USING THE VAN HIELE THEORY TO ANALYSE GEOMETRICAL CONCEPTUALISATION IN GRADE 12 STUDENTS: A NAMIBIAN PERSPECTIVE

Submitted in partial fulfilment of the requirements for the degree of

MASTER IN EDUCATION
(MATHEMATICS EDUCATION)

of

RHODES UNIVERSITY

by

MUHONGO MATEYA

DECEMBER 2008

Abstract

The study reported here utilised a theory of levels of geometric thinking. This theory was proposed and developed by two Dutch mathematics educators, Pierre van Hiele and his wife, Dina van Hiele-Geldof. The van Hiele theory enables investigations into why many students experience difficulties in learning geometry. In many nations, such as the UK, the USA, Netherlands, the USSR and to a certain extent, Nigeria and South Africa, research evidence has indicated that the overall students' mathematical competencies are linked to their geometric thinking levels. This study is the first of its kind to apply the van Hiele theory of geometric thinking in the Namibian context to analyse geometrical conceptualisation in Grade 12 mathematics students.

In all, 50 Grade 12 students (20 from School A and 30 from School B) were involved in this study. These students wrote a van Hiele Geometry Test adapted from the Cognitive Development and Achievement in Secondary School Geometry test items. Thereafter, a clinical interview with the aid of manipulatives was conducted.

The results from this study indicated that many of the School A and School B students who participated in the research have a weak conceptual understanding of geometric concepts: 35% of the School A and 40% of the School B subsamples were at the pre-recognition level. 25% and 30% of the School A, and 20% and 23.3% of the School B students were at van Hiele levels 1 and 2 respectively. An equal number of students but different in percentages, 2 (10%) in School A and 2 (6.7%) in School B, were at van Hiele level 3. Only one student from School B attained van Hiele level 4. These results were found to be consistent with those of previous similar studies in UK, USA, Nigeria and South Africa.

The findings of this study also highlight issues of how the Namibian Grade 12 geometry syllabus should be aligned with the van Hiele levels of geometric thinking as well as the use of appropriate and correct language in geometrical thinking and problem solving.

TABLE OF CONTENTS

List of tables

List of figures

Acronyms

AED-Academy for Educational Development

BES-Basic Education Support

CDASSG – Cognitive Development and Achievement in Secondary School Geometry

JMC- Joint Mathematical Council

JSC- Junior Secondary Certificate

LOLT- Language of learning and teaching

MASTEP- Mathematics and Science Teachers Extension Programme

MBE- Ministry of Basic Education

MEC- Ministry of Education and Culture

MEd- Master of Education

MoE-Ministry of Education

NIED-National Institute for Educational Development

NSSC (O/H)-Namibia Senior Secondary Certificate (Ordinary and Higher levels)

SACMEQ-Southern Africa Consortium for Monitoring Education Quality

UK-United Kingdom

USA-United States of America

USSR- Union of Soviet Socialist Republics

Acknowledgements

I would like to extend my gratitude and appreciation to the following people and institutions:

My supervisor Professor Marc Schäfer for his invaluable support and guidance;

The Rhodes University staff in the Department of Education for a well organised and fruitful week of lectures on research design;

The principals of the two schools for their permission to use their schools as my research sites;

The two mathematics teachers of the two schools for organising the students and arranging the venues;

The Grade 12 students of the two schools for their willingness to participate in the research;

Mr. Robert Kraft for his excellent lectures on referencing;

My supervisor at work, Mr. L. K. Someno for allowing me to attend all the contact sessions;

My colleagues at work for their motivations and encouragement;

Mrs Anthea Ribbink for proofreading and editing of the thesis;

My fellow MEd. Student and a great friend of mine for numbering the pages of and formatting my thesis;

NIED for the provision of accommodation during the contact sessions;

The Ministry of Education, through the Kavango Regional Training Committee, for the financial support I received to finance the second year of my study; and

My wife Anna for purchasing a lap-top for my studies, and together with our children for their love, understanding, financial support, and constant encouragement during the long and difficult process of conducting my research study.

Declaration of originality

I, **MUHONGO MATEYA**, declare that this assignment is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using complete references according to Departmental Guidelines.

Signature :

Date : 15 December 2008

Student Number : 604M5511

CHAPTER ONE

INTRODUCTION

1.1 Introduction

This chapter provides an introduction to my research study. The introduction includes the background and context of the study, rationale and purpose of the study, followed by the research questions which guided this study. It further highlights the significance and limitations of the study. It then ends with a thesis overview.

1.2 Background to and context of the study

The Namibian Government attaches great importance to the teaching and learning of mathematics and sciences. "Mathematical knowledge and mathematical methods of inquiry constitute an essential part of and contribute to all modern science and engineering" (Namibia. Ministry of Education [MoE], 2005a:2). That is why the learning of mathematics involves acquiring conceptual structures, developing strategies for problem solving and attitudes to and appreciation of mathematics.

Despite all these emphases being made by the Namibian Government, the students' performance in mathematics is still poor. For example, the results of the second report of the Southern Africa Consortium for Monitoring Education Quality (SACMEQ II) (Namibia. Ministry of Basic Education, Sport and Culture [MBESC], 2004) indicated that Namibian Grade 6 learners as well as their teachers performed poorly in the mathematics test. Out of the 12 countries that participated in this study, Namibia in relation to the mathematics performance of the Grade 6 children and teachers ranked last and second last respectively. Based on these findings, 25 upper primary mathematics teachers were identified from the nine circuits of the Kavango Education Region in Namibia and were trained with the help of the Academy for Educational Development (AED) under the auspices of the Basic Education Support (BES) Project III in Namibia. The training was necessitated by the fact that "the overall, low average scores for Namibian Grade 6 mathematics teachers and their learners indicates that there could be a problem with either the mathematics curriculum or the training of mathematics teachers and the way they teach the subject" (Namibia. MBESC, 2004:146).

During the training, a geometry test drawn from the question papers of the Grade 7 end-of-year examinations was administered to these teachers. The teachers who took this test performed very poorly. Of the 25 teachers who took the test 23 i.e. 90% scored below 50% *(*Namibia. MoE, 2006a). Two of the 23 teachers obtained zero. As one of the facilitators of this training, I realised that the majority of the upper primary mathematics teachers in the Kavango Education Region have a poorly developed proficiency in geometry. These results left me with the question, if teachers themselves have problems with understanding simple or basic geometric concepts, then what about the students who are graduating from the hands of these teachers? In my view, as a result of their poor geometric background, when these students enter the secondary phase of formal education, they will inevitably encounter problems in understanding geometric concepts.

Teppo (1991:217) states that "systematic geometry instruction in the middle grades is necessary to prevent students from entering high school at low levels of geometric concept development". Being a mathematics teacher with ten years experience at the secondary phase, I learnt that although students at Grade 12 level are familiar with some of the geometric shapes, they do not know their properties and they can hardly do even basic informal deductions. Drawing on my own experience and the findings of the SACMEQ II report (Namibia. MBESC, 2004), I found it appropriate to conduct a study to explore the geometric reasoning of the Grade 12 students in two selected senior secondary schools in the Rundu Circuit in the Kavango Region of Namibia. The difficulties encountered by students in learning geometry is not a unique problem to Namibia alone, it is a worldwide phenomenon.

The problem regarding the teaching and learning of geometry was identified in the 1950s by two Dutch mathematics educators, Pierre van Hiele and his wife, Dina van Hiele-Geldof, who due to their frustrations investigated possible reasons that could have created this problem in their classrooms. The findings of their investigations resulted in the development of a theory. The theory distinguishes five different thought levels that a student should go through when learning geometry. This theory was subsequently considered by many countries such as UK, USA and the former USSR as one of the best frameworks to assess students' geometric reasoning (Atebe & Schäfer, 2008). This is because it provides a structure for understanding how students develop geometric concepts through appropriate learning experiences (Genz, 2006).

1.3 Rationale for and purpose of the study

Geometry is regarded as a problematic learning area in mathematics around the globe (Snyders, 1995). Therefore, this study explores the geometric reasoning of selected Grade 12 students and cautiously suggests some reasons why these students experience difficulties in learning geometry in Namibia.

The purpose of this research study therefore is to gain an understanding of the application of the van Hiele levels of thinking in exploring geometrical conceptualisation in Grade 12 students in two selected schools in Namibia.

1.4 Research questions

Using the van Hiele theory in analysing the geometrical conceptualisation of Grade 12 students, I pursued the following research questions:

1. What are the van Hiele levels of thinking required by the Grade 12 mathematics curriculum in Namibia?

2. Are selected Grade 12 students in Namibia functioning at a level of geometric thinking fitting with their mathematics curriculum?

1.5 Significance of this study

The van Hiele theory has been applied to many curricula to improve geometry classroom instruction in many developed nations such as U.K. and USA (Clements, 2004). But, in Namibia, the literature appears to suggest that the theory has never been applied and researched. This study therefore is the first of its kind to test the applicability of the van Hiele theory in analysing geometrical conceptualisation in the Namibian context. The study further investigates the possibility of aligning the Namibian geometry curriculum with the van Hiele levels of thinking.

1.6 Limitations of this study

The following are considered as limitations of this study:

- The non-availability of studies conducted on the van Hiele theory in Namibia. This led to the use of only research findings obtained in international studies on the van Hiele theory to analyse the geometrical conceptualisation of the Namibian students.

- The study is limited to only two selected schools in the Kavango Region in Namibia.

1.7 Thesis overview

1.7.1 Chapter Two

This chapter deals with the literature review. The chapter starts with a brief historical overview of geometry and then follows a discussion on some reasons why geometry is regarded as an important learning area in the mathematics curriculum. It also discusses some challenges in teaching and learning geometry at Grade 12.

The research study is informed by the van Hiele theory. The theory is discussed in this chapter by looking at the levels of geometric thinking (recognition, analysis, informal deduction, formal deduction and rigour), features or properties of the learning phases (information/inquiry, directed orientation, explicitation/explanation, free orientation and integration), the implications of the levels and phases on instruction of geometry, the role of language in geometrical conceptualization and van Hiele levels, and how to determine students' levels of thinking.

The relationship between the van Hiele theory and Piaget's cognitive development theory is discussed with specific reference to their similarities and differences. The chapter further provides the results of some research studies that have used the van Hiele theory. The said results are relevant for my research study. This is followed by a discussion on the van Hiele theory and learner-centred education in the Namibian context. It ends with a critique of the van Hiele theory.

1.7.2 Chapter Three

This chapter outlines the research methodology used in this study. It describes the research orientation and the research methods – a quantitative and qualitative case study. It also describes the research site and participants as well as how the site and the participants were sampled.

It discusses the data collection process and the instruments used to collect the data. Issues of validity of the instruments used are also discussed. The chapter ends with an elaboration of research ethics.

1.7.3 Chapter Four

This chapter deals with data presentation, analysis and discussions of findings. Data analysed was generated from document analysis, the results of the pilot study, the results of the van Hiele Geometry Test and that of the clinical interviews.

1.7.4 Chapter Five

This is the last chapter of the thesis. It provides the conclusion of the study by presenting a summary of findings, significance of the study, limitations, some recommendations, avenues for further research and ends with a personal reflection.

This research study is informed by the van Hiele theory. To better understand the said theory, the next chapter reviews the related literature.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

Mathematics is a dynamic, living and cultural product. It is more than an accumulation of facts, skills and knowledge (Namibia. MoE, 2005a). That is why, consistent with good practice throughout the world, it is recognised that mathematics is one of the crucial subjects necessary for any country to realise its full potential (Namibia. Mathematics and Science Teachers Extension Programme [MASTEP], 2002).

A study carried out by the Ministry of Basic Education, Sports and Culture through MASTEP, reports that:

> Namibia has achieved a great deal since Independence in the field of education. This includes projects, papers written and research carried out in mathematics and mathematics education, many of them of the first class, but there has not been a great deal of improvement resulting from these efforts; learners still under-achieve in mathematics.
>
> (Namibia. MASTEP, 2002:3)

King (2002) in general supports the argument above by stating that dissatisfaction with the secondary school geometry curriculum and poor performance in geometry has been the topic of many discussions over the past decade or two in many parts of the world. Poor performance in mathematics in general and in geometry in particular, is not a problem unique to Namibia alone – it is a global issue. For example, Snyders (1995), as cited in Siyepu (2005), states that globally, it has been noted that geometry is problematic for both teachers and learners. This view of geometry has inspired various studies based on the van Hiele theory (e.g. Usiskin, 1982; Teppo, 1991; Mason, 2003; Siyepu, 2005; Genz, 2006; Atebe & Schäfer, 2008).

Likewise in this research study, I use the van Hiele theory to analyse geometrical conceptualisation at Grade 12 level. This is because the van Hiele theory is arguably one of the best-known frameworks presently available for studying the teaching and learning processes in geometry (King, 2002).

In this chapter, I initially present a brief historical overview of geometry. This is important because it provides a glimpse as to how and why geometry found a position in the mathematics curriculum. Then follows an assessment of why geometry is considered an important learning area in the mathematics curriculum. This is followed by a detailed discussion on the van Hiele theory. Under the theory, the following subheadings are discussed:

- The van Hiele levels of thinking;
- features or properties of the levels;
- learning phases;
- how to determine students' levels of thinking;
- implications of the levels and phases on instruction of geometry;
- and the role of language in geometrical conceptualisation and van Hiele levels.

The subheadings mentioned above provide a framework to clarify how the van Hiele theory works. A similar framework was used by other researchers, such as, Mayberry (1983); Fuys, Geddes, and Tischler (1988); Teppo (1991); and Pegg (1995).

Thereafter I will discuss the relationship between the van Hiele theory and Piaget's cognitive development theory. This relationship is discussed because both theories talk of students going through stages or levels of reasoning in geometry.

To ascertain the validity of the van Hiele theory, I briefly analyse other research studies that used the theory. The identified research studies that have used the van Hiele theory have justified the use of the theory in the study of geometry.

For example, in their studies, Usiskin (1982); Burger and Shaughnessy (1986); de Villiers (1987) and Teppo (1991) have concurred with van Hieles that the learning of geometry occurs in a hierarchical manner. That is, the learning of geometry starts from a lower level of reasoning to a higher level of thinking.

I situate my research study in learner-centred education in the Namibian context. I argue that the van Hiele theory is associated with learner-centred education, because of certain characteristics such as the emphasis on learning with understanding as opposed to rote learning, and co-operative learning. Even though the theory stands out very prominently, it has some shortcomings as well. I discuss these in a critique towards the end of the chapter.

2.2 A brief historical overview of geometry

The origin of geometry is very ancient. It is one of the oldest branches of mathematics embraced by several ancient cultures such as Indian, Babylonian, Egyptian and Chinese, as well as Greeks (Jones, 2002). These ancient cultures developed a form of geometry based on relationships between lengths, areas and volumes of physical objects. In these ancient times, geometry was used to measure the land and in the construction of religious and cultural artefacts (Jones, 2002). Mathematicians and geometers found geometry a worthwhile branch of mathematics to study, which culminated in the compilation of Euclid's *Elements* as a systematisation of the geometric knowledge in 300 B.C. (Jones, 2002). Euclid's book had a compelling influence on geometry education, first at the university level, and later at the school level (de Villiers, 1987). Shibli (1932), as cited in de Villiers 1987) elaborates that the influence of Euclid's *Elements* became particularly strong when parts of it were being used in the 14th century as prescribed books in European universities and from the 18th century in European schools. Since most of the countries in Africa, including Namibia, were colonised by European countries, the geometry education that was introduced by the colonisers had roots in Euclid's *Elements* as well. Jones (2002:15) further explains that in the 19th century, geometry went through a period of growth that was near "*cataclysmic*" in proportion. It resulted in the content of geometry and its internal diversity increasing almost beyond recognition. Due to this growth of geometry, different types of geometries were founded. Malkevitch (1991) lists some of these geometries as follows: differential geometry, hyperbolic geometry, Lobachevskia geometry, projective geometry, elliptic geometry, algebraic geometry, Euclidean geometry, analytic geometry, plane geometry, Riemannian geometry, dynamic geometry and co-ordinate geometry. These different types of geometries make geometry an important learning area in the mathematics curriculum.

2.3 Why is geometry an important learning area in the mathematics curriculum?

A well known British mathematician, Sir Christopher Zeeman, quoted in the Royal Society and Joint Mathematical Council [Royal Society/JMC] (2001), as cited in King (2002:12) explains that "geometry comprises those branches of mathematics that exploit visual intuition (which is the most dominant of our senses) to remember theorems, understand proof, inspire conjecture, perceive reality, and give global insight."

But before debating the reasons why geometry is an important learning area in mathematics, I first list the aims of teaching geometry according to the Royal Society/JMC (2001), as cited in King (2002:12):

- to develop spatial awareness, geometric intuition and the ability to visualise;
- to provide a breadth of geometrical experiences in 2 and 3 dimensions;
- to develop knowledge and understanding of and the ability to use geometrical properties and theorems;
- to develop skills of applying geometry through modelling and problem-solving in real world contexts;
- to encourage the development and use of conjecture, deductive reasoning and proof;
- to develop useful ICT (information communication technology) in specifically geometrical contexts;

- to engender a positive attitude to mathematics; and,
- to develop an awareness of the historical and cultural heritage of geometry in the society, and of the contemporary applications of geometry.

In support of this description of geometry and its aims, Jones (2002) suggests that geometry helps the students to develop the skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof. In Namibia, one of the aims of teaching mathematics is to "develop and understanding of spatial concepts and relationships" (Namibia. MoE, 2005b:2).

French (2004), as cited in Atebe and Schäfer (2008) asserts that students' general mathematical competencies have been closely linked to their geometric understanding. This implies that geometric knowledge is important for the student to perform well in mathematics in general. Sherard (1981) points out that geometry has important applications to most topics in mathematics. As a result it has a unifying dimension in the entire mathematics curriculum. It is the basis for visualisation for arithmetical, algebraic, and statistical concepts.

The knowledge of geometry remains a pre-requisite for study in such fields as "physics, astronomy, art, mechanical drawing, chemistry (for atomic and molecular structure), biology (for cell structure), and geology (for crystalline structure)" (Sherard, 1981:20). The fields of study mentioned above play a major role in the development of any given country.

Sherard (1981:23) further explains that "today, knowledge of mathematics is being called the *critical filter*, which permits entry into a wide variety of many different careers". For example, geometric skills are important in architecture and design, in engineering, and in various aspects of construction work. Namibia is striving towards the realisation of the long-term plan of Vision 2030 (Namibia. Office of the President, 2004), which puts more emphasis on careers such as architecture and design, health, engineering, and on constructions. Therefore it is important that Namibian students possess a good knowledge of geometry.

It is for these reasons that Namibian students should study geometry as part of their experience with mathematics in order for them to have a wide range of options in choosing appropriate occupations. Despite geometry being an important branch of mathematics, there are many challenges in teaching and learning it.

2.4 Some of the challenges in teaching and learning geometry at grade 12

During the ten years that I was a teacher of mathematics at grades 8–12, I learnt that students who are entering senior secondary school education have many problems with geometry. One of the aspects I have experienced for example was that, even if the students know the names of the geometric figures/shapes, they are not familiar with their properties, and are not always able to point out specific differences expressed in the definitions. This experience is supported by Clements and Battista (1992) who emphasise the difficulty of the passage from middle school to high school in USA. They hold that the major focus of standard elementary and middle school curricula is only on recognising and naming geometric shapes, writing the proper symbolism for simple geometric concepts, developing skills with measurement and construction tools such as a compass and protractor, and using formulas in geometric measurement. Clements and Battista (1992:422) further explain that the said curricula consist of a "hodgepodge" of unrelated concepts with no systematic progression to higher levels of thought, levels that are required for sophisticated concept development and substantive geometric problem solving.

Primary school teachers generally tend to spend the minimum amount of instruction time on the teaching of geometry (King, 2002). When the subject is taught, it is usually done using the traditional transmission model.

As a result students have problems with conceptual understanding in the higher standards or grades where deeper knowledge of geometric concepts is expected or presupposed (King, 2002).

The Presidential Commission on Education in Namibia reports that:

> Many teachers feel inadequate in mathematics education and are unable to give children the skills that are needed to succeed in upper primary school and at secondary level. Yet mathematics is essential for success in scientific and technical education. Unless the foundations are secured, it will be extremely difficult to build mathematical and scientific success at secondary level.
>
> (Namibia. MBESC, 1999:112)

In my view students do not receive adequate and age appropriate, high quality/proficient teaching from teachers at the primary phase. This is largely due to teachers not having the necessary knowledge of mathematics. For example, Thekwane (2001) points out that the majority of teachers have weak backgrounds in the subject matter of mathematics. As a result they commonly express a fear of or anxiety about mathematics. Teaching geometry therefore remains problematic because it requires knowledgeable and competent teachers. Due to teachers' poor mathematical backgrounds, many abstract concepts and formulas are introduced without paying much attention to aspects such as logic, reasoning, and understanding (Karnasih & Soeparno, 1999). This causes many of the students to think that geometry is very difficult to learn (Soedjadi, 1991; Kerans, 1994). In support of the latter statement, de Villiers (1996) reports that it is well known that on average pupils' performance in Matric (Grade 12) geometry is far worse than in algebra. In my personal experience, this also applies to Namibia. Where, for example, students in Namibia are often passive through out the mathematics lessons; 'chalk and talk' is the preferred teaching style; emphasis is always made on factual knowledge and questions which require only single word answers, and often answered in chorus. Consequently learning for conceptual understanding is inhibited.

The mathematics curriculum at the senior secondary phase in Namibia is divided into higher level and ordinary level. The ordinary level is further divided into *extended* and *core* syllabuses. Despite the education system advocating a learner-centred approach, the mathematics curriculum in my view is still examination driven. Therefore, to teach for conceptual understanding is difficult. Instead teachers in general tend to give students mainly what they think is important for their examinations at the end of the year.

The Kavango Education Region where I am the advisory teacher for mathematics has ten senior secondary state schools and two private schools. None of the ten senior secondary state schools offers the higher level mathematics syllabus. On the basis of my observations and experience the reason for this is that there is a shortage of trained teachers who can teach the higher level mathematics. Of the students who are doing the ordinary level syllabus of mathematics, only a small number follow the extended syllabus of this level. For example, of the 562 students in the Kavango Region who took mathematics in 2007 in the state schools, only 50 followed the extended syllabus of the ordinary level. This suggests that some teachers and most of the students are not prepared to involve themselves in the more advanced mathematics. I assert that this is brought about mainly because the mathematical background of both these teachers and students is weak. The current poor achievement of students in geometry was also encountered by the van Hieles in the 1950s in Netherlands. As a result they proposed and developed a theory that could be used in the teaching and learning processes of geometry. It inspired me to investigate and explore the van Hiele levels of geometrical conceptualisation in selected Namibian Grade 12 students.

2.5 The van Hiele theory

About 51 years ago, Pierre Marie van Hiele and his wife, Dina van Hiele-Geldof, postulated a theory of learning geometry. This theory has attracted considerable interest among researchers (Usiskin, 1982; Hoffer, 1983; Burger & Shaughnessy, 1986; Senk, 1989; Siyepu, 2005; Genz, 2006; Atebe & Schäfer, 2008).

The van Hieles developed this theory out of the frustrations both they and their students experienced with the teaching and learning of geometry (Genz, 2006). For example, van Hiele (1986:39) explains that when teaching geometry, “it always seemed as though I were speaking a different language". Usiskin (1982) further indicates that many students fail to grasp key concepts in geometry, and leave the geometry class without learning basic geometric concepts. The van Hiele theory is a learning model that describes the geometric thinking students go through as they move from a holistic perception of geometric shapes to a refined understanding of geometric proof (van Hiele, 1986). Usiskin (1982) and Teppo (1991) indicate that the theory hypothesises five levels of understanding through which students progress.

2.5.1 The van Hiele levels of thinking

Two different numbering schemes are used in the literature to identify van Hiele levels of thinking (Senk, 1989:310). The van Hieles originally referred to Levels 0 through 4, a scheme consistent with the European system of numbering floors in a building: ground, first, second, and so on (Senk, 1989). However, when Wirszup (1976) and Hoffer (1979) brought the work of the van Hieles to the attention of the American audience, they used a 1 through 5 numbering scheme (Senk, 1989). Despite the fact that the Namibian Education system is founded on the basis of the Cambridge Education system, which is from Europe, I find it fit to use the numbering system (1–5) as used in other research studies (Pegg, 1995; Mason, 1998; Siyepu, 2005; Genz, 2006; Atebe & Schäfer, 2008). This numbering scheme allows the researcher to use level 0 for students who do not function at what the van Hieles referred to as the ground or basic level. In this research study, the van Hiele levels will be discussed using the categories used by Pegg (1995); Mason (1998); and Atebe and Schäfer (2008).

- **Level 1: Recognition**

The student at this level reasons about basic geometric concepts, such as simple shapes, primarily by means of visual considerations of the concept as a whole without explicit regard to properties of its components. For example, students recognise triangles, squares, parallelograms, and so forth by their shape, but they do not explicitly identify the properties of these figures (de Villiers, 1996). "While students may make mention of the length of sides or the size of angles, when directed to focus on these aspects they will not be used spontaneously without prompts" (Pegg, 1995:89). Pegg (1995) further indicates that for students at this level, a figure is a square, cube or rectangle because it looks like one. This is because students visually recognise figures by their "global appearance" (de Villiers, 1996:2). According to Pegg (1995:90), there are at least three categories within the first level. At the first category, for example, students can identify a rectangle; they can recognise it very easily because its shape looks like the shape of a window or the shape of a door. This means that the identification of shapes is based on a certain prototype. Further examples are: a cube is like a box, or a dice; a rectangle is a long square; and parallel lines are like a door. The second category exists when students can identify certain features of a figure but not properties. These are such features as pointedness, sharpness, corners, and flatness. Students are unable to link these features to have an overview of the shape. The third category, which is the lowest, occurs when the student can focus only on a single feature.

