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## On Non-Recursive Trade-Offs Between Finite-Turn Pushdown Automata

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Nr. 2/04

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#### Abstract

It is shown that between one-turn pushdown automata (1-turn PDAs) and deterministic finite automata (DFAs) there will be savings concerning the size of description not bounded by any recursive function, so-called non-recursive tradeoffs. Considering the number of turns of the stack height as a consumable resource of PDAs, we can show the existence of non-recursive trade-offs between PDAs performing $k+1$ turns and $k$ turns for $k \geq 1$. Furthermore, non-recursive trade-offs are shown between arbitrary PDAs and PDAs which perform only a finite number of turns. Finally, several decidability questions are shown to be undecidable and not semidecidable.


## 1 Introduction

Descriptional complexity is a field of theoretical computer science where one main question is: How succinctly can a model represent a formal language in comparison with other models? Basic and early results are from Meyer and Fischer [10] from 1971. They investigated regular languages and showed that there are languages being recognized by a nondeterministic finite automaton (NFA) with $n$ states such that every deterministic finite automaton (DFA) recognizing these languages will need $2^{n}$ states. Beyond this trade-off bounded by an exponential function, Meyer and Fischer proved that between context-free grammars and DFAs there exists a trade-off which is not bounded by any recursive function, a so-called non-recursive trade-off. Additional non-recursive trade-offs are known to exist between pushdown automata (PDAs) and deterministic PDAs (DPDAs), between DPDAs and unambiguous PDAs (UPDAs), between UPDAs and PDAs and many other models. A survey of results concerning the descriptional complexity of machines with limited resources, including non-recursive trade-offs between various models, may be found in [2]. A thorough discussion of the phenomenon of non-recursive trade-offs may be found in [7].

Restricting a PDA such that the height of its stack is only allowed to increase and then to decrease, thus performing only one turn, leads to the definition of one-turn PDAs [3]. It is known that these PDAs can be grammatically characterized by linear contextfree grammars. It is an obvious generalization to consider PDAs which are allowed
to perform a finite number of turns, so-called $k$-turn PDAs [3]. If it is additionally required for a $k$-turn PDA to empty its stack up to the initial stack symbol before starting the next turn, the resulting model is called strong $k$-turn PDA [1]. Both models can be grammatically characterized by ultralinear and metalinear context-free grammars, respectively. The definition of the models will be given in the next chapter.
The intention of this paper is to show non-recursive trade-offs between finite-turn PDAs and DFAs, between PDAs performing $k+1$ and $k$ turns, and between arbitrary PDAs and finite-turn PDAs. To this end we are using a generalization of a technique which was first presented by Hartmanis [5]. A combination of this technique with some old results on ultralinear grammars [3] and some new considerations leads to the desired non-recursive trade-offs. Finally, certain decidability questions for finite-turn PDAs are shown to be undecidable and not semidecidable.

## 2 Preliminaries and Definitions

Let $\Sigma^{*}$ denote the set of all words over the finite alphabet $\Sigma, \Sigma^{+}=\Sigma^{*} \backslash\{\epsilon\}$. Let REG, LCF, CF, RE denote the families of regular, linear context-free, context-free and recursively enumerable languages. We assume that the reader is familiar with the common notions of formal language theory as presented in [6]. Let $S$ be a set of recursively enumerable languages. Then $S$ is said to be a property of the recursively enumerable languages. A set $L$ has the property $S$, if $L \in S$. Let $L_{S}$ be the set $\{<M>\mid T(M) \in S\}$ where $\left\langle M>\right.$ is an encoding of a Turing machine $M$. If $L_{S}$ is recursive, we say the property $S$ is decidable; if $L_{S}$ is recursively enumerable, we say the property $S$ is semidecidable.
In the sequel we will use the set of valid computations of a Turing machine. Details are presented in [5] and [6]. The definition of a Turing machine and of an instantaneous description (ID) of a Turing machine may be found in [6].

