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Analyzing Interest Rate Risk: Stochastic Volatility in the Term Structure of Government Bond Yields*

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Abstract:
We propose a Nelson-Siegel type interest rate term structure model where the underlying yield factors follow autoregressive processes with stochastic volatility. The factor volatilities parsimoniously capture risk inherent to the term structure and are associated with the time-varying uncertainty of the yield curve's level, slope and curvature. Estimating the model based on U.S. government bond yields applying Markov chain Monte Carlo techniques we find that the factor volatilities follow highly persistent processes. We show that slope and curvature risk have explanatory power for bond excess returns and illustrate that the yield and volatility factors are closely related to industrial capacity utilization, inflation, monetary policy and employment growth.

JEL Classification: C5, E4, G1

Keywords: Term Structure Modelling, Yield Curve Risk, Stochastic Volatility, Factor Models, Macroeconomic Fundamentals.

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1 Introduction

Much research in financial economics has been devoted to the modelling and forecasting of interest rates and the term structure thereof. Nevertheless, only a few approaches explicitly account for the time-varying risk thereof. A potential reason is the typically high (cross-sectional) dimensionality of the term structure whose multivariate volatility is cumbersome to model. Consequently, most studies focus on the riskiness of (selective) interest rates for given maturities or study aggregated volatility measures based on a common component as recently suggested by Koopman, Mallee, and van der Wel (2008) or based on bond market portfolios in the spirit of Engle, Ng, and Rothschild (1990) and Engle and Ng (1993).

In this paper, we propose capturing the riskiness inherent to the term structure of interest rates by an extended Nelson-Siegel (1987) term structure model where the underlying yield curve factors reveal stochastic volatility. We see this approach as a parsimonious alternative to a model where the individual time series of yields themselves reveal (high-dimensional) time-varying volatility. Modelling stochastic volatility rather directly in the Nelson-Siegel factors reduces the dimension of the stochastic volatility process to three and allows capturing time-varying uncertainty associated with the yield curve's level, slope and curvature. Accordingly, the so-called 'level volatility' reflects the volatility with respect to the overall level of yields, whereas the 'slope volatility' captures the time-varying riskiness in the spread between short-term and long-term yields. Correspondingly, the 'curvature volatility' is associated with the risk due to changes in the term structure curvature.

This paper contributes to the recent empirical literature on the modelling of interest rate dynamics. This literature is inspired by the fact that popular theoretical equilibrium models as, e.g., proposed by Vasicek (1977), Cox, Ingersoll, and Ross (1985), Duffie and Kan (1996), Dai and Singleton (2002) or Duffee (2002) and no-arbitrage models in the line of, e.g., Hull and White (1990) or Heath, Jarrow, and Morton (1992) successfully explain the cross-sectional term structure across the maturities of interest rates but are not very powerful in capturing interest rate dynamics. In fact, in both empirical research as well as financial practice, the latter task is dominantly addressed using factor models. The most well-known approach in this area is the Nelson and

Siegel (1987) exponential components framework which is neither an equilibrium nor a no-arbitrage model but can be heuristically motivated by the expectations hypothesis of the term structure. In this setting, the term structure is captured by three factors which are associated with the yield curve's level, slope and curvature. In a related approach, Litterman and Scheinkman (1991) propose such factors as the first three principal components based on the bond return covariance matrix. Cochrane and Piazzesi (2005) suggest a data-driven one-factor model based on a single tent-shaped linear combination of forward rates. They show that the so-called 'return forecasting factor' has more predictive power than the Litterman-Scheinkman principal components. Recently, Diebold and Li (2006) propose a simple dynamic implementation of the Nelson and Siegel (1987) model and employ it to model and to predict the yield curve. This approach is extended by Diebold, Rudebusch, and Aruoba (2006) to include macroeconomic variables and by Koopman, Mallee, and van der Wel (2008) allowing for time-varying loadings and including a common volatility component.

Motivated by the lacking empirical evidence on the role of term structure volatility, we aim to fill this gap in the literature and address the following three research questions: (i) To which extent do the yield curve factors reveal time-varying volatility? (ii) Do factor volatilities give rise to risk premia in expected bond excess returns? (iii) How are the factor volatilities linked to macroeconomic fundamentals?

We represent the Nelson-Siegel model in a state space form, where both the (unobservable) yield factors and their stochastic volatility processes are treated as latent factors following autoregressive processes. The model is estimated using Markov chain Monte Carlo (MCMC) methods based on monthly unsmoothed Fama-Bliss zero yields from 1964 to 2003. In a second step, the estimated yield curve factors and volatility factors are used (i) as regressors in rolling window regressions of one-year-ahead bond excess returns and (ii) as components of a VAR model including macroeconomic variables, such as capacity utilization, industrial production, inflation, employment growth as well as the federal funds rate.

Based on our empirical study, we can summarize the following main findings: (i) We find strong evidence for persistent stochastic volatility dynamics in the Nelson-Siegel factors. It turns out that risks inherent to the shape of the yield curve as represented by the extracted slope and curvature volatility have explanatory power for future yearly bond excess returns beyond Cochrane and Piazzesi's (2005) return-forecasting factor. In particular, including the volatility factors in rolling window regressions increases the (adjusted) R^2 from 36 percent to up to 50 percent. (ii) Our results provide evidence that the factor volatilities' explanatory power for future excess returns arises because of two effects. Firstly, it stems from a risk premium due to the uncertainty in the yield curvature. Secondly, we observe a converse effect arising from a negative relation between the slope volatility and expected excess returns. (iii) It turns out that both yield factors and factor volatilities are closely linked to macroeconomic fundamentals, such as capacity utilization, industrial production, inflation, employment growth as well as the federal funds rate. Prediction error variance decompositions show evidence for significant long-run effects of macroeconomic variables on term structure movements and volatilities thereof. Converse relations reveal a particular importance of the curvature volatility.

The remainder of the paper is structured as follows. In Section 2, we describe the dynamic Nelson and Siegel (1987) model as put forward by Diebold and Li (2006) and discuss the proposed extension allowing for stochastic volatility processes in the yield factors. Section 3 presents the data and illustrates the estimation of the model using MCMC techniques. Empirical results from regressions of one-year excess bond returns on the extracted yield factors are shown in Section 4. Section 5 gives the corresponding results when factor volatilities are used as regressors. In Section 6, the dynamic interdependencies between yield factors, factor volatilities and macroeconomic variables are investigated. Finally, Section 7 gives the conclusions.

2 A Dynamic Nelson-Siegel Model with Stochastic Volatility

Let $p_t^{(n)}$ denote the log price of an n-year zero-coupon bond at time t with $t = 1, \ldots, T$ denoting monthly periods and $n = 1, \ldots, N$ denoting the maturities. Then, the yearly log yield of an n-year bond is given by $y_t^{(n)} := -\frac{1}{n} p_t^{(n)}$. The one-year forward rate at time t for loans between time $t+12(n-1)$ and $t+12n$ is given by $f_t^{(n)} := p_t^{(n-1)} - p_t^{(n)} = n y_t^{(n)} - (n-1) y_t^{(n-1)}$. In the following we focus on one-year returns observed on a monthly basis. Then, the log holding-period return from buying an n-year bond at time $t-12$ and selling it as an $(n-1)$-year bond at time t is defined by $r_t^{(n)} := p_t^{(n-1)} - p_{t-12}^{(n)}$. Correspondingly, excess log returns are defined by $z_t^{(n)} := r_t^{(n)} - y_{t-12}^{(1)}$.

Nelson and Siegel (1987) propose modeling the forward rate curve in terms of a constant plus a Laguerre polynomial function as given by

$$f_t^{(n)} = \beta_{1t} + \beta_{2t} e^{-\lambda_t n} + \beta_3 \lambda_t e^{-\lambda_t n}. \tag{1}$$

Small (large) values of λ_t produce slow (fast) decays and better fit the curve at long (short) maturities. Though the Nelson-Siegel model is neither an equilibrium model nor a no-arbitrage model it can be heuristically motivated by the expectations hypothesis of interest rates. As Laguerre polynomials belong to a class of functions which are associated with solutions to differential equations, forward rates can be interpreted as solutions to a differential equation underlying the spot rate. The corresponding yield curve is given by

$$y_t^{(n)} = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t n}}{\lambda_t n}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t n}}{\lambda_t n} - e^{-\lambda_t n}\right). \tag{2}$$

Diebold and Li (2006) interprete the parameters β_{1t}, β_{2t} and β_{3t} as three latent dynamic factors with loadings 1, $(1-e^{-\lambda_t n})/\lambda_t n$, and $\{(1-e^{-\lambda_t n})/\lambda_t n\} - e^{-\lambda_t n}$, respectively. Then, β_{1t} represents a long-term factor whose loading is constant for all maturities. With the loading of β_{2t} starting at one and decaying monotonically and quickly to zero, β_{2t} may be viewed as a short-term factor. Finally, β_{3t} is interpreted as a medium-term factor with a loading starting at zero, increasing and decaying to zero in the limit. Showing that $y_t^{\infty} = \beta_{1t}$, $y_t^{\infty} - y_t^0 = -\beta_{2t}$, and $y_t^0 = \beta_{1t} + \beta_{2t}$ it is naturally to associate the long-term factor β_{1t} with the level of the yield curve, whereas β_{2t} and β_{3t} capture its slope and curvature, respectively. Figure 1 shows the Nelson-Siegel factor loadings with fixed $\lambda = 0.045$ stemming from our estimation results below.[1]

Denoting the yield factors in the sequel by $L_t := \beta_{1t}$, $S_t := \beta_{2t}$ and $C_t := \beta_{3t}$, we can represent the model in state-space form

$$y_t = Af_t + \varepsilon_t, \tag{3}$$

where $f_t := (L_t,\ S_t,\ C_t)'$ denotes the (3×1) vector of latent factors, $y_t := \left(y_t^{(1)},\ y_t^{(2)}, \ldots, y_t^{(N)}\right)'$ is the $(N \times 1)$ vector of yields and

$$A := \begin{pmatrix} 1 & \frac{1-e^{-\lambda \cdot 1}}{\lambda \cdot 1} & \frac{1-e^{-\lambda \cdot 1}}{\lambda \cdot 1} - e^{-\lambda \cdot 1} \\ 1 & \frac{1-e^{-\lambda \cdot 2}}{\lambda \cdot 2} & \frac{1-e^{-\lambda \cdot 2}}{\lambda \cdot 2} - e^{-\lambda \cdot 1} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda \cdot N}}{\lambda \cdot N} & \frac{1-e^{-\lambda \cdot N}}{\lambda \cdot N} - e^{-\lambda \cdot N} \end{pmatrix}$$

represents the $(N \times 3)$ matrix of factor loadings. Finally, for the $(N \times 1)$ vector of error terms ε_t we assume

$$\varepsilon_t := \left(\varepsilon_t^{(1)},\ \varepsilon_t^{(2)},\ \ldots, \varepsilon_t^{(N)}\right) \sim \text{ i.i.d. } N(0, \Sigma)$$

with

$$\Sigma = \text{diag}\left\{\left(\sigma^{(1)}\right)^2, \left(\sigma^{(2)}\right)^2, \ldots, \left(\sigma^{(N)}\right)^2\right\}, \tag{4}$$

[1] All figures and tables are shown in the Appendix.

where diag(·) captures the diagonal elements of a (symmetric) matrix in a corresponding vector. Note that we assume the decaying factor $\lambda_t = \lambda$ to be constant over time. This is in accordance with Diebold and Li (2006) and the common finding that time variations in λ_t have only a negligible impact on the model's fit and prediction power.[2]

Following Diebold and Li, the latent dynamic yield factors are assumed to follow a first order vector autoregressive (VAR) process,

$$f_t = \mu + \Phi f_{t-1} + \eta_t, \tag{5}$$

where Φ is a (3×3) parameter matrix, μ denotes a (3×1) parameter vector, and the (3×1) vector η_t is assumed to be independent from ε_t with

$$\eta_t \sim \text{i.i.d. } N(0, H_t). \tag{6}$$

Diebold and Li (2006) assume the conditional variances to be constant over time, i.e., $H_t = H$. This enables estimating the latent factors L_t, S_t, and C_t in a first step period-by-period using (nonlinear) least squares and to use them in a second step in a VAR model as given by (5).

However, given the objective of our study, we propose specifying the covariance matrix H_t in terms of a stochastic volatility process of the form

$$\text{vech}(\ln H_t) = \mu_h + \Phi_h \text{vech}(\ln H_{t-1}) + \xi_t, \tag{7}$$

where vech(·) denotes the vech-operator stacking the distinct elements of the covariance matrix, μ_h is a (6×1) dimensional parameter vector and Φ_h is a (6×6) dimensional parameter matrix. The error term vector ξ_t is assumed to be independent from η_t and ε_t and is normally distributed with covariance matrix Σ_h capturing the "covariance of covariance",

$$\xi_t \sim \text{i.i.d. } N(0, \Sigma_h). \tag{8}$$

However, fully parameterizing the matrices Φ, H_t and Φ_h leads to a complicate model which is difficult to estimate and is typically over-parameterized in order to parsimoniously capture interest rate dynamics and associated risks. Hence, to overcome the computational burden and the curse of dimensionality, we propose restricting the model to a diagonal specification with

$$\Phi = \text{diag}(\phi^L, \phi^S, \phi^C), \tag{9}$$

$$H_t = \text{diag}(h_t^L,\ h_t^S\ h_t^C), \tag{10}$$

$$\Phi_h = \text{diag}(\phi_h^L,\ \phi_h^S\ \phi_h^C). \tag{11}$$

[2] This is also confirmed by own investigations. Actually, we also allowed λ_t to be time-varying but found that this extra flexibility is not important for the model's goodness-of-fit.

