The Identification of Criteria to be utilised in Mathematical Diagnostic Tests

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DECLARATION BY STUDENT

I, Shirley Joy Wagner-Welsh, hereby declare that this thesis is my own work and that it has not previously been submitted for assessment to another University or for another qualification.

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Contents

List of Tables

List of Diagrams

Summary

School-related mistakes and low pass rates have led this researcher to perceive that some students are not adequately prepared for Mathematics 1. To address the problem of under-preparedness overseas universities use placement or diagnostic tests. Diagnostic testing identifies areas of weakness and provides information to guide the development of appropriate remedial support.

This researcher embarked on a study to identify the sub-domains (criteria) that should be included in a diagnostic Mathematics test battery at the NMMU. An analysis of first-year curricula was undertaken to determine the required Mathematical pre-knowledge and skills entry-level students should have. Thereafter, the required pre-knowledge and skills were reflected against the standard grade school syllabi. From this it was determined that the school learners should acquire the necessary pre-knowledge and skills for university success as part of the school syllabus. However, in reality this is not the case as the researcher and other Mathematics lecturers identified a number of basic errors that incoming students make. This suggests that they have not developed all the required knowledge and skills. Furthermore, their performance in the matriculation examinations does not provide an adequate measure of the requisite Mathematical pre-knowledge and skills necessary for success at university-level Mathematics.

No suitable existing diagnostic Mathematics test could be found. By utilizing both an action research as well as a test development methodology, the researcher thus proceeded to delineate the sub-domains that should be included in a diagnostic Mathematics test battery. Thereafter, test specifications were developed for two pilot tests and items were developed or sourced. The constructed response item-type was chosen for the pilot tests as it was argued that this item-type was more useful to use in a diagnostic test than a multiple-choice item format, for example. The pilot test battery, which consisted of a pilot Arithmetic and Algebra and Calculus tests, was administered to a sample of first-year students at the NMMU in 2004 and their performance in Mathematics at the end of the first year was tracked.

Tests were scored holistically and analytically to provide a rich source of information. Thereafter, the test results were analysed to obtain evidence on the content validity of the pilot tests, including the item difficulty values and the item-total correlations; to determine the predictive validity of performance on the pilot tests with respect to final first-year Mathematics marks; and their reliability was determined using the Cronbach's Alpha statistic.

These findings suggest that appropriate sub-domains (criteria) were delineated and the items appropriately covered these sub-domains (i.e. the content validity of the pilot tests is acceptable). Furthermore, the predictive validity of the pilot tests was found to be acceptable in that significant correlations were found between the pilot tests and performance in first-year Mathematics. Finally, the pilot tests were found to be reliable.

Based on the results, suggestions are made regarding how to refine the diagnostic test battery and the research related to it. The final diagnostic Mathematics test battery holds much potential to be able to assist in the early identification of at-risk students who can be timeously placed in developmentally appropriate Mathematics modules or provided with appropriate remedial intervention.

Keywords: Diagnostic testing, Sub-domain Identification, Test Development, Validity and Reliability of tests, Test battery for first year Mathematics students

CHAPTER 1 – Background to the study

1.1 Introduction

Since the political transformation in South Africa the student population at the University of Port Elizabeth - now the Nelson Mandela Metropolitan University (NMMU) - has changed from predominantly white to predominantly black. In 1996, 33% of students were black [Alum99]. In 2005 – according to official NMMU statistics – 59.7% of all students were black and in 2006, 60 percent. As a result of the influx of historically disadvantaged students [Huys00:146] as well as problems in the Education System [Cf. 2.2.3], higher education pass rates have shown a decline.

Although the students all satisfy the minimum admission requirements – a high percentage of them fail (Cf. Appendix 1). An analysis of first year mathematics curricula identified certain pre-knowledge and skills that a first year student should have (Cf. 3.3). In Chapter Three section 3.3, the university pre-knowledge identified (called the required curriculum) is compared with the standard grade school syllabus to determine whether the standard grade learner is adequately prepared for the first year (acquired curriculum). The comparison revealed that the standard grade learner should be adequately prepared for first year mathematics. However, an analysis of students' work products (Cf. 4.2.4, 4.2.5 and 4.3) revealed that many school-related mistakes were made. These mistakes have an impact on students' further learning.

The school-related mistakes made, together with the high failure rates at university (Cf. Appendix 1), have led this researcher to believe that the matriculation mathematics school symbol does not provide adequate information about the learner's pre-knowledge and skills. In view of this, it is the opinion of this researcher that first year Mathematics students should be tested to determine the pre-knowledge and skills they have (achieved curriculum).

1.2 Scope of the research

This research focused on determining the sub-domains to include in a future diagnostic mathematics test that will be developed at NMMU. The pilot test focused on determining the sub-domains for degree-type Mathematics programmes only. Since the inception of the study the University of Port Elizabeth, the Port Elizabeth campus of Vista University and the Port Elizabeth Technikon have merged to form a new institution known as the Nelson Mandela Metropolitan University (NMMU). Addressing the issue of setting criteria for diploma-type programmes in the newly merged institution will be the challenge of a follow-up study. The main reasons for this decision are as follows:

- The research was started before the merger took effect
- The diploma type courses (old Technikon Mathematics courses) are service courses only
- There always were – and still are – degree programmes where students can major in Mathematics, which was the focus of this study.

1.3 Relevance of the study

Universities are faced with the dilemma of increasing throughput rates without compromising their standards. The aim of this study was to determine the sub-domains to include in a future diagnostic mathematics test. Such a test will address students' areas of weakness and could possibly assist in the reduction of the high failure rate in Mathematics 1 courses. Diagnostic tests also provide predictive information to assist in the placement of students.

1.4 Problem Statement

Students come to university under-prepared *(*Cf. 4.2*).* The school-related mistakes made by students coupled with the high failure rate corroborate this. To overcome the problem of under-preparedness, students need to be tested. Before one can develop or purchase a test, one needs to have a clear understanding of what you want to achieve with the test. If testing achievement and streaming students are the main considerations, then a placement test will be the best choice. If identifying weaknesses and providing remedial assistance – which may also entail streaming students into development modules – are the main considerations, then a diagnostic test will be the best choice. This research aims to answer the question: Which sub-domains should be included in a future Mathematics diagnostic test battery? To address the research question a number of objectives were identified.

1.5 Research Objectives identified

- Investigate procedures followed at other universities to identify under-prepared students with a specific focus on numeracy and mathematics
- Determine the pre-knowledge and skills a first-year mathematics student requires
- Compare the required pre-knowledge and skills with school mathematics syllabi to determine the acquired curriculum
- Identify school-related mistakes made with respect to mathematics (i.e. areas of concern)
- Determine sub-domains to include in the diagnostic mathematics test using the acquired curriculum and the school-related mistakes
- Determine whether an appropriate *off the shelf* test is available
- Research test development theory and develop a pilot test based on the sub-domains

- Perform validity and reliability checks on the pilot test to determine whether it is psychometrically sound and predicative of performance in first-year Mathematics.
- Provide suggestions for remedial help to assist weaker students

1.6 Thesis Outline

This section describes how the thesis is structured.

- In Chapter 2 some of the reasons for the low pass rates will be considered. The use of placement and diagnostic tests for Mathematics at other institutions will be investigated. This chapter will highlight the under-preparedness of students as one possible reason for the low pass rates of first-year students. In the United Kingdom (UK) many universities have tried to address the problem of under-prepared students by using diagnostic testing to identify the students' areas of weakness and providing remedial support to force students to address those weaknesses (Cf. 2.5). In the United States of America (USA), the focus is on streaming prospective students and placing them according to their test results (Cf. 2.4).

- Chapter 3 determines the required pre-knowledge and skills a student should have to successfully complete a Mathematics 1 course. This pre-knowledge was compared with the standard grade FET school curriculum of the current Senior Certificate to determine the pre-knowledge the learner should have acquired at school (3.3). The chapter concludes that the standard grade learner should have most of the required pre-knowledge and skills and the higher grade learner should have all the required pre-knowledge.

- Chapter 4 investigates the school-related errors students make (Cf. 4.2-4.3). These errors can be linked to the acquired pre-knowledge and skills identified that students should have acquired. These errors led this researcher to perceive that, despite what is aspired to in the school curriculum, students do not actually have all the necessary pre-knowledge required to successfully complete a first-year Mathematics course. The researcher identified sub-domains (Cf. 4.5) that should be included in a pilot test.

- Chapter 5 focuses on why a ‘diagnostic’ and not a ‘placement’ test was required at the NMMU. A number of South African tests were considered for possible use. It became clear that a suitable diagnostic test could not be purchased. Test design was investigated from an action research perspective. Aspects of test design - pertaining to the pilot test - are described.

- Chapter 6 determines the content domain of the pilot test and gives reasons for the inclusion of specific questions in the pilot test battery. The pilot diagnostic test battery consisted of an Algebra and Calculus Test (AACT) and an Arithmetic test (AT). The administration and scoring of the pilot is also described in this chapter. The analytic scoring process identified certain errors made by students. The errors were grouped into error categories and the number of times the error was made, recorded under the refined error category.

- Chapter 7 describes the statistical analysis of the 2004 Pilot Diagnostic Mathematics Test. The content validity of the test was determined. The internal consistency reliability of the pilot test was determined using the Cronbach’s alpha statistic. Item-total correlation coefficients were calculated. Item difficulty values were determined for the test items. The correlation between the Arithmetic component and the Algebra and Calculus component of the pilot test was determined. The frequency of occurrence of the refined error categories identified was determined. Chi-square tests were performed to determine which refined error categories were predictors of success. Multiple regression analysis was used to determine if the model - using the Arithmetic and the Algebra and Calculus Pilot Tests as independent variables and NMMU mathematics marks as dependent variable - could predict students' success.

- Chapter 8 summarises the research findings and identifies future research topics. In Chapter 8 it will be shown that the sub-domains, identified by the researcher, will provide the criteria to include in a diagnostic test battery at NMMU.

CHAPTER 2 - Under-prepared students and the use of Placement or Diagnostic testing

2.1 Introduction

To decide how to deal with the problem of under-prepared students it is useful to gain insight into how other institutions deal with this problem in order to make an informed decision on how to proceed. This chapter will briefly investigate the reasons for low pass rates (Cf. 2.2). The main emphasis in this chapter will be on how overseas universities deal with the problem – in particular which types of testing and tests overseas universities use (Cf. 2.4 - 2.5) . Placement and diagnostic testing in Mathematics is relatively new in South Africa. It would be advantageous if an overseas diagnostic test could be found for use at NMMU. The content of both local and overseas tests will be discussed in section 5.5. Hereafter a decision to buy or develop will be taken. Section 2.3 considers some doubts about the validity of matriculation results.

This study did not focus on non-cognitive factors. It was beyond the scope of the study to do so. Nonetheless, the researcher is aware that a number of South African and international research studies have highlighted the role played by non-cognitive factors in determining success at higher education studies for students from disadvantaged schooling backgrounds in particular. For example, it has been observed that “Enrolment and success in higher education are determined by so many factors – differences in prior schooling and academic preparation, family and community attitudes, the student’s motivation and awareness of opportunities, the environment on campus, and economics” [Gladi99]. Gladieux comments further that “there are no simple solutions and no straight-line advancement towards equity and diversity in higher education” [Gladi99].

2.2 Reasons for the low pass rates

There are a variety of reasons for the low pass rates in higher education institutions. Some of the major reasons identified by educators and researchers are:

- Language medium and reading skills (Cf 2.2.1)
- personal circumstances (Cf 2.2.2)
- problems in the education system (Cf 2.2.3) and
- admissions criteria at universities
- school related mistakes students make (Cf. 4.2 – 4.3)

The reasons listed above will be discussed in some detail below:

2.2.1 Language Medium and Reading Skills

There is a community of international students at Nelson Mandela Metropolitan University. These students come from a variety of countries. Some of them do not have a good command of the English language. From personal experience some students have difficulty expressing themselves in English, as they have not had the opportunity to speak the language frequently. Millroy (1985:4) states that African languages also do not always describe the spatial environment scientifically and that technical terms and vocabulary are often not available in these languages [Milroy85:4].

According to Beliveau (2001) the language of Mathematics is comparable to a foreign language. Mathematics is a combination of symbols, numbers and words. She continues that it is no surprise that English second language students detest reading mathematics and that they skip straight to the problems in their textbooks [Beliv01]. “The mathematical achievement of children correlates highly with their ability to read mathematics” [Siegel89].

Fitzgerald (1995) concluded from his research that the strategies necessary to teach reading in a second language do not differ from those employed in teaching reading in a first language [Fitzg95]. According to Krussel (1998:438), mathematics is a language. Krussel believes that the skills needed to learn Mathematics may not differ greatly from those required to learn any language. This does not mean that mathematics can be learned without effort. Nor does it mean that each student's success requires the same amount of effort [Kruss98]. "Just as daily use makes it easier for young children to learn a foreign language, daily practice in reading, writing, and conversational mathematics from an early age will reap similar results" [Kruss98:438]. Reading and speaking mathematically differ from ordinary English. Mathematics has a special logic and syntax. "Teachers must take time to work with students in reading mathematics and help them to learn to understand what is required to accomplish understanding while reading. Reading for understanding is crucial" [Kruss98:438].

Krussel concludes that much of the research on problems encountered by students in courses ranging from Algebra through calculus and into courses beginning with proofs points to the language based misconceptions students develop. She suggests teachers "should encourage a balance of drill and practice of the language beginning at an early age: study of its vocabulary and structure; and practice in conversation, reading, and writing - both creatively and more formally" [Kruss98: 440].

Some learners find it difficult to attempt and solve word problems. Word problems are also known as contextualised problems. Murray (2003:39) writes: "Of course language problems and lack of reading skills are extremely important, but other factors can also act as barriers to understanding and to the learners' willingness to engage with the problem." The research done by Murray found that some mathematical structures which have been identified as suitable for early word sum problems are more difficult for learners. It also depends on the amount of exposure learners have had to a particular structure. Murray gives the example of teachers holding back on division examples, because they believe this to be more difficult and thus prevents learners from becoming familiar with the mathematical structures of the different division problems [Murra03:40].

Bohlman and Pretorius (2002) have researched the relationship between reading skills and academic performance in mathematics. Research done in 1987 showed reading proficiency in the language in which mathematics was taught was a prerequisite for mathematics achievement [Bohlm02:196]. Bohlman and Pretorius found that although language proficiency and reading ability are related, they are nonetheless uniquely specific skills that develop in distinct ways [Bohlm02:196]. Their study focused on reading comprehension and they distinguished between decoding and comprehension. Decoding is the process whereby written signs and symbols are translated into language. Comprehension refers to the overall understanding process [Bohlm02:196]. In this process meaning is assigned to the whole text. Once decoding skills have been mastered, comprehension skill can be acquired. If a learner has mastered the skill of decoding it does not imply that they have also mastered the skill of comprehension. Bohlman and Pretorius tested comprehension skills. It was assumed that at a higher education level decoding skills had already been established [Bohlm02:197].

Their research showed where students overall reading scores were less than - or equal to - 45% the student failed mathematics. Students with reading level scores equal to or exceeding 75% passed their mathematics examinations. Bohlman and Pretorius assumed that success in science and mathematics mainly requires numerical and logico-deductive skills [Bohlm02:204]. The study by Bohlman and Pretorius showed that reading is one of the variables that affect success in mathematics as they found a relationship between reading ability and performance in mathematics. However, they pointed out that “There are obviously many different variables involved, not least of which are the issues of motivation, patience, persistence and other cognitive aspects uniquely (perhaps) associated with mathematical argument [Bohlm02:204]."

The above research has emphasized the importance of reading and language skills in mathematics success. This thesis will, however, not focus on the effect that inferior language education and weak reading skills have on first-year mathematics pass rates. However, attention will be paid to the level of reading required in the pilot diagnostic mathematics test that will be developed.

2.2.2 Personal circumstances

First-year Mathematics students at NMMU come from a variety of backgrounds. Students from rural communities come from third world areas, whereas many urban students grow up in first world conditions. The South African Government has tried to redress the scholastic imbalances caused by the old apartheid system. However, many students still come from disadvantaged backgrounds. The NMMU draws the majority of its students from the Eastern Cape. A provincial profile shows that the Eastern Cape province is not only educationally disadvantaged, but also socially and economically disadvantaged [Ffacts99].

Students coming from impoverished backgrounds may not have had access to technical toys. According to Kemp (1983), technical toys such as Meccano, Lego, model building, chess and jigsaw puzzles will assist children in their development of good spatial ability. These students could experience problems with visualization in three dimensions [Kemp83]. These problems, if detected early enough in the student's university career, can be rectified [Millroy85].

The effect that family background has on pass rates falls outside the scope of this study. However, a study done in Colombia (South America) predicting college success suggests that family background and study behaviour represent significant variables in predicting college success [Ardila01].

2.2.3 Crisis in the Education System

The crisis in Education started prior to 1992. Before 1992, black learners were exposed to an inferior school system [Huys00]. Christie (1991) states that in the period 1988 –1989, 32% of black teachers had not even matriculated themselves and only five percent had university degrees. Compare this with white teachers, where all white teachers had matriculated and 32% of them held University degrees. Before 1995, the Department of Education and Training (DET) was the governing body for black schools. In 1995 all the different Education Departments merged [Chris91]. Miller (1992) warned that a mere change of departmental name would not immediately rectify the deficiencies of the former DET schools [Miller92]. As a result of inferior schooling many of these students were ill-prepared for the demands that a university education requires. Huysamen (2000) states that poor teaching in the historically disadvantaged schools cannot be remedied instantaneously [Huys00].

In Chapter 15 of the book “Marking Matric: Colloquium Proceedings”, face-to-face interviews were conducted with matric teachers teaching in township schools. Teachers from six township schools were interviewed. Three of the township schools which had done well in the 2003 matriculation examination and three township schools which had had poor results were selected for the study. The interviews were aimed at listening to Grade 12 teachers and seeking solutions to the problem of poor pass rates in some township schools. “Most of the sampled teachers argued that matric results will improve if teachers have a good knowledge of the subject they teach.” Some comments of the mathematics teachers interviewed follow [Marki06:222]:

“I think the major cause of poor results is the foundation. If the child does not have a foundation in Maths, he cannot make it in Grade 12.…”

"I think the major factor that will result in good matric passes is if the department monitors the lower Grades, because sometimes, let's say with sections like factorisation, a child is in Grade 12 and he does not know how to factorise. And remember factorisation starts in Grade 9. I cannot repeat the Grade 9 syllabus in Grade 12, so they [DoE] should monitor Math teaching in Grade 9. There is no Monitoring of Grade 9."

A national study showed that most of the teachers at township and rural schools had a poor understanding of their subjects. Without good subject knowledge teachers cannot engage learners in conceptual thinking [Marki06:221].
Teachers have also mentioned that learners are not taking responsibility for their learning [Marki06:219]. Four categories of factors that affect matric results were highlighted, namely [Marki06:219]:

- Commitment by teachers and especially learners
- The subject knowledge of teachers
- Standard of teaching in lower grades
- Socio-economic conditions of township schools and learners

Educators applauded the improvement in the Senior Certificate Examination pass rates at the end of 2002 and 2004. However, an analysis of the results of a small sample of schools in the Western Cape revealed that the successful candidates were still clustered in the middle class schools. The former Model C schools still had an advantage. Children from these schools, whether they were black, coloured or white, had a much better chance of success. Whereas children from township schools and rural schools in formerly disadvantaged areas had a very small chance of obtaining the Matriculation exemption [Marki06:178].

A study done at the University of Cape Town to assess basic Mathematics competency revealed that out of 322 students writing the test, 30 per cent failed and 20 per cent scored between 50 and 59%. Prospective first year students are not performing at the expected levels and lack the competencies required to successfully complete a university programme [Markin06:35].

Although standard grade papers were designed to be easy in the two-tier system used, a study conducted by Umalusi showed evaluators consistently found standard grade papers were easy [Markin06:24].

" … the public continue to raise eyebrows about the quality of the SCE, " (Senior Certificate Examination) "when:

- More learners pass on SG than on HG;
- The standard of question papers is not seen as acceptable;
- Qualifying learners seem to come out with apparent gaps in their knowledge and skills";

"The standardisation process is seen by outsiders as a way of manipulating results" [Markin06:23].

The continuing crisis in the education system has also received widespread media coverage. Although one cannot rely one hundred percent on the accuracy of media reports, one also cannot ignore the severity of the problems they were highlighting even before the results of official studies became known.

Professor Jansen, Dean of Education at Pretoria University, comments on the matriculation results in an article in the Sunday Times dated 4 January 2004. He writes: "more pupils are passing poorly than ever before – for two reasons". The first reason he mentions is that schools are under immense pressure to perform. Under such circumstances studies in the US and UK have revealed that schools tend to optimise their results. Schools could hold back Grade 11 pupils, whom they believe, will fail and thus negatively affect their pass rates. Many learners could be encouraged to take subjects on standard Grade for the same reason. The second reason he mentions is that learners receive up to 25 percent of their marks before they have even written the exam (from continuous assessment during the year). Professor Jansen further writes that there is no “reliable and valid protocol in place to ensure that such marks are standardised across the national education system”. Schools could be tempted to obtain maximum gain from the continuous assessment (CAS) system. Compared to other Third World countries, South Africa consistently performs worse in tests such as the international and comparative tests of learner achievement. The tests are set for Grades lower down in the education system. An example of such a test is the International Mathematics and Science Study. The article also mentions that national and provincial assessment in primary Grades points out that young learners do not know the basics.

The Third International Mathematics and Science Study conducted in 1999 showed that South African Learners performed very poorly in Mathematics. South African learners’ achievements were significantly lower compared to the other countries. “Even though performance generally differed very little between one country and the next higher- or lower-performing country, the range in performance across the 38 countries was very large” [Timss99:chapter 1:30]. The statistics showed that only the most proficient learners in South Africa approached the level of achievement of Singaporean students of average proficiency [Timss99:Chapter 1:31].

The book “Marking Matric: Colloquium Proceedings” published by the Human Sciences Research Council (HSRC) revealed “Research on performance in school Math and Science at a systemic level have indicated that performance is poor. South Africa came last out of fifty countries participating in the trends in International Mathematics and Science Study (TIMSS) 2003 study” [Marki06:140]. “South Africa also has low scores in numeracy in the Monitoring Learner Achievements (MLA)” and “Southern African Consortium for Monitoring Educational Quality (SACMEQ)” studies and the achievement scores in numeracy in the Systemic Evaluation were low [Marki06:140]”. Concern is expressed that only a small number of learners pass with Higher Grade

Mathematics. To analyse trends in Matriculation Mathematics performance schools in Gauteng and in the Free State were used. In Gauteng and in the Free State the number of schools offering Mathematics on Standard Grade only increased from 1998 to 2003. Almost half of the schools in the Free State offered Mathematics on Standard Grade only [Marki06].

In an article in "Die Burger", dated 27 September 2004, Professor Theuns Eloff encouraged more learners to take subjects on higher Grade provided the learners have the ability to do so. Professor Eloff is the Vice Chancellor of the University of the North West. In the article Professor Eloff expresses the opinion that the performance of a number of learners in Mathematics and Science is not on a par with what is expected of them. He continues that it is also important that a learner's performance on Higher Grade is good enough to ensure that they gain entry into a university [DieBur2].

On Tuesday 9 November 2004 'The Herald' reported that the Mathematics and Science pass rates were not improving. This was the finding of an independent study by the centre for development and enterprise. According to Ann Bernstein, of the centre, "Maths is the biggest constraint to African advancement. South Africa needs a dramatic improvement in higher Grade pass rates". The number of learners enrolled for higher Grade Mathematics declined from 53 631 in 1991 to under 36 000 in 2003. "The report recommended identifying and head-hunting qualified mathematics and science teachers and providing incentives for people to teach these subjects". According to the article the report also recommended that high performance schools should be supported and encouraged to expand their mathematics and science departments. The report quotes Bernstein that "Maths, science and language are South Africa's top education priority. Everything we do needs to be assessed against this priority."

The 2004 matriculation pass rates were lower than the results for 2003. Pass rates dropped from 73,3 percent to 71,4 percent. An article in 'Die Burger' (30

December 2004) mentions that some of the reasons for the drop in pass rates could be attributed to the fact that learners were either poorly prepared or that the examination surpassed their comprehension. The article also mentions that the pass rates in the six key subjects Mathematics, Science, Biology, Accounting, English Second Language and History showed a decline [DieBur3].

According to an article published in 'Die Burger' on 5 November 2005, many of the problems currently experienced in Education can be blamed on the introduction of *'outcomes-based'* education. This system was introduced after the elections, which took place in 1994. It was expected that *outcomes-based* education would redress the inequalities that existed, in the education system, because of the old "Apartheid" System. According to the article the biggest problem with *outcomes-based* education was that it turned teachers into facilitators, causing teachers to lose their ability to teach. Teachers were unsure of their role under the new system. The pace at which teachers were trained was too slow to keep up with the requirements of the new system. The *outcomes based* education system expects learners to be explorers. However, learners from previously disadvantaged communities did not have access to the same sources as the more privileged learners. This article goes further and mentions that teacher morale is low and that teachers are continually blamed that they are not doing their jobs. "It is forgotten that learners no longer have a culture of learning."

'Die Burger' (29 December 2005) reported Matriculation results were again adjusted upwards in all national subjects, including Mathematics. The 2005 group of learners were the first group to start with *outcomes-based* education (OBE), but had to revert back to the old curriculum in 2003, when the department could not implement *outcomes-based* education from Grade 10 to Grade 12. At the top end of the scale learners performed even better than in 2004, while at the opposite end of the scale more learners failed [DieBur4].

According to the book “Marking Matric: Colloquium Proceedings” published by the Human Sciences Research Council (HSRC), a consequence of the predominant trend of upward adjustments was the compromise of ‘soft’ downward adjustments. This would have contributed to the upward movement in pass rates [Markin06:26]. This could explain why learners at the top end of the scale performed even better in 2005 than in 2004.

In ‘The Herald’ (24 January 2006) Trevor Ryan, a lecturer in the physics foundation course at NMMU, writes about the factors causing poor matriculation pass rates. Learners’ apathy towards learning is cited as one of the reasons for poor learner performance. Another reason mentioned is that standards and syllabi vary from school to school. If there is more uniformity, with regards to what is being expected of learners in the matriculation final examination, learners will be more used to the level of questions asked in the final examination [Theher2].

Mr Ryan slates *outcomes-based* education (OBE). According to the article, learners do very little constructive work and acquire very little required knowledge. Teachers’ time is eroded by administration, portfolios and continuous assessments, which do not accurately reflect the learner’s knowledge. This state of affairs continues until Grade 9. In Grade 10 to 12 learners have to play catch up. According to the article some students in Grade 9 struggle to read or write and will never succeed in tertiary studies.

Promoting learners who do not meet the minimum requirements between Grade 0 and Grade 11 compounds the problem. Every year builds on the knowledge of previous years. Learners who are unduly promoted will continue to struggle until Grade 12. The opinion is expressed in the article that pass rates at school level should be raised to 50 percent to bring school pass rates in line with tertiary pass rates. Another plea is made that educators should only teach in their own home language and in their field of expertise.

Situations arise where unqualified teachers are appointed or where workloads are shared and teachers teach subjects which are not in their field of expertise. In the article, parents are discouraged from blaming the teachers and the school and encouraged to discipline their children and encourage them to work.

“Lack of curriculum control, combined with inadequate teacher preparation could be significant contributors to the apparent deficiencies in matriculating learners” [Markin06:24]. These findings by the HSRC corroborate some of the opinions expressed by Mr Ryan.

In the light of all the media reports and findings in the Human Sciences Research Council’s book “Marking Matric: Colloquium Proceedings” about the state of education and changes to the education system it is clear that higher education in South Africa will face even bigger challenges in the future. The first group of learners who will complete the new National Senior Certificate - which will replace the current Senior Certificate - will matriculate at the end of 2008. Currently many universities are using higher grade and standard grade pass rates to calculate matric points or weighted average marks which are used to determine admissions. This whole system will have to change with the introduction of the National Senior Certificate when higher grade, standard grade and lower grade are dispensed with and where learners have to choose between taking either Mathematics or Mathematical Literacy, among other things. In view of these changes, it is going to become very important that universities have new selection criteria in place.

2.2.4 Admissions criteria

Although the participation rates of previously disadvantaged groups in higher education have increased since the 1990s, the throughput and success rates of students from educationally disadvantaged backgrounds remain lower than for white students. For example, graduation rates in 2001 revealed that only 7% of African students graduated compared to the 27% of white students [Cloete02].

The opening up of access to students from educationally disadvantaged backgrounds has also caused higher education institutions to rethink their admissions criteria. Traditionally, there has been an almost exclusive reliance on matriculation performance when determining admission to universities. However, in view of the fact that matriculation performance was found to be a variable predictor for students from educationally disadvantaged backgrounds, admissions criteria had to be expanded [FoxStu05]. This has been largely achieved through introducing alternative admissions testing programmes to which applicants who do not meet the criteria for direct admission are referred to.

At the time of the study prospective first year students at the Nelson Mandela Metropolitan University (NMMU) wrote the Accuplacer computerised placement tests either for admissions or development purposes depending on whether or not they had met the matriculation admissions criteria for direct admission. This testing system is an adaptive American testing system that has been adapted for use in South Africa. The College Board developed the Accuplacer Computerised Placement tests and they were adapted for use in South Africa by the Admissions and Placement Assessment Programme at UPE. The tests are adaptive in nature. The difficulty level of the questions is based on responses to previous questions. There are ten different tests available. These tests measure reading, writing, English and mathematical ability. Three different tests are available in the Mathematics category. They are the Arithmetic test, the Elementary Algebra test and the College-level Mathematics test. Each of the tests is multiple choice with no enforced time limit [Olson03:20].

Students at NMMU wrote only the Arithmetic and Elementary Algebra tests for admissions purposes if they failed to meet the matriculation criteria for direct admission. As the tests were used for admissions and not for placement purposes, the results from these tests were not used to stream prospective first-year Mathematics students into developmentally appropriate modules. The two tests could not fulfil the needs of the Mathematics Department for a diagnostic mathematics test as they were not designed for this purpose. Research done by the University of Port Elizabeth's Admissions and Placement Assessment Programme revealed that the Elementary Algebra Test was too easy for the directly admitted students although it was useful for applicants who had a weak background in Mathematics or had studied Mathematics a few years before applying [Foxcro99]. The multiple choice test system also had drawbacks. For instance, the Elementary Algebra test gives the impression that students can factorise when in fact they are merely multiplying factors until they reach the desired polynomial.

Accuplacer has other drawbacks. If the student chooses incorrect answers early in the test this might lessen the chances of their obtaining a high score. This is a result of the adaptive nature of the test. If initial questions are answered incorrectly, future questions are easier and as such are worth fewer points. "This can lead to difficulty building up to questions of higher point value" [Olson03:22]. This is especially true of the Elementary Algebra test. …"the difference between an Accuplacer score of 72-85 and above 85 is due to one additional correct answer" [Olson03:22]. Thus "one correct guess can affect a student's chances of achieving a cut-off score" [Olson03:22].

In 2004, a student required a score of 40 percent in Mathematics higher grade or 60 percent in Mathematics standard grade as well as 40% in English First Language higher grade or 50% in English Second Language higher grade to register for any Mathematics or Applied Mathematics module at UPE. The exception was for the module General Mathematics for Foundation and Intermediate Phase. Students studying to become Foundation or Intermediate Phase Educators registered for General Mathematics. A pass in Mathematics at

Matriculation level was not a pre-requisite for General Mathematics. Students were admitted to this module as long as they passed Mathematics in Grade 9.

The low pass rates (Cf. Appendix 1) and the school-related mistakes made (Cf. 4.2) led this researcher to believe that the admission criteria used were not adequate and that more information was needed on incoming Mathematics students so that they could be placed in an appropriate Mathematics module. Diagnostic information on incoming students could assist lecturers to adapt their teaching according to the knowledge and skills needs of their students.

