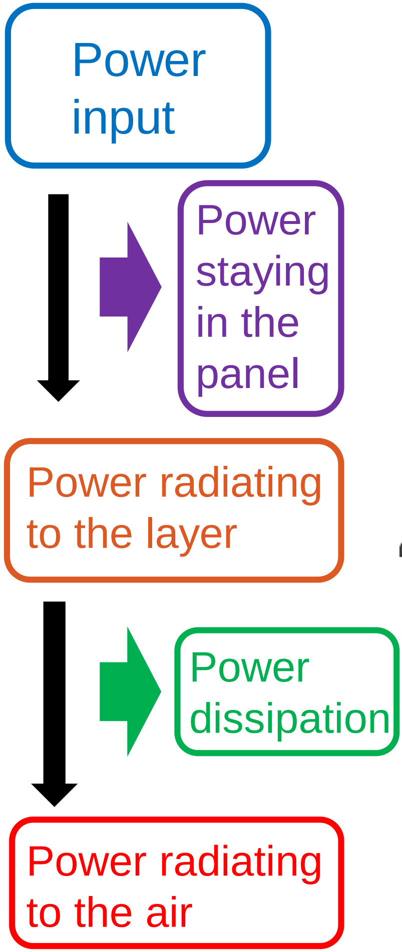
- **Power Distribution – panel + half-space air**

