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Volker Wieland

Center for Financial Studies
an der Universität Frankfurt ▪ Mertonstraße 17-21 ▪ D-60325 Frankfurt am Main
Telefon: +49 (0)69 798-30050
Fax: +49 (0)69 798-30077 ▪ http://www.ifk-cfs.de ▪ E-Mail: info@ifk-cfs.de

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Prof. Dr. Jan Pieter Krahnen

Prof. Volker Wieland, Ph.D.

Center for Financial Studies
an der Universität Frankfurt ▪ Mertonstraße 17-21 ▪ D-60325 Frankfurt am Main
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Learning, Endogenous Indexation and Disinflation in the New-Keynesian Model*

Volker Wieland[1]

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Abstract:
This paper introduces adaptive learning and endogenous indexation in the New-Keynesian Phillips curve and studies disinflation under inflation targeting policies. The analysis is motivated by the disinflation performance of many inflation-targeting countries, in particular the gradual Chilean disinflation with temporary annual targets. At the start of the disinflation episode price-setting firms' expect inflation to be highly persistent and opt for backward-looking indexation. As the central bank acts to bring inflation under control, price-setting firms revise their estimates of the degree of persistence. Such adaptive learning lowers the cost of disinflation. This reduction can be exploited by a gradual approach to disinflation. Firms that choose the rate for indexation also re-assess the likelihood that announced inflation targets determine steady-state inflation and adjust indexation of contracts accordingly. A strategy of announcing and pursuing short-term targets for inflation is found to influence the likelihood that firms switch from backward-looking indexation to the central bank's targets. As firms abandon backward-looking indexation the costs of disinflation decline further. We show that an inflation targeting strategy that employs temporary targets can benefit from lower disinflation costs due to the reduction in backward-looking indexation.

JEL Classification: E32, E41, E43, E52, E58

Keywords: Learning, Monetary Policy, New-Keynesian Model, Indexation, Inflation Targeting, Disinflation, Recursive Least Squares

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1 Goethe University of Frankfurt, CEPR, CFS and SCID, Contact: Goethe University of Frankfurt, Mertonstrasse 17, 60325 Frankfurt. Tel.: ++49-69-79825288, wieland@wiwi.uni-frankfurt.de, vwieland@stanford.edu.

1 Introduction

Developing a better understanding of the costs of disinflation has long been an important objective for macroeconomic research. Since the 1980s disinflation episodes and strategies have been studied extensively under the assumption of rational expectations. This assumption implies that central bank announcements regarding future policy plans can help achieve disinflation at little or no cost in terms of lost output in spite of the presence of price level rigidity. Many researchers consider this prediction too optimistic in light of historical experience. Thus, most models used for policy analysis today combine the rational expectations assumption with additional frictions that increase the cost of disinflation such as, for example, exogenous backward-looking indexation of wages and producer prices.

The success of many inflation-targeting countries in lowering inflation in the 1990s provides a new set of case studies that can improve our understanding of inflation-output tradeoffs and serve as a testing ground for macroeconomic modeling. On this basis, we can evaluate departures from the benchmark New-Keynesian model with rational expectations and exogenous indexation and investigate the desirability of alternative policy strategies. Chile, which was only the second country to adopt inflation targeting in 1990, constitutes a particularly interesting example as an increasing number of developing economies opt for inflation targeting. The Chilean disinflation stands out as a very gradual disinflation achieved with temporary annual inflation targets.

In light of the Chilean experience this paper examines the implications of two departures from the benchmark New-Keynesian model. First, we follow the recent literature on adaptive learning and replace the assumption of rational expectations with recursive least squares learning. Second, we introduce endogenous indexation by allowing firms to choose between backward-looking indexation and the central bank's announced target. At the start of the disinflation episode indexation is complete and price-setters expect highly persistent inflation. As price-setting firms learn over time they re-assess the likelihood of announced inflation targets and adjust indexation of contracts accordingly.

The findings in this paper indicate that learning and endogenous indexation may reduce the costs of disinflation. A gradual disinflation approach can take advantage of these favorable dynamics and achieve the long-run target at lower output costs. An interesting new result is the finding that announcing and meeting annual targets for inflation achieves lower disinflation costs relative to the announcement of a long-run inflation target that will only be met after many years of gradual disinflation. We confirm that the actual targets announced by the Central Bank of Chile during the disinflation from 1990 and 2001 induce favorable learning and indexation dynamics in our model.

The paper proceeds as follows. Section 2 shortly summarizes several aspects of the Chilean disinflation and the related literature. Section 3 compares traditional views with the New-Keynesian approach to understanding the costs of disinflation. In section 4, we introduce adaptive learning and endogenous indexation in the New-Keynesian model. Section 5 contrasts immediate to gradual disinflation strategies. In section we 6 we formulate different sequences of annual inflation targets and evaluate their performance in implementing disinflation. In section 7 we shortly discuss possible approaches to designing dynamically optimal policy while section 8 concludes.

2 Inflation targeting and disinflation: Chile 1991-2007

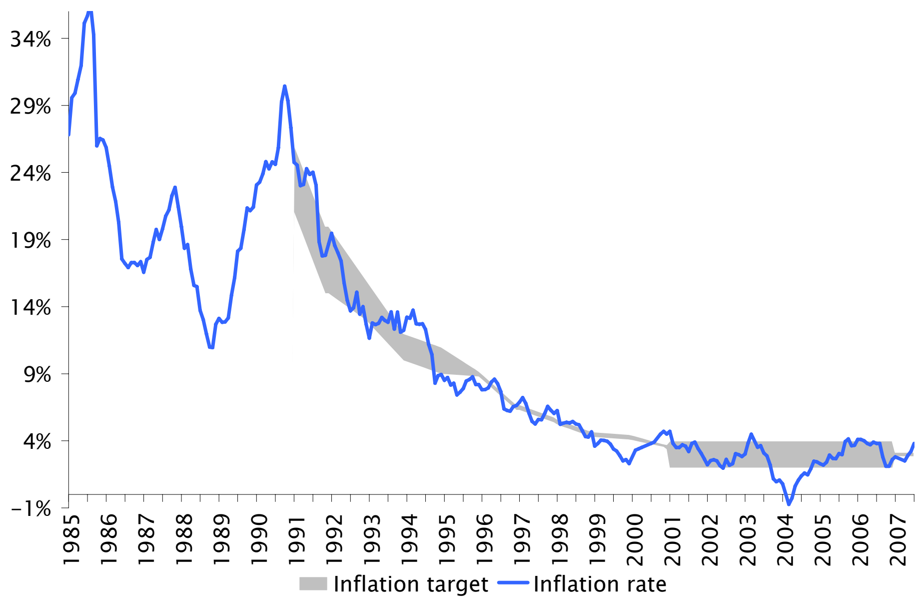
Inflation targeting started with public announcements of inflation targets in New Zealand and Chile in 1990. Since then, this monetary policy strategy has been implemented in many economies around the world. Among these economies are developed countries such as the United Kingdom, Canada, Sweden, Norway or Australia as well as an increasing number of developing countries. Many of these developing countries have been able to reduce inflation rates substantially following the adoption of inflation targeting and seem to have succeeded in stabilizing inflation at low to moderate rates. Fraga et al. (2003), Corbo and Schmidt-Hebbel (2003), Corbo et al. (2002) and Mishkin and Schmidt-Hebbel (2001) provide empirical assessments of the performance of inflation targeting in a large number of diverse economies.

Given the increasing popularity of inflation targeting in developing countries, any lessons for policy makers that can be derived from Chile's experience are particularly useful. The Chilean disinflation stands out as a very gradual disinflation. The central bank's first official target was publicly announced in September 1990. It was set for a range of 15 to 20 % for the rate of annual CPI inflation between December 1990 and December 1991. From 1991 to 1999 inflation target ranges and point targets were set on an annual basis for the following calendar year. **Figure 1** reports the inflation targets (shaded zones in light blue) along with actual inflation (dark blue line).

Initially, many observers were sceptical regarding the importance of the Chilean central bank's strategic framework in achieving disinflation. They attributed much of the improvement to good luck in form of exogenous developments concerning the exchange rate and raw material prices. Calvo and Mendoza (1999), for example, wrote: " ... *factors other than stabilization policies have played an important role in Chilean economic performance, and the dynamics exhibited by key macroeconomic aggregates can be interpreted in part as an endogenous process of adjustment triggered by exogenous shocks.*" However, the amazing success of the Central Bank of Chile in meeting its annual inflation targets during the disinflation phase from 1990 to 2001 and its continued ability to keep inflation close to or within the target zone of 2 to 4 percent suggest that its strategic framework played an important role.

Aguirre and Schmidt-Hebbel (2005) argue that the short-term annual targets announced during the disinflation phase were observationally equivalent to hard policy targets in full-fledged inflation targeting regimes. They provide some evidence in favor of this view. In spite of low initial policy credibility and widespread backward-looking price indexation in goods, labor, and

Figure 1: Inflation Targets and Actual Inflation in Chile, 1985-2007



Note: This figure extends Figure 1 in Schmidt-Hebbel and Werner (2002) with data up to 2007 and has kindly been provided by Klaus Schmidt-Hebbel, Gustavo Leyva and Fabian Gredig from the Central Bank of Chile.

financial markets, disinflation was achieved at relatively low costs in terms of associated output losses. Aguirre and Schmidt-Hebbel (2005) suggest that the central bank was able to overcome the consequences of backward-looking price indexation and related inflation inertia, and to influence private-sector inflation expectations, by pursuing a forward-looking inflation target that served as an explicit nominal anchor. Along these lines, Corbo et al (2003) draw three main lessons from Chile's experience that should be of interest to other developing countries:

> *"First, initial progress in reducing inflation toward the target was slow as the public was learning about the Central Bank's true commitment to attaining the target.*
>
> *Second, the gradual phasing in of inflation targeting contributed to declining inflation by lowering inflation expectations and changing wage and price dynamics.*
>
> *Third, with respect to the speed of inflation reduction, a cold-turkey approach would have resulted in a larger sacrifice ratio stemming from higher unemployment during the early years of inflation targeting, when credibility was gradually being built up."*

These conclusions suggest that learning by price-setting firms and changes in the degree of backward-looking indexation regarding wages and producer prices played an important role in shaping the costs of disinflation in Chile.[1]

More recently, researchers have developed and estimated sophisticated New-Keynesian dynamic general equilibrium models for policy analysis in Chile (cf. Caputo et al (2007), Caputo et al (2006), DeGregorio and Parrado (2006) and Cespedes et al (2005)). These models share the assumptions of rational expectations and exogenous backward-looking indexation with similar models developed for industrialized economies (cf. Christiano et al. (2005)). The New-Keynesian Phillips curve embedded in these models, however, does not seem to be stable. For example, Cespedes et al (2005) report evidence of structural change during the late 1990s. This change is exhibited in a higher weight of expected future inflation - and a correspondingly lower weight of lagged inflation - when producers set their prices. For a sample from 1990 to 2000 they estimate a degree of backward-looking indexation around 0.85, essentially indistinguishable from the limiting case of complete indexation. With the sample extended to 2005, however, the degree of indexation declines to around 0.66.

In the remainder of this paper, we explore departures from the standard New-Keynesian model by allowing for adaptive learning and endogenous indexation. We investigate whether the particular choice of inflation targeting strategy may influence the costs of disinflation by increasing the speed of learning and reducing the degree of backward-looking indexation.