2.3 Doubts about the validity of matriculation results

As was indicated in section 2.2, in recent years it has become apparent that universities could no longer rely solely on Matriculation examination results. After every Matriculation examination any improvement in the pass rates is emphasized. According to an article in the 'Sunday Times' dated 4 January 2004 the pass rate had improved from 54.5% in 1996 to 73.3% in the 2003 Examinations.

On 8 January 2004 an article appeared in 'Die Burger' stating that thousands of Grade 12 learners were border-line cases and that their marks were adjusted to award them passes. In Mathematics in particular, 1 800 or 5% of candidates were assisted to enable them to pass on higher grade and another 1000 who would have failed were assisted to pass on standard grade. On standard grade, 13 000 or 6% of the candidates were assisted to enable them to pass and a further 7500 or 3.4% who would have failed on standard grade were assisted to pass on lower grade. According to the statistician, who gave the statistics, this raises doubts about the quality of matriculation candidates [DieBur1].

On 27 December 2004, 'Die Burger' published an article in which Dr. Anil Kanjee of the Human Science Research Council stated that researchers and academics cannot merely compare the matriculation results from 1996 to 2003. Consideration has to be given to the fact that prior to the year 2001 no year marks were used to calculate the final mark. Furthermore, 2001 was the first year in which all the provinces wrote the same national question papers for the subjects Mathematics, English second language, Physics, Accounting and Biology. From the year 2003, all provinces were writing the same History question paper [DieBur2].

It was clear from media reports and from poor pass rates by first-year students that universities could no longer rely on matriculation marks alone to determine whether to enrol a student or not. Additional information from admissions and diagnostic testing could be of great benefit when accepting students.

2.4 Selection Procedures in the United States of America (USA)

Huysamen (1997:66) states that the Scholastic Aptitude test (SAT) was introduced in the United States as early as 1926 to provide a way in which students from unrenowned high schools could compete with students from top high schools [Huys97:66]. In 1959 a competing test battery - the *American College Test* (ACT) emerged. Huysamen continues that whereas SAT was designed to measure aspects of ability that develop both in and out of school, ACT measures academic achievement in the areas of English, Mathematics, Reading and Reasoning [Huys97:66]. Both SAT and ACT tests are not curriculum specific. Many university applicants have to take one or both of these tests.

The dynamics of the admission process at American universities have also changed in recent years. More Asian American, Hispanic and African American students are enrolling at higher education institutions. American universities attract older students, international students and also home-schooled students. The American primary and secondary education system have been going through significant reform. It has thus become important for American colleges and universities to know whether their admission measures adequately evaluate students coming from different backgrounds [Colleg01:2].

American high school students are also not ready for college or the work place. A report by the U.S. Department of Education's National Commission on the High School Senior Year (2001) states that "Although the high school diploma is a prerequisite for college admission and most jobs, students who earn one have no guarantee that they are prepared for college-level work or entry-level employment [ACT04:1]."

According to Olson (2003:14), 26 percent of the Colorado public high school graduates (or some 7507) entering public higher education in Colorado in 2002-2003 were assigned to remediation [Olson03:14]. An alarming proportion, 85 percent, of these students required remediation in Mathematics. Olson (2003) continues that proper placement and recommendation techniques are vital in ensuring that students are placed in the appropriate course commensurate with their current skill levels [Olson03:14].

College readiness refers to the level of preparation a student needs to enrol and succeed in a credit-bearing course at a two- or four-year institution [ACT04:iii]. This must happen without the need for remedial course work. According to the ACT Report, there has been a dramatic increase in the number of students at American colleges needing remedial courses. Research by Mortenson 1999 has shown that students who require substantial remediation graduated at much lower rates [EdDept99]. The ACT Report [ACT 04:11] states that it is not enough for students to take the core curriculum. The core curriculum consists of four years of English and three years of Mathematics, Science and social studies. Students who took less than the core curriculum scored lower on the American College Test than those who took the minimum core curriculum. By taking more than the four-year English core curriculum a student can improve the probability that they will meet the ACT Benchmark for College English Composition. In mathematics seventy-four percent of the students who took Trigonometry and Calculus in addition to Algebra 1, Algebra II and Geometry met the Act Benchmark College Algebra, whereas thirty-seven percent of students who took Trigonometry in addition to Algebra 1 and II and Geometry met the benchmark. The benefits of taking more than the core courses for Mathematics are evident. The finding was that Calculus takers out-scored non-Calculus takers by an average of 5.3 points on the Mathematics Test. The ACT report concludes that all students whose coursework includes courses in advanced mathematics beyond Algebra II (such as Trigonometry) as well as Biology, Chemistry and Physics are most likely to be ready for college. Hallinan (2002) found that if students were given higher level mathematics coursework their performance improved regardless of their level of prior achievement. Lowest achievers made the best progress.

Roth et al state that “taking more higher level math courses in high school is an accurate predictor of scoring well on aptitude tests commonly required for admission into four-year baccalaureate institutions” [Roth01]. Barth (2003) summarised the work of U.S. Department of Education researchers who found that students entering high school with scores in the lowest quartile grew more in college-preparation courses than they did in vocational or general courses [ACT04:24][Barth03]. ACT assessments do not measure aptitude. Instead ACT Assessments measure what students are able to do with what they have learnt at school [Act04].

Colleges and Universities in the United States recognised that they needed placement tests in Mathematics for undergraduate freshmen [Rueda04:27]. Some institutions designed their own tests, while others used placement tests in conjunction with other measurements such as the American College Test (ACT) or the Standardised Achievement Test (SAT), while other colleges and universities opted to use the Mathematical Association of America (MAA)

Placement test [Rueda04:27]. In recent years, placement exams such as Accuplacer have also been used to determine student placement [Olson03:16]. The Mathematical Association of America has discontinued its placement programme. Some individual institutions were thus forced to develop such tests [Rueda04:27]. A Study by Odell and Schumacher (1995) revealed that Mathematics SAT scores combined with placement test scores could predict success better than SAT scores alone [Odell95].

2.4.1 Placement procedure followed at St. Olaf College in Minnesota

According to Cederberg (1999:179), students are required to take one of three placement tests administered at St. Olaf College in Minnesota. The only exception is for non-degree international students and students who received a score of four or five on the College Board Calculus BC Exam. Placement exams take place during orientation. All three exams start with a survey asking students questions about:

- their motivation for taking mathematics,
- the number of terms they plan to take mathematics,
- the area in which they are going to major,
- what their last mathematics course was and the Grade they obtained in the course,
- how extensively they used calculators.

The advanced exam also asks

- how much trigonometry and calculus students had and
- which mathematics course students think they should enrol for.

All three exams carry a ninety-minute time limit. Those who want to take the advanced and regular exams are allowed to use calculators without QWERTY keyboards. No calculators were allowed on the basic exam. Students wishing to be placed beyond first semester calculus will choose to write the advanced exam. This written examination consist of twenty-five questions covering first semester calculus and another fifteen questions each on a modified version of the Mathematics Association of America (MAA) trigonometry and functions examination. Any student who wishes to take calculus at some stage during their college career will write the regular examination. This test consists of a trigonometry and functions section identical to the advanced examination as well as a calculator based MAA Algebra examination. The Algebra section consists of 32 questions. The basic test is written by students, who are hesitant to take any Mathematics. This test consists of thirty-two questions on arithmetic and basic skills and thirty two questions from a MAA Algebra examination [Ceder99:179].

The survey and the test data are scanned into the computer. The computer Grades the tests and merges the test scores with the admissions data and other relevant information to predict a Grade. The computer then assigns a recommendation to each student. The computer uses a cut-off programme that is refined from one year to the next. Borderline and special cases are considered separately. Recommendations are printed on labels and pasted on letters, which are distributed to the students' mailboxes. Results are sent electronically to academic advisors. The recommendations are computed using a large number of regression equations. The regression equations have been refined over many years [Ceder99:179].

2.4.2 Comments on the placement test used at St. Olaf

According to Rueda and Sokolowski, "Cohen, Friedlander, Kelemen-Lohnas and Elmore (1989) recommended a placement procedure that was less technically sophisticated than St. Olaf's, but still required considerable background data about students. They recommended multiple criteria methods, which included a placement test customized to an institution's curriculum. Cohen et al started with sixty variables. They found the best eight predictors to be high school graduation status, number of hours employed, units planned, age, high school Grade point average, mathematics placement test score, reading placement test score and English placement test score" [Rueda04:27].

2.4.3 Placement procedure used at the University of Arizona

Krawczyk and Toubassi (1999:181) describe the placement procedure used by the University of Arizona. The University of Arizona used two placement tests. These two tests were adapted from the 1993 California Mathematics Diagnostic Testing Project. At the University of Arizona students chose which test they felt was most appropriate for their ability and subject choice. Students had a choice between test A and test B. Test A was a 50 minute test which consisted of forty-five questions covering intermediate Algebra skills. Test B was a 90 minute test. Test B consisted of sixty question covering College Algebra and Trigonometry. Test A placed students in one of three levels of Algebra or a Liberal Arts Mathematics course. Test B placed students in Finite Mathematics, Pre-calculus or Calculus. Tests were scored electronically and results were printed the same evening. Early the following morning students could pick up their results. Students received a profile sheet indicating their mathematics placement. They also received a breakdown of their total score by topic. The computerised registration process blocks students from enrolling for courses at a higher level. Placement was initially based on placement test results. Other factors were also considered. Apart from the test they also considered Grade Point Average when a freshman's score was near a cut-off. A pilot programme was started to place freshmen into Calculus based on a high re-centered SAT or ACT score without taking test B [Krawcz99:182].

2.4.4 Placement procedure and comments from Merrimack College

At Merrimack College students are placed according to the in-house placement test. Most students placed into Math1 have had four years of high school Mathematics including pre-calculus and sometimes even calculus. The experience at Merrimack college has been that although students have had four years of high school mathematics they still did not understand the basic concepts of Algebra [Rueda04:28].

2.4.5 Placement procedure followed at the Metropolitan State College of Denver

The Metropolitan State College of Denver exempts all students with an ACT Math score of 19 or more or SAT Math score of 460 or above from writing any placement test. All other students must write the Accuplacer exam. A score of 65-79 on the College level Math exam allows the student to take Pre-Calculus, whereas a score of 80-120 on the College Level Math exam allows the student to take Calculus 1 [Olson03:26].

2.4.6 Placement procedure followed at Western State College of Colorado

Western State College of Colorado uses ACT and SAT scores and a placement test to assess incoming students. Here a student must obtain a score of 95 or higher on the Accuplacer College Level Math test before they are allowed to study Calculus 1 [Olson03:28].

2.4.7 Placement procedure followed at the University of Southern Mississippi (USM)

At the University of Southern Mississippi most students are the first in their families to attend an institution of higher learning. It is therefore important that students receive the necessary advice for academic success and retention becomes an important issue.

The Mathematics department at USM believes that advising must be comprehensive, flexible and must have a personal flavour.

“For placement the department relies on a combination of ACT scores, high school courses and Grades and self-selection based upon course catalog descriptions and discussions with the advisor” [Dobli99:185].

2.4.8 The Truman State University Predictive model

In an article in the Truman State University Grant Report a preliminary predictive model was developed to assist researchers in identifying students who may encounter academic problems. Predictive discriminant analysis was used to determine which factors from a set of four variables would enable researchers to predict which students would experience no academic problems and which students were academically at risk. The four variables used were number of high school Cs, Ds, and Fs, high school Grade point average, high school rank and American College Test scores. Further research aimed at including non-cognitive factors into the test was done. Eight factors measured by the College Success Factors Index (CSFI) were included with the traditional cognitive factors to make predictions. The traditional cognitive factors used were number of high school Cs, Ds, Fs, high school GPA, high school rank and American College Test scores. The eight non-cognitive factors used were responsibility and control, competition and collaboration, task precision, expectations, wellness, time management, college involvement and family involvement [Stefan04:1].

The number of high school Cs, Ds and Fs was a large contributor to the separation of academically strong and academically weaker students. Adding the non-cognitive variables measured by the CFSI total score to the prediction formula increased the amount of variance explained in the difference between the two groups of students. However there are some issues concerning the reliability of the CSFI sub-scale scores which require further investigation [Stefan04:12]. The researchers feel that further studies at other universities will be necessary to assess the generalizability of their findings to other student populations [Stefan04:13].

2.4.9 Placement procedures improve retention rates

Carey (2004) shows that although enrolments at colleges and universities in the United States have shown a steady increase, graduation rates have remained flat [Carey04]. Even in the United States academic failure is one of the main reasons for non-degree completion. Retention and completion rates are very low in many public colleges and universities. According to the U.S department of Education, more than one quarter of freshmen at four-year colleges and nearly half at two-year colleges do not continue to their second year [EdDept99]. Typically, admission decisions are based on a combination of standardised test scores and high school Grade point averages.

A College Board Report states that “The correlations among SAT scores, high school records, and first-year GPAs” (grade point averages), “corrected for restriction of range, variations in grading standards and criterion unreliability, can no longer be characterised as ‘small’ or even ‘moderate’”. Ramist et al. (1994) found a corrected correlation of 0.76 when first-year Grades were predicted from SAT scores and high school records [Colleg01:4].

According to Olson (2003:15), if standardised test scores are used, they are usually accompanied by a cut-off score on a placement examination. Standardised test scores are good predictors of Mathematics exam scores, but not of overall performance in Mathematics. Factors such as desire, motivation and peer study have a large effect on overall performance in class [Olson03:16].
A study during the summer of 2001 at Midwestern University showed that placement by Accuplacer score <u>alone</u> denied many students access to courses in which they would have been successful. The study “concluded that placement techniques should involve a combination of high school record and placement test score with students placed in higher-level courses” [Olson03:17].

2.5 Selection Procedures in the United Kingdom (UK)

University entrance in the United Kingdom generally is dependent on the student's performance in the General Certificate of Education Advanced (GCE A) level examination or an examination that is equivalent to it. Traditionally two or more A-levels in the GCE A examination are the minimum requirements for selection at university. At the more prestigious universities A levels are an important selection tool. In the United Kingdom 43 percent of learners from higher socio-economic backgrounds gain two or more A levels whereas only 19 percent of learners from lower socio-economic backgrounds achieve this [DeptEs03].

2.5.1 Should a test like SAT be used in the United Kingdom (UK)?

There has been a debate in the United Kingdom about using a test like SAT (American Scholastic Test) in addition to GCE A levels to make access to higher education fairer. SAT measures potential whereas GCE A levels measure achievement. Contrary to the UK there are no public examinations during secondary education in the United States. The SAT test measures verbal and mathematical skills and three hours is allowed for completion. SAT serves a similar purpose to A levels [West04]. West and Gibbs investigated whether the UK could benefit from the use of SAT. They concluded that SAT measures achievement although it is not closely related to curriculum content. Achievement is affected by factors in an individual's environment. It stands to reason that there will therefore be differences between different groups of students. West and Gibbs concluded that the use of a test like SAT would **not** be fairer than the current use of A levels. They continue: "universities are left in a dilemma: they may wish to assess potential but it is difficult to separate potential from actual test performance since tests supposedly measuring potential measure achievement at a particular point in time" [West04]. Certain American universities are using *'profiling'* in an effort to make access to higher education more equitable [West04].

2.5.2 The under-preparedness of the UK student population

In the United Kingdom the educational background of students entering University has changed. In engineering degrees at some universities students with A-levels in any subject are in the minority. The number of students with vocational qualifications has increased as well as the number of overseas students. Even in the United Kingdom it has become apparent that the competence of students with the same entry qualification has changed [Ltsn05:2].

2.5.3 The use of Diagnostic tests in the UK

Many universities in the United Kingdom administer diagnostic tests to first-year students. These tests are typically taken during the induction week or the first few weeks of the academic year. The tests vary from simple paper-based tests to computer generated multiple choice questions to intelligent diagnostic tests. The test results are used mainly to assist students who do not have the necessary pre-knowledge. The tests also inform lecturers what level their students are at [ltsn05:4]. Lecturers can thus adjust the teaching to the needs of the group. The support provided to students varies from tutoring to providing self-study materials to computer assisted learning. Some universities allow students to re-take the test. Re-testing could be either compulsory or voluntary. Where computer packages such as Diagnosys are used, re-testing can be used to test student progress throughout a course [ltsn05]. The LTSN MathTEAM project carried out a survey in which they examined the following three topics, namely [ltsn05:3]:

- Mathematical support programmes provided and resources available
- Current practices for lecturing Mathematics to engineering and science students

- Diagnostic testing

The diagnostic testing information was published in a booklet called Diagnostic Testing for Mathematics. The purpose with the booklets was to transfer knowledge between higher education institutions. The higher education community in the UK has developed a need to share knowledge and to *stop re-inventing the wheel.* A Mathematics Learning Support Centre, called the 'Mathcentre' was established. The Mathcentre aims at providing both students and professionals free access to samples of high quality learning materials aimed at alleviating the school/university interface problem [Ltsn05:3]. In June 2000 the Engineering Council in the UK recommended to all universities that students embarking on mathematics-based degree courses should take a diagnostic test on entry [Ltsn05:3]. The processes followed at some UK universities will be looked at in more detail below.

2.5.4 Diagnostic testing at the University of the West in England (UWE)

Students at the University of the West in England wrote a paper-based multiple-choice test. Students take the test during induction week. The test was marked and returned during the first week of the term. Students experiencing substantial difficulty in a topic were referred to a Mathematics and Statistics Learning Centre for additional tutoring. The system was inexpensive to set up and worked well initially. The proportion of students demonstrating weaknesses increased and it became too difficult to manage the follow-up support. Some students did not participate in the support programme [Ltsn05:10].

The paper-based test was abandoned in favour of computer-based diagnostic tests. Students have three attempts at each test. The tests must be completed by the end of the second week of term. Students receive their scores immediately. Support is initially provided through tutorials. Students with significant problems are given one-to-one tutoring or they can go to the Mathematics and Statistics

Learning Centre. After receiving help students can attempt the tests again up to a maximum of three times [Ltsn05:10].

The preliminary results for the four diagnostic tests show that the average scores for the Algebra and equations tests were higher than the scores for the trigonometry and calculus tests. This is to be expected since the Trigonometry and Calculus tests require more advanced manipulation. The Algebra and Functions tests combined the transposition of formulae, equations, trigonometric functions and other topics such as partial fractions and complex numbers. The calculus test includes the standard rules of differentiation and integration, including integration by parts and parametric differentiation. The benefit of using computer-based tests and assessments is that it is reasonably efficient in staff resources. This computer-based diagnostic test was developed from a Computer Assisted Assessment programme as well as from material imported from the Mathletics CD-ROM. To set up the initial question bank takes considerable time. UWE were fortunate to have access to material that had already been quality tested. Good in-house IT support is vital. Technical issues include the managing of the question bank database and the assigning of login usernames and passwords [Ltsn05].

2.5.5 Diagnostic testing at Bournemouth University

At Bournemouth University all design and electronics undergraduates write a forty question, paper-based, multiple-choice test during their induction week. There is no time limit on the test. All students finish within 30 to 90 minutes. The test is pitched at GSCE level. Solutions on a transparent overlay allow the tests to be marked quickly. Optical character recognition will be used for marking in the future. The test includes Numeracy, Algebra and Geometry. Students with a good overall score, but who struggle with a particular section are given directed reading. Students who do not achieve an overall fifty percent score on the diagnostic test must attend a set of one-hour-per-week “Extra Maths” classes.

The extra maths sessions are run in parallel with the students' main lectures throughout the session. They take the form of a twenty minute discussion followed by exercises for the students to complete. A register is kept for students who obtained less than fifty percent on the test score. Students scoring between fifty and sixty percent are recommended to attend, while those scoring above sixty percent do not have to attend, but can do so if they so wish [Ltsn05:12]. A diagnostic test does not help weaker students unless a follow-up support programme is in place. Diagnostic testing and support assist student retention [Ltsn05:13].

2.5.6 Diagnostic testing at Queen's University Belfast

In the autumn of 2001 the School of Biology and Biochemistry at Queen's University Belfast introduced a "Skills in Biosciences" module for all Stage 1 undergraduate students [Ltsn05]. Within the "numerical skills" section of the skills module two one-hour lectures and a three-hour practical session are used to highlight the importance of numerical skills in the biological sciences. In the first of the two lectures students are given a thirty minute paper-based practice test to assess their basic mathematical skills and knowledge without the use of any calculator. The tests are marked and returned to students before their practical class the following week.

The second lecture deals with mathematical concepts that students must understand, but often experience difficulties with. These concepts include Measuring scales and SI units, manipulating equations, logarithms, power expressions and scientific notation. During the three-hour practical session the students access, use and evaluate four computer-based learning resources that can help them develop and practise their numerical skills [Ltsn05]. The resources include Key Skills Online, Maths for Microbiology and Numbers Count. The fourth resource is selected from a list of mathematics web sites. One week after the practical class students write another test. The use of calculators is prohibited.

This test is similar to the practice test and contributes four percent towards their final module mark. The purpose of the tests is to collect data on the deficits of incoming students and to inform students about their mathematical abilities. This gives students the chance to address weaknesses early in their courses [Ltsn05].

2.5.7 Diagnostic testing at the University of York

At the University of York electronics students write a mathematics test on their second day in the department. This gives lecturers an idea of the quality of the intake and informs the lecturers of any generic areas of weakness that may require attention. The same test has been used for the last fifteen years [Ltsn05:16]. The test is a test of knowledge. Fifty multiple-choice questions must be attempted in a two-hour period. Students are required to do only simple manipulation. Guessing is discouraged. The test results show that over the last few years the ability to cope with logarithms and powers has declined. For the first few years the test was used, the “worry line” occurred at sixty percent. Students, scoring below sixty percent, were advised to revise or go for special tuition. Today the bulk of the students fall into this category. An average student with Grade B in A-level mathematics obtains a score on the University of York test that is marginally better than the score that could be obtained by random guessing [Ltsn05:17].

2.5.8 Diagnostic testing at Cardiff University

At Cardiff University students write a paper-based test during week one of lectures. The test consists of twelve multiple-choice questions. An optical reader is used to mark the questions. The results are reported back to students by their personal tutors. The support provided is part of general tutorial support. Cardiff University results revealed students encountered problems with Algebra, Elementary Calculus, Trigonometry and Complex Numbers. Support has been formalised. Support hour sessions are held with approximately twenty students.

Monitoring attendance and restricting access to resources if attendance is poor encourage students to take advantage of the available opportunities [Ltsn05:18].

2.5.9 Diagnostic testing at Coventry University

At Coventry University students write a comprehensive diagnostic test during the first week of the academic year. There are two tests at Coventry University. One test is aimed at students enrolled for courses with an A-level in Mathematics entry requirement. The other test is aimed at students enrolled for courses with a GCSE Mathematics requirement. The Maths Support Centre manages the procedure. The tests are administered by members of staff from the Mathematics department. Engineering students were given an Optical Mark Reader answer sheet and a booklet containing fifty questions. The use of calculators was prohibited. The member of staff – administering the test – collects the booklets and answer sheets and returns them to the Maths Support Centre. The tests are analysed and a computer print-out given to each student. The print-out would indicate whether the student performed satisfactorily or has areas of weakness. Follow-up support is provided by the Maths Support Centre [Ltsn05:19].

2.5.10 Diagnostic testing at Queen Mary University of London

At Queen Mary University of London students write a two-hour long paper-based multiple-choice test. The “Essential Mathematics” test covers Algebra and Arithmetic. The test is written under examination conditions. The test can be repeated up to six times but must be passed before a student is allowed to proceed to second year. If a student repeatedly fails badly, training sessions become compulsory and the student has to drop a unit from the first year programme if they fail the fourth attempt [Ltsn05:21].

2.5.11 Diagnostic testing at the University of Strathclyde

The test at the University of Strathclyde was developed in 2001. It is a paper-based test. Students write the test in class. The duration of the test is one hour. The test informs students of their mathematical knowledge at the point when they enter university. The test consists of about twenty items and tests for the basic mistakes that students make repeatedly. Students enter the correct answer in a box next to the question and hand in the entire revision test sheet. The diagnostic results are given to the tutors. Tutors can encourage students to seek help and attend clinics. A number of follow-up procedures are in place. These include Mathematics clinics during lunchtime and TRANSMATH on the web. Students are sent sample problems and exercises of the level they have to achieve. The results indicate that students' mathematical knowledge is poor compared to what is required at university level [Ltsn05:22].

2.5.12 Diagnostic testing at the University of Sussex

The University of Sussex administers a paper-based multiple-choice test to prospective first-year students. The test is administered by two postgraduate students in a lecture theatre. The duration of the test is one hour. Answers are entered on a separate answer sheet. Tests are marked manually by the two post-graduate students. Marks are collated and passed onto the sub-dean. The post-graduate students run Mathematics Skills workshops. In the workshops students work through the Algebra Refresher from LTSN Maths, Stats and OR Network and the department's thirty-two page Skills exercises. The exercises are allocated individually by the postgraduate students. The results of the test can be used to convince students to attend the workshop. In the second and third term of the year the workshops are replaced by office hours during which the postgraduates can be consulted. The results show that qualitatively students are less fluent at performing Mathematics e.g. integration [Ltsn05:23].

2.5.13 Diagnostic testing at the University of Manchester Institute of Science and Technology (UMIST)

At the University of Manchester Institute of Science and Technology students write a paper-based assessment test on their first day in the department. The use of non-graphical calculators is permitted. Students must answer forty-eight questions of the type and standard they are familiar with from A-level. The test tests a student's competence. Duration is 80 minutes and consists of twelve sections of four questions each. The test is conducted over five rooms simultaneously. Students are given a formula sheet and a calculator. The answers are filled in on an answer grid. The administrator does the marking by hand. The turn-around time is five days. The test is used for temporary streaming. The students are split for the first five weeks of term. The group that does not perform satisfactorily will do a basic Mathematics revision course and are encouraged to address the problem areas as indicated by the diagnostic test. The revision course students are taught in small groups. An attendance record is kept. After five weeks of revision another test is written [Ltsn05:24].

2.5.14 Diagnostic testing at the Anglia Polytechnic University

At the Anglia Polytechnic University a diagnostic test is administered mainly to students entering the foundation year. "The foundation year is a preparatory year for students of science and technology-related subjects. The test helps the lecturer teaching the Foundation Maths to determine the level at which to pitch the various topics. A computerised diagnostic test called Diagnosys is used. Students receive immediate feedback on their strengths and weaknesses. There is no link between the test and any remedial measures. The test results are given to tutors and the tutors take remedial measures with students. Only the "moderately able" students made use of the drop-in centre and the "office hours" to see lecturers. Students are now assigned to tutor groups of eight to ten students in size [Ltsn05:26].

Students lack knowledge of powers, scientific notation, rounding to significant figures and graphs. The students do not like the time-out on the test. Administrators feel that the test is not very “administrator friendly”. It is not easy to run the test over a network or to add or change questions [Ltsn05:26].

2.5.15 Diagnostic testing at the University of Bristol

Engineering students at the University of Bristol write two computer-based tests. Each test consists of ten multiple-choice questions. “The tests are set using the “TAL” (Teach and Learn) computer system developed at the University of Bristol”. About one hundred students can simultaneously access the test through a web-browser. A student may pass a question and return to it later. Questions have time assigned to them. Feedback on incorrect questions is provided. The test was made easier recently, because the skill level of students has declined. Students can retake the test once they have done the required revision. Walk-in sessions and support classes are provided to assist students with problems [Ltsn05:27].

The TAL tests can be accessed via the web. Use is made of a question bank. Randomisation instils confidence in the tests as the risk of plagiarism is reduced. Lack of familiarity of students with computers could be a potential problem. The speed of the system could also be a problem. This can be overcome by acquiring a dedicated server for the TAL package provided the necessary funds are available [Ltsn05:27].

2.5.16 Diagnostic testing at Brunel University

Brunel University uses diagnostic tests that are of the Computer Aided Assessment (CAA) type. The testing level required is post GCSE or post A-level. Financial computing and Mathematics students take the tests. Testing shows deficiencies have arisen in students’ knowledge. The test results as well as the students’ profiles go to the tutors. Brunel University cautioned that financial constraints may make it difficult for universities to give students the support they need [Ltsn05:28].

2.5.17 Diagnostic testing at Keele University

Students at Keele University are given revision material from the LTSN Maths, Stats and OR Network prior to their arrival at University. Students are given a computer-based test. The test consists of twenty multiple-choice questions. Students have forty minutes to complete the test. The questions come from a question bank jointly developed by Nottingham and Keele Universities. The test covers work students should be familiar with from A-level. The diagnostic test gives an immediate profile of the student’s mathematical abilities. The computer package provides a list of “Mathwise” modules that the student should study to redress deficiencies in their skills. This is an efficient way of dealing with the problem of diverse knowledge needs. Bi-weekly tutorials are held to follow up on students [Ltsn05:29].

At Keele University the curriculum was modified to take a week out and “spend it on a differentiation *blitz*”. This is going to be augmented by an integration *“blitz”* as well [Ltsn05:29].

2.5.18 Diagnostic testing at the University of Newcastle upon Tyne

The University of Newcastle upon Tyne has been using Diagnosys since October 1993. Diagnosys was developed under the Teaching and Learning Technology Programme (TLTP). It is an intelligent knowledge-based system that tests background skills in basic Mathematics or other technical subjects. The package is used to assess a student’s knowledge and to provide support where needed. The student enters their name, department and level of mathematics attained. Based on this information the package selects the initial level of questioning. At the start of the test there is a tutorial which tells students how to enter different types of answers (e.g. number, multiple-choice or Algebra). The success rate of the student determines whether the student quickly passes to more advanced topics or is taken via the slower route. The “*expert system*” approach ensures that each student follows a different path through the test which prevents cheating [Ltsn05;30].

At the end of the group test the tutor downloads the information and transfers it to a disk. The individual student profiles are printed and handed out to the students. The tutor is also provided with a group profile of all the students tested, a ranked listing of the students in terms of scores, tabulated answers to highlight common mistakes and results of all questions and skills for subsequent spreadsheet analysis [Ltsn05:30].

2.5.19 Mathletics

Mathletics is a computerised diagnostic system used at some universities. Mathlectics is a free resource. About 120 universities and colleges have received a free copy. Seeing that there is a very blurred division between school mathematics and first year mathematics, Mathletics has also been repackaged for schools [Ltsn05].

2.6 Diagnostic Testing in Australia

Mathematics Departments in Australia have come under pressure to improve their pass rates. The Australian government changed the funding formula from enrolment to completion. The introduction of a bridging course in mathematics led to an increase in pass rates. The bridging course was instituted to smooth the transition to university. Queensland University of Technology uses a diagnostic test to identify students who are at risk. Remediation or support programmes are recommended for these students. The University of Technology, Sydney has started diagnostic testing as well. Queensland University of Technology reported that optional diagnostic testing is unsuccessful – even in an online environment [Carmod06:24-25].

2.7 The importance of investigating testing procedures followed elsewhere at other universities

Investigating how other institutions deal with the problem of under-prepared students will assist this researcher in deciding how to approach the problem of under-prepared students at NMMU.

2.8 Conclusion

The process of determining admissions criteria and dealing with under-prepared students is extremely complex.

In the words of Harman (1994), "Selection is often both a highly complex technical matter and a political one. It is highly technical in terms of the choice of the method or methods used and judgements made about the utility of different methods. It is political in the sense that whatever method used can be readily contested, both on technical grounds as well as on social and economic grounds" [Harman94:316].

Significant research has been done in the USA and the UK. From the literature it is clear that there are no easy answers. Although standardised test scores are good predictors of mathematical examination scores they are not good predictors of overall mathematics success. Factors such as desire, motivation and peer study have a large effect on class performance [Olson03]. A study performed in the UK showed that the widely used SAT test is a test of achievement albeit not curriculum specific. The GCE A-level examination is also a test of achievement and curriculum specific. The study found that the UK would not really benefit from the introduction of a test like SAT.

From the literature (Cf. 2.3), it is evident that it is important to align what is measured in a diagnostic test with both the school curricula and the entry level knowledge and skills required of university students. This suggests that it is unlikely that a diagnostic test developed in another country based on another school curriculum will be suitable for use in South Africa and that local tests will have to be developed.