- **Level 2: Analysis**

Students analyse component parts of the figures, for example, opposite angles of parallelograms are congruent, but interrelationships between figures and properties cannot be explained (Teppo, 1991).This means that students analyse figures in terms of their components and relationships among components and perceive properties or rules of a class of properties of shapes empirically, but properties or rules are perceived as isolated and unrelated. According to Burger and Shaughnessy (1986), the student reasons about basic geometric concepts by means of an informal analysis of component parts and attributes.

At this level, students begin to identify properties of shapes and learn to use appropriate vocabulary related to properties, but do not make connections between different shapes, and their properties (Teppo, 1991). This implies that irrelevant features, such as size or orientation, become less important, as students are able to focus on all shapes within a class. For example, "an isosceles triangle can have two equal sides, two equal angles and an axis of symmetry but no property implies another" (Pegg, 1995:90). This means that the properties are seen as separate entities that cannot be combined together to describe a specific figure. Clements (2004:62) gives an example, "if one tells us that the figure drawn on the blackboard has four right angles, it is a rectangle even if the figure is badly drawn." But at this level properties are not yet ordered, so that a square is not necessarily identified as being a rectangle, in other words, students at this level are unable to make short deductions.

- **Level 3: Ordering**

At this level, students logically relate previously discovered properties or rules by giving or following informal arguments such as "drawing, interpreting, reducing, and locating positions" (Feza & Webb, 2005:38). Students at this level could begin to see "how one figure could be characterised by several different names" (Pusey, 2003:14). This is seen if the figures share the same properties, for example, a square is seen as a rectangle, but a rectangle is not necessarily a square. Mayberry (1983:59) states that "logical implications and class inclusions are understood". The role and significance of deduction, however, is not understood.

- **Level 4: Deduction**

Grearson and Higgleton (1996), as cited in Siyepu (2005), describe deduction as a reasoning process by which one concludes something from known facts or circumstance or from one's own observation. At this level, deduction becomes meaningful. For example, Hoffer (1981) explains that the student understands the significance of deduction and the role of postulates, axioms, theorems and proof. Pegg (1995), as cited in Schäfer and Atebe (2008), states that the students at this level should be able to supply the reason for steps in a proof and also construct their own proof while the need for rote learning is minimised. Mayberry (1983) further points out that the meaning of necessary and sufficient conditions in a definition are understood. For example, Byrkit (1971) and Krause (1975), as cited in Hoffer (1981), outline that at this level a student will be able to use the "Side Angle Side" (SAS) postulate to prove statements about triangles but not understand why it is necessary to postulate the SAS condition, or how the SAS postulate connects the distance and angle measures. Pegg (1995) gives the least amount of information, students at this level can provide: "a square is a rectangle with a pair of adjacent sides equal," or "a rectangle is a parallelogram with an angle a right angle." Pegg (1995), as cited in Schäfer and Atebe (2008) states that this level is likely to represent an upper bound on what might reasonably be expected of the students in the learning of geometry at the senior secondary school. Pegg and Faithfull (1993), as cited in Pegg (1995), state that while it is possible that students in years 9 and 10 (14–16 year-olds) might exhibit instances of level 4 thinking, it is likely that only about 25% of the 18 year-olds will feel comfortable with problems of this level. Ideally, in Namibia the 14–16 year-olds are students in the junior secondary phase (Grades 8–10) and 18 year-olds are students in their senior secondary phase (Grades 11–12). Therefore this implies that many high school (Grades 11–12) courses approach the study of geometry at this level.

- **Level 5: Rigour**

This is the highest level of thought in the van Hiele hierarchy (Teppo 1991). Students at this level can work in different geometric or axiomatic systems and would most likely be enrolled in a college or university level course in geometry (Teppo, 1991; Pegg, 1995).

Pegg (1995) states that proofs which are counter to intuition can be accepted if the argument is valid. For example, "if the postulate related to parallelism was to be modified to allow two parallel lines to meet at infinity, a logical geometry could be established" (Pegg, 1995:92). Hoffer (1981) explains that the student at this level understands the importance of precision in dealing with foundations and interrelationships between structures. Hoffer (1981) further explains that, at this level students understand, for example, how the parallel postulate (Euclidean) relates to the existence of rectangles and that in a non-Euclidean geometry rectangles do not exist. The other example is given by Krause (1986), as cited in Atebe and Schäfer (2008), that students at this level are able to establish that the locus of all points equidistant from a fixed point is a circle in Euclidean geometry, whereas, the same locus is a square in Taxicab geometry. The examples above demonstrate an advanced level of geometric thinking. Since this is the most advanced level, it is however rarely reached by high school students (i.e. senior secondary students).

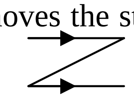
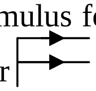
Clements and Battista (1992:429) explain that many school children exhibit thinking about geometric concepts more “primitive than, and probably prerequisite to, van Hiele level 1”. As a result they proposed the existence of Level 0, and referred to it as pre-recognition. Therefore my research study will also take the existence of level 0 into account. At this level, students notice only a subset of the visual characteristics of a shape, resulting in an inability to distinguish between figures (Mason, 1998). For example, a student may distinguish between triangles and quadrilaterals, but may not be able to distinguish between a rhombus and a parallelogram (Atebe & Schäfer, 2008; Mason, 1998).

Since this research study involves Grade 12 students, all activities are restricted to the first four levels of geometric thinking. To sum up, the van Hiele levels represent a broad structure upon which a teaching and learning program can be based (Pegg, 1995). The levels provide a window into students’ understanding and as such represent a useful tool for a teacher. Therefore it is appropriate to discuss a number of important features or properties attributed to the different levels.

2.5.2 Features or properties of the levels

According to van Hiele (1986), the theory is hierarchical in that a student cannot operate with understanding on one level without having been through the previous levels. This has been confirmed in research (de Villiers & Njisane, 1987; Fuys et al., 1988; Burger & Shaughnessy, 1989). Senk (1989) explains that the van Hiele model states that two persons reasoning at different levels may not understand each other. For example, a student who has attained level **n** may not understand thinking of level **n +1** or higher (Mayberry, 1983; Senk, 1989; Pegg, 1995). However, students can simulate higher levels by learning rules or definitions by rote or by applying routine algorithms that they do not understand (Pegg, 1995). The levels are not as discrete as suggested by the descriptions (Burger & Shaughnessy, 1986), rather it appears that students can be in transition between levels and that they will oscillate between them during the transition period. Pegg (1995) clarifies that, except perhaps when one deals with gifted or exceptional students, to move a student from one level to the next requires direct instruction, exploration and reflection by the student. This means that to succeed in moving a student from one level to the next, adequate time should be allowed for this growth to occur. This will allow students to relate information to what is already known, linking the known and the unknown (Etchberger & Shaw, 1992 as cited in Siyepu, 2005). By allowing the latter, students will collaborate when explaining, clarifying, elaborating, questioning and discussing possible solutions to the problem (Siyepu, 2005). Despite it being time consuming to move a student from one level to the next, teachers should understand that progress from one level to the next is more dependent on educational experience than on age or maturation (Pegg, 1995:93; Mason, 2003). Mayberry (1983) and Fuys et al. (1988) assert that there is also evidence that a student's level of thinking might vary across topics and according to how recently a topic was studied. As a result of this, students may differ in their conceptual understanding. This means that students on the same level may not have an identical or same understanding of concepts. However, Pegg (1995) explains that once one concept has been raised to a higher level, it will take less time for other concepts to reach that level.

Pegg (1995) therefore recommends that in moving from one level to the next, students will need to confront a personal 'crisis of thinking'. This means that students should be given challenging tasks that would require them to think and devise their own strategies that will help them to solve the given problem. Through struggling to get the solutions, students will gain insight into what the learning is all about. The activities should be designed in such a way that students are able to go through the levels sequentially. This implies that they cannot be forced to think at a higher level. Pegg (1995) warns that certain teaching strategies inhibit such growth and place boundaries on students' potential. Therefore to avoid the inhibition which may occur because of these certain teaching strategies, teachers should use a systematic approach to instructions, that is, taking progression from one van Hiele level to the next into account.

"Level reduction" refers to a state where the structures at a higher level are re-interpreted at a lower level (Pegg, 1995:93). This usually occurs by making the structures at the higher level visible. Pegg (1995) further warns that the effect of this procedure, when it is teacher-directed, can be counter-productive, because it removes the stimulus for students to attain a higher level. For example, the use of the popular  or  symbol in parallel lines helps students to identify alternate or corresponding angles respectively. Students are told that wherever they see the Z-like sign they must know that it represents alternate angles, or wherever they see the F-like sign they must know that they are dealing with corresponding angles. This leads to students not appreciating the need for learning the proofs of theorems, because the symbols enable students to easily understand the concepts through visualisation.

Language may be instrumental in the interaction between the evidence and reality (van Hiele, 1986; Clements & Battista, 1992; Pegg, 1995). Pegg (1995) further highlights that language problems can occur even among students within the same classroom. For example a rectangle might have different meanings on different levels. A student on the informal deduction level might regard a rectangle as a special kind of parallelogram, but this is not understood by a student on lower levels such as visualisation and analysis. Clements and Battista (1992) explain that each level has its own linguistic symbols. As a result a relation that is "correct" at one level can reveal itself to be incorrect at another level. Therefore, Pegg (1995) cautions that very serious communication problems can easily exist between students on one level and their fellow students, teachers, textbooks, and exercise on another level. For example, when studying number concepts, a square is a product of a number multiplied by itself, whereas in Euclidean geometry a square is a four-sided shape, with all sides equal and all angles each equal to 90°. This situation requires a teacher to make use of the appropriate language of each level in order to avoid confusion.

Each van Hiele level has its own organisation of relationships. Therefore, teachers need to be cautious that what may appear to be correct at one level may not be seen to be correct at a higher level (Pegg, 1995). For example, it is not until at van Hiele level 3, that squares and rectangles belong to the set of parallelograms. The learning process is discontinuous. That is, a student having reached a given level remains at the level for a time, as if maturing (Pegg, 1995). In support of this, Clements and Battista (1992) explain that there are jumps in the learning curve which reveal the presence of discrete, qualitatively different levels of thinking. Further, Pegg (1995) warns that forcing a student to perform at a higher level will not succeed until the maturation process has occurred.

The levels are sequential and hierarchical. Therefore, for students to function adequately at one of the advanced levels in the van Hiele hierarchy, they must have mastered large portions of the lower levels (Hoffer, 1981). Pegg (1995) however emphasises that rote learning or applying routine algorithms without understanding does not represent the achievement of a particular level. Therefore, any information or knowledge acquired without understanding cannot be regarded as the attainment of a certain level of thinking.

In designing learning units, a teacher should consider what the properties of the van Hiele levels imply in that given mathematics classroom (Siyepu, 2005). Clements and Battista (1992) state that, as postulated by the van Hieles, progress from one level to the next depends little on biological maturation or development; instead, it proceeds under the influence of a teaching and learning process. Here the learning phases, as proposed by the van Hieles, play a major role.

2.5.3 Learning phases

The learning phases are phases that a student should go through in each level in order to move from one level to the next. Progress from one level to the next involves five phases (Mayberry, 1983; Hoffer, 1983; van Hiele, 1986). Each phase involves a higher level of thinking. The students' progress from one level to the next is the result of purposeful instruction organised into five phases of "sequenced activities that emphasise exploration, discussion, and integration" (Teppo, 1991:212). Each instructional learning stage builds upon and adds to the thinking of the previous level (Genz, 2006). As a result, the instruction at each learning phase fully and clearly defines that which was implied at the previous phase. In short, the latter implies that the learning phases are useful in designing learning and instructional activities. What follows is a discussion about the phases within a level and the teacher's role in providing instruction that enables this learning:

- **Information/Inquiry**

The students become acquainted with the context domain (Clements & Battista, 1992; van Hiele, 1986). The teacher sets an environment in which the conversation takes place between the teacher and the students about the topic to be studied.

Consequently, this process causes the student to discover a certain structure (Fuys et al., 1988; Presmeg, 1991). During this phase, questions are asked and observations are made by the teacher and students about the objects of the study. This helps the teacher to evaluate students' responses and to determine students' prior knowledge about the topic. Clements and Battista (1992) further explain that the teacher learns how students interpret the language and provides information to bring students to purposeful action and perception.

- **Directed Orientation**

In this phase, students become acquainted with the objects from which geometric ideas are abstracted. The students begin to realise what direction their learning is taking. This helps the students to become familiar with "the principal connection of the network of relations to be formed" (van Hiele, 1986:177). In short, this implies that the students are becoming familiar with the structures of the topic such as the figures, vocabulary, symbols, definitions, properties and relations. The teacher's role is to direct students' activity by guiding them in appropriate explorations (Clements & Battista, 1992). This activity helps the students to explore the field of investigation using the material, for example, by folding, measuring, and looking for symmetry (Mason, 1998). Therefore teachers should choose materials and tasks in which the targeted concepts and procedures are salient.

- **Explicitation/Explanation**

In this phase, the students have gained insights in working with the structures of the topic. Students become explicitly aware of their geometric conceptualisations, describe these conceptualisations in their own language and learn some of the traditional mathematical language for the subject matter (Clements & Battista, 1992; Mason, 1998).

According to van Hiele (1986), during this phase the students learn to speak the technical language. This means that the students are supposed to make their observations explicit and begin to use accurate and appropriate vocabulary with the help of the teacher (Fuys et al., 1988; Mason, 1998).

- **Free Orientation**

In this phase, students solve problems in which the solution requires the synthesis and utilisation of those concepts and relations previously elaborated (Clements & Battista, 1992). Therefore, students prepare themselves for multi-step tasks in addition to the one-step tasks they were familiar with. Van Hiele (1986) points out that it can be said that this is the further development of the second phase in which the student, for example, learns to find his or her way in a network of relations with the help of the connections he or she has at his or her disposal. Fuys et al. (1988) and Presmeg (1991) support the above statement by stating that the field of investigation or network of relations is still largely unknown at this stage, but the student is given more complex tasks to find his or her way round this field. A student might know about the properties for a new shape, for example, a kite. The teacher’s role is to select appropriate materials and geometric problems – with multiple solution paths, to give instructions to permit various performances and to encourage students to reflect and elaborate on these problems and their solutions, and to introduce terms, concepts and relevant problem-solving processes as needed.

- **Integration**

According to van Hiele (1986), the teaching process comes to an end with this final phase indicating that the students have reached a new level of thought, and have increased their thought level in the new subject matter. This means that a student summarises all that he or she has learnt about the subject, reflects on his or her actions and thus obtains an overview of the whole network or field that has been explored, for example, summarises and synthesises the properties of a figure (Fuys et al., 1988). In this phase, the language and conceptualisations of mathematics are used to describe the network (Clements & Battista, 1992). Hoffer (1983) elaborates that the teacher provides summaries of some of the main points of the subject that are already known by the students to help this process. In other words, this phase represents the stage where the teaching-learning process is evaluated.

The van Hiele levels of thinking with the help of the learning phases put an emphasis on conceptual and procedural knowledge. A brief discussion on the implications of the levels and learning phases on the instruction of geometry follows in the next section.

2.5.4 Implications of the levels and phases on instruction of geometry

Geometry taught in the elementary school should be informal and activities should be exploratory and hands-on (Images, 2007:4). Hands-on activities will provide students with the opportunity to investigate, to build and take apart, to create and make drawings, and to make observations about shapes in the world around them (Van de Walle, 2001). This provides the basis for more formal activities at higher levels later on.

Images (2007) explains that teaching a geometry lesson at one van Hiele level when students are functioning at a lower level may hinder students learning. Teppo (1991) supports the latter statement by stating that students who are at lower levels of thinking cannot be expected to understand instructions presented to them at a higher level of thinking. This is because each level of thinking has its own language. For example, a teacher asks his or her students to play the "What am I?" game with properties of geometric figures, saying, "What am I?" To answer this question, a student must be functioning at Level 2 in the van Hiele's levels of geometric reasoning. If the students in this class are functioning at Level 1, where they recognise a figure by its appearance, they will not be able to play the game (Images, 2007). This means that if students are at different levels in one class, the teacher must use differentiated instruction to meet the needs of all his or her students.

Van Hiele (1986) warns that when mathematical language is used too early and when the teacher does not use everyday speech as a point of reference, mathematical language is often learned without concomitant mathematical understanding. This is because students may not be able to distinguish between everyday language and the mathematical language when they move to the more advanced levels.

The separation of simple straightforward tasks and those that are more difficult and open-ended is emphasised by Pegg (1995). Here, Pegg (1995) recommends that when the teacher designs tasks, he or she should ensure that the activities are in line with the levels of thinking. To succeed in moving the students from the lower level to the higher levels, more sophisticated tasks and/or activities should be introduced (Siyepu, 2005). For example, Pegg (1995) suggests that in Phase 2, the tasks should be designed to provide the students with an understanding of the breadth of the field under study. Therefore, the role of the teacher is to ensure that this occurs and it requires specific and careful teacher direction. Questions in phase 4 should not only be challenging, but should also involve multi-path strategies if possible (Pegg, 1995). Here, the role of the teacher is to encourage diversity in solving problems and to assist students in finding relationships and links between different solution paths (Pegg, 1995). This will help students to develop insight into the mathematics they are taught rather than simply be on the receiving end of knowledge in a superficial way. Consequently memorisation of rules and procedures should be discouraged. To acquire conceptual and procedural knowledge in mathematics and especially in geometry requires an appropriate usage of the correct mathematics language. The following discussion analyses the importance of language in geometrical conceptualisation as proposed in the van Hiele levels of thinking.

2.5.5 The role of language in geometrical conceptualisation and van Hiele levels

After independence in 1990, Namibia adopted English as the medium of instruction for Grades 4 to 12 (Namibia. Ministry of Education and Culture [MEC], 1993). The use of English as the medium of instruction has left many students as well as teachers in the Namibian schools struggling to cope with a language that is foreign to them.

In support of the above findings, Setati and Adler (2001) have the following to say:

> Learning and teaching mathematics in a classroom where the language of learning and teaching (LOLT) is not the learners' main language is complicated. This is because learning mathematics has elements that are similar to learning a language, since mathematics, with its conceptual and abstract forms, has a specific register and set of discourses.
>
> (Setati & Adler, 2001:247)

The quote supports the negative perception among some Namibian students and teachers that the use of English as a medium of instruction makes the learning of mathematics difficult. This is because most of the students as well as teachers in the Namibian schools are not English speakers. It further justifies the importance of a mathematical register which as yet is not properly developed in most of the indigenous languages of Namibia. Setati and Adler (2001) further explain that the challenge teachers face is to encourage movement in their learners from the predominantly informal spoken language to formal written mathematical language, and this includes both conceptual and calculational discourses.

Mathematics is a universal language. Therefore the mathematical knowledge begins with the acquisition of linguistic knowledge. This implies that it is only by local contextualisation and application that students will better understand and appreciate the uses of mathematics and thus the meaning of the new terms (Namibia. MoE, 2006b). Ernest (1991) explains that natural language includes the basis of mathematics through its register of elementary mathematical terms.

Through everyday knowledge of the uses and interconnections of the mathematical terms, and through the rules and conventions, the foundation for logic and logical truth are provided (Ernest, 1991). This demonstrates how language facilitates the learning of mathematics in general and geometry in particular. Schiro (1997) explains that when studying mathematics, students learn mathematics through the use of language, whether that language is everyday language or mathematical language. The latter explanation is in support of van Hiele (1986) who states that language is a crucial part of the learning process as students progress through the levels of thinking. Mayberry (1983), Burger and Shaughnessy (1986), and Fuys et al. (1988) warn that imprecise language plagues students' work in geometry and is a critical factor in progressing through levels. Therefore, Fuys et al. (1988) suggest that instruction should carefully draw distinctions between common usage and mathematical usage. For example, teachers are encouraged to consider the students' language when developing ideas, but there is also the need for students to be able to use correct mathematical terminology by the end of the topic (Pegg, 1995). Van Hiele (1986) believes that each level is associated with its own language. Further students may not be able to distinguish between everyday language and the mathematical language when they move to the more advanced levels. But their language and use of it will develop as they create and use their definitions and explanations. This will help students as they progress between van Hiele levels.

Language helps students learn mathematics by constructing new meanings, acquiring new understandings and information, and developing new skills (Schiro, 1997). For example, a student can examine examples and non-examples by using the material presented to him or her. Here he or she will use his or her own everyday language to describe and think about the problem (Van Hiele, 1986). In another example, the teacher directs the class to explore the object of study by means of a number of simple tasks (Genz, 2006). In the process language will allow students to express their mathematical ideas (Schiro, 1997). By so doing, they will be able to put their mathematical thoughts into words or symbols. This will result in discovering, objectifying, and confronting their meanings. As a result of the manipulation of materials and the completion of simple tasks set by the teacher, the need to talk about the subject matter becomes important (Pegg, 1995). This encourages students to better reflect on, organise, clarify, evaluate, comprehend, revise, and express their ideas.

The language will further help students to access, understand, monitor, and orchestrate their own mathematical constructs that will facilitate their enhancement, development and reconstructions (Schiro, 1997). Furthermore, through language, students learn to share the specialised language, knowledge, traditions, and affective stance of mathematics in general and geometry in particular.

When students are given a variety of activities and are expected to find their own way to a solution, language will help them relate new experiences to previous experiences in ways that facilitate assimilation and accommodation (Schiro, 1997). This personalisation will help them to accept the relevancy, meaningfulness, and usefulness of mathematics in their everyday lives (Genz, 2006).

Language helps students to formulate conjectures and convincing argument or proofs, especially in geometry. Schiro (1997) explains that language helps students to make connections. It assists them to see how mathematical ideas can be expressed in different ways, to link informal and intuitive mathematical meanings to more formal, abstract symbolism; to see connections among various forms of mathematical representation (for example, oral, written, concrete, pictorial, numerical, graphical, algebraic and geometrical).

Clarkson (2003) explains that language is vital in the learning process, because students need to discuss and share experiences and ideas and to describe, explain and record mathematics in their own language. It follows that reflecting on learning and recording mathematical ideas in written language can clarify, demonstrate understanding and prompt new thoughts.

The National Institute for Educational Development [NIED] (2003) in Namibia explains that limited language skills inhibit effective learning and teaching, whereas language proficiency facilitates learning. For example, studies of language and learning have shown that when students are engaged in collaborative learning, they negotiate partly explicitly and partly implicitly through their interactions (Namibia. NIED, 2003). This means that through language students share their different understandings of the world and find out what understandings they share with others and what differences they can all accept. With geometry perceived as one of the most difficult learning areas of mathematics, language should be well developed, so that it can help the students to conceptualise geometric concepts, especially in a country like Namibia where English, as a foreign language, is used as the medium of instruction. Thinking, reasoning and conceptualisation all depend on language acquisition. In the following discussion the assessment of students' levels of thinking is dealt with.

2.5.6 How to determine students' levels of thinking

Many researchers used various methods and techniques to assess students' van Hiele levels of thinking. Usiskin (1982) and Fuys et al. (1988) confirmed the validity of the first four levels (visualisation, analysis, abstract/informal deduction, and formal deduction), but not the fifth level, with high school students. According to Usiskin (1982), most students can be assigned a van Hiele level by giving a simple multiple-choice test, hence the construction of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) test. This van Hiele level test was a 25-item multiple choice with 5 foils per item per level. The result of the CDASSG van Hiele level test was used to assign students to the different van Hiele levels of thinking.

Mayberry (1983:59) conducted a study of 19 pre-service elementary school teachers. She designed a task using seven geometric concepts for the first four levels. These were: squares, right triangles, isosceles triangles, circles, parallel lines, similarity and congruency. The task was used to test two hypotheses. These were: “H1: For each geometric concept, a student at level N will answer all questions at a level below N to criterion but will not meet the criterion on questions above level N. And H2: A student will meet the criterion at the same level on all geometric concepts tested”. Interviews were further designed by setting questions, that were validated by 14 mathematicians and mathematics educators who had a special interest and expertise in geometry. The interviews were used to determine the students’ predominant van Hiele level of thinking. This was done to validate the students’ performance in the above explained task.

Burger and Shaughnessy (1986) designed an interview procedure that could reveal predominant levels of reasoning on specific geometry tasks. These tasks were experimentally administered to each student in an audio-taped clinical interview. The outcomes of the interviews allowed them to suggest that educators and researchers could make task assignments based on the levels with little sophisticated data, and they would be suitable to examine students’ answers and subsequently to assign students to van Hiele levels of thinking.

Gutierrez, Jaime and Fortuny (1991) used a 100-point numerical scale between levels. The assessment instrument was used as a spatial geometry test that evaluated the students’ van Hiele levels on three-dimensional geometry tasks. The numerical scale is divided into five qualitative scales: “Values in the interval” (0 – 15%) means “No Acquisition” of the level. “Values in the interval” (15 – 40%) means “Low Acquisition” of the level. “Values in the interval” (40 – 60%) means “Intermediate Acquisition” of the level. “Values in the interval” (60 – 85%) means “High Acquisition” of the level. Finally, “Values in the interval” (85 – 100%) means “Complete Acquisition” of the level.

