AN INTERVENTION FOR ENHANCING THE MATHEMATICS TEACHING PRACTICES OF GRADE FOUR TEACHERS IN THE NELSON MANDELA METROPOLITAN AREA

ADÉLE BOTHA

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by

Adéle Botha

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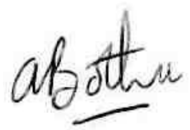
Promoter: Dr R.E. Gerber

DECLARATION

I, Adéle Botha, hereby declare that –

- The work in the dissertation is my own original work.
- All sources used or referred to have been documented and recognised.
- This thesis has not been previously submitted in full or partial fulfilment of the requirements for an equivalent qualification at any other recognised education institution.



Adéle Botha

ABSTRACT

Mathematics is regarded as a driving force in economies worldwide. The performance of South African learners in mathematics over the past decade has highlighted that problems are being experienced across all grades. This situation needs to be addressed with urgency. The South African Department of Education stated that quality learning must be the objective for all grades. The implementation of good teaching practices plays a crucial role in improving the quality of education and in guiding learners towards quality learning. To achieve quality mathematics teaching and learning it is imperative to determine what good mathematics teaching practices are. The identification of good mathematic teaching practices will provide a yard stick to measure the mathematics teaching competency of teachers.

This study identifies a set of good mathematics teaching practice indicators and evidences applicable to teachers in the Intermediate phase as a first contribution. These indicators and evidences frame the second research contribution: an assessment instrument entitled “A Classroom Observation Tool for Observing Mathematics Teaching Practices in Primary Schools”.

As a third research contribution a generic profile of a Grade four mathematics teacher has been built. This generic profile has been built through an analysis of data gathered by means of self-assessment questionnaires completed by the research sample, as well as through applying the observation tool. The value of the generic profile lies in the identification of shared strengths and shared improvement opportunities in the mathematics teaching practice of the sample and as such, it forms the basis of a theory on Grade four mathematics teaching practice.

The fourth research contribution is the design and application of an intervention that addresses the shared improvement opportunities. The research study concludes by comparing pre-intervention classroom observation data with post-intervention classroom observation data and reporting on the impact of the intervention.

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"We thank you, O God! We give thanks because you are near. People everywhere tell of your wonderful deeds. [Psalm 75:1 – New Living Translation 2007]

Dedicated to my late grandmother, Stella Bester. Without her support in earlier years of research I would not have been able to travel this road at all.

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“One never notices what has been done;

one can only see what remains to be done.”

Marie Curie - Nobel Prize Winner (1903; 1911)

Letter to her brother, 1894

CHAPTER 1 INTRODUCTION

1.1 Foreword

Mathematics (and the teaching of mathematics) impacts on both the economies and citizens of countries around the globe. According to the Programme for International Student Assessment Mathematics [PISA (2006)] and Stacey (2006), mathematics can be regarded as a driving force in economies worldwide. Mathematics also plays a pivotal role in shaping how individuals from different cultures deal with the various spheres of social, private and civil life (Walshaw & Anthony 2008; Anthony & Walshaw 2006). It is thus imperative that the question: ”What is good mathematics practice?” be addressed, as the answer to this question can help countries like South Africa, which are struggling with mathematics teaching, to gain insight into mathematics practices that have proven to be effective in a range of other countries.

1.2 Motivation for this study

The implementation of good teaching practices plays a crucial role in improving the quality of South African education. Mediocre or poor teaching practices applied by teachers in the classroom do not guide learners towards quality learning. This was the message in a statement released on 28 December 2007 by the incumbent Minister of Education, Naledi Pandor (Department of Education [DoE] 2007a). Ms Pandor said that the South African government and its partners should find ways to put effective measures in place so that better results could be achieved at all school levels. After announcing the 2007 matric results, Ms Pandor blamed the continuing bad results of poorer schools on three factors: unqualified teachers, inadequate laboratories and negligible support to schools. She said that she noted with some alarm the inadequate progress in Higher Grade passes in mathematics and science. Mrs Pandor reiterated that every child studying mathematics and science needs to have a qualified and competent teacher in their classroom. Ms Pandor continued that it was clear from various studies, as well as from the 2007 results, that quality learning needed to be the Department of Education's concrete objective for all grades. She said that problems experienced by learners in the advanced school phases could be related to a poor literacy and numeracy base being laid in the Foundation phase (DoE 2007a). This viewpoint was also expressed by Chris

Klopper, Chief Executive Officer of the South African Teachers' Union (Rademeyer 2010). Klopper stated that many 2010 Grade 12 learners had already performed poorly in the Foundation and Intermediate phases. In 2001, when these learners were in grade 3, their average pass rate for literacy was only 30% and only 15% passed numeracy on an acceptable level when tested. The average for numeracy was only 36% (Rademeyer 2010). Klopper continued that the quality of teaching experienced by the majority of learners in the Foundation and Intermediate phases was poor. Before quality learning can be achieved by learners, their teachers need to be able to deliver quality teaching through the implementation of good teaching practices. The importance of good classroom practice was stressed by Ms Pandor in her opening address delivered at the Foundation Phase Conference on 30 September 2008 (DoE 2008a). She stated that although the South African curriculum is explicit about the skills and competencies that learners should develop at different grade levels, the Department of Education realises that teachers are struggling to translate the curriculum into good classroom practice. This problem is experienced by other countries as well because the visions of curriculum designers are not always implemented by teachers in the actual classroom (Walshaw & Anthony 2008). Jansen (2008) also lists this inability to implement the curriculum as largely contributing to South Africa's education dilemma. Teachers thus need support to implement the curriculum effectively through good classroom practice. Koellner, Jacobs, Borko, Schneider, Pittman, Eiteljorg, Bunning and Frykholm (2007:275) stated that the improvement of learners' opportunities to learn mathematics fundamentally depends on their teachers' knowledge and skills. The National Council of Teachers of Mathematics [NCTM (2000)] listed multiple sources of influence on classroom practice. These include a solid mathematics curriculum; competent and knowledgeable teachers who can integrate instruction with assessment; educational policies that enhance and support learning; classrooms with ready access to technology and a commitment to both equity and excellence. Of all of the aforementioned, Koellner, et al. (2007) express the view that arguably none is more important than the teacher.

Bernstein (2007a: 14) stressed the need for competent mathematics and science teachers in South Africa in a report published by The Centre for Development and Enterprise (CDE). The CDE's second report on mathematics and science education in South Africa, Doubling for growth: Addressing the maths and science challenge in South Africa’s schools, was released on 3 October 2007. A media release titled:: “SA making inadequate progress on math and science challenge” followed (Bernstein 2007b). Bernstein, the Executive Director, revealed that findings in the report indicated that annual passes in Higher Grade mathematics have declined from 2005 to 2006. This is while the Department of Education’s stated target for 2004-2008 was to double the Senior Certificate Higher Grade maths and science passes from about 25000 to about 50000 (Bernstein 2007a: 4). The report showed that more than half of South Africa’s secondary schools failed to achieve even one HG maths pass and that 81% of schools achieved one pass each on average (Bernstein 2007b). Following the CDE report, the Director General of the Department of Education, Duncan Hindle, assured members of Parliament that although there had been a dip in Higher Grade maths passes in 2006, they still hoped to see 50000 mathematics passes on Higher Grade by 2009 (DoE 2007b). Hindle maintained that there had been considerable growth in the number of maths passes on Standard Grade and that with the necessary teaching support more students could have passed on the Higher Grade. According to Hindle 4500 mathematics and science teachers had already been tested by the Department of Education to check their competency levels in teaching those subjects. A total of 120 master teachers were also appointed in 2006 and an additional 2400 teachers were being trained.

Upon releasing the 2007 Senior Certificate results, Ms Pandor said that much attention was paid to improving the performance of Grade 12 learners in the recent past and that teachers and government alike devoted a great deal of energy to these learners (DoE 2008a). However, their performances remained poor. Ms Pandor continued that evidence indicates that the training focus should be shifted much lower than Grade 12 to improve the performance of South African learners in the future. There should be a return to basics and Foundation- and Intermediate quality education should be the focus (DoE 2008b). It is argued that many of the problems experienced by learners in the Senior phase (Grades seven-nine) and Further Education and Training phase (Grades ten-twelve) could be related to poor mathematics teaching practices applied by teachers in the Foundation (Grades R-three) and Intermediate phases (Grades four-six). Learners exposed to mediocre or poor mathematics teaching practices in the lower phases have a weak mathematics foundation. As a result these students struggle to understand mathematics concepts and many resent having to take Mathematics in the Senior- and Further Education and Training phases. A negative attitude towards mathematics, combined with a weak mathematics foundation, is a mix that can contribute to mathematics failure in the Senior- and Further Education and Training phases. Klopper (Rademeyer 2010) reiterated this when he said that the quality of teaching in the lower grades must receive attention because poor performances of Grade 12s can be attributed to the fact that many of these learners entered high school without the necessary basic literacy and numeracy skills.

The fact that South African learners are experiencing problems in terms of literacy and numeracy is confirmed through various surveys. South African learners have performed poorly and received alarmingly low scores in international and national assessments of reading and numeracy since 1994 (UNESCO 2006; DoE 2008a; DoE 2008c). Results from the surveys indicate that the majority of learners in Grade four, that is the entry grade in the Intermediate phase, do not possess the capacity to read, write and calculate at the required level. Results from surveys undertaken are discussed hereafter.

The 1999 Monitoring Learning Achievement (MLA) survey

In August 1999 the South Africa Monitoring Learning Achievement (MLA) survey was conducted under the auspices of the Department of Education with the goal of gathering data on learning achievement and outcomes at the primary school level (UNESCO 2006). Grade four learners from a random sample of 400 primary and combined schools participated in the survey. Competency instruments that were implemented for assessing literacy, numeracy and life skills learning yielded approximately 11,000 completed instruments. Results of the survey were poor. Grade four learners scored 48.1% on average in the literacy task with 13% of the learners scoring less than 25%. Results in the numeracy task were worse with 44% of the Grade four learners scoring below 25% and the average score being 30%. The results of the life skills task closely resembled those achieved in literacy with the average score being 47.1% (UNESCO 2006).

2001 and 2004 National Systemic Evaluations and the 2007 Progress in International Reading and Literacy Study (PIRLS)

According to findings from further studies the performances of South African learners in literacy and numeracy did not improve in the years that followed the 1999 survey. This statement is based on results provided by Ms Pandor in her capacity as the incumbent Minister of Education (DoE 2008c: 181). During the Fifth Session of the Third Parliament of the National Assembly Mr M.H. Hoosen of the Independent Democrats asked Ms Pandor whether “the levels of literacy and numeracy attained by learners upon completion of their primary school education (grade 7) is satisfactory; if not, (a) why not and (b) what steps has her department taken to improve this situation?”. Ms Pandor’s reply was as follows:

“National systemic evaluations, conducted by the Department in 2001 and 2004, revealed low levels of reading abilities across the country. The results of the Progress in International Reading and Literacy Study (PIRLS), released in November 2007, found that learners in our schools do not read at the appropriate level in relation to their grades and in terms of their age. Various reasons were provided for this:

- Lack of access to books in homes, at school and in their communities
- Low levels of literacy among the parents
- Ineffective teaching practices

The 2007 National Systemic Evaluations

On 30 September 2008, Ms Pandor disclosed key findings of the systemic evaluation survey conducted in 2007. The survey was conducted on a representative sample of more than 54000 Grade 3 learners (DoE 2008a). Learners who participated in the survey were tested in the written foundational skills of literacy and numeracy. The key findings were:

- Learners obtained an average overall percentage score of 36% in literacy.
- Learners obtained an average overall percentage score of 35% in numeracy.

Although this is a little higher than the baseline score of 30% in the systemic evaluation survey undertaken in 2001, the scores are still unacceptably low.

2009 National Systemic Evaluations

The evaluations were again conducted in 2009 and results obtained by grade 3 and grade 6 learners were as follows:

Grade 3's

- Learners obtained an average overall percentage score of 41% in literacy.
- Learners obtained an average overall percentage score of 44% in numeracy.

Grade 6's

- Learners obtained an average overall percentage score of 37% in literacy.
- Learners obtained an average overall percentage score of 33% in numeracy.

A major improvement was noted in the Eastern Cape Province where Grade 3 learners in 2009 obtained 45% for literacy and 51% for numeracy in comparison with 2007 when they obtained only 35% for literacy and 36% for numeracy respectively (Hendriks 2010). Dr Frank Peters, Director: Curriculum for Foundation, Intermediate- and Senior phases attributed this improvement to the pro-activeness of the Eastern Cape Department of Education in spending large amounts of time, money and energy on teacher training. Teachers were also provided with lesson plans to assist their teaching (Hendriks 2010). Notwithstanding the marked improvement in the Eastern Cape Province results in the national systemic evaluations, it should be noted that South African learners continue to perform poorly when rated internationally.

South African learners' rating according to international surveys

Results from the Trends in International Mathematics and Science Study (TIMSS) showed that South African Grade 8 learners dropped 25 places from 1995 to take the last place of the participating countries in both mathematics and science in 1999 as well as in 2003 (Reddy 2004). Since 2007 South Africa has declined further participation. Other poor performances in mathematics were in the Southern African Consortium for Monitoring Educational Quality. In 2000 the Consortium reported that only 24% of the South African Grade 6's who participated could perform at the Basic Numeracy Level (Moloi & Strauss 2005). The Education for All Development Index

(EDI) ranked South Africa 78th out of 125 countries in 2004 (UNESCO 2007). For the school year that ended in 2008 there were 127 countries placed according to the EDI, but South Africa was not included (UNESCO 2011). A report in the *Mail and Guardian* online, dated 3 August 2008 (Mohlala 2008) reiterated the poor proficiency levels of South African learners in mathematics and science. According to Mohlala (2008) the World Economic Forum rated South Africa 120th for the quality of mathematics and science teaching in 2006/2007. The situation deteriorated with South Africa dropping to 132 out of 134 countries in their quality of mathematics and science teaching for 2008/2009 in the Global Competitiveness Report published in October 2009 (HSRC 2009).

Poor performances by South African learners have continued to make headlines. Some of these include: “SA Education raises questions” headlining an article by Independent Online (IOL) on a report presented by the Organisation for Economic Co-operation and Development (OECD) on South Africa [IOL(a) 2008]; “Many SA pupils can’t read or count” (Newman 2008); “Pandor: numeracy, literacy unacceptably low” [IOL(b) 2008]; “Only 36% of Grade 3 pupils can read” (Serrao, 2008); “No improvement in matric maths” [South African Press Association (SAPA) 2009a]; “Education findings devastating” [SAPA 2009b]; “Literacy, maths shocker in SA” [SAPA 2009c]; “Cosatu: SA education in crisis” [SAPA 2009d]; “KZN pupils shocking in maths” [SAPA 2009e] as well as “Education is below standard” [SAPA 2009f].

The aforementioned discussion explains to a large extent why South African learners in the Intermediate, Senior as well as Further Education and Training phases are struggling with their schooling. Results from various studies have shown that South African learners who are exiting the Foundation phase cannot read, write, count and calculate confidently and with understanding. The poor performances delivered by learners in the Foundation- and Intermediate phases in terms of literacy, numeracy and science are especially problematic in the current South African scenario. As from 2008 all learners across South Africa follow the same national school curriculum for the first time since initial curriculum changes were introduced in 1998 (DoE 2008d). In the new curriculum for Further Education and Training (Grades ten to twelve) either mathematics or mathematical literacy is a compulsory subject. The new curriculum thus added to the demand for all students to have a sound mathematics basis laid in the Foundation- and Intermediate phases. The South African Government realised that quality education is determined in the first years that a child spends at school (DoE 2008a; DoE 2008b). According to Ms Pandor (DoE 2008a) literacy, numeracy and life skills are the essential building blocks upon which future learning takes place. To provide South African learners with a better chance of success when pursuing learning beyond the Foundation- and Intermediate phases, the Department of Education launched the Foundations for Learning Campaign on 18 March 2008 (DoE 2008b).

At the launch of both the Foundations for Learning Campaign in Cape Town on 18 March 2008 (DoE 2008b) and the Foundation Phase Conference on 30 September 2008 (DoE 2008a), Minister Pandor referred to the educationalist Maria Montessori. Montessori stressed the importance of the early learning years and believed that the foundations of human development are laid during a child’s early years. According to Ms Pandor, it is in the Foundation phase that children learn the fundamental skills and competencies that will enable them to learn and develop a clear conception of the world. Children learn to read with understanding and to understand the concept and power of numbers so that they can use the knowledge and skills in their future studies. This capacity to read, write and calculate well forms the foundation of quality education and should be learnt in the Foundation phase and thereafter consolidated in the Intermediate phase (D0E 2008b). The Foundations for Learning Campaign aims to lay a solid foundation in languages and mathematics in the Foundation- and Intermediate phases. In the four years that the campaign is to run, it aims to increase the average learner’s performance in languages and mathematics to no less than 50%. A national evaluation is to be conducted in 2011 to assess the abilities of learners in South Africa in languages and mathematics. This assessment should reflect on the effectiveness of the Foundations for Learning Campaign in increasing learners’ performances in languages and mathematics (DoE 2008b).

Requirements for success

To succeed the Foundations for Learning Campaign (DoE 2008e: 6, 7) requires that:

- every classroom has the appropriate resources for effective teaching;
- teachers plan and teach effectively;

- district teacher forums are established in all districts;
- teachers assess learner performance regularly.

Measures that should be in place to ensure that the abovementioned requirements are met include:

- Each school must ensure that every teacher teaching in the Foundation- and Intermediate phases has at least the basic minimum resources, as specified in the Government Gazette on the campaign published on 14 March 2008 (DoE 2008e), in the classroom;
- All teachers must be in their classrooms teaching planned lessons during contact teaching time. The time table must, on a daily basis, make provision for every learner in the primary school to engage in reading for 30 minutes at school; for writing a piece of extended writing appropriate to the grade that the learner is in and for engagement in mental maths for 10 minutes and written maths for 20 minutes;
- Teachers must participate in district or school forums where they can share ideas, experiences and best practices, as well as enhance their teaching strategies;
- Teachers should use the standardised assessments provided by the Department of Education. The results of these assessments must be reported to the district offices from where they will be sent to the office of the Minister of Education via the provincial offices. The Department of Education will assist teachers in managing the assessment tasks within the continuous assessment framework by providing milestones for expected attainment in mathematics and languages per term per grade. Schools will receive annual tests based on the quarterly assessments from the Department of Education.

In addition, every principal of a public school is requested to prepare a plan detailing how the academic performance of learners at his/her school will be improved. Principals should furthermore manage and support teachers in their effort to improve the ability of their learners to read, write, count and calculate at the appropriate level. The success of each principal’s plan will be measured in terms of the learners’ performances in the end of the year standardised assessment (DoE 2008e).

The importance of good mathematics practices being applied by teachers in Grade four in particular lies in the following: The Foundation for Learning Campaign has stressed the importance of providing learners with a solid foundation in reading, writing and numeracy. There are, however, a large number of learners who exit the Foundation phase and enter the Intermediate phase with a handicap in terms of their reading, writing and numeracy skills. The low attainment levels of Grade three learners in literacy and numeracy creates a problem for Grade four teachers. A large number of learners who lack the core skills required for learning effectively enter the Intermediate phase without the necessary foundation having been laid in the Foundation phase. This places an additional work load on Grade four teachers. They cannot focus their teaching practice only on consolidating the literacy and numeracy skills which learners should have mastered in the Foundation phase. Grade four teachers should, through their teaching practice, also assist learners who lack the required foundational skills in literacy and numeracy. Jansen (2008) has explained that it is the teacher's teaching practice that directly impacts on the quality of learning experienced by the learners. For South African learners to perform better in literacy and numeracy, teachers need to be more effective in their teaching practices. They must teach in ways that will improve their learners' performance (DoE 2008b).

Based on the aforementioned information, the researcher formed the opinion that an investigation into the mathematics teaching practices of Grade four teachers is important. The aim would be to identify strengths and improvement opportunities in the mathematics teaching practices of these teachers. Once identified, interventions could be designed and applied to address improvement opportunities. This is in line with the Foundations for Learning Campaign which suggests that teachers should come together to participate in school and district forums where they can share best practices and enhance their teaching strategies. Ms Pandor (DoE 2008a) has furthermore stated that teachers have difficulty in translating the curriculum into good classroom practice. Teachers thus need support in implementing the curriculum through their teaching practices.

Following the South African general elections in April 2009, the Ministry of Education has been split into two separate ministries (Bathembu 2009). One is a Ministry of Higher Education and Training that focuses on institutions of higher learning. The other is a Ministry of Basic Education that focuses on the schooling system. The new

Minister of Basic Education, Ms Angie Motshekga, was sworn in on Monday 11 April 2009 (Mbanjwa & Kgosana 2009). Ms Motshekga immediately stressed that there must be a focus on primary education and proclaimed that it is important to do the basics first and to ensure that there is competency in everything that is being done. Another priority was to improve teaching (Mbanjwa & Kgosana 2009). Ms Motshekga subsequently gave the Foundations for Learning Campaign her full support (Motshekga 2008). Ms Motshekga furthermore appointed a task team to review the National Curriculum Statements. In October 2009 the task team reported that teachers and parents whom they interviewed support the Foundations for Learning Campaign. In November 2009 Ms Motshekga announced that the Foundations for Learning Campaign must be implemented by all South African schools in the Foundation- and Intermediate phases effective from January 2010.

The research study can contribute towards an improvement in primary school mathematics teaching. The results of the investigation to determine good mathematic practices for Grade four mathematics teaching can support Grade four teachers in implementing the curriculum more effectively. Once identified, improvement opportunities in mathematics teaching practice can be addressed through applicable interventions. The development of teaching strategies forms part of the professional development of teachers according to the National Policy Framework for Teacher Education and Development released on 26 April 2007 (DoE 2007c). Interventions where good mathematics practices are demonstrated and applied by teachers can positively impact on the mathematics practice of these teachers. Improvements in mathematics practice may in turn result in Grade four learners benefiting as a result of the improved teaching. The research undertaken can furthermore contribute to the creation of a theoretical knowledge base of what is regarded internationally as good mathematics practices. It will also provide knowledge of the current teaching practices applied by Grade four mathematics teachers in South Africa.

1.3 Problem Statement

The inability of teachers in the Foundation- and Intermediate phases to implement the curriculum through their teaching practices has contributed towards the poor performance of South African learners in mathematics. To improve the teaching practices of Grade four mathematics teachers in South Africa it is imperative that improvement opportunities be identified and addressed through suitable interventions. Interventions should equip teachers with good mathematics practices and guide them towards reaching the goals set out in the Foundations for Learning Campaign.

1.4 Aim and objectives of this research

This study aims to enhance the teaching practices of Grade four mathematics teachers through interventions. Shared improvement opportunities identified in the teaching practices will be addressed and the impact of the interventions on the teaching practices of Grade four mathematics teachers will be assessed. The following objectives were pursued to achieve the aim of the research:

To study mathematics practices applied by countries internationally and to identify those teaching practices that proved effective in the teaching of primary school mathematics in a range of countries. Once identified these international good mathematics practices for the primary school could be used as a mirror for the research sample to reflect on their own teaching practice.

To use the international good practices in the compilation of self-reflective questionnaires for the sample members as well as in the design of a classroom observation tool to be used by the researcher to gather data on the teaching practice of the sample.

To analyse the data gathered by means of the questionnaires and the classroom observation tool and to compile individual teaching practice profiles highlighting the strengths and improvement opportunities in the teaching practice of each sample member.

To compare the individual profiles and determine the shared improvement opportunities in the mathematics teaching practice of the sample.

To develop interventions which address the identified shared improvement opportunities and to equip sample members with both knowledge and skills. The interventions will be attended by the sample members.

To determine the impact of the interventions on the improvement opportunities identified in the mathematics teaching practices of the sample. A second classroom observation tool focused on the improvement opportunities addressed during the intervention will be used to gather data during post-intervention observations. This data would be compared with the pre-intervention data to determine if the intervention was successful in its aim to address the identified improvement opportunities. Only then would the research objectives be met.

Below is a schematic representation of the research phases.

Table 1.1 Research phases

Phase	Description
1	Research for the development of the classroom observation tool and the three questionnaires on mathematics teaching practice
2	Application of the observation tool and questionnaires to gather the data used to build individual teacher profiles and to compile a generic profile.
3	Analysis of the generic profile to determine shared improvement opportunities in the mathematics teaching practice of the sample for intervention development.
4	Research on adult learners and mathematical thinking for intervention development. Intervention design and application to address identified improvement opportunities in the mathematics teaching practice of the sample.
5	Development and application of a second classroom observation tool to gather data after the intervention. Analysis and comparison of pre- and post-intervention data on the teaching practices of the sample to assess the impact of the intervention.

The next two subsections will discuss the delimitations and limitations impacting on the research study.

1.5 Delimitations

There were certain delimitations that had to be taken into account during this research.

- The first delimitation was that the study was limited to Grade four teachers actively teaching in schools falling under the Port Elizabeth District Office of the Department of Education.
- The second delimitation was that the focus of this study fell exclusively on mathematics teaching practice. The teaching practice of any other subject was not included in this study.

1.6 Limitations

Two limitations also impacted on the research and had to be taken into account.

- Firstly, the number of Grade four mathematics teachers who constituted the sample had to be limited. This is because the scope of the investigation would otherwise have become too large to handle effectively. Questionnaires to obtain information on Grade four mathematics teachers were initially sent to one hundred and eighty-two Primary and Intermediate schools in the Nelson Mandela Metropolitan area. A total of eighty-four questionnaires were received back. Thirty Grade four mathematics teachers indicated that they were interested in participating further in the research project. Five teachers who indicated their continued interest and who met the selection criteria were selected as the research sample.
- Secondly, the currency of the sources used for the identification of best mathematics teaching practices had a cut-off date. This was due to the literature review being limited to a specific phase of the research. Data gathered during the literature review had been analysed to identify a set of indicators and evidences of good mathematics practice. The evidences and indicators identified were used to compile the data collection tools. Literature on mathematics teaching practices that was published after the identification of the indicators and evidences were completed, could not be taken into account for the purpose of this research. The time line concerning the literature reviewed is as follows:
    - The research study commenced in 2008 and the literature review concerning best mathematics teaching practices was done in 2009.
    - The data extracted from the literature review in 2009 was used in the compilation of the questionnaires and the first classroom observation tool.
    - As explained in the foregoing paragraph more current literature on global mathematics teaching practices may since have become available, but the most current sources that were available at the particular stage of the research were used.

1.7 Defining of terminology

1.7.1 Foundations for Learning Campaign

An educational campaign launched by the South African Department of Education on 18 March 2008 to provide South African learners with a better chance of success when pursuing learning beyond the Foundation- and Intermediate phases (DoE 2008b).

1.7.2 Foundation phase

In the South African school system the Foundation phase refers to Grade R (the year before children enter Grade one to Grade three (ages five to nine years).

1.7.3 Mathematics teaching practices

In this dissertation the term mathematics teaching practices refer to effective ways and means through which teachers teach mathematics to enable learners to learn optimally. Brodie (2004) argued that mathematical knowledge and mathematical practices are mutually constitutive and that they together constitute and are constituted by mathematics teaching practices.

1.7.4 Intermediate phase

In the South African school system the Intermediate phase refers to Grades four to six (ages ten to twelve years).

1.7.5 Mathematics teacher

A mathematics teacher is someone who teaches mathematics to learners.

1.7.6 Trends

For the purpose of this dissertation a trend refers to a general tendency or inclination to teach mathematics in a particular manner.

1.7.7 Indicator

An indicator in this dissertation points to a specific mathematics teaching practice criterion that should be applied by a teacher when teaching mathematics.

1.7.8 Evidences

In this dissertation the term evidences is used to describe specific measurable outcomes of the mathematics teaching practice of a teacher.

1.8 Organization of thesis

The remaining chapters of this thesis are described below:

Chapter 2: The research design

In this chapter the research design of the thesis is discussed. Focus is placed on the conceptual framework underlying the research; the research paradigm embedded in the research methodology; the types of data that have to be collected; the data collection tools to be used and the data analysis and interpretation approaches that are followed during the research process. Aspects also included are ways in which data will be justified as evidence; ways in which evidence will be validated as knowledge and ways in which the knowledge will be communicated.

Chapter 3: Good mathematics practice indicators and -evidences

This chapter focused on what is regarded internationally as good mathematics teaching practices for the primary school level. Findings of studies on good mathematics teaching practice in different countries were studied. Good practices were compared and a coding process was developed and implemented to compile a set of good mathematics practice indicators with evidences that support each indicator. Furthermore, the composition of the three self-reflective questionnaires as well as the classroom observation tool used for data collection is explained.

Chapter 4: Building individual profiles

The data compiled via the three self-reflective questionnaires and the classroom observations is analysed. Data analysis is followed by data application and the individual mathematics teaching practice profiles of each sample member is built.

Chapter 5: Theory building

Chapter 5 explains how a generic profile of the mathematics teaching practice of the sample is built. The focus falls on the identification of shared improvement opportunities in the sample's mathematics practice, by comparing the individual profiles of the sample members.

Chapter 6: Grounding the mathematical thinking intervention

Literature pertaining to mathematical thinking is perused to substantiate what mathematical thinking is; why it is deemed important to promote mathematical thinking and how mathematical thinking can be promoted during lessons. This knowledge is essential for intervention development.

Chapter 7: The intervention sample as adult learners

An audience analysis of adult learners is undertaken. The aim of the audience analysis is to identify resistance factors to adult learning on the one hand and factors that strengthen adult learning on the other. As with chapter 6, the knowledge gained in chapter 7 is essential for intervention development.

Chapter 8: Intervention development and application

This chapter explains the design of the intervention developed to address the improvement opportunity identified during the research. The intervention application is discussed. Reflections on the intervention by both the researcher and the sample members are provided. The design and application of the second classroom observation tool is discussed. The chapter concludes with an evaluation of the impact that the intervention had on the mathematics teaching practice of the sample.

Chapter 9: Conclusion

In conclusion the outcome of the research and how the research question was answered is reviewed. Findings and recommendations arising from the research, as well as needs for further research, are also indicated.

Appendices

The appendices are included in electronic format owing to their size. The appendices contain copies of letters sent and received as part of the research; information on schools involved; copies of the questionnaires, observation tools and rubric developed. It also includes responses received to the questionnaires; observation data compiled and a description of the building of individual profiles B-E. The intervention guide and games used during the intervention; scanned copies of the responses of the sample to the intervention reflection rubric, as well as audio files of all observation lessons and the intervention are also included as appendices. The appendices are arranged as follows:

Appendix A (i): A letter, dated 27 November 2008, sent to the Port Elizabeth District Office of the Department of Education to request permission to undertake the research in Primary and Intermediate schools in Port Elizabeth.

Appendix A (ii): A letter, dated 2 December 2008, received from the Director of the Port Elizabeth District Office of the Department of Education granting permission for the research to be undertaken in Primary and Intermediate schools in Port Elizabeth.

Appendix A (iii): The covering letter, dated 15 April 2009, that accompanied a questionnaire, Appendix D(i) that was sent to principals of Primary and Intermediate schools in the Port Elizabeth District of the Department of Education.

Appendix A (iv): A letter thanking principals of Primary and Intermediate schools in the Port Elizabeth District of the Department of Education who returned questionnaires.

Appendix A (v): The covering letter sent to teachers constituting the sample with the first questionnaire on mathematics teaching practice.

Appendix B (i): The list of schools to which the background questionnaire was sent.

Appendix B (ii): The list of schools that completed and returned the background questionnaires.

Appendix C (i): The list of countries which primary school mathematics teaching practices were investigated with supporting literature.

Appendix C (ii): Scanned tables of the coding done to determine the twenty-four international good mathematics teaching practices.

Appendix C (iii): Twenty-four good mathematics teaching practices for the primary school

Appendix C (iv): Eleven indicators and eighty-six corresponding evidences of good mathematics teaching practice

Appendix D (i): The questionnaire concerned with general information on Grade four mathematics teachers in Primary and Intermediate schools in the Port Elizabeth District of the Department of Education.

Appendix D (ii): The first questionnaire sent to the sample requesting information on their implementation of evidence of good mathematics teaching practice.

Appendix D (iii): The second questionnaire sent to the sample requesting information on their implementation of evidence of good mathematics practice.

Appendix D (iv): The third questionnaire sent to the sample requesting information on how they apply good mathematics practice to specific mathematics problems.

Appendix D (v): The classroom observation instrument used by the researcher during classroom observations to assess the mathematics practice of the sample.

Appendix D (vi): A rubric for sample members to reflect on the mathematical thinking intervention.

Appendix D (vii): A classroom observation instrument to assess mathematics practices that encourage mathematical thinking.

Appendix E (i): Teacher A’s responses to Questionnaire 2.

Appendix E (ii): Evidences from Questionnaire 2 that addresses improvement opportunities in Profile 1: Teacher A

Appendix E (iii): Data compiled on Teacher A's mathematics practice through use of the classroom observation tool.

Appendix E (iv): Comparing profile 2 of Teacher A with the classroom observation tool data

Appendix E (v): A summary of improvement opportunities in Questionnaires 1-3 and the classroom observation tool: Teacher A.

Appendix F [F (i) – F (vi)]: Building Teacher B's profile

Appendix G [G (i) – G (vi)]: Building Teacher C's profile

Appendix H [H (i) – H (vi)]: Building Teacher D's profile

Appendix I [I (i) – I (v)]: Building Teacher E's profile

Appendix J: Games to stimulate mathematical thinking

Appendix K: Intervention guide

Appendix L: Rubric for sample members to reflect on the mathematical thinking intervention

Appendix M: Scanned rubrics completed by sample members to reflect on the mathematical thinking intervention.

Appendix N [(N (i) – (iv)]: Individual classroom observation data of Teachers A, C, D and E after the intervention;

Appendix N [N (v) – N (vi)]: Combined classroom observation data of the sample after the intervention and a Comparison between classroom observation data of the sample before- and after the intervention.

Appendices O: Audio files of lessons observed both before- and after the intervention. Audio files of the intervention are also included.

1.9 Conclusion

Chapter 1 provided a brief introduction to the research study. The motivation for the study was discussed aims and objectives were stated and delimitations and limitations outlined. Relevant terminology was also explained and the organization of the remainder of the thesis was delineated. Chapter 2 focuses on the research design underpinning this research study.

CHAPTER 2 THE RESEARCH DESIGN

2.1 Introduction

According to Trochim (2006) research design can be described as the glue that holds the research project together. Through the research design the research is structured. How all major parts of the research project work together to address the central research problem and sub-problems is explained. Sridhar (2008) also describes the research design as a conceptual structure or blueprint that outlines what the researcher will do. In the Berkeley-Rockefeller African Development Dissertation Workshop presented by the Institute of International Studies it was said that an effective research design will link abstract and stylized concepts with the empirical world's complexities and challenges (Watts 2001). Watts (2001) also proclaimed that there is no single research model that a researcher should follow, but that numerous alternatives must be considered and choices made throughout the research process (Watts 2001). The research design thus describes the plan that the researcher follows to get from the research questions posed at the beginning of the research study to the point where the answers are obtained to the research questions.

2.2 The conceptual framework related to the problem issue

According to Shields and Tajalli (2006) the conceptual framework connects the different aspects of inquiry, namely the problem definition, purpose, literature review, methodology, data collection and analysis. A conceptual framework thus acts as a map that gives coherence to the empirical inquiry. Mujer Sana (2003) concurs that a framework can be used as a map to assist researchers in deciding on and explaining the route that will be taken with the research project. The conceptual framework allows the researcher to explain why a specific path of action is pursued, based on the experience of others or on what the researcher personally would like to explore or discover. According to Mujer Sana (2003:3) conceptual frameworks can be explained as one, or a combination of the following:

- "A set of coherent ideas or concepts organized in a manner that makes them easy to communicate to others.

- An organized way of thinking about how and why a project takes place and about how we understand its activities.
- The basis for thinking about what we do and about what it means, influenced by the ideas and research of others.
- An overview of ideas and practices that shape the way work is done in a project.
- A set of assumptions, values, and definitions under which we all work together."