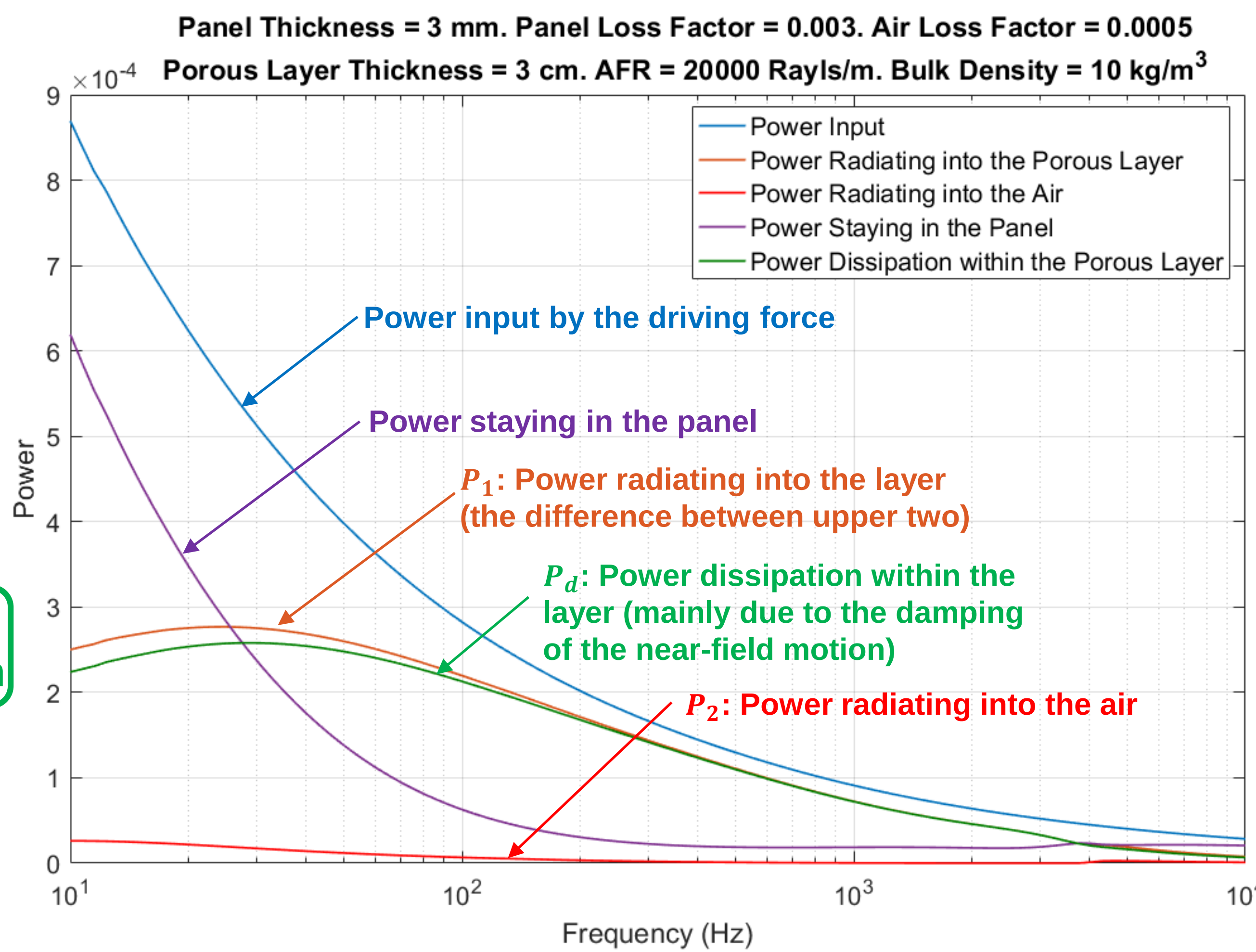




RESULTS – OBSERVATION 2

- **Power Distribution – adding limp porous layer**





- **Subsonic region attenuation due to power dissipation within the layer**

RESULTS – OBSERVATION 3

PURDUE MECHANICAL ENGINEERING

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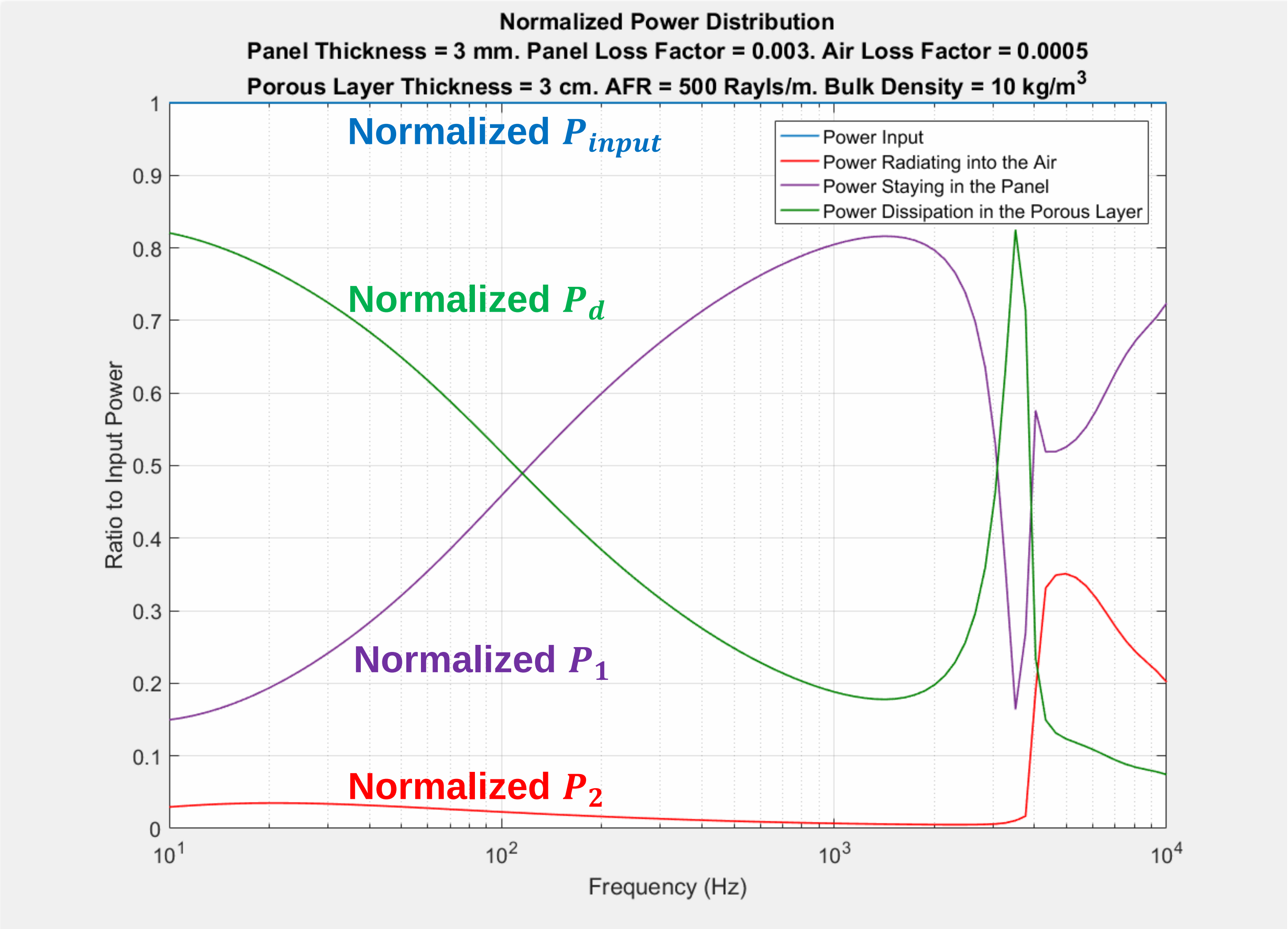
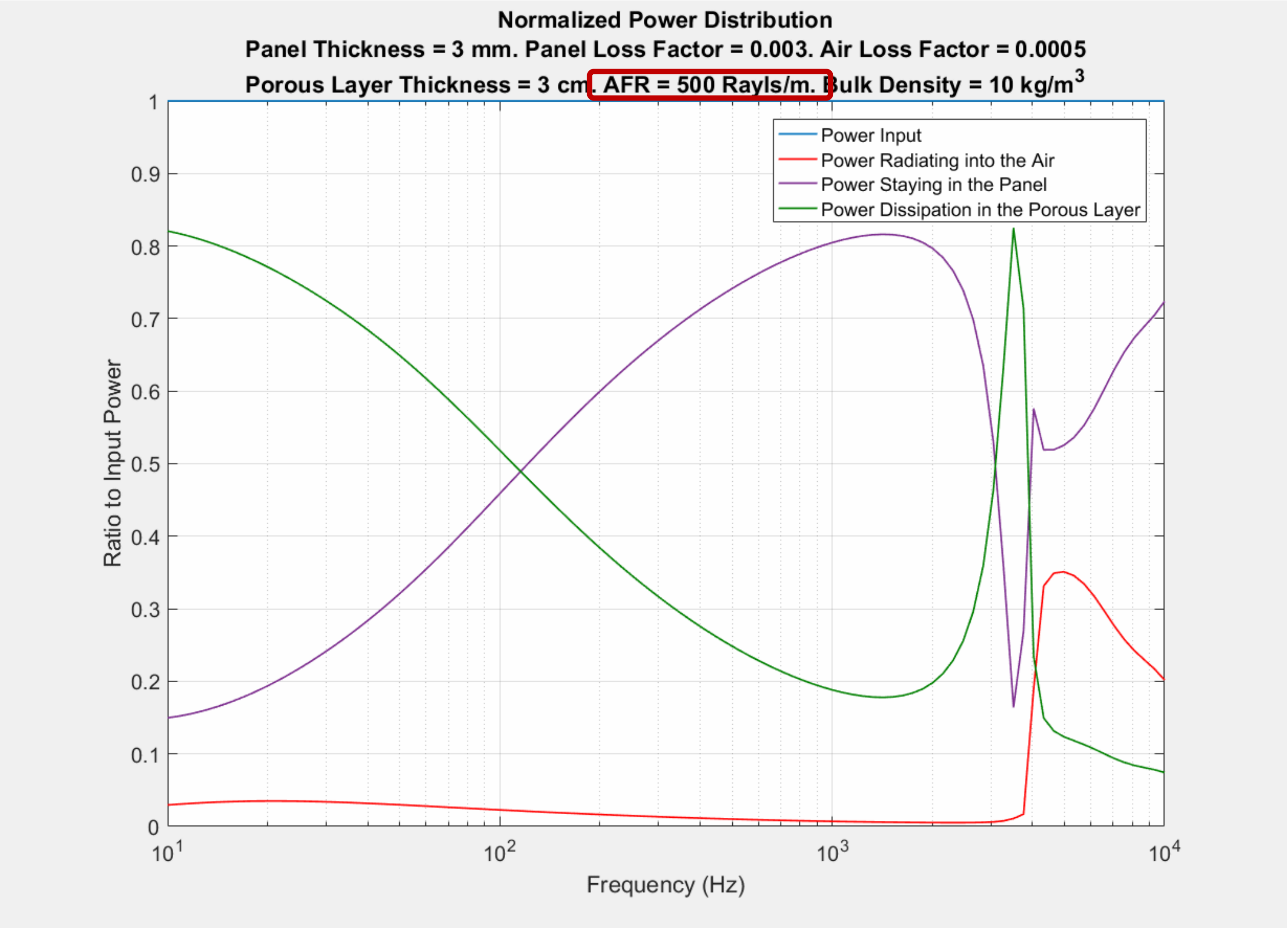
- **Limp Porous Layer Airflow Resistivity Effect on Power Dissipation**