As shown in the empirical analysis below, these restrictions are well supported by the data.[3] Then, the latent factor structure can be expressed by

$$\begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} = \begin{pmatrix} \mu^L \\ \mu^S \\ \mu^C \end{pmatrix} + \begin{pmatrix} \phi^L & 0 & 0 \\ 0 & \phi^S & 0 \\ 0 & 0 & \phi^C \end{pmatrix} \begin{pmatrix} L_{t-1} \\ S_{t-1} \\ C_{t-1} \end{pmatrix} + \eta_t \ , \tag{12}$$

where $\eta_t \ \sim$ i.i.d. $N(0, H_t)$ with

$$\text{diag}(\ln H_t) = \begin{pmatrix} \ln(h_t^L) \\ \ln(h_t^S) \\ \ln(h_t^C) \end{pmatrix} = \begin{pmatrix} \mu_h^L \\ \mu_h^S \\ \mu_h^C \end{pmatrix} + \begin{pmatrix} \phi_h^L & 0 & 0 \\ 0 & \phi_h^S & 0 \\ 0 & 0 & \phi_h^C \end{pmatrix} \begin{pmatrix} \ln(h_{t-1}^L) \\ \ln(h_{t-1}^S) \\ \ln(h_{t-1}^C) \end{pmatrix} + \begin{pmatrix} \xi_t^L \\ \xi_t^S \\ \xi_t^C \end{pmatrix} . \tag{13}$$

We refer h_t^L, h_t^S and h_t^C to as so-called "factor volatilities" capturing the time-varying uncertainty in the level, slope and curvature of the yield curve. The level volatility h_t^L corresponds to a component which is common to the time-varying variances of all yields. It might be associated with underlying latent (e.g. macroeconomic) information driving the uncertainty in the overall level of interest rates. It can be seen as a model implied proxy of the bond market volatility which is captured by Engle, Ng, and Rothschild (1990) in terms of the conditional excess return variance of an equally weighted bill market portfolio. Correspondingly, h_t^S is associated with risk inherent to the slope of the yield curve. It reflects the riskiness in yield spreads, and thus time-variations in the risk premium which investors require to hold long bonds instead of short bonds. Finally, h_t^C captures uncertainties associated with the curvature of the yield curve, which can vary between convex, linear and concave forms. Obviously, such variations mainly stem from time-varying volatility in bonds with mid-term maturities.

An alternative way to capture time-varying volatility in the term structure of interest rates would be to allow Σ itself to be time-varying. However, this would result in an N-dimensional MGARCH or SV model which is not very tractable if the cross-sectional dimension N is high. Therefore, we see our approach as a parsimonious alternative to capture interest rate risk. Note that the slope and curvature factors can be interpreted as particular (linear) combinations of yields associated with factor portfolios mimicking the steepness and convexity of the yield curve.[4] Then, the corresponding slope and

[3] Note that we also estimated models with non-zero off-diagonal elements in Φ and H_t and found that most off-diagonal parameters are indeed statistically insignificant.

[4] This interpretation is also reflected in the linear combinations of yields which are typically used to empirically approximate the underlying yield curve factors. In particular, level, slope and curvature are often approximated by $\frac{1}{3}(y_t^{(1)} + y_t^{(3)} + y_t^{(5)})$, $y_t^{(5)} - y_t^{(1)}$, and $2y_t^{(3)} - y_t^{(5)} - y_t^{(1)}$. See also Section 3.3.

curvature volatilities are associated with the volatilities of the underlying factor portfolios. In this sense, they capture time-variations in yields' variances and covariances driving the yield curve shape.

Using this structure, the unconditional moments of the yields are straightforwardly given by $\mathrm{E}[y_t] = A\,\mathrm{E}[f_t]$ and $\mathrm{V}[y_t] = A\,\mathrm{V}[f_t]\,A$, where the moments of the i-th element are given by

$$\mathrm{E}\left[f_t^{(i)}\right] = \mu^i (1-\phi^i)^{-1}, \tag{14}$$

$$\mathrm{V}\left[f_t^{(i)}\right] = \frac{1}{1-\phi^{i2}} \left[\frac{\mu_h^i}{1-\phi_h^i} + \frac{\left(\sigma_h^i\right)^2}{2(1-\phi_h^{i2})} \right], \tag{15}$$

$$\mathrm{Corr}\left[f_t^{(i)}, f_{t-k}^{(i)}\right] = \phi^{ik}, \qquad k > 0, \tag{16}$$

where $i \in \{L, S, C\}$.

Accordingly, the correlation structure in higher-order moments of de-meaned yield factors corresponds to that of a basic SV model (see Taylor (1982)) and can be approximated by

$$\mathrm{Corr}\left[a_t^{ip}, a_{t-k}^{ip}\right] \approx C\left(p, \left(\sigma_h^i\right)^2\right) \phi_h^{ik}, \qquad k > 0, \tag{17}$$

where $a_t^i := \ln\left|\eta_t^i\right| = \ln\left|f_t^{(i)} - \mu - \phi^i f_{t-1}^{(i)}\right|$ and

$$\mathcal{C}\left(p, \left(\sigma_h^i\right)^2\right) = \frac{\mathcal{A}(p, \left(\sigma_h^i\right)^2) - 1}{\mathcal{A}(p, \left(\sigma_h^i\right)^2)\mathcal{B}(p) - 1}, \tag{18}$$

$$\mathcal{B}(p) = \sqrt{\pi}\Gamma\left(p + \frac{1}{2}\right)\Gamma\left(\frac{p}{2} + \frac{1}{2}\right)^{-2}, \tag{19}$$

$$\mathcal{A}\left(p, \left(\sigma_h^i\right)^2\right) = \exp\left(p^2 \frac{\left(\sigma_h^i\right)^2}{1-\phi_h^{i2}}\right). \tag{20}$$

3 Estimating Yield Curve Factors and Factor Volatilities

3.1 Data

In order to make our results comparable to recent studies we use the same data as in Cochrane and Piazzesi (2005) consisting of monthly unsmoothed Fama-Bliss zero-coupon yields covering a period from January 1964 to December 2003 with maturities ranging between one and five years. The data is available from the Center for Research in Security Prices (CRSP) and is constructed using the method of Fama and Bliss (1987) based on end-of-month data of U.S. taxable, non-callable bonds for annual maturities up to five years. Here, each month a term structure of one-day continuously compounded forward rates is calculated from available maturities up to one year. To extend beyond a year, Fama and Bliss (1987) use the assumption that the daily forward rate for the interval between successive maturities is the relevant discount rate for each day in the interval. This allows to compute the term structure based on a step-function in which one-day forward rates are the same between successive maturities. Then, the resulting forward rates are aggregated to generate end-of-month term structures of yields for annual maturities up to five years. Summary statistics of the data are given in Panel A of Table 2.

3.2 MCMC Based Inference

The diagonal model specified above corresponds to a three-level latent hierarchical model with six latent processes. Let Θ denote the collection of the model parameters. Moreover, let $F_t := (L_t, S_t, C_t)$ and $V_t := (h_t^L, h_t^S, h_t^C)$. Then, the likelihood function of the model is given by

$$p(\Theta|Y) = \int_{F_1}\int_{F_2}\cdots\int_{F_T} p(Y|\Theta, F_1, F_2, \cdots, F_T)p(F_1, F_2, \cdots, F_T|\Theta)dF_1 dF_2 \cdots dF_T,$$

where $p(Y|\Theta, F_1, F_2, \cdots, F_T)$ denotes the (conditional) density of the data Y given the parameters Θ and the latent factors and reflects the imposed structure as given by (3) and (4). Furthermore, $p(F_1, F_2, \cdots, F_T|\Theta)$ denotes the (conditional) joint density of the latent factors, given the model parameters Θ and is determined by (5). Since the factors are unobservable, they have to be integrated out resulting in a $(3 \cdot T)$-dimensional integral. Obviously, $p(F_1, F_2, \cdots, F_T|\Theta)$ depends on a further set of unknown components as represented by the volatility factors $V_1, \ldots, V_T$. It is computed as

$$\begin{aligned} p(F_1, F_2, \cdots, F_T|\Theta) = \int_{V_1}\int_{V_2}\cdots\int_{V_T} & p(F_1, F_2, \cdots, F_T|\Theta, V_1, V_2, \cdots, V_T) \\ & \times p(V_1, V_2, \cdots, V_T|\Theta)dV_1 dV_t \cdots dV_T, \end{aligned}$$

where $p(V_1, V_2, \cdots, V_T|\Theta)$ denotes the joint density of the volatility components as determined by (7). This likelihood function cannot be computed analytically in closed form and requires numerical approximation techniques. We propose estimating the model using Markov chain Monte Carlo (MCMC) based inference. Consequently, we consider $\Omega := \{\Theta, F_1, \ldots, F_T, V_1, \ldots, V_T\}$ to be a random vector whose posterior distribution $p(\Omega|Y)$ can be arranged according to

$$\begin{aligned} p(\Omega|Y) &= p(F_1, F_2, \cdots, F_T, V_1, V_2, \cdots, V_T, \Theta|Y) \\ &\propto p(Y|F_1, F_2, \cdots, F_T, V_1, V_2, \cdots, V_T, \Theta) \\ &\times p(F_1, F_2, \cdots, F_T|V_1, V_2, \cdots, V_T, \Theta) \\ &\times p(V_1, V_2, \cdots, V_T|\Theta) \\ &\times p(\Theta). \end{aligned} \tag{21}$$

By specifying the prior distributions $p(\Theta)$ as shown in Appendix A, we utilize Gibbs and Metropolis-Hastings samplers to simulate the posterior distribution, $p(\Omega|Y)$. Then, both parameter and factor estimates are obtained by taking the sample averages of the corresponding MCMC samples.

3.3 MCMC Estimation Results

We start our analysis by estimating the model with constant volatility factors corresponding to the specification proposed by Diebold and Li (2006).[5] The estimation results are given in Panel A of Table 1. The dynamics of L_t, S_t and C_t are very persistent with estimated autoregressive coefficients of 0.98, 0.96 and 0.91, respectively. Whereas the level of interest rates is close to a unit root, the persistence of the spread component is lower but still relatively high. This finding is in strong accordance with the literature.

The model implied unconditional mean of the level factor, given by $\mu^L/(1-\phi^L)$, equals 7.96 which is close to its empirical mean of 7.12. Correspondingly, the mean value of the slope factor equals -1.96 reflecting that during the sample period the yield curve has been upward sloped on average.[6] Finally, the mean of the curvature factor is -0.28 but not significantly different from zero. Hence, on average we do not observe a strong curvature in the yield curve. The estimated decay factor λ equals 0.055, implying the curvature loading $(1-\exp(-\lambda n))/(\lambda n)-\exp(-\lambda n)$ to be maximized for a maturity of 2.72 years. The last column in Table 1 reports the Geweke (1992) Z-scores which are used to test the convergence of the Markov chains generated from the MCMC algorithm.[7] It turns out that all Markov chains have been properly converged. The

[5]Exploiting the linearity of this specification, it could be alternatively estimated using quasi maximum likelihood based on the Kalman filter, see e.g. Harvey (1990). However, to keep our econometric approach consistent, we estimate all specifications in this paper using MCMC techniques.

[6]Recall that we define the slope as the difference between short yields and long yields.

[7]For details, see Appendix A.

descriptive statistics shown in Panel B of Table 2 indicate that the dynamic Nelson-Siegel model captures a substantial part of the dynamics in the yields confirming the findings by Diebold and Li (2006). Nevertheless, remaining autocorrelations in the residuals as well as squared residuals indicate that there are neglected dynamics in the first and second moments of the process.

Figure 2 plots the resulting estimated Nelson-Siegel factors and their corresponding empirical approximations. We observe that the estimated slope factor is nearly perfectly correlated with its empirical counterpart yielding a correlation of -0.99. The corresponding correlations for the level and curvature factors are 0.90 and 0.59 indicating that level and slope factors can be easily approximated by their corresponding empirical counterparts whereas approximations of the curvature factor tend to be rather difficult.

Panel B of Table 1 shows the parameter estimates of the model with stochastic volatility components, given by equations (3) - (5) and (7). The estimated decay parameter equals 0.045 implying that the curvature loading is maximized at a maturity of 3.33 years. The estimates of the yield factor parameters are close to those of the constant volatility model. The estimated dynamic parameters in the volatility components are 0.977, 0.964 and 0.933 for the level, slope and curvature volatilities, respectively. Hence, as for the yield curve factors we also find a high persistence in the stochastic volatility processes. This is particularly true for the level and slope volatility.

