EXPLORING LEARNERS' MATHEMATICAL UNDERSTANDING THROUGH AN ANALYSIS OF THEIR SOLUTION STRATEGIES

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ABSTRACT

The purpose of this study is to investigate various solution strategies employed by Grade 7 learners and their teachers when solving a given set of mathematical tasks. This study is oriented in an interpretive paradigm and is characterised by qualitative methods. The research, set in nine schools in the Eastern Cape, was carried out with nine learners and their mathematics teachers and was designed around two phases.

The research tools consisted of a set of 12 tasks that were modelled after the Third International Mathematics and Science Study (TIMSS), and a process of clinical interviews that interrogated the solution strategies that were used in solving the 12 tasks. Aspects of grounded theory were used in the analysis of the data.

The study reveals that in most tasks, learners relied heavily on procedural understanding at the expense of conceptual understanding. It also emphasises that the solution strategies adopted by learners, particularly whole number operations, were consistent with those strategies used by their teachers. Both learners and teachers favoured using the traditional, standard algorithm strategies and appeared to have learned these algorithms in isolation from concepts, failing to relate them to understanding. Another important finding was that there was evidence to suggest that some learners and teachers did employ their own constructed solution strategies. They were able to make sense of the problems and to 'mathematize' effectively and reason mathematically. An interesting outcome of the study shows that participants were more proficient in solving word problems than mathematical computations. This is in contrast to existing research on word problems, where it is shown that teachers find them difficult to teach and learners find them difficult to understand.

The findings of this study also highlight issues for mathematics teachers to consider when dealing with computations and word problems involving number sense and other problem solving type problems.

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DEDICATION

This thesis is dedicated to my parents, the late Harry and Pat Penlington who gave me all the encouragement and support in everything that I set out to do. Their nurturing way of allowing me to be who I am, I am deeply grateful for.

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CHAPTER ONE

INTRODUCTION AND BACKGROUND TO STUDY

1.1 INTRODUCTION

The aim of this study is to identify and analyse a diversity of solution strategies employed by Grade 7 learners and their teachers.

This chapter provides an introduction to the study and explains why research of this nature was undertaken. The chapter begins with background details about the study, then gives the rationale, and places the research in a South African context, with some information about the new Outcomes Based Education approach (OBE). Further aspects dealt with in the chapter include a brief examination of the goals of the research, a brief summary of the methodology employed, some findings of the study, its limitations and significance. Finally, the chapter gives an overview of the study.

1.2 BACKGROUND TO THE STUDY

My interest in the Third International Mathematics and Science Study (1996) (TIMSS) was aroused when it was found that the performance of South African learners was placed at the bottom of a list of over 41 countries that participated in the study. Although the intention of the TIMSS study was not to gauge which countries were better or what teaching methods were more superior, my interest initially was to assess whether I would find the same results using a similar study. I have worked with many in-service teachers and their learners doing problem solving tasks. As a result I have come across a plethora of diverse solution strategies that learners use in solving problems. I have also assisted teachers in refining their own strategies. In this study I wanted to interrogate more closely whether learners and teachers use a variety of solution strategies when given problem tasks to solve.

In a further TIMSS-(R) study conducted in 1998 amongst 8 000 learners in Grade 8 in South African schools, it was found that learners still performed poorly when compared with other participating countries. What was particularly revealing was that the mean score of South African learners was even lower than two other African countries that took part, namely Morocco and Tunisia. Although South Africa had participated in both studies, there had been no real difference in performance between 1995 and 1998.

As I had helped to develop similar assessment tasks for Grade 4 learners with colleagues from the Rhodes University Mathematics Education Project (RUMEP), I felt that a study at the Grade 7 level would help teachers see whether learners were at an appropriate level to enter high school and whether learners were able to use different solution strategies when solving problems. The main research instrument to explore solution strategies that framed this study consisted of a test with twelve tasks. To construct the test, I consulted widely, adapting tasks from the TIMSS study, the Ohio proficiency tests and my own experience. In the TIMSS studies, three different types of questions were included: multiple-choice questions, short answer questions and extended answer questions. For this study both multiple-choice questions and more problem-solving, extended questions were used.

The TIMSS studies showed that learners performed poorly in fractions, number sense, data representation, geometry and algebra. I thus decided to use the same content areas in my study.

1.3 RATIONALE FOR THE STUDY

The rationale for choosing a study on solution strategies was triggered by the poor TIMSS results, the new outcomes-based approach being adopted in South Africa and my interest in the problem-centred approach that I had been using in my own class as a teacher. As I view problem solving as the vehicle to learning, (Murray, Olivier & Human 1998) and having been an advocate of the constructivist view of learning for some time, my interest in solution strategies research was heightened.

1.4 CONTEXT OF THE RESEARCH

When RUMEP was established in 1993 as a teacher development institute, it was given a mandate to develop mathematics teaching and learning skills of primary school teachers in the Eastern Cape. The primary school level was chosen as this was considered to be the level where such an intervention would make the greatest impact. I have been working at RUMEP since 1996 with a particular focus on evaluating in-service teacher workshops in the field. This study needs to be seen in the context of my work place and its mandate.

A pilot study was conducted in 2000 to investigate the solution strategies Grade 7 learners used when doing problem-solving tasks (Penlington & Michael 2000). These benchmark tasks were modelled after the TIMSS study (Beaton *et al.* 1996) and the findings show that there are large discrepancies in performance between rural and urban schools.

The actual study on which this thesis is based took place in October 2001 in nine rural and peri-urban schools in the Eastern Cape of South Africa. Some of the participating schools lacked the infrastructure for appropriate teaching and learning to take place. Basic equipment like furniture for learners, textbooks and other resources for teaching were lacking. Consequently teachers struggled to provide learners with the rich experiences needed to learn mathematics in a developmentally appropriate manner.

1.5 CURRICULUM 2005

This study is located within the spirit of Curriculum 2005. Through the implementation of Curriculum 2005, which is underpinned by an outcomes-based approach, South Africa transformed its education system from one based on a traditional, content-focussed system to one underpinned by outcomes and learner-centredness. There has been a shift to the "mastering of processes linked to intended outcomes, as well as on mastering of knowledge and skills needed to achieve the outcome" (Olivier 1998:21).

Encouraging learners to develop and use their own solution strategies is regarded as consistent with a move from being "teacher-centred" to a more process driven problem solving "learner-centred" approach (Southwood & Spanneberg 1996). The Revised National Curriculum Statement (RNCS) (2002:1) reaffirms this when it states, "The outcomes encourage a learner-centred and activity-based approach to education." With the learner at the centre of the learning process, much more emphasis is placed on learners developing conceptual understanding and learning computational skills (Bransford, Brown & Cocking 1999). Understanding *must be* the most fundamental goal of mathematics education (Hiebert *et al.* 1997). This is corroborated by the National Council of Teachers of Mathematics (1980:2) who assert that "whatever students learn, they should learn with understanding." One of the purposes of this study was to explore how learners explain their understanding, while using different solution strategies during problem solving tasks.

1.6 GOALS OF THE RESEARCH

The goal of the research was:

To identify and analyse solution strategies used by nine Grade 7 learners and their mathematics teachers in solving a given set of tasks.

Sub-goals of the research were:

a) To develop a theoretical framework for analysing solution strategies using grounded theory.
b) To classify the set of tasks according to the TIMSS Curriculum Frameworks (McNeeley 1997).
c) To investigate to what extent the learners and their teachers used similar solution strategies to solve the set of tasks.

1.7 RESEARCH METHODOLOGY

This research is qualitative in nature and is located in the interpretive paradigm. Aspects of grounded theory as a methodology, namely open coding, were used to analyse the solution strategies. Three research instruments were used:

a) A test consisting of 12 tasks.
b) Individual clinical interviews with both learners and teachers to ascertain the solution strategies they had used and their understanding of them.
c) A semi-structured teacher interview schedule that incorporated questions on the benchmark test and its relevance.

1.8 SOME FINDINGS

Although the findings of this research are documented in detail in chapter 5, I present here four of the findings.

Most of the participants in the study used solution strategies that had been 'teacher taught'. The procedures were mostly 'rule-based' with the emphasis more on procedural understanding at the expense of conceptual understanding. The solution strategies, particularly those related to whole numbers in this study, were similarly completed by both learners and teachers. However, there were instances where some participants did use their own 'constructed' solution strategies, which enhanced their conceptual understanding and their ability to 'mathematize'. Language was central to the test as the majority of tasks required careful reading with understanding. Although research has confirmed that word problems are difficult for most learners to solve, this research seems to indicate the opposite. Most of the learners interviewed appeared to have more difficulties with simple computations that required not much reading skill, than with word problems. A possible reason for learners finding word problems easier was that the test was presented to the participants in their mother tongue as well as in English.

1.9 LIMITATIONS

Subsequent to this study several limitations were identified. Firstly, the use of grounded theory as a methodology could not be implemented in its entirety. My data only lent themselves to a limited grounded theory analysis. Secondly, my sample size was too small for the making of any general inferences.

1.10 SIGNIFICANCE OF THE STUDY

I believe this study is significant for the following important reasons:

Firstly, the test was not only seen as a tool for assessing learners, but it also created opportunities for learning. Secondly, the ability of learners to construct their own solution strategies was confirmed. Research conducted by Kamii *et al.* (1993), Woods & Sellers (1996) and Fennema & Romberg (1999) has found that learners are able to construct, develop and modify their own solution strategies when given problems to solve. Another significant aspect of this study is that it has informed teacher practice by promoting the idea of encouraging teachers in all grades to encourage their learners to use a variety of their own, constructed solution strategies.

1.11 OVERVIEW OF THE STUDY

This thesis is organised into five chapters. Chapter 1 provides an overview of the thesis. In this chapter, the background of the study, the rationale, goals of the study, the new OBE approach and a brief summary of the methodology employed is discussed. The chapter concludes with an overview of the whole thesis structure.

In chapter 2 the literature pertaining to research in problem solving, the problem-centred approach and the principles underlying constructivism and social-constructivism are discussed together with a critique of both the problem-centred approach and constructivism.

In chapter 3 aspects of grounded theory as a methodology and the processes involved, together with the instruments developed, are described and used for gathering the data. Aspects dealt with include the selection of Grade 7 learners, the design of the test, the clinical interviews and the semi-structured interview schedule for teachers. Ethical issues including voluntary participation, anonymity and researcher and co-worker's identity are also described in this chapter.

In chapter 4 the analysis of each of the twelve tasks is categorised and solution strategies of learners and their teachers are compared.

In the final chapter a discussion of the findings is presented together with recommendations, implications and avenues for future research.

CHAPTER TWO

THEORETICAL PERSPECTIVE

REVIEW OF LITERATURE

2.1 INTRODUCTION

The origin of this study is derived from the new OBE approach, which was implemented in Grade 1 in South Africa in 1998. The introduction of Curriculum 2005 into Grade 7 classes, which took place in 2000, filled some teachers with excitement because many believed the new curriculum would promote better learning and help learners achieve their potential. However, many were also concerned about the implementation of Curriculum 2005.

The new curriculum was a watershed for South African schools: its outcomes-based approach represented a new paradigm which endeavoured to develop learners into citizens who would be prepared to live and work independently in a society that demanded and required skilled people in areas such as mathematics, science and technology (Brady 1996). However, Brady (1996:13) argues that:

> It places enormous demands on teachers to further individualise instruction, plan remediation and enrichment, administer diagnostic assessment and keep extensive records...Outcomes-based education will flounder if there is not appropriate high quality staff development and the provision of sufficient support... (Brady 1996:13)

The national Department of Education has committed itself to adopting and implementing a transformational outcomes-based education (Spady & Marshall 1991). This implies a move from a content-based curriculum to an outcomes-based learning programme (Spady & Marshall 1991). In this approach, the curriculum is designed:

> from future-driven exit outcomes with the emphasis falling on embedding quality problem solving skills, rather than on memorizing given information.

> The goal is to equip all students with the knowledge, competence and orientation needed for success after they leave school. (Pretorius 1998:6)

The twelve critical (generic) outcomes adopted by the South African Qualifications Authority (SAQA) are all based on the rationale of each learning area, and the specific outcomes (Olivier 1998). These would have a major influence on the kind of learning environment learners need and the kinds of activities they should engage in if they are to progress towards achieving the outcomes. However, with teachers having difficulties in coping with new curriculum documents, and the perceived inadequate OBE training, which does not address teachers' fears and answer their questions (Cele 1998), a review committee was appointed in July 2002 by the Minister of Education to look into the original policy document. The result was that this review committee recommended that the principles of outcomes-based education should be retained. This is a unanimous rejection of the apartheid education principles of Christian National Education (Ashley 1989) and of fundamental pedagogics (Kallaway 1984). The review committee proposed a National Curriculum Statement (NCS) that would supersede the previous policy document which was based on specific outcomes, performance indicators and assessment criteria (Department of Education 1997).

The new document for the General Education and Training Band (Grades R-9) was implemented in the Foundation Phase (Grades R-3) in 2004. For the mathematics learning area the NCS describes briefly the mathematical content and processes that learners are expected to learn, including learning outcomes that specify the sequence of core concepts, content and skills, together with clearly stated assessment standards.

Most American and South African teachers' conception of mathematics sees the discipline as a body of knowledge which is static, incorporating a set of rules and procedures that are applied to produce one right answer (Romberg & Kaput as cited in Fennema & Romberg 1999). To 'know' the mathematics being taught means being skilled and efficient in executing procedures and manipulating symbols without necessarily really having any understanding of what they represent. The aforementioned beliefs are consistent with mathematics learning being regarded as being 'traditional', (Department of Education 1997) with the teacher taking responsibility for transmitting the knowledge to learners and being in total control.

Research done in America, which is also applicable in my view to South Africa as far as teaching practice is concerned, is that a typical mathematics lesson starts with the teacher reviewing previous work or introducing a new procedure by providing learners with step-by-step instructions and then giving learners problems on which to practise the procedures (Gregg 1995).

In this chapter, I describe the learning theories which underpin the study. I start with an overview of the knowledge transmission model and show the move towards a social constructivist model (Vygotsky 1978). As learning mathematics is viewed as a social activity as well as an individual constructive activity, I then provide an overview of problem solving and discuss an approach referred to as the problem-centred approach (Murray, Olivier & Human 1998; Cobb *et al.* 1991). As the role of the teacher is central to the problem-centred approach, the social component and the problem component of the learning environment is further explored. To find solutions to problems, learners need to draw on their own knowledge. Through this process learners develop appropriate solution strategies for new mathematical understandings. As the concept of solution strategies is central to this study, I wove it through as a common theme.

The first step is to look at the theories of learning which inform this study.

2.2 LEARNING THEORIES

2.2.1 Introduction

Prawat (1992) mentions that the nature of teaching and learning has changed substantially from that of 20 to 30 years ago. The traditional view of knowledge, based on the idea that education is a scientific and value free endeavour transmitted from the teacher to the learner through the process of 'teacher-tell', has been criticized. The belief that an idea is true if and only if it is linked to an independent, objective reality has been looked at with a critical gaze.

2.2.2 Knowledge transmission model

The perceived view of learning in many schools in South Africa has until recently been based on the knowledge transmission model. This model implies that mathematical knowledge gained by one generation of individuals is transmitted completely to the next generation by teachers. By implication, it regards learners as passive recipients of mathematical knowledge. This transmission model is not limited to South Africa. Internationally, data gathered in other countries have shown that learners are not provided with the knowledge and skills to prepare them to tackle problem situations in effective ways (De Corte 1995).

A major change in the teaching and learning of mathematics came about *inter alia* through the work of Piaget (1972), von Glasersfeld (1991), Ernest (1998) and others. The change came in the form of a move away from the idea that knowledge is transmitted to learners who are passive to the idea that learners construct their own knowledge. Learners are no longer seen as being passive individuals receiving knowledge, but as active recipients of knowledge who interact with the environment. This shift in emphasis is referred to as constructivism (von Glasersfeld 1991; Bodner 1985).

2.2.3 Piaget's theory of cognitive development

The roots of constructivism can be attributed to the work of Piaget. Cognitive development is intellectual development, which includes a person's mental capacity to engage in thinking, reasoning, understanding and solving problems (Piaget 1972). Piaget's view is that children pass through four stages of development. The stages are:

- sensori-motor stage,
- pre-operational stage,
- concrete operational stage
- formal operational stage (Piaget 1972)

Children pass through these stages one after the other, although they may vary in the age at which they reach a particular stage. The formal operational stage from ages about 11-15 is applicable to this study. During this stage children begin to reason things out in their minds without having to see and manipulate the real objects. They also start thinking about moral and philosophical issues (Piaget 1972).

Building on Piaget's idea of how children acquire knowledge, cognitive psychologists have also shifted their perspective away from the traditional view of knowledge. The traditional view of knowledge "views the mind as a 'black box'; we can accurately judge what goes in (stimulus) and what comes out (response), but we can only guess about what is happening inside the box" (Bodner 1985:374). Bodner (ibid.) also asserts that the constructivist view of knowledge "views the environment as the 'black box', each of us knows what is going on in our minds; what we can only guess about is the relationship between our mental structures and the real world."

Cognitive psychologists have pointed out that:

> All children bring a wealth of knowledge with them into the classroom. They are not, as was previously thought, 'sponges' soaking up knowledge according to their ability levels. They have definite views about what is taught. Like adults they develop theories about nearly everything ... Children use theories to frame their interpretation of new information. Because these theories help them make sense of their world, children are often hesitant to change them.
>
> (Prawat 1992:11)

In the context of this study, the quotation above epitomizes the idea that knowledge is constructed in the mind of the learner and 'fits' reality.

It is further stated that:

> Current research ... also focuses on the role of the student. It recognises that students do not merely passively receive or copy input from the teachers but instead actively mediate it by trying to make sense of it and to relate it to what they already know (or think they know) about the topic. Thus, students develop new knowledge through a process of active construction. In order to get beyond rote memorization to achieve a true understanding, they need to develop and integrate a network of associations linking new input to pre-existing knowledge and beliefs anchored in concrete experiences.
>
> (Brophy 1992:5)

Piaget (1972) has been recognised as the creative force behind the conception of the learner as an active constructor of knowledge. The focus of this study is on learners actively using their own knowledge to construct a variety of solution strategies, which they understood and could make sense of.

2.2.4 Vygotsky and social constructivism

There are many theoretical positions within constructivism on learning, teaching, curriculum development, and the professional development of teachers (Hodson & Hodson 1998). Many of the ideas and views originated by Vygotsky who lived in the 1930s, both during and before Piaget's time, only became known to English speaking researchers after his work had been translated in the 1970s. His theory, according to Wertsch (1991), is known as socio-cultural learning because development comes about through the medium of culture that was established by special kinds of social collaboration.

The social constructivist philosophy starts from the premise that all fields of human knowledge are interconnected (Lerman 1994). Social constructivism emphasizes language, culture and the social milieu. In the social constructivist model of the teaching-learning process, four key elements interact and affect each other – the learner; the teacher; the task; and the context. As knowledge is socially constructed, the classroom is seen as an extension of the learners' environment. That knowledge which the learner knows is built on the existing knowledge gained through social interactions other than those found in the formal classroom (see page 17).

Vygotsky's emphasis is on the role of communication, social interaction and instruction in establishing development (Wood 1988). For Vygotsky, language has the strength and the power to shape future mental development; biological and natural influences are far less important than historical, cultural and social influences. Ways of thinking are not merely natural products of the mind or the one and only creation of children. Indeed these are observed as cultural interventions that need to be studied through social interaction with those who already hold and practise them (Wood 1988). Vygotsky's view is that there are two levels of development that exist simultaneously in the developing learner. Firstly, there is the actual level of development, which is found in what a learner can do on his/her own. Secondly, there is the potential level of development, which is found in a learner who has difficulty executing a task or solving a problem on his or her own, who needs help if assisted by a more knowledgeable adult. He called the gap between these two levels of development, the zone of proximal development or ZPD. The ZPD is defined by Vygotsky (1978:76) as "... the distance between the actual development level as determined by independent problem solving and the level of potential development through problem solving under adult guidance or more capable peers." Vygotsky (1978) reasoned that the mental powers to learn through instruction are itself a basic feature of human intelligence. When adults help children to achieve that which they cannot do on their own, they are furthering the development of knowledge and ability. The success of a learner lies in the greater or lesser ability to transfer from that which the child can do on his/her own to what he/she can do with an adult's help. The more experienced adult is able to provide "scaffolding" of the subject matter to support the learner's ongoing understanding (Vygotsky 1978).

While Piaget identified with the effects of both the social environment and children's development, observing that both social interaction and language are important, he did not give them the emphasis that Vygotsky provides (Knight 1993). Vygotsky's theory rests on the fundamental hypothesis that learning occurs on the social level, when the learner is engaged in co-operative activities within the cultural context. In social constructivism facts are emphasized far less than conceptual understanding and the acquisition of skills so that learners can operate as individuals in their own societies. That is why teaching as an activity cannot be separated from learning. Teaching is a process of social interaction that takes place between learners and teachers in particular contexts. This course of action is akin to what has been termed enculturation (Bishop 1988); (Schoenfeld 1992) and socialization (Resnick 1989).

According to Schoenfeld, as cited in Ernest (1991:5), enculturation is a "reconstructive and not a reproductive activity". Learners are seen as part of an immature mathematical community, who are being enculturated into the experienced mathematical community. Enculturation is concerned with learning the concepts, orientations, values and processes of the 'expert' community and seeing none of these as beyond examination and revision. Its features include learning the way ideas are examined and the systems of creating, justifying and verifying knowledge.

A strength of the social constructivist perspective is that learners are empowered by the learning process because knowledge is not seen as something 'out there' to be learned and accepted but as something to be constructed and re-constructed. Teachers require an understanding of mathematics as an activity, which unfolds in a number of social contexts. They also need to be confident with their own mathematics knowledge to allow an open interpretation of ideas (Ernest 1991). A social constructivist teacher regards himself/herself as an active participant with learners in constructing their learning. The teacher plans, designs and sets up a suitable context where learners will participate in stimulating activities that encourage and facilitate learning. The teacher will encourage, guide and respond to questions and queries and provide opportunities for learners to work with more experienced peers in order to support the learner to reach a higher level of cognitive functioning.

