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Central Counterparties*

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Abstract:
Central counterparties (CCPs) have increasingly become a cornerstone of financial markets infrastructure. We present a model where trades are time-critical, liquidity is limited and there is limited enforcement of trades. We show a CCP novating trades implements efficient trading behaviour. It is optimal for the CCP to face default losses to achieve the efficient level of trade. To cover these losses, the CCP optimally uses margin calls, and, as the default problem becomes more severe, also requires default funds and then imposes position limits.

JEL Classification: G20, G30.

Keywords: Central Counterparty; Clearing; Default; Collateral; Risk Management.

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1 Introduction

Since the 1990s, central counterparties (CCPs) have become more and more commonplace as a cornerstone of financial market infrastructures. One role of CCPs is to novate contracts. In the novation process, the original contract between a buyer and a seller is extinguished and replaced by two new contracts; one between the buyer and the CCP, and another one between the seller and the CCP. For example, clearinghouses that serve as CCP interpose themselves as the legal counterparty for trades carried out on formal security exchanges and more recently also in over-the-counter (OTC) markets.

In assuming responsibility for the terms of the trade CCPs become exposed to *replacement cost risk* - the obligation to fulfill the terms of a contract with sellers (respectively buyers) even though buyers (respectively sellers) default on their obligations.[1] Novation concentrates default risk in the hands of a single institution, the CCP. As a consequence, it has the potential to disrupt financial markets if this risk is not properly controlled for.[2]

We develop a simple model of exchange, where common trading frictions prevent investors to trade efficiently. We show that a CCP is a natural device to implement the efficient level of trade. The model features investors with a random need to trade a security. The structure of markets and preferences of traders are such that (i) trades have to be carried out by a specific time (i.e., trades are time-critical), (ii) trades cannot be fully and immediately settled at that time (i.e., there is limited liquidity) and (iii) traders have an opportunity to renege on their obligations (i.e., there is a problem of enforcing the terms of the trade). We show that these elements impose severe limitations on a delivery-vs.-payment (DvP) mechanism which can lead to welfare loss. We then introduce a CCP as a technology that can hold collateral and can commit to its promises. As such the CCP is the ideal counterparty and it arises endogenously in response to trading imperfections.

While a CCP enables trades, it faces a replacement cost risk since it guarantees the trade

[1] According to the European Central Bank Glossary on Payments and Security, this is "the risk that a counterparty to an outstanding transaction for completion at a future date will fail to perform on the settlement date. This failure may leave the solvent party with an un-hedged or open market position or deny the solvent party un-realised gains on the position. The resulting exposure is the cost of replacing, at current market prices, the original transaction."

[2] See Russo, et al. (2002). The Committee on Payment and Settlement Systems (CPSS) recently issued international principles for CCP risk management that address the three key issues for controlling systemic risk in this area: (i) the transparent and prudent way of employing risk management, (ii) the design of governance structures that balance the requirements of users and the public interest; (iii) the potential trade-off between efficiency and risk in a situation of increased competitive pressure (see CPSS (2004)).

against default. The CCP controls its risk through collateral policies. It can employ margin calls on individual transactions to secure its exposure. It can also require agents, independently of their trading needs, to participate in a default fund. Using the default fund as an insurance pool, the CCP can mutualize losses on transactions across agents participating in the CCP. We show that when the default problem is not severe, the CCP just uses margin calls. As the default problem becomes more acute, the CCP will then introduce a default fund on top of a margin call and then will use position limits as the ultimate tool against default.

To summarize, we make three main contributions. First, we provide a simple model of exchange where default is an issue, and we show that a CCP novating trades can help achieve trading efficiency. Second, we provide an explanation for the three main risk management tools that CCPs use: default funds, margin calls and position limits. Finally, we explain how investors trade in spite of default occuring in equilibrium.

The remainder of the paper is as follows. The next section lays out the basic environment. Then, we show that a CCP is necessary to obtain an efficient level of trade if liquidity is limited, trade is time-critical and there is limited eforcement. Section 4 derives the optimal collateral policies. Section 5 describes how to implement the efficient allocation. Section 6 concludes.

2 A Model of Financial Exchange

2.1 The Environment

The model is static with three subperiods, $t = 0, 1, 2$ and a unit measure of investors. There are two assets – cash and securities – in positive supply. At $t = 0$, all investors are endowed with an amount m_0 of cash and an amount q_0 of securities. Whereas cash yields a constant pay-off of 1 per unit, the pay-off of the security in terms of cash, θ, is random with a cumulative distribution function F over a support $\Theta = [0, 2]$, symmetric around its mean $E(\theta) = 1$. All pay-offs from the assets are realized at $t = 2$.

Investors are risk-neutral and value the pay-offs of the security and cash identically. However, they face an uncertain cost of holding the security. There are three types of investors, that we label as h, ℓ and s, where h types enjoy a benefit $\delta \in (0, 1]$ from holding the securities, ℓ types suffer a cost $-\delta$ from holding securities and s types get no cost or benefit from holding securities so that $\delta_s = 0$. A fraction n of investors is of type s, whereas an equal fraction $(1-n)/2$ of investors is of type h and ℓ. At $t=1$, investors learn their type and it is private information. Upon trading and once the security's payoff is realized, investors of type $i \in \{h, \ell, s\}$ enjoy utility u_i, from holding cash m and securities q which equals to

$$u_i(m, q; \delta_i) = m + (\theta + \delta_i)q,$$

where δ_i expresses the realized holding cost for investor i.

We take as given that at $t=1$ – after the holding cost of investors is known – there is a competitive market where investors can trade the security, but are not allowed to sell the security short. In the context of the financial markets infrastructure, we interpret this set-up as follows. Investors are members of an exchange trading an asset (such as stocks) or more generally a financial contracts (such as futures) on their own behalf or for their customers.[3] The need to trade the contract is expressed as the holding cost of the security which gives an intrinsic benefit from buying or selling the contract. This corresponds to the need to take a particular position in order to hedge some risk or to adjust a liquidity position among others. We label investors with such a need to trade as *fundamental traders*. Other investors are indifferent at the time of trade between buying or selling the security. To the extent that they are willing to take a position when trading occurs, they provide liquidity to the market or try to exploit a mispricing of the financial asset or contract. We thus call these investors *speculators*.

There are three assumptions on trading securities in this economy. First, while cash can be transfered in any period, the security leg of any trade in period $t=1$, can only be settled in period 1. This delay in settlement of a transaction with a payment in cash reflects the fact that on financial markets the frequency of trading is often much higher than the one of settling transactions. This feature rules out a spot trade on the basis of delivery-vs-payment (DvP). Second, while trading occurs in a centralized Walrasian market, trades are between particular investors, i.e. *at settlement* there is a single, well defined, buyer to each seller and inversely. Last, we assume that there is limited commitment in the economy, i.e. it is impossible to (fully) enforce *intertemporal* trades. In particular, investors cannot commit to give up either securities or cash at a later stage to their respective counterparty of the trade. Our first assumption implies that any trade among investors occurs at $t=1$, but

[3] Our particular assumptions on the supply and payoff structure of the security are largely irrelevant for our results as long as there is a bound on short selling. This implies that our set-up does not correspond to a particular financial instrument or market, but to a general market for trading (any) assets competitively whether on a centralized exchange or in the OTC market provided there is sufficient competition among counterparties.

must be settled in $t = 2$ after the pay-off of the security has been realized. Combined with a lack of commitment, it implies that investors can default.

2.2 Efficient Allocations

The problem in this economy is to redistribute the security and cash among investors once their holding costs have been realized. An allocation $\{(q_i, m_i)\}_{i\in\{h,\ell,s\}}$ is a distribution of the security and cash across the different types of investors. It is clearly efficient to transfer securities from low type traders and speculators to high type traders having a benefit from holding the security. The efficient distribution of cash m^* is indeterminate as all investors value it the same way. The efficient allocation is then defined by $q_h^* = 2q_0/(1-n)$, $q_\ell^* = 0$ and $q_s^* = 0$. This allocation yields expected welfare as of $t = 1$ equal to

$$W(q^*, m^*) = m_0 + (1+\delta)q_0,$$

as all securities are held by traders with a holding benefit δ.

3 Trade and Default

The efficient allocation would be feasible if agents could write fully enforceable contracts. However, we now show that our assumptions imply that default is an issue and that it unequivocally lowers welfare.