3 Disinflation and the New-Keynesian Phillips curve

A traditional perspective on disinflation

It is conventional wisdom among central bankers that conducting monetary policy so as to keep inflation constant at all times will induce fluctuations in aggregate real output. Historical experience such as the 1980s Volcker disinflation in the United States suggests that a permanent reduction in the rate of inflation cannot be achieved without a temporary decline of output below the economy's potential. Such a cost of disinflation is embedded in the traditional accelerationist Phillips curve:

$$\pi_t = \pi_{t-1} + \lambda x_t \tag{1}$$

Here, π_t denotes the rate of inflation and x_t the output gap, that is the deviation of actual output from the economy's potential.

A simple experiment serves to illustrate the cost of disinflation. Assume that inflation in period $t = 1$ is equal to 1 percent and that the central bank aims to achieve price stability, i.e. an inflation rate of zero percent, in period $t = 2$. Such a reduction in the rate of inflation requires a negative output gap of $-1/\lambda$ percent in period $t = 1$. In the absence of any future shocks that might push inflation up or down, inflation could then be held at zero from period 2 onwards by keeping the

[1] See also Herrera (2002) and Lefort and Schmidt-Hebbel (2002).

output gap closed. Thus, the cumulative output loss in absolute terms that is required to achieve a reduction in inflation of 1 percent point corresponds to $1/\lambda$ percent of total output.

In central bank circles the cumulative output loss associated with a permanent reduction of the inflation rate by one percent is often referred to as the sacrifice ratio. If equation (1), the accelerationist Phillips curve is treated as a structural relationship the associated sacrifice ratio is constant at $1/\lambda$ and invariant to policy design. In other words, no particular strategy or announcement by the central bank could help in changing the trade-off between output and inflation or in reducing the cumulative output cost of a disinflation. Nevertheless, a central bank that cares about stabilizing output and inflation would always opt for disinflating gradually and spreading the output loss over time.

The New-Keynesian perspective on disinflation

The traditional Phillips curve shown above lacks microeconomic foundations. Fortunately, the New-Keynesian paradigm offers an alternative model of inflation that is consistent with optimizing behavior and rational expectations formation by households and firms. However, the basic version of the New-Keynesian model has a very controversial property. In this model the macroeconomic policy goals of stabilizing output and inflation do not come into conflict with each other (cf. Walsh (2003), Woodford (2003)). This property is often referred to as the ”divine coincidence”. It implies that disinflation can be achieved without any reduction in aggregate output. It is somewhat surprising that a model that incorporates long-lasting nominal rigidities exhibits such a property. To understand its origins it is helpful to reiterate the elements of the model that drive price-setting and inflation dynamics.

The model is populated by a continuum of monopolistic firms that produce differentiated goods. Importantly, these firms cannot adjust product prices freely in every period. The basic version of the model relies on the mathematically convenient mechanism for modeling price rigidity due to Calvo (1983). It implies that firms have to wait for a signal to adjust prices. They receive such a signal with probability $1-\theta$. Every firm that receives a price-setting signal solves a dynamic optimization problem to set its price optimally taking into account the probabilistic constraint on future price-setting opportunities. A firm j that does not receive a price-setting signal leave its price unchanged at the zero inflation steady state. Alternatively, if the steady-state rate of inflation, π^S, differs from zero, firm j lets its price grow with that steady-state rate, i.e. $P_{j,t} = (1+\pi^S)P_{j,t-1}$. In other words, firms that are not allowed to re-optimize their price instead are assumed to index to steady-state inflation. In solving their optimization problem firms are assumed to form rational, model-consistent expectations.

A useful feature of this model is that it can be solved without explicitly tracking the distribution of prices across firms. Aggregation and log-linear approximation deliver a well-known, simple relationship between inflation, expected future inflation and the output gap - the New-Keynesian Phillips curve:

$$(\pi_t - \pi^S) = \beta E_t[(\pi_{t+1} - \pi^S)] + \lambda x_t \tag{2}$$

Here, the output gap x_t denotes the difference between actual output and the level of output that would be achieved if prices were flexible. The parameter β refers to the discount factor. The slope parameter λ is a function of θ and β.[2]

Again, a simple experiment serves to assess the cost of disinflation. Suppose the central bank enters period $t = 1$ with an inflation target, π^*, equal to 1 percent. Since equation (3) is linear, the steady-state rate of inflation must be equal to the central bank's target, $\pi^S = \pi^*$. In period $t = 2$ the central bank announces a new target rate of zero percent inflation. Market participants would immediately incorporate the new target in their expectations for period $t = 3$. It would imply zero inflation in steady state. As a result, inflation in period $t = 2$ immediately drops to the new target rate. No reduction in the output gap, x_t, is required to achieve this outcome. Disinflation is costless. It is achieved by influencing market participants' expectations.

The model's implication of costless disinflation stands in contrast to historical experience. For this reason, researchers who have estimated New-Keynesian models using data from leading industrial economies have typically assumed an additional source of price rigidity. One possible approach is to introduce firms that apply a simple rule-of-thumb in price setting as in Gali and Gertler (1999). An alternative approach assumes that some firms index prices to past inflation in those periods when they cannot adjust prices optimally (Christiano et al. (2001, 2005)).

By now backward-looking indexation has become a popular assumption embedded in many empirically estimated DSGE models used for monetary policy analysis. Firms that do not receive a Calvo-style signal to adjust prices in the current period are assumed to implement instead a pricing rule based on past inflation, i.e. $P_{j,t} = (1 + \pi_{t-1})P_{j,t-1}$. The share of firms that use backward-looking indexation, denoted by κ in the following, is treated as exogenous. Consequently, the log-linear approximation of the New-Keynesian Phillips curve takes the following form:

$$\pi_t - (\kappa\pi_{t-1} + (1-\kappa)\pi^S) = \beta E_t[(\pi_{t+1} - (\kappa\pi_t + (1-\kappa)\pi^S))] + \lambda x_t \tag{3}$$

Solving for the current inflation rate one finds that it depends on a weighted average of past and expected future inflation. The weight is a function of the share of firms that implement backward-looking indexation:

$$\pi_t = \frac{\kappa}{1+\beta\kappa}\pi_{t-1} + \frac{\beta}{1+\beta\kappa}E_t[\pi_{t+1}] + \frac{\lambda}{1+\beta\kappa}x_t + \frac{(1-\kappa)(1-\beta)}{1+\beta\kappa}\pi^S \tag{4}$$

[2]To be precise, the baseline version of the model (cf. Walsh (2003)) implies that λ is determined as follows: $\lambda = (1-\theta)(1-\beta\theta)\theta^{-1}(\sigma+\varphi)$. σ^{-1} and φ represent the constant intertemporal elasticity of consumption and labor supply elasticity, respectively.

It is useful to note that in the limiting case of complete indexation, $\kappa = 1$, the inflation equation simplifies to

$$\pi_t = \frac{1}{1+\beta}\pi_{t-1} + \frac{\beta}{1+\beta}E_t[\pi_{t+1}] + \frac{\lambda}{1+\beta}x_t \tag{5}$$

Interestingly, with complete indexation the current inflation rate is independent of steady-state inflation π^S.

Equation (4) has been estimated for many countries. Estimates for Chile have been obtained by Cespedes et al. (2005), Caputo et al. (2006) and Caputo et al. (2007). Cespedes et al (2005) took care to account for time-variation in the inflation target. In this case, the last term in equation (4) is modified to $(1-\kappa)(1+\beta\kappa)^{-1}(\pi_t^* - \beta\pi_{t+1}^*)$. They report evidence of structural change. For a sample from 1990 to 2000 they estimate a degree of backward-looking indexation around 0.85, essentially indistinguishable from the limiting case of complete indexation. With the sample extended to 2005, however, the degree of indexation declines to around 0.66.

In this paper, we relax two important assumptions of the standard model - the assumption of rational expectations and the assumption of exogenous backward-looking indexation. Relaxing these assumptions is important because of the empirical evidence regarding changes in the degree of inflation persistence during and following the disinflation in Chile. The reduction in inflation persistence may well be due to changes in price setters' beliefs or to changes in the degree of backward-looking indexation. Thus, we depart from the assumption of rational expectations by considering adaptive learning. In doing so we follow the lead of Marcet and Sargent (1987), Evans and Honkapohja (2001), Orphanides and Williams (2005, 2006) and Gaspar et al. (2005, 2006). Furthermore, we innovate by rendering the share of firms that implement backward-looking indexation endogenous. In particular, we allow firms to choose between the central bank's inflation target and past inflation as possible indices. This choice of index is made according to the likelihood that the chosen index better matches the mean of the observed inflation distribution. Thus, firms aim to choose the index that seems a better estimate of steady-state inflation.

4 Adaptive learning and endogenous indexation

Introducing adaptive learning

As shown above expectations play a key role in determining inflation dynamics. Since the 1980s research on monetary policy has relied on the assumption of rational expectations and explored its implications for policy design. A drawback of the assumption of rational expectations is that it imputes an unrealistic extent of knowledge to market participants. An interesting alternative approach is given by adaptive or least-squares learning. This approach assumes that economic agents behave like econometricians in forming expectations and estimate reduced-form inflation equations. Under certain assumptions adaptive learning may converge to rational expectations in the long-run.

Following the influential contribution of Evans and Honkapohja (2001) Orphanides and Williams (2005, 2006) and Gaspar et al. (2005, 2006) have studied monetary policy design with price-setting firms that form expectations about future inflation in a least-squares fashion. Motivated by this line of research, we assume that price-setting firms estimate the following regression for inflation:

$$\pi_t = \gamma_t \pi_{t-1} + \varepsilon_t \tag{6}$$

The parameter γ_t carries a time subscript to allow for episodes with high and low degrees of inflation persistence. We make this assumption because our model will endogenously generate a time-varying degree of inflation persistence. Incorporating this time variation in price setters' perceived inflation equation ensures that expectations formation is consistent with equilibrium outcomes. Recursive estimation then implies the following updating equations for the price setters' point estimate of the inflation persistence parameter, c_t, and its variance, Σ_t:

$$\begin{aligned} c_t &= c_{t-1} + \Sigma_{t-1}\pi_{t-1}F^{-1}(\pi_t - c_{t-1}\pi_{t-1}) \\ \Sigma_t &= \Sigma_{t-1} - \Sigma_{t-1}X_t F^{-1} X_t' \Sigma_{t-1} + \sigma^{\gamma} \\ &\qquad \text{where } F = X_t \Sigma_{t-1} X_t' + \sigma^{\varepsilon} \end{aligned} \tag{7}$$

For a derivation of these updating equations using the Kalman filter see Harvey (1992). Furthermore, the updating equations are consistent with Bayes rule under the assumption of normally distributed shocks and beliefs (see Zellner (1971)). It is typical for the adaptive learning literature that researchers choose from a variety of learning specifications. Branch and Evans (2006) provide a useful exposition of alternative approaches and investigate how well they fit survey expectations.

