

A CASE STUDY OF A PRE-SERVICE MATHEMATICS EDUCATION COURSE
TO GROW AND DEVELOP PROFICIENT TEACHING IN MATHEMATICS
IN THE INTERMEDIATE PHASE

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Abstract

This research study investigated the ways in which a mathematics module, informed by an enactivist philosophy, enabled pre-service teachers to unpack the reality of their teaching practice in terms of proficient teaching. Given the generally poor mathematics results in South Africa it is not enough for teachers to be merely proficient in Mathematics¹. They also need to be in a position to explain important mathematical concepts to children in a manner that will encourage and develop an understanding of the basic mathematical concepts.

It was my intention with this study to determine whether a mathematics education module, that embraced the underlying themes of enactivism as part of its teaching pedagogy, could have the potential to develop and increase the skills of pre-service teachers' teaching for proficiency in Mathematics. The mathematics module was underpinned by five themes of enactivism namely: autonomy, embodiment, emergence, sense-making and experience and was designed to supplement the pre-service teachers' basic skills in Mathematics in the Intermediate Phase. This mathematics module was offered to fourth year pre-service teachers completing a B.Ed. in the Foundation Phase at a private institute specialising in the training of teachers.

The theoretical framework was informed by enactivism and how the themes of enactivism could be used as a vehicle to develop teaching proficiency. The study was qualitative in nature and situated within an interpretivist paradigm. The specific perspectives of interpretivism that were used were hermeneutics, phenomenology and reflexivity. The research design was a case study that contained elements of action research and encompassed three phases of data collection. The first phase focused on the pre-service teachers' approach to teaching Mathematics and what this brought forth in terms of the reality of their teaching practice and the problems they encountered. The second phase undertook to determine what growth and development of teaching proficiency in Mathematics had emerged over the research

¹ Mathematics: when the word is used as a noun, as the subject or field of study it takes a cap and when it is used as an adjective it takes a small letter.

period. The final phase was undertaken after the pre-service teachers had graduated and were employed as full time teachers in the Intermediate Phase.

The analytical framework and lens through which the data was analysed was that of Kilpatrick, Swafford and Findell's (2001) strands of mathematical proficiency. The argument that I present is that the themes of enactivism did contribute to the growth of the pre-service teachers' teaching for mathematical proficiency. The themes of embodiment and experience were major contributions in revealing that this was a reality for the pre-service teachers from a practical perspective and was what they would be able to take away with them. However the theme of emergence stood out as the principle that generated the most awareness and growth and which, in turn, affected the participants' autonomy.

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CHAPTER ONE

INTRODUCTION

This thesis reports on a case study of an in-service mathematics education course designed to develop proficient teaching in Mathematics in the Intermediate Phase, exploring in particular how the themes of enactivism can promote teaching for mathematical proficiency. In this chapter I address briefly the purpose of this research study and its subsequent relationship to the research question, context and site of the study.

1.1 PURPOSE AND RATIONALE OF THE RESEARCH

I have had a lifelong passion for Mathematics and would like to see this passed on to children. However, from lecturing various groups of student teachers I noticed that a number of pre-service teachers and practising teachers were either not confident in Mathematics, had had a bad experience with Mathematics or do not see the value or enjoyment in Mathematics. I believe that if teachers are proficient in Mathematics and have access to strategies to teach for proficiency (Kilpatrick, Swafford & Findell, 2001), their confidence and enjoyment of Mathematics will increase which, in turn, will be passed on to the learners. It was my intention in this study to investigate whether a mathematics education module that embraced the underlying themes of enactivism as part of an enactive teaching pedagogy² would have the potential to develop and increase the skills of pre-service teachers for teaching for proficiency in Mathematics.

This research took place in the field of teacher development in mathematics education. It was the intention of this study to determine in what way a mathematics education module, informed by an enactivist philosophy, would enable the research participants to reveal the reality of their teaching practice in terms of proficient teaching. In addition, the study attempted to ascertain whether pre-service teachers embodied perceptions of their own mathematical proficiency, their development and

² Enactivism is a theory of cognition. In this research study I investigated the effect of using the themes of enactivism to underpin my pedagogic practice.

their exposure to the themes of enactivism enhanced and supported their own teaching for mathematical proficiency.

1.1.1 Research Question

In what way does a mathematics module, informed by an enactivist philosophy and teaching pedagogy, enable selected pre-service teachers to unpack the reality of their teaching practice in terms of proficient teaching?

1.1.2 Sub-questions

- What did the participants of the module bring forth with regard to teaching for mathematical proficiency?
- What problems did the participants pose with regard to teaching for mathematical proficiency?
- What evidence of growth and development of proficient teaching in Mathematics emerged?
- What evidence of growth and development of personal professional growth and proficiency emerged?

Thus it was the intention of this research study to determine, firstly, what embodied views of cognition revealed about the research participants' personal proficiency in Mathematics as well as their perceptions of their ability to teach mathematics proficiently in the Intermediate Phase. Secondly, to identify fourth year pre-service teachers' perceptions of the extent to which a mathematics module and mathematics based micro teaching tutorials, underpinned by enactivism themes, developed their confidence and skill in teaching for mathematical proficiency in the Intermediate Phase.

1.2 RESEARCH CONTEXT AND BACKGROUND

This research study was undertaken at a private Institute for Higher Education specialising in teacher education for both ECD/Foundation³ and Intermediate/Senior Primary⁴ phases.

The students who took part in this research study were fourth year pre-service teachers training to teach in the ECD/Foundation phase. Students undertaking a B.Ed. in Foundation Phase were required to demonstrate competency in the three learning programmes, namely Literacy, Numeracy and Life Skills. Since a number of the students typically find employment in the Intermediate Phase it was decided that in the fourth year of their studies the students would be required to complete a mathematics module that focused on some of the 'big ideas' in Intermediate Phase Mathematics (Van De Walle, Karp & Bay-Williams, 2010, p. xx). The module was designed to supplement their basic skills in Mathematics in the Intermediate Phase. The intention was to develop both their confidence and proficiency in Mathematics and their proficiency in teaching Mathematics in the Intermediate Phase. It was in this module that the research study was situated.

1.3 SIGNIFICANCE OF THE STUDY

Research undertaken within the South African context during the past few years (Carnoy, Chisholm, et al., 2008; Horak & Friecke, 2004), indicates that there is a strong link between a teacher's own proficiency in Mathematics and approach to teaching, and the results obtained by learners. However, a pilot study conducted in Gauteng, by Carnoy and Chisholm, et al. (2008) revealed a number of concerns. Firstly, that the number of "qualified Mathematics teachers in primary schools is small" (p. 68). Of further concern was that a large number of teachers teaching Mathematics were in fact not qualified to do so and that many teachers who were qualified, were not teaching Mathematics. Secondly, most of the teachers had diplomas and had been trained in teacher training colleges but the quality of education varied greatly between the different colleges. Thirdly, their pilot study found that learners who were "disadvantaged academically in terms of family resources including regularly using the language of instruction employed by the

³ Equivalent of Kindergarten and Elementary school (Ages 5 – 8)

⁴ Equivalent of Elementary and Middle School (Ages 9 – 14)

school” (p. 69) tended to be taught by teachers who were less proficient to teach mathematical concepts. Finally, Carnoy and Chisholm, et al. (2008, p. 69) noted that the mathematics content and pedagogical content knowledge appeared to be a major influencing factor in the quality of mathematics teaching and therefore in how much Mathematics the students were likely to learn. From their conclusions it is apparent that, firstly, pedagogical content knowledge plays an important role in improving student achievement in Mathematics in South Africa and, secondly, that further research in the pedagogy of teacher training is warranted.

Howie (2002, p. 138), when discussing the TIMSS-R⁵ study, indicates that approximately 25% of Mathematics teachers were not qualified to teach Mathematics and, of particular interest to my research, that many teachers had to re-teach topics which should have been dealt with in the lower grades. This concurs with an informal discussion that I had with Mathematical Literacy teachers that I came into contact with during Continuing Professional Teacher Development (CPTD) short courses. Howie also raises the issue that low expectations of learners could also translate directly into poor performance. The Parliamentary Monitoring Group (2008) also indicates that the challenges that the Department of Education faces are underpinned by a number of factors of which the following pertain to my research: “the lack of appropriate qualifications by teachers, the poor conceptual grasp of the learning areas, poor understanding of and/or ineffective teaching strategies, inadequate numbers of qualified teachers in particular skills areas and poor training in the implementation of the National Curriculum Statement (NCS)” (p. 1).

Keeping the South African context in mind, it is interesting to note that Kieren (2001) argues that while it may be reasonable to think that there are specific answers or responses that should match the pre-conditions of mathematical activities, this view “disembodies mathematical thinking” (p. 1). This disembodiment results, firstly, in separating mathematical reasoning from the learner’s personal history and, secondly, comes from not paying sufficient attention to the “complexities of the situations in which mathematical thinking” occurs (p. 1). This suggests that many South African classrooms are characterised by a disembodied approach to mathematical pedagogy. Kieren (2001) proposes an enactivist approach as an

⁵ TIMSS-R - Third International Mathematics and Science Study- Repeat

alternative embodied approach to the more traditional method of teaching, in that it embraces the notion that not only does the environment provide the space and opportunity for personal mathematical activities, but it also considers the mechanisms and beliefs that underpin an individual's mathematical actions. Furthermore, enactivism emphasises the interactions and conversations that occur between the different individuals within a particular environment and takes cognisance of the emerging patterns that occur between mathematical activities and understanding.

Li, et al. (2010, p. 12) indicate that enactivism as a theory of cognition requires that the teacher, or in this research study the lecturer, be the co-author of knowledge and not the source of knowledge. They refer to Jonassen (cited in Li, et al., 2010, p. 15) when asserting that "instead of focusing on a "single best sequence" of lessons for learning, teachers (lecturers) wanting enactivist classrooms build a rich learning world with abundant stimulation in their classrooms, with enough limits to guide students toward possible co-evolving patterns".

Within the South African context I believe that there is a lack of this stimulation caused in part by the scarcity of resources and qualified teachers. Kilpatrick, et al. (2001, p. 313) consider teaching and learning to be the result of the interaction that takes place between the learners, the teacher and the content. However, they indicate that for an effective outcome to these interactions, the learners' "expectations, knowledge, interests, and responses" (p. 313) must be taken into consideration since "their interpretations and actions affect what becomes the enacted lesson" (p. 314). In addition, Kilpatrick, et al. (2001, p. 312) state that to achieve mathematical proficiency, teaching pedagogy must address five intertwined strands of Mathematics namely, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. In order to achieve this, teachers need to have a sound content knowledge, the lack of which Horak and Fricke (2004) have indicated is an issue in South Africa.

This study is significant both from a teacher training and a theoretical perspective. The study investigates teacher training and the understanding of developing proficient teaching of Mathematics in the field of enactivism, particularly within the South African context. The specific focus on enactivism, embedded in a teacher

training programme, was useful in enhancing teaching proficiency and generating reflection on teaching practice, thereby adding value to the training programme. Given the poor mathematical results in South Africa and the shortage of teachers, an approach, no matter how small the scale, that enhances teaching proficiency will add value to any teacher training programme.

From a theoretical perspective, this study's contribution lies in the methodology. A matrix encompassing the themes of enactivism and Kilpatrick, et al.'s (2001) strands of mathematical proficiency, provided a new and unique lens through which to track the development of proficient teaching of Mathematics. Kilpatrick, et al.'s (2001) strands of mathematical proficiency, are known and well researched within the South African context (Adler, Ball, Krainer, Lin, & Novotna, 2005; Christiansen, 2012; Schäfer, 1999; Venkatakrishnan & Graven, 2006), however, they have not been used as a means through which to track the development of teaching for proficiency. When linked with the themes of enactivism I have two theoretical perspectives / lenses to use to reflect on and enrich the students' mathematical teaching identity. This assumes an interest and desire to improve on practice.

1.4 HOW ENACTIVISM IS LOCATED IN MATHEMATICS EDUCATION RESEARCH

The intention of this section is to give the reader a brief overview of how enactivism is located in mathematics education research since it is still, to a certain degree, an emerging theory in South African mathematics research. This synopsis will also demonstrate that this was a well-recognised and regarded theory in mathematics education research on which to base my research.

Internationally, in the greater mathematics community, enactivism has been used in a variety of research studies. Initially, Davis and Sumara (1997a, p. 122) investigated the implications of using enactivism when teaching teachers and reported that it required one to take a hermeneutic attitude. This demands that we blur the lines between school and university when 'learning to teach', so that the responsibility does not fall to either the university (learning) or the school (teaching) alone. Teacher education cannot be the sole responsibility of either the university from a theoretical or learning perspective or the responsibility of the school from a practical

or teaching aspect. Instead, teacher education should lie somewhere in between so that teaching and learning can co-emerge into an active learning space. In the same year Davis and Sumara (1997b, p. 403) investigated the relevance of enactivism for action research. Kieren (1997, p. 32) also discussed the benefits of enactivism in mathematics education research as a means to provide teachers with alternative tools and strategies for their teaching practice and observation of students.

Reid (2001, p. 1) indicated that enactivism had previously been applied to the learning of mathematics in a variety of ways. It had been used in one instance to illustrate students 'proving' as a means of explaining within a classroom context. In another study enactivism was used to explore the nature of Mathematics, as a research methodology. A further approach was to disclose the co-emergence of students in problem solving. Additionally, Fenwick (2001b, p. 1) interrogated the usefulness of enactivism, and in particular the concept of co-emergence, with regard to working- knowledge in the world of enterprise.

Frielick (2004, p. 330) revealed the implications of enactivism understanding for e-learning and a deep approach to learning, while Lozano (2005) asserted that enactivism allowed her to think of the learning of Mathematics as a complex process which encompassed the embodied structures of her students. Later, she also used enactivism to analyse themes of algebraic learning in her doctoral research in 2008.

Hamilton (2007, p. 6) investigated how the theory of enactivism could be used to empower "students to learn in a connected manner" by means of an integrated approach acknowledging "learning as a complex web of interaction", whereas Di Paolo, Rohde and De Jaegher (2007) investigated enactivism and its usefulness for artificial intelligence and more specifically their work with evolutionary robotics.

Proulx (2008b, p. 147) used the concept of structural determinism from enactivism to analyse and "make sense of the learning process". As interest has grown in the mathematics community, a forum was held at the 33rd Conference of the Psychology of Mathematics Education (PME) (Proulx, Simmt & Towers, 2009, p. 1-4) with the focus on "enactivism as a theory of mathematical knowing". Proulx (2009) in summarising the main themes and issues of enactivism that emanated from the 33rd

Conference of the International Group for the Psychology of Mathematics Education highlighted teaching as a key area where enactivism could make a significant contribution with regard to mathematics education research. He pointed out that, as yet, we do not know what enactivism means in the classroom or how learning that is dependent on teaching, but not determined by it, affects the teaching dynamic (Proulx, 2009, p. 1-22). Furthermore, Proulx indicated that one needs to consider the effect that the teacher has in both the learning process and the teaching dynamic if the teacher's actions are seen to be perturbations in the students' learning process.

Brown and Coles' (2011, p. 861) used enactivism to frame a professional development course to help novice teachers develop expertise in mathematics education through the use of enactive insights and collaborative group work. Their framework was underpinned by the enactivist links between perception and action and "working on shared experiences" (p. 871). Their findings included the importance of incorporating interaction with the wider community.

Enactivism, although not used as extensively in South Africa as internationally, has been the topic of a few research studies. Breen (2007) used the concepts of enactivism to develop an alternative teaching approach with his pre-service teachers to help them, firstly, to cope with their fear of Mathematics. Secondly, he used enactivism to create a classroom environment that supported conversation, listening and participation. Breen (2005) also used the ideas of enactivism to challenge and transform his teaching process in mathematics education by encouraging his students to play an integral part in their learning environment through co-creating the lessons. In addition, his students were encouraged to acknowledge the part that they, their previous experiences, beliefs, and environment played in their learning environment.

More recently, Mostert (2007, p. 181) adopted an enactivist approach as her methodology for her masters research in helping her to get a sense of her experiences in "coming to know maths" and her experiences as a mathematics teacher. Samson (2011, p. ii) used an enactivist perspective as an analytical tool to

explore “the inter-relationship between the embodied processes of pattern generalisation and the visualisation of pictorial cues”.

1.5 THE SOUTH AFRICAN CONTEXT

Students wishing to train as Intermediate Phase teachers in South Africa have two full time options, either to complete a degree plus a one year Postgraduate Certificate in Education (PGCE) or a four year Bachelor of Education (B.Ed.) degree.

Students who have successfully completed a three or four year undergraduate qualification may opt to undertake a one year professional Postgraduate Certificate in Education course focusing on a specific phase. These phases include the Intermediate (Grades 4 – 6, learners aged 9 – 11), Senior Phase (Grades 7 – 9, learners aged 12 – 14) or Further Education and Training (Grades 10 – 12, learners aged 15 – 17). The PGCE is a generalist qualification which is designed to “consolidate subject knowledge and develop appropriate pedagogical content knowledge, for teachers’ self-reflexivity and self-understanding, convey an understanding of teaching as a profession, and nurture commitment to the profession’s ideals” (Council on Higher Education, 2010, p. 41).

It is assumed that students who have enrolled for a PGCE programme have attained an adequate and appropriate content knowledge in their undergraduate degree, thus enabling this qualification to focus on pedagogical knowledge rather than discipline knowledge. The PGCE is an intensive programme given that approximately 8 - 10 weeks are set aside for teacher practice and the balance of the time is allocated to a variety of modules required to guide the novice teachers (Council on Higher Education, 2010).

The second option that prospective students have is to enrol for a Bachelor of Education (B.Ed.) degree. This is a four year professional programme with a core focus on initial teacher training. The regulatory framework for the B.Ed. degree was the *Norms and Standards for Educators* (NSE, 2000) (Department of Education, 2000) which was underpinned by seven guiding roles of a competent educator: Learning mediator; Interpreter and designer of learning programmes and materials;

Leader, administrator and manager; Scholar, researcher and lifelong learner; Community, citizenship and pastoral role; Assessor and Learning area/subject/discipline/phase specialist. The B.Ed. degree is a comprehensive programme that needs to meet a number of criteria, laid down by the Higher Education Quality Committee (HEQC) National Review. One of the key criteria for this programme was to “develop and consolidate both subject knowledge and pedagogical content knowledge” (Council on Higher Education, 2010, p. 75). A further component of this programme was the inclusion of work integrated practice, the duration of which was left to the discretion of the institution offering the degree. During 2010, the Draft Policy on the Minimum Requirements for Teacher Education Qualifications selected from the Higher Education Qualifications Framework (HEQF) was published for public comment. The intention of this document was to replace the *Norms and Standards for Educators of 2000* (NSE, 2000). This policy was then gazetted and published as the new *Policy on the Minimum Requirements for Teacher Education Qualifications* in 2011 to replace the NSE 2000 document. While the *Policy on the Minimum Requirements for Teacher Education Qualifications* acknowledged the usefulness of the seven roles for educators in the design of programmes the policy no longer foregrounded them and specified that the roles did not “represent the curriculum for teacher education programmes” (Department of Higher Education and Training, 2011, p. 9). Instead, the new policy now focused on Integrated and Applied Knowledge to underpin the design of a teacher qualification programme. Integrated and Applied Knowledge encompasses five types of learning: Disciplinary Learning; Pedagogical Learning; Practical Learning; Fundamental Learning and Situational Learning. The policy also stipulated that over the duration of the four year programme students should spend a minimum of 20 and a maximum of 32 weeks taking part in work integrated learning.

This study commenced at the cusp of the new curriculum policy being introduced and therefore was grounded in elements of both policies. The seven guiding roles of a competent educator, were implicitly embedded in the mathematics module (MIP 400) in particular the components of the roles of Learning mediator; Interpreter and designer of learning programmes and materials; Leader, administrator and manager; Scholar, researcher and lifelong learner; Assessor and learning area/subject/discipline/phase specialist. While the context of this study was mostly

aligned with the *Norms and Standards for Educators of 2000*, the aims and objectives of the mathematics module (MIP 400) also complemented the philosophy of the *Policy on the Minimum Requirements for Teacher Education Qualifications* that competent learning encompassed “a mixture of the theoretical and the practical” (Department of Higher Education and Training, 2011, p. 11). In this regard three of the five types of learning, namely disciplinary learning, pedagogical learning and practical learning supplemented the design of the mathematics module (MIP 400).

Currently, prospective students can choose to register at one of twenty two public Higher Education Institutions (Department Basic Education, 2013, p. 4) or one of the four accredited private Higher Education Institutions offering a Bachelor of Education degree (South African Qualifications Authority (SAQA), 2013). Both private and public Higher Education Institutions need to meet and adhere to the accreditation requirements laid down by the Higher Education Quality Council (HEQC), in addition to those of the Department of Higher Education and Training and the South African Qualifications Authority. A notable difference between private and public is that Public Higher Education Institutions receive a financial grant from the government whereas Private Higher Institutions do not. This study is located in a private institution.

1.6 MATHEMATICS AND THE B.ED. CURRICULUM

Mathematics is infused in various ways within the B.Ed. Intermediate Phase degrees offered in South Africa. A student registered for a B.Ed. Intermediate Phase degree at the Cape Peninsular University of Technology (CPUT), for example would have the option of choosing to specialise in Mathematics. All first and second year undergraduate students of the B.Ed. programme would enrol for two mathematics' modules each year. The first module, focusing on mathematical content from a primary school perspective encompasses a combination of pedagogy and content. The second, a curriculum studies module emphasises topics such as didactics, inclusion, lesson planning and assessment. Students opting to specialize in Mathematics in their third and fourth year, would continue with the mathematics module which is more content focused (at a more senior level) than pedagogy driven. The curriculum studies module has a far greater focus on assessment and

includes a research study which the fourth year students would be required to carry out (S. McAuliffe, personal communication, December 18, 2013).

The mathematics module that formed the basis of this case study was both content and pedagogically driven. It was the third mathematics module undertaken by the students during the four years of the B. Ed. Foundation Phase degree programme. The mathematics modules undertaken in their second and third year focused specifically on the Foundation Phase while this module was aimed at developing their skills in the Intermediate Phase. The key outcomes of this module included developing strategies and methods that encouraged teaching for proficiency. This also encompassed identifying and discussing diversity in South African classrooms and developing strategies to support all children mathematically. Content knowledge covered included whole numbers, fractions, decimal and percentages, ratio and proportion as well as geometric thinking and concepts. In addition to being the researcher of this case study, investigating whether underpinning this module with the themes of enactivism had any influence, I was also the lecturer of the module. This placed me in the unique position of being both the lecturer initiating perturbations for students to engage with (Section 3.2) in addition to being the observer/researcher noticing what the participants brought forth in terms of their proficiency both in teaching mathematics and “adequate conduct” (Section 2.2.3).

1.7 STRUCTURE OF THE THESIS

In this *Chapter* I discussed the research problem and the context of the study.

In *Chapter Two* I discuss the literature that is relevant to the thesis. In particular I describe the enactivist theory of cognition which forms an important theoretical component of this research study. I discuss the five underlying themes of enactivism that I used as a guideline for the “enactivist” teaching pedagogy of this module and review the literature relating to the key concepts of these themes. Having discussed enactivism and its themes, I review Kilpatrick, et al.’s (2001) framework for teaching for mathematical proficiency and developing proficiency in teaching Mathematics. In this research study I decided to use Kilpatrick’s five intertwined strands as an analytical framework for determining the perceived growth in proficiency levels through exposure to and practical experience of a pedagogy underpinned by the themes of the enactivist theory of cognition, since they formed part of the Mathematics module syllabus.

The design of this research is a case study that is qualitative in nature. *Chapter Three* discusses the suitability of this methodology and my reason for integrating elements of action research to enhance the design. I describe the methods of data collection, namely reflective tasks, interviews, questionnaires and teaching videos and the manner in which this data was analysed. This chapter concludes with a discussion of validity and generalisability and ethical considerations.

Chapter Four presents the findings and a discussion of the results that emanated from the data analysis. This follows the format of describing the perceptions of the students at the outset of the study and then tracks how these transpired over the course of the subsequent three phases.

In *Chapter Five* I conclude with a summary of my findings and recommendations.

CHAPTER TWO

LITERATURE REVIEW

In this chapter I begin with a discussion of the theoretical framework, namely enactivism and unpack the conceptual framework of the five themes that underpin this theory of cognition. The chapter concludes with an examination of Kilpatrick et al's (2001) five strands of proficiency as the lens for the analytical framework for this research.

2.1 INTRODUCTION

As a lecturer of Mathematics Education to pre-service teachers, it has always been a concern of mine that while students may demonstrate personal proficiency in Mathematics they may not be able to teach Mathematics proficiently. It had also come to my attention that a number of students would not teach Mathematics by choice opting rather to teach Literacy or Life Skills lessons. With this in mind, I decided to use the themes of enactivism, as defined by Di Paolo, Rohde and De Jaegher (2007) to determine if a Mathematics module informed by an enactivist philosophy would enable pre-service teachers participating in the Mathematics module to unpack the reality of their teaching practice in terms of proficient teaching. Since enactivism considers cognition to be a complex co-evolving process of systems interacting and affecting each other, cognition is deemed to be a producer of meaning and not a processor of information. Therefore an individual's interactions with the world and their past experiences will shape and influence the meaning that they make of their world (Lozano, 2005). I chose to use Kilpatrick, et al.'s (2001) five strands of proficiency as an analytical lens to examine the pre-service teachers' proficiency and teaching strategies since these strands formed part of the module curriculum. In this review I start with an overview of enactivism and then describe the themes of enactivism and their link to Kilpatrick, et al.'s (2001) strands of proficiency. Kilpatrick et al (2001) identified five integrated and interwoven strands to guide the teaching and learning of mathematics in school. These strands include conceptual understanding, adaptive reasoning, procedural fluency, strategic competence and productive disposition.

2.2 THEORETICAL FRAMEWORK – ENACTIVISM

2.2.1 Definition

Enactivism is a theory of cognition that has its roots in the philosophy of Merleau-Ponty, which views the body, mind and world as inseparable. Maturana, Varela, Thompson and Rosch (cited in Davis, 1996) have subsequently expanded on this theory. Enactivism considers an individual to be a complex system whose individual structure is the result of a combination of their biological structure and their personal history of interacting with the world. As a theory of cognition, enactivism “recognizes that the development of both cognition and affective growth will result in the transformation of an ecosystem, thus promoting learning that is both empowering and connected” (Hamilton, 2007, p. 6). Davis (1996, p. 9) maintains that structure is a fluid notion, since it is able to change, adapt or correspond to our experience. He also indicates that one’s structure is the outcome of our different experiences. Since enactivism “focuses on the importance of embodiment and action to cognition” (Li, et al., 2010, p. 403), the structure of the individual is considered to be adaptable as it changes to make sense of new experiences and challenges.

2.2.2 Roots of enactivism

Merleau-Ponty, a 20th century French philosopher started to look at knowledge in a more interpretive manner probably due to his strong theoretical links to phenomenology. In addition to reality being understood through analytical rational argument, he was of the opinion it could also be understood through language and descriptions of the world. Furthermore, he asserted that one’s perception of the world was influenced by social needs and cultural concerns (Davis, 2004, p. 99).

Merleau-Ponty, attempted to find an alternative way to the bipolar divisive way of thinking, namely mind and body, which was popular in the western school of thought at that time (Davis, 1996). Since enactivism views the body, mind and world as inseparable, Merleau-Ponty proposed the notion of a middle road between the inner mental and the outer physical body (Li, et al., 2010, p. 6). This, in essence, suggests that the body is the agent that makes both the mind and the world inseparable and

hence there is no demarcation between subject and object. He considered the body to be a structure that is both “of the world and of one’s self” (Davis, 1996, p. 78), in the sense that our bodies separate us from one another but similarly they allow us to interact with one another. Since our bodies are shaped by the very world that they participate in shaping, they cannot be divisive in nature. Furthermore, given that the body is simultaneously part of the individual and a part of the world it provides each individual with the means of belonging in the world. A world that Merleau-Ponty was of the opinion shaped us and that we in turn had a role in shaping (Davis, 1996, p. 9).

Because we are involved in the world we cannot remove ourselves from it and thus from a phenomenological perspective we have a hermeneutic or interpretive explanation to link our experiences and the concepts we use to explain them (Davis, 1996, p. 32). When defining hermeneutics, Davis (1996, p. 31) explains that it indicates that one’s understanding, perception, actions and experiences are in fact objects that one has already interpreted and enacted with, inside the boundaries of a context and a system of relationships. He then explains how hermeneutics encourages one to question the things that shape and structure our understanding in order to reinterpret “already interpreted phenomena” (Davis, 1996, p. 31). This is accomplished by asking ourselves what something means and how this meaning has arisen. Varela, Thompson & Rosch (1991, p. 149) describe hermeneutics as an interpretation and in this case the *bringing forth* or enactment of meaning. As phenomenology is primarily the study of experiences, and since I was examining the participants’ experiences, it was useful to supplement hermeneutics with a phenomenological approach. In addition, phenomenology also requires an individual to re-learn as they look at the world from a new experience and demands that we ask what this experience is like (p. 32). Thus it is apparent that hermeneutics and phenomenology are embedded in the roots of enactivism, since phenomena like mind and world; mental and physical are co-emergent because they are defined through interaction with one another (Davis, 1996, p. 78). Merleau-Ponty believed, firstly, that who we are, our identity, is the result of our experiences and, secondly, that we are social beings and thus “our minds and our identities emerge and evolve relationally” (Davis, 1996, p. 89).

Merleau-Ponty considered conversation and listening as a way for two systems to make sense of an experience, a phenomenon which he called structural coupling (Davis, 1996, p. 41). With regard to an individual's way of "knowing", Merleau-Ponty (cited in Davis, 1996, p. 191) explained that an individual's experiential world encompasses the way in which they know, perceive and conceive together with interaction with others. Thus a person's structure, or identity, is an ongoing self-process of transformation that encompasses and assimilates our ways of knowing different experiences, relationships and so forth.

Maturana, Varela, Thompson and Rosch, bringing with them backgrounds of Buddhism, philosophy and biology, then further developed the theory of enactivism. Varela, Thompson and Rosch (1991, p. 6) when looking at the theoretical shifts in cognitive science, sub divided them into three successive phases, namely cognitivism, emergence and enactive. They considered the enactive phase not as something different but rather as the next stage in the progression of cognitive development. In this stage they explain the belief that cognition was no longer seen as a representation of a "pregiven world by a pre-given mind" (p. 9) but instead as the enactment of the mind and a world based on a history of various actions that one needs to undertake to be part of that world. Varela, et al. (1991, p. 206) define cognition as "Enaction: A history of structural coupling that brings forth a world." They added to the inner/outer notion by stating that the concept of the inner represented an individual's "lived/experiential/phenomenological structure" and the outer characterised one's biological physical body (Davis, 1996, p. 9). An individual's structure does not always acquire information from the environment; rather one's structure will determine (structural determinism) which environmental perturbations will trigger an action.

Perturbations are actions that endeavour to trigger new ways of sense making by interrupting established habits of understanding and creating opportunities for an individual to act according to his/her structure (Davis, 1996, p. 10). The environment acts as a trigger for the structure to evolve and it is the structural make up of each individual that will determine what the change is and the extent to which the change will occur (Davis, 1996). Embodiment is seen to be a developing process of one's interaction with the real world, however, the significant fact that underpins this

potential co-emergence is that a system and its environment may interact but they do not necessarily adapt to each other, as Proulx (2004) so succinctly states “if we do not ‘see’ the triggers in the environment, we cannot be ‘affected’ by them” (p.119). Thus any perturbation will present an opportunity for an individual to act according to his/her structure and it is the structural makeup of the individual that will determine what and the extent of the change that occurs. Embodiment is a developing process through one’s interaction with the real world, encompassing sensory, motor processes, perceptions and actions. Thus enactivism encourages embodied (mindful) open-ended reflection (Li, et al., 2010, p. 7). It is important to note that a dynamic system still retains its identity even though its structure may change (Maturana, 1987, p. 67).

Branson (2009, p. 71) states that enactivism allows us to tap into our embodied awareness since it assumes that people and their environments co-emerge. He further indicates that embodied awareness means that our knowledge is provided to us through “the combined input of our cognitive capabilities, our senses, and our body’s experiences of the world as perceived through our feelings, intuitions, emotions, and physiological reactions” (p. 71). Thompson (2007, p. 13) condenses and unpacks the idea of enaction brought forth by Varela, Thompson and Rosch (1991) in their book *The embodied mind: cognitive science and human experience*, into five key points. The first point is that humans are autonomous beings who are active in the generation and maintenance of their own identities and, as a result, enact and bring forth their own knowledge and understanding. Secondly, that the human nervous system creates meaning as opposed to processing data in a computationalist sense. Thirdly, as previously discussed, cognition is viewed as an embodied action which is informed by sensorimotor coupling between individuals and their environment. The fourth point is that an individual’s world is not a “pre-specified, external realm, represented internally by its brain” (Thompson, 2007, p. 13) but rather an individual’s world is uniquely determined or brought forth through their distinct way of coupling with their environment. The final point that Thompson (2007, p. 13) identifies is that experience is not a consequence to any “understanding of the mind” instead it is a significant aspect to understanding the mind when adopting an enactive approach. Therefore, in order to use enactivism to underpin this module it was important that what the students experienced was

situated in their own life and everyday experiences, and in this case with a particular focus on their mathematical teaching practice experience.

2.2.3 Knowledge and knowing

From an enactivist perspective, knowledge is dependent on one's embodiment in the sense that one's body, language, social history and the world are inseparable and thus enactivism views "cognition as embodied understanding" (Varela, Thompson & Rosch, 1991, p. 149). This signifies that an individual's capacity to understand is rooted both in their biological embodiment and through their continual interpretation of one's lived and experienced action and history, culminating in the generation of knowledge (Varela, Thompson & Rosch, 1991, p. 149). From a phenomenological perspective these structures of embodiment, experience and history allow an individual to make sense of their world through the generation of embodied experiences which one interprets and codes in order to make meaning of the world. As complex structures we, as humans, are able to interact with other structures through the process of structural coupling. However, being autonomous structures we are able to choose which perturbations we will respond too. Varela, Thompson and Rosch's (1991, p. 173) approach to cognition indicates that "perception and action, are fundamentally inseparable". Therefore, as one interacts with the world, through our history of experiences and observation, we become more aware of how our actions are linked to our past experiences. Thus one's understanding of the world comes from one's organisation and history of their system which enables an individual to *bring forth* meaning to a given situation. Therefore cognition can be understood as an embodied action, since it is dependent on the kinds of experiences that one has and how one's structure determines how the individual will act and adapt to its environment. Similarly, the environment is shaped by the structure (Varela, Thompson and Rosch, 1991, p. 172-174). To demonstrate this Varela, Thompson & Rosch (1991, p. 205) liken embodied action to a "path laid down in walking" in the sense that it is through a lived history of viable structural coupling that cognition is enacted or brought forth. Furthermore, cognition is seen to be about something that is either missing or which is directed towards something, therefore, as an embodied action, cognition determines the problem and then lays or generates the path to walk on. The walking is in in order to establish a solution.

Maturana (1987, p. 65) introduces the notion that “everything is said by an observer”. By this he meant that if you want to determine if someone knows something, then you need to pose a question to the person. If the person is able to demonstrate ‘adequate conduct’ in response to your question, then you will know that they can do what it was that you asked of them. Therefore, in this study, ‘adequate conduct’ is behaviour that the researcher, in her role as observer, will be able to distinguish as proficient teaching. Thus it is important to identify and clarify what is meant by adequate conduct in a particular context. This, Maturana (1987, p. 66) indicates, is the only way to assess knowledge since “adequate conduct is an expression of knowledge”. In the context of my study, the question that I ask and the phenomenon that I wish to observe is teaching for proficiency. Consequently, in the context of this study, the pre-service teachers can be viewed as dynamic systems whose structure has the potential to change and the process and measures that I will use to recognise proficient teaching, or adequate conduct, will be the five strands of proficiency (Kilpatrick, et al., 2001). Hence, as an observer the researcher will notice any evidence of, or increase in, adequate conduct and a demonstration of effective teaching. In effect the students, through reflexivity, will also be an observer, noting their own and their peers’ demonstration of adequate behaviour.

2.3 CONCEPTUAL FRAMEWORK - UNDERLYING THEMES OF ENACTIVISM

Di Paolo, et al. (2007, p. 7) consider an enactivist approach to cognition to be underpinned by five intertwined themes, namely *autonomy*, *sense-making*, *emergence*, *embodiment*, and *experience* (Table 2.1). These five underlying themes of enactivism constitute the conceptual framework of this research. The intention being that, through collective knowledge and individual understanding, meaning and a common understanding of the mathematical topic would be achieved during lectures. A further objective was that the pre-service teachers could draw on this as a base from which to plan and teach mathematical concepts to their peers during practical tutorial sessions, as they endeavoured to develop skills that would enrich their teaching for mathematical proficiency. These five themes of enactivism were used to inform this case study and its methodological design, which is discussed in chapter three. Investigating the themes of enactivism in relation to mathematical proficiency within a case study, containing elements of action research, enabled me

to draw on a number of methods available to enactivism, such as phenomenology and hermeneutics. I used the five underlying themes to assist pre-service teachers to determine what embodied views of cognition reveal about their personal proficiency in mathematics, their mathematical identity and their self-development. Furthermore, the themes helped me to ascertain how the pre-service teachers' embodied perceptions of their mathematical proficiency supported their own teaching for mathematical proficiency during the practical tutorial sessions.

I now discuss the themes in greater detail, firstly, by providing a brief definition and description and, secondly, by indicating the role that each had in informing my pedagogic practice, and their practical implications. It will soon become apparent that these links are intertwined often merging into one another. Finally, I discuss Kilpatrick et al.'s (2001) framework for mathematical proficiency, which was used as the analytical framework to determine the extent to which the research participants (section 3.3.1.) teaching for mathematical proficiency had grown and developed.

Table 2.1

Themes of Enactivism that form the Foundation of the Conceptual Framework

Autonomy	Sense-making	Emergence	Embodiment	Experience
Construction of their mathematical identity.	Mode of listening	Shared activities = collective generation of knowledge and understanding	Part of a larger collective system	Emergent process
Ways of knowing	Active and participatory role in emerging conversations	Reflective techniques identifying critical incidents (CI)	Feedback gained from others	Discussing their experiences
Self-efficacy	Making meaning of the world through action	Deep reflection and understanding of the purpose of one's actions.	Identifying changes and resolving old ideas	Critical incidents of mathematics teaching
			Tracing and recording the interactions of the participants	Weekly tutorial periods
			Helping participants make sense of the emerging patterns	
			Each person had something to contribute	

2.3.1 Autonomy

Varela, et al. (1991, p. 206) describe a cognitive system as one that is comprised of many levels of “interconnected sensorimotor sub networks”. A functioning cognitive system will then be able to adapt to the existing world or shape a new one, as per evolutionary history. Living organisms are autonomous due to their ability to create their own identity which characterises them as unique entities (Di Paolo, et al., 2007). This research study focused on the student teachers’ identity as being representative of autonomy. Enactivism argues that one’s identity is enacted and therefore determined by the interplay between biology and human culture and the individual’s manner of dealing with life’s experiences. The principle of autonomy is the notion that every living organism (individual or system) is independent in the sense that they follow laws that are established by their own structure (Di Paolo, et al., 2007, p.8). This autonomy is a result of an individual’s self-generated identity as a distinct being. However, an individual must be able to construct some level of identity in order to generate their own laws. This aligns with Davis’ (2004) definition of autonomy which he describes as being structurally determined, since an individual elects how to respond to a perturbation, thus potentially enabling change to take place. Perturbations are actions that trigger new ways of sense-making by interrupting established habits of understanding, thereby creating the opportunity for an individual to act according to his/her structure (Davis, 1996). With this in mind, the case study design of this research included reflective tasks to ascertain what pre-service teachers’ perceptions of a proficient mathematics teacher were and how this influenced their mathematical identity⁶ and whether during the course of the programme these perceptions had been transformed.

Jaworski (2004, p. 19) reminds us that the conditions and ways of knowing between the systems within which the participants and I, as the researcher, work and learn are different. My way of knowing with regard to mathematics teaching and learning is based mainly on theory, in this case enactivism, and the participants way of knowing is through teaching practicum and prior school experience. Therefore, influenced by the school culture, this study hopes to raise ideas and concepts that

⁶ A pre-service teacher’s awareness of her personal proficiency and ability to teach Mathematics proficiently.

could be explored within a school context. While these two systems, namely Higher Education Institutions and schools, have an influence on the way both the researcher and the participants construct their identities, it should also provide value to the embodied knowledge that I anticipated would be generated from the research data. In addition to getting the pre-service teachers to determine the impact that their past experiences of teaching and mathematics had had on their identity, I was also mindful of the fact that the individual's identity would also be constructed through participation in the research study's mathematical community of practice.

Owens (2007) describes one's mathematical identity as encompassing both social identity and being a self-regulating learner who is displaying both cognitive and affective characteristics. Cognitive processes required by a self-regulating learner, for example planning and self-evaluations, are controlled by affective characteristics such as the engagement, confidence and the sense of ownership exhibited by the student. Thus a student's self-efficacy will impact on his/her identity. As Bandura (1994) explains, self-efficacy is one's belief in their ability to attain set goals and certain levels of performance. This in turn influences students' cognitive, motivational, affective and selection processes in terms of how they think, motivate themselves, feel and act. Bandura is of the opinion that self-efficacy can be fostered from four different sources. Firstly, the more students experience success in a task, the greater their self-efficacy. Secondly, watching a peer, who is successful in carrying out a specific task, can also boost their belief in their ability to perform a similar task. Self-efficacy can also be increased through verbal persuasion as when motivating a student or giving accurate and practical feedback to the student on his/her tasks. Finally, a student's emotional state will also affect his/her self-efficacy, thus a positive disposition will enhance self-efficacy while anxiety will undermine it. Based on the aforementioned points to foster self-efficacy, in this case study the lecture group was considered to be the mathematics community at large. The students were then formed into smaller groups or mathematics communities with six members in a group. This is discussed in detail in chapter 3.2.1. Each group received teaching tasks to prepare, with one member being selected to teach the lesson. On completion, groups received verbal feedback on their lesson from their peers. I am of the view that there is a close relationship between autonomy and self-efficacy. Autonomy allows an individual to construct his/her identity but self-efficacy

will influence how the individual will respond to the perturbation. Therefore it is imperative that teachers who wish to enhance self-efficacy create an environment or community of learning that is engaging, safe and reduces stressful situations.