Atebe and Schäfer (2008) used an adapted van Hiele geometry test and a set of manipulatives consisting of triangles and quadrilaterals of various kinds. These manipulatives were acquired on request for adoption from Feza and Webb (2005). One set of questionnaires consisting of five distinct tasks was developed and used for data collection (Schäfer & Atebe, 2008). The tasks helped them to reach the conclusion that a number of learners who participated in the study were at van Hiele Level 0, as many were only able to distinguish between triangles and quadrilaterals, but lacked the requisite vocabulary to distinguish among shapes in the same class (Atebe & Schäfer, 2008).

This research study uses only two of the above discussed methods of determining students' van Hiele levels of thinking. These are the CDASSG van Hiele geometry test (Usiskin, 1982) and the clinical interview designed by Burger and Shaughnessy (1986). The study adopts the adapted version of the CDASSG test as used in Atebe and Schäfer (2008). This was done because the adapted version is more relevant to the Namibian geometry curriculum. The other reason for using the adapted version of the CDASSG van Hiele geometry test is that it is easy to administer. The clinical interview was also adopted as it was used in Schäfer and Atebe (2008). This was adopted because it contained activities which cover the geometry content of the Namibian geometry curriculum which mainly deals with two-dimensional shapes. The activities make use of constructed triangles and quadrilaterals. These activities are more relevant because they relate physical or external world to the abstract concepts *(*Konyalioglu, Konyalioglu, Ipek & Isik, 2003). By using a visualisation approach many mathematical concepts can become concrete and clear for students to understand. The van Hiele theory advocates the learning of geometrical concepts in a more sequential order. As a result, research studies that deal with similar advocacy were reviewed. This led me to a comparison between the van Hiele theory and Piaget's theory of cognitive development, because both theories propose learning as sequentially ordered.

2.6 The relationship between the van Hiele theory and Piaget's cognitive development theory

In geometry, van Hiele traced cognitive development through a succession of increasingly sophisticated levels (Tall, 2004). The theory begins with young children perceiving objects as whole gestalts, noticing various properties that can be described and subsequently used in verbal definitions to give hierarchies of figures, with verbal deductions that designate how, if certain properties hold, then others follow, culminating in more rigorous, formal axiomatic mathematics (Tall, 2004).

Jean Piaget was a genetic epistemologist whose goal was to describe "the developmental nature of children's thinking in a variety of domains, one of which is space and geometry" (Pusey, 2003:40). The organisation of his theories about development was structured using stages of cognitive development that were typically associated with certain ages. These stages of development are *sensorimotor* (infancy), *preoperational* (early childhood through preschool), *concrete operational* (childhood to adolescence) and *formal operational* (early adulthood) (Mwamwenda, 1989). The relationship between these two theories is interesting because they both included a study about learning geometry and both propose some form of hierarchical structure.

Clements and Battista (1992) however state that there has been little research conducted on the issues of similarities and differences of Piaget's and van Hiele's theories. A few similarities as well as differences are discussed below.

- **Similarities**

Battista and Clements (1995) point out that both Piaget's and van Hiele's theories suggest that students must pass through lower levels of geometric thought before they can attain higher levels and that this passage takes a considerable amount of time.

The van Hiele theory, further suggests that instruction should help students "gradually progress through lower levels of geometric thought before they begin with a proof-oriented study of geometry" (Battista & Clements, 1995:4). This is because students cannot bypass levels or stages and achieve understanding. It further follows that prematurely dealing with formal proof can only lead students to attempt memorisation and to become confused about the purpose of proof (Battista & Clements, 1995).

According to Clements and Battista (1992), both theories emphasise the role of the students in actively constructing their own knowledge, as well as the non-verbal development of knowledge that is organised into complex systems. Therefore this type of learning makes students to not only learn facts, names, or rules, but a network of relationships that link geometric concepts and processes and are eventually organised into schemata. This emphasises the importance of students passing through levels of thinking. Battista and Clements (1995) further elaborate that both theories suggest that students can understand and explicitly work with axiomatic systems only after they have reached the highest levels in both hierarchies. This implies that the explicit study of axiomatic systems is unlikely to be productive for the vast majority of students in high school geometry because most of the senior secondary school students are operating at either van Hiele level 0 or 1 (Usiskin, 1982; Atebe & Schäfer, 2008).

Mwamwenda (1989) states that there is a good number of secondary school students who are still at concrete operational level instead of being at the formal operational level. This situation is similar to the findings of the studies about the van Hiele theory, which indicate that most of the secondary school students are operating at either the pre-recognition level or van Hiele level 1 (Teppo, 1991; Mayberry, 1983; Usiskin, 1982; Mason, 2003; Senk, 1989; Burger & Shaughnessy, 1986).

- **Differences**

Pandiscio and Orton (1998), as cited in Pusey (2003:45), state that the van Hiele theory is different from Piaget's theory in "the movement among levels or stages". Piaget's theory suggests that the movement among stages is dependent on activity while the van Hiele theory suggests that the movement through the levels of thinking is dependent on language. Pusey (2003) further states that the two theories differ in the way that the van

Hiele theory attempts to help teachers improve instruction methods by describing levels of thinking for students, whereas Piaget's theory is focussed more simply on descriptions of the progression and maturity of thinking. This, in short, implies that the van Hiele theory informs instruction while Piaget's theory informs development. Clements and Battista (1992) state that the van Hiele theory would say that students' development in their thinking about reasoning and proof is a growth that is dependent on increasing understanding of geometric knowledge and relationships. Alternatively, Piaget's theory suggests that logical operations develop in students independent of the content to which they are applied. This distinction implies that van Hiele would say a student was ready to prove something if his/her understanding of the content is at an appropriate level (e.g. formal deduction), while Piaget's theory would argue that understanding content is unrelated to a child's readiness for formal argument. Piaget's theory relates mainly to geometry as the science of space, while van Hiele's combines geometry as the science of space and geometry as a tool with which to demonstrate mathematical structure (Hershkowitz, 1990). Piaget's theory is age dependent whilst van Hiele's is dependent on systematic instruction. Numerous studies have been conducted in order to validate whether what the van Hiele theory reported was accurate and consistent. Hence, a brief discussion on some of these research studies that have used the van Hiele theory follows.

2.7 Research studies that have used the van Hiele theory

The van Hiele theory of thinking which was developed and structured by Pierre van Hiele and Dina van Hiele-Geldof in the period from 1957 to 1986 focuses on the teaching and learning of geometry.

Besides a significant amount of research studies into students' understanding of geometric proofs, the van Hiele theory stands out as one of the best recognised frameworks for the teaching and learning of geometry (Dindyal, 2007). As a result, this model is often considered as the foundation for curricula implemented in mathematics classrooms in many countries, such as Netherlands, Germany, Russia and United States of America. Since the mid 1980s there has been a growing interest in the area of teaching and learning geometry (Mayberry, 1983).

With the said interest, a number of research studies were and still are conducted based on the van Hiele theory. Below are a few examples:

Usiskin (1982) used the van Hiele theory to explain why many students have trouble in learning and performing in the geometry classroom. The finding was that the poor performance of many students either in a geometry content test or in proof writing was strongly associated with being at the lower van Hiele levels.

Mayberry (1983) conducted a study of 19 pre-service elementary school teachers. Her findings were that, 70% of the response patterns of the students who had taken high school geometry were below level 4 (formal deduction). The responses of the students implied that the typical student in the study was not ready for a formal deductive geometry course. Her conclusion strengthened the notion that the van Hiele levels of thinking are hierarchical in nature. This means that a student cannot attain a high van Hiele level of thinking before first mastering the lower ones.

Burger and Shaughnessy (1986) examined specific questions related to the van Hiele theory of learning in geometry. The first was regarding the usefulness of the van Hiele levels in describing students' thinking process on geometry tasks. The second was to find out whether the levels could be characterized operationally by students' behaviour.
A third was about designing an interview procedure that could reveal predominant levels of reasoning on specific geometry tasks. Their findings confirmed much of van Hiele's description and characteristic of the levels. According to them, the van Hiele levels are useful in describing students reasoning process for polygons.

Fuys, Geddes and Tischler (1988) pointed out that a student has to go through the levels consecutively; otherwise he or she will not be able to perform the tasks. They agreed that it was important to follow the order of the van Hiele theory's levels in geometry. They further concluded that each level had its own linguistic symbols with its own systems of relations.

Senk (1989) examined the relationship between the achievements in writing geometry proof and the van Hiele levels. For that purpose, she revisited the Cognitive Development and Achievement in Secondary School Geometry (CDASSG), on which Usiskin (1982) had previously worked. Her study reached the conclusion that there was a positive relationship between high school students' achievement in writing geometry proofs and van Hiele levels of geometric thought.

Gutiérrez, Jaime and Fortuny (1991) studied 9 eighth graders and 41 pre-service elementary teachers. The major goal of their study was to find an alternative way of assigning the van Hiele levels of thinking to students who are between two van Hiele levels. They concluded that the van Hiele levels of thinking were not discrete as the other studies suggest, instead they were of a more continuous nature than their discrete descriptions would lead one to believe.

Mason (1997) conducted a research project on the geometric understanding and reasoning of 120 mathematically talented students in the 6^{th} through 8^{th} grades. Her finding was that the performances of these gifted students were higher on the van Hiele geometry test, even if they were in grade 6, than that of those who entered a high school geometry course. She further found that the van Hiele levels were hierarchical even when one was dealing with mathematically talented students.

Siyepu (2005) conducted a research study which used the van Hiele theory to explore the problems encountered by grade 11 learners in circle geometry. His study revealed that many of the grade 11 learners were under-prepared for the study of more sophisticated geometry concepts and proofs. The study further showed that the South African high school geometry curriculum was presented at a higher van Hiele level than what the learners were operating at. His findings also supported the finding that the van Hiele levels of thinking are hierarchical.

Genz (2006) carried out a research study to determine high school geometry students' geometric understanding using van Hiele levels and to answer the question of whether there was a difference between standards-based curriculum students and nonstandards-based curriculum students. Her research revealed that students were not adequately prepared to understand the concepts of geometry, as they were presented in the high school geometry course. This also supported the view that levels of reasoning in geometry are hierarchical.

Atebe and Schäfer (2008) carried out a research study to explicate the van Hiele levels of geometric thinking of Grades 10, 11 and 12. The study involved Nigerian and South African students. Their finding was that many Nigerian and South African upper secondary school students (children) had a weak conceptual understanding of geometry and mostly operated at the pre-recognition level or van Hiele level 1.

The research studies discussed above are relevant to my study. For example, the studies carried out by Usiskin (1982), and Atebe and Schäfer (2008) are very similar to my research study.

The main results of the studies carried out by Usiskin (1982) and Atebe and Schäfer (2008) are as follows:

In Usiskin (1982:99), the study involved 2361 students from 13 high schools selected from throughout the United States of America. The study reported that 222 (9%) were found to be at the pre-recognition level; 1085 (46%) were at level 1; 671 (28%) were at level 2; 283 (12%) were at level 3; 93 (4%) were at level 4 and 7 (0%) of the students could not be assigned to a level. From the study one can clearly see that majority of students who participated in the study were found at the pre-recognition level and Level 1 respectively. Only a very small number of the students had reached level 4 of the van Hiele levels of geometric thinking.

NOTE: The percentages were rounded off to the nearest whole numbers (Usiskin, 1982:99)

In Atebe and Schäfer (2008:12-13), the study involved 144 students from Nigeria and South Africa. 72 students were drawn from each country. The study showed that 68 of the 72 students from Nigerian schools took the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) test items. Of the 68 students who took the test, 36 (53%) were found to be at the pre-recognition level; 15 (22%) were at level 1; 16 (24%) were at level 2; 1(1%) at level 3 and none at level 4.

Of the 72 students from South Africa, 71 participated in the study. From these 71 students, 29 (41%) were found to be at the pre-recognition level; 16 (22%) were at level 1; 17 (24%) were at level 2; 2 (3%) were at level 3; 4 (6%) were at level 4 and there were 3 (4%) who could not be assigned to any of the levels. From the results we can see that majority of the participants from both the two countries were found at the pre-recognition level followed by those at level 1. A very small number of students reached level 4 which is the highest level a student can reach at high school level.

All the research studies above, including the van Hiele theory, demonstrated that learning should be accompanied with understanding. This means that students are expected to construct their own meanings of the concept they learn.

In Namibia, learning with understanding is advocated through learner-centred education. Therefore, the discussion that follows is to determine the domain of the van Hiele theory in learner-centred education in the Namibian context.

2.8 The van Hiele theory and learner-centred education in the Namibian context

The education system in Namibia underwent a reform process after Independence in 1990. This reform included the change of the teaching approach which was more teacher-centred before independence to that which considers the student as the central part of the teaching-learning process.

Beukes, Visagie and Kasanda, in their paper report as follows:

> Mathematics has changed significantly all over the world from being a subject for those gifted in mathematics to a subject for all students. It has also been accepted that to do this successfully, there should be a move from teaching that is teacher-centred and relies on rote learning to one where the student is more active and involved.
>
> (Beukes, Visagie & Kasanda, 2005:11)

As a result, the approach to teaching and learning of mathematics in Namibia is now based on a paradigm of learner-centred education as described in Ministry Policy documents, curriculum guides and the conceptual framework (Namibia. MoE, 2006b). Central to learner-centred education is the view that knowledge is not a static amount of content, but is what the student actively constructs and creates from experience and interaction within the socio-cultural context (Namibia. Ministry of Basic Education [MBE], 2006). Learner-centred education puts more emphasis on co-operative learning, which is also well supported by the van Hiele theory. For example, Pegg (1995) asserts that students should be motivated to exchange ideas about what they have done and what they have found.

Another example is that of van Hiele (1986) who states that students clarify and reorganise their thoughts and understanding of geometric concepts through talking about them. Co-operative and collaborative learning should be encouraged wherever possible. This is because as students develop personal, social and communication skills, they are gradually given increasing responsibility to participate in planning and evaluating their work, under the teacher’s guidance (Namibia. MoE, 2006b). Here the teacher represents a facilitator not information provider.

Knowledge is strengthened and added to within the learning phases between each level (van Hiele, 1986). This means that learning should build upon and add to the previous knowledge learnt. Similarly, Namibia. MoE (2006b) explains that learning at school must involve, build on, extend and challenge the student's prior knowledge and experience.

Both the van Hiele theory and a learner-centred approach put emphasis on language. In support of the emphasis of the van Hiele theory and learner-centred approach, Southwood and Spanneberg (2000) elaborate that working together in small groups gives more students the opportunity to communicate and helps them to make links between language and conceptual understanding. They further support co-operative learning by stating that through the discussion and sharing of ideas, children develop the need for a common language and understand the significance of discussing and clarifying definitions and assumptions.

Despite the strengths of the van Hiele theory pointed out in the proceeding sections of this literature review, there are some weaknesses that were detected by some of the research studies. Therefore, some of these weaknesses are now discussed in the next section.

2.9 Critique of the van Hiele theory

The van Hieles claimed that the levels are discrete (Hoffer, 1981). This claim was contested by Burger and Shaughnessy (1986) who argued that the levels are not discrete. This was because their study failed to detect the discontinuity and found instead that the levels appear dynamic rather than being static and of a more continuous nature than their discrete descriptions would lead one to believe. Burger and Shaughnessy (1986) further explain that their study has found that students may move back and forth between levels quite a few times while they are in transition from one level to the next. This means that students can be in transition between these levels and that they will oscillate during the transition period (Pusey, 2003). As a result of that, there is difficulty in assigning a level to students who do not seem to fit a particular level or are in transition. Furthermore, there is evidence that a student's level of thinking might vary according to how recently a topic was studied (Mayberry, 1983; Fuys et al., 1988). Fuys et al. (1988) also found that a significant number of participants in their study made some progress toward level 2 with familiar shapes such as squares and rectangles, but encountered difficulties with unfamiliar figures. This made them conclude that progress was marked by frequent instability and oscillation between levels. The study carried out by Mason (1997) also highlights that some mathematically talented students appear to skip levels.

Gutiérrez et al. (1991:250) explain that "the levels are not as autonomous in that people do not behave in a single, linear manner, which the assignment of one single level would lead us to believe". As a result, they concluded that students can develop more than one level at the same time. For example, in their study they identified students who could be coded 100%, 85%, less than 40% and less than 15% for levels 1, 2, 3, and 4, respectively.

The van Hiele's claim that class inclusion can only be at level 3, is contested by de Villiers (1994:17) who explains that "dynamic geometry contexts can facilitate the grasping of class inclusion even as early as level 1". For example, de Villiers (1987), as cited in Siyepu (2005) states that students can see a square as a special rectangle at level 1 by simply dragging the rectangle until it becomes a square.
Senk (1989:319) states that the existence of level 0 is the subject of some controversy. Van Hiele does not acknowledge the existence of such a "non-level". Instead, he asserts that all students enter at ground level, that is, at level 1, with the ability to identify common geometric figures by sight. But Usiskin's (1982) research project has shown that level 0 exists. Usiskin (1982:99) found that '222 participants of the 2361 participants' of her study were at level 0.

Van Hiele (1986) doubts the existence or testability of levels higher than the fourth level and considered them as of no practical value. This doubt has consequently led to the reduction of the levels to three (van Hiele, 1986). This reduction was as a result of combining van Hiele levels 3 and 4.

Despite the above shortcomings of the van Hiele theory, I still find the theory a very useful framework for the teaching and learning of geometry. The theory is considered to be one of the best because of its elegance, comprehensiveness and wide applicability (Usiskin, 1982). These characteristics are briefly described in Usiskin (1982:6) as follows:

- elegance means that the theory involves a rather simple structure described by reasonably succinct statements, each with broad effect. For example, the same principles apply for movement from level 1 to 2 as from 2 to 3 and so on.
- comprehensiveness means that the theory covers the whole of learning of geometry. This means that it does not seek to explain only why students have trouble in learning but also what could be done to remove these stumbling blocks. For example, van Hiele (1973), as cited in Usiskin (1982) in his *Begrip en Inzicht* (Understanding and Insight) asserts that the theory applies to all of mathematical understanding and gives examples involving the learning of functions and other non-geometric notions.
- wide applicability means that the theory is widely applied. For example, the theory is widely applied in geometry curricula in countries as diverse as the Netherlands, the Soviet Union (now Russia) and the United States of America. And now in the Southern Africa. The theory is widely considered as the best framework because of its easy applicability.

The literature I have reviewed has shaped the path for my research study. It has helped me to establish the importance of the said theory. The literature review has further strengthened my understanding with regard to the van Hiele theory and how it can be applied in the teaching and learning processes in geometry. It further enlightened me on why geometry is considered to be a difficult branch of mathematics to deal with.

2.10 Conclusion

In this chapter, I focused my discussion on the van Hiele theory and its implications. I initially looked at a brief historical overview of geometry as a branch of mathematics. This was followed by reasoning why geometry is an important learning area in the mathematics curriculum. Due to the complexities involved in the teaching and learning process, I looked at some of the challenges encountered in the teaching and learning of geometry at Grade 12. Since this research is informed by the van Hiele theory, I provided a lengthy discussion about the said theory. The discussion further suggested some of the possible ways to make geometry a learnable branch of mathematics. The importance of language in the teaching and learning of geometry was also examined and its role in geometrical conceptualisation. The relationship between the van Hiele theory of thinking and Piaget's theory of cognitive development was examined and their similarities and differences when applied to geometry were explored.

The van Hiele theory stands out as a very appropriate framework to study geometry. This is evidenced by the many research studies that have used this theory. These were briefly summarised in this review. My research study is carried out in the Namibian context. Therefore I found it necessary to discuss the relationship between learner-centred education and the van Hiele theory. This discussion was included because the Namibian education system advocates learner-centred education/approach. The literature has shown that there is a relationship between a learner-centred approach and the approach of the van Hiele theory. Despite the theory being considered the most prominent framework for the teaching and learning of geometry, it also has its shortcomings. As a result, a critique of the van Hiele theory was also discussed.

The review of literature has provided the framework for the whole research. In the next chapter, the research design or methodology of the study is discussed.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

This chapter outlines the methodology used in this research study. The methodology is discussed in terms of orientation, design and process. It discusses the interpretive paradigm, the use of a quantitative and qualitative case study, and the selection of the research site and participants. It further explains and clarifies the tools and techniques used in collecting data for the research. The data analysis procedures and research ethics are also discussed towards the end of this chapter.

3.2 Orientation

This research study is largely situated in the interpretive paradigm and makes use of both quantitative and qualitative approaches. I thus used a mixed-method approach. Cantrell (1993), as cited in Atebe and Schäfer (2008), states that an interpretive paradigm emphasises an in-depth understanding and interpretation of the subjective experiences of the participants. This study utilises a case study approach. Yin (2003:13) describes a case study as “an empirical inquiry that investigates a contemporary phenomenon within its real-life context”. It is therefore appropriate for me to refer to this study as a case study because it studies a specific issue in mathematics-that is, the application of the van Hiele theory in selected schools in the Namibian context to determine the levels of thinking of grade 12 students. Stake (2000), as cited in Atebe and Schäfer (2008) refers to a case study that combines two research sites as a collective case study. Therefore this study is a collective case study because it utilises two schools - School A and School B - as its research sites.

3.3 Research methods

The research method employed in this study is a quantitative and qualitative case study. This section therefore briefly discusses what a case study is and its importance in educational research. The concepts of quantitative and qualitative approaches are also discussed.

Case study

Following Basey's (1999:75) ideas, "a case study is the study of a singularity which is chosen because of its interest to the researcher". Therefore, I found a case study to be helpful for me to investigate my concerns about the teaching and more specifically the level of learning geometry at Grade 12 in selected schools in Namibia. A case study approach is particularly important for individual researchers "because it gives an opportunity for one aspect of a problem to be studied in some depth within a limited time scale" (Bell, 1993:8; Leedy & Ormrod, 2005:135). This opinion is true for my research study because it explores geometrical conceptualisation in 50 Grade 12 students of the two selected schools within a limited time scale.

Quantitative approach

Leedy and Ormrod (2005:94) explain that quantitative research is used to "answer questions about relationships among measured variables with the purpose of explaining, predicting, and controlling phenomena". In this research study the use of the van Hiele Geometry Test (Usiskin, 1982) was aimed at explaining how the students can be assigned to van Hiele levels of geometric thinking. Jackson (1995:13) further explains that quantitative research "seeks to quantify, or reflect with numbers, observations about human behaviour". The results of the van Hiele Geometry Test were used to determine the number of participants at each van Hiele level. This quantification led to the observation on how the participants were assigned to van Hiele levels using their geometric reasoning.

Qualitative approach

Jackson (1995:17) states that qualitative research emphasises careful and detailed descriptions of "social practices in an attempt to understand how the participants experience and explain their own world". Leedy and Ormrod (2005:94) explain that qualitative research answers questions about the complex nature of phenomena, often with the purpose of "describing and understanding the phenomena from the participants' point of view". Therefore the use of a clinical interview in this study was aimed at allowing the participants to freely express their views with regard to the geometric concepts. The result of the clinical interview assisted in revealing the participants' conceptual understanding of the geometric concepts. The test was analysed quantitatively and the clinical interview qualitatively.

3.4 Research site and participants

In this section, sampling of the research participants and identification of the research site are discussed. The discussion further includes the description of the research site and the research participants.

Research site

This research study was conducted in the Rundu Circuit in the Kavango Region of Namibia. In total, the circuit has 22 primary and secondary schools. Only two senior secondary schools were purposefully selected from the 22 schools. McMillan and Schumacher (2001:598) explain that "purposeful sampling is a strategy to choose small groups or individuals likely to be knowledgeable and informative about the phenomenon of interest and it further refers to a selection of cases without needing or desiring to generalise to all such cases". Therefore the two schools were purposefully selected because they represent the diverse culture of the Namibian nation. School A was a former white school which is now open to all races. School B is one of the oldest schools in the

Kavango Region which accommodates students, mainly black students, from all parts of Namibia. The two schools were selected because they accommodate students with different languages, social background and learning environments. The other reason for selecting these two schools was their proximity in relation to my duty station. This assisted me in avoiding excessive travelling costs.

Research participants

The research study dealt with only one Grade 12 class from each of the two schools. The former white school (School A) has only one class of Grade 12 doing mathematics and sciences. The second school (School B) has seven Grade 12 classes. Four classes out of the seven classes at School B did mathematics and sciences. Since the class at School A accommodated students who followed both the *core* and *extended* mathematics syllabus contents, I used the same condition to select the one class from the four classes of School B. The remaining three classes of School B only followed the *core* mathematics syllabus content. The selection of these classes was prompted by the fact that these students were likely to be knowledgeable and informative about the phenomenon of interest because they were doing the same syllabus contents. The selection was also done in that way to control for the variables in the sample.

The initial statistics I received at the beginning of the first trimester, on request, from the two selected schools, indicated that the total number of the Grade 12 students I intended to use at each of the schools was 35. When the data collection process commenced, I learnt that there was a decrease in the total number of students in each intended class. 12 students from School A took transfers from the school to other schools early in the first trimester. The transfers were necessitated by the fact that they could not cope with mathematics. Instead they went to pursue their studies in the fields that did not include mathematics. Eventually there were only 23 students in this class. For the same reason, 5 of the 35 students in the class at School B, shifted within the same school to other fields of studies without mathematics. As a result, only 30 students remained in the class at School B. In total 53 students should have participated in the research study; instead only

50 students took part because 3 of the 23 students at School A were absent when the van Hiele Geometry Test was administered.

As mentioned earlier, the students followed both the core and extended mathematics syllabus contents of the Ordinary Level of the Namibia Senior Secondary Certificate. Even though these students attended the same classes, not all of them did the extended syllabus content.