4 Explaining Bond Returns Using Yield Factors

4.1 Nelson-Siegel Factors

In this section, we examine the explanatory power of the extracted Nelson-Siegel factors for future bond excess returns. In line with Cochrane and Piazzesi (2005), we regress the monthly one-year-ahead bond excess returns with maturities of two up to five years on the estimated level, slope and curvature factors, i.e.,

$$z_t^{(n)} = c + \beta_L L_{t-12} + \beta_S S_{t-12} + \beta_C C_{t-12} + \varepsilon_t^{(n)}, \quad n = 2, 3, 4, 5. \tag{22}$$

Panel A of Table 3 reports the estimation results based on alternative regressions. Two caveats should be taken into account. Firstly, because of the overlapping windows, the errors $\varepsilon_t^{(n)}$ are per construction strongly autocorrelated. In accordance with Cochrane and Piazzesi (2005) we apply the classical heteroscedasticity and autocorrelation consistent (HAC) estimators proposed by Hansen and Hodrick (1980) given by

$$Cov[\hat{b}] = E[x_t x_t']^{-1} \left[\sum_{j=-k}^{k} E[x_t x_{t-j}' \epsilon_{t+1} \epsilon_{t+1-j}] \right] E[x_t x_t']^{-1} \tag{23}$$

and the well-known (Bartlett) kernel estimator proposed by Newey and West (1987) given by

$$Cov[\hat{b}] = E[x_t x_t']^{-1} \left[\sum_{j=-k}^{k} \frac{k-|j|}{k} E[x_t x_{t-j}' \epsilon_{t+1} \epsilon_{t+1-j}] \right] E[x_t x_t']^{-1}, \tag{24}$$

where x_t denotes the vector of regressors and j denotes the order of lag truncation.

Secondly, high persistence in the yield factors used as regressors might cause spurious effects affecting the R^2. Accordingly, we support evaluations of the R^2-values using Newey-West and Hansen-Hodrick adjusted tests for joint significance as well as the Bayes Information Criterion (BIC).[8] Finally, as shown below, the explanatory power arises typically from those factors which reveal the lowest persistence. This is evidence against spurious correlation effects and confirms the robustness of our results.

In fact, it is shown that the level factor is virtually insignificant and has no explanatory power for future bond excess returns. Neglecting the latter in the regression reduces the R^2 values[9] and Hansen Hodrick HAC χ^2-statistics for joint significance only slightly. This result is mostly true for maturities longer than two years. These results are consistent with the essentially affine term structure model by Duffee (2002) that the level factor is irrelevant for bond excess returns. Though the Nelson-Siegel framework is different from Duffee's approach, the extracted yield factors behave in a quite similar way. Actually, Diebold, Piazzesi, and Rudebusch (2005) stress that the loadings in (3) are quite close to those estimated from the three factor essentially affine model.

In contrast, the coefficients for the slope and curvature factor are highly significant. We find that future excess returns decrease with the slope (defined as short minus long) and increase with the curvature. This result is consistent with, for instance, Fama and Bliss (1987) and Campbell and Shiller (1991). The positive coefficient for the curvature factor indicates that future excess returns are expected to be higher the more hump-shaped, i.e. convex or concave the current yield curve. Hence, a major factor driving future excess returns is the yield spread between mid-term and short-term bonds.

[8] For sake of brevity, these measures are not shown in the paper.

[9] Throughout the paper, the R^2 refers to the coefficient of determination, adjusted by the number of regressors.

Including all yield factors leads to an R^2 of up to 36 percent, revealing basically the same explanatory power as in Cochrane and Piazzesi (2005) using their "tent-shaped" return-forecasting factor. The corresponding χ^2-values are clearly well above the five percent critical value indicating that Nelson-Siegel factors jointly do contain significant information for future bond excess returns. Obviously, the explanatory power arises mainly from the slope and curvature factors which are statistically significant for all individual bonds. Omitting both factors from the regressions clearly reduces the R^2 and χ^2-values. This is particularly true for longer maturities and the curvature factor which turns out to be most important for explaining future excess returns.

4.2 The Cochrane-Piazzesi Return-Forecasting Factor

Cochrane and Piazzesi (2005) propose forecasting bond excess returns with the so called return-forecasting factor, ϑ_t, defined as a linear combination

$$\vartheta_t = \gamma' f_t \tag{25}$$

of five forward rates $f_t = (1, y_t^{(1)}, f_t^{(2)}, \cdots, f_t^{(5)})$ with weights $\gamma = (\gamma^{(0)}, \gamma^{(1)}, \cdots, \gamma^{(5)})$. The weights are estimated by running a (restricted) regression of average (across maturity) excess returns on the forward rates,

$$\frac{1}{4}\sum_{n=2}^{5} z_t^{(n)} = \gamma^{(0)} + \gamma^{(1)} y_{t-12}^{(1)} + \gamma^{(2)} f_{t-12}^{(2)} + \cdots + \gamma^{(5)} f_{t-12}^{(5)} + u_t. \tag{26}$$

Then, the return forecasting regression for individual bond excess returns is given by

$$z_t^{(n)} = b^{(n)} \vartheta_{t-12} + \varepsilon_t^{(n)}, \quad n = 2, 3, 4, 5, \tag{27}$$

with regression coefficients $b^{(n)}$ and the restriction $\frac{1}{4}\sum_{n=2}^{5} b^{(n)} = 1$.

Cochrane and Piazzesi (2005) show that ϑ_t contain information for future excess returns which are not captured by yield factors represented by the first three principal components of the yield covariance matrix. As suggested by Litterman and Scheinkman (1991), the latter serve as empirical proxies for the level, slope and curvature movements of the term structure. Panel A in Table 4 reports the R^2 values and Hansen-Hodrick HAC χ^2-statistics based on regressions where z_t^n is regressed on (i) the principal components (PC's), (ii) the return-forecasting factor, ϑ_t, and (iii) the Nelson-Siegel yield curve factors. It turns out that both the return-forecasting factor and the Nelson-Siegel factors have effectively the same explanatory power with $R^2 \approx 0.37$ implied by ϑ_t and $R^2 \approx 0.36$ implied by the Nelson-Siegel factors. This result is confirmed by the χ^2-statistics which are quite similar for longer maturities. The close correspondence between the return-forecasting factor and yield curve factors does not hold if the latter are constructed from principal components of the covariance matrix. Principal component factors reveal a significantly lower explanatory power with an R^2 of approximately 0.25 and clearly reduced χ^2-statistics. Figure 3 shows the Nelson-Siegel curvature loading $((1 - e^{-\lambda_t n})/\lambda_t n) - e^{-\lambda_t n}$, the return-forecasting factor loading $\gamma^{(n)}$, and the loading of the third PC factor. We observe that both the return-forecasting factor and the Nelson-Siegel curvature factor are curved at the long end of the yield curve, whereas the PC curvature is curved only at the short end. Cochrane and Piazzesi (2005) argue that in order to capture relevant information about future bond excess returns contained in the four-year to five-year yield spread, the factor loading should be curved at the long end.[10] This might explain why the Nelson-Siegel yield factors outperform the PC yield factors and why the former have similar explanatory power as the return-forecasting factor. It also stresses the importance of the curvature factor.

Corresponding results for a regression of both the Nelson-Siegel yield factors and the Cochrane-Piazzesi forecasting factor is shown in Panel B of Table 4. As expected, the explanatory power increases only slightly since both types of factors capture similar information for expected bond returns.

5 Explaining Bond Returns Using Factor Volatilities

As stressed above, the extracted factor volatilities can be heuristically interpreted as the volatilities of factor portfolios representing the level, steepness and convexity of the yield curve. A crucial question is whether riskiness in the yield curve is reflected in future bond excess returns and give rise to a risk premium.

Panel B of Table 3 shows the results of the regression

$$z_t^{(n)} = c + \alpha_L h_{t-12}^L + \alpha_S h_{t-12}^S + \alpha_C h_{t-12}^C + \varepsilon_t^{(n)}. \tag{28}$$

It turns out that the volatility factors contain significant information for future bond excess returns. Including all volatility components yields an R^2 of up to 18 percent with all factors being jointly significant.[11] The main explanatory power comes from the slope

[10] Note that the return-forecasting factor is only 'tent-shaped' when it is estimated from forward rates. If it is estimated from yields, it is curved at the long end.

[11] A potential explanation for the predictive power of volatility components for future excess returns could be that we predict *log* returns instead of simple returns. However, redoing the whole analysis

and curvature factor volatility, but not from the level volatility. Most interestingly, the impact of the slope volatility on future excess returns is negative. Hence, increasing uncertainty regarding the slope of the yield curve decreases future bond return premia. I.e., if the yield curve slope turns out to be stable, positive excess returns become more likely. This result is in contrast to the hypothesis of a positive risk premium and is rather in line with a 'stability compensation'. In contrast, we find that future bond excess returns increase with the curvature volatility. As discussed above, the latter reflects the time-varying uncertainty regarding the convexity or concavity of the yield curve, respectively, and is dominantly driven by the riskiness of mid-term bonds. Hence, our results provide evidence that the riskiness regarding yield curve slope and yield curve convexity work in opposite directions: Investors are compensated for taking risk regarding medium-term maturities and avoiding risk regarding long-term maturities. Hence, future excess returns are expected to be highest if spreads between long-term and short-term bonds are high and stable but the yield curve convexity is uncertain.

Panel C of Table 3 shows the corresponding estimation results when we control for the yield curve factors themselves. It turns out that the use of both Nelson-Siegel factors and factor volatilities yields an R^2 of about 50 percent. Hence, the inclusion of volatility factors in addition to yield factors shifts the R^2 from 36 percent to up to 50 percent. This indicates that factor volatilities have significant explanatory power for future excess returns even when we account for yield curve factors. These results are also strongly supported by a significant increase of the χ^2-statistics showing that this additional prediction power mainly stems from the slope and curvature volatility.

The regression results shown in Panel C of Table 4 show that the volatility factors have also explanatory power beyond the Cochrane-Piazessi return-forecasting factor. Actually, the R^2 increases from 36 percent to up to 42 percent if the volatility factors are added to the Cochrane-Piazzesi return-forecasting factor. This implies that the volatility factors do contain significant information on bond excess returns which is neither subsumed by yield curve factors nor by the return-forecasting factor.

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based on *simple* returns even enforces our results and indicates that our findings are not due to a predictable volatility components in the mean of log returns.

6 Yield Factors, Factor Volatilities, and Macroeconomic Fundamentals

In order to analyze in which sense yield factors and factor volatilities are connected to underlying macroeconomic fundamentals, we relate the former to the inflation rate (INF), measured by monthly relative changes of the consumer price index, manufacturing capacity utilization (CU), the federal funds rate (FFR), employment growth (EMP) as well as industrial production (IP). The choice of the variables is motivated by the results by Diebold, Rudebusch, and Aruoba (2006) who identify manufacturing capacity utilization, the federal funds rate as well as annual price inflation as the minimum set of important variables driving the term structure of interest rates. We augment the set of variables to account also for labor market activity.

To analyze the mutual correlations between yield factors and macroeconomic fundamentals we regress the yield factors on the contemporaneous (monthly) macroeconomic variables, i.e.,

$$F_t = \mu + \beta_1 INF_t + \beta_2 IP_t + \beta_3 FFR_t + \beta_4 EMP_t + \beta_5 CU_t + \varepsilon_t, \tag{29}$$

where $F_t := \{L_t, S_t, C_t, h_t^L, h_t^S, h_t^C\}$. The results reported by Table 5 show that the federal funds rate and capacity utilization are significant determinants of the level and slope factor and explain a substantial part in variations of the latter.[12] The positive signs for FFR and negative signs for CU are economically plausible and in line with theory. While the level and slope factor are closely connected to monetary policy and macroeconomic activity, only a small fraction of variations in the yield curve curvature can be explained by the latter.

Moreover, it turns out that not only the yield curve factors themselves but also their volatilities are significantly related to underlying macroeconomic dynamics. It turns out that periods of high inflation and capacity utilization are accompanied by a lower volatility in interest rate levels which might be explained by monetary policy interventions. Moreover, we find evidence for leverage effects in the sense of higher (lower) level and slope volatilities in periods of higher (lower) federal fund rate levels. This confirms the results by Engle, Ng, and Rothschild (1990) and Engle and Ng (1993)

[12] As above, one might argue that the R^2 values should be treated with caution since some of the regressors, such as CU and IP, are relatively persistent and might cause spurious correlation effects. However, robust tests on joint significance of the regressors yield the same conclusions. Moreover, the low explanatory power of the curvature factor regression indicates that spurious effects cannot be the major reason for high R^2's.

of (positive) GARCH-in-Mean effects.[13] Whereas the curvature factor is not easily explained by observable macroeconomic variables, this is not true for the corresponding volatility. Actually, we observe that particularly the federal funds rate, the employment growth rate as well as capacity utilization are significant determinants of the time-varying uncertainty in the yield curve shape yielding an R^2 of about 0.48. It turns out that periods of a high federal funds rate, low capacity utilization and negative employment growth induce higher variations in medium-term bonds and thus the term structure convexity. Overall, we can summarize that factor volatilities are even closer connected to observable macroeconomic variables than the factors themselves.