Definition: Let $M=\left(Q, \Sigma, \Gamma, \delta, q_{0}, B, F\right)$ be a deterministic Turing machine.

$$
\begin{aligned}
\operatorname{VALC}[M]= & \left\{I D_{0}(x) \# I D_{1}(x)^{R} \# I D_{2}(x) \# I D_{3}(x)^{R} \# \ldots \# I D_{n}(x) \# \mid\right. \\
& x \in \Sigma^{*}, I D_{0}(x) \in q_{0} \Sigma^{*} \text { is an initial } \mathrm{ID}, \\
& I D_{n}(x) \in \Gamma^{*} F \Gamma^{*} \text { is an accepting ID, } \\
& \left.I D_{i+1}(x) \in \Gamma^{*} Q \Gamma^{*} \text { results from } I D_{i}(x), \text { i.e., } I D_{i}(x) \vdash^{M} I D_{i+1}(x)\right\}
\end{aligned}
$$

$\operatorname{INVALC}[M]=\Lambda^{*} \backslash \operatorname{VALC}[M]$ with respect to a coding alphabet $\Lambda$.

Definition: [4] A context-free grammar $G=(V, \Sigma, S, P)$ is metalinear if all rules of $P$ are of the following forms

$$
\begin{array}{ll}
S \rightarrow A_{1} A_{2} \ldots A_{m}, & A_{i} \in V \backslash\{S\}, \\
A \rightarrow w_{1} B w_{2}, & A, B \in V \backslash\{S\}, w_{1}, w_{2} \in \Sigma^{*}, \\
A \rightarrow w, & w \in \Sigma^{*} .
\end{array}
$$

The width of $G$ is $\max \left\{m \mid S \rightarrow A_{1} A_{2} \ldots A_{m}\right\} . L$ is metalinear of width $k$ if $L=L(G)$ for some metalinear grammar $G$ of width $k$. By $\mathcal{L}(k-L C G)$ we denote the set of languages accepted by metalinear grammars of width $k$. $\mathcal{L}(M E T A-L C G)$ denotes the set of languages accepted by metalinear grammars.

It is easily observed that metalinear grammars of width 1 are exactly linear context-free grammars.

Definition: [1,3] A context-free grammar $G=(V, \Sigma, S, P)$ is ultralinear if $V$ is a union of disjoint (possibly empty) subsets $V_{0}, \ldots, V_{n}$ of $V$ with the following property. For each $V_{i}$ and each $A \in V_{i}$, each production with left side $A$ is either of the form
$A \rightarrow w_{1} B w_{2}$ with $B \in V_{i}$ and $w_{1}, w_{2} \in \Sigma^{*}$, or of the form
$A \rightarrow w$ with $w \in\left(\Sigma \cup V_{0} \cup \ldots \cup V_{i-1}\right)^{*}$.
$\left\{V_{0}, \ldots, V_{n}\right\}$ is called an ultralinear decomposition. A language is said to be ultralinear if it is generated by some ultralinear grammar. $\mathcal{L}(U L T R A-L C G)$ denotes the set of languages accepted by ultralinear grammars.

Definition: [1] Let $M=\left(Q, \Sigma, \Gamma, \delta, q_{0}, Z_{0}, F\right)$ be a pushdown automaton. A sequence of instantaneous descriptions (IDs) on $M\left(q_{1}, w_{1}, \alpha_{1}\right) \ldots\left(q_{k}, w_{k}, \alpha_{k}\right)$ is called one-turn if there exists $i \in\{1, \ldots, k\}$ such that

$$
\left|\alpha_{1}\right| \leq \ldots \leq\left|\alpha_{i-1}\right| \leq\left|\alpha_{i}\right|>\left|\alpha_{i+1}\right| \geq \ldots \geq\left|\alpha_{k}\right|
$$

A sequence of IDs $S_{0}, \ldots, S_{m}$ is called strong $k$-turn if there are integers $0=i_{0}, \ldots, i_{l}=$ $m$ with $l \leq k$ such that for $j=0, \ldots, l-1$ holds:
(1) $S_{i_{j}}, \ldots, S_{i_{j+1}}$ is one-turn
(2) $S_{i_{j}}=\left(q, w, Z_{0}\right)$ for some $q \in Q$ and $w \in \Sigma^{*}$

If only the first condition is fulfilled, then the sequence of IDs is called $k$-turn. $M$ is a strong $k$-turn pushdown automaton if every word $w \in T(M)$ is accepted by a sequence of IDs which is strong $k$-turn. A $k$-turn pushdown automaton is defined analogously. By $\mathcal{L}($ strong-k-turn-PDA) and $\mathcal{L}(k$-turn-PDA) we denote the set of languages accepted by strong $k$-turn PDAs and $k$-turn PDAs, respectively.