With a price p for the security in the Walrasian market, the budget constraint for all investors at $t = 1$ is given by

$$m_i + pq_i \leq m_0 + pq_0.$$

Since the expected utility of all investors is strictly increasing in securities or cash, or both, their budget constraint always binds. Hence, we have that

$$p\,(q_i - q_0) = m_0 - m_i. \tag{1}$$

Consider now the incentives to settle a transaction at $t = 2$. Investors do not have an incentive to default on a trade as long as the following *no-default constraints* are fulfilled

$$m_i + (\theta + \delta_i)q_i \geq m_0 + (\theta + \delta_i)q_0,$$

or

$$(\theta + \delta_i)\,(q_i - q_0) \geq m_0 - m_i. \tag{2}$$

Comparing (1) and (2), we find that speculators will default when they are buyers and $\theta < p$, or when they are sellers and $\theta > p$. Also, as high fundamental traders are buyers while low fundamental traders are sellers in any equilibrium, they have an incentive to default only if, respectively

$$\theta + \delta < p$$

and

$$\theta - \delta > p. \tag{3}$$

Hence, trades between fundamental traders will only settle if $\theta \in [p-\delta, p+\delta]$. Whereas the holding cost gives a motive for trade, it also motivate the incentives for default. Fundamental traders however default if the payoff of the security moves too far away from the trading price. Furthermore, the lower the holding cost, the larger the potential for default. As δ approaches zero, no trade would be settled.

Proposition 1. *As commitment is limited, there is no trade in equilibrium at* $t = 1$ *without a positive probability of default in period* $t = 2$.

Whenever default occurs, a trade will not settle and the two traders keep their endowments (m_0, q_0). Hence, in the case of default, the trading parties obtain utility

$$m_0 + (\theta + \delta_i)q_0$$

Notice that as default offers an option value to speculators, they will always take a position when trading occurs at $t = 1$. This implies that all fundamental traders face a positive probability that their trade will not settle. In case the trade of a high (or a low) trader does not settle, the welfare loss is proportional to the holding costs. For example, the surplus generated from a pair of traders (h, ℓ) is $m_h + (\theta + \delta)\, q_h + m_\ell + (\theta - \delta)\, q_\ell$, where feasibily requires that $m_h + m_\ell = 2m_0$ and $q_h + q_\ell = 2q_0$. If the speculator defaults, the surplus from the trade is $m_0 + (\theta + \delta)\, q_0 + m_0 + (\theta - \delta)\, q_0$, which implies a welfare loss of $\delta\,(q_h - q_\ell) > 0$. In words, the welfare implications of default play through holding costs. Speculators bear no welfare costs from default, while fundamental traders have a utility loss whenever they cannot settle their trade. Investors with high holding costs end up holding the security, while people with low holding costs hold on to cash. In conclusion, *any* default incurs a welfare loss.[4]

[4] For a fundamental trader that is a buyer, the trade will settle if the counterparty is a low type and $\theta \in [p-\delta, p+\delta]$ or if $\theta \in [p-\delta, p]$. Hence, the welfare loss per high type is given by

$$\delta(q_h - q_0)\,[1 - (F(p+\delta) - F(p))Prob(seller = \ell|seller) - (F(p) - F(p-\delta)]\,.$$

A similar condition holds for fundamental traders that sell the security.

Proposition 2. *Default lowers expected welfare.*

4 Trade and Collateral

The last result is important as it gives a reason to eliminate, or at least restrict, default. Since we study a static environment, reputation cannot dampen the incentives to default. Hence, we study first collateral requirements. To analyze which allocations is feasible with collateral, we set up a planning problem that takes the Walrasian trading mechanism, as well as the trading frictions as given. The planner objective is to maximize the expected welfare of investors at $t = 1$.

The planner operates a *collateral facility* that secures trades with collateral. We assume that storing collateral at the facility does not involve any cost. The planner therefore has the option to receive, store and disburse cash for settlement. Since the efficient allocation requires that high types get all the securities, the planner needs to give incentives to speculators to trade away their security. Hence, we assume that investors do not receive any cash back if they do not trade. Since collateral is pledged before types are revealed, speculators will always take a position, either as buyers or sellers. We denote by (w_h, w_ℓ) the settlement balances depending on whether investors buy or sell the security. Feasibility obviously requires that the sum of all settlement balances, w_0, is smaller than the overall posted collateral. Given posting collateral is costless, it is quite clear that the incentives to default are minimized when the planner requires an amount m_0 of collateral from all traders, and we will therefore assume that this is the case from now on. We also assume that the planner can impose *position limits*, which restrict the size of the investors' trades.

By itself, a collateral facility does not prevent default and we need to specify the planner's options when there is default. In this case, we assume that investors lose all their collateral to the collateral facility. The facility has then two sources of funds to cover the loss of a default. It can use the seized collateral, and it can also use the share of the collateral that is not pledged to settle trade, i.e. the weighted sum of $m_0 - w_i$. The facility uses these resources to acquire the security for its realized value θ and transfers it to the counterparty that suffered the default. We rule out that the collateral facility itself can default on this guarantee. In what follows, we label the collateral facility as *Central Counterparty* or CCP.

With a CCP, the timing of events is the following. At $t = 0$, the planner requests the transfer of cash m_0 as collateral which is costlessly stored until $t = 2$. Then, at $t = 1$, the investors' type is realized. At this stage, the collateral facility assigns settlement balances

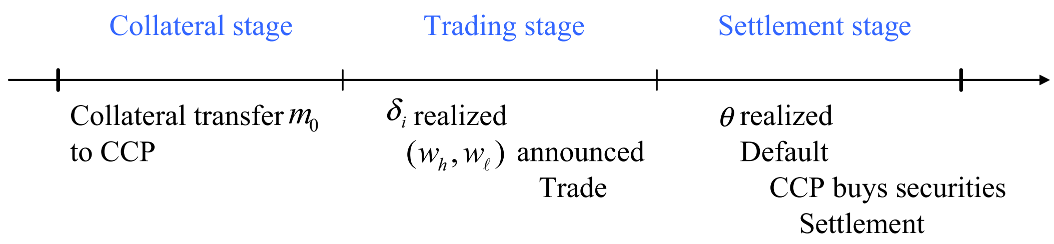

Figure 1: Timeline.

(w_h, w_ℓ) to buyers and sellers respectively. It can also set position limits, which restrict the size of the investors' trades. Taken this collateral policy as given, investors trade on a perfectly competitive Walrasian market. Finally, at $t = 2$, after the payoff θ of the security has been realized, investors either use their balances to settle their trades or default. If default occurs, the CCP uses its available resources to cover defaulted trades. Figure 1 summarizes the set-up.

4.1 Risk Management With No Aggregate Default Risk

4.1.1 No Default with Sufficient Collateral

As long as investors have sufficient collateral m_0 to cover the default exposure from trading, posting collateral can rule out default. Given an allocation (m_i, q_i), and having pledged m_0 as collateral, investors do not default as long as

$$m_i + (\theta + \delta_i)q_i \geq (\theta + \delta_i)q_0$$

for all $\theta \in \Theta$. Buyers of the security will never default as $m_h \geq 0$ and $q_h > q_0$. In equilibrium high type traders will only buy securities as they enjoy holding benefits. Also, since low type traders incur the cost of holding securities, they have less incentive to default than speculators who sold securities. Hence, default is ruled out whenever speculators do not default, or when

$$m_\ell \geq \theta q_0$$

for all $\theta \in \Theta$, where we have considered the largest sale $q_\ell = 0$. Hence, we have the following result.

Lemma 3. *If $m_0 \geq q_0$, there is sufficient collateral to rule out default.*

Proof. From the budget constraint of speculators, we have that

$$m_\ell = pq_0 + w_\ell \tag{4}$$

where w_ℓ is the balance associated with settlement for sellers. Set $w_h = w_\ell = m_0$. We first show that $p < 1$ is not an equilibrium. If $p < 1$, all speculators buy securities and therefore do not have an incentive to default. However, in this case, all buyers spend all their cash so that from the market clearing condition, it must be the case that

$$\frac{(1-n)}{2} pq_0 = nm_0 + \frac{(1-n)}{2} m_0.$$

This is equivalent to

$$pq_0 = \frac{(1+n)}{(1-n)} m_0.$$

However since we assume $m_0 > q_0$, this contradicts the fact that $p < 1$. Now assume $p \geq 1$. Then speculators are willing to sell the security and we have

$$m_\ell > q_0 + m_0 \geq 2q_0. \tag{5}$$

□

Whenever there is sufficient collateral, the CCP can rule out default *independent* of θ. In this sense, there is no aggregate default risk in the economy whenever $m_0 \geq q_0$. When there is no risk of default, the CCP also does not need to resort to position limits to restrict trade.