South Africa with its limited financial resources cannot afford to spend large sums of money on university applicants who have little or no chance of academic success [Huys97:65]. It is important that selection criteria be identified. However, tests scores alone should never be used for admission purposes. Huysamen (1997) pointed out that admission testing should also be used to identify students who will benefit from academic support programmes, remedial classes, awards or scholarships [Huys97:66].

Given the complex problems that the education system in South Africa is experiencing, the best approach for NMMU would be to test prospective first-year students to determine whether they have the necessary skills to successfully complete a first year course in mathematics. In Chapter 5 the decision between a placement or diagnostic test will be made.

Chapter 3 analyses first year modules which formed part of the study to determine the pre-knowledge (required pre-knowledge) a student requires to successfully complete Mathematics 1. The required pre-knowledge was compared to the school syllabi and in particular the standard Grade school syllabi for Grades 11 and 12 to determine whether a first-year student has all the required pre-knowledge and skills (acquired pre-knowledge).

CHAPTER 3 – Comparing the required university pre-knowledge and the acquired school knowledge

3.1 Introduction

In this chapter the necessary pre-knowledge and skills that a prospective first-year student should have were identified. This was achieved by analysing the first year curricula of the Mathematics 1 modules that formed part of the study. These modules depend entirely on secondary education pre-knowledge. The pre-knowledge identified was compared with the Department of Education's school syllabi for 2003 for standard grade and higher grade pupils to determine whether the learners are adequately prepared for first-year Mathematics.

The phrase knowledge and skills in this research study should be interpreted in a broader context. The phrase not only refers to the skills associated with the cognitive level named knowledge, but also to some of the skills associated with the other cognitive levels namely routine procedures, complex procedures and problem solving. Table 3.1 details the cognitive levels and the skills associated with the cognitive levels that were targeted in the study [DeptEd07]. Whenever reference is made to knowledge in this research, study skills at all other cognitive levels are included by implication.

Table 3.1 Cognitive levels associated skills targeted in the study.

Cognitive Levels	**Explanation of skills to be demonstrated**
Knowledge	• Algorithms • Appropriate rounding of numbers • Theorems • Straight recall • Simple mathematical facts • Knowledge and use of formulae
Routine procedures	• Perform well-known procedures • Simple applications and calculations which have many steps may require interpretation from given information
Complex procedures	• Problems do not have a direct route to the solution, but involve: o mathematical reasoning processes
Solving Problems	• Being able to break down a problem into its constituent part − identifying what is required to be solved and then using appropriate methods to solve the problem

Source: [DeptEd07].

3.2 Admission requirements at NMMU at the time of the study

The admission requirements for students to study for a Baccalaureus Scientiae degree at NMMU in 2004 were a weighted matriculation average mark of 55 percent as well as a mark of 40 percent in Mathematics higher grade or 60 percent in Mathematics standard grade. For Biological Sciences and Earth Sciences a weighted matriculation average mark of 55 percent and a pass mark in Mathematics higher grade or in Mathematics standard grade were required. To register for any Mathematics or Applied Mathematics module a learner required a score of 60% in Mathematics standard grade or 40% in Mathematics higher grade as well as 40% in English First Language higher grade or 50% in English Second Language higher grade. The current minimum requirement to enrol for a Mathematics 1 course is 70 percent on standard grade.

In setting the entry requirements, it was assumed that all the pre-knowledge required was included in the standard grade curriculum. In the next section, mathematical pre-knowledge will be identified and compared with the standard grade Mathematics school curriculum to determine whether learners have had opportunities to acquire all the pre-knowledge and skills needed for first-year Mathematics.

3.3 Pre-Knowledge and Skills Required by a First-Year Mathematics Student

The detailed syllabi for the first-year mathematics courses used in the study were analysed by the researcher. Some first year Mathematics courses use a comprehensive set of class notes, others use study guides in combination with a first-year textbook. The course material i.e. the class notes and the study guides cover the entire curriculum. The pre-knowledge and skills a first year student requires were identified from an analysis of the course material. Each chapter in the notes or study-guide was analysed to determine the pre-knowledge a student requires in order to successfully master the material in the section. The first course was analysed and the pre-knowledge required was identified and listed. Additional pre-knowledge required for the second, third and fourth courses was added to the list. The analysis identified the following list of required university pre-knowledge topics:

- Knowledge of the following number systems
- Factorisation
- Fraction Addition
- Values for which a rational function is undefined versus the values which will make a rational function equal to zero.
- Square roots can only be taken for non-negative numbers
- Sketching straight lines
- Sketching parabolas
- Sketching of circles and semi-circles
- Sketching of hyperbolas
- Graphs of trigonometric functions
- Trigonometric identities
- Knowledge on the Laws of indices

Each topic from the required university pre-knowledge list was analysed in more detail in subsequent paragraphs and compared with the school syllabi to determine whether the required university pre-knowledge was covered.

3.3.1 Knowledge of the following number systems

Knowledge of the number system is extremely important for set theory. To tabulate sets or write sets using set builder notation, students need to know the following sets:

- Natural numbers
- Whole numbers
- Integers
- Rational numbers
- Irrational numbers
- Real numbers

The Grade 11 syllabus starts with a brief review of real numbers for both standard grade and higher grade. Thus learners should have encountered the other number systems, which make up the real numbers, in earlier grades.

3.3.2 Factorisation

Factorisation is used, not only to solve for unknowns in quadratic expressions where the discriminant is a perfect square, but also for simplification. Certain limits can only be solved algebraically after they have been simplified. Students often encounter problems in calculating such a limit, because they cannot factorise. When sketching curves some students cannot calculate the $x-\text{intercepts}$, or critical numbers or inflection points, because they have not acquired the skill of factorisation. The following factorization pre-knowledge is essential:

- $c^2 + 2cd + d^2 = (c+d)(c+d)$
- $c^2 - 2cd + d^2 = (c-d)(c-d)$

- $c^2 - d^2 = (c-d)(c+d)$
- $ac + ad = a(c+d)$

Learners at the Grade 11 level must be able to find the roots of $ax^2 + bx + c = 0$ where a,b and $c \in \{\text{rational numbers}\}$ with $a \neq 0$. Apart from solving $ax^2 + bx + c = 0$ learners should know under which circumstances the equation is solvable on the set of real numbers and whether the roots are rational and equal, rational and not equal or irrational and unequal. To solve a quadratic equation learners should be able to factorise or use the formula.

3.3.3 Fraction Addition

In elementary differentiation and integration, some students often do not realise that they have to simplify the problem using fraction addition before attempting to differentiate or integrate.

- $\frac{c+d}{e} = \frac{c}{e} + \frac{d}{e}$

Learners use fraction addition to simplify a function before differentiating it and are thus familiar with the concept.

3.3.4 Values for which a rational function is undefined versus the values which will make a rational function equal to zero.

Observation during classroom exercises showed some students did not know the difference between the two concepts. When asked to supply the values for which the rational function would equal zero, some students either supplied the values for which the rational function was undefined or the values for which the rational function equalled zero as well as the values for which the rational function was undefined. The fact that division by zero was not allowed was not always remembered by students. This lack of knowledge affected the calculation of the domain of a rational function as well as how limits had to be calculated algebraically.

Functions form part of the Grade 10 syllabus and are extended in the Grade 11 syllabus to the function $y = ax^2 + bx + c$. In Grade 11 learners should know that division by zero is not allowed. At Grade 11 level, learners should know how to find the domain and range of a function, as well as the values for which the functions will equal zero.

3.3.5 Square roots can only be taken for non-negative numbers

Classroom observation showed that when a $\sqrt{\ }$ was placed in the denominator of a rational function some students did not exclude the value(s) for which the root would equal zero when stating the domain of the rational function.

In Grade 11 level learners are dealing with real numbers and should know, they cannot take the square root of a negative number. This fact impacts the determination of the domain of some functions.

3.3.6 Sketching straight lines

When the defining equation for a straight line is given to students, some first-year students cannot sketch the straight line. The researcher made the observation during a summer recess programme. The problem is even worse when students have to manipulate the defining equation into the format $y = mx + c$. The following pre-knowledge is required to sketch a straight line:

- Write the equation in the standard format $y = mx + c$
- Use c and the gradient m to sketch the graph or
- Find two points which satisfy the equation and sketch the graph

The following pre-knowledge is required to calculate the equation of a straight line:

- A point satisfying the equation has to be known and the gradient has to be calculated **or**

- Find two points $(x_0; y_0)$ and $(x_1; y_1)$ which satisfy the equation and calculate the gradient $m = \dfrac{y_1 - y_0}{x_1 - x_0}$
- Students have to know that if they are given the defining equation for one line and a point on the second line and they are told that the two lines are perpendicular to each other, then the gradient of the second line will be equal to $\dfrac{-1}{\text{gradient of first line}}$.
- Students have to know that if they are given the defining equation for one line and a point on the second line and they are told that the two lines are parallel, then the gradient of line two will be equal to the gradient of line one.

It is essential that students know how to sketch a straight line and a parabola. Students must be able to identify in the sketch the area bounded by two curves. Very often the two curves will be a straight line and a parabola. Students need to identify the bounded area to calculate the area bounded by the two curves.

At Grade 11 level learners should be able to draw a straight line. No point-by-point graphs are required for examination purposes. Learners know the following facts at this stage:

- The equation for a straight line is $y = mx + c$.
- The $y-\text{intercept}$ of a straight line is denoted by c.
- The gradient of the line is given by $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$.
- If two lines are parallel then their gradients are the same.
- If two lines are perpendicular then the product of their gradients equals -1.

- Learners should be able to sketch a straight line by calculating the $x-$ and $y-$intercepts. (Provided both points are not equal to zero, in which case a third point would have to be calculated.)
- Learners should be able to use the $y-$ intercept and the gradient to draw a straight line.

3.3.7 Sketching parabolas

Some students cannot sketch a parabola from its defining equation. The researcher made this observation during a summer school for Mata102 students. When given the equation $y = ax^2 + bx + c,\ a \neq 0,$ students should know the following facts:

- Substituting $x = 0$ into $y = ax^2 + bx + c,\ a \neq 0,$ will reveal the $y-$intercept (i.e. the $c-$value)
- Solving $ax^2 + bx + c = 0, a \neq 0,$ for x when $\Delta \geq 0$ will reveal the $x-$intercepts
- $a < 0$ indicates a maximum turning point
- $a > 0$ indicates a minimum turning point
- $x = -\frac{b}{2a}$ gives the axis of symmetry
- $y = -\frac{\Delta}{4a}$ gives the maximum or minimum value, or substitute $x = -\frac{b}{2a}$ into the original equation to obtain y
- $(-\frac{b}{2a}; -\frac{\Delta}{4a})$ gives the coordinates of the turning point

In Grade 11 learners are introduced to the function $y = ax^2 + bx + c,\ a \neq 0$ as well as alternative forms of the parabola. The alternative forms they are introduced to are:

- $y = a(x-p)^2 + q$ with $(p;q)$ as turning point
- $y = a(x-\alpha)(x-\beta)$ where α and β are the $x-$intercepts

Learners should be able to deduct the characteristics of the function $y = ax^2 + bx + c,\ a \neq 0$ and its alternative forms from their equations. Learners know the following characteristics of quadratic functions:

- $a > 0$ indicates that the parabola will have its minimum $y - \text{value}$ at $y = -\dfrac{\Delta}{4a}$
- $a < 0$ indicates that the parabola will attain a maximum $y - \text{value}$ at $y = -\dfrac{\Delta}{4a}$
- $x = -\dfrac{b}{2a}$ is the equation of the axis of symmetry of the parabola
- $(-\dfrac{b}{2a}; -\dfrac{\Delta}{4a})$

Learners should be able to sketch the graph of the quadratic function $y = ax^2 + bx + c,\ a \neq 0$. Furthermore, learners should be able to find the equation of the quadratic function under the following circumstances as well:

- when the turning point and one other point is given
- the $x - \text{intercepts}$ and one other point is given and
- the $y - \text{intercept}$ and any two points are given

3.3.8 Sketching of circles and semi-circles

- A Mathematics 1 student should know that $x^2 + y^2 = r^2$ represents a circle with centre the origin and radius r and
- A circle consists of two semi-circles given by $y = +\sqrt{r^2 - x^2}$ and $y = -\sqrt{r^2 - x^2}$ or $x = +\sqrt{r^2 - y^2}$ and $x = -\sqrt{r^2 - y^2}$

  A learner in Grade 12 should know the defining equation of a circle and should be able to sketch a circle with centre the origin and radius r. Learners should also know the equations of a semi-circle and be able to draw a semi-circle. Furthermore learners should know that the coordinates of any point on a circle should satisfy the equation of the circle.

3.3.9 Sketching of hyperbolas

- Students should know that $xy = k$ or $y = \frac{k}{x}$ represents a hyperbola
- If $k > 0$ then x and y have the same sign and the graph of the hyperbola will lie in the first and third quadrants
- If $k < 0$ then x and y have opposite signs and the graph of the hyperbola will lie in the second and fourth quadrants

Learners should have the necessary knowledge to recognise the defining equation of a hyperbola and sketch it.

3.3.10 Graphs of trigonometric functions

Students should be able to sketch the following graphs:

- $y = \sin x$
- $y = \cos x$
- $y = \tan x$

Students should know the following facts about these graphs:

- amplitude
- period

Students should know how to use the graphs of $\sin x$ and $\cos x$ to sketch the graphs of $y = a \sin x$, $y = a \cos x$ and $y = a \tan x$ as well as

$y = \sin ax$, $y = \cos ax$ and $y = \tan ax$

An analysis of the Grade 11 and 12 syllabi show that standard grade learners have to be able to draw the following functions:

- $y = \sin x$
- $y = \cos x$
- $y = \tan x$

Learners should know how to use the graphs of $\sin x$ and $\cos x$ to sketch the graphs of $y = a \sin x$, y = a $\cos x$ and $y = a \tan x$ as well as $y = \sin ax$ and $y = \cos ax$.

Higher grade learners have to be able to draw all of the above graphs including the graph of $y = \tan ax$. In addition to the graphs identified in the required pre-knowledge section higher grade pupils also have to be able to sketch the graphs of $y = a + \sin x$, $y = a + \cos x$ and $y = a + \tan x$ as well as $y = \sin(a+b)$ and $y = \cos(a+b)$

Learners are only examined on one deviation from the basic graph. Either period or amplitude will be examined.

3.3.11 Trigonometric identities

The following trigonometric identities are used regularly:

Quotient Identities

- $\tan x = \dfrac{\sin x}{\cos x}$
- $\cot x = \dfrac{\cos x}{\sin x}$

Square Identities

- $\sin^2 x + \cos^2 x = 1$
- $\sec^2 x = 1 + \tan^2 x$
- $\cot^2 x + 1 = \cos ec^2 x$

Reciprocal Identities

- $\sin x = \dfrac{1}{\csc\ x}$
- $\cos x = \dfrac{1}{\sec x}$
- $\tan x = \dfrac{1}{\cot x}$

Identities involving negative angles

- $\sin(-x) = -\sin(x)$

- $\cos(-x) = \cos x$
- $\tan(-x) = -\tan x$

Addition and subtraction formulae

- $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
- $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

Writing the function value $(90^0 - x)$ as a function value of x where $x \in [0^0; 90^0]$

- $\sin(\frac{\pi}{2} - x) = \cos x$

Double-Angle Formulae

- $\sin 2x = 2\sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
- $\cos 2x = 2\cos^2 x - 1$
- $\cos 2x = 1 - 2\sin^2 x$

Half-Angle formulae

- $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
- $\sin^2 x = \dfrac{1 - \cos 2x}{2}$

Product formulae

- $\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$
- $\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$
- $\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$

Triangles giving exact trigonometric ratios for the angles $30^0, 60^0$ and 45^0

Trigonometric identities are used in the proofs of the derivatives of trigonometric and inverse trigonometric functions and in more advanced integration techniques. Inverse trigonometric functions and advanced integration are only defined and taught in the second semester for first-year mathematics students or in the second semester of second year for Mathematics Special students. The double-angle, half-angle and product formulas were however included in the list for the sake of completeness.

An analysis of the Grade 11 and Grade 12 syllabi reveals that learners should be familiar with quotient and reciprocal trigonometric functions

- $\tan x = \dfrac{\sin x}{\cos x}$
- $\cot x = \dfrac{\cos x}{\sin x}$
- $\sin x = \dfrac{1}{\csc\ x}$
- $\cos x = \dfrac{1}{\sec x}$
- $\tan x = \dfrac{1}{\cot x}$

Learners should also be familiar with the following quadratic formulae:

- $\sin^2 x + \cos^2 x = 1$
- $\sec^2 x = 1 + \tan^2 x$
- $\cot^2 x + 1 = \csc^2 x$

An analysis of the Grade 11 and Grade 12 syllabi reveals that standard grade pupils do not deal with negative angles. Negative angles are included in the higher grade syllabus only.

- $\sin(-x) = -\sin(x)$
- $\cos(-x) = \cos x$
- $\tan(-x) = \tan x$

Learners should be able to express function values of $(90^0 - x)$, $(180^0 \pm x)$ and $(360^0 - x)$ as function values of x where $x \in [0^0;90^0]$

Higher grade pupils should be able to express a function value of $(360^0 + x)$ as a function value of x where $x \in [0^0;90^0]$.

All learners should thus be familiar with the following trigonometric equation

- $\sin(\frac{\pi}{2} - x) = \cos x$

Standard grade pupils do not deal with trigonometric addition or subtraction formulas. Higher grade pupils have to know the addition and subtraction formulas as well:

- $\cos(x \pm y)$
- $\sin(x \pm y)$
- $\tan(x \pm y)$

Standard grade pupils do not deal with double-angle, half-angle or product formulas. Higher grade pupils have to know the double-angle formulas as well as derivations of these formulas.

3.3.12 Knowledge on the Laws of indices

Students must be familiar with the following facts:

Definition: $a^n = a.a.a \ldots\ n$ factors of a where $a \in \mathbb{R}$ and $n \in \{0,1,2,\ldots\}$ and $a \neq 0$

Laws of indices:

Let $a,b \in \mathbb{R}$ and m,n $\in \{0,1,2,\ldots\}$ then

- $a^m \cdot a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$

Deductions

- $a^{-m} = \dfrac{1}{a^m}$
- $\dfrac{1}{a^{-m}} = a^m$
- $a^0 = 1$ if $a \neq 0$

Students should know the relationship between surds and indices. Both higher grade and standard grade pupils in Grade 11 should know the laws of indices. Both standard grade and higher grade pupils have to be able to simplify expressions with one term only as well as polynomials. Learners have to solve for the unknown in an exponential equation when the unknown is in the base or in the exponent.

3.3.13 Knowledge of the Laws of Surds

$\sqrt[n]{a}$ is defined as the positive n-th root of a, $a \geq 0$ and $n \in \mathbb{N}$.

$\sqrt[n]{a} = a^{\frac{1}{n}}$

Laws of surds:

- Let $a > 0$ and $b > 0$ and $n \in \mathbb{N}$ and $m \in \{0, 1, 2, \ldots\}$ then
- $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
- $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$
- $(\sqrt[n]{a})^m = \sqrt[n]{a^m} = a^{\frac{m}{n}}$
- $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

An understanding of the rules of indices and surds is required to answer questions on differentiation and integration. Students often experience problems converting surds to indices. The researcher observed this when marking questions on differentiation.

Both higher grade and standard grade pupils in Grade 11 should know the rules of surds. Both standard grade and higher grade pupils have to be able to simplify expressions with one term only as well as polynomials.

3.4 Conclusion

In Chapter 3 first-year university curricula were analysed to identify the mathematical pre-knowledge and skills a Grade 12 learner should have. Then the school syllabi were looked at to see if the pre-knowledge and skills were covered.

The comparison between the pre-knowledge a first-year student should bring to the university and the Grade 11 and Grade 12 standard grade mathematics syllabi reveals that a prospective first year comes to university with almost all the pre-knowledge except for:

- Standard grade pupils do not work with negative angles
- Standard grade pupils do not deal with trigonometric addition or subtraction formulae
- Standard grade pupils do not deal with double-angle, half-angle or product formulae.

Double-angle, half-angle or product formulae are only used in the second semester of the first year for main stream mathematics students or during the second semester of the second year for Mathematics Special students. Students should thus be able to look up the formulas and study them. An appendix in the pre-scribed first year textbook reviews trigonometry.
Mathematics special students are issued with a set of notes containing amongst others the double-angle, half-angle and product formulae. Students should be mature enough at this stage to be able to review these formulae on their own.

Negative angles are only encountered when radian measure is studied. Radian measure is seldom examined in first semester examinations and if so very often the angle is positive.

Trigonometric addition or subtraction formulae are used when derivatives of trigonometric functions are proved. The formulae are given in students’ notes in the case of Mathematics Special and in an appendix in the prescribed text book for main stream mathematics students.

A learner taking mathematics on the higher grade level has all the required pre-knowledge and has an advantage over the standard grade pupil. However, a standard grade pupil has most of the pre-knowledge and should definitely do a mathematics 1 course. This raises the very important question 'why does the student fail when – on paper – there is no reason for this to happen ?'. One possible reason is that learners rely on their continuous assessment (CASS) marks to pass the examination. Continuous assessment (CASS) contributes twenty-five percent to the student’s final mark. A student with a high continuous assessment mark does not need to perform that well in the final examination to obtain a minimum of sixty percent on standard grade or forty percent on higher grade in the examination.

The CASS average for scientific subjects is often much higher than the exam average. Adjustments are done at the school level. If the CASS average is between 5 and 10 percent above the exam average then the CASS average is not adjusted [Markin06:55-56]. Another reason for the high failure rate is that students do not retain Grade11 and Grade 12 knowledge. Some students remark verbally or on their scripts that they do not remember how to do some questions. Students are ocassionally given a tutorial test based on school work as revision during their first tutorial session. The course coordinator responsible for the course schedule will decide whether this tutorial test will be written or not.

Mathematics builds on prior knowledge. If the students cannot retain the knowledge they learnt in previous years they are going to struggle in subsequent years at university. It is vitally important that a student has the required pre-knowledge and skills before they enrol for a first-year course in Mathematics. Students should be tested before they enrol. If the test results show the student has deficiencies; the student should be sent to do compulsory remedial work before they are allowed to enrol for a first-year course in Mathematics.

The National Academy of Sciences [Nation03], in Washington DC, identified five components of Mathematical Proficiency namely:

- Conceptual understanding
- Procedural fluency
- Strategic competence
- Adaptive reasoning
- Productive disposition

Conceptual understanding refers to the comprehension of mathematical concepts, operations and relations, whereas **procedural fluency** refers to the skill in carrying out procedures flexibly, accurately, efficiently and appropriately. **Strategic competence** refers to the ability to formulate, represent and solve mathematical problems. **Adaptive reasoning** refers to the capacity for logical thought, reflection, explanation and justification, whereas **productive disposition** refers to the habitual inclination to see Mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy. The five components represent different aspects of a complex whole [Nation03].

Students who do not have the necessary conceptual understanding and procedural fluency will always struggle with their mathematical proficiency. As Mathematics impact on a variety of subject areas this could have far reaching consequences for these students. It is vitally important that a student acquires the necessary pre-knowledge and skills before they enrol for a first-year course in

Mathematics. If a student's deficiencies are not corrected before they enrol for a first-year course the student will struggle in subsequent years even if they pass the first-year course and the student may thus never reach their full potential.

Chapter 4 sections 4.2 – 4.3 will illustrate some of the school-related errors made by students. The errors made during the 2003 Mathematics Matriculation examination will also be considered in section 4.4.

CHAPTER 4 – School-related Errors made by students

4.1 Introduction

Chapter 3 identified the required university pre-knowledge that a student should have to successfully complete a Mathematics 1 degree course and compared this required pre-knowledge with the expected school knowledge (acquired curriculum). The acquired pre-knowledge and skills were identified using the Mathematics school syllabi for Grades 11 and 12. On paper it certainly looked as if the standard grade learner should be adequately prepared. However the school-related mistakes first-year students made, which will be reported in this chapter, led this researcher to believe that students' pre-knowledge and skills were suspect.

To gather data on students' areas of weakness this researcher used a qualitative approach. Methods such as observation and unstructured interviews were used to gain in-depth knowledge of students' areas of weakness. These methods are recognised data collection techniques [Devos98:70]. In Chapter 7 the areas of weakness will be analysed quantitatively.

This researcher analysed some work products of first-year students for the modules focused on in the study to determine the school-related mistakes first-year students made. Some of the errors they made were recorded and examples of the errors are illustrated in paragraph 4.3. A comprehensive list of students' mistakes was compiled in 4.2.4. The items on the error list arose as a result of observations by the researcher, work product analysis by the researcher and feedback from colleagues. The list of errors made by students during the 2003 Matriculation examination corroborates the errors identified by the researcher. The researcher's final error list corresponded to a large extent with the required pre-knowledge identified (Cf. 3.3). The researcher focused first and foremost on the errors made by students studying Algebra and Calculus courses. Arithmetic errors made were also identified. This was done using a group of education students – many with Grade 12 mathematics – enrolled for an arithmetic course.

4.2 Analysis of Some Work Products of Students

Work products of Pure Mathematics and Mathematics Special first year students were analysed to determine which school-related errors they made.

4.2.1 Mistakes Made by In-Service Educators attending Upgrading Courses

Pure Mathematics courses were presented to educators who wished to upgrade their Mathematics knowledge. Many of the educators were already teaching mathematics. They attended recess programmes that covered the first-year modules. The educators made a number of school-related Mathematics errors.

Mistakes made by educators:

- calculate a limit using factorisation to eliminate the division by zero problem
- sketch a parabola
- write a surd as an exponent
- add fractions
- factorise

There were several groups of educators attending the recess programmes. All groups did not necessarily make these errors. Some groups were mathematically weaker than others.

4.2.2 Mistakes identified from Mathematics Special Scripts

The course content of the Mathematics Special courses is in essence the same as the content of the first two Pure Mathematics courses. Mathematics Special is less theoretical than the Pure Mathematics courses. Mathematics Special is presented over two semesters and the two Pure Mathematics courses over one semester. Students enrolled for the Mathematics Special course are not necessarily mathematically strong and one could expect more errors emanating from the Mathematics Special group than from the Pure Mathematics group. The lecturer thus focussed on analysing the work products of Mathematics Special students. Commerce and some science students enrol for Mathematics Special.

An analysis of the scripts of a batch of Mathematics Special students by the researcher revealed the following mistakes:

- Some students did not know the notation for an ordered pair
- Some students were not familiar with subsets of the real numbers
- Some students made mistakes relating to the rules of indices and surds
- Some students could not solve a quadratic equation using the formula
- Some students could not factorise
- Some students could not simplify an expression
- Some students could not write a surd as an exponent
- The difference between a single term and a finite series of terms is unclear to some students
- Substitution mistakes
- Addition of fractions
- Multiplication of fractions
- Subtraction of negative numbers
- Derivative notation
- Differentiation
- Notation for open, closed and semi-closed intervals

- Some students cannot find the term T(k+1) when given a finite series

Some students in Mathematics courses need calculators to add integers when the signs differ and consequently struggle with elementary differentiation problems. A third list was compiled of the mathematical problems first-year students experienced. Items on this list arose from observations made by the researcher as well as by colleagues. The second list will be detailed in the following section.

4.2.3 School Related Mistakes Identified through Observation

The researcher observed students during tutorials and recess programmes. The researcher e-mailed colleagues teaching first-year Mathematics and Statistics and requested they list school-related mistakes. Some of the items below were also generated through informal discussion with colleagues. Experienced members of staff commented on the items listed below. These observations revealed mathematics students' struggle with:

- decomposition into powers
- fractions
- rules of indices and surds
- factorisation
- drawing of straight lines or parabolas
- calculating the constant c in $y = mx + c$ when the other variables are known
- remembering the formula to solve for x in a quadratic equation, $ax^2 + bx + c = 0$
- the sum, difference and product rule for logarithmic functions
- simplification of expressions
- writing surds as indices
- substitution into a function
- formal proofs

- limits using substitution
- differentiation
- curve sketching
- logarithmic functions
- exponential functions
- converting from logarithmic to exponential form and vice versa
- number systems
- integers
- decimals
- ratios and proportions
- notation

All errors listed were combined into one final Algebra and Calculus error list in the next section.

4.2.4 Final Algebra and Calculus Error List

This final error list was composed from the previous three error lists:

- Some students did not know the notation for an ordered pair
- Some students were not familiar with subsets of the real numbers
- Some students made mistakes relating to the rules of indices and surds
- Some students could not solve a quadratic equation using the formula
- Some students could not factorise
- Some students could not simplify an expression
- Some students could not write a surd as an exponent
- The difference between a single term and a finite series of terms is unclear to some students
- Substitution mistakes
- Operations with fractions
- Subtraction of negative numbers

- Make notation errors
- Notation for open, closed and semi-closed intervals
- Some students cannot find the term T(k+1) when given a finite series
- Cannot calculate a limit using factorisation to eliminate the division by zero problem
- Cannot sketch a parabola
- Decomposition into powers
- Cannot sketch a straight line
- calculating the constant c in $y = mx + c$ when the other variables are known
- Cannot apply the sum, difference and product rule for logarithmic functions
- Cannot answer a formal proof question
- Cannot sketch a curve
- Unfamiliar with the definition of logarithmic functions
- Unfamiliar with the definition of exponential functions
- Cannot convert from logarithmic to exponential form or vice versa
- Cannot add negative integers without using a calculator
- Cannot perform operations on decimal numbers
- Cannot construct ratios and use proportions

Some of the items on this list were also identified as problem areas at universities in the United Kingdom (Cf. 5.11). This corroborates the error list composed by the researcher and illustrates that NMMU is not the only university enrolling students with gaps in their school knowledge.

4.2.5 **Mistakes made by General Mathematics Students**

The identification of Algebra and Calculus errors was more important than Arithmetic errors. The focus of the study is on first-year courses where the content focuses on Algebra and Calculus. However students do make arithmetic mistakes (Cf. 2.4.1, 4.2.4 and 2.5.9). First-year arithmetic errors are evidenced when students have to differentiate and integrate.

The General Mathematics course is designed for foundation phase teachers. The course focuses on Arithmetic. Some students enrolling for the course have Grade 12 mathematics and others only Grade 7 mathematics knowledge and skills. The researcher decided to observe areas where the General Mathematics students err. Areas where General Mathematics students err:

- Operations on fractions (especially multiplication and division)
- Ratio and proportion
- Percentages exceeding 100
- Decimals
- Rounding
- Word sums
- Percentage discounts
- Order of operations

The errors on the error lists are corroborated by the examples of actual errors - made by students - illustrated in the next section.

4.3 Examples of Mistakes Made by Mathematics Students

The course content of the Mathematics Special courses is in essence the same as the content of the first two Pure Mathematics courses. Mathematics Special is less theoretical than the Pure Mathematics courses. Mathematics Special is presented over two semesters and the two Pure Mathematics courses over one semester. Students enrolled for the Mathematics Special course are not necessarily mathematically strong and one could expect more errors emanating from the Mathematics Special group than from the Pure Mathematics group. Commerce and some science students enrol for Mathematics Special.

An analysis of students' work products revealed that students struggle not only with Algebra and Calculus concepts, but also with basic Arithmetic. A few examples of the school-related mistakes made by a group of Mathematics Special students are listed below to illustrate students' lack of school-related knowledge:

Example 1:

$$x \times x = 4$$
$$2x = 4$$

Student confused $x \times x$ with $x + x$

Example 2:

$$4 - 1 = 4$$

Arithmetical Errors

Example 3:

$$2(-4 - 9) = 16$$

Arithmetical Errors

Example 4:

$$\frac{2-3}{2+2} = \frac{-4}{6}$$

Arithmetical Errors

Example 5:

$$\frac{1}{36} + \frac{2}{9} = \frac{1+18}{36} = \frac{19}{36}$$

Arithmetical Errors / Fraction Addition

Example 6:

Arithmetical Error

$$x^2 = 4$$
$$x = \pm 4$$

Example 7:

$$x^2 = 4$$
$$x = 2$$

Student not completely familiar with the use of the square root / factorisation technique for solving quadratic equations

Example 8:

$\frac{1}{\sqrt[3]{x}} = x^{\frac{-3}{2}}$

Student not familiar with the rules for surds

Example 9:

$\sqrt{x^3} = x^{\frac{3}{1}}$

Student not familiar with the rules for surds

Example 10:

$\sqrt[5]{5+x^2} = \sqrt[5]{5} + \sqrt[5]{x^2}$

Student not familiar with the rules for surds

Example 11:

$\frac{x}{x^{\frac{1}{2}}} = x^{-1}$

Student not familiar with the rules for indices and surds

Example 12:

$(3+x)\times x^{\frac{-1}{2}} = 3 + x \times x^{\frac{-1}{2}} = 3 + x^{\frac{1}{2}}$

Student not familiar with the use of brackets

Example 13:

Students had to sketch the parabola $y = -x^2 + 8x - 12$.