Based upon the aforementioned, the researcher regards the conceptual framework as thinking about how and why the research project should be undertaken; what activities are required to address the research questions and how concepts and ideas can be communicated clearly.

2.2.1 What does the conceptual framework of this research project look like?

In concurrence with Mujer Sana (2003) the researcher sees the conceptual framework as the basis for thinking about what we do during the research process and what it means. The researcher's point of departure for the conceptual framework lies in the matrix scheme developed by Burrell and Morgan (1979). Their matrix helped to classify and understand existing sociological theories based on four paradigms, namely the Functionalist, Interpretivist, the Radical Humanist and the Radical Structuralist (Goles & Hirscheim 1999). Hancock (2005) criticised Burrell and Morgan's paradigm by arguing that a state of incommensurability could exist between the four compartments in their matrix. Consequently, researchers working in one paradigm may be ignorant of research work being done in the other paradigms (Hancock 2005). Although acknowledging Hancock's viewpoint, the researcher supports Ching (2008) who regards each paradigm as a set of binoculars through which a researcher views and works within a specific field. As such, the researcher categorizes herself as falling under the Interpretivist paradigm. Goles & Hirscheim (1999) described that within the Interpretivist paradigm explanation is sought subjectively within the realm of individual consciousness. Furthermore, it takes as frame of reference this perspective: it is by expressing the meanings that a person attaches to his world, that social roles and institutions exist. In accordance, the researcher holds the view that human actions and interactions produce and reinforce the social world. During the research project the researcher will furthermore observe on-going processes to better understand the behaviour of individuals as described under the Interpretivist paradigm by Burrell and Morgan (1979). The conceptual framework of the research in totality and all research actions that will be undertaken will be done in a specific way because the researcher is guided by the Interpretivist paradigm. As an Interpretivist the researcher wants to understand how processes work. Furthermore occurrences will be described from a social perspective and the data gathered during the research will be utilised to generate knowledge about a theory. Finally, the researcher wants to be able to explain the theory that is developed. To be able to understand and explain what takes place throughout the research project the researcher will on an on-going basis communicate with the people directly involved in the project to ascertain how they are experiencing the project. This supports the social learning factor that is present in Interpretivist research, where meaning is created through the interaction of individuals with one another and the world they live in (Verster 2009). Verster (2009) explained that within the Interpretivist paradigm the researcher constructs meaning from everyday experiences that are viewed from the researcher’s personal background and experience. As such a researcher’s cosmological, ontological and epistemological standpoints impact on the research undertaken. The meanings of the terms cosmology, ontology and epistemology, as well as the researcher’s views with regard to each will be discussed in the next section.

2.3 Cosmological, ontological and epistemological views

2.3.1 Cosmology

Cosmology refers to a theory about an individual’s world view and includes who we are; where we come from; where we are going; how we got there; how the world came into existence; religious influences; global affiliation and values (Gerber 2008). A person’s cosmological standpoint is personal and no two persons will hold exactly the same cosmological standpoint. This provides each researcher with his or her own outlook on reality in general and on specific occurrences in particular. As such the researcher’s cosmological standpoint will influence the way in which data collected during the research project is analysed and interpreted. The researcher supports the Christian view of creation and has an educational background as a white South African going to school and university in South Africa during the late 1980’s and early 1990’s. Over the past twenty years teaching experience was gained by presenting training courses to university students as well as to adults from different work spheres.

2.3.2 Ontology

Ontology can be described as a theory of being which influences how we see ourselves in relation to others (Gruber 1993). Ontology has a long history in philosophy where the term refers to the subject of existence. Gruber (1993) uses the term ontology in the context of knowledge sharing to describe concepts and relationships that can exist for an agent or a community of agents. Voce (2004), in turn, describes the ontological question as twofold, namely: What is the form and nature of reality? and What is the nature of human beings? The researcher holds an integrated view of humans. According to this view the perspectives and beliefs of all individuals are integrated into their respective frameworks of theoretical thought (Larsson 1997). As the researcher has a long history of studying in Education, dating back to 1986, it is realised that the consequent European outlook on education may impact on the way in which data is analysed and interpreted. To prevent data from becoming compromised, data triangulation will be applied during data analysis and interpretation.

2.3.3 Epistemology

Voce (2004) stated what can be known as the most important epistemological question. In other words: What is the basic belief about knowledge?. Epistemology is thus the branch of Philosophy that studies knowledge and attempts to answer the question: What distinguishes true or adequate knowledge from false or inadequate knowledge? This question leads to another, namely How can one develop theories that are better than comparing theories? Epistemology thus refers to a theory of knowledge and includes a theory of how knowledge can be acquired (Myers 1997). According to Wallenmaier (2007:6) the problems considered in epistemology are:

- “Is genuine knowledge attainable at all?
- Is the sceptic right?

- What are the limits of knowledge?
- From what faculties of the mind does knowledge originate?
- Which method should be used to obtain valid knowledge?
- How do you justify an a priori statement?
- Where is the boundary between subjective and objective factors?
- What is the nature of truth?”

Knowledge can be distinguished from true belief by justification and much of epistemology deals with how true beliefs can properly be justified. The researcher’s epistemology is subjective in nature and as such the researcher acknowledges that researchers themselves play a role in shaping that which they study (Swanson, Watkins & Marsick 1997). To accumulate initial data literature is reviewed to determine what is internationally regarded as good mathematics teaching practices for the primary school. Based on the initial data, questionnaires for self-reflection by the sample are developed. Data is furthermore to be gathered by the researcher via a classroom observation tool. The tool is applied during lesson observations to assimilate data on the individual sample member’s teaching practice. Data acquired from the questionnaires and the observations is analysed and interpreted as part of knowledge acquisition.

To prevent the researcher’s cosmological and ontological views from influencing data interpretation, criteria and evaluation standards have been put in place against which data analysis and interpretation are validated.

2.4 The overall research approach

Based on the researcher’s underlying philosophical assumption an interpretive research approach is followed. The overall research approach is guided by the central question in the research, namely: ‘How can Grade four mathematics teachers be assisted to teach mathematics well and in so doing reach the goals set out in the Foundations for Learning Campaign? To answer the research question the researcher used qualitative research methods.

2.5 The research paradigm embedded in the research methodology

Voce (2004) defined a paradigm as being a framework within which theories are built. It influences how you see the world; determines your perspective and shapes your understanding of the connections between things. A paradigm thus is a world view that influences both your personal behaviour and your professional practice. The position that a researcher takes with regard to the research subject is also influenced by his/her view of the world. When discussing the conceptual framework of the research study it has been explained that the research paradigm embedded in the research methodology is an Interpretivist paradigm. According to Burrell and Morgan (1979), the Interpretivist paradigm presents a subjectivist approach to the analysis of the social world. As such, the Interpretivist paradigm is also in line with the researcher's ontological and epistemological standpoints. As an Interpretivist the researcher wants to get close to the phenomenon that is being researched to understand the human experience at the level that it occurs. Researchers in the Interpretivist paradigm start out from the assumption that access to reality takes place through social constructs like language, consciousness and shared meanings.

In an Interpretivist paradigm the research design is guided by two questions, namely: 'How does?' and 'How should?' (Burrell & Morgan 1979). When applying these two questions to the research study undertaken, the following questions can be asked: 'How do the mathematics teaching practices of the sample differ from the good teaching practices identified through the research?' and 'How should the shared improvement opportunities identified in the teaching practice of the sample be addressed?". In reaction to the latter of the two questions an intervention was designed and implemented to determine the impact that it had on the sample's teaching practices. Being an Interpretivist, the researcher is concerned with investigating and understanding the impact of interventions on the mathematics teaching practices of the sample. As such the researcher focused the research problem and research questions on determining how the mathematics teaching practices of the sample could be enhanced through interventions that can guide them towards reaching the goals set out in the Foundations for Learning Campaign.

2.6 Research question and sub-problems

2.6.1 Research question

This research set out to answer the following research question:

- How can Grade four mathematics teachers be assisted to teach mathematics well, based on good mathematics teaching practices that guide them towards reaching the goals set out in the Foundations for Learning Campaign?

In order to answer this question several sub-problems had to be investigated.

2.6.2 Sub-problems

The following sub-problems were investigated to illuminate the research question:

- What conceptual framework, as basis for good practice decisions and data collection, can be constructed from international good mathematics practices and the goals set out in the Foundations for Learning Campaign?
- What are the individual mathematics teaching practice profiles of the sample members and how can the profiles be used to determine strengths and shared improvement opportunities in their mathematics teaching practices?
- What theory can be built from what was learnt via the above foci?
- How can shared improvement opportunities in the mathematics teaching practices of the sample be addressed through interventions?
- What is the impact of interventions designed and applied to address shared improvement opportunities on the mathematics teaching practices of the sample?

The methodology used to conduct the research study will be discussed next.

2.7 Methodology

According to Trochim (2006), methodology is aimed at showing how the research results were achieved. Methodology answers the following two questions:

- “How was the data collected or generated?
- How was the data analysed?”

Voce (2004) describes methodology as an explanation of the way in which the researcher went about to find out what s/he believes can be known. Thus again, methodology refers to the way in which the data was collected and analysed to answer the research question. The researcher will use Trochim's two questions as a guideline to describe the methodology followed during this research study. The next two subsections will focus on data collection and data analysis.

2.7.1 Collecting the data

To obtain the desired research result each sub-problem identified has to be addressed and the research question answered. This is impossible to do without the collection and interpretation of relevant data. Trochim (2006) and Myers (1997) distinguish between two kinds of data that can be collected, namely qualitative and quantitative data. Quantitative data deals with quantities and focuses on the following areas of assessment, 'how many?', 'how much? and 'how long?'. Data are thus normally expressed in the form of numbers or percentages. Examples of quantitative data are data collected via survey methods, laboratory experiments, and formal methods, for example econometrics and numerical methods like mathematics modelling (Myers 1997). There are some qualitative studies that also involve collecting a large amount of numeric data and Trochim (2006) stresses the value of mixing qualitative research with quantitative research. This is because quantitative research is excellent for summarizing large amounts of data and for reaching generalizations based on statistical projections. Qualitative research tells the story from the participants' viewpoint thereby providing the descriptive detail that can be used to set the quantitative results into their human context. Holton and Burnett (1997) concur with Trochim (2006) that qualitative and quantitative research methods can be valuable and powerful when used together.

This research, however, is in the form of a qualitative study. As such it provides insight into the viewpoints of the sample and focuses on their teaching practices as they apply it in their natural settings. This is in accordance with Matveev (2002) who explained that qualitative researchers strive to describe, decode and interpret accurately the meanings of phenomena as they occur in their normal contexts. Matveev (2002) continued that during qualitative research the focus fell on the daily activities that people perform as part of their normal routines. The next section will focus on the aspects impacting on the collection of qualitative data. The relevance thereof to answer the research question and address the sub-problems will be discussed as well.

2.7.2 Following the qualitative research route

Matveev (2002) named three characteristics of the qualitative enquiry. Firstly, text and conversations are studied. Secondly, the interpretive principles that people use to understand symbolic activities are studied. Finally, the roles played by the research participants the situational events and the physical setting, which together constitute the contextual principles, are studied. Denzin and Lincoln (2003: 9) also accentuate the importance of the research participants in qualitative research by explaining that the qualitative researcher understands that “research is an interactive process shaped by his or her personal history, biography, gender, social class, race, and ethnicity, and by those of the people in the setting.” Furthermore researchers take a case-based position when undertaking qualitative research, which directs their attention to the specifics of particular cases (Denzin and Lincoln 2003). Generally qualitative research can be characterized as the attempt to obtain an in-depth understanding of the meanings and definitions of the situation presented by informants, rather than the production of a quantitative measurement of their characteristics or behaviour (Trochim 2006). The aim of qualitative research is to describe characteristics of the domain under investigation. Three criteria, formulated as questions, were put forward by Trochim (2006) to assist researchers in deciding whether or not they can follow the qualitative research route. These three questions are:

- “Do you want to generate new theories or hypotheses?
- Do you need to achieve a deep understanding of the issues?
- Are you willing to trade detail for generalisability?”

If a researcher answers ‘yes’ to the aforementioned questions, the qualitative research route is applicable. This is the case with the research study undertaken, as the researcher strives to achieve a deep understanding of the different issues that impact on mathematics teaching at the primary school level. Existing literature is used as one type of data source. In addition, data is also collected from a research sample consisting of Grade four mathematics teachers reflecting on their own practice. The sample provided first hand data that assisted the researcher in gaining a deeper understanding of practical issues that impact on Grade four mathematics teaching. Furthermore, by applying what was learnt in the formulation of a theory on the strengths and improvement opportunities in the mathematics teaching practices of the sample the researcher complied with Trochim's first criterion that qualitative research should generate a new theory (Trochim 2006). This point to a grounded theory approach in the analysis of data collected via the research sample. Finally detail was traded for generalisability as data on individual practice was used not only to compile individual teaching practice profiles, but also to compile a generic profile reflecting the shared improvement opportunities in the mathematics teaching practice of the sample.

As such, it has been established that the qualitative research route should be followed during this research study to answer the research question. The next section will focus on the qualitative data that was needed to address the sub-problems and answer the research question.

2.7.3 Addressing the sub-problems and answering the research question

It is through addressing each of the sub-problems that the research question is ultimately answered. The first sub-problem that needed to be addressed was:

- What conceptual framework, as basis for good practice decisions and data collection, can be constructed from international good mathematics practices and the goals set out in the Foundations for Learning Campaign?

To address this sub-problem a literature study was undertaken. Literature was perused to establish what is regarded as good mathematics teaching practices in the primary levels in a range of countries. Initially, a broad list of good mathematics teaching practices, according to the different sources perused, was compiled. Thereafter, the good teaching practices listed were grouped according to the countries in which they were successfully implemented. Subsequently, comparative tables were drawn up to establish which of the good teaching practices were shared by the different countries. The result was that twenty-four good mathematics teaching practices for the primary school were identified [Appendix C (iii)]. The framework designed by Baker and Chick (2006) for analysing Pedagogical Content Knowledge [PCK (Shulman 1986)], based on Chick, Baker, Pham and Cheng (2006), was selected for use during the triangulation of data. This particular framework was selected because it identifies key components of PCK for teaching primary mathematics (Baker & Chick 2006). The framework provides clear directives on mathematics practices for the primary school and had already been used in case studies for classroom analysis of primary mathematics teaching (Chick 2007; Chick & Harris 2007). Despite a comprehensive literature study no other framework for primary school mathematics teaching had come to the fore. The twenty-four good mathematics teaching practices were also compared to the daily activities for mathematics teaching laid out in the Foundations for Learning Campaign (DoE 2008e). Data triangulation and the comparison with the Foundations for Learning Campaign resulted in the lesser count of eleven indicators of good mathematics teaching practice for the primary school being identified. Through a second literature review eighty-six evidences of good mathematics teaching practice that reflect measurable outcomes of the eleven indicators were also identified [Appendix C (iii)].

The eleven indicators and eighty-six evidences of good mathematics teaching practice identified constitute the conceptual framework that will be used as the basis for good practice decisions and data collection during the research study. As such, the first sub-problem has been addressed.

The second sub-problem that needed to be addressed was:

- What are the individual mathematics teaching practice profiles of the sample members?

To address this sub-problem it was necessary to determine who the research participants were going to be. While the literature survey was underway, background information regarding Grade four teachers teaching mathematics in the Port Elizabeth District was collected. Primary school principals were requested to provide information on the Grade four mathematics teachers in their respective schools. The data required included data on the teachers’ academic backgrounds as well as on their teaching experience. The mediums of instruction for Grade four mathematics teaching with a breakdown of the number of Grade four learners taking mathematics in English, Afrikaans, Xhosa or another language were requested. Teachers had to indicate whether they enjoyed teaching mathematics and if they were interested in participating further in the research. The data received via the questionnaire was used to determine which teachers were willing to participate in the research study. Furthermore, it was used to select a sample of five teachers, from those who showed interest in further participation, who were representative of Grade 4 mathematics teachers in the Port Elizabeth district. The criteria used for sample selection are discussed at length in Chapter 3.

With the sample identified, the next step was to gather data on their individual mathematics teaching practices. To this end three self-reflective questionnaires were designed and distributed consecutively to the individual sample members. The first questionnaire centred on the eleven indicators of good practice the second around the eighty-six evidences of good practice; whilst the third required each sample member to explain his/her approach when teaching a given mathematical problem. The development and composition of the questionnaires are discussed at length in Section 3.10. In addition to the questionnaires, the researcher also developed a classroom observation tool that incorporated the indicators and measurable evidences. The observation tool was also used to collect data on the mathematics practice of the individual sample member in their respective classrooms (Section 3.11 refers). Data obtained via the questionnaires as well as through the classroom observation tool was analysed and used to build individual profiles of each of the sample members. During the data analysis the eleven indicators and the eighty-six evidences of good mathematics practice were used as a yardstick against which the actual data of each individual was measured. This comparative process resulted in the identification of the strengths and improvement opportunities in mathematics teaching practice of each individual sample member. With the strengths and improvement opportunities in the mathematics teaching practice of each individual sample member being identified, the individual profiles were built. Sub-problem two had been successfully addressed.

The third sub-problem that needed to be addressed was:

- How can the individual profiles of the sample be used to determine strengths and shared improvement opportunities in their mathematics teaching practice?

To determine the strengths and shared improvement opportunities in the mathematics teaching practices of the sample, comparative tables were drawn up. Individual sample members were labelled A - E for administrative ease. Columns labelled A – E, which were subdivided into strengths and improvement opportunities under each, were used. Comparisons were then drawn between the respective improvement opportunities included in the columns, to establish which of the improvement opportunities were indeed shared by more than one sample member. With the shared improvement opportunities having been identified, the strengths and remaining single sample member specific improvement opportunities were grouped together as strengths.

This led the researcher to address sub-problem four, namely

- What theory can be built from what was learnt via the above foci?

The research had so far generated two data sets on good mathematics teaching practice for the primary school. The first set of data dealt with good mathematics practices internationally. Data was obtained through the literature survey and an analysis thereof. The data was used in the compilation of questionnaires and an observation tool that were used to gather data from individual sample members on their mathematics teaching practices. A second set of data on the individual mathematics teaching practices of each sample member was generated and individual mathematics practice profiles compiled. Subsequently, the individual mathematics practice profiles of the sample were compared and the results of the comparison were analysed. This comparison generated data on the strengths and shared improvement opportunities in the current mathematics practice of the sample. This data was used to compile a generic profile portraying the strengths and the shared improvement opportunities that exist in the mathematics teaching practices of the sample. By compiling the generic profile a theory about the mathematics teaching practice of the sample was built and sub-problem four was addressed.

- How can shared improvement opportunities in the mathematics teaching practice of the sample be addressed through interventions?

The theory about the mathematics teaching practice of the sample that had been built was used as the foundation to determine the type of interventions that were needed. Interventions had to address the shared improvement opportunities in the mathematics practice of the sample. To this end, the researcher undertook a literature review to obtain information on mathematical thinking, as this was identified as the shared improvement opportunity in the generic profile. Chapter 6 focuses on grounding the mathematical thinking intervention. Literature on adult learning was also perused, as the sample consisted in totality out of adult learners. As such, chapter 7 deals with adult learners and how they should be accommodated during interventions. Once the intervention was grounded in theory and the audience analysis was completed, intervention development commenced. On completion of the intervention development, the intervention was presented with the participation of the sample. Sample members were requested to implement what they had learnt during the intervention in their individual classes. This created the opportunity for sample members to demonstrate how their learning enhanced their mathematics teaching practice.

A second data collection stage followed. Six weeks after the intervention, the researcher attended a second mathematics lesson presented by the sample members who participated in the intervention. During this observation, the researcher used an adapted version of the classroom observation tool. The second observation tool focused on the mathematics teaching practices relating to mathematical thinking only, as this had been the focus of the intervention. Whilst observing the individual mathematics lessons of each of the sample members, the lessons were recorded for data verification. Data on the teaching practice of each sample member gathered during the second observations was compared against the pre-intervention individual profile of the respective sample member. The results from the comparisons between the pre- and post-intervention teaching practices of each sample member were used to draw relevant conclusions regarding the impact that the intervention had in addressing the shared improvement opportunities.

By following the methodology described above, each of the sub-problems was addressed and by doing so the research question had been answered. The data collection tools that were used to collect the data will be discussed next.

2.7.4 Collecting the data: tools used

Data collection tools that were used during this research included the following:

- Literature reviews

Four literature reviews were undertaken. Firstly, available documents and texts on international good mathematics teaching practices for the primary school were reviewed to identify good mathematics teaching practices that had been applied with positive outcomes in a range of countries. The literature was perused a second time to identify measurable evidences for the good mathematics practices.

Thirdly, literature on mathematical thinking was reviewed. The aim of this literature review was to gain insight into what mathematical thinking is; why it is important and how it can be implemented in mathematics classrooms.

Lastly, literature that dealt with the different aspects of adult learning was perused. The goal was to ascertain the circumstances under which adults learn optimally. The literature review also highlighted pitfalls that needed to be avoided during intervention development.

- Questionnaires

Four questionnaires were distributed as part of data collection. The first questionnaire collected general data on the Grade four mathematics teachers in the Port Elizabeth District. This data was important for sample selection. Questionnaires two, three and four were directed at the sample and were used to gather data on the individual mathematics teaching practice of each sample member.

- A classroom observation tool

A classroom observation tool was used to observe a mathematics lesson of each sample member. The observation tool allowed the researcher to collect data on the individual mathematics teaching practices of the sample in each member's normal classroom environment. The researcher's impressions and reactions to the teaching practices of the sample were noted as further data collection.

A second classroom observation tool was used after the intervention. This observation included only the indicators and evidences that were addressed during the intervention. Data gathered on the teaching of each sample member via the second observations tool is used as post-intervention data during the comparison with the individual profiles of each sample member before the intervention. This allowed the researcher to determine the impact that the intervention had on the mathematics teaching practice of each sample member.

- Intervention application

A mathematical thinking intervention to address the shared improvement opportunity in the teaching practice of the sample was designed and presented. The sample participated in the intervention and was provided with the opportunity to apply what they had learnt during the intervention in their own teaching.

2.8 Analysing and interpreting the data

The qualitative data analysis and the interpretation processes followed during the research will be discussed in the following sub-sections.

2.8.1 Qualitative data analysis

According to the University of West England, Bristol (2007), qualitative analysis is the process of interpreting data in the form of words and text collected during the course of the research. Quantitative analysis, on the other hand, is the process of interpreting and presenting numerical data. As discussed under data collection tools, questionnaires were developed to accumulate data on the mathematics teaching practices of the sample. The responses of the sample to the questionnaires as well as the data obtained through the application of the classroom observations tool were analysed and interpreted as part of knowledge acquisition. Data was then interpreted in order to draw conclusions. To validate the interpretation of the data, criteria and evaluation standards were used which assured objectivity. The interpretation of the data accumulated improved the researcher's understanding of what good mathematics teaching practices in the South African context is. It further contributed to a better understanding of the extent to which the intervention had an impact in addressing the shared improvement opportunity that existed in the mathematics teaching practice of the sample. This resulted in the creation of a new body of knowledge. Through the creation of new knowledge the research questions were answered, and ultimately, so was the research problem.

2.8.2 Triangulation of data

Data was justified as evidence by the provision of related data sets and the use of validated tools for data collection. Furthermore, a representative sample was selected for data collection. Conclusions delivered as evidence were limited to those which could be justified by the data. Evidence, in turn, was validated as knowledge by using triangulation as the validation authentication method and not double coding of data collected. Multiple triangulations were used as both data collected and theory were triangulated. Data collected through the use of multiple questionnaires were triangulated against the data collected via a classroom observation tool. Theory was also triangulated because the theory developed was compared with existing and related theory from the literature.

2.8.3 Building mathematics teaching practice profiles as part of data analysis and interpretation

During profile building, the current mathematics teaching practice profiles of each individual sample member was built. Strengths and improvement opportunities in the mathematics teaching practices of each sample member were identified. Hereafter the individual profiles were compared and shared improvement opportunities in the teaching practices of the sample identified. This process resulted in the compilation of a generic mathematics practice profile of the sample. The identification of shared improvement opportunities was important as it clarified the type of interventions that had to be developed.

Selecting the sample

Questionnaires were distributed to one hundred and eighty- two primary schools in the Port Elizabeth District. The questionnaires requested principals to provide general information regarding their Grade four mathematics teachers. A list containing the names of the schools approached is attached as Appendix B (i). One question requested teachers to indicate whether they are interested in collaborating on the research study. Forty-seven schools returned a total of eighty-seven questionnaires. Thirty teachers from twenty-five different primary schools indicated their interest in further collaboration. Of these thirty teachers, a core group of five was selected as the research sample.

The sample was selected based on the following selection criteria:

Selection criteria

The four criteria used for sample selection are: teaching experience, language of instruction, school location and learner composition.

The following sub-criteria were set for teaching experience:

The sample should include teachers with:

- fewer than ten years teaching experience;
- between ten and twenty years teaching experience;
- more than twenty years teaching experience.

The following sub criteria were set for language of instruction:

The sample should include teachers who:

- use as language of instruction the mother tongue of both the teacher and the learners;
- use as language of instruction neither the teacher's mother tongue nor that of the learners;
- use their mother tongue as language of instruction to teach learners who all have a different mother tongue to the language of instruction;
- use their mother tongue as language of instruction to teach learners where the language of instruction is the mother tongue of some learners, but not of others in the class.

The following criteria were set for location:

Schools are to be included from:

- rural as well as urban areas within the Nelson Mandela Metropole;
- different parts of the Nelson Mandela Metropolitan area.

Finally, where learner composition is concerned, teachers should be selected who:

- teach a class consisting of only grade 4 learners;
- teach a class with learners split across grades, grade four included.

The sample

Based on the above criteria, a sample of five teachers was selected from the thirty respondents who had indicated a willingness to participate. These five teachers met all the selection criteria. Sample members were randomly labelled A – E for administrative ease when dealing with data. Details of the sample members are as follows:

Teacher A:

- Between 10 and 20 years teaching experience
- Teacher's home language is Xhosa; English and Xhosa are used as languages of instruction to teach Xhosa home language learners
- Rural school
- Mixed class of Grade four and five learners

Teacher B

- More than 20 years teaching experience
- Teacher's home language is Afrikaans; uses English and Afrikaans to teach Afrikaans, English and Xhosa learners
- Urban school
- Grade 4 learners only

Teacher C

- More than 20 years teaching experience
- Teacher's home language is English; uses English to teach Xhosa learners
- Rural school

- Grade 4 learners only

Teacher D

- Less than 10 years teaching experience
- Teacher's home language is Afrikaans; uses Afrikaans to teach Afrikaans learners
- Urban school
- Grade 4 learners only

Teacher E

- Between 10 and 20 years teaching experience
- Teacher is bi-lingual (Afrikaans/English); uses English to teach English and Afrikaans learners
- Urban school
- Grade 4 learners only

2.8.4 Interpreting the data collected

Initially, four sets of data were collected from the sample. Three sets were collected via self-reflective questionnaires that each sample member completed on individual mathematics teaching practices. The fourth set of data was collected by the researcher during classroom observations of each sample member. Data collection and interpretation had as aim the compilation of current mathematics teaching practice profiles of individual sample members. The individual profiles reflected the strengths and improvement opportunities in the mathematics teaching practices of each sample member. The letters A – E that had been allocated to differentiate between sample members, were used to keep the sets of data of individual sample members together. The data interpretation process that was followed to compile individual profiles A to E consists of exactly the same steps. As data interpretation is a long and tedious process, data pertaining to sample member A will be used to illuminate the process followed. As such, the process followed to interpret the data of sample member A and compile the individual teaching practice profile of A is also applicable to the processes followed to compile the individual profiles of sample members B, C, D and E respectively.

To compile the individual teaching practice profile of sample member A, data collected via the three questionnaires and the classroom observation tool were analysed and compared with the indicators and evidences of good practice (Appendix C iv). The comparative process resulted in the identification of strengths and improvement opportunities in the teaching practices of sample member A. The comparative process involved four different sets of data that were collected. The next subsections will explain the process followed to analyse each of the sets of data gathered and how they were compared against the indicators and evidences of good practice (Appendix C iv) to identify the strengths and improvement opportunities in the teaching practices of A. Once identified, the strengths and improvement opportunities were used to compile the individual teaching practice profile of sample member A.

Interpreting data from Questionnaire 1

As a first step in the compilation of profile A, the data collected via Questionnaire 1 was analysed and interpreted. In Questionnaire 1, the eleven indicators of good mathematics practice were rewritten in the form of questions. Sample member A had to answer whether the indicator was implemented in his/her own mathematics teaching or not. Questions to which the answer was in the negative indicated that the corresponding good practice indicators were lacking in the practice of the sample member. As a next step, sample member A had to discuss how the indicator is implemented, if indeed so. The responses of A to each question were analysed and the data was interpreted to determine what the implications of each response was on his/her teaching practice. Thirdly, sample member A had to provide reasons why each indicator was implemented in a specific way. The reasons provided were analysed to learn more about why sample member A applies certain teaching practices, but disregards others. During the analysis of the responses, correspondences between the eighty-six evidences of good practice and the answers of A were noted. Subsequently, a list reflecting the strengths and improvement opportunities in the mathematics practice of A was compiled. The improvement opportunities are practices included in the list of eighty- six evidences of good practice that were found lacking evidence in the responses of A to Questionnaire 1. A preliminary profile, labelled Profile 1, was compiled reflecting the evidences of good practice, named strengths and the evidences that were found to be lacking, named improvement opportunities. It should be noted that this is a preliminary profile, as there are more data gathered from the other questionnaires as well as from the observation tool that have to be taken into account.

Interpreting data from Questionnaire 2

Next, the data gathered via Questionnaire 2 were analysed and interpreted. In Questionnaire 2, sample members were requested in a table format to indicate which of the eighty-six evidences of good mathematics practice they believe that they implement. They were requested to provide examples to substantiate their responses. Where A indicated “No” against an evidence in a corresponding row, it indicated that the specific evidence is not present in the teaching practice of A. It thus pointed to an improvement opportunity in the teaching practices of A. Once the responses to Questionnaire 2 were analysed and the strengths and improvement opportunities identified, they were compared against the strengths and improvement opportunities contained in Profile 1. Where evidences were provided for previous evidences lacking, the profile was amended according. Subsequently, a second, revised profile, Profile 2, was compiled to reflect the revised strengths and improvement opportunities in the teaching practice of A after data gathered via both Questionnaires 1 and 2 had been analysed and interpreted.

Interpreting data from Questionnaire 3

In Questionnaire 3, sample members were requested to explain their approaches and methodology when having to teach a specific mathematics problem that they were presented with, to their learners. The responses to Questionnaire 3 were analysed against the list of good practice indicators and evidences. The results were used as a control measure to verify the data contained in Profile 2. In cases where the explanations of A provided evidences of teaching practices that had been included as improvement opportunities in Profile 2, the profile was adapted accordingly and a revised Profile 2 was compiled.

Interpreting data from the Classroom Observation Tool

Data accumulated by applying the classroom observation tool were analysed and interpreted to determine (1) the evidences of good practice displayed by the teacher; (2) the evidences not reflected in the teacher’s teaching practice and (3) any contraevidence reflecting bad teaching practice observed. Following the data analysis and interpretation the evidences of good practice, as well as the improvement opportunities identified were compared with Profile 2. Revisions were done where evidences for previously included improvement opportunities were provided and an adapted and final profile, Profile 3, was compiled for sample member A. This final profile highlighted the improvement opportunities identified in the mathematics teaching practices of A. The research stated as its aim the development of interventions to address shared improvement opportunities. As such, it is imperative that individual improvement opportunities be identified first as a basis for comparison later. Consequently, the process followed for sample member A was repeated for sample members B, C, D and E with their respective data as well, to build individual mathematics teaching practice profiles for B, C, D and E.

The next phase entails the compilation of a generic mathematics teaching practice profile for the sample, to identify shared improvement opportunities.

Compiling a generic mathematics teaching practice profile

To compile a generic profile of the sample and identify shared improvement opportunities, the improvement opportunities in the individual profiles of respective sample members were tabularised and compared. A generic sample profile reflecting the strengths and shared improvement opportunities in the practice of the sample was compiled. As stated in the previous subsection, the identification of shared improvement opportunities is vital for this research as it is the prerequisite for intervention design. Chapter 4 will focus on building the individual mathematics teaching practice profiles of the sample members, whilst Chapter 5 focuses on building the generic profile as part of theory building.

Interpreting data from the second classroom observation tool

As explained under data collection, sections of the classroom observation tool was used a second time. This second application of the revised classroom observation took place after the sample had attended an intervention to address the shared improvement opportunities identified. Only the relevant sections of the observation tool, which dealt with the improvement opportunities addressed in the intervention, were used. During data interpretation the data obtained from each sample member by means of the second set of observations were compared with the individual profile of the member to determine the impact that the intervention had in addressing the shared improvement opportunities.

2.9 Conclusion

This concludes the chapter dealing with the research design. The conceptual framework, the researcher's cosmological, ontological and epistemological standpoints as well as the research paradigm embedded in the research methodology were discussed. The research problem and research questions, as well as the methodology to be followed and ways in which the knowledge acquired through the research will be communicated, also received attention. Chapter 3 focuses on identifying a set of good practice indicators and evidences.

CHAPTER 3 GOOD MATHEMATICS PRACTICE INDICATORS AND - EVIDENCES

3.1 Introduction

A literature review was undertaken to investigate what is regarded internationally as good mathematics practices for the primary school. The aim was to use the data on good mathematics teaching practices as a yardstick against which the own mathematics practices of each sample member could be measured. Best mathematics practices, however, cannot merely be generalized. The specific teaching- and learning contexts which vary across and within countries need to be taken into account (Walshaw & Anthony 2008; Ellerton 2003; Clarke 2002; Cockcroft 1982). Notwithstanding that, there is much to learn about mathematics teaching practices that have been applied to good effect in other countries. Considering the best mathematics teaching practices of other countries can stimulate a critical reflection of one's own mathematics teaching practices. This critical reflection, in turn, may lead to the realisation that there are mathematics practices that work more effectively than the ones routinely employed locally. Knowledge about and consideration of mathematics practices applied effectively in other countries can enhance local mathematics practices (Clarke 2002). According to Anderson-Levitt (2002) good practices or ideas about mathematics teaching borrowed, are transformed into something new. This holds true whether the practices are borrowed from another country or from another school in the same town. As the borrowed practices are implemented by a different set of people under different circumstances they cannot stay exactly as they were. The value of comparing teaching practices lies in the contrasts discovered. It is when we see how the way in which we teach mathematics differs from how mathematics is taught in the broader universe that we understand our own teaching better and when we will attempt to make improvements. LeTrende, Baker, Akiba, Goesling and Wiseman (2002) agree with Anderson-Levitt (2002) that cross-national research is important. According to LeTrende et al. (2002) hidden levels of national or transnational culture become more visible when the teaching practices of different countries are compared.

Twenty-four good mathematics teaching practices were identified through the initial literature review. This number was reduced to eleven indicators with eighty-six evidences of good mathematics teaching practice following data triangulation and a second literature review. The indicators and evidences of good mathematics teaching practice were used to develop the data collection tools. Data collected via the collection tools were analysed and interpreted to compile the individual mathematics teaching practice profile of each sample member.

3.2 Problems in defining good mathematics practice

According to the literature (Anthony & Walshaw 2007; Marsigit 2006; Lim 2006; Watson 2004 and Askew 2000) researchers have not yet reached consensus about a single definition for good mathematics practice. The lack of consensus is attributed to different views as to what constitutes good mathematics practice. The different views can be related to cultural differences because countries and even cultural groupings within the same country have varied opinions on the goals of education and what is important for good mathematics teaching (Mosvold 2008; Clarke 2002). Culture impacts on mathematics teaching practices; thus the influence of culture cannot be ignored when investigating good mathematics practices.