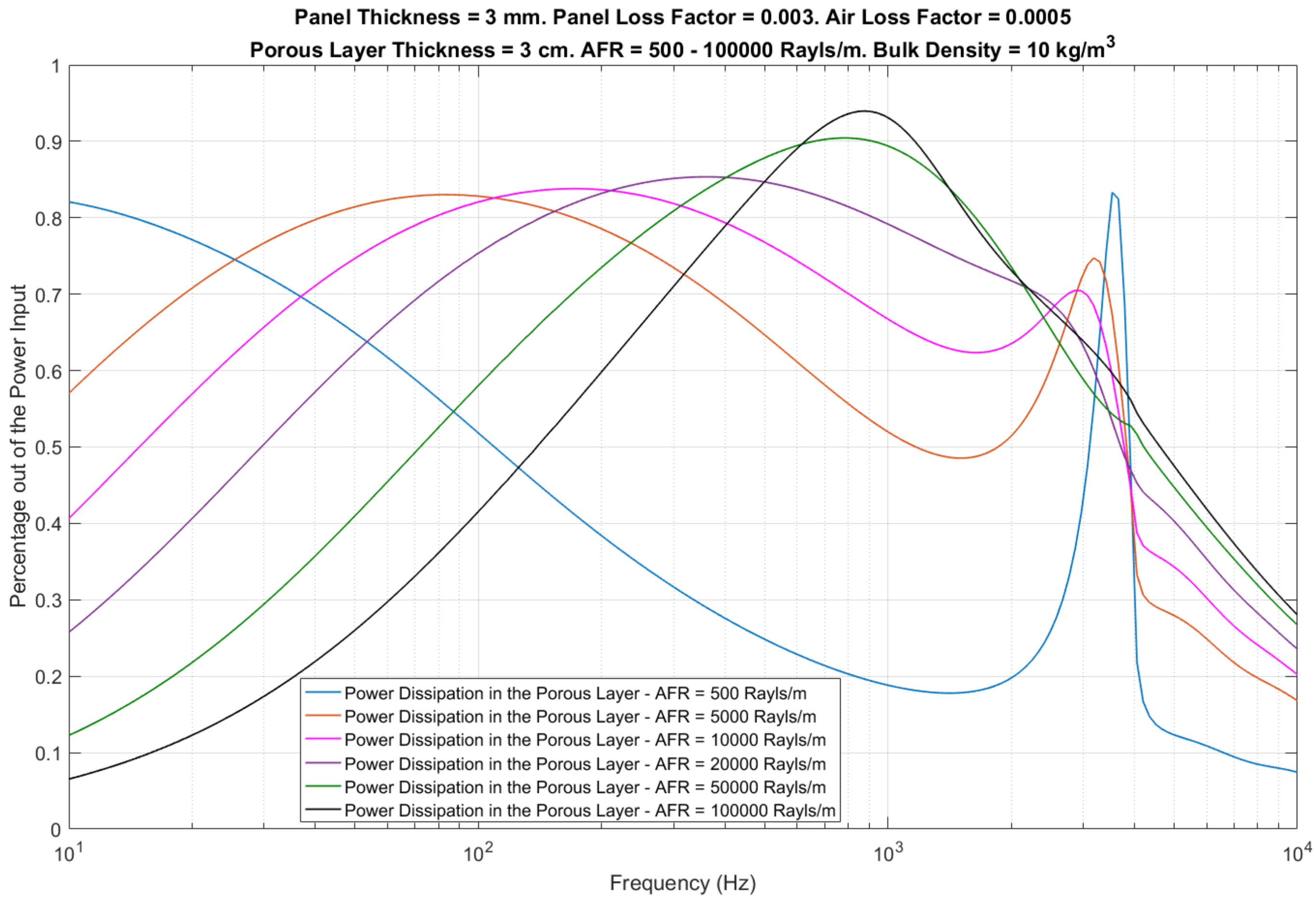
RESULTS – OBSERVATION 3

- **Limp Porous Layer Airflow Resistivity Effect on Power Dissipation**



RESULTS – OBSERVATION 3

- **Limp Porous Layer Airflow Resistivity Effect on Power Dissipation**



RESULTS – OBSERVATION 3

PURDUE MECHANICAL ENGINEERING