Given equations (6) and (7) the price setters' expectation of future inflation under least-squares learning, $E_t^{LS}[\pi_{t+1}]$, corresponds to:

$$E_t^{LS}[\pi_{t+1}] = c_{t-1}\pi_t. \tag{8}$$

Here, we follow Gaspar et al. (2006) in assuming that $E_t^{LS}[\pi_{t+1}]$ is based on the estimate c_{t-1} that does not yet incorporate the most recent inflation observation π_t.[3] Using equation (8) to substitute out expected future inflation in equation (4) one obtains the following reduced-form inflation equation:

$$\pi_t = \frac{\kappa}{1+\beta(\kappa - c_{t-1})}\pi_{t-1} + \frac{\lambda}{1+\beta(\kappa - c_{t-1})}x_t + \frac{(1-\kappa)(1-\beta)}{1+\beta(\kappa - c_{t-1})}\pi^S \tag{9}$$

[3] Alternatively, one could either use only lagged information, i.e. $E_t^{LS}[\pi_{t+1}] = c_{t-1}^2\pi_{t-1}$ or incorporate current inflation in the estimate of the persistence parameter, $E_t^{LS}[\pi_{t+1}] = c_t\pi_t$. The latter specification would require solving a more complicated fixed point problem.

Adaptive learning in form of the time-varying estimate, c_{t-1}, influences the observed degree of inflation persistence. In addition, the degree of persistence depends on central bank policy.

Introducing endogenous indexation

So far, the degree of backward-looking indexation, κ, has been treated as constant and exogenous. A novel contribution of this paper is to allow for an endogenous determination of a time-varying share of firms that apply backward-looking indexation. We assume that firms would like to pick an index that is a good estimate of steady-state inflation. They have two options. One option is the central bank's announced inflation target, π^*. If the central bank delivers on its promise, then steady-state inflation will be equal to the target. The other option is the most recent observation of inflation, π_{t-1}. If the central bank does not aim to control inflation, the inflation rate will follow a random walk and past inflation will be the best estimate of future inflation.

Every time firms obtain a new observation on inflation, they investigate whether the target or past inflation better matches the mean of the observed inflation distribution. We use $s_t = Prob(\pi^S = \pi^*)$ to denote the probability that the announced inflation target corresponds to the mean of the observed inflation distribution. When a new observation becomes available, s_t is updated as follows:

$$s_{t+1} = \frac{s_t e^{(-0.5(\pi_t - \pi^*)^2)}}{s_t e^{(-0.5(\pi_t - \pi^*))^2} + (1 - s_t) e^{(-0.5(\pi_t - \pi_{t-1}))^2}} \tag{10}$$

This updating equation is consistent with Bayes rule given normal shocks and beliefs.[4]

Firms cannot switch indices at all times. They are allowed to make a choice regarding the index at the same time as they receive a Calvo-style signal that allows them to adjust their current price optimally. The probability of such a signal is $1 - \theta$. A firm that has received such a signal will then consider whether to switch the index that will apply to its pricing rule in the periods without Calvo signals. One possibility would be to assume that firms switch from backward-looking indexation to the central bank's target as soon as the probability s_t has moved above 0.5 and switch back if this probability falls slightly below 0.5. Such an assumption would seem reasonable in the unlikely case that the index can be switched at zero cost.

Instead, we assume that firms only choose to switch the index when there is overwhelming evidence in favor of such a change. Specifically, we introduce a trigger probability $\bar{S}$. If the firm's current choice of index is π_{t-1}, it will switch to π^* once s_t exceeds $\bar{S}$. Similarly, if the current choice of indexation rate is π^*, the firm will switch back to π_{t-1} if $(1 - s_t)$, the probability of π_{t-1}, exceeds the same trigger value. We note that all firms face the same information regarding inflation. Thus, s_t is symmetric across firms. Since the probability of a Calvo signal is $1 - \theta$, a share of $1 - \theta$ firms switches the rate of indexation at any point in time given there is overwhelming

[4] See Wieland (2000).

evidence in favor of such a shift.

Finally, the degree of indexation κ_t is allowed to vary between complete indexation, i.e. $\kappa_t = 1$, and a minimal value of $\underline{\kappa}$, i.e. $\kappa_t \in [\underline{\kappa}, 1]$.[5] Thus, κ_t is governed by the following process:

$$\kappa_t = \begin{cases} \theta\kappa_{t-1} & \text{if } s_t > \bar{S} \text{ and } \kappa_t \geq \underline{\kappa} \\ \\ 1-\theta(1-\kappa_{t-1}) & \text{if } (1-s_t) > \bar{S} \\ \\ \kappa_{t-1} & \text{else} \end{cases} \tag{11}$$

Every period in which s_t exceeds the trigger probability, a share of $1-\theta$ firms switches from backward-looking indexation to the central bank's target, while a share of θ firms sticks with the past inflation rate.

Since the share of firms using backward-looking indexation varies over time, the reduced-form inflation equation (9) needs to be re-written as follows:

$$\pi_t = \frac{\kappa_{t-1}}{1+\beta(\kappa_{t-1}-c_{t-1})}\pi_{t-1} + \frac{\lambda}{1+\beta(\kappa_{t-1}-c_{t-1})}x_t + \frac{(1-\kappa_{t-1})(1-\beta)}{1+\beta(\kappa_{t-1}-c_{t-1})}\pi^S \tag{12}$$

As a short-hand we will denote the time-varying, reduced-form parameters by $\delta_{(1,2,3),t}$. Accordingly, the reduced-form inflation equation may be written as:

$$\pi_t = \delta_{1,t}\pi_{t-1} + \delta_{2,t}x_t + \delta_{3,t} \tag{13}$$

To be able to study disinflation under alternative targeting strategies we still need to describe the central bank's objectives and the determination of the output gap x_t.

5 Inflation targeting: Immediate versus gradual disinflation

Central bank objectives and policy

A central bank that has adopted an inflation targeting strategy is typically assumed to pursue a policy that minimizes the following per-period loss function:

$$l(\pi_t, x_t) = (\pi_t - \pi^*)^2 + \alpha x_t^2 \tag{14}$$

The parameter α refers to the central bank's relative preference for stabilizing output versus infla-

[5] We maintain a minimal amount of exogenous indexation to ensure that lagged inflation remains a determinant of the equilibrium inflation process under rational expectations. As a result, the learning model uses the correct reduced-form inflation equation under rational expectations.

tion.

To keep the technical analysis manageable we make two simplifying assumptions. First, the central bank directly controls the output gap, x_t. Second, it observes the key parameters of the inflation equation as well as the price setters' beliefs regarding inflation persistence, c_{t-1}. Thus, the central bank can take into account the parameters $\delta_{(1,2,3),t}$ of equation (13) in designing its policy. However, we refrain from letting the central bank exploit the dynamic learning process of the price-setters in conducting policy.[6] Under these assumptions the central bank's dynamic optimization problem corresponds to:

$$\begin{gathered} \underset{x_t}{\text{Min}}\, E_t \left[\sum_{t=1}^{\infty} \beta^{t-1} (\pi_t - \pi^*)^2 + \alpha x_t^2 \right] \\ \text{s.t. } \pi_t = \delta_{1,t}\pi_{t-1} + \delta_{2,t}x_t + \delta_{3,t} \end{gathered} \tag{15}$$

As a start we consider the extreme cases of strict inflation targeting, $\alpha = 0$, and strict output stabilization, $\alpha \rightarrow \infty$. Strict output stabilization would imply that the central bank always aims to set the output gap, x_t, equal to zero. Consequently, the dynamics of inflation would be governed exclusively by the time-varying parameter $\delta_{1,t}$, which depends in turn on the degree of backward-looking indexation and the price setters' beliefs regarding inflation persistence. If $\delta_{1,t}$ ever exceeded unity, inflation would spiral out of control.

In contrast, strict inflation targeting would ensure that the inflation target is met at all times for any perceived degree of inflation persistence. The resulting output gap policy corresponds to:

$$x_t = -\delta_{4,t}(\delta_{1,t}\pi_{t-1} + \delta_{3,t} - \pi^*) \tag{16}$$

with $\delta_{4,t} = \delta_{2,t}^{-1}$. Note, with a zero inflation target, $\delta_{3,t}$ would also be equal to zero.

Next, we consider the intermediate case with α positive but not infinite. Such central bank preferences are often referred to as flexible inflation targeting. Under this policy the output gap falls in between the two extremes implied by strict inflation targeting and strict output stabilization, i.e. $0 < \delta_{4,t} < \delta_{2,t}^{-1}$. Orphanides and Wieland (2000) provide an analytical formula for the case of $\delta_{1,t} = 1$. Dynamically optimal policies for alternative values of $\delta_{1,t}$ may be computed numerically with the algorithm provided along with that paper.[7]

Model parameterization and initial conditions

[6] We will discuss such an ambitious proposal in the last section of the paper. Gaspar et al. (2006) refer to a central bank with this capability as "sophisticated".

[7] The matlab code is available from www.volkerwieland.com.

Having specified a very stylized but complete macroeconomic model we can proceed to evaluate alternative disinflation strategies. Initial conditions for the disinflation are defined as follows: (i) initial inflation is set at 20 percent, $\pi = 0.2$, similar to the average inflation rate of Chile prior to the start of inflation targeting; (ii) initially all firms implement backward-looking indexation, $\kappa_0 = 1$; and (iii) perceived inflation persistence indicates a unit root in inflation, i.e. $c_0 = 1$.

Given these initial conditions the reduced-form inflation equation (13) simplifies to

$$\pi_t = \pi_{t-1} + \lambda x_t, \tag{17}$$

corresponding exactly to equation (1), the accelerationist Phillips curve discussed in section 2. It follows that these initial conditions represent an equilibrium if policy aims exclusively at stabilizing output, i.e. $x_0 = 0$. The parameter values used in the subsequent simulations are summarized in Table 1.

Table 1: Parameter values and initial beliefs

Parameter	Value	Economic interpretation
β	0.99	Discount factor.
λ	0.5	Slope of Phillips curve.
κ_t	$\kappa_0 = 1$	Degree of indexation to $t-1$ inflation.
c_t	$c_0 = 1$	Price setters initial belief regarding inflation persistence.
Σ_t	$\Sigma_0 = 100$	Price setters initial variance.
s_t	$\sigma_0 = 0.1$	Price/index setters initial belief regarding $Prob(\pi^S = \pi^*)$.
π_0, π^*	$0.2/0$	Initial inflation is at 0.2, long-run inflation target is 0.
$\underline{\kappa}$	0.05	Degree of minimal exogenous indexation.
θ	0.5	Probability of no price- or index-adjustment signal.
$\bar{S}$	0.8	Trigger probability for switching the rate for indexation.
σ	2^{-4}	Variance of noise (added later).
σ_γ	10	Belief regarding variability of γ.