Recent research has shown that since Piaget's early theory, other factors about children's mathematical thinking need to be taken into consideration. One such factor, which is fundamental in the learning of mathematics, is that of language (Vygotsky 1978). Vygotsky believes that language functions as a tool for thought. Thought originates in social interaction mediated by language. For Vygotsky there are two forms of child speech. Children working individually would use spoken language – what is called egocentric speech – to guide and regulate their actions in performing a task. As children mature, this speech is internalised as inner speech (Vygotsky 1978). Both are social and form a genesis of thought, so the development of thinking is from the social to the individual, rather than from the individual to the social (Vygotsky 1978). Murray (1990) also asserts that language helps the thinking process and that, as concepts get more abstract, the individual is more dependent on language.

In mathematics, learners are seen to have a strong capacity for 'making sense' of mathematical situations and particularly situations that involve interaction amongst their peers and teachers (Vygotsky 1978).

Language is also fundamental to this research study as the test that the learners completed relied heavily on understanding the language used. Although each learner completed the test individually, it is difficult for me to agree with Vygotsky when he states that the development of thinking is from the social to the individual. In this particular instance I was unable to check the development of thinking as being social as learners completed the task individually. What impacted on this study was the bilingual approach used. Learners appeared to be at an advantage in being able to read the tasks not only in English but in their mother-tongue as well. They were able "to mathematize" and to make sense of the mathematical situation (Fosnot & Dolk 2001).

2.2.5 Constructivism

Constructivists contend that knowledge is not found outside a person (Confrey 1990) but that true knowledge can only be found when it is constructed inside the mind of a human being. The understanding of any event, situation or problem happens only when relationships are made to existing understanding in a learner's mind (Cobb & Steffe 1983).

A basic tenet of constructivism as advocated by Clements and Battista (1990) is that knowledge is actively created or constructed by the child, not passively received from the environment. The ideas that learners construct are made meaningful when children integrate them into their existing structures of knowledge. This implies that teachers need to recognise and build on learners' prior knowledge by using the ideas that learners bring into the classroom to guide the direction of lessons.

As learning is a social process in which children grow intellectually in interacting with others, a supportive interactive learning environment is required. The constructivist classroom is seen as supporting a culture in which students are involved not only in discovery and invention but also in a social discourse involving discussion, explanation, negotiation, sharing and evaluation (Clements & Battista 1990). This implies that learner collaboration is being promoted by encouraging each other's conceptualisations and ideas. In a constructivist classroom, learning is viewed as a two-way flow of information between learners and teachers and implies that the teaching process should change from transmitting to facilitating the learning process (Tobin 1990). Teaching by telling learners everything should be substituted with teaching that involves discussing, reflecting, sharing, explaining and negotiating.

In a constructivist classroom the nature of mathematics changes from viewing mathematics as learning a set of procedures to a sense-making activity. Learners are encouraged to repeatedly engage in trying a particular solution strategy, reflecting on it and reviewing the strategy. In constructivist instruction, learners are encouraged to create their own methods when solving problems. They are not asked to adopt someone else's thinking but are encouraged to refine their own. As Confrey (1990:111) comments, "learners must first believe in their knowledge, since knowledge without belief is contradictory." The above reflects my intention in this study. I wanted learners and teachers to use a variety of strategies when solving the given problems. The strategy they used had to make sense to them; they had to be able to articulate what they had done and I wanted to see evidence of not just using memorised procedures given to them by their teachers with no understanding (see chapter 4, page 88).

The verb "to construct" according to Njisane, as quoted in Breen *et al.* (1992:29) implies "that the structures the child ultimately possesses are built up gradually from separate components in a manner initially different from that of an adult." Piaget believes that knowledge is acquired through individuals trying to organise, structure and re-structure their experiences by considering the available schemas.

In support of the constructivist approach, I concur with the view that children who use their own methods for solving problems and to make sense of what they are doing, will develop mathematical structures that are more complex, abstract and powerful than they currently have (Cobb 1988). Making sense implies "that the learner will reflect on the experience and what he/she already knows" (Tobin 1990:31). In this study, clinical interviews (Ginsburg 1997) were conducted with both learners and teachers in order to find out their own particular solution strategies and the thinking behind the problems presented to them (see chapter 3, page 63).

Kamii & Joseph (1989:183) notes, "encouraging children to construct knowledge from within is the diametric opposite of trying to impose isolated skills from the outside." It is apparent that the point of view that Kamii *et al.* (1989:184) subscribes to differs from that of more traditional educators who " ... assume that the job of the teacher is to put knowledge into children's heads." Like Kamii (1989), Piaget, asserts that children make ideas. Ideas are not picked up like a stone or just passed on from others like a gift (Piaget 1972). This suggests that children build new ways of thinking about the world in which they live. By reflecting on their physical and mental actions, children create new knowledge.

According to Cobb *et al.* (1991:161) using a constructivist perspective seems to advocate two major goals for mathematics instruction. The first goal is that students should formulate mathematical structures that are "complex, abstract and powerful..." so that they are increasingly capable of solving a wide variety of meaningful problems. A second goal is that they should become "autonomous and self-motivated in what they do". He further believes that they do not get mathematics knowledge from their teacher so much as from what they think, explore and participate in, and that making sense of what they are learning and communicating about mathematics is their responsibility. Learners therefore require time to experience, and to reflect on their experiences in relation to what they already know (Tobin 1990:31).

Olivier (1989) makes the following comments about the constructivist perspective:

> The student is therefore not seen passively receiving knowledge ... it is not possible that knowledge can be transferred ready made and intact from one person to another. Therefore although instruction clearly affects what children learn, it does not determine it, because the child is an active participant in the construction of his own knowledge. (Olivier 1989:11)

Constructivist teaching is concerned with learner-centred teaching. The claim made by Brookes & Brookes (1993:10) succinctly encapsulates this theory about the nature of knowledge and learning:

> ... when the classroom environment in which students spend so much of their day is organised so that student-to-student interaction is encouraged, cooperation is valued, assignments and material are inter-disciplinary and students' freedom to choose their own ideas is abundant, students are more likely to take risks and approach assignments with a willingness to accept challenges to their correct understanding. Such teacher role models and environmental conditions honour students as emerging thinkers.
> (Brookes & Brookes 1993:10)

2.2.6 Critique of constructivism

Some learners very often think that a teacher who explains things clearly is a good teacher. Constructivism therefore could undermine the view that explanations are important, unless they take place where the learner is explaining something to the teacher or learners are engaging in a group discussion. Being able to explain clearly is an important characteristic for a teacher to have, but if learners are to be given opportunities to construct their own knowledge, then it means it is an aspect which will have to be held in check (Orton & Wain 1994). A critique of constructivism that I identify with is that teachers in a constructivist classroom require considerable mathematical knowledge and pedagogical skill. In my view and experience, many teachers do not have these skills. A myth documented by Clements (1997) in constructivism is that using resources in teaching makes learners active. When teachers use resources, it does not necessarily mean that they are 'teaching constructively'. The resources that teachers use in class must serve the purpose of engaging learners in the task. Class control and management, especially for a new teacher, is sometimes seen as being more difficult when strategies other than the transmission model are being undertaken (Orton & Wain 1994). Getting learners to work in groups and applying co-operative learning also does not necessarily make teaching 'more constructivist' (Clements 1997). I agree with this statement, because very often learners sit in groups but do not participate or take part in any group discussions. They just wait for answers to develop. Assigning group roles so that everyone participates, may also distract learners' attention away from the mathematics to the group process itself (Noddings 1990).

Young teachers very often need the support and guidance of more experienced teachers to see the benefits of constructivist teaching because powerful constructions in learners can be achieved by increasing the amount of time learners spend working together (Noddings 1990). This, however, poses another difficulty. Learners who are given free rein might explore a particular problem or area, which may fall outside the content as found in the curriculum (Orton & Wain 1994). The timing of intervening in a discussion by a teacher may further exacerbate the problem. This is another area of concern for teachers. Teachers worry that too much discussion may lead to their not completing the prescribed curriculum (Orton & Wain 1994).

Orton & Wain (1994:55) further contend, "many teachers would say that approaches to learning based on constructivist beliefs do not sit easily alongside the kinds of tests and examinations, which pupils have to write." In my experience this is only partly true. However, with OBE, teachers are now using alternative forms of assessment to assess learners' understanding. By integrating assessment with instruction, the kind of authority a teacher holds in a constructivist environment differs from the traditional classroom environment where knowledge is transmitted by the teacher. A constructivist teacher has to earn respect "rather than have it accorded automatically by society" (Orton & Wain 1994:12). The teacher has to now work together with learners in setting criteria for work to be assessed. In this instance it is much more demanding for the teacher.

Another critique of constructivism is that if learning from experience is important, as constructivists claim, then in teacher education we need to provide teachers with appropriate forms of experience (Davis, Maher & Noddings 1990). Constructivist teachers also need to watch for any misconceptions, which may develop anywhere in the learning process. They need to plan activities that will lead learners to question their own faulty conceptions (Davis, Maher & Noddings 1990). Even if constructivists regard learning as an active process, it is important not to discard all the strategies recommended by theorists who favour direct instruction or alternative forms of instruction, even if we disagree with them on fundamental cognitive reasoning (Confrey 1990).

The next step is to explore the notion of problem-solving and an approach referred to as the problem-centred approach.

2.3. PROBLEM SOLVING

2.3.1 Introduction

In recent curriculum documents (Department of Education 1997:24), an important critical outcome for mathematics states:

"Identify and solve problems by using critical and creative thinking."

Further, the document describes OBE as "an activity-based approach to education designed to promote **problem solving** and **critical thinking**."

The aforementioned words 'problem solving and critical thinking' are consistent with the constructivist philosophy, with its emphasis on knowledge, skills and attitudes. A guiding principle of constructivist pedagogy is posing real-life, relevant problems for learners to solve that will help them to construct understanding, while at the same time getting them to think critically.

As far back as 1989, the National Council of Teachers of Mathematics in the United States states in its first recommendation that:

> Problem solving must be the focus of school mathematics. Fundamental to the development of problem solving activity is an open mind, an attitude of curiosity and exploration ... Mathematics teachers should create a classroom environment in which problem solving can flourish ...[it] is essentially a creative activity. (NCTM 1989:23)

This is in agreement with the Cockroft Report in Britain, which states:

> The ability to solve problems is at the heart of mathematics. Mathematics is only useful to the extent to which it can be applied to a particular situation and it is the ability to apply mathematics to a variety of situations to which we give the name 'problem solving'... The idea of investigations is fundamental both to the study of maths itself and also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems in very many fields. (Cockroft Report 1982: paragraph 249 – 250)

The focus on problem solving is supported by Great Britain (1982) and the National Criteria in Mathematics for General Certificate of Secondary Education (GCSE), which further supports the approach that includes investigative work and projects as part of the national assessment of sixteen year olds studying mathematics.

The opening statement in the recently published NCTM (2000), Principles and Standards for School Mathematics, reiterates that problem solving is a necessary part of all mathematics learning. My submission that learners make use of different strategies and look for meaningful understanding in solving problems is pertinent here. The emphasis on learners being encouraged to invent their own procedures is advocated by (Kamii *et al.* 1993; McClain & Cobb 2001) so that learners build their own meaning for themselves in order to better understand the concepts and skills of mathematics. This, in essence, is what I explore in this research (see chapter 2, page 37).

2.3.2 What is problem solving?

Problem solving has in the recent past been a powerful and resonant area of research for mathematics educators (Lester 1985). However, for Lester (1985:667) it appears that other issues have drawn attention away from problem solving. Even in the latest publication of the NCTM (2000), problem solving has been diminished to being only one of the four main foci of the curriculum. Secondly, it appears that the research community seems to be less interested in problem solving and that constructivism has taken its place as the philosophy of learning (Lester 1985). As problem solving is seen as a very complex form of human endeavour, it is concerned with more than the ability to just recall a fact or apply a well-learned procedure. The skill and ability to solve a mathematical problem unfolds slowly and as a result success is dependent on more than content knowledge. Problem solving performance according to Lester (1985:669) appears to be a "function of several interdependent categories of factors (e.g. knowledge acquisition and utilization, control, beliefs, affects and socio-cultural contexts)."

Problem solving has many different meanings. According to Branca, (as cited in Krulik & Reys 1980) the three most important interpretations of problem solving are:

- Problem solving as a basic skill
- Problem solving as a process
- Problem solving as a goal.

Branca, (as cited in Krulik & Reys 1980:7) explains:

> … considering problem solving as a **basic skill** can help us organise the specifics of our daily teaching of skills, concepts and problem solving. Considering problem solving as a **process** can help us examine what we do with the skills and concepts, how they relate to each other and what role they play in the solution of various problems. Considering problem solving as a

> **goal** can influence all that we do in teaching mathematics by showing us another purpose for our teaching. Each of these interpretations is important but they are different.

Hobden (2002) defines problem solving as follows:

> Problem solving is a multi-faceted **cognitive** activity in which we engage when we are confronted with a task in which routine action or normal thinking does not allow one to go from the given existing situation to the desired goal situation, but rather there is recourse to some form of **critical thinking.** Such critical thinking has the task of devising some action, which may overcome the perceived barrier between the existing and the goal situations.
>
> (Hobden 2002:102)

Relating Hobden's definition quoted above to the current policy documents for teachers and researchers, this definition for me sums up what problem solving is all about. It seems to incorporate the view of John Dewey's reflective inquiry (as quoted in Hiebert *et al.* 1996). I argue that using aspects of his approach will facilitate learners' understanding. I also concur with Hiebert *et al.* (1996) who claim that if the subject or task is 'problematized', it could result in the construction of understanding (see page 33).

2.3.3 Understanding the terms 'problem' and 'problem solving'.

The use of the term 'problem' is controversial within the mathematics education community. Hobden (2002) argues that students have a limited and narrow understanding of what it means to solve a problem. He believes that part of the difficulty is the plethora of meanings associated with the term that appear in curriculum documents and in teaching practice.

According to Gunter, Estes & Schwab (1995), a problem is a situation requiring much thought, which needs to be explained, but which at times does not have one correct answer (solution). As a researcher I identify with the above sentiments. This would require the person to have an open mind with clear thinking and understanding. In order not to draw incorrect inferences, authors need to furnish meanings of the terms, which apply to their own research context. They have to be clear about the use of the terms such as algorithm, procedure, real-life problems, routine or non-routine tasks.

The term 'problem solving' is frequently used in our everyday lives and is connected to thinking and higher cognitive skills (Greeno & Goldman, as cited in Malcolm & Lubisi 2002: 99). Also in many curriculum documents, critical thinking and problem solving are often referred to in the same sentence. For Hobden (as cited in Malcolm & Lubisi 2002:100) "all critical thinking is not problem solving, but all problem solving involves critical thinking." He further states that the task can only acquire the rank of a problem task and be regarded as a problem solving activity if it generates some aspect of critical thinking. The aspect of critical thinking is also endorsed in the critical outcomes adopted in our new curriculum. The focus of problem solving should be on the explanation and justification of the outcome of something and not solely on some particular pattern (Ball 1994).

2.3.4 The role of the teacher in a problem solving classroom

Cockroft (1982) comments that the teacher is the most essential ingredient in a problem solving classroom. It is suggested that the teacher in such a classroom plays a facilitative, consultative and managerial role and avoids direct teaching as much as possible. According to Simmons (1993), the teacher should demonstrate sufficient confidence in the content being presented and be able to communicate the material in an explicit and easy way to the class. Encouraging learners to react to controversial matters brought up in discussion by other students in the class and boosting their personal responses (Howe 1988) is the task of the teacher in such an environment. By engaging in realistic problems, learners learn to construct mathematical concepts. Instead of being prescriptive, the teacher needs to devise different learning opportunities that provide for the different learning styles of students.

2.3.5 Some factors that influence the problem solving process

Charles & Lester (1982:8) suggest three mental processes are required for successful problem solving. These three sets of factors, which interact with each other, are:

- Experience factors
- Affective Factors
- Cognitive factors

Experience factors include personal and environmental factors. **Affective factors** will include interest in the problem, motivation, pressure and ability to perform. **Cognitive factors** include having a sound reasoning and reading ability and adequate computational skills. These processes are seen as being 'ideal' when teaching problem solving but very often we encounter situations where one or more of the factors are missing.

Although individuals may have all the prerequisite knowledge to solve a problem, they may not be competent due to a number of factors, such as lack of familiarity with appropriate solution strategies, lack of motivation or a high level of stress. Further, two people may arrive at the same solution using different, but correct methods. This aspect of problem solving then makes it difficult to find the appropriate ways to teach problem solving (Charles & Lester 1982).

2.3.6 An overview of some problem solving research

Researchers over the past 25 years have debated many issues around the topic. In countries such as Germany, United Kingdom, Japan, Portugal, Italy and Sweden problem solving has become a particular focus of research in mathematics education (Lester 1994).

According to Lester (1994:2) a report of the National Assessment of Educational Programmes indicates that in the United States of America, the performance of students in problem solving is woefully inadequate. He quotes from Dossey, Mullis & Jones (1994) who state:

> On extended constructed-response tasks, which required students to solve problems requiring a greater depth of understanding and then explain, at some length, specific features of their solutions, the average percentage of students producing satisfactory or better responses was 16 percent at grade 4, 8 percent at grade 8 and 9 percent at grade 12.
>
> (Dossey, Mullis & Jones as cited in Lester 1994: 2)

Notwithstanding reports, textbooks and curriculum guides, which claim that problem solving is at the core of instruction at each level; the above evidence does not confirm this.

Since the formation of a review of the research on mathematics problem solving by Kilpatrick, as cited in Lester (1994), more than 25 years ago, there have been considerable reviews of the research literature by Schoenfeld (1992). In addition there have been many endeavours to understand problem solving research for classroom practice by Driscoll; Hembree & Marsh; and Kroll & Miller, (as cited in Lester 1994). The nature of problem solving has grown substantially since Kilpatrick (as cited in Lester 1994:663), "characterized the research literature on mathematics problem solving as atheoretical, unsystematic and uncoordinated, interested almost exclusively in standard text book word problems and restricted completely to quantitative measures of problem solving behaviour."

Table 2.1 provides a short summary of research in the problem solving field. It encapsulates and highlights the features of problem solving that have been of primary concern.

Table 2.1.
An overview of problem solving research emphases and methodologies: 1970-1994

Dates *	Problem solving research emphases	Research methodologies used
1970 –1982	Isolation of key determinants of problem difficulty: identification of characteristics of successful problem solvers; heuristics training	Statistical regression analysis: early "teaching experiments"
1978 – 1985	Comparison of successful and unsuccessful problem solvers (experts vs. novices): strategy training	Case studies: "think aloud" protocol analysis
1982 – 1990	Meta-cognition: relation of affects/beliefs to problem solving; meta-cognition training	Case studies: "think aloud" protocol analysis
1990 –1994	Social influences: problem solving in context (situated problem solving)	Ethnographic methods

* dates shown are only approximate

Taken from Lester (1994:664)

Table 2.1 shows that from the 1970s to the early 1980s, a considerable amount of interest was attached to the study of determinants of problem solving difficulty. The aim was mostly on the types of problems learners were required to solve at school. From the 1980s to about 1990, the emphasis was on comparing successful problem solvers with unsuccessful problem solvers, while at about the same time, interest in meta-cognition took root (see page 29 for more on meta-cognition). From the 1990s the emphasis changed to looking at problems in context and the effects of social influences.

According to Lester (1994:663-664) a study of the literature (which is not immediately apparent from looking at Table 2.1) shows four areas of inquiry where progress in teaching problem solving has been made:

(a) determinants of problem difficulty
(b) distinctions between good and poor problem solvers
(c) attention to problem solving instruction
(d) the study of meta-cognition in problem solving

(a) The problem difficulty for students

According to Lester (1994) there is general consensus today that the difficulty of a problem is not so much a function of various task variables as it is of the attributes of the problem solver. These attributes include: spatial visualisation ability; dispositions i.e. beliefs and attitudes and experiential background, such as instructional history; and familiarity with types of problems.

(b) The difference between 'good' and 'poor' problem solvers

Between the 70s and 80s problem solving was based mainly on individual problem solving competence and performance (Charles & Silver, as cited in Lester 1994:664). Much attention was given to assessing individual competence to show the differences between 'good and poor' problem solvers or 'expert and novice' problem solvers.

Lester (1994:665) makes the assertion that five important points distinguished good problem solvers (GPS) from poor problem solvers (PPS):

GPS knew more than PPS because what they knew was different, their knowledge was made up of connections and rich schemata. GPS focused more on the structural characteristics of problems while PPS focused on surface features. They were more conscious of their strengths and weaknesses than PPS. GPS were better able to regulate and check their problem solving efforts than PPS. The solutions of GPS were more refined than PPS.

Other characteristics of GPS which Suydam, (as cited in Krulik & Reys 1980:36) mention include that GPS had the ability to estimate and analyse. They scored higher on self-esteem and confidence and had good relationships with other children. GPS also had the ability to understand mathematical concepts and terms and exhibited lower scores for test anxiety.

(c) Teaching about problem solving

Lester (1994) found that the number of new problem solving programmes founded since 1975 were not firmly based on research. He further states that programmes are now grounded on the "folklore of mathematics teaching, particularly the sage advice of master teacher and problem solver – George Polya" (Lester 1994:665).

(d) Meta-cognition – the driving force in problem solving

The role of meta-cognition in mathematics activity has become an important emphasis (Lester 1985: 666). Meta-cognition, according to Flavell, as cited in Fortunato *et al.* (1991:38), refers "to one's knowledge and concerns one's own cognitive processes and products and anything related to them." In meta-cognition it is crucial that the problems chosen are challenging to learners and that there is more than one approach to the problem. Classroom discussion can benefit both learners and teachers' understanding of how learners are thinking about problem solving. Learners should be 'active' and 'doers' of mathematics, rather than only knowing mathematical facts and procedures. Teaching should be structured to develop their meta-cognition (Garofalo 1987). Lesh, Silver, and Schoenfeld (as cited in Lester 1994: 665) view meta-cognition constructs as "driving forces" in problem solving, affecting cognitive behaviour at all stages of problem solving. This area of research has flourished not only as a wedge pushing cognitive behaviour, but is also related to a number of non-cognitive factors like beliefs and attitudes.

Meta-cognition is important in this study because many teachers fail to focus on what learners should be looking at when given a problem to solve. The learners' focus should be drawn away from the problem's solution and directed towards the cognitive processes and strategies needed to solve the problem. Challenging problems given to learners enable them to reflect more on how they constructed the solution.

