

# MAXIMALITY AND MINIMALITY IN COMPARATIVES\*

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## Abstract

In this paper, I investigate more closely the contribution of modal operators to the semantics of comparatives and I show that there is no need for a maximality or minimality operator. Following Kratzer's (1981, 1991) analysis of modal elements, I assume that the meaning of a modal sentence is dependent on a conversational background *and* an ordering source. For comparative environments, I demonstrate that the ordering source reduces a set of possible degrees to a single degree that is most (or least) wanted or expected, i.e., maximality and minimality readings of comparative constructions are an effect of the pragmatic meaning of the modal.

## 1 Introduction

A maximality or minimality operator is an essential component of the meaning of comparative constructions in many recent analyses (von Stechow 1984, Rullmann 1995, Kennedy 1999, Heim 2001). Russell (1905), however, had in mind a much simpler analysis of comparative constructions.

Russell proposed for comparative sentences, as in (1-a), an analysis in terms of a comparison of definite descriptions of degrees, as in (1-b). This view presupposes an *exactly*-reading for the degree variable. That is, "Mary is *d*-tall" abbreviates that Mary has exactly the degree *d* on the tallness scale. In the *at least*-reading for the degree variable the uniqueness presuppositions would not be satisfied.

- (1) a. Mary is taller than John (is).  
 b. [the *d*: Mary is *d*-tall] > [the *d*: John is *d*-tall]

In analogy, we might assume an analysis in terms of definite descriptions for constructions with *less*, as well. Whereas the positive comparative morphemes *more* and *-er* introduce a *greater than*-relation between two degrees, the negative comparative morpheme *less* introduces a *smaller than*-relation.

- (2) a. John is less tall than Mary (is).  
 b. [the *d*: John is *d*-tall] < [the *d*: Mary is *d*-tall]

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This simple approach is, however, problematic. There are several arguments that show that an analysis in terms of definite descriptions does not make the correct predictions. They all concern constructions with a (existential or universal) quantifier in the *than*-clause. One group of critical constructions contains one of the following elements in the *than*-clause: (1) modal *can*, (2) the determiner NPI *any*, (3) the adverbial NPI *ever* or (4) the connective *or*: see von Stechow 1984. The denotation of all these elements may be captured by existential quantification. The other group of critical examples contains universal quantifiers expressing modality.

In this paper, I will concentrate on modal elements with existential and universal force.

To begin, I recapitulate the motivations for the introduction of a maximality or minimality operator and the arguments for an *at least*-reading of the degree variable. These amendments of the simple analysis known from the literature, however, will be shown not to be sufficient to explain two kinds of examples: (a) minimality readings of constructions with a modal denoting possibility and (b) maximality readings of constructions with a modal denoting necessity.

Then, I will argue that the problematic examples may easily be explained by an amendment of the semantics of the modal expression in the sense of Kratzer's doubly relative modality: see Kratzer (1981, 1990). Interestingly, the motivation for the introduction of a maximality operator or minimality operator (in the *exactly*-reading of the degree variable) disappears as well as the motivation for the *at least*-reading. For comparative environments, we demonstrate that the ordering source reduces a set of admissible degrees to a single degree that is most (or least) wanted or expected.

We might take this result as evidence for the conclusion that a simple analysis in terms of definite descriptions (the Russellian view) is adequate for comparative constructions. But as long as we do not have an explanation for the minimality/maximality effects in constructions with NPIs and *or* we are not entitled to draw this conclusion. I leave the analysis of these constructions to further research.

## 2 Previous analyses

### 2.1 Existentials

Consider an example with a modal *can* in a first step, as in (3-a). This modal is interpreted as an existential quantifier over worlds. A representation in terms of definite descriptions as in (3-b) carries a uniqueness presupposition and therefore is only defined if Bill runs equally fast in all accessible worlds, otherwise it is not defined.<sup>1</sup>

- (3) a. Jim was running faster than Bill can run.  
 b. [the  $d$ : Jim is  $d$ -fast] > [the  $d$ :  $\exists w$ [Acc( $w$ ) & Bill is  $d$ -fast in  $w$ ]]

This prediction is however intuitively not correct. Bill could run with a different speed in different accessible worlds and (3-a) may still be true. In fact, it seems that (3-a) is true if Jim runs faster than Bill's *highest* possible speed. A representation, as in (4), is defined in such a scenario. Here, the standard of comparison, i.e., the second argument of the *greater than*-relation, is the maximal degree of a set of degrees that satisfy the *than*-clause.