4.2 Constrained Efficient Allocations Without Default

Suppose now that $m_0 \geq q_0$ and that the risk management policy of the CCP rules out default at $t = 2$. If possible, the CCP chooses to distribute settlement balances w_i so that low types and speculators sell as many securities to high types. Since investors are risk neutral they will take extreme positions. They either will sell all their securities ($q_\ell = 0$) or spend all their settlement balance to buy securities ($m_h = 0$). Any allocation (m_i, q_i) that the CCP wants to achieve has then to satisfy the following incentives constraints

$$(E(\theta) + \delta)q_h = (1 + \delta)q_h > m_\ell, \tag{6}$$

$$(E(\theta) - \delta)q_h = (1 - \delta)q_h < m_\ell. \tag{7}$$

in addition, speculators will sell securities if $E(\theta)q_h = q_h < m_\ell$ and buy otherwise. If the latter condition holds with equality, speculators are just indifferent between taking any position. Hence, when α is the fraction of speculators selling the security, we have

$$\alpha = \begin{cases} 1 & \text{if } q_h < m_\ell, \\ [0,1] & \text{if } q_h = m_\ell, \\ 0 & \text{if } q_h > m_\ell. \end{cases} \tag{8}$$

To rule out default, speculators should not default for any realization of θ when they sell the security, in particular for $\theta = 2$. Hence the planner faces the constraint

$$m_\ell \geq 2q_0 \text{ if } \alpha > 0, \text{ and} \tag{9}$$

$$m_\ell \geq (2-\delta)\, q_0 \text{ if } \alpha = 0. \tag{10}$$

Finally, the market clearing conditions are then given by

$$\left(\frac{1-n}{2} + \alpha n\right) m_\ell = w_0, \tag{11}$$

$$\left(\frac{1-n}{2} + (1-\alpha)n\right) q_h = q_0. \tag{12}$$

All allocations that satisfy inequalities (6)-(10) and market clearing conditions (11) and (12) do not involve default and are feasible for the CCP. Note that these conditions satisfy the requirement that trades take place on a competitive market. Also note that the exact allocation of settlement balances only matters to the extent that it induces investors to take a particular (extreme) position of buying or selling securities and preventing default.

Due to linearity, as long as the budget constraints are satisfied and the price falls in the interval $[1-\delta, 1+\delta]$, the level of settlement balances w_i is indeterminate, and the way settlement balances are redistributed only impact the equilibrium price.[5] Still, it is optimal that the CCP returns all cash m_0 for settlement, so that we can set $w_0 = m_0$.

The allocations that are feasible for the CCP are then summarized by three conditions. The first one ensures fundamental traders take on the right position

$$(1+\delta) > \frac{1+(1-2\alpha)n}{1-(1-2\alpha)n} \frac{m_0}{q_0} > 1-\delta. \tag{13}$$

The second one describes trading by speculators and is given by

$$\alpha = \begin{cases} 1 & \text{if } \frac{1+n}{1-n} q_0 \leq m_0 \\ \frac{1}{2}\left(1 + \frac{1}{n}\frac{m_0 - q_0}{m_0 + q_0}\right) & \text{if } q_0 < m_0 < \frac{1+n}{1-n} q_0 \\ \frac{1}{2} & \text{if } q_0 = m_0. \end{cases} \tag{14}$$

[5] This is related to the second welfare theorem. A seller's budget constraint yields $m_\ell = pq_0 + w_\ell$ while a buyer budget constraint gives $q_h = q_0 + w_h/p$. Hence, given $p \in [1-\delta, 1+\delta]$, there exists a pair (w_ℓ, w_h) such that the budget constraints are satisfied. Notice that the pair has also to satisfy the feasibility constraint, $\left[\frac{(1-n)}{2} + \alpha n\right] w_\ell + \left[\frac{(1-n)}{2} + (1-\alpha)\, n\right] w_h \leq m_0$.

Notice in particular that $\alpha \geq 1/2$ to satisfy the market clearing conditions. Finally, default is ruled out as long as

$$\frac{m_0}{q_0} \geq 1 - (1-2\alpha)n. \tag{15}$$

The CCP would like to maximize total expected welfare as of $t = 1$. Since investors are risk neutral with respect to cash, only the holding costs from allocating the security matter. To see this, note that, after some manipulations, the objective function is given by

$$m_0 + E(\theta)q_0 + \frac{1-n}{2}\delta\left(q_h - q_\ell\right).$$

Using $q_\ell = 0$ and market clearing, after neglecting constants this is equivalent to choosing α to maximize

$$\frac{1}{1+(1-2\alpha)n}$$

subject to the restrictions (13)-(15) above. Clearly, the planner would like to have as many speculators sell the security as possible (or α as close to 1). Market clearing, incentives to trade and the default problem impose restrictions on the allocation he can achieve. We then obtain the following result.

Proposition 4. *If $m_0 \geq \frac{1+n}{1-n}q_0$, the CCP achieves the first-best allocation by requiring collateral m_0 and all speculators sell securities $\alpha = 1$. If $m_0 \in [q_0, \frac{1+n}{1-n}q_0)$, the CCP achieves the constrained efficient allocation $q_h = q_0 + m_0$, $q_\ell = 0$ and some speculators buy securities $\alpha < 1$. Furthermore, it is always efficient to rule out default in equilibrium.*

Proof. For the first case, set $\alpha = 1$. The no default constraint is fulfilled, as

$$m_0 \geq \frac{1+n}{1-n}q_0 \geq (1-(1-2\alpha)n)q_0$$

as $n \in (0,1)$. Also, the condition that $m_{ell} > q_h$is satisfied since $m_0 \geq \frac{1+n}{1-n}q_0$. Suppose that $(1+\delta) > \frac{1+n}{1-n}\frac{m_0}{q_0}$. Then, the trading condition on fundamental traders is also satisfied. If $(1+\delta) \leq \frac{1+n}{1-n}\frac{m_0}{q_0}$, we can still achieve the same allocation by allowing high types to have some cash $m_h = m_0 - (1+\delta)\frac{1+n}{1-n}q_0 > 0$.

For the second case, as $m_0 \geq \frac{1+n}{1-n}q_0$ is not satisfied, we can't set $\alpha = 1$. However the condition for $\alpha \in (0,1)$ gives us the values for α, and at $\alpha = \frac{1}{2}\left(1 + \frac{1}{n}\frac{m_0 - q_0}{m_0+q_0}\right)$, we have that

$$1 - (1-2\alpha)n = 1 - \frac{m_0 - q_0}{m_0 + q_0} = \frac{2q_0}{m_0+q_0} \leq \frac{m_0}{q_0}$$

so that there is no default. As $\delta > 0$ and $\frac{1+(1-2\alpha)n}{1-(1-2\alpha)n}\frac{m_0}{q_0} = 1$, the trading condition is also fulfilled.

For the last statement, suppose that all speculators sell the security ($\alpha = 1$) which could only happen in an equilibrium with default. The feasibility condition for settlement balances then becomes

$$m_\ell \left(\frac{1-n}{2} + n \right) = \left(\frac{1-n}{2} + n \right) w_\ell + \left(\frac{1-n}{2} + (1-n) \right) w_h < m_0, \tag{16}$$

where the last inequality hold as the CCP needs to secure resources to cover default losses. For speculators to have an incentive to sell, we need $m_\ell \geq q_h$. Thus,

$$q_h \leq m_\ell < \frac{2}{1+n} m_0. \tag{17}$$

Hence, buyers can obtain at most $\frac{2}{1+n} m_0$ which is less than $q_0 + m_0$. A contradiction. □

The CCP would like to transfer as many securities as possible to high types. To this end, it needs to ensure that speculators are net sellers of securities without crowding out sales by low types. In any equilibrium without default, as long as there is enough cash ($m_0 \geq (1+n)\, q_0 / (1-n)$), the CCP can induce all speculators to sell the security. Otherwise, there can be at most a fraction $\alpha < 1$ of speculators that sells the security. Ruling out default by requiring sufficient collateral is always efficient.