Some of the literature on problem solving instruction provides ambiguous messages, but according to Lester (1994) the following stand out as being important. For students to improve their problem solving ability, students must solve many problems. Problem solving ability is a slow, developmental process that takes place over a long period of time. For problem solving to be beneficial to students, it needs to be taken seriously by students and they must be certain that their teachers think it is important. The majority of students gain a lot from problem solving if the instruction is planned systematically. There is a need to teach students explicitly about problem solving strategies.

2.4 PROBLEM-BASED LEARNING AND REFLECTIVE INQUIRY

2.4.1 Introduction

New methods of learning have permeated our educational institutions over the last decade and one such approach is a method of learning referred to as problem-based learning.

2.4.2 What is problem-based learning?

In order to increase the usefulness of learners' knowledge, one response that some schools and universities have adopted is the model of problem-based learning or the problem-centred approach (Shulman as cited in Murray, Olivier & Human 1996:14). They both have the same features, but their approach may be different as regards their implementation in specific, individual classrooms (Murray, Olivier & Human 1998).

Problem-based learning (PBL)

> is a method of learning in which the learners first encounter a problem, followed by a systematic, student-centred enquiry process. Although the purpose of using problems in PBL is to stimulate learning of information and

> concepts brought out by the problem (rather than to 'solve' the problem), PBL does teach both a method of approaching and an attitude towards problem solving (Schwarz, Mennin & Webb 2001:1).

PBL is used mainly by medical and dental faculties at universities, where students work in groups of 4-6 with a tutor who acts as a facilitator of learning rather than as a direct source of information (Schwarz, Mennin & Webb 2001). The students meet the problem 'cold' not knowing much about the problem and then interact with group members to investigate their existing knowledge as it relates to the problem. They form and test problem hypotheses, establish further learning needs to solve the problem, do some self-study, then return to their groups to integrate the newly acquired knowledge to the problem.

Research by Vernon and Blake, (as cited in Schwartz, Mennin & Webb 2001:3) shows that "learners using this approach are superior to their counterparts from traditional curricula with respect to their approach to the study (being able to study for understanding rather than for short-term recall)''.

According to Boud & Feletti (as cited in Hiebert *et al.* 1996:14), PBL "is not simply the addition of problem solving activities to an otherwise discipline-centred curricula, but a way of conceiving of the curriculum which is centred around key problems in professional practice. PBL courses start with problems and as a result of working on these problems, the learners would be left with a residue of 'mathematics' rather than with the exposition of disciplinary knowledge." (see chapter 2, page 33 for different kinds of residue). As long as fifty years ago, it was claimed by critics that this approach narrowly concentrated on applications and they were concerned that important information would get lost. In the past, with the emphasis on acquiring the mechanics of mathematics, the more recent recommendations have placed more emphasis on applications and connections of mathematics in the real world (National Council of Teachers of Mathematics 1989: 1991). Learners will be able to apply the knowledge gained to a number of real-life situations. However, Hiebert *et al.* (1996: 14), still believe that this does not explain the difficulties that are implicit in the differences between acquiring knowledge and applying it. In order to understand the roots of this distinction and to develop the alternative principle of 'problematizing the subject'. Hiebert *et al.* (1996) look at John Dewey's idea of 'reflective inquiry' (see page 32 onwards).

The distinction in philosophy between 'knowing' and 'doing' builds directly on the separation between acquiring knowledge and applying it. Dewey remarks that 'knowing', resulting from reason and thought is 'potentially certain', while 'doing' is unreliable and indefinite. The difference between knowing and doing has become so prevalent and so evasive that it saturates our thinking. Dewey, (as cited in Hiebert *et al.* 1996:14) said, "We are so accustomed to the separation of knowledge from doing and making that we fail to recognise how it controls our conception of mind, of consciousness and of reflective inquiry." He accepts as true that reflective inquiry is the "key to moving beyond the distinction of knowing and doing and thereby providing a new way of viewing human behaviour" (Hiebert *et al.* 1996:14).

The fundamental features of reflective inquiry are:

- Problems are identified
- Problems are studied through active engagement
- Conclusions are reached as problems are at least partially resolved

(Dewey, as cited in Hiebert *et al.* 1996:14)

Dewey (as quoted in Hiebert *et al.* 1996:14) said, "when we treat an object as a problem to be solved and to examine it carefully, we begin to understand it, to gain more control over it and to use it more effectively for our advantage." Furthermore, he states "… were all instructors to realize that the quality of mental process, not the production of correct answers, is the measure of educative growth, something hardly less than a revolution in teaching would be worked" (Dewey, as cited in Hiebert *et al.* 1996:15). Once a conclusion has been achieved, the outcome of the process is a new situation. For Dewey, the advantage of reflective inquiry resides not in the solution to the problem but in the new relationships that are found, the new features of the situation being understood more deeply. Hiebert *et al.* (1996) links Dewey's reflective inquiry with understanding and works from the assumption that understanding is the goal of mathematics instruction. The problem-centred approach, which I am using, aims at helping children construct meaningful understandings for themselves (Olivier, Murray & Human 1990). They do this by using problem solving methods appropriate for their level of knowledge. "In fact they justify the practice of problematizing the subject by claiming that it is this activity that most likely leads to the construction of understanding," (Hiebert *et al.* 1996:14). To substantiate this assertion they look at how "problematizing" fits with the functional and structural views of mathematics understanding.

2.4.3 Functional understanding

Functional understanding means taking part in a community of people who practise mathematics (Lave 1988; Schoenfeld 1988). According to Dewey (as quoted in Hiebert *et al.* 1996: 15) "Knowing is not the act of an outside spectator but of a participator." Activity in the classroom is the focus of the functional view. Understanding is characterized explicitly by what students contribute to and share in the whole activity. Hiebert, *et al.* (1996:16) argue that the "key to shaping classroom activity that invites participation is to allow the subject to be problematic."

It is the teacher who is accountable for developing a social community of learners who problematize mathematics and who engage in looking for solutions. A significant aspect of such communities is that the gist of examination and discussion is on the methods used to find solutions. In their descriptions of classrooms, many case studies demonstrate evidence where teachers stress the open and constructive examination of methods of inquiry and solution (Cobb, Wood, Yackel & McNeal; Fennema, Franke, Carpenter & Carey; Murray, Olivier & Human, (as cited in Hiebert *et al.* 1996):16).

Hiebert *et al.* (1996) emphasize two factors of the learners' role in reflective inquiry classrooms. Firstly, the need to be accountable for sharing the results of their inquiries and for describing and accounting for their methods. This makes the learners become full participants in the community and establishes an openness that is vital for examining and improving methods. Secondly, learners need to realise and acknowledge that learning means learning from others, to look carefully at others' ideas and the results of their investigations. This supports the view taken by Murray, Olivier & Human (1993:75) that learners can and ought to learn from each other by listening to and trying to make sense of other procedures and concepts.

2.4.4 Structural understanding

Structural understanding "means representing and organising knowledge internally in ways that highlight relationships between pieces of information" (Hiebert & Carpenter 1992:17). The focus in the structural view is on what students actually take with them from the classroom. According to Davis (1992) residue provides a way of talking about the understandings that remain after an activity is over. The kind of residue will depend, in part, on the prior knowledge that learners bring to the activity and the kind of problem that has to be solved.

There are different kinds of residue. "Insights into the structure of the subject matter are left behind when problems involve analysing patterns and relationships within the subject" (Hiebert & Wearne, as quoted in Hiebert *et al.* 1996:17). An example will be in class where learners analyse the ways in which procedures work and how these procedures are the same and different. There is evidence that suggests that learners who take part in these kinds of debates have a richer structural understanding than those learners who proceed through a more traditional skills-based curriculum subject (Cobb *et al;* Hiebert & Wearne, as cited in Hiebert *et al.* 1996). Two kinds of strategies are found when working through problematic situations. One is the procedure that learners can use to solve particular problems. The second is the normal approaches or ways of thought that are required to construct the procedures. For example when a learner solves a fraction multiplication problem, they use specific procedures and techniques for solving these specific problems; in other words, specific procedures for specific tasks make up one kind of residue (see chapter 4 on analysis of solution strategies). A second strategy residue may be called 'meta-strategic'. While working through problematic situations, Hiebert *et al.* (1996) say:

> learners learn to construct strategies and learn how to adjust strategies to solve new kinds of problems. What gets left behind are the conceptual underpinnings and methods for actually working out new procedures when they are needed. (Hiebert *et al.* 1996: 17)

Evidence for this residue has shown that those learners who have been motivated to look at a situation problematically and develop and construct their own strategies are able to modify them or invent new ones to solve new problems (Fennema, Franke, Carpenter & Carey; Fuson & Briars; Hiebert & Wearne; Kamii & Joseph, as cited in Hiebert *et al.* 1996). It is these kinds of residue that I was trying to explore in the learners whom I interviewed. I was interested to see whether learners would be able to construct and develop their own solution strategies. By gaining insight into what they had constructed, could they now refine their understanding to solve new kinds of problems?

When learners develop methods for constructing new procedures they are integrating their conceptual knowledge with their procedural skill (Hiebert *et al.* 1996). This is important because research findings on learners' mathematics learning often disclose a separation between conceptual and procedural knowledge (Hiebert *et al.* 1996). From my own experience of teaching in a traditional classroom, some learners' use understanding that appears not to show their procedure and they memorise and recall procedures that they do not fully understand. In this study I was interested in seeing whether learners showed any understanding or whether they just used a memorized algorithm or learnt procedure. When learners are exposed to curricula that regard mathematics as problematic, this separation is rare (Carpenter, Fennema, Peterson, Chiang & Loef; Fuson & Briars; Fuson, Smith & LoCicero; Murray, Olivier & Human; Kamii & Joseph all cited in Hiebert *et al.* 1996). There is further evidence as quoted in Hiebert *et al.* (1996: 17) that those who participate in reflective inquiry and who treat mathematics as problematic, provide an opportunity for students to "recognise that inventiveness of their own practices" (Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Cobb *et al.* 1991).

2.5 PROBLEM-CENTRED APPROACH

2.5.1 Introduction

In the problem-centred approach, learners must be mathematically autonomous. This is what distinguishes the problem-centred approach from problem-based learning, although the theoretical principles are the same. Learners need to find out and decide for themselves how to go about solving a problem and what strategy to use (Botha *et al.* 1997). The teacher should not pressurize learners to use a specific strategy, as this will diminish the autonomy of the learner as they may just be copying out memorized procedures without the necessary understanding (Human 1993).

"As problem solving is one of the ways used to enhance pupils' construction of knowledge," as cited by Njisane, in Breen *et al.* (1992:28), this study is based on the problem-centred approach of Olivier, Murray & Human (1990). This approach is different from other problem solving approaches in that understanding is the goal of mathematics instruction. The principle is that students should be allowed to make the subject problematic. This means allowing students to search for solutions, to wonder why things are and to inquire (Hiebert *et al.* 1996).

2.5.2 Tenets of the problem-centred approach

In the early 1990s, Piet Human, Alwyn Olivier and Hanlie Murray from the University of Stellenbosch in South Africa piloted the problem-centred mathematics project, now called the problem-centred approach. An aspect of their approach is that learners construct their own knowledge, which is gained through social interaction, not necessarily found in the classroom. The intention of social interaction in the problem-centred approach is to establish openings for learners to talk about what they have done and this promotes reflection. In my study a key aspect was getting learners and teachers to communicate their thoughts about the problems presented to them. Von Glasersfeld (1991) describes this interaction as follows:

> From a constructivist point of view, there can be no doubt that reflective ability is a major source of knowledge on all levels of mathematics.... To verbalize what one is doing ensures that one is examining it. And it is precisely during such examination of mental operating that insufficiencies, contradictions or irrelevancies are likely to be spotted. By guiding learners to discuss the problem from their point of view and experimental approaches, which they might have discussed, boosts their self-confidence and equips them to reflect and display more feasible strategies. (von Glasersfeld 1991: 18-19)

It was through social interaction in the classroom that teachers and learners also construct a shared domain of "taken to be-shared mathematical knowledge that both makes possible communication about maths and served to constrain individual student's mathematical activity" (Cobb *et al.* 1991:6).

Ernest (1991: 42) contends, "The basis of mathematical knowledge is linguistic knowledge, conventions and rules and language which is a social construction." As a teacher who has taught using this approach, I can testify that it helped me to see the need for and encouraged some of my learners to interact and communicate with each other when solving problems. This was in contrast to the behaviouristic transmission model of acquiring knowledge, which I had otherwise been using. All I was interested in was getting my learners to provide me with an answer to a problem using a standard algorithm. It involved no understanding or meaning of why such an approach was used. The problem-centred learning approach represents the belief that subjective knowledge should be learned by the students "as personal construction and not re-constructed objective knowledge" Murray, Olivier & Human (1998: 171).

According to Fennema & Romberg (1999) the problem-centred approach is a programme where learners spend most of their time participating in problem solving. The emphasis on the importance of skills and understanding are an important aspect of this approach (Hiebert & Wearne 1993; Murray, Olivier and Human 1993; Carpenter, Fennema & Franke 1996). The programme has been researched and examined widely in primary classrooms (James & Tumagole 1994).

2.5.3 Cognitively Guided Instruction

The Cognitively Guided Instruction Project (CGI) and the Problem-Centred Mathematics Project (PCMP) were two other projects that inspired me to focus on learners' thinking, understanding and problem solving strategies. In both programmes children spend the majority of their time engaged in problem-solving activities. Both support a growing body of research that learning computational skills and developing understanding are important (Fennema & Romberg 1999). Teachers do not show procedures or expect learners to follow a certain algorithm. Children spend time using their own strategies (ways of solving a problem) that is consistent with the move to a more process driven problem solving approach (Southwood & Spanneberg 1996). More emphasis is now being placed on learners as they solve a variety of mathematics problems often set in story contexts, thus developing conceptual understanding and computational skills (Carroll 2000; McClain & Cobb 2001). This is consistent with Fennema & Romberg (1999) who proclaim that word problems and symbolic problems are the vehicles through which children learn mathematics concepts and skills.

The CGI project established at the University of Wisconsin by Carpenter, Fennema and Franke (1996) encourages its participants to use multiple solution strategies when tackling problem solving tasks. Research done at this institution has found that through the use of different solution strategies, children begin to feel that mathematics is an understandable body of knowledge that they can learn (Fennema *et al.* 1999). As the teacher's role is active, the goal of the CGI project "is to help teachers better understand children's thinking so that they can help children relate to what they are learning to what they already know" (Fennema & Romberg 1999: 51). Secada & Carey (2001:34) further contend, "Teaching should focus on problem solving, problem-solving processes and student understanding."

Teachers who attend CGI workshops are identified according to their various levels of belief and practice, which they exhibit. A Level 1 teacher for example is one who teaches by demonstrating the steps in a procedure and where children practise and apply the procedure. A Level 3 teacher is one who firmly believes that children can solve problems without being shown a particular strategy. Learners in such classes talk about the mathematics both to the teacher and their fellow peers. They use a variety of strategies and are able to compare and contrast them to other strategies. The distinctive difference between Level 3 and Level 4 teachers is "their use of what they learn from listening to students to make instructional decisions" (Carpenter *et al.* 1999: 108). A Level 4 teacher has a "more fluid perspective of their children's thinking; they do not apply their knowledge to assess their own students' thinking and to plan instruction, but also regard it as a framework for developing a deeper understanding of children's thinking in general" (Carpenter *et al.* 1999: 108).

Carpenter *et al.* (1999: 109) further contend, "developing an understanding of children's thinking provides a basis for change, but change occurs as teachers attempt to apply their knowledge to understand their own students." Studies have also shown that where teachers provide the environment in which children's thinking is the focus, children are able to construct their own procedures for solving problems and concepts are developed through problem solving (Carpenter *et al.* 1999).

Woods & Sellers (1996) found that when comparing children in textbook-based classes, with those in problem-centred classes, children in problem-centred classes achieved significantly higher on measures of computation and conceptual understanding and also held different beliefs and motivations about mathematics. Students in problem-centred classes believed that rather than conform to the methods shown by the teacher, it was better to find their own or different ways to solve problems (Woods & Sellers 1996).

Independent to the above is the University of Chicago Mathematics School Project (UCMSP) where learners are encouraged to develop their own solution strategies with no standard algorithm being taught as part of the curriculum up to the third grade. Findings from the study suggest that learners are capable of inventing and applying their own solution strategies when solving problems (Fennema *et al.* 1996; Kamii *et al.* 1993). It is suggested that by investigating various solutions, learners develop a larger toolbox of procedures and are more apt to choose one that best fits the problem and a solution strategy that the learner understands (Kamii 1993; Carroll 2000).

Two aspects of the problem-centred approach that require further elucidation because of their importance are the social component of the learning environment and the problem component.

2.6 THE SOCIAL COMPONENT OF THE LEARNING ENVIRONMENT

2.6.1 Introduction

In the problem-centred approach, the idea of social interaction produces opportunities for learners to talk and to discuss what they are thinking. Children are also encouraged to reflect on their different solution strategies with their peers and their teacher. By making sense of other learners' explanations, they are developing sophisticated, efficient concepts of number.

2.6.2 The role of the teacher

It is necessary to again return to the role of the teacher to explain how the teacher fits into the problem-centred approach. The idea of recognising that some learners construct their own knowledge, initially gave teachers the perception that they need not concern themselves too much in the learning process (Murray, Olivier & Human 1998). In order to clearly define the role of the teacher in the problem-centred approach, it is important to look at Piaget's classification of three kinds of mathematical knowledge, namely physical, social (conventional) and logico-mathematical knowledge, as well as two types of abstraction involved in the acquisition of each kind of knowledge (Piaget 1971; 1952); (Kamii 1985).

Physical knowledge is that knowledge which a child constructs through actions involving physical objects – for example the colour, shape or weight of counters. This forms the base for the child's knowledge about number.

Social knowledge is the knowledge teachers pass on to learners in order for them to understand the problem and to be able to put their thoughts down on paper, based on social conventions (Piaget 1971). In order for students to understand a problem, teachers need to provide students with the appropriate social knowledge (a convention for example that 1 km = 1000m). This enables them to communicate with each other, and to take possession of their thoughts in a coherent way. The tools to be used need to be shown to the students and teachers have to negotiate with learners the social norms that determine ordinary classroom behaviour and interaction. The construction of social knowledge is therefore dependent on the child's action on and interaction with other people (Wadsworth 1989).

Logico-mathematical knowledge is that knowledge which consists of relationships between objects and the ultimate source of these relationships, the human mind (Piaget 1971). For example seeing two stones on the ground, we can think of them as being different or similar. Another relationship we can see between the stone is 'two'. The stones can be observed empirically but the number 2 cannot.

The two kinds of abstraction mentioned by Piaget are empirical abstraction and constructive abstraction [also called reflective abstraction], (Kamii, Kirkland & Lewis, as cited in Cuoco & Curico 2001:26). In empirical abstraction, the focus is placed on certain properties of an object while other properties are ignored. For example, if the colour of an object is abstracted (physical knowledge), the other properties such as weight and the material with which the object is made are ignored. In contrast, constructive abstraction involves "making of mental relationships between and among objects" (Kamii, Kirkland & Lewis, as cited in Cuoco & Curico 2001:26). In the example above the relationships like similar, different, and 2 (logico-mathematical knowledge) are made by constructive abstraction.

I include the above in this research because it is essential to understand the key role that the teacher plays in such a learning environment.

2.6.3 The learning environment

In this research, I use part of the definition of culture as interpreted by Linton (cited in Haralambos & Holborn 1996:3), "culture is the skills, knowledge and accepted ways of behaving in a society in which one is found." In my view, then, the learning environment would include the nature of the tasks given to learners, the role of the teacher and the social culture of the classroom.

An important factor influencing the development of any mathematical understanding is the learning environment. A learning environment that is conducive for learners to come together to discuss, listen and take responsibility for their own learning is advocated. As learners come to the classroom with varying experiences and knowledge, their intuitions and experiences need to be explored in order for learners to develop a meaningful understanding of the topic. Learners need to be given opportunities to talk about their thinking and understanding of problems presented to them. I believe that learners need to be grouped in such a way that they can communicate with each other. I have mentioned Piaget's classification of different kinds of knowledge and agree with Murray, Olivier and Human (1998) that the physical and social knowledge, which they speak about, can be regarded as fitting

Vygotsky's view of learning (ZPD), where learners need to work with their more knowledgeable peers or with the teacher (Williams & Burden 1997).

Fennema & Franke (1992) in their research on teachers' knowledge and impact suggest that where the accent of the mathematics is placed on problem solving, with the emphasis on the 'power' of learners to do and understand the mathematics, learners become busily occupied in rich mathematical discourse with peers and teachers. In South Africa, however, research suggests that such a discourse community in the classroom is not going to be easily achieved. The work of Taylor and Vinjevold (1999) shows that much of the classroom discourse does not continue beyond the procedural level. Nickson, (as quoted in Newstead & Bennie 1999: 58) uses research to propose that the culture in classrooms depends "on its actors", in other words the teacher and the learners. Newstead and Bennie (1999) further state that "the unique culture in the classroom is the product of what the teacher and the learners bring to it in terms of knowledge, beliefs and values."

2.6.4 The social culture

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier and Human, (as cited in van Niekerk *et al.* 1999:218) identified four features of the social culture in a problem-centred classroom.

- All ideas are potentially important and should therefore be respected.
- Autonomy of methods should be encouraged. The need for every child to understand the method he or she is using must be respected. The children should also realise that a variety of methods can lead to the correct answer; therefore they should have the freedom to explore and share these methods with their peers.
- Mistakes must be seen as learning opportunities and should not be suppressed. They can lead to reasoning and discussion that might deepen learners' understanding of the problem.
- The authority for the correctness of the problem solution lies in the structure of the problem and not in the teacher or the other children. A method is not necessarily correct just because a popular child presented it.

(van Niekerk *et al.* 1999:218)

This resonates with my study and I am of the opinion that the above are essential for learners' mathematical development. In this study all ideas from the learners and teachers were taken as being important and were respected as they explained their thinking and understanding of the different solution strategies used.