This student could not find the x-intercepts of the parabola. The student also had no idea whether the parabola would open upwards or downwards. The axis of symmetry could not be calculated. The attempted sketch was thus incorrect.

Example 14:

$x - x^2 = x^2 - 2x + 1$

$\Rightarrow x - x^2 - x^2 + 2x - 1 = 0$

$\Rightarrow -2x^2 + 3x + 1 = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

Arithmetical Errors

When sketching the two parabolas the student could calculate the $x-\text{intercept}$ correctly for the parabola $y = x^2 - 2x + 1$, but could not calculate $x-\text{intercepts}$ for the parabola $y = x - x^2$ correctly. The student did not calculate the axis of symmetry for any of the parabolas. The student found the $y-\text{intercept}$ for the parabola $y = x^2 - 2x + 1$, but cannot find the $y-\text{intercept}$ for the other parabola $y = x - x^2$.

Example 15:

$3^x = 2$

$\Rightarrow \ln 3^x = \ln 2$

$\Rightarrow x \ln 3 = \ln 2$

$\Rightarrow x.1.3 = 1.2$

Definition of a logarithmic function not mastered.

Example 16:

$\ln 4x = 1$

$\therefore x = \dfrac{1}{\ln 4}$

Cannot convert from logarithmic to exponential form

Example 17:

Solve for x in each case:

$\log 10^x = -2$

$\therefore x = \log_0 -2$

Cannot convert from logarithmic to exponential form

Example 18:

$$\ln 4x = 1$$
$$\therefore \frac{\ln x}{\ln 4} = \frac{1}{4}$$
$$\therefore x = \frac{1}{4}$$

Cannot convert from logarithmic to exponential form; confuses logarithmic rules

Example 19:

$$\ln 4x = 1$$
$$\therefore x = \frac{\ln 1}{\ln 4}$$

Cannot convert from logarithmic to exponential form

Example 20:

$$\ln 4x = 1$$
$$\therefore x = 4$$

Cannot convert from logarithmic to exponential form

Example 21:

$$\frac{\ln x}{\ln 3} = \ln(x-3)$$

Unfamiliar with the quotient rule for logarithmic functions

Example 22:

$$\frac{\ln x}{\ln 3} = \ln x - \ln 3$$

Unfamiliar with the quotient rule for logarithmic functions

Example 23:

$$\log_3 x = \ln_3 x$$

Unfamiliar with the definitions of the natural logarithmic function and the logarithmic function

Example 24:

$$\log x + \log 4 = 2$$
$$x = 4$$

Unfamiliar with rules for logarithms

Example 25:

$y = \sin^4 x^2$

$= \sin x^8$

Confuses the raising of a function to a power with raising a function variable to a power.

Example 26:

$e^{3x^2} = e^{(x^2)^3} = e^{x^6}$

Confuses $x^2 + x^2 + x^2$ with $x^2 \times x^2 \times x^2$

Example 27:

Factorise : $-x^2 + 7x - 10$

Answer: $-x^2 + 7x - 10 = x^2 - 7x + 10 = (x-5)(x-2)$

Student loses the common factor;
No error checking done / Lacks the skill to handle a minus-sign

Example 28:

Let $f(x) = \dfrac{(3x^2 - 6x + x^3)}{x^2}$. Determine $f'(x)$.

Answer: $\dfrac{6x - 6 + 3x}{2x}$

$\therefore 9x - 6$

Students cannot simplify fractions and thus cannot differentiate the function

Example 29:

$\frac{3}{4} \div \frac{3}{16} = \frac{3}{4} \times \frac{3}{16} = 0.75 - 0.2\dot{2}$

Cannot divide a fraction by a fraction
Cannot write a fraction as a decimal

Example 30:

$\frac{3}{5} \times \frac{10}{3} = 0.6 \times 9.\dot{9}$

Cannot multiply a fraction by an improper fraction
Cannot write a fraction as a decimal

Example 31

Express as percentages (a) 0.003 (b)1.5

Answers

(a) 30%

(b) 15%

Cannot express a fraction as a decimal

Example 32

Make x the subject of the equation:

$$\frac{2a-3b}{3x}=b+3$$

Answer:

$$\frac{2a-3b}{3x}=x(\frac{2a-3b}{3x})=b+3$$

Multiplies the left-hand side of the equation with x instead of with $3x$ but forgets to multiply the right-hand side with $3x$

Answer:

$$3x(2a-3b)=b+3(-3x)$$

Multiplies the left-hand side of the equation with $3x$ but instead of multiplying the right-hand side with $3x$, also the student multiplies the right hand-side with the additive inverse of $3x$

Example 33

Solve for the values of x in $-2(3x+4)>10$

Answer

$$-6x-8>10$$

$$\frac{48x}{10}>\frac{10}{10}$$

Multiplies the terms on the left-hand side of the inequality instead of taking the constant term to the right.

Answer

$$-6x>10+8$$

$$\Rightarrow -6x>18$$

$$\Rightarrow -x>3$$

$$\Rightarrow x>-3$$

Does not reverse the inequality sign when dividing both sides by -1

Example 34

Mr Jay and Mr Joe start a business. Mr Jay contributes R50 000 and Mr Joe contributes R100 000 to start up the business. What percentage of the business belongs to Mr Jay and what percentage belongs to Mr Joe?

Answer :

$$\frac{100000}{50000}$$

Unfamiliar with the concept of the whole in fractions

2% each

Answer:

Mr Jay $\frac{50000}{60000}\times 100 = 83\%$

Unfamiliar with the concept of the whole in fractions / Solving a word sum

Mr Joe $\frac{10000}{60000}\times 100 = 27\%$

Example 35:

Express as a percentage: 1.5

Percentage in excess of 100% causes a problem

Answer: 50%

Example 36:

Evaluate: $(\frac{27}{8})^{\frac{1}{3}}$

Answer: $\frac{27}{8}\times\frac{1}{3}$

Cannot determine the cube root of a cubic fraction

Example 37:

The student had to sketch the line $2y = x + 3$.

The student sketched $y = x^2, x \geq 0$.

Student did not realise that the sketch asked was that of a straight line.

Example 38:

Given the equation $5x - 2y = 10$

(a) Write down the slope.

Answer m=10

Student could not write the straight line in point slope form

(b) Write down the y-intercept.

Answer

$y-$ intercept is at the axis of the slope

Example 39:

Sketch the graphs of $y = 2x + 1$ and $y = (x+1)^2$ on the same set of axes. Show clearly the $x-$ and $y-$ intercepts of the graphs. Show points of intersection of the graphs if any.

Samples of student answers to the question will follow:

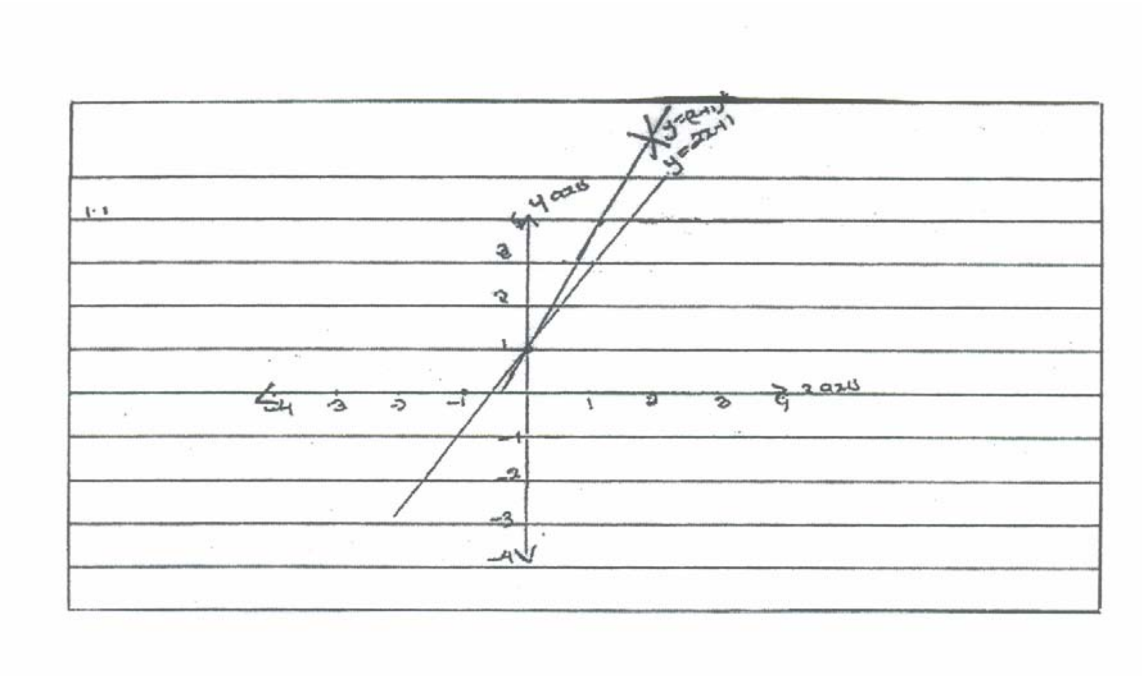


Figure 1

From figure 1 it is evident that the student did not recognise the equation $y = (x+1)^2$ represented a parabola and not a straight line. The student also did not recognise that one equation is linear and the other quadratic.

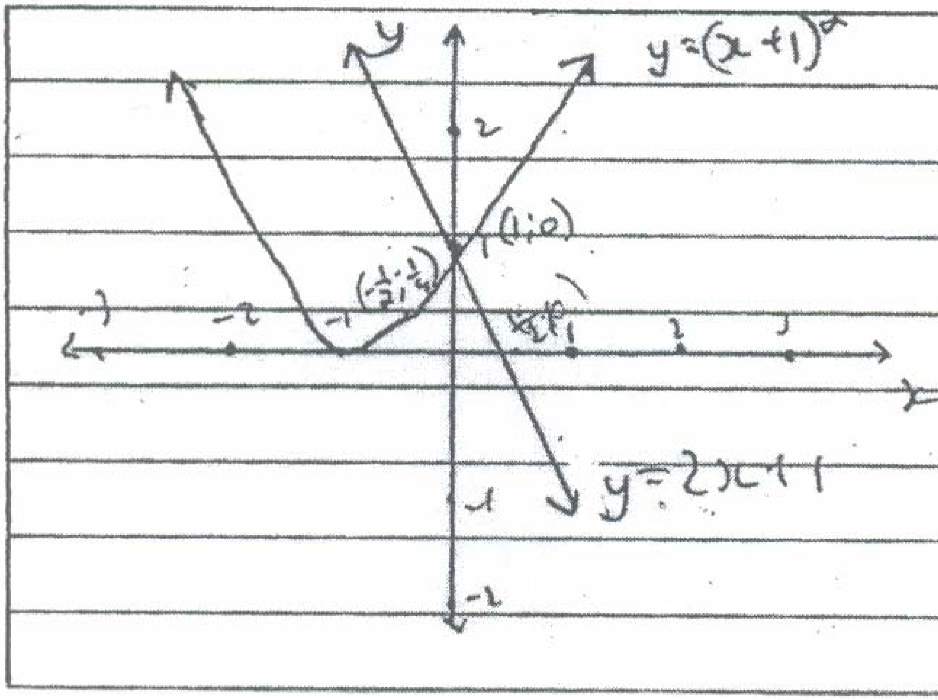


Figure 2

Figure 2 shows that the student could not draw the straight line correctly. A decreasing line instead of an increasing line is drawn.

Example 40:

Graph $y = -\frac{3}{2}x + 1$ on the given coordinate system.

Answer:

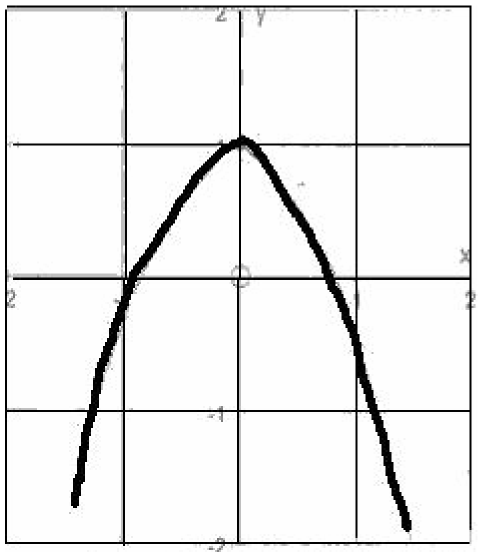


Figure 3

The student does not recognize the equation of the straight line in point slope format.

4.4 Common Mistakes Made by Learners in the Matriculation Examination

An analysis of the 2003 Matric Mathematics Examiners Report [Matri03] revealed that learners made the following mistakes:

- Sloppy use of the word “and” and “or” when dealing with inequalities
- Learners do not check that all solutions are valid
- Candidates cannot deal with fractions
- Learners lack knowledge of basic concepts such as squaring binomials
- Poor number line interpretation
- A semi-circle is sketched as a parabola, straight line or a circle
- No understanding of the concept “range” is evident
- Poor handling of negative exponents
- Index and log rules require attention
- Incorrect usage of differentiation notation
- Questions requiring an understanding of language, were answered poorly
- Learners cannot sketch the graphs of the trigonometric functions

- Learners do not learn theory
- Difference between factorising and solving is not clear
- Incorrect usage of the limit notation
- Learners have problems representing intervals using square or round brackets or when set builder notation has to be used
- Difference between the range and domain of a function is not well understood
- Questions where graphs have to be interpreted were not well answered
- Learners do not read questions carefully

These are some of the errors highlighted in the report. The researcher reported errors from the report, which corroborate the errors identified by the researcher and colleagues. The report also urges teachers to revise basic arithmetic skills such as addition, subtraction, multiplication and division. This corroborates the researcher's findings that some students cannot perform operations on fractions and integers.

4.5 Related Research

Other South African researchers identified gaps in learners' knowledge which correspond with the gaps identified by the researcher. Venter believes learners need to understand the distributive law in order to master the four arithmetic operations [Venter78:64-65]. Barnard highlights some errors made by Grade 11 learners in his research [Barnar88:96-114]:

- learners do not know the notation for indices
- as soon as an item involving a square root involves more than one thought process many learners struggle to answer it
- Learners do not know that the numerator should be treated as a unit and cancel blindly
- Factorising trinomials including negative numbers and non-standard forms made items more difficult to answer
- Learners have not mastered the principles required to manipulate formulas

- Learners are not able to perform operations on decimal numbers without a calculator
- Learners do not see a fraction as a ratio
- Learners make substitution errors as a result of regression
- Inequality signs are treated as equal signs
- Learners find it easier to sketch a line than to find the equation of the line
- Learners do not know the order of operations

Some of the errors highlighted by Barnard's research correspond with the errors identified by the researcher (Cf.7.3.1).

4.6 Conclusion

From this matriculation error report and the errors identified by the researcher it is clear that there is a correspondence between the errors that first-year students make and the errors that learners make in the matriculation examination. A first-year student should be proficient in Arithmetic and should know the pre-knowledge identified in section 3.3. However, from the matriculation examination error report and from the errors made by students during tests and examinations it is evident that students do not have the necessary pre-knowledge and skills. When a student enters a first-year course with a lack of knowledge they seldom catch up on their own and end up repeating the course in a subsequent year or doing a special summer or winter school programme. Groups attending summer or winter school recess programmes are smaller and this allows the lecturers to give more individual attention to students. Also the majority of students only do one winter or summer school programme. This allows the student to concentrate on Mathematics only. Although the lecturer can give each student more individual attention, the time constraints are such that the lecturer concerned cannot revise pre-knowledge.

Given the basic mathematical errors that students make and the deficiencies in pre-knowledge, performance in matric Mathematics, which is widely used as an admissions criterion, does not appear to provide mathematics lecturers with all the information needed to place a student in an appropriate module or to tailor a module based on the actual pre-knowledge and skills of incoming students. Consequently, the researcher contends that the additional information that can be obtained from a diagnostic mathematics test will assist in placing students and identifying aspects of mathematical pre-knowledge and skills that need to be remedied.

Chapter 5 will concentrate on what a diagnostic test is and the steps to follow when designing one. Paragraph 5.5 discusses the content of other placement or diagnostic tests used in other countries or at other higher education institutions in South Africa. Paragraph 5.6 discusses test development and in particular the development of the pilot test.

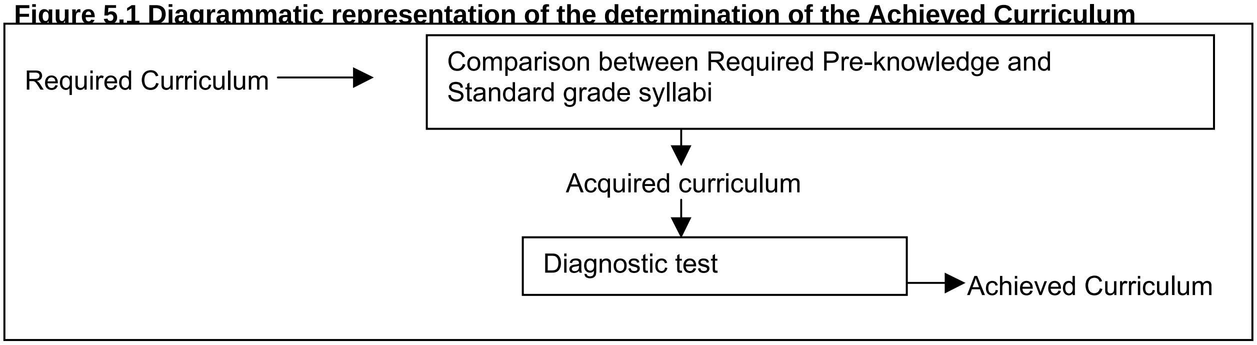
CHAPTER 5 - An Action Research Approach to Diagnostic Test Design

5.1 Introduction

This chapter will discuss the use of an action research methodology when developing a diagnostic test. In Chapter 2, admissions criteria, school-related mistakes and problems in the education system were identified as reasons for the high failure rate among first-year Mathematics students at the Nelson Mandela Metropolitan University (NMMU).

Some universities overseas have tried to overcome the problem of under-prepared students entering university by conducting placement or diagnostic testing (Cf. 2.4-2.5). This has led to the present research which aimed to establish the sub-domains that should be included in a diagnostic test and to pilot such a test. The diagnostic test needed to determine the pre-knowledge and skills (the achieved curriculum) a prospective first-year student has. In Chapter 3 the pre-knowledge and skills a prospective first-year student requires (required curriculum) were identified and compared with the school syllabi (acquired curriculum) to determine whether the standard grade Mathematics learner is adequately prepared to successfully complete a first-year Mathematics 1 course (achieved curriculum). Figure 5.1 gives a diagrammatic representation of the process. In Chapter 4 the school-related errors made by first-year Mathematics students were identified. Based on these errors and the acquired knowledge identified, sub-domains to include in the pilot test, will be identified and justified in this chapter (Cf. 5.6.2).

Figure 5.1 Diagrammatic representation of the determination of the Achieved Curriculum



This chapter focuses on:

- Why a diagnostic test and not a placement test would best serve the needs of NMMU.
- Why the test had to be an in-house diagnostic test.
- The theory behind test development from an action research framework and how the theory was applied to the pilot test.

Action research involves a number of steps. In what follows, the action research steps pertaining to the identification of the sub-domains of the diagnostic test will firstly be identified and described in detail. Thereafter, the theory behind and steps involved in the development of a diagnostic test will be described.

5.1.1 Need for diagnostic testing

Chapter 3 revealed that the higher grade mathematics pupil has all the required pre-knowledge and should not experience any problems with a Mathematics 1 course and the standard grade pupil has most of the required pre-knowledge and should be adequately prepared for first year. Despite these findings there is nevertheless a high failure rate amongst first-year students. Learners' matriculation mathematics symbols provide no diagnostic information on their areas of weakness. The pre-knowledge that a Grade 12 learner achieved at school has to be measured to identify any gaps that might exist in the learner's knowledge. This is the only way that students who are at risk can be identified and assisted. The development of a diagnostic test will make it possible to identify students who are at risk. In the opinion of the researcher this would be the fairest way of testing, given the non-homogeneity of the student population.

An added benefit of such a test is that it can assist in placing students in developmentally focused modules. The aim of diagnostic assessment is to discover what causes learning barriers. Formative assessment aims to give feedback to the student. It is an important element of teaching and learning.

Summative assessment records a judgement of the performance of the learner and gives a picture of the learner's competence at a specific point in time [NatCur03:63]. Diagnostic tests fulfil both requirements and are both formative and summative in nature. Diagnostic testing should involve compulsory support for students who are identified as having certain difficulties. A placement test provides no diagnostic information and the student's specific weaknesses can thus not be addressed. This could hamper the student's future progress.

5.1.2 Advantages of diagnostic testing

Although a diagnostic test can be used to stream or place first-year students, it is primarily an assessment tool that helps to identify and address problem areas. This understanding of the phrase "diagnostic testing" coincides with the understanding of Brian Stone [Stone95]. During a "study tour" that he conducted he came to realize that there was a wide range of understanding of the phrase diagnostic assessment. Most commonly, the phrase was understood to mean that a diagnostic assessment would indicate whether a student understood or did not understand a topic.

At the University of Western Australia an Intelligent Computer Tutor is used to allow students to assess their understanding. They do not use multiple-choice questions. An incorrect answer will result in a suggestion of the misunderstanding that could have led to the incorrect answer and appropriate help is immediately given. When a diagnostic test gives immediate diagnosis –and the diagnosis becomes part of the whole learning process – the assessment becomes formative in nature [Stone95]. A diagnostic test has many advantages [Lee05]:

- It provides information about a cohort of students
- It informs programme design
- It identifies students who are at risk of failing a mathematics course
- It helps to focus remedial support on those students in need

- It helps to focus the preparation of provision support

Based on students' weaknesses, programmes can then be designed to address their problem areas. A diagnostic test will have more value than a placement test. Barnard says diagnostic tests are more analytical in nature and allow teachers to analyze mistakes scientifically and to determine the extent of a problem experienced by a group or an individual [Barnar88:68]. Not only will such a test identify which areas students need to improve in, but it could assist with placement decisions. Support programmes can therefore be tailored according to the needs identified by the diagnostic test. The next paragraph provides a short description of action research.

5.2 Description of action research

"Action research has as a main purpose the generation of knowledge which leads to improvement of understanding and experience for social benefit" [McNiff02:17]. "Action research involves learning in and through action and reflection, and it is conducted in a variety of contexts, including the social and caring sciences, education, organisation and administration studies, and management" [McNiff02:16]. This research was not conceptualised within an action research framework from the outset. However, the research involved a number of steps, namely, the identification of a problem, the design of a plan to address the problem, an intervention phase as well as a phase reflecting on the results of the intervention. These steps are characteristic of action research.

Carol Bohlmann [Bohlm06:78] identifies eight stages that an action research project could progress through. In stage one the problem is identified, evaluated and formulated. This process involves a review of current practice and identifies possible improvements. In stage two interested parties will discuss the way forward. The results of stage two could lead to a discussion. In stage three, a literature review may be conducted. If this is done the fourth stage could involve the re-formulation of the original problem or a re-formulation of the proposal drawn up in stage two. In stage five, the research strategies would be decided on. The selection of evaluation procedures would be done in stage six. In stage seven the intervention decided upon is implemented. This stage will include methods of data collection, monitoring of tasks as well as data analysis. In the final stage the data are interpreted, a review of the intervention is done and decisions are made regarding the outcome of the project. The result of the review could be that the original plan might need to be amended and a new action phase implemented. The new phase will pass through similar stages.

One expert sees action research as a two-stage process, namely "a diagnostic stage in which the problems are analysed and the hypotheses developed; and a therapeutic stage in which the hypotheses are tested by a consciously directed intervention or experiment in situ" [Cohen00:234]. Kemmis and McTaggart (1992:10) state that "to do action research is to plan, act, observe and reflect more carefully, more systematically, and more rigorously than one usually does in everyday life" [Kemmi92].

Action research is often collaborative and could involve a team of researchers and practitioners. "To view action research solely as a group activity, however, might be too restricting. It is possible for action research to be an individualistic matter as well [Cohen00:230]". Action research could be participatory with the team members taking part directly or indirectly in the implementation of the research. This type of research is self-evaluative. Modifications are evaluated within the context of the research on an on-going basis [Bohlm06:76]. McNiff states "The methodology of action research is that people ask questions such as, How do I do this better?". "This can happen because reflection on action is an inherent part of an action research methodology [McNiff02:18]".

McNiff promotes the idea of spontaneity of action research. In theory the idea of a process is to observe, describe, plan, act, reflect, evaluate and modify. In a real life situation one could begin at one place and end up in an unexpected place [McNiff02:56]. McNiff sees action research as a “spontaneous, self-recreating system of enquiry” [McNiff02:56]. Bohlman [Bohlm06:76] states that action research can be viewed as “a disciplined enquiry, where a practitioner systematically investigates how to improve practice and how to produce evidence for the critical scrutiny of others to show how the practice can be judged to have improved”. This view is based on McNiff [McNiff02:103]. Bohlmann [Bohlm06 :77] sees the main purpose of action research as the generation of knowledge which promotes a better understanding and experience for the benefit of a particular community. This community is often an educational community.

Macintyre defines action research as follows:

“Action research is an investigation, where, as a result of rigorous self-appraisal of current practice, the researcher focuses on a ‘problem’ (or a topic or an issue which needs to be explained), and on the basis of information (about the up-to-date state of the art, about the people who will be involved and about the context), plans, implements, then evaluates an action; then draws conclusions on the basis of the findings” [Macintyre02:1].

According to Bohlman, the phrase ‘up-to-date state of the art’ could imply “the choice of the topic and possible ways of investigating the topic, should be informed by recent literature”. [Bohlm06:76]

Stephen Kemmis writes in the Handbook of Action Research [Reaso06:95] that his research group distinguished between empirical-analytic (or positivist), hermeneutic (or interpretive) and critical approaches in research theory and practice. According to Kemmis, empirical-analytic research has an interest in getting things done effectively. Interpretive research has an interest in wise and prudent decision-making in practical situations. According to Kemmis, critical research on the other hand, has an interest in liberating people from determination by habit, custom, illusion and coercion which could frame and constrain social and educational practice. Kemmis et al found that much research is of a technical nature where the aim is to bring about functional improvement. The success of technical research is measured in changing outcomes or practices. Technical research is a form of problem-solving and is regarded as successful when the goal of the project is attained. It takes a pragmatic view of its purpose in that its goals are not questioned, nor is the situation in which it is conducted questioned [Reaso06:95].

A large body of recent action research can be described as being of a practical form. The research has the technical aspirations to bring about change, but also aims to inform practical decision-making [Reaso06:95]. Practical action researchers aim at understanding and changing themselves just as much as changing the outcomes of their practice. A smaller amount of recent action research can be classified as critical or emancipatory. “This form of action research aims not only at improving outcomes, and improving the self-understandings of practitioners, but also at assisting practitioners to arrive at a critique of their social or educational work and work settings” [Reaso06:95]. The next paragraph will consider test design from within an action research framework.

5.3 Diagnostic test design within the framework of action research

The development of a diagnostic test could be regarded as an adapted form of action research, while it also has to make use of the framework within which a psychometric test is to be developed.

This research has elements of technical action research. It aims to identify students’ areas of weakness and allows intervention programmes to be introduced to assist students, but does not concentrate on why there are gaps in learners’ knowledge. This research also has elements of interpretive research in that the research will facilitate more prudent decision-making. The diagnostic test will assist students in making placement decisions. The identification of students at risk involves a number of steps. The first step was the identification of the problem, namely the under-preparedness of first-year Mathematics students. This is illustrated by the school-related mistakes made by the students (Cf. 4.2 – 4.3). The second step involved drawing up a plan to address the problem. This step involved:

- Looking at how overseas institutions have addressed the problem.
- Determining the sub-domains to include in the tests and
- Investigating whether a suitable test is available or needs to be developed.
- Deciding to develop a diagnostic test to identify students who are at risk of failing a first course in Mathematics at NMMU.

The third step comprised the development of the pilot test. The fourth step involved the administration and scoring of the test. In step five the psychometric properties of the test were determined. Step six involved the post-implementation review, which is the reflection phase. Kemmis and McTaggart state that "to do action research is to plan, act, observe and reflect …." [Kemmi92,10]. The steps followed by the researcher in her research involved the actions as spelt out by Kemmis and McTaggart. The identification of the problem and the decision to develop a diagnostic test represented the planning phase. Test development, administering and scoring represented the action phase. The determination of the psychometric properties of the test represented the observation phase. The post-implementation review represented the reflection phase. The first step in this action research framework, namely the identification of the problem, will be considered in the next section.

5.4 Identification of the problem

While marking tests and examination scripts, this researcher noticed that students made many school-related mistakes (Cf 4.2 - 4.3). These mistakes have an impact on the final mark obtained by the student. From observing students during tutorial sessions the researcher noticed that some of the weaker students required calculators to add or subtract especially when fractions were involved. This confirmed this researcher's perception that first-year students have not achieved the pre-knowledge and skills required to successfully complete first year. Earlier research by Venter and Barnard corroborates this perception (Cf.4.5).

A diagnostic test could assist in the identification of students' mathematical weaknesses, provide support and improve pass rates. Before deciding to develop a test one has to consider the advantages and disadvantages of purchasing an existing test. This decision forms part of the planning phase.

5.5 Deciding to develop or purchase a test

Once the decision has been made on which type of test to use one must decide whether to develop your own test or use an existing test. To buy an existing test has many advantages:

- Tests are immediately available.
- Selected Response Questions are easy to mark as an optical mark reader can be employed to mark the questions.

Buying an existing test also has a number of disadvantages:

- If the package consists of multiple tests, care must be exercised to buy the correct combination of tests.
- Syllabi differ and as a result packages may not completely satisfy the individual institution's needs.
- It is more difficult to determine problem areas when selected response item types are used.

To be effective a diagnostic test must be based on the curriculum and entrance requirements used at NMMU. Using an in-house test allows the researcher to tailor the test to find out what she needs to know. In South Africa placement and diagnostic testing have not been widely used. Many universities still use matriculation results as predictors of future academic success. Some universities have developed aptitude tests to test prospective students. Some of the testing programmes implemented at South African universities as well as diagnostic testing programmes elsewhere in the world will be reviewed in the next section.

5.5.1 The Alternative Admissions Research Project

The University of Cape Town (UCT) has an Alternative Admissions Research Project (AARP). The AARP developed and uses 4 types of tests. Mathematics students would write a placement test in English, a Mathematics Achievement test, a Mathematics Comprehension test and a Reasoning test. Students are advised to write the tests if they think their senior certificate results will not be good enough to gain entry or if they believe their senior certificate results will not give a true reflection of their potential. If they receive excellent marks in the tests they could receive early placement offers and offers of financial aid or scholarships based on their circumstances. The AARP tests are also used to identify students whose matriculation results do not reflect their full potential. The tests are not compulsory. If a student elects not to write the tests UCT uses their Grade 11 and/or 12 mid-year results.

The AARP Mathematics Achievement test tests students' mathematical competency. The topics that are tested are usually in the school syllabus. The test consists of 30 multiple-choice questions. Test takers have to show that they can perform basic mathematical computations and that they can solve elementary problems. The results of the test will determine if the student needs further remedial help. Students have 90 minutes to complete the test. The AARP Mathematics Achievement test is more diagnostic in nature in that it identifies students who need remedial Mathematical assistance [Aarp06]. This test however does not identify the actual problem areas where students experience difficulties and is thus not a true diagnostic test.

