AN APPROACH TO ANALYZING GOLD SUPPLY FROM THE SOUTH AFRICAN GOLD MINES

THESIS

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CONTENTS

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ABSTRACT

The gold mining firm in South Africa is viewed as a normal firm producing gold bearing ore but faced with a quality constraint (grade). Grade, however, is never uniformly distributed in a metalliferous deposit and because high grades are mined first, the quality constraint becomes increasingly severe with cumulated production. The firm will continue to mine gold bearing ore until it reaches its mining limit where the marginal cost of recovering the gold is equal to the marginal revenue received from that gold and at that point the economic deposit becomes exhausted. Because the mining limit is determined by cost/technology and price, it is not fixed and thus the point of economic exhaustion may change.

When high grades are mined first the relationship between the tonnage of gold ore and the grade describes the rate at which the grade is expected to fall with cumulated production. In this thesis, the grade for South African Witwatersrand gold producers is modelled to fall exponentially. The mining limit, determined by costs/technology and price, can be expressed in terms of grade. By predicting the decay in grade relative to the tonnage of gold ore and applying a mining limit, a life-time size of the economic deposit can be estimated. The remaining life of a producing gold mine can then be determined and the flow of gold predicted.

An empirical treatment using the disk model of a gold deposit is undertaken for a gold mine, a goldfield and the total Witwatersrand gold deposit. A dynamic econometric analysis of expected mining costs and gold prices is not attempted; however certain examples are used to illustrate the applicability of the model and the influence of the South African gold mining tax formula on the life of the mine.

LIST OF FIGURES AND TABLES

INTRODUCTION

The literature on exhaustible resources, [see, for example, Hotelling (1931), Devarajan & Fisher (1981) and Sweeney (1993)], treats the extractive firm as follows. Extraction results in depletion, and the eventual physical exhaustion, of a given [capital] stock, of a constant quality, of a natural occurring resource. Because the stock is non-renewable, depletion results in a reduction of capital thus the return from extracting [rent] must increase for flow equilibrium to be maintained in the market for the exhaustible resource. The firm will adjust the rate of output in response to various economic stimuli in order to maximise the present value of the given [remaining] capital stock. If flow equilibrium is maintained the present value from extracting will be equal in all time periods and the stock of resource will be exhausted. The treatment of the costs of extraction is obviously a major concern because it affects the rental element and therefore also the rate of extraction and the date of eventual exhaustion.

By assuming a homogenous quality for a given deposit the marginal cost of extraction might increase with cumulated production due to certain deposit-specific characteristics (stock effect). If the homogenous quality assumption is dropped however, and the best quality [low cost] areas are extracted first (Solow,1974), the deposit-specific characteristics must add to the marginal cost of extraction, over and above the normal stock effect, as extraction continues into the future. By asserting that a market for deposit-specific characteristics does not exist (Epple & Londregan,1993), the received literature abstracts from the influence of a deposit-specific quality characteristic [grade] by including it into the normal stock effect (Sweeney,1993).

Where a market for grade might not exist for oil deposits, the thesis asserts after Stollery (1982) that it does exist for the metalliferous deposits and in particular for the Witwatersrand gold deposit in South Africa. Central to the argument is a recognisable difference between oil deposits and metalliferous deposits. It is reasonable to view an oil deposit as having a homogenous grade. The metalliferous deposits should however be seen in a different light. They simply comprise a body of rock with anomalous concentrations of useful elements contained within various ore minerals. The distribution of the ore minerals, and the concentration of useful elements in the mineral, is never constant across an ore body. For the Witwatersrand gold deposit this is particularly evident from both the geological literature [see for example Pretorius (1989)] and from a historic record of recovered grades (Appendix 2).

The rate of extracting oil is measured in barrels of oil per time period, but the rate of extraction from a metalliferous deposit might be expressed either in tons of rock [ore] per time period or in terms of the metal recovered from the ore per time period. If the rate of extraction is measured in tons of ore per time period, the rate of metal output can only be known if the concentration of metal in the rock [grade] is known. On the other hand, if the rate of extraction is measured by the amount of metal output per time period, the rate of ore extracted per time period, and therefore the rate of depletion, can only be determined if the grade is known. Output measurements from metalliferous deposits are thus only meaningful if the grade is included in the measurement.

The distinction between the two measures of output from a gold deposit is made by terming the rate of extracting ore as the rate of mining and the rate of extracting gold as the rate of gold recovered. Grade is used to link the cost of mining to the cost of gold recovered and, in conjunction with the price of gold, may be employed to determine a minimum grade that can be profitably mined (the mining limit). If high grades are mined first, the deposit is depleted down the grade distribution which describes the quantity of gold ore at different grades. By applying a mining limit to the grade distribution, the life-time size of an economic deposit is calculated, and at some rate of mining the life of the deposit concluded. If a constant rate of mining is assumed, the flow of gold recovered from the deposit falls because lower and lower grades are recovered with cumulated mining.

Chapter One introduces the theory of exhaustible resources with particular attention to the treatment of mining costs, inter-deposit and deposit-specific homogeneity. With reference to gold mining in South Africa, a market for grade is argued and a simple view of the mining firm proposed. Finally, with an emphasis on the life-time size of the deposit and the remaining life of the mine, a broad framework for analyzing current and future gold supply from the South African gold mines is presented.

The purpose of Chapter Two is to provide some theoretical background applicable to the South African gold mining situation. Firstly a model (disk model) that proposes to capture the elements of a grade distribution particular to a South African gold deposit, that can be based on historic data and extrapolated into the future, is presented. Secondly, inferring that gold mines in South Africa are price takers, the expected cost of mining with cumulated production is linked to the expected cost of recovering gold through grade and is combined with an expected gold price to define a mining limit. By applying a mining limit to a grade distribution, the life-time size of an economic deposit can be observed. The effects of changing the expected price, expected cost of mining, tax and technological change on the mining limit, and therefore the life-time size of an economic deposit, are described. Finally, with the decision to mine an economic deposit, the concept of depletion and economic exhaustion is introduced.

In Chapter Three the available data necessary to estimate the variables for constructing a disk model and the resultant mining cost, and recovered gold cost, functions are discussed. A method is proposed for estimating the life-time size of the economic deposit, its total gold content and the effect of depletion.

The aim of Chapter Four is to use the available data for an empirical treatment of the disk model at different output levels (gold mine, goldfield, and the Witwatersrand gold deposit). A cost-deposit model (linking the cost of mining to the cost of recovering gold) is constructed for each level of output. Using examples of expected mining costs and expected gold prices, the effects of changing these variables are illustrated for the gold mine with regard to the remaining life of the mine and the flow of gold output. A cost-deposit model, based on the disk model of the Orange Free State goldfield, is used as an example to demonstrate the necessity for an increasing gold price with cumulative mining, declining recovered grades and a constant cost of mining, on the continued existence of the goldfield at its current rate of gold ore output. Finally, the disk model for aggregated Witwatersrand gold deposit output values is employed to examine the effect of the South African gold mining tax formula on the life-time size of an economic deposit.

CHAPTER ONE

1.1) Introduction

The purpose of this chapter is to provide an introductory view of exhaustible resource theory and its relevance for an analysis of gold supply from South Africa.

The classical economists viewed the natural resource potential of a country as the determinants of national wealth and economic growth. Resource analysis was later incorporated into the framework on this basis. Grey (1914) and Hotelling (1931) first approached the problem of exhaustibility and depletion by using a dynamic capital theory argument. The problems the theory is concerned with are the following: optimising resource flow into the markets, balancing flows between present and future and the relationship between scarcity rents and optimal depletion. These issues have been modelled for different market structures, exploration and a back-stop technology (substitutes). The more recent and current literature is mostly concerned with the impact of mining on the environment[1].

The applicability of the theory for an analysis of the determinants of supply and future supply is examined below in respect of South African gold mining output. Attention is given to the inclusion of a deposit specific characteristic, grade, in the analysis.

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[1]See for example Dasgupta (1982), Nordhaus (1982) and Heal (1984,1990).

1.2) An outline of the theory of exhaustible resources

Conventional theory on exhaustible resources is based on the principles developed by Hotelling (1931). The basic Hotelling model postulates that the market value of some resource is the present value of the total stock less the cost of mining. If the price of the commodity, less the costs of mining, rises at the social rate of discount then owners of the resource will be indifferent between mining and holding unexploited stocks. This result is known as the Hotelling rule.

The only way an unexploited deposit can yield a return to its owner (the opportunity cost of holding) is by appreciating in value and, because it does not earn a dividend, the value must grow at the interest rate. This gives rise to the simplest case of the Hotelling rule. Assuming there is no divergence between the private and social discount rate, the price of the commodity must grow at the compound rate of interest for a resource stock to remain unexploited.

$$P_t = P_0 \cdot e^{it} \tag{1.1}$$

P_t: price in period t
P_0: price in the initial period
i : interest rate equal to the social rate of discount[2].

When the deposit is exploited in a competitive resource industry, the rent received is equal to the price less the marginal cost of mining. The condition for a flow equilibrium in the

[2]I have used 'i' as opposed to the usual 'r' as notation for the interest rate because 'r' is used for deposit radius in the next chapter.

market is met when owners are indifferent at the margin between holding and mining. This can only occur when the rent received from mining (opportunity cost of holding) increases at the relevant interest rate. If prices behave in this manner, the opportunity cost or present value of a unit extracted will be the same for all periods and the owners of resources will mine at a socially optimum rate causing an efficient extraction path and a flow equilibrium in the market for resources. In this way, Hotelling (1931) showed that in competition there is no inherent tendency to overexploit natural resources.

If prices rise too slowly, that is, the rent does not increase at the compound rate of interest, then the resource will be mined more quickly because the value of the deposit decreases with time. In other words, owners of the resource will tend to shed wealth, or rapidly exploit. If, on the other hand, prices rise too rapidly, the resource will be mined more slowly as owners of resources tend to hold wealth. In a monopoly situation owners of the resource will tend to restrict output to ensure more rapidly rising prices and the resource will be mined too slowly.

Hotelling (1931:152-153) states that the costs of mining are affected by the accumulated influence on the market and increase as the mine goes deeper and the quality decreases. The treatment of mining costs in relation to the Hotelling rule, under different market structures, is a central theme in the literature, but the problem is usually broadly approached by implying the following assumptions: a) each naturally occurring mineral deposit has a uniform quality (grade), however it varies between deposits and b) the individual deposit is clearly definable and fixed in size.

Herfindahl (1967) models the optimal entry of new deposits into the market and Cummings and Burt (1969) focus on the individual firm to evaluate the rent component (opportunity cost) at the marginal rate of use by imposing a production plan that is dependant on current resource and capital stocks and future capital accumulation.

Solow (1974) simplifies the analysis by assuming output from two sources that cannot co-exist: high cost and low cost deposits. For flow equilibrium to be maintained in the asset market, the low cost producer enters the market first. As this deposit becomes depleted the market price must cover an exponentially increasing rent (opportunity cost) until eventually the high cost producer enters the market as the low cost producer exits the market. Mining [opportunity] costs thus increase with cumulated production until a 'backstop technology' (substitute) provides a ceiling for the market price of the natural resource (Solow,1974:4). Stiglitz and Dasgupta (1982) extend, and confirm, these results in different market structures and add that resource depletion precedes innovation for a back-stop technology.

Most studies assume a fixed reserve base, but it can be used as "...the basis for production and exploratory activity as a means of increasing or maintaining reserves...[therefore]...exhaustible resources can be better thought of as inexhaustible but nonrenewable" (Pindyck,1978:841-855). Pindyck (1978:845) asserts that as current deposits are depleted, increased exploration expands the reserve base but with an ever decreasing deposit size. By implication, even if the reserve base is [hypothetically] infinitely large, low cost deposits are exploited first and therefore the costs of mining increase with cumulated production. Dasgupta and Heal (1978:192) mention the adaptability of the Hotelling model to an infinite resource [reserve] base, but abandon the idea as complicated and not very useful.

Fundamentally, increasing mining costs with continued mining occur within the deposit and at an inter-deposit level. Increasing opportunity costs within the deposit, as the deposit becomes depleted [at a uniform quality], result from the decreasing capital stock and other deposit specific characteristics referred to as the stock effect. Mining costs also rise as lower quality [smaller or high cost] deposits add to the reserve base when the higher quality [larger or low cost] deposits are depleted. From the supply side therefore, by taking into account increasing opportunity costs, rent is not earned at the rate of interest, but rather at the rate of interest less the percentage increase in opportunity cost. Devarajan and Fisher (1981:69) thus propose that the rent earned is equal to the opportunity cost of deferred extraction or foregone interest or savings in future mining costs.

Arrow and Chang (1982) show that if estimated reserves expand the size of the existing reserve base, prices of a mineral commodity fail to rise at the rate of interest. Drury (1982), on the other hand, asserts that a change in the size of the reserve base has no effect on the rents or price at the initial date of working a higher cost deposit. Although their results are inconclusive, Eswaran, Lewis and Heaps (1983) re-affirm Arrow and Chang's (1982) results by showing that with initial increasing economies to scale for mining, the Hotelling rule does hold.

The notion that the level of reserves is inversely related to mining costs is challenged by Livernois and Uhler (1987) by applying an exploration incentive model and deposit specific characteristics to petroleum reserves. They contend that the general theory of increasing marginal cost for mining brought about by depletion (stock effect) leads to the misconception that "...at any period in time new discoveries fall short of extraction [mining] from earlier discoveries, so that the aggregate reserve base declines, extraction [mining] costs will tend to rise" (Livernois and Uhler,1987:155). By allowing the characteristics of individual deposits, other than their state of depletion, to affect the mining costs and dropping the assumption that deposits with lower mining costs tend to be found [mined] first, new discoveries can increase the reserve base. If the reserve base increases by including high cost deposits the mining costs will rise.

Stollery (1983) and Cairns (1986) examine the effect of increasing mining costs due to a declining grade. When high grades are exploited first, they conclude that, unlike the Hotelling model, rents decrease as progressively lower grades are mined and become zero at the mining [extraction] limit, that is, when unprofitably low grades are encountered. Thus, by dropping the implicit homogenous grade assumption, they propose that a market for grade exists, which is reflected in the asset price of mining stocks.

By taking account of a vector of input prices associated with the deposit characteristics (for example grade), an essential difference between the cost functions of a conventional commodity and an exhaustible resource is highlighted. However, traditional theory asserts that a market for these characteristics does not exist and should therefore be employed as part of the 'shadow value' of the deposit which depends on "..the current rate and mode of extraction [mining], and hence cannot be taken as a parametric price for these inputs for the purposes of cost minimisation." (Epple and Londregan,1993:1077-1080).

Sweeney (1993) allows the cost of mining to be affected by the remaining stock (stock effect) and includes this into an increasing marginal costs function for mining. He then proposes that by allowing future price expectations, cost expectations, taxes and various externalities to be reflected in the value of the deposit (opportunity cost), an analogy can be drawn to a conventional analysis of supply (Sweeney,1993:778-779). Using price plus the opportunity cost as the price [shadow price] and an increasing marginal cost for mining, the optimal rate of mining is determined. With increasing marginal costs an increase in the [shadow] price causes an increase in the rate of mining.

Harris (1993) criticises the traditional approach along two lines. Firstly, he believes that the level of abstraction and the assumptions used for the economic analysis of exhaustible resources do not capture the essential features of mineral stocks and, secondly he notes that the " magnitude of a mineral resource is seldom [in the traditional treatment] the real objective of an appraisal" (Harris,1993:1013). He proposes that by estimating potential or dynamic supply by describing the size of the stock [based on geological evidence], the flow per unit of time in relation to the stock and costs of production can be concluded (Harris,1993:1014-1020). The analysis is based on estimating, a-priori, total resource endowment [for the country or the world] using various geological and geochemical techniques.

Exhaustible or stock resource flow over time into the market is clearly dependant on the behaviour of prices in relation to the mining cost, and thus the rent or opportunity cost component. The deposit quality - costs of mining different quality deposits - impacts on the flow of the resource. It is clearly established that the low cost deposits will be mined first, but a negative relationship between reserve base size and the costs of mining cannot be inferred simply because new [high cost] discoveries can increase the reserve base.

1.3) An approach to the analysis of gold supply from SA gold mines

It is argued that metalliferous deposits should be viewed differently from oil deposits and, particularly, that a market for a deposit characteristic (grade) exists for the metalliferous deposits, at least in the South African gold mining context. Because the cost of recovering gold increases as the grade declines, there is a market for grade in the sense that share prices (and hence returns to owners) reflect the expected yields from the private ownership of South African mineral rights. The process of gold mining is outlined and the physical deposit briefly summarised with a view to providing a simple abstraction of gold mining in South Africa. Finally, an approach to analyzing gold supply in South Africa is proposed.

1.3.1) Oil and metalliferous deposits

The received view is based on the assumptions that individual deposits have a uniform grade distribution (or if there are variable grades a market for grade does not exist) and the deposit size is fixed and definable. These assumptions might hold for oil deposits but certainly not for metalliferous deposits.

Oil deposits usually form between two distinct rock strata (permeable and impermeable layers) and are definable from the surrounding rock. The deposit is assessed in terms of the size of the pool, its depth and quality. The oil is extracted [mined] simply by drilling a hole in the right place and allowing the natural pressure to force it to the surface. The quality of the oil differs from deposit to deposit, but may be safely assumed to be homogenous in regard to a particular well. As the deposit is progressively depleted, bringing the oil to the surface becomes more difficult, for example, often water has to be pumped down the well to create sufficient pressure for the oil to be forced upwards (stock effect).

The [expected] market price will influence the rate at which the oil is extracted[3] [mined] and the equipment, depth, quality of the oil and other deposit specific characteristics will determine the costs of extraction [mining]. The rent or opportunity cost is obtained simply by the [expected] price less the cost of extracting [mining] at the rate determined by this price.

Unlike oil deposits, metalliferous deposit can be viewed in terms of the concentration of useful elements contained in the minerals that make up the rock. No metalliferous deposit has a uniform concentration of elements. The deposit is usually described in terms of metal content in relation to size, how the concentration is distributed, its physical orientation in space and geologically specific characteristics that might impinge on the mining process, as well as the impact of mineralogy on the costs for the metallurgical process, amongst others.

The science of geostatistics (Van Landringham:1983) is primarily concerned with estimating, a priori, a specific deposit's metal content in relation to size and how this concentration (grade) is distributed in the ore body. The economic potential of a metalliferous deposit is assessed in these terms. Because there are always high grade areas and low grade areas and in many cases tapering concentration levels, the size of the deposit in the absence of mining costs and metal prices is often indeterminable.