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<sup>1</sup>Acc( $w$ ) abbreviates the accessibility relation for  $w$ . Whether  $w$  is accessible depends essentially on the kind of modality that is contextually salient.

- (4) [the  $d$ : Jim is  $d$ -fast] > MAX{ $d$ :  $\exists w$ [Acc( $w$ ) & Bill is  $d$ -fast in  $w$ ]}

Analogously, we may argue for cases with *ever* and *any*. *Ever* is usually interpreted as an existential quantifier over time intervals. Consider the sentence in (5-a). Most certainly, the temperature varied in the past, and the sentence is true if it was hotter yesterday than the *highest* degree measured in the past. This intuition may be captured by maximalizing the set of degrees that satisfy the *than*-clause, as in (5-b). Again, an interpretation in terms of definite descriptions would not be defined.

- (5) a. Yesterday, it was hotter than it ever was before.  
 b. [the  $d$ : it was  $d$ -hot yesterday] > MAX{ $d$ :  $\exists t$ [ $t$  < YESTERDAY & it is  $d$ -hot at  $t$ ]}

NPI-*any* is interpreted as an existential quantifier over individuals. Again, this sentence is captured correctly if we introduce maximality in the semantics of the comparative, as in (6-b). (6-a) is true if Ede is cleverer than the *cleverest* person. The representation in terms of Russellian definite description predicts the sentence to be undefined, contrary to intuitions.

- (6) a. Ede is cleverer than anyone of us.  
 b. [the  $d$ : E is  $d$ -clever] > MAX{ $d$ :  $\exists x$ [one\_of\_us( $x$ ) &  $x$  is  $d$ -clever]}

And, we encounter a similar problem if *or* is interpreted as a boolean connective with narrow scope. Consider the sentence in (7-a). If Aspen differs from Vail with respect to niceness the sentence is true if Breckenridge is nicer than the *niciest* of the two other cities. (7-b) expresses this intuition by means of a maximality operator.

- (7) a. It is nicer in Breckenridge than it is in Vail or Aspen.  
 b. [the  $d$ : it is  $d$ -nice in B.] > MAX{ $d$ : it is  $d$ -nice in V.  $\vee$  it is  $d$ -nice in A.]}

The interested reader, I refer to the work by von Stechow (1984a).

So far, we only discussed constructions with *more/-er* with a maximality reading. Rullmann (1995) challenges the view that the second argument of the comparison is uniformly a maximal degree of a set of degrees: *Less*-comparatives may have a minimality reading, as is illustrated in (8).

- (8) Students live on less money than a professor could live on.

(8) means that the amount of money that students usually live on is smaller than the *minimal* amount of money such that a professor could live on it. The correct representation seems to be one with a minimality operator instead of a maximality operator, as in (9).

- (9) [the  $d$ : students live on  $d$ -much money] < MIN{ $d$ :  $\exists w$ [Acc( $w$ ) & a professor lives on  $d$ -much money in  $w$ ]}

Not only constructions with modals show this minimality effect. Analogously, constructions with NPIs and *or* seem to call for a minimality reading. In (10) we are comparing my bicycle to the *least* expensive of all the cars.

- (10) a. My bicycle was less expensive than any car.

- b. [the  $d$ : my bicycle was  $d$ -expensive] < MIN{ $d$ :  $\exists x$ [car( $x$ ) &  $x$  is  $d$ -expensive]}

A representation with a maximality operator would make counterintuitive predictions and a variant with a definite description would not be defined.

In (11-a) Jim's *shortest* jump ever is compared to the length of his actual jump. Again, this paraphrase seems to call for a minimality reading as in (11-b).

- (11) a. Yesterday, Jim jumped less far than he ever jumped before.  
 b. [the  $d$ : Jim jumped  $d$ -far yesterday] < MIN{ $d$ :  $\exists t$ [ $t$  < YESTERDAY & Jim jumped  $d$ -far at  $t$ ]}

All these examples are problematic for the simple analysis in terms of definite descriptions and we may easily capture the minimality readings with the introduction of a minimality operator that operates on the set of degrees that satisfy a restriction imposed by the *than*-clause. This solution however necessitates the *exactly*- interpretation for the degree variable: see Rullmann 1995:77, footnote 33.

Modal existential quantifiers and negative polarity items denoting existentials seem to differ with respect to one aspect. Whereas *less*-constructions with *any*, *ever* are restricted obligatorily to a minimality reading, we may also observe maximality readings with constructions with modals.