The next subsection will explain how culture influences what is regarded as good mathematics practices.

3.2.1 The impact of culture

Mathematics teaching is a cultural activity. As such, what is regarded as good mathematics practice in one culture does not necessarily hold true for other cultures (Andrews 2007; Anthony & Walshaw 2007; Leung 2006; Lim 2006; Broadfoot 2003; Clarke 2002; Askew 2000). Different parties involved in a culture shape the notion of what good mathematics practices are. As such, the notion of good mathematics practices has become loaded with value judgement. Cultural differences have contributed largely towards the problems experienced with producing a universal definition of good mathematics practice. Leung (2006), Prediger (2005) as well as LeTrende, Baker, Akiba, Goesling and Wiseman (2001) stress that teaching practices cannot simply be transplanted from high achieving countries to low achieving ones. It implies that a country struggling with mathematics teaching cannot simply adopt the practices of another country and expect immediate success. A direct transplant of mathematics practices from one country to another is problematic as a result of their different cultures. The underlying cultural values of their country influence both teachers and their teaching practices, and cultural differences cannot be negated.

The literature review revealed that a number of studies had been undertaken to compare the mathematics teaching practices of different countries. The findings of some of these studies are relevant to the discussion on the role that culture plays in mathematics teaching. A discussion on some of the studies undertaken and the impact of their findings follows.

England and France

Comparative studies were undertaken in 1993 and 1999 respectively to compare mathematics teaching in English- and French classrooms. The studies revealed that pedagogical differences that exist between the English and the French are the result of factors outside the classroom, like differences in cultural views. It was found that English teachers have a high regard for the needs of individual learners (Askew 2000; Broadfoot, Osborn, Planel, & Sharpe 2000). The French place a high societal value on intellectual endeavour and French learners make a clear distinction between work and play (Askew 2000). When they are in class, the French learners focus on the tasks at hand. French teachers were found to be less concerned with making the work interesting than English teachers who motivate their learners through interesting activities. French teachers value instruction and there is direct interaction between teachers and learners without much interaction between the learners themselves. The goal of the lesson is that learners should master the learning material being taught. English teachers allow learners more freedom to talk among themselves and to partake in problem-solving exercises. There is less teacher-learner interaction than is the case in French classrooms (Broadfoot, Osborn, Planel, & Sharpe 2000). The studies revealed that English teachers applied learner-focused teaching practices to gain the interest of their learners. French teachers took the commitment of their learners to learn for granted and focused their teaching practices on achieving the learning outcomes rather than on making lessons interesting.

Flemish Belgium, England, Hungary and Spain

Andrews (2007) as well as Andrews, Hatch and Sayers (2005) reported on a comparative study of mathematics teaching conducted in Flemish Belgium, England, Hungary and Spain. The study revealed that Flemish teachers in the sample placed considerably less importance on problem solving than teachers from the other participating countries do. Spanish mathematics teachers, in particular, placed emphasis on problem solving and included a large portion of problem solving activities in their lessons. English teachers did not regard the structural properties of mathematics as important. English learners were rarely offered opportunities to examine mathematics structures or to make links between different mathematics concepts or entities. This is in direct contradiction to Hungarian mathematics teachers who regarded mathematical elegance as important and focused on mathematical structures and efficiency during mathematics lessons.

According to Andrews, Hatch and Sayers (2005), the differences observed in this study are in line with the findings of previous studies and the differences can be related to national behavioural patterns. This supports the notion that culture and cultural differences play an important role in mathematics teaching. Andrews (2007) stressed that although it is important to examine what mathematics teachers do in their classrooms this alone is not enough. To obtain a complete picture of the mathematics teaching practices of a teacher it is also important to determine why the teacher employs certain mathematics practices and negates others.

England and Japan

Askew (2000: 47) refers to a study by Whitburn (2000) which compared English and Japanese mathematics teaching. The study found important differences in perception between the Japanese- and the English society that impact on the respective mathematics practices of the teachers. The focus of the English teachers on meeting the needs of the individual learner in the classroom is a manifestation of their cultural perception. The Japanese, in contrast, emphasize the importance of working together in groups. Consequently, people are encouraged to work together as members in a group and to make a concerted effort to help other group members also to move forward. This enhances the chance of group success. This focus on group work as part of the Japanese culture demonstrates a difference in the balance of attention that is paid to the development of the individual as opposed to the group by the two countries. Also, in Japan perseverance is highly regarded whilst innate ability is seen as less important. Japanese teachers thus deal with the effects of ability and effort in children's learning in a different manner to the way English teachers do. As a result of the cultural differences discussed, Japanese teachers will apply mathematics teaching practices in their classrooms that are different to the practices applied by their English counterparts.

The following subsection reflects on the relevance of doing comparative studies in mathematics teaching.

Reflecting on the role of comparative studies

According to Clarke (2002) international comparative research should not only document cultural differences, but should strive to accommodate the differences identified. Cultural differences do not only impact on the way mathematics is taught in different countries. The culture of a classroom and the teaching that takes place in that classroom can differ widely within the same country and even within the same school. The value of comparative studies lies in the knowledge and insight gained in mathematical practices that are employed effectively by others. Zhang (1998) stated that we can continue to learn from comparative research and that we can absorb and where applicable, adopt useful teaching practices that have been identified.

The aforementioned discussion on culture and its impact highlighted that there is a large and diverse number of role players who compound the difficulties in defining good mathematics practices. There is a need to develop consensus on what constitutes good mathematics practices.

3.3 Consensus on good mathematics practices

Notwithstanding the difficulties experienced to define good mathematics practice as a term, there is indeed some consensus amongst role players from various countries. The first Asia-Pacific Economic Cooperation (APEC) Tsukuba International Conference on Innovative Teaching Mathematics through Lesson Study was held in Tokyo from 15 to 20 January 2006. Participants and observers from Australia, Chile, China, Hong Kong, Indonesia, Japan, Korea, Malaysia, the Philippines, Singapore, Thailand, USA and Vietnam agreed that good mathematics practices must be reflected in the outcomes (APEC 2006:308). The question that immediately came to mind was: “What kind of outcomes?” According to Anthony and Walshaw (2007) outcomes should be threefold. There should be achievement outcomes, social outcomes and cultural outcomes. Outcomes relating to affect, behaviour, communication and participation are equally important. Watson (2004) was of the opinion that a pedagogical approach can be perceived as effective when it achieves its purpose by promoting learning and development.

For the purpose of this research study, the decision was made to identify good mathematics teaching practices that have contributed towards meeting the desired outcomes in terms of achievement, affect, behaviour, communication and participation across cultural divides. This is considered to be more in line with the research objectives set than in finding a single definition for good mathematics practice. Identifying mathematics practices that reflect positive outcomes over a range of countries may result in shared good practices being determined. Shared good practices will provide ground for comparing teaching practices without having to adopt the good practices from one country directly into another. It has already been established that wholesale adoption of the teaching practices of a country is not advisable in the light of the cultural differences (Lim 2007). Identifying good mathematics practice applied across cultural barriers is supported by Anthony and Walshaw (2007). They said that there is a set of common, underlying pedagogical principles that tend to make a difference for diverse learners when teachers base their practice thereupon. This underlines the importance of identifying mathematics practices that have resulted in the desired outcomes being obtained in a number of countries. It is argued that a country like South Africa which is experiencing difficulties with mathematics teaching stands to benefit from the availability of a set of good mathematics teaching practices. The next section will focus on identifying mathematics practices that have been applied with positive outcomes in a range of other countries.

3.4 Mathematics practices in different countries

According to Askew (2000) research studies undertaken in the 1970s and 1980s concentrated on pedagogy at the enterprise level. More recent studies have, however, turned the research focus to examining the teaching of specific subject areas and even topics within the subject areas. The literature review undertaken in this research study concurs with Askew's view. Much recent and current international research has focused on mathematics teaching. Primary school mathematics teaching in particular received attention. Appendix C (i) reflects a list of sources who commented on the mathematics teaching practices applied by a number of countries. The countries linked to the different sources are also indicated. A coding process, described in the following section, was followed to compare the teaching practices. Scanned copies of the tables used during coding are attached as Appendix C (ii). The process resulted in twenty-four good mathematics practices for the primary school being identified [Appendix C (iii) refers].

3.5 Coding to determine twenty-four good mathematics practices

During coding, literature on mathematics teaching practices for the primary school by twenty-five sources spanning thirteen countries were perused. Numbers one to twenty-five were randomly allocated to the sources for coding purposes. A table containing twenty-five columns was drawn up. The good practices, as identified by each source, were written in the column of the corresponding number. The column with the most good mathematics practices were used as a starting point, as this provided the largest scope for comparison. The practices listed in the other twenty-four columns were compared to the comparative column in turn. Where a practice was found lacking in the comparative column, it was added to the bottom of the comparative column before comparing the practices listed in the next column. This process was followed until the practices in all twenty-four columns had been compared to the continuously updated comparative column.

A practice was regarded to be a shared good practice when it was listed under more than one source. A second table was designed to document the shared good mathematics practices identified during coding. The shared practices were written in the left hand column of the table with the numbers of the sources, as used in the comparative table that share the practice in the right hand column. The wording of the shared practices often differed although they had the same meaning. This is because the sources had different authors who expressed themselves in their own words. Consequently, the shared practices in each column had to be scrutinised once again after identification. This was done to compile an inclusive description of each shared good practice. The coding process culminated in the identification of twenty-four shared good mathematics teaching practices. Once the shared practices had been identified, the numbers used for coding purposes were replaced with the names of the authors and linked to the countries where the good practices were applied. The twenty-four good mathematics practices as well as the countries in which each of the practices are applied with positive outcomes, are attached as Appendix C (iii).

The twenty-four good mathematics practices identified through the research are reflected in Table 3.1 It should be noted that each of the practices was allocated a number ranging from one to twenty-four. The allocation was done randomly and is no indication of importance. Numbering was done to ease future comparisons during data triangulation and validation.

Table 3.1: The twenty four good mathematics practice indicators

Good mathematics teaching practices	Number for comparative purposes
Planning focuses on aspects of the content that must be highlighted and on organizing and orchestrating the work to suit students with varied levels of expertise.	1.
A variety of strategies and approaches for teaching mathematical concepts and for solving mathematical problems are discussed and implemented.	2.
Mathematical thinking is encouraged by accommodating, addressing and discussing students' ways of thinking about concepts and mathematical problems.	3.
Student misconceptions or less efficient responses are addressed and discussed and mistakes are treated as learning opportunities.	4.
Aspects that affect the complexity of a concept or mathematical problem are identified, explained and discussed to assist students in developing good thinking skills.	5.
Students are actively engaged in the learning process and given the opportunity to develop abilities to reason logically and to extend their own knowledge.	6.
Different resources are used to support mathematics teaching and flexible teaching methods are implemented.	7.
It is explained how topics fit into the curriculum, why certain content is included and how it can be used.	8.
The performance and progress of students are assessed during the processes of learning through activities aimed at problem solving, applying mathematical thinking, creative work and exercises with variation.	9.
An intellectual environment is created in the classroom where serious mathematical thinking is the norm.	10.
Connections are made between concepts and topics, lesson content and prior knowledge/problems or statements made earlier in the lesson.	11.
A goal for students' learning is set at the start of the lesson, with the teacher challenging and supporting the students during the lesson.	12.
Different strategies for engaging students are used and student needs are met through differentiation of work.	13.

Teaching is aimed at fostering a positive attitude towards learning mathematics and at students gaining confidence in solving problems.	14.
Opportunities for self-discovery are created by encouraging the use of multiple solutions to problems.	15.
Learning mathematics is related to the world of the child by emphasizing the application of mathematics.	16.
Individual, whole-class and group work are included appropriately to assist students in learning both individually and co-operatively.	17.
Teachers and students participate as a community of learners and the word "we" is used.	18.
Positive and well-timed teacher intervention is applied to give direction during lessons.	19.
Values including respect for others, tolerance, fairness, caring, diligence, non-racism and generosity are modelled by the teacher and expected from the students.	20.
Games and activities that emphasize mathematical thinking are applied and games are used for exercising mental calculation.	21.
Appropriate time is allowed for instruction, demonstration, questioning, exploring concepts and practice.	22.
Tasks that are mathematically challenging are included to demonstrate the high expectation of student learning.	23.
Homework and exercises are used effectively for consolidation and practice and completed homework and exercises are discussed in class.	24.

A description of measures implemented to validate the twenty-four good mathematics practices follow. Validation is done through data triangulation with the Baker and Chick framework (2006), and through a comparison with the daily teaching activities for Grade four to six mathematics teaching according to the Foundations for Learning Campaign.

3.6 Validating the twenty-four good mathematics practices

Data triangulation was applied to validate the twenty-four good mathematics practices for the primary school. The framework designed by Baker and Chick (2006) for analysing Pedagogical Content Knowledge [PCK (Shulman 1986)] was selected for data triangulation. This framework is based upon another developed by Chick, Baker, Pham and Cheng (2006). Both Brown and McNamara (2000) and Baker and Chick (2006) conceded that teachers have to draw upon a vast range of knowledge when teaching. Knowledge of content, learners, the curriculum, pedagogy and psychology have to be considered. To examine the particular knowledge required by mathematics teachers, they drew on Shulman’s definition of PCK (Shulman 1986). Shulman defined PCK as the blending of content and pedagogy into an understanding of how topics, problems and issues are organized, adapted and presented for instruction to meet the diverse interests and abilities of learners. Brown and McNamara (2000) explained that PCK is situationally and experientially grounded in and constrained by classroom practice. Classroom practice stands at the heart of PCK and it is knowledge, values and epistemological beliefs rather than initial teacher training programmes that result in PCK. Since 1987, many aspects of knowledge within PCK have been identified. These aspects of knowledge serve as criteria for good mathematics teaching practices.

3.6.1 Reasons for selecting Baker and Chick’s framework

The framework designed by Baker and Chick was selected because it identified key components of PCK for teaching primary school mathematics (Baker & Chick 2006). The framework provides clear directives on mathematics practices for the primary school. It has also been applied in case studies for classroom analysis of primary mathematics teaching (Chick 2007; Chick & Harris 2007).

During data triangulation, the mathematics practices contained in the Baker and Chick framework (Baker & Chick 2006) were compared to the good mathematics practices identified through the literature (Table 3.1). Corresponding trends in the mathematics practices in the framework on the one hand and the good mathematics practices on the other, were identified and listed. In instances where the comparison led to the identification of differences between the framework and the good mathematics practices, those differences were noted.

3.6.2 Correspondences between the Baker and Chick framework and good mathematics practices

Twelve corresponding mathematics practices have been identified as a result of the data triangulation between the Baker and Chick framework and the good mathematics practices identified through the literature review (Table 3.2 refers). It should be noted that two of the practices of Baker and Chick jointly correspond to the same good practice identified through the research. The two practices of Baker and Chick read “It is discussed how topics fit into the curriculum” and “Reasons for content being included in the curriculum or how it may be used are discussed”. Both corresponded with the good practice “It is explained how topics fit into the curriculum, why certain content is included and how it can be used.”

Data triangulation also revealed three mathematics practices which were included in the Baker and Chick framework (2006), but did not surface in the good mathematics practices identified through the research. They are:

- describe or demonstrate ways to model or illustrate a concept (can include materials or diagrams);
- identify critical mathematical components within a concept that are fundamental for understanding and applying that concept; and
- implement generic classroom practices.

It is argued that the first two practices listed above might be linked to the good mathematics practice that reads “A variety of strategies and approaches for teaching mathematical concepts and for solving mathematical problems are discussed and implemented”. The third practice, “implement generic classroom practices” is very broad and vague and does not point to a specific practice. As such, it will be ignored for the purpose of this research. Data triangulation revealed a substantial correspondence between the teaching practices in the Baker and Chick framework and the good mathematics practices identified through the research. Taking into account that thirteen out of a possible sixteen practices in the Baker and Chick framework matched the good practices of the literature study, it is accepted that all the good mathematics practices identified through this literature study are valid.

Table 3.2: Correspondences in mathematics practices between the Baker and Chick framework and good mathematics practices

Mathematics practices in the Baker and Chick Framework (2006)	Good mathematics practices according to the literature study
Different strategies or approaches for teaching a mathematical concept are used and discussed.	A variety of strategies and approaches for teaching mathematical concepts and for solving mathematical problems are discussed and implemented.
Teachers address and discuss students' ways of thinking about a concept or typical levels of understanding.	Mathematical thinking is encouraged by accommodating, addressing and discussing students' ways of thinking about concepts and mathematical problems.
Student misconceptions about a concept are discussed or addressed.	Student misconceptions or less efficient responses are addressed and discussed and mistakes are treated as learning opportunities.
Aspects of the task that affect its complexity are identified.	Aspects that affect the complexity of a concept or mathematical problem are identified, explained and discussed to assist students in developing good thinking skills.
Resources available to support teaching are discussed /used.	Different resources are used to support mathematics teaching and flexible teaching methods are implemented.
It is discussed how topics fit into the curriculum. Reasons for content being included in the curriculum or how it might be used are discussed.	It is explained how topics fit into the curriculum, why certain content is included and how it can be used.
Opportunities are created for students to exhibit deep and thorough conceptual understanding of identified aspects of mathematics.	Planning focuses on aspects of the content that must be highlighted and on organizing and orchestrating the work to suit students of varied levels of expertise.
The importance of making connections between concepts and topics, including interdependence of concepts are stressed.	Connections are made between concepts and topics, lesson content and prior knowledge/problems or statements made earlier in the lesson.
A goal for students' learning (which may or may not be related to specific mathematics content) is described;	Lessons are well-planned and a goal for students' learning is set at the start of the lesson, with the teacher challenging and supporting the students during the lesson.

Skills for solving mathematical problems (conceptual understanding need not be evident) are displayed.	Teaching is aimed at fostering a positive attitude towards learning mathematics and at students gaining confidence in solving problems.
Different strategies for engaging students are used.	Different strategies for engaging students are used and students' needs are met through differentiation of work.
A method/different methods for solving a maths problem are demonstrated.	Opportunities for self-discovery are created by encouraging the use of multiple solutions to problems.

In the next section the twenty-four good mathematics teaching practices identified through the research will be compared to the Foundations for Learning Campaign that has been implemented by the South African Department of Education (DoE 2008e). The daily teaching activities prescribed for mathematics teaching for Grades four to six in particular will be under investigation, as this is the focus area of the research study.

3.6.3 Comparing the good mathematics teaching practices with the daily teaching activities in the Foundations for Learning Campaign

The Foundations for Learning Campaign (DoE 2008e:19) highlights the following daily teacher activities as important for mathematics teaching from Grades four to six:

- developing the mental skills of learners and providing opportunities for learners to practice using their number facts both orally and mentally;
- asking questions that focus both on revising skills learned in previous lessons and on supporting the introduction of the lesson of the day;
- reviewing and correcting homework from the previous day;
- introducing the concept of the day's lesson on the board with learners listening and learning from the teacher's example;
- providing learners the opportunity to practice similar examples, going over the examples with the learners and clearing up confusion that learners may have;
- giving further examples to more able learners, whilst assisting learners who need help;

- practicing problem solving through interactive group or pair work where learners engage with a problem or challenging investigation and can apply what they have learnt in the lesson;
- encouraging learners to try out different ways to solve problems;
- giving learners the opportunity to share and explain their thinking, methods and answers;
- including questions requiring higher order thinking and the solving of word problems in particular;
- giving and explaining homework that may include doing corrections of the previous day's work as well as practicing what they have done in the day's lesson; and
- including problems with real life contexts for learners to solve.

Drawing a parallel between the good mathematics practices for the primary school identified and the teaching activities prescribed by the South African Foundation for Learning Campaign revealed that all the activities prescribed by the Foundations for Learning Campaign are in line with the good mathematics practices identified. There are good mathematics practices which are not included as daily teaching activities in the Foundations for Learning Campaign. However, these practices should be applied by mathematics teachers as part of their normal teaching practice. The practices not mentioned in the daily teaching activities are:

- The performance and progress of students are assessed during the processes of learning through activities aimed at problem solving, applying mathematical thinking, creative work and exercises with variation;
- Different strategies for engaging students are used and students' needs are met through differentiation of work;
- Teachers and students participate as a community of learners and the word "we" is used;
- Positive and well-timed teacher intervention is applied to give direction during lessons; and
- Values including respect for others, tolerance, fairness, caring, diligence, non-racism and generosity are modelled by the teacher and expected from the students.

The Foundations for Learning Campaign in its daily teaching activities for Grade four to Grade six mathematics teaching are thus in line with the good mathematics practices identified through the literature study.

The next section will focus on identifying measurable evidences for the twenty-four good mathematics practices.

3.7 Indicators and evidences of good mathematics practice

The good mathematics practices identified through the literature study were needed in the design of the data collection tools. To use the good mathematics practices in data collection design, measurable outcomes relating to each of the good mathematics teaching practices were required. It was decided that the terms indicators and evidences of good practice would be used henceforth. Indicators of good practice replaced the formerly called good mathematics practices. Evidences are used to refer to the measurable outcomes that are related to the indicators. Literature supporting the twenty-four good mathematics practice indicators was again reviewed to identify the evidences that support each of the good practice indicators.

3.7.1 Identifying evidences of good mathematics practice

During the second literature review it was found that some indicators of good practice could be regarded more as evidence supporting another indicator than as an indicator in its own right. In such cases the list of good practice indicators was amended accordingly. The second literature review and categorization process resulted in a lesser count of eleven indicators of good practice. Eighty-six evidences of good practice were identified [Appendix C (iv)] refers.

3.7.2 Tabularising the eleven indicators and corresponding good mathematics practices

A table, Table 3.3 was drawn up to explain how the twenty-four good mathematics practices correspond with the lesser count of eleven indicators of good practice.

Table 3.3: The eleven good practice indicators and the numbers of their corresponding good mathematics practices

Eleven good practice indicators	**Numbers of corresponding good mathematics practices**
Lessons are well-planned with the learning objectives and ability of the learners in mind.	1
A variety of strategies for teaching mathematical concepts and for solving mathematical problems are applied.	2; 7(b); 17
Learners are actively engaged in the learning process by encouraging mathematical thinking.	3; 5; 6; 19
A wide range of tools and activities are used to support mathematics teaching.	7(a); 21
An intellectual environment is created in the classroom where serious mathematical thinking is the norm.	10
The importance of connections in mathematics and between mathematics and real life is highlighted.	8; 11; 16
Teaching is aimed at fostering a positive attitude towards learning mathematics and at students gaining confidence in solving problems.	4; 12; 14; 15; 18
The needs of learners are met through differentiation of work.	13; 23
Positive values are modelled by the teacher and expected from the learners.	20
Good time management is practised by teachers and learners alike.	22
The prior knowledge of students, their performance and progress during the lesson and the knowledge gained/concepts formed during the lesson are assessed.	9; 24

In the table the eleven indicators of good practice have been listed in no particular order in the left hand column. The numbers that had been allocated to the good mathematics teaching practices in Table 3.1 were used to distinguish between practices. The number/s of good teaching practices that correspond with an indicator appears in the right hand column in the same row as the corresponding indicator. The good mathematics practice, identified by the number seven in Table 3.1, was split into an (a) and a (b) part as it was found to correspond with two different indicators. Table 3.3 verifies that none of the twenty-four good mathematics practices have been disregarded during the process of working with the data.

3.7.3 Categorising the indicators and evidences

The indicators of good practice and their corresponding evidences were categorized according to three categories. The categories represented the three lesson stages, namely Planning, Teaching and Assessment. A table that displays the indicators, corresponding evidences and supporting literature was drawn up. The table is attached as Appendix C (iv).

This concludes the identification of the eleven indicators of good practice and their corresponding evidences. The indicators and evidences will henceforth be used to design the data collection tools.

3.8 The development of data collection tools

With the eleven indicators and eighty-six evidences of good mathematics practice for the primary school identified, the data collection tools could be designed. The data collection tools comprised three questionnaires and a classroom observation tool. The eleven indicators of good mathematics practice were used to design the first questionnaire. The eighty-six evidences of good practice were used for the second questionnaire. In the third questionnaire sample members were given a mathematical problem to teach to their learners. The questionnaires created opportunities for sample members to reflect on their individual mathematics practices and provide relevant data on their practices. Self-reflection is supported by Pritchard and McDiarmid (2006) who said that reflecting on one's own practice is important both for effective teaching and for professional development. The classroom observation tool in its design incorporated both the indicators and evidences of good practice. The data collection tools are of cardinal importance to the research as the data collected were analysed and interpreted to compile individual mathematics practice profiles of each sample member. The compilation of the data collection tools is discussed in the following subsections.

3.8.1 Compilation of Questionnaire 1 for sample members

Questionnaire 1 was compiled to gather first hand data on the mathematics practices applied by the sample members in their respective classrooms. It has already been discussed at length how the eleven indicators of good mathematics practice were identified. The indicators will merely be used in the questionnaire without further discussion thereabout.

In Questionnaire 1 sample members were requested to reply to three questions regarding each of the indicators. The indicators had been divided into three sections. Section A contained the Planning indicators, section B the Teaching indicators and section C the Assessment indicators. The three questions were:

- Do you implement the indicator of good practice in your mathematics classroom?
- If Yes, please indicate how you implement the indicator in your mathematics teaching.
- Explain why you think it is important/not important to implement the indicator when teaching mathematics.

The aim of the questions was to determine (a) which indicators of good mathematics practice are being applied by each sample member, (b) how the good practice indicators are implemented and (c) the reasons why each sample member implements certain indicators and neglects others. Questionnaire 1 is attached as Appendix D (ii).

It should be noted that Questionnaire 2 was distributed to sample members only after all the responses to Questionnaire 1 had been returned to the researcher. This was done to prevent the responses of the sample members to Questionnaire 1 becoming contaminated by exposure to data contained in Questionnaire 2.

3.8.2 Composition of Questionnaire 2 for sample members

The process followed to identify the evidences of good practice was discussed at length in the previous section and the evidences are applied in this questionnaire without further discussion. Questionnaire 2 was designed in the form of a table. The eighty-six evidences of good mathematics practice were entered in the left hand column. Columns with Yes and No as heading followed. Sample members had to indicate by means of a tick in either the Yes or No column if they implemented the evidence in the corresponding row or not. When the answer was Yes sample members had to use the column provided on the right side of the table to explain how they know that they implement the evidence in their teaching. Questionnaire 2 is attached as Appendix D (iii).

Questionnaire 3 was also sent to the sample only after all the responses to Questionnaire 2 had been received. Questionnaire 3 focused on the application of mathematics teaching practices by the sample.

Composition of Questionnaire 3: The application of good mathematics practice by Grade 4 mathematics teachers

Questionnaire 3 requested sample members to explain their approach when teaching learners how to solve a specific mathematics problem. Sample members had to explain how they would teach their learners using good mathematics teaching practices. The sample members had to use three guidelines to explain their approach, namely

1. How would you teach to get your learners to be able to solve the mathematics problem?
2. What are the mathematical principles underlying the mathematics problem?
3. What do your learners have to know and be able to do to solve the mathematics problem?

Questionnaire 3 is attached as Appendix D (iv).

This concludes the discussion on the development of the three questionnaires that were used to collect data on the mathematics teaching practices of the sample. The next subsection discusses the development of the classroom observation tool.

3.8.3 The development of the classroom observation tool

A classroom observation tool was developed for use by the researcher during observations in the respective classrooms of the sample members. The aim was to collect first hand data on the mathematics practices of each sample member. The observations were done during mathematics lessons for Grade four learners. Both the indicators and the associated evidences of good practice were used to design the classroom observation tool. The observation tool was designed in the form of a table. The three categories Planning, Teaching and Assessment were used as subheadings. The table consisted of seven columns. The indicators were on the far left with the evidences adjacent to the corresponding indicators. Columns for Yes and No were in the centre. A Remarks column was added to the right of the Yes/No columns. The two columns on the far right were to indicate whether the evidences observed were visual, auditory or both.

During observations a tick would be made in the Yes column of the row corresponding to the evidence observed. The Remarks column was used to add supporting comments. Either the Visual or Auditory column, or both, was ticked to indicate the nature of the evidence observed. For evidences that went unobserved, the No column was ticked. The Remarks column was used to comment on negative practices, if observed. The Remarks column added value to the observations by allowing the observer not only to record whether or not evidences were observed, but also to add supporting comments. In addition to the observer observing the lesson in real-time, audio tapes were made of the lessons. This allowed the researcher to listen to the lessons again to confirm observations made and to gain additional evidence.

The classroom observation tool is attached as Appendix D (v).

3.9 Conclusion

Chapter 3 discussed how the indicators and evidences of good mathematics teaching practice for the primary school were identified. It was also explained how the indicators and evidences were applied in the design of data collection tools. The next chapter focuses on analysing and interpreting the data collected via the questionnaires and the classroom observation tool with the aim of building individual mathematics profiles of each sample member.

CHAPTER 4 BUILDING INDIVIDUAL PROFILES

4.1 Introduction

This chapter explains the process that was followed to build the individual mathematics practices profiles of the sample members. Data collected via three questionnaires and an observation tool had to be analysed and interpreted for each sample member. As such, building the individual profiles was a lengthy process. Chapter 4 focuses on building a single individual profile, the profile of sample member A, as an exemplar. Similar processes were followed to build the individual profiles of sample members B-E and they are included as Appendices F-I respectively.

4.2 Building profile 1 of sample member A

The process to build the individual mathematics practices profile of sample member A started with the analysis and interpretation of the data collected via Questionnaire 1.

4.2.1 Analysing and interpreting data from Questionnaire 1

On return, the completed Questionnaire 1 contained the responses of the sample member to eleven questions on own mathematics teaching practices. The questions related to the eleven indicators of good practice that had been identified through the research. The steps to reflect, analyse and interpret the data received in response to each of the questions follow. Firstly, the question asked in the questionnaire is provided in boldface. Secondly, the verbatim response of the sample member is given. This is followed by a sub-question, in boldface, requesting further explanation from the sample member. The answer of the sample member is given verbatim. Hereafter, the data in both verbatim responses is analysed and interpreted. The evidences of good practice that corresponded to the indicator addressed by the particular question were applied as a yardstick. Based on the comparison a list of possible strengths and possible improvement opportunities with regard to the application of the indicator by the sample member in question is compiled. The possible strengths represented evidences of good practice that were noted in the teaching practices of the sample member. The possible improvement opportunities represented evidences that were possibly lacking in the teaching practices of the sample member.

As there were eleven questions in Questionnaire 1, the result of the data analysis and interpretation was eleven lists reflecting possible strengths and another eleven lists reflecting possible improvement opportunities. The eleven lists reflecting possible strengths were combined to form one comprehensive list. This list reflected the possible strengths in the teaching practices of sample member A. In the same manner, the eleven lists of possible improvement opportunities were combined to form one comprehensive list. This second list reflected the possible improvement opportunities in the teaching practices of sample member A. By having identified both the possible strengths and the possible improvement opportunities in the teaching practices of sample member A, the first individual mathematics practices profile for sample member A had been built.

The process explained is henceforth applied to sample member A.

Question 1: Do you plan lessons with the learning objectives and ability of the learners in mind?

"Yes, I plan lessons with the learning objectives and ability of the learners in mind. An example is planning a lesson on expanded notation. I check previous knowledge about place value showing them flashcards with numbers, e.g. 2<u>1</u>, 2<u>3</u>7 and ask them what is the value of the underlined digits 1 – units; 3 – Tens. The learners must know how to split numbers according to their groups, e.g. 21 = 20 + 1 and 237 = 200 + 30 +7 so the learners would be able to expand the numbers."

Why is this important during lesson planning?

"When you make a lesson plan you must have objectives at the end of a lesson. So I must work together with the learners. They must be able to show their ability. Atmosphere in class must be conducive."

Data analysis and interpretation

Sample member A teaches according to the prescribed learning objectives for Grade 4. Planning is done with the ability of the learners in mind. Sample member A made sure that the learners possess the background knowledge required to deal with new lesson material. Background knowledge is linked to the new knowledge that sample member A wants to expose during the lesson. The learning objectives are explained to the learners in everyday language. The sample member wants to work with the learners in a class atmosphere that is conducive to learning mathematics. Sample member A indicated that learners must be provided with the opportunity to show their ability, which relates to seatwork assignments.

The comparison with the evidences of good practice that correspond with the indicator in question revealed that sample member A complied with seven evidences of good practice. They have been included as possible strengths in the profile of sample member A. The seven evidences are:

- focusing on aspects of the content that must be highlighted during the lesson;
- organizing and orchestrating work to suit learners of varied levels;
- knowing the learning material well and structuring the unveiling of the knowledge to promote learner understanding;
- helping the learners to get ready to learn by explaining the learning objectives in understandable everyday language;
- having an overall plan for the lesson and ways to make it happen;
- making connections between concepts and topics, lesson content and prior knowledge;
- preparing seatwork assignments that provide variety and challenge.

There were two evidences that were found lacking in the teaching practice of sample member A. They have been included as possible improvement opportunities in the profile of sample member A:

- identifying complex aspects in the lesson and putting plans in place to deal with expected difficulties;
- preparing probing questions to stimulate mathematical thinking.

Question 2: Do you apply a variety of strategies for teaching mathematical concepts and for solving mathematical problems?

"Yes, I apply a variety of strategies for teaching mathematical concepts and for solving mathematical problems in my mathematics classroom. I use different strategies according to the lesson and abilities. I pay attention to the slow learners and good ones giving them more work, but in my situation at my school there are 84 learners in one class Grade 4 and 5 so it is difficult for me, but I try my best."

Why do you apply a variety of strategies?

"Because in my class I have slow learners. I must pay attention to them as individuals, but I don't have more time for that because other educator step inside for his/her area."

Data analysis and interpretation

Sample member A selected different strategies based on the lesson contents and the' abilities of the learners. For slow learners alternate approaches were used to help them understand. The sample member acknowledged that time is a problem when striving to meet the different needs of the learners.

The comparison with the evidences of good practice that correspond with the indicator in question revealed that sample member A complied with five evidences of good practice. They have been included as possible strengths in the profile of sample member A. The evidences are:

- not only using whole class teaching, but also allowing for individual – and group work;
- using alternative approaches of explaining a concept which some learners have difficulty grasping;
- giving learners the opportunity for guided- and individual practice of new concepts and skills;
- using concrete illustrations to demonstrate how abstract mathematical concepts can be used to solve problems;
- using practical work and hands-on activities in addition to exposition.

There were three evidences that were found lacking in the teaching practice of sample member A. They have been included as possible improvement opportunities in the profile of sample member a:

- giving clear written and verbal instructions;

- encouraging finding multiple solutions to address a mathematical problem; and
- using innovative and creative activities like investigation projects and solving daily life problems.

Question 3: Do you engage learners actively and encourage mathematical thinking?

“Yes. Grouping them, asking questions, giving them assignments, writing tests, class work, homework and projects.”

Why is it important for you to engage learners actively and encourage mathematical thinking?

“By using these strategies I want to check their knowledge, ability and understanding. I also develop mathematical skills so that they can apply the skills in real life.”

Data analysis and interpretation

Sample member A actively engages the learners through activities like assignments and projects. The sample member wants to check the knowledge of the learners, which is related to evidences of good practice like emphasizing mental computation skills and encouraging learners to use skills like visualization. The sample member also encourages the learners to try to relate what they learn to their previous knowledge. Sample member A also wants to check the ability of the learners and their understanding. This is included in the teaching practice described as reacting to the misconceptions of the learners or to their less efficient responses by giving them guidance and directing them towards the desired response.

The comparison with the evidences of good practice that correspond with the indicator in question revealed that sample member A complied with three evidences of good practice. They have been included as possible strengths in the profile of sample member A. The evidences are:

- emphasizing mental computation skills and encouraging learners to use skills like visualization;

- encouraging learners to try to relate what they learn to their previous knowledge;
- reacting to the misconceptions of the learners or their less efficient responses by giving guidance and directing them towards the desired response.