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[3]More oil rigs are opened.

Grade in relation to the size of a metalliferous deposit plays an important role in the decision of whether to hold or to mine. An index of the opportunity cost of mining a metalliferous deposit is the cost associated with recovering the metal from the ore and is therefore bound to grade. There is an inverse relationship between the cost of recovering metal from the ore and grade. The cost of recovering metal from low grade ore is higher than recovering it from high grade ore. Because it is rational to mine low cost deposits first (Solow,1974:4), it is also rational to mine the high grade [low cost] areas of a specific metalliferous deposit before the low grade [high cost] areas. In a sense "Depletion means chipping away at the best quality ore and moving down the grade distribution." (Stollery,1983:154).

For a particular metalliferous deposit, the return from mining [rent] depends on the cost of mining in relation to the concentration of gold in the ore. If the return from mining the high grade areas is greater than the return from hoarding gold, the deposit will be exploited. With cumulated production, high grade areas become depleted and the rent diminishes. Unlike the Hotelling rule (prices must increase sufficiently so the rent increases like compound interest, that is, the present value from mining is equal in all time periods until the deposit is depleted) the rent from a metalliferous deposit becomes zero at the margin (Stollery,1983:154). In a competitive market the mining limit is determined where the marginal cost of recovering gold at a particular grade is equal to the gold price. Cost [and price], rather than physical exhaustion, will therefore provide the mining limit (Stollery,1983:155).

1.3.2) The process of gold mining

In essence, the process of mining gold in South Africa is to break the gold bearing rock by blasting and then to transport the rock to the shaft[4] where it is hauled to the surface. At the surface the rock is crushed and milled to a powder (at the mill) and finally chemically treated to yield gold (at the plant). The gold output recovered depends on the concentration of gold in the rock mined.

A number of bottlenecks can occur in the system that prevent the current rate of mining from being increased beyond a certain level. The capacity of the shaft restricts the amount of rock that can be brought to the surface and the capacity of the mill and the plant determine the total amount of rock that can be processed in a given time period, regardless of the concentration of gold in the rock. Gold ore output can be increased by building bigger mines [shafts,mills and plants] or by opening new shafts and increasing the mill and plant capacity.

The simplest view to take of the mine is that capacity is set where the long run average costs of mining gold ore [grade is not included into the calculation] are minimised. The extractive firm can be seen as a normal firm producing gold bearing rock, but faced with an increasingly severe exogenous quality constraint (grade) with cumulated production as lower grades are mined. If there is a market for deposit specific characteristics (grade) the quality constraint can be included into a cumulative model to analyze gold supply from a particular deposit.

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[4]In this case the shaft is a vertical hole in the ground up to 3km deep that serves three main purposes: To allow access to the mining area for mining personnel, to haul gold bearing rock to the surface, and as a route for ventilation ducts, power cables and water.

1.3.3) The characteristics of the Witwatersrand gold deposit

Over 99% of gold output from South Africa comes from a single deposit (Whiteside et.al.,1976:39) - the Witwatersrand gold deposit (figure 1.1). To define the precise characteristics of a market for grade, some understanding of the broad nature of the deposit is required. The geological literature is extensive and is growing. Broad descriptions are given by Whiteside et.al. (1976:39-74), Anhaeusser & Maske (1986:489-796), Guilbert & Park (1986:754-773) and Pretorius (1989). In summary, the Witwatersrand rock pile (about 11km in depth) lies in a palaeo-structural basin about 400km long (NE-SW long axis) and about 250 km wide. A simple NE-SW cross section reveals a wedge shaped structure, steeply sloping in the north west and flatter in the south east. The exploited regions are concentrated along the steep north western periphery of the deposit. The reefs (gold bearing horizons) are thin, flat, undulating and continuous and are interspersed with barren rock. There are many reefs with different characteristics and at different depths. An analogy can be drawn to a many layered cake with the icing representing the gold bearing reefs.

Most metalliferous ore deposits have a log-normal grade-volume relationship [Matheron (1983); Mickel (1983); Parker (1983); Krige (1983) and Lane (1988)]. If the high grade regions are mined first, cumulative production will result in an exponential decay in recovered grades - the grade of gold ore mined from South African gold mines has decreased over the last two decades (see appendix 3 and Chapter Three). Because the gold bearing reefs are thin and tabular, grade can be hypothetically viewed to exponentially decay along a disk shape, that is, the high grade regions are at the centre of the disk but grade worsens from the centre outward until the mining limit is reached.

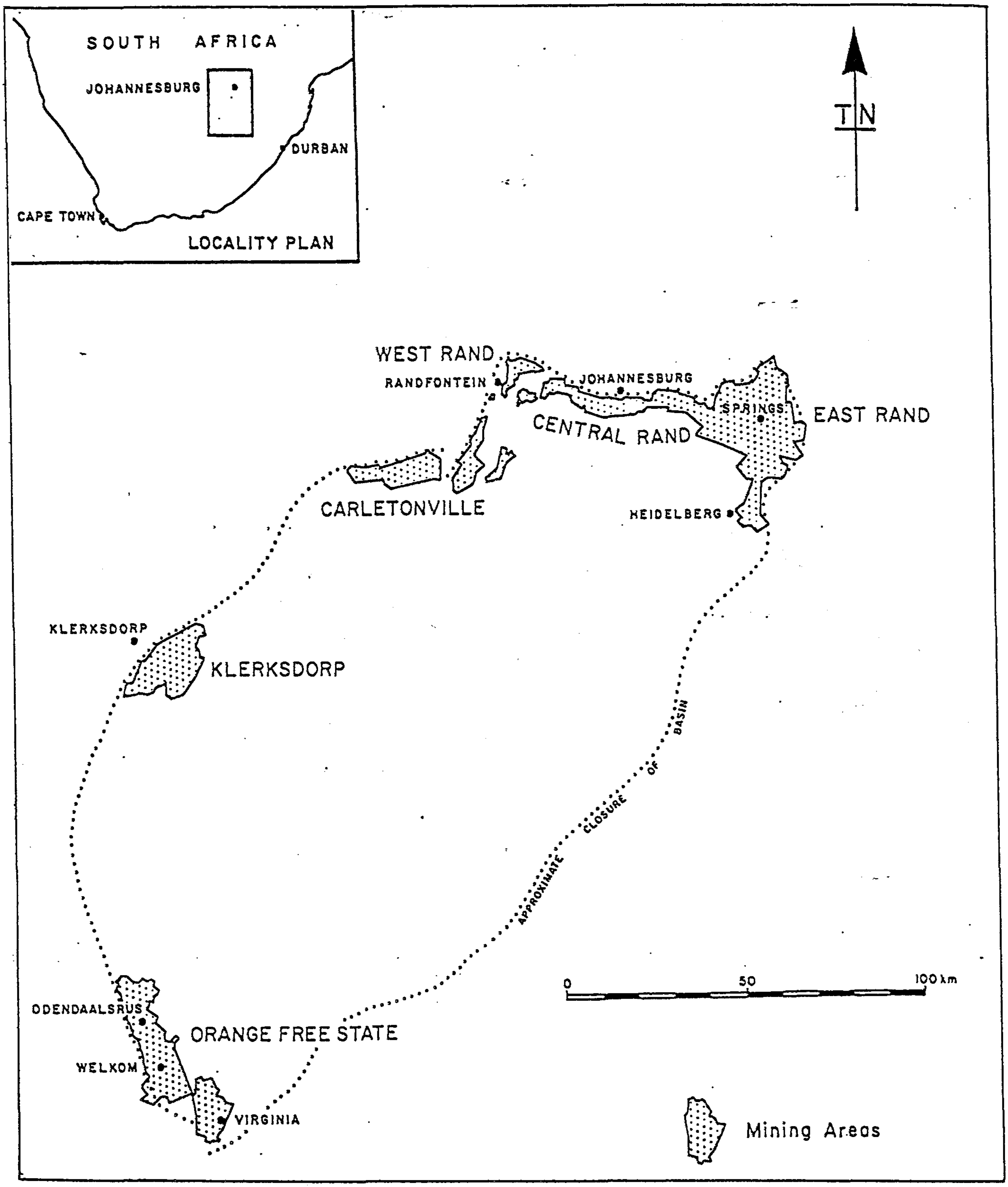

Figure 1.1: Sketch map of the approximate extent of the Witwatersrand gold deposit.
Source: Pretorius 1989.

The map only represents the approximate closure of the basin and does not show the inferred extent of any specific reef. The purpose of this map is simply to demonstrate: 1) the massive size of the deposit and 2) the small proportion of exploited regions (mining areas) to unexploited regions.

1.3.4) An approach to analyzing gold supply in South Africa

Current supply from the South African gold mines, with a constant rate of gold ore output in any time period, depends on the average grade recovered during that time period. Mines close when they exceed their mining limit. This occurs when the average recovered grade is lower than the marginal grade determined by the gold price and cost of mining the gold ore.

The future supply, and the remaining life of the economic deposit, from the South African gold mines can be estimated by determining a mining limit, that is, the minimum grade that can be economically mined.

1) If the grade distribution (how grade decays as mining continues) can be modelled, the mining limit in terms of grade can be deducted. Because cumulated production results in higher costs for gold output as the recovered grade declines, a grade is eventually reached (marginal grade or cut-off grade) where the cost of recovering the gold is equal to the price of gold (rent is zero at the mining limit).

2) The life-time size of the economic deposit is simply the volume of gold ore between the maximum and minimum grade. Total potential gold output (in grams) is the life-time size of the deposit (in tons) multiplied by the average grade (in grams per ton).

3) By treating the mining firm as a normal firm that produces gold ore at a rate determined by the capacity of the shaft, the mill and the plant, but faced with an increasingly severe quality constraint as mining continues, the life of a mine can be estimated. It will continue to mine gold ore at a constant rate until the mining limit, defined by the marginal grade, is reached.

4) Once mining has started, the size of the remaining economic ore deposit is bounded by the current recovered grade and marginal grade in relation to the grade distribution. The

remaining life of the mine is simply the size of the remaining economic deposit divided by a constant rate of mining. Total future gold output is the average grade between the current recovered grade and the marginal grade multiplied by the total remaining gold ore.

1.4) Conclusion

A gold deposit will be exploited when the present value of the total stock at a particular average grade, less the expected cost of mining, is greater than, or equal to, the discounted expected gold prices. Because grades are variable and the high grade areas mined first, the rent does not rise like compound interest but rather declines to zero at the mining limit. As a result the present value will also decline and the benefit from gold sales will be biased in favour of the past or present once mining has begun.

It is not unreasonable to view a gold mine as a normal firm, but only if output is measured in terms of rock (gold ore) brought to the surface and treated. The grade, however, determines the cost of recovering the gold in relation to the cost of mining rock. Because high grade areas are mined first, the recovered grades decline with cumulative production and the cost of producing gold from the gold ore increases. The mine encounters its mining limit when a grade is eventually reached where the cost of producing the last unit of gold is equal to the gold price.

CHAPTER TWO

2.1) Introduction

If there is a market for grade amongst deposits - the classical literature [Hotelling (1931), Solow (1974)] on exhaustible resources is in agreement that low cost [high grade][1] deposits are mined first - and if grades are not homogeneous within a deposit - Stollery (1983)[2] adds that high grade regions of a specific [metalliferous] deposit are mined first - then a market for deposit specific characteristics must exist. If a grade distribution can be defined for a particular deposit, and high grades are mined first, then some minimum grade must set the limit to the economic deposit (mining limit). The possibility of defining a grade distribution of a South African gold deposit, how the mining limit is determined and the impact it has on the physical exhaustion of the economic deposit is the aim of this chapter.

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[1]A low cost deposit also implies other characteristics i.e. depth, complexity of its geological history, the mineralogy etc.

[2]Stollery (1983) and Cairns (1986), proposed the existence of a market for deposit specific characteristics (grade) on the grounds of grade variability within the deposit. This proposal, however, was not well received in the subsequent literature, see Epple and Londregan (1993), because of the inability to clearly define it in the case of oil and natural gas and is included unexplained into the 'stock effect' Sweeney (1993). As a result I initially missed the Stollery (1983), and the Cairns (1986), papers but was forced to include [a market for] grade in order to satisfactorily explain gold production in South Africa. I arrived at similar conclusions to Stollery (1983), although our aims and market circumstances were different. I adopted the term 'mining limit' used by Stollery (1983) and removed a laborious discussion pedagogically outlining my reasoning. I have however, retained the notion that the firm behaves much like a normal firm when producing gold bearing rock, but faced with a quality constraint, and therefore the distinction between the cost of mining and the cost of recovering gold. I believe this works well, particularly in the South African case, where the costs of opening a new deep level gold mine are extremely high (see section 2.8).

2.2) A simple disk model of a gold deposit[3]

Gold exists everywhere in the earth's crust with a crustal abundance estimated at an average concentration of gold in the rocks at 3.5 parts per billion (Lee & Yoa, 1970:780), but anomalous concentrations (gold deposits) occur in specific geological environments. Although these deposits have highly complex structures some simplifying assumptions may be made that capture the essential nature of certain individual deposits.

The Witwatersrand gold deposit is made up of many sheet-like reefs (gold bearing rock) at different depths below the surface, separated by relatively extensive areas of barren rock. The shapes of the different mining lease areas are complicated and typically characterised by the existence of many reefs at different grades and at different depths. Because the reefs have different grades and the low grade reefs are far more abundant, this model provides a highly simplified but mathematically rigorous representation of the Witwatersrand gold deposit. Because high grade regions are mined first, gold concentration (grade) can be portrayed as decreasing from the centre outward along a disk as mining proceeds.

The amount of gold bearing rock/ore (T) in a disk shaped deposit is the volume of the rock multiplied by the stoping width (h) and the density of the rock (d) as follows:

$$T=\pi r^2 dh \tag{2.1}$$

T: tons of gold ore
r: radius of the deposit
d: density of the rock
h: stoping width

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[3]I am indebted to Mr Clyde Mallinson of the Geology Department at Rhodes University for the many discussions that led to the final abstraction of the deposit. I am also indebted to Dr Mike Burton of the Mathematics Department for aiding me in presenting this abstraction mathematically.

The stoping width is the distance between the rocks above the mined out area and the rocks below the mined out area. The volume of gold ore within the disk increases exponentially away from the centre of the disk as the radius (r) becomes larger.

The grade (C), or concentration of gold in the rock, decreases from the centre of the disk as mining proceeds, if high grades are mined first. This can be represented by a log-normal grade distribution as follows:

$$C_t = C_0 . e^{-kr_t} \tag{2.2}$$

C_t: grade at radius r_t
C_0: maximum grade, at radius r_0
k : grade distribution constant

The grade distribution constant (k) is a measure of the richness of the deposit (grade-tonnage relationship), for example, a high value for the grade distribution constant reflects a relatively large volume of gold bearing rock and therefore greater quantities of ore at all grades. Its value is unique to any particular deposit, goldfield, mine or reef that can be characterised as a disk. The marginal grade[4] (C_m) is the grade at the margin between gold ore that can be profitably mined and the gold ore that cannot be profitably mined, that is, the grade at the mining limit. The size of the economic deposit is the volume of rock between the maximum grade (C_0) and the marginal grade (C_m).

The total quantity of gold in an economic deposit can only be determined if the life-time average grade is known. Integrating the grade distribution function (equation 1.2) with respect

[4]The marginal grade is sometimes called the cut-off grade.

to radius and dividing the result by the radius of the economic deposit (r_m) yields the average grade (C_{avg}).

$$C_{avg}=\frac{C_0}{k}.\frac{1-e^{-kr_m}}{r_m} \tag{2.3}$$

The gold (Q_{cum}) contained in the economic deposit is the product of the total life-time tonnage of gold ore (T_m) and the life-time average grade (C_{avg}). From equations (2.1) and (2.3), and at a mining limit determined by a marginal grade (C_m) total gold output is as follows:

$$Q_{cum}=(\frac{C_0}{k}.\frac{1-e^{-kr_m}}{r_m}).(\pi r_m^2 dh)$$

$$Q_{cum}=C_{avg}.T_m \tag{2.4}$$

The life-time total gold output increases at a decreasing rate as the mining limit is extended and lower grades are mined. When mining starts, and high grades are mined first, cumulative exploitation of an economic deposit results in a decay of recovered grade and, as a consequence, decreasing gold output per ton of gold ore. This rate of decay in terms of radius, and thus tonnage, is captured by the value of the grade distribution constant (k) unique to the deposit in question.

2.3) The mining limit

To determine the mining limit of a particular economic deposit or mine it is necessary to clarify and define the following: the demand curve facing the extractive firm, the distinction between mining rock and recovering gold, and then the selection of a marginal grade. By defining the marginal grade, a mining limit is defined and, with respect to a grade distribution constant, the life-time size of an economic deposit is known.

2.3.1) The demand curve facing gold mines in South Africa

The purpose of this section is simply to show that output from individual South African gold mines is insignificant in the world market and that they may reasonably assumed to be price takers and, thus, they face a perfectly elastic demand curve.

The total gold mined each year (primary supply) adds about 2% to the existing above ground stock of gold (Gold,1993:16). Although primary supply contributes about 56% of gold that changes hands yearly, the existence of a futures market gives rise to a large speculative component which does not usually call for physical delivery. On average, between 1984 and 1992, only 9% of contract volumes called for physical delivery (Gold,1993:63). The total contribution, in terms of contract volumes, of South African primary gold is about 2% per year with, on average, 0.007% from each South African gold mine[5].

[5]Averages are derived from Gold 1993 and COMSA reports (1984 to 1992) - see Appendix 3, part 1.

2.3.2) The extractive firm

The essential point in this section is to distinguish, and link through costs, the processes of producing gold bearing rock and recovering gold from the rock. The capacity of the ore to yield gold is independent of the cost of producing it, but dependant on the grade, or concentration of gold, in the rock. However, the cost of producing gold bearing rock is linked to the cost of recovering the gold through grade. The firm will continue to produce gold bearing rock until it reaches its mining limit where the marginal cost of recovering the gold from that ore is equal to the marginal revenue from the sale of the gold.