We now study the case where collateral is scarce, $m_0 < q_0$, and derive the optimal allocation when there is no default in equilibrium. The planner then faces the following set of constraints. First the planner faces a no default constraint for speculators and fundamental investors. As before, only the default constraint for speculators is the binding constraint and since $\theta_{\max} = 2$, we can write this constraint as

$$m_\ell + 2q_\ell \geq 2q_0, \tag{18}$$

Second, the planner faces the participation constraint dictating that speculators are at least indifferent to sell their securities to high types,

$$m_\ell + q_\ell \geq q_h \tag{19}$$

with equality of $\alpha \in (0,1)$, where we have used the fact that high types spend all their cash on buying securities. Finally, the planner faces market clearing conditions given by

$$\left(\frac{1-n}{2} + \alpha n \right) m_\ell = w_0, \tag{20}$$

$$\left(\frac{1-n}{2} + (1-\alpha) n \right) q_h + \left(\frac{1-n}{2} + \alpha n \right) q_\ell = q_0. \tag{21}$$

From the previous proposition, we conclude that $\alpha < 1$, so that (19) binds. Then combined together with (21) and then (20), we obtain

$$\begin{aligned} q_h &= q_0 + w_0 \\ q_\ell &= q_0 - \frac{1 + (1 - 2\alpha)\, n}{1 - (1 - 2\alpha)\, n} w_0 \end{aligned}$$

As before, welfare is proportional to $q_h - q_\ell$, so that setting $w_0 = m_0$ is optimal, and then

$$q_h - q_\ell = \frac{2}{1 - (1 - 2\alpha)\, n} m_0.$$

Hence, the planner will set α as low as possible. However, it is constraint by (18). Using the values for m_ℓ and q_ℓ, this simplifies to $\alpha \geq 1/2$. Therefore, the planner optimally sets $\alpha = 1/2$, so that welfare is proportional to $2m_0$. We summarize this derivation in the following proposition.

Proposition 5. *If $m_0 < q_0$, the best allocation with no default is $q_h = q_0 + m_0$, $q_\ell = q_0 - m_0$, and $\alpha = 1/2$.*

In Figure 2, we show q_ℓ, α and the welfare portion originating from the exchange of the security, $q_h - q_\ell$, as a function of the available liquidity m_0. Notice that while the depicted allocation for $m_0 > q_0$ is optimal, this may not the case for the no-default allocation when $m_0 < q_0$, as we will show below.

4.3 Risk Management with Aggregate Default Risk

If sellers have an option to default, the CCP must set aggregate settlement balances below m_0 (i.e. a default fund). In this way, it insures enough resources to fulfill its promises to settle in case of default. This implies, however, that the purchasing power of buyers decreases or, equivalently, that $q_h < q_0 + m_0$. We now identify the efficient institutional features of risk management for the clearing of financial trades when there is insufficient collateral to prevent default.

4.3.1 Default with Insufficient Collateral

We introduce default risk for the economy even with a collateral facility. To build up some intuition for the following section, we first impose $q_\ell = 0$, so that sellers trade all their securities away, and derive preliminary results on default. We know from the previous

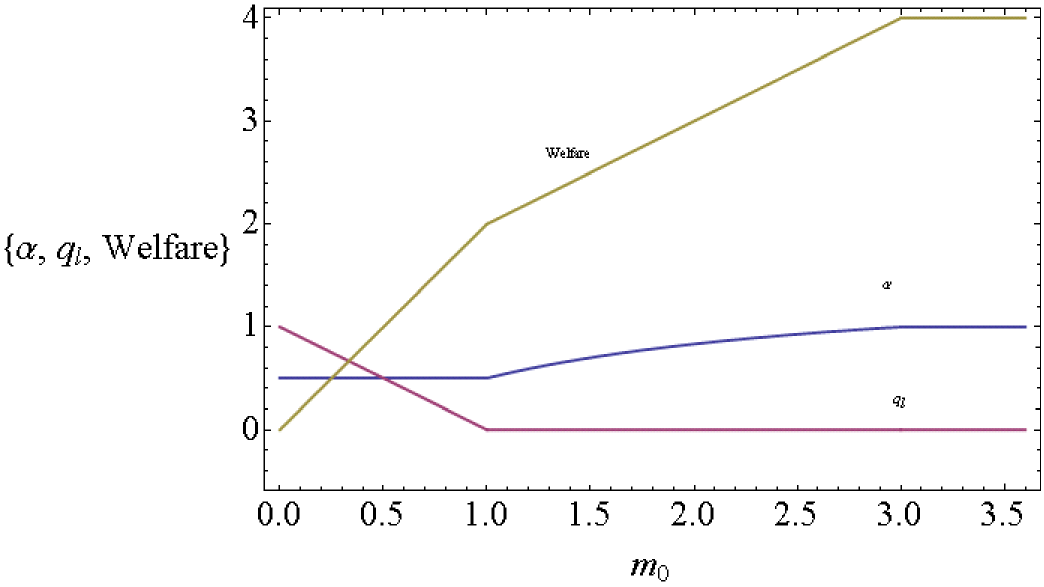

Figure 2: No default allocations, $q_0 = 1$.

section, there is not enough collateral to rule out default if investors take extreme positions when $m_0 \leq q_0$. Together with $q_\ell = 0$,

$$m_\ell \geq \theta q_0$$

for all $\theta \in \Theta$, rules out default of speculators that sell the security. As before, this implies that low type traders do not default either. However, considering speculators' incentives for trade, there cannot be trade without default. This is our next preliminary result.[6]

Lemma 6. *Let $m_0 < q_0$ and $q_\ell = 0$. If speculators are selling the security all trades involve a positive probability of default for some $\theta \in \Theta = [0, 2]$.*

Proof. First note that sellers only have an incentive to sell securities if $m_\ell \geq q_h$. Suppose there is no default. First let $m_\ell > q_h$. Then, $\alpha = 1$ as all speculators sell the security. Then, as $m_0 < q_0$, we have that $m_\ell = \frac{2}{1+n} m_0 < 2q_0 = \theta_{\max} q_0$, which is a contradiction.

Now let $m_\ell = q_h$. This implies that α is given by

$$\frac{2}{1-(1-2\alpha)n} m_0 = \frac{2}{1+(1-2\alpha)n} q_0 \tag{22}$$

[6] For the sake of completeness, after this result, we should still verify whether the allocation where all speculators are trading is dominated by an allocation where some speculators are not trading. However, a moment reflection should convince the reader that this is likely not an equilibrium, as speculators would forego the default option if they do not trade.

so that $\alpha < 1/2$. Hence, $m_\ell = q_h = \frac{2}{1+(1-2\alpha)n} q_0 < \frac{2}{1+n} q_0 < \theta_{\max} q_0$ which again is a contradiction. □

We also show that fundamental sellers also have an incentive to default if δ is small and $q_\ell = 0$. Formally we obtain

Lemma 7. *Let $m_0 < \hat{m}(\delta) < q_0$ and $q_\ell = 0$. Fundamental sellers default for some $\theta \geq \hat{\theta}(\delta)$.*

Proof. Given $q_\ell = 0$, the payoff of fundamental sellers is m_ℓ. Using the market clearing condition (11), m_ℓ is equal to $2w_0/(1-n+2\alpha n)$, where $w_0 \leq m_0$. Clearly, setting $w_0 = m_0$ minimizes the incentives to default. However, fundamental sellers prefer to default whenever

$$(\theta - \delta) q_0 > \frac{2m_0}{(1-n+2\alpha n)}.$$

Since $\alpha \in [0,1]$, fundamental sellers default whenever

$$\theta > \frac{2}{(1-n)} \frac{m_0}{q_0} + \delta \equiv \hat{\theta}(\delta).$$

Notice that $\hat{\theta}(\delta) < \theta_{\max} = 2$, when $m_0/q_0 < (2-\delta)(1-n)/2$. Hence, fundamental sellers default whenever $m_0 < \hat{m}(\delta)$, defined as

$$\hat{m}(\delta) \equiv \frac{(2-\delta)(1-n)}{2} q_0.$$

□

In particular, if $\delta = 1$, fundamental speculators will default on an allocation with $q_\ell = 0$, for some θ, when $m_0 < (1-n) q_0/2$.

