SURFACE TENSION OF NUCLEAR MATTER*

S. KNAAK, G. SÜSSMANN, E. HILF and H. BÜTTNER
Institut für Theoretische Physik der Universität, Frankfurt am Main, Germany

Received 1 November 1966

The surface tension σ and the surface density thickness t of nuclear matter have been calculated in the Fermi-gas model, the nucleons moving in a self-made shell model potential with a realistic slope and velocity dependence (parameters α and β). One gets the experimental values for σ and t with α and β agreeing with earlier data.

The semiempirical mass formula of Weizsäcker and Bethe [1] contains a term proportional to $A^{\frac{2}{3}}$. Several authors [3-9] have attempted to explain this term by a surface tension of nuclear matter. As these authors, we start with the Fermi-gas model but take a realistic, energy-dependent shell model potential. We define the surface tension σ by

$$\sigma: = \frac{\partial}{\partial S} E(A, k_f, S), \qquad (1)$$

where the total energy E of the nucleus is a function of the mass number A, the Fermi momentum k_f, and the surface area S. This definition is practically in accordance with the definition given by Swiatecki and other authors [3,6,7,9] but differs from that one used by Hill and Wheeler [4], Guruits et al. [8] and Lanzl [5] which we believe to be inappropriate to nuclear physics. (Furthermore, the method of Lanzl is entirely wrong so that with definition (1) he would get $\sigma = 0$.) The total energy E has to be calculated summing up the A lowest single particle energies ϵ_i and the corresponding expectation values $\bar{u}_i$ of the potential energy according to the formula

$$E = \sum_i \epsilon_i - \tfrac{1}{2} \sum_i \bar{u}_i . \qquad (2)$$

The second term on the right side has to be added because the nucleons are thought to move in a one particle potential U, generated by their own two particle forces. This additional term has been ignored by Guruits et al. [8], whereas this question did not arise in the other above mentioned papers, the $\bar{u}_i$ there being either zero [4-7] or not used [3,9]. The potential is assumed to depend on the x-coordinate only, i.e. curvature effects are ignored. Furthermore, we assume that the potential depends linearly on the kinetic energy. We write with reference to Perey and Buck [10]

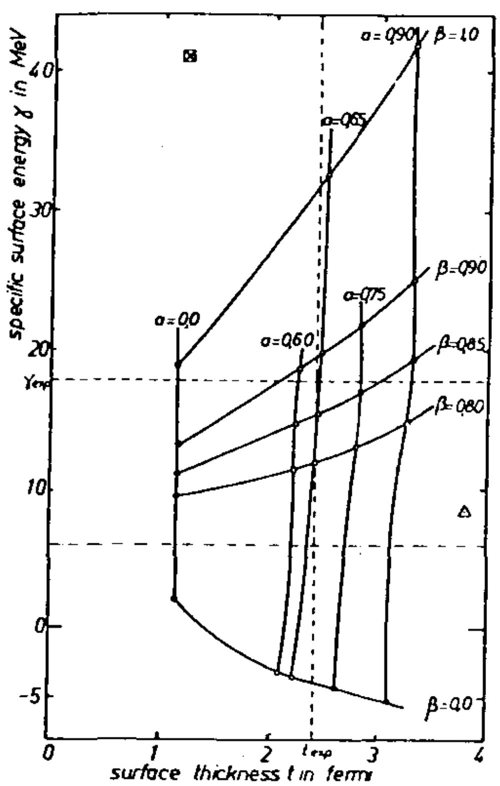

Fig. 1. Specific surface energy γ and thickness t as functions of the potential parameters α and β. The experimental values [12] are $\gamma_{exp} = 17.8$ MeV and $t_{exp} = 2.4$ f. Other theoretical results are marked by Δ(Swiatecki [3]) and ⊠ (McKellar and Naqvi [9]). The result of Guruits et al. [8] is $\gamma = 6$ MeV (marked by a dotted line).

* This work was supported in part by the Deutsche Forschungsgemeinschaft.

$$U(x) = U_0(x)\,[1 - \tfrac{1}{4}\beta^2\,(2m/\hbar^2)\,T(x)\,], \tag{3}$$

where $T(x)$ stands for the local kinetic energy $\epsilon - U(x)$ of a nucleon for the orbital in question. This is an effective mass approximation with

$$m/m_*(x) = 1 - \tfrac{1}{4}\beta^2\,(2m/\hbar^2)\,U_0(x). \tag{4}$$

For $U_0(x)$ we use a Woods-Saxon potential

$$U_0(x) = -\tfrac{1}{2}\,V_0\,[1 - \tanh\,(x/2a)]. \tag{5}$$

This x-dependence has already been taken by Guruits et al. [8] and is more realistic than the forms considered by the other authors [3-7,9].