Thus, strong $k$-turn PDAs are allowed to make a new turn only if the stack is empty up to the initial stack symbol whereas $k$-turn PDAs can make new turns not depending on the stack height. The following characterization of metalinear languages by strong $k$-turn PDAs and of ultralinear languages by $k$-turn PDAs may be found in [1] and [3], respectively.

Theorem 1 (1) $\mathcal{L}(k-L C G)=\mathcal{L}($ strong- $k$-turn-PDA)
(2) $L \in \mathcal{L}(M E T A-L C G) \Leftrightarrow \exists$ strong $k$-turn PDA $M$ such that $T(M)=L$.
(3) $L \in \mathcal{L}(U L T R A-L C G) \Leftrightarrow \exists k$-turn PDA $M$ such that $T(M)=L$.

Theorem 2 [1] Let A be a k-turn PDA. Then there are homomorphisms $h_{1}, h_{2}$ and a regular language $R$ such that $T(A)=h_{1}\left(h_{2}^{-1}\left(D_{2, k}\right) \cap R\right)$ with $D_{2, k}=D_{2} \cap\left(\left\{\left(,[ \}^{*}\{ ),\right]\right\}^{*}\right)^{k}$ and $D_{2}$ denotes the Dyck language with 2 types of balanced parentheses.

Concerning the notations and definitions of descriptional complexity we largely follow the presentation in [2]. A descriptional system $D$ is a recursive set of finite descriptors (e.g. automata or grammars) relating each $A \in D$ to a language $T(A)$. It is additionally required that exch descriptor $A \in D$ can be effectively converted to a Turing machine $M_{A}$ such that $T\left(M_{A}\right)=T(A)$. The language class being described by $D$ is $T(D)=$ $\{T(A) \mid A \in D\}$. For every language $L$ we define $D(L)=\{A \in D \mid T(A)=L\}$. A complexity measure for $D$ is a total, recursive, and finite-to-one function $|\cdot|: D \rightarrow$ I such that the descriptors in $D$ are recursively enumerable in order of increasing complexity. Comparing two descriptional systems $D_{1}$ and $D_{2}$, we assume that $T\left(D_{1}\right) \cap$ $T\left(D_{2}\right)$ is not finite. We say that a function $f: \mathbb{N} \rightarrow \mathbb{N}, f(n) \geq n$ is an upper bound for the trade-off when changing from a minimal description in $D_{1}$ for an arbitrary language to an equivalent minimal description in $D_{2}$, if for all $L \in T\left(D_{1}\right) \cap T\left(D_{2}\right)$ the following holds:

$$
\min \left\{|A| \mid A \in D_{2}(L)\right\} \leq f\left(\min \left\{|A| \mid A \in D_{1}(L)\right\}\right)
$$

If no recursive function is an upper bound for the trade-off between two descriptional systems $D_{1}$ and $D_{2}$, we say the trade-off is non-recursive and write $D_{1} \xrightarrow{\text { nonrec }} D_{2}$.

## 3 Non-Recursive Trade-Offs

In [9] the following generalization of Hartmanis' technique to establish non-recursive trade-offs is proven. Additional information on techniques to prove non-recursive tradeoffs may be found in [7].

Theorem 3 Let $D_{1}$ and $D_{2}$ be two descriptional systems. If for every Turing machine $M$ a lanyuage $L_{M} \in T\left(D_{1}\right)$ and a descriptor $A_{M} \in D_{1}$ for $L_{M}$ can be effectively constructed such that $L_{M} \in T\left(D_{2}\right) \Leftrightarrow T(M)$ is finite, then the trade-off between $D_{1}$ and $D_{2}$ is non-recursive.

Let $L=\operatorname{INVALC}[M] \subseteq \Lambda^{*}$ and $\{a, b, c\} \cap \Lambda=\emptyset$. Then we define

$$
\tilde{L}=\left\{a^{n} L c L b^{n} \mid n \geq 1\right\}
$$

Lemma 1 Let $M$ be a Turing machine and $k \geq 0$. Then the following pushdown automata can be effectively constructed:
(1) A strong $(k+1)$-turn PDA $A_{k+1}$ accepting $(L c)^{k+1}$.
(2) A strong infinite-turn PDA $A_{+}$accepting $(L c)^{+}$.
(i) A D-turn PDA $\tilde{A}$ accepting $\tilde{L}$.