There were five evidences that were found lacking in the teaching practice of sample member A. They have been included as possible improvement opportunities in the profile of sample member A:

- encouraging learners not only to provide answers, but to explain how they reached the conclusions which they did;
- asking questions requiring high level thinking skills, e.g.: "How did you work that out?"
- allowing other learners the opportunity to respond to explanations from a co-learner;
- asking the learners for alternative ways to address mathematical problems;
- intervening in a well-timed and positive way to give direction to the mathematical discussion of learners.

Question 4: Do you use a wide range of tools and activities to support mathematics teaching?

"Yes, I use a wide range of tools and activities to support mathematics teaching, e.g. textbooks, group work, peer assessment, e.g. I said to them they must bring calendars to class. Each group cut out all the month ends up on the 30, 29, 28 and 31 which is sorted. After that they must paste in the chart according to their groups. After that the learners must sort themselves according to their birth dates."

Why is it important to use a variety of tools and activities?

"It is important, because sometimes other learners participate better when they are in groups, others when they do the work alone or is good at investigating."

Data analysis and interpretation

Sample member A uses teaching aids as well as practical activities to provide variety to the learners. Teaching aids are also used to aid the learners in their learning of mathematics. The sample member uses the different teaching aids to engage all the learners and to meet their different learning needs. The sample member has little opportunity to do group work. There is a lack of time for stimulating game play or re-teaching in small groups.

The comparison with the evidences of good practice that correspond with the indicator in question revealed that sample member A complied with five evidences of good practice. They have been included as possible strengths in the profile of sample member A. The evidences are:

- using the blackboard to assist verbal explanations;
- encouraging learners to use the blackboard when explaining their mathematical thinking;
- having worksheets for learners to complete during seatwork;
- asking clear questions and generating learner talk on mathematical strategies;
- using appropriate teaching aids like scales, measuring sticks, clocks and containers.

There were two evidences that were found lacking in the teaching practice of sample member A. They have been included as possible improvement opportunities in the profile of sample member A. The two are:

- making use of games and activities for exercising mathematical thinking and mental calculation;
- using small groups when re-teaching concepts.

Question 5: Do you create an intellectual environment in your classroom?

“Yes, in class the atmosphere must be conducive. I must have relationship with the learners. I mix them – that is those learners with ability mixing them with the slow learners.”

What are the benefits of teaching in an intellectual environment?

“The slow learners would benefit from the intellectual learners when I mix them.”

Data analysis and interpretation

Sample member A strives to create a class atmosphere in which all learners get an opportunity to learn and to contribute to one another's learning. The comparison with the evidences of good practice that correspond with the indicator in question revealed that sample member A complied with three evidences of good practice. They have been included as possible strengths in the profile of sample member A. The evidences are:

- setting high standards for behaviour in the classroom and expecting learners to adhere to those standards;
- making smooth transitions between lesson activities and having efficient classroom routines in place;
- challenging learners at the start of the lesson to learn well and supporting learners to reach the learning objectives.

There were six evidences that were found lacking in the teaching practice of sample member A based on data analysed. They have been included as possible improvement opportunities in the profile of sample member A. The six are:

- explaining consequences of misbehaviour clearly to learners;
- attending to any misbehaviour quickly without disrupting the whole class;
- being firm but fair in his/her actions towards learners;
- warning learners not to be careless in their responses, but to give serious thought to the mathematical problem at hand;
- stressing that when someone is talking, the others should listen attentively without interrupting;
- allowing questions that are relevant to the subject matter to be asked, but not allowing off-the-topic talk.

Question 6: Do you stress the importance of connections in mathematics?

"Yes, I do budget in class giving them a task on how he/she spent R20 which was given by his/her parents as pocket money. By doing so, budgeting will help them when they are adults or they do business or to budget their salaries."

Why is it important to highlight connections in mathematics and between mathematics and real life?

“As they grow they would love maths and at the end they become chartered accountants, bank tellers, work in factories, engineers, doctors and auditors.”

Data analysis and interpretation

From the sample member’s responses it is clear that the relationship between classroom mathematics and the real world is highlighted during teaching. The utility value of mathematics in the everyday lives of the learners is illustrated to learners. Sample member A also shows the learners the value of understanding mathematics when they grow up and enter the work force.

The comparison with the evidences of good practice that correspond with the indicator in question revealed that sample member A complied with four evidences of good practice. They have been included as possible strengths in the profile of sample member A. The evidences are:

- indicating to learners how the proposed learning can be applied to solve problems encountered earlier in the lesson/in earlier lessons/in other subjects;
- showing how content fits in with statements made earlier in the lesson;
- making explicit links between classroom mathematics and “real life mathematics” by indicating, for example, the importance of being able to add and subtract correctly when you go shopping;
- using logical thinking and encouraging learners to use deductive thinking skills.

There were three evidences that were found lacking in the teaching practice of sample member A based on data analysed. They have been included as possible improvement opportunities in the profile of sample member A. The three are:

- highlighting connections between concepts and the topics under which the concepts belong;
- explaining to the learners how the proposed lesson content fits in with work done previously;

- explaining to learners how they can apply knowledge/skills previously acquired to the new lesson content.

Question 7: Is your teaching aimed at fostering a positive attitude towards learning mathematics?

“Yes, as a teacher I love maths and my learners must see from me that I am sure in whatever I present. Don’t be bored when learners need help. Atmosphere must be conducive too.”

Why is it important that learners have a positive attitude towards learning mathematics?

“They learnt from me that I am interested in mathematics and am trying to make them understand and love it.”

Interpretation

Sample member A has a positive attitude towards mathematics and loves teaching it to the learners. This has a positive impact on the learners as they see mathematics not as something difficult, but as something that can be enjoyed and that can add value to their lives.

The comparison with the evidences of good practice that correspond with the indicator in question revealed that sample member A complied with four evidences of good practice. They have been included as possible strengths in the profile of sample member A. The evidences are:

- intervening in a positive way and providing direction to the thinking of the learners;
- participating with the learners as a community of learners and often using the word “we”;
- emphasizing the importance of affective issues like interest, appreciation, confidence and perseverance in learning mathematics.

There were five evidences that were found lacking in the teaching practice of sample member A based on data analysed. They have been included as possible improvement opportunities in the profile of sample member A. The five are:

- allowing learners to complete their train of thought without interrupting them;
- using encouraging words, e.g. “Do continue with what you want to say.”;
- rectifying mistakes without embarrassing the learners and always thanking learners for their effort;
- complimenting learners who offer alternate solutions to learning problems;
- encouraging learners to take responsibility for their own learning by giving them practical work applications.

Question 8: Do you meet learners’ needs through differentiation in your classroom?

“I meet the learners in differentiation of work as I take care of the slow learners giving them work according to their levels. But I have already explained it is difficult for me in my classes because of too much learners (84) but I try my best.”

Why do you regard it necessary to use differentiation of work?

“It is necessary because their abilities are not the same also background at home can cause weaknesses.”

Data analysis and interpretation

The sample member has a problem with differentiation of work as there are two different year groups in the class and eighty-four learners in total. Sample member A gives special attention to slower learners to prevent them from slipping behind and not understanding the work done. Owing to the class circumstances sample member A cannot have the learners work in groups, as for group work. It is however necessary for sample member A to divide the learners into groups suitable to their learning needs for administrative purposes.

The comparison with the evidences of good practice that correspond with the indicator in question revealed that sample member A complied with five evidences of good practice. They have been included as possible strengths in the profile of sample member A. The evidences are:

- re-teaching concepts to learners who have difficulty grasping the concepts while the more advanced learners continue working on their own;

- including tasks that are mathematically challenging to encourage learners to perform at a higher level;
- using examples of varying difficulty to help the understanding of learners;
- prepares the teaching materials to suit the specific needs of the learners;
- assessing learners continuously and moving learners to a group more suited to their learning needs when necessary.

There were three evidences that were found lacking in the teaching practice of sample member A based on data analysed. They have been included as possible improvement opportunities in the profile of sample member A. The three are:

- including individual, whole-class and group work to assist students in learning both individually and co-operatively;
- placing learners in groups according to their mathematics competency;
- monitoring and assisting the different groups during group work.

Question 9: Do you model positive values and expect it from your learners as well?

“Yes, I it is in the way I present and explain the theme, relationship, tolerance, respect and so on. They must work hard and show the same.”

Why is it important to model values, e.g. tolerance of other opinions, respect and patience?

“It helps us to work together freely.”

Data analysis and interpretation

The sample member works hard with the learners under difficult circumstances. There is a relationship of mutual respect between sample member A and the learners. The comparison with the evidences of good practice that correspond with the indicator in question revealed that sample member A complied with seven evidences of good practice. They have been included as possible possible strengths in the profile of sample member A. The evidences are:

- being enthusiastic and positive when teaching;

- displaying respect for others, tolerance, fairness, caring, diligence, non-racism and generosity and encouraging learners to display the same values in their interactions;
- listening attentively when a learner talks to him or gives an explanation to the rest of the class;
- giving positive and relevant feedback to learners;
- focusing on the inappropriate behaviour and not on the personality of the learner when taking disciplinary action;
- displaying empathy with all the responses of the learners and never ridiculing a learner for an incorrect response;
- acknowledging good learner behaviour.

There were no evidences that were found lacking in the modelling of positive values of sample member A and no possible improvement opportunities had been added to the profile of sample member A.

Question 10: Do you practise good time management in your mathematics classroom?

"No, the learners and I cannot practise good time management because of the large number of learners and two different grades in one class (84) learners as I stated."

Why is good time management important when teaching mathematics?

"It is good for those who have average number of learners, ratio of 15 to 35 at least. Too much work for me even when I do marking."

Data analysis and interpretation

Owing to the large number of learners in the class the sample member is struggling with time management. This situation will only be rectified if there is a reduction in the number of learners. This will enable the sample member to give the learners who need it more attention.

The comparison with the evidences of good practice that correspond with the indicator in question revealed that there is none of the evidences with which sample member A complied. No possible strengths have been added to the profile of sample member A.

All six evidences of good practice were found lacking in the teaching practice of sample member A based on data analysed. As such they have been included as possible improvement opportunities in the profile of sample member A. The six are:

- starting the lesson quickly and purposefully;
- using class-time for learning and spending little time on non-learning activities;
- not allowing off-the-topic conversations;
- scheduling time intervals during the lesson for instruction, demonstration, questioning, seatwork and the application of new knowledge;
- achieving the lesson objectives in the time allowed;
- ensuring that learners at the end of the lesson can apply what they have learnt to solve related problems.

Question 11: Do you assess (a) the prior knowledge of learners, (b) their performance and (c) progress during the lesson and the knowledge gained/concepts formed during the lesson?

"(a) Yes, before I teach new lessons I apply concrete to abstract skills. By that I check previous knowledge in order to link it to the new topic. (b) I also pose questions giving them class work, worksheet tests, investigations i.e. giving them scores or rating scales make them see whether they can make it. (c) At the end they must know how to do their work as individuals or – must be able to do the work on their own."

Why do you ensure that the assessments are done?

"I do it (a) to check understanding and knowledge they have; (b) to assess does the learner involve him/herself in the activities; (c) I will see that my lesson is caught up when the learners is able to do the work independently and I will see they gained the concept."

Data analysis and interpretation

Sample member A does the necessary prior assessments to check the existing knowledge and pre-knowledge of the learners. The sample member also assesses learners during the lesson and ensures learner participation. The knowledge that the learners acquired during the lesson is assessed at the closure.

The comparison with the evidences of good practice that correspond with the indicator in question revealed that sample member A complied with all nine evidences of good practice. They have been included as possible strengths in the profile of sample member A. The evidences are:

- asking questions before the lesson starts to determine the prior knowledge of the learners on the topic;
- including activities aimed at problem solving, application of mathematical thinking and creative work to assess learner progress during the lesson;
- asking clear questions during the lesson to see if learners understand the concepts and ensures that all learners get the opportunity to respond;
- allowing learners to implement what they have learnt at the end of the lesson;
- assigning homework and seatwork assignments in small increments;
- checking home- and seatwork and giving prompt feedback to learners;
- giving class exercises in a variety of formats and structures to diagnose the understanding of learners of the concepts;
- testing if learners have mastered the required skills through the provision of applications and exercises;
- asking reflecting questions to help learners to focus on the work done.

There were no evidences that were found lacking and no possible improvement opportunities had been added to the profile of sample member A.

Based on the aforementioned analysis and interpretation of the data received, a first profile reflecting the possible strengths and possible improvement opportunities in the teaching practice of sample member A could be built.

4.2.2 Profile 1 – Sample member A

The possible strengths and possible improvement opportunities that have been identified in the teaching practices of sample member A form the building blocks of

Profile 1. It has been stated from the outset that the research aim is to develop interventions that address shared improvement opportunities in the mathematics teaching practice of the sample members. As such, the list containing possible improvement opportunities is especially relevant to the research.

Possible strengths

The following fifty-one possible strengths in the teaching practice of teacher A have been identified via the analysis:

- focusing on aspects of the content that must be highlighted during the lesson;
- organizing and orchestrating work to suit learners of varied levels;
- knowing the learning material well and structuring the unveiling of the knowledge to promote learner understanding;
- helping the learners get ready to learn by explaining the learning objectives in understandable everyday language;
- having an overall plan for the lesson and ways to make it happen;
- making connections between concepts and topics, lesson content and prior knowledge;
- preparing seatwork assignments that provide variety and challenge;
- not only using whole class teaching, but also allowing for individual – and group work;
- using alternative approaches of explaining a concept which some learners have difficulty grasping;
- giving learners the opportunity for guided- and individual practice of new concepts and skills;
- using concrete illustrations to demonstrate how abstract mathematical concepts can be used to solve problems;
- using practical work and hands-on activities in addition to exposition;
- emphasizing mental computation skills and encouraging learners to use skills like visualization;
- encouraging learners to try to relate what they learn to their previous knowledge;

- reacting to the misconceptions of learners or their less efficient responses by giving guidance and directing them towards the desired response;
- encouraging learners to use the blackboard when explaining their mathematical thinking;
- using the blackboard to assist verbal explanations;
- having worksheets for learners to complete during seatwork;
- asking clear questions and generating learner talk on mathematical strategies;
- using appropriate teaching aids like scales, measuring sticks, clocks and containers;
- setting high standards for behaviour in the classroom and expecting learners to adhere to those standards;
- making smooth transitions between lesson activities and having efficient classroom routines in place;
- challenging learners at the start of the lesson to learn well and supporting learners to reach the learning objectives;
- indicating to learners how the proposed learning can be applied to solve problems encountered earlier in the lesson/in earlier lessons/in other subjects;
- showing how content fits in with statements made earlier in the lesson;
- making explicit links between classroom mathematics and “real life mathematics” by indicating, for example, the importance of being able to add and subtract correctly when you go shopping;
- using logical thinking and encouraging learners to use deductive thinking skills;
- intervening in a positive way and providing direction to the thinking of learners;
- participating with the learners as a community of learners and using the word “we” often;
- emphasizing the importance of affective issues like interest, appreciation, confidence and perseverance in learning mathematics;
- re-teaching concepts to learners who have difficulty grasping the concepts while the more advanced learners continue working on their own;
- including tasks that are mathematically challenging to encourage learners to perform at a higher level;

- using examples of varying difficulty to help the understanding of learners ;
- preparing the teaching materials to suit the specific needs of the learners;
- assessing learners continuously and moving learners to a group more suited to their learning needs when necessary;
- being enthusiastic and positive when teaching;
- displaying respect for others, tolerance, fairness, caring, diligence, non-racism and generosity and encouraging learners to display the same values in their interactions;
- listening attentively when a learner talks to him or gives an explanation to the rest of the class;
- giving positive and relevant feedback to learners;
- focusing on the inappropriate behaviour and not on the personality of the learner when taking disciplinary action;
- displaying empathy with all the responses of the learners and never ridiculing a learner for an incorrect response;
- acknowledging good learner behaviour;
- asking questions before the lesson starts to determine the prior knowledge of the learners on the topic;
- including activities aimed at problem solving, application of mathematical thinking and creative work to assess learner progress during the lesson;
- asking clear questions during the lesson to see if learners understand the concepts and ensuring that all learners get the opportunity to respond;
- asking reflecting questions to help learners focus on the work done;
- allowing learners to implement what they have learnt, at the end of the lesson;
- assigning homework and seatwork assignments in small increments;
- checking home- and seatwork and giving prompt feedback to learners;
- giving class exercises in a variety of formats and structures to diagnose the understanding of the learners of the concepts;
- testing if learners have mastered the required skills through the provision of applications and exercises.

This concludes the possible strengths of teacher A. The possible improvement opportunities follow.

Possible improvement opportunities

Thirty-five possible improvement opportunities had been identified in the mathematics teaching practices of teacher A based on the analysis and interpretation of responses received for Questionnaire 1. These possible improvement opportunities are:

- identifying complex aspects in the lesson and putting plans in place to deal with expected difficulties;
- preparing probing questions to stimulate mathematical thinking;
- giving clear written and verbal instructions;
- encouraging finding multiple solutions to address a mathematical problem;
- using innovative and creative activities like investigation projects and solving daily life problems;
- encouraging learners not only to provide answers, but also to explain how they reached the conclusions which they did;
- asking questions requiring high level thinking skills, e.g.: “How did you work that out?”;
- allowing other learners the opportunity to respond to explanations from a co-learner;
- asking the learners for alternative ways to address mathematical problems;
- intervening in a well-timed and positive way to give direction to the mathematical discussion of learners;
- making use of games and activities for exercising mathematical thinking and mental calculation;
- using small groups when re-teaching concepts;
- explaining consequences of misbehaviour clearly to learners;
- attending to any misbehaviour quickly without disrupting the whole class;
- being firm but fair in his/her actions towards learners;
- warning learners not to be careless in their responses, but to give serious thought to the mathematical problem at hand;
- stressing that when someone is talking, the others should listen attentively without interrupting;

- allowing questions that are relevant to the subject matter to be asked, but not allowing off-the-topic talk;
- highlighting connections between concepts and topics under which the concepts belong;
- explaining to the learners how the proposed lesson content fits in with work done previously;
- explaining to learners how they can apply knowledge/skills previously acquired to the new lesson content;
- allowing learners to complete their line of thought without disrupting them;
- using encouraging words, e.g. “Do continue with what you want to say.”;
- rectifying mistakes without embarrassing the learners and always thanking learners for their effort;
- complimenting learners who offer alternative solutions to learning problems;
- encouraging learners to take responsibility for their own learning by giving them practical work applications;
- including individual, whole-class and group work to assist students in learning both individually and co-operatively;
- placing learners in groups according to their mathematics competency;
- monitoring and assisting the different groups during group work;
- starting the lesson quickly and purposefully;
- using class-time for learning and spending little time on non-learning activities;
- not allowing off-the-topic conversations;
- scheduling time intervals during the lesson for instruction, demonstration, questioning, seatwork and the application of new knowledge;
- achieving the lesson objectives in the time allowed;
- ensuring that learners can apply what they have learnt to solve related problems at the end of the lesson.

Profile 1 of sample member A has been completed. In the next subsection data collected via Questionnaire 2 is analysed, interpreted and applied to build Profile 2 of sample member A.

4.3 Building Profile 2 of sample member A

The process to build the individual mathematics practices profile of sample member A continued with the analysis and interpretation of the data collected via Questionnaire 2.

4.3.1 Analysing and interpreting data from Questionnaire 2

The completed Questionnaire 2 contained the responses of the sample member with regard to the implementation of each of the eighty-six evidences of good practice. Where the sample member had indicated that a particular evidence is implemented in his/her own practice, an explanation indicating how this was done was added. The data received from sample member A was captured in Appendix E (i).

As Questionnaire 2 is an extension of Questionnaire 1, the possible strengths and possible improvement opportunities that were identified in Profile 1 were compared with the evidences provided in response to Questionnaire 2. This was done to determine if evidences were provided for previous possible improvement opportunities that had been included in Profile 1. The evidences provided were analysed and interpreted to determine whether they provided acceptable evidence to negate the possible improvement opportunities [Appendix E (ii) refers]. In instances where acceptable evidences were provided, the list of possible improvement opportunities was amended accordingly. The comparison revealed that Questionnaire 2 provided evidences for twenty-five of the possible improvement opportunities that had been included in Profile 1. The total number of possible strengths has increased to seventy six. Evidences provided for these possible improvement opportunities resulted in the lesser count of ten possible improvement opportunities in the teaching practices of sample member A.

Based on the aforementioned data analysis and comparison a second profile could be built.

4.3.2 Profile 2 – Sample member A

The identification of shared possible improvement opportunities has been identified as a research objective. As such listing all the possible strengths repetitively is deemed unnecessary to the research aim. Only the possible improvement opportunities will henceforth be listed in the profiles, as it is the core of the research undertaken.

Possible improvement opportunities

The comparison between Profile 1 and data from Questionnaire 2 supported ten shared possible improvement opportunities. They have been included in Profile 2. The possible improvement opportunities that are left are:

- encouraging learners not only to provide answers, but also to explain how they reached the conclusions which they did;
- asking questions requiring high level thinking skills, e.g.: “How did you work that out?”;
- allowing other learners the opportunity to respond to explanations from a co-learner;
- asking the learners for alternative ways to address mathematical problems;
- intervening in a well-timed and positive way to give direction to the mathematical discussion of learners;
- making use of games and activities for exercising mathematical thinking and mental calculation;
- using small groups when re-teaching concepts;
- monitoring and assisting the different groups during group work;
- starting the lesson quickly and purposefully;
- achieving the lesson objectives in the time allowed.

After data from both Questionnaires 1 and 2 had been analysed and compared, Profile 2 was built. The profile consisted of seventy-six possible strengths and the ten possible improvement opportunities listed above. The next section focused on verifying the data contained in Profile 2.

4.4 Verifying Profile 2

Data received from the sample member in response to Questionnaire 3 have been analysed to determine if evidences were provided for any of the possible improvement opportunities included in Profile 2.

4.4.1 Analysing and interpreting data from Questionnaire 3

The explanations of the sample member to aspects regarding a problem presented in Questionnaire 3 (Appendix D iv) were analysed as part of data verification. In Questionnaire 3 the sample member had to explain (1) how he/she would teach to get the learners to solve the given problem – thus the approach; (2) what mathematical principles are underlying the problem and (3) what learners should know and be able to do to solve the mathematics problem.

The verbatim response of sample member A to (1)

> “The learner calculates by adding all the monies each and every day starting from Monday up to Saturday.
> So they will know how much was spent and add the 5c change.
> He/she will know that he received an allowance of R50-00 because the total money spent is R49, 95 that is if you add 5c to that you get R50-00.”

The verbatim response of sample member A to (2)

> “The learner must identify the problem;
> think about ways to solve it;
> and then apply it to solve the problem.
> The learner can apply any way of solving the problem as long as he/she gets a valid answer or reason.
> Use mathematical principles for solving problems, e.g. social problems and practical problems.”

The verbatim response of sample member A to (3)

> “They must know the rules;
> Must have critical thinking in order to solve the problem;
> Be able to identify the problem;
> Learners must know basic operations;
> They can use different methods to tackle the problem;
> Read and interpret information.”

Data analysis and interpretation

The sample member did not answer the first question put forward, namely how he/she would teach the learners to be able to solve the problem. The response merely stated one solution (the teacher's) to the problem. As such the response to (1) did not contribute towards data verification. In the response given for (2) and (3) the sample member did display an understanding of problem solving techniques that could be used. It is mentioned that learners are allowed to apply different techniques as long as they can validate their answer. This response provided evidence for one of the possible improvement opportunities that had been included in Profile 2, namely:

- encouraging learners not only to provide answers, but also to explain how they reached the conclusions which they did.

The abovementioned former improvement opportunity has been changed to a strength in the revised Profile 2 of the sample member.

4.4.2 Revised Profile 2 – sample member A

After analysing data from Questionnaire 3 one revision was made to the profile of sample member A. Evidence was provided for a former improvement opportunity and it was reclassified as a strength. The revised profile 2 comprises seventy-eight possible strengths and nine possible improvement opportunities. The list of possible improvement opportunities follow.

Possible improvement opportunities

- asking questions requiring high level thinking skills, e.g.: "How did you work that out?";
- allowing other learners the opportunity to respond to explanations from a co-learner;
- asking the learners for alternative ways to address mathematical problems;
- intervening in a well-timed and positive way to give direction to the mathematical discussion of learners;

- making use of games and activities for exercising mathematical thinking and mental calculation;
- using small groups when re-teaching concepts;
- monitoring and assisting the different groups during group work;
- starting the lesson quickly and purposefully;
- achieving the lesson objectives in the time allowed.

In the next subsection data gathered via the Classroom Observation Tool were analysed and the findings compared with those included in the revised Profile 2. Based on the comparison a third and final mathematics practice profile of sample member A was built.

4.5 Building the final profile of sample member A

The final profile was built after analysing data gathered through the application of the classroom observation tool. Possible improvement opportunities identified as a result of the classroom observation were compared with the revised Profile 2. Furthermore, a comparison was drawn between the possible improvement opportunities that had been identified following the analysis and interpretation of data gathered via all four data collection instruments.

4.5.1 Analysing and interpreting data from the classroom observation tool

Data was collected during a classroom observation of sample member A teaching mathematics. The classroom observation tool was applied as data collection instrument. The data collected during the observation was completed in table format [Appendix E (iii) refers]. An analysis of the data revealed that there were sixty-eight possible strengths and eighteen possible improvement opportunities in the teaching practices of the sample member.

As previously explained, the complete list of possible strengths is not included, as the research focus is the identification of shared improvement opportunities. The list of possible improvement opportunities identified via the classroom observation follow.

Possible improvement opportunities

- identifying complex aspects in the lesson and putting plans in place to deal with expected difficulties;
- preparing probing questions to stimulate mathematical thinking;
- using not only whole class teaching, but also allowing for individual- and group work;
- encouraging finding multiple solutions to address a mathematical problem;
- encouraging learners not only to provide answers, but also to explain how they reached the conclusions which they did;
- asking questions requiring high level thinking skills, e.g.: “How did you work that out?”;
- allowing other learners the opportunity to respond to explanations from a co-learner;
- asking the learners for alternative ways to address mathematical problems;
- intervening in a well-timed and positive way to give direction to the mathematical discussion of learners;
- encouraging learners to use the blackboard when explaining their mathematical thinking;
- making use of games and activities for exercising mathematical thinking and mental calculation;
- using small groups when re-teaching concepts;
- explaining consequences of misbehaviour clearly to learners;
- attending to any misbehaviour quickly without disrupting the whole class;
- including individual, whole-class and group work to assist learners in learning both individually and co-operatively;
- placing learners in groups according to their mathematics competency;
- re-teaching concepts to learners who have difficulty grasping concepts while the more advanced learners continue working on their own;
- monitoring and assisting the different groups during group work.

It has been noted that there was no misbehaviour in the class during the observation period. As such the inclusion of the following two improvement opportunities might be unnecessary:

- explaining consequences of misbehaviour clearly to learners;
- attending to any misbehaviour quickly without disrupting the whole class.

It stands to reason that the learners know the consequences of misbehaviour and as a result do not misbehave. The disciplined class atmosphere contributed towards this assumption. Notwithstanding the aforementioned, they were included in the table drawn up for comparison with the data from Profile 2 (revised) to establish the validity of this assumption [Appendix E (iv)].

4.5.2 Comparing data on possible improvement opportunities

Possible improvement opportunities that had been included in the revised Profile 2 were compared with the possible improvement opportunities that were identified through the analysis of the classroom observation data [Appendix E (iv)]. A second comparative table reflected the possible improvement opportunities identified through the data analysis of the four data collection instruments [Appendix E (v)]. The comparison revealed seven shared possible improvement opportunities in the mathematics teaching practices of sample member A. Together the seven possible improvement opportunities and the seventy-nine possible strengths identified in the mathematics teaching practices of sample member A build the final profile of sample member A.

4.5.3 The final profile – sample member A

With the possible improvement opportunities in the teaching practice of sample member A having been identified, the final profile is built. The profile consisted of seventy-nine possible strengths and seven possible improvement opportunities. As the research focus is the identification of shared improvement opportunities, the possible strengths receive no further attention. A complete list of the possible improvement opportunities that was included in the mathematics teaching practice profile of sample member A follows:

Improvement opportunities

- asking questions requiring high level thinking skills, e.g.: “How did you work that out?”;

- allowing other learners the opportunity to respond to explanations from a co-learner;
- asking the learners for alternative ways to address mathematical problems;
- intervening in a well-timed and positive way to give direction to learners' mathematical discussion;
- making use of games and activities for exercising mathematical thinking and mental calculation;
- using small groups when re-teaching concepts;
- monitoring and assisting the different groups during group work.

An analysis of the aforementioned list revealed that four possible improvement opportunities are concerned with encouraging mathematical thinking and the other three are concerned with teaching in groups.

4.6 Conclusion

This chapter focused on explaining the processes that were followed when working with the data that had been gathered on each of the individual mathematics teaching practices of each sample member. The data was analysed, interpreted and compared in order to build an individual mathematics teaching practices profile for each sample member. In Chapter 4 the profile of sample member A was built as an exemplar of the profile building process. Detailed accounts on the building of individual profiles for sample members B-E are attached in Appendices F- I.

Chapter 5 focuses on building a generic profile of the teaching practices of the sample as a theory of the current teaching practices of Grade four mathematics teachers in the Nelson Mandela Metropolitan Area.

CHAPTER 5 THEORY BUILDING

5.1 Introduction

A generic profile of the mathematics teaching practices of the sample is built as a theory on grade four mathematics practices in the Nelson Mandela Metropolitan Municipality. As was the case during individual profile building, the focus was placed on identifying shared improvement opportunities in the mathematics teaching practices of the sample as this was a prerequisite for fulfilling the research aim. To reflect, the research aim was the development and application of interventions that address shared improvement opportunities in the mathematics teaching practices of the sample.

5.2 Building a generic profile

Individual mathematics teaching practice profiles had already been built (Chapter 4 and Appendices E-I refer). Each of these profiles comprised possible strengths and possible improvement opportunities. The emphasis placed on the importance of identifying the possible improvement opportunities as research focus resulted in only the possible improvement opportunities being listed in the individual profiles. The possible strengths in each profile are represented by the remainder of the eighty-six evidences of good mathematics teaching practices when the possible improvement opportunities have been removed.

5.2.1 Comparing the improvement opportunities

To compile a generic teaching practice profile of the sample the possible improvement opportunities included in each of the individual profiles had to be compared. This comparison resulted in the identification of shared possible improvement opportunities in the mathematics teaching practice of the sample. A comparative table, Table 5.1 was drawn up to compare the possible improvement opportunities in the individual profiles of the sample members. A second table, Table 5.2, was drawn up to provide a global overview of the possible strengths and possible improvement opportunities in the teaching practices of the sample in relation to the indicators and evidences of good mathematics teaching practice identified through the research.

Table 5.1: A comparison between the improvement opportunities contained in Profiles A-E

Comparing the improvement opportunities in the individual profiles of sample members A-E				
Sample member A	**Sample member B**	**Sample member C**	**Sample member D**	**Sample member E**
			Organizing and orchestrating work to suit learners of varied levels.	
		Encouraging finding multiple solutions to address a mathematical problem.		
Asking questions requiring high level thinking skills, e.g.: “How did you work that out?”	Asking questions requiring high level thinking skills, e.g.: “How did you work that out?”	Asking questions requiring high level thinking skills, e.g.: “How did you work that out?”	Asking questions requiring high level thinking skills, e.g.: “How did you work that out?”	Asking questions requiring high level thinking skills, e.g.: “How did you work that out?”
Allowing other learners the opportunity to respond to explanations from a co-learner.				
Asking the learners for alternative ways to address mathematical problems.	Asking the learners for alternative ways to address mathematical problems.	Asking the learners for alternative ways to address mathematical problems.	Asking the learners for alternative ways to address mathematical problems.	Asking the learners for alternative ways to address mathematical problems.
Intervening in a well-timed and positive way to give direction to mathematical discussion of the learners.	Intervening in a well-timed and positive way to give direction to mathematical discussion of the learners.			
	Encouraging learners to use the blackboard when explaining their mathematical thinking.			
Making use of games and activities for exercising mathematical thinking and mental calculation.		Making use of games and activities for exercising mathematical thinking and mental calculation.	Making use of games and activities for exercising mathematical thinking and mental calculation.	

Using small groups when re-teaching concepts.		Using small groups when re-teaching concepts.		
	Using logical thinking and encouraging learners to use deductive thinking skills.			
	Complimenting learners who offer alternative solutions to learning problems.			
	Placing learners in groups according to their mathematics competency.	Placing learners in groups according to their mathematics competency.		
			Including tasks that are mathematically challenging to encourage learners to perform at a higher level.	
	Assessing learners continuously and moving learners to a group more suited to their learning needs when necessary.	Assessing learners continuously and moving them to a group more suited to their learning needs when necessary.		
Monitoring and assisting the different groups during group work.		Monitoring and assisting the different groups during group work.		
			Identifying complex aspects in the lesson and putting plans in place to deal with expected difficulties.	

Key for colour coding:

5 shared improvement opportunities;
3 shared improvement opportunities;
2 shared improvement opportunities;
a single improvement opportunity.