Rullmann discusses the example in (12-a) among others. (12-a) means that the *highest* amount that professors can spend exceeds the amount that students usually spend. And, we are tempted to capture this meaning by the introduction of a maximality operator, as in (12-b).

- (12) a. Students spend less money than a professor can spend.  
 b. [the  $d$ : students spend  $d$ -much money] < MAX{ $d$ :  $\exists w$ [Acc( $w$ ) & a professor spends  $d$ -much money in  $w$ ]}

Rullmann locates the ambiguity of modalized constructions with *less* in the denotation of *less* itself and proposes that *less* is (lexically) ambiguous between a maximality and a minimality reading.<sup>2</sup>

Constructions with an NP disjunction in the *than*-clause are possibly ambiguous between an minimality and a maximality reading. But my intuitions are not so clear. Consider (13).

- (13) a. Jim is less tall than Tim or Toby.  
 b. [the  $d$ : Jim is  $d$ -tall] < MIN{ $d$ : Tim is  $d$ -tall  $\vee$  Toby is  $d$ -tall}

This sentence might be true if Jim is shorter than both of the others (minimality reading) or if he is shorter than the taller one of Tim and Toby. Rullmann predicts an ambiguity.

So far, we considered only comparatives with an existential quantifier in the *than*-clause. It was shown that the Russellian approach in terms of definite descriptions is problematic for modal expressions. Furthermore, we discussed two alternative analyses, one for constructions with *-er/more* involving maximalization and one for *less* involving minimalization. In a next step we

<sup>2</sup>In his dissertation he also discusses a variant where he assumes lexical decomposition of *less* in a comparative meaning component *-er* and a adjectival component *little*: see also Heim n.d. for a split-scope account of *less*-comparatives.

consider universal quantifiers.

## 2.2 Universals

Modals denoting necessity reveal another problem with the simple analysis. Assume that Mary wants to get from A to B in at most one hour. In order to get to B she has to drive with at least 140 km/h. Now, consider the sentence in (14) in this scenario. This sentence has intuitively a minimality reading. The sentence is true in the given scenario if Mary is driving faster than the lowest admissible speed, i.e. 140 km/h. If we translate *necessary* by a universal quantifier over worlds, it turns out that the formulas in (14-a-c) are only defined if Mary has one and the same speed in all accessible worlds. In our scenario, therefore, the sentence in (14) would not be defined contrary to our intuitions. This outcome, however, hinges on the assumption that we have an *exactly*-interpretation for the degree variable  $d$  in the representations.

- (14) Mary is driving faster than necessary.
- a. [the  $d$ : M. is driving  $d$ -fast] > [the  $d$ :  $\forall w[Acc(w) \Rightarrow$  M. is driving  $d$ -fast in  $w$ ]]
  - b. [the  $d$ : M. is driving  $d$ -fast] > MIN{ $d$ :  $\forall w[Acc(w) \Rightarrow$  M. is driving  $d$ -fast in  $w$ ]}
  - c. [the  $d$ : M. is driving  $d$ -fast] > MAX{ $d$ :  $\forall w[Acc(w) \Rightarrow$  M. is driving  $d$ -fast in  $w$ ]}

If we give up the *exactly*-interpretation in favor of an *at least*-interpretation, the representation in (14-c) makes the correct predictions. Heim (2001:216) proposes an analysis for comparatives that has the effect of an “at least” interpretation for the degree variable. The maximal speed such that Mary has it in all accessible worlds is the minimally permitted speed. If we pursue the *at least*-interpretation we have to watch out for a different explanation for constructions with *less* with a minimality reading (see above). A minimality operator would give us uniformly the minimal element of the scale in the cases considered so far. Heim n.d. proposed a detailed alternative analysis for *less* in a different account with an *at least*-reading for the degree variable.

In what follows, I discuss two examples which show that the accounts discussed so far run into problems: (a) comparatives with a minimality reading and a modal denoting possibility in the *than*-clause and (b) comparatives with a maximality reading and a modal denoting necessity in the *than*-clause.

## 2.3 Two problematic cases

Imagine the following scenario. Chuck is driving a truck full of eggs on a New Jersey highway. It is true that the slower Chuck drives the less eggs break. Moreover, Chuck wants to bring as many eggs as possible to their final destination and he does not want to risk a speeding ticket. The road traffic regulations limit the minimum speed on New Jersey highways to 45 mph and the maximum speed to 65 mph. All in all, it seems desirable that Chuck is driving as slow as possible within the regulations. In this scenario, the sentence in (15) might be true if Chuck is driving with 56 mph, i.e. faster than 45 mph but slower than 65 mph. Such a reading, I will call a more-than-minimum reading.