Immediate versus gradual disinflation

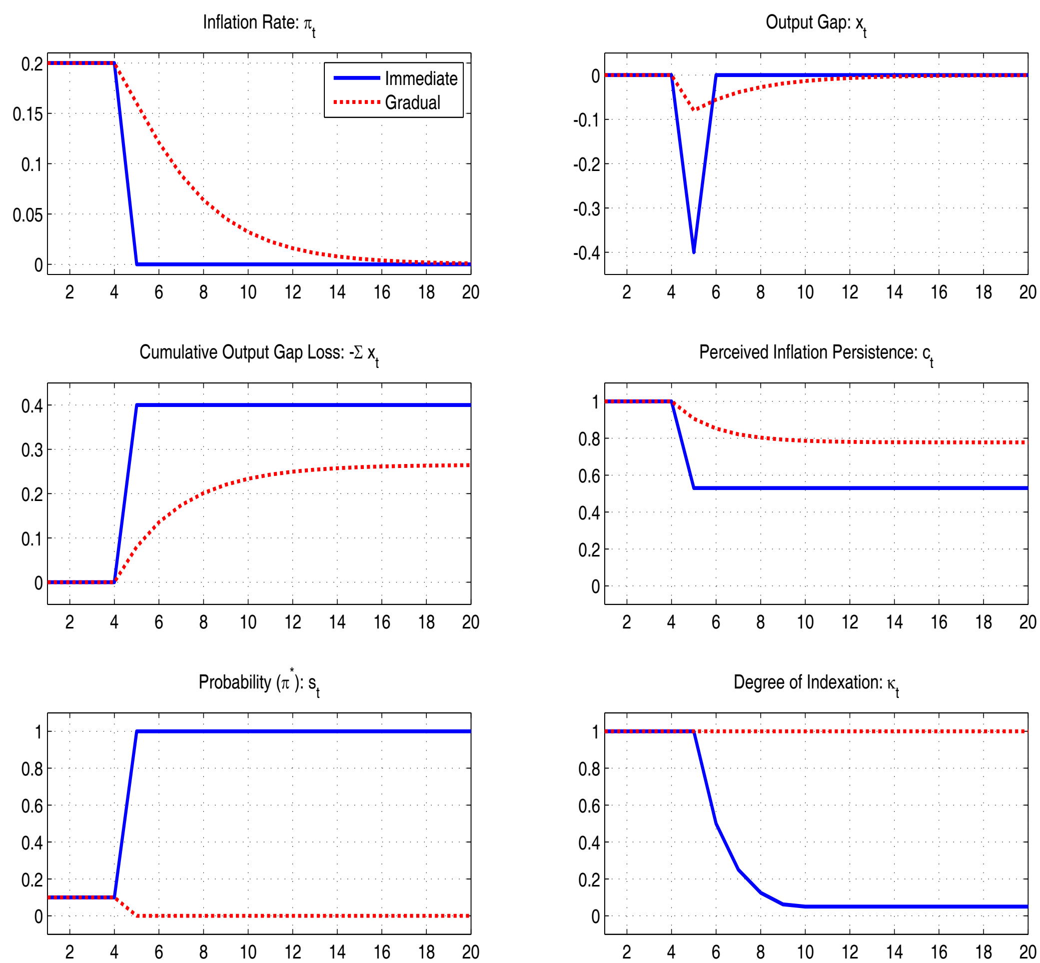
The initial conditions summarized above set the stage for the entry of an independent inflation-targeting central bank.[8] This central bank faces very high initial costs of disinflation. As a first step, we contrast the *immediate* disinflation approach that would be implemented under strict inflation targeting with a more *gradual* approach consistent with a positive weight on output in the central bank's preferences.

The optimal policy coefficient under strict inflation targeting corresponds to the inverse of the slope of the reduced-form inflation equation and equals $\delta_{4,0} = \delta_{2,0}^{-1} = 2$. In our model such a policy

[8] For a fascinating account of the implications of learning for inflation and stabilization when money growth and inflation are determined by the government's budget constraint rather than an independent central bank the reader is referred to Sargent, Williams and Zha (2007).

would achieve the inflation target of zero percent within one period. However, such an immediate disinflation would result in an output loss of 40 percent in the same period. This outcome is shown by the solid blue line in **Figure 2**. In period 5 the central bank introduces a new inflation target of zero percent. The cumulative output loss required to disinflate by 20 percentage points is also realized in period 5. While this approach can be simulated in our simple model such an immense reduction of total output would not be implementable in practice.

Figure 2: Immediate versus Gradual Disinflation



Interestingly, the dramatic experience of immediate disinflation induces price setters to revise their estimates of the inflation persistence parameter, c_t, from 1.0 to about 0.5 (middle-right panel). Furthermore, the probability s_t that is initially set at 0.1 jumps to 1.0. In other words, the immediate reduction in inflation convinces firms that the central bank's inflation target constitutes a better estimate of the mean of the inflation distribution than the past realization of inflation. Thus, from period 6 onwards the probability s_t exceeds the trigger value $\overline{S}$ (bottom left panel) and firms that receive a Calvo signal will abandon backward-looking indexation and instead choose the central bank's target as index. Since the probability of such a signal is $1-\theta$, a share of θ firms continues to implement backward-looking indexation. Thus, κ_t declines over time to the minimum exogenous degree of indexation, $\underline{\kappa}$, (bottom-right panel).

Unfortunately, a strict inflation targeting strategy fails to take advantage of the reduction in the cost of disinflation due to the decline in perceived inflation persistence and backward-looking indexation. The reason is simply that the disinflation is completed prior to these favorable developments. Instead, a gradual disinflation strategy might be able to profit from such developments and achieve disinflation at lower output costs. A gradual disinflation strategy is optimal if central bank preferences incorporate output stability, i.e. a positive weight α in the loss function, that is equation (14). In this case, the response parameter δ_4 in the policy function, equation (20), must be positive but below δ_2^{-1}.

To simulate a gradual disinflation we set $\delta_{4,t} = \delta_{2,t}/(1+\delta_{2,t}^2)$. Initially, the policy response coefficient $\delta_{4,t}$ corresponds to 0.4, that is one fifth of the policy response needed to meet the target immediately. The resulting outcome is depicted by the red dotted line in **Figure 2** with the disinflation again starting in period 5. The initial output decline is much smaller but will be sustained for a much longer time than in the case of immediate disinflation. The inflation rate declines gradually. By period 15 inflation is within 0.5 percentage points of the long-run target of zero. Treating a period in this model as a year, such a 10-year disinflation is broadly similar to the Chilean experience between 1991 and 2001.

The cumulative sum of output gap losses is much smaller under the gradual approach than under strict inflation targeting. The cumulative output loss converges to about 26% of annual output spread over more than 10 years. The reason for the decline in the sacrifice ratio from 2 in the case of strict inflation targeting to about 1.3 in the case of gradual disinflation is to be found in adaptive learning. As price-setters observe the fall of the rate of inflation they revise their estimate of inflation persistence downwards. This reduction in c_t from 1 to about 0.8 adds disinflationary impetus and reduces the costs of disinflation. While the decline in perceived inflation persistence is much smaller under gradual than under immediate disinflation, the gradual approach can take advantage of the resulting reduction in disinflation costs.

Turning to the degree of backward-looking indexation, we note that firms see now reason to switch from backward-looking indexation to the announced inflation target. The announced target is just too far way and progress towards it too slow to change the probability weights on lagged inflation versus the announced target. As a result, endogenous indexation does not come into play in terms of reducing the costs of disinflation under such a gradual disinflation strategy.

6 Inflation targeting: Temporary inflation targets

Two important aspects of the Chilean disinflation strategy have been emphasized in section 2 - its gradual nature and its use of temporary annual inflation targets. Having shown that the gradual approach helps reduce disinflation costs by taking advantage of the reduction in perceived inflation persistence we now extend the analysis to consider the effect of announcing temporary targets. In the Chilean case these temporary targets appear to have been pursued quite vigorously. Thus, we investigate whether such temporary targets, π_t^*, could have an additional beneficial effect on learning and the degree of indexation and thereby lower the costs of disinflation further.

With temporary targets the New-Keynesian Phillips curve needs to be slightly modified:

$$\pi_t = \frac{\kappa}{1+\beta\kappa}\pi_{t-1} + \frac{\beta}{1+\beta\kappa}E_t[\pi_{t+1}] + \frac{\lambda}{1+\beta\kappa}x_t + \frac{(1-\kappa)}{1+\beta\kappa}(\pi_t^* - \beta\pi_{t+1}^*) \tag{18}$$

Accordingly, the reduced-form inflation equation with adaptive learning and endogenous indexation corresponds to:

$$\begin{aligned} \pi_t &= \frac{\kappa_{t-1}}{1+\beta(\kappa_{t-1}-c_{t-1})}\pi_{t-1} + \frac{\lambda}{1+\beta(\kappa_{t-1}-c_{t-1})}x_t \\ &\quad + \frac{(1-\kappa_{t-1})}{1+\beta(\kappa_{t-1}-c_{t-1})}(\pi_t^* - \beta\pi_{t+1}^*) \qquad (19) \\ &= \delta_{1,t}\pi_{t-1} + \delta_{2,t}x_t + \delta_{3,t} \end{aligned}$$

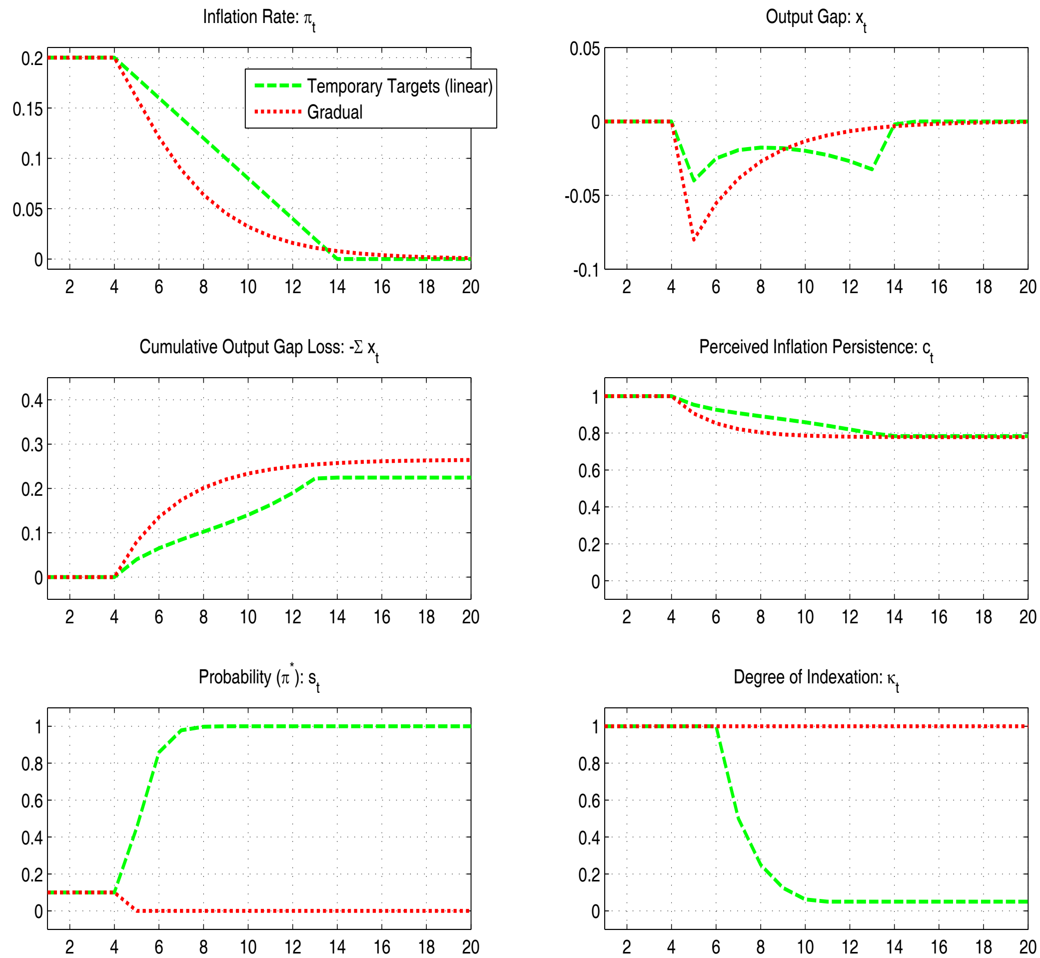
As a first example, we consider a gradual, linear reduction in the inflation target by 2 percentage points per year. Thus, the long-run target of zero percent inflation is reached in year 14, 10 years after the start of disinflation. We assume that the central bank pursues these annual targets as actively as possible. In other words, the central bank implements strict inflation targeting with respect to temporary targets. After deciding on next year's inflation target, the central bank acts in order to meet this target. Thus, it pursues the following output gap policy:

$$x_t = -\delta_{4,t}(\delta_{1,t}\pi_{t-1} + \delta_{3,t} - \pi_t^*) \tag{20}$$

with $\delta_{4,t} = \delta_{2,t}^{-1}$, and $\delta_{(1,2,3)}$ consistent with equation (19).