Teachers who exhibit a productive disposition know that “Mathematics, their understanding of children’s thinking, and their teaching practices” are interconnected in such a manner as to make sense (Kilpatrick, et al., 2001, p. 384). Furthermore, they realise that by analysing their own teaching and classroom practice they will learn more about “Mathematics, student mathematical thinking, and their own practice” (Kilpatrick, et al., 2001, p. 384) and in this way have control not only of their continued development but also their teaching practice. It is through providing the pre-service teachers with the opportunity to determine their own proficiency and its impact on their self-efficacy, together with the opportunity to teach mathematical concepts, that the study hopes to ascertain what bearing enactivism has on the pre-service teachers’ perception of their ability to teach for mathematical proficiency. Within the theory of enactivism, autonomous is defined as being ‘self-ruled’ and in control of determining your response to triggers released by the environment, which in turn mould and shape the on-going formation of your structure. This, Davis (2004) asserts, requires both an observer and an observed (p. 210). Therefore, I believe that autonomy, productive disposition and self-efficacy are intertwined and will have a degree of influence on one another in terms of responding to a perturbation.

2.3.2 Sense-making

From an enactivist perspective sense-making is an active and dynamic concept since organisms are actively involved in making meaning of their world through action, that is, “they enact a world” (Di Paolo, et al., 2007, p. 9). Thus, Di Paolo, et al. (2007) maintain that changes in one’s understanding of the meaning of one’s world, beliefs and ideas stem from the dialogue that occurs between the organism and its environment. In this research study the dialogue was between the individual, their peers and the classroom environment.

Davis (1996, p. 35) introduces the notion of listening as an embodied action that enables one to understand human communication and collective action. Listening as

an embodied action entails that one participates with intention and orientation towards a particular “object”. The nature of the “object” of listening, according to Davis (1996, p. 51), is in the interplay between the participants, the setting or context and the subject matter. Davis (1996, p. 52) identifies three types of listening, namely evaluative, interpretive and hermeneutic which he attributes to Levin. An hermeneutic mode of listening, according to Breen (2004, p. 3) encourages participants to share and discuss their assumptions relating to a common topic until a shared understanding of one another’s positions is reached. These three types of listening form part of the enactive process by both the lecturer and the participants. Breen (2001, p. 46) discusses how, through engaging with the different types of listening, participants are able to enter into one another’s space, firstly, as learners with their own understanding of a concept and, secondly, as a means to try to understand another participant’s insight with regard to the concept or process (p. 48).

Sense making will manifest in my pedagogy through encouraging and creating opportunities for the students to play an active and participatory role in emerging conversations. This will entail structural coupling resulting in deeper understanding and greater insight or the creation of new knowledge (Davis, 1996, p. 41). This, according to Davis (1996), is due to the hermeneutic nature of a conversation that encourages sense making and interpretation. Breen (2001) concurs, emphasising the importance of good conversational skills as they create the opportunity for learners to understand the issue at hand. In addition, Breen (2001, p. 44) states that participating in a Mathematics class requires that the learners not only work with their minds but with their bodies as well, bringing with them their emotions: be they fears, enthusiasm, hopes and so forth. With this in mind, Breen (2001) describes the use of various activities as a means to encourage learners to engage in conversation with one another, thus initiating the beginnings of a culture of a mathematics learning community. In South Africa, pedagogy tends to be more procedural in nature (Adler, 1994, p. 104; Sepeng, 2013, p. 635) and thus learners are not given the opportunity to participate both with their minds and bodies. If pre-service teachers have not previously been exposed to this kind of pedagogy I believe that it will be difficult for them to enact the pedagogy. Therefore by providing the students with the opportunity to practice what has been role modelled in lectures, during the tutorial

periods they can experiment with different practices and strategies to link theory to practice in a safe environment and through engaging in conversation develop confidence and strategies for teaching for mathematical proficiency.

2.3.3 Emergence

Davis (2004, p. 210) explains emergence as the notion that understanding and interpretations are generated through shared activities over a period of time, as opposed to predetermined learning objectives. Thus, it is the development of a new process or property, with its own autonomous identity, through the interactions of various existing processes and events. In addition, Davis (2004, p. 169) raises the point that learning is not the site of the individual, be it learner-centred or teacher-centred, rather he views learning as the collective generation of knowledge and understanding. Di Paolo, et al. (2007, p. 10) clarify this further as the formation of a new process or idea as a result of the interaction between different processes. In this research study a learner-centred approach was encouraged. A teacher-centred approach while not incorrect has been found to be associated more with a transmission of knowledge and places more control of learning in the hands of the teacher whereas with a learner-centred approach this responsibility lies with the learner (Brown, 2003, p. 50). This is more in line with enactivism thinking. Enactivism is described by Begg (2000) as a theory of knowing and learning that focuses on being aware of and acknowledging complementary ways of knowing. This requires one “to question the balance between academic knowing and knowing (mindfulness) through an awareness of self” (p. 10).

As part of my pedagogy, I used various reflective techniques to encourage the students to reflect on what they had learnt during a particular lecture or tutorial and to acknowledge the role that the community of practice, autonomy and sense making had played in their understanding of and culmination of this process. Li, et al. (2010) explain that the embodied experience that an individual undergoes is due to the bringing together of the mind and body by means of reflection. Thus “reflection is a form of experience itself and that reflective form of experience can be performed with mindfulness/awareness” (Varela, et al., 1991, p. 27). Lipp (2007, p. 18) defines reflexivity as “a deeper and broader dimension of reflection” which helps with the

construction of knowledge from one's experiences by examining what impact our action and position has had on a particular incident. Therefore, the participants used reflection, by means of reflective tasks and group discussions, as an enactive process to note the emergence that had taken place in their understanding or teaching experience during a particular lecture or practical tutorial. This was acknowledged by identifying critical incidents. A critical incident was any incident that at the time was considered to be significant, and upon further analysis the intention was that further learning would result from the experience (Moon, 2006, p. 155). This complements Mason's (2002, p. 30) approach of noticing as a possibility for future practice, in which one distinguishes "some 'thing' from its surroundings". The students were encouraged to try new ideas that could develop their proficiency in teaching mathematics in this particular case study and a critical incident was any moment that a student deemed to have added value to their practice of teaching for proficiency. Hamilton (2007, p. 6) is also of the opinion that enactivist learning encourages one to reflect deeply and understand the purpose of one's actions. Through the process of the three phases, different patterns of thought and mathematical proficiency that emerged were observed and noted. This was possible since "cognition is an embodied action" (Di Paolo, et al., 2007, p. 11).

2.3.4 Embodiment

Varela, et al. (1991) define embodiment as the bringing together of the mind and body through the practice of open ended reflection. However, reflection is not on an experience, but is part of the experience so that one becomes mindful and aware, thus open to other possibilities.

Embodiment means that the mind is a natural and fundamental part of the body. Therefore the mind and the body work together as a living system comprising a number of autonomous layers that assist with the sense making process. Li, et al. (2010) indicate that the embodied experience an individual undergoes is due to the bringing together of the mind and body by means of reflection. Di Paolo, et al. (2007) augment this notion stating that cognition is an "embodied action" (p. 11) in that the mind and the body form part of a living system composed of various "autonomous layers of self-coordination and self-organization" (p. 12) which allow it to interact with

the world in “sense-making” activities, with the understanding that the body is not controlled by the brain.

An embodied activity occurs not through receiving information but when a transformation takes place between the structure and its environment. From an enactivist perspective, this experience is the guiding force between how one understands and perceives an event and the knowledge of the environment. This, in essence, indicates that the more an individual practices a process or action, the more skilled one becomes and thus the experience is modified through the process of embodiment and becomes an intuitive response to a particular trigger.

Davis (2004) suggests that embodiment refers to the idea that individuals are all part of a larger collective system and cites Kauffman’s description of the relationship between the collective and the individual as one in which “the collective is enfolded in and unfolds from the individual” (p. 213). With this in mind an aspect of the pedagogical practice is to encourage the formation of a mathematical community of practice amongst the pre-service teachers to research its influence on the lesson preparation and sense making processes. Communities of practice are instituted by groups of people sharing a common interest or concern about a particular subject or area of interest, and who through interacting with one another grow their knowledge and proficiency (Wenger, McDermott, & Snyder, 2003; Li and Crichton, 2008). A key benefit of communities of practice is that they allow one the space to engage with and develop ideas through interaction with others. According to Fenwick (2001a, p. 49) cognition develops from shared interaction since knowledge cannot be confined to just one dimension of a system since it is constantly emerging and impacting on other systems resulting in an “increasingly complex system”. Therefore individuals will develop through feedback gained from others and from their participation in a community of practice. By establishing a community of practice the student teachers in my study were required to take on three roles within the community: that of communicator, story-maker and interpreter. These roles would assist pre-service teachers in identifying changes and resolving old ideas that suppress emerging ideas and possibilities. The tracing and recording of the interactions of the participants and the objects would help all participants of the community to make

sense of the emerging patterns and to understand their particular role in the patterns respectively.

Breen (2004 p. 4) used activities to establish learning communities, focusing on the process that was developed to solve a problem. This resulted in his research participants commenting that within the learning community each person had something to contribute and that most had actively tried to create a learning community. He did, however, point out that not all the communities were functional. The co-created learning environment, resulting from an enactivist approach, highlighted the fact that individuals have different understandings of a concept and the importance of listening to and building on other people's existing knowledge in order to increase one's own understanding and also to achieve a common understanding.

Varela, et al. (1991, p. 27-29) indicate that the nature of reflection when analysing an experience needs to be embodied and open ended. By this they mean that it should be a mindful bringing together of the mind and the body, so that reflection forms part of the experience and is not merely on the experience itself. In order to achieve this one needs to carry out an activity/experience with awareness and mindfulness. One also needs to be open to other possibilities and be prepared to let go of preconceived ideas and notions if need be. To a certain extent this ties in with Piaget's idea of accommodation and assimilation (Van De Walle, Karp Bay-Williams, 2010, p. 20) in the sense that one needs to be aware of the mathematical knowledge and teaching ideas that the pre-service teachers bring to the table. One should reflect on what aspects of their current experience should be assimilated into the new information and ideas and then determine how this is going to be accommodated into their present structure in order to enable them to adapt to their environment as a teacher of Mathematics while also paying cognisance to the different strands of proficiency. This Varela, et al. (1991) believe is initially a skill that needs to be developed in order to establish the link between the mental intention and the bodily act. In order to achieve this mind-body unity they are of the opinion that one needs to practise until the gap between the intention and the physical act draw closer together and eventually disappear. This will result in the act of teaching mathematics proficiently becoming a naturally coordinated and embodied action achieved through a lived experience. It was my intention with this research to provide

students with a lived experience in order to give rise to perturbations that would encourage the development of new meaning. With respect to this research study I am trying to determine if one can get to a stage with students where teaching for proficiency becomes second nature so that, with sufficient personal proficiency and a range of strategies at their disposal, instead of teaching to only one strand, all aspects of proficiency are encompassed in their approach. Should the need arise the student would be in a position to readily draw from or have at their fingertips an alternative solution, scenario or context in order to teach proficiently. So, in essence, Varela, et al. (1991, p. 32-33) say that one needs to let go of habits of mindlessness, and this is what I hope to encourage through reflective activities that ask students to think about their participation, difficulties, strengths and weaknesses. Merleau-Ponty's idea that mindfulness disrupts mindlessness and prevents one from going about a task inattentively, not realising what one is doing, is a valid one since it is only in this sense that observation can change what is being observed. This is important since the students are not being assessed on their participation in the practical tutorial session and thus it would be easy for them to carry out their task mindlessly thereby not gaining any benefit from the module. According to Varela, et al. (1991, p. 33) mindfulness is an expression of our human embodiment and thus is able to provide a bridge between cognition and experience. As one pays more attention to one's embodiment the details become more refined and clear creating awareness in one's reactions and mental attitudes. This is a valuable aspect of this study where I want to create awareness in students both of their identity as Mathematics teachers and their teaching for proficiency.

2.3.5 Experience

Di Paolo, et al. (2007, p. 13) state that experience is not just data or information but that it guides the dialogue between phenomenology and in this case teaching for mathematics proficiency. Furthermore, experience changes through embodied action and the emergent process. The theme of experience was incorporated into this research through the introduction and inclusion of weekly tutorial teaching sessions.

Pre-service teachers were given the opportunity to develop lessons for the Intermediate Phase and to teach for proficiency. It was anticipated that these

experiences would change through embodied action and the emergent process. In the initial stages of the module, activities were incorporated to encourage pre-service teachers to discuss their experiences and critical incidents in mathematics teaching. In addition, students were required to reflect on their experiences to determine the influence that they had had in determining what they understood by proficiency and their role as a teacher in trying to teach for mathematical proficiency. Through being exposed to enactivist pedagogic practices I hoped that the students would be in a position to determine how proficient they were in each of Kilpatrick, et al.'s (2001) strands of proficiency and which strands would require further attention and development, both in their personal capacity and in their teaching practice.

The foundation of this research consisted of weekly tutorial periods during which students were requested to teach mathematical concepts to a group of peers. The student teaching the concept was observed by the researcher and a different group of peers, known as the observer group, who focused on noticing critical incidents that impacted on teaching for mathematical proficiency (See section 3.2.1.). These observations assisted in tracking changes in the pre-service teachers' teaching practices.

Varela, et al. (1991, p. 23) suggest using mindfulness as a means to investigate our experiences, experiences which are part of every moment of our lives and dependent on the situations in which we find ourselves and consequently always changing. Through being mindful of an experience one is able to determine one's level of proficiency and competence in teaching in addition to adding to one's knowledge of self and mathematical identity. Varela, et al. (1991) then explain how the abstract attitude that Merleau-Ponty and Heidegger ascribe to science and philosophy can become an everyday attitude when one is not mindful and thus disconnected from one's experience.

2.4 ENACTIVISM AND TEACHER EDUCATION (MATHEMATICS MODULE)

“Effectiveness depends on enactment, on the mutual and interdependent interaction of the three elements – mathematical content, teacher, students – as instruction unfolds” (Kilpatrick, et al., 2001, p. 9). With this in mind, I chose to use the five

themes of enactivism to underpin my pedagogic practice. From an enactivist perspective the role of the lecturer is that of a “perturbator” in order to encourage and provoke learners into “thinking differently” about mathematical concepts that may not form part of their personal construct. Since enactivism emphasises understanding meaning and value it becomes an appropriate choice to underpin the Mathematics programme under consideration (Di Paolo, et al., 2007, p. 3). In this regard, enactivism views meaning as “inseparable from the whole of the context-dependent, life-motivated, embodied activity” (Di Paolo, et al., 2007, p. 6). Furthermore, it also contributes to the development of social interaction and social understanding. According to Di Paolo, et al., (2007) an enactivist approach is a synthesis of old and new ideas, which mutually support each other. This lends itself well to collaborative learning whereby students are exposed to different ideas and approaches by being part of a Mathematics community.

Hocking (2004, p.10) states that an enactivist philosophy identifies two roles for lecturers in higher education, firstly that of a co-learner and secondly the role of a facilitator. This relates not only to what they do, but also to how they perceive themselves and others relationally both in and out of the classroom. Enactivism encourages learning and the construction of knowledge by means of a collaborative process, thus, in the context of this specific study, any given learning situation is co-created by the lecturer, the participants and the particular context. The lecturer needs to be observant and to listen acutely in order to identify and gain a “fuller understanding of both a learner’s subjectivity and the learners’ collectivity” (Breen, 2004, p. 3). According to Taylor (cited in Davis, 1996, p. 193), constructivism focuses on the formulated idea of human action, whereby our “thoughts, behaviours and knowledge ... are written into the texts of our experience” either in pursuit of or to validate truths. Therefore he states that enactivism as a theory of learning complements constructivism since it focuses on the unformulated idea of human action. This relates to Maturana and Varela’s notion that “every action is an act of cognition” (Davis, 1996, p. 193) since we are naturally coupled with our context or situation. This would indicate that a great deal of what we know is unformulated since we often know something or do something instinctively. Consequently, the lecturer must be able to adapt his/her teaching approach if necessary, and, secondly, identify errors in mathematical understanding that can be used as an

opportunity to create conscious awareness about an unformulated concept (p. 3). This is achieved by encouraging the mathematics community (participants) to engage in a collaborative attempt to identify inconsistencies in various subjective conceptions and the general consensus. Breen (2004, p. 9) indicates that by working on errors that colleagues have made, participants will be able to generate a different understanding of a concept. This notion of collaborative engagement suggests that learners will have to engage in some form of listening, be it evaluative, interpretive or hermeneutic (Breen, 2001).

Davis (1995) cites Walkerdine when raising the issue that mathematics teaching practice is often viewed in a negative light and thus subject to criticism because its very nature does not invite a learner to draw from their experiences and intuitive understanding, demanding instead that these attributes be suppressed. He continues his critique of school Mathematics and education stating that the focus tends to emphasise technical proficiency and acquisition of knowledge with scant regard to human agency. In agreement, Proulx (2004, p. 116) states that learning is the result of a change in the structure of the agent and therefore it is not the environment but the individual agent who determines the change and thus the learning that takes place. He explains this further by indicating that the way an experience is understood and interpreted is determined by the agent's internal dynamics. This has led to initiatives, such as the NCTM standards project, that encourage a more learner-centred and process-orientated approach and thus "a greater awareness of subjectivity of learners" (Davis, 1995, p. 3). Thus, states Breen (2001), "the hermeneutic task of the teacher is to ask questions and challenge the student in a way that forces the student to become aware" (p. 49). Kilpatrick, et al. (2001) concur that mathematical proficiency encompasses more than just technical proficiency and acquisition of knowledge and propose the notion that mathematical proficiency is dependent on five interwoven and interdependent strands, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. These strands will be discussed in greater depth further into the literature review.

2.5 CRITIQUING ENACTIVISM

Fenwick (2001a, p. 50), in critiquing enactivism from different theoretical perspectives, raised a number of valid points that I took into consideration when planning the reflective tasks. From a constructivist perspective there was a concern that in enactivism there is a lack of recognition of individual meaning-making and identity construction. In addition, she raises the issue that although enactivism establishes a link between cognition and community interaction, constructivists would argue that there are aspects of an individual's subjective world of cognition that are not accessible through action and dialogue. With regard to my research, I believed that these concerns could be addressed through the use of personal reflective tasks or journals. However, I believe that in any research that is undertaken, a participant is only going to share or reveal what they are willing to share. Furthermore, Hamilton (2007, p. 6) raises the point that an enactivist approach does not exclude a more direct and linear approach, if it is a more appropriate approach. Thus, I believe that during the course of the research study a constructivist approach may well be adopted, where appropriate, since an embodied approach will incorporate both formulated and unformulated ideas (Davis, 2004) as previously discussed.

From a critical cultural perspective, Fenwick (2001a, p. 51) cautions that enactivism does not address power relations that exist within certain cultures nor cultural practices, for example, tools of discourse. These she believes may enable a dominant system to sustain its own interests within the complex system. I believe that as the researcher I will need to try and ascertain these power relations by engaging in conversation and listening to the participants, particularly when they present their mathematical identity task. This will enable me to raise the issue of power relations within our mathematical community so that all participants and the lecturer can come to a common understanding of the issue. In addition, at the beginning of the study both the lecturer and pre-service teachers can discuss and agree on appropriate practices and conduct within the community.

Finally, Fenwick (2001a, p. 51) queries that if enactivism "demands that the interests and identities of individual elements be surrendered to the greater community", then

what of the vulnerable individual who is manipulated by a few who wish to sustain their own power by ensuring that their own experiences are the most valued in the collective. I believe that this is where the role of the teacher educator /researcher is very important in guiding the flow of learning and reflexivity and ensuring that all role players have a fair chance, as in an ideal school context. Likewise, when the pre-service teachers graduate and have their own classes they will be placed in a similar situation and will need to have a variety of teaching strategies at hand to deal with these issues and ensure all learners are treated equally.

2.6 ANALYTICAL FRAMEWORK – KILPATRICK’S STRANDS OF MATHEMATICAL PROFICIENCY

In order to analyse the perceived growth in teaching for mathematical proficiency I decided to use Kilpatrick, et al.’s (2001) framework for mathematical proficiency as the core analytical framework for the research study. This decision was due to Kilpatrick, et al.’s (2001) cognisance of the importance of noticing and analysing learners’ interpretations and of the role that different contexts, especially environmental and situational elements, play in the teaching and learning process. These are key ideas and concepts that are also embraced by enactivism, which makes it an appropriate model. Kilpatrick et al (2001) identify mathematical proficiency as encompassing five interwoven and interdependent strands, namely, conceptual understanding; procedural fluency; strategic competence; adaptive reasoning and a productive disposition.

2.6.1 The strands of mathematical proficiency and teaching for mathematical proficiency

Kilpatrick, et al. (2001) identify mathematical proficiency as encompassing five interwoven and interdependent strands, namely:

- conceptual understanding – comprehension of mathematical concepts, operations, and relations
- procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

- strategic competence – ability to formulate, represent, and solve mathematical problems
- adaptive reasoning – capacity for logical thought, reflection, explanation, and justification
- productive disposition – habitual inclination to see Mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy. (p. 5)

Thus, they are of the opinion that, in order for an individual to develop proficiency in teaching Mathematics one needs to develop:

- conceptual understanding of the core knowledge of Mathematics, students, and instructional practices needed for teaching;
- procedural fluency in carrying out basic instructional routines;
- strategic competence in planning effective instruction and solving problems that arise while teaching;
- adaptive reasoning in justifying and explaining one’s practices and in reflecting on those practices; and a productive disposition toward mathematics, teaching, learning, and the improvement of practice (p. 10).

In the subsequent paragraphs I discuss these strands in greater detail in addition to clarifying their link to the themes of enactivism employed in this research study.

Kilpatrick, et al. (2001, p .369) are of the opinion that teaching for mathematical proficiency entails a number of factors. Firstly, teachers need to have a fundamental understanding of what the goals of instruction are and what proficiency entails for the particular mathematical content they are teaching. In addition they need to have a core understanding of the mathematics that they are required to teach and how their students will build on it. Kilpatrick, et al. (2001, p. 369) maintain that “They need to be able to use their knowledge flexibly in practice to appraise and adapt instructional materials, to represent the content in honest and accessible ways, to plan and conduct instruction, and to assess what students are learning”. Of equal importance is the ability to analyse, hear and see their students’ mathematical ideas and respond in an appropriate manner to written work, reasoning and problem solving strategies. In addition, Kilpatrick, et al. (2001, p. 369) also state that teachers have

the capacity to identify the possibilities of a task and adapt them in such a way as to meet the needs of all students. In order to support the acquisition of mathematical proficiency they will need to have a good understanding of the different trajectories along which mathematical ideas can develop.

To provide clarity regarding the link between enactivism and mathematical proficiency, it would be prudent to discuss Kilpatrick, et al.'s (2001, p.369) five strands in greater depth.

2.6.1.1 Conceptual understanding

According to Kilpatrick, et al. (2001, p. 118), to have conceptual understanding in Mathematics one must have an integrated and functional grasp of mathematical ideas. This will enable one to connect new ideas to existing ideas and to retain this new knowledge. In addition, conceptual understanding indicates that one recognises why an idea is important and in what contexts it would be useful. Characteristics of learners with good conceptual understanding are the ability to represent mathematical situations in a variety of ways and have the knowledge to identify which representations are best suited to a specific purpose. Furthermore, they are able to explain and justify the connections and consequences among concepts and procedures. This understanding is generally hierarchical in nature “with simple clusters of ideas packed into larger, more complex ones” (p. 120). Learning with understanding allows the learner to identify the relationships between their school and outside environment experiences and integrate, modify or adapt their skills to solve real problems efficiently. Therefore, in order for teachers to teach for proficiency within this strand they need to have an integrated and interconnected knowledge of Mathematics, the development of students’ mathematical understanding and a range of pedagogic practices that take into consideration the mathematical concepts being taught and how the students learn these particular concepts. Thus a teacher’s classroom practice must exhibit the effective use of both personal mathematical knowledge and knowledge of their students and how they learn (Kilpatrick, 2001, p. 381).

2.6.1.2 Procedural fluency

Procedural fluency demands a sound knowledge of mathematical procedures and the ability to discern when and how to use them appropriately. A learner who demonstrates procedural fluency will be able to carry out procedures skilfully, with a high degree of flexibility, accuracy and efficiency. These learners are able to recognise the fact that well developed and structured procedures can be used for completing routine tasks (Kilpatrick, et al., 2001, p. 121). A further characteristic of this strand is well developed estimation skills. Sound procedural fluency is necessary for learners to deepen their understanding of mathematical concepts and identify relationships and patterns which in turn will enable them to develop the other strands of proficiency.

According to Kilpatrick, et al. (2001, p. 382), a teacher who demonstrates proficiency in teaching Mathematics has a repertoire of instructional routines from which to draw on. These routines encompass both mathematical pedagogy and classroom management. To implement these routines and strategies effectively the teacher will need to have the confidence to select the appropriate strategies, apply them flexibly and adapt them to suit a particular context and/or situation as it arises. Thus the proficient teacher would know how to respond appropriately to any given situation that may arise and have a wide variety of strategies to draw from, should a particular approach not address a given situation.

2.6.1.3 Strategic competence

Kilpatrick, et al. (2001, p. 124) define strategic competence as “the ability to formulate mathematical problems, represent them, and solve them”. They emphasise that this strand entails a learner having the skill to understand a problem sufficiently well to be able to generate a suitable mathematical problem model encompassing core mathematical elements in order to arrive at a workable solution. Furthermore, competency in this strand implies the capacity to recognise shared common mathematical structural relationships that may indicate possible solutions. Characteristics of learners exhibiting strategic competence are the ability to detect mathematical relationships and develop flexibility in solving non-routine problems that require productive rather than reproductive thinking, for example, being able to

identify several approaches to solve non-routine problems. An indication of growing strategic competence is the ability to replace cumbersome procedures with more efficient concise procedures and to differentiate and select the most appropriate procedure for any given problem.

A teacher who teaches proficiently will demonstrate strategic competence through his/her ability to analyse instructional problems and to address them in a reasonable and intelligent manner (Kilpatrick, et al., 2001, p. 383).

2.6.1.4 Adaptive reasoning

According to Kilpatrick, et al. (2001, p. 129), adaptive reasoning is “the capacity to think logically about relationships among concepts and situations”. It entails the use of deductive reasoning to consider the many facets of a situation, consider all the alternatives and arrive at a justifiable solution or conclusion. Thus, the essential element of adaptive reasoning is ensuring that one’s reasoning is indeed valid. Adaptive reasoning should also encompass “intuitive and inductive reasoning based on pattern, analogy, and metaphor” (Kilpatrick, et al., 2001, p. 129). According to Kilpatrick, et al. (2001), when determining the authenticity and appropriateness of their proposed strategies, students will need to make use of adaptive reasoning.

Developing adaptive reasoning proficiency requires that a teacher engage in reflective practices. This reflexivity would need to focus on teaching strategies and how they relate to the mathematical concepts being taught and the difficulties that learners may have in the learning process. In addition, it would need to consider learners’ misconceptions and conceptions regarding Mathematics and what “representations are the most effective in communicating essential ideas” (Kilpatrick, et al., 2001, p. 384).

2.6.1.5 Productive disposition

A characteristic of an effective learner of Mathematics is a productive disposition which is the capacity to appreciate the sense and meaningfulness in Mathematics (Kilpatrick, et al., 2001). These learners have a belief in their own efficacy and are generally confident about their ability to do Mathematics, recognising that with effort

and experience they are able to learn. In order to develop productive disposition learners need to be exposed to opportunities that will enable them to make sense of Mathematics and appreciate the benefits of persevering in order to experience the resulting rewards. A contributing factor in experiencing mathematical success relates to learners' disposition and their capacity to view their ability as expandable and themselves as capable of understanding Mathematics (Kilpatrick, et al., 2001).

The role of the teacher is critical in providing a mathematics environment that develops supports and maintains in the students a positive disposition towards Mathematics. "Effectiveness depends on enactment, on the mutual and interdependent interaction of the three elements – mathematical content, teacher, students – as instruction unfolds" (Kilpatrick, et al., 2001, p. 9).

Teachers who exhibit productive disposition proficiency know that "Mathematics, their understanding of children's thinking, and their teaching practices" are interconnected in such a manner as to be meaningful and make sense (Kilpatrick, et al., 2001, p. 384). Furthermore, they realise that by analysing their own teaching and classroom practice they will learn more about "Mathematics, student mathematical thinking, and their own practice" (Kilpatrick, et al., 2001, p. 384) and in this way have control of not only their continued development but their teaching practice.

In this study the role of the lecturer is thus significant with regard, firstly, to facilitating a mathematics environment that creates an awareness of identity and develops the student teachers' positive disposition towards Mathematics and the teaching thereof. Secondly the lecturer will take on the role of the observer, as described by Maturana (1987, p. 65) in order to notice demonstrations of "adequate conduct".

Using these five strands as a lens I analysed the outcome of underpinning the mathematics module with the themes of enactivism and the perceived increase and enrichment in teaching for mathematical proficiency of pre-service teachers.

2.6.2 The relationship between enactivism and the proficiency framework

2.6.2.1 *Autonomy*

Since I am of the opinion that all five strands of proficiency would have a bearing on the participant's construction of their mathematical identity I chose to focus on their identity, how it has been constructed and what factors influenced the change and/or growth in their identity over the duration of the module. In other words, why did they decide to adopt certain methods or respond in a particular way to different perturbations and how these have affected their identity. As the lecturer and researcher it was my responsibility to put in place activities that would, firstly, allow the pre-service teachers to gain a better understanding of their identity and, secondly, provide opportunities for the participants to develop and strengthen their identities.

Teachers who exhibit a *productive disposition* know that "Mathematics, their understanding of children's thinking, and their teaching practices" are interconnected in such a way as to make sense (Kilpatrick, et al., 2001, p. 384). They realise that by analysing their own teaching and classroom practice they will learn more about "Mathematics, student mathematical thinking, and their own practice" (Kilpatrick, et al., 2001, p. 384) and in this way have control of both their continued development and their teaching practice. The key characteristics of the theme of autonomy are the pre-service teachers' identity and self-efficacy. Thus the research study sought to provide the students with would initially prompt knowledge of their mathematical identity and then provide opportunities to respond to perturbations that would positively develop their teaching for proficiency. Students were required to reflect on the triggers in their mathematical history which, through structural determinism (Varela, et al., 1991), they had autonomously chosen to respond to in a manner that had constructed their identity to date.

Through providing the students with further opportunities to determine their emerging proficiency and its impact on their self-efficacy, the study hoped to ascertain the impact it had on their perception of their mathematical identity and their ability to teach for mathematical proficiency. The study also hoped to determine and

understand the autonomous choices the students made to change and grow their identity and *productive disposition* in light of their increasing awareness of their identity. Motivation is an aspect of productive disposition and given that the pre-service teachers were aware that would be teaching Mathematics it was anticipated that a developing identity would encourage them to respond to the appropriate perturbations.

Kilpatrick, et al., (2001, p. 131) suggest that a student with a productive disposition has the capacity to appreciate the sense and meaningfulness in Mathematics and is an effective learner of the subject. In order to develop a *productive disposition* one needs to be exposed to opportunities that will enable one to make sense of Mathematics and appreciate the benefits of persevering in order to experience the rewards of making sense of it. Therefore the role of the teacher is critical in providing a mathematical environment and community for students that develops, supports and maintains a positive disposition towards Mathematics.

According to Kilpatrick, et al. (2001, p. 118), to have *conceptual understanding* one must have an integrated and functional grasp of mathematical ideas. This will enable one to connect new ideas to existing ideas and to retain this new knowledge. As a teacher one needs to have a sound knowledge of Mathematics, of the development of students' mathematical understanding and a knowledge range of pedagogic practices that take into consideration the mathematics concepts being taught and how the students learn these particular concepts. Metacognition is "knowledge about one's own thinking and ability to monitor one's own understanding and problem-solving activity" (Kilpatrick, et al., 2001, p. 118). Thus as pre-service teachers supplement and consolidate their *conceptual understanding* of the content that they are required to know so too should their mathematical identity be further developed and refined. In addition, activities (perturbations) involving *strategic competence* and *adaptive reasoning* will create an awareness of their thinking, understanding and problem solving skills.

Procedural fluency supports *conceptual knowledge*, therefore a participant who is competent in these strands will have a sound content and procedural knowledge, and the understanding of when and how to use various procedures appropriately.

2.6.2.2 Sense-making

Proficiency in *conceptual understanding* supports retention (Kilpatrick, et al., 2001, p. 118) and this, in turn, will allow pre-service teachers to play an active and participatory role in emerging conversations as they try to represent their *conceptual understanding* in different ways in order to clarify specific mathematical activities. The different modes of listening should encourage proficiency in *strategic competence* as participants recognise shared common mathematical structural relationships that may reveal possible solutions. It entails the use of *adaptive reasoning* to consider the many facets of a situation, consider all the alternatives and arrive at a justifiable solution or conclusion. Both interpretive and hermeneutic listening will encourage elements of *adaptive reasoning* to ensure that one's own reasoning is valid and to determine the authenticity and appropriateness of the other participants proposed strategies and thought processes.

2.6.2.3 Emergence

Developing *adaptive reasoning* proficiency requires that a teacher engage in reflective practices. This reflexivity would need to focus on teaching strategies and how the pre-service teachers relate to the mathematical concepts being taught and the difficulties that learners may have in the learning process. In addition, it would need to consider learners misconceptions and conceptions regarding Mathematics and what “representations are the most effective in communicating essential ideas” (Kilpatrick, et al., 2001, p. 384). Furthermore *adaptive reasoning* would require the participant to reflect on how new concepts relate or link to old conceptual knowledge. With regard to *procedural fluency*, through reflecting on shared activities and the collective generation of knowledge and understanding, the participant will be able to consolidate or develop their understanding of why a procedure is being used and if they need to adapt or refine the procedure to make it more efficient.

2.6.2.4 Embodiment

Within the mathematical community of practice the pre-service teachers will need to exhibit the effective use of both personal mathematical knowledge and an increasing

knowledge of their peers and how they learn in order to create an environment in which the exchange of ideas and understanding can result in both an individual and a common understanding of mathematical concepts as they relate to the Intermediate Phase and the task at hand. Although the pre-service teachers may have a sound *conceptual understanding*, Kilpatrick, et al. (2001, p. 118) indicate that students tend to understand content before they can verbalise their understanding. Therefore within the community of practice the pre-service teachers will be given the opportunity to verbalise their understanding and receive feedback from their peers. An essential component needed for *strategic competence* is flexibility throughout the problem solving process. As part of a mathematical community of practice participants are exposed to various strategies and representations and thus provided with opportunities to develop flexibility. In addition, participants need to justify their approach and strategies and thus advance their *adaptive reasoning* skills.

2.6.2.5 Experience

The intention of the tutorial sessions was to provide the pre-service teachers with the opportunity to practise teaching various mathematical concepts. The five strands of proficiency provided a framework for the participants to develop their lessons and guide the approach that they would take.

Therefore I, as the observer in this research study, used Kilpatrick's five strands as the framework for characteristics of proficiency and teaching for proficiency to determine evidence of the *bringing forth* or enactment of meaning and demonstrations of adequate conduct. These strands were also cross referenced with the five themes of enactivism to inform and evaluate my manifestation of enactive pedagogy.

2.6.3 Enactivism and learning

Taylor and Biddulph (2001, p. 2) indicate that enactivism views learning as ecological, and secondly that it is influenced by social, biological and historical factors. Hence enactive learning is "interactive, interrelational and interdependent" (Taylor & Biddulph, 2001, p. 2). Enactivism encourages learning and the construction of knowledge by means of a collaborative process. Consequently, any given learning

situation will encompass the lecturer, the student, the content and the context in order for some form of interaction to take place (Breen, 2001, p. 43). In a lecture situation where individuals are interacting with one another, both systems would be affected and hence their structures are said to co-emerge with an altered structure. Proulx (2008a, p. 15) describes this process as structural coupling whereby the environment and the species co-adapt to one another as they co-evolve. According to Maturana and Varela (cited in Proulx, 2008a, p. 16), structural coupling is the consequence of structural determinism, whereby the environment acts as a trigger for the species to evolve and it is the structural makeup of the system that will determine what change and the extent of the change that occurs. Although change is dependent on the environment it is not determined by the environment. Therefore, this theory serves as a sound foundation for teaching and learning since it firstly focuses on the teacher and learner working together within a community and secondly excludes the dualistic approach to teaching and learning that has traditionally been favoured. In the ensuing paragraphs I will expound on the themes of enactivism and illustrate how they effect and influence teaching and learning with particular reference to this research study.

2.6.4 Enactivism and teaching

According to Li, et al. (2010, p. 12-13) enactivism claims that the teacher, or in this research study the lecturer, is the co-author of knowledge and not the source of knowledge, “through the design of a complex learning world” that integrates biological systems and electronic media (Lozano, 2005). “Instead of focusing on a “single best sequence” of lessons for learning (Jonassen cited in Li, et al, 2010), teachers in enactivist classrooms build a rich learning world with abundant stimulation, but with enough limits to guide students toward possible co-evolving patterns” (Li, et al., 2010, p. 15). The lecturer or teacher takes on the role of a “perturbator” in order to encourage / provoke the learners into “thinking differently” (p. 4) about mathematical concepts that may not form part of their personal construct.

Hocking (2004, p. 53) notes that when we teach we do not only teach content but we also teach ourselves. Furthermore, we bring to the classroom not only ourselves as

individuals with our own identities, but ourselves as particular members of communities and cultures. He discusses a paradox of teaching; in the classroom we are required to connect with other bodies and so teach beyond ourselves often being called upon to challenge our own understandings. As a result, teachers are always in a state of flux and can be called upon to transform themselves as they interact within the classroom community. Thus the implications for teaching from an enactivist perspective are that we are part of an embodied network that shares related experiences. Therefore the classroom can be seen as a living system that invites mindful reflection “based on a commitment to education as a space of possibility” (Hocking, 2004, p. 60).

Enactivism views teaching as the quality of the relationship that we have with others (Hocking, 2004, p. 63) and recognises the value of the various connections that occur while participating in a classroom environment, for example, mind and body or environment and students connections.

According to Hocking (2004, p. 184), in order for embodied action to take place there must be occasions for interaction between theory and practice. This gives the student the chance to experience the interconnectedness between thought and action and it allows them to put their beliefs into practice before finally reflecting on how these practices shape their “values, beliefs and feelings” (p. 184). Therefore a teacher needs to have activities that “facilitate enactive ways of being” (p. 183) within the classroom, requiring students to reflect on different types of experiences in order to integrate their skills and knowledge into their identity as it continues to form/transform within a specific community of practice.

Enactivism stresses “that we are the worlds we create through our interactions with others” (Hocking, 2004, p.186) and thus teaching can be seen as a living practice. He then states that we cannot separate our individual identities from our institutional identities, and the requirements and pressures that go with that, and so we need to look for a middle path between the two by being mindful in the moment. From an enactivist perspective, self-inquiry is essential for a teacher’s development, in order that he/she may become more aware of what is important in the world view and determine “what matters as a teacher and learner”, thereby “strengthening

classroom communities through ongoing action and reflection” (Hocking, 2004, p. 192).

Hocking (2004, p. 193) identifies six values that he believes are central to an enactive view of teaching.

1. Contemplation since awareness is an essential part of self-transformation.
2. Interrelatedness because from an enactivist perspective individuals are only able to reach their potential through a network of relationships, firstly those that surround us and secondly those that we create. He then elaborates on this, explaining that an enactive learning environment must facilitate “nurturing relationships with others as an integral focus of co-learning and inquiry” (p. 193).
3. Acceptance of all participants in the classroom as integral members of that particular learning community. This entails genuine acceptance and respect for participants regardless of their diverse and perhaps imperfect knowledge, skills and backgrounds.
4. The values of spontaneity and improvisation will enable one to “enact embodied ways of living in academic settings” (p. 195). This is important since an enactive pedagogy will require one to let go and provide multiple opportunities for learning in order to trigger both personal and collective co-emergence.
5. Resilience in order to recognise that teaching and learning from an enactive perspective will be “cycles of ebb and flow, wounding and healing, pain and joy” as one engages with the space of possibilities. (p. 195).
6. Finally, the value of sustainability is important as enactivism is only possible in self-sustaining environments. Therefore the teacher needs to ensure that the basic structures of education remain intact and that democratic values are upheld. In order to do this, teachers will need to maintain and support the interconnected relationships that develop between the classroom participants thus sustaining an enactive way of being.

Finally, in order to implement enactive instructional processes, Hocking (2004, p. 197) maintains that the processes need to be dialectical, performance-oriented and self-reflexive in nature.

Since constructivism has been the mainstream theory underpinning education over the last decade or so, it makes sense to briefly compare the two theories. Constructivism is seen to have “a relativist view of knowledge” (Begg, 2000, p. 1) with each learner’s construction of knowledge being unique to the individual and is therefore an indication of their view of reality. In addition, this theory encompasses a number of classifications and when categorizing the different forms of constructivism, one sees the emergence of dichotomies, for example individual (radical constructivism) vs. social (social constructivism) (Begg, 2000, p. 2).

