PROCEEDINGS OF SCIENCE

Decay widths of resonances and pion scattering lengths in a globally invariant sigma model with vector and axial-vector mesons

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We calculate low-energy meson decay processes and pion-pion scattering lengths in a two-flavour linear sigma model with global chiral symmetry, exploring the scenario in which the scalar mesons $f_0(600)$ and $a_0(980)$ are assumed to be $\bar{q}q$ states.

8th Conference Quark Confinement and the Hadron Spectrum
September 1-6 2008
Mainz, Germany

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1. Introduction

Effective field theories provide a very efficient means to describe Quantum Chromodynamics (QCD) at low energies. They possess the same global symmetries as QCD - e.g., the chiral $SU(N_f)_r \times SU(N_f)_l$ symmetry, where N_f is the number of flavours - and are expressed in terms of hadronic degrees of freedom rather than in terms of quarks and gluons. Spontaneous breaking of the chiral symmetry leads to the emergence of low-mass pseudoscalar Goldstone bosons and their chiral partners, large-mass scalar states.

In this paper we present a linear sigma model with global chiral invariance, similar to the one of Ref. [1]. The model contains scalar and pseudoscalar as well as vector and axialvector mesons. The global invariance is motivated by the results of Refs. [2, 3] where it has been shown that a locally invariant linear sigma model fails to describe simultaneously pion-pion scattering lengths and some important decay widths. For the globally invariant model additional terms appear in the Lagrangian which introduce new coupling constants that can in principle be adjusted to improve the agreement with the experimental data. In this paper we show the first results from this approach for the case of $N_f = 2$.

As outlined in Ref. [3], there are two possibilities to interpret the scalar $\sigma$and a_0 fields contained in the model where they are $\bar{q}q$ states [$\sigma = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$, $a_0^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$]: (*i*) they are identified with $f_0(600)$ and $a_0(980)$ which form a part of a larger nonet that consists of $f_0(980)$, $a_0(980)$, $k(800)$ and $f_0(600)$ (resonances below 1 GeV); (*ii*) they are identified with the $f_0(1370)$ and $a_0(1450)$ resonances forming a part of a nonet that consists of $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $a_0(1450)$, $K_0(1430)$ - i.e., resonances above 1 GeV (see Ref. [4]). In the assignment (*ii*), scalar mesons below 1 GeV, whose spectroscopic wave functions possibly contain a dominant tetraquark or mesonic molecular contribution [5], may be introduced as additional scalar fields.

In this paper, we describe briefly the consequences of assignment *(i)*; the consequences of assumption *(ii)* as well as more detailed results in assignment *(i)* may be found in Ref. [6].

2. The Linear Sigma Model with Global Chiral Symmetry

The Lagrangian of the globally invariant linear sigma model with $U(2)_R \times U(2)_L$ symmetry reads [2, 3, 7]:

$$\begin{aligned}\mathscr{L} =\ & \mathrm{Tr}[(D^\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\ & -\frac{1}{4}\mathrm{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2}\mathrm{Tr}[(L^\mu)^2 + (R^\mu)^2] + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] \\ & +c(\det\Phi + \det\Phi^\dagger) - 2ig_2(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & -2g_3\{\mathrm{Tr}[(\partial_\mu L_\nu + \partial_\nu L_\mu)\{L^\mu, L^\nu\}] + \mathrm{Tr}[(\partial_\mu R_\nu + \partial_\nu R_\mu)\{R^\mu, R^\nu\}]\} + \mathscr{L}_4 \\ & +\frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}[(L^\mu)^2 + (R^\mu)^2] + h_2\mathrm{Tr}[(\Phi R^\mu)^2 + (L^\mu\Phi)^2] + 2h_3\mathrm{Tr}(\Phi R_\mu \Phi^\dagger L^\mu), \quad (2.1)\end{aligned}$$

with $\Phi = (\sigma + i\eta_N)t^0 + (\vec{a}_0 + i\vec{\pi})\cdot\vec{t}$ (scalar and pseudoscalar mesons; our model is valid for $N_f = 2$ and thus our eta meson η_N contains only non-strange degrees of freedom); $L^\mu = (\omega^\mu - f_1^\mu)t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu)\cdot\vec{t}$ and $R^\mu = (\omega^\mu + f_1^\mu)t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu)\cdot\vec{t}$ (vector and axialvector mesons), where t^0, $\vec{t}$ are the generators of $U(2)$; $D^\mu\Phi = \partial^\mu\Phi + ig_1(\Phi L^\mu - R^\mu\Phi)$, $L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu$, $R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$

and $\mathscr{L}_4$ containing all the vertices with four vector and axialvector mesons. Explicit breaking of the global symmetry is described by the term $\mathrm{Tr}[H(\Phi+\Phi^\dagger)] \equiv h\sigma(h = const.)$. The chiral anomaly is described by the term $c\,(\det\Phi + \det\Phi^\dagger)$ [8].