The AARP Mathematics Comprehension test has been designed to determine the ability of students to learn and apply mathematical knowledge. Topics fall outside the scope of the school syllabus. Test takers answer two different items. Forty-five minutes is allowed per item. Students have to study the text carefully to show that they can interact with new material. The text design is such that a student will not be disadvantaged if they are not very competent with the language in which the test is written. Test takers have to demonstrate that they have the basic arithmetic and algebraic skills required to study Mathematics at a tertiary level. The AARP Mathematics Comprehension test is an aptitude test [Aarp06]. The AARP tests are not diagnostic tests and will not meet the requirements of this researcher.

5.5.2 The Port Elizabeth Technikon (PET) Test

The former Port Elizabeth Technikon (PET), now part of the Nelson Mandela Metropolitan University developed an entry-level Mathematics test to select students for admission to the Academic Support Programme run by the then Port Elizabeth Technikon. The test consists of 30 multiple-choice questions. Learners had forty-five minutes to complete the test. Currently, the test forms part of a battery of tests used to stream prospective first-year engineering students at the Nelson Mandela Metropolitan University when an admissions decision needs to be made for the student. This test is an achievement test and has no diagnostic component to it. This test focused initially on foundation level students and not on first-year students. This test was obtained from the admissions and testing department at NMMU.

5.5.3 The University of South Africa Test

The practice of pre-registration diagnostic assessment at UNISA has been stopped. The test used for this purpose has remained confidential. The researcher has obtained a copy of the foundation level test – a different and more mathematical test – that is undertaken as an assignment. This test is an achievement and not a diagnostic test. This test is aimed at foundation level students and not first-year students.

5.5.4 The Accuplacer Tests used at NMMU

At the time of the study, students at NMMU had to take two Accuplacer tests (Cf. 2.2.4). One test was a Linear Algebra test and the other an Arithmetic test. Both tests are multiple choice tests. These tests are achievement tests and have no diagnostic component to them. Copies of these tests were obtained from the admissions and placement department at NMMU.

5.5.5 National Benchmark Tests being developed by Higher Education South Africa

There is work under way to create a set of national benchmark tests for South African Universities. Higher Education South Africa has published the specific aspects that they intend to assess in their Cognitive Academic Mathematical Proficiency (CAMP) test. The aspects they wish to assess include [Hesa06]:

- Operations with fractions and decimals
- Operations involving abstract relationships such as ratios, percentages and powers; interpretation of scientific notation; orders of magnitude; number sense and quantitative comparisons
- Spatial perception
- Functions represented by tables, graphs and symbols; distinction between dependent and independent variables; relationships between graphs and algebraic equations and inequalities; functions and their inverses; recognising

- and applying functional relations; understanding above/below; translation between different methods of representation of functions.
- Operations with surds
- Circle geometry
- Basic trigonometry, including graphs of trigonometric functions
- Solving for unknown quantities in single and simultaneous linear, quadratic equations, and simple polynomial expressions and inequalities
- Using common statistical measures (mean, median, mode, range)
- Pattern recognition (as in sequences and series)
- Conversion from language to symbolic form

These tests will be criterion-referenced and will not be diagnostic in nature.

5.5.6 The John Barnard Test

The only example of a South African diagnostic test that the researcher could find was a test developed by John James Barnard. John James Barnard designed his diagnostic test, which was administered to 4635 Grade 11 learners from 106 schools, in the late eighties. The test was the property of the Human Sciences Research Council and test items were confidential. The diagnostic model that he used consisted of three categories [Barnar88:94]

- Expression in mathematical language
- Use of fundamental concepts, principles and skills
- Testing whether answers make sense

The sixteen topics in the category 'Use of Fundamental Concepts, Principles and Skills' will be considered in more detail in Table 5.1. Many of these topics coincide with topics included in the pilot test [Barnar88].

Table 5.1 Topics included in the John James Barnard diagnostic test.

Topic
Priority of operations
Graphs
Inequalities
Substitution
Surds
Factorization
Formulas
Decimal numbers
Impossible Problems
Arithmetic
Indices
Fractions
Solving equations
Percentages
Ratio and Proportion
Surplus Information

Source: [Barnar88]

There is a big overlap between the topics included in Barnard's diagnostic test and the errors identified by the author (Cf. 4.2.3 and 4.2.4). Barnard published his research in 1988. Some of the problem areas that learners struggle with are definitely not new. This researcher could also not use this test. The test is too old and does not cover all the pre-knowledge and skills identified by the researcher (Cf. 3.3). The next section looks at the content of diagnostic tests used in the United Kingdom.

5.5.7 The Achievement in Mathematics tests developed in New Zealand

A new series of mathematics tests are being developed in New Zealand. The tests are currently called the Achievement in Mathematics tests. The AIM tests may eventually replace the Progressive Achievement Tests (PAT). The Constructed Response item type has been used in the AIM tests. The reason for this is that constructed responses have a stronger formative dimension [Neil02:4]. The tests are designed to examine all objectives within an individual strand. Five strands are in different mathematical content areas and the sixth strand, namely Mathematical Processes will impact on the other five strands. The five strands are [Neil02:3]:

- Numbers
- Measurement
- Geometry
- Algebra
- Statistics

The analysis of the results will provide information which can identify individual students' strengths and weaknesses. The data can be aggregated at different levels. Feedback can thus be given at the level of the individual student, group of students, the whole class or the whole school [Neil02:4]. Each question will provide diagnostic information. The diagnostic information for the question will reveal the misconceptions that students, who answered the question incorrectly, have. Some questions may also look at the correct answers that students produce. This test although diagnostic in nature is not intended for first-year students.

5.5.8 Diagnostic Tests used in England

The course content of diagnostic and placement tests used elsewhere in the world differ from the required pre-knowledge and skills identified. Many universities in the UK have either developed their own diagnostic tests or have opted to use Mathletics or Diagnosys software to develop tests. The Mathletics test used in the United Kingdom also tests both Algebra and Calculus but the content differs [Ltsn05:8]. The selected content of Mathletics given below covers topics that a prospective first-year student at NMMU will study in statistics or in the first and/or second year of Mathematics:

- Numbers: BODMAS, General Arithmetic, Arithmetic models, Terminology, Decimals and Scientific Notation, Fractions, Powers of numbers, Percentages, Numerical Sequences, Units and Dimensions, Surds, General Complex numbers, Add Complex, Multiply and Divide Complex, Argand Diagram, Complex Polynomials, DeMoivre, Definitions and notation, Divisibility and Prime Numbers, Euclidean Algorithm, Modular Arithmetic, U(n) Groups 1 and 2.
- Probability and Statistics: Basic Probability, Combinations, Permutations, Probability trees, Data Types, Data Display, Analysis of data, Shapes of data, Definitions of Measure, Measures of Location, Measures of Dispersion, Correlation, Binomial Distribution, Cumulative Binomial Distribution, Poisson Distribution, Normal distribution 1 and 2, Central Limit Theorem, Confidence Intervals, Hypothesis testing 1,2 and 3, Basic Queuing, Little's Laws.
- Algebra: Proportionality, Linear Equations, Modelling, Coordinates, Sequences 1 and 2, Expanding Brackets, Factorisation, Flow Charts, Rearranging Equations, Indices 1 and 2, Simplification, Solving Equations, Using Formulae, Sigma Notation, Completing the Square, Inequalities and Simultaneous equations, Growth/Decay, Pascal Triangle, Binomial Theorem, Polynomial Multiplication, Polynomial Division, Maximum/Minimum Values of Quadratics, Partial Fractions 1 and 2.
- Functions: Recognising graphs, Limits of functions, Domains and ranges 1 and 2, Inverse Functions, Function Manipulation, Symmetry of functions, Degrees <> Radians, Trigonometry Definitions, Special Trigonometric Values, Trigonometry Equations 1 and 2, Trigonometry graphs 1 and 2, Reciprocal Trigonometry Functions, Sine Rule, Cosine Rule, Combining Signals, General

Trigonometry Solutions, Trigonometry identities, Logarithms and Exponentials.

- Differentiation: General Differentiation, Differentiation of powers, Differentiation of Products, Differentiation of quotients, Chain Rule, Differentiation of logarithms and exponentials, Differentiation of Trigonometry, Differentiation of Hyperbolic Functions, Differentiation of Inverse Trigonometric Functions, Differentiation of Inverse Hyperbolic Functions, Parametric differentiation, Implicit Differentiation Series and Expansions, Differentiation of Functions of two variables.
- Integration: Integration of Polynomials, Integration of Algebraic Functions, Integration of Rational Functions, Integration of Trigonometry Functions, Integration of Hyperbolic Functions, Integration by Substitution, Integration by partial fractions 1 and 2, Integration by Parts 1 and 2, Double Integration, etc.
- Laplace Transformations.
- Numerical methods.
- Statics.
- Vectors and Matrices: General Vectors, Vector Addition, Dot and Cross products, Triple Products, Lines and planes, 2x2 and 3x3 Numeric and Algebraic Determinants, Applications of Determinants, Matrix Addition and Scalar Multiplication, Matrix Multiplication, Inverse Matrix, Non-square systems, Translation, Rotation and shear, 2x2 and 3x3 Eigenvalues and Eigenvectors.
- Graph Theory: Basics, Digraphs, Colourings, Networks, Paths and Walks 1 and 2.

5.5.8.1 The Diagnostic Test used at Loughborough University in 2002

The faculty of Engineering at Loughborough University focused their diagnostic testing in 2002 on numerical skills and algebraic manipulation only. Table 5.2

below gives a more detailed breakdown of the Loughborough test of 2002 [Lee05:156].

Table 5.2 Loughborough diagnostic test of 2002

	Ques.	**Category**
Qu. 1-12	1,2	BODMAS
number	3,4	Scientific Notation
	5,6	Indicies
	7,8	Degrees/Radians
	9,10	Addition/Multiplication of Fractions
	11,12	Percentages
Qu. 13-40	13,14	Removing Brackets
Algebra	15,16	Evaluating expressions for given values of x
	17,18	Indices
	19,20	Factorising
	21,22	Addition/Subtraction of Fractions
	23,24	Fractions Equivalents
	25,26	Changing subject of the formula
	27,28	Quadratic Equations
	29,30	Equation of a straight line
	31,32	Simultaneous Equations
	33,34	Inequalities
	35,36	Laws of Logarithms
	37,38	Laws of Exponentials
	39,40	Partial Fractions

Source: [Lee05]

Areas of concern at other universities are identified in subsequent sub paragraphs.

5.5.8.2 Problem Areas Identified by the University of Bristol Diagnostic Test

The diagnostic test used at the University of Bristol covers topics such as Arithmetic, Algebra, Geometry, Functions, Calculus and Probability. At this university they have highlighted logarithms, probability and Trigonometry as areas of concern [Ltsn05:27].

5.5.8.3 Problem Areas Identified by the Anglia Polytechnic University Diagnostic Test

The Anglia Polytechnic University identified that students lack knowledge about powers, scientific notation, rounding to significant figures and graphs [Ltsn05:26].

5.5.8.4 Problem Areas Identified by the University of Manchester Institute of Science and Technology Diagnostic Test

The University of Manchester Institute of Science and Technology included questions on Arithmetic and Algebra through logs to differentiation, integration and questions on vectors in their diagnostic test [Ltsn05:24].

5.5.8.5 Problem Areas Identified by the University of Strathclyde Diagnostic Test

The diagnostic test of the University of Strathclyde covers fractions, quadratic equations, powers, trigonometric equations and simplification of equations[Ltsn05:22].

5.5.8.6 Problem Areas Identified by the Queen Mary University of London Diagnostic Test

The Queen Mary, University of London includes inter alia the following topics in their diagnostic test: decomposition into powers of primes, long division, fractions, surds, elementary function definition and equalities [Ltsn05:21].

5.5.8.7 Problem Areas Identified by the Bournemouth University Diagnostic Test

The School of Design, Electronics and Computing at Bournemouth University includes Numeracy, Algebra and Geometry in their diagnostic test [Ltsn05:12].

5.5.8.8 Problem Areas Identified by Queens University Belfast Diagnostic Test

At Queens University Belfast the School of Biology and Biochemistry tests their students to determine their mathematical skills and/or lack of skills and to inform students of their mathematical abilities. The results of their test indicate that students are experiencing problems with fractions, logarithms, problems involving conversion between units of measurement and SI units. They have identified that many students "fail to appreciate the importance of numerical skills, not only in their elected discipline, but also in selection procedures used by employers, their future profession, as well as in their everyday lives, and their reliance on calculators for even the simplest procedure is alarming". The introduction of computer-based learning had little effect in improving students' mathematical abilities. Reasons given for this were a possible lack of time before the second test or reluctance on the part of the students to address their mathematical weaknesses [Ltsn05:14].

5.5.8.9 Problem Areas Identified by Cardiff University Diagnostic Test

Cardiff University included basic Algebra, logarithms, integration, differentiation, Trigonometry and approximation in their diagnostic test. Deficiencies noticed by Cardiff University are Algebra, Elementary Calculus, Trigonometry and Complex Numbers [Ltsn05:18].

5.5.8.10 Usefulness of Existing Diagnostic Tests

Although some of the diagnostic tests developed at universities in the United Kingdom cover similar topics one cannot merely adopt such a test for use, because of syllabus differences. For example, although there are similarities between the in-house test used by Loughborough University in 2002 and the pilot test used at NMMU, one has to take into consideration that students in the UK have already been introduced to the exponential function $e^{f(x)}$, for example. Students at NMMU will be introduced to this function only in the second semester of the first year.

Locally developed tests also do not fulfil the requirements at NMMU. Universities have different programmes and the content of the tests will differ to suit the needs of the institution developing the test. Table 5.3 lists the shortcomings of existing tests. The quality of education varies across the country. The Eastern Cape area has had weaker pass rates than most of the other provinces in the country. According to an article which appeared in “Die Burger” on 30 December 2004 the Eastern Cape had a pass rate of 53,5 per cent compared to a pass rate of 85 per cent in the Western Cape and a pass rate of 76,8 per cent in Gauteng. On 29 December 2006, “The Herald” reported that the pass rate for the Eastern Cape was 59,3 per cent compared to 84,7 percent in the Southern Cape. Students in different provinces could have different weaknesses. This researcher thus decided to pilot her own diagnostic test.

Table 5.3 Shortcomings of Existing Tests

Test	Major Shortcomings of Existing Tests
AARP Tests	Not true diagnostic tests ; No follow-up research on test improvements available.
PET Test	Foundation Level Achievement Test; No further research has been conducted on this test
UNISA Test	Foundation Level Achievement Test; No further research has been conducted on this test
Accuplacer Tests	Placement tests; Olson has conducted further research on this and other tests [Olson05]
CAMP Test	Criterion-referenced and not diagnostic in nature; Still under development
John Barnard Test	Designed to test Grade 11 course work only ; Once-off test
AIM Test	Inappropriate Level − Not suitable for first-year students; No further research available.
UK Diagnostic Tests	Appropriate Level - Content differs; No follow-up research available

Source [Own Construct]

5.6 Development of the pilot test

Before a diagnostic test can be developed one has to investigate what the theory tells us about test development. The literature spells out the steps involved in developing a test. Developing a psychological measure involves planning, item construction, determining item effectiveness, choosing final items, administering the test to determine its validity, determining the reliability and the norm of the test, compiling a test manual, classifying the measure, publishing and marketing the measure and then lastly refining and updating the measure [Foxcro05:70-71]. The pilot test will not involve the compilation of a test manual, classification of the measure or any marketing. These steps of test design will be omitted.

5.6.1 Planning the development of a test

During this phase the developer must decide what the purpose of the test is [Foxcro05:70]. The purpose of the overall research project is to identify the sub-domains, which are predictors of success in Mathematics, and that should be included in a diagnostic test at NMMU. The diagnostic test that will ultimately be developed could be used as a decision-making tool to determine whether a prospective first-year student is ready to enrol for a Mathematics 1 course and will determine areas where the student is mathematically weak. A pilot version of the diagnostic test will be developed and researched in this study.

5.6.2 Determining the content domain of the test

The content domain (construct) can be defined "by undertaking a thorough literature study of the main theoretical viewpoints regarding the construct that is to be measured. The dimensions of the construct identified in the theoretical review could be used as a basis for operationalising the construct more concretely" [Foxcro05:72]. A brief overview of the sub-content domains of the two tests to be developed will be provided here. While the specific test specifications to operationalise each sub-domain of the pilot diagnostic tests will be outlined in Chapter 6.

The pilot test battery consisted of an Arithmetic Test and an Algebra and Calculus Test. Paragraph 5.6.2.1 gives the sub-domains to include in the Arithmetic Test and paragraph 5.6.2.2 gives the sub-domains to include in the Algebra and Calculus Test. Reasons for the inclusion of these sub- domains are also given.

5.6.2.1 Sub-domains to include in the Pilot Arithmetic Test (AT)

The researcher looked at the required pre-knowledge identified (Cf. 3.2), the content of placement and diagnostic tests used at other universities as well as at the school-related mistakes made by first-year students to decide on the sub-domains to include in the research test. The errors were compared with the required university pre-knowledge to determine if it corresponds. The researcher wanted to establish whether there was a link between the mistakes students made and their success in a Mathematics 1 course.

Many universities in the United Kingdom (Cf. 5.11) include an arithmetic component in their diagnostic tests. Arithmetic errors also appeared on the list of common mistakes (Cf. 4.2 and from 4.3 Examples 2 to 6,29-31 and 34) made by students at NMMU. The Arithmetic test consisted of the following sub- domains:

- Operations with decimals (Cf. 4.2.5)
- Rounding of decimals (Cf. 4.2.5)
- Operations with fractions (Cf. 4.3 Examples 29 and 30 and 4.2.4, 4.2.5)
- Order of operations in the absence of brackets
- Ratio related problems (Cf. 4.3 Example 34 and 4.2.4 and 4.2.5)
- Percentages exceeding 100 percent (Cf. 4.3 Example 35 and 4.2.5)
- Percentages (Cf. 4.2.5)
- Word sums (Cf. 4.3 Example 34, 4.2.5)
- Distributive law
- Finding the square root of the sum of two natural numbers
- Finding the cube root of a cubic ratio (Cf. 4.3 Example 36)
- Make x the subject of the equation (Cf. 4.3 Example 32)

5.6.2.2 **Sub-domains to include in the Pilot Algebra and Calculus Test (AACT)**

The sub-domains of the Algebra and Calculus test were based on the required university pre-knowledge identified, the errors made by students as identified in 4.2 and 4.3 and the school syllabi. Only pre-knowledge and skills that standard grade learners should be familiar with were included in the test as the minimum entrance requirement was based on the standard grade. The following sub-domains were identified:

- Factorisation (Cf. 4.2.4 and 4.3 Examples 7 and 27)
- Solving a quadratic equation using the formula (Cf. 4.2.4)
- Functions
- Squaring binomials (Cf. 4.4)
- Rules of indices and surds (Cf. 4.2.4 and 4.3 Examples 8-11)
- Graphing a straight line (Cf. 4.2.4 and 4.3 Example 39-40)
- Graphing a parabola (Cf. 4.2.4 and 4.3 Examples 13 and 39)
- Knowing the product of the gradients of two perpendicular lines
- Knowing when two lines are parallel
- Sketching a semi-circle (Cf. 4.4)
- Domain and range (Cf. 4.4)
- Calculating a limit using substitution to avoid the division by zero problem (Cf. 4.2.4)
- Differentiation (Cf. 4.23.4)
- Application of log rules (Cf. 4.2.4 and 4.3 Examples 15 and 21 to 24)
- Solving x by converting from logarithmic to exponential form (Cf. 4.3.3 and 4.3 Examples 16-20)
- Laws of exponentials (Cf. 4.3.3)
- Simplification of functions (Cf. 4.3 Example 28)

Once the content domain has been established the item type to be used has to be decided upon.

5.6.3 The item type(s) decision

Care has to be exercised when the decision is made on which question format to use. If the test is to be administered to large numbers of people, ease of marking plays an important role. The test developer should also consider the time available to administer the test and how quickly the results are required.

According to Harding, assessment questions can be divided into two formats, namely Constructed Response Questions (CRQs) and Provided Response Questions (PRQs). As the name suggests Constructed Response Questions requires the student to formulate their own responses. Provided Response Questions allow the student to choose “between a selection of given responses” [Hardin05]. Included in Constructed Response Questions are “open-ended paper questions, essays, projects, short answer questions (paper-based or online) and paper assignments” [Hardin05]. Provided Response Questions include multiple-choice questions, multiple response questions, matching questions and “hot-spot” questions [Hardin05]. Non-multiple-choice provided response items are sometimes referred to as *other objective answer formats*. There are several non-multiple-choice provided response item types such as true/false, multiple true/false, matching, grid-in and computer-based item types. Computer-based item types include things such as highlighting text at request and '*dragging and dropping'* [Sireci03].

Many paper-based achievement and aptitude tests use multiple-choice as the chosen item type. An advantage of using this item type is that it is easy to mark and the process of marking is totally objective. Students can however sometimes work backwards to obtain an answer. It is then perceived that the student knows the concept when in fact they do not. Although this shows some insight on the part of the student, it is misleading for the person analyzing the test when areas of weakness have to be determined.

Provided response questions often test cognitive skills at a lower level [Hardin05]. There are however Provided Response questions that test the understanding of important Mathematical ideas [Hardin05]. Harding and Engelbrecht found in their research that unless a sufficient number of quality distracters are available guessing can be a real problem when using multiple-choice as an item type. They found that students obtained higher marks for paper-based constructed response questions than for online constructed response questions. One possible reason for this is that when students are writing they are forced to write down all the steps and this could help them to eliminate errors [Hardin05].

Harding and Engelbrecht have done some research on assessment format in an online Mathematics environment. Some of the criticism levelled against Provided Response Questions is that it measures “discrete bits of information, rather than an overall understanding of the topic”. There are examples of Provided Response Questions that integrate more than one mathematical concept [Hardin05]. Further criticism against the use of Provided Response Questions is “the rigidity of the marking system”. Students often choose the right option in a multiple question for the wrong reason [Tamir90]. Multiple-choice questions make no provision for allocating partial credit. A minor mistake is penalised in exactly the same way as a major mistake.

Guessing is another concern when use is being made of Provided Response Questions. When an assessment uses true/false questions, guessing alone could result in an average score of 50 percent. Multiple-Choice questions requiring a student to choose one option from the given 5 options give an average of 20 percent to the student who has guessed all the answers. “Guessing can be counteracted by negative marking” [Hardin05]. Not all institutions allow negative marking. Where distracters are based on misconception; it is advisable to give immediate feedback if the test is formative in nature [Hardin05].

Nearly every standardised paper-based achievement/diagnostic test in South Africa, the United Kingdom and the United States uses the multiple-choice format, including elementary and high school achievement tests and college and graduate-school admissions tests. There are a variety of reasons for the popularity of the multiple-choice item type. Haladyna and Downing (1989) list the following reasons [Halady89]:

- The sampling of content is better than that obtained using other formats
- By using sufficient high quality multiple-choice items the reliability of the test scores increases
- It is easy to test multiple-choice items
- Item-banking systems make it easy to store, use and re-use multiple-choice items
- Tests can be scored quickly and objectively
- It is easy to obtain a diagnostic sub-score
- Most types of content can be tested including higher level thinking

Harding and Engelbrecht have found in their study that students perform better on average in online Provided Response Questions than in online Constructed Response Questions. Students also perform better in paper-based Constructed Response Questions than in online Constructed Response Questions. Even when partial credit was discarded, students performed better in paper based Constructed Response Questions than in online Constructed Response Questions.

Short constructed response items have a number of advantages. They can be used in Mathematics and other fields to measure synthesis and evaluation. Some of the advantages are:

- It removes the guessing problem that is present in multiple-choice and true/false items
- These item types are easy to construct
- Constructed response tests are easy to mark

Constructed Response tests do have drawbacks.

- It is not always possible to score the tests with a machine
- A scoring rubric has to be developed to assist in the scoring process

Researchers, who developed the Achievement in Mathematics (AIM) tests in New Zealand, used a mixture of selected response and constructed response items. In formative testing the constructed response item type was the preferred item type although selected response also played a role. Neil gives the following reasons for this preference.

- When constructed response is used the method that the student uses to find the answer is clear. Constructed response has a strong diagnostic dimension, because an incorrect answer will often reveal the student's misconception. The multiple-choice distracter on the other hand provides only weak clues as to why the student does not understand the principle being asked. The multiple-choice question type therefore has a much weaker formative-diagnostic potential.
- Selecting the correct response in a multiple-choice question does not imply that the student understands the concept. The correct response can be selected by test-wise strategies or purely by chance. A high multiple-choice score indicates that the student has strong skills, but that does not mean the student has every skill. Multiple-choice questions suit summative rather than formative testing.
- Students will guess in selected response questions [Neil02:16].

From the arguments presented above, constructed response appears to be the item type of choice when developing a diagnostic test. The pilot test battery developed and administered to most first-year students at the University of Port Elizabeth (now NMMU) at the beginning of 2004 thus used constructed response as its item type. The reason for this was to force students to write down all the steps in the solution to enable the researcher to determine where the problem areas were and to determine whether the students had acquired the necessary university pre-knowledge and skills during their high school careers. In this case the provided response item type would not have provided the same diagnostic information. With the multiple-choice item type students can guess the answer. Especially in a situation where they have nothing to lose. The next section will describe the method followed to write the items.

5.6.4 Item Writing

Once the developer has determined the content domain, the item format to use and has decided on the length of the test and the number of items to use, item writing can begin. Various sources can be consulted to "get ideas for items" [Foxcro05:74]. Sources include existing tests, textbooks, curricula, theories, self-descriptions and so on. Test developers will include considerably more items in the pilot test than in the final version of the test [Foxcro05:74]. In order to write good quality items the test developers will have to consider the guidelines for the item type of their choice. The item type used for this pilot diagnostic test was constructed response.

The sub-domains for the Arithmetic test were identified in section 5.6.2.1 and will be detailed in Table 6.1. The questions for the pilot Arithmetic test were based on questions from textbooks, old papers, and other arithmetic tests as well as a pool of questions that the researcher has developed over the years. The items used in the pilot test were selected from this pool of questions. The textbook used was Mathematics for Elementary Teachers A Balanced Approach [Krause91].

Section 5.6.2.2 identified the sub-domains that had to be included in the pilot Algebra and Calculus test. Table 6.2 will give a detailed breakdown of these sub-domains. The Algebra and Calculus questions focused on the acquired pre-knowledge where students err and the actual items included in the test came from a self-help guide designed to assist standard grade scholars with their examination preparation. The minimum enrolment requirement at the time of the study was 60 percent on standard grade. All questions thus had to be on the standard grade level. The self-help guide used was Study and Master Mathematics Standard Grade by E.A. Bester, J. Ham, K. Loots and A. Stark. The pilot test battery will appear at the back of the thesis as Appendices 2 and 3.
Once the items have been written the items have to be reviewed. The next step in the test development process deals with item review.

5.6.5 Item Review

According to the literature, all items – after being written – should be reviewed by a panel of experts [Foxcro05:75]. The experts will then determine whether the content domain has been covered. The items will also be assessed for cultural, linguistic and gender appropriateness. The items could also be administered to a sample of the target population to get qualitative information regarding the items and to determine if the test takers clearly understood the items. Based on the feedback from the panel, some items might have to be revised or re-written [Foxcro05:75].

The reason for using the guide 'Study and Master Mathematics Standard Grade', by E.A. Bester, J. Ham, K. Loots and A. Stark is because the writers are Senior Subject Advisors and Educators. They are experienced examiners and sub-examiners and are therefore familiar with the content of the Grade 11 and 12 syllabi.

A colleague, who is an educationist and experienced lecturer reviewed the pilot test items. He is also involved with various projects to upgrade the knowledge of Grade 11 and 12 learners and has been assisting teachers who want to upgrade their qualifications. The reviewer was handed a list of the common mistakes made by students. This enabled the reviewer to check that the full spectrum of mistakes was covered.

Once the items have been reviewed the length of the test has to be determined.

5.6.6 Length of the test

The amount of reading involved has to be taken into consideration when determining the length of the test especially if the test has to be completed within a limited time frame. This could result in some items being discarded. Where the test has a time limit attached to it; another alternative would be to reduce the amount of reading required [Foxcro05:76].

The tests could only be written during the first lecture or first tutorial period of each course included in the study. Each lecture was 75 minutes long with a 10-minute break between lectures. Students had 35-40 minutes to complete the Algebra and Calculus Algebra pilot test. The number of items in the test had to be limited as a result of the limited amount of time available. For the same reason students had 30-35 minutes to complete the Arithmetic test. The number of items in this test had to be limited as a result of the time constraint.

5.6.7 Answer Protocol

Students had to answer the questions in the allocated spaces provided on the question paper. This made it easier to administer the test, because each student only had to be given one question paper. Providing spaces for the answers forced students to be more organised when answering the questions and made it easier to score the tests. Once the answer protocol has been decided upon, the administration instructions can be finalised.

5.6.8 Development of administration instructions

Instructions that must appear at the beginning of the test include instructions for recording responses, time limits, and general strategies for answering the test.
Sireci (2003) gave some advice on the layout of the test. He advised that space should be left between items. Items should not be placed on top of each other. Also to use horizontal lines to separate sections, especially if the sections contain different item types. Items should be numbered sequentially starting at one. Material associated with an item such as a graph must be placed above or adjacent to the item. Ideally, the item and the associated material must appear on the same page or at worst on adjacent pages [Sireci03].

The following written instructions were given to students writing the pilot test:

- Answer all questions
- Use of a calculator is prohibited

The time limit was indicated on the front page. A block was provided where students could fill in their student numbers

5.7 Piloting of the Test

The test administration phase as well as the item analysis phase of test design could be seen as similar to an intervention step in the action research cycle. The administering and scoring of the pilot test battery will be discussed in more detail in Chapter 6.

5.7.1. Item Analysis phase

During the item analysis phase each item will be analysed to determine whether it serves the purpose it was developed for. The characteristics of the item will be statistically determined. The statistics will assist test developers to choose the items to include in the test. Item analysis allows the test developer to determine the difficulty level of an item and whether it is a good discriminator between good and poor performers. It also reveals the deficiencies of an item. After completing the item analysis the test developer can choose the best and most appropriate items to include in the test [Foxcro05:77].

The pilot test battery was administered to a large sample of the target population. After scoring the tests and capturing the data, information on the items was determined. The findings will be reported in Chapter 7.
The items used in the test were all aimed at the knowledge level of the standard grade pupil. All prospective first-year mathematics students should have been able to complete and pass both tests. Item difficulty values for each test item will be calculated in Chapter 7 and results provided.

5.8 Determining aspects of the Reliability and Validity of a test

According to Linn and Gronlund (2000), reliability refers to the consistency of assessment results. Although one needs consistency (reliability) to obtain valid results, it is possible to have reliability without validity. Consistent measures can provide the incorrect information. Reliability estimates are given in the form of a reliability coefficient or the standard error of measurement. Reliability is thus a statistical concept. Different methods are used to determine reliability coefficients. Each coefficient gives a different measure of consistency. The test-retest method requires that the same test be given twice to the same students with an interval in between tests. The calculated coefficient measures stability. The equivalent forms method requires that two forms of a test be given to the same group with an interval in between or in close succession. “The equivalent forms method provides a rigorous evaluation of reliability, because it includes multiple sources of variation in the assessment results” [Linn00]. Another method that can be used to determine reliability is by administering a test once and correlating the scores on two halves of the assessment. Reliability can also be determined by using the standard error of measurement. The standard error of measurement is computed from the standard deviation and the reliability coefficient. The standard error of measurement gives an indication of the band of error surrounding each score [Linn00].