Table 5.2: A global overview of the strengths and improvement opportunities of the sample in relation to the indicators and evidences of good mathematics teaching practice

Compiling the generic teacher profile

Category 1: Planning

Indicators	Evidences	Strength	Improvement opportunity	Improvement opportunity % of sample
Lessons are well-planned with the learning objectives and ability of the learners in mind.	The teacher: • focused on aspects of the content that must be highlighted during the lesson;	5	0	0%
	• organized and orchestrated work to suit learners of varied levels;	4	1	20%
	• identified complex aspects in the lesson and put plans in place to deal with expected difficulties;	5	0	0%
	• knew the learning material well and structured the unveiling of the knowledge to promote learner understanding;	5	0	0%
	• prepared probing questions to stimulate mathematical thinking;	5	0	0%
	• helped the learners get ready to learn by explaining the learning objectives in understandable everyday language;	5	0	0%
	• had an overall plan for the lesson and ways to make it happen;	5	0	0%
	• made connections between concepts and topics, lesson content and prior knowledge;	5	0	0%
	• had prepared seatwork assignments that provide variety and challenge.	5	0	0%

Category 2: Teaching

Indicators	**Evidences**	**Strength**	**Improvement opportunity**	**Improvement opportunity % of sample**
A variety of strategies for teaching mathematical concepts and for solving mathematical problems are applied.	The teacher: • does not only use whole class teaching but also allows for individual – and group work;	5	0	0%
	• gives clear written and verbal instructions;	5	0	0%
	• encourages finding multiple solutions to address a mathematical problem;	4	1	20%
	• uses alternate approaches of explaining a concept which some learners have difficulty grasping;	5	0	0%
	• gives learners an opportunity for guided - and individual practice of new concepts and skills;	5	0	0%
	• uses concrete illustrations to demonstrate how abstract mathematical concepts can be used to solve problems;	5	0	0%
	• uses innovative and creative activities like investigation projects and solving daily life problems;	5	0	0%
	• uses practical work and hands-on activities in addition to exposition.	5	0	0%
Learners are actively engaged in the learning process by encouraging mathematical thinking.	The teacher: • encourages learners not only to provide answers, but also to explain how they reached the conclusions which they did;	5	0	0%
	• asks questions requiring high level thinking skills, e.g.: "How did you work that out?";	0	5	100%

Indicators	Evidences	Strength	Improvement opportunity	Improvement opportunity % of sample
	• allows other learners the opportunity to respond to explanations from a co-learner;	4	1	20%
	• reacts to misconceptions of learners or less efficient responses by giving guidance and directing them towards the desired response;	5	0	0%
	• asks the learners for alternative ways to address mathematical problems;	0	5	100%
	• intervenes in a well-timed and positive way to give direction to mathematical discussion of learners;	3	2	40%
	• emphasizes mental computation skills and encourages learners to use skills like visualization;	5	0	0%
	• encourages learners to try to relate what they learn to their previous knowledge.	5	0	0%
A wide range of tools and activities are used to support mathematics teaching.	The teacher: • uses the blackboard to assist verbal explanations;	5	0	0%
	• encourages learners to use the blackboard when explaining their mathematical thinking;	4	1	20%
	• has worksheets for learners to complete during seatwork;	5	0	0%
	• asks clear questions and generates learner talk on mathematical strategies;	5	0	0%
	• makes use of games and activities for exercising mathematical thinking and mental calculation;	2	3	60%
	• uses small groups when re-teaching concepts;	3	2	40%

Indicators	Evidences	Strength	Improvement opportunity	Improvement opportunity % of sample
	• uses appropriate teaching aids like scales, measuring sticks, clocks and containers.	5	0	0%
An intellectual environment is created in the classroom where serious mathematical thinking is the norm.	The teacher: • sets high standards for behaviour in the classroom and expects learners to adhere to those standards;	5	0	0%
	• explains consequences of misbehaviour clearly to learners;	5	0	0%
	• makes smooth transitions between lesson activities and has efficient classroom routines in place;	5	0	0%
	• attends to any misbehaviour quickly without disrupting the whole class;	5	0	0%
	• is firm but fair in his/her actions towards learners;	5	0	0%
	• challenges learners at the start of the lesson to learn well and supports learners to reach the learning objectives;	5	0	0%
	• warns learners not to be careless in their responses but to give serious thought to the mathematical problem at hand;	5	0	0%
	• stresses that when someone is talking, the others should listen attentively without interrupting;	5	0	0%
	• allows questions that are relevant to the subject matter to be asked but off-the-topic talk is not allowed.	5	0	0%
The importance of connections in mathematics and between mathematics and real life is highlighted.	The teacher • highlights connections between concepts and topics under which the concepts belong;	5	0	0%

Indicators	Evidences	Strength	Improvement opportunity	Improvement opportunity % of sample
	• explains to the learners how the proposed lesson content fits in with work done previously;	5	0	0%
	• indicates to learners how the proposed learning can be applied to solve problems encountered earlier in the lesson/in earlier lessons/in other subjects;	5	0	0%
	• shows how content fits in with statements made earlier in the lesson;	5	0	0%
	• explains to learners how they can apply knowledge/skills previously acquired to the new lesson content;	5	0	0%
	• makes explicit links between classroom mathematics and “real life mathematics” by indicating, for example, the importance of being able to correctly add and subtract when you go shopping;	5	0	0%
	• uses logical thinking and encourages learners to use deductive thinking skills.	4	1	20%
Teaching is aimed at fostering a positive attitude towards learning mathematics and at students gaining confidence in solving problems.	The teacher: • participates with the learners as a community of learners and the word “we” is often used;	5	0	0%
	• allows learners to complete their line of thought without disrupting them;	5	0	0%
	• uses encouraging words, e.g. “Do continue with what you want to say.”	5	0	0%
	• rectifies mistakes without embarrassing the learners and always thanks learners for their effort;	5	0	0%

Indicators	Evidences	Strength	Improvement opportunity	Improvement opportunity % of sample
	• compliments learners who offer alternate solutions to learning problems;	4	1	20%
	• intervenes in a positive way and provides direction to learners' thinking;	5	0	0%
	• emphasizes the importance of affective issues like interest, appreciation, confidence and perseverance in learning mathematics;	5	0	0%
	• encourages learners to take responsibility for their own learning by giving them practical work applications.	5	0	0%
Learners' needs are met through differentiation of work.	The teacher: • includes individual, whole-class and group work to assist students in learning both individually and co-operatively.	5	0	0%
	• places learners in groups according to their mathematics competency;	3	2	40%
	• re-teaches concepts to learners who have difficulty grasping the concepts while the more advanced learners continue working on their own;	5	0	0%
	• includes tasks that are mathematically challenging to encourage learners to perform at the higher level;	4	1	20%
	• uses examples of varying difficulty to help learners' understanding;	5	0	0%
	• prepares the teaching materials to suit the specific needs of the learners;	5	0	0%
	• assesses learners continuously and moves learners to a	3	2	40%

Indicators	**Evidences**	**Strength**	**Improvement opportunity**	**Improvement opportunity % of sample**
	group more suited to their learning needs when necessary;			
	• monitors and assists the different groups during group work.	3	2	40%
Positive values are modelled by the teacher and expected from the learners.	The teacher: • is enthusiastic and positive when teaching;	5	0	0%
	• displays respect for others, tolerance, fairness, caring, diligence, non-racism and generosity and encourages learners to display the same values in their interactions;	5	0	0%
	• listens attentively when a learner talks to him or gives an explanation to the rest of the class;	5	0	0%
	• gives positive and relevant feedback to learners;	5	0	0%
	• focuses on the inappropriate behaviour and not on the personality of the learner when taking disciplinary action;	5	0	0%
	• displays empathy with all learners' responses and never ridicules a learner for an incorrect response;	5	0	0%
	• acknowledges good learner behaviour.	5	0	0%
Good time management is practiced by teachers and learners alike.	The teacher: • starts the lesson quickly and purposefully;	5	0	0%
	• uses class-time for learning and spends little time on non-learning activities;	5	0	0%
	• does not allow off-the-topic conversations;	5	0	0%
	• schedules time intervals during the lesson for instruction,	5	0	0%

Indicators	Evidences	Strength	Improvement opportunity	Improvement opportunity % of sample
	demonstration, questioning, seatwork and the application of new knowledge;			
	• achieves the lesson objectives in the time allowed;	5	0	0%
	• ensures that learners at the end of the lesson can apply what they have learnt to solve related problems.	5	0	0%

Category 3: Assessment

Indicators	Evidences	Strength	Improvement opportunity	Improvement opportunity % of sample
Prior knowledge of students, their performance and progress during the lesson and the knowledge gained/concepts formed during the lesson are assessed.	The teacher: • asks questions before the lesson starts to determine the prior knowledge of the learners on the topic;	5	0	0%
	• includes activities aimed at problem solving, application of mathematical thinking and creative work to assess learner progress during the lesson;	5	0	0%
	• asks clear questions during the lesson to see if learners understand the concepts and ensures that all learners get the opportunity to respond;	5	0	0%
	• asks reflecting questions to help learners focus on the work done;	5	0	0%
	• allows learners to implement what they have learnt at the end of the lesson;	5	0	0%

Indicators	Evidences	Strength	Improvement opportunity	Improvement opportunity % of sample
	• assigns homework and seatwork assignments in small increments;	5	0	0%
	• checks home- and seatwork and gives prompt feedback to learners;	5	0	0%
	• gives class exercises in a variety of formats and structures to diagnose learners' understanding of the concepts;	5	0	0%
	• tests if learners have mastered the required skills through the provision of applications and exercises.	5	0	0%

Key for colour coding:

5 shared improvement opportunities;
3 shared improvement opportunities;
2 shared improvement opportunities;
A single improvement opportunity

5.2.2 Data analysis and conclusions

An analysis of the data contained in Tables 5.1 and 5.2 revealed a total of sixteen improvement opportunities among the five sample members. The percentages represented by sample members that shared specific possible improvement opportunities were also indicated in Table 5.2. The improvement opportunities have been identified as:

- Organizing and orchestrating work to suit learners of varied levels.
- Encouraging finding multiple solutions to address a mathematical problem.
- Asking questions requiring high level thinking skills, e.g.: "How did you work that out?"
- Allowing other learners the opportunity to respond to explanations from a co-learner.
- Asking the learners for alternative ways to address mathematical problems.
- Intervening in a well-timed and positive way to give direction to mathematical discussion of learners.
- Encouraging learners to use the blackboard when explaining their mathematical thinking.
- Making use of games and activities for exercising mathematical thinking and mental calculation.
- Using small groups when re-teaching concepts.
- Using logical thinking and encouraging learners to use deductive thinking skills.
- Complimenting learners who offer alternate solutions to learning problems.
- Placing learners in groups according to their mathematics competency.
- Including tasks that are mathematically challenging to encourage learners to perform at the higher level.
- Identifying complex aspects in the lesson and putting plans in place to deal with expected difficulties.
- Assessing learners continuously and moving them to a group more suited to their learning needs when necessary.
- Monitoring and assisting the different groups during group work.

Eight of the aforementioned improvement opportunities, however, were limited to one sample member only. As such they could not be regarded as shared improvement opportunities for inclusion in the generic profile of the sample. The improvement opportunities that were excluded were:

- Organizing and orchestrating work to suit learners of varied levels.
- Encouraging finding multiple solutions to address a mathematical problem.
- Allowing other learners the opportunity to respond to explanations from a co-learner.
- Encouraging learners to use the blackboard when explaining their mathematical thinking.
- Using logical thinking and encouraging learners to use deductive thinking skills.
- Complimenting learners who offer alternative solutions to learning problems.
- Including tasks that are mathematically challenging to encourage learners to perform at the higher level.
- Explaining consequences of misbehaviour clearly to learners.
- Warning learners not to be careless in their responses but to give serious thought to the mathematical problem at hand.
- Identifying complex aspects in the lesson and putting plans in place to deal with expected difficulties.

The eight remaining improvement opportunities were classified as shared improvement opportunities as they represented evidences that were lacking in the teaching practices of more than one of the sample members. The extent to which each of these improvement opportunities was shared by the sample are discussed in the next subsection.

5.2.3 Shared improvement opportunities

After discarding the improvement opportunities related to single sample members, eight shared improvement opportunities remained. The shared improvement opportunities as well as the number of sample members that share each of the improvement opportunities have been indicated in table 5.2.

Table 5.3: Shared improvement opportunities and related number of sample members

Shared improvement opportunities	**Number of sample members shared by**
Asking questions requiring high level thinking skills, e.g.: “How did you work that out?”	5
Asking the learners for alternative ways to address mathematical problems.	5
Intervening in a well-timed and positive way to give direction to mathematical discussion of learners.	2
Making use of games and activities for exercising mathematical thinking and mental calculation.	3
Using small groups when re-teaching concepts.	2
Monitoring and assisting the different groups during group work.	2
Assessing learners continuously and moving learners to a group more suited to their learning needs when necessary.	2
Placing learners in groups according to their mathematics competency.	2

In the next subsection the relationship between the eight shared improvement opportunities, listed in Table 5.3, and the eleven indicators of good mathematics teaching practice is investigated.

5.2.4 Investigating the shared improvement opportunities in relation to the indicators of good mathematics practice

The eighty-six evidences of good mathematics teaching practice have been used as a yardstick during profile building to identify the shared improvement opportunities. These eighty-six evidences represented the measurable outcomes of the eleven indicators of good practice identified through the research. The relationship between the shared improvement opportunities and the eleven indicators of good practice was investigated to determine under which indicator each of the shared improvement opportunities is found. Correspondences between shared improvement opportunities and a specific indicator of good practice would indicate that the sample shares an improvement opportunity where that specific indicator is concerned.

Table 5.2 provided an overview of the improvement opportunities of the sample in relation to the indicators and evidences of good mathematics teaching practice. The data reflected in Table 5.2 was analysed and interpreted to draw conclusions on the relationship between the shared improvement opportunities and the indicators of good mathematics teaching practice.

Conclusions on the relationship between the shared improvement opportunities and the indicators of good practice

Indicators 1 and 2

No shared improvement opportunities were found under Indicators 1 and 2. ("A variety of strategies for teaching mathematical concepts and for solving mathematical problems are applied" and "Lessons are well-planned with the learning objectives and ability of the learners in mind." As such Indicators 1 and 2 were included as strengths in the generic profile of the sample.

Indicator 3

Three shared improvement opportunities that were identified were found under Indicator 3, "Learners are actively engaged in the learning process by encouraging mathematical thinking." Two of the shared improvement opportunities were included in the individual profiles of all five sample members. They were:

- "Asking questions requiring high level thinking skills, e.g.: "How did you work that out?" and
- "Asking the learners for alternative ways to address mathematical problems."

The third improvement opportunity, shared by two sample members, was:

- "Intervening in a well-timed and positive way to give direction to learners' mathematical discussion."

Based on the aforementioned it was concluded that the sample needed to improve their teaching practices where actively engaging their learners in the learning process by encouraging mathematical thinking is concerned. Indicator 3 has been identified and included as an improvement opportunity in the generic mathematics teaching practice profile of the sample.

Indicator 4

Two shared improvement opportunities were found under Indicator 4, “A wide range of tools and activities are used to support mathematics teaching.” They were:

- “Using small groups when re-teaching concepts”; and
- “Making use of games and activities for exercising mathematical thinking and mental calculation”

The first improvement opportunity listed was shared by two sample members. Owing to the circumstances under which the two sample members affected by this improvement opportunity teach, it was argued that the improvement opportunity was directly related to the classroom limitations and learner composition of the classes of the specific sample members. Both classrooms had been overcrowded with too many learners per class to effectively manage group work. In the one instance the sample member had more than eighty learners that represented three grades in one classroom.

The sample provided ample evidence that they indeed applied a wide range of tools and activities to support mathematics teaching. As such, the second shared improvement opportunity, namely “Making use of games and activities for exercising mathematical thinking and mental calculation” reflected a lack of evidence in using activities that exercised the mathematical thinking of learners as such. This shared improvement opportunity could thus be regarded as support to the inclusion of Indicator 3 as an improvement opportunity in the generic profile.

Based on the aforementioned arguments, Indicator 4 was included as strength in the generic profile of the sample.

Indicators 5, 6 and 7

As no shared improvement opportunities were identified in Indicator 5, “An intellectual environment is created in the classroom where serious mathematical thinking is the norm”; Indicator 6, “Using logical thinking and encouraging learners to use deductive thinking skills” or Indicator 7, “Teaching is aimed at fostering a positive attitude towards learning mathematics and at students gaining confidence in solving problems" these three indicators were included as strengths in the generic profile of the sample.

Indicator 8

Three shared improvement opportunities were found under Indicator 8, "Learners' needs are met through differentiation of work". They were:

- "Monitoring and assisting the different groups during group work."
- "Assessing learners continuously and moving them to a group more suited to their learning needs when necessary."
- "Placing learners in groups according to their mathematics competency."

As had been the case with Indicator 4, the improvement opportunities were limited to the same two sample members who, as a result of overcrowded classrooms and learner composition, had difficulties in implementing group work. It was not possible to address their specific classroom situations through an intervention. As such the performance of the sample in the five evidences remaining under Indicator 8 was scrutinised to judge whether the indicator should be included as strength or improvement opportunity. Ample evidence was found that the sample met the needs of their learners through differentiation of work. The sample included individual- and whole class work; re-taught learners who struggled with concepts used examples of varying difficulty and included teaching material that suited different' needs of the learners. Based on the aforementioned evidence Indicator 8 was included as a strength in the generic profile of the sample.

Indicators 9, 10 and 11

As no shared improvement opportunities were found under Indicator 9, "Positive values are modelled by the teacher and expected from the learners"; Indicator 10, "Good time management is practiced by teachers and learners alike" or Indicator 11, "Students' prior knowledge, their performance and progress during the lesson and the knowledge gained/concepts formed during the lesson are assessed" these three indicators were included as strengths in the generic profile of the sample.

5.2.5 The generic profile

Based on the aforementioned comparison and findings a generic profile of Grade 4 mathematics teachers in the Port Elizabeth district has been built. The generic profile consisted of ten strengths and one improvement opportunity in terms of the indicators of good mathematics teaching practice. The strengths identified were:

- Lessons are well-planned with the learning objectives and ability of the learners in mind.
- A variety of strategies for teaching mathematical concepts and for solving mathematical problems are applied.
- A wide range of tools and activities are used to support mathematics teaching.
- An intellectual environment is created in the classroom where serious mathematical thinking is the norm.
- The importance of connections in mathematics and between mathematics and real life is highlighted.
- Teaching is aimed at fostering a positive attitude towards learning mathematics and at students gaining confidence in solving problems.
- The needs of learners are met through differentiation of work.
- Positive values are modelled by the teacher and expected from the learners.
- Good time management is practised by teachers and learners alike.
- Prior knowledge of students, their performance and progress during the lesson and the knowledge gained/concepts formed during the lesson are assessed.

The shared improvement opportunity that was identified was:

- Learners are actively engaged in the learning process by encouraging mathematical thinking.

Four evidences that were identified to be lacking in the teaching practices of two or more sample members, were found under this shared improvement opportunity. They were:

- "Asking questions requiring high level thinking skills, e.g.: "How did you work that out?"
- "Asking the learners for alternative ways to address mathematical problems."
- "Intervening in a well-timed and positive way to give direction to learners' mathematical discussion."
- "Making use of games and activities for exercising mathematical thinking and mental calculation"

The building of the generic mathematics teaching practice profile of the sample has been concluded with the identification of the shared improvement opportunity that needed to be addressed.

5.3 Conclusion

In this chapter a generic profile of the mathematics teaching practices of Grade 4 teachers in the Nelson Mandela Metropolitan Area was built. Shared improvement opportunities were identified and the relationship between the improvement opportunities and the eleven indicators of good practice were investigated. As a result of the investigation, Indicator 3 that read "Learners are actively engaged in the learning process by encouraging mathematical thinking." was identified as the shared improvement opportunity in the generic mathematics teaching practices profile of the sample.

With the shared improvement opportunity having been identified, the intervention development could follow. However, before the intervention could be developed, theoretical grounding had to be done. The theoretical grounding done was twofold. Firstly, it focused on mathematical thinking in Chapter 6 and secondly, it focused on adult learners in Chapter 7.

CHAPTER 6 GROUNDING THE MATHEMATICAL THINKING INTERVENTION

6.1 Introduction

The theory built in Chapter 5 was used to determine the type of intervention that needed to be developed. Chapter 5 highlighted that the intervention sample required knowledge and skills with regard to how to encourage learner engagement through mathematical thinking. As such the focus of the intervention needed to be mathematical thinking. Prior to intervention development, the intervention had to be grounded in theory. To this end, a literature review was undertaken. The literature review had two focus points. The first was mathematical thinking, as it had been identified as the shared improvement opportunity that needed to be addressed in the intervention. The second was adult learners, as they constituted the research sample. Mathematical thinking is the focus of chapter 6, whilst chapter 7 focuses on the adult learner.

A literature review on mathematical thinking was undertaken to establish (i) what mathematical thinking is, (ii) why it is regarded as important and (iii) how teachers could promote mathematical thinking in their classrooms.

The findings of the literature review on mathematical thinking follow.

6.2 Mathematical thinking

6.2.1 What is mathematical thinking?

The literature review highlighted that there are different views on what mathematical thinking entails. Schoenfeld (1992) mentions the lack of a coherent explanatory frame that explains how the varied aspects of mathematical thinking and problem solving fit together. He does identify five important aspects of cognition that impact on mathematical thinking and problem solving. These are (i) the knowledge base, (ii) problem solving strategies, (iii) monitoring and control, (iv) beliefs and affects and (v) practices (Schoenfeld 1992:348). Dunlap (2001:1) defines mathematical thinking as "a cognitive approach to a problem that is both logical and mathematically sound". This definition allows teachers to approach mathematical thinking in a manner conducive to solving mathematics problems without limiting teachers or learners only to one correct solution. Furthermore, it does not prescribe that the shortest and quickest method possible should be used. Another definition of mathematical thinking reads: “Applying mathematical techniques, concepts and processes, either explicitly or implicitly, in the solution of problems.” (Henderson, Baldwin, Dasagi, Dupras, Friz, Ginat, Goelman, Hamer, Hitchner, Lloyd, Marion, Riedesel & Walker 2001:117). Henderson-, et al. point out that mathematical thinking is the application of mathematical modes of thought to solve problems in different domains. Learners should first and foremost learn and be able to use the fundamental mathematics. Without fundamental mathematics learners will be unable to apply mathematical modes of thinking effectively. This view is supported by Brodie (2004) who stressed that there is no conceptual understanding without procedural fluency. Katagiri (2004:12) states that mathematical thinking is like an attitude or a state of “attempting to do” or “working to do” something. This type of thinking is not determined by the problem alone; the person and the strategy applied must also be considered. Mathematical thinking thus acts as a guiding force to elicit the knowledge and skills required to solve problems.

Katagiri (2004) furthermore explains that as mathematical thinking is used during mathematical activities, it is related both to the contents and the methods of mathematics and arithmetic. The importance of basic mathematical knowledge as a prerequisite for mathematical thinking was also highlighted in a discussion on the Online Math Forum (2010). In the forum entitled “What is Mathematical Thinking?” it was communicated that mathematical thinking involves making connections in order to construct some sort of mathematical understanding. The view that a basic or fundamental knowledge of mathematics is a prerequisite for solving problems and thinking mathematically is underlined. However, it is imperative that teachers should not only pay attention to their learners gaining procedural knowledge. Conceptual knowledge must also be granted equal attention (Koellner, Jacobs, Borko, Schneider, Pittman, Eiteljorg, Bunning & Frykholm, 2007; Brodie, 2004; NCTM, 2004a).

Lim and Hwa (2006) also attempted to answer the question “What is mathematical thinking?” Their research led them to concur that a well-defined meaning or explanation of mathematical thinking was lacking. Lim and Hwa (2006:82) thus proposed their own definition, namely that mathematical thinking is “a mental operation supported by mathematical knowledge and a certain kind of predisposition, toward the attainment of a solution to a problem”. Their definition also supports the notion that mathematical knowledge is a prerequisite for mathematical thinking and that mathematical thinking is aimed at solving a problem. Frederickson and Ford (2004:190) explained that “students could imitate, but they didn’t seem to be able to extend their thinking or look for their own methods for solving problems.” This lack of mathematical thinking can be addressed by scaffolding problems and changing the teacher’s role from demonstrator of how problems can be solved to coach who allows learners to figure out problems on their own. As such they support the notion that encouraging learners to find their own ways to solve problems is central to mathematical thinking. Zarinnia and Romberg (1992:257) stated that “if one considered mathematics as problem solving” the following categories would emerge: inquiring; designing/modelling; working through/solving and explaining/interpreting. The National Council of Teachers of Mathematics (NCTM) also uses the words problem solving, reasoning, connections and communicating skills when referring to mathematical thinking (NCTM 2004a). Koelner, et al. (2007) also list activities such as exploring, proving, justifying, generalising ideas, critiquing, making representations and explaining the procedures of their solution strategies as important actions for mathematics learners when engaging in mathematical thinking.

Tall (1991:14) states that “the nature of mathematical thinking is inextricably interconnected with the cognitive processes that give rise to mathematical knowledge.” Teachers should encourage learners to work hard on a mathematical problem to stimulate mental activity, but then allow them the opportunity to relax so processes can carry on subconsciously. New ideas about possible solutions may thereafter arise.

Teacher beliefs and affect were identified as factors that impact on the teaching of mathematical thinking by Schoenfeld (1992:348). Kilpatrick, Swafford and Findell of the National Research Council (2001:338) expressed the view that “Teachers’ selection of tasks and their interactions with students during instruction are guided by their beliefs about what students need to learn and are capable of learning.” Teachers may tend to direct less cognitively demanding questions to learners of who they have lesser expectations. Knowing their learners’ current mathematical thinking levels is important for teachers who want to meet their learners' learning needs. Thompson (1985:286) presented two case studies that demonstrated how teacher beliefs play out in the classroom and noted that "There is research evidence that teachers' conceptions and practices, particularly those of beginning teachers, are largely influenced by their schooling experience prior to entering methods of teaching courses." Thompson cautions against making any conclusive statements because of the complex relationship that exist between teachers' conceptions of mathematics and mathematics teaching. However she does state that the observations made during the case studies suggest that teachers' views, beliefs, and preferences about mathematics do influence their instructional practice. Raymond (1997) says that to help future teachers work toward practices that are consistent with beliefs, mathematics educators cannot ignore the fact that teachers are exposed to many factors that may influence their teaching practice. Teachers may indeed benefit from self-examining the relationships that exist between their beliefs and their teaching practice.

Kilpatrick, Swafford and Findell (National Research Council 2001) stress the importance of a sound knowledge base when teachers teach mathematics. They argue that teachers with a strong mathematical knowledge at a wider span and greater depth more likely can foster their learners' ability to problem-solve, reason, and conjecture, thus promote mathematical thinking. Kilpatrick, Swafford and Findell (National Research Council 2001:313) continue that attaining mathematical proficiency must be the aim for all learners. They describe mathematical proficiency as "the integrated attainment of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition". Effective teaching practices will attend to all these strands of mathematical proficiency. They conclude by saying that in every grade in school, learners can demonstrate mathematical proficiency in some form and proclaim that "All young Americans must learn to think mathematically, and they must think mathematically to learn" (National Research Council 2001:409). This implies that they should understand mathematical ideas, must be able to compute fluently, must be able to solve problems, and to engage in logical reasoning.

Ball (2000:243) in turn agrees with Schoenfeld that teaching practices impact on mathematical thinking when stating: "It is not just what mathematics teachers know, but how they know it and what they are able to mobilize mathematically in the course of teaching." Cobb (1986:7) supports Ball and Schoenfeld by stating that knowledge of mathematics for teaching is embedded in the practice of teaching mathematics: "in mathematics, how teachers hold knowledge may matter more than how much knowledge they hold." To promote mathematical thinking it is important that teachers should understand how their learners reason and employ strategies for solving mathematical problems. Teachers should furthermore take into account how their learners apply or generalize problem-solving methods to various mathematical contexts and they have to provide opportunities for their learners to share their thinking and problem-solving processes, justify and formulate conjectures publicly, and evaluate multiple solution strategies (Franke, Kazemi, & Battey 2007). Hiebert and Carpenter (1992) contend that the depth of knowledge depends on learners' prior knowledge on the one hand and their ability to access and connect complex mathematical ideas within a broad network of understanding on the other. Learners with connected knowledge structures are able to engage in reasoning and problem solving. They can furthermore transfer learning and adapt understandings to new contexts. Teachers who want to promote mathematical thinking should include mathematical tasks that facilitate and support their learners' conceptual understanding of mathematics and foster deep connections among mathematical ideas.

Sierpinska encapsulates the importance of teachers taking their learners' pre-knowledge and level of understanding into account when teaching mathematics when she starts her book on understanding in mathematics with: "How to teach so that students understand? What exactly don't they understand? What do they understand and how?" (Sierpinska 1994: XI). She furthermore raises a crucial issue regarding mathematics research by asking:" What is the use, for didactics of mathematics, of *general* models of mathematical thinking and understanding, which are not specific to some concrete mathematical contents?" (Sierpinska 1994:388). She presented examples of learners' work in linear algebra, and showed that although successful learners have a sense of what it means to work within a theoretical system, they are also practical in moving about in the theory, finding conceptual shortcuts, and picking exactly what is needed from the theory. These learners solve the problem at hand; they do not develop a theory of solving all problems of a kind. Following her analysis of fifty five Research Reports published in Volume 4 of the Psychology of Mathematics Education Proceedings Sierpinska (2003:31) offers the opinion that for all the theory production in mathematics education, conclusions from research remain shaky or weak. She asks if studies to obtain more empirical evidence for people's difficulties with abstract mathematical thinking is really necessary and suggests that we should rather use what we already know about the distribution of thinking styles among learners in designing curricula, textbooks, and teaching methods so that they reflect this distribution. Sternberg (1996) mentions that mathematical thinking usually is defined as a set of mathematical and mental activities. These activities include abstracting, problem solving, conjecturing, generalizing, reasoning, deducting and inducting. Sternberg (1996:303) concludes that "there is no consensus on what mathematical thinking is because scholars define the term depending on their own perspective". The literature perused to determine what mathematical thinking is, lead to a concurrence with the conclusion made by Sternberg, namely that a single universally accepted definition of mathematical thinking has yet to be reached. For the purpose of this dissertation mathematical thinking is defined as follows:

Mathematical thinking entails the application of fundamental mathematical knowledge to solve new problems through reasoning, explaining and justifying possible solution strategies whilst fostering deep connections among mathematical ideas.

6.2.2 Why is mathematical thinking important?

Takahashi (2007) stated that mathematics education should not only enable learners to acquire knowledge and skills, but that it should also foster mathematical thinking. The question that arose was: "Why should mathematics education foster mathematical thinking?" The literature provided a number of reasons. According to Katagiri (2004) teachers should work hardest at cultivating mathematical thinking in arithmetic and mathematics courses as it will enable learners to think and make judgements independently. Mathematical thinking is furthermore important because it encourages mathematical discourse between teachers and learners and between learners and other learners. This notion of mathematical discourse is relevant as it fits in with the notion of what is expected of mathematics learners today. A list of notions of what mathematics teaching should enable learners to do was provided by Marcut (2005) and the NCTM (1998). Marcut (2005:62) and the NCTM (1998) state that teaching, doing and understanding mathematics entails that learners will be able to do the following:

- Organise and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Analyse and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

Mathematics learners are thus no longer required only to reach an answer to a problem posed. They must also be able to communicate their thinking to the teacher and to other learners so that together they can reflect on possible solutions to the problem posed (Stein, Engle, Smith & Hughes 2008; Walshaw & Anthony 2008; Frederickson & Ford 2004; Dunlap 2001). Walshaw and Anthony (2008) refer to a large body of empirical and theoretical evidence in which researchers demonstrated that effective instructional practices in the mathematics classroom demand mathematical talk. Learners must be able to both show their solution process and justify their answer. This allows them to see and evaluate their own thinking as well as the thinking of their fellow learners. In so doing, learners learn to weigh both the strengths and the limitations of different approaches and they become critical thinkers about mathematics (Marcut 2005; NCTM 2004a & 1998; Dunlap 2001).

The importance of gaining insight into the thinking of learners is highlighted by Koellner, et al. (2007) who expressed the view that teaching is not a process of transmitting a set of procedures but rather a dynamic process of inquiry into student reasoning. Brodie (2007) concurs by expressing the view that genuine participatory classrooms can be liberating and empowering for both teachers and learners. Koellner et al. (2007) continues that the role of a mathematics teacher is not to provide learners with new, disconnected pieces of information, but to build on the existing mathematical knowledge of the learners. Teachers should guide and support the thinking of their learners (Shimizu, 2007). It is through encouraging mathematical thinking and by allowing learners to express their ideas, that the mathematical reasoning of the learners becomes visible. This expression of ideas becomes a source of valuable information for teachers and learners alike. For teachers it provides information on what the learners already know and what they still need to learn. On the side of the learners it becomes a resource that enables them to challenge, stimulate and extend their own thinking (Walshaw & Anthony 2008). Encouraging mathematical thinking thus has an important role to play in the teaching of mathematics. The contributions made to the learning of the learners are vast. It allows learners to think about ideas, to engage in discussions, to reflect, to fine tune their mathematical thinking through the use of language and to articulate their thinking. Mathematical thinking contributes towards mathematical argumentation and towards participatory classrooms where learners and teachers alike engage with mathematics and together create new knowledge. Dunlap (2001:6) goes so far as to say that he believes that “mathematics is a way of thinking”. Teachers should not give instructions to learners on how to solve a problem. This limits learners into accepting one specific way of solving the problem as the only right way and prohibits their mathematical thinking.

The aforementioned discussion highlighted the importance of including mathematical thinking in the classroom. It showed that teaching mathematics is about more than a regurgitation of rules and algorithms. Although it is important that learners have procedural knowledge, there is more to teaching mathematics. Learners should be allowed the opportunity to apply their procedural knowledge to solve new problems through mathematical thinking. The literature review revealed various ways in which mathematical thinking in the classroom can be promoted. Through the literature review four aspects that can contribute towards promoting mathematical thinking were identified. They are:

- enhancing the environment;
- using appropriate questioning techniques;
- applying classroom practices that encourage mathematical thinking; and
- using games to advance mathematical thinking in learners.

The next sections in turn discuss each of the aforementioned ways in which teachers can promote mathematical thinking.

6.3 Enhancing the environment

If teachers want to encourage mathematical thinking and learner engagement they must start with the environment in which teaching and learning takes place, which is the classroom (Marcut 2005; Watson, De Geest & Prestage 2003; Henningsen & Stein 1997; Potts 1994).

According to Potts (1994) teachers who want to promote mathematical thinking should ensure that their classrooms portray a physical and intellectual environment that encourages a spirit of discovery in learners. Marcut (2005) supports this by highlighting the importance of learners feeling free to express their ideas and to listening attentively to their fellow learners while showing an interest in their ideas. Two suggestions are made with regard to the mathematics classroom. The first deals with seating arrangements. It is suggested that learners should not all sit facing towards the teacher, as it creates a passive, receptive mode that is detrimental to mathematical thinking. Learners should rather be placed in such a way that they can all see and interact with the teacher and with one another. Watson, De Geest and Prestage (2003) suggest a horseshoe or boardroom style where learners can see each other when they talk. This seating arrangement contributes to the teacher not being seen as the centre of all interactions and promotes real discussion.

The second suggestion deals with visual aids in the classroom. It is suggested that mathematical thinking can be encouraged by displaying posters on the walls of the mathematics classroom that say, for example: “Why do I think that?” or “Is there a different way to solve the problem?” Marcut (2005:64) reports that learners who are encouraged to express their ideas freely, to think, to formulate hypotheses, to listen to one another and to estimate their judgements mutually not only found mathematics classes very enjoyable, but also performed better. Engaging in mathematical thinking does, however, also demand a higher level of personal risk for learners than when they are merely busy with procedural tasks (Henningsen & Stein, 1997). Teachers should have classroom management practices in place that support mathematical thinking. These practices include the classroom norms and the physical organization of the environment, as stressed by Marcut (2005) and Watson, De Geest and Prestage (2003). It also includes the way in which order is established in the classroom; the amount of time allotted to activities; the ways in which discipline interventions are handled and the establishment of accountability structures (Henningsen & Stein 1997).

6.4 Using effective questioning techniques

Effective questioning techniques can be used to encourage mathematical thinking in the classroom. The importance of using questioning techniques to stimulate mathematical thinking is well documented (NCTM 2010; Franke, Webb, Chan, Ing, Freunf & Battey 2009; McCosker & Diezmann 2009; Stein et al 2008; Way 2008; Koellner et al. 2007; Ban Har 2007a; Ban Har 2007b; Brodie 2007; Takahashi 2007; Brodie 2004; Katagiri 2004; Sanchez & Ice 2004; Dunlap 2001; NCTM 1998; Potts 1994). The use of questions features prominently amongst the techniques identified by Sanchez & Ice (2004) and the NCTM (1998) to encourage mathematical thinking in the classroom. Some of these techniques include:

- Promoting effective discourse with and between learners by asking non-leading and open-ended questions;
- Using reflective questions that assist with pinpointing important lesson areas;
- Paraphrasing the answers of learners to help them think about their answers;
- Allowing wait time to give learners more time to think their reasoning through and to explain it.

The aforementioned is in line with what Katagiri (2004) suggested that teachers should do to assist learners in appreciating and gaining the ability to use mathematical thinking. Katagiri said that teachers should provide assistance to learners who get stuck by asking questions, rather than by directly providing them with knowledge and skills. He stressed that teachers should consider in advance the type of questions that should be asked to elicit mathematical thinking and as such, he supports the notion of scaffolding. Scaffolding entails that teachers ask a hierarchy of questions, leading learners to higher level thinking and reasoning. Scaffolding as a questioning technique to promote mathematical thinking, is also supported by Way (2008), Walshaw and Anthony (2008) and Ban Har (2007a). Scaffolding questions can furthermore provide learners with the opportunity to develop their self-confidence, independence and sense-making skills while they are working mathematically (McCosker & Diezmann 2009). Way (2008:23) identified four main categories of questions that can be used by mathematics teachers to stimulate their mathematical thinking of their learners. The categories are:

- Starter questions
- Questions to stimulate mathematical thinking
- Assessment questions
- Final discussion questions

Some examples and a short discussion on each category of question according to Way (2008:24-27) follow:

6.4.1 Starter questions

Examples of starter questions include:

- How could you sort these…?
- How many different… can be found?
- How many ways can you find to…?
- What are some examples of…?
- What can be made from…?
- Of what does this remind you?
- What happens when we…?

When looking at the aforementioned examples it is evident that starter questions are questions that provide learners with a starting point to think about the problem at hand. They are asked to focus the thinking of the learners in a general direction. A starter question is formulated as an open-end question to encourage the learner to communicate their mathematical thinking to the teacher and fellow learners.