The extractive firm is viewed as a normal firm but one that is faced with a stock effect in the production of gold bearing rock plus a quality constraint (grade) in the production of gold from the ore. As mining proceeds, increased depth and greater distances from the working face to the haulage shaft result in a stock effect that adds to the marginal cost of producing gold bearing rock (MCE). The marginal cost of recovering gold (MC) from the gold bearing rock increases as the recovered grade declines and is linked to the MCE (equation 2.5) in the following way[6]:

$$MC_r = \frac{MCE_r}{C_r} \qquad (2.5)$$

MCE_r: Marginal cost of producing gold ore at radius r
MC_r : Marginal cost of recovering gold at radius r
C_r : Grade at radius r

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[6]Stollery (1983:155) derives a similar result, but in a form that is used to estimate a falling resource rent with declining grades, i.e. the firm's discount rate is included into the equation.

As the recovered grade decreases with cumulative production the MC increases, over and above the influence of the stock effect, in proportion to the decline in recovered grades. The value of the grade distribution constant (k) for the deposit will determine the rate of increase in MC with cumulated mining because it links the tonnage of gold ore to the grade.

Thus, by introducing a vector of input prices associated with the deposit characteristics - declining grade as mining proceeds - the difference between the cost function facing a normal firm [for example, increasing, constant or decreasing marginal cost of producing gold bearing rock or oil] and a firm producing gold [from the gold bearing rock] is highlighted.

2.3.3) The mining limit

Because high grades are mined first, the life-time size of the economic deposit is bounded by the lowest grade that can be profitably mined at the mining limit. Assuming that firms are price takers - their marginal revenue from gold sales is equal to the gold price - the mining limit (equation 2.6) is set where the expected marginal cost of recovering the gold (MC_m) at the marginal grade (C_m) is equal to the expected gold price at the mining limit(P_m).

$$MC_m = P_m \tag{2.6}$$

MC_m: Expected marginal cost of producing gold ore at the mining limit
P_m : Expected gold price at the mining limit

By applying the link between the marginal cost of producing gold bearing rock to the marginal cost of recovering gold (equation 2.5), equation 2.6 may read:

$$\frac{MCE_m}{C_m} = P_m \qquad (2.7)$$

C_m : Marginal grade (at the mining limit)
MCE_m: Expected marginal cost of recovering gold at the mining limit.

Rearranging equation 2.7, the marginal grade at the mining limit (C_m) is thus:

$$C_m = \frac{MCE_m}{P_m} \qquad (2.8)$$

The mining limit, given an expected gold price (P_m) and the expected cost of mining gold bearing rock (MCE_m), is defined by the marginal grade (C_m). The expected life-time size of the economic deposit (T_m) is set by the mining limit but dependant on the value of the grade distribution constant (figure 2.1).

2.4) The effect of price

An expected change in the gold price will alter the mining limit, through the marginal grade, and therefore the size of the economic deposit and potential flow of gold output. For example, an expected increase in the price of gold lowers the marginal grade and extends the mining limit. The size of the economic deposit is expanded and potential gold output from the deposit is increased (see figure 2.2) and vice-versa for an expected decrease in the price of gold.

The long-run supply curve for [primary] gold production is positively sloped because an expected increase in the gold price extends the mining limit of all firms [if all firms mine high grades first] and thus potential [primary] gold output is expanded. Also, if the mining limit is reduced, certain low grade gold deposits may become economic and begin producing gold.

In the short-run, however, the ability to produce gold bearing rock from the existing mines in South Africa is limited by the capacity of the shaft, mill and plant. It takes about eight years to develop a new deep-level mine (Dowsley,1993). Although high grades are mined first, most mines will to some extent, in the short-run, control the recovered grade by blending different grade ore. With an increase in the price of gold, ceteris paribus, the revenue per ton of gold bearing rock at a constant grade increases. The revenue per ton of gold bearing rock may, however, be kept constant by blending ore that yield lower grades. At a higher price a market exists for lower grade ore. If the rate of output of gold bearing rock is held constant, and lower grades are recovered, in response to a higher gold price, the total gold output will decline. The concentration of gold is lower and if the amount of gold bearing rock treated is constant, total gold recovered must be less. In other words, there could be a negatively sloped supply curve for gold output in the short-run.

2.5) The effect of mining costs

A change in the cost of mining, ceteris paribus, influences the cost of gold output by shifting the MC curves upwards or downwards. For example, a real increase in the wage rate shifts the MCE curve upwards in proportion to the increase and therefore also the MC curve. If the gold price remains unchanged the marginal grade is increased, the mining limit is reduced and the size of the life-time economic deposit is smaller (figure 2.3).

2.6) The effect of a tax

The two basic forms of tax on primary gold output that can be levied are: a per unit of output tax and a tax based on profits or rent. Because all primary gold producers are assumed to be price takers, the entire burden of the tax falls on the producer.

A per unit of output tax does not take into account the increment in costs of gold output as mining continues and recovered grades decline. In essence, it simply adds to the cost of mining gold bearing rock and therefore shifts the MCE curve, and the MC curve, upwards in proportion to the tax. The economic life of the deposit is reduced and the unextracted gold lost to society forever.

On the other hand, a profits (rent) tax takes into account the incremental costs of gold output as mining proceeds. Although it adds to the cost of mining the additional tax decreases as the recovered grade drops. This effect may be introduced into the model by decreasing the slope of the MCE curve. The ideal risk free case, at a constant MCE and constant gold price, is when the rent tax decreases in direct proportion to the incremental increase in the cost of recovering gold as lower and lower grades are mined. This will result in a flat MC curve equal to the gold price. All rents may be taxed away and the tax has no effect on the extraction rates. The life-time size of the economic deposit is not affected by the imposition of the tax (figure 2.4).

2.7) The effect of technological change

Technological change will always reduce the cost of gold output either by reducing the cost of mining through increased productivity of capital and labour, or via the recovered grade due to improved mining methods (for example, a smaller stoping width) or metallurgical methods (for example, the cyanidation process).

A reduction in the cost of mining by increased productivity causes a shift downwards of the MCE curve and therefore also a shift downwards of the MC curve. At a constant gold price, the mining limit is expanded by a lower marginal grade and the life-time size of the economic deposit is increased.

A smaller stoping width in the South African sense simply means that less barren rock is incorporated into the gold bearing rock and thus recovered grades are improved. Holding the cost of mining constant[7], the MC curve will shift downwards extending the mining limit at a particular gold price and therefore the life-time size of the economic deposit is expanded.

Metallurgical advance generally takes the form of more efficient gold recovery from the gold ore - more gold can be recovered more cheaply per ton of ore. Increased gold recovery shifts the MC curve downwards. This movement can be compounded by a further downward movement of the MCE curve if recovery is cheaper. As a result the life-time size of the economic deposit is increased.

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[7]The cost of mining will most likely increase due to more difficult accessibility to the mining/stoping face. The net effect of instituting this type of technology will depend on the value added from a higher recovered grade.

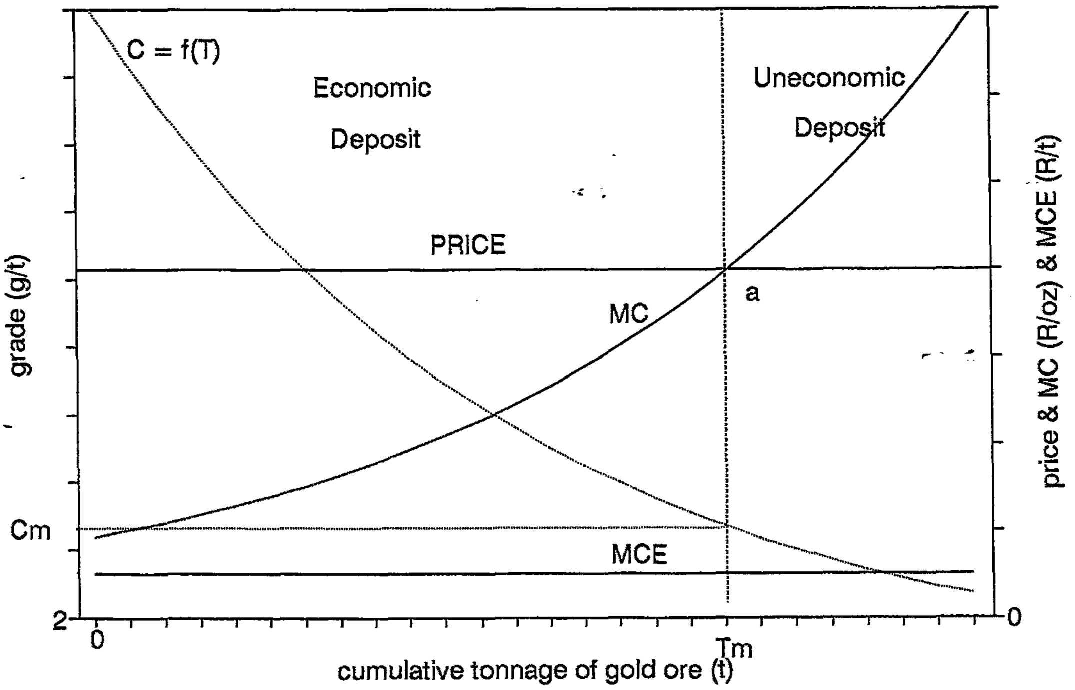

Figure 2.1: The mining limit.

With reference to the above figure: For purposes of illustration the expected marginal cost of mining (MCE), in rand per ton (R/t), is held constant along the proposed intertemporal mining path. The grade distribution when high grades are mined first, with cumulative production of gold ore, may be written as a function of cumulated tonnage, C = f(T). The marginal cost of recovering gold (MC) increases in direct proportion to the decay in grade as mining proceeds. Although grades are quoted in grams of gold per ton of gold ore (g/t), the expected gold price and MC are converted from rand per gram (R/g) to rand per ounce (R/oz) because the international price is usually quoted in ounces of fine gold. The conversion factor used (COMSA,1970) is:
1000 g = 32.15103 oz.

The mining limit is determined at point 'a' on figure 2.1 where the expected marginal cost of recovering gold, at a constant marginal cost of producing gold ore, is equal to the expected price of gold (MC=P). At the marginal grade, Cm, the life-time size of the economic deposit is Tm. The shape of the grade distribution curve has an important bearing on the size of the economic deposit, for example, a flatter slope (larger value of the grade distribution constant) will result in a larger life-time size of the economic deposit.

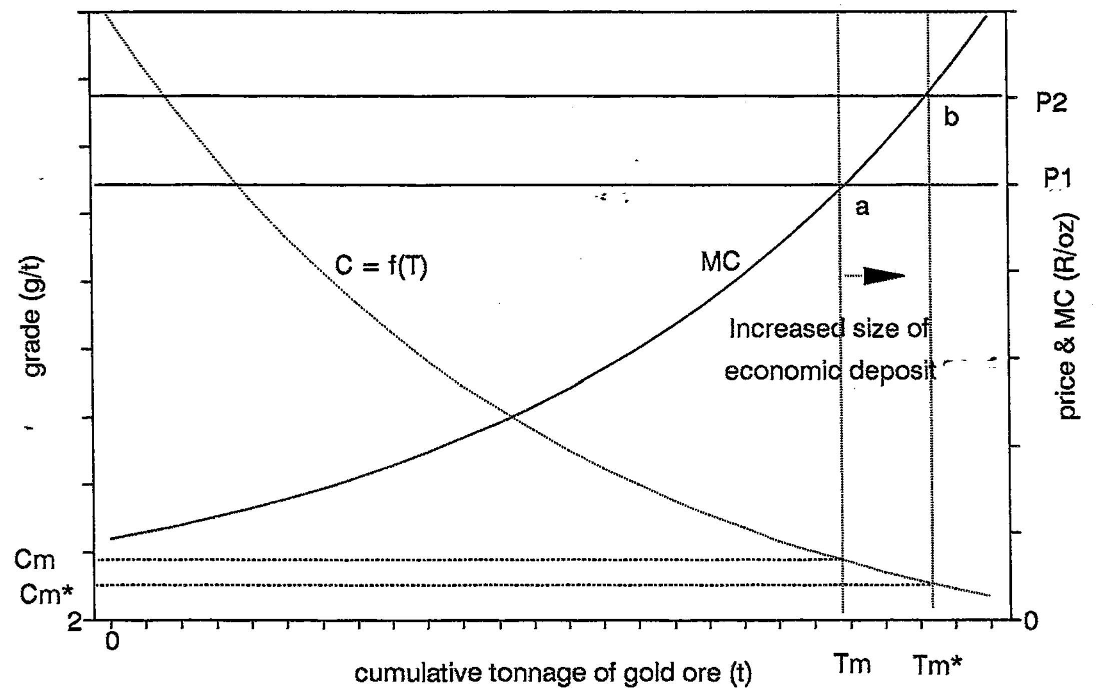

Figure 2.2: The effect of price expectations on the mining limit.

The mining limit at an expected price (P1) and a constant marginal cost of mining (not shown on the figure) is shown at point 'a'. The marginal grade is Cm and the size of the economic deposit, with respect to the grade distribution, is Tm. Holding the MCE constant, if the expected price per ounce of gold increases to P2 rand per ounce, the position of the mining limit on the MC curve will change - a movement from point 'a' to point 'b' and the marginal grade (Cm) is reduced to Cm*. The life-time size of the economic deposit, with respect to the grade distribution, is increased (from Tm to Tm*).

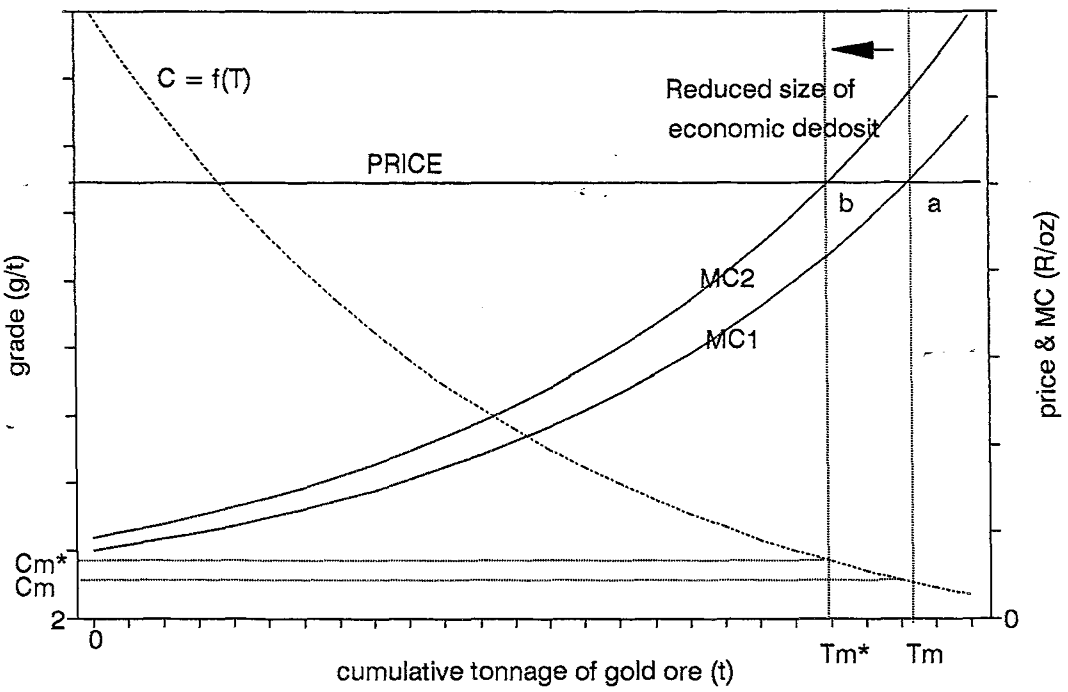

Figure 2.3: The effect of mining cost expectations on the mining limit.

An expected increase in the cost of mining shifts the MCE curve upwards in proportion to the increase (not shown on figure). The MC curve is likewise moved upwards from, for example, MC1 to MC2. At constant price expectations, the mining limit is changed from point 'a' to point 'b' via a decrease in the marginal grade from Cm to Cm*. The life-time size of the economic deposit is reduced from Tm to Tm*.

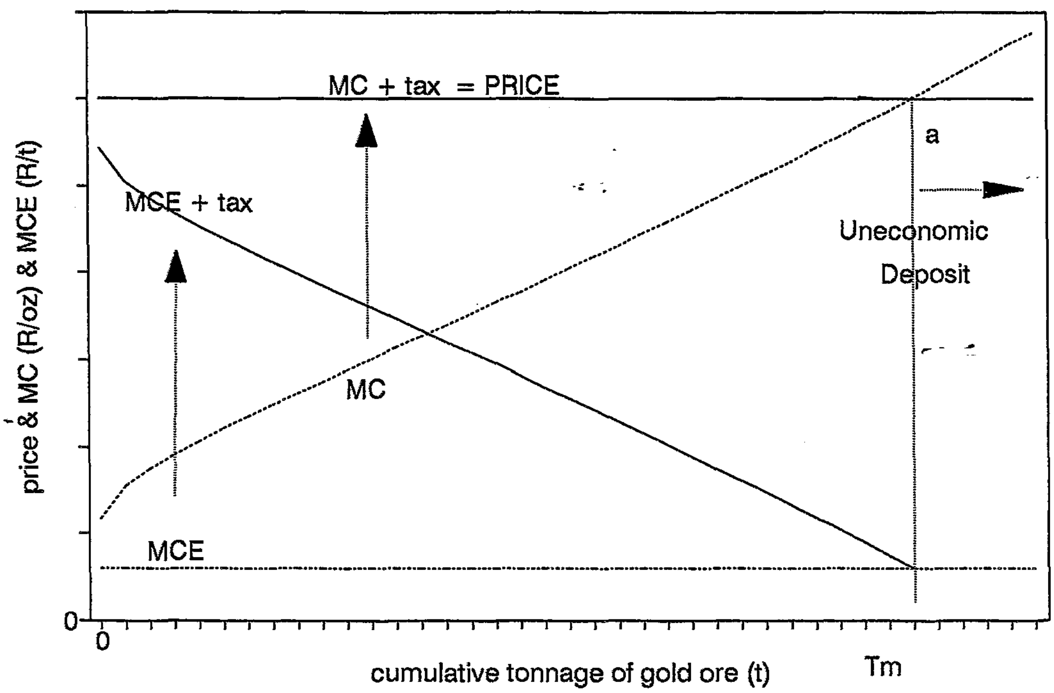

Figure 2.4: The effect of an ideal risk free rent tax.