Only 4 of the 20 students of School A and 12 of the 30 students at School B did the extended syllabus content. The rest of the students in these classes followed the core syllabus content. This indicates that the two classes accommodated students with mixed abilities in mathematics.

The mean age profile of the participants is presented in Table 3.1.

Table 3.1 Number and mean age of the participants

School	**Number of participants**			**Mean age (in yrs)**
	M	F	Total	
A	10	10	20	17.75
B	23	7	30	18.5
Totals	33	17	50	18.2

The table indicates that in total, 33 boys and 17 girls participated in the study. The total of 33 boys is made up of 10 boys from School A and 23 from School B. The total of 17 girls constitutes 10 girls from School A and 7 from School B. The mean age of the participants from School A is 17.75 years, while that for School B is 18.5 years. The mean age of all the research participants is 18.2 years.

3.5 Data collection

This section describes the instruments used and the process followed in collecting data. Data collection is the process of gathering quantitative or qualitative information with an intention of answering the research question(s) (McMillan & Schumacher, 2001). For that reason, three instruments were used in this research study. These were:

- document analysis
- test
- clinical interview

The data collection process was divided into three phases as follows:

Phase 1: Document analysis

Documents refer to “records of past events in the form of letters, diaries, anecdotal notes, and documents usually preserved in collections” (McMillan & Schumacher, 2001:598). For this research study, the broad curriculum, mathematics syllabi for both the junior and senior secondary phases and past Grade 12 geometry end-of-year national examination questions were the documents analysed. The broad curriculum was analysed to determine what the Namibian Education System requires the Grade 12 students to know with regard to geometry. More importantly, I wanted to establish at what van Hiele levels of geometric reasoning the mathematics syllabus is pegged. The high school geometry builds on the elementary school geometry which traditionally emphasises measurement and informal development of the basic concepts (Dindyal, 2007). These basic concepts are very important in the development of geometry at the high school level. Therefore, the geometry syllabus contents (see Section 4.2.1, Tables 4.1 and 4.2) for both the junior and senior secondary phases were analysed to determine how geometric concepts are developed from Grade 8 through to Grade 12. Geometry questions from the 2007 end-of-year national examination papers (compare Section 4.2.3 Figures 4.1 and 4.2) were analysed to establish the highest possible van Hiele level at which the Grade 12 students in Namibia are assessed. The 2007 end-of-year geometry questions were selected because they were based on the current Namibia Senior Secondary Certificate (Ordinary level) [NSSC (O)] mathematics syllabus.

Phase 2: Test

Cohen, Manion and Morrison (2000:317) explain that in tests "researchers have at their disposal a powerful method of data collection, an impressive array of tests for gathering data of a numerical rather than verbal kind". For this study, the van Hiele Geometry Test, as constructed by staff of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) (Usiskin, 1982), and adapted by Atebe and Schäfer (2008) (see Appendix C) was adopted with their permission. The reason for adopting the adapted version of the van Hiele Geometry Test was because it was appropriately content specific. Van Hiele (1986) and Senk (1989) suggest that studies that seek understanding of the thinking processes that characterised the van Hiele levels of geometric reasoning should be content specific. The other reason for using the adapted version of the CDASSG test was because it was relevant to the Namibian situation. The adapted test contained the aspects of the themes/topics prescribed for geometry in the mathematics syllabus for the Namibia Senior Secondary Certificate (Ordinary and Higher levels) [NSSC (O/H)]. These themes/topics are: geometrical terms, geometrical construction, symmetry, angle properties and locus (Namibia. MoE, 2005a:6–7).

The initial CDASSG test in Usiskin's (1982) project was developed to assess or determine all the five van Hiele levels of geometric reasoning, but the adapted version contained test items that could only investigate the attainment of the first four van Hiele levels. This is because many researchers (van Hiele, 1986; Burger & Shaughnessy, 1986; Senk, 1989; Pusey, 2003; Siyepu, 2005; Genz, 2006; Atebe & Schäfer, 2008) suggest that the highest van Hiele level attainable by a student in formal education is ideally van Hiele level 4. The reduction in levels is further supported by van Hiele (1986), as cited in Atebe and Schäfer (2008) as he disavows the existence of the fifth level, and said:

> Some people are now testing students to see if they have attained the fifth or higher levels. I think this is only of theoretical value… so I am unhappy if, on the ground of my levels of thinking, investigations are made to establish the existence of the fifth and higher levels.
>
> (van Hiele, 1986:47)

Van Hiele clearly argues that the fifth level is not appropriate at Grade 12 level.

The van Hiele Geometry Test consisted of four subtests, each with five multiple-choice items based on each of the van Hiele levels. That is, there were 20 items in all, with numbers 1–5 testing the attainment of van Hiele level 1(recognition/visualisation), 6–10 level 2 (analysis), 11–15 level 3 (informal deduction) and 16–20 level 4 (formal deduction).

Process of test administration

An arrangement was made with the Grade 12 mathematics teacher at each school to inform the students about the writing of the test. This was done three days before the test was taken. The mathematics teachers arranged the venues where the test would be taken. When I arrived at these research sites I found that the venues were well prepared and my task was only to distribute the test papers.

The test was administered in the afternoon on the same day (22 May 2008) at each of the two schools. That is, at half past two in School A and at half past three in School B. I invigilated in each school where the test took place.

Since the number of participants who took the test was reasonably small, I did the marking of the scripts manually. The marking of the scripts started in the evening of the same day the test was written and I completed the process the next day (23 May 2008).

After the test was marked, I gave the scripts together with the marking scheme to one of the mathematics teacher to check for any possible marking errors. This was done in one day and then both the scripts and the marking scheme were brought back to me for analysis.

Phase 3: Clinical interview

Gutiérrez and Jaime (1998:27) opine that "most people agree that a clinical interview is the most accurate way of assessment of the van Hiele levels, since it provides more information about the student's way of reasoning than other procedures".

In conjunction with the interviews I used a set of manipulatives (see Appendix D). The manipulatives were designed in such a way that they engaged the participants and allowed them to express their opinions freely. This instrument was freely acquired from Schäfer and Atebe (2008). I adopted the set of manipulatives because they consisted of triangles and quadrilaterals of various kinds, and this fitted well within the Namibian geometry curriculum. The other reason for adopting the manipulatives was because of their rich variety. They included triangles such as scalene, isosceles, right-angled, and equilateral triangles and quadrilaterals such as squares, rhombi, rectangles, parallelograms, kites and trapeziums. The latter are problematic to many students with regard to identification, description, definition and classification.

3 students from School A and 4 from School B were selected on the basis of their performance in the van Hiele Geometry Test and were asked to participate in a clinical interview. The selection was done as follows: From School A one student at the pre-recognition level, one at van Hiele level 2 and one at van Hiele level 3, while from School B one student at the pre-recognition level, one at van Hiele level 2, one at van Hiele level 3 and one from the group of "no fit". I did not choose any student at van Hiele level 1 because the pre-recognition level and van Hiele level 1 have almost the same features. In support of using manipulatives, van Hiele (1999), as cited in Schafer and Atebe (2008) suggests that giving learners ample opportunity for playful exploration of hands-on manipulatives gives teachers a chance to observe and assess informally learners' understanding of and thinking about geometric shapes and their properties.

Therefore, the use of hands-on manipulatives allowed the students to demonstrate what they know and think about geometric concepts.

Process of conducting the clinical interview

One week after the van Hiele Geometry Test was analysed, the 7 selected students were informed through their mathematics teachers about the clinical interview.

One afternoon at each school was assigned to this activity. A clinical interview was conducted in School A on (28 May 2008), while in School B it was on (29 May 2008). A clinical interview was conducted on two different days because the activities lasted for two hours at each school. The activities in both schools started at half past two and ended at half past four in the afternoon. This activity was carried out in their own classrooms.

One set of questionnaires consisting of five distinct tasks together with the pack of concept cards numbered 1 to 30 was given to each participant. A new complete mathematical set was given to each participant. The instructions on how to carry out the tasks were explained to the students. The five tasks were as follows:

Task 1: Identifying and naming shapes. This task required the students to identify each shape by stating the correct names of the shapes. This required each student to justify his or her naming.

Task 2: Sorting of shapes. This task required the students to sort all 30 shapes into two groups. That is, groups of triangles and quadrilaterals. The students were further required to state the criteria for their groupings, and to state the general/common or collective name of the shapes in either group.

Task 3: Sorting by class inclusion of shapes. This activity required the students to make a further sorting of the shapes in either group into smaller sub-groups of shapes that were alike in some way. Students were requested to state how the shapes in each sub-group were alike. This activity was intended to assess students' knowledge of class inclusion or lack of it.

Task 4: Defining shapes. This task required the students either to state a definition of a shape or list the defining properties of a shape.

A sample question from this task is as follows: *What would you tell someone to look for in order to pick out all the parallelograms from among these shapes?* This question was repeated for rectangles, rhombi, squares, trapeziums and isosceles triangles.

Task 5: Class inclusion of shapes. Students were required to state with justification whether a given shape belongs to a class of shapes with more general properties. A sample question from this task is: *Is shape No.23 a rectangle? How do you know?* Shape No.23 was a concept card of a square in this study. Similar questions were asked for other shapes.

Source (Schäfer & Atebe, 2008:6)

When the students were busy performing the tasks in the classroom, I made some observations. Through observation, I picked up some common responses from the students. I made use of these common responses to conduct unstructured interviews with some of the participants. Notes were taken during these interviews.

3.6 Data analysis

This section discusses the process of how the data collected was analysed. Data analysis is the way of looking for "underlying themes and other patterns" that characterise the case more broadly than a single piece of information can reveal (Leedy & Ormrod, 2005:136).

Document analysis

The geometry syllabus content of the Junior Secondary Certificate (JSC) was analysed to establish how geometric concepts are developed from Grade 8 through to Grade 12. This was done by comparing the themes/topics of the junior secondary geometry syllabus with that of the senior secondary phase.

The analysis of the geometry syllabus content of the NSSC (O/H) was done in order to determine the van Hiele levels of thinking required by the Grade 12 mathematics curriculum in Namibia. Even though that the NSSC (O/H) does not talk about van Hiele levels of geometric reasoning, I used the general objectives (Namibia. MoE, 2005a) in relation to the features of the van Hiele levels as suggested by de Villiers (2003) to determine the highest possible van Hiele levels required by the Grade 12 mathematics curriculum.

Van Hiele Geometry Test

The analysis of the CDASSG test, used in this study, was done using the success criteria suggested by Usiskin (1982:23). There were many success criteria suggested by Usiskin, but for this study, I used the “3 of 5 criterion”. This criterion means that if a student answered correctly at least 3 out of 5 items in a given subtest, he or she was considered to have mastered that level. Usiskin (1982:22) further developed a grading system to assign weighted sum scores for each student. This grading system was used and comprises:

- 1 point for satisfying criterion on items 1–5 (level 1).
- 2 points for satisfying criterion on items 6–10 (level 2).
- 4 points for satisfying criterion on items 11–15 (level 3).
- 8 points for satisfying criterion on items 16–20 (level 4).

Thus, the maximum score obtainable by any student was 1+ 2+ 4+8 = 15 points.

I worked out the weighted sum score for each participant using the grading system above, and then used the weighted sum scores to assign the participants to van Hiele levels of geometric thinking. The following is an example of how the weighted sum scores were worked out: for a student to be at the pre-recognition level, the weighted sums should be (0, 2, 4, or 8). This means a weighted sum of 0, a student did not get at least 3 out of 5 in any of the subtests of the van Hiele Geometry Test.

For the weighted sum of 2, the student got at least 3 out of 5 only at level 2. But because of skipping level 1, the student is classified under the pre-recognition level. For the weighted sum of 4 the participant has obtained at least 3 out of the 5 only at level 3, and for the weighted sum of 8 the participant has obtained at least 3 out of the 5 only at level 4. The student with a weighted sum of 4 or 8 is at the pre-recognition level because of skipping levels 1 and 2 for the weighted sum of 4 or levels 1, 2 and 3 for the weighted sum of 8. This process was continued with the weighted sums for van Hiele levels 1, 2, 3 and 4.

Assignment of levels: Using the 3 of 5 correct success criterion, two methods were used to assign students to levels as follows:

- Classical and modified van Hiele levels: A student's van Hiele level was defined to be the highest consecutive level (beginning from level 0) he or she has mastered. If, for example, a student satisfies the criterion at levels 1, 2 and 4, he or she would be assigned to van Hiele level 2. Usiskin (1982) would only assign modified van Hiele level 2 to such a student, but would not classify the student – for skipping level 3 – under the classical theory (Atebe & Schäfer, 2008).

- Forced van Hiele levels: Usiskin (1982:34) assumed that the "fixed sequential nature of the levels is valid", and therefore believed that a student whose responses "do not fit the sequence is probably demonstrating random fit". As a result, a method was developed for assigning levels to such students as follows: A student is assigned to level n if "(a) the student meets the criterion at levels n and n - 1 but perhaps not one of n - 2 or n - 3, or (b) the student meets the criterion at level n, all levels below n, but not at level n + 1 yet also meets the criterion at one higher level" (Usiskin, 1982:34). This criterion allows for more students to be assigned into levels.

Clinical interview

The tasks of the manipulatives were aimed at establishing the patterns of geometric reasoning of the selected participants up to van Hiele level 3. The results of the clinical interview were not used to assign the seven selected participants to van Hiele levels of geometric thinking. Instead they were used to establish whether or not the participants could portray the geometric reasoning they had shown in the van Hiele Geometry Test; hence, to determine their conceptual understanding of the geometric concepts they have dealt with. To do this, analytic method was used (Schäfer & Atebe, 2008). This method involved the following aspects:

- recognition of types and families of geometric figures.
- definition of a geometrical concept using properties.
- classification of geometric figures or concepts into different families or classes.

The three aspects mentioned above represent the descriptions of van Hiele levels 1, 2 and 3. Since the tasks of the manipulatives were based on these aspects, it was reasonably easy to determine the line of reasoning each of the participants had shown.

For example, to establish whether the student had portrayed the line of geometric reasoning at the pre-recognition level or van Hiele level 1, the following was taken into account: (i) a student who was either not able to or was only partly able to sort the shapes into groups of triangles and quadrilaterals was deemed to be at the pre-recognition level, whereas (ii) a student who was able to sort the shapes into distinct groups of triangles and quadrilaterals and was able to state the correct criterion for sorting was considered to be at van Hiele level 1 (Schäfer & Atebe, 2008). This process was continued in order to determine which participants had shown the lines of geometric reasoning of van Hiele levels 2 and 3.

The responses of the students on each of the five tasks entailed in the manipulatives as well as that of the unstructured interviews were analysed.

3.7 Validity

This section outlines the validity of the research instruments I used to collect data for my research. These instruments are document analysis, the van Hiele Geometry Test and a clinical interview.

Validity is an important key to effective research; if a piece of research is invalid then it is worthless. Therefore the instruments used in this study were checked for validity. The documents analysed were found to be valid because they were all consistent with the Namibian education system.

The van Hiele Geometry Test was first developed by Usiskin (1982) to test the geometric reasoning of the American students. Atebe and Schäfer (2008) adapted this test for their study with the Nigerian and South African students. My study is similar to the studies of Usiskin (1982) and that of Atebe and Schäfer (2008). Therefore I utilised the adapted test as it was used by Atebe and Schäfer (2008). Knowing that the adapted version of the test was based on the mathematics curricula of Nigeria and South Africa, I decided to pilot it in order to check for its suitability in the Namibian context. The pilot study was carried out with students of a school different from the schools I used in the actual research study. Results of the pilot study were discussed and analysed with Atebe and Schäfer during the April 2008 MEd contact session and were presented in a table (see Chapter four, Section 4.3). The discussion had helped me to determine the relevancy and validity of the test in the Namibian context.

The manipulatives were not piloted because I found the manipulatives tasks to be very specific and straightforward in relation to the Namibian curriculum.

The three methods used in collecting and analysing the data, helped me with the triangulation of the findings. Triangulation is defined as "the use of two or more methods of data collection in the study of some aspects of human behaviour" (Cohen, Manion & Morrison, 2000:112).

3.8 Ethical issues

This section discusses the ethical issues in my research study.

"… and whenever human beings are the focus of investigation, we must look closely at the ethical implications of what we are proposing to do" (Leedy & Ormrod, 2005:101). The above warning means that a researcher has the obligation to protect the anonymity of his or her research participants and to keep research data confidential (Frankfort-Nachmias & Nachmias, 1992).

To comply with the warning given above in my research, I heeded the following ethical recommendations made by McNiff (1996:35):

Negotiate access: The inspector of education of the Rundu circuit was approached at the beginning of the first trimester of this year (2008). The aim was to seek permission to use the two selected schools as my research sites. Then, the inspector of education referred me to the principals of the two schools. The two principals were approached in February 2008. I explained the aim of my research to them and then asked whether I could use their schools as my research sites. They positively agreed. After obtaining the permission from the principals, I approached the students and discussed the aim of my research project with them as well. The students agreed to participate in the research. Later in the course of the first trimester I wrote consent letters to the principals of the two schools (see Appendix A). Since the students were minors, I wrote a consent letter to the parent(s) or guardian(s) of each student (see Appendix B). The letter was given to each student to take to his or her parent(s) or guardian(s). I received a positive response from the parent(s) or guardian(s) of each student.

Anonymity: During the discussions with the principals and the students, and in the consent letters, the issue of anonymity was pointed out. Both the principals and the participants were informed that the schools and the names of the participants would be kept anonymous. As a result, the research sites were referred to as School A and School B. Codes were used to refer to the research participants, that is, participants from School A were coded from NNAS 01 – NNAS 20, while the participants from School B were coded as RNAS 01– RNAS 30.

Right to withdraw: It is the researcher's duty to inform the participants that their participation in the research is strictly voluntary (Dane, 1990). Therefore in the consent letter to the parent(s) or guardian(s) and during the discussion with the participants, they were informed that the participation in the research was on a voluntary basis. They had the right to withdraw from the research at any time.

3.9 General issues

This section sheds some light on some of the problems encountered during the process of collecting data. The problems were as follows:

During the administration of the van Hiele Geometry Test in School A, 3 participants were absent. When I enquired from the mathematics teacher about the whereabouts of these students, I was told that they had permission to be excused from the afternoon studies because all three of them had problems to solve at their homes. As a result, these students were excluded from the entire research process.

The other problem encountered was the duration of the clinical interview. The activities/tasks took longer than anticipated at both schools. As a result, the participants stayed longer at school. The last participant was interviewed up to half past six and I offered to take the student home.

3.10 Conclusion

This chapter described the whole research design of my study. It started with an outline of the orientation in which the study was located. The study was oriented in an interpretive research paradigm which used quantitative and qualitative approaches. The study was a case study and its meanings and importance were discussed.

The chapter talked about the research sites and participants. The sites as well as the participants and the way they were sampled were described. This study used three research instruments – documents analysis, van Hiele Geometry Test and a clinical interview. These instruments were discussed and described in detail.

The process of data collection was also discussed in detail. Aspects of validity were also analysed. The chapter ended with the discussions on ethical issues as well as some general issues that came up during the process of data collection.

The research design explained the strategies of collecting data for the research study. In the next chapter, the data collected is presented, analysed and discussed.

CHAPTER FOUR

DATA PRESENTATIONS, ANALYSIS AND DISCUSSIONS

4.1 Introduction

This chapter focuses on the process of presenting and analysing data and thereafter discusses the findings. Data presentation refers to the manner in which the collected information is displayed to the reader of the report. Leedy and Ormrod (2005) recommend that the data should be presented thoroughly and, of course, accurately. They further state that it is helpful to organise some of them into tables, figures, and other concise presentations. Therefore in this study, I have used tables, figures and a graph. The data presented is that of the documents perused, the test, as well as the clinical interview results. Data analysis involves “working with data, organising it, breaking it down, synthesising it, searching for patterns, discovering what is important and what is to be learned, and deciding what you will tell others” (Southwood & Spanneberg, 2000:60).

The analysis started off with an analysis of various documents. These documents are the Namibian geometry syllabus contents for the junior secondary and senior secondary phases. The two geometry syllabi were studied and analysed to establish how geometric concepts are developed from the junior secondary phase through to the senior secondary phase. Thereafter, an analysis was carried out in order to determine the relationship between the senior secondary phase geometry syllabus and the van Hiele levels of geometric reasoning. A further document analysis was carried out on the national 2007 end-of-year geometry examination questions. This was done in order to determine the highest proposed van Hiele level at which the Grade 12 students in Namibia are assessed in geometry in the end-of-year national examinations. This was followed by a brief presentation of the results of the pilot study.

The third instrument used was the van Hiele Geometry Test (CDASSG). The results of this test were analysed in order to assign the research participants to the van Hiele levels of geometric thinking. This was followed by an analysis of the individual item of the van Hiele Geometry Test to find out how the research participants tried to answer each item. The latter was followed by analysis of the results of the clinical interview. The discussions on some of the findings follow towards the end of this chapter.

4.2 Document analysis

4.2.1 The relationship between the Junior Secondary (JSC) and the Namibia Senior Secondary Certificate [NSSC (O/H)] geometry syllabus contents

Table 4.1 below shows the geometry themes/topics and the learning objectives at the Junior Secondary Phase in Namibia. This information is extracted from the Mathematics Syllabus (Namibia. MoE, 2006b:6). The whole mathematics curriculum constitutes the themes/topics, the learning objectives and basic competencies (see Appendix E for the geometry curriculum). The themes/topics, learning objectives and basic competencies are arranged as follows in the syllabus: For example Grade 8:

Theme/Topic 2: Constructions
Learning objective: know how to perform geometrical constructions using straightedges
Basic competencies:

- measure lines and angles
- construct triangles, given three sides; two sides and the included angle; a right angle, and any two sides; or two angles and a corresponding side
- construct other simple geometrical plane figures from given data

From the example above it can be seen that the learning objective is derived from the theme/topic; it represents the general knowledge and understanding, and demonstrates skills on which students in Grade 8 are assessed with regard to the theme/topic of constructions. The basic competencies, on the other hand, are derived from the learning objective and are used to develop activities for the students that are aimed at achieving the learning objective. For the students to realise the learning objective in the example above, they are required to perform certain activities. These activities involve measuring of lines and angles and the construction of triangles when sufficient or adequate information is provided. Therefore the learning objective and the basic competencies are interrelated in such a way that they show a gradual development of the geometric concepts.

Table 4.1 Geometry syllabus content: Junior Secondary Phase: Grades 8–10

Theme/Topic	Grade 8	Grade 9	Grade 10
1.Geometrical terms and relationships	Use terminology of lines, angles and triangles	Understand and apply the Theorem of Pythagoras	Apply the properties of similar triangles
2. Constructions	Construct and measure lines and angles; construct triangles	Construct parallel and perpendicular lines and angle bisectors.	Perform geometrical constructions, for example: accurate scale drawings of maps, nets of cubes, cuboids, triangular prisms and cylinders
3. Symmetry and transformations	Find line and rotational symmetry in plane figures	Interpret and draw reflections and rotations of plane figures	Construct and describe enlargements with positive whole numbers as scale factors
4. Angle properties	Apply the properties of angles on lines and of angles in triangles	Know and understand angle properties of quadrilaterals to solve problems	Know and understand angle properties of polygons, and of angles in circles

Table 4.2 below displays the themes/topics and the general objectives of the geometry content at the Senior Secondary Phase in Namibia. The information is extracted from the Namibia Senior Secondary Certificate Syllabus of the Ordinary and Higher levels

(Namibia. MoE, 2005a:6-7). The whole mathematics curriculum contains themes/topics, general objectives and specific objectives (see Appendix F). The following example outlines how the theme/topic, general objective and specific objectives are related.

Theme/topic: geometrical terms and relationships
General objective: know and use geometrical terms and the vocabulary of simple plane figures and simple solids
Specific objectives:

- use and interpret the geometrical terms: point, line, parallel, intersecting, bearing, right angle, acute, obtuse and reflex angles, perpendicular, similarity, congruency
- use and interpret vocabulary of triangles, quadrilaterals, circles, polygons and simple solid figures, including nets
- use the relationships between areas of similar triangles, with corresponding results for similar figures and extension to volumes and surface areas of similar solids

The example above shows that the general objective is derived from the topic; it represents the general knowledge and understanding, and demonstrates the skills on which students are assessed in that specific theme/topic. The specific objectives are the detailed and specific content of the syllabus that are derived from the general objective. These specific objectives are used to develop activities on which the students are assessed (Namibia. MoE, 2005a). The activities developed from the specific objectives are aimed at the achievement of the general objectives. For the students to realise the general objective in the example above, they need to carry out activities that involve the use and interpretation of the geometrical terms as listed, the use and interpretation of the vocabulary of triangles, quadrilaterals, circles, polygons and simple solid figures, including nets, and use the relationships between areas of similar triangles, with corresponding results for similar figures and extension to volumes and surface areas of similar solids. The explanation above demonstrates how the theme/topic, general objective and specific objectives are interrelated.

Table 4.2 Geometry Syllabus Content: Senior Secondary Phase (Grades 11–12)

Themes/Topics	**General Objectives**
1. Geometrical terms and relationships	Know and use geometrical terms and the vocabulary of simple plane figures and simple solids.
2. Geometrical constructions	Measure lines and angles and construct simple geometrical figures using straight edges, compasses, protractors and set squares.
3. Symmetry	Recognise properties of simple plane figures directly related to their symmetries.
4. Angle properties	Calculate unknown angles using the geometrical properties of intersecting and parallel lines and of simple plane figures [reasons may be required but no formal proofs].
5. Locus	Determine the locus (path) of point under certain conditions

The tables 4.1 and 4.2 above are displayed in order to try and establish how geometric concepts are developed from Grade 8 through to Grade 12.