To study the dynamic interdependencies between yield factors and macroeconomic variables, we estimate a VAR(1) model of monthly yield factors and the macroeconomic fundamentals,

$$F_t = \mu + A\, F_{t-1} + \varepsilon_t, \tag{30}$$

where $F_t := \{L_t, S_t, C_t, INF_t, IP_t, FFR_t, EMP_t, CU_t\}$.

Based on the results shown in Table 6 we can summarize the following results: Firstly, the yield factors primarily depend on their own lags but not on those of the other factors which confirms the diagonal specification of Φ in (9). Secondly, we observe that the yield factors are not (short-term) predictable based on macroeconomic fundamentals. This is particularly true for the level and the curvature factor whereas for the curvature factor slight dependencies from lagged inflation rates, federal fund rates, and employment growth rates are observable. Thirdly, it turns out that level and slope factors have significant short-term prediction power for nearly all macroeconomic variables. In particular, rising interest rate levels and yield spreads predict increases in industrial production, federal funds rates, the growth rate of employment as well as the capacity utilization. In contrast, the term structure curvature contains no information for one-month-ahead macroeconomic variables. Overall these results generally confirm those by Diebold, Rudebusch, and Aruoba (2006).

Table 7 shows the results for VAR(1) regressions where we include the factor volatilities, i.e. F_t is chosen as $F_t := \{h_t^L, h_t^S, h_t^C, INF_t, IP_t, FFR_t, EMP_t, CU_t\}$. It turns out that most of the (short-term) dynamics are driven by process-own dependencies confirming also the assumption of a diagonal structure of H_t in (10). Moreover, we observe

[13] In preliminary studies we found evidence for SV-in-Mean effects for the level factor. Given the close relation between the federal funds rate and the level of interest rates this effect is now obviously reflected in the present regressions. The results are not shown here but are available upon request from the authors.

that the level volatility is dominantly predicted by past level and slope volatilities but not by macroeconomic variables. Similar relations are also observed for the slope volatility where the latter also significantly (positively) depends on the lagged federal funds rate. In contrast, the curvature volatility depends solely on its own history. Hence, we can conclude that in the short run term structure volatilities are not predictable based on macroeconomic factors. Conversely we observe a weak predictability of the level volatility for future macroeconomic fundamentals. In particular, higher level volatilities predict increases in industrial production, employment growth rates as well as decreasing inflation rates and manufacturing capacity utilizations. Similar effects on inflation rates and capacity utilization is observed for the slope volatility. Interestingly, the strongest impact on future macroeconomic variables stems from the curvature factor which has significant prediction power for all macroeconomic factors. This finding illustrates again the importance of term structure curvature risk confirming our results above.

Long-term relations between the individual variables are analyzed based on prediction error variance decompositions (see e.g. Hamilton (1994)) implied by the VAR estimates discussed above. The corresponding plots are shown in Figures 5 to 12. We observe that not only in the short run but also in the long run macroeconomic variables virtually do not contribute to the prediction error variances in yield curve levels and curvatures. Only for the yield curve slope, particularly capacity utilization and industrial production can explain about 25% in prediction error variances after 100 months. Conversely, we observe significantly higher long-run forecasting ability of yield term factors for macroeconomic fundamentals. This is particularly apparent for the federal funds rate whose prediction error variance is dominated by the level and slope factor (by nearly 80%). For CU, EMP and IP we observe that yield curve factors - predominantly level and slope - can explain around 40% in long-run prediction error variances. Hence, in line with Diebold, Rudebusch, and Aruoba (2006) we conclude that level factors serve as long-run predictors of future industrial utilization, employment growth and short-term monetary policy. A notable exception is the inflation rate which is not predictable based on yield curve factors, neither over the short run nor the long run.

Figures 9 to 12 show the corresponding variance decompositions implied by the VAR estimates for $F_t := \{h_t^L, h_t^S, h_t^C, INF_t, IP_t, FFR_t, EMP_t, CU_t\}$. It is evident that macroeconomic fundamentals explain a major part in long-term prediction error variances of level and slope volatilities. Particularly capacity utilization and industrial production explain approximately 50% and 40% in long-term prediction error variances of the level volatility and slope volatility, respectively. In contrast, long-term prediction error variances of curvature volatility can be explained by less than 20%. Vice versa, we again observe an important role of curvature volatility for the prediction of future macroeconomic activity. This is particularly true for capacity utilization, employment growth and industrial production whose prediction error variation after 100 months is significantly influenced by the current curvature volatility. In contrast, virtually no long-run explanatory power of level and slope volatilities for future macroeconomic variables can be identified. Hence, we observe that particularly the uncertainty with respect to the shape of the yield curve has long-term consequences for capacity utilization and employment growth.

Further insights into the role of the extracted factor volatilities can be gained by Figure 4 which plots the former over the sample period. It turns out that the slope volatility peaks in April 1974, April 1980 and March 2001 corresponding to three major economic recession periods in the U.S. as identified by the National Bureau of Economic Research (NBER). Viewing the slope factor as a short-term factor, its high fluctuations in these periods might be attributed to monetary policy reflected in short-term yields during economic recessions. The same pattern is observed for the curvature volatility capturing mainly the uncertainty in medium-term yields and significantly peaking during all recessions periods. Hence, we observe that interest rate risk during economic recessions is dominantly reflected in the shape of the yield curve but not in the overall level.

7 Conclusions

We propose a dynamic Nelson-Siegel type yield curve factor model, where the underlying factors reveal stochastic volatility. By estimating the model using MCMC techniques we extract both the Nelson-Siegel factors as well as their volatility components and use them to explain bond return premia and to relate them to underlying macroeconomic variables. This approach allows us to link the approaches by Diebold and Li (2006), Diebold, Rudebusch, and Aruoba (2006), Cochrane and Piazzesi (2005) on factor-based term structure modeling with the GARCH-in-Mean models by Engle, Ng, and Rothschild (1990) and Engle and Ng (1993) capturing interest rate risk premia.

We can summarize the following main findings: (i) We find that the slope and curvature factors extracted from the dynamic Nelson-Siegel model describe time-variations in future yearly bond excess returns with an R^2 of up to 36 percent. (ii) The Nelson-Siegel yield factors have basically the same explanatory power as the return-forecasting factor proposed by Cochrane and Piazzesi (2005). This result arises mainly because of a close similarity between the loadings of the "tent-shaped" return-forecasting factor and that of the slope and curvature Nelson-Siegel factor. This forecasting performance is not achieved when principal components are used as predictors. (iii) We show that the time-varying volatility associated with the level, slope and curvature factors have significant explanatory power for future excess returns beyond the factors themselves. Including the extracted factor volatilities in rolling window regressions increases the (adjusted) R^2 to approximately 50 percent. It turns out that the explanatory power in the volatility factors mainly stem from the risk inherent to the yield curve's slope and curvature. (iv) We document that riskiness regarding the yield curve shape (convexity) but not the riskiness regarding the slope induces a positive risk premium in excess returns. Actually, we find that slope uncertainties decrease future bond return premia revealing a compensation for stability in term structure slopes. (v) Yield term factors and - to an even larger extent - factor volatilities are closely connected to key macroeconomic variables reflecting capacity and production utilization, employment growth, inflation and monetary policy. (vi) We observe that macroeconomic variables have more long-run predictability for term structure volatilities than for the term structure itself. It turns out that capacity utilization and industrial production are important long-term predictors for risk inherent to the level and slope of the yield curve. Conversely, we observe that yield factors have significant forecasting ability for capacity utilization, employment growth and industrial production but only negligible impacts on the volatilities thereof. Nevertheless, we identify an important role of the curvature volatility for long-term predictions of macroeconomic variables. These results provide hints that risk inherent to the shape of the yield curve is relevant and seems to be effectively captured by a stochastic volatility component in the curvature factor.

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A MCMC based Bayesian Inference

Let Ω collect all model parameters including the latent variables, and let Y denote the observed data. Applying Clifford-Hammersley's theorem (see Hammersley and Clifford (1971), Besag (1974)), the posterior distribution

$$p(\Omega|Y) \propto p(Y|\Omega)p(\Omega) \tag{31}$$

can be broken up into a complete set of conditional posteriors, $p(\Omega_i|\Omega_{-i}, Y), i = 1, \ldots, N$, where $p(\Omega)$ denotes the prior distribution of Ω, N is the number of blocks, Ω_i denotes the i-th block and Ω_{-i} denotes all the elements of Ω excluding Ω_i. Then, the elements Ω_i can be sampled according to the following Markov chain:

- Initialize $\Omega^{(0)}$.
- For $i = 1, \ldots, G$:
  1. draw $\Omega_1^{(i)}$ from $p(\Omega_1|\Omega_2^{(i-1)}, \Omega_3^{(i-1)}, \cdots, \Omega_N^{(i-1)}, Y)$,
  2. draw $\Omega_2^{(i)}$ from $p(\Omega_2|\Omega_1^{(i)}, \Omega_3^{(i-1)}, \cdots, \Omega_N^{(i-1)}, Y)$,

     $\vdots$

  N. draw $\Omega_N^{(i)}$ from $p(\Omega_N|\Omega_1^{(i)}, \Omega_2^{(i)}, \cdots, \Omega_{N-1}^{(i)}, Y)$,

where G is the number of MCMC iterations. In dependence of the form of the conditional posteriors we employ Gibbs or Metropolis-Hastings samplers as implemented in the software package BUGS (see Spiegelhalter, Thomas, Best, and Gilks (1996)). The procedure works well, is easily implemented but is relatively inefficient in the given context. In order to guarantee a proper convergence of the Markov chain we run 2,500,000 MCMC iterations with a burn-in period of 500,000 iterations.[14]

All model parameters are assumed to be a priori independent and are distributed as follows:

- Σ is the variance-covariance matrix with zero off-diagonal elements of equation (3). We assume that each of its elements follows an Inverse-Gamma(2.5,0.025) distribution with mean of 0.167 and standard deviation 0.024.
- For λ we assign a uniform distribution on the interval $[0, 1]$.
- For the persistent parameters of the yield curve factors ϕ^i, $i = L, S, C$, we assume their transformations $(\phi^i + 1)/2$ to follow a beta distribution with parameters 20 and 1.5 implying a mean of 0.86 and a standard deviation of 0.11.

[14] More efficient estimation algorithms for the model are on the future research agenda but are beyond the scope of the current paper.

- μ^i, $i = L, S, C$ in (5) are assumed to be independently normally distributed with mean 0 and variance 10.
- h^i, $i = L, S, C$ in (5) are assumed to follow an Inverse-Gamma(2.5,0.025) distribution.
- For ϕ_h^i, $i = L, S, C$ in (7), we assume their transformations $(\phi^i + 1)/2$ to follow a beta distribution with parameters 20 and 1.5 implying a mean of 0.86 and a standard deviation of 0.11.
- μ_h^i, $i = L, S, C$ in (7) are assumed to be independently normally distributed with mean 0 and variance 10.
- σ_h^i, $i = L, S, C$ in (7) are assumed to follow an Inverse-Gamma(2.5,0.025) distribution.
- d^i, $i = L, S, C$ are assumed to be normally distributed with mean 0 and variance 10.

To test for the convergence of the generated Markov chain, we use the Z-score by Geweke (1992). Let $\{\Omega^{(i)}\}_{i=1}^{G}$ denote the generated Markov chain with

$$\bar{\Omega}_1 = \frac{1}{G_1}\sum_{i=1}^{G_1}\Omega^{(i)}, \quad \bar{\Omega}_2 = \frac{1}{G_2}\sum_{i=p^*}^{G}\Omega^{(i)}, \quad p^* = G - G_2 + 1, \tag{32}$$

and let $\hat{S}^1(0)$ and $\hat{S}^2(0)$ denote consistent spectral density estimates (evaluated at zero) for $\{\Omega^{(i)}\}_{i=1}^{G_1}$ and $\{\Omega^{(i)}\}_{i=p^*}^{G}$, respectively. If the sequence $\{\Omega^{(i)}\}_{i=1}^{G}$ is stationary, then as $G \to \infty$,

$$(\bar{\Omega}_1 - \bar{\Omega}_2)/[G_1^{-1}\hat{S}^1(0) + G_2^{-1}\hat{S}^2(0)] \xrightarrow{d} N(0,1) \tag{33}$$

given the ratios G_1/G and G_2/G are fixed, and $(G_1 + G_2)/G < 1$. Geweke (1992) suggests using $G_1 = 0.1G$ and $G_2 = 0.5G$.

Table 1: MCMC estimation results for dynamic Nelson-Siegel models. Based on monthly observations of unsmoothed U.S. Fama-Bliss zero coupon yields from January 1964 to December 2003 with maturities of one to five years. 480 observations.