Without a tax the mining limit is at point 'a'. The MCE is assumed to be constant along the intertemporal mining path (no stock effect) and the MC increases with lower recovered grades as mining continues. The imposition of an ideal risk free rent tax adds to the MCE by an amount that swivels the MC curve upwards to equal price - all the rents are taxed away. The mining limit is unaffected by the tax because no tax is levied at the marginal grade ($P=MC_m$). The size of the deposit is unaltered and the tax has no effect on the extraction rates

2.8) Depletion and exhaustion

In this section the concept of depletion, once mining has begun, is explained in terms of removing gold bearing rock from the economic deposit and, at a constant mining limit, reducing the size of the economic deposit until it is finally exhausted. However, after Stollery (1983), when the mining limit is allowed to vary, the physical exhaustion of the economic deposit is determined by cost, technology, price and taxes and better thought of as economic exhaustion.

As stated in chapter one: a gold deposit will be exploited when the present value of the total stock, less the expected cost of mining, is greater than the expected return from gold that has already been mined. The total stock of gold in an economic deposit is defined by the mining limit in conjunction with the grade distribution constant (k) of that particular deposit. In other words, if the amount of gold ore that exists at various grades (k) and the mining limit can be determined, the present stock of gold can be estimated. Therefore the expected price, expected costs/technology and taxes not only determine the present value of the economic deposit but also define the mining limit and the total stock. If, for example, the gold price was expected to drop the present value of waiting would fall and the life-time size of the economic deposit decrease. The present value declines further via a smaller total stock of gold and, depending on the expected return from hoarding gold, the rent may rise but will eventually tend toward zero as grades decline with cumulated mining.

A simpler way to view the decision to mine a deep-level South African gold deposit is: the present value of the total stock must be greater than the discounted value of development costs[8] and the size, and value, of the remaining stock is determined by applying a management or shareholder utility function. The treatment of the rent from the remaining stock is then viewed in a different light because when mining has begun the option of whether to mine or wait is removed, at least until the mining limit is reached. Once the mine is in operation, costs/technology, price and tax, and not the value of the total remaining stock, dictate the mining limit and therefore also determine the state of depletion and eventual exhaustion. In other words, once mining has begun and full production of gold bearing rock is underway, the firm behaves much like a normal firm but faced with an increasingly severe quality constraint (grade) with cumulated mining that will eventually cause the closure of the mine when the mining limit (marginal grade) is reached.

With full production, the process of mining is to remove the gold bearing rock in order to win the gold. If a constant rate of mining is assumed, then the size of the economic deposit, ceteris paribus, will be steadily depleted in each period (figure 2.5). As the size of the economic deposit becomes smaller with continued mining [depletion] the recovered grade falls, as the high grades are mined first, and the flow of gold from the deposit declines. The mine will, however, continue to produce gold until the mining limit is reached, at which stage it becomes marginal, and finally the economic deposit is exhausted when the gold bearing ore at the marginal grade is depleted. By holding the mining limit constant, the economic deposit is steadily depleted along an intertemporal mining path until it is eventually

[8]These are the costs incurred before the mine reaches full production and include the cost of setting up the plant and mill, sinking the shafts and providing access to the ore body by making development tunnels. The South African gold mining tax allows for an accelerated depreciation of these costs - see Chapter Four - which are considerable " A deep level gold mine scheduled to produce 120 000 tons [of gold bearing rock] per month...cost[s] approximately R2.5 billion in July 1993 money terms...[and] takes approximately 8 years to develop (exploration excluded)" (Dowsley,1993).

exhausted. When the mining limit is allowed to change by varying costs, technology, price and taxes, the point at which economic exhaustion may occur is dynamically determined.

2.9) Conclusion

The behaviour of the grade distribution particular to the South African Witwatersrand gold deposit could be characterised as a disk and if high grades are mined first then the recovered grade will decline as mining proceeds. The mining limit, in terms of a marginal grade and the grade distribution, defines the size of the economic deposit. Gold price, cost/technology and tax expectations influence the marginal grade and thus the mining limit, the size of the economic deposit and the total stock of gold.

Once mining has begun the economic deposit becomes depleted from the centre of the disk outwards, but physical exhaustion only occurs when the mining limit is reached. Because the mining limit is defined in terms of economic criteria, physical exhaustion is better referred to as economic exhaustion.

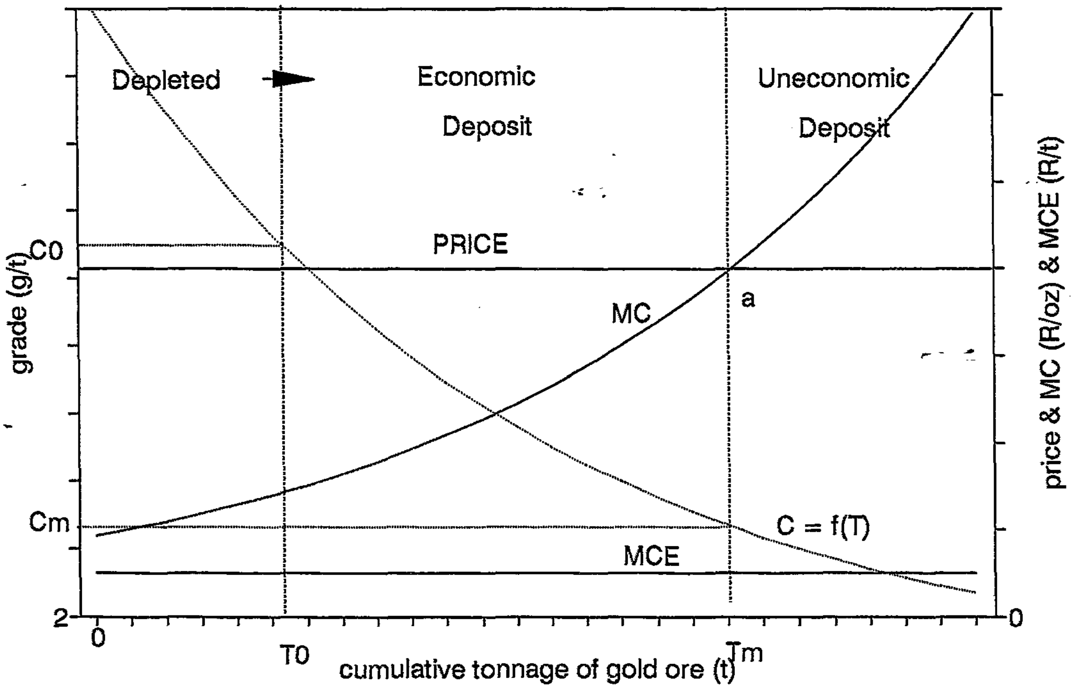

Figure 2.5: Depletion and economic exhaustion of an economic deposit

Before mining begins, at constant price expectations and cost/technological expectations, the mining limit is at point 'a' with a marginal grade of Cm and the size of the economic deposit is Tm. Once mining has begun gold ore is progressively removed, the intertemporal mining path moves down the grade distribution curve and the economic deposit is depleted. With reference to the figure above, at some period in time after mining has begun and at a constant mining limit, the size of the economic deposit is reduced through depletion to T0-Tm and bounded by a new maximum grade C0. When the marginal grade Cm is eventually mined the mine becomes marginal and finally exhausted when no more ore at the marginal grade is left (T0=Tm).

Because the mining limit can change (figure 2.2 and figure 2.3) the point of economic exhaustion is uncertain unless there is perfect knowledge of all prices and costs.

CHAPTER THREE

3.1) Introduction

To build a cost deposit model that can be empirically tested, certain variables must be estimated from the available data. The aim of this chapter is to provide an outline of the data available, how certain variables can be estimated from the data, and finally the use of a cost-deposit model for an empirical treatment.

3.2) Available Data

Three main sources of data are used to estimate and test the model in the South African context. There are statistical tables of working results from the Chamber of Mines of South Africa, yearly average gold prices from the Gold Fields Mineral Services publication Gold and deflators from the South African Reserve Bank publications.

3.2.1) Gold production and costs data

Gold production and cost data are available in quarterly analysis of working results and annual statistical tables published by the Chamber of Mines of South Africa. Although this data only includes member mines, the excluded gold output from non-members is generally only about 3% of the total South African gold output per year.

i) Production data.

a) Yearly totals of ore milled are taken as the rate of mining. Adjustments should be made for stockpiling of ore either underground or above ground, but the ore milled is a fair proxy for the rate of mining if taken over a number of years.

b) The recovered grade is measured after milling and processing and thus it takes into account both the stoping width and the metallurgical process used to recover the gold from the ore. In situ grades measure the grade of the gold bearing rock at the working face. Because the width of the gold bearing reef is usually smaller than the stoping width, some of the barren hanging wall and/or footwall rock is included into the ore that is then hauled to the surface for milling and treating. The recovered grade is as a consequence usually lower than the in situ grade. The grades quoted in the Chamber of Mines working results are always the recovered grades and show the following relationship:

$$C_t(g/t) = \frac{Q_t(kg)}{T_t(000tons)} \tag{3.1}$$

C_t: grade in time t
Q_t: gold output in time t
T_t: gold ore output in time t

c) The influence of surface dump mining on total gold output and recovered grade is very small and can largely be removed by excluding the Anglo American O.F.S. Joint Metallurgical Production Scheme, and subsequently the Free State Consolidated Metallurgical Scheme.

ii) Cost data

Yearly average working costs per ton milled and per kilogram fine gold output are taken as the variable average costs of mining (ACE) and gold recovered (AC) respectively. Capital replacement costs generally contribute about 10% to total costs. The South African Chamber of Mines data reflects the link between mining gold bearing rock and recovering gold as follows:

$$C_t(g/t) = \frac{AC_t(R/g)}{ACE_t(R/t)} \tag{3.2}$$

C_t : grade in time t
ACE_t: average cost of mining in time t
AC_t : average cost of recovering gold in time t

Because the gold price is usually quoted per ounce the cost per kilogram of gold is accordingly adjusted - there are 32.15103 ounces of gold per kilogram (COMSA, 1970).

The annual average grade can be used as proxy for the marginal grade if there are many time periods and the grade-tonnage possibilities open to the firm (assuming high grades are mined first) in each time period are exhausted (the average grade becomes bounded by the marginal grade from the previous period).

3.2.2) The gold price

The current gold price in dollars, pounds, Duitse mark, Swiss franc, Yen, Australian dollar, Canadian dollar and Rand is published in Gold. The prices are the annual averages in terms of the London PM fix and equivalents.

3.2.3) Deflators

Costs can be deflated using a general mining deflator from the South African Reserve Bank (SARB,1994). However, the gold price deflator that is relevant should link the price of gold to the general price trend working in the South African economy as a whole, that is, a GDE deflator.

Current costs and current gold prices can be used to estimate the current mining limit in terms of a marginal grade. This mining limit is, however, only useful for an analysis of marginal mines.

Expected mining costs, including a stock effect, and expected gold recovery costs can be econometrically determined. Various methods can be used to estimate the expected return from holding or mining gold (semi-variograms[1], econometric and portfolio analysis). This analysis is, however, not approached in this thesis.

[1]A semi-variogram is a statistical tool that can be used to analyze the frequency and variations of cyclical fluctuations in any deflated time series data.

3.3) Estimating the grade distribution

Estimating, post priori, the grade distribution with cumulated mining and predicting the expected decay in recovered grades is necessary to construct the cost function for gold production for a gold mining firm. Because the grade is modelled to fall exponentially as the radius increases (equation 1.2), the grade distribution constant (k) measures the amount of gold ore available at various grades and the rate that the recovered grades are expected to fall as mining proceeds. The grade distribution constant can be estimated using observed data (discrete values of recovered grade and cumulated tonnages of gold ore) and thus an ideal grade distribution for a disk shaped deposit may be constructed and tested against the observed grades.

a) The average stoping width used to estimate a grade distribution is taken at 1.5 meters (Mallinson,1994), and the average density of the rock at 2.65 (Moore,1994)[2] times that of pure water at sea level.

b) The disk model employs radius (r_{cum}) to estimate the grade distribution. Because there are no observations of radius, the cumulated discrete annual observations of tonnage of gold

[2]The density of the Witwatersrand rocks is taken as the density of quartz, i.e., 2.65 (Deer,Howie & Zussman,1983:340).

bearing rock (T_{cum}) are used as an instrumental variable and converted to cumulated radii (r_{cum}) in the following way:

$$r_{cum}=\sqrt{T_{cum}.\pi.d.h)} \qquad (3.3)$$

d: density of the rock
h: stoping width

c) The disk shaped deposit is modelled in such a way that the grade decays exponentially from the centre. The rate of decay or the grade distribution constant (k) describes the relationship between cumulated radius (equation 3.3) and grade (equation 2.2). Estimating the grade distribution constant (k) requires grade-cumulated radii observations.

For every observation of tonnage of gold ore, there is a corresponding observation for average recovered grade. Thus for each discrete observation of a cumulated T (T_{cum}), and because average grade can be used as proxy for the marginal grade (see section 3.2.1), a historical grade-tonnage relationship can be derived. Converting cumulative tonnage observations to radii (equation 3.3), an observed grade-cumulated radii relationship is constructed. Employing discrete values, the grade distribution constant (k) may then be estimated as follows:

$$k=\frac{1}{r_t}.\ln\frac{C_0}{C_t} \qquad (3.4)$$

C_0: maximum grade mined in the initial period
C_t: grade mined at time t
r_t: radius at time t

The minimum grade mined (C_t) is taken from the most recent non-anomalous observation. Anomalous observations could result from mining the shaft pillar[3], using a stock pile of high grade ore or the late exploitation of high grade high cost reefs.

d) Using the estimated grade distribution constant (k) for each observed radius (r_t), a corresponding estimated grade (C_t^*) can be calculated as follows:

$$C_t^* = e^{-k.r_t} \tag{3.5}$$

C_t^*: estimated grade at time t
r_t : radius at time t
k : estimated grade distribution constant

The goodness of fit between the estimated grade distribution for a disk shaped deposit and the observed recovered grades can be tested using an ordinary least squares linear regression and an r^2 correlation co-efficient. Plotting the observed grades against the estimated grades should yield a straight line if the deposit is disk shaped.

3.4) <u>The cost-deposit model</u>

By estimating the grade distribution of a specific deposit, a gold output cost function (cost-deposit model) can be derived. The cost deposit model, with [expected] mining costs and an

[3]The shaft pillar is a stabilising body of rock of a specific circumference that surrounds the shaft. Unexploited high grade reefs immediately surrounding the shaft in conjunction with the incidence of shafts usually being sunk into the high grade regions of a mining lease area result in high recovered grades when it is eventually mined.

[expected] gold price, can be used to determine the life-time size of the economic deposit and total gold content. With depletion the remaining life and gold content can be calculated.

3.4.1) The mining limit

The mining limit is based on the marginal grade and is therefore dependant on the expected costs of gold output (ACE_{exp}[4]) and the expected gold price (P_{exp}), but not the grade distribution of the specific deposit. The grade distribution in conjunction with the mining limit is used to estimate the life-time size of the economic deposit. The set of all marginal grades at various prices can be empirically determined using average mining costs and calculated as follows[5]:

$$C_{ij}=\frac{ACE_i}{P_j} \tag{3.6}$$

C_{ij} : set of marginal grades
ACE_i: vector of expected average costs of mining
P_j : vector of expected gold prices

The mining limit is where the mine exploits the marginal grade. The exact timing when the mine reaches the marginal grade will depend entirely on the grade distribution of the deposit (described by the grade distribution constant) and the rate of mining per time period.

[4]Average costs may reasonably approximate the marginal values because of many time periods and the high grade areas are assumed to be progressively exhausted with cumulated production.

[5]See Appendix 1 for a table of marginal grades that determine the mining limit.

3.4.2) The cost function for gold output

The estimated grade distribution function for a specific deposit is used to derive a cost function for gold output (AC_{cum}) as follows:

$$AC_{cum}=\frac{ACE}{C_{cum}=f(r_{cum})} \tag{3.7}$$

AC_{cum}: cumulative cost function of gold output
C_{cum} : estimated grade distribution function
ACE : estimated mining cost function

A stock effect for the mining cost function can be econometrically estimated using deflated time series analysis and adjusting for capital and working cost (variable cost) elasticities - in conjunction with a changing capital labour ratio - derived from a production function for gold mining. However, this type of analysis is not undertaken in this thesis - the purpose is simply to provide an illustrative guide to an analysis of the future of the various mines and the gold mining industry in South Africa.

3.4.3) The life-time size of the economic deposit

The radius of the life-time size of the economic deposit (r_m) may be expressed as a function of the grade distribution constant (k), the maximum grade (C_0) and the marginal grade (C_m) as follows:

$$r_m=\frac{1}{k}.\ln(\frac{C_0}{C_m}) \tag{3.8}$$

Both the marginal grade and the grade distribution constant are estimated (section 3.4.1 and section 3.3), and the maximum grade is empirically observed.

The radius is then converted into tonnage using equation 2.1 giving the life-time size of the economic deposit in terms of the volume of gold ore (T_m). The average grade (C_{avg}) for the life-time size of the economic deposit is calculated using equation 2.3 and finally the gold content (Q_m) of the economic deposit is simply the amount of gold bearing rock in the economic deposits multiplied by the life-time average grade (equation 2.4).

3.4.4) Depletion

With depletion the remaining size of the economic deposit at any time period is the life-time size of the economic deposit (T_m), defined by the mining limit and the grade distribution, less the cumulated tonnage of gold ore removed (T_{cum}). The remaining life of the deposit, at a constant rate of mining per time period, is the remaining size of the economic deposit divided by the constant rate of mining.

Because high grades are mined first, the grade distribution is bounded by the recovered grade mined in the previous period (C_{t-1}) and the marginal grade (C_m). The average grade (C_{avg}) of the remaining deposit is thus:

$$C_{avg}=\frac{1}{r_m-r_{t-1}}.\ln(\frac{C_{t-1}}{C_m}) \qquad (3.9)$$

r_m-r_{t-1}: size of the remaining economic deposit in terms of radius

The gold contained in the remaining economic deposit is the product of its average grade and the residual economic deposit.

3.5) Conclusion

The available data may be used to analyze the future of the gold mines, and the gold industry, in South Africa. In particular, the historic grade distribution can be estimated and the future grade distribution extrapolated. From the grade distribution, the incremental increase in the cost of recovering gold is calculated. If the expected mining cost and expected gold price are correctly estimated, the size of the economic deposit with depletion, and the potential flow of gold output from the gold mining industry in South Africa, can be predicted.