The importance of asking open-ended questions to stimulate mathematical thinking was also accentuated by Sanchez and Ice (2004). Another advantage of open-ended starter questions lies in the very nature of the question as it puts learners at ease and reduces their anxiety of giving a "wrong" answer. Consequently learners are more willing to participate and to become involved in the mathematical thinking process.

The second category that Way (2008) proposed is questions to stimulate mathematical thinking.

6.4.2 Questions to stimulate mathematical thinking

Examples of this type of questions include:

- Can you find more examples?
- Can you see a pattern? Can you explain it?
- How can this pattern help you find an answer?
- How can you group these… in some way?
- Is there a way to record what you have found that might help us see more patterns?
- What are some things you could try?
- What do you think comes next? Why?
- What is the same? What is different?
- What would happen if…?

These questions can assist learners to connect to previous experiences and to see patterns and relationships. The establishment of patterns and relationships contributes towards the conceptual understanding of mathematics of the learners (Koellner, et al. 2007). Teachers can use these questions as a prompt when learners become stuck and they can also help learners to focus on possible strategies that they can use. Dunlap (2001) also referred to the value of asking leading questions when teachers want to stimulate mathematical thinking. He suggested the use of the Socratic Method. This method is based on the theory of Socrates that it is more important to enable students to think for themselves than merely to fill their heads with the right answers (Socratic Seminar Study Guide 2010; Regina Public Schools and Saskatchewan Learning 2003; Dunlap 2001). To this end, Socrates taught his students by asking leading questions that ultimately lead them to a discovery of the knowledge sought. He often responded to questions of the students with another question instead of providing an answer. The Socratic Method encourages learners to think for themselves rather than being told what to think.

The third category is assessment questions.

6.4.3 Assessment questions

Examples of assessment questions include:

- What have you discovered?
- How did you find that out?
- What made you decide to do it that way?
- What makes you confident it is correct?
- Why do you think that?

Assessment questions require learners to explain what they are doing or how they arrived at a solution. By asking assessment questions teachers can see the way that their learners are thinking, what they understand and at what level they are operating. Assessment questions should be asked after the learners have had some time to make progress in dealing with the problem. Learners may already have recorded some findings and have achieved a possible solution when the teacher uses assessment questions to get them to explain their thinking. These questions can furthermore be used to prompt learner reflection and self-assessment, thereby helping learners to prepare for their contribution to a class discussion.

The last category presented by Way (2008) is final discussion questions.

6.4.4 Final discussion questions

Examples of final discussion questions include:

- Is everybody's result the same? Why/why not?
- Who has the same answer/pattern/grouping as this?
- Who has a different solution?
- Have you thought of another way this could be done?
- Have we found all the possibilities? How do we know?
- Do you think we have found the best solution?
- What new questions/problems have you thought of?

Way (2008) describes final discussion questions as questions that pull together the efforts of the whole class. These questions prompt the sharing and comparing of strategies and solutions by learners. It is a vital phase in the mathematical thinking process as it provides learners with a further opportunity for reflection and for the realisation of mathematical ideas and relationships. Learners are encouraged to evaluate their own work and to appreciate the thinking of their fellow learners.

Using questions to stimulate the mathematical thinking of learners not only holds value for the learners, but also for teachers.

Evaluating the thinking of their learners can help teachers gain an understanding of the mathematical constructs embedded in the problem that they otherwise may not have discovered.

6.4.5 The questions of learners

Watson, De Geest and Prestage (2003) use a different perspective when stressing the importance of using questions to develop mathematical thinking. According to them, it is very important to allow learners the opportunity to create questions about the mathematics for each other. When learners have to create questions about mathematics for their fellow-learners they have to engage in mathematical thinking and thereby become participants in the creation of mathematics in the classroom.

From the aforementioned scenario, it became evident that the application of effective questioning techniques holds value for both learners and teachers when advancing mathematical thinking. McCosker and Diezmann (2009), Walshaw and Anthony (2008) and Brodie (2007), however, warn that not all question- and- answer types of interactions between teachers and learners support the mathematical thinking of learners. There are cases where the scaffolding of questions by teachers may be ineffective because of their own actions. The next section discusses some of the threats to effective questioning.

6.4.6 Threats to effective questioning

When scaffolding questions in support of mathematical thinking, teachers should refrain from the following as it could influence the effectiveness of their questioning (McCosker & Diezmann 2009:27-30; Walshaw & Anthony 2008:522; Brodie 2007:3):

- Pressing for meaning;

- Offering misleading guidance;
- Offering encouraging comments;
- Influencing learners' responses and
- Funnelling or narrowing of questions.

A short explanation of each of the aforementioned threats to effective questioning follows:

Pressing for meaning

Teachers should refrain from making comments that limit their learners' level of mathematical thinking and communication. Comments like "Explain how you solved this problem step by step" or "Can you tell me what you are thinking?" are good examples of scaffolding and pressing for meaning. On the other hand a comment like "What an interesting idea" ends a learner's mathematical thinking process and renders the scaffolding ineffective (McCosker & Diezmann 2009:28).

Offering misleading guidance

Teachers should refrain from making misleading suggestions in an attempt to scaffold learners' mathematical thinking (McCosker & Diezmann 2009). They should rather work from the learners' ideas than their own pre-suppositions and progress through scaffolding questions. Hereby they will scaffold learners' thinking by developing their understanding of the problem. Three ways to develop learners understanding of the problem were suggested by Jacobs and Ambrose (2009:261). They proposed that teachers can (i) use questions, e.g. "What are you trying to find out?"; "Can you explain what you already know about the problem?" and "What is confusing to you?" or (ii) rephrase or elaborate the problem or (iii) re-contextualise the problem to make it more familiar to the learners. These actions will promote a better understanding of the problem from the learner's side and enhance the mathematical thinking process.

Offering encouraging comments

Teachers should remember that the offering of encouraging comments is different to scaffolding. Remarks like "That is good" or "Keep going, something might come to you" do not constitute scaffolding as they do not promote mathematical thinking and reasoning (McCosker & Diezmann 2009:30; Walshaw & Anthony 2008:527). For learners to achieve more with scaffolding than they would have without, teachers should offer comments that support and encourage their mathematical thinking.

Influencing learners' responses

Teachers should not ask questions to which they already know the answer and then press the learners' responses towards the answer that they want (Walshaw & Anthony 2008; Brodie 2007). This limits the space for real learner contributions and prohibits their mathematical thinking.

The aforementioned discussion has illuminated the value that asking the right questions at the right time holds for the advancement of mathematical thinking in the classroom. There are, however, a number of other classroom practices that teachers can apply to encourage mathematical thinking. The next section investigates classroom practices that encourage mathematical thinking. Factors impacting on classroom practice are also discussed.

6.5 Applying classroom practices that can encourage mathematical thinking

Teachers can include interactive tasks that are open-ended and multimodal to inspire mathematical thinking (Makofsky 2009). Stein et al. (2008:317-321) and Makofsky (2009 & 2010) also suggest that teachers use problem-solving activities like brainstorming and discussion groups to encourage learners to explore different strategies that can be used for solving mathematical problems instead of letting learners focus only on supplying an answer. When stimulating mathematical thinking it is important to remember that the idea is not finding one method that works, but finding as many ways as possible (Walshaw & Anthony 2008; Watson, De Geest & Prestage 2003; Dunlap 2001). As such mathematical thinking and extended exploration are encouraged. When a number of different methods are posed for finding one answer, the teacher can further promote mathematical thinking by getting learners to evaluate and discuss the power of each of the methods.

When promoting mathematical thinking in the classroom it is important to generate both concentration and participation from the learners. To attain this goal, teachers can facilitate situations in the classroom where learners have to share their ideas and elaborate on their thinking or the thinking of a fellow learner. The teacher can, for example, ask learners to repeat another learner's answer or idea in their own words. Being aware that they can be asked to respond to a fellow learner's comment will motivate learners to focus and "think with". When learners question their peers, seek clarifications, are allowed opportunities to defend their responses and to resolve conflicting views it is found to be conducive to mathematical thinking (Watson, De Geest & Prestage 2003; Ban Har 2007b; Lim 2007b). Learners can also be asked to generate problems, examples, reasons or methods and then to share them with the class. When learners are creating their own examples, their teachers should encourage them to make up hard examples to present fellow learners with new challenges and to encourage both their reasoning and justification skills. Fellow learners can be invited to comment on what they do not understand and to offer other possibilities.

In addition to asking learners to comment on fellow learners' answers, teachers themselves can revoice or repeat learners' answers or explanations as part of stimulating mathematical thinking. They can also describe the strategies that they thought were used by their learners to solve a particular problem and highlight the mathematical ideas underlining their learners' explanations. Learners can then in turn react to the teacher's thinking and interpretation (Franke, Webb, Chan, Ing, Freund & Battey 2009). Walshaw and Anthony (2008) and Stein, et al. (2008) stress the importance of teachers knowing when to intervene in the mathematical discussion and when to transfer the responsibility back to the learners. Thorough planning will enable teachers to anticipate possible learner contributions and prepare responses that will enhance the learners' mathematical thinking. Teachers who know when to step in and out of the mathematical interaction tend to foster learner motivation and spark their learners' curiosity. Learners who experience that their teachers listen actively to their ideas and value their thinking are willing and eager to participate in mathematical thinking activities and to share their thinking in class.

The aforementioned classroom practices are in line with the Advancing Children's Thinking framework of Fraivillig, Murphy and Fuson as described by Walshaw &

Anthony (2008:536). This conceptual framework was developed to describe the ways in which teachers can fine-tune learners' mathematical thinking through the cognitive structure that they provide. The three main components of the framework is 'eliciting', which entails promoting and managing of classroom interactions, 'supporting' which is concerned with assisting individual learner's thinking and 'extending' which refers to the practices that work to advance learners' knowledge. The notion of 'extending' leads to another way in which mathematical thinking can be promoted, namely the use of games. Sanchez & Ice (2004), Seay (2009); Sun (2009) and Watson, De Geest and Prestage (2003) also advise the use of games to encourage mathematical thinking.

The next subsection discusses factors that impact on classroom practice whilst the one that follows investigates some interventions undertaken to improve mathematical thinking through classroom practice.

6.5.1 Factors impacting on classroom practice

A number of factors that impact on classroom practice had been identified through the literature (Walshaw & Anthony 2008; Brodie 2007; UNESCO 2006; Lim & Hwa 2006; Marcut 2005; Watson, De Geest & Prestage 2003; Henningsen & Stein 1997; Potts 1994) and via the sample's reflection on their own teaching practice. These factors include:

Large classes

When there are a large number of learners in a single class or learners from more than one grade in the same class a teacher's classroom practice may be influenced negatively. Problems with limited instruction time and a lack of physical space in the classroom may result in the teacher having difficulty with activities like group work, mathematical games or open-ended questioning. UNESCO (2006:2) acknowledges that large classes is a challenge, but states that it can have a positive impact on classroom practice as well.

Large classes can give teachers the opportunity to:

- improve their organizational and managerial skills;
- improve their interpersonal skills;

- improve their teaching and presentation skills;
- use many different, active, and fun ways of teaching because of the diversity of students and learning styles;
- improve their evaluation skills as a variety of ways to tell whether students have really learned the material can be devised;
- track the students successes;
- involve their students in their learning and in assessing;
- encourage students to discuss and learn from each other;
- bring variety and speed up the work; and
- give students responsibility and the opportunity to help each other through group activities. Students have to listen, have patience, and express themselves within a diverse group of people – skills that will be valuable for them throughout their lives.

The classroom environment

Teachers have control over the classroom environment in which they teach as they have the opportunity to arrange their assigned classroom as they see fit. How they arrange their classrooms are important as the environment affects how well students can learn. Teachers should manage the classroom environment so that it is a comfortable space in which to teach and learn. The classroom should encourage learners to participate and awaken a spirit of discovery. The classroom environment encompasses the physical environment, the learners' desks and seating, the teacher's work space as well as the learning resources required for lessons. Learners should be placed so that they can see and interact with one another as well as with the teacher. The classroom environment also includes the psycho-social environment which entails ways to promote learning as a community and dealing effectively with misbehaviour. To prevent misbehaviour teachers should change the classroom environment as soon as they sense that their students are becoming bored with sitting in the classroom. The teacher's ability to create well-managed physical and psycho-social environments can make all the difference between a functioning classroom where teaching and learning takes place and a dysfunctional classroom where chaos reigns (UNESCO 2006; Watson, De Geest & Prestage 2003; Henningsen & Stein 1997; Potts 1994).

Curriculum and the workload

The content that needs to be covered within a specific time frame according to the curriculum may limit teachers' classroom practice. Lim and Hwa (2006:87) talk about the "finish the syllabus syndrome" that causes many teachers to use procedural teaching and merely "teach to test" with a disregard for their learners' understanding and needs. Even if some teachers want to include more stimulating classroom practices in their classrooms they often can't because there is no time due to the workload attached to merely covering the curriculum. Walshaw and Anthony (2008:520) conclude that "teaching is a complex activity – some teaching and classroom arrangements produce differential results from one setting to another." Teachers should strike a balance between teaching the necessary content, whilst still meeting their learners' needs.

Teachers' beliefs and their expectations of their learners

If the teacher feels uncertain or is uncomfortable with a certain practice, for example the use of open-ended questions, he or she will steer away from that practice. Teachers with low learner expectations will not demand excellence and effort from their learners. In conclusion Walshaw and Anthony (2008:522) found that quality teaching and learning require classroom practices that are founded on material, systems, human and emotional support as well as a collaborative effort of teachers who want to make a difference in the lives of their learners.

Professional development

Finally teachers should focus on their own professional development to stay abreast of educational developments and better their own skills and classroom practices. In the UNESCO report (2006:50) the following is said about good teachers: "Good teachers think about their teaching – all of it, their own classroom behaviour, the plans they have, the activities they use, the backgrounds and experiences of their students, what and if their students are learning, why and why not. And good teachers do more than think about their teaching; they use whatever means possible to improve upon it." In these words the importance of teachers not neglecting their personal development is encapsulated. The attitude of the sample members who despite heavy workloads and time limits were prepared to partake in the research to become more competent at teaching mathematics reflects that they realise the importance of their own professional development and are willing to make sacrifices to gain new knowledge and skills. The next subsection discusses some interventions that were undertaken to improve mathematical thinking in the classroom.

6.5.2 Interventions to improve mathematical thinking through classroom practice

Koellner, et al. (2007) refers to a vignette on developing pedagogical content knowledge where at the end of the workshop one of the teachers called Kim commented: "Initially when we work out a math problem, we might see the mathematics involved only to a certain degree. Then we gave the problem to a bunch of kids and we see a lot of strategies we haven't thought about" and another teacher, Kristin shared what seemed to be a common feeling in the group: "Their thinking, obviously, was just as good as ours." Another teacher in the workshop, Ken, said: "I will be able to help kids more because I now understand these two ways to solve the problem. I understand how one connects to the other. So I would probably ask better questions." Teachers found that exploring the thinking of learners were to their advantage as it could help them with future decision-making processes, like preparing learners for problem-solving and capitalising on the developing of the ideas of the learners.

When reporting on The Improving Attainment in Mathematics Project (Watson, De Geest & Prestage 2003) it was noted that establishing working habits is very important. Working habits that are especially relevant in the context of mathematical thinking included the following:

- To establish the expectation that everyone will be thinking about the mathematics problem at hand, learners must be aware that, after some thinking time was allowed, anyone in the class might be called upon to answer.
- To establish the expectation that all learners will be talking about mathematics, learners will be asked to discuss mathematics in pairs. All pairs will have to report back on their findings.

It was decided to incorporate the notion of working habits into the intervention design. Sample members in the role of learners would be requested to work together in pairs while discussing aspects of mathematical thinking and to report back on their findings. This would stimulate the mathematics discussion and keep the focus on thinking about mathematics throughout the intervention.

Jacobs and Ambrose (2009:260, 261) studied teacher-student conversations conducted by 65 teachers interviewing 231 children solving 1018 story problems and identified eight categories of intentional teacher actions that stimulate learners' mathematical thinking. The following four teacher actions support a learner's thinking before a correct answer is given:

- Ensure that the learner understand the problem;
- Change the mathematics in the problem to match the learner's level of understanding;
- Explore what the learner has already done; and
- Remind the learner to use other strategies.

The next four teacher actions are aimed at extending learners' mathematical thinking after a correct answer is given:

- Promote reflection on the strategy the learner has just completed;
- Encourage the learner to explore multiple strategies and their connections;
- Connect the learner's thinking to symbolic notation; and
- Generate follow-up problems linked to the problem the learner just completed.

Another intervention aimed at improving mathematical thinking is the Jet South African Mathematics Challenge. It is an annual event for Grade 4 to Grade 7 learners aimed at enhancing the quality of teaching and learning of mathematics. Questions are aimed at knowledge application in new situations, conceptual knowledge, problem solving, reasoning, communication and general mathematical thinking. It has a wide reach with about 60000 learners partaking annually (SAMF 2011).

Interventions can also take place in E-learning format. Jennifer Seay, Middle School Mathematics Professional Development Coach, Wicomico County Public schools in Salisbury, Maryland presents Math Games for the Classroom Grades 6-8 in online workshop format. It is done under the auspices of the National Council of Teachers of Mathematics. The workshop discusses why Number and Operations are studied, it covers the major themes of the Number and Operations in grades PK-12 and lists the Number and Operations Expectations for Grades 6-8. The main focus is on games. It explains the importance of games in mathematics and highlights its value for highlighting concepts and stimulating mathematical thinking. Criteria for selecting games to used are provided as well as “Do’s” and Don’ts” when including games in your mathematics lesson. Material required, objective and rules as well as questions to ask learners that can stimulate mathematical thinking are discussed. Teachers who participate online can ask questions and share their mathematical game ideas. Teachers are requested to implement something learnt during the E-Workshop with at least one group of learners. There is a follow-up session where teachers can discuss their experiences when they introduced the games in their classrooms. Electronic copies of learners’ work, activities created and insights generated by the learners can be shared electronically. The session concludes with a reflection on Teaching and Learning and teachers who participated are informed that they will receive a short evaluation survey to complete electronically. The workshop is offered by the National Council of Teachers of Mathematics as part of their professional development offerings (Seay 2009).

Games aimed at stimulating mathematical thinking in learners that can be used to test procedural knowledge as well as conceptual insight, will be explained next.

6.6 Using games to advance mathematical thinking in learners

Teachers can with positive effect also make use of mathematical games to stimulate mathematical thinking. The games can be used as a starting point of enquiry that motivates the learners to become involved in mathematical thinking (Sanchez & Ice 2004; Seay 2009; Mahoney 2005). There are many advantages to the inclusion of games in the mathematics classroom. Seay (2009) says that mathematics teachers should make use of games for the following reasons:

- It increases learners’ curiosity and their motivation
- It establishes a sense of community amongst learners

- It creates a learner cantered learning environment
- It reduces learner anxiety in the mathematics classroom
- It allows for cooperative learning environments

To accentuate the importance of including games in the mathematics classroom Seay (2009) quotes Sherri Martinie ("Games in the Middle School", Mathematics Teaching in the Middle School, 94-95, September, 2005) who had said "When carefully selected, games can highlight specific mathematics concepts, activate strategic thinking, and create an opportunity to develop logical reasoning skills. The format of a game enables students to get immediate feedback. The value of games should not be underestimated." There are, however, important questions that teachers need to answer when selecting the game to be used. These include:

- What is the purpose of getting the learners to play the game – what will they practice or develop by playing this particular game?
- Is the game on the learners' ability level?
- What do the learners need to play the game effectively – boards, pens, etc.?
- How will I be able to assess the effectiveness of the game?

Prior to playing the game, the teacher should also inform the learners of the objectives of the game. When they finish playing, time should be allowed for reflection as well (Seay, 2009). A list of games and an explanation why each is well suited to encourage mathematical thinking on the grade 4 level follows. Background information, instructions on how to play the games as well as examples of game boards are provided in Appendix J. The games is located in the National Council of Teachers of Mathematics web site, categorised under Paper-based games, and is aimed at providing fun ways to sharpen learners' mathematic skills. The relevant web site is

http://www.nctm.org/resources/content.aspx?id=23220.

The games are:

- Game of 9 Cards (versions 1 and 2)
- Number Mastermind
- Contig (elementary and advanced)

- Number Neighbourhood (similar to Contig advanced)
- The 24 game
- Game of Nim
- The 100 game

Game of 9 Cards: version 1 - Impact on mathematical thinking

In version 1 the game is played to sum 15. Though the game in itself is fun and challenging for grade 4 learners, the teacher can add questions to stimulate and direct the learners' mathematical thinking. These questions can be:

- Do you think there will always be a winner in this game? If you answer Yes/No, why do you think so?
- Who is more likely to win – the player who goes first or second? Why?
- Why do we use a sum of 15? Do you think there is another number that is better to use?
- Are there ways to make sure that you do not lose in this game?
- Is there a card that gives you an advantage when you choose it? Why do you think so?

Game of 9 Cards: version 2 - Impact on mathematical thinking

Questions that can be asked to stimulate mathematical thinking when playing the game to sum 99 are:

- Is there a number that will give you an advantage over your opponent? If you answer Yes, how do you propose finding that number?
- Does someone always win?
- Are you at a disadvantage if you are not the player to start the game?

Number Mastermind: Impact on mathematical thinking

Interpreting the hints correctly stimulates learners' mathematical thinking. Teachers can also ask the following questions to encourage mathematical thinking:

- If player 2 has no dots and no x's in his/her first attempt is that good or bad? Why?

- If you got three x's on a guess, what do you think is the most number of attempts that you will need to figure out the number correctly? And the least number?

Contig: easy version - Impact on mathematical thinking

Working out the best possible play to earn the maximum number of points stimulates learners' mathematical thinking. Learners also try to outdo each other's plays to earn points which keep them thinking mathematically. Teachers can furthermore ask questions like:

- Why do you think that there are gaps between numbers on the board?
- Why is the first number left out on the board 13?
- Can you explain why the highest number on the board is 36?
- How were the numbers on the Contig board selected?
- How many ways are there to cover the number 6 on the board?

Contig (advanced version) – Impact on mathematical thinking

Impact on mathematical thinking

It is extremely challenging to work out the best possible play to earn the maximum number of points using three dice and the process stimulates learners' mathematical thinking. Learners also try to outdo each other's plays to earn points which keep them thinking mathematically. Teachers can stimulate their mathematical thinking by asking questions like:

- Why do you think the numbers on the Contig board were selected in the way that they were?
- Why are some numbers between 1 and 216 not on the Contig board?
- Why is the number 43 not on the board?
- Can you explain why there are no numbers between180 and 216 on the board?
- What is the probability to cover 216 on your first throw?
- How many ways are there to cover the number 96 on the board?
- And how many ways to cover 216?

- With 5 players why will numbers on the Contig board still be uncovered when everyone has made three plays?
- Would it be possible to use all the numbers from 1-216 on a Contig version 2 board if the dice went from 1 to 10?

Number Neighbourhood: impact on mathematical thinking

The game is similar to advanced Contig. The only difference being that the numbers on the Number Neighbourhood are not arranged in numerical order on the board like it is the case with Contig. The game has the same advantages for mathematical thinking as Contig (advanced).

Another game that has a large following amongst the teaching community and that has proven its value to improving learners' mathematical competencies is the 24 Game (Lewis, 2006).

The 24 game: impact on mathematical thinking

Impact on mathematical thinking

The 24 game requires learners to apply mathematical thinking throughout their plays. Participants do not only have to calculate how to get to 24 with the numbers given to them, but they also have to focus on the number of points accumulated by their opponents as this impacts on their choice of difficulty level of the next card. If teachers in their schools do not have a 24 game set of cards they can devise their own set of cards by asking learners to think out as many combinations of 4 numbers that can be used to calculate 24 as they can. Each learner can write his/her 4 number combinations down on flashcards. The teacher can then gather all the different flashcards and use this to play the 24 game with the learners. The deck of cards could be extended as learners think of new combinations of 4 numbers that can be used to calculate 24. This activity in itself is an exercise in mathematical thinking that can be used by teachers with great effect.

Game of Nim – impact on mathematical thinking

There are several questions that the teacher can ask to stimulate learners' mathematical thinking about the game. These include:

- Is the game of Nim fair regardless of who gets to start the game?
- How many circles do you need your opponent to leave you to ensure that you will be the winner?
- If you are left with four circles when it is your turn, how many would you cross out to ensure that you will win the game?
- What strategy do you have to follow in the game to win?
- If you were allowed to mark only one circle at a time, how would that have influenced your strategy?
- How would you play the game differently if you could pick 1, 2 or 3 circles to cross out during a turn?

The impact of the last game, included in this section, on learners' mathematical thinking follows.

The 100 game – impact on mathematical thinking

The teacher can ask questions about the game to stimulate learners' mathematical thinking. Questions that can be asked include the following:

- Does it matter if you get to go first or second?
- Do you have a fair chance at winning the 100 game regardless of who starts?
- If your opponent says "81" what is the best choice for you?
- If your opponent says "89" can you still win?
- What should be your strategy when you reach the "80"s in order to win?
- If you were limited to 5 numbers higher than the previous what would be the best number to aim for to ensure that you win the game of 100?

This completes the section dealing with mathematical games and their impact on learners' mathematical thinking. Teachers can also present mathematical tasks which require extended thought and reasoning in the form of games as explained in the next subsection.

6.6.1 Tasks presented as games

Mathematical tasks can also be presented to learners as games and positively stimulate their mathematical thinking. An example from Watson, De Geest and

Prestage (2003) is for teachers to write a set of numbers on the board, for example 21; 4; 1; 5; 5 and 3 and to challenge learners to find as many ways as possible to get 21 by using the basic operations (+, -, x and /) and all six numbers. Learners can work alone or in teams. Learners can be presented with a different set of numbers and a different target number to attain when all possible combinations have been identified.

Learners can also be presented with a task in the form of a pyramid structure. When presented with a pyramid structure in which some cells are left blank, learners can be asked to complete the missing cells. They could also be asked to complete the next level of the pyramid. They should also identify the rule that governs the pyramid structure. When learners have succeeded in completing the structure and know the rule according to which the cells were filled, they can be asked to develop some pyramid challenges of their own. These pyramid challenges developed by the learners can be used to stimulate mathematical thinking and class discussion.

Example of a pyramid structure

22	?	13
29	20	
29		

The aforementioned discussion on games that can be used to stimulate mathematical thinking highlights that learners' can have fun doing mathematics and at the same time acquire essential higher level mathematical thinking skills.

To summarise this section a list of actions that teachers should keep in mind during lesson planning and presentation to actively engage their learners in mathematical thinking follows (Makofsky 2009 & 2010; McCosker & Diezmann 2009; Seay 2009; Walshaw & Anthony 2008; Way 2008; Ban Har 2007a; Lim 2007b; Mahoney 2005; Marcut 2005; Sanchez & Ice 2004; Watson, De Geest & Prestage, 2003; Dunlap, 2001; Henningsen & Stein 1997; Potts 1994):

- Encourage learners to participate
- Create opportunities for learners to justify their answers
- Promote both logical and creative thinking

- Challenge learners to suggest alternative methods to solve problems
- Encourage learners to demonstrate their solutions/methods to the rest of the class
- Use questions effectively
- Include variation
- Introduce games
- Make the classroom setting conducive to mathematical thinking

The aforementioned list of teacher actions is in line with the daily teaching activities prescribed for South African teachers teaching grades four to six in the Foundations for Learning Campaign (as described in section 3.6). The following daily teaching activities expected from South African teachers can be linked to the advancement of mathematical thinking:

- including problems with real life contexts for learners to solve;
- practicing problem solving through interactive group or pair work where learners engage with a problem or challenging investigation and can apply what they have learnt in the lesson;
- encouraging learners to try out different ways to solve problems;
- giving learners the opportunity to share and explain their thinking, methods and answers;
- including questions requiring higher order thinking and the solving of word problems in particular.

The data collected from the sample has revealed that they experience problems with the advancement of mathematical thinking as part of their daily teaching activities. As such the next subsection will investigate possible obstacles that may hinder teachers when they want to encourage learner engagement through mathematical thinking.

6.7 Obstacles that hinder mathematical thinking

According to Lim (2007b), Ban Har (2007a), Lim and Hwa (2006) and Katagiri (2004) several obstacles that hinder teachers in their attempts to promote learner engagement through mathematical thinking have been identified. These include the following:

- a lack of knowledge and understanding from the teachers' side as to what mathematical thinking is;
- a lack of understanding from teachers regarding the importance of mathematical thinking;
- a lack of knowledge on how teachers are supposed to implement mathematical thinking in their classrooms;
- the use of learning material, for example worksheets, that requires prompt responses;
- pressure experienced by teachers to complete the prescribed work that the syllabus demands from them in the time allowed;
- administrative tasks that add to the demand on teachers' time;
- a feeling from the teachers' side that lessons aimed at promoting mathematical thinking takes extra preparation time and also uses up more lesson time due to the participative nature thereof than teacher-centred content-driven lessons.

The obstacles raised here will be kept in mind during intervention development.

6.8 Conclusion

The grounding of the mathematical thinking intervention is hereby concluded in so far as the mathematical thinking component is concerned. However, as stated in the introduction to Chapter 6, a second investigation had to be undertaken as part of the grounding of the intervention. This focus of the second investigation is adult learners, as they constitute the research sample. Chapter 7 thus focuses on the intervention sample as adult learners.

CHAPTER 7 THE INTERVENTION SAMPLE AS ADULT LEARNERS

7.1 Introduction

The audience, being the sample for which the mathematical thinking intervention is designed, is constituted in totality of adult learners. As such, this chapter focuses on the intervention sample as adult learners. A second literature review was required to gain insight into adult learners and their learning. Various aspects concerning adult learning, including possible resistance factors that had to be taken into account during intervention development, needed to be identified. This detailed information on adult learning is central to intervention development.

According to Cynthia Starks, executive speech writer from Starks Communications LLC, the undertaking of an audience analysis is the key to effective presentations/interventions (Starks 2010a). The next section focuses on the audience analysis.

7.2 The audience analysis

Starks explained that an audience analysis entails that the developer/presenter of an intervention must determine the characteristics of the intended audience prior to designing the intervention. The audience impacts on both the contents and style of the presentation. Starks (2010b) provided the following list of questions that can be applied as guidelines when performing an audience analysis:

1. Who is your audience?
2. What is the knowledge of the audience on the subject?
3. What is the age, gender and education background of the audience?
4. Why will they attend the presentation/intervention?
5. Where will the presentation/intervention take place?
6. What are the needs of the audience that would be addressed in the presentation/intervention?
7. What can the audience expect to learn from the presentation/intervention?

When the aforementioned list of questions was applied to the intervention sample, it was found that questions 2 to 7 were easy to answer as the information was known. The answers are provided in the following subsections.

Question 2 - audience knowledge on the subject

The research study identified promoting mathematical thinking through learner engagement as the improvement opportunity that needs to be addressed in the current teaching practice of the sample. As such, the knowledge of the sample on mathematical thinking and the promotion thereof was found lacking.

Question 3 - age, gender and education background

The sample is made up of adult women with degrees/diplomas in education who are working full-time in the education profession.

Question 4 – reason for attending

The sample expressed a desire to improve their mathematics teaching practice and volunteered their participation.

Question 5 - intended location of the intervention

The intervention was planned to take place at the Bird Street Campus of the Nelson Mandela Metropolitan University in Port Elizabeth.

Question 6 – needs of the audience to be addressed

The research study found that the sample needed to improve both their knowledge and skill levels on engaging their learners in mathematical thinking. As such, the intervention focused on equipping the sample with knowledge and skills on how to promote mathematical thinking through learner engagement in their respective classrooms.

Question 7 – what the audience can expect to learn

The intervention had dual purposes. Firstly, it aimed to explain mathematical thinking and the relevance thereof in the classroom to the sample. The sample would thus learn the theory and relevance of mathematical thinking. Secondly, the intervention would provide practical examples that could be applied by the sample to encourage mathematical thinking in their respective classrooms. The sample would be equipped with practical tools that they could apply in their respective classrooms to enhance mathematical thinking.

This leaves the all-important first question, namely "**Question 1 - Who is the audience?"** unanswered. Question 1 is answered in the subsection that follows.

The audience

The audience had been identified as adult learners, but clarity on what constitutes the adult learner had not been provided. As such, the question remained: "Who are adult learners and what characterises their learning?" It is therefore imperative that, before intervention development could commence, a further literature study had to be undertaken with the adult learner as focus. During the review it was ascertained how adult learners are defined; how they learn, as well as what aspects impact positively and negatively on their learning. This knowledge is essential to the researcher, in the capacity of intervention developer during the intervention development process. The knowledge would enable the intervention developer to accentuate the factors that draw adults towards learning whilst simultaneously avoiding factors that distract adult learners from learning.

The literature review revealed that there are a number of characteristics displayed by adult learners in general (Blake 2009; Conner 2007; Li 2004; Newman & Peile 2002; Holmes & Abington-Cooper 2000; Dewar 1999; Knowles, Holton & Swanson 1998; Billington 1996; Brookfield 1995; Lieb 1991; Carlson 1989; Zemke & Zemke 1984). Five characteristics that might impact on intervention development are discussed in the following subsections, namely:

- Adult learners have reached maturity;
- Adult learners have a wealth of experience;
- Adult learners have learning intentions;
- Adult learners have competing interests; and
- Adult learners have their own set pattern of learning.

7.2.1 Adult learners have reached maturity

Adults are biologically mature and can reproduce; they have legal capacity and can vote, drive and marry; they fulfil socially responsible roles, for example they work, are spouses and parents (Zemke & Zemke 1984; Brookfield 1995; Knowles, Holton & Swanson 1998:64). Psychologically, adults have formed their self-concepts and know that they themselves are responsible for their decisions and actions in life (Holmes & Cooper 2000; Knowles, Holton & Swanson 1998). As such, adults feel that they are self-directed, autonomous and have the right to make their own decisions (Blake 2009; Lieb 1991). When adults have not participated in training or attended interventions for some time, they may at first feel safe if the presenter treats them as children. This feeling, however, soon passes because they are adults by nature who want to act autonomously (Newman & Peile 2002; Knowles, Holton & Swanson 1998). While initially they may feel dependent and seek definite direction with regard to aspects concerning the learning situation, they will soon seek greater participation and become involved in decision-making, as adults naturally strive for independence.

7.2.2 Adult learners have a wealth of experience

It has been ascertained in the previous subsection that adult learners are mature human beings. As such, they have accumulated a package of experience through their work, previous education and family responsibilities (Zemke & Zemke 1984). Adult learners have, however, not reached the end of a process. They are continuously growing and developing; constantly making sense of their experiences and consistently searching for meaning (Connor 2007). Learning can be described as an ongoing life-long process that is influenced by every other facet of the lives of the adult learners. This includes their physical well-being, their intellectual development, their emotions, relationships and cultural interests (Dewar 1999). By virtue of their adulthood, adult learners possess experience in the form of knowledge and skills that they have acquired throughout their lives and careers (Lieb 1991; Zemke & Zemke 1984). They seek recognition and credit for their prior experience and they also want to assess their own knowledge and skills to discover any inadequacies for themselves (Dewar 1999). There is a downside to this wealth of experience though. Should their experience be devalued or ignored, adult learners might become disappointed, feel rejected and withdraw their further participation. This package of experience that adult learners possess should be utilised effectively during learning interventions. When new learning material can be linked to the existing knowledge and experience of adult learners it becomes easier to see possible relationships and to understand where the new knowledge fits into the scheme of work. The integration of new knowledge with existing knowledge thus makes the learning process easier and more understandable (Holmes & Cooper 2000; Lieb 1991; Zemke & Zemke 1984).