2.7 THE PROBLEM COMPONENT OF THE LEARNING ENVIRONMENT

2.7.1 Introduction

In the problem-centred approach, the choice of the problem is crucial if learners are to understand concepts, different sub-constructs and different meanings. If learners are not subjected to these sub-constructs, "limiting constructions" are formed. This phrase, coined by D'Ambrosio and Mewborn (as cited in Olivier & Newstead 1998: 177), shows the misconceptions that occur when learners have only had a limited opportunity to develop a certain concept. For example the misconception that multiplication only makes bigger. This occurs when multiplying whole numbers, for example, 6 by 7 is 42. This restricts learners when they have to deal with fraction operations. For example, multiplying $\frac{2}{7}$ by $\frac{3}{6}$ equals to $\frac{1}{7}$, is pertinent here.

2.7.2 The kind of problem posed

Murray, Olivier & Human (1998: 177) use the words "learning trajectories", which refers to the focus or goal of a learner's problem solving experience, and mention that problems chosen should be realistic, meaningful and related to real life situations. In this study I have tried to make the problems as meaningful as possible and have used real life situations with which the learners can identify (see chapter 3 on analysis of solution strategies).

Problems serve different functions or are used for various purposes. Any problem chosen should be based on thorough content analysis and a sound understanding of how learners develop concepts and misconceptions. It is imperative that learners interact mathematically with a variety of problems and be given opportunities to make sense of and discover meanings about the new knowledge (Murray, Olivier & Human 1998:176). The problems that learners are given to solve may stay the same or be similar for a certain period of time. However, the solution strategies used by the learners should change and develop towards becoming more refined during this time.

The question should be asked whether a learning trajectory should end with a particular algorithm (for example division). I am of the opinion that learners who show flexibility in acquiring the knowledge of properties of numbers and operations and can make sense of what they are doing should be encouraged to continue exploring.

2.7.3 The role of time in the development of concepts

Research conducted by Murray, Olivier and Human (1998), shows that learners who have been exposed to the problem-centred approach for a few weeks to several months, and who appear to be weak at mathematics, can and do construct mathematical concepts. Some learners are able to construct mathematical ideas if not hindered in their thought processes. I can testify to this idea when I was teaching grade 5 learners in 1995. Some of the learners (after a period of time) were able to make sense of and understand mathematical concepts about the new knowledge they had learnt.

Learners very often appear to change a given activity into smaller sub-activities by transforming them into parts that they can cope with. For example, 29×46 can be written as $(29 \times 20) + (29 \times 20) + (29 \times 3) + (29 \times 3)$

In the above computation, digits are transformed into numbers that are easier to recall through the use of doubling and tripling. The ability to transform numbers like this is dependent on the learners' intuitive feeling for numbers and on the awareness of certain properties of operations. This was evident when I was analysing learners' solution strategies (see chapter 4, page 89).

2.8 Critique of the problem-centred approach

The problem-centred approach to learning may not achieve its goals if, firstly, the problems put forward are not selected for their mathematical structures and the order in which the problems are introduced are not well planned. Secondly, the development of routine skills like multiplication tables and bonds, for example, should not be abandoned but ways should be sought to encourage learners to see patterns and relationships (Murray, Olivier & Human 1998). Thirdly, in order to use the problem-centred approach successfully, teachers should "let students struggle towards solutions without suggesting procedures, yet provide sufficient scaffolding to keep students interested and on the task" (Clarke 1997: 282). I endorse this sentiment fully and try to implement it in my own teaching. Fourthly, a problem solving approach is not likely to be beneficial to learners if the prerequisite content has not been taught (Murray, Olivier & Human 1998).

The emphasis on getting learners to discover and refine their solution strategies when solving problems seems to be gaining momentum. The 'negative' effects of teaching only one written standard algorithm have been documented by Kamii; Plunkett (as quoted in Zarzycki 2001). The latter teaching strategies discourage a learner's logical mathematical thinking and have led to a poor success rate of large numbers of learners in solving such algorithms. Kamii & Dominick, (as cited in Morrow & Kenney 1998) found that learners using their own calculating procedures were able to solve more problems correctly than children who had been taught the standard algorithm. Fosnot & Dolk (2001) also found that teaching algorithms gets in the way of children's sense making. They further state that some children perform a series of memorised steps in getting to the solution, rather than trusting in their own mathematical sense.

This study has shown that learners are able to construct their own solution strategies and are willing to share them with their peers and teachers.

2.9 CONCLUSION

In this chapter I briefly discussed the transmission model, which initially had a big influence on my own teaching. The idea of learning procedures and memorizing information without understanding was a trademark of the approach. As a theory about the nature of knowledge and the nature of learning, the chapter's change in focus to a constructivist model showed how teachers interacted with children by questioning and discussion. They provided learners with problems to solve and responded to their ideas by getting them to discover relationships and predict future events. The aim of the teacher was therefore on children's thinking, rather than on their ability to write correct answers (Kamii & Joseph 1989).

I then engaged the reader with a section on problem solving giving an overview of problem solving research emphases and methodologies as well as some factors that influence the problem solving process.

As my own teaching paradigm shifted to a problem-centred approach with its emphasis on learners finding out how to go about solving a problem and using various solution strategies in the process, I explored the tenets of the approach and the various levels of belief that teachers undergo while participating in CGI workshops. As the teacher is central to the success of the problem-centred approach, I elaborated more on the role of the teacher, the learning environment and the problem component.

This chapter emphasised that learners should see mathematics as a problem posing, problem solving and investigative activity in which they build on their existing achievements to formulate new understandings (Southwood & Spanneberg 1996). In order to have control over their learning, learners need to take an active role in verifying their mathematical ideas and to reflecting on their mathematical experiences.

In chapter three, I examine the process of my research and focus on the qualitative methods that were used for collecting and analysing the data.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1. INTRODUCTION

The TIMSS study, one of the largest international comparative studies of its kind, found that South Africa's results showed the lowest overall improvement from Grade 7 to Grade 8. Further, it suggested that South African students seem to lack adequate problem solving techniques (Beaton *et al.* 1996). The TIMSS – Repeat (Howie: 2000) conducted in 1998/9 with more than 8000 learners in 200 schools in South Africa shows that the performance of Grade 8 learners in mathematics remained unchanged. Comparative results for fractions and number sense, for example, show that South African students achieved a significantly lower score than all other countries.

The main aim of this research was to explore Grade 7 learners' and their teachers' mathematical understanding through an analysis of their solution strategies (see chapter 4, page 78 onwards).

3.2. CONTEXT OF RESEARCH

The Rhodes University Mathematics Education Project (RUMEP) is a non-governmental, independently funded organisation, linked to the university with the specific aim of improving the quality of teaching and learning mathematics in primary schools. By assisting teachers in disadvantaged, rural schools, the focus of RUMEP is on in-service professional development and curriculum reform. My role at RUMEP is to evaluate teacher mathematics workshops out in the field and to co-ordinate the Advanced Certificate in Education course.

Cole & Flanagan (1995) recommend in a policy document that primary education should be the investment priority target for South Africa, as it will yield the highest social and private return of any level of education. Prompted by the design of developing a test instrument for use at Grade 4 level, (Mboyiya 2000), I embarked on a pilot study in 2000. In the context of my RUMEP work, the pilot study set out to develop tasks aimed at testing the problem solving skills and mathematical understanding of Grade 7 learners.

This pilot study investigated the solution strategies of Grade 7 learners. The assessment of the tasks shows discrepancies in performance between peri-urban schools in Grahamstown and urban schools in Port Elizabeth. This could have been language related as the schools all contained English second language speakers and the tasks were only written in English.

To ascertain the extent of the problem solving ability of learners, and their ability to use different solution strategies at Grade 7 level, the actual study of this thesis was based on a new test consisting of 12 tasks undertaken in the year 2001, in nine schools in the Northern region of the Eastern Cape. Learners from government schools with limited resources, such as a lack of stationery and classroom furniture, in the remote districts of Sterkspruit, Whittlesea and Bolotwa, near Idutywa, took part.

This chapter describes and explains the research procedure that was followed in the study.

3.3. THE CHOSEN PARADIGM

This study is best described as qualitative in nature and lies in the interpretive paradigm. According to Cohen & Manion (1994:36) this "is characterized by a concern for the individual. The central endeavour in the context ... is to understand the subjective world of human experience." Cantrell (1993:84) on the other hand states, "... interpretive researchers seek to understand phenomena and to interpret meaning within the social and cultural context of the natural setting." Mwira & Wamahiu (1995) assert that researchers adopting the interpretive paradigm:

> attempt to produce data that is holistic, contextual, descriptive, in-depth, rich in detail. They are concerned with discovering the inner meanings of social action rather than just their outward form. They are interested in describing processes instead of simply the outcome or end results.
>
> (Mwira & Wamahiu 1995:116)

A dimension that was important to this study was that knowledge in the interpretive paradigm was seen as constructed, rather than discovered (Denzin & Lincoln 1994). As I used a constructivist approach (see chapter 2, page 8), (Wellington 2000), the participants were encouraged to seek, understand and articulate their interpretations of their own solution strategies. A qualitative approach to data analysis was seen as appropriate. The focus was on action, especially on observing, describing and interpreting the action as it occurs within the context. Data was gathered by using in-depth interviews. The interpretive paradigm allowed the research design to be sufficiently flexible for the perusal of emerging categories and strategies.

Although I could have constructed a quantitative study, like the TIMSS study, my overarching purpose was to seek understanding, to explain and interrogate 'the what and the how' of learners' solution strategies. Hence, my commitment to the interpretive paradigm (Yin 1989). In this study, learners were encouraged to explain in depth 'what' and 'how' they attempted to solve a set of tasks. I was interested in exploring their own meaning and interpretation of solving the tasks. The interpretive paradigm was seen to be suitable for this study because I wanted to look for any constructs to which meaning could be assigned.

I acknowledge that my findings in this research are more tentative than assertive. The findings will hopefully encourage and facilitate further inquiry.

This study was interested in exploring and describing particular solution strategies. It was empirical in nature as it depended on interaction with learners and teachers in a natural setting i.e. in the classroom, and so fits the interpretive paradigm.

Maykut & Morehouse (1994) point out that the purpose of qualitative research is to gather enough knowledge to lead to understanding. I have tried to do this in this study by acquiring data from two sources, teachers and learners.

Quantitative approaches could have been used in this study if it had focussed on actual results of the tasks only. A qualitative approach was however seen as more appropriate because I wanted to explore learners' understanding by doing an interpretive analysis of their solution strategies.

3.4. GROUNDED THEORY AS A METHODOLOGY

3.4.1 Introduction

Grounded theory was the underpinning methodology selected for this study. As a form of qualitative research, the grounded theory perspective is " by far the most used qualitative interpretive framework in the social sciences today" (Denzin & Lincoln 1994: 508). Its applications are extensive as it caters for a specific set of steps a researcher can follow. Grounded theory is often referred to in the literature as the constant comparative method (Strauss & Corbin 1990). The methodology was seen as appropriate for the following reasons. Firstly, my study was a small-scale investigation into the solution strategies of learners. Secondly, the study was interpretive because it aimed at understanding the actions of individuals and was interested in looking at the thoughts of human beings. Lincoln & Guba (1985) refer to this as the use of 'human instruments'. Thirdly, it was a study where I wanted to explore, describe and classify the specific solution strategies of both teachers and learners in categories.

For this study I have used certain aspects of grounded theory. Some background information on grounded theory is necessary to understand the principles that I adopted in this study.

3.4.2 Beginnings of grounded theory research

In 1967 two sociologists, Barney Glaser and Anselm Strauss, first postulated grounded theory research, which is a general methodology, and later expanded on it through their books (Glaser & Strauss 1967; Strauss 1987; Strauss & Corbin 1990). They hold the view that hypotheses should be "grounded" in data from the field, especially in the actions, interactions and social processes of people.

3.4.3 Definitions of Grounded Theory

Various definitions of grounded theory exist. According to Rubin & Babbie (1993:55) grounded theory as a term is "based more on an observation than on deduction". This suggests that grounded theory provides a framework for taking observations, intuitions and understandings to a conceptual level. By this (conceptual level) is meant looking at the data carefully and taking a sentence, paragraph or observation and giving each individual event or occurrence a name. The name given to each event must represent a phenomenon (Strauss & Corbin 1990).

Schuerman (1983:111) states that the grounded theory approach "is concerned exclusively with the generation rather than the testing of theory."

The definition of grounded theory given by Strauss & Corbin (1990:24) best fits the methodological underpinnings of this study:

> Grounded theory is a qualitative research method that uses a systematic set of procedures to develop an inductively derived grounded theory about a phenomenon. That is, it is discovered, developed and provisionally verified through systematic data collection and analysis of data pertaining to that phenomenon. (Strauss & Corbin 1990:24)

In the grounded theory approach data collection, analysis and theory are in a "reciprocal relationship", (De Vos 1998:260). This implies that the researcher begins with an area of investigation and what is central to that area is allowed to unfold.

Glaser & Strauss (as cited in Wells 1995), further contend that it is vital that the theory is discernible and that the social world of the source needs to be clear, giving enough detail to the research process.

3.4.4 Coding

The coding of data into categories is central to grounded theory. Coding has been defined by Kerlinger (as cited in Cohen, Manion & Morrison 2000:283), "as the translation of question responses and respondent information to specific categories for the purpose of analysis." Coding can be very tedious, yet it is critical as it immediately raises data to a conceptual level (Strauss & Corbin 1990).

In this study the tasks given to the learners and teachers and the interviews were connected. How the tasks were answered helped the researcher to frame the questions to be asked.

The three major types of coding in grounded theory are open coding, axial coding and selective coding (Strauss & Corbin 1990).

Open coding

Open coding is the first step in the coding process as outlined in the grounded theory literature and involves looking at the following:

Labelling phenomena

The first step in analysis is conceptualising the data. By this Strauss & Corbin (1990:63) mean pulling apart an observation, a sentence or a paragraph and then assigning a name to each distinct occurrence, idea or event. The name given must represent or portray a phenomenon. Using the interviews and tasks, data of both learners and teachers were classified into various strategies. Strategies are the various ways employed to solve mathematical problems like decomposing a number, adding vertically or horizontally, and looking for the lowest common denominator. In the context of my research, I use the word **strategies** and not **phenomena**.

The strategies are now grouped together so as to lessen the number of units the researcher had to work with. The combination of concepts that seem to pertain to the same phenomenon (in my case strategies) are at this stage still considered "provisional" according to grounded theory (De Vos 1998:272).

When developing a category, it is the properties that are first developed. According to De Vos (1998:272) "properties are the characteristics or attributes of a category." He further mentions that once properties have been developed, these are then dimensionalized. Dimensions according to De Vos (1998:272) "represent locations of a property along a continuum. Dimensions were not considered in this study, as they did not fit the data collected.

Variations in ways of doing open coding

The process of open coding can be done in different ways. One way might be to take the first interview and observe and analyse it line by line. In my study, I pasted each interview and each question onto large sheets of paper, then took each line, examined phrases, sometimes even single words, to find the strategies, properties and categories. I then wrote them down next to the learner or teacher's work.

The other two types of coding in grounded theory, namely axial and selective coding (Strauss & Corbin 1990), were not used in this study as the relevant material, the solution strategies, did not lend itself to further coding. The limitations of grounded theory in the context of my research are discussed below in 3.5.

3.5. THE CONSTANT COMPARATIVE METHOD AND TIMSS CURRICULUM FRAMEWORKS

Grounded theory is based upon the joint coding and analysis of data to systematically generate theory – also referred to as the constant comparative method (Glaser & Strauss 1967). My original aim was to adopt this methodology. However, it was not possible to use all aspects of this approach, as my data did not lend itself entirely to such analysis. Creating and suggesting many categories, properties and hypotheses, is the crux of the constant comparative approach.

I was only able to code the tasks into categories, but was not able to compare the categories with previous developments in the same category, as my research instrument consisted of only one interview. My data were such that I could not compare the different strategies that learners used and produce theoretical properties of the category, which is characteristic of the constant comparative method (Glaser & Strauss 1967). I therefore agree with Lincoln & Guba (1985) who argue that the use of grounded theory in its entirety must fit the situation that is being researched. My research project only partly lent itself to the constant comparative method.

The initial justification for using this research method was that I intended to develop categories of children's solution strategies and come up with a theoretical framework.

For this reason, I needed to be flexible. I believed this approach would help me find answers to issues and that it was geared towards action and process. However, when examining the solution strategies that learners and teachers used, I found that the level of each task only allowed me to go as far as the first type of coding, namely, open coding.

In order to deconstruct the individual benchmark test, I used the TIMSS Curriculum Frameworks (McNeeley 1997) to classify each task into three classes: content category, performance expectation and perspectives (see chapter 4).

3.6. DESIGN OF THE RESEARCH INSTRUMENTS

3.6.1 The test

The Grade 7 test upon which this study rests is included in Appendix B. It consisted of 15 multiple-choice type and 12 tasks and comprised two parts: Section 1 - multiple-choice items, section 2 - problem-type word problems as described in section 3.8.1.

For this research only section 2 of the test was relevant. The main reasons for this were that I was not particularly interested in final solutions to a particular problem (although the schools asked for all the results of all the learners). For this study, the most interesting and important aspect was to find out how the learners and teachers confronted the problems and what strategies they had used to solve the problems. Further, I was also interested to see whether the solution strategies adopted by the learners were the same or different from those their teachers had used.

I had intended using section 1 of the test as well. However, some learners did not make use of the space provided to show exactly how they arrived at a solution. Hence, I decided to leave out this section and concentrate only on section 2.

3.6.2 Individual clinical interviews

In order to analyse the solution strategies in more depth, I interviewed nine learners and nine teachers. The interview schedule was semi-structured in nature (Bogdan & Biklen 1992) and contained a checklist of suggested questions. However, not all questions were pre-determined. This kind of interview allowed me to have more flexibility and freedom to explore the solution strategies adopted by the participants. The goal and emphasis of using the semi-structured interview was to probe for (learner's) understanding. Before each interview, I obtained permission from the learners and teachers to record the interview. The first question asked of all respondents was the same, namely to explain carefully and exactly how they (the learners and teachers) went about solving the tasks. Each respondent was then asked to elaborate on strategies in solving the tasks. Some learners and teachers were asked more questions than others. What I explored in the interviews was to find out how the learners and teachers had answered the tasks. I was also interested in the solution strategies that each one adopted and whether they could explain with understanding what they had done. None of the responses were exactly the same. I had to approach each task in a different way, according to the responses that the learners and teachers gave me.

3.6.3 Teacher interview schedule

A short additional structured interview schedule was drawn up asking the participating teachers to comment on a few aspects of the benchmark test. (see chapter 4, page 142). The intention behind doing this was to gauge whether the tasks were too difficult for their classes and whether having them translated into their mother tongue had any effect on how learners approached them. I was also interested to find out what the teachers thought about using multiple-choice questions as a form of assessment and whether they had covered all the work required of them in the curriculum prior to writing the test.

According to Cohen *et al.* (2000:273) a structured interview "is one in which the content and procedures are organised in advance." There is less flexibility over the range and order of questions in the schedule. It is also characterised by being a closed situation and may provide an easier framework for analysis (Wellington 2000). This I found to be applicable in this situation. Lincoln & Guba (1985: 269) suggest,

> that the structured interview is useful when the researcher is aware of what he/she does not know and therefore is in a position to frame the questions, that will supply the knowledge required.
>
> (Lincoln & Guba 1985)

3.7 SELECTION AND CHARACTERISTIC S OF PARTICIPANTS

3.7.1 The choice of the grade, criteria and schools

Grade 7 was chosen for this study because it is the beginning of the Senior Phase in the primary school. The TIMSS study in 1995 used learners aged 13-14 years, which is about Grade 7 level in South Africa. Learners at this age level are able to articulate their thought processes. Their communication skills are also sufficiently developed at this stage to articulate their solution strategies. From previous experience as a classroom teacher at this level, and from my RUMEP experience of working with many Grade 7 teachers in the Eastern Cape who are upgrading their qualifications, I felt that the teachers would be able to relate to this study.

The central criteria used to select the sample of learners for this study were competency to articulate in their mother tongue (isiXhosa), and a willingness to participate as a volunteer. Interviews would be conducted in mother tongue (or if the learners so wished in English). This was made very clear at the beginning of the interview.

The schools in which the pilot study took place were all government schools situated in Grahamstown, Port Elizabeth and Queenstown. Two were ex-model C schools, while the rest were township schools.

The Northern Region of the Eastern Cape was the chosen region for the actual study of this thesis because Department of Education (DoE) officials provided me with the names of schools in the districts and ensured access for me to visit schools. They even showed willingness to accompany me on my journey to the schools and were very supportive and positive in their approach. The schools chosen were rural in nature, some of them situated in very impoverished communities. Some schools lacked the necessary infrastructure, with a number of schools having several broken windows and doors, insufficient desks and chairs and inadequate curriculum materials like textbooks and other resources.

Towards the end of the first term in the year 2001, permission had been granted by the DoE in the Northern Region of the Eastern Cape to do the study. Twelve schools were approached and letters were sent to each of the schools in the Whittlesea, Queenstown and Sterkspruit districts.

Nine schools volunteered to take part. Three schools were situated in Sterkspruit, one in Bolotwa, near Idutywa, and five schools in Whittlesea. I visited all of these schools, before the study was to take place. The principals and teachers were informed about the purpose of the study. All information would be kept strictly confidential between the school and the researcher.

As this study involved working with children and teachers, ethical issues had to be taken into consideration. I discuss these in section 3.12.

3.8 DATA COLLECTION

According to Maykut & Morehouse (1994:174), the purpose of qualitative research is "to accumulate sufficient knowledge to lead to understanding". This has methodological implications for my study. In this study 'sufficient knowledge' was gathered using three data collecting techniques.

They included:

a) a test (see page 54)
b) individual, semi-structured clinical interviews (see page 55)
c) structured teacher interviews (see page 55)

The qualitative approach was appropriate in this study as it allowed me to analyse learners and teachers' work and hear each individual explain to me how they solved each benchmark task. I was able to cross-check by looking at their solutions, whether correct or incorrect, and probe further. This was important because there was the possibility that they may have given responses which they wanted me to hear, but could not explain.

The study progressed through three stages.

3.8.1 Stage one: The pilot study

Development of the pilot test

The pilot test referred to on page 48 in section 3.2, was modelled after the TIMSS study (Beaton, *et al,* 1996). The conceptual framework centred on the intended curriculum (the curriculum which learners are supposed to learn as laid out in official documents), the implemented curriculum (the learners' opportunity to learn) and the attained curriculum (what the learners actually achieve) (Long 1998). The pilot study was useful as it helped to establish baseline data of where Grade 7 learners were in terms of mathematical content and understanding in their schooling career and for the construction of the test for the actual study.