CHAPTER FOUR

4.1) Introduction

As lower grades are mined with cumulated production an ability to predict future grades, and to construct a cost-deposit model, is obviously an important consideration for the gold mining sector in South Africa. Estimating a grade distribution constant, and therefore a grade distribution, at different levels of production (mine, goldfield and aggregated output) using the available data is a central concern in this chapter. A good fit between the historic fall in recovered grades and the estimated grade decay implies that the disk model is applicable to the mining of gold from the Witwatersrand gold deposit. Further reductions in the recovered grade can therefore be predicted and thus the incremental increase in the cost of recovering gold in relation to the cost of mining may be inferred.

Using available historic data between 1970 and 1992, and applying the disk model, a grade distribution constant (k) - and the resultant grade distribution - is constructed for a gold mine, the Orange Free State goldfield and aggregated data for Chamber of Mines gold mining members of South Africa. A predicted grade distribution is projected into the future and a cost-deposit model is derived using examples of expected mining costs and expected gold prices[1]. The marginal grade, and thus the mining limit, is then calculated for these examples and the remaining life of the mine (goldfield or aggregated output) is deducted for a constant rate of mining.

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[1]A rigorous econometric treatment to include a stock effect and labour policy on the expected mining cost function is not undertaken. Comparative gold price analyses are also not included in the thesis.

Applications of the cost-deposit model are illustrated for: 1) the effect of a change of the expected gold price and a change of expected mining costs on the mining limit and the subsequent increase in the size of the economic deposit for an individual gold mine, 2) the effect of different gold price and mining cost combinations necessary for the continuation of mining, at a constant rate, from the Orange Free State goldfield and 3) a simple application of the cost-deposit model on the effect of the South African gold mining tax formula. The application used at each output level may likewise be applied to the other output levels.

4.2) The Blyvooruitzicht gold mine

Because high grades are mined first, the recovered grade worsens with cumulated mining and the differential between the cost of producing gold ore (ACE) and recovering gold (AC) increases (Figure 4.1). The applicability of the disk model to predict future decay in grade as mining proceeds is tested using the Blyvooruitzicht gold mine. A cost-deposit model is then constructed, from the disk model, and is used to illustrate the effect of an expected increase in the gold price, and an expected decrease of the costs of mining, on the added life of the mine.

4.2.1) The disk model

The estimated grade distribution constant for Blyvooruitzicht is $2*10^{-4}$ with an r^2 correlation between the recorded historic grades and estimated grades of 0.99 (figure 4.2). Extrapolating the Blyvooruitzicht disk model to the year 2000 and using a rate of mining of 2 million tons of gold ore milled per year results in a predicted decline in the flow of gold output from 1.23

million ounces in 1992 to 640 000 ounces in 2000. The predicted total gold output in this period is 7.88 million ounces of fine gold.

4.2.2) The cost-deposit model

At a constant average cost of mining (R120 per ton of gold ore) and using the example of an expected increase in the gold price from R1000 per ounce to R1100 per ounce the marginal grade at the mining limit is decreased from 3.73 g/t to 3.39 g/t[2]. The mining limit is moved forward and the economic deposit is expanded by 2.4 million tons of gold ore. At a rate of mining of 2 million tons per year the life of the mine is increased by one year and two months (figure 4.3). The total predicted increase in gold output is 265 000 ounces of fine gold with a value of R291.5 million at R1100 per ounce. The export value in dollars, assuming an exchange rate of $0.35 per rand, is $102 million.

At a gold price of R1000 per ounce, an expected real decrease in the cost per ton of gold ore milled from R120 per ton to R100 per ton will lower the marginal grade from 3.73 g/t to 3.11 g/t. At a rate of mining of 2 million tons of gold ore per year, the size of the economic deposit is increased by 4.5 million tons and the life of the mine will be increased by two years and three months (figure 4.4). Predicted total gold output will increase by 483 000 ounces of fine gold with a dollar value of $169 million (R1000 ounce and an exchange rate of $0.35 per rand). Total saving in costs over the extended period is R90 million.

The cost-deposit model, based on the disk model, is a useful tool to isolate and analyze the effect of different economic variables on the continued existence and the flow of gold from

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[2]Appendix 1 provides a table of marginal grades at various gold prices and mining costs.

South African gold mines. The cost of recovering the gold (AC or MC) in relation to the gold price is important because it determines the profitability of the firm (figure 4.5). However, the cost of mining the gold ore (ACE or MCE) is particularly relevant because the grade mined [exogenous quality constraint] in conjunction with this cost controls the cost of recovering the gold [cost-deposit model].

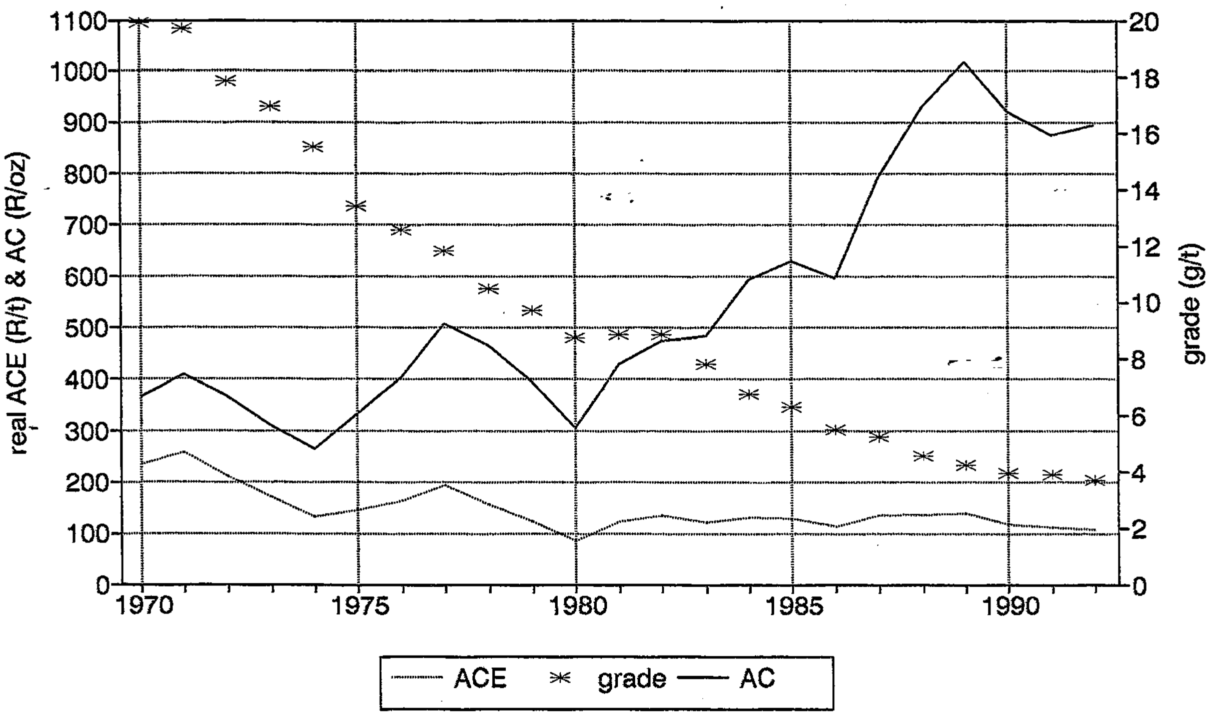

Figure 4.1: Time series of real average costs and grade from the Blyvooruitzicht gold mine.
Data source: COMSA 1970-1992, SARB 1993.

Figure 4.1 illustrates the relationship between mining costs (ACE), gold production cost (AC) and grade (C). The data for recovered grades after milling and treating is recorded in grams per ton of gold ore, the average costs of mining (ACE) in rand per thousand tons of gold ore milled, and the average cost of recovering gold in rand per kilogram (COMSA,1970-1992) such that:

$$C_t(g/t)=\frac{AC_t(R/kg)}{ACE_t(R/000t)}$$

Cost data are adjusted to real values using a general mining deflator (SARB,1993). The ACE is illustrated in figure 4.1 in rand per ton and AC is converted from rand per kilogram to rand per ounce as follows:

$$ACE_t(R/oz)=\frac{ACE_t(R/kg)}{32.15103}$$

As the recovered grades decline with continued mining, the differential between the cost of producing gold bearing rock (ACE) and recovering the gold (AC) rises in proportion to the decay in grade.

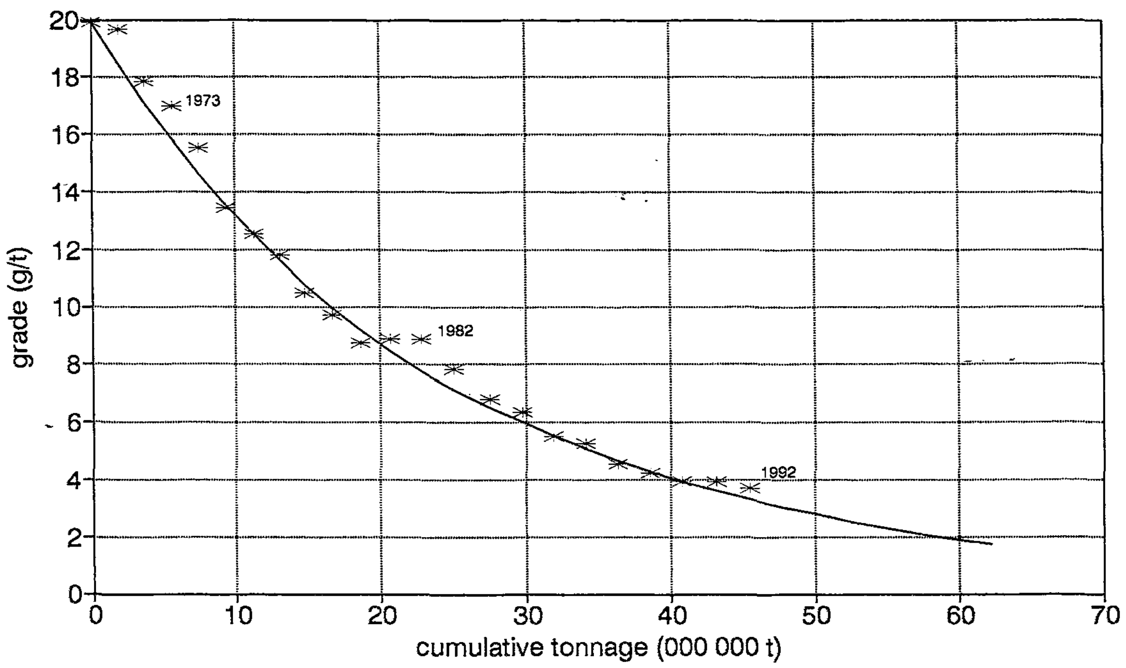

Figure 4.2: The disk model for the Blyvooruitzicht gold mine.
Data source: COMSA 1970-1992.

The disk model for the Blyvooruitzicht gold mine is illustrated in figure 4.2. It is represented by the line and may be written as follows:

$$C_t = e^{-0.0002 * r_t}$$

The radius at some time period t (r_t) is a function of the cumulated tonnage at the same time period (T_{cum}) as follows:

$$r_t = \sqrt{T_{cum} . \pi . d . h .}$$

Observed recovered grade is represented by an asterick. The density of the data points reflect the rate of mining per year at approximately 2 million tons per year.

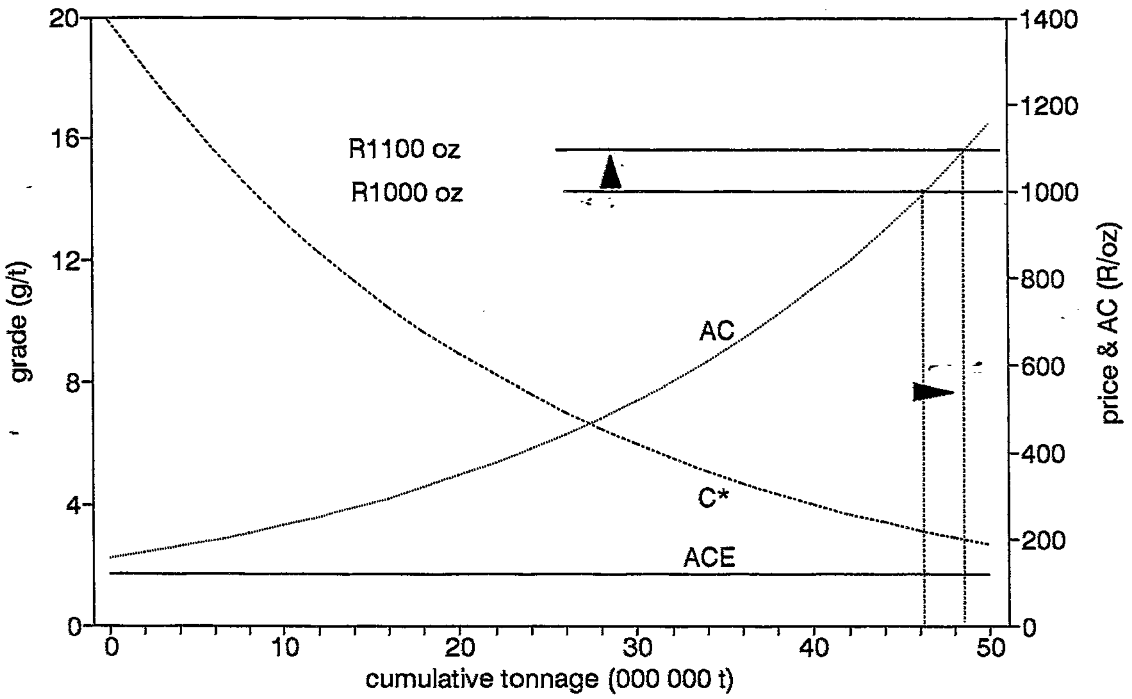

Figure 4.3: The effect of an expected increase of the gold price on the Blyvooruitzicht gold mine.

The cost deposit model for Blyvooruitzicht gold mine is constructed from the grade distribution estimated by the disk model. The average cost of mining (ACE) is held constant and the average cost of recovering gold (AC) increases proportionally to the predicted decay in grade (C^*). Using an example of R1000 per ounce for the expected gold price and an average cost of mining at R120 rand per ton milled and treated, the mining limit is set at a marginal grade equal to 3.73 g/t. For an expected gold price increase to R1100 per ounce, the mining limit is expanded (marginal grade falls) and the size of the economic deposit is increased.

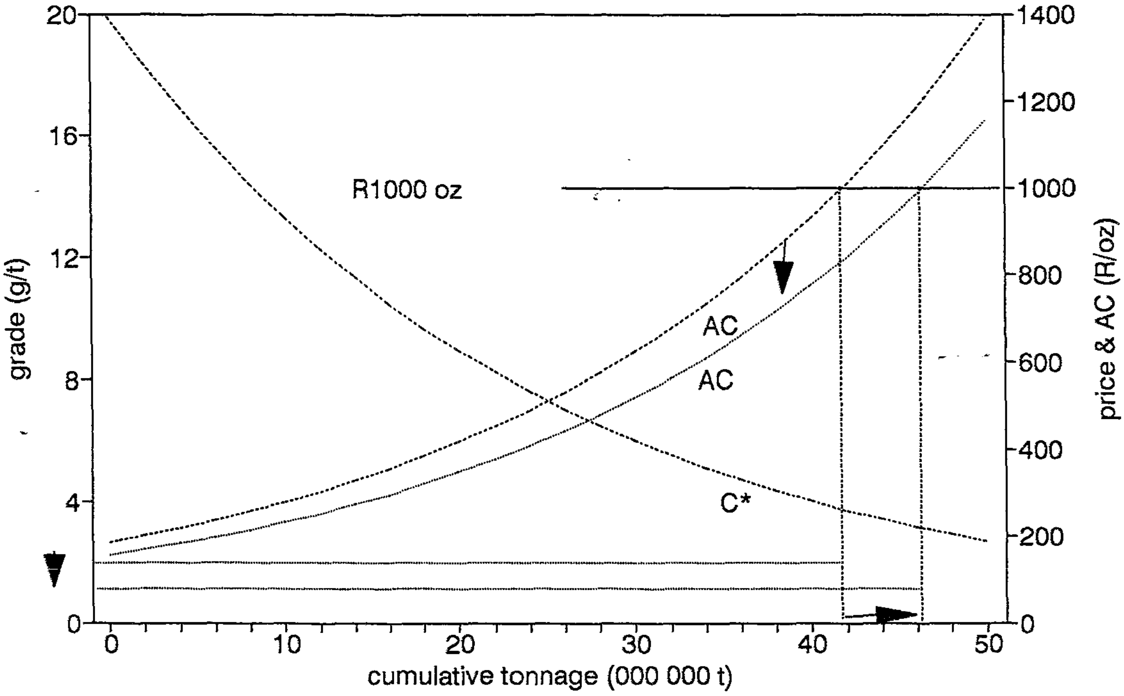

Figure 4.4: The effect of an expected decrease of mining costs on the Blyvooruitzicht gold mine.

An expected fall in the average cost of mining (ACE) shifts the average cost of recovering gold curve (AC) down in proportion to the change. The mining limit is increased (lower marginal grade) and the size of the economic deposit is expanded.