- (15) Chuck is driving faster than he is allowed to drive.

A representation in terms of definite descriptions of degrees would not make the correct predictions since the speed in the regulation worlds may vary. But, the repair strategy by introduction

of a maximality operation does not make the correct predictions either.

(16) [the  $d$ : Chuck is  $d$ -fast] > MAX{ $d$ :  $\exists w$ [Acc( $w$ ) & Chuck is  $d$ -fast in  $w$ ]}

In the accessible worlds no violations of the law in the actual world occur. Therefore everybody is driving with a speed between 45 and 65 mph. The maximal permitted speed is 65 mph. We predict (16) to be true only if Chuck is driving faster than 65 mph. This reading is possible but not intended in the above mentioned scenario of Chuck the egg truck driver. Consequently, we predict (16) to be false if Chuck is driving 56 mph, contrary to the intuition.

Note that this outcome is not an effect of the *exactly*-interpretation or the *at least*-interpretation of the degree variable. Both versions make the same predictions in this case.

There are two possible ways out of this situation. Either we follow Rullmann's strategy for *less* and assume that *more/-er* are ambiguous, as well.<sup>3</sup>

Or we locate the unexpected reading in the denotation of the modal elements and not in the denotation of the comparative morphemes. I will pursue the second approach.

Modals denoting necessity show a similar behaviour. They can not only trigger a more-than-minimum reading but also a more-than-maximum reading. We already discussed an example with a more-than-minimum reading: see above.

A more-than-maximum reading is preferred if we assume a situation where John applies for a job in an asylum for the elderly. In order to get the job he has to be friendly but not too friendly. Unfortunately he didn't get the job because of the fact in (17).<sup>4</sup>

(17) He was more helpful than he should have been.

(17) means that John's helpfulness exceeds the maximally admissible helpfulness. A representation as in (18) cannot express this meaning neither in the *at least*- nor in the *exactly*-interpretation of the degree variable. In the *at least*-interpretation the sentence is predicted to have only a more-than-minimum reading. And in the *exactly*-interpretation the sentence is not defined in the given scenario.

(18) [the  $d$ : J. was  $d$ -helpful] > MAX{ $d$ :  $\forall w$ [Acc( $w$ )  $\Rightarrow$  J. was  $d$ -helpful in  $w$ ]}

In general, it seems to be possible that constructions with *can*, *could* and *be allowed to* (possibility operators), but also constructions with *should*, *must* and *have to* (necessity operators) allow ambiguities with respect to the characteristics of the standard of comparison. Further investigation has to reveal whether there are lexical restrictions with respect to the use of the modals.

An analysis in the spirit of von Stechow with an *exactly*-interpretation of the degree variable has difficulties to predict the available readings in several cases: comparative constructions with an

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<sup>3</sup>Rullmann (1995:95) himself rejects that constructions with *more/-er* in combination with a positive polar adjective are ambiguous between a more-than-minimum and a more-than-maximum reading. But he mentions constructions with negative polar adjectives that allow an ambiguity without giving an explanation.

(i) The helicopter was flying lower than a plane can fly.

<sup>4</sup>This example is adapted from Rullmann (1995:80). Rullmann does not consider that examples of this kind (with a modal denoting necessity) in the *than*-clause are not defined under his own approach.

embedded necessity operator are undefined in an *exactly*-interpretation of the degree variable if the relevant degrees vary in the accessible worlds. An analysis with an *at least*-interpretation does slightly better since it explains the more-than-minimum readings of constructions with a necessity operator and the more-than-maximum readings of constructions with a possibility operator. The interpretation of constructions with universal modals, therefore, seems to call for an *at least*-interpretation of the degree variable. The *at least*-interpretation of the degree variable is, however, not compatible with an account involving a minimality operator.

But, no version can predict the more-than-minimum reading in constructions with an embedded possibility operator and the more-than-maximum reading in constructions with an embedded necessity operator.<sup>5</sup> How could we fix these problems?

As far as constructions with modals are concerned, I will approach this question as follows: I will propose a repair strategy in order to derive the correct readings. This repair strategy consists essentially in a revision of the semantics of the modals. Instead of interpreting them only with respect to some accessibility relation, I am interpreting modal elements dependent on a modal base *and* an ordering source. If we do that, the maximality operator or minimality operator in the classical analysis of comparatives becomes obsolete. Maximality and minimality readings depend on the kind of ordering source salient in the context. To begin, I am working with an approach to comparison with two maximality operators. And, I show that the set of degrees that the maximality operator in the standard of comparison is applying to is always a singleton (in the *exactly*-interpretation of the degree variable) if the *than*-clause is modalized.