We make two facilitating assumptions. First, the CCP induces (some) speculators to sell the security and all speculators to trade. Second, θ is uniformly distributed over the interval $[0,2]$.

4.3.2 Constrained Efficient Allocations with Default

The CCP would like to transfer as many securities as possible from low types and speculators to high types. We already showed that whenever θ is too high, there is a risk of default, leading to a misallocation of securities. There are two ways for the CCP to remedy this problem. First, it could impose position limits on the sale of the security, implying a minimum amount of securities holdings for low type traders, $q_\ell \geq \underline{q} > 0$. Second, it could only return $w_0 < m_0$ to investors for settlement purposes, using the remainder $m_0 - w_0$ to purchase the security at $t = 2$ at a price equal to the realized pay-off θ, and transfer to the buyer whose counterparty defaulted.

Clearly, there must be sufficient funds available to cover the default risk for all realizations of θ. There are two cases to consider. The first the case occurs when $m_0 > \hat{m}$, defined above. Then fundamental sellers never default. However, speculators who sold securities might default. That is, whenever $\alpha > 0$ and $m_0 > \hat{m}$, the CCP faces the following budget constraint

$$m_0 - (1 - \alpha n)\, w_0 \geq \alpha n \theta_{\max}(q_0 - q_\ell) = 2\alpha n(q_0 - q_\ell), \tag{23}$$

If there is default, the CCP confiscates the collateral of those αn speculators who default. However, it still needs to transfer w_0 to those who do not default. Hence, the CCP has $m_0 + \alpha n w_0 - w_0$ available to cover the worst possible loss. This loss occurs at the best pay-off for the security and is due to all speculators who sold securities up to the potential position limit $q_0 - q_\ell$. When $\alpha = 0$, there is never default, and the planner does not need to secure funds.

The second case is when $m_0 < \hat{m}$. Then both speculators and fundamental sellers might default. Then the CCP faces the following budget constraint instead of (23):

$$m_0 - (1 + (1 - 2\alpha)\, n)\, \frac{w_0}{2} \geq (1 - (1 - 2\alpha)\, n)\, (q_0 - q_\ell). \tag{24}$$

We proceed as in the previous section. The CCP has to maintain the incentives for trade. High types buy and low types sell the security as long as

$$\begin{aligned} (E(\theta) + \delta)\, (q_h - q_\ell) &= (1 + \delta)\, (q_h - q_\ell) > m_\ell \\ (E(\theta) - \delta)\, (q_h - q_\ell) &= (1 - \delta)\, (q_h - q_\ell) < m_\ell \end{aligned}$$

Here we have taken into account that (i) low types traders do not default when selling their securities and (ii) that high types traders can always settle their trade either with the sellers of the security or the CCP, as (23) guarantees. Also, as settlement is guaranteed for buyers, they will spend all their settlement balances on buying securities so that $m_h = 0$. Next, we have the two market clearing conditions which are now given by

$$\left(\frac{1-n}{2} + \alpha n\right) m_\ell = w_0 \tag{25}$$

$$\left(\frac{1-n}{2} + (1-\alpha)n\right) q_h + \left(\frac{1-n}{2} + \alpha n\right) q_\ell = q_0. \tag{26}$$

There is now less cash available for the CCP to distribute as settlement balances for investors' trade, as the CCP has to cover its exposure against default. Hence, total cash balances for trading w_0 are all held by sellers in equilibirum. Furthermore, whenever the CCP imposes position limits, sellers cannot sell all their securities.

Finally, we have to take into account that speculators have an incentive to trade. As they can default if the security pay-off is sufficiently high, they have an option value of default, while they enjoy guaranteed settlement through the CCP when buying the security. Hence, speculators will be indifferent between buying or selling the security as long as

$$E(\theta)q_h = \int \max\{q_\ell\theta + m_\ell, \theta q_0\} dF(\theta) \tag{27}$$

The right-hand side takes into account that speculators who sell the security default whenever

$$\theta > \frac{m_\ell}{q_0 - q_\ell}.$$

Since we have assumed that δ is large enough, fundamental traders will always have an incentive to trade. Taking into account that θ is uniformly distributed, we can rewrite the indifference conditions for speculators (27) to obtain

$$m_\ell = 2\sqrt{(q_h - q_0)(q_0 - q_\ell)}. \tag{28}$$

Rewriting the market clearing condition for securities (26), we obtain

$$q_h - q_0 = \frac{1-(1-2\alpha)n}{1+(1-2\alpha)n}(q_0 - q_\ell). \tag{29}$$

Finally, we have to check whether speculators have an incentive to trade rather than not which is equivalent to the condition

$$E(\theta)q_h \geq E(\theta)q_0,$$

where we have used the fact that investors who do not trade lose their posted collateral. This condition is clearly always satisfied.

The CCP again chooses to maximize total expected welfare at $t = 1$ which is proportional to $q_h - q_\ell = (q_h - q_0) + (q_0 - q_\ell)$. This allows us to express the problem for the CCP in terms of α, the number of speculators selling the security and q_ℓ, the position limit on sales of the security. The CCP then solves the following problem

$$\max_{\alpha\in[0,1], q_\ell \geq 0} W(\alpha, q_\ell) = \frac{2}{1+(1-2\alpha)n}(q_0 - q_\ell)$$
$$\text{subject to: (23) if } m_0 \geq \hat{m} \text{ and } \alpha > 0, \text{ (24) if } m_0 < \hat{m},$$

(23) combined with (25), (28) and (29) gives us

$$\frac{m_0}{q_0 - q_\ell} \geq 2\alpha n + (1-\alpha n)(1-(1-2\alpha)n)\sqrt{\frac{1-(1-2\alpha)n}{1+(1-2\alpha)n}}. \tag{30}$$

While (24) combined with (25), (28) and (29) gives us

$$\frac{m_0}{(q_0 - q_\ell)} \geq 1 - (1-2\alpha)\,n + \frac{\left(1-(1-2\alpha)^2 n^2\right)}{2}\sqrt{\frac{1-(1-2\alpha)n}{1+(1-2\alpha)n}}. \tag{31}$$

Proposition 8. *Suppose $m_0 > \hat{m}$ and the planner faces default constraints. (i) For $m_0 \in [\underline{m}, q_0)$ it is optimal to set $\alpha \in (0,1)$ and $q_\ell = 0$ so that there is a positive probability of default. (ii) For $m_0 \in (\hat{m}, \underline{m})$ it is optimal to set $\alpha = 0$ and $q_\ell = 0$, so that there is no default.*

Proof. The objective function of the planner is decreasing in q_ℓ and increasing in α, but $q_\ell = 0$ and $\alpha = 1$ is not feasible as this violates the constraint. Suppose first $\alpha = 0$, so that the constraint is irrelevant. Then the planner sets $q_\ell = 0$, and its objective function is

$$W\,(0,0) = \frac{2q_0}{1+n}.$$

Suppose now $\alpha > 0$, so that the planner faces the resource constraint (30). If (30) does not bind, it is always optimal for any given $\alpha \in (0,1]$ to decrease q_ℓ until it binds. Using (30) to get an expression for $q_0 - q_\ell$, and replacing it in the objective function of the planner, we get that the planner seeks to *minimize*

$$(1+(1-2\alpha)n)2\alpha n + (1-\alpha n)\,(1-(1-2\alpha)n)\sqrt{1-(1-2\alpha)^2 n^2}.$$

with respect to α. This function is concave and unimodal in α for all values of n. Since it is increasing at $\alpha = 0$, we first need to compare the value of this function at $\alpha = 0$ and $\alpha = 1$. At $\alpha = 0$, it is $(1-n)\sqrt{1-n^2}$ while at $\alpha = 1$, it is $(1-n)2n + (1-n^2)\sqrt{1-n^2}$, clearly greater than $(1-n)\sqrt{1-n^2}$. Hence, the planner would prefer to set α as low as possible. The restriction that $q_\ell \geq 0$, however imposes that

$$2\alpha n + (1-\alpha n)\,(1-(1-2\alpha)n)\sqrt{\frac{1-(1-2\alpha)n}{1+(1-2\alpha)n}} \geq \frac{m_0}{q_0}. \tag{32}$$