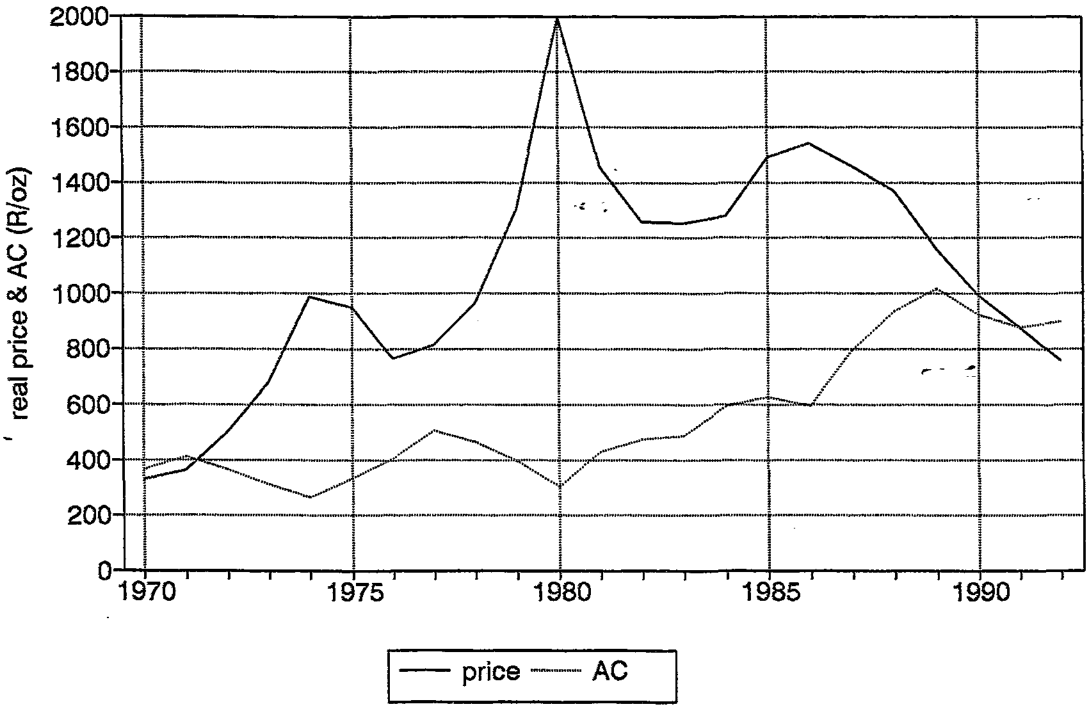

Figure 4.5: Time series of real average costs of recovering gold from the Blyvooruitzicht gold mine and the real domestic gold price.
Data Source: COMSA 1970-1992, Gold '93, SARB 1993.

The gold price (Gold,1993) is deflated using a GDE deflator (SARB,1993) and the average costs of recovering gold (AC) are converted to rand per ounce from Chamber of Mines data (COMSA,1970-1992) and adjusted to real values using the general mining deflator (SARB,1993). With reference to figure 4.1: The AC has increased in response to declining recovered grades and adjusted to the changing costs of mining (figure 4.1). Mining rent is the differential between the gold price and the cost of recovering the gold. The mine reaches its mining limit, at a particular cost of mining, when the gold price is equal to the cost of recovering gold.

$$AC_m = P$$

Because AC is a function of the cost of mining (ACE) and grade (C), rearranging the equation above yields the marginal grade at the mining limit as follows:

$$C_m = \frac{ACE_m}{P}$$

Figure 4.5 shows that the gold mine became marginal (reached its mining limit) in 1991 (AC=P) and would require state assistance, or a ring-fencing arrangement, to continue producing gold.

4.3) The Orange Free State goldfield

Aggregated data is used in this section to test the applicability of the disk model to the Orange Free State goldfield. Although various reasons are offered, the data suggests that the model might be incorrect particularly from 1986 onwards. Assuming that the disk model is correct, however, the effect of an increasing cost of recovering gold - as lower and lower grades are mined - on the continued existence of the Orange Free State goldfield, at its current rate of mining, is illustrated.

Data from the Virginia gold mine is not included in the aggregated figures because the mine consolidated with Harmony gold mine in 1973 (COMSA, 1974) which is not part of the Orange Free State goldfield. The Anglo American OFS Joint Metallurgical Production Scheme, and subsequently Free State Consolidated Metallurgical Scheme, is excluded from gross gold output because it consists of surface mine dump reworking operations. The consolidation of various mines throughout this period is taken into account by aggregating their gold ore output and gold output. Two major mining companies (Free State Consolidated, south region, and Free State Consolidated, north region) dominated production from 1986 onwards[3]. An average grade for the goldfield is constructed by weighting average grades from individual mines by their level of gold ore output. The aggregated average grade is verified simply by dividing aggregated gold output by aggregated gold ore output.

[3]See Appendix 2.

4.3.1) The disk model

The estimated grade distribution constant for the Orange Free State goldfield is $3.79*10^{-3}$ with an r^2 correlation between recorded historic grades and estimated grades of 0.98 (figure 4.6). The predicted grade distribution may however be unreliable because the data suggests a flattening out of the recovered grades after 1986 which is not reflected in the disk model.

No new principal gold mines have opened in the Orange Free State goldfield since 1961 (COMSA,1992:11). However, new shafts on the individual mines, improved mining methods and development of new higher grade reefs may have occurred. The aggregated data since the formation of the two dominant Consolidated Free State mining companies in 1986 indicates an increased rate of mining by 2 million tons of gold ore per year that coincides with a flattening out of the recovered grades.

Recovered grades that are higher than the predicted grades, particularly from aggregated data, could result from a number of factors. Once the area that is serviced, decreasing real costs, for example, through labour shedding (Viruly,1994), might result in higher grade ore at a previously unprofitable depth being mined due to the opening up of a new shaft or sub-level shaft[4]. Improved metallurgical techniques or mining the shaft pillar from an economically exhausted shaft (the area that is serviced by the shaft has exceeded its mining limit) could also result in higher than predicted grades.

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[4]A sub-level shaft is sunk from some depth below the surface and linked to a development tunnel. Ore at the deeper level is hauled to a shallower level and then transported to the principal shaft where it is hauled to the surface for milling and treatment.

Ring-fencing tax provisions in South Africa allow profitable mines to combine their taxable earnings with unprofitable mines enabling a mine that has exceeded its mining limit to continue producing. The provision could thus enable the mining of high grade but unprofitably deep reefs resulting in a higher than predicted average grade for the combined data.

The existence of many reefs at different depths has the potential to complicate the predictive capability of the disk model particularly in the face of decreasing real mining costs, technological advance and ring-fencing provisions. If a late exploitation of higher grade reefs occurs a flattening out of the grade distribution will result.

4.3.2) The cost-deposit model

Assuming that the disk model for the Orange free State goldfield is valid, requisite mining cost and price combinations can be calculated that will ensure the continued existence of the Orange Free State goldfield at its current rate of mining (no mines or shafts open or close).

The gold price must increase [or the cost of mining must decrease] to compensate for the reduction in recovered grades and if gold ore is removed from the ground at a rate of 25 million tons per year, the disk model may be used to predict the intertemporal decay in grade with cumulated production. The ratio of the expected mining cost to the expected gold price provides a marginal grade at the mining limit[5]. For production to continue at a constant rate the mining limit cannot be exceeded. Combinations of necessary gold prices, at various costs,

[5]See Appendix 1.

at the mining limit dictated by the intertemporal decay in grade are illustrated in table 4.1.

If the US dollar gold price does not increase sufficiently, a deteriorating rand/US dollar exchange rate can maintain the profitability of the mines. A time series showing the recovered grades from the Orange Free State goldfield, and aggregated and averaged South African grades, compared to the rand/US dollar gold exchange rate is illustrated in figure 4.7. Although no causation is implied, the declining value of the rand has been fortuitous to the gold mining industry in South Africa by inflating the domestic gold price. This, in combination with declining real costs of mining due to labour shedding (Viruly,1994), a low import propensity for the gold mines (13.6% for gold mining and 17.8% for manufacturing) and a high output to input value added for gold mining (237% for inputs in gold mining and 41.9% in manufacturing) (Rustomjee,1993:6-8), has expanded the size of the economic deposit. These factors enabled the continued existence of many marginal mines in South Africa resulting in a considerable earning in foreign exchange.

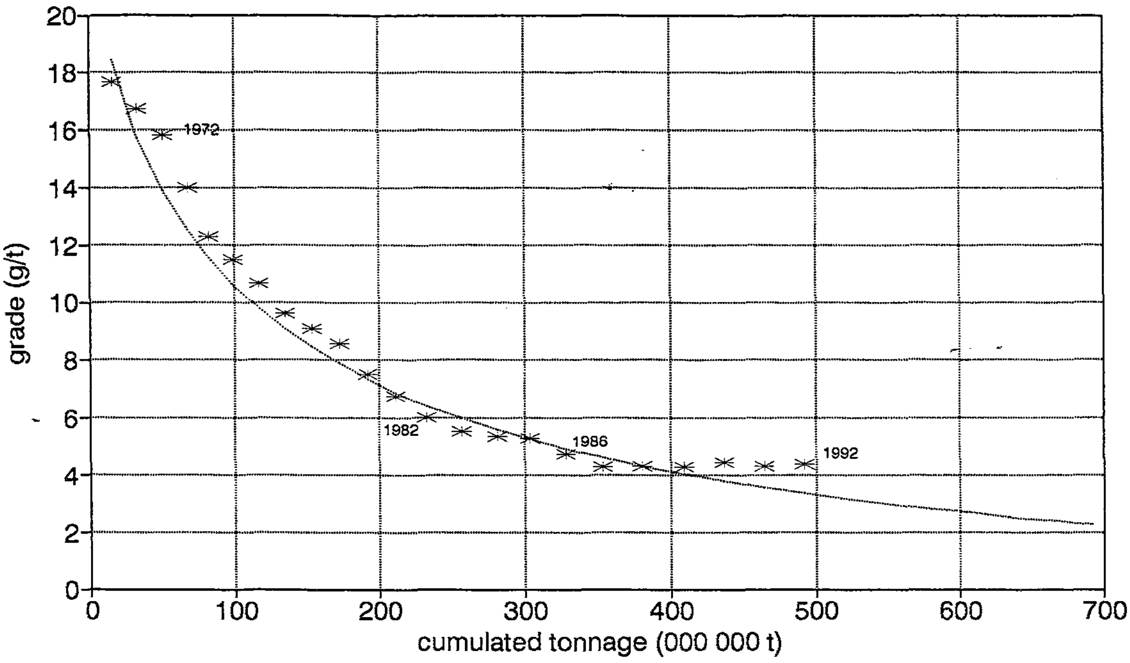

Figure 4.6: The disk model for the Orange Free State goldfield.
Data source: COMSA 1970-1992.

The function for the disk model is written as follows:

$$C^* = e^{-0.00379*r}$$

The recorded historic average grades flatten out after 1986 and are higher than the grades predicted by the disk model. This occurrence coincides with the conglomeration of most of the gold mines on the Orange Free State goldfield into two large consolidated companies and might be the result of the late exploitation of high grade reefs.

Table 4.1: Predicted gold prices necessary for the continued exploitation of the Orange Free State goldfield at a constant rate of mining.

	1995 2.87 g/t	1996 2.74 g/t	1997 2.61 g/t	1998 2.49 g/t	1999 2.38 g/t	2000 2.27 g/t
R100/t	R1083.74	R1135.15	R1191.69	R1249.12	R1306.86	R1370.19
R110/t	R1192.11	R1248.67	R1310.86	R1374.04	R1437.54	R1507.20
R120/t	R1300.48	R1362.18	R1430.03	R1498.95	R1568.23	R1644.22
R140/t	R1517.23	R1589.21	R1668.37	R1748.77	R1829.60	R1918.26
R160/t	R1733.98	R1816.25	R1906.71	R1998.60	R2090.97	R2192.30

The predicted decay in grade, from the disk model of the Orange Free State goldfield, with cumulative mining, is tabulated beneath the respective years on table 4.1. The gold price (P_m), in rand per ounce, that is necessary for the predicted grade (C^*) to equal to marginal grade (C_m)at each date is calculated for various costs of producing gold bearing rock (ACE) as follows:

$$P_m(R/oz)=\frac{ACE(R/t)}{C^*(g/t)=C_m(g/t)}*\frac{1000}{32.15103}$$

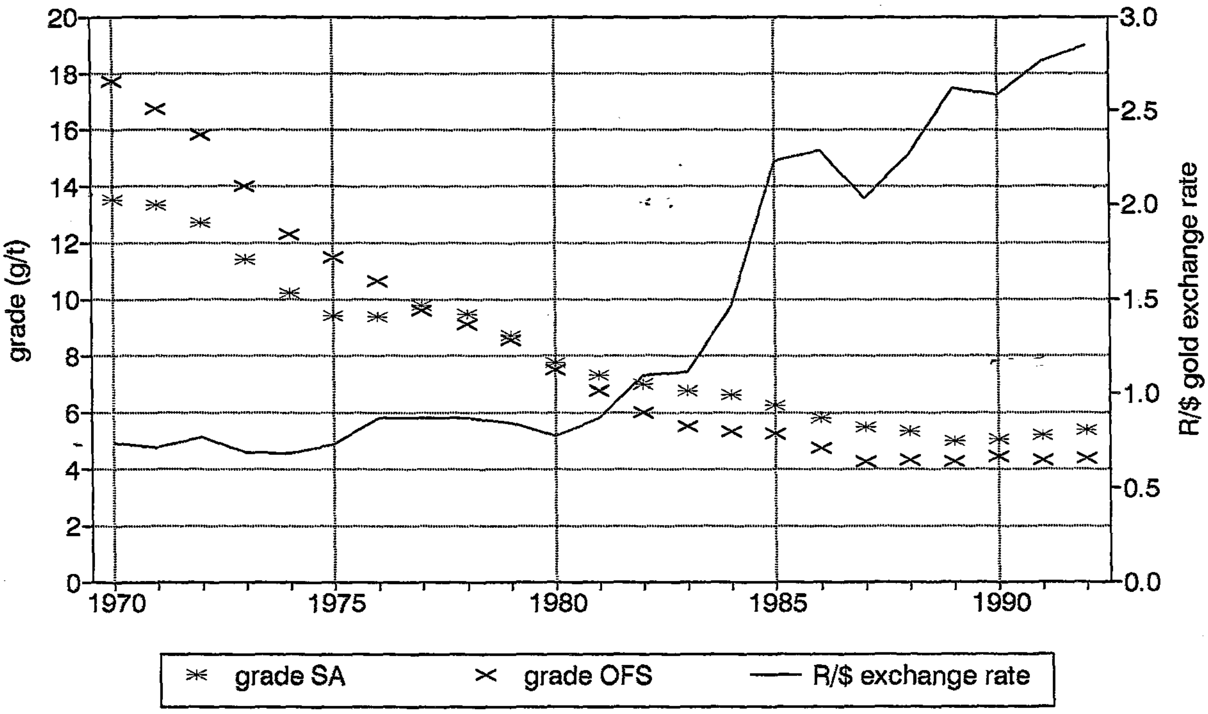

Figure 4.7: Time series of aggregated average grades and the rand/US dollar gold exchange rate.
Data source: COMSA 1970-1992, Gold,1993 .

The rand US dollar gold exchange rate is determined by dividing the yearly average rand price for gold (Gold,1993) by the yearly average US dollar gold price (Gold,1993). The time series displays a deteriorating exchange rate as the recovered grade falls. Particularly after the early 1980's the dollar gold price remained relatively stable (see Appendix 2, part 1), but a deteriorating exchange rate inflated the domestic price of gold thus expanding the economic [Orange Free State and Witwatersrand] gold deposit.

4.4) The Witwatersrand gold deposit

The central concern in this section is to analyze the effect - using a cost-deposit model based on the available data between 1970 and 1992 - of the South African gold mining tax formula on the size of the economic deposit.

4.4.1) The disk model

Analytical problems similar to those for the Orange Free State goldfield occur for the aggregated data of the Witwatersrand gold deposit. The estimated grade distribution constant for the aggregated data from Chamber of Mines of South Africa gold mining members is $3.187*10^{-2}$ with an r^2 correlation coefficient between historic grades and estimated grades of 0.92 (figure 4.8).

An average of 100 million tons of gold ore is produced each year[6]. As in the previous sections the life of the gold industry can be estimated. However, it is important to note that as the grade decays, or costs of extraction increase, from mine to mine, certain mines may close down and if the domestic gold price increases sufficiently some mines, or shafts, may open. When a mine closes total gold output decreases but the industry adjusts to a slower rate of mining and the life of the industry may not necessarily be decreased.

[6]The Orange Free State goldfield produces about 25 million tons of gold ore per year.

4.4.2) The South African gold mining formula tax

The major issues surrounding the formula tax are the admissible expenditure deductions used to calculate taxable income, the marginal rate and the depletion allowance (tax tunnel). In this section a very broad overview of the South African gold formula tax will be described. The cost-deposit model for the aggregated data is used to illustrate the effect of the tax on potential gold output from South African mines.

The tax is a progressive revenue (rent) tax based on the following structure[7]:

$$y = a - ab/x \qquad (4.1)$$

y: tax rate to be determined (average rate)
a: the marginal tax rate
b: the portion of tax free revenue (tax tunnel)
x: the ratio of taxable income to revenue

The conventional interpretation of the formula is that "... it imposes a tax of a%, but only to the extent that the mine's taxable income from gold mining exceeds an amount equal to b% of the mine's total income [revenue]..." (Van Blerck:1992,8-3).

The formula factor values (marginal rate and tax tunnel) have been adjusted since the inception of the gold mining formula tax in 1947 (Van Blerck,1992:C5). The Marais Committee (1988:1.10) recommends that the values be standardised (the 'b' factor or 'tax tunnel' at a constant 5% and the 'a' factor or marginal rate at 61%) and phased in to take effect from 1995. The collections, under these factors, are expected to generate approximately the same revenue as a flat 50% rate applicable to normal company tax (Van Blerck,1992:C6-C15).

[7]The lease formula is identical in construction, but it will be phased out by 1995 (Marais Committee,1988:1.8).

Although the calculation of taxable income from gold mining is complicated[8] some generalizations may be drawn:

1) The revenue from mining is the total income from the sale of gold recovered from South African ground (source basis).

2) The determination of taxable income from gold mining is in accordance with general tax principles regarding admissible expenditure deductions[9], but with some modifications. These modifications are explained in detail by Van Blerck (1992;6.2-6.3). However, the more important ones to the gold mining industry are:

a) "Capital redemption deduction[s] and capital allowance[s] are claimable before the mine commences production [development stage - shaft sinking and establishing access to the ore body from the shaft]" (Van Blerck,1992;12.3), which amounts to an important accelerated depreciation provision. These provisions provide a necessary incentive for the opening up of new gold mines which requires a capital expenditure of approximately R2.5 billion in July 1993 prices (Dowsley,1993).

b) Ring-fencing provisions are the " isolation for tax purposes of certain types of activities, income or losses" (Van Blerck,1992;15.2). In essence, ring-fencing allows profits from one mine to be offset against losses from another, but does not allow loss offset between mining and non-mining activity[10]. In other words, a ring-fence allows mines to operate beyond their mining limit and make a loss because they can be subsidised by mines still making a profit. This provision therefore extends gold output from South Africa beyond an

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[8]See Van Blerck (1992) for a complete accounting treatment of mining tax.

[9]Mining expenditure is primarily concerned with non-capital expenditure accrued after prospecting and the preliminary stages of mine development are over (Van Blerck,1992;11.2).