### 3 Theoretical assumptions

#### 3.1 Comparative

In order to interpret comparative constructions, I am using a version of a degree-quantifier analysis. Comparatives are relations between sets of degrees. I assume that the comparative morphemes *-er*, *more*, and *less* respectively, are quantifiers that relate two sets of degrees where one set satisfies the requirements imposed by the *than*-clause (Q) and one set satisfies the requirements imposed by the matrix (P). Negative polar adjectives induce an order reversal.

$$(19) \quad \llbracket \text{-er/more} \rrbracket = \lambda Q_{\langle d, t \rangle} \lambda P_{\langle d, t \rangle} \text{MAX}\{d : P(d) = 1\} > \text{MAX}\{d^* : Q(d^*) = 1\}$$

$$(20) \quad \llbracket \text{less} \rrbracket = \lambda Q_{\langle d, t \rangle} \lambda P_{\langle d, t \rangle} \text{MAX}\{d : P(d) = 1\} < \text{MAX}\{d^* : Q(d^*) = 1\}$$

#### 3.2 Modality

In the framework developed in Kratzer (1981), the interpretation of modal elements does not only depend on (a) the kind of modal relation (i.e. possibility and necessity) and (b) information that characterizes the accessibility relation but also (c) on an ordering source that induces an ordering on the accessible worlds: see also Kratzer (1991) for arguments for doubly relative modality.

I adopt the definitions in (21) and (22). Modals are interpreted with respect to an accessibility relation (*Acc*) and an ordering ( $\leq$ ). Both parameters are supplied contextually. Modals denoting

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<sup>5</sup>It is however a fact that minimality readings are indeed preferred over maximality readings with necessity operators and maximality readings are preferred over minimality readings in construction with possibility operators.

necessity like *must*, *should*, etc. have universal force. Modals denoting possibility like *can*, *be allowed to* etc. have existential force. Accessibility and ordering restrict the quantificational domain of the quantifiers.

$$(21) \quad \llbracket \text{NEC} \rrbracket = \lambda w \lambda p \forall w' [w' \in \text{Acc}(w) \ \& \ \neg[\exists w^* \in \text{Acc}(w) \ \& \ w^* <_w w']] \Rightarrow p(w') = 1]$$

$$(22) \quad \llbracket \text{POSS} \rrbracket = \lambda w \lambda p \exists w' [w' \in \text{Acc}(w) \ \& \ \neg[\exists w^* \in \text{Acc}(w) \ \& \ w^* <_w w']] \ \& \ p(w') = 1]$$

The versions differ from Kratzer's original version of human necessity and human possibility in two respects. I assume simplifying that there *is* a world that comes maximally close to the world  $w$ . And, ordering sources have originally a more complex form than in this version. I will come back to this point.

Consider the interpretation of the sentences that were problematic for the analyses considered so far under the new perspective where we interpret the modals doubly relative.

## 4 Sample applications

I am going to show that it is possible to derive three relevant cases:

- (23)    a. *Less* + possibility with a minimality reading  
           b. *More/-er* + possibility with a minimality reading  
           c. *More/-er* + necessity with a maximality reading

Recall that case (a) was problematic because it motivated the minimality operator. But a minimality operator was shown not to be compatible with *at least*-readings for the degree variable. The *at least*-readings on the other hand were shown to be requisite for more-than-minimum readings of a construction with a modal denoting necessity. The cases (b) and (c) are still unexplained in the literature so far.

### 4.1 *Less* + possibility

An example as in (24) might be uttered in a situation where a jet fighter is trying to chase a helicopter. And the helicopter escaped the jet fighter by hiding below a bridge. One reason the helicopter could escape may be (24). (Once more, the example is adapted from Rullmann 1995.)

(24)    The helicopter was flying less high than a jet fighter can fly.