Proof: It is shown in [ 6 ] that INVALC[ $M$ ] is a context-free language. Taking a close look at the construction we can show that INVALC[M] is the union of languages which are accepted by finite automata or 1 -turn PDAs. Since the linear contextfree languages are effectively closed under union, we can construct a 1-turn PDA $A_{1}$ such that $T\left(A_{1}\right)=\operatorname{INVALC}[M] c=(L c)^{1}$. For $k \geq 1$ the language $(L c)^{k+1}$ can be represented as the marked concatenation of languages which are accepted by oneturn PDAs. Thus, it is easy to construct a strong $(k+1)$-turn PDA $A_{k+1}$ accepting $(L c)^{k+1}$. Analogously, a strong PDA $A_{+}$making infinite turns can be constructed accepting ( $L c)^{+}$. The language $L c L$ is accepted by a 2 -turn PDA. Thus, a 2 -turn PDA $\tilde{A}$ accepting $\tilde{L}$ can be easily constructed.

Theorem 4 (Ginsburg, Spanier [3]) Let $\Sigma$ be a finite alphabet, and let $c \notin \Sigma$. Let $S \subseteq \Sigma^{*}$. Then $(S c)^{+} \in \mathcal{L}(U L T R A-L C G) \Leftrightarrow S$ is regular.

Lemma 2 Let $M$ be a Turing machine and $k \geq 1$. Then
(1) $L c \in R E G \Leftrightarrow T(M)$ is finite
(2) $(L c)^{k+1} \in \mathcal{L}(k$-turn PDA $) \Leftrightarrow T(M)$ is finite
(3) $(L c)^{+} \in \mathcal{L}($ finite-turn $P D A) \Leftrightarrow T(M)$ is finite
(4) $\bar{L} \in \mathcal{L}($ strong infinite-turn $P D A) \Leftrightarrow T(M)$ is finite