Irrespective of $\mathscr{L}_4$, the model contains 13 parameters - 12 parameters from the Lagrangian (2.1), plus the wave function renormalisation constant of the pseudoscalar mesons [9], Z. However, only seven of those (Z, $g_{1,2}$, $h_{1,2,3}$, λ_2) are relevant for the decays that will be considered in the following. The parameters g_1, h_3 and λ_2 are expressed in terms of Z:

$$g_1 = g_1(Z) = \frac{m_{a_1}}{Z f_\pi}\sqrt{1-\frac{1}{Z^2}},\;\; h_3 = h_3(Z) = \frac{1}{Z^2 f_\pi^2}\left(m_\rho^2 - \frac{m_{a_1}^2}{Z^2}\right),\;\; \lambda_2 = \lambda_2(Z) = \frac{Z^2 m_{a_0}^2 - m_{\eta_N}^2}{Z^4 f_\pi^2},$$

and thus the number of independent relevant parameters is decreased to four. Additionally, m_σ (which is a function of m_0, λ_1, λ_2, c and Z) is taken as a parameter that can be determined from the pion-pion scattering lengths yielding five independent parameters for the meson decay modes and scattering lengths described below.

2.1 Relevant Decay Modes and $\pi\pi$ Scattering Lengths

The following decay modes of two-flavour low-energy mesons have been taken into account [parameter dependence in brackets]: $\rho \to \pi\pi$ [Z, g_2], $f_1 \to a_0\pi$ [Z, h_2], $a_1 \to \pi\gamma$ [Z], $a_0 \to \eta\pi$ [Z, h_2], $\sigma \to \pi\pi$ [Z, h_1, h_2], $a_1 \to \sigma\pi$ [h_1, h_2, Z], $a_1 \to \rho\pi$ [g_2, Z]. We have also considered the pion-pion scattering lengths $a_0^0(h_1, h_2, Z, m_\sigma)$ and $a_0^2(h_1, h_2, Z, m_\sigma)$.

Given that the decay widths for the channels $\sigma \to \pi\pi$ [Z, h_1, h_2], $a_1 \to \sigma\pi$ [h_1, h_2, Z] and $a_1 \to \rho\pi$ [g_2, Z] are poorly known, we have not taken any numerical values for these decay widths to fit our parameters - these decay widths will be calculated as a consistency check on the basis of the results obtained from the other decay widths and the scattering lengths.

Here, we will present formulas that have been used to fit the parameters; for all other formulas, see Ref. [6].

Decay width for $\rho \to \pi\pi$. The decay width reads

$$\Gamma_{\rho\to\pi\pi} = \frac{m_\rho^5}{48\pi m_{a_1}^4}\left[1-\left(\frac{2m_\pi}{m_\rho}\right)^2\right]^{\frac{3}{2}}\left[\left(g_1-\frac{g_2}{2}\right)Z^2+\frac{g_2}{2}\right]^2.$$

The experimental value is (149.4±1.0) MeV [10].

Decay width for $f_1 \to a_0\pi$. The following formula for the decay width is obtained:

$$\Gamma_{f_1\to a_0\pi} = \frac{g_1^2 Z^2}{2\pi}\frac{k^3}{m_{f_1}^2 m_{a_1}^4}\left[m_\rho^2 - \frac{1}{2}(h_2+h_3)\phi^2\right]^2, k = \frac{1}{2}\sqrt{m_{f_1}^2 - 2(m_\pi^2 + m_{a_0}^2) + \frac{(m_{a_0}^2 - m_\pi^2)^2}{m_{f_1}^2}}$$

where $\phi \equiv Z f_\pi$ is the vacuum expectation value of the σ field.