A. Model without volatility factors

	μ^L	μ^S	μ^C	ϕ^L	ϕ^S	ϕ^C	h^L	h^S	h^C	λ
Mean	0.028	-0.053	-0.028	0.994	0.956	0.906	0.307	0.597	0.757	0.055
SD	0.014	0.030	0.036	0.003	0.013	0.023	0.022	0.024	0.060	0.006
95% CI, lower	0.007	-0.116	-0.099	0.987	0.929	0.860	0.266	0.552	0.637	0.046
95% CI, upper	0.062	-0.001	0.042	0.998	0.980	0.947	0.351	0.645	0.871	0.069
Z-score	-0.171	1.198	1.615	0.196	1.352	-0.507	-1.737	-0.381	1.418	0.172

B. Model with stochastic volatility factors

	μ^L	μ^S	μ^C	ϕ^L	ϕ^S	ϕ^C	μ_h^L	μ_h^S	μ_h^C	ϕ_h^L	ϕ_h^S	ϕ_h^C	$(\sigma_h^L)^2$	$(\sigma_h^S)^2$	$(\sigma_h^C)^2$	λ
Mean	0.023	-0.008	-0.025	0.994	0.981	0.879	-0.082	-0.083	-0.038	0.977	0.964	0.933	0.246	0.343	0.269	0.045
SD	0.011	0.008	0.034	0.002	0.006	0.024	0.053	0.037	0.023	0.016	0.017	0.034	0.078	0.070	0.083	0.002
95% CI, lower	0.005	-0.028	-0.092	0.988	0.966	0.830	-0.214	-0.169	-0.096	0.936	0.923	0.851	0.134	0.225	0.042	0.042
95% CI, upper	0.049	0.004	0.042	0.998	0.993	0.925	-0.017	-0.024	-0.005	0.996	0.992	0.987	0.439	0.498	0.048	0.048
Z-score	-1.647	-0.902	-1.085	1.615	0.878	1.463	-0.594	0.772	-1.196	-0.486	0.532	-1.383	0.378	-0.493	0.454	0.568

"95% CI" denotes the 95% credibility interval of the posterior distribution. The Z-score statistic is the Geweke (1992) test statistic for the convergence of MCMC samples, see Appendix A.

Table 2: Summary statistics of the data and model residuals

A. Zero yields from January 1964 to December 2003

				Autocorrelation of residuals										Autocorrelation of squared residuals									
	Mean	SD		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
1	6.520	2.719		0.975	0.945	0.919	0.895	0.876	0.854	0.832	0.818	0.800	0.780	0.963	0.920	0.889	0.865	0.852	0.829	0.808	0.802	0.786	0.769
2	6.741	2.633		0.979	0.953	0.929	0.908	0.890	0.870	0.851	0.834	0.816	0.798	0.973	0.938	0.911	0.890	0.874	0.854	0.835	0.823	0.808	0.792
3	6.914	2.538		0.980	0.957	0.936	0.917	0.900	0.882	0.863	0.846	0.829	0.811	0.977	0.948	0.926	0.908	0.893	0.875	0.855	0.840	0.824	0.808
4	7.049	2.484		0.980	0.959	0.941	0.922	0.906	0.889	0.870	0.854	0.836	0.818	0.978	0.953	0.934	0.917	0.903	0.885	0.864	0.849	0.831	0.814
5	7.127	2.439		0.982	0.963	0.945	0.928	0.913	0.897	0.880	0.864	0.848	0.832	0.981	0.958	0.940	0.924	0.911	0.894	0.875	0.861	0.846	0.831

B. Residuals of the model with constant volatility factors

				Autocorrelation of residuals										Autocorrelation of squared residuals									
	Mean	SD	MAE	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
1	0.002	0.049	0.0355	-0.114	-0.069	0.169	-0.077	0.026	0.055	-0.142	0.088	0.040	-0.012	0.223	0.135	0.158	0.179	0.129	0.042	0.137	0.150	0.202	0.145
2	-0.006	0.066	0.0487	0.360	0.257	0.316	0.130	0.073	0.076	0.086	0.105	0.198	0.147	0.221	0.214	0.078	0.241	0.191	0.116	0.141	0.115	0.252	0.208
3	-0.001	0.046	0.0307	0.114	0.073	0.194	0.038	0.036	0.171	-0.005	-0.008	0.091	-0.007	0.140	0.050	0.248	0.126	0.036	0.022	0.118	0.013	0.046	0.035
4	0.011	0.066	0.0463	0.283	0.157	0.386	0.159	0.124	0.188	0.017	0.084	0.006	-0.089	0.196	0.142	0.329	0.163	0.078	0.149	0.057	0.089	0.181	0.246
5	-0.003	0.036	0.0267	0.049	0.042	0.042	0.005	0.002	0.069	0.015	-0.026	0.096	-0.107	0.288	0.208	0.236	0.144	0.171	0.155	0.105	0.241	0.121	0.156

C. Residuals of the model with stochastic volatility factors

				Autocorrelation of residuals										Autocorrelation of squared residuals									
	Mean	SD	MAE	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
1	0.001	0.035	0.027	-0.108	-0.100	0.089	0.006	0.021	0.031	-0.115	0.007	0.088	0.020	0.065	0.019	0.056	0.032	0.009	0.000	0.010	0.065	0.134	0.033
2	-0.009	0.071	0.053	0.346	0.249	0.327	0.129	0.085	0.088	0.076	0.104	0.196	0.139	0.198	0.204	0.064	0.176	0.207	0.078	0.134	0.121	0.220	0.193
3	-0.003	0.046	0.032	0.141	0.086	0.182	0.036	0.023	0.164	-0.021	-0.004	0.085	-0.010	0.155	0.063	0.226	0.131	0.045	0.019	0.119	0.020	0.063	0.016
4	0.009	0.065	0.045	0.301	0.182	0.375	0.162	0.127	0.174	0.020	0.090	0.005	-0.090	0.203	0.167	0.329	0.172	0.079	0.159	0.052	0.100	0.183	0.255
5	-0.005	0.037	0.029	0.087	0.052	0.081	0.038	0.018	0.033	-0.003	-0.014	0.075	-0.105	0.139	0.151	0.124	0.103	0.012	0.009	-0.001	0.091	0.086	0.081

The individual rows are associated with the corresponding maturities of the underlying data. The columns give the corresponding lags. MAE denotes the mean absolute error defined as MAE= $\frac{1}{T}\sum_{t=1}^{T}|y_t^{(n)} - \hat{y}_t^{(n)}|$.

Table 3: Monthly regressions of one-year-ahead bond excess returns on Nelson-Siegel yield factors and factor volatilities. Yield factors and factor volatilities extracted from monthly observations of unsmoothed U.S. Fama-Bliss zero coupon yields from January 1964 to December 2003 with maturities of one to five years.

A. Regression: $z_t^{(n)} = c + \beta_L L_{t-12} + \beta_S S_{t-12} + \beta_C C_{t-12} + \varepsilon_t^{(n)}$

	All yield factors							No level factor						No slope factor						No curvature factor					
n	c	β_L	β_S	β_C	Adj.R^2	HH	NW	c	β_S	β_C	Adj.R^2	HH	NW	c	β_L	β_C	Adj.R^2	HH	NW	c	β_L	β_S	Adj.R^2	HH	NW
2	−0.92	0.14	−0.25	0.51	0.31	43.8	49.9	0.18	−0.26	0.57	0.29	34.4	39.8	−0.68	0.15	0.53	0.25	25.3	29.2	−1.78	0.24	−0.27	0.17	12.2	15.3
	(−1.31)	(1.54)	(−2.37)	(4.41)				(0.61)	(−2.09)	(4.10)				(−0.73)	(1.22)	(3.80)				(−1.88)	(1.84)	(−2.21)			
3	−0.92	0.15	−0.51	1.01	0.32	41.3	46.9	0.19	−0.52	1.07	0.32	36.7	42.5	−0.44	0.17	1.06	0.24	23.4	26.7	−2.63	0.35	−0.57	0.15	9.8	12.5
	(−0.70)	(0.82)	(−2.48)	(4.62)				(0.36)	(−2.19)	(4.36)				(−0.25)	(0.71)	(4.01)				(−1.46)	(1.35)	(−2.31)			
4	−1.40	0.18	−0.79	1.42	0.35	40.6	47.4	0.02	−0.81	1.50	0.34	37.0	43.8	−0.65	0.23	1.49	0.25	23.1	26.5	−3.80	0.48	−0.87	0.17	10.9	13.9
	(−0.77)	(0.74)	(−2.78)	(4.66)				(0.03)	(−2.45)	(4.46)				(−0.26)	(0.64)	(3.92)				(−1.53)	(1.31)	(−2.57)			
5	−1.86	0.21	−1.01	1.78	0.36	44.6	51.6	−0.26	−1.02	1.87	0.36	40.8	47.9	−0.92	0.26	1.88	0.25	25.3	28.6	−4.87	0.57	−1.10	0.18	11.5	14.7
	(−0.84)	(0.69)	(−2.93)	(4.95)				(−0.34)	(−2.57)	(4.76)				(−0.31)	(0.60)	(4.13)				(−1.62)	(1.30)	(−2.69)			

B. Regression: $z_t^{(n)} = c + \alpha_L h_{t-12}^L + \alpha_S h_{t-12}^S + \alpha_C h_{t-12}^C + \varepsilon_t^{(n)}$

	All volatility factors							No level volatility factor						No slope volatility factor						No curvature volatility factor					
n	c	α_L	α_S	β_C	Adj.R^2	HH	NW	c	α_S	α_C	Adj.R^2	HH	NW	c	α_L	α_C	Adj.R^2	HH	NW	c	α_L	α_S	Adj.R^2	HH	NW
2	−1.37	3.50	−2.95	3.11	0.18	22.1	22.3	−1.22	−2.21	3.52	0.14	13.9	14.4	−0.81	0.84	1.46	0.06	2.18	2.77	0.14	4.69	−1.59	0.07	2.54	3.18
	(−2.30)	(1.25)	(−2.18)	(4.02)				(−2.00)	(−1.9)	(3.76)				(−1.08)	(0.35)	(1.44)				(0.24)	(1.58)	(−1.15)			
3	−2.25	5.42	−5.52	5.60	0.16	20.6	21.7	−2.02	−4.37	6.23	0.14	15.9	16.7	−1.19	0.44	2.50	0.04	1.53	1.93	0.48	7.56	−3.08	0.06	2.16	2.71
	(−2.15)	(1.04)	(−2.35)	(3.93)				(−1.91)	(−2.23)	(3.98)				(−0.85)	(0.09)	(1.25)				(0.44)	(1.37)	(−1.26)			
4	−2.91	7.37	−7.95	7.54	0.16	20.6	21.5	−2.59	−6.39	8.40	0.11	16.0	16.9	−1.38	0.21	3.07	0.03	1.11	1.39	0.77	10.26	−4.67	0.07	2.32	2.92
	(−2.09)	(1.01)	(−2.51)	(3.77)				(−1.80)	(−2.41)	(3.99)				(−0.71)	(0.03)	(1.06)				(0.533)	(1.34)	(−1.39)			
5	−3.59	8.95	−9.72	8.95	0.16	19.3	20.2	−3.21	−7.82	9.99	0.13	15.3	16.2	−1.73	0.19	3.49	0.02	0.95	1.19	0.77	12.38	−5.82	0.07	2.35	2.95
	(−2.15)	(1.01)	(−2.49)	(3.65)				(−1.85)	(−2.41)	(3.90)				(−0.73)	(0.02)	(0.98)				(0.44)	(1.33)	(−1.42)			

C. Regression: $z_t^{(n)} = c + \beta_L L_{t-12} + \beta_S S_{t-12} + \beta_C C_{t-12} + \alpha_L h_{t-12}^L + \alpha_S h_{t-12}^S + \alpha_C h_{t-12}^C + \varepsilon_t^{(n)}$

	All factors									
n	c	β_L	β_S	β_C	α_L	α_S	α_C	Adj.R^2	HH	NW
2	−4.51	0.56	0.01	0.53	−6.30	−3.13	4.65	0.47	114.8	120.4
	(−5.82)	(2.73)	(0.10)	(6.56)	(−1.87)	(−2.90)	(4.88)			
3	−7.31	0.87	−0.04	1.03	−11.1	−5.54	8.29	0.46	105.9	109.4
	(−4.88)	(2.37)	(−0.20)	(6.67)	(−1.86)	(−2.91)	(4.57)			
4	−10.4	1.33	−0.11	1.45	−17.2	−7.74	10.9	0.49	112.2	117.0
	(−5.04)	(2.87)	(−0.40)	(6.81)	(−2.26)	(−3.13)	(4.32)			
5	−12.7	1.65	−0.17	1.80	−21.5	−9.31	13.1	0.50	118.8	122.8
	(−5.09)	(3.11)	(−0.49)	(7.22)	(−2.45)	(−3.05)	(4.13)			

$z_t^{(n)}$ denotes the n-year one-year ahead bond excess return. L_t, S_t and C_t denote the estimated level, slope and curvature factors, respectively. Their corresponding volatility factors are h_t^L, h_t^S and h_t^C. Both yield curve factors and volatility factors are extracted from model (3), (5) and (7). HH and NW are χ^2 statistics for joint significance tests using Hansen-Hodrick and Newey-West corrections, respectively. The 5-percent critical values for $\chi^2(2)$, $\chi^2(3)$ and $\chi^2(6)$ are 5.99, 7.82 and 12.59. The robust t-statistics based on HH corrections are reported in parentheses "( )".

