Equilibrium and non-equilibrium effects in relativistic heavy ion collisions.

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The hypothesis of local equilibrium (LE) in relativistic heavy ion collisions at energies from AGS to RHIC is checked in the microscopic transport model. We find that kinetic, thermal, and chemical equilibration of the expanding hadronic matter is nearly reached in central collisions at AGS energy for $t \geq 10$ fm/c in a central cell. At these times the equation of state may be approximated by a simple dependence $P \cong (0.12 - 0.15)\,\varepsilon$. Increasing deviations of the yields and the energy spectra of hadrons from statistical model values are observed for increasing bombarding energies. The origin of these deviations is traced to the irreversible multiparticle decays of strings and many-body ($N \geq 3$) decays of resonances. The violations of LE indicate that the matter in the cell reaches a steady state instead of idealized equilibrium. The entropy density in the cell is only about 6% smaller than that of the equilibrium state.

The assumption of the creation of a locally equilibrated (LE) hadronic state in ultra-relativistic heavy ion collisions has been subject of theoretical and experimental efforts during the last decades. Despite the long history the question remains still open. The present analysis employs the Ultra-relativistic Quantum Molecular Dynamics (UrQMD) model [1] to examine the approach to local equilibrium of hot and dense nuclear matter, produced in central heavy ion collisions at energies from AGS to SPS and RHIC.

First, the kinetic equilibration of the system is examined. In order to diminish the number of distorting factors we choose a cubic cell of volume $V = 125\,\mathrm{fm}^3$ centered around the origin of the CM-system of the colliding nuclei. Due to the absence of a preferential direction of the collective motion, the collective velocity of the cell is essentially zero. The longitudinal flow in the cell reaches its maximum value at times from $t = 2$ fm/c (RHIC) to $t = 6$ fm/c (AGS). Then it drops and converges to the transverse flow. Disappearance of the flow implies: **(i)** isotropy of the velocity distributions, which leads to **(ii)** isotropy of the diagonal elements of the pressure tensor, calculated from the virial theorem,

$$P_{\{x,y,z\}} = \sum_{i=h} p^2_{i\{x,y,z\}}/3V(m_i^2 \ + \ p_i^2)^{1/2} \quad , \tag{1}$$

containing the volume of the cell V and the mass and the momentum of the i-th hadron, m_i and p_i, respectively. The time evolution of the pressure in longitudinal and transverse

directions shows (Fig. 1) that kinetic equilibration in the central zone of the reaction takes place at $t \cong 10$ fm/c (AGS), 8 fm/c (SPS), and 4 fm/c (RHIC). Note that the pressure given by the SM [see Eq. (5)] is in a good agreement with microscopic results.
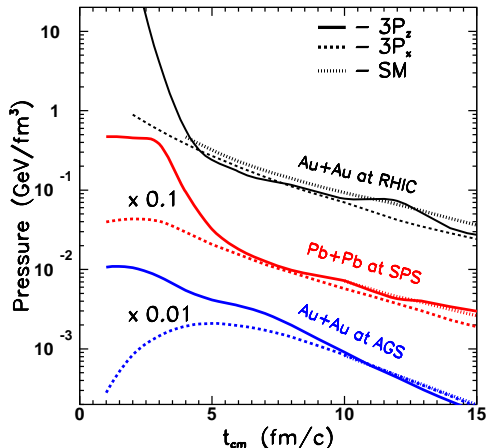

Figure 1. The longitudinal (solid) and the transverse (dashed) diagonal components of the pressure tensor P in the central cell of heavy ion collisions compared to the SM results (dotted lines).

Having the criterion for *kinetic* equilibrium fulfilled, one may address the question on *thermal* and *chemical* equilibrium in the cell. Both criteria read: **(iii)** the distribution functions of hadrons obey Bose-Einstein or Fermi-Dirac statistics (thermal equilibration) with the unique temperature T

$$f(p, m_i) = \left[\exp \left(\sqrt{p^2 + m_i^2} - \mu_i \right)/T \pm 1 \right]^{-1} \quad (2)$$

(where μ_i is the chemical potential of ith particle, " $+$ " sign stands for fermions and " $-$ " for bosons), and **(iv)** the yields of hadrons are calculated via $f(p, m_i)$ with $\mu_i = \mu_{\mathrm{B}} B_i + \mu_{\mathrm{S}} S_i$ (chemical equilibrium). The latter condition assumes that any inverse reaction proceeds with the **same rate** as the direct reaction, i.e., that the detailed balance is fulfilled.

The standard procedure is to compare the snapshot of particle yields and spectra in the cell at given time with those predicted by the statistical thermal model of a hadron gas [2,3]. Three parameters, namely the energy density ε, the baryon density ρ_{B}, and the strangeness density ρ_{S}, extracted from the analysis of the cell, are inserted into the equations for an equilibrated ideal gas of hadrons. Then all characteristics of the system in equilibrium, including the yields of different hadronic species, their temperature T, and chemical potentials, μ_{B} and μ_{S}, can be calculated. If the yields and the energy spectra of the hadrons in the cell are sufficiently close to those of the SM, one can take this as indication for the creation of equilibrated hadronic matter in the central reaction zone.