Enactivism, on the other hand, views knowledge as an embodied action in which the learner is viewed as a structure that continually changes in order to adapt to its environment in contrast to constructivism which sees that learner as active in the process of constructing his or her subjective knowledge (Holton, 2010, p. 1). Furthermore, enactivism considers cognition to include both formulated and unformulated knowledge. Formulated knowledge entails a conscious act by the learner while unformulated knowledge is considered to be an instinctive action carried out by the learner. Holton (2010, p. 5 -6) further indicates that unformulated knowledge also encompasses emotion and mindfulness which are interrelated with a learner’s way of knowing. Enactivism defines knowing as that which is known intuitively in addition to incorporating the role of emotions in learning.

Constructivism considers learning as coming to know something. The emphasis is on human constructed knowledge and is regarded as something which fits with an experience of the knower. Knowledge is seen as an object that is shaped by context and interaction with others which results in individual sense making (Begg, 2000). This process is considered to be an active process whereby an individual constructs his “own subjective knowledge” (Holton, 2010, p. 1) and is shaped by the learners individual experiences of the world (Proulx, 2008a, p. 14). This knowledge is then either assimilated or accommodated in the existing knowledge of the individual. These experiences then act as a filter through which the learner will understand new experiences. Thus, when defining knowledge, “the key for constructivists is an adequate fit, not an optimal match” and knowing as “*fitting to*, and compatibility with, the experiential world” (Proulx, 2008a, p. 15).

In comparison, enactivism defines knowledge as “an effective (or adequate) behavior in a given context” (Maturana & Varela, 1998, p. 174). As Proulx explains (2004, p. 115) since enactivism considers the mental and physical process to be one, “knowledge is adequate action in the world: behaviour IS knowledge” or, as Hamilton (2006) indicates, it is effective behaviour within a given cultural context. Enactivism views learning as co-evolving and knowledge as an action with an emphasis on knowing. As such, knowledge is embedded in the world and structures (individuals) through co-emerging in different systems, for example within a classroom, are able to make sense of embodied cognition (Begg, 2000, p. 8). The ensuing collective action results in shared understanding and meaning (Davis, 1996). To make a distinction between knowledge and knowing through the lens of enactivism: learners interpret and bring forth their knowing through interaction with the environment and use this to develop the best possible strategy to solve their problem. This process of determining a strategy is an example of the learners’ knowing. Knowledge therefore is a demonstration of adequate action for a particular situation as observed by the person posing a problem (Proulx and Simmt, 2013).

With regard to education, from a constructivist perspective, teaching and learning is student centred, in which active participation and cooperative learning is encouraged. The teacher co-constructs knowledge together with the learner (Holton, 2010).

Hamilton (2006, p. 6) suggests that education from an enactivist position encourages “teachers and students to reflect deeply on their practice to understand the purpose of all actions”. Furthermore, it encourages classroom research in order to develop students holistically and support the use of processes that exclude dualistic thinking, resulting in flexible pathways for student learning. Lastly, an enactivist education is supportive of both self-assessment and assessment practices that take into consideration all the domains in which a student can operate. The importance of listening and dialogue is emphasised. With regard to the teaching of Mathematics, it is essential that the teacher needs to anticipate and be open to the variety of directions that a particular lesson may take, in order to form an embodied understanding (Davis, 1996, p. 238).

2.7 CONCLUSION

For this study I chose to underpin the Mathematics module with the themes of enactivism since I wanted to try a different approach which would develop teaching for mathematical proficiency. I chose enactivism above constructivism, since although similar in many respects, I believed it to be a more embodied approach.

The next chapter on methodology describes what this looked like in practice, and discusses the research methods that were employed in this study. In addition, I explain what the themes of enactivism looked like in my practice.

CHAPTER THREE

METHODOLOGY

In the previous chapter, I reviewed the literature relating to enactivism and mathematical proficiency. In this chapter I present the theoretical and methodological framework that structured this research study. As will be detailed in this chapter this interpretive study draws on both a case study approach and elements of action research as its methodology. The focus is a one year mathematics module (MIP 400) which was underpinned by the themes of enactivism. The data comprised of questionnaires, observations, video footage, focus and individual interviews, and reflective tasks.

3.1 ORIENTATION OF THE STUDY

It was the intention of this study to determine how a mathematics module, underpinned by an enactivist philosophy and teaching pedagogy, would help students to unpack the reality of their teaching practice in terms of proficient teaching. This research study is qualitative in nature and situated within an interpretivist paradigm. Qualitative research attempts to understand the behavioural patterns of people or systems within their own environment together with their understanding and interpretation of a given context (Nieuwenhuis, 2007a, p. 54). It is often associated with small scale projects encompassing researcher involvement from a holistic perspective (Denscombe, 2010, p. 237). The study was a small scale project that focused on a mathematics module that was carried out over the period of one year.

My theoretical framework was informed by enactivism and how the themes of enactivism, discussed in the previous chapter, could be used as a vehicle to develop teaching proficiency. In addition, reflexivity by the students played an extensive role in integrating action and research within this study and in shaping and informing the phases of the research. Coghlan and Brannick (2010, p. 41) describe reflexivity as the concept that is “used to explore and deal with the relationship between the researcher and the object of research”. Reflexivity was observed in the emergence theme of enactivism and helped me to track the perceived growth in proficiency.

All research takes an ontological position. Merriam (2009, p. 8) describes ontology as being “what one believes about the nature of reality”. I positioned the ontological perspective in this study as the process of reflexivity that took place by both the researcher (observer) and the research participants in analysing their mathematical identity and beliefs with regard to their role either in training future mathematics educators or the role of an educator teaching for mathematical proficiency. Having established a mathematical identity pertaining to a specific role and reality, the researcher and participants needed to reflect on and determine whether in fact their identity or belief was being carried through in epistemological issues and if not, why not. Davis (1995) indicates that when using enactivism a person should develop their understanding of what it means to do Mathematics. Thus rather than following a syllabus rigidly we should be open to exploring “what it means to behave mathematically” (p. 6) and encourage children to participate in the shaping of the mathematics that they are learning. He considers education to be “an ontological issue in which we question our existence, being and identity” (p. 6) but he is of the opinion that generally mathematics educators focus on “epistemological issues such as knowledge, programs, learning and instruction” (p. 6). Davis (1996, p. 89) justifies this by saying that we are “the product of our experiences” and thus our minds and identities will evolve in relation to one another. Since enactivism brings knowledge and action together and its perspective regarding cognition is that an individual affects the formation of the world and vice versa, Davis positions enactivism as an ontology (1996, p. 191).

Since I was interpreting the research participants’ responses to the module and practical tutorials, and their perception of how it affected their personal proficiency in Mathematics and their teaching for mathematical proficiency through exposure to themes of enactivism, my research is situated in an interpretivist paradigm. Within an interpretivist paradigm, the researcher tries to make meaning of people’s beliefs, values and intentions, with particular reference to why and what meaning they make of a given situation or context (Henning, 2005, p. 20). Thus, my understanding was informed by the research group in the study and required that I, as the observer, was especially sensitive to my role within the given context and how the team dealt with this experience. In order to gain a better understanding of the role that the themes of enactivism (See section 2.3) played in enriching teaching for proficiency, questions

and tasks were supported by linking them to one of the five themes of enactivism. Merriam (2009, p. 9) indicates that interpretive research yields multiple observable realities, which is consistent with enactivism philosophy in that, as autonomous beings, we respond (or not) to perturbations differently.

Two perspectives of interpretivism that I made use of were hermeneutics and phenomenology. Hermeneutics is the practice whereby we try to recognize current practices and understanding through asking questions such as, “What is it that we believe? and How is it that we came to think this way?” (Davis, 2004, p. 10). I used hermeneutics as a means to understand the study in terms of developing and encouraging teaching for proficiency and I used phenomenology as a means to understanding the nature of teaching for proficiency within the context of the study (Davis, 1996, p. 32). According to Varela, et al. (1991, p. 149) hermeneutics is a form of interpretation and is thus “the *enactment* or *bringing forth* of meaning from a background of understanding”.

Phenomenology was one of the most influential movements in the development of the notion of an embodied mind (Davis & Sumara, 2006, p. 71). Phenomenology requires that an individual be in closer contact with their world in order to construct their reality, therefore phenomenology asks “What is this experience like?” (Davis, 1996, p. 32). It does not give a theoretical account of an experience rather it requires that individuals give an account as they live the experience. In order for individuals to describe or explain a given experience they would need to use language, which infers that it is interpretive in nature. Furthermore, phenomenology allows the researcher to “describe phenomena dispassionately” (Davis 2004, p. 208) which made it suitable for this study, given my role as an observer. Varela, et al. (1991, p. 27) discuss the link between mindfulness and phenomenology indicating that it influences the nature of reflection moving it from being disembodied to embodied. By this they mean that an individual does not merely reflect on an experience, the reflection is in fact a form of the experience.

Lakoff and Johnson (1999) identify three levels of embodiment, of which one is phenomenological. At this level one is aware of everything that there is to be aware of, in respect of our minds, bodies, experiences and environment. This, they say, is

the same as phenomenological reflection. They then discuss the idea that embodied truth means that there is not one unique description of a given situation, instead there can be a variety of correct descriptions of the truth that an individual has. This is due to the individual's structure encompassing different levels of embodied understandings of a situation (p. 109). It is through these levels that one is able to consider conceptually what is required in order to survive, achieve a workable understanding and/or function successfully in a given situation.

3.2 METHODOLOGY

My research study is an in depth study of a mathematics education course designed to grow and develop proficient teaching in mathematics in the Intermediate Phase, making a case study the most appropriate choice of research design. Furthermore, this was a qualitative enquiry that focused on the perceptions of a small number of students registered for the module. The aim was to analyse whether the themes of enactivism had any effect in their development in teaching for mathematical proficiency (Mentor, Elliot, Hulme, Lewin & Lowden, 2011, p. 42). A "case study assumes that 'social reality' is created through social interaction" (Stark & Torrance, 2005, p. 33) which made the design a good fit with enactivism, since from an enactivist perspective learning occurs through co-emergence with our world (Begg, 2013, p. 82). In addition, the use of a case study meant that I could study the relationships and social processes that ensued within the context of the module as well as the co-emergence of the research participants as a result of structural determination (Denscombe, 2010, p. 55).

In line with Stark and Torrance's (2005, p. 33) thinking, this was "a 'case' of innovative training" as it related to a mathematics module at a private institution that specialised in training teachers. In addition, I was observing and studying the research participants' from the position of asking "What does 'the case' look like ... (from the) participant's point of view?" Stark and Torrance (2005, p. 35) indicate that when employing research methods in a case study it is often helpful "to ask respondents to identify and reflect on a 'critical incident' in their ... situation". This corresponded well with the reflective tasks that the participants were asked to

complete whereby a 'critical incident' was viewed as a perturbation for the purposes of this study.

In this research, I investigated a self-contained entity with distinct boundaries thereby meeting the criteria for a case study (Denscombe, 2010, p. 56; Merriam, 2009, p. 41). The mathematics module (MIP 400) that served as the case study was a year long and I had two units of analysis for my study, namely proficient teaching and the students' perceptions thereof. The mathematics module provided me with an ideal opportunity to investigate the extent to which the themes of enactivism enhanced the participants teaching for proficiency (Cohen, Manion & Morrison, 2000, p. 181; Denscombe, 2010, p. 53). Merriam (2009, p. 43) talks of the researcher uncovering the "interaction of significant factors characteristic of the phenomenon" when undertaking a case study. This was my intention in selecting Kilpatrick, et al.'s (2001) five strands of proficiency as the lens through which to identify the most significant themes of enactivism for enriching teaching for proficiency.

Although my research design took the form of a case study, elements of action research were embedded in it since I wanted to investigate whether the mathematics module (MIP 400), underpinned by the themes of enactivism, would help the students to unpack their levels of proficiency, in three different phases. I used the first phase of the case study to determine students' perception of their levels of proficiency, the second phase to promote the use of all the strands of mathematical proficiency and the final phase to determine what the students learnt from the module and applied to their practice as qualified teachers.

Cohen, et al., (2000, p. 226) state that action research is "a small-scale intervention in the functioning of the real world and a close examination of the effects of such an intervention" making it the ideal approach to embed in a case study. Furthermore, action research is undertaken with the specific purpose of producing knowledge together with the participants who are going to be affected by that knowledge (Bhana, 2006, p. 430). Each phase was influenced by elements of Kolb's cycles of experiential learning, "consisting of four major moments: plan, act, observe and reflect" (Zuber-Skerritt, 1992). Bhana (2006, p. 430) states that "[Participatory action research] aims to produce knowledge in an active partnership with those affected by

that knowledge, for the express purpose of improving their social, educational, and material conditions". Ideally, at the conclusion of this module a student should be proficient in all five of Kilpatrick, et al.'s (2001) strands (section 2.6) and the objective of this study was to determine how the five themes of enactivism enriched the various strands. Therefore I continually referred to the lens of Kilpatrick's five strands of proficiency when evaluating the module as part of the research process to ascertain the effect of the enactivism themes underpinning the module. The use of elements of action research led to the research being undertaken in three phases, described later (section 3.2.2.). Lessons learnt from the initial phase (such as the students focusing on only two of Kilpatrick's five strands, see section 3.2.2.) were thus used to develop subsequent phases. Therefore, the elements of action research produce useful knowledge that informs the case study about the levels of proficiency at each phase.

An additional feature of action research is that both the researcher and the students are involved in decisions relating to the research process. This approach was used as a means to empower the students to unfold their teaching proficiency (Struwig & Stead, 2001, p. 15). Indeed the initial focus of action research is usually to find "practical, workable solutions to problems", generating knowledge to attend to the concerns (Bhana, 2006, p. 439). Similarly, enactivism recognises that any given learning situation is co-created by the lecturer and the participants. This was of particular interest to me, because, as educators, we are aware of the problems and difficulties in Mathematics in South Africa, but we need practical and workable solutions to these problems.

Bhana (2006, p. 432) identifies three key tensions that action research raises, namely, the tension between science and practice, the tension resulting from trying to mediate between individual and collective needs and, thirdly, the ensuing tension in the relationship between those being researched and the researcher. With regard to the tension pertaining to science and practice, it is the belief of action researchers that authentic knowledge can only occur if there is a shift in the knowledge base of the participants. It was therefore important that this study led not only to the research documented in this thesis, but also to meaningful learning by the participants. The tension between the individual and the collective stems from a desire to ensure that

knowledge resulting from the knowledge creation process does not become the property of an individual or small interest group. Bhana (2006, p. 432) does raise the concern that there is a danger that the needs of the individual could be surpassed by the needs of the collective. Bhana (p. 432) clarifies the tension between those being researched and the researcher as the researcher tries “to know *with* others, rather than *about* them, and to reconceptualise and foster knowledge as something that exists among people, rather than as some sort of barrier between them”. Therefore, as the researcher I needed to develop an open relationship with the participants and encourage them to become fully involved in all aspects of the research process. In order to achieve this I encouraged students’ participation through various means, namely reflective tasks, observation, group feedback and group participation in determining teaching strategies, with the underpinning thread of “How does this relate to my mathematical autonomy/identity?”

Bhana (2006, p. 438) identifies the fundamental goal of the relationship between participants and the researcher as being structured transformation and an improvement in the lives of all involved in the study. He elaborates on this by saying that a successful action research study will result in empowerment, specifically in a “raised awareness in people of their own abilities and resources” (Bhana, 2006, p. 430). I was able to align this element of action research in my case study to the enhancement of the participants’ awareness of their proficiency in order to gather data pertaining to the undertaking of this research project. Empowerment helps the participants to begin to view themselves as educators who can make a difference and who are self-confident when articulating their opinions and thoughts. The third phase of the case study tried to determine if this did indeed occur as the research participants were by then qualified teachers and had taught in the Intermediate Phase for a year. Furthermore, Bhana (2006, p. 439) points out that empowerment results in changes to the ecology of knowledge in that the participants may have learnt new skills and they could now be consulted on issues where previously their voices were not heard. In terms of this study, the perceptions that emerged through the data were used to inform the conceptualisation of the module, and in fact some practices were incorporated into other mathematical modules offered at the Institute based on feedback that was generated in this study.

Winter (1996, p. 13 -14) identified six principles of action research namely, reflexive critique, dialectical critique, collaboration, risking disturbance, creating plural structures and theory and practice internalised. From his definitions of these principles, and on closer inspection, it was evident that these six principles were important guides for this case study. He describes *reflexive critique* as a process whereby a person becomes aware of their perceptual bias. Reflexivity had an important role in this research as students were required to reflect on their personal proficiency in Mathematics, their identity and self-development, and how these embodied perceptions of their mathematical proficiency supported their own teaching for mathematical proficiency. *Dialectical critique* refers to the manner in which one understands the relationships between the elements that a specific context is composed of. This encompassed the enactivist notion of a person's structure co-evolving with other structures in any given situation and the individual's response to the perturbations emanating from that particular experience. The principle of *collaboration* was addressed from an enactivist perspective through the view that cognition is a result of systems interacting and affecting one another. Thus every pre-service teacher's point of view needed to be taken into consideration in order to arrive at a common understanding and the creation of the *plural structures* which resulted from the various accounts and critiques. Winter's (1996) definition of *risking disturbance*, is similar in nature to the manner in which the perturbations provided the students with the opportunity to evolve by choosing to relook at their current understanding of a topic or mathematical concept. In addition, through structural coupling with other students they were able to submit their individual understanding of a concept for critique. This also applied to their teaching approach which was held up to their peers for comment, critique and further development. Lastly, Winter suggests that *theory and practice is internalised* when they are seen as "two interdependent yet complementary phases of the change process" (cited in Cohen, et al., 2000, p. 229). Thus it was important that both my participants and I were able to use the learning from the study.

Enactivism views cognition as a complex co-evolving process of systems interacting and affecting each other, not as processors of information but as producers of meaning. Lozano (2005, p. 7) explains this as the manner in which an individual's interactions with the world and their past experiences shape and influence the

meaning that they make of their world. The students were asked to reflect on their experiences at each tutorial session and identify what aspects or critical incidents they believed had influenced their proficiency. By incorporating reflexivity into the module I wanted the students, firstly, to become more mindful of their own and other's teaching practices and, secondly, to develop a discipline of 'noticing' when teaching. Begg (2009, p. 207) refers to 'noticing' as thinking that results from living mindfully, in the sense that we become aware of what we know from an experience, analysing our behaviour as we anticipate future possibilities or experiences. Relating this to teaching he gives the example that through the discipline of noticing we are able to give consideration to different strategies and interventions.

Fenwick's (2001a, p. 50) suggestion that in order to assist with the flow of learning in systems the teacher educator tracks, identifies and points out individual changes, ties in well with an action research approach, an aspect that I was able to draw on in the case study when planning the reflective tasks linked to the tutorial sessions (Craig, 2009; James, Milenkiewicz & Bucknam, 2008). Furthermore, the teacher educator or participants could either draw attention to the perturbations or introduce perturbations that could create learning opportunities and further intensify their potential by naming, focusing or highlighting their significance. This in essence means that through structural coupling both the teacher educator and participants act as reciprocal triggers to one another in the learning process as in when "teacher educators *get in the way, provoke* the learning process, *orient* ideas and *influence* what is learned, *create* opportunities, *influence* teachers" and then act upon the participants responses to these triggers. (Proulx, 2010, p. 61) .This notion served both as feedback to the individual systems and the community of practice system as a whole in addition to generating hermeneutic listening and conversation about the perturbation (Section 2.3.2.).

3.2.1 Teaching Module

The research took the form of a case study of a fourth year Mathematics module⁷ (MIP 400) for B.Ed. degree students training to teach in the Foundation Phase. The duration of the module was a year and it is aimed to equip the Foundation Phase students with the basic skills and content to teach Mathematics with confidence and proficiency in the Intermediate Phase, since a number of exiting students were appointed to teach in the Intermediate Phase, although trained and qualified as teachers in the Foundation Phase. The aim of this Mathematics module was to develop and cultivate students who would be able to act as role models for their learners and instil in their learners a love and interest in Mathematics. Thus the module needed to firstly, augment the students' basic skills in Mathematics and, secondly, provide them with opportunities to become competent and proficient in basic Mathematics and pedagogic skills for the Intermediate Phase. The curriculum of this module focussed on and encompassed the following nine units:

1. Mathematics proficiency and teaching for proficiency
2. Mathematics anxiety and teaching for diversity
3. Understanding the foundations for learning Intermediate Phase
4. Strategies for whole number computation
5. Developing fraction concepts and computation with fractions
6. Developing concepts of decimal and percentages
7. Developing concepts of ratio and proportion
8. Geometric thinking and concepts
9. Algebraic thinking: generalisations, patterns and functions

The module comprised a weekly ninety minute lecture (a double period), underpinned by the five themes of enactivism, namely autonomy, sense-making, emergence, embodiment and experience, in which mathematical theory and content was discussed. In addition to the lectures, weekly practical tutorials were held during which the students were given the opportunity to develop their Mathematics teaching skills and thus participate in the experience theme. Although the practical tutorials were held on a weekly basis, they were attended fortnightly by Group A and Group B (See Figure 3.1). The themes of enactivism and indeed the fact that they

⁷A module is the same as a course and can be either a semester or year in duration.

underpinned the module, was not discussed with the students and were incorporated within the module by means of various activities (Table 3.1).

Table 3.1

Activities that were used to encompass the themes of enactivism within the mathematics module (MIP 400)

Theme	Activity
<p>Autonomy This theme related to the construction of the students mathematical identity, their ways of knowing mathematics and their self-efficacy.</p>	<p>The students were required to complete reflective questions/ tasks after practical teaching tutorials (<i>theme of experience</i>) relating to identity issues, resulting from their participation in their group.</p> <p>All students were expected to contribute and participate actively in group activities.</p>
<p>Sense-making This theme referred to the student's mode of listening and active and participatory role in the emerging conversations about Mathematics, and the teaching of mathematical concepts.</p>	<p>In order to create an awareness of their role in sense-making and the process thereof, students were required to complete:</p> <ul style="list-style-type: none"> • An observation sheet during practical tutorial sessions. • A self-reflective task on completion of the preparation and delivery of a Mathematics lesson. <p>In addition, the students were expected to actively participate in their group to reach a common understanding of the task and to determine an appropriate teaching strategy.</p>
<p>Emergence, related to the collective generation of knowledge and understanding by the students allocated to each group and to use reflective techniques to identify critical incidents. In addition, students were required to reflect on and understand the purpose of their actions.</p>	<p>Examples of reflective tasks that the students were required to complete after group work, included:</p> <ul style="list-style-type: none"> • Identifying a critical incident that related to their proficiency or teaching for proficiency • "What listening did I do today" – identify the incident, how did this contribute to their understanding for proficiency
<p>Embodiment incorporated being part of a larger collective system, namely their mathematical group, and as a member of the lecture group, and realizing that each person had something to contribute. The intention was to create an awareness of the feedback gained from others and identifying changes and resolving old ideas with regard to mathematical concepts and the teaching thereof. Through interacting with other students, participants were encouraged to make sense of the emerging patterns. As such, participants took on three roles within their particular mathematical community, that of communicator, story-maker and interpreter.</p>	<p>The students were required to identify, through reflective practice,</p> <ul style="list-style-type: none"> • what they had contributed to the group in terms of understanding and teaching strategies • an awareness of their mathematical proficiency and understanding from the feedback gained from others • changes in understanding and resolution of old ideas with regard to mathematical concepts and the teaching thereof • and interacting with other students, and to make sense of new and any emerging patterns
<p>Experience was an emergent process in which students were required to take part in fortnightly tutorial periods. This allowed students to discuss their experiences and identify critical incidents relating to their preparation for and delivery of Mathematics lessons and teaching practice.</p>	<p>Practical tutorial sessions took place weekly and were attended on a fortnightly basis by Group A and B. The sessions provided an environment for the students to link theory to practice and develop their skill in teaching mathematics proficiently, receiving feedback without any formal assessment.</p>

The lectures followed the traditional format of presenting content and theory, and included discussions on pedagogical practices to ensure all nine units of the module

were covered. Each section was discussed within the framework of Kilpatrick, et al.'s (2001) five strands of proficient teaching.

The fifty students were divided into two tutorial groups and each group was expected to attend the tutorial sessions on a fortnightly basis. Both tutorial groups, A and B, were further subdivided into five smaller groups of five students, namely three *teaching* groups; one *learner* group and one *observation* group (Figure 3.1).

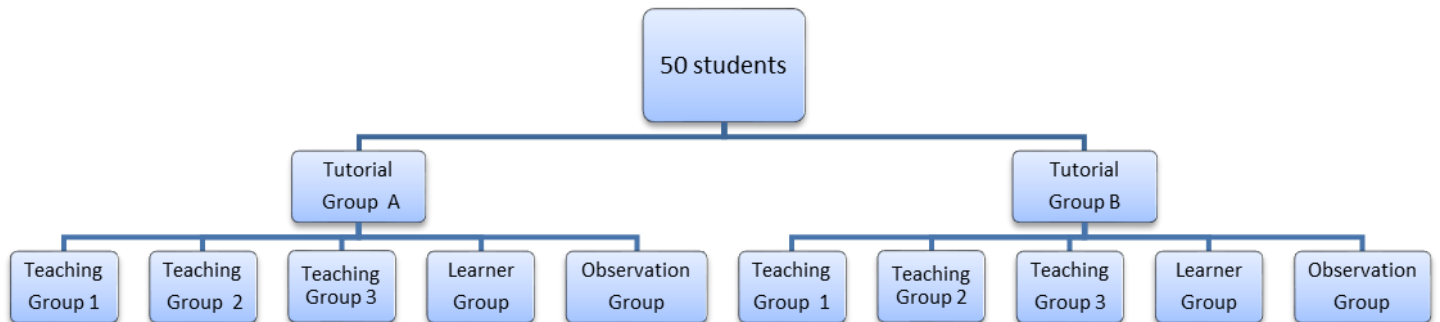


Figure 3.1 Structural diagram showing subdivision of tutorial groups

At each tutorial session the three *teaching* groups taught a mathematical concept, the *learner* group took on the role of the learners and the *observation* group observed and analysed the teaching process. Over the course of the year the groups in each tutorial session rotated so that each research participant had the opportunity to be in a teaching, learner or observation group.

The three *teaching* groups were given prior warning of the content that they were required to teach and each group then developed a 15 minute micro lesson that encompassed one or more of the strands of mathematical proficiency, as discussed in section 2.6. All the members of the *teaching* groups were expected to contribute towards the preparation of the lesson and resources, although only one member from each group taught the lesson at the tutorial session. However, this student was able to draw on their group members for support and assistance if and when the need arose. In some instances where students were nervous or unsure they were given the opportunity to co-teach rather than not teach at all. Most of the students had at least one opportunity to teach a lesson.

At the conclusion of each micro lesson, the students entered into a conversation about the outcome of the lesson with regard to teaching for proficiency, with the *observation* group and the *learner* group identifying what strands of proficiency they believed had been addressed. The role of the *observation* group was to notice and identify which strands of proficiency had been addressed, what pedagogic strategies had had a positive effect and where and what difficulties had arisen in the lesson delivery and activities. The *observation* group then raised critical incidents that they deemed had an impact on and affected teaching for proficiency. This same process was then repeated for the second and third *teaching* group. Following the last lesson, the final 15 minutes was allocated to a group discussion with the students, relating to the perceived growth and development of teaching for mathematical proficiency and its contributing factors. The students in the *teaching groups* were given the opportunity to engage with and process new information through activities and tasks that were planned for them through some form of reflective activity. This activity endeavoured to ascertain what the student had gained both individually and collaboratively from their experiences of the lecture's activities and how these had shaped their mathematical identity, self-efficacy and proficiency. All lessons were videotaped so that the researcher could revisit the tutorial and further analyse the critical incidents. As well as the weekly lectures and fortnightly tutorials, the mathematics module (MIP 400) also included fortnightly reflective tasks. The reflective tasks comprised four or five questions that were either relevant to one of the themes of enactivism or were linked to Kilpatrick's five strands of proficiency. For example, the students were asked to respond to the following question "*An aspect of sense-making in learning mathematics is playing an active and participatory role in emerging conversations. Discuss the role that you played in helping your group come to a common understanding of the concept/task you were planning to teach*". Analysing the participants' responses to the reflective tasks gave me insight into their thinking and how it related to the themes of enactivism and the strands of proficiency.

3.2.2 Three Phases of the Case Study

This case study was sub-divided into three phases, the first two of which followed elements of an action research cycle, to allow me to assess the levels of teaching for proficiency using Kilpatrick, et al.'s (2001) five strands of proficiency and thereafter to

adjust the reflective tasks and activities in a phase, if need be, to better inform my data collection. For example, after phase one I noticed that all the groups had focused on conceptual understanding and procedural fluency. Since I wanted to investigate the role of the themes of enactivism in all five strands of proficiency, this allowed me to specify the teaching focus in phase two.

At the beginning of the module all 50 students in the mathematics module (MIP 400) were asked to complete an initial questionnaire comprising both open and closed questions [Appendix One] to determine their definition of mathematical proficiency and teaching for proficiency. The purpose of the questionnaire was, firstly, to create awareness in the students regarding their personal level of proficiency and growth, and how it related to their mathematical identity. Secondly, the questionnaire enabled me to elicit initial attitudes and perceptions of the students regarding their personal proficiency and their perception of their ability to teach mathematics proficiently in the Intermediate Phase.

3.3 DATA COLLECTION

In order to increase the reliability of my data, the same research instruments were used for all the participants (Denscombe, 2010, p. 300). To demonstrate the reliability and dependability of my study, I now discuss in more detail the data collection methods that I employed.

Table 3.2
Methods of Data Collection for each Phase

Phase One 2011	Phase Two 2011	Phase Three 2013
Observations and video analysis of research team.	Observations and video analysis of research team.	Individual interviews with three members of the research team, working in the Intermediate Phase.
Focus group interview with research team.	Individual interviews with each member of the research team.	
Research team's reflective tasks.	Research team's reflective tasks.	

3.3.1 PARTICIPANTS

There were 50 students who participated in the module. Eight of these students volunteered to be part of the research team. The research participants played a very specific role in that they were asked, in some cases, to present more than one lesson. They also took part in individual interviews and their reflective tasks were analysed by the researcher. This eight participant sample group was a non-probability sample as this module was intended for students training to be teachers (Cohen, et al., 2000, p. 102). A non-probability sample is used when the sample group is selected for a particular purpose and is therefore not representative of the wider population. This kind of sample, according to Cohen, et al., is especially suitable for a small scale case study that does not necessarily seek to generalise its findings. More specifically, I made use of purposive sampling (Cohen, et al., 2000, p. 103) since the participants were selected for a particular purpose and need, namely training to teach and supplying me with input on their teaching practice in terms of proficient teaching. The research participants were purposively selected on the basis of relevance and knowledge (Denscombe, 2010, p. 35). In respect of relevance to the philosophy being investigated, the research participants were in a position to share their personal knowledge of having experienced the themes of enactivism through their involvement in the module. From this, I was then in a position to analyse the data to determine the research participants' perceptions of the development of teaching for mathematical proficiency and after phase three, the extent to which this had been carried through to their practice as qualified teachers.

3.3.2 Phase One

The first phase was a pilot phase that lasted for two weeks. The intention of this phase was to determine the students' thoughts on mathematical proficiency and teaching for proficiency, in addition to observing which of Kilpatrick, et al.'s (2001) five strands of mathematical proficiency emerged in the lessons taught.

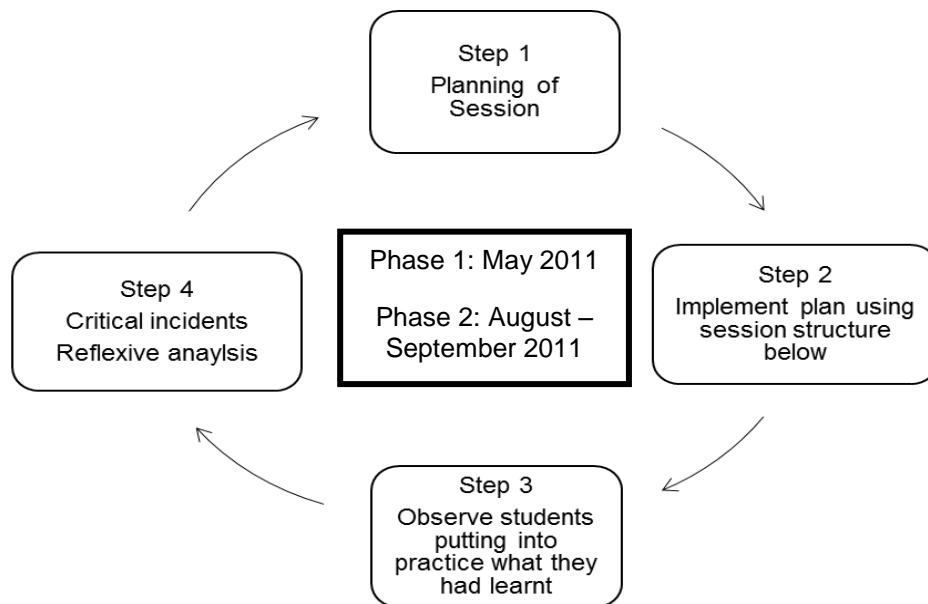


Figure 3.2 Conceptualisation of the first two phases

Step one of the first phase (Figure 3.2) entailed planning for the tutorial sessions. This needed to allow the students to link the mathematical content taught during lectures to practical teaching experience, underpinned by and indirectly incorporating the themes of enactivism. Step two consisted of lectures and practical tutorial sessions. During step three I observed the research participants and students over the course of the pilot phase and gathered data by means of reflective tasks, observation, video footage and a focus group interview.

Since class group interviews are “useful for developing themes, topics, and schedules for subsequent interviews” (Morgan & Krueger, cited in Cohen, et al., p. 288), each of the tutorial sessions closed with an informal class group discussion. During these discussions the students and I discussed and reflected on the lessons that had been observed and how they had drawn on the five strands of proficiency, both from a personal and professional perspective. It was from this interaction between the students that some of the data emerged (Cohen, et al., p. 288) and which I, as the observer, was able to reflect upon ‘adequate conduct’ (section 2.2.3.). During the class group discussion the researcher was able to guide the discussion to ensure a productive direction (Koshy, 2009, p. 88).

All the lessons were videotaped. As the observer, I made use of non-participant observations since they are less subjective and entail observing the actions and interactions of the participant silently without the researcher being actively involved in the lesson (Koshy, 2009, p. 92). These observations allowed me the flexibility to make comments and notes. The students who formed the observation group for each tutorial session were given an observation checklist to complete [Appendix Two]. The checklist had points relating to the kind of listening that took place, the structure of the mathematical community that the students had created and the type of learning and knowing that was taking place. The list also included points that related to different aspects of the five strands of mathematical proficiency.

As the first phase was only two weeks long I felt that a follow-up focus group interview was the best choice to gather information and feedback from the members of the research team. The profile of the research participants fits the criteria of a focus group in that they were a small group of specifically selected participants from whom I wanted to elicit detailed information about the first phase of the study (Mentor, Elliot, Hulme, Lewin & Lowden, 2011, p. 148). The fact that this interview was not representative in the broad sense was fine, as I required feedback and input on this particular case study in order to inform the next phase of the research (Nieuwenhuis, 2007b, p. 91).

Step four entailed analysing the data, identifying critical incidents, and drawing conclusions and recommendations that influenced the development of the second phase of the module and practical tutorial experience.

3.3.3 Phase Two

Phase two took place over eight weeks and followed the same planning and reflective process as phase one (Figure 3.2). A similar data collection process as for phase one was followed with each research participant except that the focus group interview was replaced by individual interviews with the research participants.

On completion of phase two I interviewed the research participants. The intention of the interviews was to gather richer and more informative data regarding the module, the practical tutorials and the participants' experience of having to teach to their

peers (Koshy, 2009, p. 85). These interviews took the form of a semi-structured interview, in which I devised a set of questions and sub-questions in order to gather further information and probe ideas and issues that participants raised (Koshy, 2009, p. 85).

The interview questions [Appendix three] were designed to elicit information as to whether the research strategies implemented in the various lectures helped create an awareness of their personal proficiency in Mathematics, their mathematical identity and their self-development. The questions also tried to ascertain how the enactivist themes influenced mathematical teaching during the tutorial sessions within Kilpatrick's framework for mathematical proficiency. The first set of questions, were general and designed to discover the students' perception of what proficient teaching looks like, the development of their mathematical teaching for proficiency and their mathematical identity. In addition, I wanted to gain an understanding of their thoughts on the value and impact of the module. These questions were also phrased to gain an understanding of the students' awareness of Kilpatrick's five strands of mathematical proficiency and whether or not they had taken cognisance of these in the planning and implementation of their lessons.

Finally, I used the interviews to determine the growth (if any) in teaching for proficiency.

This data was then analysed during step four of the second phase and the information was used to inform the questions in interviews with three of the research participants who were employed to teach mathematics in the Intermediate Phase.

3.3.4 Phase Three

One and a half years after qualifying as B.Ed. Foundation Phase teachers, three of the research participants were interviewed to determine the influence that the Mathematics module had had on their teaching practice. These participants were selected specifically as they were currently teaching in the Intermediate Phase, which was the focus of this study. The data collected for the final phase of the case study was in the form of a semi-structured interview with open-ended questions. The

intention of the interview was to determine the effect the module had had on the participants' current practice as qualified teachers. I was curious to discover if the themes of enactivism that had underpinned the mathematics module were being implemented into either their personal or their teaching practice.

3.4 DATA ANALYSIS

For this research study there were three data sets that required analysing, namely the individual and focus group interviews, the videos and the reflective tasks.

Merriam (2009, p. 175) talks of data analysis as being both inductive and comparative indicating that this makes the constant comparative method a widely used tool for analysis without it necessarily being used to develop grounded theory. Terre Blanche, Durrheim & Kelly (2006, p. 323) refer to an inductive approach as being bottom-up, in the sense that having immersed yourself in the data, the researcher then determines what the key principles/themes are that underpin the data.

Using an interpretative paradigm I combined open coding and selective coding and the resultant categories (Corbin & Holt, 2005, p. 50) with my predetermined codes of Kilpatrick's strands of mathematical proficiency. Interpretative phenomenological analysis examines a participant's experience and their perception of an event (Smith & Osborn, 2007, p. 53-55). Smith and Osborn (2007) explain that this is a suitable approach if one is trying to understand how the participants perceive and make sense of a particular situation. In addition, these kinds of studies have a small sample size in order to discern the perceptions and understanding of the research participants rather than make general claims. Thus, a combination of the constant comparative method and interpretative phenomenological analysis appeared to be a good fit for this study. Rapley (2011, p. 275) explains the process of interpretative phenomenological analysis as "moving from 'themes' to 'superordinate themes'". I opted to expand on the process and used open coding to code the incidents or events that emerged through analysis of the data in order to identify the themes. A selective code is a core or central category that encompasses other codes, in this

study the five strands of proficiency formed the basis of my selective coding (Saldaña, 2009, p. 45).

Saldaña (2009, p. 45) speaks of first cycle and second cycle coding when undertaking data analysis. In the first cycle of analysis I used open coding to elicit and code the key ideas, incidents and events that emerged from the interviews and reflective tasks. I first coded and categorised the data without using the themes of enactivism as a framework in order to ascertain what other underlying points and images had materialised.

During the second cycle of coding, I made use of selective coding based on the strands of proficiency, which were the lens through which I would determine the effect of the themes of enactivism. This enabled me to determine the link that the other codes and categories had to the selected codes. In addition, focused coding allowed me to identify the frequency of the codes of the themes of enactivism and their link to Kilpatrick's strands of proficiency (Saldaña, 2009, p. 155). Drawing up a data analysis matrix seemed to be the most efficient way of looking at my data from the two key perspectives, namely the strands of proficiency and the themes of enactivism. This led me to establishing an enactivism/strands of proficiency data analysis matrix (Table 3.4). Hence, these became my two main categories (selective coding) of analysis. In order to make the matrix functional (Klopper, Lubbe & Rugbeer, 2007, p. 269), I devised a system to reveal the data I was after. The top row represented, Kilpatrick et al's (2001) strands of teaching for proficiency and was further sub-divided into the five strands. The left column representing enactivism was likewise sub-divided into the five themes of enactivism. I then unpacked what I believed to be the key points of each cell. I first coded the data according to Kilpatrick, et al.'s (2001) strands of proficiency. Since there were a number of points pertinent to each of the strands of proficiency I drew up a checklist to monitor similar points which I referred to when analysing the different sets of data (Table 3.3).

Table 3.3
Teaching for Mathematical Proficiency Checklist

TEACHING FOR MATHEMATICAL PROFICIENCY [Kilpatrick et al (2001) Chapter 10]

CONCEPTUAL UNDERSTANDING

Conceptual understanding of the core knowledge required in the practice of teaching.
Knowledge must be connected so that it can be used intelligently.
Make connections within and among their knowledge of mathematics, students, and pedagogy.
An elaborated, integrated knowledge of mathematics,
A knowledge of how students' mathematical understanding develops, and
A repertoire of pedagogical practices that take into account the mathematics being taught and how students learn it.
Connections between the different forms of knowledge.
Know how to use math knowledge and knowledge of students effectively in the context of their work.

PROCEDURAL FLUENCY

Fluency in carrying out basic instructional routines.
A repertoire of instructional routines to draw upon them as they interact with students.
Some routines e.g. Classroom management: e.g. how to get the class started each day and procedures for correcting and collecting homework.
Some routines grounded in mathematical activity: e.g. how to respond to a student who gives an answer the teacher does not understand or who demonstrates a serious misconception or how to deal with students who lack critical prerequisite skills for the day's lesson.
Has several ways of approaching teaching problems.
Can apply a range of routines flexibly, know when they are appropriate, and can adapt them to fit different situations.