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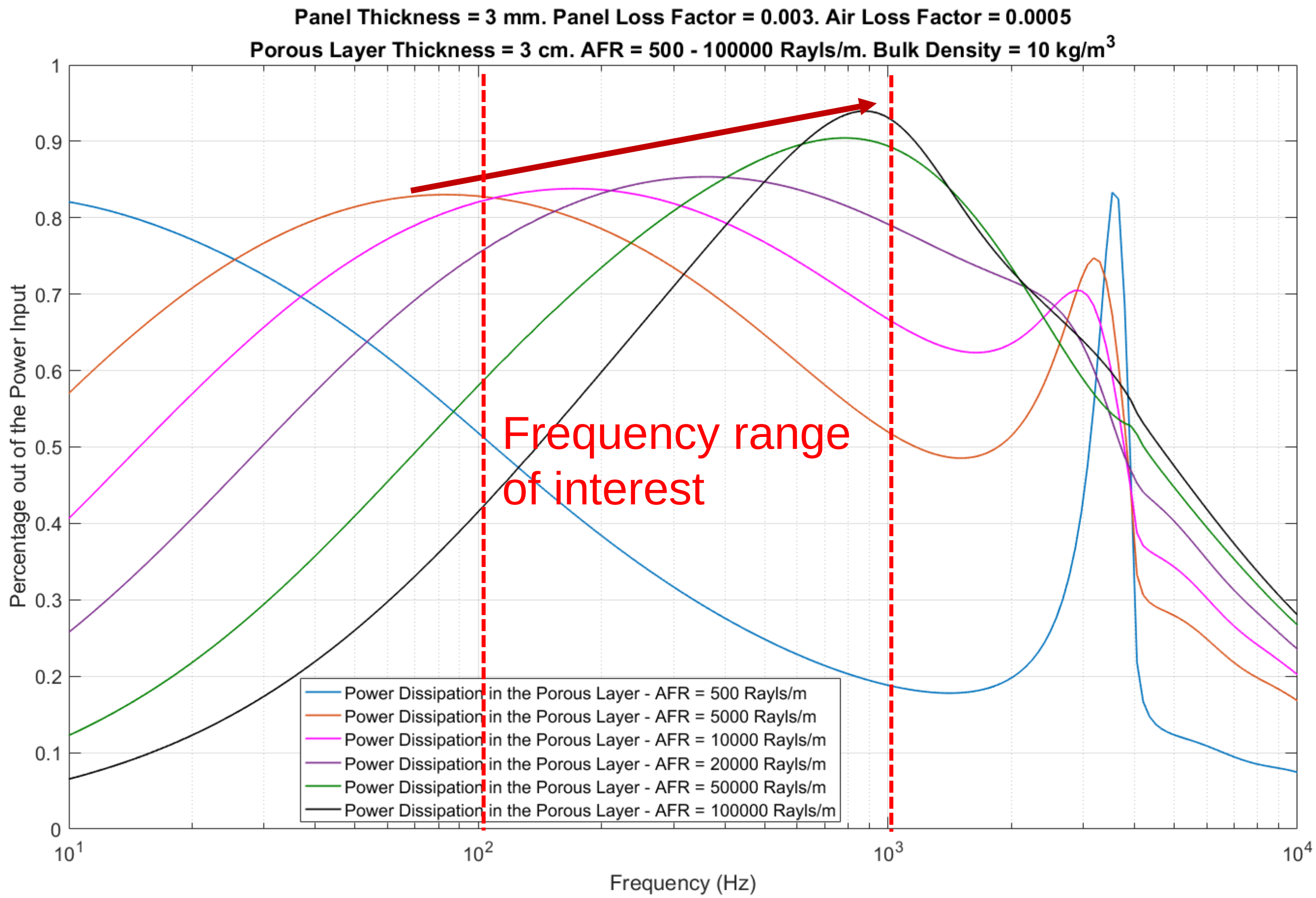
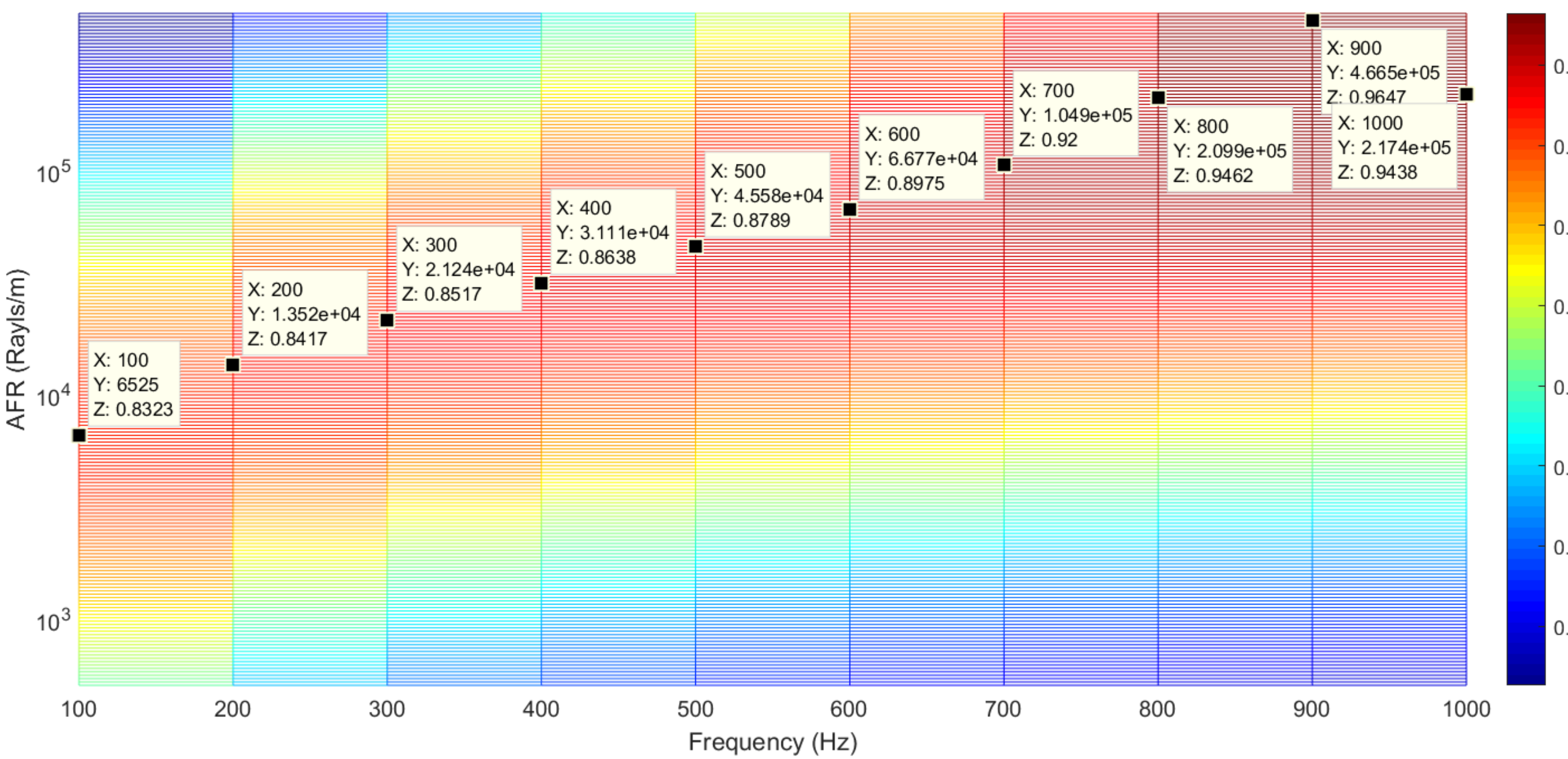
- **Limp Porous Layer Airflow Resistivity Effect on Power Dissipation**



- **Optimal damping corresponds to different optimal AFRs at different frequencies**