4.2.2 The Namibia Senior Secondary Certificate Ordinary and Higher levels [NSSC (O/H)] and the van Hiele levels of geometric thinking

Table 4.3 below shows the possible correlation between the [NSSC (O/H)] and the van Hiele levels of geometric reasoning. The van Hiele theory is not mentioned anywhere in the Namibian mathematics curriculum or more specifically in the geometry syllabus. The correlation was done by relating the specific objectives of the geometry syllabus with the properties/features of the van Hiele levels of geometric reasoning as suggested by de Villiers (2003:12). The table only shows the themes/topics, general objectives and the van Hiele levels. The specific objectives could not be included in this table because they are too numerous and would have taken up too much space.

Table 4.3 The correlation between the NSSC (O/H) geometry content and the van Hiele levels of geometric reasoning

Themes/Topics	General Objectives	Van Hiele Levels			
		1	2	3	4
1. Geometrical terms and relationships	Know and use geometrical terms and the vocabulary of simple plane figures and solids	x	x	x	
		1	2	3	4
2. Geometrical constructions	Measure lines and angles;	x			
	Construct simple geometrical figures using straight edges, compasses, protractors and set squares	x	x		
		1	2	3	4
3. Symmetry	Recognise properties of simple plane figures directly related to their symmetries	x	x	x	
		1	2	3	4
4. Angle properties	Calculate unknown angles using the geometrical properties of intersecting and parallel lines and of simple plane figures **[reasons may be required but no formal proofs]**	x	x	x	
		1	2	3	4
5. Locus	Determine the locus (path) of points under certain conditions	x	x		

NOTE: An **x** proposes the van Hiele level at which each geometry theme/topic is presented at Grades 11–12 level in Namibia.

To determine the possible van Hiele levels, the specific objectives for each theme/topic were extracted and compared with the features/properties of the van Hiele levels. For example, the following are the specific objectives of theme/topic 3 and how they were related to the van Hiele levels of geometric reasoning:

- recognise line and rotational symmetry (including order of rotational symmetry) (**van Hiele level 1)**
- recognise properties of triangles, quadrilaterals and circles directly related to their symmetries **(van Hiele level 2)**

- use the following symmetry properties of circles:
  - equal chords are equidistant from centre
  - the perpendicular bisector of a chord passes through the centre of a circle
  - tangents from an external point are equal in length

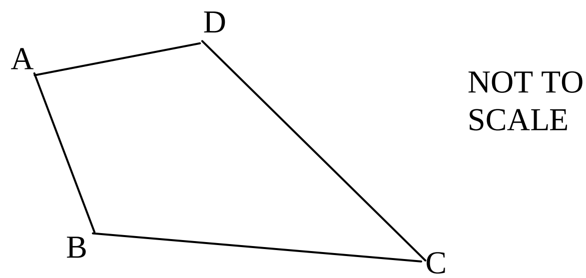
  (van Hiele level 3)

In the first specific objective, students are required to identify the number or type of symmetry a given shape has. This qualifies for van Hiele level 1. The second specific objective requires the students to recognise properties of the two-dimensional shapes. This is the type of geometric reasoning that prevails at van Hiele level 2. Students are required to discuss and know the properties of shapes at van Hiele level 2. In the third specific objective, students are required to use the symmetry properties of the circles and informally deduce that equal chords are equidistant from centre, the perpendicular bisector of a chord passes through the centre of a circle and tangents from an external point are equal in length. This type of geometric reasoning is required at van Hiele level 3. After all these relationships between the specific objectives and the van Hiele levels were established, I indicated it in the table. That is why the table above shows that the highest possible van Hiele level at which theme/topic 3 can be taught is level 3.

4.2.3 Past geometry examination questions and the van Hiele levels of geometric thinking

Past geometry examination questions were extracted from papers 1 and 2 of the 2007 end-of-year Namibian Grade 12 national examinations. These questions are presented in figures 4.1 and 4.2 respectively. This section strives to establish the highest van Hiele level at which the Namibian Grade 12 students are assessed at national level.

Question 4



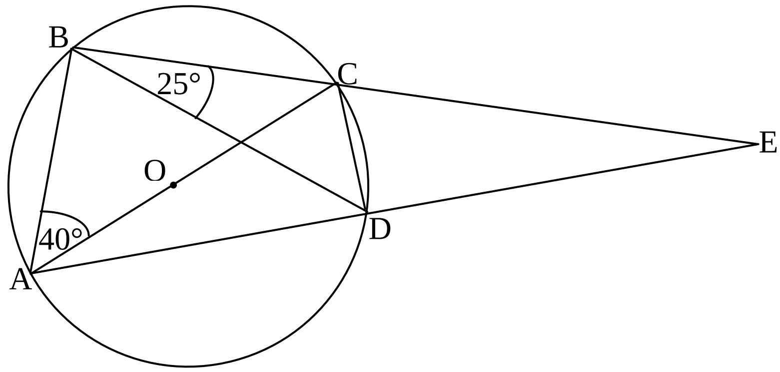
In quadrilateral ABCD, AB = AD and BC = CD.
(a) What is the special name given to ABCD?
(b) How many lines of symmetry does ABCD have?

Figure 4.1 A sample of geometry questions from Paper 1 (Ordinary Level-Core syllabus) of the 2007 end-of-year Namibian Grade 12 national examinations (Namibia. MoE, 2007a:2**)**

Question 4(a) required students to identify the quadrilateral ABCD and state its name. The question was even made easier by stating that the adjacent sides are equal in length (AB = AD and BC = CD). With the provided information, the quadrilateral is identified as a kite. Question 4(b) tested the students' knowledge about the concept of symmetry which is also one of the properties of a kite. The students were required to state the number of lines of symmetry. In my view, the two sub-questions were aimed at assessing van Hiele levels 1 and 2 respectively.

Question 16

NOT TO SCALE



A circle with centre O has chords BC and AD which are extended to meet at point E. Angle BAC = 40^0 and angle CBD = 25^0. AOC is a straight line.

(a) Give a reason why angle CAD = 25^0.
(b) Find the size of angle BCD, giving a reason for your answer.
(c) Find the size of
    (i) angle BDA
    (ii) angle BED
(d) Give the special name for triangle BDE.

Figure 4.2 A sample of geometry questions from Paper 2 (Ordinary Level-Extended syllabus) of 2007 end-of-year Namibian Grade 12 national examinations (Namibia. MoE, 2007b:8).

In sub-question 16(a), the students were required to know the theorem "*angles subtended by the same chord are equal*". Therefore this sub-question assesses the student's knowledge on van Hiele level 3 as it requires skills characterized by informal deduction. Sub-question 16(b) also assessed the students at van Hiele level 3. This is because for the students to solve this type of question they needed to carry out an informal deduction. Sub-questions 16(c)(i)-(ii) required the knowledge of van Hiele level 2. This is because the students were required to know the properties of the given triangles in order to solve the questions. The last sub-question assessed the students' van Hiele level 1 of geometric thinking as it required the students to only identify the triangle BDE. Students were expected to use their answer in sub-question 16(c)(ii) together with the given value of angle CBD to identify triangle BDE. From these observations, it can be seen that the end-of-year national examinations for Grade 12 in Namibia are set up in line with the objectives as stipulated in the syllabus. It further shows that the highest van Hiele level of geometric reasoning at which the students were assessed in geometry in Namibia in 2007 was level 3.

4.3 Analysis of the pilot study results

In this section the results of the pilot study carried out using the van Hiele Geometry Test are presented and briefly discussed. The analysis of these results was done using the (3 out of 5) criterion as proposed by Usiskin (1982). The criterion means that a student is considered to have attained a van Hiele level if he or she has correctly answered at least three items of the five items that make up the subtest of each van Hiele level. The modified van Hiele theory was used because level 5 did not form part of the van Hiele Geometry Test that was used in this study.

Table 4.4 Number and percentages of participants at each van Hiele level of geometric reasoning

Classical/modified van Hiele theory 3 of 5		
Van Hiele Level	**N**	**%**
0	25	78.1
1	4	12.5
2	3	9.4
3	0	0
4	0	0
Total fitting	32	100
No fit	0	0
Totals	32	100

Table 4.4 indicates that 25 (78.1%) of the 32 students who participated in the pilot study were found at the pre-recognition level. 4 (12.5%) and 3 (9.4%) of the participants were respectively found at van Hiele levels 1 and 2. None of the participants was either at van Hiele level 3 or 4. The table further shows that all the participants were assignable to van Hiele levels of geometric reasoning using the 3 of 5 van Hiele modified theory.

4.4 Analysis of the van Hiele geometry test results

4.4.1 Assignment of van Hiele levels of geometric thinking

Usiskin (1982:99) proposed a schematic description of 32 possible profiles of meeting or not meeting the criteria at the five van Hiele levels together with the corresponding weighted sum and assignment of forced van Hiele levels. For this study, the schematic description was adapted to give 15 profiles as presented in table 4.5A and table 4.5B. This was done because the research participants are Grade 12 students, ideally who are expected to function up to van Hiele level 4. The meaning of the weighted sums is explained in Chapter 3, Section 3.2

Table 4.5A. Schematic description and number of students at each of forced van Hiele assignment, School A subsample.

Weighted sum	Level 1	2	3	4	3 of 5 criterion	Total (%) at level
Forced VHL0 = 0C0, M0					4	
2		x			2	
4			x		0	
8				x	1	7(35)
Forced VHL1= 1C1, M1	x				5	
5	x		x		0	
9	x			x	0	5(25)
Forced VHL2 =3C2, M2	x	x			5	
11	x	x		x	1	6(30)
Forced VHL3= 6		x	x		0	
7C3, M3	x	x	x		2	2(10)
Forced VHL4 =13	x		x	x	0	
14		x	x	x	0	
15C4, M4	x	x	x	x	0	0(0)
Forced No Fit =10		x		x	0	
12			x	x	0	0(0)
Totals						20(100)

NOTE: An x indicates that the student has met the criterion at that level.

Table 4.5A indicates that of the 20 (3 students were absent) School A subsample that participated in this research study, 7 (35%) were at the pre-recognition level of geometric thinking. 5 (25%), 6 (30%) and 2 (10%) were respectively found at van Hiele levels 1, 2 and 3. None of the students reached van Hiele level 4.

Table 4.5B. Schematic description and number of students at each level of forced van Hiele assignment, School B subsample.

Weighted sum	Level 1	Level 2	Level 3	Level 4	3 of 5 criterion	Total (%) at level
Forced VHL0= 0C0, M0					1	
2		x			7	
4			x		2	
8				x	2	12(40)
Forced VHL1= 1C1, M1	x				5	
5	x		x		1	
9	x			x	0	6(20)
Forced VHL2= 3C2, M2	x	x			6	
11	x	x		x	1	7(23.3)
Forced VHL3 = 6	x	x			0	
7C3, M3	x	x	x		2	2(6.7)
Forced VHL4 =13	x		x	x	0	
14		x	x	x	1	
15C4, M4	x	x	x	x	0	1(3.3)
Forced No Fit =10		x		x	1	
12			x	x	1	2(6.7)
Totals						30(100)

NOTE: An x indicates that the student has met the criterion at that level.

Table 4.5B depicts that of the 30 (all students were present) School B students who participated in this research study, 12 (40%) were found at the pre-recognition level of geometric thinking. 6 (20%), 7 (23.3%), 2 (6.7%) and 1 (3.3%) were respectively functioning at van Hiele levels 1, 2, 3 and 4 of geometric thinking.

Table 4.6 Number and percentages of students at each forced van Hiele level

	School A		School B	
Level	**N**	**%**	**N**	**%**
0	7	35	12	40
1	5	25	6	20
2	6	30	7	23.3
3	2	10	2	6.7
4	0	0	1	3.3
Total Fitting	20	100	28	93.3
No Fit	0	0	2	6.7
Totals	20	100	30	100

From the table above it can be seen that all 20 (100%) of School A participants are assignable to van Hiele levels, compared to the participants of School B where 28 (93.3%) are assignable and 2 (6.7%) do not fit the criteria for classification. A further analysis was carried out to determine the distribution of the School A and School B students into the van Hiele levels according to the classical/modified van Hiele levels.

Table 4.7 Number and percentage of School A and School B students at each classical/modified van Hiele level

Classical/Modified van Hiele levels				
Levels	**School A**		**School B**	
	N	**%**	**N**	**%**
0	7	35	12	40
1	5	25	6	20
2	6	30	7	23.3
3	2	10	2	6.7
4	0	0	0	0
Total Fitting	20	100	27	90
No Fit	0	0	3	10
Totals	20	100	30	100

Table 4.7 shows that using the (3 out of 5) modified van Hiele theory, all (100%) of the School A students are assignable to the Hiele levels. The table further indicates that using

the same theory, only 27 (90%) of the School B students are assignable to van Hiele levels of geometric thinking, with none of them reaching van Hiele level 4.

And the number of students in School B who could not fit the classification criterion increased to 3 (10%) compared to the 2 (6.7%) in Table 4.6. The information in the table above is presented in the figure below.

Assignment of the participants to van Hiele levels using the modified van Hiele levels (3 out of 5).

The bar graph in the figure below shows how the participants were assigned to van Hiele levels using the modified van Hiele levels. Usiskin (1982:79) refers to the van Hiele theory with level 5 as the “classical theory”, while the theory without level 5 is called the “modified theory”. This study uses the modified theory for the reason that level 5 was not part of the test that was used. The representation below indicates the modified van Hiele levels and the percentage of students at each level.

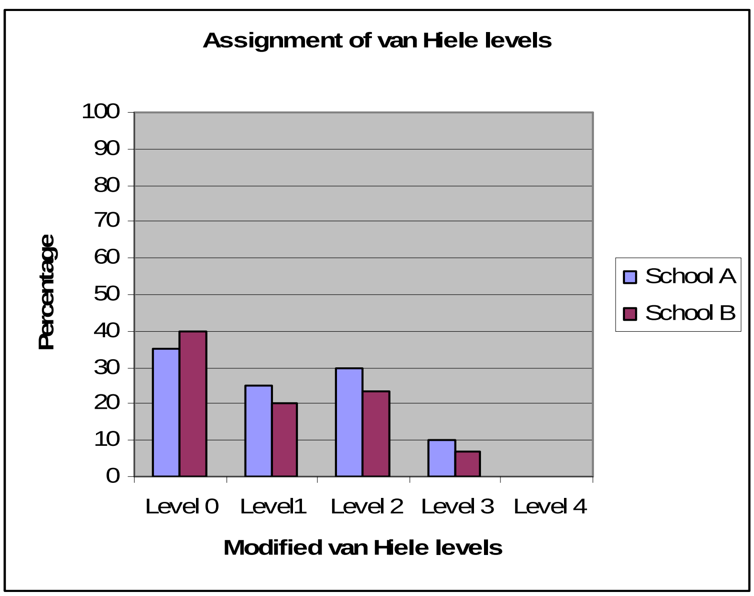


Figure 4.7 Bar graph of the assignment of participants to modified van Hiele levels.

The figure shows that there were (40%) of the participants from School B, compared to the (35%) of the participants from School A who were at the pre-recognition level.

The graph reveals that the percentage of participants at level 2 in both schools is more than that at level 1. In School A, 25 % of participants were at level 1 and 30% were at level 2. In School B, 20% were at level 1 and 23.3% were at level 2. This finding is consistent with those of Atebe and Schäfer (2008) and Usiskin (1982). In the study of Atebe and Schäfer (2008), 24% of the students from Nigeria were at level 2 with 22% at level 1. 24% of the students from South Africa were at level 2, while 22% were at level 1. Also Usiskin's (1982:105) results showed that "13% and 23% of her research participants were respectively at levels 1 and 2". This study could not establish the reason for this finding. But I opined that perhaps the items for van Hiele level 2 of the test were straightforward and biased towards the identification of properties of shapes, whereas the items for van Hiele level 1 were about recognising shapes by means of visual considerations of the concept as a whole without explicit regard to properties of its components which is more difficult for some of the students.

From the graph it can further be seen that 10% of participants in School A and 6.7% of participants in School B were at level 3. And none of the participants from the two schools was at level 4 as per the 3 of 5 of the modified van Hiele levels.

The findings presented in the figure concur with those of the previous research studies (Usiskin, 1982; Burger & Shaughnessy, 1986; Senk, 1989; Pusey, 2003; Siyepu, 2005; Atebe & Schäfer, 2008). The findings of the studies mentioned here, indicated that the majority of their research participants were found to be operating at the pre-recognition level, and that a very small number of students operated at van Hiele levels 3 and 4. This is problematic, as in Namibia, level 3 skills are required to successfully complete the Grade 12 syllabus. Teaching and learning in geometry is mainly focused on van Hiele levels 1 and 2, with a small amount of geometry work being done at level 3. My results show that most participants only operate on the pre-recognition, 1 and 2 levels.

I now provide a detailed analysis of each item of the van Hiele Geometry Test.

4.4.2 Analysis of items

In this section students' performance per item is looked at.

Table 4.8 Van Hiele Geometry Test: Item analysis for each level per school

Level	Choice	Item	1A	1B	2A	2B	3A	3B	4A	4B	5A	5B
1	A		0	0	**15**	**22**	6	6	1	6	12	26
	B		4	6	1	0	1	0	6	13	0	1
	C		1	0	3	3	**11**	**19**	**12**	**5**	5	3
	D		0	0	1	3	2	3	0	4	1	0
	E		**15**	**24**	0	2	0	2	1	2	**2**	**0**
2		Item	6A	6B	7A	7B	8A	8B	9A	9B	10A	10B
	A		1	2	0	2	4	0	1	2	**15**	**21**
	B		**12**	**20**	2	1	1	7	11	16	2	4
	C		3	5	**16**	**20**	**6**	**11**	2	4	0	2
	D		0	2	1	4	1	7	2	2	2	2
	E		4	1	1	3	8	5	**4**	**6**	1	1
3		Item	11A	11B	12A	12B	13A	13B	14A	14B	15A	15B
	A		0	1	5	9	0	2	4	10	3	5
	B		**6**	**13**	0	1	5	10	7	5	**5**	**10**
	C		6	3	2	1	0	4	**5**	**9**	2	2
	D		3	3	**4**	**0**	1	0	2	3	3	6
	E		5	10	9	19	**14**	**14**	2	3	7	7
4		Item	16A	16B	17A	17B	18A	18B	19A	19B	20A	20B
	A		**3**	**4**	**7**	**11**	2	5	**2**	**10**	6	12
	B		0	4	1	0	1	3	5	13	2	2
	C		7	3	2	12	7	7	2	2	**5**	**9**
	D		2	8	4	4	0	1	7	2	2	4
	E		8	11	6	3	**10**	**14**	4	3	5	3

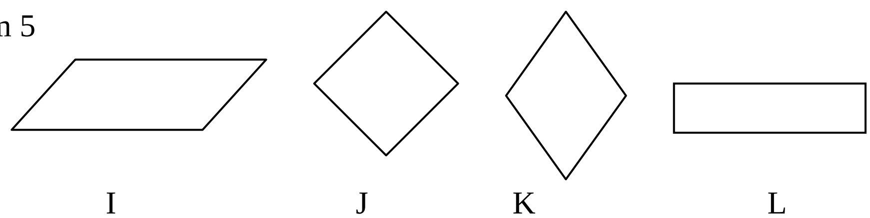
NB: The figures in bold represent the total number of students who answered that item correctly. The A and B represent School A and School B.

Table 4.8 indicates how the students tried to answer each item in each subtest of the van Hiele Geometry Test. The item analysis was done per subtest as follows:

Subtest 1: van Hiele level 1

The students of School A performed well only in the first four items of subtest 1. The table shows that 15 (75%), 15 (75%), 11 (55%) and 12 (60%) of School A managed to answer items 1, 2, 3 and 4 in that order, compared to the students of School B, with 24 (80%), 22 (73.3%) and 19 (63.3%) who managed to answer the first three items as reflected. School B did not do well in item 4 with only 5 (16.7%) managing to answer the item correctly. Both schools did poorly in item 5.

Item 5



Which of these are parallelograms?
A. I only
B. L only
C. I and K only
D. J and L only
E. All are parallelograms

Figure 4.3 A sample of the items for Subtest 1.

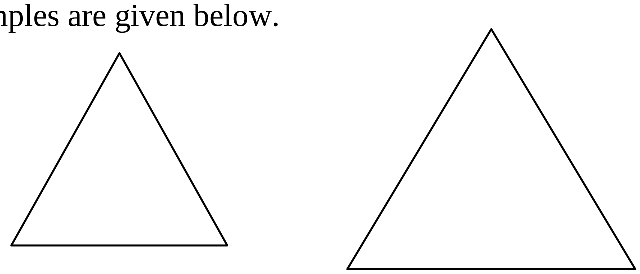
The correct answer for the item in Figure 4.3 is choice E. Table 4.8 indicates that only 2 (10%) of School A subsample knew that all the given quadrilaterals can be referred to as parallelograms, while none from School B subsample got the answer correct. This shows lack of knowledge about 'class inclusion' in 18 (90%) and 30 (100%) of the students who participated in this research study.

Subtest 2: van Hiele level 2

Students in both schools performed fairly well on items 6, 7 and 10. Of the 20 students of School A, 12 (60%), 16 (80%) and 15 (75%) respectively answered items 6, 7 and 10 correctly, while 20 (66.7%), 20 (66.7%) and 21 (70%) of the students from School B managed to answer the same items correctly. Participants from both schools did not do well in items 8 and 9. Of the 20 students of School A, 6 (30%) compared to 11 (36.7%) of the 30 students from School B managed to answer item 8 correctly. Both schools performed poorly in item 9.

Item 9.

An equilateral triangle is a triangle with all the three sides equal in length. Two examples are given below.



Which of (A) – (D) is true in every equilateral triangle?
A. Each angle is an acute angle.
B. The measure of each angle must be 60^0.
C. Each angle bisector is a line of symmetry.
D. Each angle bisector must also bisect the opposite side perpendicularly.
E. All of (A) – (D) are true.

Figure 4.4 A sample of the items for Subtest 2.

Of the given choices for the item in Figure 4.4, choice E is the correct answer. Table 4.8 indicates that only 4 (20%) of School A participants and 6 (20%) of School B participants answered the item correctly. This means that 80% of the participants from each school incorrectly answered the item. This reveals students' lack of knowledge about the properties of an equilateral triangle.

Subtest 3: van Hiele level 3

In general, the performance of the participants for both schools in subtest 3 was very poor, except for School A in item 13, where 14 (70%) of the participants correctly answered the said item. Subtest 3 is about students knowing the properties of given figures and using these to place figures with common properties in one class. Of the five items of the subtest, item 12 was extremely poorly attempted by both subsamples. This item is presented in the figure below.

Item 12

Which is **true**?

A. All properties of rectangles are properties of all parallelograms
B. All properties of squares are properties of all rectangles.
C. All properties of squares are properties of all parallelograms.
D. All properties of rectangles are properties of all squares.
E. None of (A) – (D) is true.

Figure 4.5 A sample of items for Subtest 3

The correct choice is D. From Table 4.8 it can be seen that only 4 (20%) of participants in School A correctly answered the item, while none (0%) of the participants in School B answered the item correctly. This situation means that 80% of School A and 100% of School B research participants did not know that rectangles have common properties with squares. This suggests that students have difficulties in understanding 'class inclusion'.

Subtest 4: van Hiele level 4

The research participants performed poorly in this subtest. Only 10 (50%) of the participants in School A answering item 18 correctly, compared to 14 (46.7%) of the participants in School B. The figure below presents a sample of the items for subtest 4.

Item 20

Here are three properties of a figure.
Property P: It has diagonals of equal length.
Property Q: It is a square.
Property R: It is a rectangle.

Which is **true**?

A. P implies Q which implies R.
B. P implies R which implies Q.
C. Q implies R which implies P.
D. R implies Q which implies P.
E. R implies P which implies Q.

Figure 4.6 A sample of items for subtest 4.

The correct answer for this item is Choice C. Table 4.8 shows that only 5 (25%) of the participants in School A correctly answered item 20 compared to 9 (30%) of the participants in School B. The item wanted the students to establish the relationship between a square and a rectangle that is brought about by the concept of diagonals. The item further wanted the students to establish that a square is a special rectangle. I suggest that the use of the concept 'implies' might have caused a problem to the students. The notion of 'implication' may be unfamiliar to students.

Participants' performance at each van Hiele level

The bar graph in Figure 4.8 below presents the performance of the research participants in each subtest of the van Hiele Geometry Test. The representation indicates the van Hiele levels and the percentage of the scores obtained by the students per item per level.

The bar graph shows that School A performed better in subtests 1 and 3. The participants from School A obtained 55% and 34% in the subtests of levels 1 and 3 compared to 45.3% and 31.3% obtained by the participants from School B in the same subtests. On the other hand, School B outperformed School A in subtests 2 and 4. The graph shows that School B obtained 54.7% and 32% in subtests 2 and 4 respectively, while School A obtained 52% and 27%.

A further revelation made by the graph is that the participants answered the subtests of van Hiele levels 1 and 2 far better than the subtests for van Hiele levels 3 and 4. This is because the first two van Hiele levels are about visualisation and analysis of the geometric shapes, while at van Hiele levels 3 and 4 the students are required to carry out proofs.