Internal consistency reliability is a measure of “the item-to-item consistency of a subject’s responses within a single test” [Person05]. The reliability coefficient ranges from 0 to 1. Zero indicates no reliability and one indicates perfect reliability. The Cronbach’s Alpha statistic is normally used to assess the internal consistency of a test. The reliability coefficient should be 0.8 or higher. The internal consistency reliability was established for the pilot Arithmetic Test and the Algebra and Calculus Test. The results will be reported in Chapter 7.

Validity determines “how well the test measures what it purports to measure” [Person05]. There are various aspects of validity such as construct, content and criterion-related (predictive validity). For the research that this thesis focused on, content validity is important. A brief investigation of content validity and its evaluation was thus undertaken. “Evaluating inferences derived from test scores begins with evaluating the test itself” [Sireci1998]. Sireci continues, “Content validity refers to the degree to which a test measures the content domain it purports to measure. To argue that a test is valid for a particular testing purpose, it must be shown that the items and tasks composing the test are representative of the targeted content domain” [Sireci1998]. The test must adequately measure the content domain. Content validity is not measured statistically, but is judged by expert opinion.

For the pilot Arithmetic Test as well as the Algebra and Calculus Test, the items were selected from a self-help matriculation examination guide. The writers are experts in the field of education and are curriculum experts (Cf. 5.6.5). Items were reviewed by an experienced lecturer who is familiar with the Grade 11 and 12 syllabi.

When establishing the criterion-related or predictive validity of a test the aim is to establish how well a test score predicts future performance with respect to a particular criterion. A Chi-square analysis will thus be done to determine whether there is a relationship between the error categories of the pilot tests and the student’s final Mathematics 1 mark. Furthermore, a multiple regression analysis will be performed to determine the extent to which the total scores on the pilot Arithmetic and Algebra and Calculus tests predicted performance in first-year mathematics. The results of the Chi-square and multiple regression analyses will be provided in Chapter 7.

5.9 Conclusion

Diagnostic testing at Mathematics departments at South African Universities is still in its infancy. Testing will become very important in 2008 when the first students matriculate with the new National Senior Certificate (NSC). In the NSC subjects are not offered on different levels. Currently, these levels serve as entry requirements at many universities.

Some universities, such as the Nelson Mandela University draw students from unique feeder areas. It is therefore not possible for these Universities to use a common diagnostic test or to buy a diagnostic test from overseas as the quality of education and the syllabi might be vastly different. In the light of this – this researcher decided to develop her own in-house diagnostic test. Not many mathematics diagnostic tests have been developed in South Africa.

The reason why a diagnostic test and not an achievement or placement test is required at NMMU is because a diagnostic test determines students' areas of weakness. This will allow for the development of the necessary remedial programmes and material to bring prospective students up to the required level of pre-knowledge and skills. Secondly, the test could assist with the streaming of prospective first-year students. This could reduce the failure rate amongst first-year students.

Chapter 6 will provide more detail on the test specifications developed to operationalise the content sub-domains of the pilot test battery. This chapter will also give reasons for the inclusion of the chosen items and discuss the administering and scoring of the pilot test battery.

Chapter 6 – Determination of the test specifications and developing items for the pilot test battery

6.1 Introduction

The aim of this study was to identify the content sub-domains to include in a future Mathematics diagnostic test to be used at the NMMU and to develop a pilot test battery in this regard. This chapter focuses on:

- Determining the specifications for the two content sub-domains to be included in the pilot test battery.
- Developing items for and giving reasons for the inclusion of specific items in the pilot tests.
- The administration of the pilot test battery.
- The scoring of the pilot test battery.

The pilot test battery consisted of both an Arithmetic test and an Algebra and Calculus test. The identification of the content sub-domains of these two tests was discussed in 5.6.2.1 and 5.6.2.2 respectively. Based on these content sub-domains, test specifications were developed by the researcher for each of the pilot tests. The pilot Arithmetic test will be discussed first.

6.2 Identification of the Test Specifications of and the Development of Items for the pilot Arithmetic Test

To decide what the test specifications for the pilot Arithmetic Test should be, the delineation of the content sub-domain (Cf. 5.6.2.1), student mistakes (Cf. 4.3), feedback from colleagues as per the list of mistakes identified in section 4.2.4, as well as other tests (Cf. 2.4 and 5.5) were considered.
More than one question was included on selected topics based on either the importance of the topic or the high frequency of student errors with respect to the topic. The test specifications, developed by the researcher, for the pilot Arithmetic Test are provided in Table 6.1.

Table 6.1 Test Specifications for the pilot Arithmetic Test

Sub-domain	Topic	No. of Questions Included
Operations with decimals	Decimal subtraction	2
	Decimal division	2
	Decimal addition	1
	Relationship between multiplication and division	1
Rounding of decimals	Rounding	1
Operations with fractions	Fraction multiplication	1
	Fraction division	1
	Fraction subtraction	2
	Mixed numeral addition	1
	Equivalence of fractions	1
Order of operations in absence of brackets	Order of operations	1
Fractions and percentages	Calculating the whole when the percentage and value of a part are given	1
Percentages	Percentages exceeding 100%	1
	Calculating percentage discounts	1
Ratio and proportion	Ratio-related problems	2
Meaning of “of” in mathematics	Meaning of “of” in mathematics	1
Distributive law	Distributive Law	1
Roots	Square root of the sum of two natural numbers	1
	Find the cube root	1
Word Sums	Word Sums	2

Source: own construct

Based on the above test specifications, items were developed or sourced by the researcher for the pilot Arithmetic Test. Examples of some of the items and reasons for their inclusion are provided in the next sub-section:

6.2.1 Examples of the items included in the pilot Arithmetic Test

This section gives examples of the items included and the reasons for their inclusion. The items have been grouped according to the sub-domains of the Arithmetic Test. Consequently, the order in which the examples of the items are presented differs from the order in which the items appeared in the pilot test.

Decimal Operation Examples (Cf. 4.2.5):

Calculate the following:

$23.34 - 22.56$

Calculate the following:

$0.11 + 11 + 1.1$

Calculate the following:

$12.5 \div \frac{1}{4}$

Rounding Example (Cf.4.2.5):

What are 2.3988 rounded to the nearest hundredth?

Fraction Operation Examples (Cf. 4.3 Examples 29-30, 4.3.1,4.2.4 and 4.2.5):

Calculate the following:

$36 \times 3\frac{3}{4}$

Calculate the following:

$\frac{5}{8} \div \frac{1}{2}$

Calculate the following:

$3\frac{1}{4} + 2\frac{4}{5}$

Calculate the following:

$\frac{4}{5} - \frac{3}{15}$

Order of operations/Fractions (Cf. 4.2.5):

Calculate the following:

$$\frac{3}{\frac{1}{4}+\frac{1}{12}}$$

Ratio Examples (4.3 Example 34, 4.2.4 and 4.2.5):

The ratio of men to women is 5 is to 6 and the ratio of adults to children is 3 is to 2. What is the ratio of women to children?

(This question is from the textbook Mathematics for Elementary Teachers A Balanced Approach, 2nd Edition by Eugene F. Krause. The researcher noticed that the education students who were mathematically strong experienced no problems with this question. However the weaker students did. The question was included in the pilot test to determine whether this was true for students in general and whether this type of question would be a possible predictor of success.)

The ratio of girls to boys is 2 is to 3. If there are 24 boys, how many girls are there?

(This question is from the textbook Mathematics for Elementary Teachers A Balanced Approach 2nd Edition by Eugene F. Krause. The researcher noticed that most education students could cope with this question and wanted to see if this was true in general.)

Percentage and Discount Examples (Cf.4.2.5 and 4.3 Example 35):

Two toys are on sale. The first toy is a toy truck and was selling for R120 before it was marked down by 40%. The second toy is a doll and was selling for R80 before it was discounted by 20%. Which toy is the now the cheapest?

An item costs twice as much as it did last year. What percentage is the new price of the old price?

Mathematical meaning of “of” Example:

What is $\frac{2}{5}$ of 30?

Distributive Law Example (Cf. 4.5):

Calculate the following:

21 864 880 456 678 $\times$ 986 – 985 $\times$ 21 864 880 456 678

Square and Cube Root Examples (Cf. 4.5 and 4.3 Example 36):

Calculate the following:

$\sqrt{9+16}$

Calculate the following:

$\sqrt[3]{\frac{250}{54}}$

Word Sum Examples (Cf.4.3 Example 34 and 4.2.5):

On a train trip from Port Elizabeth to Johannesburg 136 of the 240 available seats are occupied. What fraction of the available seats is occupied? (Simplify the fraction as far as possible.)

John spends his pocket money as follows: $\frac{1}{4}$ on sweets, $\frac{3}{5}$ on entertainment and $\frac{1}{10}$ on gifts. How much money does he have left?

The content validity of the pilot Arithmetic test will be addressed in the next chapter in sub-section 7.2.1.

Section 6.3 covers the test specifications, developed by the researcher, of the pilot Algebra and Calculus Test as well as providing examples of items developed for it.

6.3 Identification of the Test Specifications of and the Development of Items for the pilot Algebra and Calculus Test

To determine the specifications for the pilot Algebra and Calculus Test, consideration was given to the delineation of the Algebra and Calculus sub-domain (Cf. 5.6.2.2), the pre-knowledge that a first-year student requires (Cf. 3.3), first-year students' mistakes (Cf. 4.3), and the list of mistakes that arose from observations by the researcher and colleagues (Cf. 4.2.4). The test specifications for the pilot Algebra and Calculus Test are provided in Table 6.2. It should be noted that the pilot Algebra and Calculus Test contained 80 percent Algebra items and 20 per cent Calculus items.

Table 6.2 Test Specification of the pilot Algebra and Calculus Test

Sub-domain	Topic	No. of Questions Included
Factorisation	Factoring	2
	Solving a trinomial using the formula	1
Functions	Values for which a rational function equals zero	1
Squaring binomials	Expanding binomials	1
Indices and Surds	Rules of indices and surds	1
Graph sketching	Sketching a straight line	1
	Sketching a parabola	1
	Sketching a semi-circle	1
Domain and range	Domain and range of a semi-circle	1
Analytical Geometry	Knowing the product of the slopes of two perpendicular lines and calculating the equation of the new line when a point on the new line is given in addition to the equation of the perpendicular line	1
Limits	Calculating a limit using substitution	1
Differentiation	Differentiation involving simplification	1
	Differentiation involving roots and negative exponents	1
	Differentiation involving simple sum of functions rule	1
	Differentiation using first principles	1
Logarithmic Functions	Rules for Logarithmic functions	2
	Converting from logarithmic to exponential form	1
Exponential Functions	Exponential functions	1

Source: own construct

Based on the above test specifications, items were developed or sourced by the researcher for the pilot Algebra and Calculus Test. The next sub-section will provide examples of the items included in the pilot test and reasons for their inclusion.

6.3.1 Examples of the items included in the pilot Algebra and Calculus Test

Factorisation Examples:

Factorise the following:

$(a)\ x^2 - x - 6$

$(b)\ 3x^2 + 6x$

Solve for x in $3x^2 + 2x - 4 = 0$

The reason for the inclusion of items on factorisation is that students' inability to find a limit when algebraic manipulation is required, stems from an inability to factorise. Factorisation also affects curve sketching. In Paragraph 4.3, examples 13 and 27 are examples of the mistakes made by students when factorising and paragraph 4.2.4 also identifies factorisation as an area of weakness.

Values for which a rational function evaluates to zero:

Determine the x- value(s) for which $\dfrac{(x+1)^2}{x^2 - x - 6} = 0$

The researcher wanted to establish whether the students knew the difference between the x- value(s) for which a rational function evaluates to zero and the x- value(s) for which the rational function is undefined. The distinction becomes important when the domain of the rational function has to be determined.

Examples involving the rules of indices and surds and the expansion of a quadratic expression involving a surd:

Simplify the following:

$(a)\ (2+\sqrt{3})^2+(2-\sqrt{3})^2$

$(b)\ \left(\dfrac{\sqrt{a}}{a^{\frac{-3}{2}}}\right)^{\frac{1}{2}}$

Students often experience problems when they have to convert from a surd to an exponent (Cf. 4.3 examples 8-11). The researcher wanted to establish whether students could correctly expand a quadratic expression involving a surd. Example (a) also identified a number of factorisation misconceptions. Paragraph 4.2.4 corroborate that students find it difficult to convert a surd into a power.

Graph Sketching Examples:

Sketch the following:

$(a)\ 3x+3y=1$

$(b)\ y=-2x^2+2x+12$

$(c)\ y=-\sqrt{4-x^2}$

Students often cannot find the area between two curves correctly because of an inability to sketch the area. Examples 13 and 39 (Cf. 4.3) show errors relating to the sketching of parabolas and examples 37,38 and 40 (Cf. 4.3) show errors relating to the sketching of straight lines. These examples also illustrate that students could even mistake one for the other. Paragraph 4.2.4 identifies an inability to sketch parabolas as a problem experienced among first-year students.

Domain and range of a function:

The domain and range of the function $y=-\sqrt{4-x^2}$ has to be determined.

Domain and range is introduced at school. Although the topic is re-introduced in first year, the researcher wanted to establish the pre-knowledge the learner comes to university with.

Limit Calculation Example:

Calculate the following limit:

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$$

Although the topic is re-introduced in first year of university studies, the researcher wanted to establish the learner's knowledge of the topic. The researcher also wanted to establish whether the learners could factorise the numerator.

Calculation of Derivative Examples:

(a) Let $f(x) = -3x^2$. Use the definition of the derivative of a function to determine $f'(x)$.

(b) Determine $\frac{dy}{dx}$ if $y = \frac{3x^2 - 2x + 2}{x}$

(c) Determine $\frac{dy}{dx}$ if $y = \sqrt[3]{x} - \frac{2}{x^2}$

(d) Determine $\frac{dy}{dx}$ if $y = 3x^4 - 2x^3 + x - 1$

Roth et al state that "taking more higher level math courses in high school is an accurate predictor of scoring well on aptitude tests commonly required for admission into four-year baccalaureate institutions" [Roth01]. The researcher wanted to establish whether there is a relationship between performance on the differentiation questions and successful completion of Mathematics 1 courses at the NMMU. Section 4.3 example 28 highlights that students cannot simplify the rational function and thus cannot differentiate it. Section 4.3 examples 8 and 9 illustrate students' unfamiliarity with the rules of indices and surds hamper their ability to differentiate a function.

Logarithm Rule Examples:

Calculate the following:

$(a)\ 5\log_4 2 - \log_4 0.125 - 2\log_4 8$

$(b)\ \log_3 27 + \log_3 3$

Chapter 4 section 4.3 examples 21-24 illustrates students make mistakes when they have to apply the rules for the natural logarithmic function. The researcher wanted to establish whether the problem existed just for the natural logarithmic function or whether the inability to apply log rules also applied to other logarithmic functions introduced at school.

Exponential function Examples:

Solve for x in each case:

$(a) \log x^3 = 6$

$(b)\ 2^{\log x} = 4^{-1}$

Chapter 4 section 4.3 examples 16-20 highlight students' inability to convert from the natural logarithmic function to the exponential function. The researcher wanted to establish whether the problem existed just for the natural logarithmic function or whether the inability to convert from logarithmic to exponential function also applied to logarithmic functions introduced at school.

The items for the pilot Algebra and Calculus Test were selected from a self-help guide *Study & Master Mathematics Standard Grade*. The reason why the *Study and Master Mathematics Standard Grade* by E.A. Bester, J. Ham, K. Loots and A. Stark guide was used is that the writers are Senior Subject Advisors and educators. They are experienced examiners and sub-examiners and thus should have a good knowledge of the Grade 11 and 12 standard grade school syllabi.

The researcher felt that this would ensure that all questions asked were on the standard grade level and thus within the content domain. Item review was discussed in paragraph 5.6.5.

The content validity of the pilot Algebra and Calculus Test will be discussed in section 7.2.1 in Chapter 7.

The next section considers the administration instructions given to the students when the pilot test battery was administered.

6.4 Administration Instructions

For the pilot tests, the researcher acted as the assessment practitioner and was present at all test sessions to assist with possible queries.

Students were given two written instructions, namely, to answer all questions and not to use calculators. These instructions appeared on the first page of the test booklet underneath the student number block. Students had to fill in their student numbers to enable the researcher to link pilot test results to students' academic performance. The time limit for the test also appeared on the first page. A message was also printed on the first page of the test booklet to inform students that the pilot tests would not be taken into consideration when determining their first-year mathematics marks.

6.5 Administering the pilot tests

The pilot test battery was administered at the beginning of 2004. Although the lecturer responsible for each group (class) administered the pilot tests, the researcher was present at each session to ensure consistency. At the beginning of the lecture or tutorial period, students were welcomed by the administrator and were verbally informed that they were going to write two surprise tests to assess how much pre-knowledge they had. They were informed that the results of the test would not in any way affect their marks. The question paper also stated that the pilot test was to be used for research purposes and that all results would be treated as being confidential. Students were reminded of the instructions and the time limit. The administrator also explained why they had to show all workings. The students were thanked for their co-operation and then proceeded to write the test.

6.6 Scoring the Pilot Test

Both the pilot Arithmetic Test and the pilot Algebra and Calculus Test were scored holistically and analytically. The holistic score provided a student's overall level of knowledge on the topics tested. The minimum holistic score per question was zero (no appropriate effort) and the maximum score per question was five (correct answer). The analytic scoring assessed a student's areas of weakness, i.e. what pre-knowledge the student was lacking at the start of first year. Each student's paper was scored twice; once holistically and once analytically. These two scoring methods will be discussed in more detail in sub-sections 6.6.1 and 6.6.2 below.

6.6.1 Holistic scoring

A holistic score gives an overall impression of the test-taker's attempt. Holistic scoring is quicker and easier to perform than analytic scoring [Sireci03]. When assessing a student's work, one must assess both the thinking involved and the final product. The holistic marking was done using the unaltered Otis and Offerman holistic scoring scale. Randall Charles developed a Focused Holistic Scoring Point System to evaluate a student's solutions [Bosho02]. Randall's scale assessed the student's solution using a five-point scale in which points ranging from zero to four are given for a student's solution [Bosho02]. Otis and

Offerman modified Randall's five-point scale and obtained the six-point scale given in Table 6.3 [Bosho02], which was used in this study.

Table 6.3 Holistic Scoring Scale of Otis and Offerman

Number of points	Observed characteristics of the student's solution
0	▪ Blank paper ▪ Numbers from problem recopied – no understanding of problem evidenced ▪ Incorrect answer and no work shown
1	▪ Inappropriate strategy started – problem not finished ▪ Approach unsuccessful – different approach not tried ▪ Attempt failed to reach a subgoal
2	▪ Inappropriate strategy – but showed some understanding of the problem ▪ Appropriate strategy used – did not find the solution or reached a subgoal, but did not finish the problem ▪ Correct answer and no work shown
3	▪ Appropriate strategy, but ▪ Ignored a condition in the problem ▪ Incorrect answer for no apparent reason ▪ Thinking process unclear
4	▪ Appropriate strategy or strategies ▪ Work reflects understanding of the problem ▪ Incorrect answer due to a copying or computational error
5	▪ Appropriate strategy or strategies ▪ Work reflects understanding of the problem ▪ Correct answer

Source: [Bosho02]

It was important to be consistent when marking students' solutions. To achieve consistency when marking students' tests, the students' solutions as well as the holistic mark allocated to the attempt according to the Otis Offerman Scale were recorded initially. This served as a reference during the marking process.

6.6.2 Analytical scoring

Analytic scoring is more complicated. The Scorer has to identify all aspects of the model answer and record the mistakes made per question or sub-section thereof. In the case of the pilot study each mistake was numbered. Each mistake identified was checked against a list of errors to determine whether it was already on the list or had to be added. When the scoring was completed the errors were categorised to meet the requirements for statistical analysis and to simplify the reporting process. The final categories are referred to as refined categories in this text. Diagram 6.1 gives an example of the categorisation process:

Diagram 6.1 Example illustrating the categorisation of errors

Example of an error list for operations with fractions:
1. Cannot add fractions
2. Cannot multiply a proper fraction with a proper fraction
3. Cannot divide a proper fraction by a proper fraction
4. Cannot subtract two fractions when their denominators differ.

These errors were grouped under the appropriate category:
5. Errors relating to operations with fractions

Table 6.4 details the refined Arithmetic Error Categories. All reporting in Chapter 7 will take place in terms of the refined error categories used when performing the analytical scoring of performance in the pilot Arithmetic Test.

Table 6.4 Refined Arithmetic Error Categories

Category Number	Category Description
1	Calculation Mistakes and Copying Errors
2	Ratio-related problems
3	Finding the square root of the sum of two natural numbers
4	Errors relating to percentages
5	Errors relating to operations with fractions
6	Errors related to operations with decimals
7	Disregard for order in the number system
8	Cannot find the cube root of a given ratio
9	Cannot apply the distributive law
10	Simplification of fractions or expressions
11	Reading problems
12	Mathematical meaning of "of"
13	Making x the subject of the equation
14	Construct a fraction from a word sum problem
15	Ranking of arithmetical operations

Source: own construct

The initial Algebra and Calculus Error Categories were also very detailed. To meet the requirements of statistical analysis and to streamline the reporting process the initial categories were grouped into broader categories. For instance all errors related to differentiation were grouped together and similarly all errors related to notation were grouped together. This gave rise to the refined error categories in Table 6.5.

Table 6.5 Refined Algebra and Calculus Error Categories

Category Number	Category Description (Algebra and Calculus)
1	Factorization
2	Copying and Calculation mistakes
3	Determines the values for which a rational function evaluates to zero
4	Errors made when expanding perfect squares especially if square roots are involved
5	Cannot apply the rules of indices and surds involving division and roots
6	Recognition of and drawing of a straight line
7	Recognition of and drawing of a parabola
8	Knowing the product of the slopes of two perpendicular lines and calculating the equation of the new line when a point on the new line is given
9	Recognition of and drawing of a semi-circle
10	Notation
11	Finding the domain and range of a semi-circle
12	Cannot calculate a limit
13	Writing power functions as negative powers or as roots or vice versa
14	Problems performing elementary differentiation
15	Arithmetical errors
16	Cannot convert from logarithmic to exponential form
17	Unfamiliar with log rules
18	Unfamiliar with rules for exponential functions(equal bases implies equal exponents)
19	Simplification especially of rational functions
20	Language problems

Source: own construct

On the answer script next to the holistic score for the question, the error category was also recorded during the marking of scripts. A forward slash separated the holistic and analytic scores. The holistic mark was recorded in black and the analytic in red. An example (diagram 6.2) illustrates the process.

Diagram 6.2 Illustration of the recording of the holistic and analytic mark on students' scripts

Q11(b) 3 / ***17***

Q11(b) 4 / ***2***

Holistic mark in black
Analytic mark in red

The next two sub-sections will provide an example of the holistic and analytical scoring methods.

6.6.3 Examples of holistic scoring

To illustrate the scoring process, Question 11(b) of the pilot Algebra and Calculus Test will be used.

Question 11(b) required students to calculate $\log_3 27 + \log_3 3$

Table 6.6: Illustration of holistic scoring process

Attempt	**Holistic Mark Allocation and Reasons for the Allocation**
$\log_3 30$	Inappropriate strategy started. No sub-goal reached. Mark Allocated: 1
$\log_3 81$	Appropriate strategy started. Sub-goal reached, but did not finish the problem. Mark allocated: 2
$\log_3 3^3 + \log_3 3$ $= 3\log_3 3 + \log_3 3$ $= 4\log 1$	Appropriate strategy, but thinking becomes unclear. Mark allocated: 3
$= \log_3(27+3)$ $= \log_3 30$ $= \dfrac{\log 30}{\log 3}$	Inappropriate strategy started. No sub-goal reached. Mark Allocated: 1

Attempt	**Holistic Mark Allocation and Reasons for the Allocation**
$\log_3 3^3 + 1$ $= 2$	Appropriate strategy, but thinking becomes unclear. Mark allocated: 3
$9 + 1 = 10$	Inappropriate strategy started. Shows some understanding. Mark allocated: 2
$\log_3 3^3 + \log_3 3$ $= 3\log_3 3 + \log_3 3$ $= 3 \times 1$	Appropriate strategy started. Sub-goal reached, but did not find the solution. Mark allocated: 2
$= 27 \times 3$	Inappropriate strategy started. Shows some understanding. Mark allocated: 2
$\log_3 (27 \times 3)$	Appropriate strategy started. Sub-goal reached, but did not find the solution. Mark allocated: 2
$3 + 3 = 6$	Appropriate strategy started. Sub-goal reached, but did not find the solution. Mark allocated: 2
$\log_3 (27 \times 3)$ $\log_3 81$ $\log_3 3^5$ $= 5$	Appropriate strategy started. Work reflects understanding. Incorrect answer due to computational error. Mark allocated: 4
$3\log_3 3 + \log_3 3$	Appropriate strategy started. Sub-goal reached, but did not find the solution. Mark allocated: 2
$= \log_3 7 \times 4 + 1$ $= \log_3 7 \times 2^2 + 1$ $= 2\log_3 7 \times 2 + 1$	Appropriate strategy started. Sub goal reached, but did not find the solution. Mark allocated: 2
$\log_3 27 + \log_3 3$ $\log 9 + \log 1$ $\log 10$	Inappropriate strategy started. No sub-goal reached. Mark Allocated: 1
$\log_3 27 \times 3$ $\log_3 8 = 81$	Appropriate strategy started. Sub-goal reached, but did not find the solution. Mark allocated: 2
$2\log_3 31$	Inappropriate strategy started. No sub-goal reached. Mark Allocated: 1
$\log_2 27 + 1$	Inappropriate strategy started. Shows some understanding. Mark allocated: 2

Attempt	Holistic Mark Allocation and Reasons for the Allocation
$\log_3 3^3 + \log_3 3$ $= 81$	Inappropriate strategy started. Shows some understanding. Mark allocated: 2

Source: own construct

6.6.4 Examples of analytic test scoring

Analytic scoring involved the categorization of errors made by the students. Examples were:

- The analytic error category associated with question 11(b), i.e. the calculation of $\log_3 27 + \log_3 3$, was “Unfamiliar with the product and/or power rule for logarithmic functions”.
- All mistakes related to unfamiliarity with rules for logarithmic functions were grouped together under “Unfamiliar with log rules” which is category 17 in Table 6.5.
- Where a student made an arithmetical error, the error was recorded as the analytic category error “Copying and Calculation mistakes” which is category 2 in Table 6.5.

6.7 Recording and verification of the data in Excel

The student’s holistic and analytic marks were recorded in a Microsoft Excel® spreadsheet. The conditional format feature in Excel was used for the identification of invalid values, e.g. a holistic mark greater than 5, or inconsistencies, e.g. a holistic mark of 5 (correct answer) with an analytic mark implying that an error was made. Other information and data recorded for each student were:

- Student number
- Gender
- Age
- Home Language

- Matriculation English Mark
- Matriculation Mathematics Mark
- The average mark for Matriculation Accountancy, Science and Biology
- Algebra and Calculus Pilot Test Score – the sum of the Algebra and Calculus holistic marks as a percentage of the maximum possible (5 times the number of questions)
- Arithmetic Pilot Test Score – calculated identically as explained in the previous bullet
- Final University Mathematics 1 Mark

Student numbers and matric and university examination results were extracted from official university records and matched with the captured pilot test data.

6.8 Conclusion

When developing a diagnostic test, once the sub-domains to include have been delineated, test specifications need to be developed to guide the item development process. In Chapter 5 the sub-domains were delineated, while in this chapter a detailed breakdown of the test specifications and examples of the items that were developed or sourced for the pilot tests were provided. In addition, the administration instructions for the pilot tests and the scoring methods used were discussed.

In Chapter 7 the statistical analyses performed to explore the item difficulty, validity and reliability of the pilot tests will be presented.

CHAPTER 7 – Results: Validity and Reliability of Pilot Diagnostic Test Battery

7.1 Introduction

As explained in Chapter 6, a pilot diagnostic Mathematics test battery was administered in 2004 to determine its validity and reliability. This test battery consisted of:

1. A pilot Arithmetic Test and
2. A pilot Algebra and Calculus Test.

The aim of the study was to initially delineate the sub-domains to include in a diagnostic Mathematics test and to pilot a test battery in this regard. This chapter describes the results of the statistical analysis of the pilot test battery. The purpose of the statistical analysis was to determine aspects of the validity and reliability of the tests included in the pilot test battery.

In the first section of the chapter, aspects of the sample will be described. In the subsequent sections evidence related to the verification of the validity and reliability of the two pilot tests will be provided. Results of the content validity analysis will be reported. Item difficulty levels and item-total correlations will be presented for each test item. Thereafter, results that provide evidence related to the predictive validity of the pilot tests will be presented. Finally, to determine the internal consistency of the pilot tests, the Cronbach's Alpha findings will be presented.

7.2 Sample Description and Descriptive Statistics

A total of 331 first-year Mathematics students at the NMMU (formerly UPE) wrote the pilot Algebra and Calculus Test and 340 wrote the pilot Arithmetic Test. The reason for the differing number of students that wrote the two pilot tests was that some students could only complete one test in the time allocated.

For 41% of the sample the gender indicator was not available. Using the available information, 54% of the sample was male and 46% female. The mean age of the students sampled was 19.22 years with a standard deviation of 2.87. For 41% of the sample there was no indication whether they were local or foreign students. Using the available information, 84% of students were South Africans and 16% international students. As the pilot tests were diagnostic in nature, only international students who had school marks comparable to the South African matric marks were included in the sample. Information on home language was only available for 59% of the sample. Table 7.1 provides a breakdown of the language distribution of the students for whom information on home language was available.

Table 7.1 Home Language Distribution

Language	**n**	
Afrikaans	36	17%
English	98	46%
Xhosa	46	22%
Other African languages	26	12%
Other European languages	3	1%
Chinese	4	2%
Total	213	100%

Information on culture group was only available for 59 percent of the sample. Table 7.2 details the breakdown of the ethnic groups represented in the sample, where information was available.

Table 7.2 Ethnic Group Distribution

Ethnic Group	**n**	
Black	81	38%
Chinese	10	5%
Coloured	25	12%
Indian	10	5%
White	87	41%
Total	213	100%

As can be seen in Table 7.2, the predominant culture groups in the sample were white and black. This mirrors the fact that black and white students comprise the two largest culture groups in the Faculty of Science. Actual enrolments per ethnic group for Mathematics 1 courses in 2004 – as reflected in the pass rates table in Appendix 1 – corroborate the fact that black and white were the two predominant groups in the Mathematics department in 2004.

The samples used were thus sufficiently reflective of students who take Mathematics in degree programmes at the NMMU and were also sufficiently large [Hair92: 227] to conduct the statistical analysis that needed to be performed.

Table 7.3 gives the midpoint of the range specified for each matriculation symbol. "Standard-grade marks were therefore equalized relative to higher grade marks by multiplying the standard grade mark by 0.75" [Neale03:72]. For example a standard grade 'A' symbol is equivalent to a 'C' higher grade symbol. Standard-grade marks were rounded to the nearest percent [Neale03:72].

Table 7.3 Matriculation Symbol conversion

Matriculation Symbol	Midpoint	HG Mark	SG Mark equalized
A	85	85	64
B	75	75	56
C	65	65	49
D	55	55	41
E	45	45	34

Table 7.4 provides the descriptive statistics for the Mathematics marks for the sample obtained in the matriculation examination as well as the mean school performance score.

Table 7.4 Matriculation marks

Statistic	Matriculation Mathematics (n=193)	School Performance Score (n=195)
Missing records (%)	47	46
Mean	58.23	59.77
SD	13.78	9.30
Minimum	25.00	41.04
Quartile1	48.75	53.08
Median	56.25	58.75
Quartile 3	63.75	65.83
Maximum	85	81.46

From Table 7.4 it follows that 75% of first-year students scored less than or equal to 63.75 per cent for Mathematics in the matriculation examination and had a school performance score of less than or equal to 65.83 per cent.

Table 7.5 provides the descriptive statistics for the Arithmetic and the Algebra and Calculus Test marks for the sample.