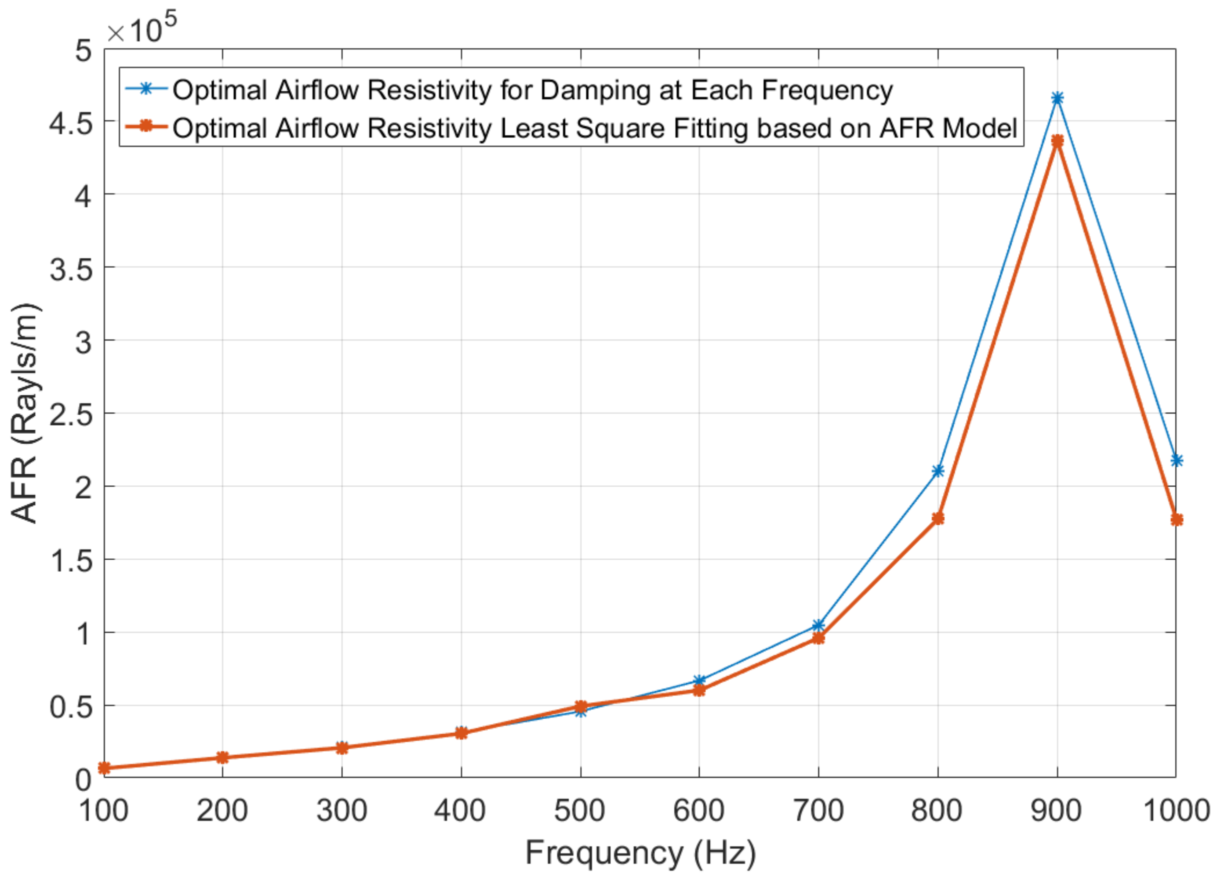
RESULTS – OBSERVATION 3

- **Finding Optimal Fiber Size for Optimal Damping – identifying optimal AFRs**
  - Aluminum panel thickness = 3 mm; Loss factor = 0.003; Air loss factor = 0.0005
  - Polymer fibrous layer thickness = 3 cm; Bulk density= 10 kg/m^3; Tortuosity = 1.2; Porosity = 99%



RESULTS – OBSERVATION 4

- **Finding Optimal Fiber Size for Optimal Damping – least square fitting AFRs**
  - Aluminum panel thickness = 3 mm; Loss factor = 0.003; Air loss factor = 0.0005
  - Polymer fibrous layer thickness = 3 cm; Bulk density= 10 kg/m^3; Tortuosity = 1.2; Porosity = 99%
  - Fiber inputs: $\rho_1 = 910\ \mathrm{kg/m^3}$; $\rho_2 = 1380\ \mathrm{kg/m^3}$; $X_1 = X_2 = 50\%$; $r_2 = 13\ \mu\mathrm{m}$; r_1 → design target



RESULTS – OBSERVATION 4

PURDUE MECHANICAL ENGINEERING
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- **Finding Optimal Fiber Size for Optimal Damping – translating into optimal fiber sizes**
  - Aluminum panel thickness = 3 mm; Loss factor = 0.003; Air loss factor = 0.0005
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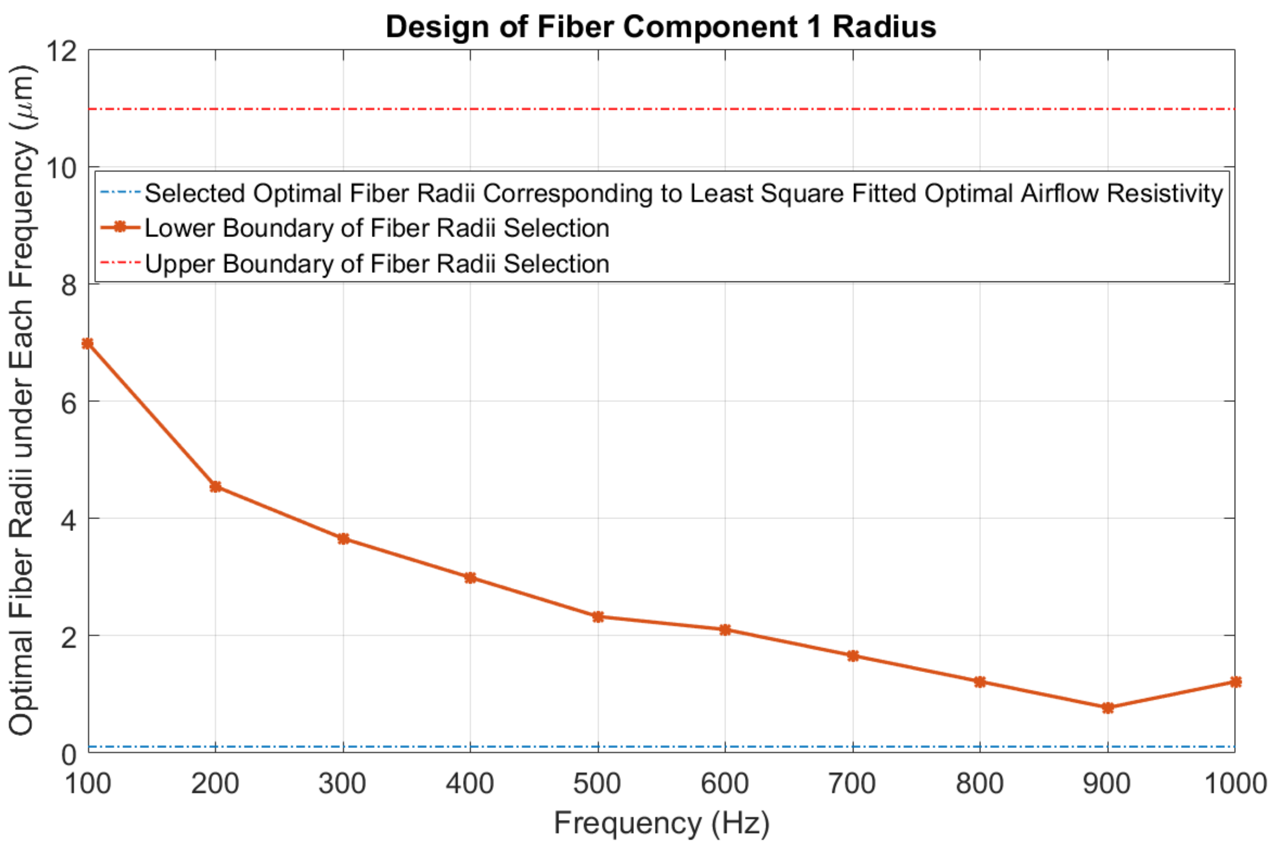