Decay width for $a_1 \to \pi\gamma$. The Lagrangian leading to the formula for the decay width $\Gamma_{a_1\to\pi\gamma}$ is obtained from the Lagrangian (2.1) by coupling the photon to the relevant part of the axial current $J_\mu = -ig_1 Z^2 f_\pi (a^+_{1\mu}\pi^- - a^-_{1\mu}\pi^+) - Zw(a^+_{1\mu\nu}\partial^\nu\pi^- - a^-_{1\mu\nu}\partial^\nu\pi^+)$, where $w = \frac{g_1\phi}{m^2_{a_1}}$, and reads $\mathscr{L}_{a_1\pi\gamma} = eJ_\mu A^\mu$.

The decay width reads

$$\Gamma_{a_1\to\pi\gamma} = \frac{e^2}{96\pi}(Z^2-1)m_{a_1}\left[1-\left(\frac{m_\pi}{m_{a_1}}\right)^2\right]^3.$$

Note that the sole dependence of the $a_1 \to \pi\gamma$ decay width on the parameter Z may in principle lead to an accurate determination of this parameter. However, the experimental value of the $a_1 \to \pi\gamma$ decay width is not very precise ($\Gamma_{a_1\to\pi\gamma} = 0.640 \pm 0.246$ MeV [10]) and thus we have used the χ^2 method to determine all the parameters from the decay widths and scattering lengths.

Decay amplitude for $a_0 \to \eta_N\pi$. The mass of the η_N meson can be calculated using the well-known mixing of strange and non-strange contributions in the physical fields η and $\eta'(958)$ yielding $\eta = \eta_N \cos\varphi + \eta_S \sin\varphi$; $\eta' = -\eta_N \sin\varphi + \eta_S \cos\varphi$, where η_S denotes a pure $\bar{s}s$ state and $\varphi \simeq -36^\circ$ [11]. Then we obtain $m_\eta = 716$ MeV.

Note that we have used the decay amplitude for the $a_0 \to \eta_N\pi$ decay instead of the decay width as quoted by the PDG [10] in order to fit the parameters of the model. The experimental value of the decay amplitude is known from the Crystal Barrel data: $A_{a_0\eta\pi} = (3330 \pm 150)$ MeV [12] which for our purposes has to be divided by $\cos\varphi$; the formula for the decay amplitude obtained from Eq. (2.1) is

$$A_{a_0\eta\pi} = \frac{m^2_\eta - Z^2 m^2_{a_0}}{\phi} + \frac{g_1^2\phi}{m^2_{a_1}}\left\{\left[1-\frac{1}{2}\frac{Z^2\phi^2}{m^2_{a_1}}(h_2-h_3)\right](m^2_{a_0}-m^2_\pi-m^2_\eta)+Z^2m^2_{a_0}\right\}.$$

Scattering length a_0^0. The formula for a_0^0 is calculated using the partial wave decomposition [13] which leads to

$$\begin{aligned} a_0^0 = \frac{1}{4\pi}\Bigg\{ & 2g_1^2Z^4\frac{m^2_\pi}{m^4_{a_1}}\left[m^2_\rho+\frac{\phi^2}{16}(12g_1^2-2(h_1+h_2)-14h_3)\right]-\frac{5}{8}\frac{Z^2m^2_\sigma-m^2_\pi}{f^2_\pi} \\ & -\frac{3}{2}\left[2g_1^2Z^2\phi\frac{m^2_\pi}{m^2_{a_1}}\left(1+\frac{m^2_\rho-\phi^2(h_1+h_2+h_3)/2}{2m^2_{a_1}}\right)-\frac{Z^2m^2_\sigma-m^2_\pi}{2\phi}\right]^2\frac{1}{4m^2_\pi-m^2_\sigma} \\ & +\left[g_1^2Z^2\phi\frac{m^2_\pi}{m^4_{a_1}}\left(m^2_\rho-\frac{\phi^2}{2}(h_1+h_2+h_3)\right)+\frac{Z^2m^2_\sigma-m^2_\pi}{2\phi}\right]^2\frac{1}{m^2_\sigma}\Bigg\}. \end{aligned}$$

We are using the result $a_0^0 = 0.233 \pm 0.023$ (normalised to the pion mass) in accordance with data published by the NA48/2 collaboration [14].