STRATEGIC COMPETENCE

Strategic competence in planning effective instruction and solving problems that arise during instruction.
Be effective in solving instructional problems.
Teaching as a problem-solving activity.
Decisions in planning instruction, implementing those plans, and interacting with students.
Figuring out what to teach when, how to teach it, how to adapt material so that it is appropriate.
How much time to allow for an activity.
Find out what a student knows, choose how to respond to a student's question or statement,
Decide whether to follow a student's idea.
Take into account the mathematics that students are to learn; what their students understand and how they may best learn it; and representations, activities, and teaching practices that have proven most effective in teaching the mathematics in question.

ADAPTIVE REASONING

Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them.
Learn from their teaching by analysing it.
Difficulties their students have encountered in learning a particular topic.
What the students have learned;
How the students responded to particular representations, questions, and activities;
Become reflective practitioners - is essential in improving their practice
Making more visible the goals, assumptions, and decisions involved in the practice of teaching.
Engage not only in learning methods of teaching but also in reflecting on them and justifying and explaining them in relation to:-
 The mathematics being taught,
 The goals for students,
The difficulties they have in learning it, and the conceptions and misconceptions that students have about the mathematics,
The representations that are most effective in communicating essential ideas.
Examine familiar artefacts from practice, to help them focus their attention and develop a common language for discussion.

PRODUCTIVE DISPOSITION

Productive disposition toward mathematics, teaching, learning and the improvement of practice.
Think that mathematics, their understanding of children's thinking, and their teaching practices fit together to make sense.
Think they are capable of learning about mathematics, student mathematical thinking, and their own practice themselves.
Analysing what goes on in their classes.
Learn by listening to their students and by analysing their teaching practices.
Develop more elaborated conceptions of how students' mathematical thinking develops.
Learn mathematical concepts and strategies from their interactions with students,
 Comfortable with mathematical ideas.
Lifelong learners who can learn from studying curriculum material.
In control of their own learning.

I then re-coded the data apportioned to each strand of proficiency in terms of best fit according to the themes of enactivism (Table 2.1). Having established the emerging categories from cycle one and cycle two of coding, I made use of axial coding to “make meaningful connections between the categories” (Craig, 2009, p. 190).

Having already analysed the data using Kilpatrick, et al.’s (2001) five strands, I then superimposed the five themes of enactivism on the five strands of proficiency. This enabled me to complete the analysis by highlighting the connections that had emerged between each of the strands of proficiency with the themes of enactivism and thus I could allocate the data to the appropriate cell of the matrix (Table 3.4). A completed matrix is included at the conclusion of the data analysis discussion of each phase in Chapter 4. As far as I know this study is the first to bring these two frameworks together, in order to unpack the role that each theme of enactivism played in developing proficiency in the teaching of mathematics.

Table 3.4
Data Analysis Matrix

		Strands of proficiency (Kilpatrick, et al., 2001, Chapters 9 & 10)				
		Conceptual understanding Conceptual understanding of the core knowledge required in the practice of teaching.	Procedural fluency Fluency in carrying out basic instructional routines.	Productive disposition Productive disposition toward mathematics, teaching, learning, and the improvement of practice.	Adaptive reasoning Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them.	Strategic competence Strategic competence in planning effective instruction and solving problems that arise during instruction.
Themes of Enactivism	Autonomy Construction of their mathematical identity. Ways of knowing Self-efficacy					
	Sense-making Mode of listening Active and participatory role in emerging conversations					
	Emergence Shared activities = collective generation of knowledge and understanding Reflective techniques identifying critical incidents Reflect deeply and understand the purpose of one's actions.					
	Embodiment Part of a larger collective system Feedback gained from others Educator: three roles within the community, that of communicator, story-maker and interpreter. Identifying changes and resolving old ideas Trace and record the interactions of the participants Help participants make sense of the emerging patterns Each person had something to contribute					
	Experience <ul style="list-style-type: none"> • Emergent process • Discuss their experiences • Critical incidents of mathematics teaching • Weekly tutorial periods 					

In order to ensure the credibility of this research study I also needed to address a number of factors, such as validity, positionality, reliability, generalisation and objectivity (Denscombe, 2010, p. 298).

3.4.1.1 Validity

In terms of ensuring that the data was not only accurate but appropriate in terms of my research question, I triangulated the data by means of different data collection

methods, namely video recordings, interviews and reflective tasks. According to Denscombe (2010, p. 348) triangulation provides the researcher with the opportunity to understand the data from different viewpoints. This in turn provides a means of validating the data through increased accuracy and authenticity as data is cross checked between the different methods to corroborate findings and check for consistency.

3.4.1.2 Positionality

In line with qualitative research this study emphasised the “role of the researcher in the construction of the data” (Denscombe, 2010, p. 237), in order to understand the factors that influenced the development of proficiency. As the researcher, I assumed the role of the observer (Maturana, 1987) [Refer to Chapter 2.2.3], with the purpose of determining the students’ perception of the extent to which the module, underpinned by an enactivist philosophy and pedagogy, had helped them to unpack the reality of their teaching practice in terms of proficient teaching. Furthermore, I used my position as observer to analyse the data, in terms of teaching for proficiency through the lens of Kilpatrick, et al.’s (2001) five strands, namely conceptual understanding, procedural fluency, adaptive reasoning, productive disposition and strategic competence.

The researcher in this study was both curriculum designer and lecturer of the module and had to manage these roles as the researcher striving for an unbiased perspective. Issues of positionality can be a concern. Thus, when considering my positionality within this study I needed to consider the implication of my role as the researcher within my own module and the influence that this would have on my construction of knowledge. According to Herr & Anderson’s continuum of positionality, I identified myself as an insider researcher in this particular study, since it was my intention to study “the outcomes of a program or actions in (my) own setting” (2005, p. 31). Herr & Anderson raise the point that as an insider researcher one does “not have direct access to the “truth” of the setting. There is merely one truth among many” (Herr & Anderson, 2005, p. 32). This aligns with my position as the researcher. I was the lecturer and designer of the module, who was researching the perceived value of the module in developing teaching for mathematical

proficiency, and thus I was aiming to determine the 'other truths' from the research participants. This data was obtained by means of interviews, reflective tasks and video footage and the implementation of triangulation was used to circumvent any bias.

As an insider I had a "better initial understanding of the social setting" as well as the module since I had designed it and was thus aware of the context of the environment and the end goals (Mercer, 2007, p. 6). Being the lecturer for the module afforded me the opportunity of having access to and an established rapport with my students which made it easier to discuss the shared experiences of the study.

Hopkins (2007, p. 388) raises the point that it is important to recognise the similarities and differences between the research and the participants in order to work with them more effectively rather than focusing on overcoming differences. He goes on to say (p. 391) that identity is an aspect of positionality and therefore we need to consider our positionality in respect to our different identities in order to carry out ethical research. Since my positionality in this study was determined by where I stood, as both lecturer and researcher, "in relation to 'the other'" namely the students (Merriam, 2001, p. 411) I was conscious of the power relationship that might result. Thorkildsen (2013, p. 21) moves away from the concept of power as something that is localised in an institution or an entity who has the power and instead suggests that we need to consider how this power is exercised. Taking this perspective into consideration, I am of the opinion that since the lecturer and participants were both of a common understanding to determine if the module had enhanced teaching for proficiency that it negated the fact that I was their lecturer. In addition, as there was no assessment attached to the practical teaching experience there was no need for bias or feelings of vulnerability from the participants. At best, the participants would perceive the module to have helped them develop teaching for mathematical proficiency and at worst they would have felt that their teaching practice had not been significantly improved.

3.4.1.3 Generalisation

Since this was a small scale research intervention the findings are not generalisable. With small scale research although there is a problem with the generalisability to a

broader context, it does allow for more in depth detailed data to be collected and a richer analysis to be undertaken.

Williams (2000, p. 222) asserts that interpretivism must have some form of generalisability which he refers to as *moderatum* generalisability. These are everyday generalisations that are derived from the researcher's understanding when undertaking interpretivist research and "Can form the basis of theories about process or structure". Using *moderatum* generalisability, some aspects of this study could be recognised within broader characteristics and thus generalised in *moderatum* to similar situations, for example other teaching training programmes. Denscombe (2010, p. 301) asks the question "to what extent *could* the finding be transferred to other instances?" It is my opinion that people involved in teacher education would regard the findings of this study useful and could incorporate teaching practice tutorials into their modules.

Quality indicators of validity, positionality and generalizability, derive from other paradigms and are arguably of less relevance to enactivism studies. However, since this research study was qualitative in nature and situated in an interpretivist paradigm it is appropriate to make mention of these measures of credibility.

3.4.1.4 *Ethics*

Before undertaking any form of research it is imperative to consider the benefits and risks of the study (Zeni, 1998, p. 14). Since this study was undertaken to investigate if the themes of enactivism would assist in the development of teaching for proficiency and that this would be determined by the research participants and, thus, inform the module going forward, I do not believe that the students were at any risk. However, to ensure that my students were not at risk, I explained to the entire lecture group the nature of the research study, what I hoped to determine with regard to teaching for proficiency and what would be expected of them. All the students were asked to complete informed consent documentation. The consent form allowed students to indicate what data they were willing to allow me to use. Further consent was gained from the eight research participants since they would be taking part in focus group and individual interviews and their research tasks, questionnaires and video footage would contribute to my data. The research participants were informed

that the information they provided would be kept anonymous and confidential through the use of pseudonyms. In addition, the participants were informed that they could withdraw from the study at any stage.

Having discussed in depth the methodology and methods that were implemented in this study, I analyse the results of the data in Chapter 4.

CHAPTER FOUR

ANALYSIS AND DISCUSSION OF RESULTS

In this chapter I discuss my analysis of the research team's findings about the reality of their teaching practice in terms of proficient teaching and what problems were posed after each phase. Finally I present an argument for the value of the themes of enactivism that played a key role in the process of developing mathematical teaching proficiency.

I proceed by discussing my analysis of each phase, in terms of my research question and sub-questions. I then conclude the chapter with an overview of the development of mathematical teaching proficiency from the first to the last phase (Table 4.1).

Table 4.1
A Brief Overview of each of the Research Study Phases

Summary of Phases

Initial Questionnaire	To determine what knowledge of proficiency the students brought with them to the Mathematics module
Phase One	A two week phase in which the students were divided into small groups and were required to prepare a lesson to teach, based on word problems given to them by the lecturer, relating to the mathematical concepts being lectured in the module at that particular time, thereby linking practice to theory.
Phase Two	An eight week phase in which the students remained in their same small group mathematical communities and were required to prepare a lesson to teach, based on word problems given to them by the lecturer, relating to the mathematical concepts being lectured in the module at the particular time, thereby linking practice to theory.
Phase Three	A year and a half after the research participants had qualified as B.Ed. Foundation Phase teachers, I interviewed three of the team members who had been employed to teach in the Intermediate Phase, to determine how the mathematics module had influenced and impacted on their current mathematical teaching practice.

My interest in this research project stemmed from a personal belief that while a person may have personal mathematical proficiency they may not be able to teach the subject proficiently. As I lecture pre-service teachers in Mathematics, I wanted to determine if I underpinned my module with the themes of enactivism would it enrich their capacity to teach proficiently.

This resulted in my overarching research question:

In what way does a mathematics module informed by an enactivist philosophy and teaching pedagogy enable selected pre-service teachers to unpack the reality of their teaching practice in terms of proficient teaching? This gave rise to the following sub-questions to inform and give greater clarity to my research question:

- What did the participants⁸ of the module bring forth with regard to teaching for mathematical proficiency?
- What problems did the participants pose with regard to teaching for mathematical proficiency?
- What evidence of growth and development of proficient teaching in Mathematics emerged?
- What evidence of growth and development of personal professional growth and proficiency emerged?

My discussion starts with the analysis of the initial questionnaire that the 50 students registered for the mathematics module (MIP 400) students⁹ were asked to complete. The questionnaire was administered so that I, as the researcher, could ascertain what their initial understanding of mathematical proficiency and teaching for proficiency was. I then discuss the results that were brought forth from each of the three phases in this research project. Proulx and Simmt (2013, p. 66) explain the term *bringing forth* as identifying what is significant and understood in a person's world; "Moment by moment we bring forth our worlds of significance" (p. 67). Thus it was my intention to determine what was significant in the pre-service teachers' world of teaching Mathematics and how they would bring forth this significance from phase to phase. I conclude with an overview of the results of the research project. My

⁸ Research participants' – this refers to the 8 fourth year students in the mathematics module who agreed to be part of the research study and from whom I obtained my data.

⁹ Students – this relates to all the students who were part of the mathematics module and who had given me permission to analyse their questionnaire.

analytical framework for determining the perceived growth in proficiency levels through exposure to and practical experience of the themes of enactivism was through the lens of Kilpatrick, et al.'s (2001) framework for teaching for mathematical proficiency and developing proficiency in teaching Mathematics. I used these strands as an analytical lens for phase one, two and three but not for the initial questionnaire as this was merely a tool to establish the pre-service teachers' ideas and perceptions of teaching for mathematical proficiency. Table 4.2 gives a breakdown of the different lenses of the enactivist themes, personal mathematical proficiency and teaching for mathematical proficiency that I used to analyse the data.

Table 4.2

Description of strands of proficiency and enactivism themes used as data analysis lens

THEMES OF ENACTIVISM (Di Paolo, Rohde, & De Jaegher, 2007)

<p>Autonomy Construction of their mathematical identity. Ways of knowing Self-efficacy</p>	<p>Sense-making Mode of listening Active and participatory role in emerging conversations. Making meaning of the world through action.</p>	<p>Emergence Shared activities = collective generation of knowledge and understanding. Reflective techniques identifying critical incidents (CI) Reflect deeply and understand the purpose of one's actions.</p>	<p>Embodiment Part of a larger collective system Feedback gained from others Identifying changes and resolving old ideas Trace and record the interactions of the participants. Help participants make sense of the emerging patterns. Each person had something to contribute.</p>	<p>Experience Emergent process Discuss their experiences Critical incidents of mathematics teaching Weekly tutorial periods.</p>
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STRANDS OF MATHEMATICAL PROFICIENCY (Kilpatrick et al, 2001)

<p>Conceptual Understanding Comprehension of mathematical concepts, operations, relations. Understand why concept is NB and contexts in which it is useful. Can make connections between ideas. Can represent math situations in different ways/representations. Have knowledge clusters of interrelated facts & principles.</p>	<p>Procedural Fluency Skill in carrying out procedures: flexibly, accurately, efficiently, appropriately. Quick mental strategies Ability to modify & adapt procedures.</p>	<p>Strategic Competence Ability to formulate & generate mathematical representation. Represent in a variety of ways e.g. graphs Solve mathematical problems Flexibility of approach to solve non-routine problems.</p>	<p>Adaptive Reasoning Capacity for logical thought, reflection, explanation, justification. Determine whether procedure is appropriate.</p>	<p>Productive Disposition Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. Confident in own knowledge and ability.</p>
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TEACHING FOR MATHEMATICAL PROFICIENCY (Kilpatrick et al, 2001)

<p>Conceptual Understanding Understand core knowledge required in the practice of teaching. Make connections within and between knowledge of mathematics, learners mathematical understanding, and pedagogy. Have range of pedagogical practices. Know how to use both knowledge of math and how pupils learn effectively in the context of the classroom.</p>	<p>Procedural Fluency Fluency in carrying out basic instructional routines. Able to respond to a learner who gives an answer the teacher does not understand. Able to respond to a learner who demonstrates a serious misconception. Able to deal with learners who lack critical prerequisite skills for a specific lesson. Knows several ways of approaching teaching problems.</p>	<p>Adaptive Reasoning Justify and explain instructional practices. Reflecting on practices in order to improve them Analyze teaching difficulties their learners encounter in learning a particular topic; what the learners have learnt; how the learners responded to particular representations, questions, and activities Able to provide evidence to justify claims and assertions</p>	<p>Productive Disposition Toward mathematics, teaching, learning, and the improvement of practice. Perceive themselves to be in control of their own learning Perceive themselves as capable to have an integrated practice based on pt. 1. Comfortable with mathematical ideas See themselves as lifelong learners are in control of their own learning, student mathematical thinking, and teaching practice.</p>
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4.1 INITIAL QUESTIONNAIRE PRIOR TO THE COMMENCEMENT OF THE RESEARCH PROJECT

The questionnaire was divided into three sections (Appendix one). Section A consisted of open ended questions whereby I tried to ascertain the students' knowledge of mathematical proficiency and what they thought teaching proficiently in Mathematics required and entailed. The first four questions focussed on what the students understood by the term mathematical proficiency, the skills and tools they thought they would need to teach for mathematical proficiency in the Intermediate Phase, how they would know whether their teaching strategies had achieved mathematical proficiency in their learners and to identify the types of teaching strategies that they would use to effectively teach for mathematical proficiency.

The research participants' responses to the first question relating to what they understood by the term mathematical proficiency, predominantly indicated that they viewed proficiency as being linked to understanding basic concepts and operations, namely conceptual understanding. Strategic competence and adaptive reasoning were acknowledged as being aspects of mathematical proficiency while surprisingly procedural fluency, an ability to carry out these operations accurately and skilfully was not acknowledged to be that important. I was not surprised to note that productive disposition was scarcely identified as a characteristic of mathematical proficiency since in my experience a number of students suffer from mathematics anxiety and admit to being hopeless at Mathematics which would have an impact on their self-efficacy.

The responses to the question regarding the skills and tools that are required to teach for mathematical proficiency in the Intermediate Phase were analysed using Kilpatrick's model namely:

- conceptual understanding of the core knowledge required in the practice of teaching;
- fluency in carrying out basic instructional routines;
- strategic competence in planning effective instruction and solving problems that arise during instruction;

- adaptive reasoning in justifying and explaining one’s instructional practices and in reflecting on those practices so as to improve them; and a
- productive disposition toward Mathematics, teaching, learning, and the improvement of practice (Kilpatrick et al. 2001, p. 380).

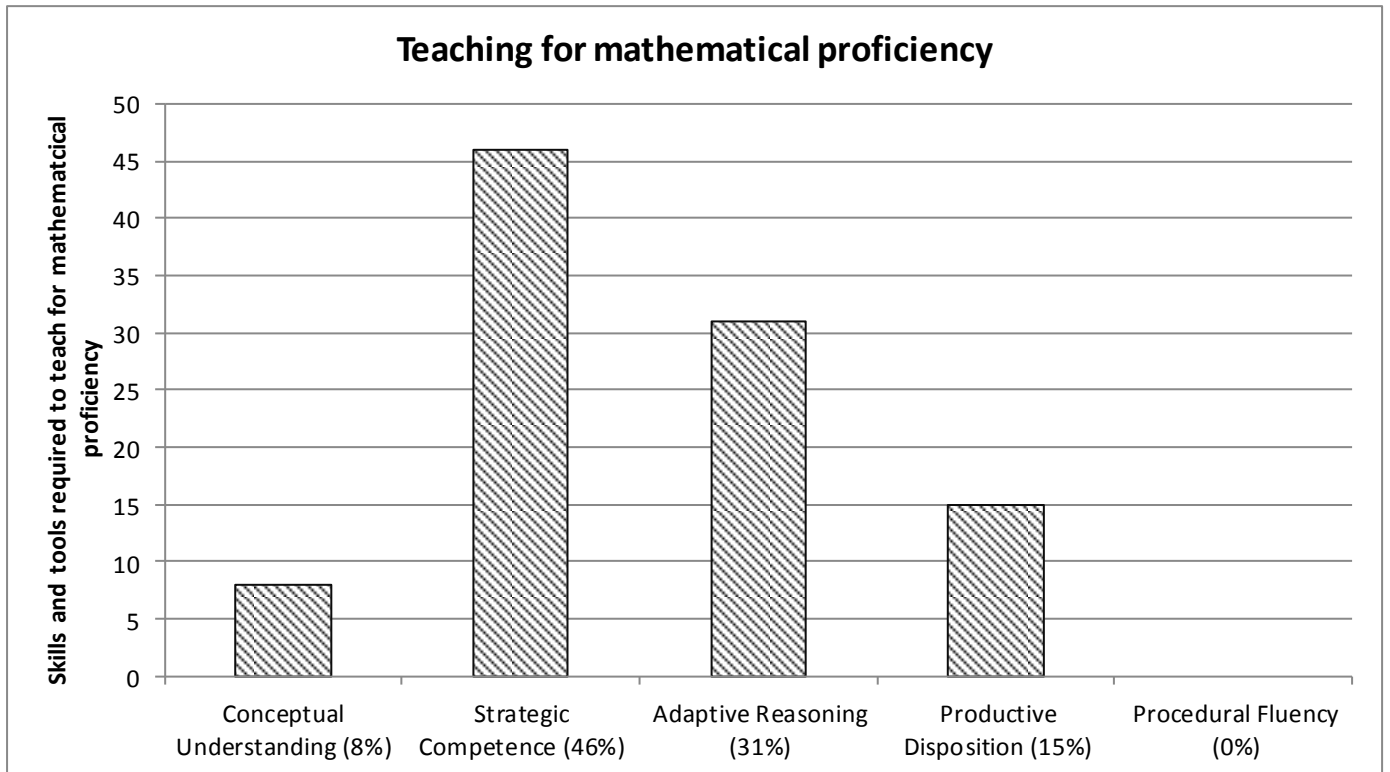


Figure 4.1 Perceived skills and tools required to teach for mathematical proficiency

As can be seen from the graph in Figure 4.1 strategic competence was viewed as the predominant strand with all of the responses referring to the importance of resources. However, it needs to be noted that using resources and models does not necessarily mean that an effective lesson has taken place nor that strategic competence has been demonstrated. This raises the question as to whether this particular result should be considered an outlier. The responses referring to adaptive reasoning were that a teacher should demonstrate creativity and openness towards new ideas, be on the level of the pupils and incorporate a variety of appropriate teaching strategies to encompass different learning styles.

Through the process of analysis it became evident that the productive disposition strand was alluded to more frequently as a trait for teachers and teaching, with

factors like self-efficacy, patience, passion, attitude, discipline and mathematically rich environment being acknowledged as important. These were the qualities that students felt were positive characteristics displayed by teachers with whom they had had a good mathematical experience or were the characteristics that they would have liked to have seen modelled by their teachers.

Only one of the comments related to conceptual understanding and none to procedural fluency. This raised three points for me to consider, firstly, whether students felt they did not necessarily have to have conceptual understanding and know the rules of Mathematics in order to teach successfully, that other traits were more important. Secondly, that students believed that reading through the rules and procedures was sufficient when preparing a mathematics lesson and thirdly, that using one method or approach was sufficient to teach a mathematical concept.

In the opinion of the students, adaptive reasoning was the most recognised strand considered to be effective in measuring if a teacher's instructional strategies had achieved mathematical proficiency in the learners. Although I acknowledge that successful adaptive reasoning would encompass conceptual understanding and procedural fluency, I find this prominence interesting given that students are not given many opportunities to explain, justify and reflect at school. This was the outcome of the influence of the pedagogic theory that they had been exposed to during their degree.

The balance of the questions in Section A focused on the students'¹⁰ mathematical experiences to date, their thoughts on teaching Mathematics in the Intermediate Phase and some of their achievements and challenges prior to starting the mathematics module. Through the initial questionnaire I was able to determine that some of the challenges that the students experienced relating to teaching and learning Mathematics were their personal proficiency in Mathematics, that is, their personal ability and confidence to do Mathematics, although this generally related to secondary school Mathematics as well as their self-efficacy. The issue of their own

¹⁰ These students were currently undertaking a B.Ed. Foundation degree. Please refer to page 3 for a clarification of the different phases of the General Education and Training (GET) system within the South African context.

understanding of basic concepts and rules was raised, as was their level of confidence to teach mathematics. Mathematics anxiety was also identified and the impact thereof, as the following response indicates *“Having to put my fear of math aside and learn to enjoy it so that the learners do too”* (Student,¹¹ April 2011). In addition, a lack of teaching strategies to use to teach learners who did not easily understand concepts presented a challenge. This was influenced by the fact that the students own exposure to different teaching strategies was limited to a single method or that in some cases they do not have the conceptual understanding and adaptive reasoning necessary to develop a repertoire of alternative methods. This would also affect their ability to relate to the learner’s level of understanding. This is a concern given that the students were in their final year and had already completed four years of teaching practice in the classroom in addition to completing modules in mathematics teaching. Perhaps the modules were too theory based, and the students had insufficient opportunity to practice and develop their skills for teaching Mathematics proficiently or alternatively, the students were not being exposed to good role models in the teaching of Mathematics during teaching practice. In some cases students said that they were not allowed to try new methods in the classroom during teaching practice. Herein lay the value of the tutorials, aligned to the enactivism theme of experience, by providing students with the opportunity to experiment with and practise various techniques and strategies within a mathematics community (the embodiment theme).

With regard to the students’ personal mathematics experiences and history, three key areas were identified as having a bearing on their ability to teach for proficiency in Mathematics, namely self-efficacy, teaching approach and empathy which has a noticeable link to productive disposition. Of the responses to the Likert scale statements relating to this topic five research participants agreed or strongly agreed that past experiences impacted on their conceptual understanding and their current practice as indicated, *“I feel determined to be more positive about Mathematics, and to be understanding when it comes to the mathematical needs and levels of each learner”* (Research Participant 6, April 2011).

¹¹ Quote taken from students’ questionnaire, who was not part of the research team, but who had given me permission to use their responses.

Generally the negative experiences that emerged from the questionnaire addressed the effect on the self-efficacy of the students. This resulted in two main influences, firstly, a negative influence where students were not confident to teach Mathematics. A second, positive stimulus, was that the negative approach would be replaced by a teaching strategy that the students would have preferred in order to avoid experiencing mathematics anxiety or feelings of inferiority. In addition they did not want to transfer their negative feelings of Mathematics to the learners. Some had chosen to rise above these negative experiences in *“order to grow and enjoy math”* (Student, April 2011). While others recognized that additional planning would be required on their part in order to teach successfully. Key elements that related to their teaching practice was that they would make use of various methods and resources to suit the learners’ needs and also go the extra mile to explain to learners who did not grasp concepts quickly. Empathy was a notable area identified by students who themselves had battled with Mathematics and saw themselves as teachers who would be understanding with regard to the learners needs and abilities and who would take the time to assist learners to grasp the foundations of Mathematics, while embodying qualities of patience and encouragement.

In section B of the questionnaire I tried to ascertain the students’ thoughts on teaching Mathematics by asking them to complete Likert scale statements. The statements, in some cases, were underpinned by the themes of enactivism and the strands of mathematical proficiency. Responses to the Likert scale statements to determine the perception of their personal mathematical proficiency rendered the following results. With regard to conceptual understanding three of the research participants believed that they had sound personal content knowledge of Intermediate Phase mathematical concepts, operations, and relations, four were unsure and one student felt that she did not. This was a concern given that these are fourth year students who were going out to teach. Furthermore, in order to teach proficiently an educator must have personal mathematical proficiency. In contrast, half the research participants felt that they had the necessary skill to carry out Intermediate Phase mathematical procedures flexibly, accurately, efficiently, and appropriately. This raised the question for me, relating to procedural knowledge without a solid foundation of conceptual understanding, *“were they teaching by rules and not engaging the learners?”* More than half, five, of the respondents believed

that they were proficient in the areas of adaptive reasoning and strategic competence. It was pleasing to note that with regard to productive disposition all the research participants believed Mathematics to be sensible, useful and worthwhile.

In responding to statements regarding teaching for mathematical proficiency five of the participants were in agreement that they had a good understanding of the instructional practices needed for teaching Mathematics and the procedural fluency required to carry out basic instructional routines in Intermediate Phase Mathematics. Furthermore, they were confident that they could plan effective lessons and solve problems that might arise while teaching Mathematics in the Intermediate Phase and could justify and explain their teaching practice. They also indicated that they regularly reflected on their teaching practice in order to improve it. This is a requirement in their Teaching Practice files.

In comparing the perception of the students of their personal mathematical competency and their teaching for mathematical proficiency the discrepancy between their perception of their personal conceptual understanding and their ability to teach Mathematics was evident. However, I was concerned that the Likert scale statement dealing with conceptual understanding could not represent the definition in its entirety thus raising the question as to the validity of that particular strand. In most cases students' perceptions of their ability to teach proficiently were higher (6% - 19%) than what they perceived their personal proficiency to be. (See Figure 4.2)

Participants perceptions of their personal mathematical proficiency versus their ability to teach for mathematical proficiency

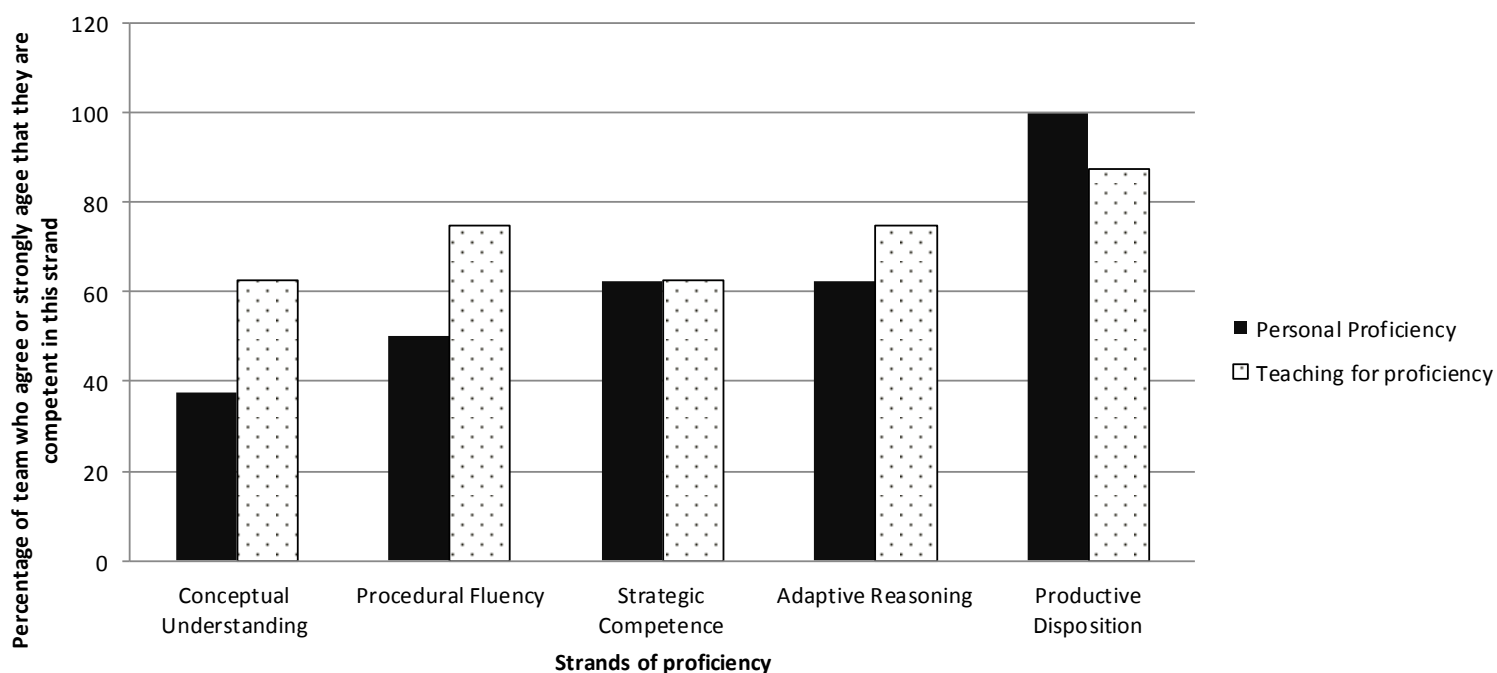


Figure 4.2 Personal proficiency vs. Teaching for proficiency

After analysing the initial questionnaire the research participants brought forth the following, with regard to the reality of their teaching practice in terms of proficient teaching. Firstly, relating to their personal proficiency, conceptual understanding was the strand where the students perceived themselves as least competent and while they all agreed that Mathematics was valuable (productive disposition) it was not necessarily an indication of their belief in their own self-efficacy. In contrast, when looking at their ability to teach the five strands proficiently, they were far more confident that they would be able to do this, although conceptual understanding and strategic competence were the strands they believed would present the most difficulties.

Furthermore, it was evident that the research participants agreed that their past mathematical experiences had had an impact on the way in which they viewed their mathematical identity and their ability to teach proficiently. Good experiences and success in Mathematics had resulted in a confident and positive mathematical identity. On the other hand, negative experiences or being unsuccessful at

Mathematics had led to uncertainty with regard to their mathematical identity but this was coupled with a sense of purpose to change the status quo. The participants had not heard of Kilpatrick's five strands of mathematical proficiency and tended to understand them as focusing on a more traditional approach to teaching based on conceptual understanding and procedural fluency.

Enactivism considers problem posing as problems or incidents that a person would encounter interacting with their environment. These would act as a trigger or perturbation for them to make meaning of their world (Varela, et al., 1991, p. 205; Proulx and Simmt, 2013, p. 66). This would require an embodied action in order to find solutions as problems arose by purposefully engaging in some form of activity or strategy to address the issue in order to create meaning, allowing the person to bring forth an adequate response in their context. The initial problem posing that emerged in terms of teaching for proficiency included the following:

- anxiety due to a lack of confidence and being a little out of practice
- coming *“down to a child's level of thinking as although she knows the answer and how she would arrive at the answer she does not know how she would explain the concept”* (Research Participant 2, April 2011)
- teaching their peers
- *“trying to teach something that has already been taught, as the learners do it one way but you are trying to do another way”* (Research Participant 5, April 2011)
- *“explaining problems to learners who really don't understand what is going on!”* (Research Participant 7, April 2011)
- solving problems
- catching up on mathematical concepts that were neglected at school and
- a lack of teaching strategies.

At the conclusion of the initial questionnaire I realised that a focus on conceptual understanding and skills was required. There was also the assumption that when lessons, were taught, they would be more procedurally driven, in that the steps of an algorithm would be explained in a rote manner. Since the first phase was two weeks in length I decided to evaluate what emerged from the practical tutorial sessions and focus group interview before planning any further perturbations. It was also evident that I needed to be sensitive to the anxiety and nervousness that some

of the students were experiencing with regard to their perceived proficiency and teaching their peers. This entailed making the environment in which they conducted their practical tutorial as secure as possible. To this end, I reiterated at the beginning of the lessons that the person teaching was representing and presenting the ideas and teaching strategies of their group, a small mathematics community within the mathematics module (MIP 400) class group, and not their personal approach. The development of and exposure to different teaching strategies was also identified as an end goal that required attention for the mathematics module (MIP 400). As this was before the practical tutorials had started I decided to wait and see what teaching strategies emerged as a result of the different groups' responses to the perturbation presented to them.

With regard to the pre-service teachers' ideas and perceptions of teaching for mathematical proficiency, the initial questionnaire established three key points. Firstly, there was the need to focus on and develop conceptual understanding of mathematical concepts. Secondly, the pre-service teachers needed opportunities to grow and develop their confidence in Mathematics and the teaching thereof. The third point was to expose the pre-service teachers' to different teaching strategies and approaches to teach the same mathematical concept. These three points play an important role in the pre-service teachers' ability to teach mathematics proficiently and helped determine what perturbations needed to be implemented to trigger problem posing.

4.2 PHASE ONE

Phase one was carried out over a two week period. The groups were asked to choose an aspect of whole number development that they had covered during lectures and to prepare a lesson to teach this concept to their peers the following week. This was the prompt that I as the observer, offered them in order to determine what problem posing would emanate from the trigger and what path each student would take to address this. Following the lesson they had to complete and submit a reflective task based on the preparation and delivery of the lesson. The students were never made explicitly aware of the enactivism themes that underpinned the mathematics module. However, I used these themes to formulate the reflective task

questions [Appendix Four]. The themes being autonomy (mathematical identity; self-efficacy); sense-making (listening, active and participatory role); emergence (collective generation of knowledge and understanding, reflective techniques); embodiment (identifying changes and resolving old ideas, help other students make sense of the emerging patterns) and experience (Table 3.1).

The data for phase one was collected by means of reflective tasks, observation and video footage of some of the research participants and a focus group interview with the research participants. In analysing the data collected from phase one I focused on the influence that each of the themes of enactivism had had on the research participants with regard to helping them unpack the reality of their teaching practice in terms of proficient teaching of Mathematics. I conclude by identifying what the team brought forth after phase one and what problems they posed.

4.2.1 Analysis of Videos Phase One

It was my intention to give each research participant at least two chances to teach a lesson. Unfortunately this was not possible due to participants missing lectures as a result of ill health. In addition, as the module progressed more and more students, who were not members of the research team wanted to teach lessons to their peers. Two videos of the research participants teaching were taken in the first week of phase one. It was the students' first experience of teaching their peers and they were not sure what to expect.

4.2.1.1 Phase One Tape 1 (Grade 4: Place value and word problems using landmark numbers)

This research participant demonstrated conceptual understanding of the concept that she taught, however, she did not take the opportunity to make connections between the different forms of knowledge, for example grouping tens and units as a strategy for addition. The lesson started with a basic counting activity with no inclusion of extension work. For example, she only used counting 'from' a range of numbers when she could have used counting 'between' a range of numbers and from more challenging starting points, to extend the activity and the meaning of the vocabulary

used. In the second and third activity there was very little adaptive reasoning or strategic competence evident. Although the level of the second activity was pitched a little low, clear instructions and a visual demonstration was given on the board while students carried out the activity. In a classroom situation this would be useful to pupils who were not following the development of the lesson. All the students were involved in a hands-on activity but when the students identified that $\frac{1}{2}$ the ten-frame was filled the research participant did not take the opportunity to ask where students had placed their beans on the ten-frame and whether the beans representing a $\frac{1}{2}$ had to be placed in the same specific blocks. The research participant asked questions but did not allow students sufficient time to answer before giving the correct answer herself. In the third activity although the research participant used some mathematics vocabulary the lesson would have been enhanced by the inclusion of more vocabulary, especially when asking questions so as to familiarise the students with relevant terminology for example: 'mathematical operation'. Nonetheless, the activity had a strong link to developing the learners' conceptual understanding and procedural fluency.

4.2.1.2 Phase One Tape 2 (Grade 5: Whole numbers using base ten models)

This was not a particularly challenging activity that was confidently presented. The research participant demonstrated procedural fluency in terms of the routines that Foundation Phase teachers would engage in, for example counting activities. In carrying out the main activity for the lesson she demonstrated both conceptual understanding and procedural fluency of the mathematical concept but did not encourage strategic competence or adaptive reasoning. The research participant demonstrated the activity with students using 1 or 2 examples before asking them to do an example on their own. The lesson would have been enhanced by asking higher order questions when working on the board.

4.2.2 Analytical observation of the themes of enactivism that emerged in Phase One

4.2.2.1 Themes of enactivism that emerged

Autonomy

With regard to the theme of autonomy and the notion that we form our own structure, the construction of the students' mathematical identity is associated with their way of knowing and self-efficacy within this research project. I first undertook to understand the influence that their Mathematics teachers and their experience of teachers at school had had on their autonomy. Bad experiences with teachers at school in some cases put students off Mathematics and in other instances made them determined to learn from the "bad" experiences and change them in their own approach to teaching, for example *"to start at the basics now, so that everybody's at the same level"* (Research participant ¹², June 2011). Some of the bad experiences that the research participants raised included the fact that teachers had not ensured that the learners were at the same level or had a similar understanding of the basics so some felt that they had been left behind. Another factor was that groups were graded according to marks and the possibility of being dropped to a lower group if you did not maintain your grades led to stress and a fear of looking like a 'loser' if you were dropped to a lower group. This linked with the expectation that most of the research participants believed was placed on them, to do well academically in Mathematics. Thus participants' autonomy and productive disposition, the way that they constructed their identity as a mathematics teacher, and their ability to teach for proficiency, was influenced by their past mathematical experiences. This is in line with the enactivist philosophy in that a person's structure is determined through their past histories and interactions. As evidenced by comments such as the following, *"Obey the rules and those are the rules, you didn't ask questions, you just did ..."* (Research participant, June 2011), and considering the research participants mathematical histories, a pattern of procedural fluency as a teaching approach was evident whereas alternative methods or strategies and discussion were neither encouraged nor entertained.

¹² Quote from focus group interview with research participants'

The approach that they were exposed to during the lecture and tutorial sessions based on the strands of proficiency raised the comment *“You actually show us how it’s done and not just if you multiply these fractions you’ll get this answer and it’s just like that. But I find it difficult to understand but if we’d been doing that since, like intermediate, since we were in the Intermediate Phase, it would have been much better”* (Research participant, June 2011). This response indicated a clear distinction between a procedural fluency approach versus a holistic use of the strands of proficiency approach. Furthermore it was apparent that the other strands of proficiency received very little focus. There was a notion that there was only one correct approach and that sense-making was not an idea that was afforded much time could be seen in comments such as *“And I hate that (you are asked) ‘what don’t you understand?’, if I knew what I didn’t understand I could probably fix it so”* (Research participant, June 2011). This has influenced the participant’s autonomy as a Mathematics teacher and her self-efficacy and productive disposition.

“Reply: I remember she – I did some method, I didn’t do it the right way, and she tore my pages out the book and I remember crying in the bathroom about it, like it was horrible. And I’ve never forgotten that. She said if we do it but didn’t do it the right way she’ll tear a page out and have to do it again.

Researcher: And did she tear them out in the middle of the class?

Reply: Ja, in front of everyone

Researcher: How did it make you feel about maths then?

Reply: Oh horrible. Like it was really horrible – I hated maths. I hated maths throughout high school. I think my attitude changed back in college, it was like a fresh start.” (Research participant, June 2011).

This participant further discussed the importance of teachers ensuring that they know the level of each learner and that they confirm that the learners actually understand and are not just saying that they do. A useful practice that she experienced was *“there’d be three or four groups or whatever, while the three groups are busy with other work...., she (the teacher) would work with one group, going over and consolidating the concept and all of that, working hands on, working, you know, doing the basic things with them and getting to their levels, with each of the groups”* (Research participant, June 2011). Often the students wanted to address their negative experience by making their learners’ mathematical experiences more positive, as

Research Participant 1 (May, 2011) indicated: *“I will always remember to make sure my learners don’t feel ‘lost’ like I was”* as well as *“will make time to help learners”*. This raised two points, firstly had the negative experience affected the students’ personal proficiency and if so how would this impact on their desire to make their learners’ experience more positive. Secondly, would this desire in turn make them respond positively to the perturbations that they were presented with during lectures and tutorials in order to develop their own teaching practice.