(24) is true if the helicopter is flying below the minimal height possible for the jet. The accessibility relation, I take to pick worlds that conform to the dispositions of jet fighters. Assume for convenience that causal laws determine that jet fighters fly at a height between 60 and 10'000 meters. Worlds where a jet is flying lower or higher do not conform to the causal laws. Formally, we might capture the set of accessible worlds as in (25).

$$(25) \quad \forall w : \text{Acc}(w) \subseteq \{w^* : \forall d \forall x [x \text{ is a jet-fighter in } w^* \ \& \ x \text{ flies } d\text{-high in } w^*] \Rightarrow 60\text{m} \leq d \leq 10\text{km}\}$$



How could we capture a suitable ordering source? Intuitively, it should single out the accessible worlds where the jet fighter is flying lowest. This would explain the minimality reading of the sentence in (24). In other words, accessible worlds where the jet fighter is flying lowest (= with a height of 60 meters above the ground) come closest to the ideal. But why should a world  $w'$  where the jet fighter is flying with a height of 60 meters be preferred to a world  $w''$  where he is flying at a higher height?

Let us assume that the pilot of the jet fighter wants to fly at a height of at most 35 meters. This wish could be a consequence of the pilot's commission to fight the helicopter and the helicopter is hiding below a bridge that has a height of 40 meters. The wish of the jet pilot determines a set of propositions as in (26).

$$(26) \quad A = \{p : \exists n[0 < n \leq 35 \text{ meters} \ \& \ p = \text{the jet fighter is flying at a height of at most } n \text{ meters}]\}$$

No accessible world may make this wish true. Thus, it is not possible to establish an order on the accessible worlds with this set of propositions. But, 'what is wanted' ( $A$ ) determines a set of relevant propositions that are *not* wanted ( $\bar{A}$ ).<sup>6</sup> Note that the set  $\bar{A}$  is equivalent to the set  $\{\neg p : p \in A\}$ . So,  $\bar{A}$  is not the complement of  $A$ , as could be implied by the notation. In our case these are propositions that are implied by the fact that the jet fighter is flying with at least 35 meters, as in (27).

$$(27) \quad \bar{A} = \{p : \exists n[35 < n \leq \infty \text{ meters} \ \& \ p = \text{the jet fighter is flying at a height of at least } n \text{ meters}]\}$$

Worlds where the jet fighter is flying lower make less propositions of what is not wanted true than worlds in which he is flying higher or, equivalently, worlds in which the jet fighter is flying lower make more propositions of what is not wanted false than worlds in which the jet fighter is flying higher. With these clarifications in mind, we might correctly capture the relevant ordering source in form of a three place relation among worlds, as in (28).

$$(28) \quad \forall w, w', w'' : w' \leq_w w'' \text{ iff } w' \text{ conforms } \textit{less} \text{ than } w'' \text{ to what the pilot does } \textit{not} \text{ want in } w.$$

Therefore, a world in which the jet fighter is flying at a height of 60 meters is closer to the ideal than a world in which the jet fighter is flying with a higher height. Note that the construction scheme of this ordering source does not follow the usual practice! In the classical version of Kratzer's, a world comes closer to an ideal than another world if the first makes more propositions of the ideal true than the second. Kratzer's original ordering sources, we could call *positive ordering sources*.

$$(29) \quad \forall w', w'' \in W \text{ and for any } A \in D_{\langle s, t \rangle} : \\ w' \leq_A^+ w'' \text{ iff } \{p : p \in A \text{ and } w'' \in p\} \subseteq \{p : p \in A \text{ and } w' \in p\}$$

In our case, however, a world comes closer to an ordering source than another world if the first makes more propositions of the NOT-ideal *false* than the second. Such ordering sources, we may call *negative ordering sources*.

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<sup>6</sup>Note that the pilot may have other wishes that are irrelevant for the cruising height of his jet. These wishes do not count.

$$(30) \quad \forall w', w'' \in W \text{ and for any } A \in D_{\langle s, t \rangle}: \\ w' \leq_{\bar{A}} w'' \text{ iff } \{p: p \in \bar{A} \text{ and } w'' \notin p\} \subseteq \{p: p \in \bar{A} \text{ and } w' \notin p\}$$

The accessibility relation in (25) and the ordering in (28) indeed determine what we want in order to derive the less-than-minimum reading as represented in (31). Our sentence is true if the maximal height that the helicopter was flying in the actual world of utterance (@) is below the maximal height  $d^*$  such that there is an accessible world  $w$  in which the jet fighter is flying  $d^*$ -high and that comes closest to the ordering source established in (28).

$$(31) \quad \text{MAX}\{d: \mathbf{H. was flying } d\text{-high in } @ \} < \\ \text{MAX}\{d^*: \exists w[w \in \text{Acc}(@) \ \& \ \neg \exists w^{**}[w^{**} \in \text{Acc}(@) \ \& \ w^{**} <_{@} w^*] \\ \& \ \mathbf{J. is flying } d^*\text{-high in } w]\}$$