Instead of determining the individual eigenvalues ϵ_i and expectation values u_i we approximate the sums in (2) by integrals in momentum space using the expression

$$\frac{\partial}{\partial k_x}\,n_x\,(k_x, k_\rho) = \frac{2L}{\pi} + \frac{2}{\pi}\frac{\partial}{\partial k_x}\;\theta(k_x, k_\rho)$$

for the density of one particle levels derived by one of us [11], c.f. (7). The result is that the total energy E can be written as a sum of two terms, one proportional to the volume Ω of the nuclear matter the other proportional to its surface area S, and with an additional term proportional to A, so $E = \epsilon_0(\Omega + a_\mathrm{E}\,S) - \tfrac{1}{2}\,V_0 A$. In the same way the mass number A can be shown to be of the form $A = \rho_0(\Omega + a_\mathrm{N}\,S)$. After elimination of the volume Ω the definition (1) yields the expression

$$\sigma = \epsilon_0(a_\mathrm{E} - a_\mathrm{N}). \tag{6}$$

The values of ϵ_0, a_E and a_N are given by the following formulae. Putting

$$W(x,k_x\,,k_\rho):\, = [1 - \tfrac{1}{4}\,\beta^2\,(k_x^2 + k_\rho^2)]\,\frac{U_0(x) + V_0}{1 - \tfrac{1}{4}\beta^2\,(2m/\hbar^2)\,U_0(x)}$$

with the restrictions $k_x \geq 0$ and $k_x{}^2 + k_\rho{}^2 \leq k_\mathrm{f}{}^2$ we define a function $\Psi(x, k_x, k_\rho)$ by the equations (primes indicate differentiation with respect to x)

$$\Psi''\,(x, k_x, k_\rho) + [k_x^2 - (2m/\hbar^2)\;W(x, k_x, k_\rho)]\Psi(x, k_x, k_\rho) = 0;$$

$$\Psi(x, k_x, k_\rho) \rightarrow +0, \qquad x \rightarrow +\infty;$$

$$k_x^2\,\Psi^2\;(x, k_x, k_\rho) + [\Psi'(x, k_x, k_\rho)]^2 \rightarrow k_x^2, \qquad x \rightarrow -\infty.$$

Now, by the asymptotic equation

$$\Psi(x, k_x, k_\rho) \sim \sin[\,-k_x x + \theta\,(k_x, k_\rho)], \qquad x \rightarrow -\infty, \tag{7}$$

a continuous function $\theta(k_x, k_\rho)$ which vanishes for $k_x = 0$ is uniquely defined [11,15]. With the abbreviations $t_\mathrm{f}:\, = (\hbar^2/2m)k_\mathrm{f}{}^2$, $\alpha:\, = \tfrac{1}{4}\beta^2\,(2m/\hbar^2)\,V_0$, and $\chi_\rho:\, = (k_\mathrm{f}{}^2 - k_\rho{}^2)^{\frac{1}{2}}$ we then get

$$\epsilon_0 = (1 + \tfrac{1}{2}\alpha)\;(2/5\pi^2)\;k_\mathrm{f}^3\,t_\mathrm{f}, \tag{8}$$

$$a_\mathrm{E} = \tfrac{5}{16}\pi\;k_\mathrm{f}^{-1} + 5k_\mathrm{f}^{-3}\int_0^{k_\mathrm{f}} \mathrm{d}k_\rho k_\rho\,\theta(\chi_\rho, k_\rho) - 10\,k_\mathrm{f}^{-5}\int_0^{k_\mathrm{f}} \mathrm{d}k_\rho k_\rho\int_0^{\chi_\rho} \mathrm{d}k_x k_x\;\theta(k_x, k_\rho)\;+$$

$$-\,5(1 + \tfrac{1}{2}\,\alpha)^{-1}\;k_\mathrm{f}^{-3}\;t_\mathrm{f}^{-1}\int_0^{k_\mathrm{f}} \mathrm{d}k_\rho k_\rho\int_0^{\chi_\rho} \mathrm{d}k_x\;[1 - \tfrac{1}{4}\beta^2\,(k_x^2 + k_\rho^2)]\int_{-\infty}^{+\infty} \mathrm{d}x\,\Psi^2\,(x, k_x, k_\rho)\,[U_0(x) + V_0], \tag{9}$$

$$a_\mathrm{N} = -\tfrac{3}{8}\;\pi\;k_\mathrm{f}^{-1} + 3k_\mathrm{f}^{-3}\int_0^{k_\mathrm{f}} \mathrm{d}k_\rho k_\rho\;\theta\,(\chi_\rho, k_\rho). \tag{10}$$

If one (incorrectly) puts $\beta = 0$ in (3) and omits in (2) the second term on the right side, our formula for σ gives an expression that would have been obtained by Guruits et al. [8] if they had used our definition

(1) of the surface tension. Their approach is different from ours.