7.2.3 Adult learners have learning intentions

Adult learners have learning intentions. This means they want to know why they have to learn something and what the relevance is of what they are being taught (Blake 2009; Li 2004; Newman & Peile 2002; Holmes & Cooper 2000; Knowles, Holton & Swanson 1998:65; Zemke & Zemke 1984). Lieb (1991) said that interest and selfish benefits are some of the best motivators for adult learners. Li (2004) also listed interest and enjoyment as good motivators for adult learners. They are prepared to learn that what they need to know in order to cope with the demands of their own day to day life situations. Adult learners generally have work responsibilities and family lives that also place a demand on their time. The intervention or training session is not the only item on their agenda. Consequently, they are not prepared to spend time on learning things where they do not see its relevance. Application of the learning material is equally important to adult learners. It can be said that the time perspective of adult learners is one of immediacy of application (Newman & Peile 2002). Adult learners also value instant feedback on their progress and are encouraged by the feedback that they receive (Li 2004). Some adult learners wish to improve their skills and knowledge to become more marketable and competitive in the job market. Others want to advance within their own organisation and value learning as a tool for progression (Blake 2009). The majority of adult learners can thus be described as being task- or goal orientated (Knowles, Holton & Swanson 1998:65).

There are, however, a number of adult learners who pursue learning with different intentions. They are labelled activity-orientated and learning-orientated learners respectively. Activity-orientated learners engage in learning activities for the enjoyment of social contact with other adults who share their interest in a certain topic. They enjoy the atmosphere of the adult training environment, find it fulfilling and thus attend educational interventions for the enjoyment of the activity of learning itself. To them the content of what they learn is not as important as participating and sharing in the learning experience together with other adults. Learning-oriented learners participate in learning because they want to obtain knowledge or to acquire a skill for its own sake. They have inquiring minds and will continue to pursue knowledge of their own accord (Lieb 1991). These learners are not pressured into learning by job demands and are relaxed in the learning situation.

7.2.4 Adult learners have competing interests

For many adults learning takes place on a part-time basis. They experience it as secondary in importance to other aspects in their lives such as their jobs, their families and their social lives (Blake 2009; Newman & Peile 2002; Zemke & Zemke 1984). What adult learners learn during an intervention should add value to their daily lives and should fit into their total scheme of life to be worth the time and effort that they spend. These competing interests that adults have may sometimes become barriers to adult learning. For example, a learner might be worried about the ill health of a family member or might be experiencing financial problems or problems in the work place. Factors such as the aforementioned may distract the attention of adult learners from the learning at hand. Section 7.3 will focus in more detail on resistance factors to act as barriers to adult learning.

7.2.5 Adult learners have their own set patterns of learning

Over the duration of their lifespan adults have developed learning styles that suit their individual needs and purposes (Blake 2009; Clark 2008; Lieb 1991). They have their own mindsets with regard to what helps them to learn most easily, most effectively and in the shortest possible time. This implies that within any group of adult learners, there may be a variety of learning styles represented (Li 2004). Learning styles, in brief, include auditory, visual and kinaesthetic styles Auditory refers to hearing; visual to seeing and kinaesthetic to movement and touch. Learners use different styles for different tasks, but usually show a preference for one style, which is considered to be the dominant style (Clark 2008). Clark (2008) concurs with

Constantinidou and Baker (2002) when stating that there are times when a learner may prefer to use a combination of styles or to switch from one style to another owing to the nature of the task. Having to cater for different learning styles may sound problematic to designers and presenters of adult training interventions. A study undertaken by Constantinidou and Baker (2002:309), however, found that people are able to switch between strategies regardless of their learning style preferences if the task at hand requires it. The importance of visual stimuli in presentations or interventions is also stressed as the inclusion thereof was found to be advantageous to learners. Learners benefited from the inclusion of visual images in presentations irrespective of having a preference for the visual learning style.

Apart from the differences with regard to their learning styles, adult learners also have different paces of learning. This implies that different adult learners are comfortable with learning at different tempos. Whilst for some adult learners the pace at which knowledge is being unveiled may be too slow, others may struggle to keep up. The differences with regard to the pace at which knowledge is understood and at which knowledge unveiling should occur, is an important factor that intervention developers and presenters should be aware of at all times. Proceeding at a pace that is too fast for some adult learners may result in attendees being left behind and experiencing knowledge gaps, whilst proceeding too slowly, on the other hand, can result in learners who become bored and disruptive. With these possible problems in mind, trainers of adult learners have to judge the reactions of their learners to what is being done during the intervention and adjust the pace at which knowledge is unveiled when necessary.

This concludes the discussion on the characteristics of the adult learner. The next subsection focuses on resistance factors that might hamper adult learning.

7.3 Resistance factors influencing adult learners

Factors that could contribute towards a resistance to learning being experienced by some adult learners were broadly divided into the following three categories:

- Emotional factors
- Situational factors
- Personal factors

Within each of the abovementioned categories there are a variety of factors that can individually, or in combination with other factors, serve as barriers to adult learning (Maple 2008; Lieb 1991). Atherton (1999:84) stated that an event or occurrence which might trigger resistance in one adult learner might present no problem to other learners. Resistance factors to adult learning are thus highly personal. A discussion on some of these possible barriers to adult learning follows.

7.3.1 Emotional factors

Emotional factors encompass aspects relating to the attitudes of adult learners and to their perceptions of their own abilities as learners. Some adult learners have barriers to learning that originated in their past (Maple 2008). They might have had a previous negative learning experience in school or at an institution of further learning like university or college. Adult learners with past negative learning experiences tend to underestimate their abilities and, because they overemphasize past negative learning experiences, perform below their real abilities. Fear of failure and damage to their status are other emotional factors that might contribute towards adults having a negative attitude towards training interventions (Atherton 1999:79, 84). This is especially problematic when supervisors and subordinates attend the same training intervention. Supervisors might fear that some lack of knowledge is exposed in front of their subordinates or that their subordinates might outperform them in some way. This, they believe, can be detrimental to their status when they return to the work place. This fear of failure can lead to some adult learners withdrawing, if not physically then emotionally and intellectually from the intervention by withholding their contributions and participation in the process of learning.

7.3.2 Situational factors

Situational factors include the administrative procedures, learning material as well as the physical environment in which the intervention takes place. Administrative procedures refer to all the prior arrangements before attendees arrive for the intervention, the registration process as well as arrangements for teatimes and, if necessary, accommodation. Should adults find any of the aforementioned lacking, they might become apprehensive and develop a negative mindset towards the intervention (Atherton 1999:86). Fellow intervention attendees may sometimes be a cause of frustration as late-comers who interrupt a session can contribute towards feelings of apprehension and negativity. As part of the administrative procedures, the time schedule relating to the intervention should be made available from the outset. The presenter should request that the attendees commit themselves to keeping to the schedule as a sign of respect to both the presenter and fellow attendees.

Where the learning material is concerned, it should be applicable to the aim of the intervention, developed and ready and presented professionally. It has already been established in earlier paragraphs that adult learners are goal-orientated and as such, the presenter should be knowledgeable with regard to the learning material. Preparation on the part of the presenter is of utmost importance as any display of ill-preparedness may result in frustration from the adult learners. They might feel that their time is being wasted and may consequently withdraw themselves emotionally or even physically from the intervention. The contents of the learning material could, in particular, contribute towards learner resistance. Atherton (1999:78) identified the supplantive nature of some learning material as one of the most defining characteristics of resisted learning in adults. The supplantive nature refers to learning material that threatens or that replaces the existing knowledge or skills of the adult learner.

Finally, the physical environment in which the intervention takes place should be conducive to learning. Li (2004) stressed that a positive learning environment in which they feel comfortable is extremely important to adult learners. The necessary electronic equipment like data projectors or laptops should be present and working properly. Seating should be comfortable, lighting adequate and the temperature in the room should neither be freezing nor too hot. Factors like the aforementioned are important to adult learners as they are used to working in such environments and do not like to experience discomfort when attending interventions or training sessions.

7.3.3 Personal circumstances

Personal circumstances relate to matters that might complicate the intervention attendance of an individual adult. Factors like financial difficulties in meeting obligations relating to the intervention; problems with transport or arrangements that have to be made to cater for dependents while attending the intervention, are some examples of personal circumstances that can hinder adult learners. Some of these factors might be especially problematic if the intervention is presented away from the home town of the adult and he/she has to stay over where the intervention takes place.

This concludes the audience analysis of the intervention sample as adult learners. Knowledge gained through the audience analysis will henceforth be applied to the intervention development process.

7.4 Application

The knowledge gained through the literature review and audience analysis will be applied during the intervention development process. The following guiding questions will be used as self-reflection to assess knowledge acquisition:

- What was learned about the prospective audience?
- What that was learned can be applied in the intervention?
- What are possible hindrances that can negatively impact on the application?
- How can planning be made to avoid possible hindrances/problems?

Each of the aforementioned questions will be answered in turn in the following subsections.

7.4.1 What was learned about the prospective audience?

It was learned that adults want to make their own decisions and do not like to be pressured. When pressured, some adults may practise their adulthood by withholding their participation during interventions. This may especially be the case where it is expected from adults to attend work-related training sessions or interventions. Although these adult learners may physically attend the intervention they will be "absent" in their minds. This could be problematic during interventions as it is impossible to force a learner to participate who prefers to stay uninvolved.

Adult learners also bring maturity that could manifest itself in the confidence that they show, in a sense of self awareness and through their problem solving skills. It has furthermore been learned that adult learners are in a continuous process of growth.

They have life experiences, knowledge and pre-conceived ideas that they bring with to training sessions and interventions.

It is furthermore important to distinguish between the different intentions that adult learners have. Goal-oriented learners may experience more pressure and can display anxiety in the learning situation. In contrast, activity and learning-orientated learners who attend interventions of their own accord and for personal pleasure will be much more relaxed. Most adult learners are goal-orientated and partake in interventions because they are experiencing some sort of inadequacy that needs to be corrected. They thus want to be able to apply the knowledge that they gain in the intervention immediately.

It has also been learned that adults have competing interests that also demand their time. Some adult learners may be going through difficult changes in their personal lives or may be experiencing physical or emotional problems that can impact on their learning. The attention of adult learners may sometimes be distracted because other problems overshadow their need to learn.

Finally it has been learnt that adults have different learning style preferences. Some may learn best whilst listening to a lecture being presented to them, whilst others may learn better by seeing images presented to them or participating in physical activities. Although adults may have learning style preferences it has also been learnt that they are capable of switching between learning styles and adapting to the demands placed on them by the intervention. Adult learners also learn at different paces and each learner has a learning pace at which he or she feels comfortable.

7.4.2 What material that was learned can be applied in the intervention?

It was learnt that adults want to make decisions and participate in the teaching-learning process. As such the intervention needed to be an interactive, working together session. Although the intervention activities need to be planned, one has to be flexible and open to the needs and the contributions of the adult learners. One must refrain from being prescriptive, but one has to work with the sample towards the discovery of knowledge and the acquisition of skills. The intervention has to take place in a positive learning environment that motivates the learners; increases their opportunities to learn and helps them maintain appropriate behaviour.

Contributions have to be valued and comments handled with the necessary sensitivity. The knowledge and abilities of the learners needs to be acknowledged and used to the advantage of the whole group. Learners have to be given ample opportunity to ask as well as to answer questions from the facilitator as well as from fellow intervention attendees. Discussions on problems that they expect when returning to their respective classrooms and possible solutions to these problems have to be encouraged. Whilst planning the intervention time needs to be allowed for these discussions. It is important that the intervention activities are planned in such a way that the time of the sample is used both efficiently and effectively.

It is known that the sample consists of goal-orientated learners who are attending the intervention to address an improvement opportunity in their teaching practice. As such they have to be informed at the start of the intervention what they can expect to learn and how the intervention means to improve their teaching practice. They have to know from the outset how the intervention could benefit them on their return to their respective classrooms.

While planning arrangements regarding the intervention, the fact is also acknowledged that there are various other interests that compete for the attention of the sample. Prior to setting the date for the intervention the researcher will meet with the sample and their respective school principals and will, together with them, ascertain the most suitable date for the intervention. The sample will receive due notice of all the arrangements and administrative details surrounding the intervention. During the intervention the researcher will allow for learning style preferences by including auditory, visual and kinaesthetic activities. Asking questions to the sample as well as allowing the sample in turn to put forward their questions would allow the researcher the opportunity to judge the pace at which knowledge should be unveiled. Finally, activities will be prepared to stimulate the interest of the learners and exercises will be devised that can assist both the slower and quicker learners in mastering the intended knowledge and skills.

7.4.3 What are possible hindrances that can impact negatively on the application?

Each sample member participated voluntarily in the research study. No-one was forced to participate at any stage. Consequently it is not foreseen that any sample member will attend the intervention in a negative frame of mind. There are, however, unforeseen personal circumstances that may serve as hindrances, for example illness or worries on the day of the intervention. It is impossible to prepare beforehand for such eventualities and such situations will be handled with care, should they arise.

A problem that needs to be catered for is over-eager participants. One or more of the sample may try to force their views on the whole group or attempt to dominate the intervention by displaying their advanced knowledge. This can have a negative impact on the intervention as other sample members may refrain from offering their input and sharing their experiences. When interventions run longer than the expected time, some of the sample members may become agitated and inconvenienced. Exceeding the time stipulated for the training should be avoided.

7.4.4 How can the researcher plan to avoid possible hindrances/problems?

The sample will be informed well in advance of when and where the intervention will take place. The date will be set only after proper consultation between the sample, their school principals and the researcher. The sample will be advised as to what they can expect from the intervention and what, in turn, will be expected of them. The start and finish times of the intervention will be provided and the intervention facilitator will plan effectively and allow for possible interruptions so as to ensure that the intervention finishes at the time stipulated. The provision of clear directives concerning the intervention will enable the sample to make the necessary arrangements and to prepare them for the intervention.

The required course material needed for the intervention will be developed and printed in time for the intervention. All course material will be waiting for the attendees when they enter the intervention venue. The necessary arrangements with regard to the electronic equipment needed, as well as the venue to be used, will be made in time to ensure the smooth running of the intervention.

During the intervention the following will be done if problems with attendees arise. If it is experienced that a participant acts in a domineering fashion during the intervention, that person will be thanked tactfully for their contribution. Then another participant will be called upon to make a contribution or to comment on the input of the first speaker. Considering that the sample consists of adults with relevant educational experience who have offered their assistance of their own accord the intervention facilitator would feel comfortable to ask them, by name, for their contributions, if they have any to offer. This will allow all sample members contribute to the discussion/intervention whilst simultaneously stemming possible domineering actions. The discussion on how adults learn best and the implications that it has on intervention design is hereby completed.

7.5 Conclusion

A literature review was undertaken to gain insight into two different aspects that both impact on the intervention to be developed. Firstly, a literature review on mathematical thinking was undertaken in Chapter 6 as mathematical thinking forms the focal point of the intervention to be developed. This first literature review provided insight as to what mathematical thinking is and the importance of stimulating mathematical thinking in the mathematics classroom. In short, engaging learners in mathematical thinking holds the following advantages: it values the ideas of learners; explores their answers and provides them with the opportunity for justification; incorporates their background knowledge and encourages communication about mathematics between learners and teachers amongst learners themselves. The literature review furthermore offered practical advice for teachers on what they can do, say and ask to promote mathematical thinking.

Secondly, in Chapter 7, literature on adult learning was perused to ascertain how adult learners learn. This is particularly relevant to the intervention development as the intervention attendees are all adults. How adults learn and what impacts both positively and negatively on their learning are important factors that need to be taken into account during the intervention development process. The learning acquired through the literature studies is applied during intervention development and application.

The next chapter, Chapter 8, focuses on the intervention development process and on the application of the intervention.

CHAPTER 8 INTERVENTION DEVELOPMENT, APPLICATION AND REFLECTION

8.1 Introduction

Chapter 8 focuses on the mathematical thinking intervention. Intervention development is discussed, followed by the intervention application and finally the intervention reflection. The purpose of the intervention was twofold. Firstly, it had to generate awareness under the intervention sample of what mathematical thinking is and stress the importance thereof when teaching mathematics. Secondly, it had to provide the intervention sample with knowledge, skills and tools, including games, that they could apply in their classrooms to encourage learner engagement through mathematical thinking. During intervention development the knowledge gained in the previous chapter on both mathematical thinking and adult learners was applied.

8.2 Intervention development

As designer and presenter of the intervention I had the responsibility to create the ideal climate in which the sample could master the required knowledge and skills. Taking cognisance of the knowledge gained in the previous chapter on adult learners and how they learn optimally, the intervention was designed as an interactive workshop. This format allowed the sample the opportunity to participate and contribute towards knowledge acquisition. As a starting point to the workshop a PowerPoint presentation was developed in conjunction with an Intervention Guide that structured the intervention. The PowerPoint slides were designed to display the broader and most important aspects that were covered during the intervention whilst the Intervention Guide contained detailed information.

8.2.1 The Intervention Guide

The Intervention Guide was designed to guide the intervention process throughout, to explain all activities and document conclusions reached during the intervention. The Intervention Guide could furthermore be used as a reference for revision upon the sample's return to their classrooms. As such the guide contains detailed information pertaining to mathematical thinking and the promotion thereof, questions to be answered during the intervention as well as instructions of tasks to be completed. Ample space is made available for the sample to make notes and write down their own as well as fellow participants' contributions in the guide book. The Intervention Guide is attached as Appendix K.

8.2.2 The PowerPoint presentation

The PowerPoint presentation was designed to visually represent the main aspects dealt with during the intervention. During the arrival and welcoming of the participants to the intervention a slide (slide 1) displaying the topic, presenter and location of the intervention was displayed. Hereafter the PowerPoint presentation was initiated with a humorous slide (slide 2). This slide, displaying a cartoon strip, illustrates how differently a teacher and a learner can look at the same problem. The decision to use a cartoon strip as an ice breaker was made because the sample members met each other for the first time as a group at the intervention. Prior training experiences and literature have taught that sharing a laugh together helps course attendees to feel more comfortable in unfamiliar surroundings (Exforsys Inc., 2011). The ice breaker slide was furthermore used to focus the participants' thoughts on mathematical thinking, it being the intervention topic. They were requested to look at the situation depicted in the cartoon strip and to then answer the question typed underneath the cartoon strip namely "What does the cartoon strip tell you about mathematical thinking?" The participants had to write their personal views in the allotted space where after they were given the opportunity to share their views. The words "We look at problems differently..." were added as heading to the slide and participants were asked if they concur with the view or not. Time was allowed for sharing of difference or concurrence.

The next two slides (slides 3 and 4) displayed information pertaining to the practical aspects of the intervention, namely the materials needed to partake in the intervention and the agenda for the day. It is important to show these slides at the start of the intervention, as the participants feel more at ease when they know exactly what they need to partake and what is expected of them during the intervention. Knowing the structure of the intervention contributes to the participants feeling safe and at the same time provides them direction, like a road map, of where they are heading and what the ultimate destination or goal is.

As introduction to the intervention the next slide (slide 5) proceeded to address the question “Why is mathematical thinking the intervention topic?” The answer, that encouraging mathematical thinking was identified as the one shared improvement opportunity in the sample’s mathematics teaching practice profile, was displayed as follows in red on the slide.

Shared Improvement opportunity:

Indicator 3: “Learners are actively engaged in the learning process by encouraging mathematical thinking.”

To clarify the identification of Indicator 3 as the single shared improvement opportunity a detailed explanation is provided in the intervention guide. In addition to the written explanation, a copy of the table in Table 5.2, displaying the improvement opportunities in a tabularised fashion, was included at the back of the intervention guide.

The next slide (slide 6) focused on gaining a conceptual understanding of mathematical thinking. Three questions that served as guidelines directing participants towards acquiring this conceptual understanding were displayed on the slide, namely:

1. “What is Mathematical Thinking?”
2. “Why is it important to promote Mathematical Thinking”
3. “How can Mathematical Thinking be promoted in the classroom?”

Slide 7 focused on the first question and had “What is Mathematical Thinking?” as header. The body of the slide consisted of three points that directed the participants thinking and contained instructions that they had to follow. The three points were:

- Brainstorm what you regard as Mathematical Thinking;
- Evaluate literature definitions and compare with your own definition;
- Compile an informed definition of Mathematical Thinking

The Intervention Guide provided detailed instructions concerning each of the three points. In dealing with the first point, “Brainstorm what you regard as Mathematical Thinking” the Intervention Guide explained that the participants had to brainstorm, that is provide all possible responses that they could think of. All contributions would be written down without exception. When no more responses were forthcoming, the participants had to discuss the contributions written on the flipchart by the facilitator and agree on a cohesive answer of what they as a group regarded as mathematical thinking. The researcher/facilitator gave guidance to the discussion only when requested and mainly acted as the scribe. In particular the researcher/facilitator refrained from pushing any of her views on the participants. The concurred definition that the group arrived at was written on the flip chart.

To deal with the second point on Slide 7 “Evaluate literature definitions and compare with your own definition”, a number of definitions taken from the literature were displayed in the Intervention Guide. The participants were requested to compare their definition with the definitions from literature. They then had to compile an informed definition of Mathematical Thinking.

Slide 8 displayed the researcher/facilitator’s definition of Mathematical Thinking that was compiled after reviewing literature on Mathematical Thinking as part of the comprehensive research study. The slide has “Mathematical Thinking?” as header. The body of the slide contains the researcher’s definition, namely

Mathematical Thinking is:

- the application of fundamental mathematical knowledge
- to solve new problems
- through reasoning, explaining and justifying possible solution strategies.

Slide 9 had “Why should we promote mathematical thinking?” as header and the body of the slide consisted of a list of seven reasons. Slide 10’s header read “Ways to promote mathematical thinking” and the body contained suggestions that teachers can use to promote mathematical thinking. The Intervention Guide in both instances specified group discussions to be held. The groups consisted of two participants per group and the aim was to determine the participants’ viewpoints with regard to the questions (2) “Why do you think it is important to stimulate mathematical thinking”; and (3) “How can we stimulate mathematical thinking?” respectively. One member from each group received the opportunity to report back and share their group’s responses with the larger group. After both groups had reported back, all responses offered were discussed. Information from the literature that provided insight into both questions was included in the Intervention Guide to clarify matters for the participants. This enabled them to compare their own views with that contained in the literature and to arrive at clear answers for the questions why it is important to stimulate mathematical thinking as well as how it can be done. The Intervention Guide also contained a set of example questions that teachers can use to promote mathematical thinking.

The next slide, slide 11, and corresponding section in the Intervention Guide, dealt with mathematical thinking relevance with regard to the Daily Teacher Activities during mathematics time (grades 4-6) as specified by the Foundations for Learning Campaign. Special attention was paid to the fourth activity listed, namely Problem solving. The attendees were requested to individually study the contents included in the activity and to comment on the correlations (and differences if there were any) that they noticed between the said activity and the suggestions on how to stimulate mathematical thinking. Comments regarding correlations and differences had to be written in a table provided in their notes. Once the individual tables had been completed, each attendee gave feedback. From the feedback a summative table was drawn up to illustrate the correlation between the expected teacher activities and activities that stimulate mathematical thinking. This concluded the section of the workshop that focused on providing the attendees with the necessary knowledge to gain a sound conceptual understanding of mathematical thinking.

The next section of the intervention focused on a sampling of mathematics games that could be used by the participants to stimulate mathematical thinking in their classrooms. The rules pertaining to each game, as well as why the particular game was good for promoting mathematical thinking, is contained in the Intervention Guide whilst the PowerPoint slide, slide 12, only displayed the names of the games to be sampled. The sample was provided with game boards, dice, pens and paper necessary to play each of the games. Whilst playing the different mathematics games the participants could ask any questions that arose and offer suggestions that they had.

Slide 13 deals with expected obstacles and challenges on return to the classroom. Participants are invited to offer possible solutions. The final slide discusses the observation that is to follow in a month's time to measure the impact, if any, of the intervention on the sample's teaching practice.

The presentation is attached for perusal in electronic format. In addition rules of play for various games that can be used to stimulate mathematical thinking were also provided in electronic as well as paper format. The game boards were developed and all additional material needed for each of the games were acquired. During the planning of the time frame of the intervention ample opportunity was allowed for the sample to offer their input and to ask questions. As stated in the previous paragraph time was also set aside for a practise session with the mathematical games provided to the sample.

8.3 Intervention application

Four of the five sample members attended the intervention at the Bird Street Campus of the Nelson Mandela Metropolitan University on Friday 11 March 2011. The intervention started at 9:00 and was finished by 16:00. A lack of personnel at her school due to illness prevented one sample member from attending. This was unfortunate and unplanned as all sample members had confirmed their intention to attend the day before the intervention.

As previously stated the intervention took the form of an interactive workshop. A PowerPoint presentation was used to display the main points dealt with during the intervention whilst an intervention guide served as both a source of information and a work book for the sample to make notes. After the sample was welcomed and the administrative details concerning the intervention explained, the ice breaker followed. The sample was requested to look at a cartoon strip displayed in their intervention guides as well as on the screen. The question that they had to answer was typed underneath the cartoon strip namely "What does the cartoon strip tell you about mathematical thinking?" The sample wrote their answers in the allotted space where after they were asked to share their views with the whole group. The researcher as facilitator wrote their individual responses on the flip chart and a group discussion followed. The responses were as follows:

Sample member 1

The teacher wants the answer to a mathematical concept, while the learner is looking at the problem in a wider way.

Sample member 2

The teacher wants the answer to an abstract concept, but the learner did not understand what the mathematics described in the problem is.

Sample member 3

The teacher starts at the abstract level and is missing the concrete. The learner does not understand the problem and needed things like real counters to work with.

Sample member 4

The teacher is missing the concrete when putting forward a mathematics problem and the learner did not understand what the teacher wanted at all.

The flip chart sheets were put up on the wall and the discussion that followed led the group to the realisation that the teacher in the cartoon strip looked at the problem in a different way to the learner. They did not see the same thing when they looked at the same problem. The heading "We look at problems differently" was then added to the PowerPoint slide. The sample agreed that this is indeed so and that because people (teachers and learners) look at problems differently there will be different ways to solve a given problem.

Next the sample individually answered the question: "How does the fact that people look at problems differently impact on mathematics teaching?" in the space provided in their intervention guides. They verbally expressed their opinions and the following responses were noted:

Sample member 1

Applying knowledge to the learners so that they can think; reason and investigate using different methods to solve the problem. There are slow learners and quick learners and easy ways and more difficult ways to solve problems. The teaching should cater for slow and faster learners.

Sample member 2

Intellectual abilities differ, some kids may confuse numbers, e.g. 6 and 9. Others want to finish the task quickly without thinking about what they write.

Sample member 3

Children have different backgrounds, gender, age, teachers should try to present lessons keeping this in mind also intellectual ability, language barriers and terminology.

Sample member 4

- Ask the same question in a variety of ways;
- Get feedback from the learners to understand how they are thinking about the problem;
- Ask the weaker learners to see if they grasp the meaning of the question.

A discussion of the individual responses lead the sample to the realisation that when they are teaching mathematics it is important to acknowledge that what seems to be the “best” or “easiest” way to solve a problem for one learner will not necessarily be the “best” or “easiest” way for all the other learners. As such teachers should encourage their learners to offer different solutions to a given problem. Problems should also be reworded to assist learners who do not understand.

The next part of the intervention focussed on the identification of the intervention topic. The results of the data analysis, that identified mathematical thinking as the indicator to be addressed, was explained verbally and backed-up by the data presented in the table in pp. 51-58 of the intervention guide (Appendix K refers). The sample did not have any further questions.

After tea the focus fell on a conceptual understanding of mathematical thinking. Three questions guided the process, namely “What is mathematical thinking?”; “Why is it important to promote mathematical thinking?” and “How can mathematical thinking be promoted in the classroom?”

The sample brainstormed the first question “What is mathematical thinking?” and their responses were written on the flip chart. Ten possible responses were offered, namely:

Mathematical thinking is:

- “getting the learner to actually think about what they were asked”;
- “something that requires more than a one word answer”;
- “exposing learners to different ways to get to an answer”;
- “bring real life to the classroom: relate maths to real life”;
- “making learners to think abstractly”;
- “bringing a fun element to maths”;
- “being flexible and not too rigid”;
- “taking away the fear factor of maths”;
- “letting learners investigate things on their own” and
- “bring up contentious questions where learners have different opinions”.

The flip chart sheets were put up on the wall and the sample discussed the opinions offered and came up with the following group definition of mathematical thinking:

What is mathematical thinking?

To stimulate the learner to think about a concept in a variety of ways (out of the box) without being fearful and allowing flexibility and a fun element. Mathematical thinking must stem from the practical application of lessons.

Definitions of mathematical thinking from the literature included in the intervention guide were discussed in the group. The sample was then granted the opportunity to revise their definition with hindsight of what they learnt from the literature. Their revised and final definition reads as follows:

Sample's final definition

Mathematical thinking must stem from the practical application of lessons and a fundamental understanding of the concepts to stimulate the learner to solve problems in a variety of ways (out of the box) without being fearful and allowing flexibility and a fun element.

The focus next shifted to why mathematical thinking must be promoted. The sample was divided into two smaller groups of two members each. Within each small group reasons why teachers should promote mathematical thinking were written on the flip chart sheets provided. As soon as the groups had finished writing down their reasons, one group member put the flip chart sheet up on the board and presented their comments. The reasons provided (verbatim) were as follows:

Group 1: Why should we promote mathematical thinking?

- To encourage learners to have a positive attitude in maths.
- Create awareness that it involves our daily lives.
- They must know their mathematical concepts.

Group 2: Why promote mathematical thinking?

- To encourage learners to be able to do reasoning, problem solving and critical thinking of any mathematical concept
- so to be able to express themselves adequately and thereby solve the problem.

Hereafter reasons to promote mathematical thinking from the literature perused were discussed. The total sample was then granted the opportunity to judge both the literature contributions and their groups' reasons for promoting mathematical thinking with the aim of clarifying why they, as sample group, regard the promotion of mathematical thinking in the classroom important.

The revised and final reasons for promoting mathematical thinking

To create awareness that mathematical thinking involves our daily lives and to encourage learners to be able to reason, solve problems and think critically of any *mathematical concept so as to be able to express themselves adequately and thereby solve the problem.*

Attention shifted to the third and final aspect of gaining a conceptual understanding of mathematical thinking, namely "How can mathematical thinking be promoted in the classroom?" The sample was again divided into two smaller groups of two members each. The group members had to differ from the previous group discussion. Within each small group ways in which teachers could promote mathematical thinking were written on the flip chart sheets provided. When the groups had finished writing their contributions, one group member put the flip chart sheet up on the board and presented their thoughts. The different groups' contributions (verbatim) were as follows:

Group 1

- By discussing verbally each step of the practical implementation of the lesson by playing games, etc.
- By giving a variety of answers to the problem by brainstorming with each other and in groups.
- By expressing their answers in their groups in a logical way.

Group 2

- Use different teaching strategies. Group work children learns better from their peers/using mathematical language.
- Use different teaching aids/tools to do practical activities.
- Allow learners to ask logical questions.
- Teacher should ask open-ended questions.
- Mathematics should be "FUN", e.g. use memory games/number games.

Examples of how mathematical thinking can be promoted according to literature perused were discussed. The sample hereafter evaluated both the suggestions made in the literature as well as by the two groups and concluded that as the sample group they regarded the following as ways to promote mathematical thinking in their classrooms. One member wrote the final suggestions on the flip chart and it was put up for display on the whiteboard.

Sample - How can we promote mathematical thinking in the classroom

- By discussing verbally each step of the practical implementation of the lesson by playing games, etc.
- Teacher should ask open-ended questions and allow learners to ask logical questions.
- By giving a variety of answers to the problem by brainstorming with each other and in groups.
- To express their answers in their groups in a logical way.
- Mathematics should be "FUN", e.g. use memory games/number games.

Hereafter examples of questions that teachers can ask to promote mathematical thinking in their classrooms received attention. This is important as asking open-ended questions were included in both the literature and by the sample as a way in which mathematical thinking can be promoted. A selection of possible questions to ask were listed in the Intervention Guide and these questions and their abilities to encourage mathematical thinking were discussed by the sample.

The final session before lunch focussed on the relationship between the Daily Teacher Activities during Mathematics time for grades 4 to 6, as prescribed by the Foundations for Learning Campaign and mathematics thinking. For the benefit of one sample member who has since the beginning of 2011been moved to grade 3 the Daily Teacher Activities during Numeracy time grades 1-3 was also provided. The sample was requested to individually compare the actions prescribed by the Daily Teacher Activities, as provided in a table in the Intervention Guide, with the knowledge that they had acquired on mathematical thinking. In relevant columns in the table sample members had to indicate whether the specified Daily Teacher Activities promote mathematical thinking or not. When the individual sample members reported back to the larger group it was found that they were unanimous in agreeing that the Daily Teacher Activities as prescribed do indeed support mathematical thinking.

The sample concluded: As intervention participants we concur that mathematical thinking has a part to play in the Daily Teacher Activities for grade 4 mathematics.

This concluded the theoretical component of the intervention. The session after lunch focussed on games that can be used to promote mathematical thinking. The sample was provided the opportunity to practically play the different mathematics games and had a thoroughly enjoyable time doing so. Sample members were granted the opportunity to play each game a couple of times to gain confidence and to help them in explaining the rules of the game to their learners. After they were confident that they understood the game, ways in which each game could be used to enhance mathematical thinking were discussed. Questions that can be asked to enhance mathematical thinking are listed after the games in the Intervention Guide. Discussions were held to answer each of these questions as the sample needed to know the relevance and answers to questions if they are to ask them to their learners. The different games played and discussed were:

- Game of 9 cards (version 1 and version 2);
- Contig (elementary and advanced);
- Number Neighbourhood;
- Game of Nim;
- The 100-game; and
- Number Mastermind

The sample could ask questions about the games played at any stage and were confident that they could play and explain the games at the end of the session.

The intervention concluded with a session where expected obstacles, that could hamper the sample in applying what they had mastered during the intervention, were identified and pre-emptive solutions suggested. The sample was afforded the opportunity to think about the individual circumstances in which they teach mathematics and to identify possible obstacles. When all sample members were finished, a list of obstacles was written on the flip chart for discussion. The complete list of possible obstacles included:

- The number of learners in the class
- Space and overcrowding
- Noise levels
- Language barriers

- Listening skills
- Teaching aids
- Absenteeism
- Time factor
- Transport

Problems related to space, noise levels and time were general, whilst problems related to the number of learners in class, language barriers and listening skills, teaching aids, absenteeism and transport were limited to the sample members with exceedingly large classes and who have learners speaking a different mother tongue than the language of instruction.

After a discussion of the problems possible solutions were suggested by the sample. To address the problems with the number of learners in the class, space and overcrowding as well as high noise levels it was suggested that the learners be divided into groups based on competency – while more competent learners can play mathematics games, teachers can focus their attention on learners who are experiencing problems with the mathematics concepts being taught. Limiting the number of learners who participate in game play at a particular time will address all of the aforementioned problems as with only some learners participating at a particular time, space will be less of a problem and the noise levels will also be reduced. As the teacher will be instructing some learners, the other will have to keep the noise level down. Before learners commence playing mathematics games, the teacher should warn them that making a noise would result in game time being stopped. A desire to also play mathematics games can also serve to motivate some learners to master the basic mathematics concept being taught in order to move to a more competent group and being allowed to play mathematics games.

Where there are language barriers it was suggested that a learner or another teacher who is fluent in both the learners' home language and the language of instruction be asked to assist the teachers in explaining how the mathematics games work.

Problems experienced with learners due to a lack of listening skills can be addressed by asking learners to explain how they understand a specific question put to them.

They can also be asked to rephrase mathematics questions in their own words. This will assist the teacher in determining whether they have listened and understood the initial question. To help learners focus and listen when fellow learners are explaining their thinking, learners can be asked to restate the explanation in their own words.

Absenteeism and transport problems experienced at some schools are general problems that hamper the learners' total learning experience. Suggesting solutions to these type of problems fall outside the scope of this research study.

In conclusion the teachers were encouraged to, on their return to their classrooms, apply the knowledge, skills and attitudes that they acquired during the intervention in their mathematics lessons. They were each given a complete set of the mathematics games used during the intervention as well as dices and markers. The teachers were requested to provide their learners with the opportunity to play the mathematics games that were handed to them during mathematics lessons where appropriate.