Since the left hand side is strictly increasing in α, the planner will set α so that (32) binds and $q_\ell = 0$. With $q_\ell = 0$, the planner's payoff is

$$W\,(\alpha, 0) = \frac{2}{1+(1-2\alpha)n} q_0 > W\,(0,0)\,.$$

At $\alpha = 0$, we have that

$$(1-n)\sqrt{\frac{1-n}{1+n}} \geq \frac{m_0}{q_0}, \tag{33}$$

which may not be satisfied, if m_0 is too close to q_0. Define $\underline{m}$ as

$$\underline{m} = q_0(1-n)\sqrt{\frac{1-n}{1+n}}$$

Then, for all $m_0 > \underline{m}$, (33) is violated, and α is optimally set such that (32) binds. Then, $\alpha > 0$ and $q_\ell = 0$. for all $m_0 < \underline{m}$ we have that (33) is satisfied, and the planner optimally sets $\alpha = 0$. As there is then no default, the planner also sets $q_\ell = 0$. □

Notice that the set $(\hat{m}, \underline{m})$ can be empty if $\underline{m} < \hat{m}$. However to be concise, we did not make the distinction. When the planner faces default, there needs to be sufficient funds available to cover default losses. Losses can be mitigated when $q_\ell > 0$. This is however costly as this means fundamental sellers have to keep some of their securities. Given $q_\ell = 0$, the planner prefers to limit losses by reducing the number of sellers. As the default problem becomes more acute, α eventually reaches zero, so that all speculators are buyers and therefore do not default. Given fundamental buyers incur a relatively large cost of holding securities, there is a (posibly empty) region where they do not default. Once fundamental sellers have an incentive to default, the planner sets $q_\ell > 0$ to limit default losses, as we now show.

Lemma 9. *Suppose $m_0 < \hat{m}$ and the planner faces default constraints. Then it is optimal to set $\alpha = 0$ and $q_\ell > 0$.*

Proof. The proof is in three steps. First, we show that $\alpha = q_\ell = 0$ is not feasible. Second, we show that at $q_\ell = 0$, $\alpha > 0$ is not feasible. Therefore, we conclude that $\alpha \geq 0$ and $q_\ell > 0$. Finally, we show that there exist $q_\ell > 0$ feasible such that $\alpha = 0$.

Step 1. We first show that if $m_0 < \hat{m}$,, then $\alpha = q_\ell = 0$ is not feasible. At $\alpha = 0$, the budget constraint (31) is

$$\frac{m_0}{(q_0 - q_\ell)} \geq 1 - n + \frac{(1-n^2)}{2}\sqrt{\frac{1-n}{1+n}}.$$

$$m_0 < \hat{m} = \frac{(2-\delta)(1-n)}{2} q_0.$$

$$\underline{m} = q_0(1-n)\sqrt{\frac{1-n}{1+n}}$$

There is a δ such that $\hat{m} > \underline{m}$. If $\delta = 1$, this depends on n. With $\delta = 1$ (this is the best case scenario in terms of no-default):

$$m_0 < \hat{m} = \frac{(1-n)}{2} q_0.$$

Hence, if $q_\ell = 0$, the constraint is

$$m_0 \geq q_0(1-n) + q_0 \frac{(1-n^2)}{2}\sqrt{\frac{1-n}{1+n}}$$

which can't be the case since $m_0 < (1-n)\, q_0/2$. Hence, we can't have both $\alpha = 0$ and $q_\ell = 0$.

Step 2. We now show that at $q_\ell = 0$, $\alpha > 0$ is not feasible. Indeed, the right hand side of (31) is strictly increasing in α, and we showed in Step 1 that $\alpha = q_\ell = 0$, is not feasible. Hence, at $q_\ell = 0$, increasing α will not be feasible. So, either $\alpha \geq 0$ and $q_\ell > 0$, or there is no trade.

Step 3. The planner seeks to maximize $W(\alpha, q_\ell) = \frac{2}{1+(1-2\alpha)n}(q_0 - q_\ell)$ subject to (31). As before, (31) will bind and replacing the expression for $(q_0 - q_\ell)$ in the planner's objective function, we get that the planner seeks to *minimize*

$$1 - (1-2\alpha)^2 n^2 + \frac{1-(1-2\alpha)^2 n^2}{2} \sqrt{1-(1-2\alpha)^2 n^2}.$$

This is a strictly increasing function of α and therefore the planner optimally sets $\alpha = 0$ and

$$q_\ell = q_0 - \frac{m_0}{\left[1 - n + \frac{(1-n^2)}{2}\sqrt{\frac{1-n}{1+n}}\right]}$$

Given $m_0 < \hat{m} = (1-n)\, q_0/2$, we obtain

$$q_\ell \geq q_0 - \frac{(1-n)\, q_0}{\left[2\,(1-n) + (1-n^2)\sqrt{\frac{1-n}{1+n}}\right]} > 0,$$

so that this q_ℓ is feasible. □

It is instructive to see that as soon as a default problem appears there is a discrete jump downwards in α and the positive mutualized default fund is introduced. This is intuitive. As speculators are still selling they have an incentive to default for some θ. To cover for losses, less cash is transferred to fundamental buyers for settlement, and they can buy relatively less securities. Therefore, to compensate for the lower demand, more speculators must buy securities (α must be less than one). As losses are strictly positive, there is a discrete jump downward in α as well as in welfare. At some stage, the optimal allocation implies that speculators all buy the security. Hence, they do not have an incentive to default anymore and CCP's guarantees are not any longer needed. Eventually, fundamental sellers, however, have an incentive to default and guarantees in form of a mutualized default fund may be needed again. However, for this case, position limits are needed, too.

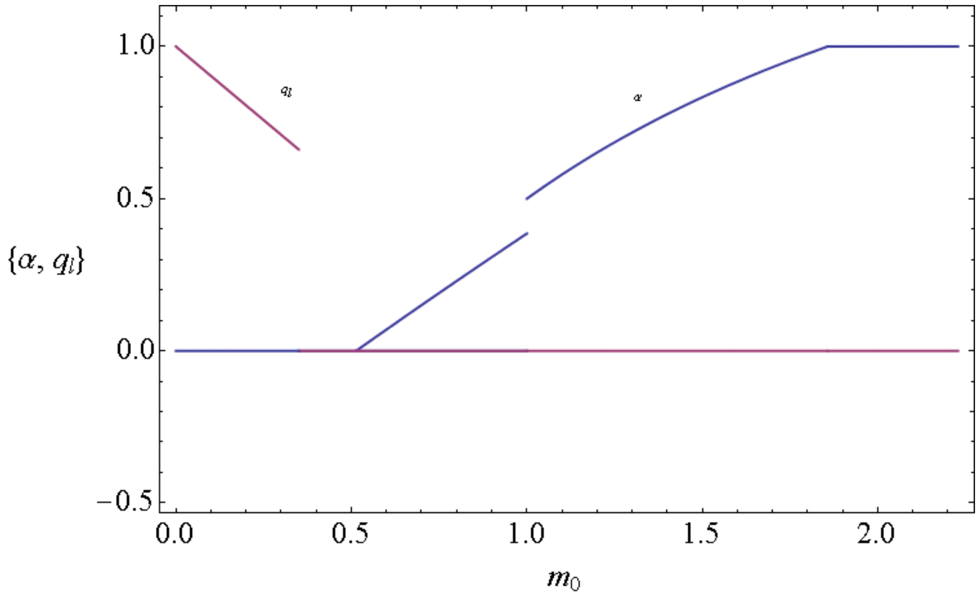

Figure 3: Optimal (α, q_ℓ) (with default when $m_0 < q_0$) as a function of available collateral m_0.