The specific surface energy $\gamma = \sigma S A^{-\frac{2}{3}}$ can be compared with a value known from analysis of experiments [12]. If one assumes that the nucleus is spherical and has the saturation density $\rho_0 = (2/3\pi^2)k_f^3 \approx A/\Omega$, the specific surface energy is given by $\gamma = (3/\rho_0)^{\frac{2}{3}}(4\pi)^{\frac{1}{3}}\sigma$. We have calculated γ for different values of the thickness $t_a = 4a \log 3$ of the potential $U_0(x)$ in (5) and for different values of the parameter β in (3). Moreover, each time we have evaluated the thickness t (90%-10% definition) of the density slope in the surface region. As fixed input data we have taken the Fermi momentum $k_f = 1.45\,f^{-1}$ corresponding to $\gamma_0 = \frac{1}{2}(9\pi)^{\frac{1}{3}}k_f^{-1} = 1.05$ f and the Fermi energy [13] $\epsilon_f = -15.5$ MeV. The results are plotted in fig. 1. It shows that one gets the experimental values of γ and t if $a = 0.64$ f and $\beta = 0.88$ f. From these values we derive $V_0 = (-\epsilon_f + t_f)(1-\frac{1}{4}\beta^2 k_f^2)^{-1} = 99.5$ MeV. For comparison we should note that for example Perey and Buck [10] have obtained $a = 0.65$ f, $\beta = 0.85$ f, and $V_0 = 71$ MeV by fitting the scattering data of slow neutrons. Meldner et al. [14] have proposed $a = 0.65$ f, $\beta = 0.90$ f, and $V_0 = 76$ MeV for calculation of nuclear ground state energies. Their parameters refer to a non-local potential, which is well approximated by our local energy-dependent version. Fig. 1 shows that taking $\beta = 0$ one gets a negative surface tension, as if the nucleus were not stable. Therefore these results give new evidence for the necessity of introducing a velocity dependent potential in shell model calculations.

References

1. C.F.v.Weizsäcker, Die Atomkerne (Akademische Verlagsgesellschaft m.b.H., Leipzig, 1937) § 10.
2. P.Gombas, Die statistische Theorie des Atoms und ihre Anwendungen (Springer Verlag, Wien, 1949).
3. W.J.Swiatecki, Proc. Phys. Soc. (London) A64 (1951) 226.
4. P.J.Hill and J.A.Wheeler, Phys. Rev. 89 (1953) 1125.
5. G.Lanzl, Dissertation (1962) Göttingen, Germany.
6. E.Hilf, Diplomarbeit (1963) Frankfurt a.M., Germany.
7. E.Hilf and G.Süssmann, Physics Letters 21 (1966) 654.
8. S.A.Guruits, A.A.Migdal and A.M.Polykow, Soviet Phys. JETP 19 (1964) 149.
9. B.H.J.McKellar and M.A.Naqvi, Nucl. Phys. 71 (1965) 161.
10. F.Perey and B.Buck, Nucl. Phys. 32 (1962) 353.
11. G.Süssmann, to be published.
12. R.Hofstadter, Rev. Mod. Phys. 28 (1956) 214.
13. R.Muthukrishnan and M.Baranger, Physics Letters 18 (1965) 160.
14. H.Meldner, G.Süssmann and W.Ulrici, Z. Naturforsch. 20a (1965) 160.
15. S.Knaak, Diplomarbeit, Frankfurt a.M., Germany, to be published.

* * * * *

NONLOCAL POTENTIAL BARRIER AND THE PEREY-EFFECT IN ALPHA DECAY

Gy. BENCZE *

Research Institute for Theoretical Physics, University of Helsinki, Helsinki, Finland

Received 12 November 1966

It is shown that nonlocality in the alpha-nucleus potential increases the barrier penetrability. The range of nonlocality of the alpha-nucleus potential is estimated by comparing experimental and theoretical alpha decay rates.

In recent years a satisfactory description of the relative values of alpha decay rates has been achieved. However, the absolute values still differ considerably from the experimental ones. Several arguments suggest that the barrier penetrabilities are responsible for the discrepancy, since the potential barrier is rather vaguely defined. In the

* On leave of absence from Central Research Institute for Physics, Budapest, Hungary.