## Proof:

(1) If $T(M)$ is finite, then VALC[ $M$ ] is a finite set. This implies that the complement $L=\operatorname{INVALC}[M]$ and thus $L c$ are regular. In [0] it is proven that $\operatorname{VALC}[M] \in$ $\mathrm{CF} \Leftrightarrow T(M)$ is finite. Then, the first claim is easy to show.
(2) If $T(M)$ is finite, then $(L c)^{k+1}$ is a regular language and thus can be accepted by a $k$-turn PDA. We next show that $(L c)^{k+1} \notin \mathcal{L}(k$-turn $P D A)$ provided that $T(M)$ is infinite. If $T(M)$ is infinite, then INVALC $[M] \in \mathcal{L}(L C G) \backslash \mathrm{REG}$. By the definition of the rank $r$ of a ultralinear language [3] we obtain that $r($ INVALC $[M])=1$. Applying Corollary 1 from [3] results in $r\left((L c)^{k+1}\right)=k+1$. We now assume that $(L c)^{k+1} \in \mathcal{L}(k$-turn PDA). Thus there is a $k$-turn PDA $A$ such that $T(A)=(L c)^{k+1}$. Due to Theorem $2(L c)^{k+1}$ then has a representation as $h_{1}\left(h_{2}^{-1}\left(D_{2, k}\right) \cap R\right)$ with homomorphisms $h_{1}, h_{2}$ and a regular set $R$. It can be easily shown that $r\left(D_{2, k}\right) \leq k$. Thus $r\left((L c)^{k+1}\right) \leq k$, since the operations homomorphism, inverse homomorphism and intersection with regular languages do not increase the rank of a language due to Theorem 4.2 in [3]. This is a contradiction to the above fact that $r\left((L c)^{k+1}\right)=k+1$.
(3) This claim follows easily from (1) and Theorem 4.
(4) If $T(M)$ is finite, then $\tilde{L}$ is a linear language and a strong infinite-turn PDA accepting $\tilde{L}$ can be easily constructed. We next show that the fact that $T(M)$ is infinite implies that $\tilde{L} \notin \mathcal{L}$ (strong infinite-turn PDA). We first assume that $\tilde{L} \in$
$\mathcal{L}$ (strong finite-turn PDA). Then $\check{L}$ can be generated by a metalinear grammar of width $k$. Thus, each $w \in \dot{L}$ has a derivation $S \Rightarrow A_{1} A_{2} \ldots A_{m} \Rightarrow^{*} w$ with $m \leq k$ where each $A_{i}(1 \leq i \leq m)$ generates a linear language. There exists at least one non-terminal $A_{i}$ from which words containing infinitely many $a$ 's can be derived. This $A_{i}$ generates a linear language. Let $n$ be the constant number resulting from Ogden's lemma for $L\left(A_{i}\right)$ where all $a$ 's are marked. We now choose a word $w \in \tilde{L}$ such that $w$ contains a subword $w^{\prime} \in L\left(A_{i}\right)$ with $\left|w^{\prime}\right|_{a} \geq n$. Applying Ogden's leruma to $L\left(A_{i}\right)$ we obtain that either $a$ 's and $b$ 's or $a$ 's and no $b$ 's are pumped. This leads, in the latter case, to words in $\tilde{L}$ with different numbers of $a$ 's and $b$ 's which is a contradiction. If $a$ 's and $b$ 's are pumped, then $L\left(A_{i}\right)$ generates a linear language which is a subset of $\{a\}^{*} L c L\{b\}^{*}$. We now consider the set $N$ of all non-terminals $A_{i}$ from which words containing infinitely many $a$ 's can be derived. By the preceding considerations we obtain that each $A \in N$ generates a linear subset of $\{a\}^{*} L c L\{b\}^{*}$. Thus, $\bigcup_{A \in N} L(A)=M_{1} L c L M_{2}$ with $M_{1} \subseteq\{a\}^{*}$ and $M_{2} \subseteq\{b\}^{*}$. Since the set of linear languages is closed under union, left and right quotient with regular languages and concatenation with regular languages, we obtain that $L c L e$ is a linear language and thus accepted by a 1 -turn PDA. Applying (2) we have that $T(M)$ is finite which is a contradiction.
We next show that $\tilde{L}$ is not accepted by a strong infinite-turn PDA. If $\tilde{L}$ is accepted by a strong infinite-turn PDA $A$, we can conclude that the number of turns needed to accept an input increases with the length of the input. Otherwise, the mumber of turns could be bounded by a fixed number and thus $\tilde{L}$ would be acepted by a strong finite-turn PDA which is a contradiction. Let $n$ be an arbitrary natural number. If we choose a word $w \in L c L$ large enough with $u^{n} w b^{n} \in \tilde{L}$, then a combination of some state $q$ and the initial stack symbol is attained during A's course of computation at least two times. The subword $v$ read between these two occurrences then can be repeated arbitrarily often without affecting the acceptance of the input. If $v$ contains $a$ 's, $b$ 's or both $a$ 's and $b$ 's, then $A$ accepts inputs with a different number of $a$ 's and $b$ 's or inputs with the wrong format which is a contradiction. If $v$ contains no $a$ 's and $b$ 's, then $v$ also contains no $c$ and w.loog. it can be assumed that $v$ is located in the first $L$ of $L C L$. We are now using an incompressibility argument. More general information on Kolmogorov complexity and the incompressibility method may be found in [8]. Let $a^{n} w^{\prime}$ be the subword read until the combination of the state $q$ and the initial stack symbol occurs for the first time. Then $a^{n} w^{\prime} \# \# c b^{n} \in \tilde{L}$. But this implies that $n$ can be described by a program simulating $A$ starting in state $q$ with the initial stack symbol and reading the input \#\#cb* until an accepting state in $A$ is attained. Thus, $n$ is the number of $b$ 's read until the input is accepted. The Kolmogorov complexity $C^{\prime}(n)$ of $n$, i.e. the minimal size of a program describing $n$, is then bounded by the description sizes of $A, q$ and the above program. Obviously, these sizes are bounded by a constant number $c$ not depending on $n$. Thus, $C(n) \leq c$. Due to [8] there exist natural numbers such that $C(n) \geq \log n$. If we choose such a number and consider a word $a^{n} w b^{n} \in \tilde{L}$ heing large enough, we get a contradiction.

Combining the results of Theorem 3, Lemma 1 and Lemma 2 we get the following non-recursive trade-offs which are pictorially summarized in Fig. 1.