In reviewing the influence of teaching practice and personal experiences as a pre-service teacher in some cases *“the little bit of maths that I saw in grade 2 was very rote learned, like it was written on the board, now copy this down, and, um, if I did any maths with the pupils there was no such thing as using different methods . . . , the teacher wanted them to all do the same.”* (Research participant, June 2011). This is symptomatic of teaching for procedural fluency and disregarding the other four strands of proficiency. In addition the issue was raised that it is often difficult to introduce new approaches as a pre-service teacher *“because you’re in someone else’s classroom, you can’t do exactly what you want, and how you think it should be done.”* (Research participant, June 2011). This demonstrates that it is imperative that pre-service teachers are given an opportunity to link theory to practice and develop their teaching for proficiency in an environment that is supportive and encourages them to experiment, without fear of failure in terms of a practical assessment, and where constructive feedback and co-emergence are supported.

Generally, where students had a negative attitude or fear of Mathematics, it was due to three factors. Firstly, having had a bad experience with a teacher who was unapproachable, impatient or unsupportive and feeling that the teacher did not take the time to help or make sure that all the learners understood the concept. Secondly, students found the lessons to be teacher centred with few or no resources used and no hands on activities. Thirdly a lack of confidence and belief that they would ever be good enough to understand Mathematics was a factor that influenced their autonomy resulting, in some cases, of them feeling stupid.

The positive outcome of this was that some believed that it would not impact on their teaching instead it encouraged them to want to make their learners love and

understand Mathematics and never feel stupid. This is in line with the enactivist idea of structural determination in that the manner in which the pre-service teachers were taught was the perturbation but they determined their response to the trigger and the resultant change with regard to proficient teaching and their autonomy (Maturana & Varela, 1998, p. 96). For some this entailed becoming creative and approachable teachers with a positive attitude to encourage learners to become excited about, love and understand Mathematics. However, the fear was expressed that a lack of confidence could affect their teaching for mathematical proficiency.

Half of the research participants had a positive mathematical identity, in that they were confident about their mathematical proficiency and had enjoyed mathematics' lessons. They had, in most cases, had a positive experience at school and generally done well, thereby creating a good relationship with Mathematics. Furthermore their relationship with the subject had helped them solve problems and look at situations more critically. The outcome of this experience was that students believed that it would have a positive impact on their teaching Mathematics proficiently and they were encouraged to share their knowledge with other learners.

Sense making

Sense making from an enactivist perspective is to change the meaning of one's world through dialogue with others. The tutorial sessions played a big role in sense making as members of the various groups contributed ideas and then determined which ideas would work best. Research Participant 1 (May, 2011) indicated that *"We definitely wanted the learners to be hands-on, so we decided to use manipulatives that they could use to solve the problem"*. Furthermore they tried to think how they understood the concept before developing and completing their lesson plan, as demonstrated in *"Our whole group came together well as we sat down and listen to all ideas on whole numbers. We then discussed which ideas would work best and tried to think of how we ourselves understood it completing our lesson plan"* (Research Participant 2, May, 2011). Taylor and Biddulph (2001, p. 6) found a similar "co-involvement and co-emergence aspect of enactivism" using a process of collaboration approach.

Emergence

The theme of emergence was noted as well as the acknowledgment of co-emergence between the different students as a research participant demonstrated, *“As I am quite “afraid” of teaching maths, I asked questions on how to teach the concept”* (Research Participant 1, April 2011). Key factors of co-emergence that were identified were through the questioning the participants did in order to firstly understand the concept to be taught. Generally groups tended to ensure that all members understood the concept in some cases *“repeat(ing) what others girls said but refined and simplified their explanations”* (Student Reflective task, May 2011). This was followed by listening to all group members’ ideas on the topic and discussing which ideas would work best and determining how the members themselves understood the concept before completing their lesson plan. Further evidence of co-emergence was revealed by members indicating that they *“. . . explained fully the lesson that we had chosen and everyone seemed to understand the concept”* (Student Reflective task, May 2011). It was noted that after group discussion and conversation some participants felt that there was no need for further practice.

Through the reflective tasks regarding their personal proficiency and teaching, students gained insight into how to improve and develop their proficiency. As one participant reflected *“I realised I need to do more research and ask others on new ideas and concepts of how to teach rounding off”* (Research Participant 2, April 2011).

Embodiment

Smaller mathematics communities (section 2.3.4) were established to determine the impact they would have on the students. Looking through the lens of enactivism the theme of embodiment was evident given that in most cases groups worked well together and were found to be beneficial. This mathematics community was generally deemed to be helpful as members contributed ideas with varying levels of input and involvement. As one respondent commented *“Working with a group with only one person teaching you know you have support if you get stuck – helps with anxiety”*. (Student Reflective task, May 2011). In instances where the group did not work well

together the reasons given were: no appointed leader and lack of communication which resulted in either rushed meetings or poor attendance at meetings.

In this research study a deliberate attempt was made to establish smaller mathematical communities within the broader mathematical community of the lecture group. Wenger, McDermott and Snyder (2002, p. 4) describe communities of practice as groups of people who meet to “deepen their knowledge and expertise in this area by interacting on an ongoing basis”. The area referred to could be a concern, a problem or a passion. During the meeting they would share ideas, understanding and insight thereby learning together. Furthermore, over time this interaction “may even develop a common sense of identity” (p. 5). Using the enactivist theme of embodiment as a stimulus I created a similar type of mathematical community of practice. The team members felt that being part of a group made them comfortable sharing their ability in teaching Mathematics and they did not feel pressurised if they were not competent. In some cases it had the effect of improving their self-efficacy to see that there were other students who did not understand or who had less confidence than they did or suffered from mathematics anxiety. This in turn created an opportunity for these students to help their peers until the group came to a common understanding of the problem and the way forward to teach it. Thus, as enactivism predicts, the sense of the task co-emerged for the students through structural coupling. Being part of a mathematical community took the pressure off planning as through conversation and listening the students were able to gauge if they were on the right track. It was seen as a positive that the students had been placed in groups and would remain in the groups for the duration of the module as it created an awareness of one another’s strengths and taught them that *“that you can’t always be the leader ...you have to sit down and listen to other people and they guide you and you guide them”* (Research participant, June 2011). Li, et al. (2010, p. 7) describe an enactivist view of cognition as a complex co-evolving process of systems interacting and affecting each other, not as a processor of information but as a producer of meaning. An illustration of this is evident in the comment that *“I think also working in our groups, in our tut groups, it helps, because I mean we’re working together through this journey ...and we’re understanding it together, we’re not changing groups and ... in our group, we know the level of each of the members in our group, we’ve become comfortable in”* (Research participant, June 2011). Through

the act of co-emergence, the pre-service teachers were able to elucidate their understanding and give meaning to the task. Similarly McGann (2008) indicates that through coupling with their environment a living being is able to make sense of their environment.

In some instances during the tutorial sessions students needed to understand the concept that they were required to teach. Often being part of a mathematics community assisted the students' firstly with understanding the concept and secondly with how to explain the concept. In some cases trying to determine how to teach the concept proved to be quite stressful even though the students knew the concept. The fact that students had been, in most cases, taught in a procedural manner meant that conceptual understanding, adaptive reasoning and strategic competence skills had not always been adequately developed. This in turn would affect their productive disposition.

Being members of both a larger (4th year mathematics group) and smaller (groups of 6 members) mathematics community students were able to work together within a system to assist with the sense making process and responding to specific triggers determined on an individual basis. For example, the group would explain to those members who were less sure, so that everyone understood the concept. *"Working with a group, you know you have support if you get stuck – helps with anxiety"* (Research participant, June 2011). The sharing of ideas by group members helped with the development of the lesson, which a number of students felt they would have struggled to do on their own.

Experience

The tutorial sessions filled a gap that helped to address the balance between theory and practice when it came to teaching for mathematical proficiency as demonstrated by the comment *"We were never really taught how to actually do it. I feel fine in English . . . I don't know about teaching maths"* (Research participant, June 2011). Watching their peers teach during tutorials was considered valuable in that they were exposed to different teaching approaches and could see where different groups chose to place the emphasis of the lesson. In addition it built confidence which increased the more

opportunities a student had to teach. The fact that the tutorial sessions were not assessed meant that groups were more innovative and creative in the teaching approach that they took, knowing that *“it’s okay if we make a mistake because we’re learning from it; it’s not a pass or fail”* (Research participant, June 2011).

It was noted that that the groups only focussed on the conceptual understanding and procedural fluency strands of proficiency. During the focus group interview students were receptive to the idea of groups being allocated specific strands so that they could encompass the different strands into their own teaching practice.

On the one hand students found teaching peers difficult, expressing the opinion that pupils don’t judge. One participant expressed the opinion that *“Teaching to your peers is daunting but it is a “safe” environment if you make a mistake it is okay whereas in the school it’s not”* (Research Participant, May 2011). During the first phase, six students were given the opportunity to teach a lesson to their peers. Two interesting comments that emerged with regard to this experience were firstly *“I was the first person to do the maths lesson and was not sure what was expected of me! While teaching, I actually felt a bit threatened”* and secondly *“was very afraid to teach – tried my best”* (Research Participant 1, April 2011).

In addition the students felt they were getting far more feedback from the tutorial group than from a teacher as well as guidance instead of just a teaching assessment mark. They also expressed the view that they needed to start teaching Mathematics earlier and not only in the third and fourth year of teaching practice, so that they could develop confidence.

During the practical tutorial sessions, which were included to incorporate the experience theme of enactivism, students were able to identify incidents that were opportunities that could have been used more effectively or differently. As a larger mathematical community the students discussed different approaches that could have been taken. Groups who had taught lessons indicated that it helped to get ideas from their peers, acknowledging that sometimes it takes an *“outside constructive view to make your lesson even better”* (Research Participant, May 2011). There were also comments indicating that students felt that they would not have known how to teach a concept but having observed a lesson they now had alternative strategies

and approaches to use. The students who were observing the lesson found that it helped them to clarify their thoughts and ideas about a given concept. The perceived benefit of the teaching experience was that *“Having watched the lesson(s) I think that I could teach each but if I had had to come up with a lesson by myself I would have struggled.”* (Research Participant, May 2011).

At each tutorial session a number of students were allocated the role of observer. Their role included identifying and commenting on critical incidents that may have arisen during the lesson as well as giving feedback to the students teaching the lesson. Observers did a thorough task and generally raised very valid points. The observation group was also very mindful of not hurting the presenter’s feelings and thus discussed both positive and negative points giving suggestions where applicable. A noticeable difference during the practical tutorial sessions was that, as one group followed on after another, it was evident that they had taken cognisance of the observers’ comments to the previous groups and tried to implement these suggestions or avoid the perceived errors. For example, if a lesson was particularly teacher centred the following group tried to engage the learner group more. In observing the different lessons, it was evident that in theory students often have the right idea but have difficulty in translating the idea into practice. This is where the practical tutorials provided an opportunity for pre-service teachers to develop these skills within a mathematical community environment.

Having analysed the data that emerged during phase one through the lens of enactivism, in which autonomy, embodiment and experience were the significant themes in developing proficiency, I now re-analyse the data through the five strands of mathematical proficiency.

4.2.3 Analytical observation through the lens of the five strands of teaching for proficiency for Phase One

4.2.3.1 Conceptual Understanding

An important aspect of this strand according to Kilpatrick, et al. (2001, p. 381) is that “teachers need to make connections within and among their knowledge of

Mathematics, students, and pedagogy” as well as a range of pedagogical practices. From this perspective the analysis revealed that the students’ repertoire of teaching strategies was limited and often their choice of method was the approach demonstrated during the lecture. This in itself is not to be discouraged since in some cases it helped to consolidate the students’ personal conceptual understanding and *“it would benefit us to try it out in a lesson and get more understanding of it”* (Research Participant, May 2011). One needs to factor in the realisation that these were pre-service teachers and so a limited repertoire of strategies is to be expected. In addition, I also took into consideration the possibility that some students did not have the confidence or courage to try new ideas in case they were unsuccessful in front of their peers. In some cases students were not too sure of the level at which to pitch the lessons.

4.2.3.2 Procedural Fluency

In some instances the instructions were not always clear and more explanation was needed for clarity. I noticed that the teaching strategy tended to be very procedural in nature and teacher centred, As a result of their previous experience they did not always have an alternative strategy to employ. However they did know that although this was an artificial situation, the expectation was to teach a lesson aimed at Intermediate Phase learners.

4.2.3.3 Strategic Competence

According to Kilpatrick, et al. (2001, p. 383) strategic competence requires that the teacher needs to determine what the learner knows, then decide firstly, how to respond to learners’ ideas and, secondly, whether or not to follow these ideas. In essence they see teaching as a form of problem solving in itself. This was a more difficult strand to implement, however, one group decided to anticipate the difficulties some learners might experience and created a story that would overcome these difficulties. *“We thought it would help the learners to understand the concept more easily and remember it”* (Participant, May 2011). This was not a strand that was very well embodied. However, it was noted that if the student ran into difficulties during a

lesson, the balance of her group readily supported her with ideas or answers, in this way bringing the enactivism theme of embodiment to the fore.

4.2.3.4 Adaptive Reasoning

There was far more evidence of adaptive reasoning firstly, in the conversations generated within the groups, to explain and justify the concept and teaching approach that should be adopted and secondly, in the reflective tasks that the teaching groups had to complete. Nevertheless, during the discussion at the completion of each session many students, including the teaching groups, contributed thoughtful and constructive comments and suggestions. Furthermore in some cases students engaged in discussion justifying the intention of the lesson, particularly if this had not been clearly demonstrated. With regard to the lessons taught, students seldom required their 'learners' to explain how they got to their answers. As one member of the observation team group pointed out, she "*never required learners to explain their process just accepted their answers*" (Student, May 2011). This could have been a confidence issue if the student herself did not have the necessary conceptual understanding. Alternatively, because they were teaching their peers they may have taken it for granted that they all understood.

4.2.3.5 Productive Disposition

Kilpatrick, et al. (2001, p. 384) indicate that to develop a productive disposition a teacher needs to ensure that the Mathematics to be taught, the teaching strategy to be used and their understanding of the learners thinking, needs to be synchronised. This is a process that can be broadened by listening to the learners and how they engage with the concepts and the strategies they employ.

Again this strand was not well addressed since most of the lessons were teacher centred making it difficult to ascertain the learners' mathematical thinking. In addition, the tutorial sessions were contrived so that the 'learners' were their peers and not 10 and 11 year old pupils.

What the students brought forth

Enactivism views knowing as knowledge in action and so from phase one the following points were brought forth by the research participants in terms of what they know to be teaching for proficiency. It is, as Proulx and Simmt (2013, p.72) point out, “what they know, how they are, and who they are”.

- Firstly, mathematical histories and experiences had had an impact on the team’s mathematical identity. Through the lens of Kilpatrick’s strands of proficiency it became evident that past experiences had affected *productive disposition* which had resulted in *procedural fluency* becoming the dominant strand when teaching for proficiency. As the observer I reflected on what past histories had brought to the lecture room, and to what extent lecturers need to deal with this influence. Is it enough to merely draw the students’ attention to the impact of their past experiences in an attempt to get the students to respond to the perturbation offered during lectures? The students need to take steps to address their mathematical identity, consolidate areas of strength and develop and remediate areas of weakness.
- Their previous experience was generally teacher centred not student centred.
- Students had to become accountable for their personal conceptual understanding in the event they would be required to teach during the tutorial session.
- Co-emergence ensued between the different students as a result of having to engage in conversation in order to teach a mathematical concept.
- Reflective tasks regarding their personal proficiency and teaching gave students insight into what was required to improve and develop their proficiency.
- Embodiment had an influence on productive disposition. As they were working in a smaller mathematics community students were able to support those who were anxious and less confident by helping them to understand the problem and share ideas on teaching strategies and approaches.
- Tutorial sessions provided students with an opportunity to link the theory of mathematical concepts and pedagogy when it came to teaching for mathematical proficiency.

- Experience in the form of teaching tutorials resulted in students being introduced to different teaching strategies and approaches.
- Students were most comfortable and confident with the conceptual understanding and procedural fluency strands of proficiency with regard to their teaching practice.
- An unthreatening environment that did not include assessment, encouraged creativity and experimentation in teaching approaches.
- The practical tutorial experience allowed the students' to make mistakes in their choice of teaching approach and strategies as there were no repercussions.
- The pre-service teachers received more feedback on their teaching than they would ordinarily from teaching practice. This was deemed to be helpful and constructive.
- They had more confidence to teach certain concepts having listened to feedback and through observation of the lessons taught.
- They knew from past experience what qualities they believed a teacher should have for proficiency.

Problems the participants posed in terms of teaching for proficiency

Firstly, in many instances conceptual understanding needed to be strengthened. Regarding the practical tutorial experience, students had difficulty in addressing the productive disposition, adaptive reasoning and strategic competence strands when teaching for proficiency. They also had difficulty in translating their ideas into teaching practice and often the practical sessions tended to be more teacher centred and procedural in nature. Lastly the students found teaching their peers created an element of anxiety as it seemed as though they were being judged.

On completion of the analysis of phase one the following points emerged with regard to the influence that the different themes of enactivism had on particular strands of teaching for mathematical proficiency (Table 4.3). Three themes of enactivism were revealed to be more significant in informing the students practice in proficient teaching. The theme of autonomy revealed that the students past experience affected their level of confidence to teach mathematics proficiently and tended to

result in a more teacher centred and procedural approach to teaching. Embodiment provided support in terms of understanding and the growth of confidence. Finally, the experience theme provided a safe environment in which to put theory into practice in addition to introducing students to various approaches to teaching the same concept. It is important that lecturers involved in the training of pre-service teachers take into consideration the impact that past experiences have on students and introduce reflective tasks and triggers to generate self-awareness of their mathematical identity in the students thus giving them the opportunity to problem pose and react to these perturbations.

Table 4.3

Matrix indicating the points raised during analysis of the influence of the themes of enactivism on the five strands of mathematical proficiency in Phase One

	Autonomy	Sense-Making	Emergence	Embodiment	Experience
Conceptual Understanding	Accountable for their personal conceptual understanding.	No points emerged	No points emerged	Accountable for their personal conceptual understanding.	Linked theory of mathematical concepts and pedagogy. Introduced to different teaching strategies and approaches.
Procedural Fluency	No points emerged	No points emerged	No points emerged	No points emerged	Dominant strand when teaching for proficiency - generally teacher centred.
Strategic Competence	No points emerged	No points emerged	No points emerged	Received feedback deemed to be helpful and constructive.	Encouraged creativity and experimentation in teaching approaches.
Adaptive Reasoning	No points emerged	No points emerged	No points emerged	No points emerged	No points emerged
Productive Disposition	No points emerged	No points emerged	Insight into what was required to improve and develop their proficiency.	Received feedback deemed to be helpful and constructive. Gained confidence from feedback and observation of lessons taught.	Past experiences have an impact on identity and way of knowing.

4.3 PHASE TWO

Phase two encompassed a period of eight weeks, which was sub divided into four sessions of two weeks each. Due to the number of mathematics students attending the module the class was divided into two groups of twenty-five students with each group attending the practical tutorial experience on alternate weeks. Therefore, the

same concept/s were taught over a two week period, using different word problems, so that all the groups (mathematics communities) were given the same opportunity to put their lecture content into practice. After reviewing the lessons taught in phase one, I decided to allocate specific strands of proficiency that needed to be addressed during their lessons, as I wanted to encourage the pre-service teachers to try a more learner centred and holistic approach to their teaching practice. In the analysis of phase two, I discuss the findings of each of the four sessions separately before concluding the analysis of this phase.

4.3.1 Session One

The emphasis of session one was the teaching of fractions. The students were required to focus on the strategic competence and productive disposition strands of teaching for mathematical proficiency. The reflective task activities plus preparation and planning was the same for all six groups that taught during this session. [Appendix Five]. The reflective tasks required that the students discuss the active and participatory role that they had played in helping their group come to a common understanding of the concept/task that they were planning to teach. To encourage an awareness in the types of listening that occur in the classroom, the students had to rank the mode of listening (Evaluative, Interpretative, Hermeneutic) that they and their peers engaged in. Finally, they were asked to identify an incident (or discussion) during the planning of the lesson that had contributed to their understanding of the topic. They then had to explain how the incident contributed to their understanding for teaching for proficiency and in particular the strands that they were assigned to focus on.

4.3.1.1 Analysis of Videos Phase Two Session 1

Tape 1: (Grade 6: Equivalent Fractions)

In the first lesson there was evidence of structure and the intentional inclusion of developing strategic competence and adaptive reasoning. Even though the research participant was comfortable with presenting the ideas her pedagogy was still fairly procedural. The first activity was a revision of finding equivalent fractions which was

very procedural in nature with regard to methodology, merely a demonstration of procedural fluency. There was also no expectation of the learners to demonstrate conceptual understanding or questions asked to ascertain whether the learners had a conceptual understanding of the equivalent fractions. Equivalent fractions were found without any explanation as to the link between the two fractions, this only became evident in activity two, namely the addition of fractions. Activity two was an example of addition of fractions using the same denominator. The learners were never asked about their conceptual understanding of the concept; the activity was purely the application of a procedure. The research participant was quick to answer questions without giving the learners an opportunity to respond. The third activity was a word problem. As learners worked in a group there was evidence of interaction and both strategic competence and adaptive reasoning, however, the research participant group essentially explained the problem to the learners. This activity would have been more appropriate as an individual or paired activity as some of the students did not participate and could rather have been involved with the balance of the activity and not run over time. When a student was asked to give a report back of the activity she did not include an in depth explanation of solutions and no questions were asked by the research participant to encourage adaptive reasoning. There was evidence of structure to the lesson and the intentional inclusion of developing strategic competence and adaptive reasoning. The research participant was comfortable with presenting the ideas and concepts to the students although she did not really interrogate the solution that the students arrived at which would have provided an opportunity for developing adaptive reasoning skills.

Tape 2: (Grade 4: Fractions as part of a collection of objects)

The second lesson presented at the practical tutorial session was a very learner centred lesson. The research participant was unclear about the goal of the lesson, as were the students, therefore it would have been more useful had she recapped what the concept was, how it is solved conceptually and finally demonstrated how it could be translated into an efficient procedure. This was a confident presentation and the research participant had a good conceptual understanding and procedural fluency of what she was teaching. The research participant was successful in incorporating opportunities to utilise skills associated with the different proficiency strands into her lesson in addition to creating the space for the students to demonstrate their

command of the five strands of proficiency. One of the activities required the students to work in pairs and to represent their solution by means of a drawing; this was an effective manner to get the students to show conceptual understanding. When a student was asked to report back on her solution, the research participant was able to translate and represent the student's solution from manipulatives to a visual diagram on the board. The development of adaptive reasoning and strategic competence skills was evident, as the research participant asked for alternative strategies to finding the solution and requested the students to explain their solution. In the next activity the research participant did not always use the correct mathematical vocabulary, for example, she spoke about the number on the top as opposed to the numerator. The research participant was quick to get students to suggest strategies to solve the problem rather than giving them the opportunity to try on their own first. She then asked the students to try to find a different strategy to solve the problem, which gave the students an opportunity to develop their skills in strategic competency. What was evident in this lesson was that the students were confident when explaining their solutions to their peers and that they used different models of fractions to solve the problem, for example the set model. Although the type of model chosen was not always in the appropriate context there was an increase in the variety and type of model rather than the area model which tends to be overused as a means of explanation.

Tape 3: (Grade 5: Addition of fractions with unlike denominators)

In the third lesson, based on a fraction word problem, there was far more evidence of the strands of conceptual understanding and adaptive reasoning. The research participant read the problem to the group, which would have been a useful strategy for second language learners. A confident presentation was given by the participant with evidence of interpretive listening to what students said. The research participant involved the students and expected them to explain how they had arrived at the answer. There were three questions to this problem and the research participant's group opted to guide the learners through the process by giving them a worksheet with fraction rectangles from which to work. As the questions became more difficult, the participant could have asked the students to shade in $\frac{1}{4}$ of the 'wall' to confirm their calculation and consolidate conceptual understanding rather than resorting to procedural knowledge to check their answer. There was evidence that the students,

both in the mathematics module and the research participants, were starting to link conceptual understanding with procedural fluency by, for example, linking the use of resources to a procedure/process to develop conceptual understanding.

4.3.1.2 Themes of enactivism that emerged

Embodiment

Being part of a smaller mathematics community encouraged the development of all five strands for teaching for proficiency for the students at the conclusion of session one.

Conceptual understanding was predominantly demonstrated by the members of the research team, who had sufficient understanding of the core knowledge required to assist their peers, to arrive at a common understanding. Some took on a leadership role. These participants were able to make connections between their own understanding and the requirements of the task and then to link them to a teaching strategy to complete the task. They were then able to respond to their peers who did not understand, demonstrating strategies to assist them, which indicates a degree of procedural fluency and adaptive reasoning. Breen (2004, p. 3) describes hermeneutic listening as “an approach where listeners open themselves to others without holding on to their own assumptions. In this form of listening, both parties enter into a shared project of coming to a joint understanding of each other’s position”. The prominence of hermeneutic listening within the mathematics groups indicates that adaptive reasoning played a role in that, in order to decide on a teaching plan, justification and explanation of the teaching strategies needed to take place. Furthermore, it also suggests the presence of conceptual understanding; procedural fluency and strategic competence.

Being part of a mathematics community, procedural fluency was evident in that students were able to respond to peers within the group who did not have the necessary skills to understand and complete the task. There was also evidence of strategic competence since, as members of a group, the students’ were able to discuss and determine whose idea was the best teaching strategy to implement for

the lesson. Productive disposition started to emerge as the research participants undertook further research on the five strands in order to better understand and engage with them. The fact that the students in the groups had to teach a task, and in some instances learn the concepts, led to an improvement of their practice and, in addition, of developing an integrated practice. For example, Research participant 6 identified an incident that stood out when trying to solve the problem as a group. She explained that *“This incident helped me to understand strategic competence in more depth, as I realised that children use their own systematic steps and procedures to understand and solve problems. For example, (student one) and (student two) drew pictures while I used manipulatives, and yet we both reached the same conclusions”*.

Students who were involved in the observation group evaluating their peers demonstrated the use of strategic competence and adaptive reasoning in that they were able to point out both weaknesses and strengths in the teaching approach of their peers and substantiate their point of view. Mason (2002, p. 33) refers to this as *marking* and refers to it as “a heightened form of noticing”. This is valuable because it indicated that the incident observed was significant for recall and further reflection and discussion.

Autonomy

An incident that triggered a great deal of stress emerged when no one in the group understood the problems that they were required to teach. Research Participant 8 explains that *“we started stressing about it (the task) because we felt if we don’t understand this then how are we supposed to teach it”*. However having realised that they needed to be in control of their own learning, since they were required to teach the task the following week, the team then asked for and received help from the lecturer. This enabled them to discuss what they had learnt and successfully continue with the task. This response to a perturbation relates well to the themes of autonomy and productive disposition.

Only the themes of *autonomy* and *embodiment* were evident during the first session of phase two. With regard to *embodiment*, *being* actively involved and participating in a mathematics community created awareness among the members that the

experience of having to solve the problems and then teach them is modified as they engage with one another. The groups worked as a team with everyone contributing where they could and this lightened the individual load and helped with the sense-making process. Engaging with one another influenced individual autonomies as members had to deal with and respond to the sharing and accepting of ideas and the compliments and criticisms of the ideas.

What the students brought forth

After the first session, comprising two weeks, the research participants brought forth four critical incidents that contributed to their understanding of teaching for proficiency. Two themes of enactivism were prevalent, namely embodiment and autonomy.

Firstly, the development of a repertoire of strategies that the students understood, having worked through them with their peers within their mathematical community, thereby enhancing their conceptual understanding. *“Trying to solve the problem as a group using pictures then concrete manipulatives”* (Research Participant 6, August 2011) drew attention to the notion of there being a variety of ways to approach the teaching of problems (procedural fluency) and still arriving at the same answer. The use of both manipulatives and drawings by different members helped students to understand strategic competence in more depth. Initially students battled to understand the strands and how they translated into practical application.

The second incident was the importance and benefit of *“planning and working as a team”* (Research Participant 4, August 2011). It was felt that with more people contributing ideas the work load was lightened and became more manageable.

The third incident related to autonomy and the need for accountability and control of one’s learning by responding to a perturbation. This incident, which resulted in a reaction to the perturbation, was caused by the students not understanding how to solve the word problem that they were required to teach as part of their task. Thus, in order to teach the task the students needed to take action in order to successfully address the perturbation.

The final incident mentioned was the point that *“when a teacher teaches for proficiency it is important that she understands that not all the learners in her class will all understand something at the same time”* (Research Participant 1, August 2011). This entails a teacher being patient and taking into account the differences of each learner when explaining a concept which is preferably linked to everyday life to make Mathematics useful, sensible and worthwhile.

Problems the participants posed in terms of teaching for proficiency

In terms of teaching for proficiency, the problem that students posed was that of having to solve problems or tasks when they were unable to draw on their collective experience in order to determine a solution.

Including mathematical communities into a mathematics module is important in the training of pre-service teachers as it creates opportunities for discussion and listening which improve conceptual understanding. Conceptual understanding is further developed through the micro teaching incidents that emanate from these discussions in addition to exposure to the different strategies embodied to solve a problem or come to an appropriate solution. This enactive approach of including mathematical communities encourages participants to respond to perturbations and triggers thus increasing student accountability for the development of their teaching for proficiency. This in turn affects their mathematical identity positively. Therefore it is imperative in training pre-service teachers that the autonomy and embodiment strands of enactivism underpin the lecturing approach.

4.3.2 Session Two

In session two the pre-service teachers continued to focus on fractions, this time with the emphasis on fraction operations. The strands of proficiency that the students were asked to incorporate were strategic competence and adaptive reasoning. [Appendix Six]. For the reflective tasks, the students needed to discuss the role that they had played in helping the group come to a common understanding of the concept/task they were planning to teach. Furthermore, they were required to identify an incident (or discussion) during the planning phase that had contributed to their

understanding of the topic. They also had to unpack how the incident contributed to their understanding for teaching for proficiency and discuss how old ideas that they may have had regarding the concept were changed and/or resolved. A third task was to discuss how, working as part of a group, both in the mathematics lecture and in preparing for the tutorial sessions, had changed their mathematics identity with regard to their personal mathematical proficiency and teaching for mathematical proficiency. This was in response to Leatham and Hill's (2010) definition of mathematical identity as being one's relationship with Mathematics with regard to the way one learns, does, thinks about, retains or chooses to associate with the subject. The final task was to explain what assumptions they had about the concept/task and how to teach it and how these assumptions changed through working with their group.

4.3.2.1 Analysis of Videos Phase Two Session 2

Tape 1: (Grade 5: Multiplication of fractions)

The first lesson was a very confident presentation, the participant was able to engage the students with her initial activity as it was something that they all could relate to, namely making French toast and she encouraged participation through various mathematics questions. Students were asked to explain how they arrived at an answer, thereby developing their adaptive reasoning and strategic competence skills. Her activities were very learner centred and she demonstrated good classroom management, for example, if a student's attention wandered she brought them back to task by asking an appropriate mathematical question. The activities generated discussion and participation and the explanations often included visual representations. Again there was a good balance of incorporating the five strands of proficiency into the development of the lesson. There was evidence in part of the lesson that her conceptual understanding was not as strong as it could have been.

Tape 2: (Grade 6: Decimals – problem solving)

The second lesson was confidently presented by the research participant who was able to use different methods to explain and at the same time to demonstrate the link between the alternative methods. The participant encouraged learner participation through encouraging their strategic competence skill in determining a strategy to

arrive at the solution and their adaptive reasoning by way of an explanation. Her conceptual understanding and procedural fluency was evident in the way she was able to explain the concepts using a variety of strategies and demonstrate the link between the alternative methods used. There was also effective use of board work which was linked with a table that the students had to complete. The participant was also comfortable working with the mathematical ideas that the students raised; an indication of a positive disposition and conceptual understanding. The activities that she used scaffolded one another.

4.3.2.2 Themes of enactivism that emerged

Emergence

There was an indication of *emergence* as students started to reflect on their own practice and that of others in addition to acknowledging that working in a mathematics community “*takes the pressure off*” (Research Participant 7, August 2011). An awareness of what an individual needed to do from a personal perspective to develop their own proficiency and improve their practice and approach to teaching was evident. For some students *conceptual understanding* of the topic was required before they could begin trying to teach the topic. However having understood the concept they were then happy to put forward ideas with an increase in the focus on linking mathematical concepts to real life. An increase in conceptual understanding emerged with students extending their pedagogical practice to include linking concepts to everyday ideas that the students were familiar with, for example making French toast as part of their introductory activity to build a mathematical concept. This extended to their *procedural fluency* as indicated by a member of the team “*I feel that I have always taught in my own way and now can use other strategies*” (Research Participant 2, August 2011). The fact that each session in phase two has been allocated specific strands on which to focus meant that the students continued to develop their repertoire of strategies, encompassing all five strands of proficiency. Furthermore students were recognising that learners think in different ways and that they needed to know how to approach this. As a team member pointed out, “*I have always done math in one way and it was how I was taught – with our lessons I have how to*

teach all ways or to see how pupils would try to do the sums and therefore also let them participate” (Research Participant 2, August 2011).

Embodiment

There is evidence of embodiment as students indicated that being part of a group contributed to their understanding, and having understood the concept they were then able to contribute to the perturbation, the task, and to teach it. It has also resulted in developing confidence in the students. Initially students were reluctant to teach in front of their peers but by the end of the semester we had two or three students teaching together because they all wanted to try their hand at teaching their peers and we had run out of tutorial sessions. Furthermore participating in a group and watching other groups teach similar concepts exposed students to different approaches to teaching concepts, again building up their repertoire of strategies. In many cases the discussion during meetings to prepare for the forthcoming tutorial session contributed to the conceptual understanding and thus sense-making as well as the instructional approach of the students. There is more evidence of creativity in the planning of lessons since students were required to focus on strategic competence and adaptive reasoning for example having an introductory activity that captured the learners’ attention and that they could relate to. An increase in the use of invented strategies as a means of developing strategic competence and adaptive reasoning in the learners became apparent. Van De Walle, et al. (2010, p. 215) describe invented strategies as “personal and flexible strategies” used for computation that differ from the traditional algorithm.

Autonomy

Productive disposition started to emerge by this session in that students indicated that they were more confident to teach in the Intermediate Phase at the same time acknowledging the importance of attending group meetings and taking part in the discussions to avoid feeling insecure about what needed to be taught. Furthermore, in session two there was a noticeable increase in autonomy as the team indicated an increase in confidence to teach the content. This was brought about by “*teaching and planning in a group*” (Research Participant, August 2011) as all members had come to

a common understanding. Some students felt more proficient at teaching Mathematics to the Intermediate Phase as the lectures and tutorials had created an awareness “*of the different learners in the class and their needs and the different way each concept can be taught*” (Research Participant 5, August 2011). This resulted in some students feeling less anxious about teaching in the Intermediate Phase.

Sense making

Students noted the benefit of playing an active and participatory role in emerging conversations within the group in the sense-making process as it assists in finding a solution to the problem and determining how to teach the concept. Davis, Sumara & Luce-Kapler (2008, p. 69) describe conversations as coupling that which occurs between individuals at the level of brain activity, resulting in a single cognitive system with enhanced capabilities. This is due to the various experiences and interpretations available.

What the students brought forth.

There was growth in conceptual understanding and procedural fluency in addition to an increase in their repertoire of teaching strategies encompassing the strands of proficiency. Furthermore there was an observable creativity in the planning of lessons as well as an increase in the use of invented strategies as a means of developing strategic competence and adaptive reasoning in the learners. Students indicated that they were feeling more proficient and less anxious about teaching Mathematics to the Intermediate Phase.

Problems the participants posed in terms of teaching for proficiency

No problems were identified.

The themes of embodiment and autonomy continued to be significant in increasing confidence, conceptual understanding and in developing an increasing repertoire of teaching strategies. This research study demonstrates that including the themes of enactivism, and in particular the experience theme, assists in the growth and

development of teaching for mathematical proficiency in pre-service teachers by creating an awareness of identity and a sense of accountability when responding to perturbations.

4.3.3 Session Three

The focus of the third session was the concept of decimal fractions underpinned by the strands of conceptual understanding and procedural fluency. [Appendix Seven]

4.3.3.1 Themes of enactivism that emerged

Emergence

During session three the students were required to focus on the strands of conceptual understanding and procedural fluency. The students were required to design and write a postcard. On the front they needed to draw something that captured their imagination during the practical tutorial session or the lectures. On the back, they had to explain why they chose it and how it contributed to the development of their teaching for proficiency. On analysing the various postcards there was a good distribution of all the strands. From the research participants who completed this task, one member identified the importance of conceptual understanding when teaching in that the teacher needed to encourage and promote the use of invented strategies. Invented strategies allow pupils to solve a problem in a manner that is meaningful to them and they are then able to explain and justify how they got to their solution. This would entail the teacher allowing learners sufficient time to work out the problems for themselves *“in order for meaningful understanding to take place”* (Research Participant 7, September 2011). Strategic competence was represented by a clock with the student indicating that teachers need to take time when explaining a new concept but also take time to listen to the learner’s questions and ideas. Towards the end of the second phase there was a postcard that represented productive disposition and how the student had grown from being anxious about teaching to a new-found confidence.

Students were also asked to reflect on the connections they saw between the lectures and what they could apply to their forthcoming practical tutorial lesson. The key points raised related to conceptual understanding and procedural fluency, in that they had a better understanding of the concepts and were gaining knowledge on how to teach these concepts, as well as being exposed to different strategies. In addition they were being exposed to diverse ideas and approaches, e.g. the decimal face, which enabled them to *“portray or teach my knowledge in a more meaningful way”* (Research Participant 7, September 2011). There was also the recognition that they could no longer teach concepts based on personal procedural knowledge, so well captured in a team member’s comment, *“but when we got back together we all seemed to not understand and all wanted to grab a calculator and we couldn’t do that as we need to teach it using conceptual understanding and procedural fluency”* (Research Participant 8, September 2011). From the perspective of productive disposition it was acknowledged that there was an expectation that they had to do *“further researches into the topic to be able to try teach it proficiently in the tutorials”* (Research Participant 8, September 2011).

Autonomy

Some of the postcards linked to and demonstrated a positive impact on the autonomy of the students, as illustrated in the ensuing two quotes. In the second last week of the second phase the larger math community now had *“a confident teacher who has done research and understands what she’s teaching. This has captured me as I have been worried to teach maths and now feel, after I have researched and understand what we are doing, I can teach maths successfully”* (Research Participant 2, August 2011). A second comment that links to both autonomy and emergence demonstrates how this experience has influenced the development of the student’s proficiency. *“At first my thoughts and feelings with decimals, percentages and ratio was all confusing and I didn’t understand at all during lectures. Tutorials let my imagination flare and it helps me understand these concepts better”* (Research Participant 2, September 2011).

Embodiment

Embodiment is referred to and acknowledged as being beneficial as watching other groups teach the same concept, using different approaches, reinforced the concept from a personal perspective affecting both sense-making and conceptual understanding.

What the students brought forth

The students highlighted the importance of conceptual understanding when teaching and the need for a teacher to encourage and promote the use of invented strategies. Another opinion was that teachers should allow learners sufficient time to work out the problems for themselves in order for meaningful understanding to take place in addition to taking the time to listen to the learner's questions and ideas. There was an increase in the belief that they could teach Mathematics and a greater awareness of the measures they needed to take in order to develop proficient teaching.

Problems the participants posed in terms of teaching for proficiency

No problems were identified.

4.3.4 Session Four

In the final session the students were required to plan a lesson based on the van Hiele Levels of Geometric Thought. The focus of the lessons was on one of the four content goals for geometry, namely shapes and properties, location, transformation and visualization of the lesson at a van Hiele level 1 activity. In addition the students were required to encompass all five strands of mathematical proficiency in their lesson. [Appendix Eight]

4.3.4.1 Themes of enactivism that emerged

Experience

The final reflective task required the students to identify three facets of the practical tutorial sessions that the students attended in the first and second phase that had helped to develop their skills in teaching for mathematical proficiency. They were also asked to reflect on the link between lectures, tutorials and their teaching practice. From the perspective of conceptual understanding the team pointed to the use of resources and real-life objects to make the mathematics concepts clearer and to enhance their practice. In addition it was noted that *“I have realised that teaching for proficiency requires planning and preparation on the part of the teacher, in order to ensure successful learning”* (Research Participant 6, September 2011).

With regard to procedural fluency the team indicated they were more flexible in their teaching and acknowledged the need to be prepared in order to deal with any problems the learners might pose. Strategic competence was recognised by the team, who were of the opinion that the learners should be given the opportunity to explore problems for themselves and come up with their own methods (invented strategies) since *“the exploration of the problems is very important as it encourages understanding and independence”* (Research Participant 4, September 2011). *“I have found that letting the learners come up with their own methods to be one of the strongest connections between lectures and the tutorial sessions. I battled with this concept in the beginning as I was always taught to follow the methods the teacher used, especially in maths. I find coming up with your own methods fun, rewarding and it makes complicated things seem manageable and easier to remember. I will definitely use this in my teaching practice as I think the learners will thoroughly enjoy the chance at coming up with their own methods and showing the class their way”* (Research Participant 5, September 2011), indicating the importance of adaptive reasoning.