Table 7.5 Pilot Test Battery Marks

Statistic	Arithmetic Test (n=361)	Algebra and Calculus Test (n=330)
Missing records (%)	6	8
Mean	69.55	40.91
SD	18.05	19.49
Minimum	16	0
Quartile1	59	27
Median	72	38
Quartile 3	52	83
Maximum	100	100

From Table 7.5 it can be deduced that while performance on the pilot Arithmetic Test can be described as acceptable, students performed poorly on average in the pilot Algebra and Calculus Test

7.3 Determining the Validity of the Pilot Test

A psychometric measure must meet two requirements. These requirements are reliability and validity [Foxcro05:28]. Section 7.3 will address the issue of validity.

7.3.1 Content Validity

In developing the pilot diagnostic tests, cognisance was taken of the content domains that were delineated in sections 5.6.2.1 and 5.6.2.2 and the test specifications in Tables 6.1 and 6.2. For a diagnostic test based on content domains covered in the school and university curricula, it is essential to establish its content validity [Foxcro05:33]. "To demonstrate this form of validity the instrument must show that it fairly and comprehensively covers the domain or items that it purports to cover. It is unlikely that each issue can be addressed in its entirety simply because of the time available… [Cohen00:109]". In Table 7.6 the abbreviations used in Tables 7.7 and 7.8 are supplied.

Table 7.6 Abbreviations required for Tables 7.7 and 7.8

Name of the test	**Abbreviation used**
Pilot Test	P
John James Barnard Test	B
Loughborough Test	L
P.E. Technikon Test	T
Unisa Foundation Level Maths Test	U

Source: own construct

The sub-domains identified by the researcher (Cf. 5.6.2.1 – 5.6.2.2) and included in the pilot test battery were compared with the sub-domains included in a number of local tests and one overseas tests. The John James Barnard and the

Loughborough tests are diagnostic tests. The other tests are achievement tests. The John James Barnard test was designed to test Grade 11 learners. Some of the sub-domains used in this test will overlap with the content domains identified by the researcher. The Loughborough test was designed to test first-year students. Although there are similarities between this test and the pilot test battery there are also differences as a result of curriculum differences. The P.E. Technikon Test and the Unisa Foundation Level Maths Test were aimed at testing foundation phase students. The sub-domains included in these two tests will overlap the sub-domains included in the pilot test battery to some extent. The sub-domains will not correspond totally due to the fact that the pilot test battery was designed to test first-year students. These tests, found by the researcher, were the only tests corresponding to a large extent with the pilot test sub-domains identified by the researcher.

Table 7.7 Content sub-domains: Pilot diagnostic Arithmetic Test compared to other tests

Sub-domains	**P**	**B**	**L**	**T**	**U**
Operations with decimals	yes	yes			
Rounding of decimals	yes				
Operations with fractions	yes	yes	yes	no	yes
Ratio-related problems	yes	yes			yes
Percentages exceeding 100 percent	yes				
Percentages	yes	yes	yes		yes
Word Sums	yes			yes	yes
Distributive law	yes				
Square root of sum of two natural numbers	yes			yes	
Finding the cube root of a cubic ratio	yes				

Table 7.8 Content sub-domains: Pilot diagnostic Algebra and Calculus Test compared to other tests

Sub-domains	P	B	L	T	U
Factorisation	yes	yes	yes		
Solving a quadratic equation using a formula	yes		yes		
Values for which a rational function equals zero	yes				
Squaring binomials	yes			yes	
Rules of indices and surds	yes	yes		yes	yes
Straight lines	yes	yes	yes		
Parabolas	yes	yes			
Knowing the product of the slopes of two perpendicular lines and calculating the equation of a line when a point on the new line is given in addition to the equation of the perpendicular line	yes				yes
Semi-circles	yes				
Domain and range	yes				
Calculating a limit using substitution	yes			yes	
Differentiation	yes			yes	
Application of log rules	yes		yes	yes	yes
Converting from logarithmic to exponential form	yes				
Laws of exponentials	yes		yes		

There are many similarities between the pilot tests and other available tests that they were compared with. However, from the tables it is also clear that none of the other tests tap all the sub-domains identified by this researcher to include in her pilot diagnostic test battery. One of the reasons for this is that the content sub-domains for the pilot diagnostic tests were closely linked to the pre-knowledge and skills that a prospective first-year degree student in Mathematics at NMMU should have (Cf. 3.3). In the next sub-section item difficulty values for the test items will be reported.

7.3.2 Item Difficulty Values and Item-total Correlations for individual Items

7.3.2.1 Item Difficulty Levels and Item-total Correlations for the pilot Arithmetic Test

The item difficulty level is normally calculated as the proportion of individuals who answered the item correctly or

$$p = \frac{\text{Number of people who answered the item correctly}}{\text{Sample size}}$$

The item difficulty index was calculated based on the proportion correct, but also on the proportion of students who scored between 3 and 5 marks on the item. This was done to take into consideration the fact that holistic scores varied between 0 and 5 and not merely between right and wrong. A student obtaining a score of 3 or more on the item obtains a pass mark. The difficulty index based on the mean holistic score (DIM) was also calculated. The mean holistic score was divided by five to obtain the DIM. The item difficulty guidelines used to assess items are stated in Table 7.9 [Kruger80]. Table 7.10 contains the Item Difficulty Indices for the pilot Arithmetic Test.

Table 7.9 Item Difficulty Guidelines

Difficulty Value	Interpretive Description
0.00 — 0.20	Unacceptably Difficult
0.21 — 0.30	Difficult
0.31 — 0.69	Average
0.70 — 0.80	Easy
0.81 — 1.00	Unacceptably Easy

Table 7.10 Item Difficulty Levels for the Arithmetic Pilot Test

Item Number	Mean Score	Difficulty Index based on Mean	Difficulty Index proportion score 3 to 5	Difficulty Index proportion correct
9	0.51	.102	.062	.062
23	1.08	.215	.191	.179
21	1.94	.388	.350	.350
10	2.26	.453	.426	.426
16	2.50	.500	.500	.500
7	2.60	.521	.468	.412
25	3.17	.634	.685	.503
2	3.35	.670	.668	.668
3	3.56	.713	.706	.659
24	3.66	.731	.694	.647
14	3.65	.731	.712	.659
13	3.89	.778	.753	.700
8	3.90	.779	.756	.741
4	3.95	.791	.782	.629
19	3.99	.798	.788	.765
22	4.01	.802	.774	.774
11	4.11	.821	.806	.738
18	4.16	.832	.829	.806
1	4.19	.839	.832	.818
15	4.24	.848	.841	.721
5	4.34	.868	.847	.818
6	4.37	.874	.868	.853
12	4.44	.887	.879	.868
20	4.45	.891	.891	.865
17	4.60	.920	.918	.918

The item difficulty index values of the Arithmetic Test using the DIM show that items 1,5,6,11,12,15,17,18 and 20 were unacceptably easy and item 9 was unacceptably difficult. The fact that about a third of the items of the pilot Arithmetic Test were found to be easy could be one of the reasons why a relatively high mean score of 69.55 was obtained by the sample on this test. Furthermore it is expected that a first-year student should find Arithmetic easy.

"One of the purposes of item analysis is to discover which items best measure the construct or content domain that the measure aims to assess [Foxcro05:52]." Consequently, item-total correlations were calculated for the pilot Arithmetic test items. "A positive item-total correlation indicates that the item discriminates between those who do well and those who do poorly on the measure

[Foxcro05:5].” A score close to zero indicates the item does not discriminate well between high and low scores. An item must have an item-total correlation value of at least 0.2 before it can be selected. Table 7.11 contains the Item-total Correlation statistics for the pilot Arithmetic Test. All items in the pilot test have item-total correlation values exceeding 0.2 and all the correlation values are positive. This suggests that the items have the potential to discriminate between students who perform well and poorly in the measure, which is essential in a diagnostic test.

Table 7.11 Item-Total Correlation statistics for the pilot Arithmetic Test

Item Number	Item-Total Correlation
1	.258
2	.267
5	.332
9	.332
10	.344
17	.349
22	.354
11	.371
23	.389
6	.396
12	.408
3	.416
16	.419
19	.426
8	.433
7	.437
20	.455
21	.458
4	.471
18	.490
13	.497
24	.498
14	.510
25	.515
15	.538

7.3.2.2 Item Difficulty Levels and Item-total Correlations for the pilot Algebra and Calculus Test

Table 7.12 contains the Item Difficulty Indices for the pilot Algebra and Calculus Test per item.

Table 7.12 Item Difficulty Levels for the pilot Algebra and Calculus Test

Question Number	Mean Score	Difficulty Index based on Mean	Difficulty Index proportion score 3 to 5	Difficulty Index proportion correct
11a	0.82	.163	.060	.051
7b	0.84	.169	.133	.051
12b	0.88	.176	.109	.100
12a	1.01	.202	.121	.109
10	1.08	.215	.178	.082
9a	1.18	.237	.136	.097
9b	1.48	.295	.287	.133
2	1.86	.373	.278	.224
6	1.88	.376	.317	.202
8	1.89	.378	.363	.196
7a	1.93	.387	.369	.215
4b	1.95	.389	.296	.251
11b	2.05	.410	.332	.290
9c	2.08	.416	.498	.239
5b	2.17	.434	.453	.109
3	2.27	.453	.281	.254
4a	2.76	.552	.498	.366
5a	3.32	.663	.604	.495
1b	4.68	.937	.934	.894
1a	4.79	.958	.952	.946

The item difficulty index values of the pilot Algebra and Calculus Test using all three score possibilities reveal that items 1(a) and 1(b) were unacceptably easy and 7(b) and 11(a), 12(a) and 12(b) were unacceptably difficult. Items which are unacceptably easy or difficult will be reviewed and either rewritten or excluded.

An item must have an item-total correlation value of at least 0.2 before it can be selected. Table 7.13 contains the Item-Total Correlation statistics for the Pilot Algebra and Arithmetic Test. All items in the pilot test have item-total correlation values exceeding 0.2 with the exception of question 1(b), which would round off to 0.2. This suggests that the items in the pilot test are able to discriminate between those who perform well and poorly on the test, which is essential in a diagnostic test.

Table 7.13 **Item-total Correlation statistics for the Pilot Algebra and Calculus Test**

Question Number	Item-Total Correlation
1b	.198
1a	.285
3	.354
5a	.387
8	.435
4a	.437
2	.454
4b	.464
11b	.507
7a	.523
5b	.548
12a	.600
10	.612
12b	.617
6	.618
9c	.621
11a	.645
7b	.656
9a	.681
9b	.715

In summary, evidence has been gathered supporting the content validity of the two pilot tests as well as the fact that the difficulty levels of the items are reasonably acceptable and they are also able to discriminate between good and poor performers.

7.3.3 Predictive validity

To gather evidence regarding the predictive validity of the two pilot tests, performance on the tests was correlated with performance in first-year Mathematics. As performance on the pilot tests was scored using both the holistic and the analytical scoring methods, the analyses were conducted for each scoring method separately.

7.3.3.1 Predictive validity analyses: Holistic scoring

The descriptive statistics for the holistic scores obtained in the pilot tests as well as first-year Mathematics marks are reflected in Table 7.14. According to these results, 75% of the students had a score of 62 or less. The corresponding values for the Pilot Algebra and Calculus Test and the Arithmetic Test were 52 and 83 respectively.

Table 7.14 Descriptive Statistics for holistic scores obtained in the pilot tests and Mathematics 1 marks

Statistic	**Arithmetic**	**Algebra and Calculus**	**Mathematics 1**
Missing records (%)	6%	8%	0%
Mean	69.55	40.91	50.80
SD	18.05	19.49	16.99
Minimum	16.00	0.00	8.00
Quartile1	59.00	27.00	40.00
Median	72.00	38.00	50.00
Quartile 3	83.00	52.00	62.00
Maximum	100.00	100.00	96.00

Source: own construct

Correlation coefficients of 0.29 and 0.35 were obtained between students' first-year NMMU Mathematics marks and the pilot Arithmetic Test and the pilot Algebra and Calculus Test. The correlation coefficients were moderate and significant. Considering the sample size (n=361) a correlation coefficient of 0.3 or higher can be deemed to be significant.

Correlation coefficients between a criterion (first-year Mathematics marks) and predictors (performance in the pilot tests) provide an indication of the predictive validity of a test. "A test may appreciably improve predictive efficiency if it shows any significant correlation with the criterion, however low. Under certain circumstances, even validities as low as .20 or.30 may justify inclusion of the test in a selection program" [Anasta97:144]. "Values of 0.3 or even 0.2 are acceptable if the test is used for selection purposes" [Foxcro05:37]. The moderately significant correlation coefficients obtained between first-year Mathematics marks and performance on the two pilot tests thus provide evidence regarding the predictive validity of the two pilot tests.

7.3.3.2 Predictive validity analyses: Analytical Scoring Method

This section will provide information about the relatedness of the error categories and the performance of students in the Mathematics modules included in the study. Frequency tables were drawn up for each error category. Cross-tabulations were also compiled to uncover possible relationships between each category error and the students' final first-year Mathematics marks. These tables appear in the Appendices 5 and 6. Table 7.15 details the number of times a Refined Arithmetic Category error occurred. The table has been arranged so that the more prevalent error categories are at the top of the table.

Table 7.15: The occurrence of errors in the Pilot Arithmetic Test according to error category

Refined Category (n=340)	None (%)	Once (%)	Twice (%)	Thrice or more (%)
Ratio-related problems	6	70	24	0
Cannot find the cube root of a given ratio	20	80	0	0
Errors relating to operations with fractions	26	26	12	35
Errors relating to percentages	27	38	26	9
Errors related to operations with decimals	33	29	18	20
Cannot apply the distributive law	35	65	0	0
Calculation Mistakes and Copying Errors	52	30	12	7
Simplification of fractions or expressions	69	25	4	2
Construct a fraction from a word sum problem	76	24	0	0
Finding the square root of the sum of two natural numbers	79	21	0	0
Reading problems	83	16	1	0
Mathematical meaning of "of"	84	16	0	0
Making x the subject of the equation	99	1	0	0
Ranking of arithmetical operations	99	1	0	0
Disregard for order in the number system	100	0	0	0

Source: own construct

From Table 7.15 it is evident that 67 per cent of the students who took the test made errors related to operations with decimals. Seventy-four per cent of students taking the test made errors related to operations with fractions. Operations with fractions have an impact on the study of integration, and differentiation. Operations with decimals and fractions are taught in primary school and illustrates that students' areas of weakness are not confined to secondary school topics.

Table 7.16 details the number of times a refined Algebra and Calculus category error occurred. The table has been arranged so that the more prevalent error categories are at the top of the table.

Table 7.16: The occurrence of errors in the Pilot Algebra and Calculus Test according to error category

Refined Category (n=331)	**None (%)**	**Once (%)**	**Twice (%)**	**Thrice (%)**
Recognition of and drawing of a parabola	21	60	15	3
Finding the domain and range of a semi-circle	21	79	0	0
Cannot calculate a limit	24	32	44	0
Problems performing elementary differentiation	25	15	12	4
Factorisation	25	62	13	1
Determining the values for which a rational function evaluates to zero	27	72	0	0
Unfamiliar with log rules	30	28	31	11
Cannot apply the rules of indices and surds involving division and roots	30	37	33	0
Recognition of and drawing of a semi-circle	31	69	0	0
Knowing the product of the slopes of two perpendicular lines and using it to calculate the equation of the new line when a point on the new line is given	34	38	28	0
Notation	38	22	11	30
Copying and Calculation mistakes	41	34	15	10
Arithmetical errors	45	55		0
Cannot convert from logarithmic to exponential form	49	39	12	0
Errors made when expanding perfect squares especially if square roots are involved	51	49		0
Recognition of and drawing of a straight line	56	39	5	0
Writing power functions as negative powers as roots or vice versa	60	37	2	0
Unfamiliar with rules for exponential functions	60	40	1	0
Simplification especially of rational functions	99	1		0
Language problems	100	0	0	0

From Table 7.16 it is evident that although students satisfy the minimum entrance requirements for a first-year Mathematics course; they lack essential university pre-knowledge. This is illustrated by the fact that seventy-nine per cent of students who took the test experienced problems when they had to sketch a parabola and seventy per cent of students found the rules of indices and surds taxing. Familiarity with the topics listed in Table 7.16 is vital for the success of a first-year student.

The Chi-square test of independence was utilized to determine whether there was a relationship between the error categories and performance in first-year Mathematics [Black07:479].

Students' Mathematics marks were categorized into three categories namely, "Less than 40%", "40 to 59%" and "60% and above". The refined error categories consisted of two groups namely, the "No" group representing the students who did not make that particular error and the "Yes" group representing those who did make the error.

The null hypothesis and alternative hypothesis for each error category were as follows.

H_0: There is no relationship between the category errors that students made and their final Mathematics marks

H_A: There is a relationship between the category error the students made and their final Mathematics mark

For the test to be significant it is required that both p<.05 and Cramér's V > 0.1 be satisfied. The significance statistics for the pilot Arithmetic Test as well as the Algebra and Calculus Test are reported in Table 7.17 while the contingency tables appear in the Appendices 5 and 6.

Table 7.17: Chi-square test of independence - Results for statistically significant refined Arithmetic Categories

Refined Categories	χ^2-value (d.f. = 2)	p-Value	Cramér's V (d.f.=1)
Ratio-related problems	6.13	.047	0.13
Finding the square root of the sum of two natural numbers	15.54	<.0005	0.21
Errors relating to percentages	6.79	.034	0.14
Errors relating to operations with fractions	12.13	.002	0.19
Errors related to operations with decimals	15.07	.001	0.21
Cannot find the cube root of a given ratio	23.5	<.0005	0.26
Cannot apply the distributive law	15.54	<.0005	0.21
Mathematical meaning of "of"	6.82	.033	0.14
Construct a fraction from a word sum problem	11.46	.003	0.18

For the Algebra and Calculus Test, the refined error categories listed in Table 7.18 were significantly related to performance in Mathematics 1.

Table 7.18: Chi-square test of independence - Results for statistically significant refined Algebra and Calculus Categories

Refined Categories	χ^2-value (d.f. = 2)	p-Values	Cramér's V (d.f.=1)
Factorization	7.36	.025	0.15
Determining the values for which a rational function evaluates to zero	24.02	<.0005	0.27
Errors made when expanding perfect squares especially if square roots are involved	13.77	.001	0.18
Cannot apply the rules of indices and surds involving division and roots	22.61	<.0005	0.26
Recognition of and drawing of a straight line	6.73	.035	0.12
Recognition of and drawing of a parabola	6.34	.042	0.14
Knowing the product of the slopes of two perpendicular lines and using it to calculate the equation of the new line when a point on the new line is given	6.07	.048	0.14
Cannot calculate a limit	9.12	.010	0.17

It can thus be concluded that in the case of the eight and nine significant relationships found between final Mathematics mark and the Algebra and Calculus Test categories and the Arithmetic Test categories respectively, the relevant null hypothesis can be rejected. It is evident that there is a relationship between the errors made in certain categories and the students' final Mathematics marks.

Closer inspection of the contingency tables in Appendices 5 and 6 reveals a consistent trend in that for all the refined error categories, the group of students who made one or more errors of that particular type performed worse in Mathematics 1 compared with students who did not make that type of error, even though they were not all statistically significant. This strengthens the conclusion that there is a relationship between the error categories identified and the final Mathematics marks obtained by the students.

Having established that there is a relationship between the predictor variables (performance on the two pilot tests) and the criterion variable (performance in first-year Mathematics), **multiple regression analysis** was performed to determine whether performance on each test contributes to the linear prediction of performance in first-year Mathematics.

7.3.3.3 Multiple Regression Analysis

Multiple regression analysis was used to determine if student performance in first-year Mathematics (dependent variable) can be predicted from performance in the pilot Arithmetic and the Algebra and Calculus Tests (independent variables). The resultant analysis revealed that performance on the two pilot tests explains 14% (adjusted R^2=0.14) of the variance in students' final Mathematics marks. The F-test of overall significance yielded a F-test value of 27.082 with p<.0005. This shows that there is at least one significant predictor of final

Mathematics marks included in the analysis. The t-tests revealed that both predictor variables contribute significantly ($p<.0005$) to linearly predicting the dependent variable [Black07: 601].

The various analyses conducted thus provide evidence for the predictive validity of the pilot Arithmetic Test and the pilot Algebra and Calculus Test as scores and certain error categories were found to correlate significantly with performance in first-year Mathematics.

7.4 Reliability of the pilot tests

"Reliability is linked to consistency of measurement. Thus: The reliability of a measure refers to the consistency with which it measures whatever it measures [Foxcro05:28]."

While there are many types of reliability, only the internal consistency coefficients were determined for the pilot tests. This type of reliability is a measure of the item-to-item consistency of a test-taker's responses within the pilot test. The value of the reliability coefficient varies between 0 (no reliability) and 1 (perfect reliability) [Person05:507].

Cronbach's Alpha statistic was used to compute the reliability coefficients for the pilot tests. Cronbach's Alpha values of 0.86 and 0.90 were observed for the pilot Arithmetic Test and the pilot Algebra and Calculus Test respectively. This suggests a high level of internal consistency and hence reliability for both of the pilot tests.

7.5 Conclusion

This chapter has investigated the validity and reliability of the pilot tests. For both tests there was evidence of appropriate content validity, predictive validity and reliability. This suggests that a promising foundation has been laid in the pilot tests for the implementation of a diagnostic Mathematics test battery at the NMMU once further refinements to the items have taken place. One such refinement will be to introduce a few more difficult items into the Arithmetic test as there are currently too many easy items on this test.

Chapter 8 will focus on reflecting on the findings and contributions of the present study as well as to provide suggestions for future research.

CHAPTER 8 - Conclusion and Future Research

8.1 Introduction

This chapter contains a summary of the findings with respect to the two pilot diagnostic mathematics tests that were developed. Possible refinements to the two pilot tests will be proposed and areas of possible future research will be highlighted.

8.2 Findings related to the pilot tests

8.2.1 Sub-domains and their possible refinement

This researcher identified certain sub-domains in Mathematics which were essential for successful completion of mathematics first-year modules (Cf. 5.6.2.1 and 5.6.2.2). A total of seven students scored full marks in the Arithmetic Test and only one student scored full marks in the Algebra and Calculus Test. The fact that some of the students performed quite poorly on the pilot tests, and that performance on the pilot tests correlated significantly with performance in first-year mathematics (Cf. 7.3.3.) suggests that the sub-domains were appropriately identified and delineated. These sub-domains thus provide the criteria to include in a diagnostic test battery at NMMU.

The analytic scoring method used identified certain refined error categories. Post hoc analysis confirmed the sub-domains identified in section 5.6.2. Tables 8.1 and 8.2 summarise the results of analyses to determine which Refined Arithmetic and Algebra and Calculus Error Categories are related to performance in Mathematics 1.

Table 8.1: Refined Arithmetic Error Categories significantly related to performance in Mathematics

Refined Error Category	Included in Sub-domains
Calculation Mistakes and Copying Errors	No
Ratio-related problems	Yes
Finding the square root of the sum of two natural numbers	Yes
Errors relating to percentages	Yes
Errors relating to operations with fractions	Yes
Errors related to operations with decimals	Yes
Disregard for order in the number system	No
Cannot find the cube root of a given ratio	Yes
Cannot apply the distributive law	Yes
Simplification of fractions or expressions	Yes
Reading problems	No
Mathematical meaning of "of"	Yes
Making x the subject of the equation	Yes
Construct a fraction from a word sum problem	Yes
Ranking of arithmetical operations	Yes

Source: own construct

Table 8.2: Refined Algebra and Calculus Error Categories significantly related to performance in Mathematics 1

Refined Error Category	Included in the Sub-domains
Factorisation	Yes
Copying and Calculation mistakes	No
Determining the values for which a rational function evaluates to zero	Yes
Errors made when expanding perfect squares especially if square roots are involved	Yes
Cannot apply the rules of indices and surds involving division and roots	Yes
Recognition of and drawing of a straight line	Yes
Recognition of and drawing of a parabola	Yes
Knowing the product of the slopes of two perpendicular lines and using it to calculate the equation of the new line when a point on the new line is given	Yes
Recognition of and drawing of a semi-circle	Yes
Notation	No
Finding the domain and range of a semi-circle	Yes
Cannot calculate a limit	Yes
Writing power functions as negative powers as roots or vice versa	Yes
Problems performing elementary differentiation	Yes
Arithmetical errors	Yes
Cannot convert from logarithmic to exponential form	Yes
Unfamiliar with log rules	Yes
Unfamiliar with rules for exponential functions	Yes
Simplification especially of rational functions	Yes
Language problems	No

Source: own construct

Not only did the comparisons provided in Tables 8.1 and 8.2 corroborate the importance of the identified sub-domains but they also highlighted a few sub-domains that could be added when the diagnostic test battery is finalized (e.g., notation). However, adding further sub-domains and items will increase the length of the test battery and the time taken to complete it.

8.2.2 Reducing the sub-domains or the number of items to develop a shortened version of the diagnostic tests

A balance needs to be achieved between the extent of the content coverage of a test and the time allocated to complete the test. Students completed the pilot diagnostic tests during a lecture period, but due to their length, not all students were able to complete both pilot tests. As it is likely that the final diagnostic mathematics test battery will also be completed during a lecture period at the start of an academic year, consideration may have to be given to shortening rather than lengthening (as suggested in 8.2.1) the test battery. One way of shortening the battery would be to include only refined error categories that correlated significantly with students' final Mathematics marks as the core of the final diagnostic test battery. Consideration can then be given to which of the remaining sub-domains should also be included or to reducing the number of items that tap the remaining sub-domains.

Performance in first-year mathematics was found to correlate significantly with certain of the arithmetic error categories, which are listed in Table 8.3.

Table 8.3 Refined Arithmetic Categories that correlated significantly with first-year performance Source: own construct

Category Number	**Category Description (Arithmetic)**
2	Ratio-related problems
3	Finding the square root of the sum of two natural numbers
4	Errors relating to percentages
5	Errors relating to operations with fractions
6	Errors related to operations with decimals
8	Cannot find the cube root of a given ratio
9	Cannot apply the distributive law
10	Simplification of fractions or expressions
12	Mathematical meaning of "of"
14	Construct a fraction from a word sum problem

These statistically significant Arithmetic error categories could be the most essential ones to include in a shortened version of an arithmetic diagnostic test. The other sub-domains should be critically evaluated to determine whether it is necessary to include all of them in the final test and whether it is possible to reduce the number of items that tap them.

Similarly, first-year performance was found to correlate significantly with certain of the Algebra and Calculus categories, These Algebra and Calculus error categories are highlighted in Table 8.4.

Table 8.4 Refined Algebra and Calculus Categories that correlated significantly with first-year performance

Category Number	Category Description (Algebra and Calculus)
1	Factorization
3	Determines the values for which a rational function evaluates to zero
4	Errors made when expanding perfect squares especially if square roots are involved
5	Cannot apply the rules of indices and surds involving division and roots
6	Recognition of and drawing of a straight line
7	Recognition of and drawing of a parabola
8	Knowing the product of the slopes of two perpendicular lines and using it to calculate the equation of the new line when a point on the new line is given
12	Cannot calculate a limit

Source: own construct

These statistically significant Algebra and Calculus error categories could be the most essential ones to include as part of the core of a shortened version of an Algebra and Calculus diagnostic test. However, the researcher feels strongly that the other Algebra and Calculus sub-domains should also be included in the final diagnostic test as they also represent important required pre-knowledge and skills. There could possibly be less items that tap the sub-domains that were not found to be statistically significant if a shortened version of the diagnostic test battery is developed.

8.2.3 Item revision

The results of the analysis of the items included in the two pilot tests identified that certain items were too difficult or too easy (Cf. 7.3.2). These items will either have to be re-written or excluded and replaced by other items. Should additional sub-domains be added, as suggested in section 8.2.1 above, additional items will have to be developed to tap these sub-domains.

The next section will highlight further research that needs to be undertaken as part of the ongoing development of a diagnostic mathematics test battery.

8.3 Future research

The way in which this study can be expanded in subsequent research phases will be outlined in this section.

8.3.1 Confirming the Sub-domains against the New NSC Curriculum

When this study was initiated, school learners were still following the differentiated mathematics curriculum of the Senior Certificate. Consequently, when the pre-knowledge and skills required for mathematics at a university level were analysed in relation to what the school syllabus should provide, the syllabus that is part of the Senior Certificate was consulted. However, since 2006, school learners now follow the new national curriculum of the National Senior Certificate (NSC). The first school-leaving (matriculation) examinations for the NSC will be written in the latter part of 2008. Consequently, it will be important to replicate this study using the mathematics curriculum in the NSC to see if the same findings are obtained. By way of introduction to such a study, in Tables 8.5 and 8.6, the sub-domains identified in this study are linked to the new national curriculum statement (NCS) for Mathematics. In this table, “LO” is used as an abbreviation for “Learning Outcome” and “AS” as an abbreviation for “Assessment Standard”.

Table 8.5 Comparing the identified sub domains for the pilot Arithmetic Test with the National Curriculum Statement for Mathematics

Sub Domains	**NCS**
Operations with decimals	**LO1,AS8.1.3**
Operations with fractions	**LO1,AS8.1.3**
Ratio-related problems	**LO1,AS8.1.6**
Errors relating to percentages	**LO1,AS8.1.3**
word sums	**LO1,AS6.1.6**
Distributive law	**LO1,AS8.1.9**
Finding the square root of the sum of two natural numbers	**LO1,AS8.1.3**
Finding the cube root of a cubic ratio	**LO1,AS8.1.3**
Simplification of fractions or expressions	**LO1,AS7.1.4**
Make x the subject of the equation	**LO2,AS8.2.2**

Table 8.6: Comparing the identified sub-domains for the pilot Algebra and Calculus Test with the National Curriculum Statement for Mathematics

Sub-domains	**NCS**
Factorisation	**LO2,AS 10.2.5**
Solving a quadratic equation using the formula	**LO2,AS 11.2.5**
Values for which a rational function equals zero	**LO2,AS11.2.2***
Squaring binomials	**LO2,AS 9.2.8**
Rules of indices and surds	**LO1,AS 11.1.2**
Sketching a straight line	**LO2,AS 10.2.2**
Sketching a parabola	**LO2,AS 11.2.2**
Knowing the product of the slopes of two perpendicular lines and calculating the equation of the new line when a point on the new line is given in addition to the equation of the perpendicular line	**LO3,AS 11.3.3**
Sketching a semi-circle	**LO2,AS 12.2.1***
Domain and range of a semi-circle	**LO2,AS 11.2.3**
Calculating a limit using substitution to avoid the division by zero problem	**LO2,AS 12.2.7**
Differentiation	**LO2,AS 11.2.7**
Application of log rules	**L02,AS 12.1.2**
Solving x by converting from logarithmic to exponential form	**LO2,AS 12.2.2**
Laws of exponentials	**LO2,AS 12.2.2**
Simplification of functions	**LO2,AS 10.2.4**

Note:

AS11.2.2* states learners should be able to generate the graph of $y = \frac{a}{x+p} + q$. It is thus implied that learners should be familiar with rational functions. In the Study and Master guide [Vander07:76] based on the new curriculum the natural domain of a rational function is determined.

AS 12.2.1:* states learners should be able to work with various types of functions and relations. It is thus implied that learners should have knowledge of circles and semi-circles. In the Study and Master guide [Vander07:83] based on the new curriculum the concept semi-circle is introduced.

The information provided in Tables 8.5 and 8.6 suggest that all the important sub-domains identified in the present study could be linked to the new Mathematics curriculum and thus it should be assumed that school learners would acquire the essential mathematical pre-knowledge and skills. Nonetheless, a more in depth investigation is necessary and the first cohort of NSC learners who enter university should be tested on the diagnostic mathematics test battery to see whether they have indeed acquired the requisite mathematical pre-knowledge and skills.

8.3.2 Determining Sub-domains for Diploma Type Programmes

As a result of the merger of institutions to form the new NMMU, diploma and degree type programmes are now housed within the same Mathematics department. As the present study was initiated before the merger, only degree modules and degree students were used when the sub-domains were delineated and the pilot tests were developed. Consequently, further studies need to be conducted to replicate the present study for diploma students. It will be interesting to see whether similar sub-domains emerge and whether one diagnostic mathematics test can be used for degree and diploma students.