The disinflation performance with temporary annual targets is shown by the dashed green line in **Figure 3**. It is compared to the gradual disinflation, that is the dotted red line, shown previously in Figure 2. In both cases, the parameter governing the perceived degree of inflation persistence, c, declines towards a value of 0.8 (middle-right panel). This decline occurs slightly faster under the gradual disinflation because initially inflation is reduced more quickly than the linear reduction implied by the annual targets.

Figure 3: Temporary Inflation Targets



An important difference arises with respect to the degree of backward-looking indexation. By announcing and meeting the temporary annual inflation targets the central bank succeeds in convincing firms that they are better off by choosing the central bank's target as an index for the pricing rule applied in those periods without Calvo-style optimal price-adjustment signals. The probability s_t that indicates the likelihood of the central bank's target(s) representing the mean of the inflation distribution rises quickly (bottom-left panel). It exceeds the trigger probability $\overline{S}$ of 0.8 by the second year of the disinflation. Every year from then on a share of $1-\theta$ of the firms that previously applied backward-looking indexation switches to using the central bank's targets. As a result the degree of backward-looking indexation declines fairly rapidly and approaches the minimum level $\underline{\kappa}$ by year 11.

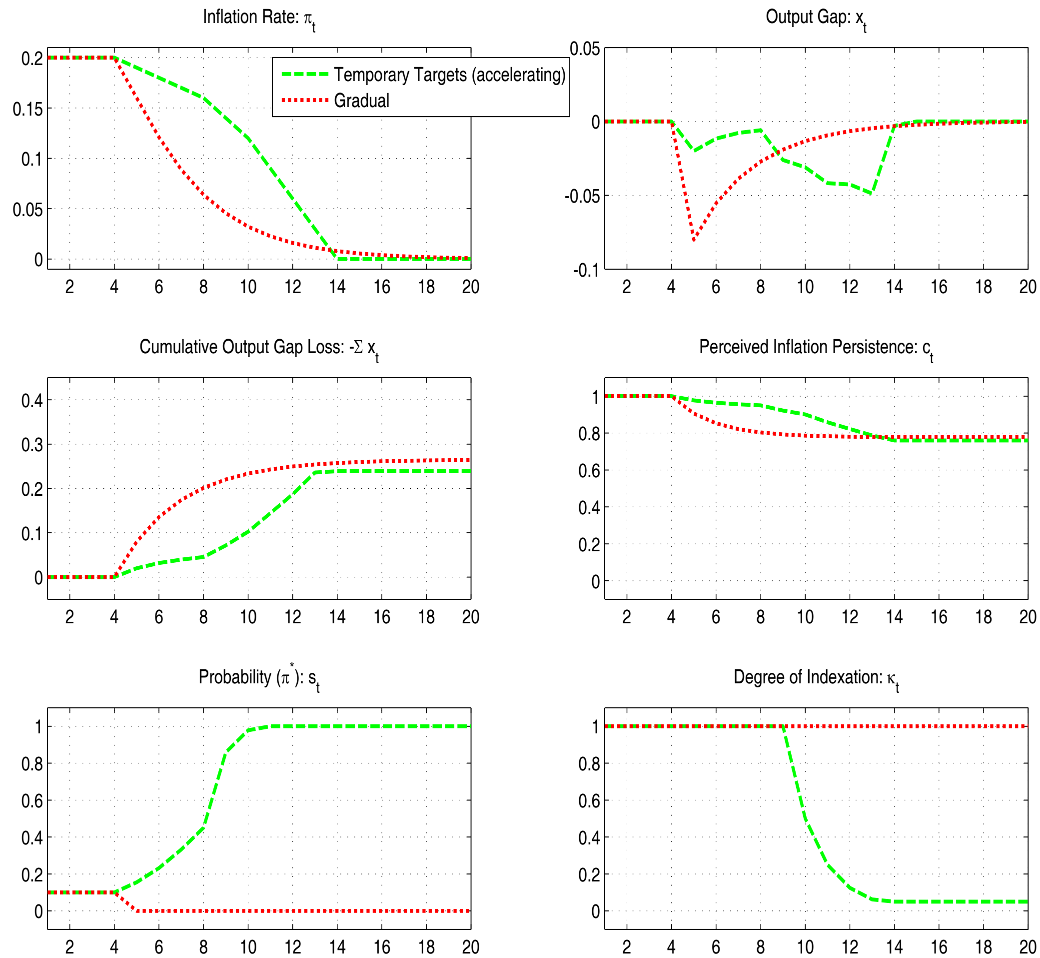
Compared to the gradual disinflation strategy with a long-run target the strategy with temporary, annual targets allows the central bank to take advantage of the endogenous reduction in backward-looking indexation. Firms change their behavior because they can already observe during the first few years of the disinflation that the central bank means to achieve its announced targets. As a result, the output losses associated with disinflation are lower with annual targets. The cumulative output loss, (middle-left panel) converges to 22 percent of output, that is 4 percent lower than in the case of the gradual disinflation. The sacrifice ratio is reduced to 1.1. Further substantial gains in terms of stabilization performance will accrue in the future. Given the substantial reduction in backward-looking indexation the central bank will be able to reduce variations in inflation in the event of unexpected shocks at much lower cost in terms of output variability.

Next, we consider three alternative parameterizations of the sequence of annual inflation targets: (i) targets that imply accelerating disinflation; (ii) targets that imply decelerating disinflation; and (iii) the annual targets set in Chile from 1991 to 2001.

In the first case, shown in **Figure 4** the reduction in the central bank's annual targets accelerates over time (dashed green line). Initially, the central bank lowers the inflation target by 1 percentage point per year. Starting in year 9, the fifth year of the disinflation, the inflation target is lowered by two percentage points per year. From year 11 onwards the target is lowered by 3 percentage points per year. The long-run target of zero percent is reached in year 14, after a 10-year long disinflation process. Compared to the disinflation with linearly-declining targets, accelerating targets initially imply a slower decline in inflation. The output gap incurred during the disinflation increases over time in absolute value. The total cost of disinflation, that is the cumulative output gap, remains smaller than with the gradual disinflation strategy (dotted red line) but larger than with linearly-declining targets. The cumulative output gap reaches 24 percent relative to the 22 percent with linearly-declining targets. Due to the slow pace of disinflation during the first few years, price-setting firms take longer to become convinced that they are better off with using the central bank's target as an index for their pricing rules in periods without Calvo-style signals. The probability s_t, (lower-left panel), rises slowly and takes five years to exceed the trigger value of 0.8. Only from year 10 onwards those firms that receive Calvo signals start switching from backward-looking indexation to the central bank's targets.

Figure 5 shows the simulation with decelerating targets. In the first year of disinflation, year 5, the central bank aims to lower inflation by 4 percentage points to 16 percent. In subsequent years the speed of disinflation declines. These annual inflation targets (dashed green line) are set to be identical to the path for inflation that is realized under the gradual disinflation with a long-run target (dotted red line). Thus, the actual path of inflation (top left panel) coincides under these two scenarios. This parameterization is particularly interesting because it provides a ceteris-paribus assessment of the reduction in disinflation costs that is achieved by announcing temporary annual targets. As shown in the top-right panel, the output gap associated with the disinflation with temporary targets (dashed green line) is at all times equal or smaller in absolute value than

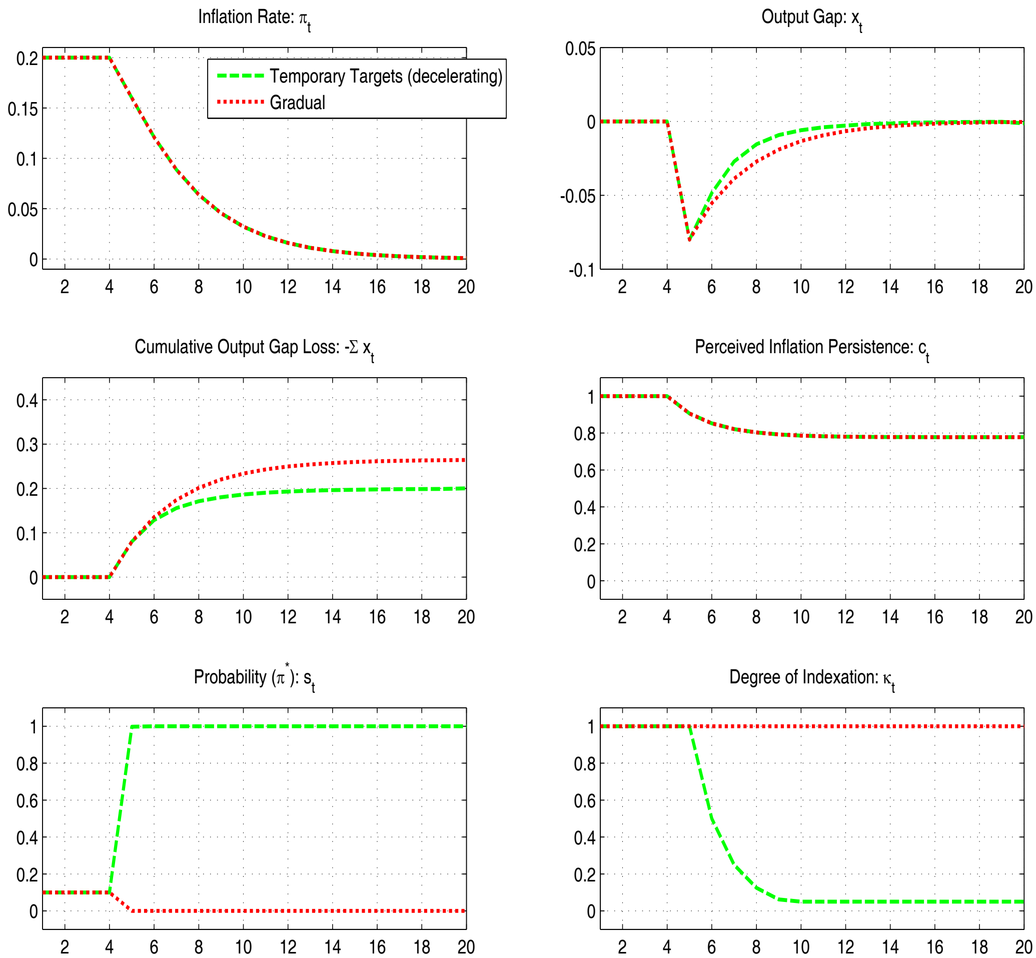
Figure 4: Accelerating Disinflation with Temporary Targets



in the case of the gradual disinflation with a long-run target. The total cost of disinflation comes to 20 percent of output, that is another 2 percent lower than with linearly-declining targets. The sacrifice ratio associated with a disinflation from 20 percent to zero inflation is unity. Announcing and achieving the reduction of inflation by 4 percentage points in the first year of the disinflation convinces price-setting firms that the central bank means business. As a result, the probability s_t rises rapidly and firms soon start to abandon the practice of backward-looking indexation.