RESULTS – OBSERVATION 4

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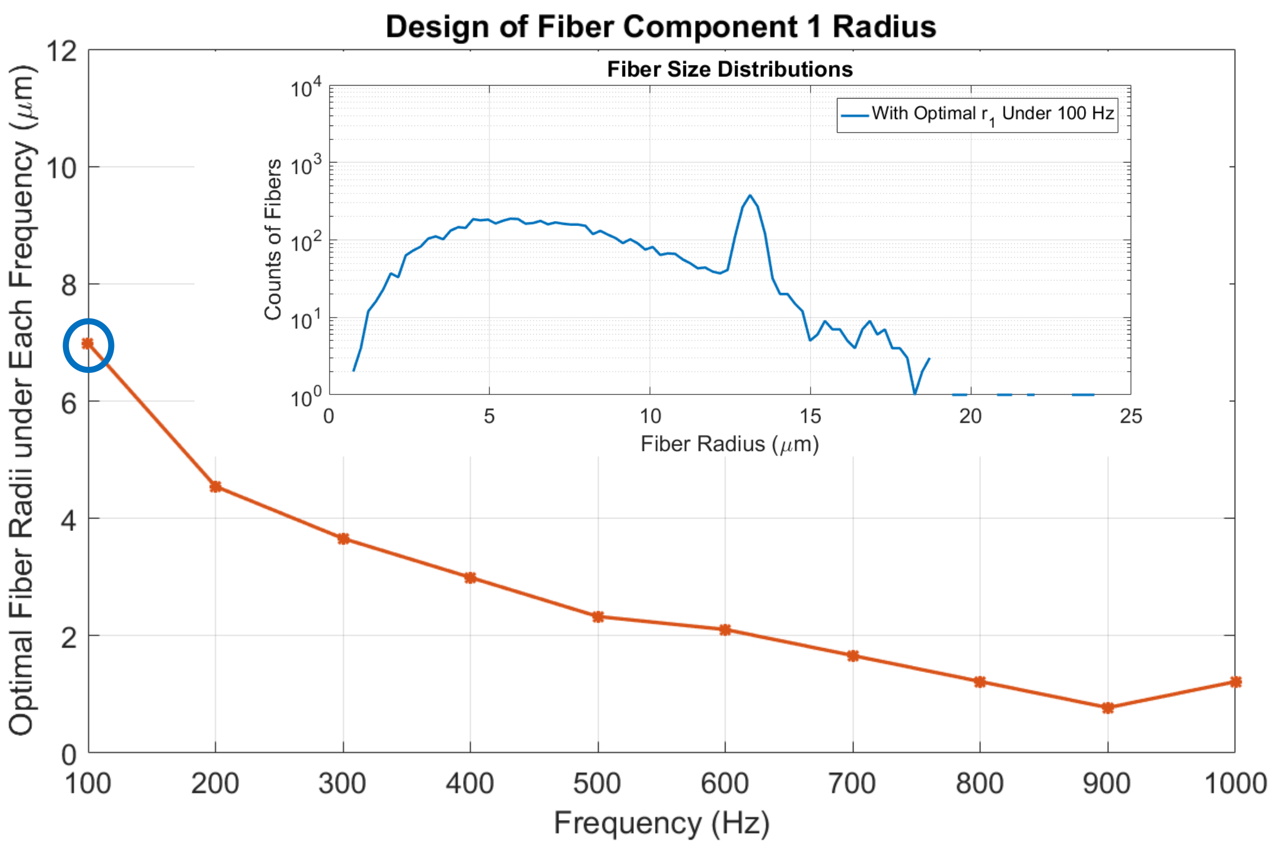

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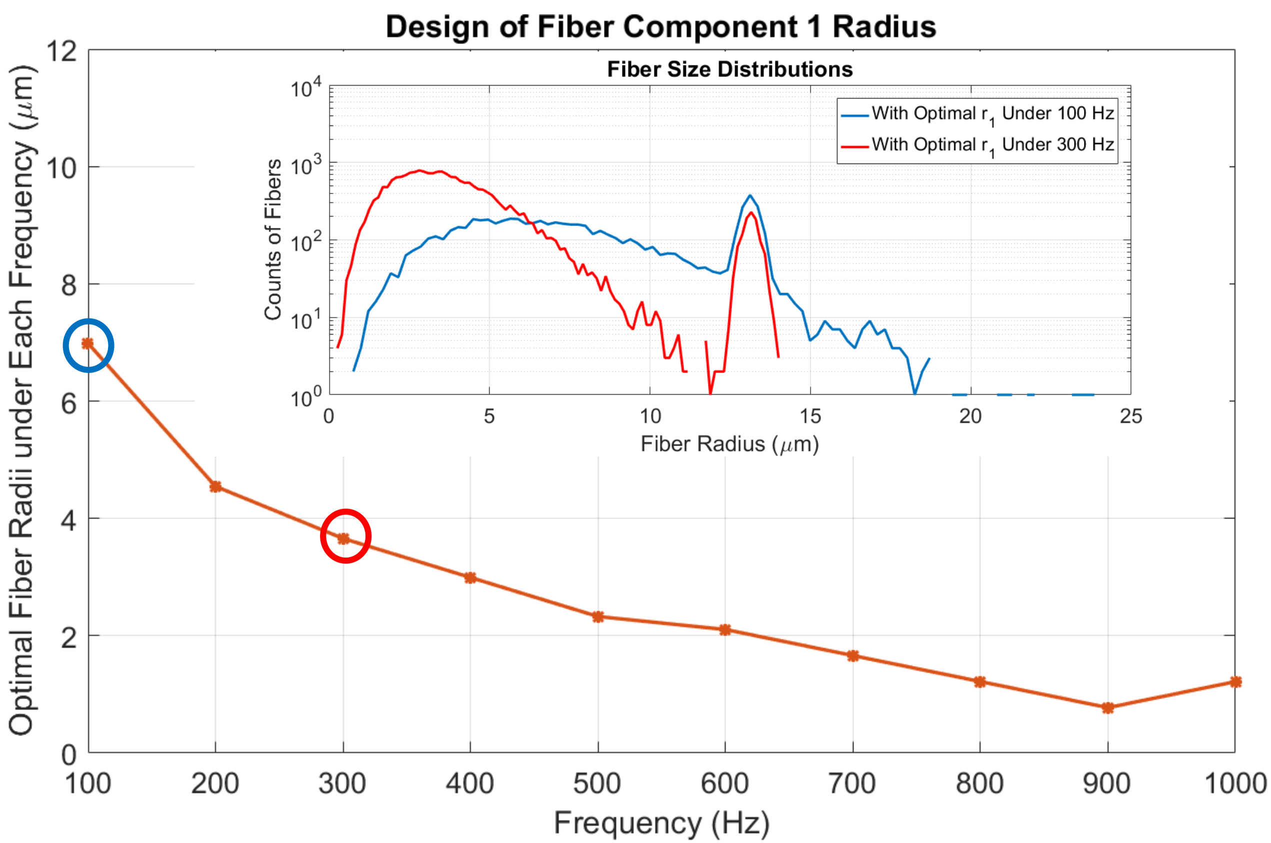