The particle yields, N_i^{SM}, and total energy, E_i^{SM}, of the hadron species i read

$$N_i^{\mathrm{SM}} = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 f(p, m_i) dp \quad , \qquad (3)$$

$$E_i^{\mathrm{SM}} = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 \sqrt{p^2 + m_i^2} \, f(p, m_i) dp \quad . \qquad (4)$$

Here g_i is the degeneracy factor, and the distribution function $f(p, m_i)$ is given by Eq. (2).

The hadron pressure and the entropy density are calculated within the SM as

$$P^{\mathrm{SM}} = \sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 \frac{p^2}{3(p^2 + m_i^2)^{1/2}} \, f(p, m_i) dp \quad , \qquad (5)$$

$$s^{\mathrm{SM}} = -\sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty f(p, m_i) \left[\ln f(p, m_i) - 1 \right] p^2 dp \quad . \qquad (6)$$

Figure 2 shows the energy spectra of hadronic species, obtained from the microscopic calculations together with the predictions of the SM. At AGS energy the difference between the UrQMD and SM results for baryons lies within the 10%-range of accuracy. With

the rise of initial energy from AGS to SPS the agreement between the models becomes worse. Moreover, even at 10.7 AGeV the deviations of pion spectra in UrQMD from those of the SM are significant.
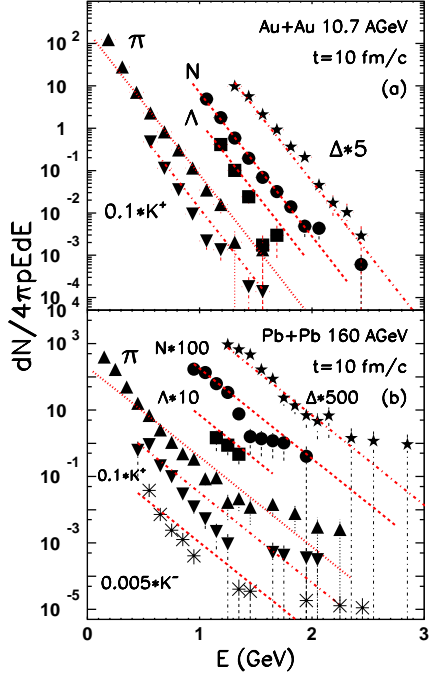

Figure 2. Energy spectra of hadrons in the central cell of heavy ion collisions at AGS **(a)** and SPS **(b)** energies at $t=10$ fm/c. Dashed lines are the results of SM fit.
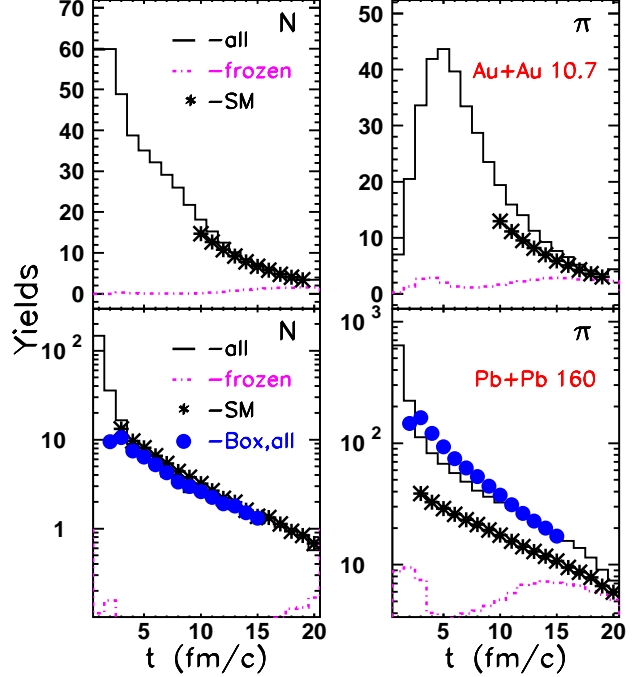

Figure 3. Yields of nucleons and pions in the central cell at AGS (upper row) and SPS (lower row) energies as a function of time, obtained in UrQMD model (histograms), in SM ($*$), and in box calculations (circles).

The Boltzmann fit to pion and nucleon energy spectra from the central cell at 160 AGeV shows [3] that the nucleon (pion) "temperature" is about 30 (50) MeV below the T^{SM}. The subtraction of pions [3] does not decrease the temperature in the SM fit, but leads to the increase of chemical potential of strange particles.