Table 4: Monthly regressions of one-year-ahead bond excess returns on PCA factors, the Cochrane-Piazzesi forecasting factor and Nelson-Siegel yield factors. Yield factors extracted from monthly observations of unsmoothed U.S. Fama-Bliss zero coupon yields from January 1964 to December 2003 with maturities of one to five years.

A.

	PCA factors			Return-forecasting factor			Nelson-Siegel factors		
n	Adj. R^2	HH	p-value	Adj. R^2	HH	p-value	Adj. R^2	HH	p-value
2	0.204	36.090	0.000	0.309	65.241	0.000	0.313	49.937	0.000
3	0.212	35.984	0.000	0.336	60.443	0.000	0.321	46.971	0.000
4	0.241	34.375	0.000	0.370	55.896	0.000	0.348	47.469	0.000
5	0.247	35.338	0.000	0.343	46.686	0.000	0.361	51.607	0.000

B. Forecasting regressions: $z_t^{(n)} = \beta_L L_{t-12} + \beta_S S_{t-12} + \beta_C C_{t-12} + \varphi\vartheta_t + \varepsilon_t^{(n)}$

n	β_L	β_S	β_C	φ	Adj. R^2	HH	NW
2	0.028	−0.046	0.283	0.297	0.336	104.524	99.876
	(0.917)	(−0.530)	(2.024)	(3.760)			
3	0.022	−0.137	0.533	0.563	0.358	90.910	89.637
	(0.402)	(−0.795)	(2.172)	(3.862)			
4	0.000	−0.245	0.717	0.830	0.390	81.581	85.855
	(0.007)	(−1.016)	(2.173)	(4.167)			
5	−0.030	−0.472	1.130	0.786	0.381	67.406	73.079
	(−0.356)	(−1.618)	(2.782)	(3.110)			

C. Forecasting regressions: $z_t^{(n)} = \alpha_L h_{t-12}^L + \alpha_S h_{t-12}^S + \alpha_C h_{t-12}^C + \varphi\vartheta_{t-12} + \varepsilon_t^{(n)}$

n	α_L	α_S	α_C	φ	Adj.R^2	HH	NW
2	−1.039	−0.956	1.135	0.442	0.343	113.181	121.356
	(−0.552)	(−0.993)	(2.114)	(6.730)			
3	−3.451	−1.849	2.394	0.870	0.381	122.700	131.127
	(−1.025)	(−1.117)	(2.394)	(6.747)			
4	−5.411	−2.758	3.361	1.278	0.421	135.324	145.272
	(−1.202)	(−1.254)	(2.569)	(6.967)			
5	−6.198	−3.514	3.795	1.509	0.393	109.265	121.594
	(−1.094)	(−1.290)	(2.364)	(6.524)			

$z_t^{(n)}$ denotes the one-year-ahead bond excess return of n-year bonds. L_t, S_t and C_t denote the estimated level, slope and curvature factors, respectively. Their corresponding volatility factors are h_t^L, h_t^S and h_t^C. Both yield curve factors and volatility factors are extracted from model (3), (5) and (7). ϑ_t denotes the return-forecasting factor of Cochrane and Piazzesi (2005). HH and NW are χ^2 statistics for joint significance tests using Hansen-Hodrick and Newey-West corrections, respectively. The 5-percent critical value of $\chi^2(4)$ is 9.49.

Table 5: Linear regressions of monthly yield factors L_t, S_t, C_t, and factor volatilities h_t^L, h_t^S, h_t^C, on log changes of the consumer price index (INF), capacity utilization (CU), employment growth rate (EMP), the federal funds rate (FFR) and industrial production (IP). Yield factors and factor volatilities extracted from monthly observations of unsmoothed U.S. Fama-Bliss zero coupon yields from January 1964 to December 2003 with maturities of one to five years. Robust standard errors in parantheses.

	CONST	INF	CU	EMPLOY	FFR	IP	R^2
L	27.781 (3.355)	−0.258 (0.160)	−0.298 (0.044)	27.120 (17.756)	0.495 (0.035)	5.324 (4.787)	0.77
S	−29.404 (3.302)	0.159 (0.184)	0.322 (0.044)	−13.601 (19.192)	0.357 (0.029)	−7.225 (4.749)	0.69
C	10.248 (4.302)	−0.697 (0.378)	−0.147 (0.050)	38.976 (18.406)	0.104 (0.091)	−1.060 (6.277)	0.12
h^L	0.905 (0.206)	−0.040 (0.014)	−0.012 (0.003)	0.041 (0.827)	0.035 (0.002)	0.465 (0.285)	0.76
h^S	−0.058 (0.461)	−0.029 (0.037)	0.002 (0.006)	−3.207 (1.929)	0.071 (0.008)	−1.202 (0.846)	0.63
h^C	1.879 (0.530)	0.063 (0.038)	−0.014 (0.006)	−7.449 (2.316)	0.029 (0.010)	1.371 (0.767)	0.48

Table 6: VAR(1) estimates of the monthly yield factors L_t, S_t, C_t, log changes of the consumer price index (INF), capacity utilization (CU), employment growth rate (EMP), the federal funds rate (FFR) and industrial production (IP). Yield factors and factor volatilities extracted from monthly observations of unsmoothed U.S. Fama-Bliss zero coupon yields from January 1964 to December 2003 with maturities of one to five years. Robust standard errors in parantheses.

	L_t	S_t	C_t	INF_t	CU_t	FFR_t	IP_t	$EMPLOY_t$
L_{t-1}	0.966 (0.028)	0.085 (0.085)	−0.096 (0.072)	−0.011 (0.0264)	0.311 (0.059)	0.508 (0.133)	0.004 (0.001)	0.001 (0.000)
S_{t-1}	−0.010 (0.023)	0.982 (0.069)	−0.078 (0.063)	0.005 (0.023)	0.231 (0.055)	0.466 (0.127)	0.003 (0.001)	0.001 0.000)
C_{t-1}	−0.011 (0.009)	0.020 (0.016)	0.894 (0.023)	−0.018 (0.012)	−0.013 (0.020)	−0.031 (0.0213)	0.000 (0.000)	0.000 (0.000)
INF_{t-1}	0.046 (0.046)	−0.026 (0.108)	−0.188 (0.113)	0.211 (0.072)	0.091 (0.077)	0.083 (0.078)	0.001 (0.002)	0.000 (0.000)
CU_{t-1}	−0.004 (0.004)	0.026 (0.013)	−0.018 (0.016)	0.001 (0.006)	0.973 (0.015)	0.006 (0.012)	0.000 (0.000)	0.000 (0.000)
FFR_{t-1}	0.023 (0.020)	−0.066 (0.074)	0.098 (0.059)	0.007 (0.022)	−0.278 (0.050)	0.572 (0.114)	−0.004 (0.001)	−0.001 (0.000)
IP_{t-1}	−0.222 (0.465)	−0.978 (1.272)	−1.359 (2.014)	0.949 (0.609)	4.190 (1.752)	1.466 (1.198)	1.017 (0.029)	0.039 (0.008)
$EMPLOY_{t-1}$	0.975 (1.463)	3.674 (3.991)	8.901 (5.210)	2.381 (1.768)	−2.675 (3.702)	−4.390 (4.483)	−0.168 (0.074)	0.908 (0.022)
$CONST$	0.412 (0.355)	−2.428 (1.072)	1.225 (1.410)	−0.138 (0.501)	1.945 (1.249)	−0.812 (0.977)	0.006 (0.021)	−0.001 (0.006)

Table 7: VAR(1) estimates of the monthly factor volatilities h_t^L, h_t^S, h_t^C, log changes of the consumer price index (INF), capacity utilization (CU), employment growth rate (EMP), the federal funds rate (FFR) and industrial production (IP). Yield factors and factor volatilities extracted from monthly observations of unsmoothed U.S. Fama-Bliss zero coupon yields from January 1964 to December 2003 with maturities of one to five years. Robust standard errors in parantheses.

	h_t^L	h_t^S	h_t^C	INF_t	CU_t	FFR_t	IP_t	$EMPLOY_t$
h_{t-1}^L	0.981 (0.010)	−0.116 (0.033)	−0.058 (0.038)	−0.491 (0.246)	−4.829 (3.014)	1.138 (0.732)	0.019 (0.012)	0.005 (0.003)
h_{t-1}^S	0.014 (0.003)	0.987 (0.028)	0.019 (0.014)	−0.145 (0.070)	−2.614 (1.106)	−0.055 (0.416)	0.003 (0.005)	0.001 (0.002)
h_{t-1}^C	0.007 (0.003)	0.019 (0.014)	1.003 (0.016)	0.212 (0.102)	5.321 (1.469)	0.458 (0.286)	−0.011 (0.006)	−0.004 (0.002)
INF_{t-1}	0.001 (0.001)	0.016 (0.010)	0.004 (0.005)	0.190 (0.075)	0.888 (0.339)	0.121 (0.087)	0.002 (0.002)	0.001 (0.001)
CU_{t-1}	0.000 (0.000)	0.001 (0.001)	0.000 (0.000)	0.007 (0.006)	0.586 (0.114)	0.013 (0.015)	0.000 (0.000)	0.000 (0.000)
FFR_{t-1}	0.000 (0.000)	0.005 (0.002)	0.001 (0.002)	0.023 (0.011)	0.361 (0.156)	0.945 (0.026)	−0.001 (0.000)	0.000 (0.000)
IP_{t-1}	0.017 (0.020)	−0.064 (0.083)	−0.069 (0.098)	0.634 (0.620)	−31.098 (11.181)	−1.487 (1.568)	1.025 (0.027)	0.044 (0.008)
$EMPLOY_{t-1}$	0.095 (0.056)	0.274 (0.219)	0.325 (0.283)	2.436 (1.708)	143.696 (36.878)	9.511 (3.923)	−0.162 (0.076)	0.904 (0.024
$CONST$	−0.027 (0.014)	−0.074 (0.058)	−0.016 (0.073)	−0.751 (0.511)	25.893 (8.430)	−1.460 (1.315)	0.030 (0.027)	0.010 (0.008)

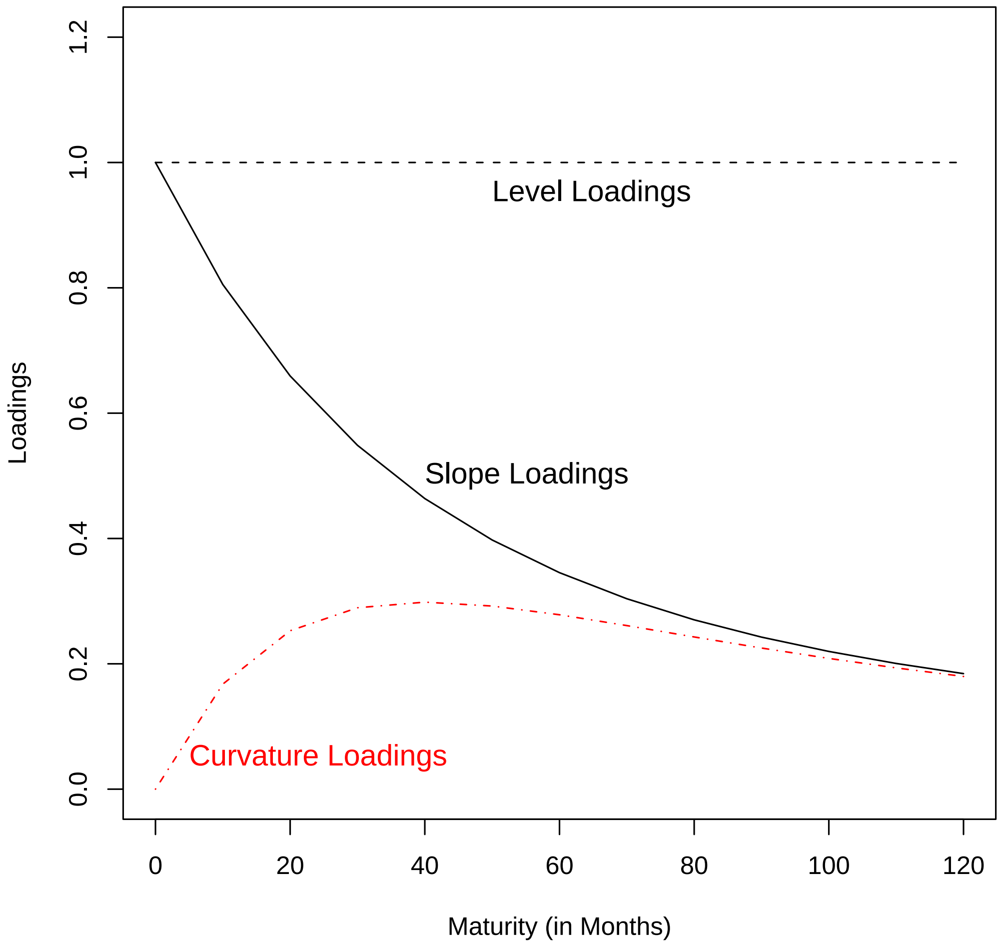


Figure 1: Plot of the Nelson-Siegel factor loadings. $\lambda = 0.045$.