The annual targets set by the Chilean central bank between 1991 and 2001 also implied a decelerating disinflation. In 1990 inflation was substantially above 20 percent. Thus, the announced target for 1990 of 15-20 percent indicated a significant reduction with the start of the inflation targeting strategy. **Table 2** reports the announced target ranges and point targets (top row) as well as the mid-points of these ranges (bottom row). From 2001 onwards, the central bank has aimed

Figure 5: Decelerating Disinflation with Temporary Targets

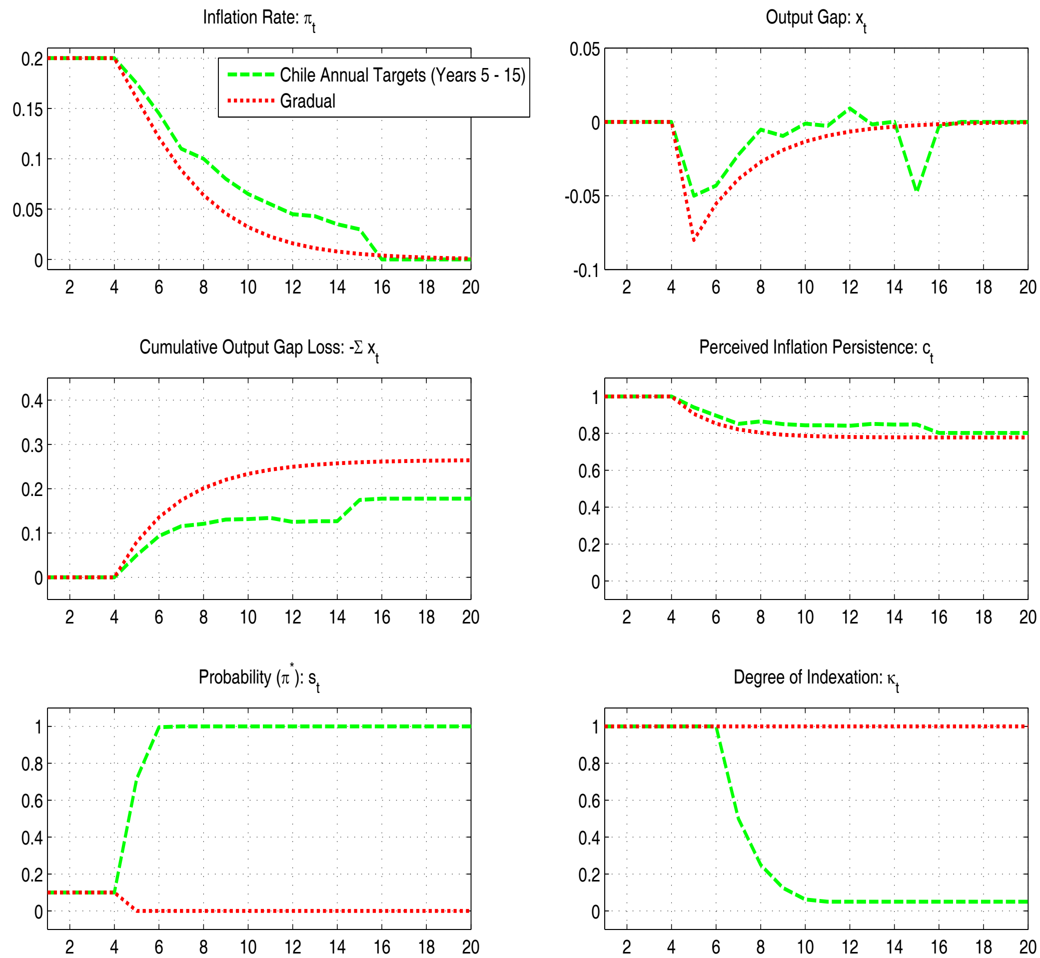


to keep inflation within a target range of 2 to 4 percent.

We have used the midpoints[9] of the target ranges from 1991 to 2001 to simulate a disinflation in the New-Keynesian model with adaptive learning and endogenous indexation developed in this paper. The initial conditions are the same as the preceding simulations shown in Figures 2 to 5. The midpoints of the Chilean target ranges are implement starting in year 5 till year 15. To render the cost of disinflation incurred by the pursuit of the Chilean targets in our model comparable to the preceding simulations, we added a further reduction in the inflation target. In period 16 the target is reduced by an additional 3 percentage points so as to reach a long-run target of zero inflation.

[9]We disregard the potential effects of target ranges and refer the reader instead to the analysis of such nonlinearities in Orphanides and Wieland (2000).

Figure 6: Chile's Inflation Targets: 1991 - 2001 (Year 5 to 15)



Note: From 2001 onwards the Central Bank of Chile pursued an inflation target zone of 2 to 4 percent with a midpoint of 3 percent. For comparability with the preceding evaluation of disinflation costs we have added a further 3 percent disinflation step in year 16 to achieve a long-run target of zero inflation.

Table 2: Chile's Inflation Targets: 1991-2001

Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
in Model	5	6	7	8	9	10	11	12	13	14	15
Range	15-20	13-16	10-12	9-11	8.0	6.5	5.5	4.5	4.3	3.5	2-4
Midpoint	17.5	14.5	11	10	8	6.5	5.5	4.5	4.3	3.5	3

The total cost of disinflation in terms of the cumulative output gap loss amounts to 18 percent of GDP spread over 12 years (middle left panel). The sacrifice ratio is 0.9. It is lower than in the simulation with decelerating targets shown in Figure 5. This reduction is possible for the following reasons. The initial disinflation steps in years 5, 6 and 7 are vigorous enough to reduce the perceived degree of inflation persistence (middle right panel) and to raise the probability s_t beyond the trigger level $\bar{S}$. Thus, the degree of backward-looking indexation declines over the course of the disinflation. However, the disinflation stretches out for a longer period than in Figure 5 and thereby benefits even more from the reduction in inflation persistence and indexation.

The baseline-version of the New-Keynesian model does not include structural shocks in the inflation equation. However, such shocks are often added either to capture the presence of measurement error or to reflect missing variables or other sources of rigidity. Thus, we now proceed to introduce random shocks in the New-Keynesian Phillips curve:

$$\pi_t = \frac{\kappa}{1+\beta\kappa}\pi_{t-1} + \frac{\beta}{1+\beta\kappa}E_t[\pi_{t+1}] + \frac{\lambda}{1+\beta\kappa}x_t + \frac{(1-\kappa)}{1+\beta\kappa}(\pi_t^* - \beta\pi_{t+1}^*) + \varepsilon_t \tag{21}$$

The shocks are denoted by ε_t and normally distributed with zero mean and variance $\sigma = 2^{-4}$. The timing of expectations formation, policy actions and shocks is such that the shocks are realized after time t expectations have been formed and policy has been set. Thus, the shocks introduce noise in inflation that cannot be avoided by contemperaneous policy actions. However, in the period following the shock the central bank will act to minimize further consequences from these variations that would occur due to the intrinsic persistence of inflation. To this end, the central bank induces offsetting variations in the output gap.

The fluctuations of inflation and output that result from random shocks and subsequent policy responses have an important influence on the dynamics of learning and endogenous indexation. On the one side, such shocks imply that the central bank never meets its target exactly. Thus, firms may find it more difficult to assess whether it is better to use past inflation or the central bank's target as an index for their pricing rules in periods without Calvo signals. On the other side, the fact that the central bank will set policy to counter the consequences of unforeseen shocks to inflation will generate information regarding the degree of inflation persistence and induce adaptive learning. In this manner, fluctuations may increase the speed of learning and reduce inflation persistence. As a consequence, the costs of disinflation may decline further.

Figure 7: Shocks Accelerate Learning and Perceived Persistence Declines

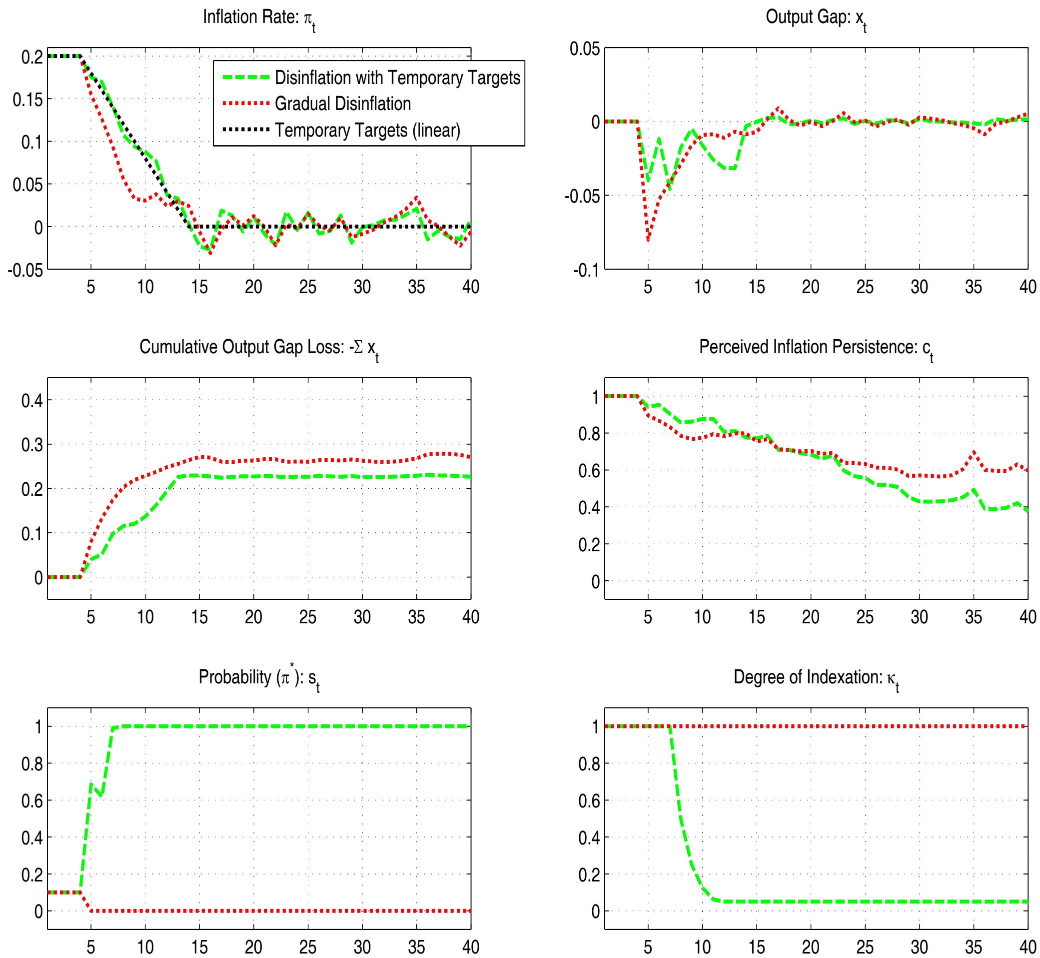


Figure 7 shows dynamic simulations with a particular draw of random shocks ε. The length of time shown is 40 rather than 20 years as in the preceding figures. Figure 7 compares the outcome under a gradual disinflation with a long-run target (dotted red line) to a disinflation with linearly-declining annual targets (dashed green line). The top left panel reports the actual inflation rates, which exhibit some random fluctuations. The annual targets are shown by the black-dotted line.