The set of tests were piloted in two township schools in Grahamstown, two ex-model C schools, one in Grahamstown and one in Port Elizabeth, and two township schools in Queenstown and consisted of 258 Grade 7 learners.

Nature of the pilot test

The content areas of the test were based on the TIMSS report (Beaton *et al.* 1996) which focussed on application and understanding (see chapter 4, page 73)

The set of tasks consisted of two sections: Section 1 consisted of 16 multiple-choice type questions where learners had to choose the appropriate answer from a list of four possibilities. Space was provided for any working out which might be required by the learner. Section 2 comprised 5 questions, which was a combination of calculation and word problems. In these tasks all thinking and working out had to be shown.

Table 3.8 indicates the breakdown and distribution of the content areas of the pilot test.

Table 3.8 Content areas, number of tasks and the percentage of total pilot benchmark test

Content area	Number of tasks	Percentage of total
Number Sense	13	62%
Measurement	1	5%
Geometry	3	14%
Data Handling	0	0%
Patterns	4	19%
TOTAL	21	100%

The test was heavily loaded on number sense, as this was one of the content areas where the findings of the TIMSS study indicate that South African learners performed poorly. I was specifically interested in exploring further solution strategies in this area of content.

Administering the tasks

Two staff members from RUMEP, both English first language speakers, were responsible for ensuring that the test was handed out to the learners in an orderly and organised manner. The instructions given to the two staff members are found in Appendix C. The instructions were necessary to ensure uniform participation conditions in the different schools.

Assessment of tasks

The test was assessed as follows:

In section 1 of the test, one mark was allocated for each of the multiple-choice questions. Table 3.8.1 shows the criteria used to evaluate the benchmark test of Section 2. In this marking scale, marks are not only allocated to an answer, but marks are also given for the correct operation and the process used in getting to the solution.

Table 3.8.1 Assessment rubric for tasks in Section 2

Marks	Criteria
0	Blank, nothing is shown. Problem is just re-copied. An incorrect answer. No working out is shown
1	Work shows some understanding of the problem. Uses correct operation sign, but implemented incorrectly. Answer is not shown or is incorrect.
2	Correct strategy shown, but makes errors along the way. Answer is not correct.
3	Correct strategy chosen with correct answer and correctly labelled.

adapted from Charles, Lester and O'Daffer (1987: 35)

The schools requested percentages for each learner and these were made available to each school. The assessment was regarded as confidential.

Capturing the data and results

All the scripts were marked and the raw data was captured from the learners' response booklets. The average for each school was recorded and all information was communicated back to the participating schools. The data from each school was analysed to see which areas in the mathematics curriculum each school found problematic.

Lessons learnt from the pilot study

As a result of the pilot study, certain aspects needed to be taken into consideration before the actual study took place. There were two main problems:

- Difficulties with language. Tasks were only written in English and this was a problem for most learners. Learners appeared to have difficulty in understanding exactly what was required, especially in the problem-type word problems.

- The participating schools had not covered some mathematical content knowledge. I then decided to look carefully at the tasks to see where changes needed to be made. One task that required learners to draw a reflected image was removed and substituted with a similar task that did not require a reflected image.

It was then decided that in order to give the learners a better opportunity of showing their understanding, the set of tasks would also be translated into their mother tongue. This was to address the language limitations as expressed by Howie (1997). Together with two other RUMEP staff members, questions were reviewed, rephrased or omitted and substituted by more relevant questions. The calculations and problem type tasks were carefully scrutinised for any misunderstandings like ambiguity and level of difficulty.

Five experts in the teaching of mathematics and the language field were requested to review the new version of the set of tasks. These included a primary school principal who taught mathematics, two practising mathematics teachers, the co-ordinator of the collegial cluster project at RUMEP, and an experienced mother-tongue education lecturer. Discussions between the above 'experts' and myself were held to check the appropriateness of the tasks in terms of language, format and context.

I also learnt that setting a set of tasks is not an easy thing to do. As the tasks had to be translated into mother tongue, it required that I be very clear and unambiguous in stating what was wanted in each task. It also showed me how difficult it must be for learners to learn mathematics in a second language. This was evident from the different ways that the set of tasks had been translated by the various mathematics 'experts'.

3.8.2 Stage two: The main study

Nature of the main test

The content areas for this test remained the same as the pilot test. Appropriate adjustments were made as discussed on page 60.

The test still consisted of two sections. Section 1 consisted of 15 multiple-choice type questions where learners were given four and sometimes five possibilities to choose the correct answer. Space was provided for any working out which the learner needed to do. Section 2 comprised 12 questions. This was a combination of computation and problem-type word problems. Learners had to show all thinking and working out in these tasks. Table 3.8.2 below indicates the total number of tasks asked and the total percentage of each content area.

Table 3.8.2 Content areas, number of tasks and percentages of the main total of the benchmark tasks

Content area	Number of tasks	Percentage of total
Number Sense	16	59%
Measurement	3	11%
Geometry	4	15%
Data Handling	1	4%
Patterns	3	11%
TOTAL	27	100%

Administering the test

A co-worker from RUMEP and I administered the test. A total of 341 learners participated and learners were given approximately an hour and a half to complete all the activities. After the learners had completed the test, I requested that the nine mathematics teachers also complete the test.

The notes to the administrators were the same as in the pilot study (Appendix C) except that a mother tongue speaker now read the tasks to the learners in isiXhosa. A study by Long (1998) had shown that although items had been translated into isiXhosa, the reading level of some of the items might have been too difficult. In order to overcome the difficulty of language, I also read each task in English.

Assessment of test and capturing of data

I used the same marking scheme as that used for the pilot study (see page 59). It was not my intention to collect quantitative data in this study, but schools asked specifically that I send the mean for their school and a short summary of the findings to them.

3.9 INTERVIEWS

One learner from each school was chosen by the teacher, using the selection criteria articulated on page 55, to be interviewed. Each interview took approximately 40 minutes. I divided the interview into three stages: beginning stage; questioning stage; and intervention stage (Ginsburg 1997). I initiated the interview by starting to ask the learners and teachers non-threatening questions about their families and favourite activities. I then told them that the purpose of the interview was find out how each task was solved and that I was interested in their thinking. During the questioning stage, I said the following: "I am going to read the problem to you and then I want you to tell me how you went about solving the problem." As the learners and teachers explained, I had to get them to elaborate on their responses and support their responses with explanations. The interview became a verbal means for me to listen carefully and gain a better understanding of how they approached each task. After the learners had been interviewed, I then interviewed the teachers. All interviews were captured on a tape recorder for further analysis

.

Hopkins (1993) points out that interviewing children can be problematic because they might encounter difficulties in explaining their thoughts and feelings. Children may also feel intimidated by the one-to-one nature in the interview, as they do not know the interviewer. In my experience in working with children, I have found similar observations. I also found that some children have a short attention span and if questioned too closely about a particular task, they refuse to talk or say, "I don't know".

My co-worker and I took cognisance of the fact that the respondents would place the interviewer on the same or on a higher level of authority as the teacher. Prior to the interview, I therefore tried to make the learners and teachers feel relaxed, as mentioned above. Clandinin & Connelly (1994) comment that the way in which the interviewer questions and responds to the answers will have an effect on the way the children and the teachers respond.

In order to address the above, the clinical interviews were kept as conversational as possible. The purpose of a clinical interview is to probe for understanding (Ginsburg 1997). Although I remained focussed and despite the careful planning, my co-worker, who was a Xhosa mother tongue speaker, and I still experienced problems like forgetting to probe further or not writing down something to return to later.

Ginsburg (1997) remarks that a clinical interview is difficult, as it takes skill and insight to reword questions and to do this on the spot. Fontana and Frey (1994: 361) make the assertion that "asking questions and getting answers is a much harder task than it may seem at first." We both found this to be true. I found it difficult to listen and to make notes, (see page 64) especially as my learners were second language speakers of English who would from time to time continually 'code switch' from isiXhosa to English. I had to force myself not to interrupt the learners or the teachers while they were talking, when I wanted to probe for more detail. Yin (1989) emphasises that it is important for the interviewer to make notes during an interview. We had great difficulty with that because we had to continually bring the child or the teacher back to the problem when they were derailed for some reason. The taped interview and the participants' written work was all that I could rely on for gathering my information.

The purpose of using clinical interviews as a data collecting technique was that I wanted to gain more insight into the 'why' and the 'how' of learners and teachers' solution strategies, through a careful analysis of the solutions to the tasks confronted by them. I also wanted to get learners and teachers to elaborate on and support their responses with clear explanations. Clinical interviews were chosen because using them would also enable me to do a comparison between learners and teachers' responses.

As I had chosen the clinical interview as my most important data collecting technique, I had to consider the following very carefully. I had to be really interested in each child and each teacher's response. This I did by carefully getting each one to elaborate on responses that seemed unclear or vague. I had to be flexible in my approach and allow the interviewee enough time to complete each question. I did this by giving the interviewees time to pause. My focus had to be clear of what was required of each teacher and learner. I had to refrain from interrupting them as they explained a problem, but rather allow them the opportunities to clarify their thinking.

Being a teacher myself, I found it quite difficult to not give any clues. I was able to ask the teachers and children to repeat a step if I could see that he/she had made an error. Very often when teachers and learners discovered an error in their mathematics on their own, they were able to correct it on their answer sheet.

When interviewing, I was particularly interested in the processes the respondents had using in solving the tasks, and wanted to gain insight into these processes. Another aspect I had to consider in the construction of the interviews was taking gestures (see page 63) and displays of emotion into account. Although some interviewees struggled from time to time, it was important for me to offer encouragement and to listen to their responses.

I also had the difficult task of encouraging both the teacher and the learner to verbalise everything that they were thinking. This was quite demanding and drained one emotionally (Ginsburg 1997).

Ginsburg (1997) lists some of the major features of the clinical interview that should be considered. Firstly, I should be open and allow the interviewees to explore, but at the same time try tò establish the learner's/teacher's competence by continuing to probe and record questions. Secondly, it was difficult trying to continually consider hypotheses about why the learner/teacher does or says what he/she is saying, without asking for more explanation. Thirdly, I had to always think, or theorize about what children at this age are capable of and had to try to make them feel comfortable and at ease. As all the learners were second language speakers, I was able to use my co-worker's help with the interviews. It was also important for me to take cognisance of the behaviour of both the child and the teacher for whatever clues they may offer about their thinking and understanding. For example, looking at their facial expressions and noting pauses as they spoke. One particular learner was very excited about answering the tasks. At the end of the interview, he asked me what I was going to give him. Another learner fidgeted continually with the cuff of his shirt and, from the expression on his face, he appeared quite nervous. I therefore needed to make him feel at ease.

As I examined the child's and the teacher's thinking, I also looked at the following aspects, as suggested by Ginsburg (1997). The asking of fundamental questions like 'how did you know that', or 'explain to me how you worked out that sum' happened often during the interviews. I had to be careful not to discourage the child or the teacher's way of solving the problem, but accept and value their strategies even if it was unorthodox or even incorrect. At times, it was necessary not to assume that a wrong answer is wrong or that a right answer is right. I needed to delve beneath the answer to understand the thought that produced it.

According to Ginsburg (1997) in order to establish credibility in the clinical interview, one has to often rephrase the task to avoid misunderstanding. Sometimes in the interviews it was necessary to change the problem by first substituting it with a simpler problem. In order not to make the child or the teacher feel uncomfortable, I had to sometimes repeat the question and ask them to explain it more carefully or slowly by saying, "I didn't quite understand what you meant, so let's do it again and you tell me what you are doing?"

3.9.1 Reflections on the individual clinical interviews

As I reflect on the experience of conducting an interview, I can say that I learnt that one should never go into an interview 'cold', without thorough planning and preparation. The tasks presented to the respondents had to be complex enough to engage the child or teacher in thinking.

Both teachers and learners were unfamiliar with the one-to-one nature of the interview situation and both groups had great difficulty expressing themselves. As these interviews took place in the respective schools, it was sometimes difficult to find a suitable, quiet spot to conduct the interview as interrupting noises were heard.

The learners were at first fascinated by the micro-cassette tape recorder, but nevertheless responded willingly and generously. I found the tape recorder was sometimes a source of distraction to both the interviewer and some of the interviewees. Some spoke very softly and sometimes, in my haste, I was worried that I might have pressed the incorrect button and not recorded the interview. At times I did, however, experience a 'dead end' when getting no responses from some of the learners. However, on the whole the teachers felt more intimidated and inhibited in answering than the learners did.

The original taped interviews and their corresponding transcripts are housed in the RUMEP resource centre at Rhodes University, Grahamstown.

3.10. ANALYSIS OF DATA

Some interviews, which were recorded in isiXhosa, were first translated into English, then transcribed and photocopied. Large sheets of cardboard were used where each task was given a page number and was coded, for example Q1.L1 or Q1.T1. (Question 1, Learner 1 or Question 1, Teacher 1). All the solution strategies for task 1 were then pasted onto separate sheets of cardboard. The solution strategies of each task were indicated in pencil in the margin together with the properties and possible categories. The learner and teacher sheets were then placed side by side and compared for similarities and differences. The interview scripts were then reread a number of times to substantiate the properties and the category (See chapter 4, from page 77).

3.11. VALIDITY AND RELIABILITY

Validity is a prerequisite for both quantitative and qualitative research. In qualitative research validity is concerned with description and explanation and asks the question whether a given explanation fits a given description (Denzin & Lincoln 1994). According to Gronlund, (as cited in Cohen Manion & Morrison 2000:105), validity should be seen "as a matter of degree, rather than an absolute state".

In order to assess the methodology which I used, I refer to Campbell & Stanley (as cited in Le Compte, Millroy & Preissle 1992). They list eight factors that threaten internal validity and four factors that threaten external validity. Internal validity has to do with how accurately the findings describe the phenomena being researched

(Cohen, Manion & Morrison 2000). External validity "is the degree to which results can be generalised to the wider population" (Cohen, Manion & Morrison 2000:109).

The two threats affecting the internal validity of my study are: subject selection and changes in instrumentation. Although all learners in Grade 7 took the benchmark test, I left it up to the teachers to choose the learners whom I could interview. Although there were no direct changes to the instrumentation as such, it was necessary to translate the tasks into isiXhosa and change some of the problems, because teachers had not covered certain sections of the curriculum.

I think a threat to external validity included subject selection effects. That the subjects selected for this study come from only one racial group, may affect the validity. Other threats to validity, which may have had an impact on this study, were ensuring that the level of the benchmark tasks were appropriate for Grade 7 learners, i.e. neither too easy nor too difficult.

During the stage of data analysis, a threat to validity may have been my not making enough inferences and poor coding, as I used only aspects of grounded theory.

I am of the opinion that my interviews are validated because when I compare the interview transcripts with the actual written work of the learners and teachers, the two achieve the same result. This comparison is termed convergent validity.

A criticism of qualitative research has been the concern with bias. Bias may have crept into this study through the respondents behaving differently when they were placed in the new situation, like the interview situation. Another bias may have been the attitude, gender, race, age, personality, dress, comments, replies and non-verbal communication of the researcher. I tried by to minimize the amount of bias as much as possible by doing clinical interviews, which puts tremendous pressure on the interviewer rather than on the respondent (Ginsburg 1997). However, I agree with Hitchcock and Hughes (as cited in Cohen, Manion & Morrison 2000) who argue that because humans are interacting with humans, it is unavoidable that the researcher will have some effect on the respondents and, consequently on the data.

Research in the interpretive paradigm recognises that the findings stress a subjective interpretation, which may be biased. Subjectivity is therefore accepted and valued as a vital aspect of understanding. This is an essential part of the paradigm, although I have tried to be conscious of my own subjectivity and my position in this research. That is why a co-worker was used to help with the mother tongue interviews and their transcription.

Reliability is concerned with precision and accuracy (Denzin & Lincoln 1994). According to Bogdan & Biklen (as cited in Cohen, Manion & Morrison 2000:119), reliability "is regarded as a fit between what researchers record as data and what actually occurs in the setting that is being researched."

I believe that the reliability of my interviews and benchmark tasks have been enhanced by the careful piloting which was done the previous year and which resulted in changes and revisions being made. On the aspect of reliability, I concur with Wellington (2000:13) that "reliability is linked to the idea of 'replicability', i.e. the extent to which a piece of research can be copied or replicated in order to give the same results in a different context with different researchers."

3. 12 RESEARCH ETHICS

It is asserted by many researchers that ethical issues need to be given careful thought before, during and after the research process (Dane 1990). There are several distinguishing issues, according to Dane (1990), which the researcher needs to take into consideration. The following were important for this study: voluntary participation, informed consent, researchers and co-worker's identity and anonymity of participants.

3.12.1 Voluntary participation and informed consent

For the actual study, informed consent was obtained firstly from the DoE in Queenstown and Aliwal North who gave me the names and addresses of schools. Individual letters were then written to each school requesting them to indicate whether they wanted to take part in the study or not. A mother tongue co-worker and myself then visited schools that indicated a willingness to participate. I explained to the principal, teachers and learners of each school, the purpose of the study and what was to take place. They were then asked to indicate whether they still wanted to participate or not. It was explained to the teachers and learners that the purpose of the task was not to test for right or wrong answers, but rather to look at the processes involved in the solving of problems using different solutions. According to Dane (1990: 59), "research participants have the right to voluntary participation, the right to be aware that they are participating and the right to information that may affect their decision to participate."

3.12.2 Researcher's and co-worker's identity

As the researcher and co-worker, it was our responsibility to portray ourselves accurately to the school, the principal, the teachers and the learners (Dane 1990). I made every effort to be sincere, honest and to gain the trust of all those involved. In a qualitative study it is often taken for granted that research is not value- or bias-free, with the researcher early in the study describing and identifying his own biases and tenets (Janesick 1994).

The mathematics teachers and the learners were kept fully informed about our visits. A letter was sent to all principals informing them about the exact date of our visits. They were also informed that if they felt uncomfortable about taking part, they could withdraw at any time.

No school objected and principals and teachers welcomed us with open arms each time. They were grateful for any feedback on what we had found.

After the pilot study, all schools were sent letters informing them about the results and the general trends which were found (Penlington & Michael 2000). Principals wanted to know where their learners stood as compared to other schools in the study. They were not that much interested in what I had set out to do, namely to explore the solution strategies of learners and their understanding of these strategies.

3.12.3 Anonymity

All learners and teachers who participated in this research were volunteers and their names or schools have not been mentioned. All information has remained strictly confidential with the researcher. Transcripts of raw data and interview schedules are available in the RUMEP archives.

3.12.4. The role of the mathematics teachers

I was completely open with each mathematics teacher who participated in the study by carefully describing the purpose of the study and the point of the research. All the teachers (except for one teacher, who appeared nervous at the start, but nevertheless still took part) showed their willingness and volunteered to participate. They were eager to see how their learners performed. I explained that the mathematics teacher could be present at all times when the benchmark tests were being administered and that they could act as an assistant interpreter should the need arise. All the teachers readily took up their role as assessment interpreters.

3.13. CONCLUSION

This chapter has located the study in an interpretive paradigm. More specifically, it has described and justified the methodology used. It was a small-scale investigation, which used only one level of grounded theory as a methodology. Some limitations of the methodology were discussed. Further, the design of the research instruments used to collect the data was explained. Issues of validity and reliability were addressed, as were ethical considerations.

The next chapter describes the analysis of the solution strategies used by learners and teachers.

CHAPTER FOUR

ANALYSIS AND INTERPRETATION OF SOLUTION STRATEGIES

4.1 INTRODUCTION

In this chapter, the data gathered using the methodology discussed in chapter 3 are analysed. In the context of South Africa being placed at the bottom of all countries that participated in the last two TIMSS studies (Beaton *et al.* 1996 and Howie 2000), my study on solution strategies used some of the original TIMSS items. The tasks required learners to solve problems and computations using their own solution strategies.

The analysis procedure for each task will follow the same format:

a) Description of task: Here I show the task as it was presented to the learners.
b) Categorisation of task using the TIMSS Curriculum Frameworks: Here I use the TIMSS curriculum framework as discussed in 4.2. to categorise each task into categories, subcategories and subordinate categories.
c) Open coding: Here I analyse the solutions of the tasks, identifying strategies used and labelling the properties. I have replaced the term 'phenomena' (as used in grounded theory) with 'strategies'
d) Discussion of solution strategies: Here I describe and discuss the solution strategies based on learners and teachers' transcripts inferred from the interviews.
e) Comparison of teacher and learner solution strategies: Here I look for comparisons and similarities between teachers and learners' solution strategies. I examine whether learners adopted similar strategies as teachers and vice-versa.

The test (only section B was analysed) consisted of 12 tasks on numbers, measurement, geometry, proportionality, functions and data handling for Grade 7 learners (see chapter 3). The test was administered after adaptations had been made to it on the basis of the pilot study. The solution strategies and complementary data were supported by learners and teachers' clinical interviews (see chapter 3). Their responses to the tasks were coded using aspects of grounded theory and the referred TIMSS Curriculum Frameworks (McNeeley 1997).

4.2. TIMSS CURRICULUM FRAMEWORK

The TIMSS Curriculum Framework was used in this study to categorise each task into categories, subcategories and subordinate categories related to learning goals comparing mathematics education across countries. The frameworks as used in the TIMSS study of 1996 examined three aspects of the curricula: Firstly, content or subject matter, secondly, performance expectations, e.g. problem solving and thirdly, perspectives, such as a positive attitude towards mathematics (McNeeley 1997).

4.2.1 Content

According to Mc Neely (1997:28), "any curriculum component in this classification system can be described by a 'signature' involving categories and subcategories of the three aspects." The content component of the TIMSS Curriculum Frameworks shows the content categories of mathematics used in the American curriculum. The content categories are comparable to the curriculum found in South Africa. The 10 content categories (Figure 4.1) are further subdivided into 29 subcategories and 20 subordinate subcategories (Only subcategories and subordinate subcategories applicable to this study are found in Appendix A).