The set of degrees that fulfill the requirements of the standard of comparison, i.e. the second argument of the comparative, is a singleton. Therefore the following equations are valid, as shown in (32).

$$(32) \quad \text{MAX}\{d^*: \exists w[w \in \text{Acc}(@) \ \& \ \neg \exists w^{**}[w^{**} \in \text{Acc}(@) \ \& \ w^{**} <_{@} w^*] \\ \& \ \mathbf{J. is flying } d^*\text{-high in } w]\} \\ = \text{MIN}\{d^*: \exists w[w \in \text{Acc}(@) \ \& \ \neg \exists w^{**}[w^{**} \in \text{Acc}(@) \ \& \ w^{**} <_{@} w^*] \\ \& \ \mathbf{J. is flying } d^*\text{-high in } w]\} \\ = \text{the } d^*: \exists w[w \in \text{Acc}(@) \ \& \ \neg \exists w^{**}[w^{**} \in \text{Acc}(@) \ \& \ w^{**} <_{@} w^*] \\ \& \ \mathbf{J. is flying } d^*\text{-high in } w]\} \\ = 60 \text{ meters}$$

We may state the following observations: (a) If ordering sources are used to restrict the existential quantifier over worlds, the maximality operator introduced by the comparative morpheme is redundant. (b) There is no need for a minimality operator (contra Rullmann). This fact seems to have been overlooked so far. And (c), the minimality reading is a pragmatic effect of the interpretation of the modal.

## 4.2 *more/-er* + possibility

In much the same way we may derive the more-than-minimum reading for the truck-driver example, as in (33).

$$(33) \quad \text{Chuck was driving faster than he is allowed to drive. } = (15)$$

Assume that the modal *be allowed to* in this example is associated with two contextual parameters as follows: (a) the law, i.e., the road traffic regulations for New Jersey, in addition to some kind of complicated causal law that describes the proportional dependency of speed and damage to a truck load, and (b) an ideal, i.e., what Chuck wants, in particular, that he is driving with at most 35 mph.

$$(34) \quad \forall w: \text{Acc}(w) \subseteq \{w^*: \forall n[\text{Chuck is driving } n\text{-fast in } w^* \Rightarrow 45\text{mph} \leq n \leq 60\text{mph}]\}$$

Furthermore, the ordering source is required to reduce the regulation worlds to worlds where Chuck is driving slowest (= with a speed of 45 mph). 'What is wanted' determines the negative

ordering source in (35).

$$(35) \quad \forall w, w', w^* : w' \leq_w w'' \text{ iff } w' \text{ falsifies more propositions that Chuck does } \textit{not} \text{ want in } w \text{ than } w''.$$

The ordering source induces a ranking on the worlds that conform to the modal base. The slower Chuck is driving the less propositions of what he does not want become true. With these definitions in mind we state the truth conditions for our sentence as in (36).

$$(36) \quad \text{MAX}\{d : \mathbf{Ch. is driving } d\text{-fast in } @\} > \text{MAX}\{d^* : \exists w[w \in \text{Acc}(@) \ \& \ \neg \exists w^{**}[w^{**} \in \text{Acc}(@) \ \& \ w^{**} <_{@} w] \ \& \ \mathbf{Ch. is driving } d^*\text{-fast in } w]\}$$

Therefore, the set of degrees  $d$  such that there is a world that conforms the least well to what Chuck does not want given the traffic regulations and Chuck is driving  $d$ -fast in that world is a singleton. And, we arrive at the following equivalency for the standard of comparison.

$$(37) \quad \begin{aligned} &\text{MAX}\{d^* : \exists w[w \in \text{Acc}(@) \ \& \\ &\neg \exists w^{**}[w^{**} \in \text{Acc}(@) \ \& \ w^{**} <_{@} w] \ \& \ \mathbf{Ch. is driving } d^*\text{-fast in } w]\} \\ &= 45 \text{ mph} \end{aligned}$$

### 4.3 *More/-er + necessity*

By similar reasoning, we may solve the problems in analyzing the more-than-maximum reading in constructions with a necessity modal. The key to the solution is an additional condition on the restriction of the quantifier introduced by the modal. For illustration, consider again the example with John, the geriatric nurse in (38).

$$(38) \quad \text{John was more helpful than he should have been.} = (17)$$

*Should* is a universal quantifier and plausibly associated with a modal base that regulates the behaviour of the nursing staff in an asylum for the elderly. For simplicity, I assume that helpfulness is measurable on a scale from 0 to 10. John gets the job if he is helpful to degree 4 or higher and less helpful than 7. Being too helpful could spoil the patients. If the staff is not helpful enough, the elderly could possibly harm themselves. Under these assumptions, the accessible worlds may be characterized as in (39).