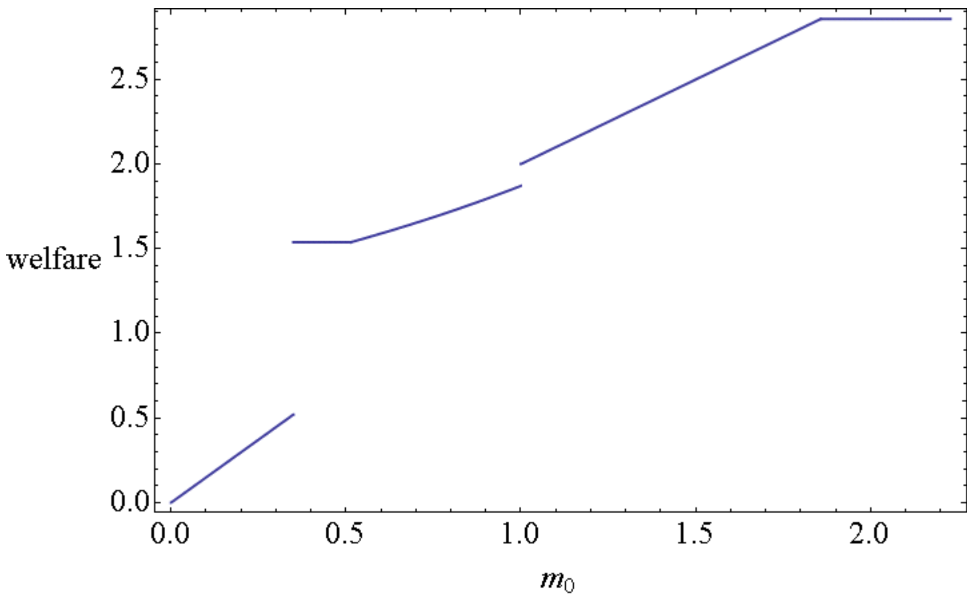

Figure 4: Welfare with default when $m_0 < q_0 = 1$.

4.4 Optimal Allocations

We now merge our characterization of the optimal allocations with and without default to find the optimal allocations irrespective of whether default is allowed or not.

Proposition 10. *There exists $\bar{m} < q_0$, such that the optimal allocation is the no-default allocation for all for $m_0 > \bar{m}$. If $n \geq \bar{n}$, the optimal allocation is the default allocation for all $m_0 \leq \bar{m}$. If $n < \bar{n}$, the optimal allocation is the default allocation for all $m_0 \in (\hat{m}, \bar{m})$, and the no-default allocation for all $m_0 \leq \hat{m}$.*

Proof. Let us first consider the case where $m_0 \in [\underline{m}, q_0)$. Then, welfare with default is (proportional to)

$$q_h - q_\ell = \frac{2}{1 + (1 - 2\alpha)\, n} q_0,$$

where α is defined by (32) holding with equality. Since (32) does not hold with equality when $\alpha = 1/2$, we know that the optimal value for α is strictly less than $1/2$. Hence, welfare with default is strictly lower than $2q_0$. However, welfare with no default is proportional to $2m_0$. Hence, when m_0 is close to q_0, the optimal allocation is the no-default allocation.

Now, let us consider the case when $m_0 \in (\hat{m}, \underline{m})$, where $\underline{m} = q_0\,(1 - n)\sqrt{\frac{1-n}{1+n}}$. Then welfare with default is (proportional to)

$$q_h - q_\ell = \frac{2}{1 + n} q_0.$$

At $m_0 = \underline{m}$, welfare with no default is

$$q_h - q_\ell = 2m_0 = 2q_0\,(1 - n)\sqrt{\frac{1 - n}{1 + n}}$$

which is strictly lower than the one with default. Welfare with default is continuous and strictly increasing in m_0 from $\underline{m}$. Hence, by continuity, there exists $\bar{m}$, such that for all $m_0 \in (\underline{m}, \bar{m})$, default is preferred to no default. We also strongly suspect that $\bar{m}$ is unique.

Finally, consider the case where $m_0 < \hat{m} = (1 - n)q_0/2$. Then allowing for default, it is optimal to set $\alpha = 0$ and $q_\ell > 0$ and welfare is (proportional to)

$$q_h - q_\ell = \frac{m_0}{(1 - n^2)\left(1 + \frac{1}{2}\sqrt{1 - n^2}\right)}.$$

While welfare is proportional to $2m_0$ in the case with no default. Therfore, there is a unique $\bar{n}$, such that for all $n \geq \bar{n}$, default is preferred to no default, and inversely for $n < \bar{n}$. The rest of the Proposition follows directly from our previous results. □

The two following figures illustrate the different cases.

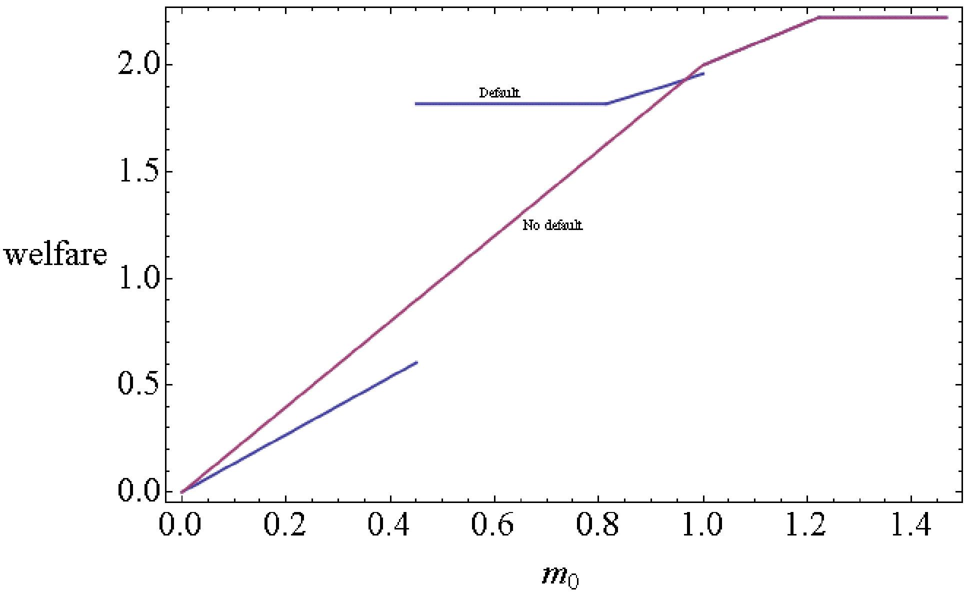

Figure 5: Welfare with and with no default. $n = 0.1$, $q_0 = 1$.

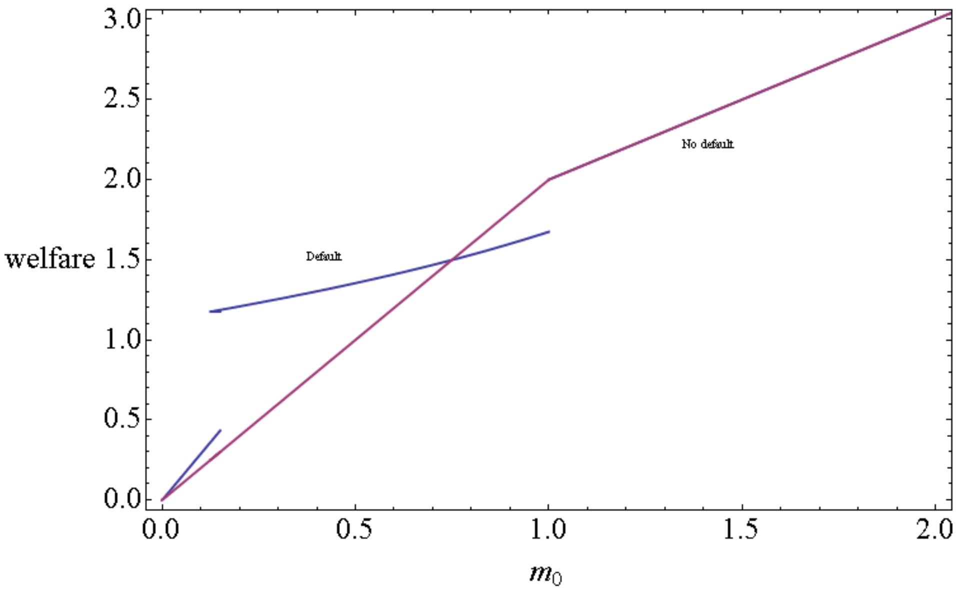

Figure 6: Welfare with and with no default. $n = 0.7$ and $q_0 = 1$.

5 Implementation

We investigate now, how actual risk management employed in practice by CCPs achieve the optimal allocation we have described in Section 4. Risk management involves the choice of guarantees against default exposure and collateral policies – margin calls, default fund and trading limits. We will show that commonly employed risk management tools can achieve the optimal allocation by influencing investors' trading behavior in equilibrium. The combination of of risk management tools employed will depend on the amount of liquidity and collateral (m_0) relative to overall trading needs (q_0) and the overall potential default risk given by $\theta_{\max}$. Our exposition here again assumes that $\theta_{\max} = 2$.