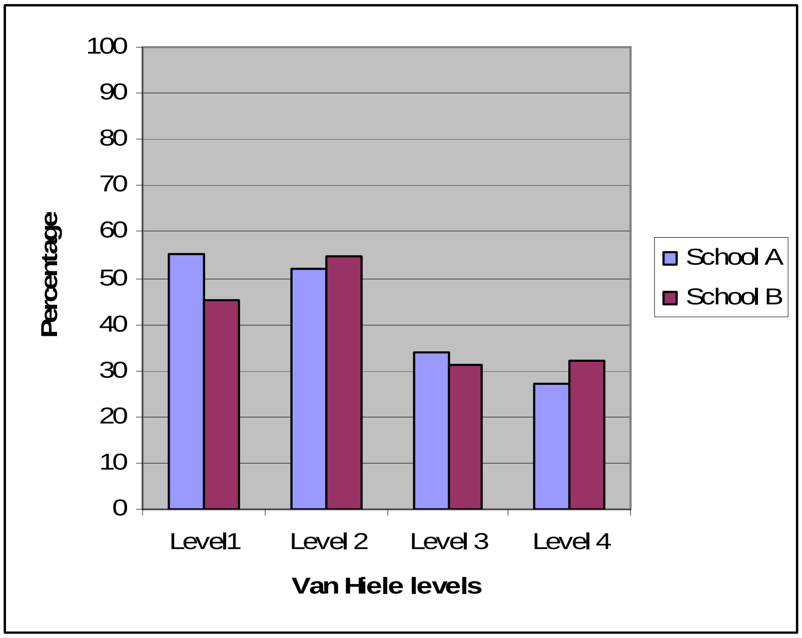


Figure 4.8 Bar graph of the performance of the participants at each van Hiele level in percentages

From the graph it can be seen that the performance of the participants from School B was inconsistent. The said participants performed better at van Hiele level 2 than level 1; this is still observed with regard to their performance in the subtests of van Hiele levels 3 and 4. I therefore opined that because the van Hiele Geometry Test was in the form of multiple choice, the participants from School B might have guessed to obtain the answers. This type of performance might have led to the majority of the participants from School B being assessed as operating at the pre-recognition level, and to three of them not fitting the classification criterion. Finally, the graph shows that participants from both schools performed better in the subtests of van Hiele levels 1 and 2 than in van Hiele levels 3 and 4. This is because the items for van Hiele levels 1 and 2 are straightforward compared to that of levels 3 and 4 where informal and formal deductions are required.

4.5 Analysis of the clinical interview

Clinical interviews were used by Burger and Shaughnessy (1986) and Feza and Webb (2004) as their tool for collecting data. The results were used to assign their research participants to van Hiele levels. For this research study, I have used the clinical interview to help me establish whether the seven selected participants would confirm similar patterns of geometric reasoning as they did in the van Hiele Geometry Test. The seven selected participants were purposefully drawn from the larger population of the research participants who were already assigned to van Hiele levels using their results for the van Hiele Geometry Test performance.

Exploratory Analysis

To analyse the results of the clinical interview, I adopted an exploratory analytic method used by Mayberry (1983), Genz (2006) and Schäfer and Atebe (2008) (see Chapter 3, section 3.6). The exploratory analytic method enabled me to establish patterns in the way the participants geometrically reasoned about the geometric concepts they have dealt with.

In order to explore the patterns of geometric reasoning that could possibly be displayed by the participants, students carried out certain activities. These activities included identifying and naming of geometric shapes, defining shapes and sorting geometric shapes by class inclusion. This meant that the participants were first given geometric shapes to identify and name. Thereafter, the participants were asked to define the given shapes by using certain properties. These activities culminated in the participants being expected to sort the geometric shapes by class inclusion. The results of the activities were used to establish the type of reasoning each of the participants had shown. The result of each activity was recorded in a table. The responses of the participants were further used to conduct unstructured interviews. This was done in order to gain a deeper understanding of how these participants reasoned about the given geometric concepts.

Identifying and naming shapes task:

Table 4.9 Number of students who named geometric shapes correctly and who stated the correct reason for naming each shape.

Shape No.	Name of shape	No. correctly naming shape		No. stating correct reason	
		School A (n = 3)	**School B** (n = 4)	**School A** (n = 3)	**School B** (n = 4)
1	Rhombus	3	2	3	3
2	Isosceles trapezium	2	3	2	3
3	Rectangle	3	3	3	3
4	Obtuse-angled scalene triangle	2	4	1	4
5	Rectangle	0	3	0	3
6	Square	2	4	3	4
7	Isosceles trapezium	2	3	1	4
8	Kite	3	3	2	2
9	Rhombus	2	2	2	1
10	Isosceles triangle	3	3	2	3
11	Parallelogram	3	2	3	3
12	Equilateral triangle	1	3	1	3
13	Rhombus	2	1	2	1
14	Isosceles triangle	1	2	0	2
15	Rectangle	3	4	3	3
16	Isosceles trapezium	2	3	1	1
17	Rectangle	2	3	2	3
18	Equilateral triangle	1	2	2	2
19	Rectangle	1	2	1	1
20	Right-angled trapezium	2	3	1	2
21	Right-angled isosceles triangle	2	1	1	1
22	Right-angled isosceles triangle	1	2	0	2
23	Square	3	3	3	3
24	Right-angled isosceles triangle	2	2	1	3
25	Parallelogram	3	3	3	4
26	Right-angled trapezium	0	2	1	2
27	Scalene triangle	2	4	2	4
28	Kite	3	4	3	3
29	Parallelogram	3	4	2	3
30	Right-angled scalene triangle	2	3	2	3

Table 4.9 shows some important misconceptions about geometric concepts among the research participants. Most of the participants identified shapes by the property of sides. This finding is consistent with that of Schäfer and Atebe (2008) and that of Burger and Shaughnessy (1986). Six out of the seven students identified the shapes by the property of sides. The table shows that students can easily identify a shape when it is in an orientation that they are familiar with. For example, shapes No. 3, 5, 15, 17 and 19 are rectangles. It was easy for the students to recognise shape No.3 as a rectangle (all three students of School A and 3 of the four students from School B). This was different with shape No.5 where none of the seven students realised that it is also a rectangle. This observation is evidenced in an interview with student NNAS13.

The interview took place as follows:

Researcher: *I have seen that your answer for shape No.5 is a square. Are you sure that it is a square?*

Student: *Yes, sir.*

Researcher: *Why do you refer to it as a square*?

Student: *I have measured the angles. The angles are* 90^0 *each.*

Researcher: *Is that enough to justify your answer?*

Student: *Ja…ja. I think so!*

Researcher: *Did you measure the sides of the shape?*

Student: *No. Sir, I only measured the angles.*

Researcher: *Try to measure the sides.*

Student: *I am trying…. Sir the side on top is equal to the one at the bottom. And the one on the right is equal to the one on the left. Then I think it is not a square.*

Researcher: *Now, what name do you give to shape No.5?*

Student: *Sir, I think, hmmm…it is a rectangle.*

Researcher: *Thank you very much for participating in this interview.*

The interview with student NNAS13 shows that some students are likely to use only one property of a given shape for its identification and naming. This type of reasoning mainly prevails in students who are operating at either the pre-recognition level or van Hiele level 1. This is because at those levels students make use of the physical appearances only to identify and name shapes.

All of the seven students recognised shape No.15 as a rectangle. Six of the seven students managed to use the property of sides to justify why shape No.15 is a rectangle.

The following are some of the common reasons used when quadrilaterals were identified: "it's having four sides", "the opposite sides are parallel", and "all sides are equal". One student RNAS18 explained that "a trapezium is almost like a triangle but its top is not pointed and its base is wider than the top". This shows that students are used to a certain orientation of specific shapes. For example, in a right-angled trapezium some students were unable to recognise that it is also a trapezium due to its orientation. For example, shape No.20 (a right-angled trapezium), only two of the students from School A and three of the students from School B were able to recognise that it correctly.

The most commonly given reason(s) for the identification of triangles is/are "it is having three sides", "three sides are equal", "two sides equal", and "no side is equal". The first reason refers to a triangle in general. The second and third reasons refer to equilateral and isosceles triangles respectively, while the last reason refers to a scalene triangle.

Another issue revealed by this task was that none of the students spoke about "isosceles trapezium", "right-angled trapezium", "right-angled isosceles triangles", "right-angled scalene triangles", "obtuse-angled triangles" and "acute-angled triangles". Instead, the students just indicated "trapezium" and, "isosceles, equilateral, and right-angled triangles" in general. This type of revelation shows that despite the students being provided with mathematical sets, they did not use them effectively to establish these extra properties that could help them to distinguish trapeziums among themselves. These findings are consistent with that of Schäfer and Atebe (2008).

Table 4.10 Number of students who successfully sorted shapes into groups of triangles and quadrilaterals.

Activity	**School A (n = 3)**		**School B (n = 4)**	
No. correctly sorting shapes into 3-sided and 4-sided shapes	1		0	
No. sorting shapes by property of sides	2		4	
Concept	Triangles	Quadrilaterals	Triangles	Quadrilaterals
No. stating the correct name of the group of shapes	1	2	3	1

Table 4.10 indicates that none of the students from School B managed to correctly sort the shapes into two groups of 3-sided shapes and 4-sided shapes. For example, one student, RNAS18, from School B tried to list down shapes under "Group A" as triangles but included shapes No. 8 and No. 13 which are kites. To answer the question "What would you tell someone to look for in order to pick out, from among these shapes, a shape that belongs to: Group A?" the student RNAS18 explained that "*all the shapes in Group A are most triangles only two kites are also there and they have three sides*". Student RNAS30 did not list down the shapes that belong to Group A, instead the student indicated that all the shapes under Group A are "Right angles triangles". Student NNAS19 included shape No.0 (pentagon) in Group A and shape No. 28 (kite) in Group B respectively. This student, in Question 2(ii) (a) said "*look at how many sides does the shape have. Which shows that all angles from Group A have 4-sides*". The student made this remark without knowing that shape No.0 that was included in Group A (4-sided shapes) has five sides. For Question 2 (ii) (b), the same student said "*look at the sides must have 3 sides*" but shape No.28 is a kite and it has four sides.

Even though none of the participants from School B managed to correctly list/sort the shapes under Group A and Group B, all managed to state that triangles have three sides while quadrilaterals have four sides. Only two students from School A managed to correctly sort shapes by property of sides.

Only one student from School A stated that shapes with 3-sides are called triangles and two stated that all 4-sided shapes are quadrilaterals. In School B, three students indicated that shapes with 3-sides are triangles, while only one student indicated that 4-sided shapes are quadrilaterals.

Defining shapes task

This activity expected the participants to define the given shapes by means of their properties.

Table 4.11 Defining shapes task.

Shapes	**Number stating the correct definition**	
	School A (n = 3)	**School B (n = 4)**
1. Parallelograms	1	0
2. Rectangles	2	1
3. Rhombuses	2	0
4. Squares	3	4
5. Trapeziums	3	1
6. Isosceles triangles	2	3

Table 4.11 shows how the students performed in the task of defining shapes. It reflects that all the seven students managed to define a square. This shows that most of the students are more familiar with a square than the other shapes. The table further shows that students of School A had a better understanding of defining the given shapes than students of School B. There is a weak understanding among students with regard to the concept of parallelogram. As can be seen from Table 4.11, only one student from School A managed to define a parallelogram, while there was none who could do so in School B.

The following are a few examples of the definitions from the students about trapeziums:

Student NNAS15: "*Only one pair of side on top must be parallel*".

From the definitions, student NNAS15 has tried to be precise with regard to the definition of a trapezium-hence the use of the phrase "only one pair of side".

This student's definition was spoiled by the inclusion of the phrase "on top". The definition implies that the "only one pair of side" which must be parallel should be on top. The question is "on top of what?" Leaving out the phrase "on top", the definition is still imprecise and incorrect. This is because the student left out the concept "opposite". The correct definition should have been "Only one pair of opposite sides must be parallel".

Student NNAS19: "*When you look at a trapezium it has two sides moving a bit like towards one another but on top of these two equal lines there is a shorter length than the bottom side, which is longer*".

Student NNAS19 has tried to describe an isosceles trapezium because it has two equal adjacent sides. The student lacks geometrical conceptualisation of the concepts "isosceles trapezium and adjacent sides". He used the physical appearance to describe the shape. This type of reasoning is likely to place this student either at the pre-recognition level or van Hiele level 1. The student further made use of phrases such as "on top", "a shorter length", and "the bottom side". These demonstrate that the student has the knowledge that when a trapezium is in the orientation as described by the student, there are bottom and top sides that are related. The description could not explicitly make it clear how the top and the bottom sides are related. This means that the definition is incomplete. The other aspect revealed by this student's definition is the use of physical appearance to describe a shape. This is a result of the student's lack of knowledge about the precise properties of a trapezium.

Student RNAS05: "*A shape which have two parallel diagonals, but one parallel diagonals are shorter than the other*".

Student RNAS05 demonstrates a grave lack of geometric conceptual understanding. The student confuses diagonals with sides. The student talks of the diagonals of a trapezium being parallel which is never true.

Student RNAS30: " *Trapezium it have greater base on the bottom of it and lower base on top on it and this base are parallel and two sides equal*".

Student RNAS30 also described an isosceles trapezium. The student talked of "greater base on the bottom of it", "lower base on top on it" and "this base are parallel and two sides equal". Despite using confusing phrases, one has the understanding that the student was talking about an isosceles trapezium. This student does not posses the knowledge about the meaning of a 'base". That is why the student stated that the "base are parallel". This shows that this student does know that a base refers to the bottom side of the shape (in this case a trapezium).

All the definitions above reveal that even though that the research participants are Grade 12 students, they still have a problem in using the appropriate descriptions to describe most of the geometric shapes. The definitions further show that most of the research participants lack fundamental geometrical conceptualisation.

Sorting by class inclusion task

This task required the participants to identify common properties among the given shapes and sort them by class inclusion.

Table 4.12 An example of possible class inclusion: Quadrilaterals

Quadrilaterals	**No. stating the correct possible class inclusion**	
(Class)	**School A (n = 3)**	**School B (n = 4)**
1. **Parallelograms:** Squares, Rhombuses, Rectangles	0	0
2. **Rectangles:** Squares, Rhombuses, Parallelograms	0	0
3. **Squares:** Rhombuses, Rectangles	0	0

The table shows examples of possible class inclusions with regard to quadrilaterals. As it can be seen, none of the seven students managed to form the classes as shown. This means that students sorted the shapes so as to prohibit class inclusions. This finding is consistent with that of Schäfer and Atebe (2008). All seven students, (3 from School A and 4 from School B) excluded rectangles, squares and rhombuses from the class of parallelograms. They did not perceive squares as rectangles, or squares as rhombuses.

Table 4.13 An example of a possible class inclusion: Triangles

Triangles	Number of students stating the class inclusion	
(Class)	School A (n = 3)	School B (n= 4)
1. Acute-angled triangles: (12; 20; 27)	0	0
2. Right-angled triangles: (21; 22; 24; 30)	0	0
3. Equilateral triangles: (10; 12; 14; 18)	1	1
4. Isosceles triangles: (10; 21; 22; 24)	2	1
5. Scalene triangles: (4; 27; 30)	2	1

Table 4.13 indicates that all the seven students excluded isosceles triangles (shapes No. 21, 22 and 24) from the class of right-angled triangles (shapes No. 21, 22, 24 and 30). None of the seven students realised that the scalene triangle (shape No.27), and the equilateral triangle (shape No. 12) belong to the class of acute-angled triangles (shape No. 12, 20 and 27). Only two of the students from School A recognised that the class of isosceles triangles included triangles (shapes No.10, 21, 22, and 24) compared to one from School B. Two of the students from School A realised that the class of scalene triangles included obtuse-angled triangle (shape No. 4), acute-angled triangle (shape No.27) and right-angled triangle (shape No.30), while only one student from School B managed to state that class inclusion.

Table 4.14 Students' responses to the class inclusion task.

Question posed	No. with correct response	
	School A (n =3)	School B (n =4)
Is shape No.23 a rectangle?	0	1
Is shape No. 17 a parallelogram?	0	1
Is shape No. 6 a rhombus?	1	0
Is shape No. 1 a parallelogram?	1	0
Is shape No. 30 a scalene triangle?	2	2

NOTE: A student was considered to have answered correctly if he/she responded in affirmative and gave a correct reason to justify his/her answer (Schäfer & Atebe, 2008). From the table, it is clear that class inclusion is one of the major problems as far as students' geometrical conceptualisation is concerned. None of the three students from School A had the knowledge that shape No. 23 (a square) belongs to the set of rectangles, while there was only one student from School B who knew about this class inclusion. A rectangle belongs to the set of parallelograms, but none of the three students from School A appreciated that rectangles belong to the set of parallelograms, while only one student from School B could indicate that. Only two students from each school knew that the right-angled triangle (shape No.30) is a scalene triangle.

The outcomes of the manipulatives showed that none of the seven students correctly identified and named all the 30 shapes. Most of them showed the line of reasoning at either the pre-recognition level or van Hiele level 1, because when they were asked to describe the given shapes, they only described them by the property of sides. Most of the seven participants did not know the properties of the given shapes they dealt with. They lack the knowledge required by van Hiele level 2. The participants were unable to form class inclusions. This showed that the participants were not familiar with the concept of class inclusion which is one of the features or properties of van Hiele level 3. Therefore, this indicated that none of the participants displayed the line of reasoning at van Hiele level 3. The results of the manipulatives are consistent with that generated with the van Hiele Geometry Test. The majority of the participants were mainly operating at the pre-recognition level, and a smaller but significant number at van Hiele levels 1 and 2.

4.6 Discussion

During data analysis, certain issues emerged. Some of these issues were: how geometric concepts are developed through to Grade 12; some of the challenges in learning geometry at Grade 12; the relationship between the van Hiele theory and the NSSC (O/H) geometry curriculum; using the van Hiele levels to determine students' geometric reasoning; language issues and class inclusion. Therefore the discussions in this section are based on these topics.

4.6.1 How geometric concepts are developed through to Grade 12

Tables 4.1 and 4.2 indicate that the geometry content at Junior Secondary phase is strongly linked to the geometry content of the Senior Secondary Phase in Namibia. The only exception is the addition of theme/topic 5 to the Senior Secondary Phase. In my view, theme/topic 5 in the Senior Secondary Phase geometry content is a consolidation of the work done in theme/topic 2 of both phases. This is because theme/topic 5 involves constructions that are mainly done in theme/topic 2. Sherard (1981:23) states that "geometric skills are important in architecture and design, in engineering, and in various aspects of construction work" (see Chapter 2, Section 2.3). Therefore, I consider that the inclusion of theme/topic 2 in the geometry syllabi of both the junior and senior secondary phases asserts the importance of geometry. Through constructions, students can develop skills of applying geometry through modelling and problem-solving in real world contexts (see Chapter 2, Section 2.3). The other slight difference in the geometry curricula of the two phases is the inclusion of the concept of transformation in theme/topic 3 of the junior secondary while it is not the case with the senior secondary phase.

In my view, geometrical concepts are gradually or sequentially developed from the junior secondary phase through to the senior secondary phase. For example, in theme/topic 4 in Grade 8, students are expected to "apply the properties of angles on lines and of angles in triangles"; in Grade 9, students should "know and understand angle properties of quadrilaterals to solve problems"; while in Grade 10, students should "know and understand angle properties of polygons, and of angles in circles. In Grade 12, theme/topic 4 requires the students to "calculate unknown angles using the geometrical properties of intersecting and parallel lines and simple plane figures – triangles, quadrilaterals, other polygons and circles (reasons may be required but no formal proofs). The discussion above shows the gradual development of concepts, that is, students should be able to know about triangles, quadrilaterals, and other polygons while in Grades 8, 9 and 10. This knowledge is expected to be used by these students while in Grade 12 to solve problems that concern the stated geometric concepts.

4.6.2 Some of the challenges in learning geometry at Grade 12

Despite the apparent gradual development of concepts, Tables 4.1 and 4.2 only to a small extent support Clements and Battista's (1992:422) claim that geometry curricula consist of a "hodgepodge" of unrelated concepts with no systematic progression to higher levels of thought, levels that are required for sophisticated concept development and substantive geometric problem solving (see Chapter 2, Section 2.4). For example, in Grade 9, students are expected to "apply the Theorem of Pythagoras". This theorem is not mentioned in either Grade 8 or 10. It is also not mentioned under the geometry curriculum in Grade 12. Now, the question is when did the students in question know about the "Theorem of Pythagoras" in order for them to apply it?" Are the students only expected to know about the theorem in Grade 9 and not further than that?

Tapson (2006:90) defines a polygon as "a plane shape completely enclosed by three or more straight edges". Therefore the introduction of the concept polygon in Grades 8 and 10 causes the students in Grade 9 to believe that triangles and quadrilaterals are not also part of the class of polygons. This is because the Grade 9 syllabus only talks about the concept of quadrilaterals with no reference to the concept polygon. If the syllabus (Namibia. MoE, 2006b) required the students to know about the concept of polygon, the concept should have been used consistently throughout the Junior Secondary phase. This would make students understand the concept polygon from Grade 8 and carry that understanding through to the higher grades. The concept of polygon is still confusingly used in Grade 12. That is, students should be able to calculate unknown angles using the following geometrical properties: "angle properties of triangles and quadrilaterals, and angle properties of regular and irregular polygons". In my view, this is confusing because "an equilateral triangle and a square" are also regular polygons. And the remaining triangles and quadrilaterals are simultaneously irregular polygons. Again, the question is which of the shapes are referred to as regular and which are irregular polygons? Another challenge learnt from the results of the van Hiele Geometry Test and the clinical interview is that, even if the students know the names of geometric figures, they are not familiar with their properties, and are not always able to point out specific differences expressed in the definitions (see Chapter 2, Section 2.4). This finding was evidenced by responses to Figures 4.4 and 4.5, and the results shown in Table 4.11.

Figure 4.4 tested the students' knowledge about the properties of an equilateral triangle. The results from this item showed that 80% of each school's participants did not know that all the listed properties are for an equilateral triangle. Figure 4.5 asked the students to establish how a square is related to a rectangle. The outcome showed that 80% of the participants from School A and 100% of the participants from School B did not know about the relationship between a square and a rectangle. Table 4.11 reveals how students attempted to define the given shapes. It emerged that none of the students from School B could define or describe a parallelogram and a rhombus by using their properties. Therefore, if these are typical of geometric reasoning patterns demonstrated by Grade 12 students, learning geometry is problematic.

4.6.3 The relationship between the van Hiele theory and the NSSC (O/H) geometry curriculum

Table 4.3 shows that none of the general objectives clearly depicts the features of van Hiele level 4. None of the learning objectives show that students should be able to carry out any formal proof; instead the general objective for theme/topic 4 states that students, when calculating unknown angles, may be required to "give reasons, but no formal proofs". This shows that van Hiele level 4 is not required by the geometry curriculum at the senior secondary phase in Namibia. Therefore, I propose that the highest van Hiele level that a student can attain at senior secondary phase in Namibia is level 3.

Consequently, Table 4.3 answers the research question 1 of this research study. Answering of research question 1 is further supported by the fact that the geometry curriculum does not talk about concepts such as "axioms, postulates and theorems" that are dominantly spoken about at van Hiele level 4 of geometric reasoning. Previous studies (Usiskin, 1982; Mayberry, 1983; van Hiele, 1986; Senk, 1989; Teppo, 1991; King, 2002; Pusey, 2003; Siyepu, 2005; Genz, 2006; Atebe & Schäfer, 2008), suggest that the learning and teaching of geometry at senior secondary phase should reach van Hiele level 4 (formal deduction). In my view, the geometry content at the senior secondary phase in Namibia is learnt and taught at van Hiele levels lower than van Hiele level 4. As a result, the Grade 12 students in Namibia are not compatible with Grade 12 students in other countries as far as geometry knowledge is concerned.

4.6.4 Using van Hiele levels to determine students' geometric reasoning

Tables 4.5A and 4.5B indicate that the majority of the participants in this research study were found at the pre-recognition level. The tables reveal that 7 (35%) of School A participants and 12 (40%) of School B participants were at the pre-recognition level. In both schools, the number of participants who were found at van Hiele level 1 was less than the number of participants at van Hiele level 2. The results established that 5 (25%) and 6 (30%) of the participants in School A were at van Hiele levels 1 and 2 respectively. In School B, 6 (20%) of the participants were at van Hiele level 1 and 7 (23.3%) were at level 2. Only 2 participants from each of the schools were found at van Hiele level 3. Table 4.6 shows that one participant from School B had reached van Hiele level 4. This table displays the result of the forced van Hiele theory. But Usiskin (1982:35) warns that "forcing a van Hiele level is tantamount to assuming that the theory does hold and that those students who do not fit would have fit if there had been more items or better items to minimize random error misclassification". Therefore I do not accept the result of School B which shows that there was one student who reached level 4. This is because I do not want to assume that the theory holds, for the reason that the levels were determined on the basis of the forced van Hiele theory. Instead I consider the results of Table 4.7 to be more appropriate than Table 4.6. Therefore I suggest that Table 4.7

answers research question 2 of this research study. This is due to the fact that Table 4.7 uses the classical and modified van Hiele theory (see Chapter 3, Section 3.2 for the explanation of this theory). This means that the Grade 12 students in the sample function at a level of geometric thinking fitting with their mathematics curriculum. The result of the CDASSG test is consistent with the first part of Wirzup's (1976) claim as cited in Usiskin (1982:35–36) that the majority of our high school students are at the first level of development in geometry. On the other hand Table 4.3 does not concur with the last part of the said claim which says that "while the course they take demands the fourth level of thought". This is because according to the geometry content at the senior secondary phase (Table 4.3), the highest proposed van Hiele level of geometric thought is level 3.