[10]A mining activity ring-fence of sorts does however exist: "...non-mining tax losses may reduce mining income, and mining losses caused by working losses may reduce non-mining income." (Van Blerck,1992:15.4).

economically determined limit, but at the expense of government revenue collections. The benefit of these provisions from added foreign exchange earnings due to the extra gold output should be weighed against the internal domestic cost of protecting the industry and loss of government revenue.

The 'b' factor or 'tax tunnel' has been called a depletion allowance by many authors [Kotze (1933); Krige (1968); Bason & Varon (1977); Krige (1979); Krige (1983); Marais (1988) and Van Blerck (1992)]. Dasgupta and Heal (1979;368) justify a depletion allowance on the grounds that "... the depletion of reserves is a depreciation of the company's capital assets [stock of ore in the ground] and should thus be tax deductible" and implemented at every stage of mining. The tax tunnel reduces the marginal rate by b% of total income, in effect treating b% of total income as an allowance equivalent to the depletion of the mines reserves.

When the mine is in a tax paying position, the tax adds to the costs of mining (ACE + tax), and therefore the cost of recovering gold (AC + tax), but at a declining rate proportional to the rate of decay in grade, that is, it changes the shape of the ACE and AC curves (figure 4.9) depending on the marginal tax rate and the depletion allowance.

If revenue is the quantity of gold (Q) multiplied by the domestic price of gold (P) and taxable income is revenue less tax deductible costs (ACE/C) the value of the 'x' factor is:

$$x = \frac{(P*Q)}{(P*Q) - (\frac{ACE}{C})} \tag{4.2}$$

As the grade decays (C decreases), holding ACE constant, the value of the 'x' factor (relative profitability) decreases and the proportion of tax collected to total revenue declines. Taxable income becomes zero before the deposit is exhausted, when the 'x' factor is equal to b%, allowing the full extent of the economic deposit to be exploited - the mining limit is unaffected by the imposition of the tax. The South African gold mining formula tax is like a tax on rent with a depletion allowance.

The distribution of revenue from gold sales is divided between government revenue [tax], costs, depletion allowance and net profit. With cumulated production and decreasing recovered grades the total share accruing to costs rises, and the respective share accruing to government and the firm fall. If the depletion allowance is viewed as the difference between the profit calculated at a marginal rate of a% with no depletion allowance and the profit calculated at a marginal rate of a% with a depletion allowance of b% according to the formula tax, the size of the depletion allowance increases with cumulated production (figure 4.10).

The depletion allowance (tax tunnel) also reduces risk by providing a buffer between a changing size of the economic deposit due to domestic gold price fluctuations and domestic costs because it exempts the mine from paying tax at low levels of relative profitability. For example, without the depletion allowance a fluctuation in the domestic price might result in mine closure of existing marginal mines and some gold output lost to society forever. The size of the depletion allowance should therefore reflect the risk associated with domestic gold price fluctuations. It should, for example, approximate the medium to long run variance in the domestic gold price.

An accelerated depreciation allowance provides an incentive for opening new deep-level gold mines in South Africa. Once mining has begun and capital expenditure has been depreciated, the gold mining tax formula is like a tax on rent but with a depletion allowance. Rents are taxed away at a rate determined by a combination of the depletion allowance and the marginal rate. Finally, toward the end of the life of the mine, the depletion allowance works to exempt the mine from paying taxes, thus leaving the mining limit unaffected. The size of the economic deposit, and therefore total gold output, is not affected by the tax. Ring-fencing provisions, however, might expand gold output beyond the mining limit but at the expense of government revenue collection.

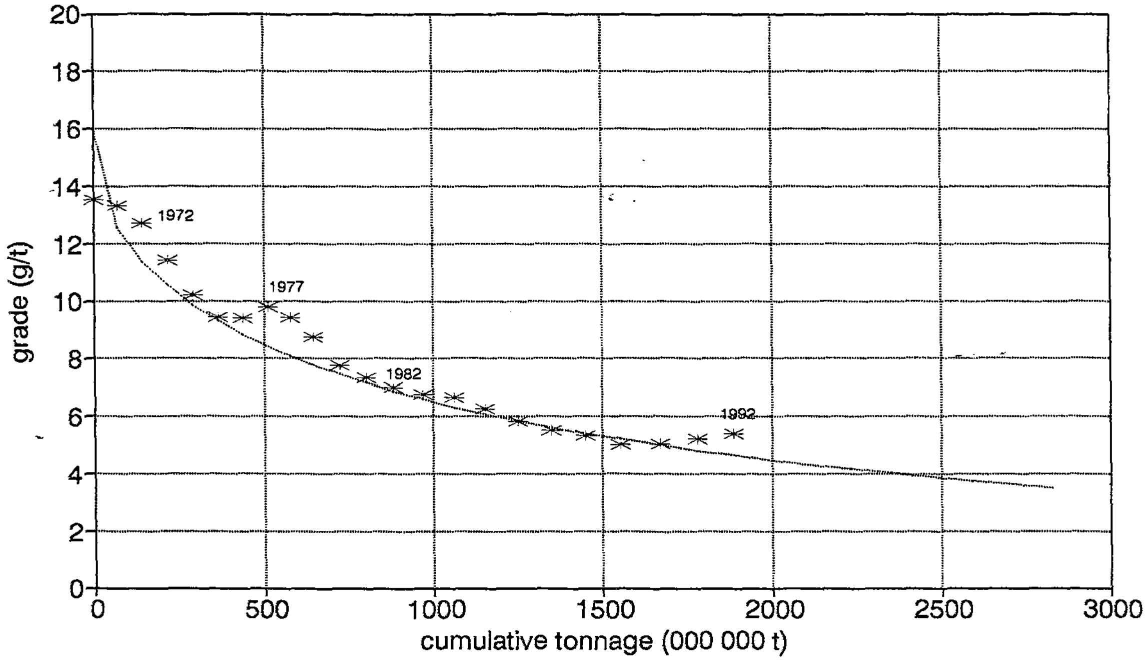

Figure 4.8: The disk model for the Witwatersrand gold deposit.
Data source: COMSA 1970-1992.

The function for the disk model is written as follows:

$$C^* = e^{-0.03187 * r}$$

The higher than predicted grades recovered from 1991 are probably a result of the late exploitation of high cost high grade reefs.

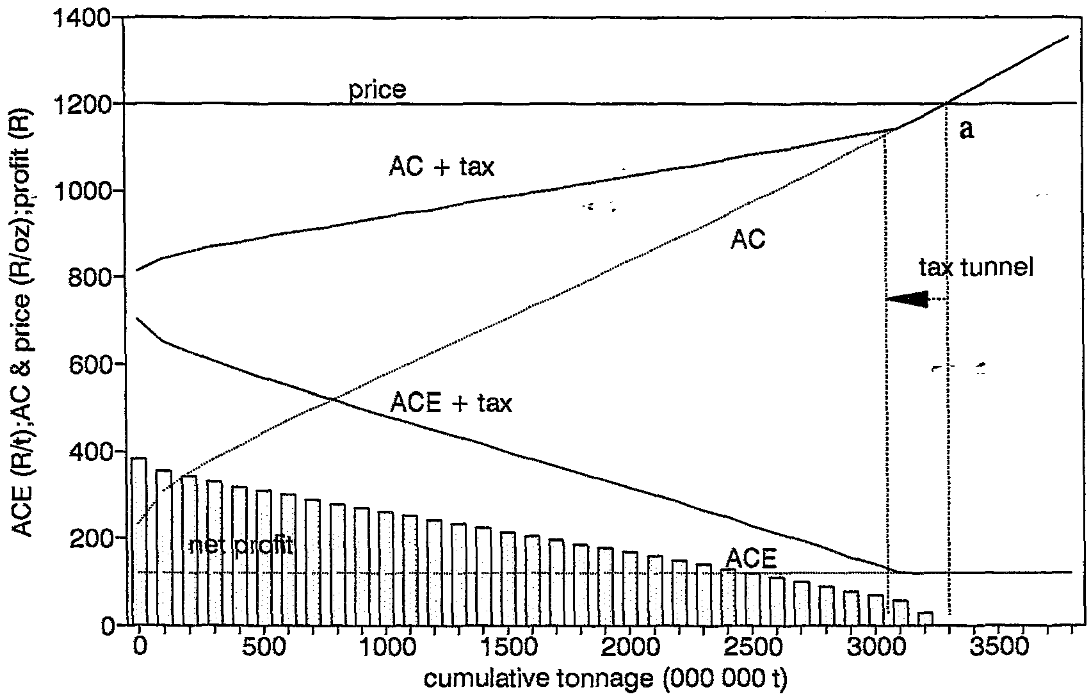

Figure 4.9: The effect of the gold mining tax formula, with cumulated production, on the life of the economic deposit.

Without a tax the average cost of producing gold ore (ACE) is assumed constant for illustrative purposes, and the average cost of recovering gold (AC) increases in proportion to the decay in grade as mining continues. The mining limit is determined where the AC is equal to the gold price (point a). The imposition of a tax adds to the cost of producing gold ore and therefore also gold; i.e. it changes the slope of the ACE and AC functions. By including a depletion allowance the marginal rate is decreased and the firm stops paying tax at a level of relative profitability equal to the percentage depletion allowance determined (tax tunnel).

A constant gold price of R1200 per ounce, and a constant average mining cost of R120 per ton, is assumed for the entire intertemporal mining path in Figure 4.9. The 'x' factor at some time period t for each ounce of gold sold, is calculated as follows:

$$x_t = \frac{R1200*1}{(R1200*1)-(\frac{R120}{C_t^*}.\frac{1000}{35.15103})}$$

The average cost of recovering gold is converted from rand per gram to rand per ounce. As the recovered grade decreases with cumulative mining the 'x' factor also falls. The average rate (y) is calculated using the formula factor values 61% and 5% for the marginal rate ('a' factor) and the depletion allowance ('b' factor) respectively:

$$y = 61 - (61*5)/x$$

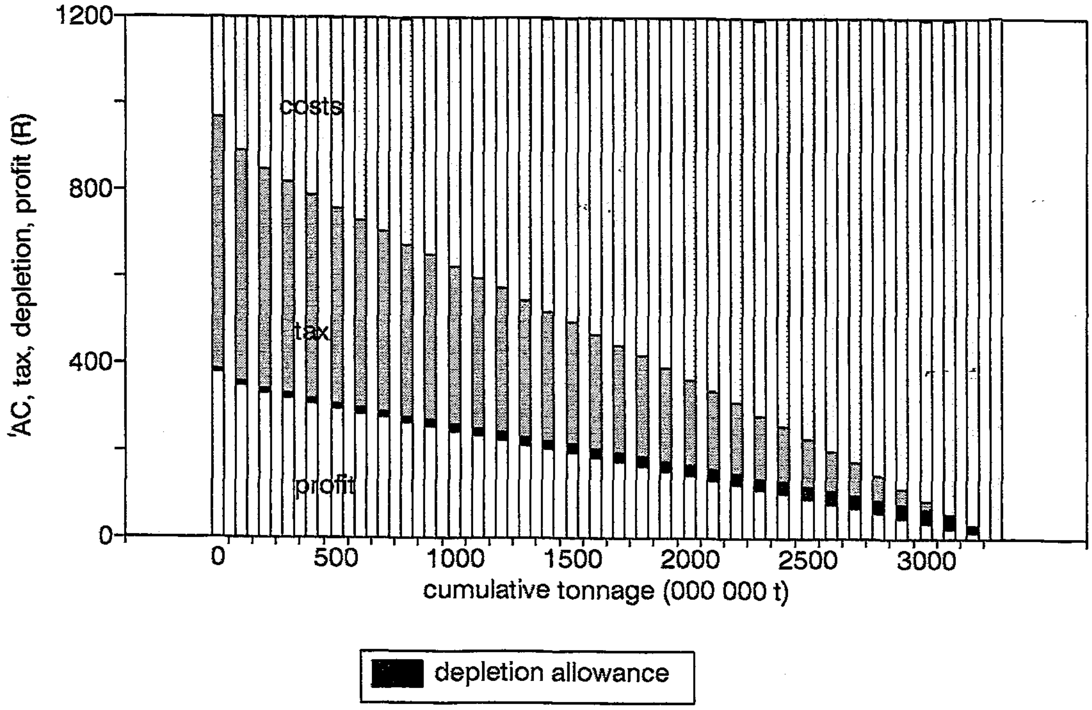

Figure 4.10: The distribution of gold sales revenue from gold mining in South Africa.

Once the mine is in a tax paying position, gold mining revenue from gold sales is distributed between costs, tax, depletion and profit. The share accruing to costs increases with cumulated production as the grade recovered declines. The tax share is governed by the gold mining tax formula and the applicable formula factor values. The remainder after costs and tax is divided into a depletion allowance and profit or rent. The depletion allowance out of net profit (depletion and rent) may be calculated by taking the differential between a tax without a depletion allowance and a tax with a depletion allowance in the following way:

If 'a' is the marginal rate (61%) and 'b' the depletion allowance (5%), the average rate (y^*) without a depletion allowance is:

$$y^* = 0.61 * (\text{revenue - cost})$$

The gold mining formula tax includes a depletion allowance and the average rate (y) is calculated as follows:

$$y = 61 - (305)/x$$

The value of the depletion allowance (V_d) is thus:

$$V_d = y^* - y$$

Because the 'x' factor decreases with lower recovered grades and cumulated production, the value of the depletion allowance must increase as the mine progressively depletes its capital stock (ore). The value of the depletion allowance using a constant gold price of R1200 and a constant cost of mining of R120 per ton is illustrated on the above figure.

4.5) Conclusion

A good fit between recorded grades and expected grades, from the disk model, is evident (r^2 values of 0.99, 0.98, and 0.92 for Blyvooruitzicht, Orange Free State goldfield and the Witwatersrand gold deposit respectively). However, for the aggregated data, recorded grades fall less rapidly at the tail than the grades expected from the disk model. A number of reasons for this occurrence are offered but a decomposition of the available data from other data (collected on a mine to mine basis), and extending the time series, is necessary to prove conclusively the validity and robustness of the applicability of a disk model to predict the fall in recovered grades as mining continues into the future.

The cost-deposit model, based on the disk model, is however a useful tool to isolate and analyze the effect that certain economic variables have on the gold mining industry in South Africa. The deposit-specific characteristics captured by the grade distribution constant combined with the gold price, the cost of mining and the exchange rate influence the mining limit and thus the size of the economic deposit. The gold mining tax formula does not affect the size of the economic deposit and thus has no associated excess burden.

CONCLUSION

The received literature does not make an analytical distinction between oil and metalliferous deposits. The difference is, however, central to the thesis because the rate of gold output cannot be linked to the rate of mining gold ore, and therefore the rate of depletion, without including grade into the analysis. Similarly, the rate of mining does not determine the rate of gold output unless grade is factored into the equation.

The thesis provides an analytical framework and a deposit model (disk model) used to predict the flow of gold from the South African gold mines. A simple view of the gold mining firm is adopted. It is a normal firm producing gold bearing rock faced with, firstly, a stock effect as the depth of mining increases and the working face moves further from the shaft with depletion and, secondly, an increasingly severe quality constraint (declining grade) as mining continues. An econometric analysis of the stock effect on the costs of mining is not approached; rather the thesis focuses on the influence of the quality constraint (grade) on the cost of recovering gold with cumulated mining.

The disk model is tested against historic data for the Blyvooruitzicht mine, the Orange Free State goldfield and aggregated Witwatersrand deposit data. A good fit between the data and the model is evident but the predictive capacity is brought into question because of the occurrence of higher than predicted average grades at the tail. Some reasons are offered but the data set needs to be extended backwards in time, and de-aggregated probably to shaft level in some cases, before any conclusive statements regarding the robustness of the disk-model can be implied.

The method of analysis does however provide a good framework for analyzing the effect of economic criteria, price and costs, on the continued existence of a gold mine and gold mining sector in general. For example, the incidence of a rising real wage on the continued existence and the reduction in the size of the economic deposit can be isolated. The effect of a worsening exchange rate, and a higher domestic gold price, on the flow of gold from the South African gold mines can be analyzed. The gold mining tax formula depletion allowance works in a such way that the life of the mine or industry is not affected by the imposition of the tax.

Although the results from the study are not conclusive it is clear that grade has an important influence on the costs of mining gold and, after Stollery (1982), should be treated separately from the normal stock effect. A rigorous treatment of the time dynamic is an area for further work. The firm's discount rate and expected revenue flows relative to gold flow, which includes an analysis of price, may be formally included into the model. The demand curve facing aggregated primary gold producers and the influence of primary gold production from South Africa on the world price may be analyzed with respect to an increasingly severe quality constraint.

APPENDIX 1: MARGINAL GRADES

The mining limit is expressed in terms of a marginal grade which is the minimum grade that a mine can exploit (cut-off grade or pay-limit grade). The marginal grade (C_m) is determined where the marginal cost of extracting a gram of gold from the ore (MC_m) at the mining limit is equal to the price received for that gram of gold (P_m).

$$P_m = MC_m$$

$$P_m = \frac{MCE_m}{C_m}$$

$$C_m = \frac{MCE_m}{P_m} \qquad \text{(A1.1)}$$

where:

P_m : the expected price of gold at the mining limit
MC_m : the marginal cost of recovering gold at the mining limit
MCE_m : the marginal cost of mining gold ore at the mining limit
C_m : the marginal grade

The set of all marginal grades at various prices and mining costs can be calculated as follows:

$$C_{ij} = \frac{MCE_i}{P_j} \qquad \text{(A1.2)}$$

where:

C_{ij} : set of marginal grades
MCE_i : vector of marginal costs of mining
P_j : vector of gold prices

The set of marginal grades (in grams per ton) at various gold prices (adjusted to rand per fine ounce) and mining costs (in rand per ton) are calculated and presented in Table A1.1. The current average cost of mining (ACE) published for all member mines by the Chamber of Mines, or an expected average cost of mining, together with an expected gold price, can be used to determine the mining limit in terms of a marginal grade.

If the grade distribution constant and a marginal grade is known the life-time size of the economic deposit and total gold content can be calculated (section 3.4.3). With depletion a change of the remaining size of the economic deposit due to an expected change in the costs of mining or the expected gold price, may also be calculated (section 3.4.4).