- Numbers
- Measurement
- Geometry: Position, Visualisation and Shape
- Geometry: Symmetry, Congruence and Similarity
- Proportionality
- Functions, Relations and Equations
- Data Representation, Probability and Statistics
- Elementary Analysis
- Validation & Structure
- Other Content

Figure 4.1. TIMSS content categories

4.2.2 Performance expectations

The performance expectation component refers to the cognitive dimension and describes the many kinds of performances or behaviours that might be expected of learners. The expectations refer to what learners were able to do with the knowledge they obtained. The performance expectations (Figure 4.2) vary from being able to understand information, to applying it in theory, problem solving and investigations.

- Knowing
- Using routine procedures
- Investigating and problem solving
- Mathematical reasoning
- Communicating

Figure 4.2. TIMSS performance expectation categories

Figure 4.2.1 below highlights the performance expectation categories listed above which are further subdivided.

Knowing		
Representing	Recognising equivalents	Recalling mathematical objects and properties

Using routine procedures		
Using equipment	Performing routine procedures	Using more complex procedures

Investigating and problem solving				
Formulating and clarifying problems and situations	Developing strategies	Solving	Predicting	Verifying

Mathematical reasoning					
Developing notation and vocabulary	Developing algorithms	Generalising	Conjecturing	Justifying and proving	Axiomatising

Communication			
Using vocabulary and notation	Relating representations	Describing and discussing	Critiquing

Figure 4.2.1 Categories and subcategories of performance expectations

The performance expectation categories and subcategories were used as a guide to frame the discussion of the solution strategies.

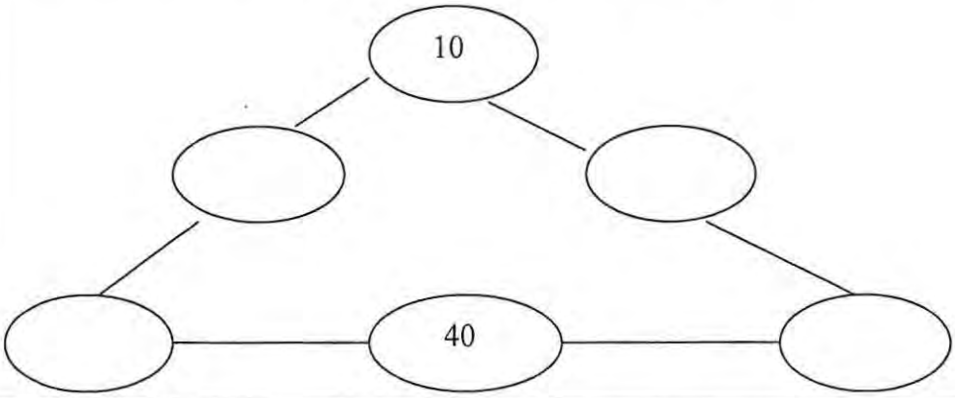
4.2.3 Perspectives

The aim of the perspectives aspect of the TIMSS Curriculum Framework is to identify clear goals for teaching mathematics. The perspectives aspect concentrates on the development of learners' attitudes, interest and motivation. This aspect will not be discussed in the study.

4.3. TASK 1

4.3.1 Description of the task

In the magic triangle all the numbers along each edge must add up to 90. Put all the numbers 20, 30, 50 and 60 in the circles to make correct totals.



4.3.2. Categorisation of the task

1.Content Area Category	Number
2.Subcategory	Whole numbers
3.Subordinate Subcategory	Operations
4.Performance Expectation Category	Investigating and problem solving
5.Subcategory	Developing strategies

Figure 4.3 TIMSS Curriculum Framework categorisation for task 1

The task as illustrated in Figure 4.3 relates to the addition of whole numbers, which is part of number sense. It incorporates investigating whole numbers and requires learners and teachers to investigate and develop a strategy in order to solve the problem.

4.3.3. Open coding

Table 4.1 Summary of strategies used by both learners and teachers in solving task 1

CS = Correct solution IS = Incorrect solution

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1	✓		Trial and error	1	✓		Sequence of numbers
2		×	Addition	2	✓		Trial and error
3	✓		Trial and error	3	✓		Consecutive multiple strategy
4		×	Trial and error	4	✓		Trial and error
5	✓		Trial and error	5	✓		Consecutive multiple strategy
6	✓		Addition	6	✓		Trial and error
7	✓		Trial and error	7	✓		Addition strategy
8		×	Trial and error	8	✓		Trial and error
9	✓		Trial and error	9	✓		Trial and error
67 % correctly answered by learners				100% correctly answered by teachers			

Table 4.1 refers to the strategies used to solve task 1 by the learners and teachers. The following strategies were identified:

Trial and error strategy

The trial and error strategy is a strategy where any numbers are placed in the circles in a random way in order to get to a total of 90.

Sequence of numbers strategy

The sequence of numbers strategy is a strategy starting with a particular number and following a sequence to get a total of 90.

Consecutive multiple strategy

The consecutive multiple strategy involves starting with the first multiple of 10 and placing numbers 20-60 in the appropriate positions to arrive at a sum of 90.

Addition strategy

The addition strategy uses the numbers given which are then added to the other multiples of 10.

The property (see chapter 3, page 52) I have identified in task 1 is 'using multiples of 10 from 10 to 60 to get to a total of 90'.

4.3.4. Discussion of solution strategies

The strategy used by six of the learners in task 1 was trial and error. In the interviews, learners mentioned using random numbers to get to a total of 90 along one side before applying the other numbers to the other two sides. The three learners, who were unsuccessful in the task, failed to check the sides of the triangle to get to a sum of 90 and consequently ended up with a total of more than 90.

L 1 refers to learner 1 and T 1 refers to teacher 1.

L 1 started using the given numbers 10 and 40.

> L 1: These numbers are 10 and 40. I put 50 here and it makes 60 and 30 here and it makes 90. And here it's 50 and here it's 20. I took 40 and put 30 to this and it makes 90. There it is 60 and 30, makes 90. I add 30, this 40, it makes 70 plus 20 and it makes 90.

L 9 used one side to complete the triangle before moving onto the other two sides.

> L 9: It is said that we must add these numbers to make 90. We were given 20, 30, 50, 60 and on this side 10 and on this side 40. And then on this side I took 60 and 20 equals 80 and add 10 to make 90. And I also added 40 + 30 = 70 and 20 + 70 = 90 and on this side 50 + 30 = 80 and add 10 = 90.

Five out of nine teachers also used the trial and error strategy by looking at the numbers and then randomly apportioning them to one side and its opposite side.

T 1 made use of a clue by ordering the numbers in a sequence.

> T 1: I found that there is a clue here because if you see 10 there, then the corners. You make, I , I decided to put them all to their order 10, 20 , 30. Then after finding those solutions, then I looked for a number that is to make the sum of 90 and then I decided on 60 on this side and 40 on that side.

T 4 who used the trial and error strategy said:

> T 4: It asks that we have to get the answer 90 so we are given it here at the top, 40 at the bottom and I looked for both for giving a number. I was also looking at the right hand side and the bottom part. Whenever I put a number here, it must also give me the answer here. It was the right hand side. It was easy to get to the answer.
>
> I: So basically what strategy did you use?
>
> T 4: I used the trial and error strategy.

Gray (as cited in Nickson 2000), found that children who were asked to rearrange numbers visually, used their knowledge of pattern and were forced to complete the task, using a specific method. However, when numbers were given to them vertically, they resorted to algorithmic 'rule-like' behaviour, which many did not understand. In this task learners and teachers were free to choose any strategy as long as they arrived at a total of 90.

The different solution strategies used by learners is emphasized by de Lange (as cited in Nickson 2000), who comments that besides the different models that should be made available to learners to help them solve problems, learners should also be allowed to use their own strategies.

4.3.5. Comparison between learner and teacher solution strategies

The teachers used more sophisticated approaches to solve the magic triangle, like looking at a sequence of numbers or looking for a relationship between numbers on the sides of the triangle. Starting from one side of the triangle and using consecutive numbers was another strategy used. There were two instances where the teachers' strategies matched the learners' strategies like in the trial and error approach. This was very common among the learners and was used by more than half of all the participants.

T 3 used a consecutive multiple strategy and said:

> T 3: I started with the corner numbers, using consecutive multiples of 10, (10, 20 and 30). It was easy to fill the middle numbers since they are also consecutive multiples of 10 (40, 50 and 60). The three all add up to 90.

The learner in this teacher's class started with the number 10 that was given.

> L 3: I add 10 + 60 + 20 and I got 90. And 20 + 40 + 30, I got 90 and 10 + 50 + 30 and I got 90.
>
> I: So how did you know that you have to put 60 on this side and not on that side.
>
> L 3: I thought if I put it this side, I won't be able to get 90.

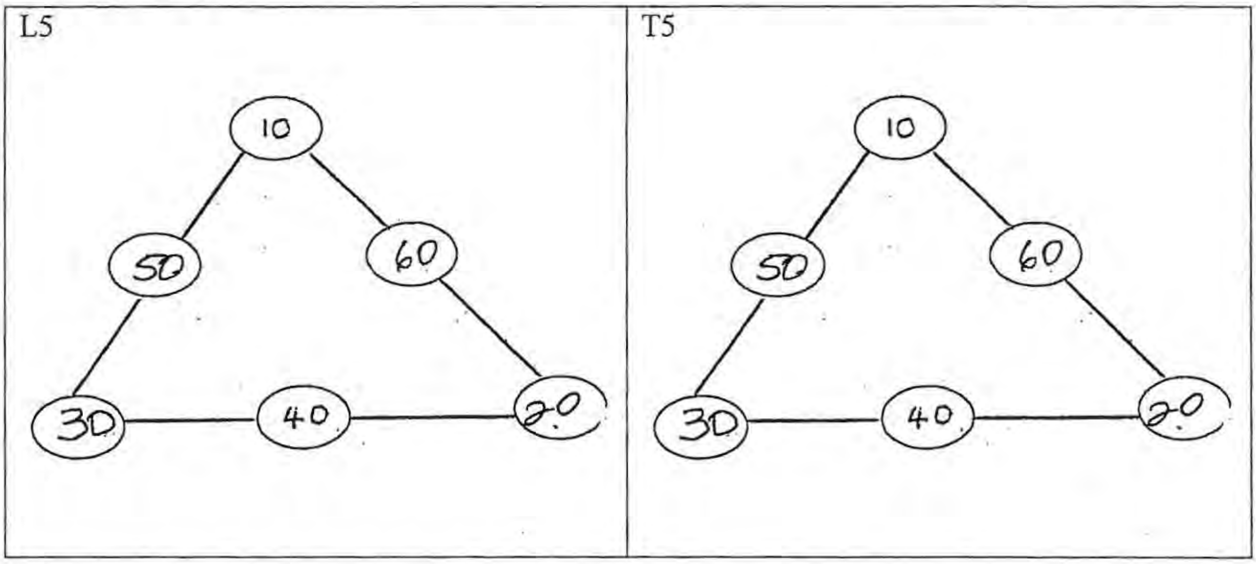


Figure 4.4 Identical solution strategies of a learner and teacher

Figure 4.4 shows identical solutions, although the strategies used by both learner and teacher differed. L 5 used random numbers to solve the problem, while T 5 used a consecutive multiple strategy.

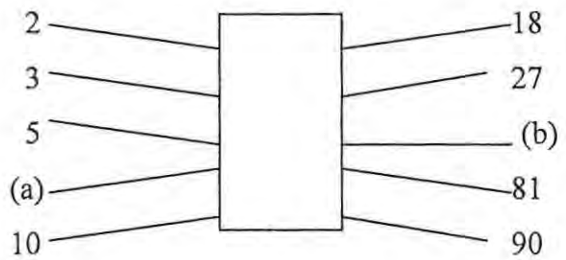
> L 5: I took some numbers from here 50 + 30 = 80 + 10 = 90. That is my first answer. And I add 30 + 40 and it became 70 and I add 20 and it became 90. That is my second answer. I add 10 + 60 and it became 70 and I add 20 and it became 90.

> T 5: Now when looking at this triangle, we have been given, if I am correct this magic number and we have got this 40 and 10 and that's 50. Now eh, I asked myself having been given that magic number, what would I possibly have to add to get 40. Now not only looking at this in order for that 90. Now this 30, I added 30 and 40 to get 70. So here now I said 40 + 30 = 70. 90 minus 70 gives 20. As well as this 40 + 30. And I subtracted this from 90 and got 50.

4.4 TASK 2

4.4.1 Description of the task

Fill in the missing numbers in (a) and (b)



4.4.2. Categorisation of the task

1.Content Area Category	Functions, relations and equations
2.Subcategory	Patterns, relations and functions
3.Subordinate Subcategory	Not applicable
4.Performance Expectation Category	Knowing
5.Subcategory	Recognizing equivalents

Figure 4.5 TIMSS Curriculum Framework categorisation of task 2

Figure 4.5 shows the reason for placing this task in the content category of functions. It is because learners at this stage are learning to develop the concepts of variable relationship and function. The understanding of these relationships will allow the learner to describe the rules generating the patterns. Learners had to process the information and look for connections between input and output numbers.

4.4.3 Open coding

Table 4.2 Summary of strategies used by both learners and teachers in solving task 2

CS = Correct solution IS = Incorrect solution

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1		×	Skip counting	1	✓		One-to-one correspondence
2		×	Addition strategy	2	✓		Trial and error
3		×	Counting	3	✓		Generalised pattern
4	✓		Multiplication pattern	4	✓		Multiplication pattern
5		×	Addition strategy	5		×	Difference between numbers
6	✓		Multiplication pattern	6	✓		Multiplication pattern
7	✓		Multiplication pattern	7	✓		Trial and error
8	✓		Multiplication pattern	8	✓		Multiplication pattern
9	✓		Multiplication pattern	9		×	Counting strategy
56 % correctly answered by learners				78 % correctly answered by teachers			

Table 4.2 shows the strategies used by learners and teachers in solving task 2. The strategies include:

Skip counting strategy

In this strategy digits are counted but some are skipped out. For example counting digits like 2, 3 but then skipping out 4 and then continuing with 5, 7 and skipping out 6.

One-to-one correspondence strategy

The one-to-one correspondence strategy involves relating digits on the left hand side with digits on the right hand side, like $2 \rightarrow 18$; $3 \rightarrow 27$ and looking for a pattern. In this example , the relationship was found by multiplying the digits on the left hand side by 9.

Generalised pattern strategy

A generalised pattern strategy is looking for a relationship between the input and the output numbers. This strategy is similar to the one-to-one correspondence strategy except that the generalised rule, 9n was used.

Multiplication pattern strategy

In the multiplication strategy, input numbers are multiplied by a constant (in this case 9) in order to get the output numbers.

Trial and error strategy

In this strategy random numbers are chosen together with one of the four basic operations in order to get the output number.

The *counting strategy* and the *difference between numbers strategy* did not yield the correct solutions. These strategies were used by learners and based on simple counting and looking for a difference between numbers shown on the input and output sides.

The properties identified were pattern recognition, flow diagrams and looking for a generalisation between numbers.

4.4.4. Discussion of solution strategies

According to the Principles and Standards (2000), the idea of using variables where learners explore patterns and note relationships should be developed. In the spider diagram (task 2), learners were being challenged to use the ideas of a variable as they thought about how to describe a rule.

L's 4, 6, 7 and 8 spotted a pattern by linking the input numbers and output numbers.

> L 6: Here I said, 2 × 9 and I got 18 and 3 × 9 and I got 27; 5 × 9 and I came up with 45 and 9 × 9 and I got 81 and 10 × 9 and I got 90.

T 6 also sees a pattern.

> T 6: Number machine. It was multiplication. You multiply by 9. So when you multiply by 9 to get 45.
>
> I: How did you get that 9 there?
>
> T 6: I divided by 9.
>
> L1 looks at the numbers vertically at first and does not see a pattern.
>
> L 1: It's 2, 3, 5. You skip 1 it's 2, you skip 4, don't skip here, you skip 4 here and you skip 5 and you skip 6, 7 and here is 10.

I: And this 36 here. How did you get that?

L 1: Here it is 18. Here I added the number 9 and it makes 27 plus 9, it makes 26, I mean 36.

I: Let us get back to the 7. You said here that you added 1 and here you added 2: Now what did you add here to get this, to get the (a) part?

L1: I added 1 to 2 to get 3 and add 2 and that is 5 + 2 is 7.

L 9 strategizes by looking at the output numbers to help work out the input numbers.

L 9: Here I said that we must put 9 and $9 \times 9 = 81$. In (b) we have $5 \times 9 = 45$.

I: How did you know to put 9 here?

L 9: I looked for the answer because they are multiplied by 9.

L 2 and L 5 had not encountered such functions before.
L 2 said that he counted the first two input numbers 2 + 3 and got 5 and then said 5 + 5 = 10 and placed 5 at position (a). Working with the output numbers he said that he added 18 + 9 to get 27 and 27 + 9 to get 36 for his missing number in (b).

L 5 said that this problem was very difficult and that she did not know how to do it and just wrote down 1 because before 3 is 2 and before 2 is 1. In the output number she said, " I just add 9." When prompted by the researcher that the problem showed a machine with numbers that go into the machine and numbers that come out of the machine, she very easily explained that what happened inside the machine was that each of the numbers was multiplied by 9.

Four of the nine teachers mentioned that they looked for a relationship between the input numbers and the output numbers. Teacher 3 looked for a rule to help her arrive at the correct solution while teachers 6 and 8 spotted a pattern.
L 4 and T 4 looked for a relationship between the numbers.

L 4: (Pause......) I saw that if you multiply 2 by 9, you get 18 and 3 by 9 you get 27 and I thought that all these numbers would be multiplied by 9. Then in (b) I said (Pause.....) 5 times 9, I got 45 and (a) I said 81 divided by 9 and I got 9.

T 4: Okay, I just looked for a relationship between the input and output number. The output is 18 and the input is 2 and then when the input is 3, the output is 27. So these are multiples of 9 so they should be multiplied by 9.

I: So how did you get to the input?

T 4: I also looked for the output. The output is 81. Each number is multiplied by 9 to get the input.

T 5 said that he found the problem difficult and got 2 for (a) and 11 for (b). When prompted by the interviewer, he then saw his error and was able to correct his first answer. However, he was unable to explain how he arrived at 11 for (b)

4.4.5. Comparison between learner and teacher solution strategies

This question elicited a number of different strategies with only three teachers and learners showing the same strategy of using a multiplication strategy or pattern. Skip counting was a strategy used by one learner while trial and error, and one-to-one correspondence were other strategies which evolved.

T 7 and L 7 below both described how they found the pattern, which was to multiply the input numbers by 9.

T 7: With this one I can see the spider pattern. I, I quickly detect that 2 and 18, 3 and 27. Immediately I knew that it is multiplication and I knew that I have got to multiply by 9 first to get 18. I started off with the given number so I multiplied 2 by 9 and I got 18, so I've got to think of (a) and then I can see that ehhh, I, I took the pattern here 5 times 9, 6 times 9 I could see that. I tried that one. I used trial and error.

L 7: Here I have multiplied $9 \times 9 = 81$ and $5 \times 9 = 45$.

4.5. TASK 3

4.5.1. Description of the task

25 learners go on an outing to the beach. They each buy an ice-cream which costs R3,50. How much must they pay altogether?

4.5.2. Categorisation of the task

1.Content Area Category	Number
2.Subcategory	Decimal fractions
3.Subordinate Subcategory	Properties of operations
4.Performance Expectation	Using routine procedures
5.Subcategory	Performing routine procedures

Figure 4.6 TIMSS Curriculum Framework categorisation of task 3

Figure 4.6 shows the task being placed in the content area of number. It is a multiplication word problem, which is part of number (sense). It has been further subdivided into the subcategory of decimal fractions.

4.5.3. Open coding

Table 4.3 Summary of strategies used by both learners and teachers in solving task 3

CS = Correct solution IS = Incorrect solution

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1	✓		Vertical algorithm	1	✓		Vertical algorithm
2		×	Counting strategy	2	✓		Decomposition of the multiplier
3		×	Vertical algorithm	3	✓		Vertical algorithm
4	✓		Use of a mathematical model	4	✓		Decomposition of the multiplier
5	✓		Vertical algorithm	5	✓		Vertical algorithm
6		×	Vertical algorithm	6	✓		Fraction multiplication
7		×	Fraction multiplication	7	✓		Decomposes multiplier
8		×	Counting strategy	8	✓		Vertical algorithm
9	✓		Vertical algorithm	9	✓		Short method of ×
44 % correctly answered by learners				100% correctly answered by teachers			

Table 4.3 shows the various solution strategies used by teachers and learners. The following strategies were identified:

Decomposition of the multiplier

In this strategy the multiplier, 25 is decomposed into (20 + 5) and then multiplied by R3,50.

Vertical multiplication algorithm

In this strategy, the multiplicand and multiplier are placed vertically underneath each other and are multiplied starting with the ones (units) digit.

Use of a mathematical model

In this strategy a learner used his prior knowledge that 25 is the same as 5 times 5 to simplify the problem and then multiplied.

Counting strategy

In the counting strategy, the learner counted 5 groups of 350 at a time and found the total.

Short method strategy

In this strategy prior knowledge of multiplying the multiplicand by 100 and dividing by 4 was used.

Fraction multiplication

In this strategy R 3,50 was converted into 3 ½ and then ½ was multiplied by 25.

The properties I identified in the task were decomposition of the multiplier e.g. 25 = (20 + 5) and the short method of multiplying by 25

4.5.4. Discussion of solution strategies

Five learners used the traditional vertical multiplication algorithm with L 1, 5 and 9 successfully solving the problem.

L 1 was a little nervous and had to be asked to repeat what she had done. She solved the problem like this:

> L 1: I said 25, 350 times 25. I got R 87,50.
>
> I: Can you explain that piece in the middle?
>
> L 1: 5 times 0 is 0, 5 × 5 is 25. I took 5, I put it here, I carried 2 ; 2 x 3 is 6 ; 2 x 5 is 10 plus 2.
>
> I: Let us start again.
>
> L 1: 5 times zero is 0 , 5 x 5 is 25, put 5 here, carry 2. 5 x 3 is 15 plus 2 is 17; 5 x 2 is 10 , 2 times 5 is 10 and put 0 here and 2 x 3 is 6 plus 1 is 7. And all these is number 8.
>
> I: And the total is?
>
> L 1: R 87,50
>
> L 2 used a counting strategy of adding 350, twenty five times and got R78,50 instead of R87,50. In the interview, he counted twenty-five threes but got lost when adding the fifties.

L 3 had a problem computing 5 × 0 by saying it equalled 5, and then corrected himself, but then made further multiplication errors.