We now define what we mean by implementation. A collateral policy is summarized by a vector $w = (w_h, w_\ell, w_s)$, where w_k is the cash in hand of type k traders at the time of settlement. A collateral policy is feasible if w_0, the weighted sum of w_k, is less than m_0. An allocation (q_h, q_ℓ) is then implementable if there is a feasible collateral policy such that the resulting market equilibrium allocation is (q_h, q_ℓ). In the sequel, we will show how to relate the collateral policy w to margin call, default funds and position limits.

5.1 Novation and Margin Requirements

As our normative analysis showed, in general the CCP has to require collateral to rule out default. The reason is that exchanging collateral ex-ante between trading parties does not alleviate the default problem. Hence, it is crucial that the collateral facility operates as an independent third-party that novate trades. The collateral facility then interposes itself between the trading parties and becomes the buyer of every seller and the seller of every buyer. It is also important that the contract be legally binding, i..e that the resources of the collateral facility by sufficient to cover any default loss.

Collateral policies depend on the amount of collateral available. When the asset eligible as collateral (here cash) is sufficiently plentiful relative to trading needs and the exposure to default, i.e. $m_0 \geq (1+n)\, q_0/\,(1-n)$, collateral policies (default funds or margin calls) are indeterminate. In this case, the efficient allocation is implementable *even if* collateral is not available for settling the trade. When cash is scarce, but sufficient to fully collateralize all trades, two issues emerge. First, investors must be able to use some of the cash posted as collateral for settling their trade. Second, the distribution of settlement balances influences the price at which the security is traded in equilibrium.

When $m_0 \in [q_0, (1+n)\, q_0/\,(1-n))$, the optimal allocation is given by $q_h = q_0 + m_0 = m_\ell$. As there is no default and speculators must be indifferent between buying and selling, we need that

$$\frac{w_h}{q_h - q_0} = p = \frac{m_\ell - w_\ell}{q_0}. \tag{34}$$

Hence, for $w_h = w_\ell = m_0$, we have that $p = 1$ in equilibrium. The CCP then achieves the optimal allocation by requiring collateral at the trading stage $t = 1$ equal to m_0, which is used to directly settle the trade at $t = 2$. Requiring collateral in the form of *margin calls* – i.e. conditionally on taking a particular trading position – implements the optimal allocation. Using margin calls, investors can never trade more than their settlement balances as otherwise they would violate the margin requirement. In this sense, settlement balances and margin calls are in fact identical and all trades are settled fully against the margin.

Proposition 11. *Suppose* $\frac{1+n}{1-n} q_0 \geq m_0 \geq q_0$. *A collateral facility novating trades can achieve the (constrained) efficient allocation, by requiring a margin call equal to* m_0 *and allowing posted margins to be used for settling trades.*

Note that as liquidity falls, the ratio between collateral and trade size stays the same. The only feature that changes in equilibrium is the position that speculators take. As m_0 declines, more and more speculators become buyers until the number of sellers and buyers are the same at $m_0 = q_0$. Margin calls equal to m_0 do not influence the price which is constant at $t = 1$ and, hence, independent of the amount of collateral relative to the size of the market as given by q_0.

5.2 Guarantees, Mutualized Default Fund and Position Limits

In principle, the CCP would like to rule out default and have the highest trading volume possible. If collateral is sufficiently scarce ($m_0 < q_0$), these two objectives are incompatible. The CCP has then to guarantee against default and back this guarantee up by using a default fund.

The CCP can guarantee no default for buyers by setting a default fund at $t = 0$ equal to $f = m_0 - w_0$, where w_0 is the total amount of settlement balances available to investors. This default fund is mutualized to cover all default exposure. Furthermore, margin calls set equal to w_0 allow for maximum trade and achieve the optimal allocation as long as $m_0 \geq \hat{m}$. If $m_0 < \hat{m}$, the CCP uses also position limits. It restricts the amount of securities than can be sold, but allows for as many securities to be bought as possible.

What is important for implementing the optimal allocation, is the total amount of collateral earmarked for guaranting settlement and for direct settlement. The CCP for example could also rely only on the default fund, but earmark a sufficient share $(m_0 - w_0)/m_0$ as a guarantee. The rest of the fund could be used for settlement. Again, returning the default fund only for settlement, implements the optimal allocation uniquely for a profit-oriented CCP. Hence, it is not the distinction between margin calls and default fund per se what matters, but whether these funds mutually guarantee settlement or not.

Proposition 12. *Let $m_0 \in (0, q_0)$. A CCP achieves the optimal allocation by using a default fund to mutually guarantee settlement of trades, but allows all other collateral to be used for settlement. Furthermore, if $m_0 < \hat{m}$, the CCP imposes position limits on selling the security.*

It is instructive to look at the prices at which trading takes place for $m_0 < q_0$. Prices are always below 1, if we assume equal settlement balances – or, equivalently, margin calls – and take into account that we might need a mutualized default fund. Prices are then given by

$$\frac{w_0}{q_h - q_0} = p. \tag{35}$$

This implies for $m_0 \in (\underline{m}, q_0)$,

$$\frac{w_0}{q_h - q_0} = \frac{1}{1 - \alpha n} \frac{1 + (1 - 2\alpha)n}{1 - (1 - 2\alpha)n} \left(\frac{m_0}{q_0} - 2\alpha n \right) = p. \tag{36}$$

Using the definition of α, we obtain

$$p = \sqrt{1 - (1 - 2\alpha)^2 n^2} < 1. \tag{37}$$

Hence, sellers have still a preference to sell the security, as they have an option value of default. In fact, the price just balances the incentives to buy the security, as it makes the security cheap to purchase. The price declines with m_0, as α – the number of speculators that sell – also decreases.

If m_0 falls into the region $(\hat{m}, \underline{m})$, we do not have to default and $m_0 = w_0$. With margin calls, the price is then given by

$$\frac{m_0}{q_h - q_0} = \frac{m_0}{q_0} \frac{1 + n}{1 - n} = p. \tag{38}$$

Using the definition of $\underline{m} = (1 - n)\sqrt{\frac{1-n}{1+n}} q_0$, we obtain

$$p < (1 + n)\sqrt{\frac{1 - n}{1 + n}} < 1. \tag{39}$$

Hence, speculators have a strict preference to buy the security ($\alpha = 0$) as the price is below their expected valution of the security and there is no default. Also note that the price declines with m_0.

Finally, for $m < \hat{m}$, we have that

$$\frac{w_0}{q_h - q_0} = \frac{1+n}{1-n}\frac{m_0}{q_0 - q_\ell} = \frac{1+n}{1-n}\frac{m_0}{w_0}\left((1-n) + \frac{1-n^2}{2}\sqrt{\frac{1-n}{1+n}}\right) = p. \tag{40}$$

Using the definition of w_0, we get

$$p = (1-n)^2\sqrt{1-n^2} < 1. \tag{41}$$

Here again, prices are less than one and all speculators are sellers. Sellers obtain a low price, so for θ high enough, they have an incentive to default and a default fund to finance a guarantee is needed. Interestingly, the price stays constant for this region, as position limit successively restrict trading as m_0 declines. In conclusion, the clearing arrangments through a central counterparty lowers trading prices and – despite incurring a default risk – improves the allocation the market achieves. Without such an arrangement, avoiding default would have required position limits and trade at $p = 1$.

6 Conclusion

We provide a simple model of a Central Counterparty. The frictions we consider (trades are time-critical, limited liquidity and limited commitment) are common in stock exchanges, and we show that they are sufficient frictions to explain why CCPs novate trade. The model is also able to explain current collateral practices of operating CCPs.

There are many issues that remain to be studied. First, the willingness of the CCP to take on risk might depend on its governance structure. Currently CCPs operate under two main governance structures. The first structure is the mutual ownership of the CCP among members. We will refer to such institutions as user-oriented CCPs. The second type of institutions is operated on a for-profit basis, rather than optimizing the provision of services for the majority of its users. Traditionally CCPs were user-oriented institutions, but lately many CCPs have demutualized and switched their objectives toward profit-maximization. Explaining the reasons and the implications of this shift is obvioulsy an important question. Also, we have left aside issues of competition and the consequences of CCPs failure in case of a systemic event. These are also crucial questions and we hope that this paper provides a useful first benchmark to study these and other issues on CCPs.

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