8.3.3 Motivation for the development of a Computer-based Diagnostic Test

The constructed response item type was used in the paper-based pilot tests and tests were scored manually. This process is time consuming and delays feedback to students. When one takes into account that at least five hundred students will have to be tested annually it is clearly not feasible to use a paper-based diagnostic test battery.

Instead, consideration should be given to computerizing the diagnostic mathematics test battery. Computer-based diagnostic testing is widely used in many countries. For example, the University of Western Australia uses an

Intelligent Computer Tutor [Stone95]; the Anglia Polytechnic University in the UK uses a computerised diagnostic test – developed using Diagnosys – which gives students immediate feedback on their strengths and weaknesses [Ltsn05:24]; and Mathlectics is a computer-based diagnostic system used in the higher education sector in the UK, which is also available for use in schools in the UK.

Locally, there is no *off-the-shelf* computerised diagnostic mathematics test available. It would thus be beneficial if the diagnostic mathematics test battery that was piloted in this study could be computerised as it will fill a gap in the marketplace. The fact that the diagnostic test currently under development uses a constructed response item type makes it ideally suited to being computerised as this item type has been the subject of much research in recent times. The outcome of this research has led to innovations in the types of items that can be included in computer-based tests, with constructed response items being one such innovation [Parsha02]. The development of a computerized test battery also provides an ideal opportunity for collaboration with the Department of Computer Science and Information Systems, and specifically its research group at the NMMU that focuses on computers in education. Whether a product such as Diagnosys should be purchased to develop the computer based diagnostic mathematics test battery at the NMMU or whether the NMMU's interdisciplinary research team will develop their own testing system remains to be seen. However, the software system that is used or developed should be able to produce profiles of individual as well as groups of students on the various aspects tapped in the tests and should highlight the knowledge and skill areas where there are deficiencies. This type of diagnostic information will be required to best assist at-risk students.

8.3.4 Expanding research into the predictive validity of the diagnostic test battery

The current study only correlated performance in the pilot test battery with performance at the end of first-year mathematics for students in 2004. This cohort of students should be tracked and their second and final year mathematics marks should also be correlated with performance on the pilot tests. This will enable researchers to see whether the pilot test battery has predictive power beyond the first year. Furthermore, the battery should be administered to further intakes of students to see whether similar low to moderately significant correlations can be obtained, which would add further evidence to support the predictive validity of the battery. Consideration could also be given to including various matriculation marks (e.g., Mathematics, English, Biology and Science marks) together with performance on the diagnostic tests in the prediction model to improve the prediction of first-year mathematics performance and the early identification of students who will require assistance if they are to succeed in their first-year mathematics modules.

8.3.5 Testing the reading levels of Mathematics students at the NMMU

In Tables 8.1 and 8.2 above two error categories were identified which related to either language or reading problems. No language testing was done in the present study. However, studies at the University of South Africa (UNISA) highlighted that unless students improve their reading level to achieve a reading test score of at least sixty percent or more, they will not be able to successfully study Mathematics at a distance education university [Bohlm03]. It would thus be very interesting to do a similar study at the Nelson Mandela Metropolitan University to determine the reading test score that a non-distance education student requires to successfully complete a first-year mathematics course. Scores on the reading test could also be related to scores obtained on the mathematics diagnostic test battery to see whether the addition of information about the reading level of students helps to identify high-risk students more effectively. If is does, then the developmental interventions planned for high-risk students will have to include reading and language enhancement.

8.3.6 Development of support programmes and centres

Where the diagnostic testing identifies that entry level mathematical pre-knowledge and skills are lacking, consideration should be given to developing an appropriate support programme that could even be offered at a dedicated support centre. For example, some universities in the UK have opened drop-in centres for students that are manned by post-graduate tutors. Students can also be provided with study materials to aid their development. For example, at the University of Newcastle on Tyne they initially used material supplied by the mathcentre project (with their permission) but they are now expanding this by developing their own additional material [Foste05].

At the NMMU a number of strategies can be followed:

- Web-based courses could be designed where students could work through course material and perform self-help tests to test their progress.
- Support programmes can be developed based on the weaknesses identified
- A support centre can be established. Given the fact that the vision for the Missionvale campus of the NMMU is to enhance development in Mathematics and Science, plans are under way to develop a Maths-Science Centre on this campus. It is interesting to note that it has been found that an educational support centre should be marketed directly to the students. Universities in the UK have reported that students perceive such an approach to be non-judgemental and they are more likely to use the centre if it is marketed in this way.

- Some first-year modules can be repeated in the second semester to allow weaker students who followed a remedial programme to join the main stream.

8.4 Limitations of the study

The study was subject to a number of limitations

- The results presented in the study is from 2004 and not current
- Students did not have unlimited time to complete the tests
- As a result of the time limitation the number of items testing each sub-domain had to be limited.

8.5 Concluding Remarks

This study has made the following contributions:

- It has been established that while the school syllabi cover all the important mathematical pre-knowledge and skills required to succeed at studies in Mathematics at university, incoming students have not all acquired sufficient knowledge and skills at school.
- Based on the essential mathematical pre-knowledge and skills required at entry to university, the errors made by students, observations of mathematics lecturers, and previous research and existing Mathematics achievement and diagnostic tests, essential sub-domains in Arithmetic and Algebra and Calculus were identified. Thereafter, test specifications were developed by the researcher and items were written or sourced for a pilot Arithmetic Test and a pilot Algebra and Calculus Test. The pilot tests were administered to a sample of first-year degree students in 2004. From the resultant data, it was established that there was sufficient evidence regarding the content and predictive validity as well as the reliability of the two pilot tests. The results also revealed areas where the pilot tests can be refined so that the diagnostic Mathematics test battery can be finalised. This battery has the potential to be

widely used at the NMMU and possible elsewhere in South Africa as well, especially at comprehensive universities.

- Valuable suggestions were made regarding the need to develop a computer-based version of the diagnostic mathematics test battery.
- Suggestions were also made on how to expand the scope of the content coverage and research with the diagnostic Mathematics test battery.

Appendix 1: Pass Rates for first–year Mathematics modules (2003-2005)

Pass Rates for Mathematics first year subjects for the years 2003-2005

			2003			2004			2005		
Subject Code	Subject Name	Race	Enrol-ments	Number Passed	Pass Rate	Enrol-ments	Number Passed	Pass Rate	Enrol-ments	Number Passed	Pass Rate
MAPM101	GRAPH THEORY & SC PROGRAMMING 101	BLACK	27	23	85.2%	37	22	59.5%	48	26	54.2%
		COLOURED	7	7	100.0%	11	6	54.5%	7	6	85.7%
		INDIAN	1	1	100.0%	2	2	100.0%	1	0	0.0%
		WHITE	15	14	93.3%	17	15	88.2%	28	22	78.6%
	GRAPH THEORY & SC PROGRAMMING 101 Total		50	45	90.0%	67	45	67.2%	84	54	64.3%
MAPM102	MECHANICS 102	BLACK	19	16	84.2%	20	11	55.0%	33	16	48.5%
		COLOURED	7	6	85.7%	5	5	100.0%	4	3	75.0%
		INDIAN	1	0	0.0%	3	2	66.7%	0		0.0%
		WHITE	14	12	85.7%	13	12	92.3%	23	18	78.3%
	MECHANICS 102 Total		41	34	82.9%	41	30	73.2%	60	37	61.7%
MAPM103	MATHEMATICAL MODELLING 103	BLACK	37	22	59.5%	35	19	54.3%	46	21	45.7%
		COLOURED	7	3	42.9%	7	6	85.7%	5	3	60.0%
		INDIAN	1	0	0.0%	4	3	75.0%	0	0	0.0%
		WHITE	16	16	100.0%	14	11	78.6%	32	21	65.6%
	MATHEMATICAL MODELLING 103 Total		61	41	67.2%	60	39	65.0%	83	45	54.2%
MATA101	MATHEMATICS SPECIAL A 101	BLACK	143	82	57.3%	180	116	64.4%	181	112	61.9%
		COLOURED	31	13	41.9%	35	26	74.3%	39	26	66.7%
		INDIAN	12	6	50.0%	15	11	73.3%	18	11	61.1%
		WHITE	130	79	60.8%	161	113	70.2%	167	115	68.9%
	MATHEMATICS SPECIAL A 101 Total		316	180	57.0%	391	266	68.0%	405	264	65.2%
MATA102	MATHEMATICS SPECIAL 102	BLACK	125	77	61.6%	179	64	35.8%	221	90	40.7%
		COLOURED	31	10	32.3%	43	14	32.6%	47	27	57.4%
		INDIAN	13	9	69.2%	12	6	50.0%	17	7	41.2%

		WHITE	112	69	61.6%	170	76	44.7%	191	95	49.7%
	MATHEMATICS SPECIAL 102 Total		281	165	58.7%	404	160	39.6%	476	219	46.0%
MATH101	ALGEBRA 101	BLACK	32	10	31.3%	44	14	31.8%	58	11	19.0%
		COLOURED	8	2	25.0%	11	4	36.4%	13	7	53.8%
		INDIAN	2	0	0.0%	4	2	50.0%	3	3	100.0%
		WHITE	32	27	84.4%	37	22	59.5%	41	28	68.3%
	ALGEBRA 101 Total		74	39	52.7%	96	42	43.8%	115	49	42.6%
MATH102	DIFFERENTAL CALCULUS 102	BLACK	29	13	44.8%	42	7	16.7%	62	20	32.3%
		COLOURED	9	6	66.7%	10	2	20.0%	11	6	54.5%
		INDIAN	1	1	100.0%	3	1	33.3%	1	1	100.0%
		WHITE	31	29	93.5%	41	20	48.8%	42	30	71.4%
	DIFFERENTAL CALCULUS 102 Total		70	49	70.0%	96	30	31.3%	116	57	49.1%
MATH103	CALCULUS 103	BLACK	29	15	51.7%	34	17	50.0%	45	16	35.6%
		COLOURED	12	6	50.0%	7	3	42.9%	9	6	66.7%
		INDIAN	1	1	100.0%	2	1	50.0%	1	1	100.0%
		WHITE	33	27	81.8%	30	25	83.3%	37	30	81.1%
	CALCULUS 103 Total		75	49	65.3%	73	46	63.0%	92	53	57.6%
MATH104	ALGEBRA 104	BLACK	32	10	31.3%	45	11	24.4%	67	14	20.9%
		COLOURED	10	5	50.0%	8	2	25.0%	11	6	54.5%
		INDIAN	1	1	100.0%	3	2	66.7%	2	0	0.0%
		WHITE	31	25	80.6%	35	23	65.7%	38	26	68.4%
	ALGEBRA 104 Total		74	41	55.4%	91	38	41.8%	118	46	39.0%
Grand Total			1042	643	61.7%	1319	696	52.8%	1549	824	53.2%

Source: Official NMMU Statistics

Appendix 2: Arithmetic test

Office Use Only	Math1E1 G	Math1E1 PE: ✔	Math101	Mata101	Matc101	Other

Student Number	

Highest Grade for which you teach Mathematics	

<u>RESEARCH ARITHMETIC PRE-TEST : January 2004</u>
(PLEASE NOTE ALL TEST RESULTS ARE HIGHLY CONFIDENTIAL)

TIME / TYD: 30 MINUTES

- **ANSWER ALL QUESTIONS – SHOW ALL WORKINGS**
- **USE OF A CALCULATOR IS PROHIBITED**
- **THANK YOU FOR CO-OPERATION**

1. Calculate the following

23.34 – 22.56	

2. What is 2.3988 rounded to the nearest hundredth?

3. Calculate the following

$60.04 \div 1000$	

4. Calculate the following

$36 \times 3\frac{3}{4}$	

5. Calculate the following

$\frac{5}{8} \div \frac{1}{2}$	

6. What must be added to $\frac{5}{8}$ to give $1\frac{3}{8}$?

7. If 38% of a number is 19, what is the number ?

8. The ratio of boys to girls is 2 is to 3. If there are 24 boys, how many girls are there ?

9. The ratio of men to women is 5 is to 6 and the ratio of adults to children is 3 is to 2. What is the ratio of women to children ?

10. An item costs twice as much as it did last year. What percentage is the new price of the old price ?

11. Two toys are on sale. The first toy is a toy truck and was selling for R120 before it was marked down by 40% . The second toy is a doll and was selling for R80 before it was discounted by 20% . Which toy is now the cheapest ?

12. Calculate the following

0.11 + 11 + 1.1	

13. John spends his pocket money as follows : $\frac{1}{4}$ on sweets, $\frac{3}{5}$ on entertainment and $\frac{1}{10}$ on gifts. How much money does he have left to save ?

14. Calculate the following:

$12.5 \div 0.25$	

15. Calculate the following:

$3\frac{1}{4} + 2\frac{4}{5}$	

16. Multiplying a number by 0.01 gives the same result as dividing the number by what number ?

17. Given the fractions $\frac{6}{9}$, $\frac{8}{12}$, $\frac{10}{15}$, $\frac{12}{21}$. Which fraction is not an equivalent fraction of $\frac{2}{3}$?

18. What is $\frac{2}{5}$ of 30 ?

19. Calculate the following :

12.35 – 3.4	

20. Calculate the following :

$\frac{4}{5} - \frac{3}{15}$	

21. Calculate the following :

21 864 880 456 678 × 986 – 985 × 21 864 880 456 678

22. Calculate the following :

$\sqrt{9+16}$	

23. Calculate the following :

$\sqrt[3]{\frac{250}{54}}$	

24. Calculate the following :

$\dfrac{3}{\frac{1}{4}+\frac{1}{12}}$	

25. Calculate the following :
On a train trip from Port Elizabeth to Johannesburg 136 of the 240 available seats are occupied. What fraction of the available seats are occupied ?
(Simplify the fraction as far as possible)

THANK YOU FOR TAKING PART IN OUR RESEARCH PROJECT
BEST WISHES FOR 2004

Appendix 3: PRE-TEST: ALGEBRA and CALCULUS

PRE-TEST: ALGEBRA and CALCULUS

Office Use Only	Math1E1 G	Math1E1 PE: ✔	Math101	Mata101	Matc101	Other

Student Number	

Highest Grade for which you teach Mathematics	

RESEARCH ALGEBRA and CALCULUS PRE-TEST : January 2004
(PLEASE NOTE ALL TEST RESULTS ARE HIGHLY CONFIDENTIAL)

TIME / TYD: 35 MINUTES

- **ANSWER ALL – SHOW ALL WORKINGS**
- **USE OF A CALCULATOR IS PROHIBITED**
- **THANK YOU FOR CO-OPERATION**

1. Factorize the following:

(a) $x^2 - x - 6$	

(b) $3x^2 + 6x$	

2. Solve for $3x^2 + 2x - 4 = 0$	

3. Determine the x - value(s) for which:

$\dfrac{(x+1)^2}{x^2-x-6}=0$	

4. Simplify without using a calculator

(a) $(2-\sqrt{3})^2+(2+\sqrt{3})^2$	

(b) $\left(\dfrac{\sqrt{a}}{a^{-3/2}}\right)^{1/2}$	

5. Sketch the following graphs:

(a) $3x+2y=1$	

(b) $y = -2x^2 + 2x + 12$	

6. Determine the equation of the line that is perpendicular to the line $y = 2x + 1$ and that runs through the point (4;1).

7. Sketch:

(a) $y = -\sqrt{4 - x^2}$	

(b) Determine the domain and range of the above.	

8. Calculate the following limit:

$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$	

9. Determine: $\frac{dy}{dx}$ if:

(a) $y = \frac{3x^2 - 2x + 2}{x}$	

(b) $y = \sqrt[3]{x} - \frac{2}{x^2}$	

(c) $y = 3x^4 - 2x^3 + x - 1$	

10. Let $f(x) = -3x^2$. Use first principles (use definition of the derivative of a function) to determine $f'(x)$.

11. Calculate the following:

(a) $5\log_4 2 - \log_4 0.125 - 2\log_4 8$	

(b) $\log_3 27 + \log_3 3$	

12. Solve for x in each case:

(a) $\log x^3 = 6$	

(b) $2^{\log x} = 4^{-1}$	

Appendix 4: Correlation Tables for the Pilot Test battery

Correlation was used to measure the degree of relatedness of the variables refined error category and the variable first-year performance in mathematics.
A widely used sample coefficient of correlation is the Pearson product-moment correlation coefficient, r. This coefficient measures the linear correlation between two variables. The number r range from –1 to 1. A high correlation indicates that a straight line approximates the relationship between the variables. This straight line is called a regression line or a least squares line. A negative correlation indicates that an inverse relationship exists between the variables. A zero correlation indicates that there is no linear relationship present between the variables.

If the Pearson r coefficient is squared, the resulting value r^2 represents the proportion of shared variation in the two variables. The coefficient r^2 is known as the coefficient of determination. This coefficient gives the magnitude of the relationship between the variables. The aim with the correlation analysis was to establish whether there is a relationship between the category mistake the students made and their first-year marks.

Table 1 will detail the correlation between the Arithmetic categories and student performance in Mathematics 1. A negative correlation is indicative of the fact that mistakes made by students negatively impact their marks. Most of the Arithmetic categories showed a zero correlation or a significant negative correlation. Category 8 (in bold) shows a significant negative correlation with NMMU first-year Mathematics 1 marks.

Table 1: Correlation between the refined category error and first-year mathematics marks (n=340)

Category No.	Problem Category	Correlation
1	Calculation Mistakes and Copying Errors	-0.07
2	Ratio related problems	-0.16
3	Finding the square root of the sum of two natural numbers	-0.16
4	Errors relating to percentages	-0.24
5	Errors relating to operations with fractions	-0.23
6	Errors related to operations with decimals	-0.18
7	Disregard for order in the number system	-0.02
8	Cannot find the cube root of a given ratio	**-0.32**
9	Cannot apply the distributive law	-0.21
10	Simplification of fractions or expressions	0.00
11	Reading problems	-0.09
12	Mathematical meaning of "of"	-0.11
13	Making x the subject of the equation	0.00
14	Construct a fraction from a word sum problem	-0.17
15	Ranking of arithmetical operations	-0.01

The results of the descriptive statistical analysis revealed that most of the refined Algebra and Calculus categories (see Table 2) show either a zero correlation or a significant negative correlation with the exception of Categories 2 (0.07) and 19 (0.09). Category 2 dealt with copying and calculation mistakes and category 19 dealt with simplification especially of rational functions. It is possible for a student to have a positive correlation in these two categories and to still pass a Mathematics 1 course. Table 2 will detail the correlation between the Algebra and Calculus categories and student performance in Mathematics 1. Three refined Algebra and Calculus categories, namely categories 3 (-0.27), 4 (-0.19) and 5 (-0.27) shows a significant negative correlation with NMMU first-year Mathematics marks. Category 3 required students to determine the values for which a rational function evaluates to zero. Category four dealt with errors made when expanding perfect squares especially if square roots are involved. Category 5 dealt with the application of the rules of indices and surds involving division and roots. These categories appear in bold. Other significant correlation values are shaded.

Table 2: Correlating Refined Algebra and Calculus Categories with NMMU first-year marks (n=331).

Category	Problem Category	Correlation
1	Factorization	-0.15
2	Copying and Calculation mistakes	0.07
3	Determining the values for which a rational function evaluates to zero	**-0.27**
4	Errors made when expanding perfect squares especially if square roots are involved	**-0.19**
5	Cannot apply the rules of indices and surds involving division and roots	**-0.27**
6	Recognition of and drawing of a straight line	-0.16
7	Recognition of and drawing of a parabola	-0.11
8	Knowing the product of the slopes of two perpendicular lines and using it to calculate the equation of the new line when a point on the new line is given	-0.14
9	Recognition of and drawing of a semi-circle	-0.09
10	Notation	0.00
11	Finding the domain and range of a semi-circle	-0.06
12	Cannot calculate a limit	-0.15
13	Writing power functions as negative powers or as roots or visa versa	-0.10
14	Problems performing elementary differentiation	-0.12
15	Arithmetical errors	-0.10
16	Cannot convert from logarithmic to exponential form	-0.02
17	Unfamiliar with log rules	-0.07
18	Unfamiliar with rules for exponential functions(equal bases implies equal exponents)	-0.05
19	Simplification especially of rational functions	0.09
20	Language problems	-0.01

Appendix 5: Contingency Tables: Refined Arithmetic Error Categories

Where a category was statistically significant the chi-squared values, p-values and Cramér's V values were displayed in boldface type. The No-label in the table represents the group of students who did not make the mistake. The "No" row indicates into which group the "No-label" student's NMMU marks placed the student. The "Yes-label" represents the group of students who made the mistake. The "Yes-Label" row indicates into which group the "Yes-label" student's NMMU marks placed the student. A detailed Contingency Table for all Arithmetic Error categories will follow.

Table 1: Contingency Table for Arithmetic Error Categories

Category	**Maths NMMU**			**Total**
ArFC1	<40	40-59	60+	
No	42	76	59	177
	23.73%	42.94%	33.33%	100%
Yes	43	81	39	248
	26.38%	49.69%	23.93%	100%
Totals	85	157	98	425
	20%	37%	23%	100%
Chi²(df=2)=3.68; p=.159		Cramér's V(df=1): 0.09		
ArFC2	<40	40-59	60+	Total
No	4	6	11	21
	19.05%	28.57%	52.38%	100%
Yes	81	151	87	248
	25.39%	47.34%	27.27%	100%
Totals	85	157	98	269
	32%	58%	36%	100%
Chi²(df=2)=6.13; p=.047		**Cramér's V(df=1): 0.15**		
ArFC3	<40	40-59	60+	Total
No	62	118	87	267
	23.22%	44.19%	32.58%	100%
Yes	23	39	11	248
	31.51%	53.42%	15.07%	100%
Totals	85	157	98	515
	17%	30%	19%	100%
Chi²(df=2)=15.54; p<.0005		**Cramér's V(df=1): 0.17**		

Category	Maths NMMU			Total
ArFC4	<40	40-59	60+	Total
No	18	38	36	92
	19.57%	41.30%	39.13%	100%
Yes	67	119	62	248
	27.02%	47.98%	25.00%	100%
Totals	85	157	98	340
	25%	46%	29%	100%
Chi²(df=2)=6.79; p=.034		**Cramér's V(df=1): 0.14**		
ArFC5	<40	40-59	60+	Total
No	19	31	38	88
	21.59%	35.23%	43.18%	100%
Yes	66	126	60	248
	26.19%	50.00%	23.81%	100%
Totals	85	157	98	336
	25%	47%	29%	100%
Chi²(df=2)=12.13; p=.002		**Cramér's V(df=1): 0.19**		
ArFC6	<40	40-59	60+	Total
No	30	37	46	113
	26.55%	32.74%	40.71%	100%
Yes	55	120	52	248
	24.23%	52.86%	22.91%	100%
Totals	85	157	98	361
	24%	43%	27%	100%
Chi²(df=2)=15.07; p=.001		**Cramér's V(df=1): 0.20**		
ArFC7	<40	40-59	60+	Total
No	85	156	98	339
	25.07%	46.02%	28.91%	100%
Yes	0	1	0	248
	0.00%	100.00%	0.00%	100%
Totals	85	157	98	587
	14%	27%	17%	100%
Chi²(df=2)=1.17; p=.557		Cramér's V(df=1): 0.04		

Category	Maths NMMU			Total
ArFC8	<40	40-59	60+	Total
No	8	24	35	67
	11.94%	35.82%	52.24%	100%
Yes	77	133	63	248
	28.21%	48.72%	23.08%	100%
Totals	85	157	98	315
	27%	50%	31%	100%
Chi²(df=2)=23.5; p<.0005		**Cramér's V(df=1): 0.27**		
ArFC9	<40	40-59	60+	Total
No	24	45	50	119
	20.17%	37.82%	42.02%	100%
Yes	61	112	48	248
	27.60%	50.68%	21.72%	100%
Totals	85	157	98	367
	23%	43%	27%	100%
Chi²(df=2)=15.54; p<.0005		**Cramér's V(df=1): 0.21**		
ArFC10	<40	40-59	60+	Total
No	63	106	66	235
	26.81%	45.11%	28.09%	100%
Yes	22	51	32	248
	20.95%	48.57%	30.48%	100%
Totals	85	157	98	483
	18%	33%	20%	100%
Chi²(df=2)=1.33; p=.515		Cramér's V(df=1): 0.05		
ArFC11	<40	40-59	60+	Total
No	68	129	86	283
	24.03%	45.58%	30.39%	100%
Yes	17	28	12	248
	29.82%	49.12%	21.05%	100%
Totals	85	157	98	531
	16%	30%	18%	100%
Chi²(df=2)=2.2; p=.333		Cramér's V(df=1): 0.06		

Category	**Maths NMMU**			**Total**
ArFC12	<40	40-59	60+	Total
No	70	125	90	285
	24.56%	43.86%	31.58%	100%
Yes	15	32	8	248
	27.27%	58.18%	14.55%	100%
Totals	85	157	98	533
	16%	29%	18%	100%
Chi²(df=2)=6.82; p=.033		**Cramér's V(df=1): 0.11**		
ArFC13	<40	40-59	60+	Total
No	84	155	97	336
	25.00%	46.13%	28.87%	100%
Yes	1	2	1	248
	25.00%	50.00%	25.00%	100%
Totals	85	157	98	584
	15%	27%	17%	100%
Chi²(df=2)=0.03; p=.983		Cramér's V(df=1): 0.01		
ArFC14	<40	40-59	60+	Total
No	58	113	86	257
	22.57%	43.97%	33.46%	100%
Yes	27	44	12	248
	32.53%	53.01%	14.46%	100%
Totals	85	157	98	505
	17%	31%	19%	100%
Chi²(df=2)=11.46; p=.003		**Cramér's V(df=1): 0.15**		
ArFC15	<40	40-59	60+	Total
No	85	154	98	337
	25.22%	45.70%	29.08%	100%
Yes	0	3	0	248
	0.00%	100.00%	0.00%	100%
Totals	85	157	98	585
	15%	27%	17%	100%
Chi²(df=2)=3.53; p=.171		Cramér's V(df=1): 0.08		

Appendix 6: Contingency tables: Refined Algebra and Calculus Error Categories

Where a category was statistically significant the chi-squared values, p-values and Cramér's V values were displayed in boldface type. The No-label in the table represents the group of students who did not make the mistake. The "No" row indicates into which group the "No-label" student's NMMU marks placed the student. The "Yes-label" represents the group of students who made the mistake. The "Yes-Label" row indicates into which group the "Yes-label" student's NMMU marks placed the student. A detailed Contingency Table for all Algebra and Calculus Error categories will follow.

Table 1: Contingency Table for Algebra and Calculus Error Categories

Category	**Maths NMMU**			**Total**
AlFC1	<40	40-59	60+	
No	15	34	34	83
	18.07%	40.96%	40.96%	100%
Yes	67	117	64	248
	27.02%	47.18%	25.81%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=7.36; p=.025		**Cramér's V(df=1): 0.15**		
AlFC2	<40	40-59	60+	Total
No	35	66	36	137
	25.55%	48.18%	26.28%	100%
Yes	47	85	62	194
	24.23%	43.81%	31.96%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=1.27; p=.531		Cramér's V(df=1): 0.06		
AlFC3	<40	40-59	60+	Total
No	18	28	45	91
	19.78%	30.77%	49.45%	100%
Yes	64	123	53	240
	26.67%	51.25%	22.08%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=24.02; p<.0005		**Cramér's V(df=1): 0.27**		

Category	Maths NMMU			Total
AIFC4	<40	40-59	60+	Total
No	30	76	63	169
	17.75%	44.97%	37.28%	100%
Yes	52	75	35	162
	32.10%	46.30%	21.60%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=13.77; p=.001		**Cramér's V(df=1): 0.20**		
AIFC5	<40	40-59	60+	Total
No	15	37	47	99
	15.15%	37.37%	47.47%	100%
Yes	67	114	51	232
	28.88%	49.14%	21.98%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=22.61; p<.0005		**Cramér's V(df=1): 0.26**		
AIFC6	<40	40-59	60+	Total
No	37	86	63	186
	19.89%	46.24%	33.87%	100%
Yes	45	65	35	145
	31.03%	44.83%	24.14%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=6.73; p=.035		**Cramér's V(df=1): 0.14**		
AIFC7	<40	40-59	60+	Total
No	12	30	29	71
	16.90%	42.25%	40.85%	100%
Yes	70	121	69	260
	26.92%	46.54%	26.54%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=6.34; p=.042		**Cramér's V(df=1): 0.14**		

Category	**Maths NMMU**			**Total**
AIFC8	<40	40-59	60+	Total
No	20	51	41	112
	17.86%	45.54%	36.61%	100%
Yes	62	100	57	219
	28.31%	45.66%	26.03%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=6.07; p=.048		**Cramér's V(df=1): 0.14**		
AIFC9	<40	40-59	60+	Total
No	24	43	36	103
	23.30%	41.75%	34.95%	100%
Yes	58	108	62	228
	25.44%	47.37%	27.19%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=2.06; p=.356		Cramér's V(df=1): 0.08		
AIFC10	<40	40-59	60+	Total
No	31	63	31	125
	24.80%	50.40%	24.80%	100%
Yes	51	88	67	206
	24.76%	42.72%	32.52%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=2.57; p=.276		Cramér's V(df=1): 0.09		
AIFC11	<40	40-59	60+	Total
No	20	26	25	71
	28.17%	36.62%	35.21%	100%
Yes	62	125	73	260
	23.85%	48.08%	28.08%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=2.98; p=.225		Cramér's V(df=1): 0.09		

Category	**Maths NMMU**			**Total**
AIFC12	<40	40-59	60+	Total
No	17	28	34	79
	21.52%	35.44%	43.04%	100%
Yes	65	123	64	252
	25.79%	48.81%	25.40%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=9.12; p=.010		**Cramér's V(df=1): 0.17**		
AIFC13	<40	40-59	60+	Total
No	47	88	64	199
	23.62%	44.22%	32.16%	100%
Yes	35	63	34	132
	26.52%	47.73%	25.76%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=1.58; p=.453		Cramér's V(df=1): 0.07		
AIFC14	<40	40-59	60+	Total
No	20	32	32	84
	23.81%	38.10%	38.10%	100%
Yes	62	119	66	247
	25.10%	48.18%	26.72%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=4.18; p=.124		Cramér's V(df=1): 0.11		
AIFC15	<40	40-59	60+	Total
No	35	64	49	148
	23.65%	43.24%	33.11%	100%
Yes	47	87	49	183
	25.68%	47.54%	26.78%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=1.58; p=.455		Cramér's V(df=1): 0.07		

Category	Maths NMMU			Total
AIFC16	<40	40-59	60+	Total
No	42	71	49	162
	25.93%	43.83%	30.25%	100%
Yes	40	80	49	169
	23.67%	47.34%	28.99%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=0.44; p=.804		Cramér's V(df=1): 0.04		
AIFC17	<40	40-59	60+	Total
No	25	45	29	99
	25.25%	45.45%	29.29%	100%
Yes	57	106	69	232
	24.57%	45.69%	29.74%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=0.02; p=.991		Cramér's V(df=1): 0.01		
AIFC18	<40	40-59	60+	Total
No	48	90	59	197
	24.37%	45.69%	29.95%	100%
Yes	34	61	39	134
	25.37%	45.52%	29.10%	100%
Totals	82	151	98	331
	25%	46%	30%	100%
Chi²(df=2)=0.05; p=.974		Cramér's V(df=1): 0.01		
AIFC19	<40	40-59	60+	Total
No	82	149	96	327
	25.08%	45.57%	29.36%	100%
Yes	0	2	2	4
	0.00%	50.00%	50.00%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=1.59; p=.452		Cramér's V(df=1): 0.07		

Category	Maths NMMU			Total
AIFC20	<40	40-59	60+	Total
No	82	150	98	330
	24.85%	45.45%	29.70%	100%
Yes	0	1	0	1
	0.00%	100.00%	0.00%	100%
Totals	82	151	98	331
	24.77%	45.62%	29.61%	100%
Chi²(df=2)=1.2; p=.550		Cramér's V(df=1): 0.06		

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