$$(39) \quad \forall w : \text{Acc}(w) \subseteq \{w^* : \forall n[\text{John is } n\text{-helpful in } w^* \Rightarrow 4 \leq n \leq 7]\}$$

The ordering source ranks the accessible worlds with respect to their closeness to a personal ideal of helpfulness. Intuitively, worlds in which John is more helpful come closer to this ideal than worlds in which he is less helpful. We might express this intuition on the basis of what John does not want in technical terms as in (40). John does not want to be helpful to a degree lower than 10.

$$(40) \quad \bar{A} = \{p : \exists n[n < 10 \ \& \ p = \text{John is at most helpful to degree } n]\}$$

With this NOT-ideal in mind we might express the ordering relation as in (41).

(41)  $\forall w, w', w^* : w' \leq_w w^*$  iff  $w'$  conforms less than  $w^*$  to what John does *not* want in  $w$ .

An accessible world is closer to the wishes of John if it falsifies more propositions of (40) than another accessible world. Therefore, the degree of helpfulness that John has in the accessible worlds in which he is most helpful determines the standard of comparison. Under this conception, the interpretation of the modal in the *than*-clause is defined since the ordering source restricts the worlds that the quantifier operates on to worlds that do not vary with respect to the helpfulness of John.

(42)  $MAX\{d : \text{John is } d\text{-helpful in } @\} >$   
 $MAX\{d^* : \forall w[w \in Acc(@) \ \& \ \neg\exists w^{**}[w^{**} \in Acc(@) \ \& \ w^{**} <_@ w^*]]$   
 $\Rightarrow \text{John is } d^*\text{-helpful in } w\}$

The standard of comparison is a maximal possible degree of helpfulness according to the rules. Note that this reasoning shows that the ordering source may not be omitted in order to make the correct predictions.

## 5 Conclusions

I argued that we may derive minimality readings and maximality readings of comparative constructions in the classical approach to comparison with a maximality operator if we interpret the modals that participate in the constructions not only relative to an accessibility relation but also relative to an ordering source. In order to predict the correct readings, we needed *negative ordering sources*. This line of reasoning showed in the first place that there is no motivation for an analysis with a minimality operator. Second, we observed that whenever we interpret the modals as doubly relative to an accessibility relation and an ordering source, the maximality operator becomes redundant. And finally, there seemed to be no need for an *at least*-interpretation of the degree variable in order to predict the meaning of constructions with a modal denoting necessity.

These results suggest that we could make correct predictions for the meaning of comparative constructions in terms of a comparison of definite descriptions of degrees. And we are back to the simple analysis attributed to Russell at least for one group of sentences that motivated the more complex interpretation with a maximality operator.

An analysis of comparatives in terms of definite descriptions is, however, difficult to defend. Ordering sources are contextual parameters and it is not clear, so far, how the accommodation of a suitable ordering source is restricted. Furthermore, the simple account requires that we analyze elements like *any*, *ever* and *or* not in terms of simple existential quantifiers but something else. The most interesting observation concerning these constructions is that they exhibit some kind of a free choice effect. The relevant constructions have in common that they seem to express some sort of wide scope universal quantification.

(43) a. Jim is taller than Tim or Toby.  
 b.  $\forall x[[x = \text{Tim} \vee x = \text{Toby}] \Rightarrow [\text{the } d: \text{Jim } d\text{-big}] > [\text{the } d: x \text{ is } d\text{-big}]]$

(44) a. 2 is greater than any rational lower approximation of  $\sqrt{2}$ . Pinkal 1989  
 b.  $\forall x[x \text{ is a lower approximation of } \sqrt{2}] \Rightarrow [\text{the } d: 2 \text{ is } d\text{-great}] > [\text{the } d: x \text{ is } d\text{-great}]]$

(45) a. Yesterday, it was hotter than it ever was before.

- b.  $\forall t[t < \text{YESTERDAY} \Rightarrow [\text{the } d: \text{ it was } d\text{-hot yesterday}] > [\text{the } d: \text{ it was } d\text{-hot at } t]]$

There are a couple of promising proposals around in the literature that try to capture this effect akin to these elements: see Zimmermann 2000 for FC-*or*, Zepter 2001 for *any* and *ever* in comparatives and most recently Kratzer n.d. for so-called indeterminate pronouns.

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