Theorem 5 Let $k \geq 1$ :

- (strong) 1-turn PDA $\xrightarrow{\text { nonrec }}$ NFA using $L_{M}=L c$
- (strong) $(k+1)$-turn PDA $\xrightarrow{\text { nonrec }}$ (strong) $k$-turn PDA using $L_{M}=(L c)^{k+1}$
- $(k+1)$-turn $P D A \xrightarrow{\text { nonrec }}$ strong $(k+1)$-turn PDA using $L_{M}=\tilde{L}$
- (strong) finite-turn PDA $\xrightarrow{\text { nonrec }}$ (strong) $k$-turn PDA using $L_{M}=(L c)^{k+1}$
- finite-turn $P D A \xrightarrow{\text { nonrec }}$ strong finite-turn $P D A$ using $L_{M}=\tilde{L}$
- (strong) infinite-turn PDA $\xrightarrow{\text { nanrec }}$ (strong) finite-turn PDA using $L_{M}=(L c)^{+}$
- infinite-turn $P D A \xrightarrow{\text { nonrec }}$ strong infinite-turn PDA using $L_{M}=\tilde{L}$

| 0 -turn PDA |  |  |
| :---: | :---: | :---: |
| $\nearrow$ |  | 「 |
| strong 1-turn PDA | $=$ | 1-turn PDA |
| $\dagger$ |  | $\dagger$ |
| strong 2-turn PDA | $\longleftarrow$ | 2-turn PDA |
| $\uparrow$ |  | $\uparrow$ |
| ; |  | : |
| $\dagger$ |  | $\uparrow$ |
| strong $k$-turn PDA | $\square$ | $k$-turn PDA |
| $\dagger$ |  | $\dagger$ |
| strong ( $k+1$ )-turn PDA | $\leftarrow$ | $(k+1)$-turn PDA |
| $\dagger$ |  | $\uparrow$ |
| ! |  | $\vdots$ |
| $\dagger$ |  | $\dagger$ |
| strong finite-turn PDA | $\leftarrow$ | finite-turn PDA |
| $\dagger$ |  | $\dagger$ |
| strong infinite-turn PDA | $\leftarrow$ | infinite-turn PDA |

Figure 1: Non-recursive trade-offs between PDAs with different numbers of turns allowed

Remark: It should be noted that the non-recursive trade-offs between strong $(k+1)$ turn PDAs and strong $k$-turn PDAs could have been shown using a result from [4] which states that $(L c)^{k+1} \in \mathcal{L}(k-L C G)$ if and only if $L \in \operatorname{REG}$. The approach presented in this paper extends the non-recursive trade-offs to arbitrary $k$-turn PDAs.

## 4 Decidability Questions

The faet that the set of invalid computations can be recornized by a 1 -turn PDA allows us to simply prove that certain decidability questions for strong $k$-turn PDAs are not deridable and not even semidecidable. The results obviously hold for $k$-turn PDAs anl arhitrary PDAs as well.

Lemma 3 Let M be a Turing machine. It is not semidecidable whether $T(M)=\emptyset$ or $T(M)$ is finite.

Proof: The lemma can be easily seen using Rice's theorem for recursively enumerable index sets [6].

Theorem 6 Let $k, k^{\prime} \geq 1$ be two integers. It is not semidecidable for arbitrary strong $k$-turn PDAs A and strony $k^{\prime}$-turn PDAs A' whether
(1) $T(A)=\Sigma^{*}$
(2) $T(A)=T\left(A^{\prime}\right), T(A) \subseteq T\left(A^{\prime}\right)$
(3) $T(A) \in R E G$
(4) $T(A) \in \mathcal{L}($ strong $(k-1)$-turn PDA)

Proof: Let $M$ be an arthitrary Turing machine. By Lemma 1, we can construct a 1-tum PDA A accepting INVALC[M]. Suppose that the first question is semidecidable. Then we can semidecide whether $\operatorname{INVALC}[M]=\Sigma^{*}$, or equivalently, whether $\operatorname{VALC}[M]=\emptyset$. Thus, we can semidecide whether an arbitrary Turing machine accepts the empty langnage which is a contradiction to the above lemma. The questions of (2) can be easily reduced to the first question. If we could semidecide question (3), we could semidecide whether $M$ accepts a finite language due to Lemma 2(1). This again contradicts the above lemma. The non-semidecidability of (4) is shown similarly considering Lemma 2(2).

It can be learned from the proofs of (3) and (4) that the existence of non-recursive tradeoffs implies that it is not semidecidable for a PDA with a certain number of turns allowed whether its languge accepted could be accepted by any other PDA with a smaller number of turns. For example, it is not semidecidable whether a language described by an infinite-turn PDA can be accepted by a finite-turn PDA. Thus, the minimal number of turns needed to accept a context-free language cannot be determined algorithmically.

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