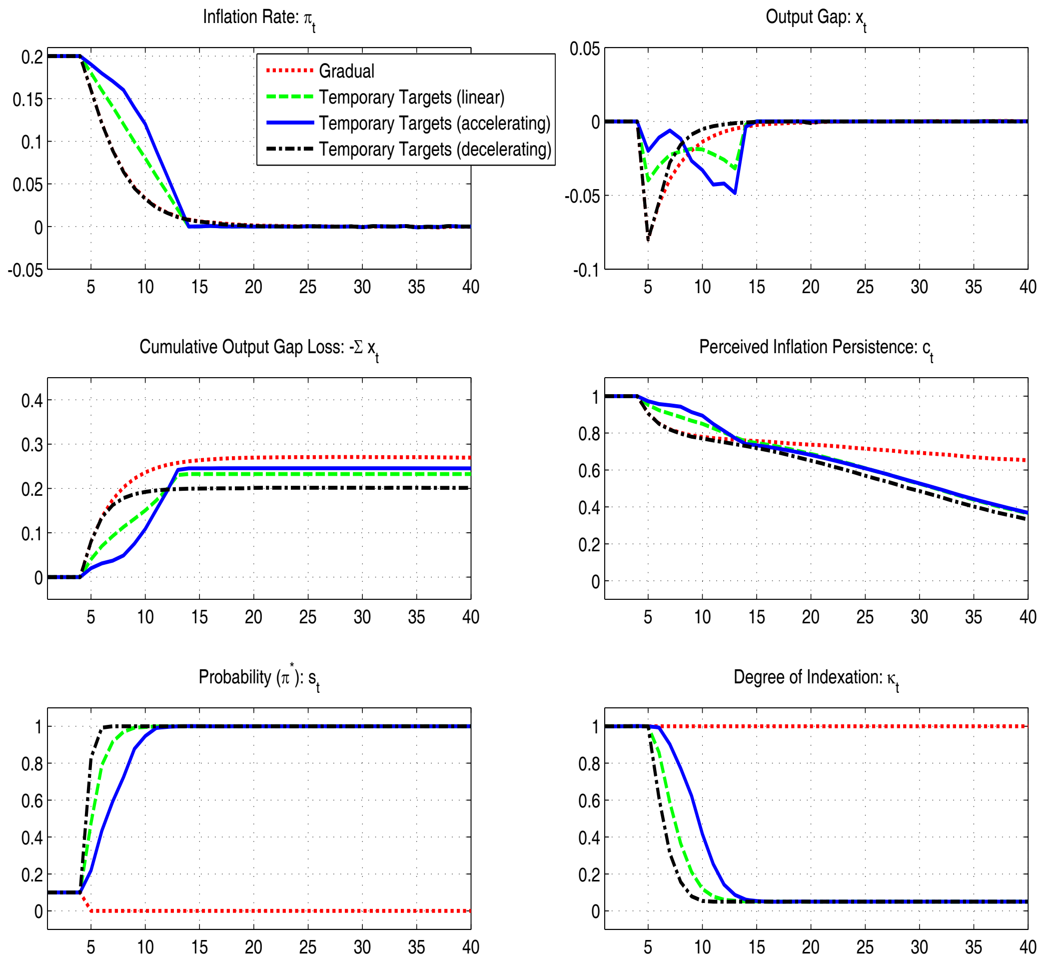
Two aspects of these stochastic simulations are of particular interest. The middle-left panel shows that the perceived degree of inflation persistence continues to decline even after the disinflation process has been completed. It is the policy response to the consequences of unforeseen shocks that stabilizes inflation fluctuations and drives down price setters' estimates of the persistence parameter, c_t. This decline is much more pronounced in the simulation with annual targets. By year 40 it reaches 0.4 while it is still at 0.6 in the gradual disinflation with long-run target.

The reason is that the structural persistence due to indexation is ultimately much smaller in the simulation with annual targets. The central bank's announcement and achievement of these targets has convinced firms to switch from backward-looking indexation to using the target rates. It is noteworthy that the probability s_t that measures the usefulness of central bank targets for indexation does not increase as smoothly as in the absence of unforeseeable random shocks. In Figure 3, lower left panel, the probability s_t rises rapidly and smoothly above the trigger level in the simulation with linearly-decline targets. In Figure 7, lower left panel, it moves up and down a little bit before rising further above the trigger level. This finding shows that the switch from backward-looking indexation to the central bank targets is influenced by the particular series of shocks.

Of course, Figure 7 only reports the outcomes for a single draw of shocks. The strategy with temporary inflation targets need not always outperform the gradual disinflation strategy in terms of output losses. To shed further light on the likely outcomes we simulate 1000 series of shocks drawn from a normal distribution and compute averages across these 1000 simulations. These averages are reported in **Figure 8**. We show averages for the gradual disinflation with long-run target (red dotted line), with linearly-declining annual targets (dashed green line), with decelerating targets (dashed-dotted black line) and with accelerating targets (solid blue line).

The results are quite similar to the simulation without shocks though not the same due to the nonlinearity resulting from adaptive learning and indexation. The ranking of speeds of disinflation (top left panel) and cumulative output losses (middle left panel) remains unchanged. The perceived degree of inflation persistence reaches 0.4 for all three types of temporary targets by year 40. After many more years it converges to a small but positive value consistent with the persistence implied by the minimum degree of backward-looking indexation under rational expectations. The increase in the probability s_t, (lower left panel) is fastest with decelerating targets and slowest with accelerating targets. As a result, the degree of backward-looking indexation declines most quickly with decelerating targets and most slowly with accelerating targets. In the case of a gradual disinflation with long-run targets backward-ward looking indexation remains complete.

Figure 8: Averages over 1000 Simulations



7 A ”sophisticated” central bank versus one that learns

A ”sophisticated” central bank

Our findings suggest that the performance of monetary policy could be improved further by allowing the central bank to observe and exploit the nonlinear dynamics due to adaptive learning and endogenous indexation, i.e. equations (7), (10) and (11), in the design of dynamically optimal policy. Gaspar et al (2006) have studied such an optimal policy problem with adaptive learning but without endogenous indexation. They introduced the label ”sophisticated” for a central bank that is capable of exploiting learning dynamics. In our model, such a sophisticated central bank would solve the following dynamic optimization problem:

$$\underset{x_t}{\text{Min}} E_t \left[\sum_{t=1}^{\infty} \beta^{t-1} (\pi_t - \pi^*)^2 + \alpha x_t^2 \right] \tag{22}$$

$$\text{s.t. } \pi_t = \delta_{1,t}\pi_{t-1} + \delta_{2,t}x_t + \delta_{3,t}$$
$$\text{and s.t. equations } (7), (8), (10) \text{ and } (11).$$

The optimal policy is nonlinear because it takes into account the nonlinearities arising from recursive estimation of the degree of inflation persistence, i.e. equations (7) and (8), and endogenous indexation, i.e. equations (10) and (11).

Following Gaspar et al (2006) the central bank's choice variable is assumed to be the output gap and the central bank is assumed to aim at a long-run inflation target. An alternative approach, inspired by our paper, would be to use annual inflation targets as the choice variable of the central bank. A particular choice of temporary target would then automatically imply a given output gap according to the strict inflation targeting policy shown by equation (20).

The optimization problem defined by (22) corresponds to a nonlinear dynamic programming problem with four state variables: $(\pi_{t-1}, c_{t-1}, \Sigma_{t-1}, s_{t-1})$. Numerical approximation of such a problem is complicated but within reach of current methodology. However, optimal policy design in this manner relies on rather courageous assumptions regarding the central bank's knowledge of private sector expectations formation. The central bank is not only assumed to observe the private sector's beliefs, it is also assumed to know the exact learning dynamics. Perhaps, the policy that could be implemented by such an extremely knowledgeable central bank forms a useful benchmark for model-based comparison but it does not represent a strategy that could be implemented in practice. We propose instead an alternative approach to policy design under uncertainty that can be pursued under more realistic informational assumptions.

A central bank that learns

Optimal policy design that could be implemented with the information available to central banks in practice takes recourse to learning. In this case, the central bank would learn about inflation dynamics by recursively estimating the relevant parameters of the reduced form inflation equations (13) or (19). Contrary to the price-setting firms in our model that were assumed to simply estimate a regression of inflation on its own lag, the central bank can spend more resources on learning. Certainly, central bank econometricians estimate Phillips curves on a regular basis that include the effect of policy on inflation via the output gap x_t.

In the model studied in this paper, central bank learning could be applied to the reduced-form inflation equation consistent with adaptive learning and endogenous indexation, that is,

$$\pi_t = \delta_{1,t}\pi_{t-1} + \delta_{2,t}x_t + \delta_{3,t} \tag{23}$$

Following Wieland (2006)[10] central bank beliefs regarding the three time-varying parameters may be summarized by the vector $d_t = (d_{1,t}, d_{2,t}, d_{3,t})$ and associated covariance matrix $\Sigma_{d,t}$.

$$Var\left[\begin{pmatrix} d_{1,t} \\ d_{2,t} \\ d_{3,t} \end{pmatrix}\right] = \Sigma_d = \begin{pmatrix} v_t^1 & v_t^{12} & v_t^{13} \\ v_t^{12} & v_t^2 & v_t^{23} \\ v_t^{13} & v_t^{23} & v_t^3 \end{pmatrix}. \tag{24}$$

The vector of state variables that characterize central bank beliefs contains nine variables, the three means, three variances and three covariances. The associated updating equations for recursive least squares with time-varying parameters correspond to:[11]

$$\begin{aligned} \begin{pmatrix} d_{1,t} \\ d_{2,t} \\ d_{3,t} \end{pmatrix} &= \begin{pmatrix} d_{1,t-1} \\ d_{2,t-1} \\ d_{3,t-1} \end{pmatrix} + \Sigma_{t-1}X_tF^{-1}(\pi_t - d_{1,t-1}\pi_{t-1} - d_{2,t-1}x_t - d_{3,t-1}) \\ \Sigma_{d,t} &= \Sigma_{d,t-1} - \Sigma_{d,t-1}X_tF^{-1}X_t'\Sigma_{d,t-1} + \sigma_d \text{ where } F = X_t\Sigma_{d,t-1}X_t' + \sigma^\eta \end{aligned} \tag{25}$$

where F refers to the conditional variance of inflation.