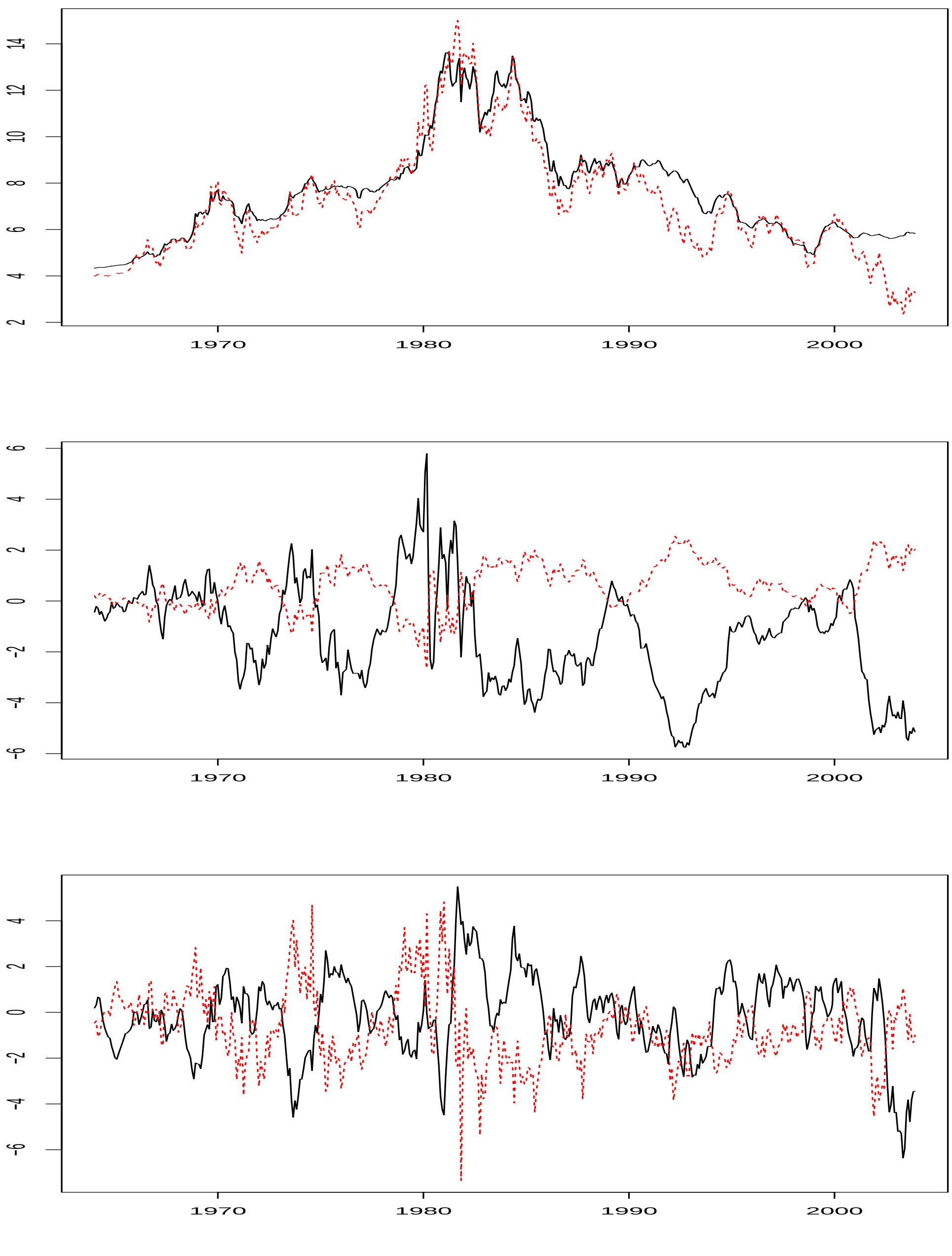


Figure 2: The estimated yield factors (solid lines) and their empirical approximation (dotted lines).

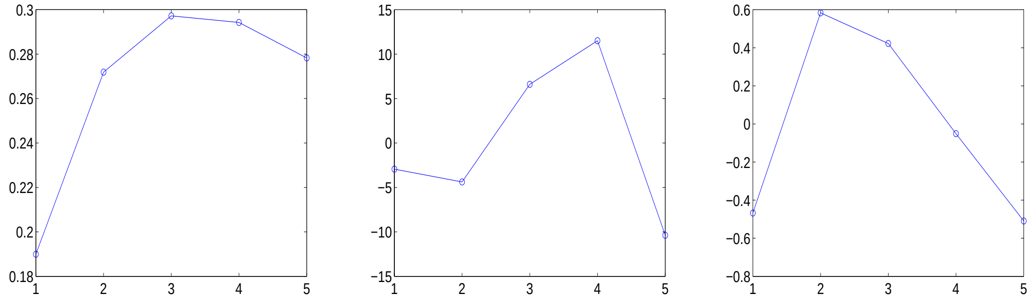


Figure 3: Loadings on the Nelson-Siegel curvature factor (left), $\lambda = 0.045$, the return-forecasting factor (middle) and the PC curvature factor (right).

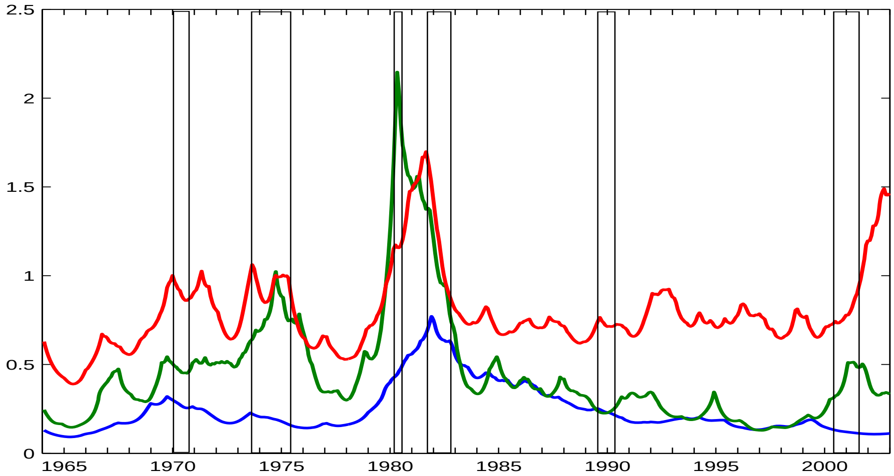


Figure 4: The estimated level volatility factor (blue line, top), the slope volatility factor (green line, middle) and curvature volatility factor (red line, bottom).

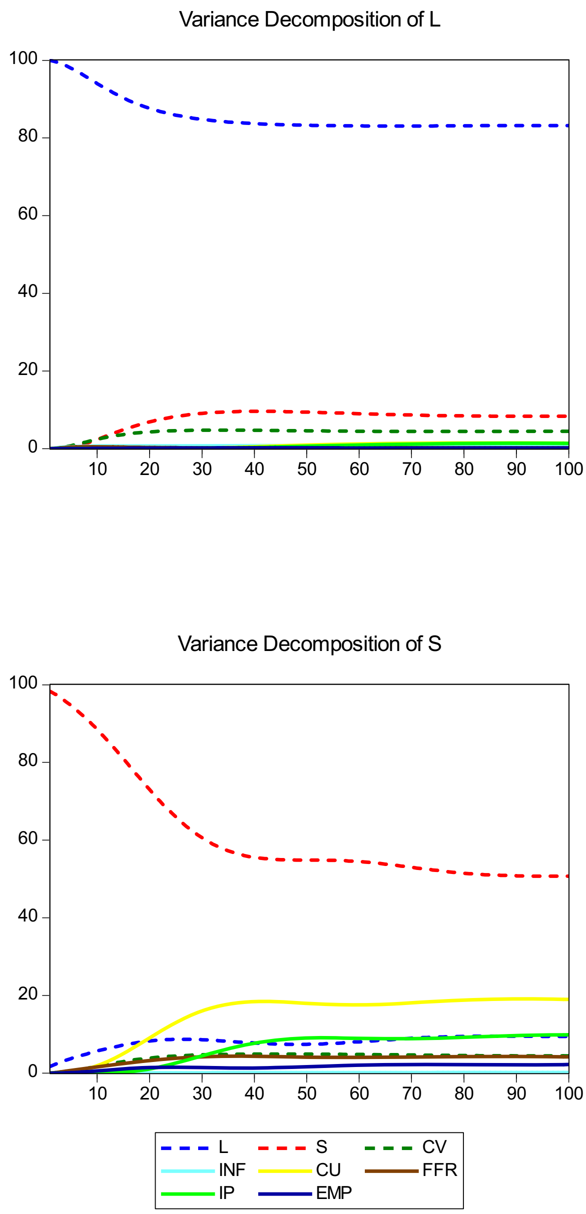


Figure 5: Prediction error decompositions of the level and slope factor. Based on a VAR(1) model of yield factors and macro factors using a Cholesky decomposition of the covariance.

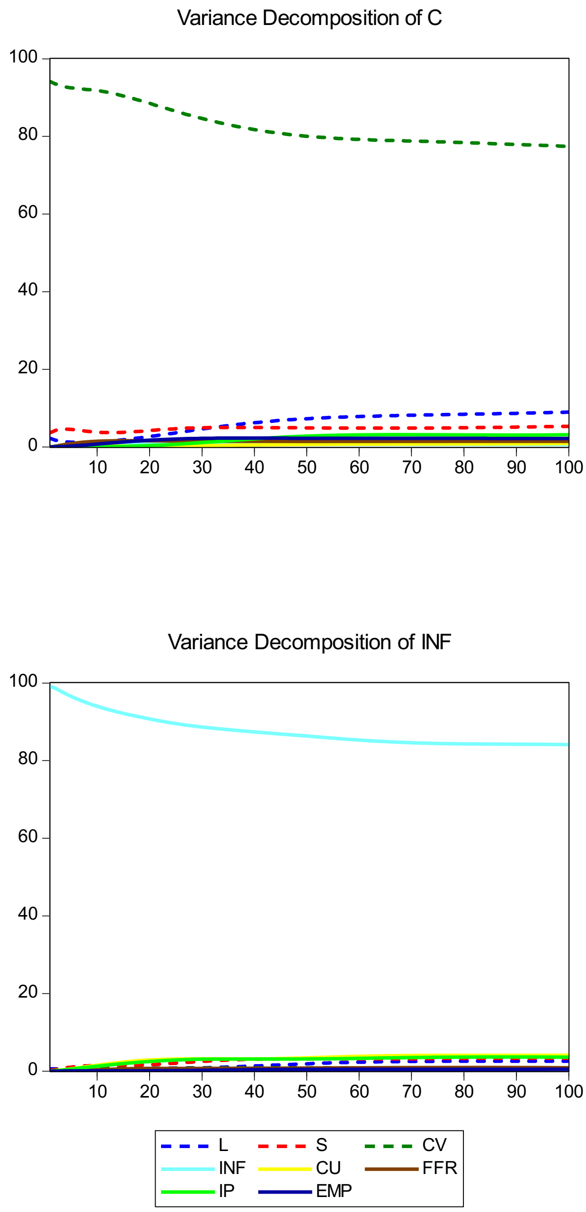
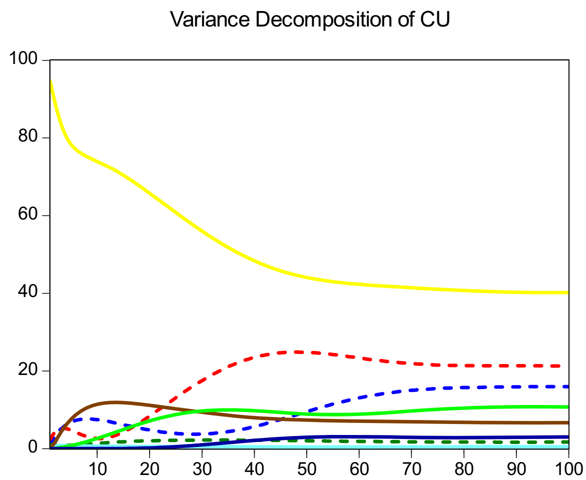


Figure 6: Prediction error decomposition of the curvature factor and inflation. Based on a VAR(1) model of yield factors and macro factors using a Cholesky decomposition of the covariance.