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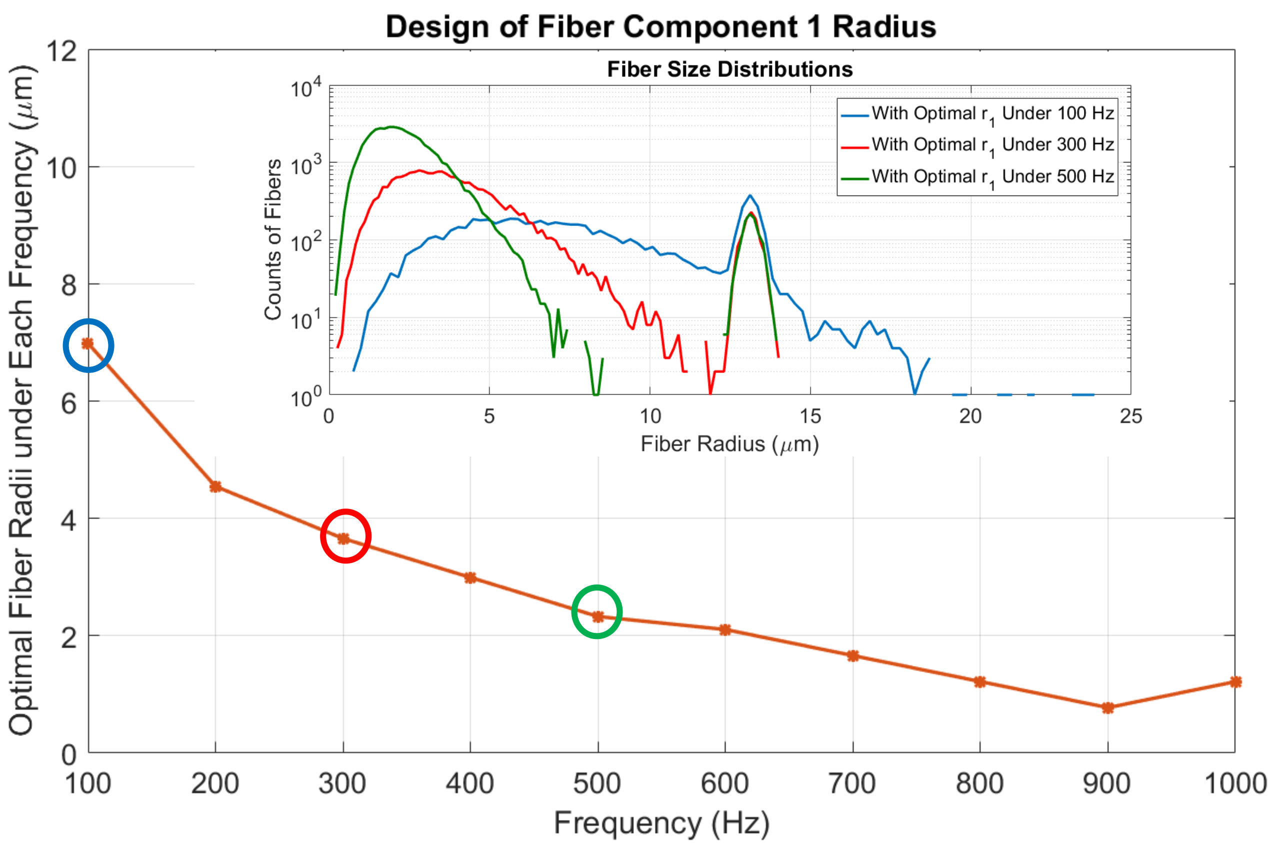

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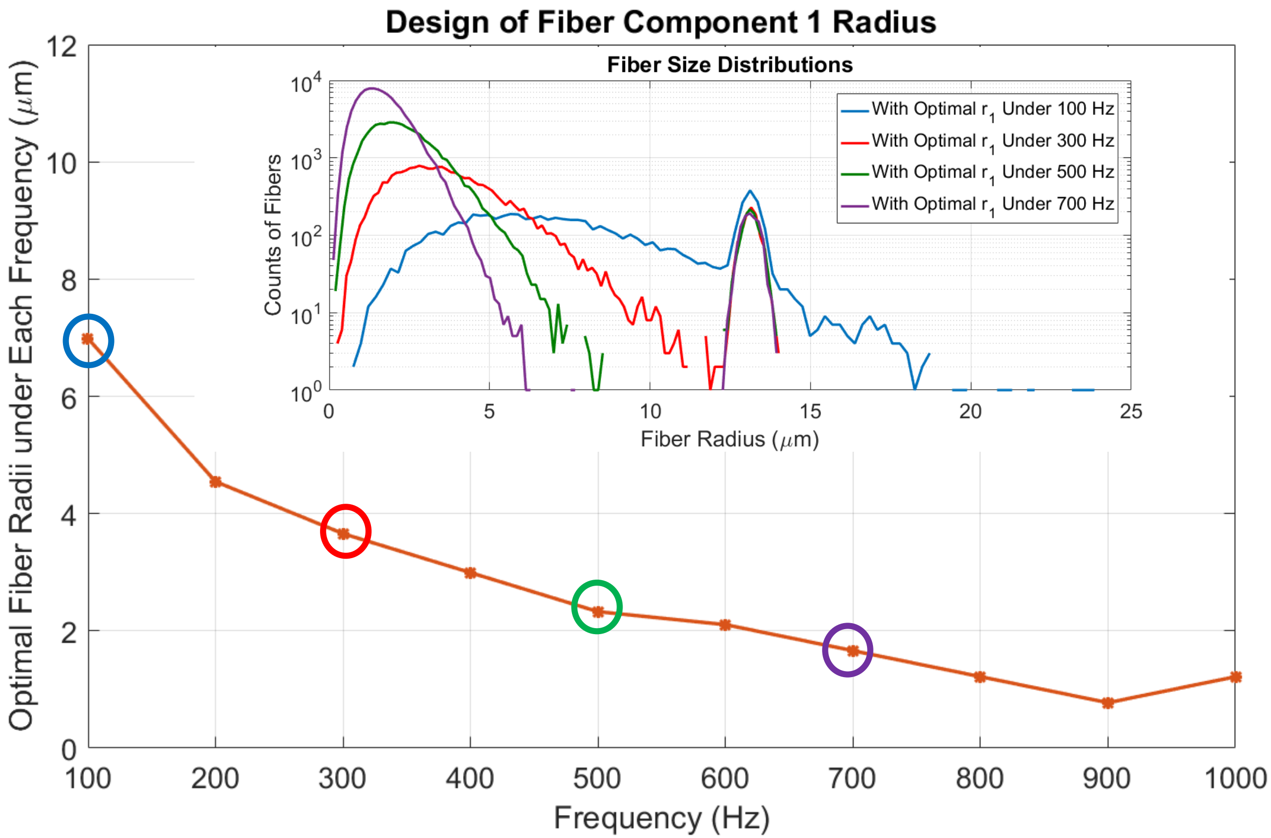