In contrast to the ”sophisticated” central bank discussed above, the information requirements for such a learning central bank are much less stringent. Only inflation and output observations are needed. Potential output could be subsumed in the time-varying intercept. Thus, an area of fruitful future research would be to re-assess the disinflation policies in the preceding section under the assumption that the central bank learns about the time-varying parameters governing the inflation process in this manner. Wieland (2000, 2006) and Beck and Wieland (2002) compute optimal learning policies for such problems with up to two unknown parameters and compare their performance to passive learning policies that do not take into account the central bank’s own updating equations in optimization. At least, policy design under passive learning could be applied to the policy problem in this paper.

[10]Other related work on central bank learning that is of interest in this context includes Cogley, Colacito and Sargent (2005), Ellison (2006), Svensson and Williams (2006) and Wieland (2000a,b).

[11]For a derivation of the updating equations using Bayes rule or Kalman filter see Zellner (1971) and Harvey (1992) respectively.

8 Conclusions and extensions

In this paper, we have shown that inflation targeting strategies can lower the costs of disinflation and future inflation stabilization. We have explored two channels through which such a reduction may take place: adaptive learning and endogenous indexation. Arguably, both channels may have played an important role in Chile's disinflation experience.

If market participants learn adaptively rather than form rational expectations history matters. As the central bank acts to bring inflation under control, market participants will observe the consequences of these actions and revise their beliefs regarding the degree of inflation persistence. Over time, adaptive learning lowers the cost of disinflation. A gradual approach to disinflation can take advantage of this beneficial effect.

Endogenous indexation implies that price-setting firms are allowed to choose between past inflation and the central bank's target as index for their pricing rule in periods without Calvo-style signals to set prices optimally. Firms assess the likelihood that announced inflation targets determine steady-state inflation and adjust the indexation of contracts accordingly. A strategy of announcing and achieving short-term targets for inflation is able to influence the degree of backward-looking indexation. It implies that firms are able to observe fairly soon whether the central bank acts to meet the targets it proclaims. Following up on words with the appropriate deeds raises the likelihood that firms switch from backward-looking indexation to the central bank's announced targets. Short-term annual targets that are pursued aggressively help reducing the degree of indexation more effectively than a strategy with a long-run target that is achieved only gradually.

Our analysis suggests that dynamic general equilibrium models estimated under the assumptions of rational expectations and a exogenous, constant degree of backward-looking indexation may misjudge the costs of disinflation in two ways. On the one hand, the assumption of rational expectations may overstate the power of the central bank to influence the costs of disinflation by words alone whether they be announcements or verbal commitments. Learning implies that announcements need to be followed by action to convince market participants. The resulting reduction in inflation persistence is influenced by policy actions as well as economic shocks. The assumption of exogenous indexation, on the other hand, may lead to model estimates that overstate the cost of disinflation and inflation-output tradeoffs. Endogenous reductions in the degree of backward-looking indexation as inflation rates decline to a low level consistent with announced targets would present the central bank with more favorable trade-offs.

There are a number of interesting and potentially important possible extensions of this research. These extensions concern the optimal design of monetary policy, the formation of expectations, the role of the interest rate, the role of the exchange rate and the degree of openness of the economy.

With regard to dynamically optimal policy design, two possible approaches have been proposed in section 7 of the paper. It would be of interest to derive the dynamically optimal policy that takes into account the nonlinear learning dynamics present in the model. Although such a policy relies on unrealistic informational assumptions it would form a useful benchmark for comparison with simpler, practically implementable policies such as the policy with central bank learning proposed in section 7.

As to the formation of expectations it would be useful to evaluate the implications of alternative adaptive learning specifications (cf. Branch and Evans (2006), Milani (2007)) for the cost of disinflation. Also, it would be interesting to study endogenous indexation under rational expectations. In this manner, the quantitative effects due to endogenous indexation could be studied separately from those due to adaptive learning.

The model consider so far is very stylized. The central bank has been assumed to control the output gap directly. Instead, the transmission from the central bank's primary policy instrument, that is the nominal short-term interest rate, and the output gap could be modeled explicitly. In other words, the model can be extended to include the log-linearized Euler equation of the households, i.e. the New-Keynesian IS curve. This extension would allow us to pose a host of new questions regarding the design of interest rate rules and the conditions for stability under learning (see also Llosa and Tuesta (2007)).

Finally, Chile as well as many other inflation targeting countries are small open economies. During the disinflation in Chile favorable shocks to the exchange rate and the terms of trade may have played an important role in cushioning the economy. These effects could be examined by extending the analysis of learning and endogenous indexation conducted in this paper to a small open economy. In an open economy further practical questions arise such as whether to target domestic inflation or CPI inflation and how to account for the exchange rate in interest rate policy.

References

Aguirre, Alvaro and Klaus Schmidt-Hebbel, 2005, ”Monetary Policy in Transition from Partial to Full-Fledged Inflation Targeting in Chile”, working paper.

Branch, William, and George Evans, 2006, ”A Simple Recursive Forecasting Model”, *Economics Letters*, 91, 2, May.

Caputo, Rodrigo, Felipe Liendo, and Juan Pablo Medina, 2007, ”New Keynesian Models for Chile in the Inflation-Targeting Period”, in, Mishkin, Frederic and Klaus Schmidt-Hebbel (eds.), Monetary Policy under Inflation Targeting, Central Bank of Chile, Santiago.

Caputo, Rodrigo, Juan Pablo Medina, and Claudio Soto, 2006, ”Nominal Rigidities, Indexation and Inflation Persistence in Chile: A Structural Investigation”, working paper, Central Bank of Chile, Santiago.

Calvo, Guillermo, 1983, ”Staggered Prices in a Utility-Maximizing Framework”, *Journal of Monetary Economics*, 12, 383-98.

Calvo, Guillermo and Enrique Mendoza, 1999, ”Empirical Puzzles of Chilean Stabilization Policy”, in, Perry and Leipziger (eds), Chile: Recent Policy Lessons and Emerging Challenges, World Bank, Washington D.C..

Christiano, Lawrence, Martin Eichenbaum and Charles Evans, 2005, ”Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy”, *Journal of Political Economy*, 113, No. 1, pp 1-45. (2001 working paper).

Cespedes, Luis, Ochoa, Marcelo, and Claudio Soto, 2005, ”The New-Keynesian Phillips Curve in an Emerging Market Economy: The Case of Chile”, Central Bank of Chile Working Paper No. 355, Central Bank of Chile, Santiago, December.

Cespedes, Luis, and Claudio Soto, 2007, ”Credibility and Inflation Targeting in Chile”, in, Mishkin, Frederic and Klaus Schmidt-Hebbel (eds.), Monetary Policy under Inflation Targeting, Central Bank of Chile, Santiago.

Cogley, Timothy, Riccardo Colacito and Thomas Sargent, 2005, ”The Benefits from U.S. Monetary Policy Experimentation in the Days of Samuelson and Solow and Lucas”, working paper.

Corbo, Vittorio, Oscar, Landerretche, and Klaus Schmidt-Hebbel, 2002, ”Does Inflation Targeting Make a Difference?”, in Loayza, Norman and Raimundo, Soto, (eds.), Inflation Targeting: Design, Performance, Challenges, Central Bank of Chile.

Corbo, Vittorio, and Klaus Schmidt-Hebbel, 2003, ”Inflation Targeting: The Latin American Experience”, in Gonzalez, Jose-Antonio, Vittorio, Corbo, Ann O., Krueger and Aaron, Tornell, (eds.), Latin American Macroeconomic Reform: The Second Stage, The University of Chicago Press.

De Gregorio, Jose and Eric Parrado, 2006, ”Overshooting meets inflation targeting”, Central Bank of Chile Working Papers, December 2006.

Ellison, Martin, 2006, ”The Learning Cost of Interest Rate Reversals”, *Journal of Monetary Economics*, Vol. 53 No. 8, November.

Milani, Fabio, 2007, ”Expectations, Learning and Macroeconomic Persistence”, *Journal of Monetary Economics*, 54, 7, 2065-2082.

Evans, George, and Seppo Honkapohja, 2001, Learning and Expectations in Macroeconomics, Princeton University Press.

Fraga, Arminio, Ilan Goldfajn, and Andre Minella, 2003, ” Inflation Targeting in Emerging Market Economies”, NBER Working Paper 10019, October.

Gaspar, Vitor, Frank Smets and David Vestin, 2006, ”Adaptive Learning, Persistence and Optimal Monetary Policy”, *Journal of the European Economic Association* , 4, 376-385.

Gaspar, Vitor, Frank Smets and David Vestin, 2006, ”Optimal monetary policy under adaptive learning”, working paper.

Gali, Jordi, and Mark Gertler, 1999, ”Inflation Dynamics: A Structural Econometric Analysis”, Journal of Monetary Economics, Vol. 44 No. 2, pp.195-222.

Harvey, A., Time Series Models, 1992, 2nd edition, MIT Press.

Herrera, Luis Oscar, 2002, Indexation, Inflation Inertia and the Sacrifice Coefficient, in, Lefort, Fernando and Klaus Schmidt-Hebbel (eds.), Indexation, Inflation and Monetary Policy, Central Bank of Chile, Santiago.

Lefort, Fernando and Klaus Schmidt-Hebbel, 2002, Indexation, Inflation and Monetary Policy: An Overview, Central Bank of Chile, Santiago.

Llosa, Luis-Gonzalo and Vicente Tuesta, 2007, Learning about Monetary Policy Rules when the Cost Channel Matters, Working Paper 2007-014, Banco Central de Reserva del Per

Mishkin, Frederic and Klaus Schmidt-Hebbel, 2001, One Decade of Inflation Targeting: What Do We Know and What Do We Need to Know?, NBER Working Paper 8397, July 2001.

Orphanides, Athanasios and John C. Williams, 2006, ”Monetary Policy with Imperfect Knowledge”, *Journal of the European Economic Association*, 4(2-3), 366-375

Orphanides, Athanasios, and John C. Williams, 2005, ”Inflation Targeting Under Imperfect Knowledge”, working paper.

Sargent, Thomas, 1999, The Conquest of American Inflation, Princeton University Press.

Sargent, Thomas, Williams, Noah and Tao Zha, 2006, ”The Conquest of South American Inflation”, NBER Working Paper 12606, National Bureau of Economic Research, Boston, October.

Schmidt-Hebbel, Klaus, and Alejandro Werner, 2002, ”Inflation Targeting in Brazil, Chile and Mexico: Performance, Credibility and the Exchange Rate”, *Economia*, Spring.

Svensson, Lars, E.O., and Noah Williams, 2007, ”Bayesian and Adaptive Optimal Policy under Model Uncertainty,” NBER Working Papers 13414, NBER.

Walsh, Carl, 2003, Monetary Theory and Policy, 2nd edition, The MIT Press.

Walsh, Carl, 2007, ”Announcements and the Role of Policy Guidance”, working paper.

Wieland, Volker, 2006, “Monetary Policy and Uncertainty about the Natural Unemployment Rate: Brainard-Style Conservatism versus Experimental Activism”, *Advances in Macroeconomics*, 6 (1), Article 1.

Wieland, Volker, 2000a, “Learning by Doing and the Value of Optimal Experimentation”, *Journal of Economic Dynamics and Control*, 24, pp. 501-534.

Wieland, Volker, 2000b, “Monetary Policy, Parameter Uncertainty and Optimal Learning”, *Journal of Monetary Economics*, 46, pp. 199-228.

Woodford, Michael, 2003, Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton University Press.

Zellner, A., Introduction to Bayesian Inference in Econometrics, 1971, Wiley.

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