In terms of productive disposition both from a personal and teaching for proficiency perspective the team revealed that there had been an improvement in their practice and that in one case it was acknowledged that despite being able to do the Mathematics it was more challenging trying to teach the concepts, than was originally anticipated. The need for teachers not to *“assume that they know everything*

about a subject; just because they have done the subject themselves as a learner at school”, but the importance of doing further research for “math related lessons” (Research Participant 6, September 2011) was also identified. There was also recognition of the value in linking mathematics to the real world and the need for additional research in order to develop proficiency. A third member expressed the connection succinctly in the following comment *“during the lectures, we discuss themes and concepts, as well as teaching strategies, learning styles, and the importance of age appropriate teaching [conceptual understanding]. During the tuts, we take what we have learnt during the lectures, and apply it into practice [procedural fluency]. I can apply this to my teaching practice by making sure I do research and planning before I teach my lessons [conceptual understanding/productive disposition]”* (Research Participant 6, September 2011).

Emergence

In each session the theme of *emergence* was developed since the teaching groups were required to complete questions in order to reflect on and create awareness with regard to their proficiency.

Autonomy

Autonomy featured more prominently in the final session’s reflection with the team recognising an improvement in their confidence, to varying degrees, and capability to teach Mathematics, with one member indicating that *“the math tuts have helped me to transcend my own boundaries, and this will be beneficial for me as I have accepted an Intermediate Phase post for next year”* (Research Participant 6, September 2011). The characteristics of flexibility and patience were mentioned as the team had either recognised these qualities in themselves or as qualities that they needed to develop. This is consistent with McGann (2008) and Thompson’s (2007) view that as autonomous beings we have the ability to generate ourselves in order to determine and bring forth our cognitive ability. Similarly the pre-service teachers were able to bring forth their cognitive ability with regard to teaching for proficiency and what they had gained from the module.

Embodiment

Finally embodiment was identified through the value of the groups and the discussion and active participation that these generated. Since students had been placed in groups, not of their choosing, they had indicated that they felt more accountability toward the group members, with some taking on new roles like leadership, which they would not have done if they had been a group with their friends. The group discussions helped to maintain focus and provided clarity on the task. In addition, groups were required to think about the hermeneutic questions that they could ask the learners. This requirement had been included in an attempt to encourage strategic competence and adaptive reasoning.

What the students brought forth

The research participants acknowledged the benefit of using resources and real-life objects to make the mathematics concepts clearer and to enhance their teaching practice. They also recognised that teaching for proficiency and successful learning required planning and preparation on the part of the teacher. They were of the opinion that flexibility and patience is an important characteristic of teaching.

The participants indicated that learners should be given the opportunity to explore problems for themselves and come up with their own methods (invented strategies). This was based on their experience of lectures and the tutorial sessions. *“I battled with this concept in the beginning as I was always taught to follow the methods the teacher used, especially in maths. I find coming up with your own methods fun, rewarding and it makes complicated things seem manageable and easier to remember”* (Research Participant 5, September 2011). They all felt that there had been an improvement in their teaching practice and that there had been an improvement in their confidence and ability to teach Mathematics. They also remarked that teaching the concepts was more challenging than was originally anticipated and recognised the value in linking Mathematics to the real world. Finally the participants recognised the need for additional research in order to develop proficiency.

Problems the participants posed in terms of teaching for proficiency

No problems were identified

Following session three it was evident that the inclusion of the five themes of enactivism in pre-service teacher education in a mathematics module develops flexibility in solving mathematical tasks and teaching for proficiency. This increases the ability to respond appropriately to different methods of approaching a problem or task put forward by pupils. Creating an awareness of mathematical identity through the theme of autonomy and reflective tasks (emergence theme) illustrates an increase in mathematics flexibility which in turn affects mathematical confidence positively and encourages an appropriate response to further perturbations and triggers.

4.3.5 Analytical observation through the lens of the five strands of teaching for proficiency for Phase Two

With reference to the research question, "In what way does a Mathematics module informed by an enactivist philosophy and teaching pedagogy enable pre-service teachers to unpack the reality of their teaching practice in terms of proficient teaching", the following points stand out after the analysis of phase two.

4.3.5.1 Conceptual understanding

There was a need to make connections between the student's understanding and the requirements of the task and then link these to a teaching strategy and in some cases they needed support from their group to do this. Using resources and manipulatives for teaching was beneficial as it helped with their personal conceptual understanding and provided clarity on certain concepts. The participants found teaching a concept proficiently can be challenging even if you have conceptual understanding, recognising that learners think in different ways and that they needed to know how to approach this. The participants felt that they had increased expertise in teaching different mathematical concepts as well as being exposed to different strategies which developed their practice.

4.3.5.2 Procedural fluency

This had been developed through having to respond to their peers who did not have the necessary skills to understand and complete the task. This resulted in the development of a repertoire of strategies that they understood having worked through them with their peers within their mathematical community. They also saw the value of explaining a concept which was preferably linked to everyday life to make Mathematics useful, sensible and worthwhile. The participants acknowledged that they could no longer resort to teaching concepts based on personal procedural knowledge if the intention was to develop conceptual understanding which shows a more flexible approach.

4.3.5.3 Strategic competence

The participants identified the benefits of using invented strategies as a means of developing strategic competence and adaptive reasoning in the learners.

4.3.5.4 Adaptive reasoning

Deciding on a teaching plan and having to justify and explain a teaching strategy helped to develop adaptive reasoning. Adaptive reasoning improved as they were able to point out both weaknesses and strengths in the teaching approach of their peers and substantiate their point of view in presenting their own strategies.

4.3.5.5 Productive disposition

Having to teach a task and learn the concepts in some instances led to an improvement in their teaching and in developing an integrated practice. It also created an awareness of the need to be in control of their own learning since they were required to teach the task the following week and to do research if need be in order to teach it proficiently. Finally the participants indicated that they were more confident to teach in the Intermediate Phase at the same time acknowledging the importance of attending group meetings and taking part in the discussions.

Having analysed the data gathered during phase two the points that emerged regarding the influence of the themes of enactivism on teaching for mathematical proficiency have been tabulated. (Table 4.4) It is evident that the five themes of enactivism played a key role in growing and developing proficient teaching. The theme of autonomy encouraged a sense of accountability in the participants as they took control of their own learning so that they could actively participate in their mathematical community. The increase in connections made by the participants within and among their mathematical knowledge, the theoretical knowledge that they were introduced to and their pedagogical practice came out of the emergence theme. Embodiment continued to develop conceptual understanding which further developed the other strands of proficiency. Working in a mathematical community informs participants of different instructional practices and approaches to mathematics which promotes proficient teaching. The most important theme is that of experience. By providing the students with a safe environment to micro teach mathematical concepts without fear of a poor lesson affecting their assessment the participants were encouraged to try new ideas and practices.

Table 4.4

Matrix indicating the Strands of Proficiency accentuated by the Themes of Enactivism in Phase Two

	Autonomy	Sense-making	Emergence	Embodiment	Experience
Conceptual Understanding	Take into account the mathematics being taught and how students learn it. Conceptual understanding of the core knowledge required in the practice of teaching.	No points emerged	Demonstrated conceptual understanding of the core knowledge they were required to teach. Made connections within and among their knowledge of mathematics, students, and pedagogy. Developed a knowledge of how students' mathematical understanding develops. Developed a repertoire of pedagogical practices.	Demonstrated conceptual understanding of the core knowledge they were required to teach. Able to make connections within and among their shared knowledge of mathematics, students, and pedagogy. A repertoire of pedagogical practices that take into account the mathematics being taught and how students learn it.	No points emerged
Procedural Fluency	Developing fluency in carrying out basic instructional routines.	No points emerged	Among the group there was a repertoire of instructional routines to draw upon as they interact with students. Several ways of approaching teaching problems.	Among the group there was a repertoire of instructional routines to draw upon them as they interact with students.	Apply a range of routines flexibly, know when they are appropriate.
Strategic Competence	Teaching is a problem- solving activity.	Planning effective instruction.	Figured out what to teach when, how to teach it, how to adapt material so that it is appropriate. Time to allow for an activity. Find out what a student knows.	In the group with regard to planning effective instruction and solving problems that arise during instruction. Effective in solving instructional problems while planning their lesson. Figured out what to teach when, how to teach it, how to adapt material so that it is appropriate.	Planning instruction, implementing those plans, and interacting with students. Teaching is a problem- solving activity.
Adaptive Reasoning	No points emerged	Justifying and explaining one's instructional practices and in reflecting on the practices of others.	Determine what the students have learned. Consider how the students respond to particular representations, questions, and activities.	Justifying and explained their instructional practices. Reflected on their own as well as observed practices to improve them. Learnt from their teaching by analysing it. Used representations that were most effective in communicating essential ideas to their peers. Focused their attention and developed a common language for discussion.	Justifying and explaining instructional practices. Become reflective practitioners. Engage not only in learning methods of teaching but also in reflecting on them and justifying and explaining them.
Productive Disposition	In control of their own learning to ensure they understood. Productive disposition toward mathematics, teaching, learning and the improvement of practice. Think they are capable of learning about mathematics, student mathematical thinking, and their own practice by themselves. Learn by listening to their peers.	No points emerged	They were capable of learning about mathematics, student mathematical thinking, and their own practice themselves. Learnt mathematical concepts and strategies from their interactions with their peers. In control of their own learning to ensure they understood. Becoming more comfortable with mathematical ideas.	Learnt by listening to their peers and by analysing their teaching practices. Were in control of their own learning in order to be able to teach the lesson.	Productive disposition toward mathematics, teaching, learning and the improvement of practice. Lifelong learners. In control of their own learning.

4.4 PHASE THREE

A year and a half after they had graduated with a B.Ed. Foundation Phase degree I interviewed three of the research participants who had found employment as Intermediate Phase teachers. I wanted to investigate to what extent they believed that the module had contributed to their teaching practice. This investigation took the form of unstructured interviews. In analysing the data for the third phase I provide a profile of the participant and then identify which aspects of the five themes of enactivism were revealed. I then analysed this data through the lens of Kilpatrick, et al.'s (2001) strands of teaching for mathematical proficiency to determine which themes of enactivism emerged as the most significant.

4.4.1 Phase Three Research Participant One

4.4.1.1 Profile

This participant is currently teaching a class of 40 Grade 4 pupils. The mathematics planning is done by one of the Grade 4 teachers who specifies what work needs to be covered in a particular week. Teachers meet weekly to reflect on the previous week's Mathematics and to discuss the mathematics content for the upcoming week.

Extra mathematics lessons were offered to pupils after school, these were not compulsory and should a teacher feel that a child would benefit from them, a letter was sent to the parent. The participant also helped pupils who struggled with the content during their break.

4.4.1.2 Themes of enactivism that emerged

Autonomy

The participant is not comfortable with Mathematics, which she considered to be a personal matter, and was of the opinion that this was something that she will need to overcome by herself stating that *"I still feel like I'm afraid and there's not much you can do*

unless you have overcome that fear of maths" (Research Participant 1). Furthermore, she believed that until a person had overcome the fear, there was nothing that an outside influence could do to assist. This clearly impacted on her teaching for proficiency, specifically productive disposition, as could be seen by the following comment *"I'm not a very maths person so I don't really promote it as well as I should - which is a bad thing I know - um, ja the learners just do what I teach and that's about it"* (Research Participant 1). This fear, *"I'm afraid that I'll get myself into a situation I can't get out of ...so if they ask me a question and I'm not able to answer, I think my self-confidence will actually take a bit of a knock"* and *"because of that situation I might land myself into"* (Research Participant 1) impacted on her approach to teaching and thus proficiency. An additional impact on her teaching approach was what she acknowledged as *"I'm not a very adventurous person when it's with maths"*. (Research Participant 1). Evidence of emergence was reflected by the comment *"But I do know that I need to put in a little bit more effort"* (Research Participant 1). Although, there was an indication that her confidence had grown, it was noticeable that she did not have much self-efficacy, given the following statement *"also feel like I'm not yet - um, like confident enough to do that. Um, ja it's bad, I'm a bad teacher"* (Research Participant 1).

Her teaching approach had in some respects reverted to the way that she was taught at school because she was nervous, which she acknowledged as *"which I think isn't a really good thing, but I'm very nervous with venturing out"* (Research Participant 1) but even this familiar and traditional approach did not make the participant comfortable. Although she admitted to having been introduced to different strategies during the module she chose not to react to the perturbation believing that the methods that she was taught at school were *"I think it was so drilled into me at school that this is how maths is done and that's just the way it is"* (Research Participant 1). Thus it was evident that although she had a choice to respond to the perturbations of increasing her toolbox of mathematical strategies, without the safety of her mathematical community she chose not to, preferring to remain in her comfort zone.

Interestingly she did view the module as being beneficial but indicated that one year could not undo years of being exposed to one approach. *"I find maybe, ja, I think maybe this is it. I've had years of maths being taught to me that this is the way it should be, whereas one year at college didn't really make a difference. So maybe from the beginning of*

college, so in first year ... at least that will give you four years to instil other methods and confidence, actually. Whereas I've had, however many years, of this is how it should be done" (Research Participant 1). Had a similar Mathematics module been compulsory for the four years of the degree it might have given her sufficient time to develop the confidence to respond to the perturbations and to strengthen her conceptual knowledge.

Embodiment

Due to the nature of her identity, that of mathematical anxiety and a low level of confidence, this participant drew on the mathematics community of her fellow teachers and in some instances the pupils within her environment to help her develop new meaning with regard to mathematical concepts and thus further her conceptual understanding. However there was not a "formal mathematical community" as such that she felt comfortable with to exchange mathematical ideas as well as teaching approaches and strategies. However, she was able to draw on her colleagues to make sense of concepts and believed that *"I think when I find my feet and have been teaching for a few years, then I would have more of a voice"* (Research Participant 1) which would allow her to contribute her opinion regarding different teaching approaches. It was evident from interviewing this participant that some form of embodiment had developed during her practice as a qualified teacher. The mathematical community of her classroom environment and colleagues together with an awareness of her mathematical proficiency and understanding had been created from the feedback gained from others. She had responded to this perturbation by indicating that with more experience she would be in a position to contribute to the community. In this particular case, the participant's intuitive response to mathematical triggers or perturbations was to fall back on the support provided by the mathematical community in which she found herself.

Emergence

The participant is aware of and acknowledged that there are various ways of knowing, and when referring to the pupils in her class shares that *"I have some who follow my methods in my class and then I have, well maybe two or three learners who do*

their own thing, and I always encourage it, because you can only teach it your way and understand it your way, but I want them to understand it as a personal way for themselves, so that's what I think proficiency is" (Research Participant 1). In planning her Mathematics lessons and marking, she reflects on the extent to which the pupils understand the current work. This is influenced by the difficulties arising from the demands of the Curriculum Assessment Policy Statements (CAP's). This document affects teaching for mathematical proficiency indicating that teachers are under pressure due to the time framework in which certain content needs to be covered.

Experience

The increased practice and experience has increased her confidence, which in turn appeared to have a positive effect on her productive disposition. *"I think maybe because I've had a practice run ...now I understand it better"* (Research Participant 1). Now in her second year of teaching, she asked for assistance less frequently and indicated that the practical tutorial session during the mathematics module (that is, the experience theme of enactivism) and having to answer questions posed by her peers had helped in dealing with the pupils, *"... so that when you're faced with the pupils, it's not as bad"* (Research Participant 1).

In discussing her teaching approach, her comment *"I find that the maths we did in college was far more difficult in a way, whereas the maths now isn't that difficult ...I find like with the maths that we did in college, you had to explain that using manipulative and all of that ... whereas now in grade 4 you don't really have to, they're understanding it."* (Research Participant 1) raised the question of the extent to which procedural teaching was taking place. Due to the class size, the lessons appeared to be more teacher centred, with individual interaction between teacher and pupil rather than group work. When undertaking problem solving examples, adaptive reasoning and strategic competence were encouraged, by asking the pupils to participate and explain their solutions to the rest of the class. Having taught the lesson, pupils who did not understand were encouraged to come up to the desk to be taught on a one-on-one basis, the comment *"I have a long line."* (Research Participant 1) raised the question of the level of proficient teaching. This was corroborated when she commented that *"(I) think its maybe my memory, I need to remember how I used to work it*

out, because it was I don't know how many years ago, I wouldn't like to say ...so I had to remember how I did it at school ...so I think that's, that's what made me a bit afraid" (Research Participant 1). This indicated that she relied on procedural fluency since there is no or little conceptual understanding to draw on to explain a concept. With regard to productive disposition and linking Mathematics to the pupils' everyday lives, the participant tends to use examples such as sweets or chips because *"I think you also have to be very, um, like careful how you do it, because a lot of the pupils don't have all these nice things ... you know you can't use examples like cell phones and all of that because of their economic backgrounds as well"* (Research Participant 1). When planning a lesson, she acknowledged not taking the pupils' disposition towards Mathematics into consideration and felt that the clever pupils were possibly disadvantaged by the weaker pupils, in that *"they finish their work quite quickly, the cleverer ones, um, and then they basically just sit and do nothing while they're waiting for the weaker ones to finish"* (Research Participant 1). Of concern for teaching for proficiency, was the comment *"I just find that you come into a school with all these like hopes and everything ...and then they're sort of like zapped and then you just teach like a robot, basically"* (Research Participant 1).

Sense-making

Her identity, which was one that encompassed a fear of Mathematics, was such that it allowed her to be open to help and explanations from students thus encouraging sense-making by means of active and participatory conversations and listening. *"I find that sometimes, like if I don't understand a maths problem, I'll actually ask my learners to explain it ... So, I feel that I'm more open to, like, suggestions from anybody, than me standing up and this is the way it needs to be done ...you know, like as a dictatorship, so I ask my learners sometimes to help me out"* (Research Participant 1). Thus using Kilpatrick's framework this means that the pupils who participated were making use of their conceptual understanding, strategic competence, adaptive reasoning and productive disposition skills.

Curriculum Assessment Policy Statements (CAPS) DBE (2010)¹³

CAPS DBE (2010) are strictly followed and it is seen to be time consuming by the participant and therefore impacts on the approach that is taken for teaching. It is seen to be very structured, *“like this is what you have to cover and that’s that”* (Research Participant 1). The impact of the CAPS DBE (2010) document on time was raised a number of times.

Impact of the mathematics module (MIP 400)

The module had influenced her teaching practice and increased her confidence with regard to teaching as seen by the comment *“I found that the maths in fourth year was quite difficult, and then knowing that I could overcome that, Grade 4 maths is easier, so in a way it like boosted my confidence, ja, quite a bit actually”* (Research Participant 1). Although she had enjoyed group work and had intended to use this knowledge in her teaching practice, current circumstances dictated otherwise, *“so my intentions to work in smaller groups are there ... but it’s just hard as one teacher faced with 40 learners, so, ja, it really is difficult”* (Research Participant 1). Establishing smaller mathematics communities within the classroom, in the form of small groups, although beneficial to her, was perceived as difficult, *“so I would like to work in smaller groups but because you’re working with this one, the other ten are busy running around and, you know, doing their own thing, so I think you lose focus on what’s actually wanting to be taught”* (Research Participant 1).

The use of manipulatives as well as the module content was viewed as beneficial to the development of her personal proficiency and conceptual understanding however their use within her classroom practice was perceived to be difficult to implement given the class size and lack of finances. However, her strategic competence and adaptive reasoning were not as developed and so she shied away from problem solving preferring to use methods that she could recognise and identify with.

Although acknowledging that when working with word problems *“I also find that I need to take time myself to sit down and then work out the answers and you know, like first*

¹³ A policy statement for learning and teaching in South African schools.

internally understand it ... before I can teach it . . . so I think I do need to make more time in understanding things myself, um, so that I'm not placed in the situation of not knowing what to do" (Research Participant 1). Due to time constraints resulting from marking, school administrative work as well as the ANA paper work and extra mural commitments, developing personal proficiency in areas where difficulties are experienced means that " *I sort of wing it most of the time, ja, I do*" (Research Participant 1).

4.4.2 Reflection on Research Participant One through the lens of the five strands of teaching for proficiency

4.4.2.1 Conceptual Understanding

The strength of a person's conceptual knowledge will influence the risks they take and the strategies they use which in turn will have an impact on their ability to teach Mathematics proficiently. Embodiment that assists with the sense making process can be augmented through participation in a mathematical community. Conceptual understanding is sometimes sacrificed due to the pressure of meeting the demands of government policies and assessments.

4.4.2.2 Procedural Fluency

Her lack of confidence resulted in her adopting a more procedural teacher centred approach to teaching Mathematics. This was often a fall-back approach and was not necessarily based on a strong conceptual understanding.

4.4.2.3 Strategic Competence

The fact that the participant indicated that she was not very adventurous and was nervous about getting into situations that she could not control suggested that although she encouraged the pupils to participate in discussions and making sense of mathematical concepts she may not have had the conceptual understanding to pick up fundamental errors or misconceptions.

4.4.2.4 Adaptive Reasoning

Recognising the importance of adaptive reasoning when teaching for mathematical proficiency the participant was willing to let her pupils share their methods, even though this may have placed her in an uncomfortable situation based on her personal proficiency.

4.4.2.5 Productive Disposition

A person's belief, or lack thereof, in their self-efficacy has an effect on their productive disposition regardless of whether or not they view Mathematics as worthwhile and valuable. She did not take the students productive disposition into consideration and did not appear to emphasise the value and usefulness of Mathematics in her lessons.

What the participant brought forth

- Working in different mathematical communities provides support, helps develop conceptual understanding and increases a person's confidence.
- A fair amount of time is spent during breaks to help pupils to understand Mathematics.
- While acknowledging that she enjoyed working in smaller groups in the module the participant was unable to carry this out in her own teaching practice and classroom.

Problems the participant posed in terms of teaching for proficiency

- The constraints imposed by adhering strictly to the CAPS DBE (2010) requirements impacted on her ability to teach for mathematical proficiency.
- Time constraints imposed by CAPS DBE (2010) and ANA in addition to administrative work affected her ability to further her personal proficiency in Mathematics and thus her teaching for mathematical proficiency.

Reflecting on the data from this participant this research has added to mathematics teacher education by highlighting the following important points brought to the fore by the themes of enactivism. Drawing the students' attention to their mathematical autonomy and identity to create an awareness of their strengths and weaknesses allows them to recognise the kind of response required to a perturbation in order to build a comprehensive mathematical foundation from which to draw on when teaching. It highlights the importance of a mathematical community (embodiment theme) to develop conceptual understanding and proficient teaching skills and emphasises the continued usefulness of this supportive community of practice as a qualified teacher.

4.4.3 Phase Three Research Participant Two

Profile

Participant two was teaching a Grade 6 class of 21 children. There were two teachers in this grade and the other teacher planned the mathematics file and work for the year. This small community of practice met once a week to "*discuss what maths will be done for that week, and any problems, because in CAPS we're supposed to spend so many hours on each thing*" (Research Participant 6).

The participant's breaks were often spent marking or assisting pupils with work. In addition to providing one-on-one assistance in class, pupils who did not finish their work were required to go to the quad during breaks to finish their tasks according to school policy. "*They're sent to the quad for, um, most unfinished work. It's not – it's not um, it's not sort of a discipline, they're not in trouble. There's a grade 7 monitor on duty, the quad is a tarred area with, um, outdoor tables and chairs ... so they go there to finish their work, they take their lunch and the grade 7s help them, so if they're struggling with whatever they're doing they can ask*" (Research Participant 6). Furthermore, to combat the pressure of time, the teacher often provides assistance to pupils who are not coping after school.

This participant arrived at school at 6h30 and used the time before the start of the formal lessons to organise herself and carry out last minute preparations. Some difficulties that she had experienced with teaching Mathematics in the Intermediate Phase she explained as *“I think which is what I felt in my first year as well, I felt that I wasn’t knowledgeable enough...I felt that I needed to know more than the pupils did, and I didn’t always ... I feel much better this year, knowing it so if the pupils are doing it that way I can help them”* (Research Participant 6). This links with what Carnoy and Chisholm, et al. (2008) said regarding teachers being untrained to teach in certain grades.

4.4.3.1 Themes of enactivism that emerged

Autonomy

Her first year as a qualified teacher resulted in her identity adapting to encompass a great deal of responsibility, as could be seen by the following comment, *“I just sort of felt a lot of responsibility, not with my other subjects, English, all the other subjects I didn’t feel that same responsibility. With Mathematics I felt very responsible for how much they understood, how well I was teaching it, um, so the first year was quite stressful, um, and in fact at the end of the first year I requested to only teach English,.. ... then when I started this year I changed my mind and I decided to keep the maths and I, in fact, really enjoy it, I find that its one of the subjects that there’s the most um, interaction and I enjoy maths a lot more, I feel more confident in it, um, I see it being far more purposeful than I did and I find that I’m not sort of focusing on what I was taught but I’m drawing on it. Now, with what I’m doing in the classroom I remember a lot from the module ... and it now makes more sense on how I’m supposed to be using it”* (Research Participant 6). With experience and practice, her identity continued to adapt from being stressed and holding herself responsible and accountable for how well the pupils did, to one of confidence and enjoyment with a sense of purpose and more realistic expectation of accountability. It was also evident that her identity was empathetic towards the learners who lacked confidence, *“I sort of try to be ... sort of caring ... and help them. I’ve had a few cry in fact. Not in any other lessons, just maths. It can get so difficult, they cry, so I know which pupils struggle and after I’ve explained I go around and check, okay, what didn’t you understand, let’s go through it again, so while the stronger ones are working I can go to the weaker ones ...”* (Research Participant 6).

In comparison to the teaching approach she experienced as a child, which was just do the Mathematics, this participant made use of discussion and was of the opinion that more problem solving took place in her lessons. *“We talk more about it before we actually do it. In fact we have such long discussions sometimes I have to say okay, we actually have to start doing some work on paper, so I try and make it as learner-centred as I can and I also try to be quite sensitive about the maths because I think that is one subject that some pupils really, really find frightening”* (Research Participant 6). She did ask the pupils to refer back to what they had learnt in the previous year, particularly when introducing new concepts. Her teaching practice also demonstrated elements of support, empathy and a productive disposition relating to her personal autonomy in that she considered the pupils’ disposition towards Mathematics when preparing her lessons stating that, *“I do see certain things that we’re doing on that day that I think might be difficult and I make sure I don’t leave them alone on those ones, we need to do them together, discuss it, so I do think about the pupils’ disposition when I’m planning, not the content but how we go about teaching it and how we do it in the lesson”* (Research Participant 6). She was also very aware of the abilities and levels of the pupils and especially their productive disposition and self-efficacy, *“my pupils seriously lack confidence in maths”* (Research Participant 6). *“We do want them to use maths purposefully and to have a positive attitude towards maths, we really try and build that up. I think my colleague and I both really enjoy teaching maths”* (Research Participant 6) indicating evidence of productive disposition.

With regard to teaching for proficiency the participant indicated that her idea of what this looked like changed, *“and it was different to what I’d – to the idea I’d had before we started the module”* (Research Participant 6). She believed that *“a proficient teacher, um, shouldn’t just focus on one thing. You yourself need to be proficient in those give strands of maths ...you need to be willing to research further because as the conceptual understanding curriculum changes you need to be able to incorporate different teaching methods and different ideas and to sort of not focus on one thing, and I do have an idea of what a proficient teacher should be, um and I think that I’ve tried my best to be a proficient teacher but I still feel that you can become more proficient with maths”* (Research Participant 6). Furthermore she believed that the module contributed to her growth and development in teaching proficiently *“because in second year we did a lot of theory. In this fourth year module we practised teaching maths, it wasn’t just about learning how to do maths. If I even think of the exam that we wrote, it was how would you teach this*

concept? How would you use manipulatives to deal with this concept? um so it was theory and practice. I think without that I would have struggled more. Um, I think that the fourth year module helped give us an idea of how we would have to behave as a maths teacher, um, how we would have to present lessons, how we'd have to deal with concepts, by doing maths ourselves as learners, by teaching it to our colleagues as teachers" (Research Participant 6). Looking at her identity as a Mathematics teacher, she had chosen to adapt and construct it to encompass the term proficiency, "think there is a bigger focus now on proficiency in maths and I know me and my colleague, we definitely focus on that when teaching maths" (Research Participant 6).

Embodiment

Her approach to teaching was to initially teach the whole class and then work with smaller groups, encouraging pupils to develop adaptive reasoning and strategic competence skills, "but if they are getting to an answer in a different way than what you've explained or what you expected, if they're getting to the right answer, let them explain how they did it" (Research Participant 6). Pupils were encouraged to take part in discussions and to demonstrate and explain their approaches to a problem, which again encouraged adaptive reasoning and strategic competence, "Um, I think the other day we discussed a problem-solving question and I was sort of – we did it – I did it with them and we did it the long way, and then one of the kids said to me 'well why don't you just times this by two and then divide it and you'll get the answer easier, and I said yes, come and show us, so some of them are quite confident, others I have to sort of draw them in" (Research Participant 6). Thus her approach was to establish one mathematical community comprising of the whole class rather than smaller communities within the class.

Although the participant believed that group work would help the pupils to build conceptual knowledge, she did not always make use of it because of the time factor, "think it would, but once again it's the time factor, unfortunately, because to build that kind of understanding together you can't give, okay ten minutes and then you need to know what's going on....It sort of will take its own time and unfortunately we haven't been given that time in the document, um, but perhaps there is a way of incorporating it, because I know when we did it in the fourth year, it was very helpful" (Research Participant 6). Furthermore, to include group work you need to be aware of discipline and participation issues

therefore “you have to be on your toes. You have to be looking at everybody and finding out what they’re doing, listening in, giving some input, you know, or else they won’t really get much done” (Research Participant 6).

Emergence

With regard to emergence I was looking for evidence of a collective generation of knowledge and understanding through shared activities and reflective skills. This was apparent in the weekly Mathematics meeting of the participant and the other Grade 6 teacher to reflect on their pupils’ progress and any difficulties experienced. In addition, when preparing and planning for the following day’s lesson, the participant reflected on the pupils in her class, considering various factors, as can be seen “sometimes it might only be two pages in the textbook but if I read through those pages its going to take a lot long then it looks. I look at how much do they already know, how much do they not yet know, what needs to be explained, how many examples are we going to have to do before we can start the work, um, I look at the difficulty level. Um, if it’s not appropriate, I take out things and I add my own, um, so I have to look at those before we actually do it in class” (Research Participant 6).

Experience

This participant’s teaching approach differed from what she had been exposed to as a learner. When discussing teaching for proficiency she was of the opinion that you needed to use manipulatives to teach proficiently, expressing that “. . . manipulatives definitely help, and also they’ll be using those things in, you know, their daily lives so it is um, it does help them to be proficient. Um, I would also say, um, group work” (Research Participant 6). Although the strands of proficiency were not explicitly referred to there was evidence that they underpinned her approach to teaching, as she revealed that “we do focus on – sometimes they do need to just know, you know what is rate and ratio, you have to learn it to be able to differentiate between them, what does medium mean, your average, they need to know what’s the answer. There is that sort of theory that conceptual knowledge that they need to know, then procedural, they need to be able to um, work on solving problems without their step-by-step. They need to be able to read it and understand it and work it out sort of logically” (Research Participant 6).

Sense-making

Her teaching approach entailed a great deal of discussion, with listening as well as active and participatory conversations being an important part of the sense-making process. There was limited use of group work, with pupils sitting in pairs or on their own, often as a result of parental request. In some cases students were paired together to help with the sense-making process, *“I usually put a stronger child with a weaker child, because the stronger pupils enjoy sort of teaching and being, um, the leader and I think they – and they respect that role ... and the weaker child needs somebody to sort of guide them”* (Research Participant 6). To help a child who did not understand, she would explain the work again on an individual basis or in small groups, using a variety of strategies. There was an element of adaptive reasoning involved in the lessons drawing the other pupils into the sense-making process, for example, if she did not understand a child’s explanation she would *“ask somebody else to help – sometimes the pupils understand each other ... and they will – there’s one child in particular I keep thinking of who, who – he’ll say okay, I know what he’s saying, and he’ll explain for me, so if I don’t understand then the other pupils help”* (Research Participant 6).

4.4.3.2 Curriculum Assessment Policy Statements (CAPS) DBE (2010)

This participant felt that the CAP’s document, which pre-service teachers needed to study thoroughly, had a bigger focus on problem solving. Her opinion was that the CAPS DBE (2010) document was *“quite specific, the CAPs tells you exactly what you’re supposed to do”* (Research Participant 6). She thought that the document puts pressure on a teacher who would like to adopt a more learner centred approach due to the time factor. *“The one thing is time. I find with the CAPs we are on quite a tight schedule. In fact, I know with next term, we actually had to start this term’s work just a few lessons earlier”* (Research Participant 6). She also considered the CAPS DBE (2010) document to have high expectations *“quite enjoy it. Um, I do think its forcing you to look at all aspects of maths. Um, sometimes it is a bit ideal ...in that you’re supposed to get this much done in this amount of time, but um, the work that they’re doing now is sort of covering all spectrums and expecting different abilities from the pupils in maths”* (Research Participant 6).

4.4.3.3 Impact of mathematics module (MIP 400)

Although trained to teach in the Foundation Phase she was appointed to teach in the Intermediate Phase. The participant said that the mathematics module (MIP 400) had helped her with her teaching of the Grade 6 classes. *“During teaching practice, I actually found maths quite frightening. I actually tried to avoid teaching maths lessons as much as possible, but in the lectures (Math module), I would say it’s not that much different, um, because we did a lot of problem-solving”* (Research Participant 6). *“Now that I’m with Intermediate Phase and we did study Intermediate Phase, a lot of the problem-solving and the different approaches have now come up, and even with the CAPs textbook that I’m using, we are expected to teach them at least four different approaches”* (Research Participant 6).

The problems that she had encountered in terms of teaching for proficiency related to the fact that *“some of the pupils do like to sort of have a method (rather than) use their initiative to find their own answer (in problem solving)”* (Research Participant 6). A further difficulty identified was *“the different levels of understanding ... the sort of higher, the medium and the sort of – the pupils who struggle more. It’s difficult to teach for proficiency across all those um, sort of, across that spectrum,”* (Research Participant 6). When doing problem solving, the participant referred back to the mathematics module (MIP 400), *“Um, in fact in our last year, with the maths module we focused a lot on problem-solving and not following procedures but trying to solve things logically in your own mind, the way you see it...and I draw on that a lot when we do problem-solving. Problem-solving does come up in all the sort of math themes, the maths, the distance and everything and so I sort of try to draw on that, I try to remember that there are different methods. Some pupils can – will work with some, some pupils will want to draw pictures, um, so I keep that in mind when we’re doing work like that. I also try to remember how difficult I found maths and how off-putting it is when somebody sort of gets irritated or just moves on when you don’t understand and I try to be very accommodating with the maths, as best I can”* (Research Participant 6). She found the problem solving aspect to be most useful in that, *“if we hadn’t covered problem-solving in the fourth year I don’t think I’d be so comfortable with it as I am now and, um, it also in fourth year made us look at all the different ways that something can be seen. I know we were expected to come up and show how we solved problems, and three people might have each done it in a completely different way ...so I’m always keeping that in mind with my own pupils now with my class. They all have different sort of, um, levels*

of thought and different ways of reasoning, and they might be seeing the problem in a different way. So I try to keep that in mind still, now” (Research Participant 6).

Her teaching approach generally involved the use of the board, however for more abstract concepts or to develop understanding, manipulatives were used, often with a problem solving approach when linking Mathematics to their everyday lives.

The module has impacted on her approach towards her teaching practice. *“Definitely with making sure that both procedural and conceptual are important, um, because I think that a lot of – especially with maths there can be a danger of just focusing on the procedures. You need to know this and this and this and you need to do it this way, especially with long division and long multiplication, its fine to use the method as long as you actually understand how to multiply and how to divide first. . . . Because there’s really little understanding if you follow those methods, so I realise that they both have a place and they’re both important and I keep remembering that in my planning and in my class work . . . I would say I have, especially with the balancing the different kinds of maths that are important. Um, I try not to focus – it’s very easy to focus on known methods and procedures and its quite nice actually, it’s sort of comfortable ... but I try to teach the pupils to use their own minds and to first, to not be afraid to try on their own and come up with the wrong answer, um, and then try again” (Research Participant 6).* In addition, the module created awareness of her personal proficiency and in particular the theme of embodiment, in preparing her for teaching an Intermediate Phase class. *“I think when we worked in groups and we planned those lessons and we did those lessons in front and then we also did the homework tasks but then presented them to the class the next day, there were a lot of things that I can remember that we did that I can use now ...and sometimes even in class I wouldn’t understand until somebody sort of did it the way I understood, which I keep in mind now” (Research Participant 6).*

4.4.4 Reflection on Research Participant Two through the lens of the five strands of teaching for proficiency

4.4.4.1 Conceptual Understanding

In her discussion of her planning and preparation approach, there was evidence of conceptual understanding as she demonstrated knowledge of her pupil's abilities and what would work best within her classroom context.

4.4.4.2 Procedural Fluency

There was evidence of procedural fluency in her approach to classroom management and what she needed to do to carry out particular instructional practices effectively, for example, group work. To a certain extent there was also a routine to her teaching practice, but she was flexible in her choice of approach.

4.4.4.3 Strategic Competence

She used the strand of strategic competence in her classroom practice by allowing and encouraging the pupils to explain their approach and strategies. This was also evident in the decision that she had taken with regard to how she was going to approach her teaching and interaction with students, especially with regard to the pupils who battled with Mathematics.

4.4.4.4 Adaptive Reasoning

This was evident in her comment that she drew on her previous experiences and reflected on her current practice in making her instructional practice more purposeful.

4.4.4.5 Productive Disposition

With experience and practice, productive disposition and efficacy grew in addition to a more realistic expectation of accountability for student proficiency. She felt the need to develop in her pupils the recognition of the value and purposefulness of

Mathematics and she was empathetic with regard to the pupils' abilities and problems. Productive disposition was also evident in that the participant analysed her practice in order to improve it and 'recognition' in that learning was an ongoing process and therefore she was willing to research further should the need arise. A willingness to listen to her pupils' explanations and allowing them to assist with providing clarity on their peers' explanations demonstrated her productive disposition. She was also comfortable with her instructional approach and her willingness to reflect on her pupil's abilities and to adapt her strategy accordingly. This demonstrated a productive disposition. The participant was aware of her own ability and the need to strengthen and develop her knowledge.

What the participant brought forth

She was drawing on the mathematics module in her current teaching practice, recognising its purpose and value in teaching for proficiency. The module had unpacked proficiency for her, and she incorporated this approach in her teaching practice. She also believed that this was an on-going process, which is in accordance with the seven roles of a teacher identified by the norms and standards¹⁴ for educators (Department of Education, 2000). The module contributed to her growth and development in teaching proficiently. The CAPS DBE (2010) document although well received, was considered to be idealistic and not taking into account the varying degrees of pupils' mathematical ability. The module had given her a basic grounding in problem-solving and "*not following procedures but trying to solve things logically in your own mind*" as well as introducing her to different approaches to teaching Mathematics to use in her current practice. The module had also created awareness of the different approaches that pupils might adopt and to keep this in mind in her teaching practice in addition to underpinning her practice with the strands of proficiency. A great deal of time was spent on administrative tasks and providing additional support to pupils who were not keeping up.

¹⁴ Norms and Standards for Educators (Department of Education, 2000) – Roles and criteria for teachers

Problems the participant posed in terms of teaching for proficiency

The CAPS DBE (2010) document puts pressure on teachers who would like to include a more learner centred approach, due to the time-frames within which the teacher needs to cover work. This is confining and places negative restrictions on strategic planning. Another factor was the varying levels in the pupils' abilities in addition to some pupils' preference for a more procedural approach to Mathematics rather than conceptual understanding.

The third phase of this research study with this participant revealed that the contribution that the research has made to mathematics teacher education is to make evident the value of including the themes of enactivism in a mathematics module in order to develop proficient teaching. With positive practical experience in the mathematics classroom comes confidence and a more defined sense of mathematical identity. Therefore by including practical experience within a mathematics education module the pre-service teachers will, on graduation, be better equipped to implement various teaching approaches within their context and deal with any challenges that may arise. By encouraging embodiment through the development of either a class mathematical community or a small group community, guided correctly through appropriate triggers and perturbations, pre-service teachers will grow and develop strategic competence and adaptive reasoning. The theme of emergence underpins the other themes with reflection becoming part of the inherent process undertaken by the participant as did the theme of sense-making through the encouragement and use of dialogue and listening.

4.4.5 Phase Three Research Participant Three

Profile

The third participant was teaching a Grade 6 class. Of the 24 pupils only 6 spoke English at home, the rest being second language pupils. Some of the pupils had come from an isiZulu school to an English school in Grade 6, with the participant commenting that *"when I'm speaking to them, they don't even understand – some of them"* (Research Participant 3). This will affect teaching for proficiency and although

covered theoretically in the module was not unpacked practically given that the majority of the students spoke English. Van De Walle, et al. (2010) point out that learning Mathematics as an academic language is difficult because it is not part of the pupils' everyday world and there may not be words in their native language for the content that they are learning. Like the context of the previous two participants, in this school the participant did not have anything to do with the planning of Mathematics. Other teachers planned the different sections of the work and gave it to the teachers in the grade. The participant was responsible for the planning of English and Social Science. Extra mathematics lessons are offered at the school; however it is optional to attend. Pupils are also welcome to meet the teacher during breaks for help it *"just depends on the child. Um, most of the time I will – I give up a lot of my breaks ...and they'll come to me, because also some of them just need a one-on-one and without distractions"* (Research Participant 3).

4.4.5.1 *The themes of enactivism that emerged*

Autonomy

This participant's identity as a mathematics teacher had not changed since university, she loved Mathematics and had tried to incorporate what she was exposed to at university into her own teaching, *"because there was not so much pressure...it was more relaxed and there was more hands-on ... fun, so that's what I try and bring in for them, because I know that if you have a maths anxiety it's so difficult to get over that...So I try and bring in the fun for them, because I know how it felt when it was just teaching and doing, teaching and doing, so that's ...Um, definitely, college has definitely helped me with the confidence due to maths and I've so many ideas, I mean from doing normal maths in school, to the maths we did at college, is completely different. There was a lot more hands-on activities and fun ... so that's definitely changed and now, teaching, I try to bring in the stuff from college"* (Research Participant 3).