It was agreed that the researcher would make the necessary arrangements with the respective school principals for a follow-up classroom observation. The aim of this second classroom observation was to establish the impact, if any, that the intervention had in addressing the improvement opportunity previously identified. This second classroom observation necessitated the adaptation of the earlier classroom observation tool. Where the initial observation tool included eleven indicators and eighty six evidences of mathematics teaching practice, the second observation tool had to focus on the indicator identified as the shared improvement opportunity and the related evidences. Subsection 8.4 deals with the development of the second classroom observation tool.

8.4 Developing the second classroom observation tool

In Chapter 5 it was explained that when the five individual profiles had been built, a total number of eighteen improvement opportunities were identified. Ten of these were limited to one sample member's profile. Consequently they had been excluded as shared improvement opportunities. That left eight shared improvement opportunities. Four of these dealt with issues relating to group work. As they were circumstance-specific, the result of overcrowded classrooms and difficult learner compositions, it was decided that an intervention would have no impact in addressing these improvement opportunities. Consequently the focus was shifted to the remaining four shared improvement opportunities. Three of them were directly linked to one indicator, whilst the fourth also supported the same indicator. A single Indicator, “Learners are actively engaged in the learning process by encouraging mathematical thinking” was thus identified as the shared improvement opportunity. This became the focus of the intervention. Once the intervention had been completed, there was a need to assess its impact on the shared improvement opportunities that it strove to improve.

To determine the intervention impact the sample needed to be assessed against the shared improvement opportunities addressed in the intervention. The four shared improvement opportunities that supported the indicator constitute the criteria in the classroom assessment tool. They are:

- “Asking questions requiring high level thinking skills, e.g.: “How did you work that out?”
- “Asking the learners for alternative ways to address mathematical problems.”
- “Intervening in a well-timed and positive way to give direction to learners’ mathematical discussion.”
- “Making use of games and activities for exercising mathematical thinking and mental calculation”.

The observation tool titled “A classroom observation instrument for observing mathematics teaching practices that encourage mathematical thinking in primary schools” was developed. The tool is attached as Appendix D (vii). Space was created where information regarding the topic of the lesson observed, the lesson duration and the date of the observation could be completed. The tool was designed in the form of a table consisting of seven columns. Starting from the far left the first column was titled Indicator, with adjacent to it the Evidences column. The one shared indicator where the sample needed improvement was entered in the Indicator column, namely “Learners are actively engaged in the learning process by encouraging mathematical thinking.” The Evidences column consisted of the four shared improvement opportunities listed in the aforementioned paragraph. In the centre of the table there were two columns with the headings “Yes” and “No” respectively. These columns were used to indicate whether the evidence was observed, in which case a check mark was made in the “Yes” column, or not observed, which resulted in a check being made in the “No” column. Should evidence contrary to that which would encourage mathematical thinking be noticed, “No” was also checked. The column to the right of “No” was labelled Remarks. This is where the lesson observer wrote down actions and words used by the sample member that substantiated either the “Yes” or the “No” that had been ticked. The two columns on the far right were titled Visual and Auditory. Here the observer could indicate whether the actions/words observed were in the form of auditory comments, visual actions or both.

This concludes the development of the second observation instrument, designed to assess the sample’s teaching practices that are used to encourage mathematical thinking. The next section discusses the second classroom observations undertaken and analyses the data collected. The sample members’ mathematical thinking teaching practices before the intervention was attended is compared with their teaching practices after they participated in the mathematical thinking intervention. A reflection on the intervention as experienced by both the researcher and the sample follows.

8.5 Intervention reflection

The reflection is twofold. On the one hand the researcher’s experience of and observations on the intervention are described. On the other hand the sample members’ experiences and comments are reviewed. The sample members’ responses were obtained via a rubric that they completed during the second observation visit. A discussion on the rubric design is included in the subsection dealing with the sample’s reflection on the intervention.

8.5.1 The researcher’s reflection

The researcher experienced the intervention as being meaningful and adding value to the attendants. It was a disappointment that despite all five sample members confirming that they would indeed attend the intervention, one sample member was unable to attend on the day as there was a teacher shortage at her school. The shortage was due to unexpected illnesses and it was unforeseen that she would not be able to attend. Nevertheless the other four sample members did attend the intervention and were able to learn from the material presented during the intervention as well as share from their teaching experiences. As the researcher I am of the opinion that there were the following positives forthcoming from the intervention:

The sample gained both knowledge and skills with regard to mathematical thinking and the promotion thereof in their classrooms that they did not possess before attending the intervention. Knowledge included what is meant by the term mathematical thinking, if and why it is important to promote mathematical thinking as well as practical examples of what can be done in the classroom to promote mathematical thinking. It is important to note that this knowledge was not only arrived from literature sources, but also from the sample's own contributions based on their teaching experiences. Skills mastered during the intervention include learning to play different games that can be used to promote mathematical thinking as well as the ability to ask questions about each game that stimulate mathematical thinking.

Throughout the intervention the sample was allowed opportunities to share their personal knowledge and teaching experience. This was done in line with teaching principles pertaining to adult learners that indicate that adult learners want to participate and want to contribute from their wealth of knowledge and experience. During the intervention sample members mentioned that they valued the fact that the intervention was not a one-sided affair where they just had to sit and listen to a lecture. The participative nature of the intervention was definitely experienced as positive by the researcher as well. There were different aspects pertaining to mathematical thinking that sample members addressed in two small groups. They had to talk to one another, explain their thinking and also write down and present their group's answer to the intervention participants as a whole.

The sample's thinking was enhanced throughout the intervention as they first had to formulate and present their own answers to problems, thereafter they were presented with information from the literature and finally they had to compare their initial responses with the new knowledge gained from the literature and make adjustments where necessary. This process also stimulated discussion and led sample members to motivate their views.

The sample realised that teaching mathematics could also be fun. During the game play sessions the sample members themselves played different games that stimulate mathematical thinking and enjoyed it thoroughly. Game play led the sample to realise the importance of learners possessing the basic numeracy skills, thus being able to add, subtract, multiply and divide correctly, as it is impossible to play the games without the basic knowledge being in place.

Finally the sample was allowed the opportunity to voice their personal concerns on things that could prevent them from fully implementing what they had learnt during the intervention in their own classrooms. This created the opportunity for sample members to share their own classroom situations, however problematic it might be due to large learner numbers or language problems. Just the fact that they could share their situations with one another led to some sample members realising that their situation is not as bad or as unique as they thought it was. Sample members attempted to assist one another by offering advice to address some of the concerns mentioned. They also shared their telephone numbers and said that they wanted to keep in touch after the intervention. This in my view as researcher was a very positive, if unexpected, outcome of the intervention. The intervention brought together four teachers from different schools in different areas of Port Elizabeth who had no previous contact. During the intervention they learned and shared together and this led to them sharing phone numbers and expressing the desire to keep in contact and to motivate each other after the intervention.

In so far as the administrative arrangements pertaining to the intervention were concerned, I was satisfied with all arrangements made and how the intervention played out. Sample members received directions to the venue beforehand, and the four sample members that attended the intervention arrived at the designated venue. The lecture room was well equipped and all learning material was ready and waiting when the first sample member arrived. A separate section of the venue was used during game play and here also everything was ready before sample members arrived. The PowerPoint presentation was used effectively during the ice breaker, to display the main points throughout the intervention as well as to highlight the aspects up for discussion during participative sessions. It is my experience that the intervention guide could be used effectively throughout the intervention and that it contained enough activities to keep the sample members interested. Enough white space was included where the sample members could write their responses. The duration of the intervention was adequate and allowed enough time for the sample to play and thus practice the different mathematical games on offer. Providing a complete set of the mathematics games played during the intervention with the necessary equipment like dice and white board markers to each sample member for use in the classroom is in my view another positive outflow of the intervention. This concludes my reflection on the intervention from a researcher's point of view. A discussion on the sample's reflection on the intervention will follow.

8.5.2 The sample's reflection

It was important to allow the sample time to implement what they had learnt during the intervention in their respective classrooms before asking them to comment on the value thereof. A rubric was designed with the following reflection criteria:

- I enjoyed the participative nature of the intervention.
- The intervention provided me with new knowledge on mathematical thinking.
- The mathematics games that I played during the intervention were enjoyable and can help me to promote mathematical thinking.
- The intervention provided me with questions that I can ask to promote mathematical thinking during maths lessons.
- The intervention provided me with the opportunity to share my knowledge with my peers.
- The intervention created an opportunity for me to learn from my peers.
- The learning material was applicable to the intervention.
- I enjoyed working in small groups and sharing my knowledge.
- I think that the intervention was worth the time that I spent to attend.
- I think that the intervention was a good balance of theory and practice.

Three descriptor levels were used in the rubric, namely 1: Fully agree; 2: Partially agree and 3: Do not agree. The sample members were requested to use the statements contained in the rubric as guidelines to reflect on the intervention. The rubric used for the sample member's individual reflection on the intervention is included as Appendix L. Sample members were asked to complete the rubric anonymously during each of the second round classroom observations which took place six to eight weeks after the intervention. The classroom observations of three sample members were done after six weeks, but due to transport problems impacting on learner attendance the fourth and final classroom observation could only be done when the learners returned to school after the Easter long weekend.

The samples individual responses to the statements contained in the rubric were scanned and are included as Appendix N. A summary of the sample's responses to each of the statements contained in the rubric is displayed in Table 8.1. From the unanimous positive responses of the sample to the intervention it can be deduced that the intervention not only provided sample members with knowledge and skills on mathematical thinking and ways to promote it in their respective classrooms, but also created a platform for sample members to share their teaching experience and to converse with their peers about the challenges that they as mathematics teachers face on an ongoing basis.

Table 8.1: A summary of the sample's reflection on the intervention

Reflection criteria	Fully agree	Partial agree	Not agree
I enjoyed the participative nature of the intervention.	100%		
The intervention provided me with new knowledge on mathematical thinking.	100%		
The mathematics games that I played during the intervention were enjoyable and can help me to promote mathematical thinking.	100%		
The intervention provided me with questions that I can ask to promote mathematical thinking during maths lessons.	100%		
The intervention provided me with the opportunity to share my knowledge with my peers.	100%		
The intervention created an opportunity for me to learn from my peers.	100%		
The learning material was applicable to the intervention.	100%		
I enjoyed working in small groups and sharing my knowledge.	100%		
I think that the intervention was worth the time that I spent to attend.	100%		
I think that the intervention was a good balance of theory and practice.	100%		

Although it is acknowledged that teachers carry a heavy workload, the sample's response to the intervention and the opportunities that it created spoke of a need from teachers' side to connect and share with fellow teachers. The sample is represented by teachers who come from schools in different geographical parts of the Nelson Mandela Metropolitan area. There are differences in terms of learner language composition in their respective classes, the number of learners per class differs and some teach in a language that is not their mother tongue, but despite these differences they connected as mathematics teachers who look similar difficulties in the eye. More opportunities should be created for teachers to learn from one another and to share both problems and successes.

8.6 Conclusions on intervention outcome

The adapted observation instrument [Appendix D (vii)] was used during the observation of a mathematics lesson presented by each of the sample members in their respective classrooms. The lesson contents presented during the observations were a continuation of work that sample members were focussing on as part of their normal mathematics year planning. No special requests were made for lessons that focussed on mathematical thinking. The data collected during these observations is reflected in Appendices N (i) – (iv). Appendix N (v) portrays the combined data relating to the sample's observations. In order to compare the combined pre-intervention and post-intervention data a revised table with the pre-intervention data of the four sample members who actually attended the intervention had to be drawn up [Appendix N (vi)]. This was required due to the one sample member not turning up on the day of the intervention. Consequently there was no after-intervention observation of the sample member's teaching practices available. The pre-intervention data had been removed to prevent skewed data analysis. Comparative before and after intervention data is displayed in a table in Appendix N (vii). The comparison revealed a significant improvement in the mathematics teaching practices of the intervention sample in their engagement of the learners in the learning process by encouraging mathematical thinking.

Conclusions regarding the impact of the intervention on the sample's mathematics teaching practices, based on an analysis of the data assimilated, are drawn in the next subsection.

8.6.1 Intervention impact

An analysis of the data compiled during the classroom observations after the intervention revealed noticeable improvements in teaching practice. There was improvement in the teaching practices in terms of all four shared improvements opportunities identified before the intervention. The improvements are discussed in turn, starting with the improvement opportunities that showed the biggest improvement.

Firstly, “asking questions requiring high level thinking skills” had been identified as an improvement opportunity in all sample members’ teaching practice prior to the intervention [Table 5.2 and Appendix N (vi) refer]. During the observation lessons after the intervention it was observed that all sample members asked questions that required high level thinking skills. This provides evidence that the improvement opportunity had indeed been addressed by the sample’s participation in the intervention. In Section 5 of the Intervention guide the focus fell on questioning techniques and the type of questions that teachers can ask to stimulate mathematical- and higher level thinking in their learners. The sample made use of the following questions that require high level thinking skills during their lessons:

- *“Why do you say that it is a quarter?”* (Lesson on fractions)
- *”Why do you say this is not a square?”* (Lesson on three dimensional shapes)
- *“Now that the fifty card is gone think if there are any other cards on the board that you can use that also gives you fifty?”* (Lesson on breaking up numbers)
- *”Do you agree that Keenan (learner) and I (teacher) are the same age because we have the same birthday month? Why not?* (Lesson on working with a calendar)

Secondly, “asking the learners for alternative ways to address mathematical problems” had also been identified as an improvement opportunity that existed in all sample members’ teaching practice [Table 5.2 and Appendix N (vi)]. However, after the intervention this was no longer an improvement opportunity in any of the sample members. Examples of sample members asking learners to give alternative ways to address a mathematical problem observed during the classroom observations include:

- *“Do you agree with Apiwe? What do you think is the denominator? Why do you think that?*” (Lesson on fractions)

- *"Please explain why this shape cannot be called a cube?"* (In a lesson on shapes learners had to comment why a particular 3D object could not be a cube, but had to be a rectangular prism)
- Asking learners to suggest alternative ways in which numbers can be built, e.g. *"Can you suggest other ways in which we can build the number 50/200/500?"* Learners participated eagerly and supplied a variety of options. (Lesson on breaking up numbers)
- *"Can we only use months to determine who are the same age? Who of you still says Yes." Please explain why you say Yes.* (Lesson on working with a calendar)

Thirdly, seventy five percent (3/4) of the pre-intervention sample had as improvement opportunity in their teaching practice "making use of games and activities for exercising mathematical thinking and mental calculation" before the intervention. In the observation that followed the intervention all four sample members provided evidence to negate this as an improvement opportunity in their teaching practice.

Examples of how sample members made use of games and activities for exercising mathematical thinking and mental calculation during lesson observed include the following:

- *Learners in their groups physically had to fold a paper into a half, discuss what is meant by the term half, e.g. one out of two equal parts, and they had to write the fraction on one of the halves. They then folded the paper into quarters and mastered the concept of what a quarter is, the same with one eight. Learners learnt what the fractions meant by means of the physical actions and they could see that two quarters are the same as one half*. (Lesson on fractions)
- *Flash cards were used in the beginning of the lesson to test the learners' multiplication and division. The sample member made use of an activity that lead to learners folding a rectangular prism out of a piece of paper to assist learners in discovering what a rectangular prism is*. (Lesson on three dimensional shapes)

- *The sample member made use of number sheets, head counting and dice for practicing calculation skills. She also used number cards that were pasted on the blackboard to practice the breaking up of numbers with learners. Learners had to select cards and build the number requested with the cards*. (Lesson on breaking up numbers)
- *In this lesson a calendar chart was put up on the board. Learners were asked when it was each one's birthday. Their names, ages and their dates of birth were written in under the respective months. The sample member used this real data to explain to the learners how a calendar works. She also asked questions that challenged learners' thinking, e.g. "If two learners that have their birthdays in the same month but were born in different years, are they the same age?" Learners also used a printed calendar with a year displayed on it to perform calculations.* (Lesson on Working with a calendar)

In the fourth place "intervenes in a well-timed and positive way to give direction to learners' mathematical discussion" had been included as a shared improvement opportunity in the generic profile. This improvement opportunity was shared by a sample member who attended the intervention as well as by the sample member who did not attend the intervention. Subsequently Appendices N (vi) and N (vii) show that the improvement opportunity is related to a single sample member. However, when the generic profile of a grade 4 teacher was compiled, it was a shared improvement opportunity and as such was included in the profile. The intervention also addressed the improvement opportunity and it was included in the observation instrument for after-intervention assessment of the intervention sample' mathematics teaching practice. Examples of how sample members intervened and gave direction to learners' mathematical discussion include the following:

- *Learners were asked to fold the paper according to instructions from the teacher, e.g. "fold the paper in two equal parts" or "fold the paper into four equal parts" in their groups. While they were busy the teacher walked between the groups as they worked on folding the paper into different fractions. She monitored what was done and directed learners throughout. When group representatives reported back to the whole class she summed at what they said to the other learners.* (Lesson on fractions)

- *The teacher guided the discussions on the different 2D and 3D shapes and their characteristics by asking the learners questions regarding the shapes "Why would we call this a sphere and not a circle? What is the difference between a sphere and a circle?* (Lesson on three dimensional shapes)
- *The teacher asked learners to suggest alternative ways in which numbers could be built. They responded very well to questions like "Can you suggest other ways in which we can build 200?" Throughout the discussions and suggestions the teacher maintained control and directed learners' thoughts towards the mathematics problem.(Lesson on breaking up numbers)*
- *The teacher used phrases like "do you agree?" and asked other learners to respond on their fellow learner's comments. She also repeated what was said to keep learners focused. She asked many questions to direct learners thinking and also took them back to the calendar to keep the aim of what were investigating in mind.* (Lesson on working with a calendar)

In general it was observed that the after-intervention lessons were participative in nature. This is in contrast to the pre-intervention lessons which were more instructional with learners being knowledge receivers rather than participants in knowledge creation, Another noticeable difference that brought the sample's mathematics teaching in line with the aims set out by the Foundations for Learning Campaign is the focus placed on addition, subtraction, multiplication and division. In three of the sample member's classes the mathematics lesson started with ten minutes allocated to practicing and testing the learners' procedural skills. The mathematical thinking intervention stressed that procedural competency is a prerequisite for higher levels of mathematical and conceptual thinking. In one case the teacher made use of a game involving two dice to help the learners test their multiplication tables. The learners enjoyed this so much that they applauded when she said that would test their knowledge of tables in a similar way the following day. In the second case learners were shown flash cards to test their multiplication and division. It was also noted that in all three of the aforementioned observation classes mathematics lessons were now an hour long – this is also in line with the Foundations for Learning Campaign directives. The intervention sample also made use of activities that involved the learners with the content and kept them interested. Examples of the activities used included the following:

Lesson on fractions

Large paper sheets were handed to learners and they had to fold the sheets as prescribed, e.g. into halves/quarters/eights. This lead to learners not only hearing the terms theoretically but physically experiencing what a half/quarter/one eighth is.

Lesson on shapes

Learners brought containers of different shapes to class. These different shapes and their properties were discussed and the differences between two- and three dimensional objects that have the same form were noted. Examples used include a flat 2D circle cut out from paper in contrast to a 3D ball; a flat 2D paper triangle in contrast to a 3D triangular prism (Brie cheese box) and a square (2D) in contrast to a cube (3D). However, when the correspondences between the names of the 2D and 3D objects were discussed, one was deliberately left out by the teacher (a 3D rectangle). She wanted the learners to create this particular 3D object and discover its name based on association with previously discussed 2D and 3D objects. To reach this objective the teacher handed each learner a sheet of paper. They then received step-by-step instructions from their teacher that lead to each learner folding his/her own rectangular prism. They were amazed at what they folded and when ask what this particular 3D shape is called, there was one learner that made the deduction that it is a rectangular prism – this was correct. She used the knowledge that she had of rectangles (two opposing sides are the same length) and the 3D triangular prism named earlier in the lesson to arrive at this correct deduction. For this she received warm applause and admiration from both teacher and fellow learners.

Lesson on breaking up numbers

In this lesson the learners sat down on the floor of the classroom and acted as physical counters. Learners practised counting in twos; fives and tens. The learner who was given a task e.g. “How many fives do you need to get 25?” calculated the answer by touching the fellow learners’ heads while counting, 5 – 10 – 15 – 20 – 25. The learner realised that because he/she touched five learners’ heads to get to 25 it means that 5 X 5 = 25. Learners also used a dice during multiplication exercises and they all participated eagerly. To practise the breaking up of numbers the teachers pasted number cards on the black board, e.g. 1; 2; 3 ...9; 10; 20...90 and 100; 200...900. Learners were asked to build numbers on the board using the number cards, for example if 624 had to be built, the learner picked the cards showing 600; 20 and 4 respectively. Learners participated eagerly and enjoyed working on the black board.

Lesson: Working with a calendar

In this lesson learners received a calendar handout each and had to work out certain answers using the calendar. Question included “How many months have 31 days?; “Is it true that all months that do not have 31 days have 30 days? Why not? Which month is different?”

Learner’s own birthdays as well as that of the teacher was written on a year calendar and put on the blackboard. The teacher used the learners’ own data to keep their interest and to ask stimulating questions that lead to learners having to think critically. One such question was: “Two learners were born in the same year, 2001. One was born in February 2001 and the other in November 2001. Are these two learners the same age because they are born in the same year? Learners had to explain their answers and a lively discussion followed. Another question that challenged learners’ thinking was: “Two learners in the same class have their birthdays in the same month, but the one is a year older than the other one. What can be the reason? Again learners had to explain their answers, which included one can be a year older because he/she was born a year earlier than the other one. Another explanation was that one of the two already had his/her birthday during their birthday month, but that the other one’s birthday was still coming up in the days left in the month. Both of these explanations are valid ones. The learners also enjoyed this discussion because it was real examples from their own lives with which they could associate.

8.7 Conclusion

This chapter focussed on the intervention development, the application of the intervention and concluded with a reflection on the impact that the intervention had on the sample's mathematics teaching. In so far as the intervention development and application were concerned, sample members have unanimously indicated in anonymously completed assessments that the intervention in contents as well as presentation fulfilled their needs. Not only did it provide them with knowledge and skills, but also with the opportunity to share their expertise and experience with fellow teachers and to learn from one another.

Reflection on the intervention impact from the researcher's side was done by means of observations of the sample teachers presenting a mathematics lesson as part of their normal teaching curriculum. An analysis of the observation data compiled during these classroom observations provided evidence that the intervention succeeded in its aim to provide sample members with knowledge and skills to promote mathematical thinking in their respective classrooms. Sample members asked questions and used scenarios that involved their learners to stimulate mathematical thinking. Where the lessons observed before the intervention were more teacher-centred and instructional in nature, the observations after the intervention revealed a more participative approach in all four sample members' teaching. Learners were actively engaged in the learning process by having to, for example, fold papers to form rectangular prisms as part of a lesson on three dimensional shapes, throw dice when practicing tables, paste number cards on the board when doing break up of numbers, fold sheets of paper in halves, quarters and one eighths when doing fractions and using their own birthdays written in on a calendar to work out problems. Learners were actually eager to continue learning tables and to focus on mastering a next set of tables because it was perceived and enjoyed as a game.

The observations also highlighted that there may be lessons that due to its content are not suited to some aspects of encouraging mathematical thinking. In the introductory lesson on fractions, for example, it was difficult to address aspects like finding multiple solutions to a problem or finding alternative ways to address mathematical problems. However, the way in which the lesson was presented with learners folding sheets of paper and writing the fractions where appropriate lends itself to a follow-up lesson that could focus on finding multiple solutions to a problem or finding alternative ways to address mathematical problems. Learners could, for example, be asked to explain the relationship between four eights, two quarters and one half using the sheets of paper that they designed in the earlier lesson. Based on the sample's feedback and the data analysis of the after intervention classroom observations the researcher is of the opinion that the intervention succeeded in meeting its objectives.

The findings and conclusion of this research as well as the needs for further research and recommendations follows in Chapter 9.

CHAPTER 9 CONCLUSION

9.1 Introduction

Poor performances in national as well as in international assessments have indicated that South African learners across grades are experiencing problems with mathematics.

The implementation of mediocre or poor mathematics teaching practices has contributed to these problems. Concerns were raised regarding the mathematics teaching practices of Grade four mathematics teachers. The objective of the research was to develop interventions that can address shared improvement opportunities in the mathematics teaching practices of teachers. However, before interventions could be designed, the shared improvement opportunities had to be identified.

Chapter 3 focused on identifying mathematics teaching practices that had proven to be effective in a number of countries and over varied circumstances. These good mathematics teaching practices were transformed into a set of indicators and measurable evidences of good mathematics teaching practice. The indicators and evidences were applied to assess the current mathematics teaching practices of each sample member. Individual mathematics teaching practice profiles of each sample member were compiled.

Shared improvement opportunities were identified and a theory on Grade four mathematics teaching practices was built. The theory built was in the form of a profile of a Grade four mathematics teacher teaching in the Nelson Mandela Metropolitan Area. With the shared improvement opportunities identified, an intervention to address the identified improvement opportunities was developed. The research sample participated in the intervention. After the intervention an observation instrument was designed and applied to ascertain the impact that the intervention had in addressing the improvement opportunities that had existed in the mathematics teaching practices of the sample.

A short discussion on the sub-foci, addressed through the research, follows.

9.2 Sub-foci addressed

The outcome of this research study is summarised by discussing how the five sub-foci were addressed.

What conceptual framework, as basis for good practice decisions and data collection, can be constructed from international good mathematics practices and the goals set out in the Foundations for Learning Campaign?

The third chapter of the research study was devoted to answering this question. A literature study provided data on international good mathematics teaching practice for the primary school. A comparison between the mathematics teaching practices of different countries resulted in the identification of twenty-four global mathematics teaching practices. A framework for analysing Pedagogical Content Knowledge (Shulman 1986; Baker & Chick 2006; Chick, Baker, Pham & Cheng 2006) was used to triangulate data. The twenty-four global mathematics teaching practices were compared to the daily teaching activities for Grade four mathematics, as outlined in the Foundations for Learning Campaign. This resulted in the identification of eleven indicators and eighty-six evidences of good mathematics teaching practice for the primary school. The indicators and evidences served as the conceptual framework that was used as a yardstick for good practice decisions.

The indicators and evidences were also used in the compilation of self-assessment questionnaires and a classroom observation tool that was used for data collection during classroom observations performed by the researcher.

What are the individual mathematics teaching practice profiles of the sample members and how can the profiles be used to determine strengths and shared improvement opportunities in their mathematics teaching practices?

Chapter 4 was devoted to explaining how the individual profiles of the sample members were built. Individual profile building is a lengthy process involving the interpretation and comparison of three sets of self-assessment data as well as data gathered through a classroom observation. Some of the data collected, as well as comparisons between the data, are reflected in tables. These tables are contained in Appendix E.

The process followed to build the individual profile of sample member A is explained in Chapter 4. As the profile building process had to be repeated for sample members B – E, and individual profile building was a lengthy process, the detailed explanations on how their profiles were built, were included in Appendices E-I.

Through building individual profiles that identified the strengths and improvement opportunities of each sample member in mathematics teaching practice, the second sub-focus was addressed.

What theory can be built from what was learnt via the above foci?

The knowledge gained on the individual strengths and improvement opportunities of sample members was used to build a theory on the mathematics practice of Grade four teachers. The theory was built in the form of a generic profile of Grade 4 mathematics teachers in the Port Elizabeth district. The generic profile was built by tabularising and comparing the improvement opportunities contained in each of the individual profiles (Table 5.1, pp. 96, 97 and Table 5.2, pp. 98-106). This comparison enabled the researcher to determine which of the improvement opportunities were shared by more than one sample member. These shared improvement opportunities would form the focus of interventions to follow. Through the identification of shared strengths on the one hand, and shared improvement opportunities on the other hand, a theory on the mathematics teaching practices of Grade four teachers was built.

By building a generic mathematics teaching practice profile of the sample and identifying shared improvement opportunities, the third sub-focus was addressed.

How can shared improvement opportunities in the mathematics teaching practices of the sample be addressed through interventions?

In Chapter 5 it was established that the sample should improve on actively engaging their learners in the learning process by encouraging mathematical thinking. An intervention to address this improvement opportunity was designed in Chapter 8. However, before the intervention could be designed, the mathematical thinking intervention had to be theoretically grounded. This was done in Chapter 6. In intervention design, the audience at which the intervention is directed is also important. Chapter 7 focused on adult learners, as they constitute the intervention sample. Taking cognisance of the theoretical aspects of promoting mathematical thinking as well of the needs and abilities of adult learners, a Mathematical Thinking Intervention was designed and attended by sample members.

Through the design and implementation of a Mathematical Thinking Intervention the fourth sub-focus was addressed.

What is the impact of interventions designed and applied to address shared improvement opportunities on the mathematics teaching practices of the sample?

Six weeks after the intervention, classroom observations were done to re-assess the mathematics teaching practices of the intervention sample. An adapted observation tool that focused on the shared improvement opportunities addressed during the intervention was used during these observations. As with the pre-intervention observations, these second observations were also done in the respective classrooms of the sample members. During these observations data on how the intervention sample encouraged mathematical thinking through their respective teaching practices was gathered. The post-intervention data was compared with corresponding pre-intervention data to determine the impact of the intervention on the shared improvement opportunities that had existed in the mathematics teaching practices of the sample [Appendix N (vii)].

The comparison revealed that the intervention had a positive impact on the mathematics teaching practices of the sample. Participating in the intervention had effectively addressed the shared improvement opportunities that were identified prior to the intervention. By determining the impact of the intervention, the final sub-focus of the research was addressed.

9.3 An observation and responsive suggestion

At the conclusion of the mathematical thinking intervention, sample members agreed that the games included in the intervention guide were both fun and stimulating. However, they also commented that the games took quite a long time to complete. This could especially present a problem for learners in the lower grades. Teachers are often under pressure to complete the syllabus and the intervention sample suggested that it might be difficult to put enough time aside for these mathematical games. Sample members did highlight an important advantage of having these games available though. They indicated that the games would be handy tools for stimulating mathematical thinking in learners who are quick to grasp mathematical concepts. Whilst they play the games, the teacher can repeat and re-explain concepts to learners experiencing difficulties.

During the second round of classroom observations it was noticed how much learners enjoyed mathematical activities that simply involved throwing two die and performing calculations with the numbers shown. Sample members used the die to let their learners practice their multiplication tables. This spurred the idea of using a well-known game and adapting it to encourage mathematical thinking. The result was an adapted version of the Snakes and Ladders game.

The game is called **MADS**nakes after the four basic operations used in mathematics. The M stands for multiplication, the A for addition, the D for division and the S for subtraction. Any normal snakes and ladders board and two die can be used for game play. The game is well suited to encourage mathematical thinking because learners are presented with a choice between four operations to perform on the two numbers shown on the die. For each throw learners have to weigh up their options and the consequences of their choices before moving their markers. Learners thus have to calculate the outcome of the different operations and evaluate what operation would suit them best. Basic procedural knowledge of multiplication, division, addition and subtraction is practised, but simultaneously mathematical thinking is exercised. Take, for example, a throw of the die that resulted in the numbers six (6) and three (3) shown. With **M**ultiplication, the marker would be moved eighteen spaces, with **A**ddition nine spaces, with **D**ivision two spaces and with **S**ubtraction three spaces. The learner has to decide on the best move to avoid snakes and advance up a ladder, if possible, based on the calculation results.

As the idea for the game only originated after the second round of observations was completed, it has not been tested in the classrooms of the intervention sample. Permission would be requested from the Port Elizabeth District Office of the Eastern Cape Department of Education to introduce the game in the primary schools that participated in the research. It is envisaged that the game can even be used in the Foundation phase to stimulate mathematical thinking. A smaller game board with numbers one (1) to fifty (50) can be used. Young learners can be encouraged to choose between adding or subtracting the two numbers on the die. Whilst learners will practise procedural knowledge in a fun way, they will at the same time start to exercise mathematical thinking. Comparing results and making decisions based on the implications of each result will have an invaluable impact on the development of the mathematical thinking of young learners.

9.4 Limitations revisited

The two limitations noted on page 14 of Chapter 1 are still relevant. In addition a further reduction in the sample number during the intervention impacted negatively on the data produced.

9.5 Shortcoming

A shortcoming that came to light during the study is the decision to only use triangulation of data and not doing double coding of data as validation instrument.

The next subsection describes the needs for further research.

9.6 Needs for further research

During this research study a classroom observation tool for observing mathematics teaching practices in primary schools was developed. This observation tool was applied to a limited sample of five Grade four mathematics teachers in the Nelson Mandela Metropolitan area. An analysis of the data resulted in the identification of shared improvement opportunities and the design of an intervention to address the improvement opportunities. The researcher identified the following six needs for further research:

- To apply the classroom observation tool to a wider scope of teachers. The observation tool was designed to assess the mathematics teaching practices of primary school mathematics teachers. This means that the observation tool can be applied to all mathematics teachers from Grades four to six (the Intermediate phase) in South Africa.
- To compile generic mathematics teaching practice profiles for Grade 5 and 6 mathematics teachers.

- To compare the data generated on the mathematics teaching practices of primary school mathematics teachers across grades.
- To refine the classroom observation tool by monitoring the indicators and evidences of good practice. As literature on globally effective mathematics teaching practices becomes available, the classroom observation tool must be adapted accordingly.
- To apply the observation instrument that focuses on encouraging mathematical thinking in learners to a large number of mathematics teachers.
- To present the Mathematical Thinking Intervention to teachers who experience difficulty with encouraging mathematical thinking in their learners. The impact of the intervention developed during the research can then be assessed on a broader scale.

9.7 Recommendations

The researcher makes the following recommendations:

- That indicators and evidences of good mathematics teaching practice also be identified for the Foundation-, Senior- and Further Education and Training phases.
- That classroom observation tools be developed to assess mathematics teaching practices in the Foundation-, Senior- and Further Education and Training phases.
- That interventions be designed and presented to address improvement opportunities identified in the teaching practice of mathematics teachers in the Foundation-, Intermediate- and Senior- and Further Education and Training phases.

9.8 Conclusion

This research has identified a set of eleven indicators and eighty-six evidences of good mathematics teaching practice for the primary school. The indicators and evidences were applied in a classroom observation tool. The ideal to strive for would be that all measurable evidences be present in the mathematics teaching practices of every teacher in every mathematics lesson. The mathematics teaching practices of teachers should be assessed to determine improvement opportunities that need to be addressed.

The research did this on a limited scale. The classroom observation tool was used to assess the current mathematics teaching practices of a sample of Grade four mathematics teachers. Data analysis revealed that the mathematics teaching practices of the sample needed improvement insofar as engaging learners by encouraging mathematical thinking. A mathematical thinking intervention was developed to address the improvement opportunity. After the sample members participated in the intervention, their mathematics teaching practices showed improvement in the areas addressed by the intervention. The intervention sample, through their participation in the research, showed that teachers can improve their mathematics teaching practices.

Competency in each of the mathematics teaching practices is important to mathematics teachers. The relevance of the research study, as argued in Chapter 1, page 2 (Bernstein 2007) is reaffirmed by the following current findings published in an in depth report of the Centre for Development and Enterprise:

- “This study has confirmed that the poor performance of many teachers is a major reason for the dismal results achieved by large sections of South Africa’s schooling system.” (Bernstein 2011:5); and
- “CDE’s research confirms that the poor performance of teachers is a major reason for the poor performance of the South African schooling system.” (Bernstein 2011:27).

The report concluded that “teachers are at the centre of South Africa’s struggling school system” (Bernstein 2011:29). As such the identification of improvement opportunities in the mathematics teaching practices of teachers should be encouraged and interventions should be developed to address those improvement opportunities.

"If we teach children everything
we know, their knowledge is
limited to ours.
If we teach children to think,
their knowledge is limitless."

Michael Baker
President: The Critical Thinking Company (May 2005)

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