L 6 used the traditional multiplication algorithm, was successful up to a point, but his lack of mastery in his multiplication tables let him down and he made other simple computational errors. An understanding of 'place value' is important as it involves both knowledge of the significance of the position of a digit and also the relationship between digits in the same number.

L 4 used an interesting mathematical modelling strategy by using his prior knowledge that 25 was made up of 5 × 5. (Figure 4.6)

> L 4: The learners were 25 so I divided 25 by 5, then 5 and I said R3,50 multiplied by 5 and it was R17,50 and then I multiplied this R17,50 by 5 and I got R87,50.

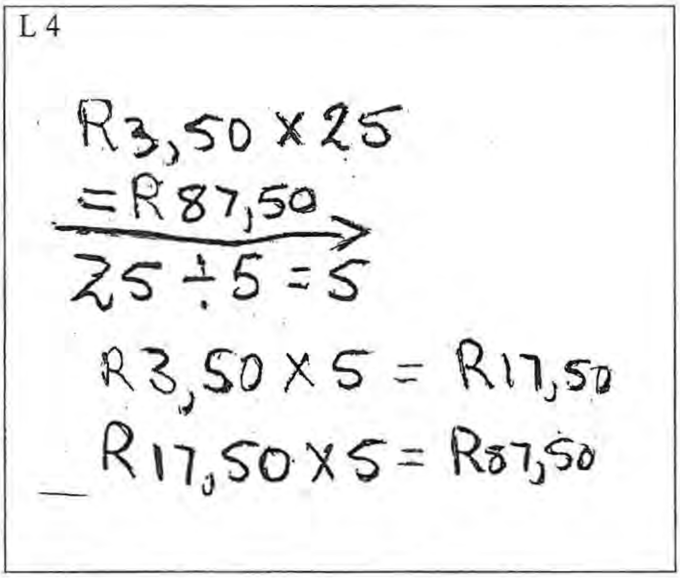


Figure 4.7 shows an individually constructed solution strategy

Whether the strategy illustrated in Figure 4.7 done by L 4 was taught or not, is unclear, but it appears this learner used his own constructed knowledge to fathom out how to solve the problem (see page 44, chapter 2). The use of the mathematical model was very efficient in this case and says a great deal about the learner's number sense. It appears that he had built his understanding of manipulating numbers and made sense of the procedure used.

Mathematical models "are mental maps of relations that can be used as tools when solving problems" (Fosnot & Dolk 2001:88). According to Fosnot & Dolk (2001:88), "models are representations that a learner constructs over time as they reflect on how one thing can be done or changed into another" (see the strategy used by learner 4). Dabell (2002:22) contends that "multiplication is difficult to learn and hard to teach", with a number of adults and children avoiding multiplication and substituting it with addition. However, he feels that we need to extend the repertoire of strategies we teach, as some children are unsure that there are different ways to multiply.

L 7 used fraction multiplication by saying that R3, 50 is $3\frac{1}{2}$ and then multiplied it by 25. He managed to get 175 divided by 2 and finally got $89\frac{1}{2}$.
Except for the error in the final calculation, this learner also used a strategy which made sense to him and which he understood.

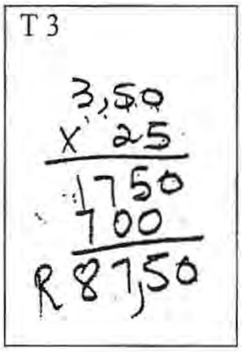


Figure 4.8 illustrates a procedurally directed algorithm

Four of the nine teachers used the traditional vertical algorithm to arrive at the correct solution. Figure 4.3.2 shows that the teaching may be procedurally directed and the teacher's knowledge of the algorithm emphasised more procedural understanding. Procedural understanding has been described as a step-by-step sequence where rules are memorized and followed, often without real understanding.

Another three teachers used the decomposition strategy. They decomposed 25 into two multiples of 10 and a multiple of 5. The strategy used by T 2 in Figure 4.9 is conceptually directed as it shows how the teacher applied the skills of decomposition (breaking up of numbers) to make sense of the task. Their knowledge of the algorithm involved suggests both conceptual and procedural understanding. Conceptual understanding according to McNeeley (1997:143) means:

> that students make sense of the maths operations they perform. They not only know how to apply skills but also when to apply them and why they are being applied. Conceptual understanding provides students with the basis for seeing relationships between skills and

problem solving and among mathematical ideas. Students with conceptual understanding see the structure and logic of maths more flexibly and appropriately and are able to recall or adapt rules because they see the larger pattern.

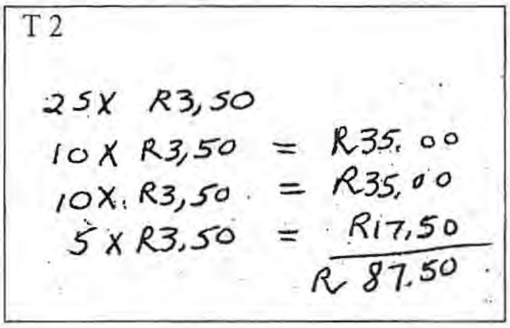


Figure 4.9 conceptual understanding of an algorithm

T 9 used the short method of multiplying by 25. She explained that she multiplied 350 by 25 by adding two zeros to 350 and then dividing by 4 to get the answer. This strategy appears to be a learnt strategy.

4.5.5. Comparison between learner and teacher solution strategies

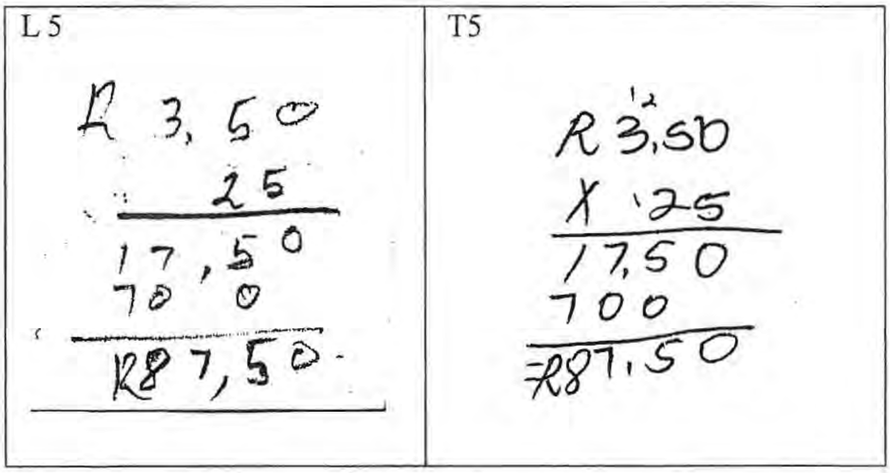


Figure 4.10 a traditional vertical algorithm

By far the most common strategy used was the traditional vertical method as illustrated in Figure 4.10. Nine learners and teachers used this strategy. This suggests that most of the teachers appeared to teach this algorithm.

Figure 4.10 indicates how L 5 and T 5 solved the task.

L 5: Okay, I multiply 350 by 25 and it became R 87,50.

I: Explain how you multiply here.

L 5: I multiply 5 x 0 =0 and 5 x 5 = 25 and I put it 2 here and I put I said 5 x 3 = 15; 15 + 2 and it became 17 and 2 x 0 = 0 and 2 x 5 = 10 and I carry 1 and I put 0 and 2 x 3 = 6 + 1 = 7 and I add 0 and becomes equal to 0 and 5 + 0 = 5 and 7 + 0 = 7 and 1 + 7 = 8.

T 5: 20 learners go on an outing. So now this R 3,50, I multiplied by 25 learners. I get R 87,50.

I: How did you get it?
T 5: I did it the traditional multiplication way.

Ma (1999) found that the way Chinese and American teachers teach the multiplication algorithm differs dramatically. The majority of Chinese teachers teach the algorithm in a conceptual way by using the distributive property and breaking up the problem into its component parts. Only after the conceptual understanding has been developed, do they compare the problems in the algorithm with the component parts. Ma (1999) found that 70% of American teachers teach the algorithm as a set of procedures. Learners are reminded of the 'rules', to use zero as a place value holder and that when multiplying by tens, they must move their answer to the next column.

Kamii *et al.* (1993) found that when children are asked to develop their own ways of decomposing a problem, like when the algorithm has not been taught to them, most children use a form of the distributive property. Clark & Kamii (1996) also found that multiplicative thinking in learners develops slowly. They found that only 28% of fourth graders and 49% of fifth graders demonstrated solid multiplicative thinking. They further suggest that the pedagogical implication of this is that when multiplication problems are given to learners, they should be allowed to solve them in their own way – some learners using multiplication and others addition.

Fosnot and Dolk (2001) caution that algorithms should not be the primary goal of computation instruction. Solving number sense calculations means that the numbers should be looked at first and then a strategy decided that is flexible, efficient and fitting. According to Fosnot and Dolk (2001:102) "children need to mathematize, to think like mathematicians, to look at the numbers before they calculate."

4.6. TASK 4

4.6.1. Description of the task

Calculate: $\frac{3}{4} - \frac{1}{3}$

4.6.2. Categorisation of the task

1.Content Area Category	Number
2.Subcategory	Fractions
3.Subordinate Subcategory	Common fractions
4.Performance Expectation Category	Knowing
5.Subcategory	Recognising equivalents

Figure 4.11 TIMSS Curriculum Framework categorisation of task 4

Figure 4.11 refers to the content category in which the fractional computation was placed. The task is part of number (rational numbers), with its subordinate subcategory being common fractions.

4.6.3. Open coding

Table 4.4 Summary of strategies used by both learners and teachers in solving task 4

CS = Correct solution IS = Incorrect solution

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1		×	Drawing	1	✓		Fraction algorithm
2		×	Drawing	2		×	Multiplies denominator
3		×	Drawing	3	✓		Fraction algorithm
4		×	Uses whole number subtraction	4	✓		Fraction algorithm
5	✓		Fraction algorithm	5	✓		Fraction algorithm
6	✓		Equivalency	6	✓		Equivalency
7	✓		Fraction algorithm	7	✓		Equivalency
8	✓		Fraction algorithm	8	✓		Fraction algorithm
9		×	Uses whole number subtraction	9	✓		Fraction algorithm
44 % correctly answered by learners				89 % correctly answered by teachers			

Table 4.4 shows the strategies teachers and learners used in solving the computation. The strategies were:

Fraction algorithm

This strategy used finding the lowest common denominator and was used by 44 % of the participants.

Equivalent fraction strategy

In this strategy the same equivalent fraction was found for both fractions to be subtracted.

Drawings

Representations were used to show the two fractions and then subtracted.

Use of whole number subtraction

In this strategy, whole number subtraction was used where either the two numerators were subtracted or both numerators and denominators were subtracted.

Multiplies denominators

A lowest common denominator was found for the two fractions and then the two fractions were simply subtracted.

The following properties were identified when analysing the solution strategies: making use of equivalent fractions; finding the lowest common denominator and making use of sketches.

4.6.4. Discussion of solution strategies

L's 5, 6, 7 and 8 were all able to complete this computation by explaining in detail that in order to subtract both fractions, they had to first look for the lowest common multiple of 4 and 3 which is 12. Then once both fractions were the same, by multiplying $\frac{3}{4}$ by $\frac{3}{3}$ and $\frac{1}{3}$ by $\frac{4}{4}$ the two fractions could then be subtracted from one another.

L 1 appeared very confident when trying to explain with a drawing but got lost along the way.

> L 1: Here is our whole, I draw it. I draw a line to show these are quarters. There are $\frac{3}{4}$ and I subtract one and two are left and there are $\frac{2}{3}$ left ($\frac{3}{4}-\frac{1}{3}=\frac{2}{3}$)

The interviewer then asked her what would happen if the sum were $\frac{1}{2}-\frac{1}{4}$?

> L: It is $\frac{1}{4}$
>
> I: What is left over?
>
> L: $\frac{1}{4}$ will be left.
>
> I And if it was $\frac{2}{4}-\frac{1}{8}$
>
> L: That would be a $\frac{1}{2}$ and $\frac{1}{8}$. (Long pause) I am stuck.

The interviewer ended the interview here.

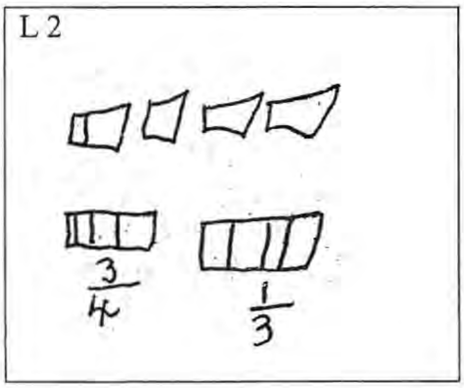


Figure 4.12 Drawings used by L 2

Figure 4.12 shows the drawings used by L 2. She mentions that $\frac{3}{4}$ is the first picture and $\frac{1}{3}$ is the second one. When asked what $\frac{3}{4}-\frac{1}{3}$ would be, the learner promptly writes down $\frac{2}{4}$. When prompted to explain further, there were inaudible mumbles and the learner said, "I don't understand."

L 3 appeared confused and said that he drew $\frac{3}{4}$ and subtracted $\frac{1}{3}$ and was left with $\frac{2}{4}$ and $\frac{1}{3}$. The interview went as follows:

> I: How did you get $\frac{2}{4}$?
> L: I minus $\frac{1}{3}$ from $\frac{3}{4}$.
> I: How did you then get this $\frac{2}{4}$?
> L: It is because a quarter is made up of two one thirds and I minus $\frac{1}{3}$ in this and left with $\frac{1}{3}$.
> I: Are you sure a quarter is made up of two one thirds?
> L: Yes, two one thirds.
> I: Which is the bigger a quarter or a third?
> L: A quarter. I "killed" this third and I showed it here, so was left with two quarters and $\frac{1}{3}$.

It appears the learner subtracted the two numerators $3 - 1 = 2$ and then put this over the denominator of 4. He then took the two denominators and subtracted them $(4 - 3)$ and put the answer over 3. Some learners approach the computation from a whole number point of view, using the whole number subtraction process, while not taking into consideration the fractions, as fractions. The common error above has been documented by Carpenter; Coburn; Reys & Wilson (1976) and Howard (1991). Both Carpenter *et al.* (1976) and Howard (1991) found that the main reason for learners doing this, especially in addition of fractions, is the introduction of multiplication of fractions before addition. When learners have learnt the 'top × top' over 'bottom × bottom' strategy, they tend to transfer that process to addition. Based on my experience, I disagree with the findings of the above but agree with Lukhele *et al.* (1999) who believe that the main reason learners make such errors is because of their lack of understanding of the concept of a fraction.

This study shows that some learners view the fractions as two separate whole numbers (mentioned above). When two fractions are used in a computational or word problem, learners see the fractions as four separate whole numbers that need to be added or subtracted to give a whole number (see L 2 mentioned above). The strategies used by learners in the above examples show that learners have learnt the operations and algorithms of whole numbers. They think that the same processes can be used when calculating fractions. D'Ambrosio & Mewborn (1994) state that fraction concepts should come from learners' own constructions and these then need to be used to develop the algorithms. Research conducted by Lukhele *et al.* (1999:96) found that "by showing learners different algorithms, teachers impose arbitrary definitions on learners that make no sense to them."

L 4 subtracted the numerators from each other $\frac{3}{4}-\frac{1}{3}=\frac{2}{4}$ and kept the denominator of four. When questioned about how he got 2 as a numerator and 4 as the denominator, he was unable to answer. The interviewer then asked:

I: What is $\frac{1}{2}-\frac{1}{4}$?
L: It's a quarter
I: How did you get a quarter?
L: A half is made up of two quarters.
I: Good, what about $\frac{1}{2}+\frac{1}{4}$?
L: (long pause) ...... It is $\frac{2}{3}$
I: How did you get it?
L: No answer
I: Let's make it easier for you. What is $\frac{2}{4}+\frac{1}{4}$?
L: It's $\frac{3}{4}$
I: So $\frac{1}{2}+\frac{1}{4}$
L: It's $\frac{3}{4}$

Again the learner was doing the same as he had done at the beginning of the interview, adding the numerators and then keeping one of the denominators. This shows that the learner has not been exposed to other fraction types although he was able to subtract $\frac{1}{4}$ from $\frac{1}{2}$ and add fractions with the same denominators. However, he still could not compute $\frac{1}{2}+\frac{1}{4}$. L's 2, 3 and 4 see the sum and denominator as a measure (which is a description of the size of the measure) and not as a count (Kieran 1993). It is important for learners to realise that one can never add sizes of measures.
L 9 subtracts the numerators from each other and also the denominators.

L 9: I minus $\frac{3}{4}-\frac{1}{3}=\frac{2}{1}=2$.
I: What happens if you had $\frac{1}{2}-\frac{1}{4}$?
L 9: Get a $\frac{1}{4}$
I: How did you get a quarter?
L 9: Because $\frac{1}{2}$ has $\frac{2}{4}$ and minus $\frac{1}{4}$ and you have a quarter.
I: What will you do then with $\frac{3}{4}$ and $\frac{1}{3}$?

L 9: I am not sure.

This common error has been found by a number of researchers. Hart (1981) for example found that the error happened more when the question was stated in computational form than in word problem form. I have found this to be the same in my research with only 44% of the sample interviewed being able to solve the computation. I agree with Hart (1981:46) when she says, "the emphasis must change from algorithmic learning to understanding the structures of the operations themselves and how and when they should be applied." She further states that maybe the traditional standard algorithms taught should be given up completely in favour of other strategies which may be less efficient, but more akin to children's own informal methods which are easier to remember, like equivalent fractions and using drawings.

Research conducted by Baroody & Hume (1991); Streefland (1991 ; D'Ambrosio & Mewborn (1994) and Murray *et al.* (1996) has shown that there appear to be three main possible causes why learners in the primary school have a poor understanding of common fractions:

- The environment in the classroom where, through lack of opportunity, the everyday conceptions of fractions are not consolidated and incorrect intuitive reasoning is not checked.
- The way and order in which fractions are taught using pre-partitioned materials with teachers stressing halves and quarters.
- Improper application of whole number schemas, like seeing the numerator and the denominator as distinct whole numbers.

In the fraction computation, five learners had limited constructions probably arising from their own intuitions and real life experiences or it might be the direct result of the teaching approach used by the teacher.

Baroody and Hume (1991) argue that learners' errors in fractions may be the result of poor understanding of underlying concepts, as well as not being able to recognise accurate visual representation. The research done found that it is imperative for teachers to plan their fraction activities in such a way that enables learners to build conceptual understanding rather than to teach fractions in the 'traditional' way.

Researchers such as Mullis, Dossey, Owen and Philips (1991) and Groff (1994) maintain that middle grade learners persist in doing poorly on fraction work regardless of their being given broad and continuous instruction. These researchers also believe that learners find fractions very complex and problematic to grasp.

Gabb (2002) found that some children rely heavily on memorisation when doing fraction work and often have weak strategies when manipulating the ideas. The result is that learners' perceptions of fractions tend to discourage conceptual development and the sense-making process of mathematics.

Hanson (2001) acknowledges that year in and year out, learners learn fractions but forget how to add, subtract, multiply and divide with them. This becomes terribly frustrating to both learners and teachers. The main reason learners had difficulties was because they would memorise formulas or algorithms without any real understanding. This study concurs with Hanson's (2001) findings.

Seven of the nine teachers had good procedural knowledge and used the formal traditional algorithm of finding the lowest common multiple (LCM).

> T 7: Immediately I jumped to equivalency because I even taught my learners that it's difficult to do maybe subtraction or division of fractions which are totally different. We should make them the same although, they still have the same value so I changed $\frac{3}{4}$ into 9. I looked for a common denominator, which became 12. So I changed $\frac{3}{4}$ to become $\frac{9}{12}$ and $\frac{1}{3}$ to become $\frac{4}{12}$ and then I've minussed 9-4, then I got $\frac{5}{12}$.

Figure 4.13 shows how T 2 solved the computation.

> T 2: The fractions are not the same so I made them to be the same. I multiply the denominators by the first then by 4 and 3 and 3 and 4 by the other side. Then I get the denominator as a 12. Then I subtract $\frac{1}{12}$ from $\frac{3}{12}$. I get $\frac{2}{12}$.

T 2

$$\frac{3}{4}-\frac{1}{3}$$
$$\frac{3}{4}\times\frac{3}{3}-\frac{1}{3}\times\frac{4}{4}$$
$$\frac{9}{12}-\frac{4}{12}=\frac{5}{12}$$

Figure 4.13 shows what T 2 did

4.6.5. Comparison between learner and teacher solution strategies

Three learners applied the same strategy using a drawing to try and solve the computational sum, but were unsuccessful. The four learners and six teachers who used the traditional fraction algorithm successfully were confident and relaxed while explaining what they had done.

L 5	T 5
$\frac{3}{4}-\frac{1}{3}$	$\frac{3}{4}-\frac{1}{3}$
$\frac{3}{4}\times\frac{3}{3}-\frac{1}{3}\times\frac{4}{4}$	$\frac{3}{4}\times\frac{3}{3}-\frac{1}{3}\times\frac{4}{4}$
$\frac{9}{12}-\frac{4}{12}=\frac{5}{12}$	$\frac{9}{12}-\frac{4}{12}$
	$\frac{5}{12}$

Figure 4.14 illustrates how L 5 and T 5 solved the task.

Figure 4.14 shows the exact strategy used by L 5 and T 5.

> L 5: Number 4 is difficult because I said $\frac{3}{4}$ x $\frac{3}{3}$ – $\frac{1}{3}$ x $\frac{4}{4}$ and I said and 4 x 3 = 12 and I put it 12 and I put it there the basic operation and I said 1 x 4 = 4 and multiply 3 x 4 = 12 and I minus 9-4 = 5 and (indistinct)
>
> I: If you have a number, let's say you've got two quarters and then you take away a quarter. What do you get?
>
> L 5: You get a quarter.
>
> I: Alright, so if you've got $\frac{9}{12}$ and take away $\frac{4}{12}$, what do you get?
>
> L 5: I get $\frac{5}{12}$
>
> I: Ja, okay.
>
> T 5: Now this is how I said. Now because this has different denominators. Now the common denominator should be 12. So we say we cannot minus or add fractions that are different so we have to make them common. So now $\frac{3}{4}$ multiplied in order to get the denominator being that 12 multiplied by $\frac{3}{3}$ minus $\frac{1}{3}$ multiplied by $\frac{4}{4}$. Then I said 3 x 3 is $\frac{9}{12}$ minus 4 x 1 = 4 ; $\frac{4}{12}$ and then subtracted 4 from 9, got 5 so $\frac{5}{12}$.