The use of both the CDASSG and the clinical interview in the form of manipulatives shows an appropriate way of assessing students' geometric reasoning. Therefore the two instruments complemented each other.

4.6.5 Language issues

Van Hiele (1986) states that language is a crucial part of the learning process as students progress through the levels of thinking (see Section 2.5.5). This means that students should use appropriate language in order to move from one level through to the next level. In support of the latter statement, Mayberry (1983), Burger and Shaughnessy (1986), and Fuys et al. (1988), in the same section, warn that imprecise language plagues students' work in geometry and is a critical factor in progressing through levels.

The following are some of the examples of imprecise terminologies extracted from the responses of the participants in the clinical interview: "sguare, pallalelograms, rhomas, reactangle, pallilegram, rhombas". Schäfer and Atebe (2008) warn that such use of imprecise terminologies, should not be taken for granted and considered to be mere spelling mistakes; instead they should be taken seriously because they may lead to an inhibition of conceptual understanding of geometric concepts. For example, the description given by the student who referred to a rhombus as "rhombas" is "two sides are pallallel". In this case, the name of the shape is wrong as well as the word parallel.

Even though teachers are encouraged to consider the students' language when developing ideas, there is a need for students to be able to use correct mathematical terminology by the end of the topic (see Chapter 2, Section 2.5.5).

The three methods of data collection used in this study have helped in answering the research questions. The results of the van Hiele Geometry Test and the clinical interview revealed that the students have a lack of conceptual understanding in geometric concepts. It further emerged from the results that most of the participants in the study could describe shapes by the property of sides. The CDASSG test and the clinical interview have shown that class inclusion is a big problem even at Grade 12 level.

4.6.6 Class inclusion

Subtest 3 for van Hiele level 3 contained items that tested the participants' knowledge about class inclusion. From Table 4.8 it can be seen that the research participants did not do well in subtest 3. Tasks 4 and 5 of the manipulatives were about class inclusion. The outcomes of the said tasks showed that the students who participated in the clinical interview did not perform well.

From the findings above it can be seen that class inclusion is a problem. Therefore I suggest that, because the students could only sort geometric shapes by class inclusion when they knew their properties, the research participants had problems with the properties of geometric shapes. When students know the properties of each geometric shape, they will be in a position to identify properties that are common among them. This will further help them to sort the shapes by class inclusion considering the common properties.

4.7 Conclusion

This chapter dealt with my data analysis. This was done by first perusing documents such as the Namibian mathematics junior secondary and senior secondary phase syllabi. From the document analysis it emerged that the junior secondary phase geometry content is strongly linked to that of the senior secondary phase.

The senior secondary phase geometry syllabus was further analysed in order to determine the highest possible van Hiele level required by the Namibian mathematics curriculum. Even though the van Hiele theory is not mentioned anywhere in the mathematics curriculum, features of the van Hiele levels were used to align the general objectives of the NSSC (O/H) geometry content with the van Hiele levels of geometric reasoning. This alignment resulted in the van Hiele level 3 being the highest possible level a Grade 12 student in Namibia can attain. This finding was further supported by the analysis made on the 2007 geometry Grade 12 end-of-year examination questions. It also revealed that the highest possible van Hiele level a Grade 12 student can be assessed at in the national examinations is level 3. After this, the analysis of the van Hiele Geometry Test followed. The results of the van Hiele Geometry Test concurred with the findings of other previous researchers that used the van Hiele theory. These results of the CDASSG test revealed that the majority of students who participated in this study were operating at the pre-recognition level, with a significantly smaller number attaining levels 1 and 2. There was a very small number of students who attained van Hiele level 3. And by using the 3 of 5 classification criterion, with specific reference to the classical and modified van Hiele levels, it emerged that none of the students in this study had attained van Hiele level 4.

To support the results of the CDASSG test, a clinical interview with manipulatives was also used as an instrument to collect data. The results of the clinical interview were also analysed. It emerged that of the students who participated in the study, class inclusion remains a big challenge in the learning of geometry.

Finally, the results of both the CDASSG test and the clinical interview showed that students can be in the same grade but operate at different van Hiele levels of geometric reasoning.

The next chapter concludes the whole research project.

CHAPTER FIVE

CONCLUSIONS

5.1 Introduction

This chapter provides the conclusion of the whole research project. It includes a summary of the findings and highlights the significance of the study. It further outlines some of the limitations, recommendations and avenues for further research studies. As a novice researcher, I gained valuable experience in the process of conducting this research. Therefore, this chapter ends with a reflection of the experience I have gained from conducting this piece of research.

5.2 Summary of findings

Findings of this research study are discussed in detail in chapter four. These findings were generated by using three instruments - document analysis, the van Hiele Geometry Test and the clinical interview.

5.2.1 How geometric concepts are developed through to Grade 12

The analyses of the junior secondary and senior secondary phase geometry content revealed that there is a strong link and coherence between the two syllabi contents. The two syllabi contain the same themes/topics. It further emerged that the importance of geometry is emphasised in the curriculum. This is illustrated by the inclusion of the theme/topic such as “constructions”. Constructions are fundamental to train students to gain basic geometrical skills that they can use to solve problems in the real-life world.

5.2.2 Some of the challenges in learning geometry at Grade 12

The main challenge found by this study was that the participants lacked conceptual understanding of geometric concepts. I found that even though that the participants knew the names of most or all geometric figures or shapes, could not state all or some of the properties. The participants could define the given shapes by properties of sides only. This indicated that even if the students are in Grade 12, they are unable to use other properties of the given shapes to define them adequately. This is problematic as far as identification of shapes is concerned. This kind of geometric reasoning displays the features of a student who is operating at either the pre-recognition level or van Hiele level 1.

Sequencing of the learning objectives is not appropriately done. For example, the learning objectives of theme/topic 1 (Geometrical terms and relationships) from Grade 8 through to Grade 12 are as follows:
Grade 8: use terminology of lines, angles and triangles
Grade 9: understand and apply the Theorem of Pythagoras
Grade 10: apply the properties of similar triangles
Grades 11-12: know and use geometrical terms and the vocabulary of simple plane figures and simple solids

There should be explicit links between these objectives. But the way the "Theorem of Pythagoras" is introduced for example creates a problem. This is because the theorem is not mentioned elsewhere in Grades 8 and 10 or in the syllabus of the senior secondary phase. As a result, students would think that the theorem is only applicable in Grade 9. It is clear that the theorem is used to establish relationships between triangles, but the concern is that the students are required to apply the theorem in Grade 9 without previous knowledge of it. For the students to apply the theorem in Grade 9, the theorem should have been introduced in Grade 8.

5.2.3 The relationship between the van Hiele theory and the NSSC (O/H) geometry curriculum

Since one of the research questions was to determine the van Hiele levels required by the mathematics curriculum, with specific reference to geometry, the NSSC (O/H) geometry syllabus was analysed. Despite the Namibian mathematics curriculum not referring to the van Hiele theory *per se*, by comparing the specific objectives that are linked to the learning objectives of the Grade 12 geometry syllabus with the features of the van Hiele levels of geometric reasoning, it emerged that there is a possible relationship between the two. From the findings it was suggested that the highest possible van Hiele level of thinking required by the mathematics curriculum is level 3. This finding was further supported by the analysis made on the past geometry questions extracted from the 2007 end-of-year Namibian national Grade 12 mathematics question papers (see Chapter 4, Section 4.5.3).

5.2.4 Using van Hiele levels to determine students' geometric thinking

The use of the van Hiele Geometry Test relates my research study to other previous research studies. It was found that the majority of the research participants of this study are at the pre-recognition level and van Hiele levels 1 and 2. The results of the test indicated that of the 50 students who participated in this study, 19 (38%) were at the pre-recognition level, 11 (22%) at van Hiele level 1, 13 (26%) at van Hiele level 2 and 4 (8%) at van Hiele level 3. These results were analysed by using the 3 of 5 classification criterion. Research question 2 of this study was answered with the results generated from the van Hiele Geometry Test. The results showed that the students who participated in the study are functioning at a level of geometric thinking not fitting with their mathematics curriculum. The results of the van Hiele Geometry Test showed that the CDASSG is one of the appropriate instruments to assess students' geometric reasoning. The results further support the finding of previous research that students' geometric thinking is hierarchical in nature. This means that for the student to understand, for example, the geometric concepts at van Hiele level 3, he or she should go through van Hiele levels 1 and 2.

5.2.5 Language issues

The problem of language was very prominent when the participants were asked to name and define geometric concepts. The language used was inappropriate in most instances. The findings of the clinical interview revealed that wrong spelling of and incorrect uses of geometrical terminologies were very widespread in the participants' responses. I suggest that this may lead to a lack of conceptual understanding of geometric concepts in students or students' misconceptions in geometry. As a result, students may not do well in geometry in particular and in mathematics in general.

5.2.6 Class inclusion

The results of both the van Hiele Geometry Test and the clinical interview concur with the findings of the previous research studies. For this study, the results suggest that the students are not or rarely taught about class inclusion. Knowledge on class inclusion is essential, because it enables students to establish family of shapes, for example, family of quadrilaterals with common properties. With this knowledge at their disposal students will be able to do some proofs. The results further demonstrate that even though the research participants were Grade 12 students, the majority could competently operate at van Hiele levels that are below level 3. As a result of this, students would find it difficult to reach van Hiele level 4.

5.3 Significance of the study

The purpose of this study was to explore the application of the van Hiele theory to analyse geometrical conceptualisation in Grade 12 students. This study is the first of its kind to be conducted in Namibia. It therefore attempted to establish whether or not the application of the van Hiele theory applies to the Namibian context.

The other reason for conducting this research study was to try and identify some of the problems encountered by the students in the learning of geometry and use these findings to make recommendations where necessary. Furthermore, this study was conducted in order to determine the compatibility of the Namibian geometry syllabus with the van Hiele theory.

The findings of this study are intended be used to train mathematics teachers, especially in the Kavango Region, on how to use the van Hiele theory in order to determine the geometric reasoning of students in their classes.

5.4 Limitations of the study

The results of this research study could not be generalised due to the following set of reasons:

- The research sites of this study were schools from one urban area only. The results can therefore not be generalised as they may be different for example in a rural school with a lack of resources, or a school in a different city like Windhoek.

- The non-existence of information regarding the van Hiele theory in the Namibian mathematics curriculum was also a limitation. I was unable to draw from local examples and knowledge.

- The small number of participants remained another limitation. The 50 participants who participated in the study was just a small portion of the total population of the Grade 12 students in the whole of Namibia.

- The highest possible van Hiele level required by the NSSC (O/H) geometry syllabus is level 3. This remained a limitation because it causes the teaching and learning of geometry to be only up to van Hiele level 3.

5.5 Recommendations

From the results of the document analysis, it emerged that the highest possible van Hiele level attainable by a Grade 12 student in Namibia is level 3. This situation leaves the Grade 12 students in Namibia not being compatible with other students in the rest of the world where geometry is required to be taught and learnt up to van Hiele level 4. Mathematics at post grade 12 level is more abstract and theoretical, and heavily founded on the basis of proofs. Therefore, any student who enters these institutions of higher learning is expected to think at van Hiele level 4. Curriculum developers and policy makers should therefore consider revisiting the NSSC (O/H) mathematics curriculum, with specific reference to the geometry van Hiele level 4. These aspects include proofs of theorems, conjecturing and working with axioms in Euclidean geometry.

Since the van Hiele theory forms the foundation of mathematics curricula for countries such as USA, Britain, Netherlands, Russia, etc., I recommend that the Namibian mathematics curriculum should also align itself with the said theory.

The teaching and learning of geometry should involve more hands-on activities that will actively engage the students. This will enhance students' conceptual understanding of geometric concepts.

When teaching about geometric concepts, teachers should ensure that students understand and know the properties of all geometric shapes. By knowing the properties of the geometric shapes, students will be able to establish class inclusion, which according to this study is sorely lacking. Students can only recognise, describe and distinguish geometric shapes from each other by knowing their properties.

When teaching about geometric shapes and concepts, teachers should ensure that the proper geometric terminologies are used by both the teachers and students. This will address language barriers in students who use English as a second language. This involves correct spelling of the concepts, proper pronunciations and using the correct names of the geometric shapes.

5.6 Avenues for further research

This case study forms a useful platform for future research in the following areas:

- the possibility of aligning the Namibian geometry curriculum with the van Hiele levels of geometric thinking.
- to establish why students perform better in the subtest of the van Hiele level 2 than in the subtest of the van Hiele level 1.
- to establish the link between language and the progression from one van Hiele level to the next with specific reference to the English Second Language speakers.

5.7 Personal reflection

As a novice researcher, I found the research process a fascinating and enriching experience. When I started with the identification of my research topic and questions, I thought the process was easy. But, when the writing of the research proposal started, I realised that doing research needed much of someone’s attention and concentration. Although the writing of the research proposal did not cause major challenges, I found it difficult to source relevant literature in Namibia. This difficulty was exacerbated when I wrote the literature review chapter.

The process of data collection was also found to be challenging, because it was difficult to secure the full participation of people. At first the participants were uncertain and afraid to contribute.

5.8 Conclusion

This chapter summed up the whole research project. It first discussed the summary of findings by discussing how geometric concepts are developed through to Grade 12, some of the challenges in learning geometry at Grade 12, the relationship between the van Hiele theory and the NSSC (O/H) geometry curriculum, and the use of van Hiele levels to determine students' geometric thinking and language issues.

The significance and limitations of the study were also discussed. This was followed by some possible recommendations and the reasons why they were made. Possible avenues for future research were proposed. The chapter ended with a discussion on the experience I gained from conducting the study.

The results of the van Hiele Geometry Test attested that the van Hiele theory holds and is a useful tool to determine students' geometric reasoning.

This study supports the claim that the van Hiele theory is one of the best frameworks in exploring students' geometric reasoning. The findings also support the claim for the theory's wide applicability.

REFERENCES

Atebe, H. U., & Schäfer, M. (2008). Van Hiele levels of geometric thinking of Nigerian and South African mathematics learners. In M. C. Polaki, T. Mokulu & T. Nyabanyala (Eds.), *Proceedings of the 16th Annual Conference of the Southern Africa Association for Research in Mathematics, Science and Technology*. Maseru: SAARMSTE.

Basey, J. M. (1999). Foraging behavior of beavers, plant secondary compounds, and management concerns. In P. E. Busher & R. M. Dzieciolowski (Eds.), *Beaver protection, management and utilisation in Europe and North America.* New York and London: Plenum Press.

Battista, M. T., & Clements, D. H. (1995). Geometry and proof. *Mathematics Teacher, 88*(1), 48–54.

Bell, J. (1993). *Doing your own research project: A guide for first-time researchers in education and social science* (2nd ed.). Philadelphia: Open University Press.

Beukes, F., Visagie, A., & Kasanda, C. D. (2005, June). *Mathematics teaching and teacher education in changing times in Namibia.* Paper presented at the ICMI, 1st Africa Regional Congress, University of Witwatersrand, Johannesburg.

Burger, E. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education, 17*(1), 31–48.

Byrkit, D. R. (1971). Taxicab geometry – a non-Euclidean geometry of lattice points. *Mathematics Teacher, 64*, 418–22.

Cantrell, D. (1993). Alternative paradigm in environmental educational research: The interpretive perspective. In R. Mrazek (Ed.*), Alternative paradigms in environmental education research* (pp. 81–104). Ohio: North America Association for Environmental Education.

Clarkson, P. C. (2003). Language, logical thinking and communication in school mathematics: Whose responsibility. In H. S. Dhindsa, L. S. Bee, P. Achleitner & M. A. Clements (Eds.), *Studies in science, mathematics and technical education* (pp.99–116). Brunei Darussalam: Universiti Brunei.

Clements, D. H. (2004). Perspective on the child's thought and geometry. In T. P. Carpenter, J. A. Dossey & J. L. Koehler (Eds.), *Classics in mathematics education research* (pp 60–66). Reston: National Council of Teachers of Mathematics.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp.420–464). New York: Macmillan Publishing Company.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education* (5th ed.). London: Routledge.

Dane, F. C. (1990). *Research methods*. Pacific Grove: Brooks/Cole.

De Villiers, M. D. (1987). *Research evidence on hierarchical thinking, teaching strategies and the van Hiele theory: Some critical comments*. Research Units for Mathematics Education (RUMEUS). Stellenbosch, University of Stellenbosch.

De Villiers, M. D. (1994). The role and function of a hierarchical classification of quadrilaterals. *For the Learning of Mathematics*, *14*(1), 11–18.

De Villiers, M. D. (1996). The future of secondary school geometry. *For the Learning of Mathematics, 1*(1), 1–7.

De Villiers, M. D., & Njisane, R. M. (1987). The development of geometric thinking among high school pupils in Kwazulu. In J. C. Bergeron, N. Herscovics, & C. Kieran (Eds.), *Proceedings of the 11th International Conference for the Psychology of Mathematics Education, 3*, 117–123. Montréal, Canada, Université de Montréal.

Dindyal, J. (2007). The need for an inclusive framework for students' thinking in school geometry. *The Montana Mathematics Enthusiast, 4*(1), 73–83.

Ernest, P. (1991). *Mathematics, education and philosophy: An international perspective*. London: The Falmer Press.

Etchberger, M. L., & Shaw, K. L. (1992). Teacher change as a progression of transitional images. A chronology of a developing constructivist teacher. *Journal of Research in Science and Mathematics Education, 92*(8), 411–417.

Feza, N., & Webb, P. (2005). Assessment standard, van Hiele levels, and grade seven learners' understanding of geometry. *Pythagoras, 62*, 36–47.

Frankfort-Nachmias, C., & Nachmias, D. (1992). *Research methods in social sciences*. London: Edward Arnold.

French, D. (2004). *Teaching and learning geometry*. London: Continuum International Publishing Group.

Fuys, D., Geddes, D., & Tischler, R. (1988). The van Hiele model of thinking in geometry among adolescents. *Journal of Research in Mathematics Education. Monograph, 3.* Reston: National Council of Teachers of Mathematics.

Genz, R. (2006). *Determining high school geometry students' geometric understanding using van Hiele levels: Is there a difference between standards-based curriculum students and nonstandards-based curriculum.* Unpublished master's thesis, Brigham Young University, Rexsburg ID.

Grearson, P., & Higgleton, E. (1996). *South African students dictionary.* Swaziland: Macmillan Boleswa Publishers.

Gutiérrez, A., & Jaime, A. (1998). On the assessment of the van Hiele levels of reasoning. *Focus on Learning Problems in Mathematics, 20*(2 & 3), 27–46.

Gutiérrez, A., Jaime, A., & Fortuny, J. M. (1991). An alternative paradigm to assess the acquisition of the van Hiele levels. *Journal for Research in Mathematics Education, 22(*3), 237–251.

Hershkowitz, R. (1990). Psychological aspects of learning geometry. In P. Nesher & J. Kilpatrick (Eds.). *A research synthesis by the International Group for the Psychology of Mathematics Education* (pp. 71–93). Cambridge,UK: Cambridge University Press.

Hoffer, A. (1979). *Geometry.* Teacher's edition. Menlo Park, CA: Addison-Wesley.

Hoffer, A. (1981). Geometry is more than proof. *The Mathematics Teacher, 74*(1), 11–18.

Hoffer, A. (1983). Van Hiele based research. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 205–227). New York: Academic Press.

Images. (2007). *Van Hiele levels of geometric reasoning*. Retrieved July 08, 2007 from images.rbs.org/cognitive/van_hiele.shtml

Jackson, W. (1995). *Doing social research methods*. Montreal: Prentice Hall.

Jones, K. (2002). Issues in the teaching and learning of geometry. In L. Hag arty (Ed.), *Aspects of teaching secondary mathematics: Perspectives on practice* (pp.121–139). London: Routledge Falmer.

Karnasih, I., & Soeparno. (1999). *Teaching mathematics has to focus on logic*. Indonesia, Kompas May 17, 2000.

Kerans, D. S. (1994). *Teaching geometry at primary schools in Kupang, East Nusa Tengara*. Indonesia: Surabaya State University.

King, L.C.C. (2002). Assessing the effect of an instructional intervention on the geometric understanding of learners in a South African primary school. In C. Malcolm and M. Lubisi (Eds.), *Proceedings of the 10th Annual Southern Africa Association for Research in Mathematics, Science and Technology Education Conference* (pp. 178–181). Durban: University of Natal.

Konyalioglu, S., Konyalioglu, A. C., Ipek, A. S., & Isik, A. (2003). *The role of a visualization approach on student's conceptual learning*. Erzurum: University of Erzurum.

Krause, E. F. (1975). *Taxicab geometry.* Menlo Park: Addison-Wesley Publishing Co.

Krause, E. F. (1986). *Taxicab geometry: An adventure in non-Euclidean geometry*. New York: Dovers Publications.

Leedy, P. D., & Ormrod, J. E. (2005). *Practical research: Planning and design* (8th ed.). New Jersey: Pearson Prentice-Hall.

Malkevitch, J. (1991). *Geometry's future.* Arlington, MA: COMAP.

Mason, M. M. (1997). The van Hiele model of geometric understanding and mathematically talented students. *Journal for the Education of the Gifted, 21*(1), 38–53.

Mason, M. (1998). The van Hiele levels of geometric understanding. In L. McDougal (Ed.), *Professional handles for teachers: Geometry* (pp. 4–8). Boston: Houghton-Mifflin.

Mason, M. (2003). *The van Hiele levels of geometric understanding*. Virginia: University of Virginia.

Mayberry, J. (1983). The van Hiele levels of geometric thought in undergraduate pre-service teachers. *Journal for Research in Mathematics Education, 14*(1), 58–69.

McMillan, J. H., & Schumacher, S. (2001). *Research in education: A conceptual introduction*. London: Evans.

McNiff, J. (1996). *You and your action research project*. London: Routledge.

Mwamwenda, T. S. (1989). *Educational psychology: An African perspective.* Durban: Butterworth.

Namibia. Mathematics and Science Teachers Extension Programme. (2002). *Improving mathematics in Namibia: Report of a national consultative process with recommendations*. Windhoek: MBESC.

Namibia. Ministry of Basic Education, Sport and Culture. (1999). *Presidential commission on education, culture and training: Report*. Windhoek: Gamsberg Macmillan.

Namibia. Ministry of Basic Education, Sport and Culture. (2004). *The SACMEQ II Report: A study of the conditions of schooling and the quality of primary education in Namibia*. Windhoek: Directorate of Planning and Development.

Namibia. Ministry of Basic Education. (2006). *The pre-service Basic Education Teacher Diploma: Broad curriculum.* Okahandja: NIED

Namibia. Ministry of Education and Culture. (1993). *Toward education for all: A development brief for education, culture and training.* Windhoek: Gamsberg Macmillan.

Namibia. Ministry of Education. (2005a). *Mathematics syllabus ordinary level grades 11–12*. Okahandja: NIED.

Namibia. Ministry of Education. (2005b). *Mathematics syllabus higher level grades 11–12.* Okahandja: NIED.

Namibia. Ministry of Education. (2006a). *Basic education support III project*. Rundu: Academy for Education Development

Namibia. Ministry of Education. (2006b). *Mathematics syllabus grades 8-10.* Okahandja: NIED.

Namibia. Ministry of Education. (2007a). *Namibia Senior Secondary Certificate: Mathematics ordinary level.* Windhoek: Directorate of National Examinations and Assessment.

Namibia. Ministry of Education. (2007b). *Namibia Senior Secondary Certificate: Mathematics ordinary level.* Windhoek: Directorate of National Examinations and Assessment

Namibia. National Institute for Educational Development. (2003). *Learner-centred education: A conceptual framework.* Okahandja: NIED.

Namibia. Office of the President. (2004). *Namibia Vision 2030: Policy framework for long-term national development (summary).* Windhoek: Office of the President.

Pandiscio, E., & Orton, R. E. (1998). Geometry and metacognition: An analysis of Piaget's and van Hiele's perspectives. *Focus on Learning Problems in Mathematics, 20*(2–3), 78–87.

Pegg, J. (1995). Learning and teaching geometry. In L. Grimison (Ed.), *Teaching secondary school mathematics: Theory into practice (*pp.87–103). London: Harcourt Brace.

Pegg, J. E., & Faithfull, M. (1993). Analysing higher order skills in deductive geometry. In A. R. Baturo (Ed.), *New directions in research on geometry and visual thinking*. Brisbane: Queensland University of Technology.

Presmeg, N. (1991). Applying van Hiele theory in senior primary geometry: Use of phases between the levels. *Pythagoras, 26,* 9–11.

Pusey, E. L. (2003). *The van Hiele model of reasoning in geometry: A literature review*. Unpublished master's thesis, North Carolina State University.

Royal Society & Joint Mathematical Council. (2001). *Teaching and learning geometry 11–19*. London: Royal Society/Joint Mathematical Council.

Schäfer, M., & Atebe, H. U. (2008). "As soon as the four sides are all equal, then the angles must be 90° each". Children's misconceptions in geometry. In M. C. Polaki, T. Mokulu & T. Nyabanyala (Eds.), *Proceedings of the 16th Annual Conference of the*

Southern Africa Association for Research in Mathematics, Science and Technology. Maseru: SAARMSTE.

Schiro, M. (1997). *Integrating children's culture and mathematics in the classroom: Children as meaning makers, problem solvers, and literacy critics*. New York: Teachers College Press.