Table A1.1: Table of marginal grades

Cost per ton of gold ore milled (R/t)

Gold price (R/oz)	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175	180	185	190	195	200
900	3.46	3.63	3.80	3.97	4.15	4.32	4.49	4.67	4.84	5.01	5.18	5.36	5.53	5.70	5.88	6.05	6.22	6.39	6.57	6.74	6.91
920	3.38	3.55	3.72	3.89	4.06	4.23	4.40	4.56	4.73	4.90	5.07	5.24	5.41	5.58	5.75	5.92	6.09	6.25	6.42	6.59	6.76
940	3.31	3.47	3.64	3.81	3.97	4.14	4.30	4.47	4.63	4.80	4.96	5.13	5.29	5.46	5.63	5.79	5.96	6.12	6.29	6.45	6.62
960	3.24	3.40	3.56	3.73	3.89	4.05	4.21	4.37	4.54	4.70	4.86	5.02	5.18	5.35	5.51	5.67	5.83	5.99	6.16	6.32	6.48
980	3.17	3.33	3.49	3.65	3.81	3.97	4.13	4.28	4.44	4.60	4.76	4.92	5.08	5.24	5.40	5.55	5.71	5.87	6.03	6.19	6.35
1000	3.11	3.27	3.42	3.58	3.73	3.89	4.04	4.20	4.35	4.51	4.67	4.82	4.98	5.13	5.29	5.44	5.60	5.75	5.91	6.07	6.22
1020	3.05	3.20	3.35	3.51	3.66	3.81	3.96	4.12	4.27	4.42	4.57	4.73	4.88	5.03	5.18	5.34	5.49	5.64	5.79	5.95	6.10
1040	2.99	3.14	3.29	3.44	3.59	3.74	3.89	4.04	4.19	4.34	4.49	4.64	4.79	4.93	5.08	5.23	5.38	5.53	5.68	5.83	5.98
1060	2.93	3.08	3.23	3.37	3.52	3.67	3.81	3.96	4.11	4.25	4.40	4.55	4.69	4.84	4.99	5.13	5.28	5.43	5.58	5.72	5.87
1080	2.88	3.02	3.17	3.31	3.46	3.60	3.74	3.89	4.03	4.18	4.32	4.46	4.61	4.75	4.90	5.04	5.18	5.33	5.47	5.62	5.76
1100	2.83	2.97	3.11	3.25	3.39	3.53	3.68	3.82	3.96	4.10	4.24	4.38	4.52	4.67	4.81	4.95	5.09	5.23	5.37	5.51	5.66
1120	2.78	2.92	3.05	3.19	3.33	3.47	3.61	3.75	3.89	4.03	4.17	4.30	4.44	4.58	4.72	4.86	5.00	5.14	5.28	5.42	5.55
1140	2.73	2.86	3.00	3.14	3.27	3.41	3.55	3.68	3.82	3.96	4.09	4.23	4.37	4.50	4.64	4.77	4.91	5.05	5.18	5.32	5.46
1160	2.68	2.82	2.95	3.08	3.22	3.35	3.49	3.62	3.75	3.89	4.02	4.16	4.29	4.42	4.56	4.69	4.83	4.96	5.09	5.23	5.36
1180	2.64	2.77	2.90	3.03	3.16	3.29	3.43	3.56	3.69	3.82	3.95	4.09	4.22	4.35	4.48	4.61	4.74	4.88	5.01	5.14	5.27
1200	2.59	2.72	2.85	2.98	3.11	3.24	3.37	3.50	3.63	3.76	3.89	4.02	4.15	4.28	4.41	4.54	4.67	4.80	4.92	5.05	5.18
1220	2.55	2.68	2.80	2.93	3.06	3.19	3.31	3.44	3.57	3.70	3.82	3.95	4.08	4.21	4.33	4.46	4.59	4.72	4.84	4.97	5.10
1240	2.51	2.63	2.76	2.88	3.01	3.14	3.26	3.39	3.51	3.64	3.76	3.89	4.01	4.14	4.26	4.39	4.51	4.64	4.77	4.89	5.02
1260	2.47	2.59	2.72	2.84	2.96	3.09	3.21	3.33	3.46	3.58	3.70	3.83	3.95	4.07	4.20	4.32	4.44	4.57	4.69	4.81	4.94
1280	2.43	2.55	2.67	2.79	2.92	3.04	3.16	3.28	3.40	3.52	3.64	3.77	3.89	4.01	4.13	4.25	4.37	4.50	4.62	4.74	4.86
1300	2.39	2.51	2.63	2.75	2.87	2.99	3.11	3.23	3.35	3.47	3.59	3.71	3.83	3.95	4.07	4.19	4.31	4.43	4.55	4.67	4.79
1320	2.36	2.47	2.59	2.71	2.83	2.95	3.06	3.18	3.30	3.42	3.53	3.65	3.77	3.89	4.01	4.12	4.24	4.36	4.48	4.59	4.71
1340	2.32	2.44	2.55	2.67	2.79	2.90	3.02	3.13	3.25	3.37	3.48	3.60	3.71	3.83	3.95	4.06	4.18	4.29	4.41	4.53	4.64
1360	2.29	2.40	2.52	2.63	2.74	2.86	2.97	3.09	3.20	3.32	3.43	3.54	3.66	3.77	3.89	4.00	4.12	4.23	4.35	4.46	4.57
1380	2.25	2.37	2.48	2.59	2.70	2.82	2.93	3.04	3.16	3.27	3.38	3.49	3.61	3.72	3.83	3.94	4.06	4.17	4.28	4.40	4.51
1400	2.22	2.33	2.44	2.55	2.67	2.78	2.89	3.00	3.11	3.22	3.33	3.44	3.55	3.67	3.78	3.89	4.00	4.11	4.22	4.33	4.44
1420	2.19	2.30	2.41	2.52	2.63	2.74	2.85	2.96	3.07	3.18	3.29	3.40	3.50	3.61	3.72	3.83	3.94	4.05	4.16	4.27	4.38
1440	2.16	2.27	2.38	2.48	2.59	2.70	2.81	2.92	3.02	3.13	3.24	3.35	3.46	3.56	3.67	3.78	3.89	4.00	4.10	4.21	4.32
1460	2.13	2.24	2.34	2.45	2.56	2.66	2.77	2.88	2.98	3.09	3.20	3.30	3.41	3.52	3.62	3.73	3.83	3.94	4.05	4.16	4.26
1480	2.10	2.21	2.31	2.42	2.52	2.63	2.73	2.84	2.94	3.05	3.16	3.26	3.36	3.47	3.57	3.68	3.78	3.89	3.99	4.10	4.20
1500	2.07	2.18	2.28	2.38	2.49	2.59	2.70	2.80	2.90	3.01	3.11	3.21	3.32	3.42	3.53	3.63	3.73	3.84	3.94	4.04	4.15
1520	2.05	2.15	2.25	2.35	2.46	2.56	2.66	2.76	2.86	2.97	3.07	3.17	3.27	3.38	3.48	3.58	3.68	3.79	3.89	3.99	4.09
1540	2.02	2.12	2.22	2.32	2.42	2.52	2.63	2.73	2.83	2.93	3.03	3.13	3.23	3.33	3.43	3.53	3.64	3.74	3.84	3.94	4.04
1560	1.99	2.09	2.19	2.29	2.39	2.49	2.59	2.69	2.79	2.89	2.99	3.09	3.19	3.29	3.39	3.49	3.59	3.69	3.79	3.89	3.99
1580	1.97	2.07	2.17	2.26	2.36	2.46	2.56	2.66	2.76	2.85	2.95	3.05	3.15	3.25	3.35	3.44	3.54	3.64	3.74	3.84	3.94

APPENDIX 2: DATA TABLES

Part 1: Gold contracts

The table below illustrates the insignificant volume of contracts that require physical delivery. The market is dominated by speculators who attempt to take advantage of price occilations. The contribution to contract volumes by individual South African gold mines is small.

Table A2.1: Gold contracts.
Data source: Gold (1993), COMSA 1984-1992.

	Futures Contracts	Total Supply	Mine Supply	S.A. Supply
1984	29061	1849	1170	619
1985	24993	1995	1239	604
1986	27348	2238	1300	577
1987	35126	2273	1387	541
1988	31997	2683	1552	554
1989	34045	2853	1682	570
1990	37298	2898	1746	566
1991	25776	2852	1775	562
1992	22899	3182	1841	574

Futures contracts: Contracted volumes on the world future markets for gold.
Total supply: The amount of physical gold that changes hands (undergoes physical delivery) and is equal to the demand to hold.
Mine supply: The increase to the world's above ground stocks of gold by mining.
S.A. supply: The total contribution by South African gold mines to the world mine supply.

Part 2: Gold price and gold exchange rate

The rand and US dollar gold prices are current yearly averages and the gold exchange rate is the ratio of current yearly average rand gold price to the current yearly average US dollar gold price.

Table A2.2: Rand and US dollar gold prices, R/US dollar gold exchange rate and recorded grades.
Data source: Gold (1993), COMSA 1970-1992.

	GOLD PRICE			AVERAGE GRADE	
	$/oz	R/oz	e-rate	SA	OFS
1970	35.96	26.50	0.74	13.51	17.68
1971	40.81	29.17	0.71	13.32	16.73
1972	58.20	44.85	0.77	12.69	15.80
1973	97.22	67.09	0.69	11.42	13.99
1974	159.13	108.27	0.68	10.19	12.26
1975	161.05	118.07	0.73	9.42	11.47
1976	124.83	108.55	0.87	9.37	10.64
1977	147.71	128.46	0.87	9.78	9.60
1978	193.29	168.02	0.87	9.43	9.08
1979	305.85	257.50	0.84	8.71	8.54
1980	614.63	476.84	0.78	7.73	7.50
1981	460.13	400.08	0.87	7.32	6.77
1982	375.64	408.76	1.09	7.00	6.03
1983	423.68	471.59	1.11	6.76	5.53
1984	360.72	526.45	1.46	6.64	5.34
1985	317.32	708.44	2.23	6.28	5.28
1986	368.03	843.74	2.29	5.81	4.73
1987	446.53	908.65	2.03	5.49	4.28
1988	436.84	991.76	2.27	5.33	4.31
1989	381.05	997.26	2.62	4.99	4.26
1990	383.66	991.97	2.59	5.05	4.43
1991	362.30	1000.09	2.76	5.20	4.30
1992	343.91	979.81	2.85	5.37	4.37

Gold price:
$/oz: The US average gold price based on London AM & PM fixes.
R/oz: The rand gold price based on the PM fix.
e-rate: The ratio of the rand gold price to the US$ gold price.

Average grades:
SA: The aggregated average grades for Chamber of Mines members.
OFS: The aggregated average grade for the Orange Free State goldfield (see part 5).

Part 3: GDP, GDE and general mining deflators

The deflators are implicit deflators calculated from the South African Reserve Bank statistics (1993), employing 1990 as the base year. The 1980 deflators in all cases is too low for gold mining, probably resulting from the abnormally high gold price experienced during that year.

Table A2.3: GDP, GDE and general mining deflators.
Data Source: SARB (1993).

	DEFLATORS		
	GDP	GDE	MINING
1970	13.027	12.819	27.200
1971	12.380	12.364	27.560
1972	11.200	11.800	19.861
1973	9.419	10.099	13.287
1974	8.033	9.125	8.601
1975	7.347	8.018	7.926
1976	6.706	7.009	7.482
1977	6.065	6.334	6.600
1978	5.453	5.717	5.045
1979	4.741	5.056	3.567
1980	2.457	4.191	2.073
1981	3.414	3.640	2.488
1982	3.042	3.077	2.498
1983	2.611	2.659	2.103
1984	2.361	2.429	2.011
1985	2.045	2.104	1.567
1986	1.759	1.826	1.257
1987	1.534	1.603	1.262
1988	1.347	1.380	1.145
1989	1.162	1.161	1.062
1990	1.000	1.000	1.000
1991	0.879	0.874	0.923
1992	0.772	0.769	0.902

Part 4: Blyvooruitzicht gold mine

Table A2.4: Data for the Blyvooruitzicht gold mine.
Data source: COMSA 1970-1992.

BLYVOORUITZICHT	T	Q	G	C*	ACE	AC
1970	3171	60413.0	19.05	19.05	7.74	406
1971	3174	58757.1	18.51	18.11	8.58	464
1972	2831	54232.5	19.16	17.27	8.24	430
1973	2880	49655.9	17.25	16.46	9.76	566
1974	2507	39466.6	15.74	15.74	13.34	848
1975	3095	47736.0	15.42	14.98	20.73	1344
1976	2941	44531.9	15.14	14.27	24.70	1631
1977	2977	43479.3	14.61	13.59	29.48	2018
1978	3223	45657.2	14.17	12.91	32.67	2306
1979	3241	47889.7	14.78	12.27	37.15	2514
1980	3329	45620.8	13.70	11.65	44.17	3223
1981	3156	39012.9	12.36	11.08	55.53	4492
1982	3499	39475.9	11.28	10.51	62.47	5537
1983	3522	39454.8	11.20	9.97	67.83	6055
1984	3576	36046.1	10.08	9.45	71.91	7133
1985	4076	37467.7	9.19	8.92	79.49	8646
1986	5527	37200.1	6.73	8.35	78.98	11734
1987	5826	34296.2	5.89	7.80	95.99	16305
1988	6579	39223.2	5.96	7.25	104.54	17534
1989	6584	40568.9	6.16	6.74	124.83	20258
1990	6598	38487.0	5.83	6.27	136.93	23474
1991	6623	41110.0	6.21	5.83	144.93	23349
1992	6459	38354.0	5.94	5.43	154.16	25960

T : Total tonnage of gold ore milled in 000's tons.
Q : Total gold output in kilograms.
G : Recovered average grade in grams per ton.
C* : Disk model; expected grade in grams per ton.
ACE: The average cost in rand per ton of gold ore milled.
AC : The average cost in rand per kilogram of gold produced.

Part 5: The Orange Free State goldfield

The data excludes the Harmony gold mine and gold output from the Anglo American OFS Joint Metallurgical Scheme, subsequently Free State Consolidated Metallurgical Scheme. Taking into account the mergers that took place from 1970 to 1992, culminating in Free State Consolidated (South Region and North Region), Chamber of Mines data is aggregated from Freddies Consolidated, Free State Gedult, Free State Saaiplaats, President Brand, President Steyn, St Helena, Welkom and Western Holdings. Freddies North and Freddies South last declared gold in May 1954, and Merriespruit in October 1956. A weighted average grade for the goldfield is calculated by weighting each mine's average grade by its contribution to the total tons milled for the goldfield.

Table A2.5: Data for aggregated gold mining output in South Africa.
Data source: COMSA (1970-1992).

ORANGE FREE STATE GOLDFIELD

	T	Q	G	C*	ACE	AC
1970	16507	291799.4	17.68	18.42	7.20	406
1971	16760	280493.6	16.73	15.69	7.79	566
1972	16859	266349.7	15.80	13.87	8.47	536
1973	16979	237559.4	13.99	12.49	10.31	736
1974	15045	184546.5	12.26	11.51	13.97	1139
1975	16940	194368.7	11.47	10.59	16.58	1445
1976	17606	186486.3	10.64	9.77	18.83	1778
1977	18142	174087.1	9.60	9.06	21.85	2276
1978	18214	165366.7	9.08	8.43	25.23	2780
1979	18620	159103.8	5.54	7.87	28.04	3281
1980	19914	149306.4	7.50	7.34	32.13	4285
1981	20079	135968.4	6.77	6.87	39.34	5809
1982	21638	130214.2	6.03	6.42	44.95	7461
1983	23949	132365.5	5.53	5.97	48.55	8784
1984	23629	126281.7	5.34	5.58	57.10	10683
1985	22995	121575.9	5.28	5.24	69.26	13099
1986	24571	116231.9	4.73	4.91	77.56	16396
1987	24987	106971.8	4.28	4.61	88.04	20563
1988	27086	116722.2	4.31	4.31	100.20	23251
1989	28053	121787.0	4.26	4.03	111.11	25593
1990	27724	125412.3	4.43	3.79	193.93	28653
1991	28029	120515.7	4.30	3.56	125.05	29328
1992	27701	120970.0	4.37	3.35	127.59	29217

T : Total tonnage of gold ore milled in 000's tons.
Q : Total gold output in kilograms.
G : Recovered average grade in grams per ton.
C* : Disk model; expected grade in grams per ton.
ACE: The average cost in rand per ton of gold ore milled.
AC : The average cost in rand per kilogram of gold produced.

Part 6: The Witwatersrand gold deposit

Aggregated data for gold mining in South Africa is published by the Chamber of Mines with weighted average grades. Excluded data from non-members is generally about 3% of total gold output in South Africa per year.

Table A2.6: Data for aggregated gold mining output in South Africa.
Data source: COMSA 1970-1992.

WITWATERSRAND GOLD DEPOSIT

	T	Q	G	C*	ACE	AC
1970	72696	981957.6	13.51	16.00	7.34	544
1971	71877	957585.6	13.32	12.55	7.88	591
1972	70370	893063.5	12.69	11.36	8.79	693
1973	73296	936807.8	11.42	10.53	10.51	921
1974	73009	743695.4	10.19	9.86	13.18	1294
1975	74409	701203.6	9.42	9.31	16.71	1774
1976	74445	697258.1	9.37	8.82	19.30	2061
1977	66636	651783.7	9.78	8.40	23.87	2441
1978	69703	657451.1	9.43	8.07	27.14	2877
1979	74600	649879.0	8.71	7.75	30.18	3464
1980	80318	620924.6	7.73	7.44	35.53	4587
1981	82109	601125.5	7.32	7.14	41.89	5719
1982	87278	610750.6	7.00	6.86	47.25	6751
1983	92132	622371.8	6.76	6.58	51.88	7680
1984	93264	619044.7	6.64	6.31	58.94	8861
1985	96145	604117.2	6.28	6.07	68.76	10938
1986	99297	577218.4	5.81	5.83	80.22	13799
1987	98570	540900.6	5.49	5.61	94.90	17294
1988	103373	554267.4	5.33	5.40	106.77	20040
1989	113690	569833.4	4.99	5.20	116.67	23340
1990	111175	565653.7	5.05	4.99	130.34	25733
1991	107352	562023.0	5.20	4.81	136.05	26136
1992	106400	574318.7	5.37	4.68	141.82	26373

T : Total tonnage of gold ore milled in 000's tons.
Q : Total gold output in kilograms.
G : Recovered average grade in grams per ton.
C* : Disk model; expected grade in grams per ton.
ACE: The average cost in rand per ton of gold ore milled.
AC : The average cost in rand per kilogram of gold produced.

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