The yields of nucleons and pions in the central cell are shown in Fig. 3. The agreement between the SM and UrQMD nucleon yields is reasonably good for $t \geq 10$ fm/c. Compared to UrQMD, the statistical model significantly underestimates the number of pions, especially at 160 AGeV. The conditions **(iii)** and **(iv)** are not satisfied. Despite the occurrence of a state in which hadrons are in kinetic equilibrium and collective flow is rather small, the hadronic matter is neither in thermal nor in chemical equilibrium. However, the hadron multiplicities in the cell are in a good agreement with those of the equilibrated infinite matter [4], simulated within the UrQMD (Fig. 3, circles).

The detailed balance in relativistic heavy ion collisions is broken because of the irreversibility of multiparticle processes and non-zero lifetimes of resonances. Thus, the matter in the cell is in a steady state [5,6] rather than in idealized equilibrium. The entropy per baryon ratio stays remarkably constant during the expansion at the quasi-equilibrium stage [3]. This fact supports the applicability of hydrodynamics, which assumes an isentropic expansion of the relativistic hadron liquid. The partial entropy densities, carried separately by hadron species in the cell (Fig. 4) are close to those in the SM. The total

entropy density is only about 6% smaller than the SM total entropy density.

The evolution of the central cell in T-μ_{B} plane (Fig. 5) indicates that the extraction of temperature by performing the SM fit to hadron yields and energy spectra is a very delicate procedure. Although the temperatures of hadrons in the steady state are limited to $T_{lim} \leq 145$ MeV, the "apparent" temperature obtained from the fit may occur high enough to hit the zone of quark-hadron phase transition or even pure QGP phase. To study the heavy ion collisions at high energies one has to apply the non-equilibrium thermodynamics of irreversible processes, and not the equilibrium thermodynamics!
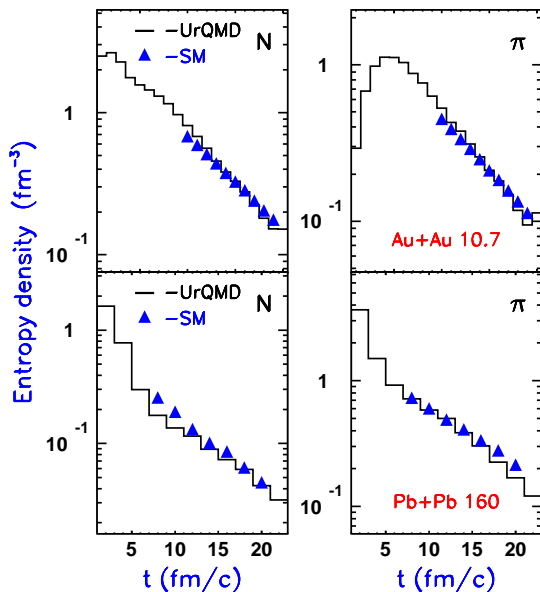

Figure 4.   The same as Fig. 3 but for the entropy densities of nucleons and pions. Histograms indicate UrQMD results, symbols are the SM predictions.

Figure 5.   The plot T vs μ_{B} in the central cell of heavy ion collisions. Open symbols correspond to the nonequilibrium stage, full symbols indicate the stage of equilibrated matter.

Conclusions. The results of the present study may be summarized as follows.

1. There is a kinetic equilibrium stage of hadron-string matter in the central $V = 125$ fm^3 cell of relativistic heavy ion collisions at about $t \geq 8$ fm/c.

2. Entropy per baryon ratio remains constant during the time interval $8 \leq t \leq 18$ fm/c. This result supports the application of relativistic hydrodynamics.

3. The differences between the UrQMD and SM results increase with rising bombarding energy, i.e., thermal and chemical equilibrium is not reached. But: hadron spectra and yields in the cell are consistent with the UrQMD infinite matter calculations.

4. We call this quasi-equilibrium state steady state. Its origin is traced to the irreversible multiparticle processes and many-body decays of resonances.

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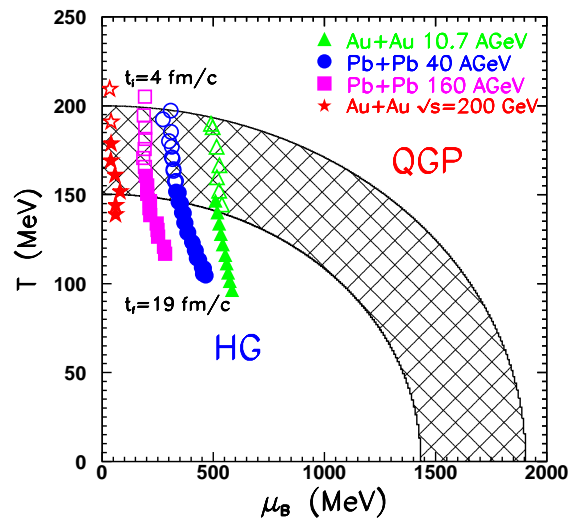6. E.E. Zabrodin, L.V. Bravina, H. Stöcker, and W. Greiner, preprint hep-ph/9901356.