She was compassionate in her approach to teaching and revealed that *"I bring empathy into a lot, like if I know that I battled with it, I'm a lot more relaxed with the concept ..."* (Research Participant 3). Although she tried to link her lessons to everyday life to help the pupils see the value of Mathematics, she admitted that it could be a struggle

in that *“they know they have to do maths and some of them genuinely enjoy maths, but most of them are very apathetic...So it’s a bit frustrating when you’re going to all the effort of trying to teach them and they actually don’t care”* (Research Participant 3). However she put this sense of apathy down to their age.

With regard to her class, teaching for proficiency meant that although *“normally maths is supposed to be numbers and not language but, um, agh, lots of hands-on. You have to be very hands-on with maths, within my grade, Grade 6 ...um, but more um, very hands-on in group work. You can’t just stand in front of a class and just do a sum any more ...it has to be ... things, you know, like concrete”* (Research Participant 3). She indicated that *“It’s very difficult to teach second language learners”* (Research Participant 3), which resulted in her using manipulatives and resources *“and that, I find, helps them a lot because they’re enjoying it as well, the hands-on activity is a lot more enjoyable than me just going and teaching... So that, I think, like just helps them a little bit more. But I still find it’s very difficult to teach them”* (Research Participant 3). Conceptual understanding is more difficult to gain if there is a language barrier to take into consideration. Her advice to pre-service teachers is that, in order to be proficient teachers, they need to plan in advance and *“you have to try and get a balance of teaching and fun for them to understand. I mean then they grasp the concepts a lot easier when it’s to their level ...and when it’s interesting and excites them, then it’s a lot easier”* (Research Participant 3).

Embodiment

Given the language barrier, there was considerable focus on conceptual understanding and working within a community in the form of group work. She asserted that *“you can’t just do maths and teach and get on with it and I find I’m stuck on a subject for weeks in advance, like I have to go on and go over and go over because, to move on, I’d rather them know, you know, five things as opposed to not know 25 things, because they – I don’t know what, the maths for them is very difficult...I think it is a language barrier because, especially when it comes to word problems. I mean obviously if they don’t understand the language, the word problems are going to be a problem ...and then um, but the maths, I don’t know because of the ANA¹⁵ exams, we have to teach so many different ways of adding ...and I think that also gets confusing”* (Research Participant 3). In this

¹⁵ Annual National Assessments - South African National Assessments designed to track the Literacy and Numeracy performance of learners annually.

regard she had tried to establish a mathematics community that encouraged a community of practice, as was evident in the comment *“also I say as soon as they come into my class, this is a team, so if someone – one person doesn’t understand then everyone has to like stop and help, unless its going on too long ...”* (Research Participant 3). She also taught problem solving, *“I try and encourage problem-solving, I mean group work and hands-on. I mean I think it helps them a lot more than anything else does”* (Research Participant 3).

The teachers had also formed a mathematical community in which they helped one another in the sense-making process, which she felt was similar to what she had experienced in the mathematics module (MIP 400). At the weekly meetings *“so then we would sit down and suggest different approaches, different things and we would obviously discuss the kids in our class who are battling and then get ideas from each other to see how we can help them. We do that every week on a Wednesday – we have an hour ...But often we will swap and go to each other and say listen, we’re not understanding how to get this through to the class, do you have ideas and – because we’re very supportive and we help each other a lot, so it’s very helpful... We often discuss things and then give each other ideas and so that – it’s very helpful. I mean I couldn’t imagine teaching with four other people, three other people and not being able to go to them for advice”* (Research Participant 3).

Emergence

There was evidence of emergence and productive disposition in that she had reflected on her pupils’ abilities and challenges and recognised that *“they don’t have the confidence in themselves to do the maths, they get so despondent ...and every time I do a test, if they’re not doing well, then it just brings them further and further down, and then that anxiety will never go away... So it will continue with them for the rest of their lives”* (Research Participant 3). This was further indicated in her comment, *“so if they have like a maths anxiety then I will try and make it more fun for them, or I’ll make them come up and do things, because a lot of them, they like to be the centre of (attention) they like to come up and do it and even if they are shy and they’re nervous and anxiety – like I said, most of the learners in the class are very ...like ... accepting”* (Research Participant 3).

Experience

Her teaching experience was very different from both her years at school as a pupil which took the form of rote learning and her own teaching practical when there was time to do extra things. This participant appeared to be the most creative in her approach to developing conceptual understanding. This impacted on her timeframes with regard to the CAPS DBE (2010) expectations, which then resulted in teacher directed lessons with the occasional use of acronyms to assist with the procedures that pupils needed to know. There was evidence of this procedural teaching, for example *“when changing it from a mixed number to an improper number, I’ll say to them they must take the bottom number times by the top number, but I’ll make it like a little story for them and they’ll – some of them never forget but most of them do”* (Research Participant 3). However, her main approach seems to be group work and manipulatives. Hence it appeared that this participant tried to take a learner centred approach but when under pressure resorted to a more teacher directed approach. *“It’s very unfortunate, but you know, there’s just so much I can do”* (Research Participant 3). She tended to use the government textbook for homework and consolidation as it encompassed mainly one word answers and was repetitive in nature.

She motivated the pupils through games and rewards since *“they love that, and I mean they want to do it more because there’s like almost like a pride there...So that’s – for them there always has to be something to gain out of it...the learning isn’t the gaining for them anymore, it’s like the treat at the end, so ...Oh, either extra break or when we go to PE they can play soccer for ten minutes extra, that kind of stuff, its more – not necessarily sweets... Reply - But, ja, just extra time for their fun, soccer or extra break or whatever, or colouring in – some of them, I mean a lot of them are so talented with drawing”* (Research Participant 3). Her approach was *“at least once a week to recap with those games at the end of activities, ..., now with this age group and this language barrier you have to have extra manipulatives and like make fun games but often there’s not enough time to do all that, unfortunately”* (Research Participant 3). Her teaching approach also includes games, quizzes, board work and using a variety of approaches based on the concepts being taught. *“I try and use hands-on things as much as I can but obviously some days there’s just no time”* (Research Participant 3). Games are also used to consolidate work.

With regard to her teaching approach, the language barrier made teaching for proficiency difficult. However she resorted to a number of different strategies to help the pupils understand, often linking her strategy to something they could relate to and understand, for example soccer. She got the pupils to come up to the chalk board and explain their work and *“just for them also to gain the confidence and if they don’t understand then someone will come and help them on the board and so they’re not left there ...by themselves because that’s a bit daunting. Some of them are not very confident in themselves, let alone in maths, so they get a bit scared to come up but then – I mean we’re quite supportive in the class. A lot of them help them, so we just try and get as much as we can together. I don’t like let - I don’t like to let them just do their own thing”* (Research Participant 3). This is the same practice that was used in the mathematics module (MIP 400) to develop confidence.

Sense-making

She encouraged discussion which helped develop adaptive reasoning and strategic competence, for example *“we do a lot of reasoning in the class and because there’s so many different ways now with the three different topics of maths, uh, addition, subtraction and then when we discuss them, I always say to them why are we doing this, why would you choose to do it this way, is it easier for you and there’s a lot of discussion...”* (Research Participant 3). However, the issue of second language meant that *“often, I don’t understand what they’re saying because they’re not able to express themselves correctly. I think it’s maybe the language or they just – they understand and yet they’re not able to bring it across the way they’re understanding it... So I’m very – I think I’m very patient with them. Um, obviously if it comes to a time when its taking up too much of the lesson, I’ll say just hang on to the question and come back to me or write it down and drop it on my desk and I’ll look through it and come back to you the next day. So it just depends, but – it depends on the child”* (Research Participant 3).

4.4.5.2 Curriculum Assessment Policy Statements (CAPS) DBE (2010)

This participant also raised the impact that the CAPS DBE (2010) document with its deadlines and timeframes had on the teaching time needed to develop conceptual understanding. She explained *“So I try. It’s very difficult that our tongue is such a huge*

factor ...that I, sometimes I can't do it and I can see them getting more and more despondent but there's nothing that I can do, because there's so much that you have to fit in within a term ..." (Research Participant 3). She then discussed the pressure of the CAPS DBE (2010) document, "It's very strict. You have to have a certain amount of hours per week and – actually it's a little unrealistic because there's so much to do ..." (Research Participant 3). In an effort to combat this, the pupils are divided into four mathematics classes, an A group and three mixed ability groups. Her opinion of the CAPS DBE (2010) document was that "I find it very unrealistic, there's way too much" (Research Participant 3) and it impacted on teaching for proficiency in that, "And then it becomes difficult because then you're almost like just teaching something ... , just teaching to get it done, which is obviously not what you – where you want to be. But I mean the CAPS document doesn't take into account the interruptions you're going to have in the lessons and the fact that some pupils aren't going to understand, that there are behaviour problems" (Research Participant 3).

The expectations and preparation of the pupils for ANA also impacted on her teaching approach "because that's just what they understood and with um, with the ANA exam there's only two other ways and they understand the one way they've done, so I have to focus on those two, so I suppose maybe close to the end of the year if there's time I could teach them that, but to teach them like so many concepts when they've already got to learn two more than they already know is a bit much" (Research Participant 3). "So we do base quite a lot of the maths on ANA, which we probably should – not that we should but there's so much else that they need to know, but when it comes close to the exams, ja, we do focus a lot on the ANA exams" (Research Participant 3).

4.4.5.3 Impact of mathematics module (MIP 400)

She noted that the mathematics module (MIP 400) had played a role in her teaching approach, "... because of my school, when I was in school, the way it was just teacher-directed, and then when we were in fourth year it was very So to have all the different types of people that we had in fourth year, some were like creative, others, you know, there were just so many different types of people in that year, to have them all do different ways, it was definitely a lot and it was great . . . and just take it all in and do – like get different ideas of how you can teach one concept. I mean, like, you know what I mean, there were like five

different groups there but one concept but so completely different and the different takes, and then you would think okay well that might work for this learner, but this would be better for that child didn't have that I probably would still be doing like teacher-directing, almost, so that helped me a lot and also because of my experience from school, being so anti-maths, not anti-maths but um, not very confidence in maths ...to coming here and doing it in a group and being able to discuss it and talk about it and then doing different strategies and also looking at other people's different strategies, it gave you a better understanding and a lot more ideas, so I took that into the classroom because I knew that if this is how I was . . . I mean at least half of them must feel that way because I mean, unfortunately, there were a lot of us that didn't weren't too excited about maths" (Research Participant 3).

It also developed her personal proficiency as was evident in the comment *"jeepers that makes a lot of sense and it's a lot more – I understand it – often, when they were doing their lessons I mean I understood them far better than for the last four years of my high school years" (Research Participant 3).* The module helped to prepare her teaching approach in that, *"I found that box of manipulatives was very good and then just the way I now approach maths, because of my experience in fourth year... That has helped, helped me become more confident, and helped me have a better understanding of how they might be understanding it, so I think that is, that – I brought that into consideration a lot of times" (Research Participant 3).*

Regarding her identity, the module developed and introduced her to more strategies for teaching different concepts, *"So to have all the different types of people that we had in fourth year, some were like creative, others, you know, there were just so many different types of people in that year, to have them all do different ways, it was definitely a lot and it was great . . . and just take it all in and do – like get different ideas of how you can teach one concept. I mean, like, you know what I mean, there were like five different groups there but one concept but so completely different and the different takes, and then you would think okay well that might work for this learner" (Research Participant 3).* She then went on to say, *"If I didn't have that module I think I would, honestly, I'd be like teacher-directed lesson, definitely. I mean obviously I would bring in other concepts, but it just like opened me up to a whole different way of teaching... depending on who it was teaching, but most of them were actually very good. .. that maths module was great. Like I'm very grateful for that. It's, like I said it opened up for so many different ways of teaching. Like you become so set in your ways, even – because of the way you've been taught, you become set in your ways and saying that's how I was taught so I'm going to do it, just a continuation of that, but being*

– seeing all the other different ways, being exposed to different people and the dynamics and the way they teach just gives you so many more ideas” (Research Participant 3).

4.4.6 Reflection on Research Participant Three through the lens of the five strands of teaching for proficiency

4.4.6.1 Conceptual Understanding

There was a suggestion of growth in conceptual knowledge in her discussion about her teaching practice and her confidence in teaching. This was also reflected in her teaching approach and how she integrated the content knowledge that the pupils required into their context and environment, for example soccer. This was based on her understanding and knowledge of the pupils in her class. This participant also used her knowledge of the pupils to adapt her pedagogy in order to make connections with the mathematical content that the pupils needed to know. For example, with language being an area of difficulty, she tried to develop the pupils' confidence by getting them to come up to the board to explain their solutions. However, she is cognisant of the fact that they have difficulty in expressing themselves in their second language and so allows the pupils to come to the board in pairs, for support.

4.4.6.2 Procedural Fluency

There is evidence of procedural fluency since the participant introduces different ideas and instructional approaches, for example, community of practice, hands-on approach, group work, problem solving.

4.4.6.3 Strategic Competence

The participant demonstrated strategic competence in her planning of lessons to include hands on activities to help second language children develop conceptual understanding. This was also evident in her understanding of the children and her effort to find a balance and include something that would excite the children and capture their interest when planning lessons. She demonstrated her strategic

competence in the planning of her instruction and interaction with the pupils to address the issues of apathy and the challenges of second language through the use of manipulative, fun activities and real life examples suitable to the pupils' environments. To deal with the challenges that arose through instructional practices the participant demonstrates strategic competence in setting up solutions, for example, establishing a community of practice. This is further demonstrated through her understanding of the pupils and her use of incentives, such as allowing them free time for drawing, in order to motivate the pupils.

4.4.6.4 Adaptive Reasoning

There was demonstration of adaptive reasoning in her response to the pupils' view of Mathematics, for example apathy in some cases, and her justification as to why she was adopting a particular approach or being empathic. Again she demonstrated adaptive reasoning in recognising the challenges of learning Mathematics in a second language and her choice of using concrete materials to develop conceptual understanding. Her practice of reflecting on the pupils responses to the Mathematics currently taught and the difficulties they experienced showed her adaptive reasoning ability. This was further indicated by the instructional decisions that she took and the justification for this. She decided to focus on a few concepts in depth, in order for the pupils to attain conceptual knowledge and procedural fluency than on a number of concepts and achieving only a surface knowledge.

4.4.6.5 Productive Disposition

She drew on her past experiences at higher education and the mathematics module to improve her practice by understanding the pupils' anxiety and linking this to the teaching practice that she adopted. Through being part of a mathematical community she demonstrated an interest in being in control of her learning and in being part of the sense making community in order to inform instructional approaches that would assist in teaching the curriculum.

What the participant brought forth

Through analysing the data I was able to identify a number of points relating to teaching for mathematical proficiency raised by this participant. Firstly she had recognised the importance of using manipulatives and resources in the development of conceptual understanding, especially with second language learners. The research participant's teaching approach and strategies had been influenced by her experiences in the module, particularly the use of manipulatives. Being exposed to a variety of teaching approaches for the same concept had also helped her to become more learner centred in her approach to teaching. Furthermore, her conceptual understanding had improved as a result of the mathematics module and her observation of her peers teaching different mathematical concepts. Through her knowledge and understanding of the pupils she was able to use incentives to motivate and encourage learning. Finally, in this instance, teaching for proficiency was hampered by the demands of the CAPS DBE (2010) document and ANA requirements.

Problems the participant posed in terms of teaching for proficiency

There were a number of problems that emerged in terms of teaching for proficiency. Firstly there were the challenges of trying to teach and develop conceptual understanding in Mathematics for pupil's learning in their second language and the subsequent time taken to develop conceptual understanding and proficiency. This was compounded by the apathy of some of the children in terms of engaging with the mathematics lesson and how to respond to this situation. The requirements of the ANA exams resulted in time constraints that could have been used to develop a greater depth of conceptual understanding. Finally, the unrealistic expectations of CAPS DBE (2010) were identified as a problem in terms of teaching for mathematical proficiency due to the quantity of content and the timeframes under which one had to teach the content.

It is evident that the themes of enactivism contribute to the growth and development of proficient teaching in pre-service mathematics teacher education and in particular for generalist teachers who are not majoring in Mathematics. The theme of

autonomy and the recognition of her mathematical identity enabled this participant to take what she had learnt as a pre-service teacher, and to respond confidently to the perturbation of having a 75% second language speaking class, in an efficient manner with the inclusion of mathematical communities (embodiment).

A summary of the findings from phase three are presented in table 4.5

Table 4.5

Summary matrix indicating the Strands of Proficiency accentuated by the Themes of Enactivism in Phase Three

	Autonomy	Sense-making	Emergence	Embodiment	Experience
Conceptual Understanding	<p>Participant One No evidence brought forth.</p> <p>Participant Two Know how to use math knowledge and knowledge of students effectively in the context of their work.</p> <p>Participant Three No evidence brought forth.</p>	<p>Participant One No evidence brought forth.</p> <p>Participant Two Knowledge of how students' mathematical understanding develops.</p> <p>Participant Three No evidence brought forth.</p>	<p>Participant One A knowledge of how students' mathematical understanding develops.</p> <p>Participant Two No evidence brought forth.</p> <p>Participant Three Use math knowledge and knowledge of students effectively in the context of their work.</p>	<p>Participant One No evidence brought forth.</p> <p>Participant Two No evidence brought forth.</p> <p>Participant Three Uses community of practice to make connections between knowledge of mathematics, students and pedagogy.</p>	<p>Participant One Increased conceptual understanding.</p> <p>Participant Two Range of pedagogical practices that take into account the mathematics being taught and how students learn it.</p> <p>Participant Three Connections within and among their knowledge of mathematics, students, and pedagogy. Repertoire of pedagogical practices. Use math knowledge and knowledge of students effectively in the context of their work</p>
Procedural Fluency	<p>Participant One Revert to procedural teacher centred lesson when conceptual understanding was limited.</p> <p>Participant Two No evidence brought forth.</p> <p>Participant Three No evidence brought forth.</p>	<p>Participant One Being open to help and explanations from the learners.</p> <p>Participant Two Grounded in mathematical activity e.g. How to respond to a student who gives an answer the teacher does not understand or who demonstrates a misconception.</p> <p>Participant Three No evidence brought forth.</p>	<p>Participant One No evidence brought forth.</p> <p>Participant Two No evidence brought forth.</p> <p>Participant Three No evidence brought forth.</p>	<p>Participant One No evidence brought forth.</p> <p>Participant Two Routines regarding classroom management and grounded in mathematical activity.</p> <p>Participant Three Community of practice encouraging the flexibility in a range of strategies and routines.</p>	<p>Participant One More teacher centred lessons and procedural in nature – less emphasis on conceptual understanding.</p> <p>Participant Two No evidence brought forth.</p> <p>Participant Three Several ways of approaching teaching problems.</p>

	Autonomy	Sense-making	Emergence	Embodiment	Experience
Strategic Competence	<p>Participant One No evidence brought forth.</p> <p>Participant Two Teaching is a problem- solving activity. Planned instruction, implementing those plans, and interacting with students.</p> <p>Participant Three She is able to determine the best approach to teach the appropriate material to the pupils.</p>	<p>Participant One Being open to help and explanations from the learners.</p> <p>Participant Two Effective in solving instructional problems.</p> <p>Participant Three Effective in solving instructional problems. Interacts with students. Find out what a student knows.</p>	<p>Participant One No evidence brought forth.</p> <p>Participant Two Figuring out what to teach when, how to teach it, how to adapt material so that it is appropriate.</p> <p>Participant Three No evidence brought forth.</p>	<p>Participant One No evidence brought forth.</p> <p>Participant Two No evidence brought forth.</p> <p>Participant Three Problem solving as an approach to teaching.</p>	<p>Participant One Some evidence of this.</p> <p>Participant Two Decisions in planning instruction, implementing those plans, and interacting with students.</p> <p>Participant Three Adapt material so that it is appropriate for learners. Taken into account how her pupils learn best.</p>
Adaptive Reasoning	<p>Participant One No evidence brought forth.</p> <p>Participant Two Conceptions and misconceptions that students had about the mathematics.</p> <p>Participant Three Recognises difficulties the pupils have and how they respond to activities.</p>	<p>Participant One Being open to help and explanations from the learners.</p> <p>Participant Two How the students responded to particular representations, questions, and activities.</p> <p>Participant Three Use representations that are most effective in communicating essential ideas. How the students responded to particular questions and activities.</p>	<p>Participant One Reflects on learner's current understanding when planning.</p> <p>Participant Two No evidence brought forth.</p> <p>Participant Three Analyse difficulties their students have encountered. Analyse how students responded to particular activities and tasks.</p>	<p>Participant One No evidence brought forth.</p> <p>Participant Two Justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them.</p> <p>Participant Three Able to justify her rationale for her teaching approach. Reflective about her teaching strategy with regard to the pupils</p>	<p>Participant One Some evidence of adaptive reasoning.</p> <p>Participant Two Being a reflective practitioner.</p> <p>Participant Three A repertoire of instructional routines to draw upon. Classroom management. Apply a range of routines flexibly.</p>
Productive Disposition	<p>Participant One Anxious with low self-efficacy. Little confidence.</p> <p>Participant Two Great sense of responsibly for the learners' success and mathematical understanding. Productive disposition toward mathematics, teaching, learning and the improvement of practice. Analysed what goes on in her classes.</p> <p>Participant Three Positive attitude towards the teaching of mathematics, which has influenced her approach to teaching. Confident.</p>	<p>Participant One No evidence brought forth.</p> <p>Participant Two Learn by listening to their students.</p> <p>Participant Three Learn by listening to their students.</p>	<p>Participant One No evidence brought forth.</p> <p>Participant Two No evidence brought forth.</p> <p>Participant Three Productive disposition toward mathematics teaching. Analyse what goes on in their classes. Learn by listening to their students.</p>	<p>Participant One Learnt by listening to colleagues and occasionally pupils. Learnt mathematical concepts and strategies through interaction with pupils.</p> <p>Participant Two No evidence brought forth.</p> <p>Participant Three Lifelong learner. In control of her learning through participation in mathematical community.</p>	<p>Participant One Increased confidence.</p> <p>Participant Two No evidence brought forth.</p> <p>Participant Three Analysed what went on in her class. Was capable of learning about mathematics, student mathematical thinking, and their own practice themselves ,</p>

4.5 CONCLUSION

In this chapter I described the data collected according to the three phases as it related to the different themes of enactivism, namely autonomy, sense-making, emergence, embodiment and experience. I then analysed this data through the lens of Kilpatrick, et al.'s (2001) five strands of mathematical proficiency in order to determine which themes were perceived to have the most influence on developing proficient teaching. In chapter five I argue that each of the themes played a role in developing proficiency.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

The purpose of my research study was to determine in what way a mathematics module informed by an enactivist philosophy and teaching pedagogy enabled selected pre-service teachers to unpack the reality of their teaching practice in terms of proficient teaching. The eight research participants were final year students undertaking a B.Ed. Foundation Phase degree.

The research was undertaken as a case study which encompassed elements of action research. The first two phases of data were collected over a six month period and the third and final phase of data was collected a year and a half after the students had qualified in order to determine how the mathematics module (MIP 400) had enhanced their teaching practice.

I begin by responding to the sub questions, namely:

- What did the participants of the module bring forth with regard to teaching for mathematical proficiency?
- What problems did the participants pose with regard to teaching for mathematical proficiency?
- What evidence of growth and development of proficient teaching in Mathematics emerged?
- What evidence of growth and development of personal professional growth and proficiency emerged?

I conclude with the overarching research question.

5.1 SUMMARY OF FINDINGS

5.1.1 What the participants of the module brought forth with regard to teaching for mathematical proficiency

We bring forth our world, as we experience it, through structural coupling and in co-existence with others at a given moment. Knowledge is recognised as effective or

appropriate behaviour in a particular context (Maturana & Varela, 1998, p. 174). With this in mind, I discuss what the participants revealed with regard to teaching for mathematical proficiency over the three phases.

After analysing the *initial questionnaire* the research participants brought forth the following in terms of the reality of their teaching practice in terms of proficient teaching. One theme of enactivism that became apparent was that of autonomy. The students admitted that conceptual understanding and strategic competence were the most difficult strands to address in their teaching. It also emerged that there were degrees of self-efficacy regarding their personal mathematical ability and their capacity to teach it proficiently. Having not heard of Kilpatrick, et al.'s (2001) five strands of mathematical proficiency, students tended to understand these to mean a more traditional approach to teaching based on conceptual understanding and procedural fluency.

The other theme of enactivism that emerged was experience. The students were in agreement that their past mathematical experiences had affected the way in which they unpacked their mathematical identity and their ability to teach proficiently. As to be expected, good experiences and success in Mathematics resulted in a confident and positive mathematical identity. In contrast, negative experiences or being unsuccessful at Mathematics resulted in a mathematical identity that encompassed uncertainty, a little anxiety and doubt. In spite of this students exhibited a sense of purpose to change the status quo.

Although *Phase One* was only two weeks in length, observation of the research participants' teaching, in addition to the focus group interview, presented a number of points relating to each of the themes of enactivism.

The theme of autonomy highlighted how mathematical histories and experiences had influenced the research participants' mathematical identity. Although they knew from past experience what qualities they believed a teacher should have for proficiency, their lessons tended to be mainly teacher-centred, focusing on teaching a mathematical procedure in a more routine approach rather than engaging the learners in finding solutions. This was in line with their early mathematical

experience. After listening to feedback and through observing lessons, the participants said they had increased confidence in teaching certain concepts. An additional outcome would have been the subsequent growth in their personal mathematical understanding as they had to become accountable for their own conceptual understanding in case they were required to teach during the tutorial session.

Sense-making was apparent, through the co-emergence between the different students, as they engaged in conversation with one another to unpack the task at hand and agree on a teaching approach and strategy.

The reflective tasks at the conclusion of the lessons revealed the theme of emergence regarding their personal and teaching proficiency, giving the participants an insight into what was still required to improve and develop their mathematical proficiency.

Working in a small group (mathematical community) to complete the given task gave rise to the theme of embodiment. Through interacting with one another, the small mathematics community influenced productive disposition in terms of confidence. This was because they were all in agreement on the approach to be used and that the necessary support was at hand. Students were also more comfortable and confident with the conceptual understanding and procedural fluency strands of proficiency with regard to their teaching practice.

The theme of experience, in the form of practical teaching tutorials, provided students with the space to link the theory of mathematics with mathematical concepts and pedagogy when it came to teaching for mathematical proficiency. Moreover, it resulted in students being introduced to different teaching strategies and approaches. The practical teaching tutorials were considered an unthreatening environment that encouraged creativity and experimentation in teaching approaches. It also provided the freedom to make mistakes. The fact that they received more feedback with regard to their teaching practice than in formal teaching practice experience was considered helpful and constructive.

On completion of *Phase Two*, which encompassed four two week sessions, a number of points pertaining to the five themes of enactivism emerged.

The first notable theme was that of embodiment. The participants observed that working in a pre-selected group of students as part of a mathematical community was important and beneficial. One benefit was the easing of the workload as a consequence of working with their peers and, another was enhanced conceptual understanding. Moreover, it resulted in the development of a repertoire of strategies that the students understood and the recognition that there were a variety of ways to approach the teaching of problems and still arrive at the same answer. Initially, students found it difficult to understand the strands and how they translated to practical application but through practice and observation of their peer's lessons they became more fluent in addressing the strands.

Relating to the theme of autonomy, the need for accountability and control of one's learning by responding to a perturbation was noted. Traits that were considered to be linked to proficient teachers started to emerge. These were patience, flexibility and taking into account the differences of each learner when explaining a concept. Participants also identified the need for a teacher to both encourage and promote the use of invented strategies and to link Mathematics to everyday life as important. In considering the individual structures, namely the research participants' mathematical identities, they expressed the feeling that they were more proficient and less anxious about teaching Mathematics in the Intermediate Phase. This is attributed to the fact that there had been growth in their conceptual understanding and procedural fluency. There was an increase in the belief that they, the participants, could teach Mathematics and had a greater awareness of the measures they needed to take in order to develop proficient teaching.

Reflecting on the participants' developing teaching practice, a number of points were revealed. In addition to an increase in the creativity of the lessons planned, there was also growth in their repertoire of teaching strategies to address the different qualities and characteristics of the five strands of mathematical proficiency. The participants started to use invented strategies more regularly as a means of developing strategic competence and adaptive reasoning, by allowing the learners'

time to work out the problems for themselves “*in order for meaningful understanding to take place*” (Research Participant 7, September 2011). The use of resources and real-life objects to make mathematics concepts clearer was viewed as beneficial and as a means of enriching their teaching practice. Finally, the point was raised that teaching for proficiency and successful learning required planning and preparation on the part of the teacher.

Upon reflection and through the theme of emergence, the participants noted an improvement in their teaching practice as well as an improvement in their confidence and capacity to teach Mathematics. They also disclosed that it was more challenging trying to teach the concepts than originally anticipated and noted the need for additional research in order to develop proficiency. The value of linking Mathematics to the real world was recognised as indicated by Research Participant 6 (September 2011) “*I have come to realise how possible and important it is to relate all aspects of Mathematics to the real world, in terms of teaching aids and examples used to teach concepts*”.

In *Phase Three* of the study the three participants, now qualified as B. Ed Foundation Phase teachers and employed to teach in the Intermediate Phase, acknowledged the following aspects relating to their mathematical teaching practice. From the perspective of the embodiment theme the participant who was least confident in teaching Mathematics had noted the value of working in different mathematical communities. These were seen to provide support, develop conceptual understanding and increase confidence and self-efficacy. In spite of this being a useful tool for her, she revealed that she had not carried this out in her own teaching practice and classroom to any great extent.

The two more confident participants maintained that from a personal perspective the module had unpacked the notion of mathematical proficiency and their conceptual understanding had improved as a result of the module and observing their peers teaching different mathematical concepts. With regard to their teaching approach and strategies, these had been enriched by their experiences in the module and were incorporated into their teaching practice, indicating its purpose and value in teaching for proficiency. Being exposed to a variety of teaching approaches for the

same concept helped them to become more learner-centred in their approach to teaching, and provided a base from which to draw ideas.

All the participants revealed that they spent additional time outside of lessons assisting their pupils with Mathematics. A further point raised was the importance of using manipulatives and resources in the development of conceptual understanding, especially with second language learners.

From the perspective of autonomy, the two more confident participants presented as more proficient teachers who had knowledge and understanding of their pupils and the self-assurance to meet these needs. The third student was aware of her areas of difficulty and was, where possible, taking steps to address them.

Having described what the students brought forth with regard to teaching for mathematical proficiency, it is evident that this research study has contributed to the future implementation of the theory of enactivism, especially within the South African context. As lecturers of pre-service teachers it is important to take into consideration how the students' mathematics identity impacts on their approach to learning and teaching Mathematics. Reflective tasks and activities should be designed to create self-awareness of weakness and strengths with regard to the five strands of proficiency. Thereafter suitable triggers and perturbations should be presented firstly, to encourage student accountability and secondly so that the lecturer and students can play active and dynamic roles in the learning environment to grow and develop proficient teaching skills. This will contribute towards consolidating a well-developed and resourced foundation in Mathematics encompassed and underpinned by the five strands of proficiency. The development of small mathematics communities should become an essential feature of all mathematics modules. This will allow the development of all strands of proficiency. Students will not necessarily naturally form a community in which they are held accountable for their growth and the development of their teaching skills thus, by making it a compulsory component of a module students will be required to respond to this perturbation. The use of embodiment could be enhanced by the inclusion of case studies to deal with situations that may arise in the classroom environment.

5.1.2 Problems the participants posed with regard to teaching for mathematical proficiency

Our problems emerge through our interaction with the environment (Varela, et al., 1991; Proulx, 2013). The problems are not already there for us to assume, instead we pose or bring forth these questions, issues or problems at a specific moment in our lives, in response to our co-determination with the environment as we try to make meaning of our world. These questions/problems/issues that are brought forth are triggered by the environment and thus are contextually specific and relevant to the person in question. In this research study, 'the person in question' refers to the students and research participants registered for the mathematics module. Proulx (2013, p. 315) states that: "We interpret events as issues to address; we see them as problems to solve". This then required an embodied action in order to find solutions as they arose since it was relevant to the students and research participants in this research study.

The problems that were identified at the outset of the mathematics module in the *initial questionnaire* fell mainly under the autonomy theme of enactivism. Within the theme of autonomy and relating to the students' mathematical identities and productive disposition, it emerged that some students were anxious due to a lack of confidence to teach Mathematics in the Intermediate Phase and being out of practice with the subject. Teaching their peers was also an area of apprehension, in addition to catching up on mathematical concepts that were missed at school.

From a teaching perspective there was concern about a lack of teaching strategies in addition to coming "*down to a child's level of thinking as although she knows the answer and how she would arrive at the answer she does not know how she would explain the concept*" (Research Participant 2, April 2011). A further concern was "*trying to teach something that has already been taught, as the learners do it one way but you are trying to do it another way*" (Research Participant 5, April 2011). A few students indicated that they were worried about solving problems and "*explaining problems to learners who really don't understand what is going on!*" (Research Participant 7, April 2011).

On completion of *Phase One*, it was felt that conceptual understanding needed to be strengthened. In the practical tutorial experience the students had difficulty in addressing the productive disposition, adaptive reasoning and strategic competence strands when teaching for proficiency. They also found difficulty in translating their ideas into teaching practice and often the practical sessions tended to be more teacher-centred and procedural in nature. Lastly, the students found teaching their peers created an element of anxiety as it seemed as though they were being judged.

At the conclusion of *Phase Two* the participants were more confident to teach their peers and try new ideas and approaches. This was evident given that fewer problems and issues were identified as triggers that they needed to respond to in order to adapt to the environment that had emerged from the mathematics module.

In the third and *Final Phase* of the research study a new set of problems emerged as the participants had moved on to a new environment/world, that of a qualified teacher. The participants were now responsible for the development of their own smaller mathematical community, namely the pupils in their classroom as well as their co-emergence with fellow mathematical teachers within their school environment. The key problem that emerged from all three research participants who were currently teaching in the Intermediate Phase, albeit in different contexts, was that of the CAPS DBE (2010) document. All the participants found that adhering strictly to the CAPS DBE (2010) document time frames impacted on their ability to teach for mathematical proficiency as they did not have the time to include a more learner centred approach in their teaching, which would encourage the development of conceptual understanding.

For the less confident participant, CAPS DBE (2010) together with ANA and administrative work affected her ability to further her personal proficiency in Mathematics and thus her teaching for mathematical proficiency. Another research participant agreed that the time taken in preparation for and meeting the requirements of the ANA could be better utilised to develop a greater depth of conceptual understanding amongst her pupils.

One participant, within her particular context, faced the issue of varying levels in her pupils' abilities, in addition to some pupils' preference for a more procedural approach to Mathematics rather than conceptual understanding. Teaching pupils who were learning Mathematics in their second language was another specific problem/issue. Another problem that emerged for this participant was how to respond to the apathy displayed by some of her pupils in terms of engaging with the mathematics lesson and how she, as the teacher, should respond to this situation.

Table 5.1 comprises a summary of the data gathered relating to the growth and development of proficiency from the outset of the module to phase three. I use this to draw from when responding to research sub – question 5.1.3 and 5.1.4.

A number of key problems were posed over the three phases that encompassed the research study. These were mathematics anxiety and confidence, lack of teaching strategies, conceptual understanding, translating idea into practice, dealing with curriculum documentation and requirements in addition to administrative work.

These are real issues that need to be included in and addressed through the mathematics module. Some of these issues were starting to be addressed in the module. As lecturers of Mathematics, particularly where students are required to take mathematics modules it would be advisable to do diagnostic tests of personal proficiency to gauge where additional tutorials and development of skills are needed in order to meet a minimum standard of personal proficiency. As this study has shown, this will impact on anxiety and confidence as proficiency develops. There is a need to include more micro teaching opportunities and case studies to develop and have exposure to alternative teaching strategies and practices and to encourage debate and discussion as to the best strategies to deal with context specific difficulties.

This research study has shown that the themes of enactivism, with input from both the lecturer and students, have the capacity to address the problems posed by a cohort of students, over a period of time. The lecturer and students' feedback provides the direction as to the focus of the triggers and perturbations in order to address individual needs.

Table 5.1

Table indicating the perceived development of observed growth in the various strands of teaching for mathematical proficiency.

Strands of proficiency	Phase One	Phase Two	Phase Three
Conceptual Understanding	Limited repertoire of teaching strategies.	Making connections between own understanding and requirements of the task in addition to determining a teaching strategy – sometimes needed support. Resources and manipulatives for teaching beneficial to their personal conceptual understanding. Teaching a concept proficiently can be challenging. Learners think in different ways and that they need to know how to approach this. Gained knowledge on how to teach different math concepts using different strategies.	At risk due to demands on time by CAPS DBE (2010) and ANA. This increased through participation in a Mathematical community. The better the conceptual understanding the greater the variety of teaching strategies. Knowledge of her pupils' abilities and what would work best within her classroom context. This has started to underpin the teaching approach resulting in the integration of a number of aspects, e.g. context, experiences the learner has brought with him/her.
Procedural Fluency	Unclear instructions. Teaching strategy procedural in nature. Teacher centred.	Development of a repertoire of strategies that they had understood. The value of explaining a concept linked to everyday life. More flexible in their teaching approach and strategies.	Fear of Mathematics and weaker conceptual understanding resulted in more procedural teacher-centred approach to teaching Mathematics. Good classroom management and flexibility regarding instructional practices. An increase in the variety of strategies that are implemented.
Strategic Competence	Tried to anticipate difficulties learners may have experienced.	Benefits of using invented strategies as a means of developing strategic competence.	Encouraged pupil participation but may not have picked up errors/misconceptions. Learner-centred problem solving encouraging active participation. Reflective thought during her planning in order to best meet the needs of the pupils. Effective in addressing instructional challenges. Considers pupils backgrounds and mathematical knowledge in her planning.
Adaptive Reasoning	Indication of this in reflective tasks and during tutorial discussions. Not required of the learners during the lessons taught.	Benefits of using invented strategies as a means of developing adaptive reasoning. Justify and explain a teaching strategy. Justify and substantiate feedback.	Encouraged the sharing and justification of methods. Reflection on practice to make instructional practice more purposeful. The justification and explanation of instructional decisions. Understanding of how the pupils responded to certain activities. Understanding of the difficulties that some of the pupils experienced.
Productive Disposition	Not addressed.	Teaching a task had resulted in an improvement of teaching and in developing an integrated practice. Need to be in control of their own learning. More confident to teach in the Intermediate Phase.	Not a key focus (Participant 1) Analysis of teaching practice. Lifelong learner. Productive disposition toward mathematics, teaching, learning and the improvement of practice. Analysed what goes on in her class. Learn mathematical concepts and strategies from their interactions with colleagues and students. Lifelong learners who can learn from studying curriculum material. In control of their own learning.

5.1.3 Evidence of growth and development of proficient teaching in Mathematics that emerged

Enactivism talks of autonomy as being representative of the composition of a person's structure, over which one has total control due to structural determination. In this study, I considered this from the perspective of how the participants constructed their mathematical identity with regard to their personal mathematics proficiency and their ability to teach mathematics proficiently. I also took into account their self-efficacy relating to Mathematics and finally their ways of knowing. In responding to this question I report on the growth and development of their teaching mathematics proficiently and in 5.1.4 I discuss the perceived growth in their personal mathematics ability.

At the outset of the module the students were confident about their ability to teach Mathematics proficiently, although they had doubts about conceptual understanding and strategic competence. This was most likely due to the fact that their experience to date consisted mostly of a teacher-centred approach. After the two week *Phase One* period, the participants used a teacher-centred approach when teaching their tasks in the practical tutorial sessions. Participants were more comfortable focusing their teaching on the conceptual understanding and procedural fluency strands of proficiency. This was evident as all the students unpacked the mathematical concept in a more procedural manner focusing on the concept and the steps to be followed. During the eight week *Phase Two* sessions, evidence in growth and development of proficient teaching in Mathematics emerged. From the perspective of their autonomy the participants acknowledged that they were feeling more proficient at teaching Mathematics to the Intermediate Phase and less anxious about teaching.

5.1.3.1 Conceptual Understanding

In the context of teaching, Kilpatrick, et al. (2001) suggest that teaching for proficiency requires that a teacher has a conceptual understanding of the core knowledge required in the practice of teaching.

Within the theme of *autonomy* the growth and development of proficient teaching began as a more teacher-centred approach focusing on only two strands, namely conceptual understanding and procedural fluency. This progressed in phase two to taking into account the Mathematics being taught and how students learn it. In phase three, as practising teachers, the more confident participants in particular started to embody a conceptual understanding of the core knowledge required in the teaching of Mathematics. In addition, they started to use their mathematical knowledge in conjunction with their knowledge of the pupils more effectively in the context of their classroom. Only in the third phase was there greater evidence of the *sense-making* theme as participants revealed a more embodied knowledge of how students' mathematical understanding develops. This is plausible given that in the first two phases they were teaching their peers who would have had a better conceptual understanding and more experience than the pupils in their classes. There was no evidence of *emergence* in phase one. However, in phase two the participants started to demonstrate conceptual understanding of the core knowledge that they were required to teach, making connections within their knowledge of mathematics and pedagogy and developing and linking this to their knowledge of how students' mathematical understanding develops. In addition, the participants started to use different strategies and teaching approaches. By phase three the participants had a more practical knowledge of how students' mathematical understanding develops and were structuring their teaching approach to adapt more effectively and efficiently to their specific contexts.

In phase one the focus of *embodiment* centred on developing their personal conceptual understanding in order to be able to teach their peers. In phase two this progressed from being a more personal focus to a pedagogical approach that encompassed different strategies through the sharing of ideas and experiences within the mathematics community. As practising teachers in phase three, the *embodiment* theme came to the fore as the mathematical community now encompassed their fellow colleagues and/or the pupils, as a means of support to embody teaching practices that would develop conceptual understanding.