Senk, S. L. (1989). Van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education, 20*(3), 309–321.

Setati, M., & Adler, J. (2001). Between languages and discourses: Language practices in primary multilingual mathematics classrooms in South Africa. *Educational Studies in Mathematics, 43*, 243–269.

Sherard, W. H. (1981). *Why is geometry a basic skill?* Greenville: Furman University.

Shibli, J. (1932). *Recent developments in the teaching of geometry*. Pennsylvania: Maple Press.

Siyepu, S. W. (2005). *The use of van Hiele theory to explore problems encountered in circle geometry: A grade 11 case study*. Unpublished master's thesis, Rhodes University Grahamstown.

Snyders, M. (1995). Geometry in senior secondary class. *Proceedings of the 4th Amesa Conference* (pp 5–13). Port Elizabeth: University of Port Elizabeth.

Soedjadi, R. (1991). *The feature of mathematics education at our primary school*. Indonesia, Surabaya State University.

Southwood, S., & Spanneberg, R. (2000). *Rethinking the teaching and learning of mathematics.* Pretoria: Mc Printer.

Stake, R. E. (2000). Case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 435–454). London: Sage Publications.

Tall, D. (2004). Thinking through three worlds of mathematics. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 4*, 281–288.

Tapson, F. (1991). *Oxford mathematics study dictionary*. New York: Oxford University Press.

Teppo, A. (1991). Van Hiele levels of geometric thoughts revisited. *Mathematics Teacher*, *84*(3), 210–221.

Thekwane, B. K. (2001). *A guide to the teaching of learner-centred mathematics*. Retrieved March 09, 2008 from **www.nied.edu.na/publications/journals/journal14/14art2.pdf.**

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry.* (Final Report of the Cognitive Development and Achievement in Secondary School Geometry Project). Chicago: University of Chicago.

Van de Walle, J. A. (2001). *Elementary and middle school mathematics: Teaching developmentally* (4th ed.) (pp. 12–43). Boston: Allyn & Bacon.

Van Hiele, P. M. (1973). *Begrip en Inzicht (Understanding and insight)*. Purmerend: Muusses

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education.* Netherlands: Academic Press.

Van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics*, *5*(6), 310–16.

Wirszup, I. (1976). *Breakthroughs in the psychology of learning and teaching geometry: Space and geometry*. Ohio: Eric Smeac.

Yin, R. K. (2003). *Case study research: Design and methods*. Thousand Oaks: Sage Publications.

APPENDICES

Appendix A: Consent letter to the principals of schools A and B

New Millennium Park
P.O. Box 1049
Rundu
13 May 2008

The Principal
……Secondary School
P.O. Box..
Rundu

Dear Sir/Madam

I am registered as a part-time student at Rhodes University, Grahamstown (**Student No: 604M5511)**. I have been studying for a Master's degree in Mathematics Education since February 2007. I would be most grateful if you would allow me to use the Grade 12 class as the research site for the research report which I am required to write.

The aim of my research is to use the van Hiele theory in order to analyse geometrical conceptualisation in Grade 12 students. Data analysis will be collected from, van Hiele Geometry Test (the whole class) and a clinical interview (with three or four students, depending on their performance in the test) from your school.

The parents or guardians will be asked for permission to use the Grade 12 students as subjects for my research study. The data collection process will last approximately for two weeks (**19-23 and 28-31 May 2008**). And it will not interfere with the normal school activities, because the activities will be executed in the afternoons.

The school and students will be assured of anonymity in the final report and will be invited to proofread drafts of the report to ensure that details are accurately recorded and reported.

Should you have any concerns or questions about this request, you can contact me at **+264813132619 or 066-255770.**

Yours Sincerely

Mateya, M. (Mr)

CONSENT FORM

Muhongo Mateya is hereby granted permission to use the Grade 12 class of……..Secondary School as the research site for the research report he is required to write for the completion of his Master's degree. I understand that data for analysis will be collected from test (whole class) and a clinical interview (three or four students) and that information from these may be used in the final report. I have been assured that my school and my students will have anonymity in that report.

……………………………………. ……………………………

Principal's Signature **Date**

Appendix B: Consent letter to the parent(s)/guardian(s) of the students

New Millennium Park
P.O. Box 1049
Rundu
13 May 2008

Dear Parent(s) or Guardian(s)

I am registered for a Master's degree in Mathematics education with the Department of Education at Rhodes University, Grahamstown. To qualify for my Master's degree, I am required to write a research report on a topic that is linked to an aspect of the work undertaken in the coursework component of the Master's programme. My research study is on the use of the van Hiele theory to analyse geometrical conceptualisation in Grade 12 students. I have chosen this topic because I have learnt from my past experience as a teacher, that students have problems with geometry. And the recommendations that will be made will help to improve the performance of students in geometry.

The whole class will take a geometry test and then six students will be selected for the clinical interview. The six students will be selected based on their performance in the test. I will attempt to answer the following questions:

- what are the van Hiele levels of thinking required by the Grade 12 mathematics curriculum?
- are the Grade 12 students in Namibia functioning at a level of geometric thinking fitting with their mathematics curriculum?

Please complete the attached consent form if you are willing to assist me with this research:

- by allowing me to test your child
- by allowing your child to participate in the clinical interview

Yours Sincerely

Muhongo Mateya (Mr)

CONSENT FORM

I hereby agree to assist **Muhongo Mateya** in his research. I understand that he will be:

* administering a geometry test to my child to use in his final research report
* conducting a clinical interview with my child and use the information in the research report.

Parent(s) or Guardian(s)

Signature: Date:

………………………………………. …………………………..

Appendix C: The van Hiele Geometry Test

APPENDIX C

THE VAN HIELE LEVELS OF GEOMETRIC THINKING: VISUALIZATION, DESCRIPTION, ORDER AND DEDUCTION.

THE VAN HIELE GEOMETRY TEST
(Grade 12).

Date: …………………

INSTRUCTIONS:

1. Do not start until you are told to do so.
2. While you are waiting, please fill the appropriate information in the spaces below.

Name (Surname first): ……………………………………………………………

Name of school: ……………………………………………………………………

Grade: ……………………………………………………………………………

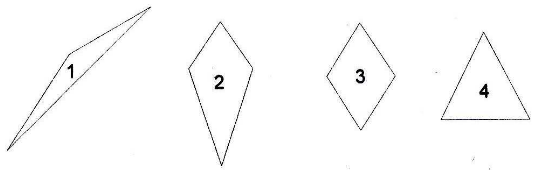
Age (in years): ……………………………… **Sex:** ……………………………………

3. This paper will last for **30 minutes.**
4. It is an objective test, consisting of **20 multiple- choice** questions. Provide your answers on the **computer answer sheet** that you are given. **Do not mark your answers on the test booklet.**
5. Each question is followed by **five** options lettered **A to E**. There is only one correct answer to each question. Choose the correct option for each question and **shade in pencil** on your computer answer sheet the answer space which bears **the same letter** as the option you have chosen. **Give only one answer to each question.**
6. Use an **HB** pencil throughout.

NOTE: The **diagrams** in this test are **not necessarily** drawn to scale.

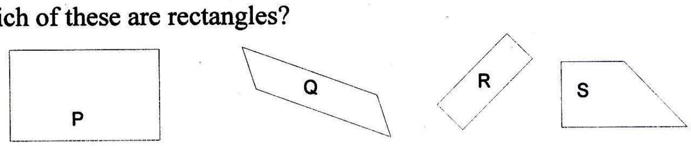
Time Allowed: 30 Minutes

1. Which of these are triangles?



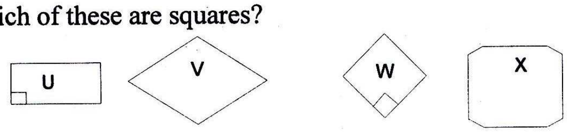
A. All are triangles
B. 4 only
C. 1 and 2 only
D. 3 only
E. 1 and 4 only

2. Which of these are rectangles?



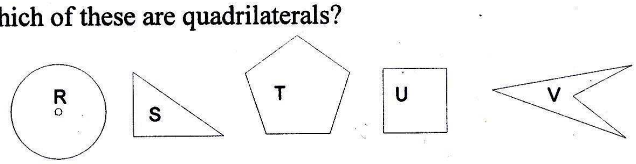
A. P and R only
B. Q and S only
C. P only
D. R only
E. All are rectangles

3. Which of these are squares?



A. X only
B. U and W only
C. W only
D. V and X only
E. U only

4. Which of these are quadrilaterals?

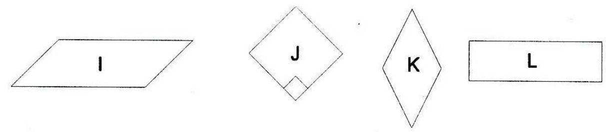


A. None of these are quadrilaterals
B. T only
C. U and V only
D. R only
E. S and V only

1

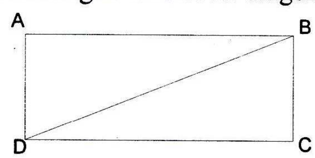
The Van Hiele Levels of Geometric Thinking.
9 April 2008: 10:51:24 AM

5. Which of these are parallelograms?



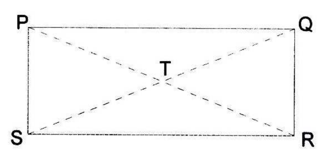
A. I only
B. L only
C. I and K only
D. J and L only
E. All are parallelograms

6. ABCD is a rectangle. One of its diagonals, BD is drawn.



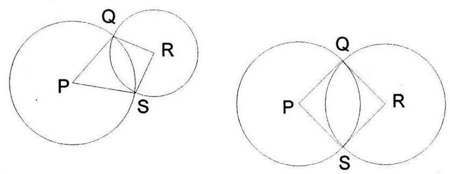
Which is **true in every** rectangle?
A. BD bisects ∠B and ∠D.
B. BD divides ABCD into two congruent right-angled triangles.
C. BD is a line of symmetry.
D. BD divides ABCD into two congruent isosceles triangles.
E. BD and DC have the same measure.

7. PQRS is a rectangle. The diagonals, PR and SQ intersect at T.



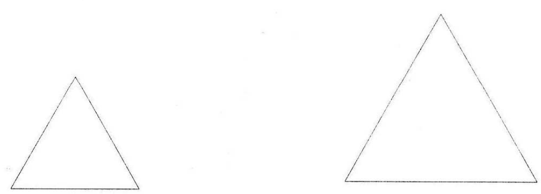
Which of the following is **not true** in **every** rectangle?
A. PR and QS have the same measure.
B. T is the midpoint of both PR and QS.
C. PR and QS have different measures.
D. There are four right angles.
E. PS and QR are parallel.

8. P and R are the centres of two circles which intersect at Q and S to form a 4-sided figure PQRS. Two examples are given below.



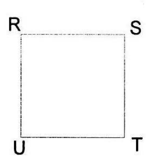
Which of (A) – (D) is **not** always **true**?
   A. PQRS will have at least two angles of equal measure.
   B. PQRS will have two pairs of sides of equal length.
   C. Angles P and R will have the same measure.
   D. The lines PR and QS will be perpendicular.
   E. All of (A) – (D) are true.

9. An equilateral triangle is a triangle with all the three sides equal in length. Two examples are given below.



Which of (A) – (D) is **true in every** equilateral triangle?
   A. Each angle is an acute angle.
   B. The measure of each angle must be 60^0.
   C. Each angle bisector is a line of symmetry.
   D. Each angle bisector must also bisect the opposite side perpendicularly.
   E. All of (A) – (D) are true.

10. RSTU is a square. Which of these properties is **not true** in all squares?

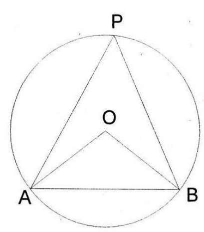


   A. RS and SU have the same measure.
   B. The diagonals bisect the angles.
   C. RT and SU have the same measure.
   D. RT and SU are lines of symmetry.
   E. The diagonals intersect at right angles.

11. What do all rectangles have that some parallelograms **do not** have?
    A. Opposite sides are parallel.
    B. Diagonals are equal in length.
    C. Opposite sides are equal in length.
    D. Opposite angles have equal measure.
    E. None of (A) – (D).

12. Which is **true**?
    A. All properties of rectangles are properties of all parallelograms.
    B. All properties of squares are properties of all rectangles.
    C. All properties of squares are properties of all parallelograms.
    D. All properties of rectangles are properties of all squares.
    E. None of (A) – (D) is true.

13. In the diagram, O is the centre of the circle. AB is a chord and P is any point on the circumference. Which relationship is **true in every** circle?

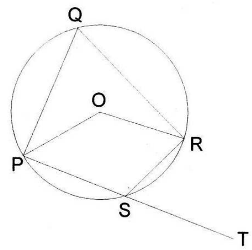


    A. ΔAOB is isosceles.
    B. AP and BP have equal measure.
    C. OA and OB have equal measure.
    D. ∠ AOB = ∠ APB.
    E. (A) and (C), above are true.

14. Consider these two statements.
    Statement R: In ΔABC, ∠A and ∠C are complementary.
    Statement S: ΔABC is a right-angled triangle.
    Which is **true**?
    A. If **R** is true, then **S** is false.
    B. If **R** is false, then **S** is true.
    C. If **R** is true, then **S** is true.
    D. **R** and **S** cannot both be false.
    E. **R** and **S** cannot both be true.

15. Here are two statements.
    Statement S: Figure **1** is congruent to Figure **2**.
    Statement T: Figure **1** is similar to Figure **2**.
    Which is **true**?
    A. If **S** is true, then **T** is false.
    B. If **S** is true, then **T** is true.
    C. If **S** is false, then **T** is false.
    D. **S** and **T** cannot both be true.
    E. None of (A) – (D) is true.

16. PQRS is a cyclic quadrilateral. O is the centre of the circle. Line PS is produced to a point T, outside the circle.



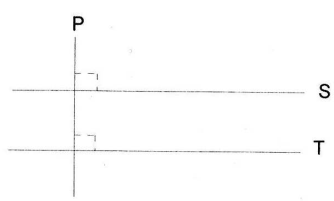
From this diagram, one can prove that ∠Q = ∠RST. What would you conclude from this proof?

A. Given any cyclic quadrilateral PQRS with PS produced to T, then ∠Q = ∠RST.
B. Only in this cyclic quadrilateral can we be sure that ∠Q = ∠RST.
C. Given any quadrilateral, PQRS with PS produced to T, then ∠Q = ∠RST.
D. Only when the quadrilateral, PQRS looks like a kite can we be sure that ∠Q = ∠RST.
E. Only in some, but not all cyclic quadrilaterals PQRS, can we prove that ∠Q = ∠RST.

17. Examine these three statements.

(i). Two lines perpendicular to the same line are parallel.
(ii). A line that is perpendicular to one of two parallel lines is perpendicular to the other.
(iii). If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines S and P are perpendicular and lines T and P are perpendicular.



Which of the above statements could be the reason that line S is parallel to line T?

A. (i) only
B. (ii) only
C. (iii) only
D. Either (ii) or (iii)
E. Either (i) or (ii)

Appendix D: The Manipulatives: Sorting activities

THE VAN HIELE LEVELS OF GEOMETRIC THINKING: VISUALIZATION, DESCRIPTION AND INFORMAL DEDUCTION.

The Manipulatives: Sorting Activities.

Date:……………………..
Time Allowed: 90 minutes

Instructions:

1. Do not start until you are told to do so.
2. While you are waiting, fill in the appropriate information in the spaces below.

Name (Surname first):…………………………………………………………….

Name of school:……………………………………………………………………

Grade:……………………………………………………………………………..

Age (in years):……………………………………………… Sex:……………….

3. This activity involves the identification, description and classification of some geometrical shapes in the form of cut-outs.
4. You are allowed to make use of straightedges (rulers), protractors, dividers, or any other mathematical instruments in the mathematical set that you are supplied.
5. Answer the questions in the spaces provided for each question.

Question 1

Placed before you are a set of numbered geometrical shapes. You are required to identify each of the shapes by writing the name of each shape and giving a reason to explain how you know it is that shape. An example is given.

Shape No.	**Name of shape**	**Reason**
0.	Regular Pentagon	It is a polygon having five equal sides
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		

9.		
10.		
11.		
12.		
13.		
14.		
15.		
16.		
17.		
18.		
19.		

20.		
21.		
22.		
23.		
24.		
25.		
26.		
27.		
28.		
29.		
30.		

Question 2.

(i) Sort the shapes on your desk into two broad groups. List the shape numbers only.

Group A shapes	**Group B shapes**

(ii) What would you tell someone to look for in order to pick out, from among these shapes, a shape that belongs to:

(a) Group A shapes

(b) Group B shapes

Answer.

(a) For Group A shapes: ……………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(b) For Group B shapes: ……………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(iii) (a). What is the general/common name for all the shapes in Group A?

Answer: …………………………………………………………………………

(b) What is the general/common name for all the shapes in Group B?

Answer: …………………………………………………………………………

Question 3.

(i) Now, make a further sorting of the shapes in Group A into as many smaller groups as you can, putting together shapes that are alike in some way, by completing the following table.

NOTE: A shape may belong to two or more groups.

Group	**Shape Nos.**	**Common name for these shapes**	**How are these shapes alike?**
I.			
II.			
III.			
IV.			
V.			
VI.			
VII.			

(ii) Now, make a further sorting of the shapes in Group B into as many smaller groups as you can, putting together shapes that are alike in some way, by completing the following table.

NOTE: A shape may belong to two or more groups.

Group	Shape Nos.	Common name for these shapes	How are these shapes alike
I.			
II.			
III.			
IV.			
V.			
VI.			
VII.			
VIII.			

Question 4.

(i). What would you tell someone to look for in order to pick out all the parallelograms from among these shapes?

Answer: ……………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………

(ii). What would you tell someone to look for in order to pick out all the rectangles from among these shapes?

Answer: ………………………………………………………………………………
………………………………………………………………………………………
………………………………………………………………………………………

(iii). What would you tell someone to look for in order to pick out all the rhombuses from among these shapes?

Answer: ....
………………………………………………………………………………………
………………………………………………………………………………………

(iv). What would you tell someone to look for in order to pick out all the squares from among these shapes?

Answer: …………………………………………………………………………………
....
....

(v). What would you tell someone to look for in order to pick out all the trapeziums from among these shapes?

Answer: …………………………………………………………………………………
....
....

(vi). What would you tell someone to look for in order to pick out all the isosceles triangles from among these shapes?

Answer: …………………………………………………………………………………
………………………………………………………………………………………
………………………………………………………………………………………

Question 5.

(i) (a). Is shape No.23 a rectangle?

Answer: …………………………………………………………………………………
………………………………………………………………………………………
………………………………………………………………………………………

(b). How do you know?

Answer: …………………………………………………………………………………

…………………………………………………………………………………………
…………………………………………………………………………………………

(ii)(a). Is shape No.17 a parallelogram?

Answer:……………………………………………………………………………….
…………………………………………………………………………………………
…………………………………………………………………………………………

(b). How do you know?

Answer: ……………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

(iii) (a). Is shape No.6 a rhombus?

Answer: ……………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

(b). How do you know?

Answer: ……………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

(iv) (a). Is shape No.1 a parallelogram?

Answer: ……………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

(b). How do you know?

Answer: ……………………………………………………………………………

(v) (a). Is shape No.30 a scalene triangle?

Answer: ……………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

(b). How do you know?

Answer:……………………………………………………………………………….
…………………………………………………………………………………………
…………………………………………………………………………………………

Appendix E: The geometry syllabus for the Junior Secondary Phase

JUNIOR SECONDARY CERTIFICATE (JSC) GEOMETRY CONTENT

Theme/Topic 1: **Geometrical terms and relationships**

Learning objectives and Basic competencies

Grade 8	Grade 9	Grade 10
Learning objective: Understand and apply geometrical terms and relationships **Basic competencies:** - use and interpret the geometrical terms: point, line, diagonal, parallel, perpendicular, vertical, horisontal - name angles as right, acute, obtuse, straight or a revolution - identify pairs of angles as complementary or supplementary - use the term “congruent” for plane figures that are the same in all aspects	**Learning objective:** Understand and apply the Theorem of Pythagoras **Basic competencies:** - calculate the third side of a right-angled triangle, if two sides are given - apply the Theorem of Pythagoras to prove that an angle is a right angle	**Learning objective:** Understand the concept of similarity **Basic competencies:** - recognise plane figures that are similar by referring to the shape and size - *show that triangles are similar* - *calculate unknown sides of similar triangles*

Theme/Topic 2: **Constructions**

Learning objectives and Basic competencies

Grade 8	Grade 9	Grade 10
Learning objective: know how to perform geometrical constructions using a straightedge, a compass and a protractor **Basic competencies:** - measure lines and angles - construct triangles, given	**Learning objective**: know how to perform geometrical constructions of parallel and perpendicular lines and of angle bisectors **Basic competencies:** - using a straight edge and a pair of compass only,	**Learning objective**: Perform geometrical constructions **Basic competencies:** - make accurate scale drawings of maps, plans

three sides; two sides and the included angle; a right angle and any two sides; or two angles and a corresponding side - construct other simple geometrical plane figures from given data	construct: parallel lines; the perpendicular from a point to a line; the perpendicular from a point on the line; the perpendicular bisector of a line segment; the bisector of an angle	and journeys, which include directions given as three-figure bearings - construct nets of cubes, cuboids, triangular prisms and cylinders

Theme/Topic 3: **Symmetry and transformations**

Learning objectives and Basic competencies

Grade 8	Grade 9	Grade 10
Learning objective: Know line and rotational symmetry in plane figures **Basic competencies:** - identify the number and position of lines of symmetry in simple plane figures and polygons - locate the centre of rotation and state the order of rotational symmetry of given plane figures and polygons	**Learning objective:** Understand how plane figures are reflected and rotated **Basic competencies:** - draw and describe reflections of plane figures in vertical and horisontal lines - draw and describe rotations of plane figures around the origin, a vertex or the midpoint of a line and through angles which are multiples of 90^o	**Learning objective:** Understand how plane figures are enlarged **Basic competencies:** - construct and describe enlargements with positive whole numbers as scale factors

Theme/Topic 4**: Angle properties**

Learning objectives and Basic competencies

Grade 8	Grade 9	Grade 10
Learning objective: Know and understand angle properties to solve problems **Basic competencies:** - identify and use angle properties to solve problems - calculate unknown angles	**Learning objective:** Know and understand angle properties of quadrilaterals to solve problems **Basic competencies:** - identify and use angle properties of quadrilaterals	**Learning objective:** Know and understand angle properties of polygons **Basic competencies:** - identify and use angle

using the following geometrical properties: angles at a point; adjacent angles on straight line; angles formed at intersecting lines; angles formed within parallel lines; angles in triangles	to solve problems - calculate unknown angles using the geometrical properties of the parallelogram, rectangle, rhombus, kite and square	properties of polygons - calculate the sizes of the interior and exterior angles of regular polygons - *calculate the sizes of interior and exterior angles of irregular polygons* - *calculate unknown angles using the following properties: the sum of the angles of irregular polygons; an angle in a semi-circle; the angle between a tangent and a radius of a circle; the angle at the centre of a circle is twice the angle at the circumference; angles in the same segment are equal; angles in opposite segments are supplementary*

Appendix E: The geometry syllabus for the Senior Secondary Phase

THE NAMIBIA SENIOR SECONDARY CERTIFICATE ORDINARY AND HIGHER LEVELS [NSSC (O/H)] GEOMETRY CONTENT

Themes & Topics	General Objectives	Specific objectives
1. Geometrical terms and relationships	Know and use geometrical terms and the vocabulary of simple plane figures and simple solids	- use and interpret the geometrical terms: point, line, parallel, intersecting, bearing, right angle, acute, obtuse and reflex angles, perpendicular, similarity, congruence - use and interpret vocabulary of triangles, quadrilaterals, circles, polygons and simple solid figures, including nets - use the relationships between areas of similar triangles, with corresponding results for similar figures and extension to volumes and surface areas of similar solids
2.Geometrical constructions	Measure lines and angles and construct simple geometrical figures using straightedges, compasses, protractors and set squares	- measure lines and angles - construct a triangle given the three sides using a straight edge and compasses only - construct other simple geometrical figures from given data using protractors and set squares as necessary - construct angle bisectors and perpendicular bisectors using straight edges and compasses only - read and make scale drawings
3. Symmetry	Recognise properties of simple plane figures directly related to their symmetries	- recognise line and rotational symmetry (including order of rotational symmetry) in two dimensions - recognise properties of triangles, quadrilaterals and circles directly related to their symmetries - use the following symmetry properties of circles: *equal chords are equidistant from the centre; the perpendicular*

		bisector of a chord passes through the centre of a circle; tangents from an external point are equal in length
4. Angle properties	Calculate unknown angles using the geometrical properties of intersecting and parallel lines and of simple plane figure	- calculate unknown angles using the following geometrical properties (reason may be required but no formal proofs): angles at a point; angles on a straight line and intersecting straight lines; angles formed within parallel lines; angle properties of triangles and quadrilaterals; angle properties of regular polygons; angle in a semi-circle; angle between tangent and radius; *angle properties of irregular polygons; angle at the centre of a circle is twice the angle at the circumference; angles in the same segment are equal; angles in opposite segments are supplementary*
5. Locus	Determine the locus (path) of a point under certain conditions	- use the following loci and the method of intersecting loci for sets of points in two dimensions: which are at a given distance from a given point; which are at a given distance from a given straight line; which are equidistant from two given points; which are equidistant from two given intersecting straight lines