Scattering length a_0^2. An analogous calculation as in the case of the scattering length a_0^0 leads to

$$a_0^2 = -\frac{1}{4\pi}\left\{\frac{1}{4}\frac{Z^2m^2_\sigma-m^2_\pi}{f^2_\pi}+g_1^2Z^4\frac{m^2_\pi}{m^4_{a_1}}\left[m^2_\rho-\frac{\phi^2}{2}(h_1+h_2+h_3)\right]\right.$$

$$-\left[g_1^2 Z^2 \phi \frac{m_\pi^2}{m_{a_1}^4}\left(m_\rho^2 - \frac{\phi^2}{2}(h_1+h_2+h_3)\right) + \frac{Z^2 m_\sigma^2 - m_\pi^2}{2\phi}\right]^2 \frac{1}{m_\sigma^2}\Bigg\}.$$

The result for a_0^2 from the NA48/2 collaboration [14] is $a_0^2 = -0.0471 \pm 0.015$.

3. Results

In order to fit the relevant parameters of our model (Z, g_2, $h_{1,2}$, m_σ) to experimental data (for the aforementioned decay widths, $a_0 \to \eta\pi$ decay amplitude and scattering lengths) we have used the χ^2 method. The error for the mixing angle φ is neglected in this first case study.
Our best fit yields the minimal value of $\chi^2_{\text{min.}} = 0.752516$ per degree of freedom which leads to the following values of parameters: $Z = 1.5217$, $g_1 = 6.59$, $g_2 = 0.3365$, $h_1 = -100.7$, $h_2 = 106.045$, $h_3 = -2.63$, $m_\sigma = 330$ MeV.
It is interesting to note that, although new parameters have been introduced in the globally invariant model, the values of $Z = 1.5217$, $g_1 = 6.59$, and $m_\sigma = 330$ MeV are virtually the same as those obtained in the locally invariant model where the corresponding values were $Z = 1.586$, $g_1 = 6.51$, and $m_\sigma \simeq (315-345)$ MeV [3]. Note also that the value of h_1 does not appear to be large-N_C suppressed, although the parameter $h_1/2$ is the prefactor to a term consisting of a product of two traces $\text{Tr}(\Phi^\dagger\Phi)\text{Tr}[(L^\mu)^2 + (R^\mu)^2]$ - in fact, the modulus of the corresponding prefactor $h_1/2$ is by about a factor of ten larger than the prefactor to the term $\text{Tr}(\Phi R_\mu \Phi^\dagger L^\mu)$ (i.e., $2h_3 = -5.26$).

Using the parameters above leads to the following consequences: (*i*) $\Gamma_{a_1\to\sigma\pi} = 90.163$ MeV; (*ii*) given that in the globally invariant model the ρ mass term is $m_\rho^2 = m_1^2 + \phi^2(h_1+h_2+h_3)/2$, it is possible to calculate the contribution of the bare mass (m_1^2) to the total mass m_ρ^2 - the result $m_1 \simeq 758$ MeV is obtained, leading to a very small contribution of the quark condensate to the ρ mass; (*iii*) the $\sigma \to \pi\pi$ decay width has a value of less than 10 MeV - it is thus too small - and the $a_1 \to \rho\pi$ decay width has the value of 1.4 GeV - it is thus too large.

Hence, in the light of our results we conclude that the $\bar{q}q$ assignment of the light scalar mesons leads to contradictions to experiment. For a definite conclusion, the errors of the parameters in the model should be evaluated (see Ref. [6]), but it is already clear from our current results that the assignment of $f_0(600)$ and $a_0(980)$ as $\bar{q}q$ states may be problematic.

A possible way to resolve the aforementioned problem is to redefine σ and a_0 mesons in the model as $f_0(1370)$ and $a_0(1450)$, respectively, and hence assign the scalar meson states to the energy region above 1 GeV [6]. Then, the mixing of quarkonia and tetraquark states [16] needs to be examined.

4. Conclusions and Outlook

A globally invariant linear sigma model with vector and axial vector mesons and its consequences for low-energy meson decay channels and pion-pion scattering lengths have been presented. Results obtained in the assignment in which scalar mesons are identified as states under 1 GeV indicate contradictions to experimental data, hence raising questions about the justification of the mentioned assignment. Thus, a detailed study of the other possible assignment for scalar mesons (in which those states are located in the energy region above 1 GeV) is necessary. In the future, other relevant issues in connection with vacuum phenomenology will be addressed such as the inclusion of the nucleon field together with its chiral partner [15] as well as extending the work of Refs. [9, 16] to consider chiral symmetry restoration at nonzero temperature.

Acknowledgments

The authors thank H. Leutwyler and S. Strüber for valuable discussions during the preparation of this work.

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