Initially the theme of *experience* saw participants in phase one try to link the theory and their conceptual understanding of the concepts covered in the lecture into

practice during the practical tutorial sessions. It was in this phase that they were introduced to different approaches and strategies as they and their peers discussed how they had solved the same problem. This grew and developed over the phases until in the third phase the participants were able to incorporate this into their classroom practice using different context-specific strategies.

From the perspective of developing proficiency in teaching for conceptual understanding the themes of *experience*, *embodiment* and *emergence* contributed the most meaningfully. Having to prepare lessons to teach to their peers on a regular basis meant that the students had to participate in their mathematical communities, namely the group that they had been assigned to. This encouraged interaction and a sharing of ideas and experiences in order to understand the task, in the event that they were asked to teach.

5.1.3.2 Procedural Fluency

Through the lens of being procedurally fluent, Kilpatrick, et al. (2001) indicate that this type of teacher would exhibit fluency in carrying out basic instructional routines. In phase one the only theme to emerge with regard to procedural fluency was that of *experience*, in which a more procedural and generally a teacher-centred approach was adopted. During the second phase of the study procedural fluency started to emerge within the themes of *autonomy*, *emergence* and *embodiment* as the participants started to develop their fluency in carrying out basic appropriate instructional routines. Within their communities of practice there were various instructional routines to draw from as they interacted with one another in preparation for the teaching of their peers. In phase three, within their classroom environment, from the perspective of *autonomy* the less confident participant would revert to a more procedural teacher-centred lesson when her conceptual understanding was limited.

The strand of *sense-making* saw participants being open to the idea of getting help and explanations from the learners where fluency was not strong or in the case of second language learners getting assistance with explanations. The participants made use of their mathematical communities of practice to encourage procedural

fluency and flexibility in a range of strategies and routines as well as to establish routines regarding classroom management. This was underpinned by the theme of *embodiment*. The theme of *experience* showed that the least confident participant resorted to teacher-centred lessons which were more procedural in nature with less emphasis on conceptual understanding. The other participants were starting to use different ways to approach teaching problems.

Thus, through the lens of Kilpatrick, et al.'s (2001) stands of proficiency, the themes of *sense-making*, *experience* and *embodiment* emerged as contributing the most to the development of proficient teaching.

5.1.3.3 Strategic Competence

A teacher exhibiting strategic competence when teaching Mathematics is able to plan effective instruction and solve problems that may arise during instruction (Kilpatrick, et al., 2001).

In phase one the only theme to emerge that demonstrated some evidence of strategic competence was experience, with the acknowledgement by participants that the practical tutorial sessions were a space that encouraged and allowed creativity and experimentation in their teaching approaches. In phase two evidence of engaging with this approach is noticeable in all themes of enactivism. The participants viewed teaching as a problem-solving activity. In their small mathematical communities they were helping one another figure out how to teach the task, plan effective instruction and consider how to solve problems that might arise during instruction. The participants and students were also effectively solving instructional problems while planning their lessons, as very few groups approached the lecturer for additional help. Phase three saw the emergence of a more embodied teacher.

The participants who had taught for a year and a half, revealed the following traits in line with Kilpatrick's framework for strategic competence. From the perspective of their *autonomy* they all saw the value in implementing a problem-solving approach to their teaching practice, however, the less mathematically confident participant found

it more difficult to implement. All the participants found that the lack of time was an inhibiting factor. In planning their instruction, they were able to determine the best approach to teach the appropriate material to their pupils and then, in most instances, implement those plans. This is evidence of time spent interacting with their pupils both during the formal lesson and informally by means of setting time aside for additional help.

Sense-making underpinned their teaching practice through their interaction with their pupils which enabled them to discover their pupils' prior knowledge. This enabled them to try and solve their instructional problems, for example using games to engage second language learners and consolidate their conceptual understanding and procedural fluency. The fact that the participants were open to explanations and assistance, in the case of the less confident participant from the pupils, meant that they were engaging with their pupils.

The participants showed evidence that the themes of *embodiment*, *emergence* and *experience* underpinned their practice in that they embodied problem solving as an approach to teaching and drew, where possible, from their experience in the mathematics module (MIP 400). This meant that they were making an attempt to link the requirements of CAPS DBE (2010), with their specific strategies and adapt this to be appropriate and in the best interests of the pupils within their particular context.

Of the five themes of enactivism, *sense-making*, *embodiment* and *experience* contributed the most to the growth in strategic competence.

5.1.3.4 Adaptive Reasoning

Teachers who demonstrate adaptive reasoning when they are teaching Mathematics proficiently, are able to justify and explain their instructional practices and reflect on those practices, so as to improve on their teaching approach (Kilpatrick, et al., 2001).

The theme of enactivism that contributed most to the growth and development of adaptive reasoning was that of *embodiment*. This was followed closely by those of *sense-making* and *emergence*. While there was no evidence of adaptive reasoning

during the first phase, in the second phase, encompassing eight weeks, the participants had to justify and explain their instructional practices to the other members in their mathematical community. The students also made use of various representations in order to communicate the essential conceptual ideas when teaching their peers. The establishment of the mathematical communities was underpinned by the theme of embodiment.

Further evidence of this was their reflecting on and analysing their practice as required by the reflection tasks. These reflective tasks were supplemented by their observation of team members' lessons, in addition to feedback from the observation team and peers registered for the mathematics module (MIP 400).

As practising teachers, in phase three *embodiment* was evident during the interviews as participants justified and explained their instructional practices with particular reference to their classroom context. In discussing the rationale for their different approaches it was evident that they had reflected on their strategies and were adapting them when the need arose within their specific context.

The theme of *sense making* was exposed in phase two as the research participants and students had to understand one another's approach to a task in order to decide which approach to use for the practical tutorial session lesson. In phase three the participants promoted pupil participation by encouraging their pupils to explain their strategies and solutions in addition to observing their response to different questions and activities. This resulted in the theme of *emergence* coming to the fore as participants took cognisance of the pupils' understanding at the time, by analysing and reflecting on their responses and the difficulties they were experiencing. This then informed their lesson planning and instructional approach.

5.1.3.5 *Productive Disposition*

Through analysis of the lens of productive disposition toward Mathematics teaching and learning, and the improvement of practice, the theme *autonomy* emerged as having the most influence on teaching for mathematical proficiency. The participants were able to draw from a base of *emergence*, *embodiment* and *experience* from

phase one. In this phase they received feedback on their practice thereby giving them insight into what was required to improve and develop their proficiency. This enabled them to consciously construct their mathematical identities and determine what perturbations or triggers to respond to as presented during phase two.

Phase two resulted in the participants listening to and engaging with their peers and taking control of their own learning to ensure they understood the mathematical concepts to be taught. Their productive disposition toward Mathematics grew as the fortnightly practical tutorial sessions progressed. This resulted in an increase in confidence in teaching the Intermediate Phase. In phase three, teaching the Intermediate Phase and not having the same access to their mathematical community as they had had during the mathematics module (MIP 400) resulted in different teaching experiences. The less confident participant experienced anxiety and low self-efficacy. The other two participants were more confident with a positive attitude towards their competence to teach Mathematics. Moreover, one participant experienced a great sense of responsibility for her pupils' success and mathematical understanding. The more confident participants' approaches to teaching were influenced by their analysis of what was happening in their classrooms and how best to address these needs.

The third sub question looked at the growth and development of proficient teaching over the three phases. This study unravelled the five intertwined themes of enactivism and investigated the influence that each theme contributed to the development of proficient teaching. Allocating relevant tasks to each theme provided continuous information and a better understanding of the problems that pre-service teachers experience in trying to qualify as teachers as they related to each strand of proficient teaching. The constant feedback of information be it positive or negative, enabled both the students and lecturer to play active roles in responding to and addressing problems posed in the teaching and learning programme. This is significant for teacher education in that the lecturer is able to make the content of the module relevant to individuals in the programme at the same time as dealing with the difficulties normally associated with large classes by encouraging students to take on the role of 'teacher' within their smaller mathematics communities. Having to put theory into practice either when teaching a lesson or to their peers, within their

community, gives students a better understanding of the practicalities of teaching for proficiency in addition to providing experience in deciding on appropriate triggers to help their peers and/or pupils.

5.1.4 Evidence of growth and development of personal professional growth and proficiency that emerged

An aspect of professional growth and proficiency that emerged was that, in addition to the mathematics module (MIP 400) supporting participants in linking theory to teaching practice, it also helped them to consolidate their understanding of the mathematical concepts that were unpacked during the lecture. Research Participant 7 (September 2011) states that, *“While working through the textbook together during lectures, we gain knowledge about how to teach concepts in the classroom. Beneficial to work on concepts during lectures to better understand content – discussing how would teach in different ways – work through any confusion may have”*. The module also *“... allow(ed) the learners the opportunity to come up with their own methods. This helps them to understand the work better”* (Research Participant 4, September 2011), as well as introducing the participants to different strategies and teaching approaches. Research Participant 2 (September 2011) indicated that, *“Our math lectures are a great help in finding different strategies to do sums or calculations and when a ‘learner’ in the tutorials comes up with a different way to solve a problem I am not thrown in the deep end and unable to comment or praise”*.

The last session required students to reflect on and identify what they had learnt about themselves during the practical tutorial sessions with regard to teaching Mathematics. There was evidence of growth in their productive disposition both personally and professionally. Research Participant 1 (September 2011) indicated that, *“I have learnt that I am capable of teaching maths (even though it was quite scary teaching to your peers! I still, however, don’t feel too confident but this will come with more practice”*. Research Participant 6 (September 2011) had a similar experience, acknowledging that, *“At first, I was not completely confident in my ability to teach mathematics, especially Intermediate Phase mathematics, however, I now not only feel more confident teaching mathematics, but I have also realised that I prefer and feel better suited too Intermediate Phase teaching, as opposed to Foundation Phase!”*

The mathematics module (MIP 400) also revealed characteristics about themselves that they had not been aware of previously. For example, Research Participant 4 (September 2011) learnt that “... *during the sessions, that I am very patient as I don't mind explaining things over and over when someone doesn't understand*”. Research Participant 5 (September 2011) realised that, “*I have always enjoyed both doing and teaching maths. I thought that teaching in the Intermediate Phase would be more fun and easy than the younger grades, but I have surprised myself by being challenged by it. I enjoy it thoroughly, but I have had to use my brain and really think about suitable activities and methods for older grades*”.

5.2 SIGNIFICANCE AND CONTRIBUTIONS OF THE RESEARCH PROJECT

This research study has made two significant contributions. Firstly, from the perspective of teacher training, underpinning the mathematics module (MIP 400) with the five themes of enactivism generated accountability and a reflective awareness of the link between theory and pedagogy. Secondly, from a theoretical aspect, this study used two frameworks, prominent in mathematics education research, and merged them together to investigate if the one framework enriched the development of the other. In the subsequent paragraphs I discuss these two contributions.

Regarding teacher training, the mathematics module (MIP 400) aimed to develop the skills of undergraduate students and to empower them to become proficient mathematics teachers.

Students do not necessarily have the capacity to allocate a great deal of the time allocated to work integrated learning (formal teaching practice), to enhance their mathematics skills and develop proficient teaching, since there are so many aspects of teaching that need to be developed and honed during this period. In addition, the students are assessed by both their school-based mentor teachers and the institute through which they are training and thus tend to present a safe ‘formulaic’ mathematics lesson. Therefore, I believe that the scheduling of practical tutorial sessions focussing purely on the development of mathematical pedagogy as part of the lecturing programme provided an invaluable alternative approach to teacher

training. The benefit and contribution of this approach is threefold. Firstly this approach provides the support of a mathematical community to strategize various teaching approaches in a safe environment and secondly it builds and consolidates personal conceptual understanding and thirdly, the space to embody mathematical knowledge both theoretically and practically.

From a theoretical aspect, enactivism, a theory of cognition, has mostly been used as an overarching concept without being unpacked into its constituent themes, namely autonomy, sense-making, embodiment, emergence and experience. Therefore the first theoretical contribution this research study made was to engage in a deep analysis of each of the five enactivism themes with regard to the contribution each theme made to the teaching and learning of a mathematics education module. It extended the theory of enactivism by taking the five interconnected themes of enactivism and linked them directly to the five intertwined strands of Kilpatrick's model of proficiency. As explained in section 2.3, this study draws on the work of Di Paolo, Rohde and De Jaegher (2007) to develop five clear themes of enactivism that could allow for specific analysis of data as opposed to mere descriptive accounts of the pedagogy. There is a danger of reductionism by spelling out five constituent themes and a danger that each theme will be considered in isolation, however this does allow for a more sophisticated account of how enactivism is actively realised. This study showed conclusively that each theme contributed significantly to teacher education by providing clarity and awareness as to the triggers and responses required to develop proficient teaching. Despite the emphasis on each strand's contribution, the evidence still revealed the interdependency of the strands on each other. However, it stills remains the decision and choice of an individual student to respond to a perturbation.

The second theoretical contribution of the study was the significance of bringing together these five themes of enactivism with Kilpatrick, et al.'s (2001) five strands of mathematical proficiency to develop a matrix of analysis. Firstly, to my knowledge, this has not been done before and, secondly, both frameworks play a considerable role in the teaching and learning of Mathematics. Kilpatrick, et al. (2001) has influenced and provided a framework for mathematics curricula and education and enactivism has been researched extensively in the field of Mathematics. Therefore,

with this study I was able to consider my data through the combined lenses of two significant frameworks in mathematics education research, to determine to what extent the one had enriched and developed the other. Taking Kilpatrick, et al.'s (2001) as a benchmark for teaching for mathematical proficiency, I was able to use this as a measure of proficiency. Then from an alternative perspective, this matrix allowed me to determine which of the five themes of enactivism had the most influence in the perceived growth in teaching proficiently in meeting the criteria as laid down by Kilpatrick, et al. (2001). The implication of aligning these two frameworks in mathematics education is that the student has to participate and contribute towards activities which provoke self-reflection and self-awareness in terms of the strands of mathematical proficiency. This awareness then places the responsibility of accountability onto the students as areas of proficiency that need to be developed are highlighted. Secondly, it enables the lecturer to have an overview of what the general strengths and weaknesses within a particular group are and what triggers need to be put forward to help them respond to the problems posed. Finally, with enactivism supporting the notion of the teacher and pupils being actively involved in the learning process, the concept of the matrix model is role-modelled to the pre-service teacher. The pre-service teacher in turn is having the opportunity to try the same model in a controlled environment by teaching mathematical concepts to their peers. The pre-service teachers thus gain valuable experience before implementing the model as qualified teachers in their classroom practice.

5.3 CONCLUSIONS AND IMPLICATIONS

I conclude by responding to my over-arching research question *to Determine in what way a mathematics module informed by an enactivist philosophy and teaching pedagogy enabled selected pre-service teachers to unpack the reality of their teaching practice in terms of proficient teaching.*

A module underpinned by enactivism created awareness in participants, in response to a perturbation, as to what their past history brought forth about their ability to teach proficiently. It then provided a number of perturbations for the students to respond to in order to assume accountability for their learning and teaching for mathematical proficiency. It was determined that working in a smaller mathematical

community had a positive effect on self-efficacy and autonomy as students were able to see that often other students experienced similar difficulties which they found reassuring and encouraging. An understanding of their mathematical identity resulted in an awareness of the qualities and characteristics a teacher should have in order to teach mathematics proficiently. Furthermore, practical tutorial sessions were recognised as developing confidence and exposure to different teaching techniques and approaches, which in turn impacted positively on self-efficacy. Teaching mathematical tasks to their peers and receiving feedback, resulted in an improvement in their teaching practices, which in turn resulted in an improvement in their productive disposition. Reflecting on and understanding their mathematical identity created awareness in the students of the need to be in control of their own learning and the need to research further, if need be, in order to teach a concept proficiently. This culminated in an overall increase in confidence to teach Mathematics. The measure of growth varied according to the participants' initial mathematical capacity and disposition, for example the participant who had a fear of Mathematics and was not as confident, did experience growth in proficiency particularly in the second phase as she engaged and interacted with members of her mathematical community. However, as a qualified teacher she was still anxious and had a low self-efficacy with regard to Mathematics. There is also the need to be realistic about what can be achieved in one year given the mathematical experiences and histories that the participants bring with them. If the necessary skills can be developed that enable the students in this research to recognise triggers that will develop their teaching practice, then they can implement these and strengthen the strands of proficiency in which they perceive themselves to be weak or inadequate. For example, a response to a trigger could result in a newly qualified mathematics teacher starting a mathematical community of practice in her area.

Subsequent to this study, additional practical tutorial periods were incorporated into all the mathematics modules offered at the institute to provide the space for theory to be linked with practical experience.

The three themes that most influenced the measure of growth and development in proficient teaching were *autonomy*, *embodiment* and *experience*. I also noted that the themes of *sense-making* and *emergence* significantly enriched the participants'

capacity to teach. These changes were not overtly perceived by the participants, but were noted by the observer as knowledge brought forth, through a demonstration of adequate conduct, with regard to proficient teaching. A person's autonomy and structural makeup influenced the way they responded to perturbations and the extent to which they engaged in the mathematics module (MIP 400). Experience and embodiment provided the opportunity to try out new ideas and link theory to practice despite the difficulties these entailed. The value and support gained from being part of a small mathematical community was often communicated but was not easy to implement as a teacher. Including a unit into the mathematics module (MIP 400) that deals with the implementation of small group teaching would enhance and provide value for this module in the future.

5.4 LIMITATIONS OF THE STUDY

I acknowledge that the generalisability of my research is limited, since this was a small scale case study that encompassed a small participant sample, and thus the results from this group of participants would not necessarily be the same as those of another group. In addition, these students were training as Foundation Phase teachers and were required to take a module focusing on some elements of Intermediate Phase mathematics which could be considered as a limitation, as Intermediate Phase training was not their area of specialisation. However, I think that given that a number of students are employed to teach in the Intermediate Phase, in spite of their qualifications, that it could be considered as a benefit and an advantage. A further limitation was that all students are required to take the mathematics module. Therefore, a lecture group will encompass a range of mathematics abilities which will influence the quality and type of conversations, engagement and thus the sense-making process. A module which students have purposely selected would most likely generate a different set of results as they might have better conceptual understanding and a passion for mathematics. A different set of results per sample group would be in line with enactivism philosophy, since our identity is built on the foundation of our experiences and our responses and reactions to perturbations in our world. Therefore, each new sample of participants could conceivably have a different way of bringing forth mathematical proficiency.

5.5 SUGGESTIONS FOR FURTHER RESEARCH

It would be interesting if further studies were carried out with students who had chosen to teach and/or major in Mathematics. These students would have different mathematical experiences and abilities, and might respond differently to the perturbations raised from the themes of enactivism.

The issue of CAPS DBE (2010) and ANA was raised by each of the research participants who are qualified educators teaching in the Intermediate Phase. Further research needs to be conducted to determine how teachers, especially those new to the profession, can meet the demands of government requirements, but at the same time have the space to teach for mathematical proficiency.

If we consider a teacher who, firstly, embodies the knowledge of their field of interest and expertise and, secondly, is able to facilitate lessons that are underpinned by enactivism, embedded with considered perturbations, it would be valuable to research similar studies focusing on other subjects.

5.6 SOME REFLECTIONS

Having completed the study and after reflecting on both the process and the outcome, I have a few comments.

Firstly, allocating the students to specific groups, not of their own choosing, required them to work with other people in an effective and efficient manner. This is important when moving into a new environment, which is the case when the students qualify as teachers and take up positions in different school environments. As a member of a school and their own classroom community, it is imperative that they know how to engage with a variety of personalities in a productive manner. A further benefit was that it created the space for each student to assume a different role, for example, a leadership role, which they may not have been able to do had they chosen their own group of friends that already encompassed strong leader figures. This is important for personal growth and self-efficacy and to expand one's perception of one's abilities.

Secondly, observing what the students brought forth about their teaching for proficiency, as a lecturer I wondered to what extent I needed to determine the impact that the students' past histories and interactions had on their self-efficacy and autonomy in relation to Mathematics and teaching Mathematics. This would provide an opportunity to put measures in place, for example, journals and reflective practice in order to assist the students to relook at their mathematical identity. In both the initial questionnaire prior to the students being introduced to the strands of proficiency and in phase one of the research, productive disposition was not a factor that was identified as being important to teaching for proficiency, yet it became apparent that the research participants mathematical history did influence the lens through which they perceived their personal proficiency in Mathematics and their ability to teach it.

Thirdly, I found that including the practical teaching sessions was very beneficial in assisting the students to link theory and practice in a meaningful way. The inclusion of the tutorials enhanced and enriched the mathematics module (MIP 400). As a result, and recognising that one year is too short a period of time to build a solid foundational knowledge of Mathematics, the practical teaching sessions have been included in all three years of the mathematics modules offered. South Africa is in need of competent mathematics teachers who can teach proficiently, while including practical tutorial sessions as part of the module is not the only solution, it is definitely a step in the right direction.

From a personal perspective, this research journey raised two key ideas for me to reflect on, namely perturbations and the theme of experience. This prompted a more in depth reflective process than I would normally have engaged in. As a lecturer, focusing on mathematics education research, the enactivist notion of triggers and perturbations resonated strongly with me as a more focused means of encouraging students to engage with theory and practice on various levels. For example, a student firstly had to choose whether or not to respond to a trigger and secondly, there was the structural coupling that emerged during the sense-making process and participation in their mathematics communities. As a person who would not normally choose to participate in group work, this journey reinforced, after each phase, the value that working in a mathematical community can add, especially for students and

teachers who are new to the profession. It is a point that I will emphasise and reiterate in my practice in future.

The second key idea, relating to the theme of experiences, allowed me to address an area that I believe is central to teacher education and that is the ability to link theory to practice. I feel emphatically that it is not sufficient to be solely proficient in Mathematics to be a good mathematics teacher it is more useful that a person is able to link their mathematical knowledge and theory in a manner that demonstrates and has as its end result, proficient teaching. Thus including the practical tutorial sessions as part of my mathematics module created the space for me to offer the students a chance to gain experience and be introduced to different ideas and approaches. This enabled the students to embody their teaching approach while they experimented with different strategies and methods to link theory to practice and to determine which best demonstrated proficient teaching. This experience was not only beneficial to the students but for me as well as it afforded me the opportunity to advance my own practice.

Finally, I found that this journey made me a far more reflective lecturer as I observed different responses to the triggers and in turn needed to assess what these meant for my practice. Completing this journey required that I, as the researcher, reflect on the extent to which each theme of enactivism, namely autonomy, sense-making, embodiment, emergence and experience, enriched and enhanced, firstly, my mathematical identity as a lecturer and, secondly, my practice in training teachers for Mathematics.

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7. APPENDICES

7.1 APPENDIX ONE: INITIAL QUESTIONNAIRE

Questionnaire – Teaching for proficiency in the intermediate phase.

Section A

Please answer the following questions to the best of your knowledge and experience in the space provided.

1. What do you understand by the term mathematical proficiency?
2. What skills and tools do you think that you need, to teach for mathematical proficiency in the Intermediate Phase
3. How will you know that your teaching strategies have achieved mathematical proficiency, in your learners?
4. List the types of teaching strategies that you would use to effectively teach for mathematical proficiency.
5. What has been your greatest accomplishment related to teaching and learning mathematics?
6. What has been your biggest challenge related to teaching and learning mathematics?
7. Explain how you use mathematics in your life.
8. If you think of your mathematics experiences and history, what impact do you think that they will have on own mathematical proficiency and development.
9. If you think of your mathematics experiences and history, what impact do you think that they will have on your ability to teach for proficiency in mathematics?
10. What strategies would you use to encourage active learning in your mathematics lessons?
11. List 10 words that describe someone who is proficient at mathematics
12. List 10 words that describe teaching mathematics proficiently

Section B

Please rate each of the following statements by drawing a circle around the rating of your choice. If possible please provide additional comments in the space provided.

- 1** Strongly Disagree
- 2** Disagree
- 3** Unsure
- 4** Agree
- 5** Strongly Agree

1. Teaching mathematics to Intermediate Phase learners makes me anxious.

- 1** **2** **3** **4** **5**

Additional Comments:

2. I feel confident that I can teach for mathematical proficiency in the intermediate phase.

- 1** **2** **3** **4** **5**

Additional Comments:

3. I think that past mathematics teaching and learning experiences impact on one's current teaching practice.

- 1** **2** **3** **4** **5**

Additional Comments:

4. My lessons encourage interaction between the learner, myself and the mathematical content.

- 1** **2** **3** **4** **5**

Additional Comments:

5. If required to teach a mathematics lessons to an Intermediate Phase class I would tend to use familiar traditional practices that I was exposed to as a learner.

- 1** **2** **3** **4** **5**

Additional Comments:

6. I think that it is important for learners to draw from their own experiences in mathematics lessons. If you agree how would you encourage this?

- 1** **2** **3** **4** **5**

Additional Comments:

7. I have a sound personal content knowledge of Intermediate Phase mathematical concepts, operations, and relations (you may refer to the CAPS document to see what content is covered).

1 2 3 4 5

Additional Comments:

8. I have the necessary skill to carry out Intermediate Phase mathematical procedures flexibly, accurately, efficiently, and appropriately.

1 2 3 4 5

Additional Comments:

9. I have the ability to formulate, represent, and solve Intermediate Phase mathematical problems

1 2 3 4 5

Additional Comments:

10. I have the capacity for the logical thought, reflection, explanation, and justification required by the Intermediate Phase mathematics curriculum

1 2 3 4 5

Additional Comments:

11. I believe that mathematics is a sensible, useful, and worthwhile subject

1 2 3 4 5

Additional Comments:

12. I always ensure that I have a good understanding of my learners, their ability, fear and mathematical needs.

1 2 3 4 5

Additional Comments:

13. I have a good understanding of the instructional practices needed for teaching mathematics

1 2 3 4 5

Additional Comments:

14. I have the procedural fluency required to carry out basic instructional routines in Intermediate Phase mathematics.

1 2 3 4 5

Additional Comments:

15. I am confident that I can plan effective lessons and solve problems that may arise while teaching mathematics in the Intermediate Phase

1 2 3 4 5

Additional Comments:

16. I am able to justify and explain my teaching practice with regard to mathematics

1 2 3 4 5

Additional Comments:

17. I regularly reflect on my teaching practice with regard to mathematics, in order to improve my practice

1 2 3 4 5

Additional Comments:

Section C

Leatham and Hill (2010) define mathematical identity as one's relationship with mathematics with regard to the way one learns, does, thinks about, retains or chooses to associate with the subject. Reflect on this definition and write a short reflective report on your mathematical identity and how you believe it has impacted on your personal mathematical proficiency and how it has/will impact on your teaching for mathematical proficiency.

7.2 APPENDIX TWO: TUTORIAL OBSERVATION SHEET

Tutorial Observation Sheet

Observer's Name:
Pre-service teacher's name:
Lesson taught:

Please rate each of the following statements by drawing a circle around the rating of your choice. If possible please provide additional comments in the space provided

- 1 **Strongly Disagree**
- 2 **Disagree**
- 3 **Unsure**
- 4 **Agree**
- 5 **Strongly Agree**

TEACHER AND LEARNERS	1	2	3	4	5	Not noticed	Evidence/Comment
Evidence of high expectations of learners' capabilities							
Evidence of low expectations of learners' capabilities							
Responds well to challenges presented by learners							
Appears confident when teaching and interacting with learners							
Interacts positively with learners							
Value of what was being taught was explicit							
Evidence of motivation. If agree please indicate							
The teacher modelled motivation in her classroom discourse							
Evidence of evaluative listening							
Evidence of interpretive listening							
Evidence of hermeneutic listening							
TEACHER AND CONTENT	1	2	3	4	5	Not noticed	
The tasks are academically challenging							
The tasks focus on sense making							
The tasks are designed to develop and build skills							
Teacher tried to keep student engagement at a high level							
Time allocated to tasks suitable for maintaining high level of student engagement							
Task had multiple solutions							
Learners expected to explain the development of meaning /sense making							
Evidence of active and consistent support of learners							
The teacher was able to represent mathematical ideas in a variety of ways							

The teacher appeared confident							
The teacher was able to explain and justify the connections and consequences among concepts and procedures.							
The teacher demonstrated a sound knowledge of procedures							
When required the teacher was able to carry out procedures skilfully, with a high degree of flexibility, accuracy and efficiency							
The teacher was able to reconstruct important procedural steps							
The teacher was able to explain mathematical relationships							
The teacher demonstrated flexibility and creative thinking							

Identify and discuss one or more critical incidents that you observed today

In the following section, please tick the appropriate boxes.

This lesson focused on developing the following strand/s of mathematical proficiency

Conceptual understanding		Productive disposition	
procedural fluency		Adaptive reasoning	
Strategic competence			

7.3 APPENDIX THREE: PHASE TWO INTERVIEW QUESTIONS

SECTION A: QUESTIONS	
What influence have the lectures and tutorial sessions had on your views of teaching for proficiency?	
Has your approach to teaching mathematics changed as a result of the lectures and tutorial sessions? Please describe.	
To what extent do you think teaching for proficiency has been role modelled in the lectures? Please explain and give examples.	
Do you think that your ability to teach mathematics in the IP has improved through the lectures and tutorial sessions - please explain/give examples?	
To what extent was the lecturer a participant in your learning? Please explain.	
Were you aware of the 5 strands prior to module - did you ever focus on them intentionally when planning for a lesson?	
Did you start to focus on them for planning?	
Will you next year when planning your lessons?	
Do you consider any one strand more important than the others?	
At every meeting / lecture you entered with ideas or attitudes as to what to expect (concept e.g. fractions)... after the lecture/meeting/tutorial you would have left a slightly different person. Can you describe what changed ...your understanding, vocabulary, confidence, attitude to maths and what brought about this change?	
Interaction with lecturer, peers, and what caused to become involved or to react to a particular trigger ...you want to learn, achieve good results, love maths, fear.	
Has the module developed and grown your confidence and skills in teaching for mathematical proficiency in the Intermediate Phase?	
What effect has your prior experience had on your view of teaching for proficiency? Has this changed through your participation in the module?	
What impact has your personal proficiency had on your mathematical identity?	
What do you think schools see as the characteristics of a proficient maths teacher?	
SECTION B: QUESTIONS	Underpinning Enactivism Theme
1. How valuable was the practical tutorial with regard to the lecture?	Autonomy Embodiment Sense making Emergence Experience
2. What aspects of the module and practical tutorials have influenced your mathematical identity?	Autonomy
3. How has your mathematical identity changed during the course of the year and what has brought about these changes?	Autonomy
4. What characteristics do you think you need to teach for proficiency?	Autonomy Embodiment Sense making Emergence Experience

5. Given the 5 strands when you started the module were you proficient in all strands?	Autonomy
6. How has this changed through the year?	Autonomy
7. What support or value has being part of a group provided you with?	Embodiment
8. Did working in a group expose you to (a) different ways of knowing/understanding a mathematical concept & (b) different ways of teaching the concepts?	Embodiment
9. Did working in a group affect (a) your confidence to teach and (b) your personal proficiency?	Embodiment
10. Within your group did people take on different roles and did these roles change during the course of the year?	Embodiment
11. What have you found valuable in your participation in the tutorial groups?	Embodiment
12. Did each member of your group have something to contribute?	Embodiment
13. Were you actively involved in making meaning of the different problems - what was your role?	Sense making
14. How did you go about trying to solve the problem / task?	Sense making
15. When you engaged in conversation with your group members, were you and the other members able to express your concerns, fears?	Sense making Autonomy
16. Were you about to help each other make sense of the Mathematics task through talking and listening to one another?	Sense making
17. Did your confidence and variety of teaching strategies grow/increase as the year progressed . . . were you more confident to try new ideas?	Sense making Autonomy
18. Did the reflective task contribute to your understanding of the concepts?	Emergence
19. Did the reflective task and critical incidents make you think about your personal proficiency of a topic and how you would go about teaching it proficiently?	Emergence
20. Were you able to determine / identify what you still needed to learn in order to teach the concept for proficiency?	Emergence
21. From this module have you been able to unpack your personal proficiency and your teaching for proficiency in the Intermediate Phase?	Emergence
22. Did not knowing who was going to teach on a particular day make you take ownership of your own learning, did you make sure that you were proficient?	Experience
23. Did you find that the teaching tutorials were a non-threatening environment in which to practice teaching for proficiency?	Experience
24. Compare how you have previously taught maths at prac and what you have discovered during tutorial lessons - any similarities or differences?	Experience
25. Do you think this module has developed and increased your skills for teaching for proficiency in Mathematics? Please describe.	Experience
26. Did you find the feedback received from the observation group and the rest of your peers useful/valuable?	Experience
27. What were the positives and negatives of being a part of the observation team?	Experience
28. Do you think that your ability to teach maths in the IP has improved since the beginning of the year - please explain / give examples?	Experience
29. Do you think that if you are personally proficient at math that you can teach it proficiently?	Experience
30. Are you more comfortable to learn the theory of math and then revert to your own methods of teaching or what you have observed in the classroom?	Experience
31. What do you model your approach to teaching maths (how you were taught, what you have learnt, how they teach in the schools)?	Experience
32. Did the tutorial sessions allow you to move from how you were taught at school to the different strategies that you have learnt during your 4 years?	Experience

7.4 APPENDIX FOUR: TEACHING GROUP REFLECTIVE TASK – MAY

You are required to complete this reflective task and submit it to your lecturer at the MIP 400 lecture on Friday. Please complete this task on your own, do not discuss with group members, as it is your reflective work.

1. Reflect on the planning that went into developing your lesson and discuss the following:
 - How did your group/mathematics community work together
 - did all group members contribute to understanding the concept
 - what was your contribution to the sense making process
 - Discuss what you thought the learners thinking would be during this lesson
 - Identify what difficulties you thought the learners may have or may experience and how you would overcome them
 - Discuss your rationale for the approach that the group took, the representations and tasks that were selected and
 - Having taught the lesson and listened to the feedback from the observation group what changes would you make to the lesson?
2. Choose a key concept from your lesson topic and using the Frayer model, demonstrate your understanding of the concept.
3. Using the problem that was taught in your lesson, write down the steps for an alternative solution to the problem. Then give a written explanation for your strategy and why you chose it. Describe what tips you would give a learner trying to solve this problem.
4. Complete the attached continuum task for each member of your group including yourself and submit with your reflective task sheet. Complete the continua by placing where you think each person is situated with regard to the statement below. Then discuss and justify your reasons for choosing that specific position.

(Name/I) is proficient or not proficient to teach this topic relating to whole numbers.

PROFICIENT = I am able to make sense of the topic that we planned and am confident that I could teach the lesson in a manner that would facilitate learner understanding taking into account different learners' mathematical thinking.

NOT PROFICIENT = I was not able to make sense of the topic that we planned and I doubt that I could teach the lesson in a manner that would facilitate learner understanding taking into account different learners' mathematical thinking.

Each group is required to submit a lesson plan as well as a description of the various solutions to their tasks and a discussion on how these could be used to encourage meaningful discussion during the lesson.

CONTINUUM TASK

Student Name: _____

NOT PROFICIENT

PROFICIENT



Student Name: _____

NOT PROFICIENT

PROFICIENT



Student Name: _____

NOT PROFICIENT

PROFICIENT



Student Name: _____

NOT PROFICIENT

PROFICIENT



Student Name: _____

NOT PROFICIENT

PROFICIENT



Student Name: _____

NOT PROFICIENT

PROFICIENT



7.5 APPENDIX FIVE: TEACHING GROUP REFLECTIVE TASK – AUGUST

Tutorial Session One

Group A: Group 1

Date: August

You are required to teach the task below in your 15 minute session.

During your preparation and planning, you must:

- focus on strategic competence and productive disposition.
- write down 3 questions that would encourage hermeneutic listening
- identify the concept that you are teaching
- discuss how you have addressed strategic competence and productive disposition
- determine what grade you think this problem would be suitable for and justify your decision
- identify what concepts the children may learn

Grandmother's 80th Birthday Party

There are 24 people at Grandmother's 80th birthday party. Timothy counts all the people and writes down the following puzzles. Can you solve them?

1. Half of the people at the party are related to Grandmother. How many people are related to her?
2. A quarter of the people at the party are children. How many children are there?
3. Two thirds of the people at the party are female. How many females are there?
4. Grandmother's eldest son is $\frac{3}{4}$ of her own age. How old is he?

[MALATI and the Open Society Foundation for South Africa]

Reflective task (to be submitted on Friday 5 August)

1. An aspect of sense-making in learning mathematics is playing an active and participatory role in emerging conversations. Discuss the role that you played in helping your group come to a common understanding of the concept/task you were planning to teach.
2. Rank the mode of listening (Evaluative, Interpretative, Hermeneutic) that you and your peers engaged in from
1 = most used; 2 = 2nd; 3 = least used
3. Identify the active and participatory role that each member played.

Identify an incident (or discussion) during your planning that contributed to your understanding of the topic. Then explain how the incident contributed to your understanding for teaching for proficiency and in particular the strands that you were assigned to

7.6 APPENDIX SIX: TEACHING GROUP REFLECTIVE TASK

Tutorial Session 2

Date: August

You are required to teach the task below in your 15 minute session. You need to give a brief explanation of how you would introduce and conclude the lesson.

During your preparation and planning, you must:

- focus on strategic competence and adaptive reasoning.
- write down 3 questions that would encourage hermeneutic listening
- identify the concept that you are teaching
- discuss how you have addressed strategic competence and adaptive reasoning
- determine what grade you think this problem would be suitable for and justify your decision
- identify what concepts the children may learn

Making Large Candles

Themba and Xolile are making large candles. They are making

- large round candles (They use exactly $1\frac{1}{4}$ an ordinary candle to make one of these.)
 - large square candles (They use exactly $1\frac{3}{5}$ of an ordinary candle to make one of these.)
1. How many ordinary candles do they have to buy to make 15 large round candles and 15 large square candles?
 2. How many large round candles can be made out of one packet with 25 ordinary candles?
 3. How many large square candles can be made out of one packet with 25 ordinary candles?
 4. Calculate:
 - a. $4 \times 1\frac{1}{4}$
 - b. $25 \div 1\frac{3}{5}$

[MALATI and the Open Society Foundation for South Africa]

Reflective task (to be submitted on Friday 19 August)

1. An aspect of sense-making in learning mathematics is playing an active and participatory role in emerging conversations. Discuss the role that you played in helping your group come to a common understanding of the concept/task you were planning to teach.
2. Identify an incident (or discussion) during your planning that contributed to your understanding of the topic. Then explain how the incident contributed to your understanding for teaching for proficiency and in particular the strands that you were assigned to focus on. In your explanation discuss how old ideas that you may have had regarding the concept were changed and/or resolved.
3. Leatham and Hill (2010) define mathematical identity as one's relationship with mathematics with regard to the way one learns, does, thinks about, retains or chooses to associate with the subject. Discuss how working as part of a group, both in the mathematics lecture and in preparing for the tutorial sessions to date, has changed your mathematic identity with regard to your personal mathematical proficiency and your teaching for mathematical proficiency.
4. What assumptions did you have about this concept/task and how to teach it? How did these assumptions change through working with your group?

7.7 APPENDIX SEVEN: TEACHING GROUP REFLECTIVE TASK

Tutorial Session 3
August/September

Date:

You are required to teach the task below in your 15 minute session.

During your preparation and planning, you must:

- Focus on conceptual understanding and procedural fluency.
- write down 3 questions that would encourage hermeneutic listening
- identify the concept that you are teaching
- discuss how you have addressed conceptual understanding and procedural fluency
- determine what grade you think this problem would be suitable for and justify your decision
- identify what concepts the children may learn
- plan lesson in such a way that multiple solutions are given and learners come to a consensus as to what the correct answer is

Which One is Bigger?

1. In each case, say which decimal fraction you think is biggest and why.
 - a. 0,03 or 0,3
 - b. 5,31 or 5,13
 - c. 3,5 or 3,412
 - d. 4,09 or 4,1
 - e. 0,76 or 0,760
 - f. 0,89 or 0,089

[MALATI and the Open Society Foundation for South Africa]

Reflective task (to be submitted on Friday 2 September)

1. You are required to design and write a postcard. On the front draw something that captured your imagination during the practical tutorial session or the lectures. On the back, explain why what you have drawn captured your imagination during the practical tutorial session or the lectures and how it has contributed to the development of your teaching for proficiency
2. What connections do you see between the lectures on and how you can apply it to your forthcoming practical tutorial lesson?
3. Identify an incident (or discussion) during your planning that contributed to your understanding of the topic. Then explain how the incident contributed to your understanding for teaching for proficiency and in particular the strands that you were assigned to focus on

An aspect of sense-making in learning mathematics is playing an active and participatory role in emerging conversations. Discuss the role that you played in helping your group come to a common understanding of the concept/task you were.

7.8 APPENDIX EIGHT: TEACHING GROUP REFLECTIVE TASK

Tutorial Session 4

Date: September

You are required to design a van Hiele level 1- Transformation activity, to teach in your 15 minute session.

During your preparation and planning, you must:

- focus on all 5 strands (one of which may be planned as a homework task but explain it to us)
- write down 3 questions that would encourage hermeneutic listening
- identify the concept that you are teaching
- discuss how you have addressed the five strands of proficiency
- identify what concepts the children may learn

Reflective task (to be submitted on Friday 30 September)

1. Identify three facets of the practical tutorial sessions that you attended in the first and second semesters that helped develop your skills in teaching for mathematical proficiency
2. What have you learnt about yourself during the practical tutorial sessions with regard to teaching mathematics?
3. What connections do you see between your lectures and practical tutorial sessions and how you can apply it to your teaching practice.
4. Identify an incident (or discussion) during your planning that contributed to your understanding of the topic. Then explain how the incident contributed to your understanding for teaching for proficiency and in particular the strands that you were assigned to focus on
5. An aspect of sense-making in learning mathematics is playing an active and participatory role in emerging conversations. Discuss the role that you played in helping your group come to a common understanding of the concept/task you were planning to teach.