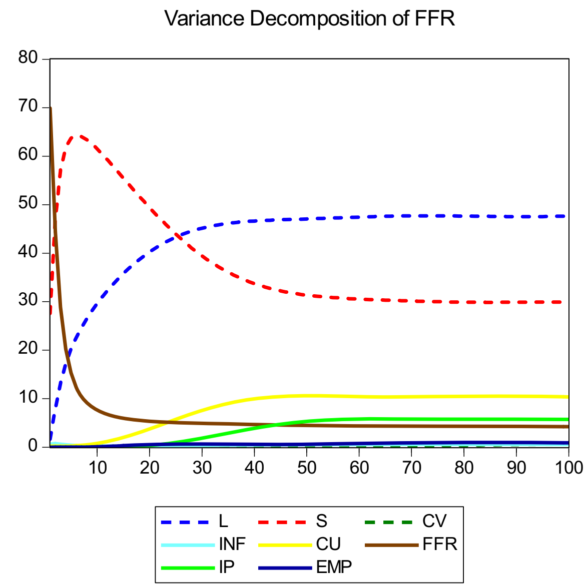


Figure 7: Prediction error decomposition of capacity utilization and of the federal funds rate. Based on a VAR(1) model of yield factors and macro factors using a Cholesky decomposition of the covariance.

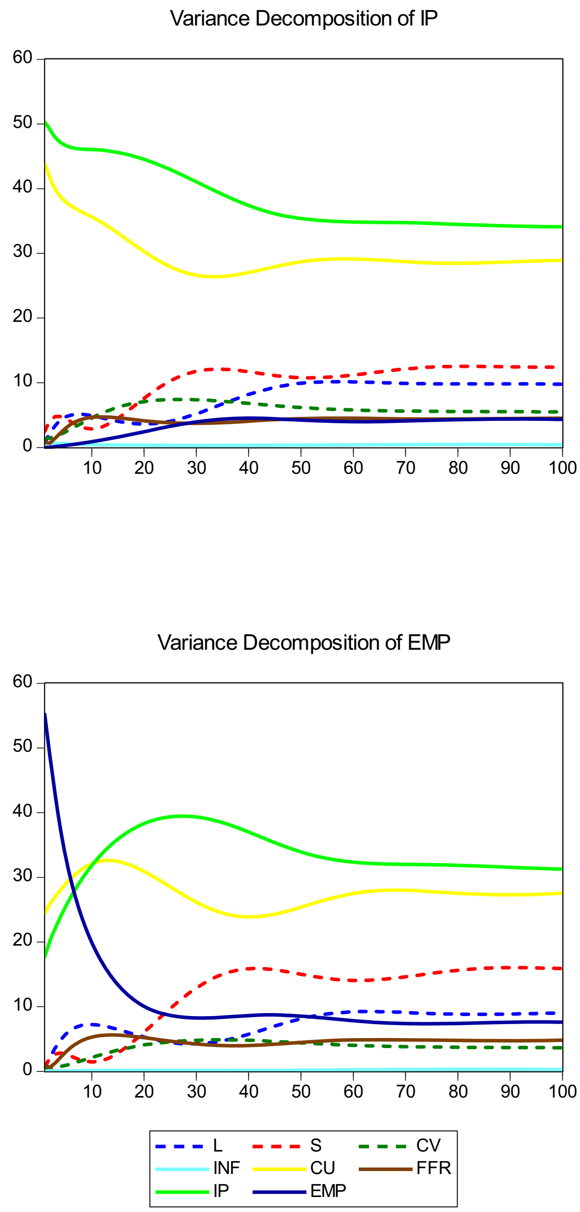


Figure 8: Prediction error decomposition of industrial production and of the employment growth rate. Based on a VAR(1) model of yield factors and macro factors using a Cholesky decomposition of the covariance.

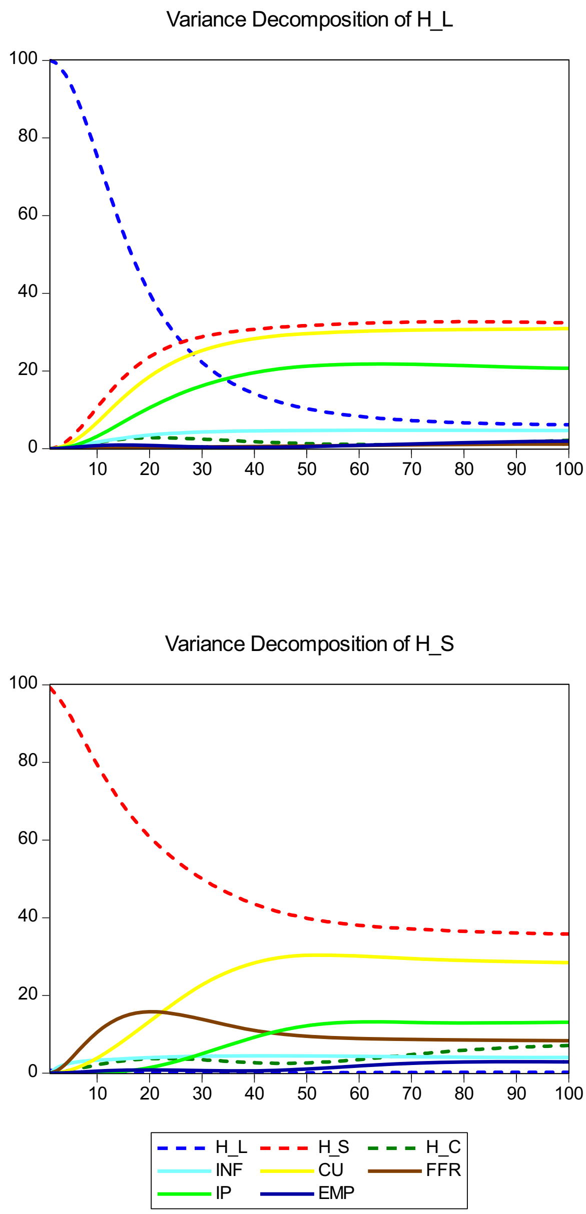


Figure 9: Prediction error decomposition of the level and slope volatility. Based on a VAR(1) model of volatility factors and macro factors using a Cholesky decomposition of the covariance.

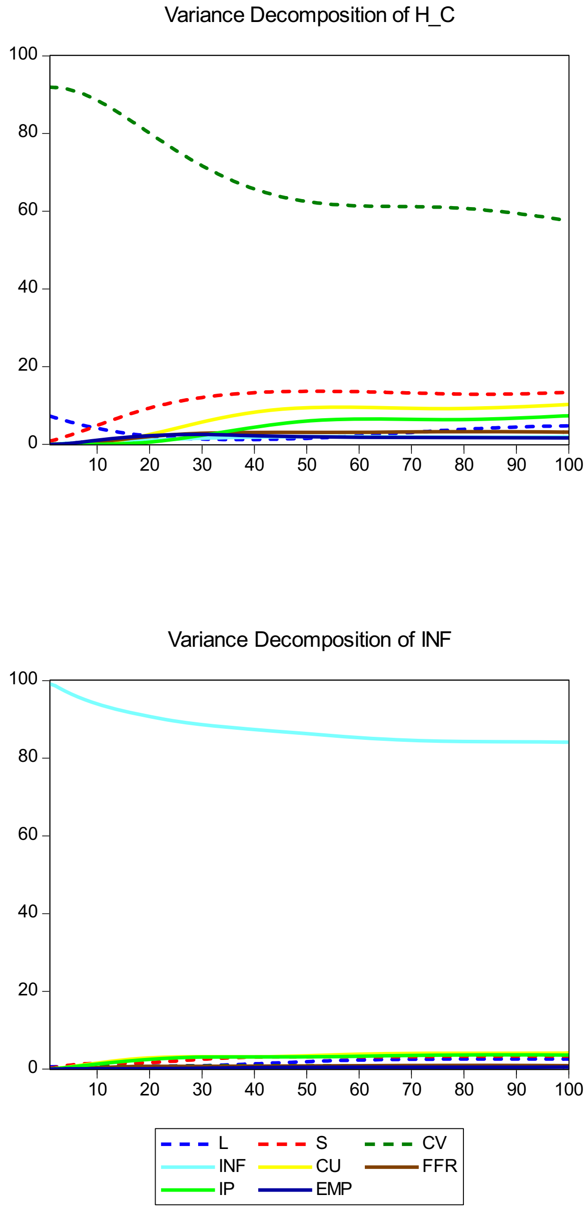


Figure 10: Prediction error decomposition of the curvature volatility and inflation . Based on a VAR(1) model of volatility factors and macro factors using a Cholesky decomposition of the covariance.

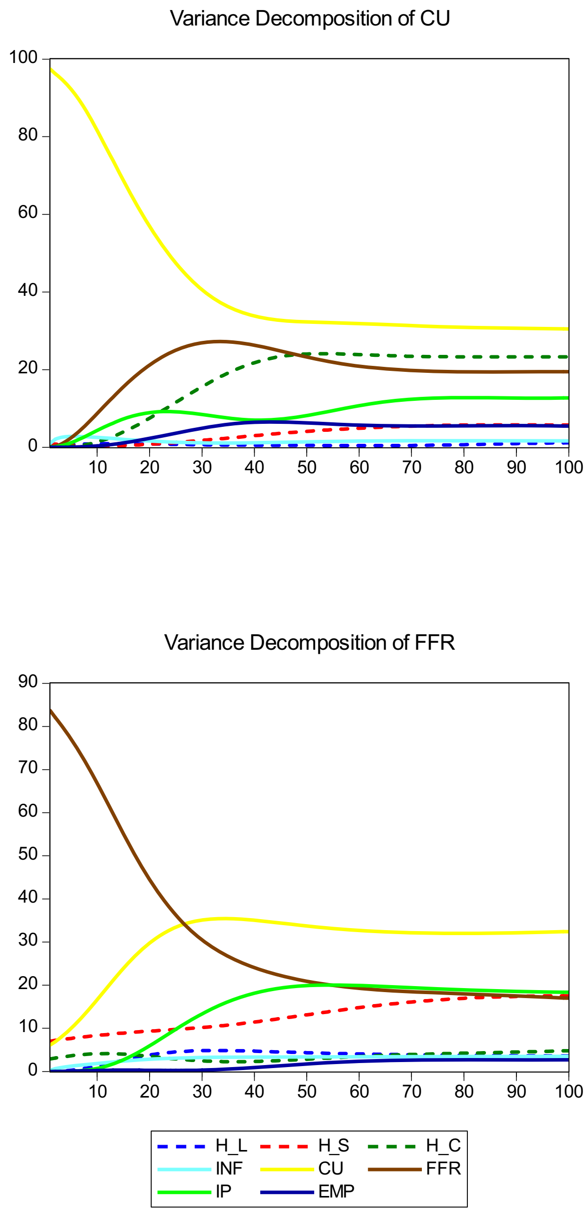


Figure 11: Prediction error decomposition of capacity utilization and of the federal funds rate. Based on a VAR(1) model of volatility factors and macro factors using a Cholesky decomposition of the covariance.

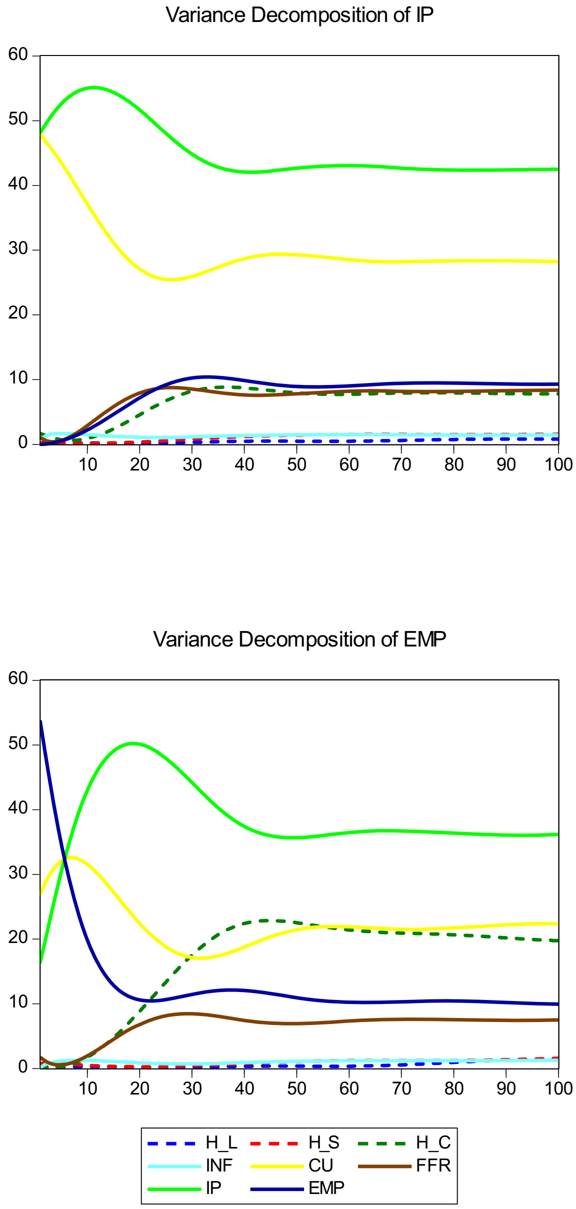


Figure 12: Prediction error decomposition of industrial production and of the employment growth rate. Based on a VAR(1) model of volatility factors and macro factors using a Cholesky decomposition of the covariance.

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