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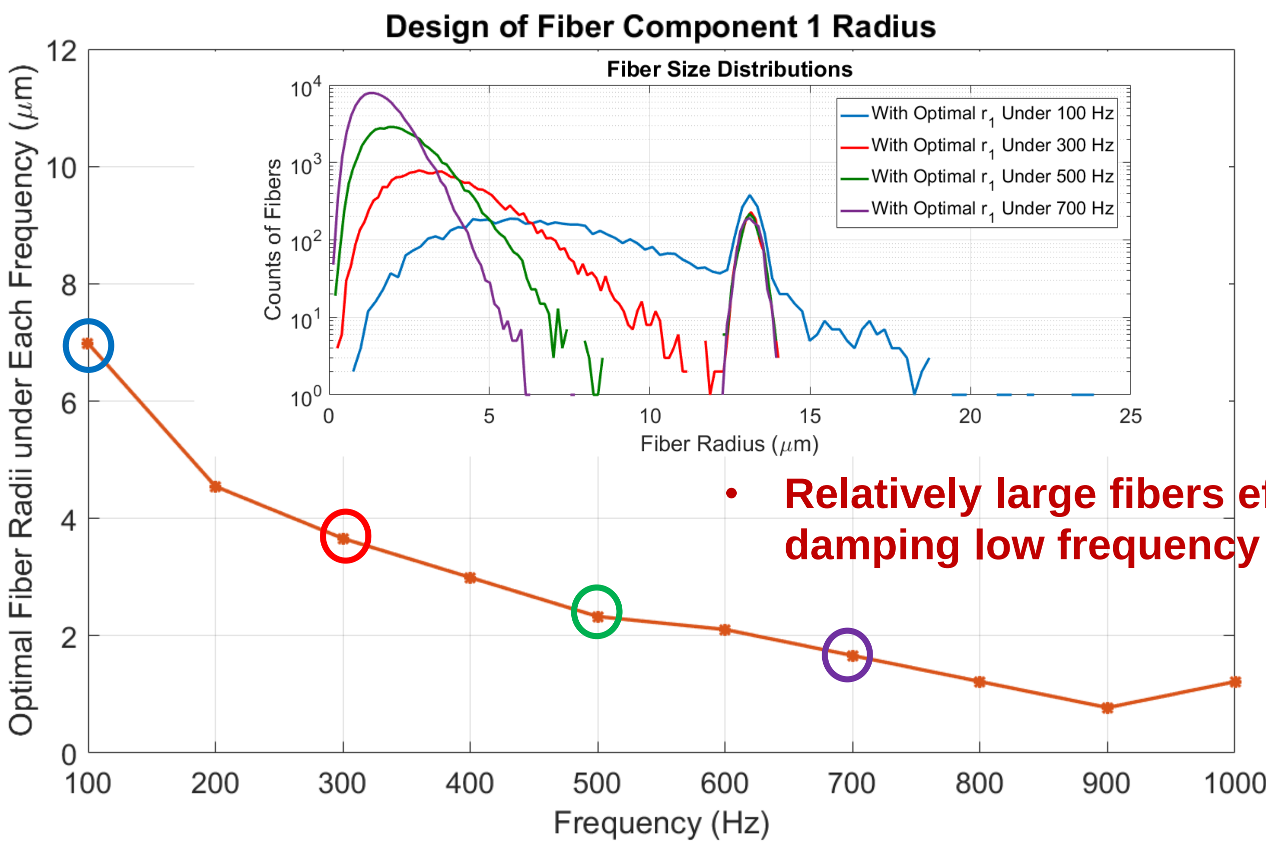

- **Relatively large fibers effective at damping low frequency vibration**

CONCLUSIONS

PURDUE MECHANICAL ENGINEERING

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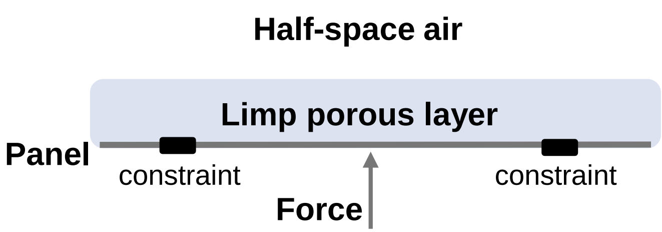
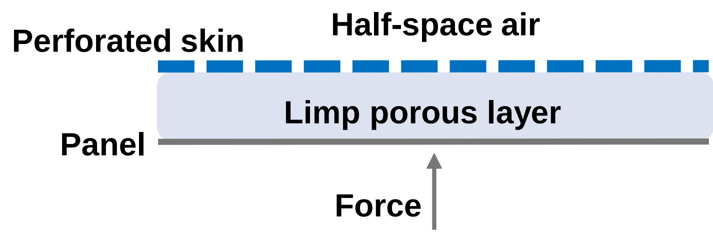
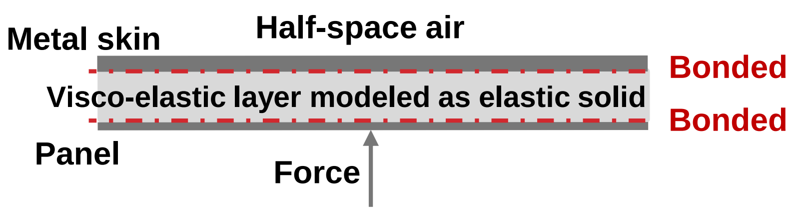
- **Transfer Matrix Method (TMM) and Near-field Damping (NFD) model based on Inverse Discrete Fourier Transform (IDFT) provide a powerful tool connecting fibrous materials' airflow resistivity with their damping performance**
- **Modified Airflow Resistivity (AFR) model connects fibrous materials' airflow resistivity with their microstructure (*i.e.*, various fiber sizes)**
- **For a limp porous layer attached to a stiff panel, an optimal airflow resistivity can be found to provide optimal damping performance (subsonic region power dissipation within the fibrous layer) under each frequency based on TMM and NFD**
- **Corresponding to the optimal airflow resistivity, an optimal fiber size then can be found to provide optimal damping performance under each frequency based on AFR and numerical optimization method**
- **Relatively large fibers are effective at damping low frequency vibration**

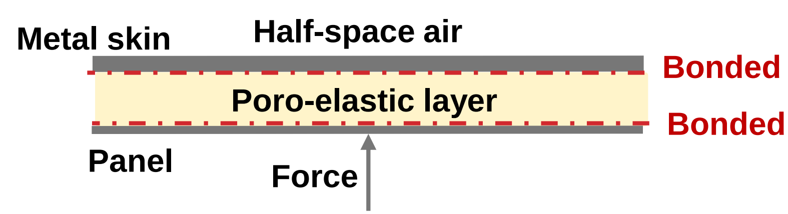
EXTENSION

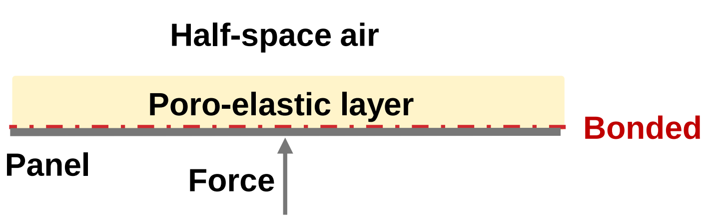
- Other cases that have been built by the "TMM + NFD + AFR" model

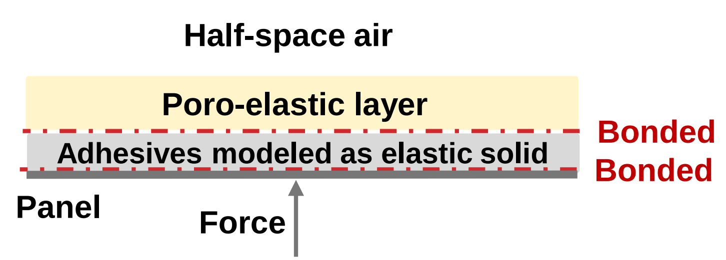












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