L 6 and T 6 were very similar in their explanation.

> L 6: I said here is the LCD of 4 and 3 is 12 and I said, how many 4s in 12 and they are 3 and I said $3 \times 3 = 9$ and how many 3s in 12 and I got 4 and I said $1 \times 4 = 4$ and $9 - 4 = \frac{5}{12}$
>
> T 6: In terms of the fraction, I actually looked at the LCM but the quickest way is through multiplication so I decided to multiply by 3 where the denominator is 4 and by 4 where the denominator is 3.
> I: So you got $\frac{9}{12}$ minus $\frac{4}{12}$ and then $\frac{5}{12}$.

There were more matches of learners using the same strategy as teachers in this question. This flexibility of teachers using both equivalent fractions and the LCM to teach this aspect of number sense is apparent from their work.

4.7. TASK 5

4.7.1. Description of the task

Look at the following two shapes:

A B

What do we call shape A?

What do we call shape B ?

Write one thing that is the same about both shapes.

4.7.2. Categorisation of the task

1.Content Area Category	Geometry: position, visualisation & shape
2.Subcategory	Two-dimensional geometry: Polygons
3.Subordinate Subcategory	Not applicable
4.Performance Expectation Category	Communication
5.Subcategory	Describing and discussing

Figure 4.15 TIMSS Curriculum Framework categorisation of task 5

Figure 4.15 shows that geometry is the content category used to categorise the task. My justification for including a question on geometry was to see whether learners could accurately identify and describe a two-dimensional shape.

4.7.3 Open coding

Table 4.5 Summary of strategies both learners and teachers used to solve task 5.

PC = Partially Correct TC = Totally Correct TI = Totally incorrect

Learner	PC	TC	TI	Strategies	Teacher	PC	TC	TI	Strategies
1	✓			Identifies shape	1		✓✓		Identifies shape
2	✓			Identifies shape	2		✓✓		Identifies shape
3	✓			Identifies shape	3		✓✓		Identifies shape
4		✓✓		Identifies shape	4		✓✓		Identifies shape
5	✓			Identifies shape	5	✓			Identifies shape
6	✓			Identifies shape	6		✓✓		Measurement
7	✓			Identifies shape	7		✓✓		Identifies shape
8		✓✓		Identifies shape	8		✓✓		Identifies shape
9			×	Identifies shape	9	✓			Identifies shape
22 % of learners totally correct 67% of learners partially correct 11% of learners totally incorrect					22% of teachers partially correct 78% of teachers totally correct				

Open coding was not used in this task (Table 4.5) as learners had to only identify two shapes and find one similarity between them.

The property I have used to describe this task is 'identifying polygons'.

4.7.4. Discussion of solution strategies

L 1 was able to correctly identify the parallelogram but called the rectangle a square. Because of this she said that the sides were the same but the sizes of the angles were different. L 2 appeared to have very little idea of any of the shapes.

This was how the interview went:

> I: What can you tell me about these two shapes?
> L 2 : Angles
> I: Are the angles the same?
> L: yes
> I: Angles the same, but what else is the same?
> L: Straight lines
> I: How many lines?
> L: 4 lines
> I: What do you call this shape? What is it called?
> L: length and breadth.
> I: Okay, what else can you tell me about the two shapes? Anything else.
> L: I don't see anything else.

The interviewer decided to halt the interview at this point as it was obvious the learner had not been exposed to the properties of the shapes.

L's 3 and 4 appeared very confident and were able to deduce the names of the quadrilaterals and compare them. L 5 confused the parallelogram with a rectangle and the rectangle with a square. He was able to mention that both shapes have four straight lines and four corners.

Both L's 6 and 9 confused the parallelogram with a rhombus and the rectangle with a square. The possibility existed that they had been exposed to the new vocabulary and just got the names of the shapes mixed up. I am not sure what the reason for this could be.

This was what L 8 had to say:

> L 8: I made a mistake here.
> I: What is it supposed to be?
> L: Parallelogram
> I: And here.
> L: Square
> I: What is a square?
> L: Something with equal sides.
> I: Are these sides equal?
> L: No
> I: So what is it if the sides are not equal?
> L: Rectangle.
> I: Okay, what is it which is the same about the two shapes?
> L: They have two short and two long sides.

All teachers except T 5 and T 9 answered this question correctly. T 9 confused a rhombus with a parallelogram. She was able to conclude that opposite sides in both shapes are parallel, that the angles add up to 360^0 and that they have four sides. T 5 could not identify either of the two shapes but was able to see that both shapes have four sides. He identified shape A as a rhombus and shape B as a square.

4.7.5 Comparison between learner and teacher solution strategies

This task required learners and teachers to apply their social knowledge (see chapter 2) to identify the shapes and then to use this knowledge to find something similar about the two shapes. The learners and teachers' strategies were all the same as all the task required was to identify the shape and recognise a similar property.

T 6 used measurement as a strategy to decide on the shape.

He said the following:

> T 6: Well, I've thought of it. Due to the fact that I measured it so it's going to be a parallelogram. And this one I've thought is a rectangle.
>
> I: What is common about the two shapes?
> T 6: Well in terms of sides, they both have 4 sides.
> I: What else?
>
> T 6: Angles as well, Uhhh, well if I could talk in terms of the diagonals, they would both have two diagonals.

L 4 and T 4 were clear in their descriptions.

> L 4: A is a parallelogram and B is a rectangle.
> I: How are they the same?
>
> L 4: The two opposite sides are equal in both of the shapes.
> I: Anything else?
>
> L 4: They have four corners. They have four sides.
> I: What is the word we give to these shapes with four sides?
> L 4: Quadrilaterals
>
> T 4: I said this is a parm and this is a rectangle. What is common? They have one pair of parallel sides. This one has one and this has two pairs.
> I: How many pairs has this one got?
>
> T 4: Only two pairs that is A, and 2 here in B.
> I: What else can you tell me that is common about those two shapes?
> T 4: They are all quads and also they are also rectangles.

According to the van Hiele levels of geometric understanding, some of the learners and teachers appear to be at level 1 where most were able to recognise the shape and make decisions based on perception (van Hiele 1986). However, it was not possible to verify whether a teacher was at level 2 of geometric understanding. For effective geometric learning to take place, learners need to engage with the geometry being taught in appropriate contexts leading to discussion and reflection.

4.8. TASK 6

4.8.1 Description of the task

Calculate : 2806 ÷ 8

4.8.2. Categorisation of task

1.Content Area Category	Number
2.Subcategory	Whole numbers
3.Subordinate Subcategory	Operations
4.Performance Expectation Category	Knowing
5.Subcategory	Representing

Figure 4.16 TIMSS Curriculum Framework categorisation of task 6

Figure 4.16 shows the content category for the computation which is number with its subcategory whole numbers.

My justification in including a computation on division was to find out whether learners were able to come up with a variety of solution strategies to solve the computation.

4.8.3. Open coding

Table 4.6 Summary of strategies used by both learners and teachers in solving task 6

CS = Correct solution IS = Incorrect solution

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1		×	Vertical algorithm	1		×	Vertical algorithm
2		×	Unclear strategy	2		×	Partitioned dividend
3		×	Unclear strategy	3	✓		Partitioned dividend
4		×	Factors of 2 and 4	4	✓		Partitioned dividend
5		×	Partitioned dividend	5		×	Vertical algorithm
6		×	Vertical algorithm	6		×	Vertical algorithm
7		×	Partitioned dividend	7		×	Partitioned dividend
8		×	Guess work	8		×	Partitioned dividend
9	✓		Vertical algorithm	9	✓		Vertical algorithm
11 % correctly answered by learners				33 % correctly answered by teachers			

The following strategies (See Table 4.6) were identified.

Formal vertical algorithm

This is the traditional division algorithm of dealing separately with the digits of the dividend, first dividing, multiplying, subtracting and then carrying down in a vertical way.

Partitioned dividend

In this strategy the dividend is partitioned into multiples taken from the divisor.

Using factors of the divisor

The divisor of 8 was decomposed into 2 and 4 and then divided.

The properties which describe the task were counting, the traditional vertical algorithm and decomposing the dividend.

4.8.4. Discussion of solution strategies

According to Fosnot & Dolk (2001:102) an algorithm is a "structured series of procedures that can be used across problems, regardless of their numbers."

Maurer (as cited in Morrow & Kenney 1998) argues that an "algorithm is a precise systematic method for solving a class of problems [It] takes input, follows a determinate set of rules and in a finite number of steps gives output that provides a conclusive answer."

Tirosh and Graeber (1991) suggest two models of division that children use. The first model involves partitioning or fair sharing and the other involves measurement or repeated subtraction. Because no context (it was a straight forward calculation) was provided for this computation, it could not be classified as 'measurement' or 'partitioning'. The solution therefore depended on being able to use a variety of strategies. To understand multiplication and division, learners need to understand the relationship between them. Learners need to learn the meaning of a remainder by modelling division problems and exploring the size of the remainders.

Steinberg (as cited in Nickson 2000) examined the strategies and algorithms used by children in CGI classes (see chapter 2, page 37) when doing division. The approach used gave the learners the chance to solve problems and to construct, present and discuss their strategies. Learners in this class solved a variety of problems using different solution strategies, which they were able to explain and discuss with their teacher and peers. In this study I had hoped to see more variety in the solution strategies presented. It was disappointing to see that more than 50% of the teachers used the vertical algorithm, which incurred the most mistakes.

L 1

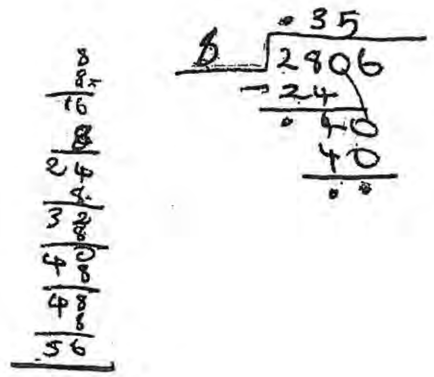


Figure 4.17 shows the traditional vertical algorithm. It is, however, incomplete

In Figure 4.17, L 1 gave a clear explanation of having being taught the traditional algorithm, but does not know what to do with the 6 in the dividend and ends up with a quotient of 35.

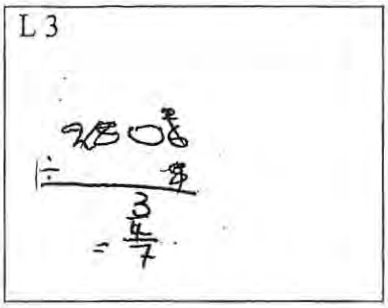


Figure 4.18 shows an incorrect division strategy

Figure 4.18 shows how L 3 attempted to solve the division computation.

> L 3 : I asked how many 8s in 28, 3 times and left with 2 and I put 2 on top and how many 8s in 26, 4 times and I add 3 plus 4 and I got 7.
>
> I: What is 7 × 8?
>
> L3: 56
>
> I: If you can get 7 there and it means that this is 56.
>
> L 3: How many 8s in 24? Three times and put 3 .... The rest becomes inaudible.

Fosnot & Dolk (2001:116) contend that, "it's an insufficient model to teach division as a 'goes into' and treating digits separately, not only confuses children, makes little sense to them." I tend to agree with these authors because, in the above example, the learner needed to know that 300 × 8 = 2 400 and 50 × 8= 400 and that there was a remainder of 6. If one used 'the goes into' strategy, learners will not see the connection between how many 8s in 2 400 or how many 8s in 24.

L 4 divided the dividend by 2 and got 1403 and then divided this answer by 4 and got 350,10. She was unable to explain how she got the 3 in her answer. When asked to explain how she got 0,10 there was just a long pause and the learner said, "I don't know."

> L 5: I think I've got the wrong answer here and it was very difficult.
>
> I: What is the question?
>
> L 5: The question is I must divide 2806 by 8.
>
> I: What did you do?
>
> L: I said 8 × 100 = 800 and I said 8 x 200 = 1600.

In this strategy the child decomposed 2806 into 800 and 1600 but then could not go any further.

In Figure 4.19, L 6 used the traditional formal algorithm and good procedural understanding. She explained what she was doing in a simple manner. From the interview it was not clear though whether she had any conceptual understanding of the strategy used.

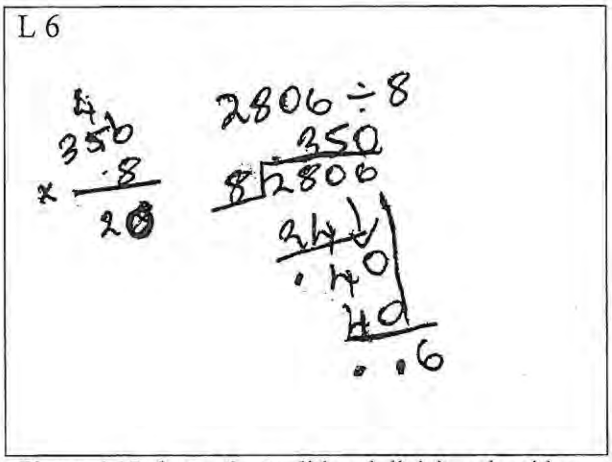


Figure 4.19 shows the traditional division algorithm

L 7 did the following: $2000 \div 8 = 200$ remainder 40

$800 \div 8 = 100$

$6 \div 8 =$

305

This learner used the decomposition strategy and although he made errors, seemed to be thinking about what he was doing.

L 9 used the traditional formal algorithm but made a mistake when getting to the remainder 6. She forgot to insert a decimal comma after the 6 and ended up with a remainder of 4.

$$
\begin{array}{r|l}
 & 357 \\
8 & 2806 \\
 & -24 \\
\hline
 & 40 \\
 & -40 \\
\hline
 & 60
\end{array}
$$

-56

4

T's 1, 5, and 9 used the traditional formal division algorithm. T 9 being the only one to complete it successfully but with a faulty explanation:

> T 9: I divided by 8, I write 2806 ÷ 8. How many 8s in 28, is 3, 3 x 8 =24. Subtract now 24 from 28 then it's 4. Then I take down the zero and then now it's 40. How many 8s in 40, then it's 5, 5 x 8 = 40. Then 40 take away 40 is 0 and then take down the 6 then it's 6. Now 2806÷ 8 becomes 350 remainder 6.

T 1 stopped after getting 35 and T 5 got up to 35, forgot about putting down a zero and then continued to work out the number of eights in 60. It was interesting to discover the number of teachers who partitioned the number 2806 into 2000 + 800 + 6 and then divided each number by 8. Only T 8 used multiples of 8 but made a mistake after being left with having to divide 6 by 8. It was only T 3, 4 and 9 who were successful in partitioning and arriving at a correct solution.

T 4	T 9
2000 ÷ 8 = 250 800 ÷ 8 = 100 6 ÷ 8 = 350 rem 6	2806 ÷ 8 2400 ÷ 8 = 300 320 ÷ 8 = + 40 86 ÷ 8 = 10 rem 6 350 rem 6

Figure 4. 20 shows the strategy of partitioning the dividend used by T 4 and T 9

4.8.5. Comparison between learner and teacher solution strategies

There were seven instances where the teachers and learners used the same strategy, namely the traditional division algorithm, with five teachers and two learners partitioning the dividend 2806 into (2000 + 800 + 6) or multiples of 8 (2400 + 400 + 6). It is interesting to note from Figure 4.20 that some teachers were using flexible strategies when solving this computation. The strategy of partitioning the dividend into multiples of 8 is clear from Figure 4.20. Learners who used the traditional algorithm often failed to complete the strategy, suggesting that they had tried to memorize a particular strategy, but lacked understanding in completing it.

I agree with Scharton (2004) who argues that providing opportunities for learners to create, explain and analyse their computational methods will result in their developing efficient, accurate and flexible strategies for computations. She further supports this statement by saying:

> giving students experience with solving problems and allowing them to communicate their problem-solving strategies to others are essential components to developing understanding and benefits students in a variety of ways… exposure to a variety of computation strategies allows students access to methods that they may not have considered on their own … their knowledge of different strategies grows, so does their computational flexibility. Scharton (2004:278)

The above quotation reiterates the need for teachers to expose their learners to a variety of solution strategies as everyone benefits in some or other way.

Scharton (2004:280) further emphasises that the goal of arithmetic instruction for her is, "to help students build computational fluency by inventing their own computation procedures, effectively explaining these procedures to their peers, and analysing procedures for relatedness, efficiency and effectiveness." This sentiment has been corroborated by Carpenter *et al.* (1997) and Fuson *et al.* (1997) who found that elementary students are able to construct computational procedures.

4.9 TASK 7

4.9.1 Description of the task

Half a cake is shared fairly amongst four children. What fraction should each child get?

4.9.2 Categorisation of the task

1.Content Area Category	Number
2.Subcategory	Fractions
3.Subordinate Subcategory	Common fractions
4.Performance Expectation Category	Investigating and problem solving
5.Subcategory	Solving

Figure 4.21 TIMSS Curriculum Framework categorisation of task 7

The above word problem was included to see whether learners would be able to understand a simple real life problem involving division of fractions. Figure 4.21 shows its classification under the content category of number with common fractions as its subordinate subcategory.

4.9.3 Open coding

Table 4.7 Summary of strategies used by both learners and teachers in solving task 7

CS = Correct solution IS = Incorrect solution

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1	✓		Drawing	1	✓		Mental strategy
2		×	Drawing/guesswork	2	✓		Drawing
3		×	Drawing	3	✓		Multiplication
4	✓		Multiplicative strategy	4	✓		Invert & multiply
5	✓		Drawing	5		×	Invert & multiply
6	✓		Mental strategy	6		×	Invert & multiply
7	✓		Invert & multiply	7	✓		Invert & multiply
8		×	Drawing	8	✓		Drawing
9	✓		Using the fraction family	9	✓		Using the fraction family
67 % correctly answered by learners				78 % correctly answered by teachers			

The following strategies were identified in Table 4.7:

Invert and multiply strategy

This is the traditional strategy used to teach division where the second fraction is inverted.

Mental strategy

This strategy involves doing calculations 'in your head' without paper and pencil.

A mental strategy is any procedure that involves calculating something in your head without the use of paper and pencil McChesney & Biddulph (as cited in Neyland 1993).

Use of a drawing

This model of representation includes drawing a picture of the problem and then making inferences.

Multiplicative strategy

In this strategy participants made use of '½ a cake of ¼' because there were four people involved.

Using the fraction family

This strategy uses the 'family of halves and quarters' to arrive at a solution.

In this task the properties were sharing, the traditional division strategy of invert and multiply, the use of drawings and equivalent fractions.

4.9.4 Discussion of solution strategies

Fractions and division of fractions in particular are considered to be the most complex numbers in primary school mathematics, with most teachers using the 'invert and multiply' method when teaching this concept (Bezuk & Armstrong 1993).

In this study it is interesting to note that of the five learners who used a drawing when thinking about the problem, only two were able to correctly solve the problem. According to Woleck (as cited in Cuoco & Curico 2001:215), "representations are not static products, rather they capture the process of constructing a mathematical concept or relationship. Representations can take on many forms like drawings and models, to graphs and symbolic expressions." In this question, drawings served as a representation to support mathematical learning. It was a tool learners used to support and process their mathematical communication. Drawings show to the teacher the ideas held by the learner and contain much information. They enable the teacher to see and the learner to display qualities of understanding that are obscured from other procedures.

When division problems are given in the context of a situation, Newstead & Murray (1998) found that learners achieved better and made better sense of the unfamiliar situations. L 1 was very clear in her explanation as she explained her drawing. She was able to rectify her first mistake when saying that each learner should get a quarter. When she realised it was half a cake and not a whole cake, she was able to say each child should get one eighth.

Figure 4.22 shows a series of drawings and guesswork, with L 2 just saying "$\frac{1}{4}$". When the interviewer started probing by asking how a quarter was found the learner became silent for a moment and then repeated, "Each gets one, one quarter." My impression was that he probably took the half cake to be a whole cake and then divided it into four pieces, so each piece would be a quarter.

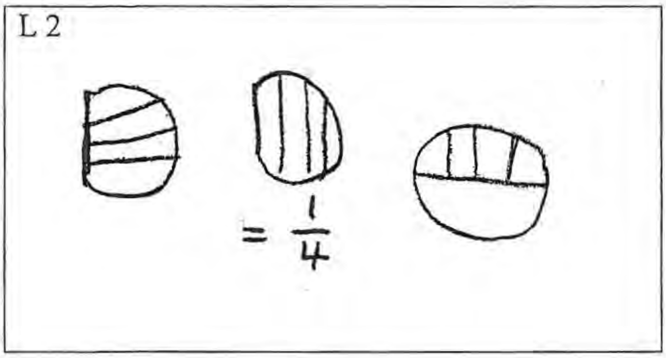


Figure 4.22 shows a series of drawings with some guesswork

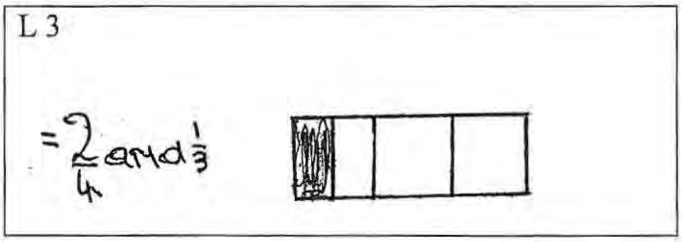


Figure 4.23 shows a drawing of the whole cake

Figure 4.23 shows how L 3 interpreted the fraction task.

> L 3: I shared a half into four and had two quarters and I shared again a quarter and it is made of two one thirds. So each child gets a third.
>
> I: What is $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ equal to?
>
> L: $\frac{3}{3}$
>
> I: And that one in your drawing
>
> L: $\frac{4}{3}$
>
> I: And what is it here?
>
> L: A whole
>
> I: If you say this $\frac{1}{3},\frac{1}{3},\frac{1}{3}$ makes a whole, because this is $\frac{3}{3}$. Three thirds is the name for the whole so you can't get this $\frac{4}{3}$ as this is more than a whole.

It then was obvious that the learner was having difficulty in answering this and no further probing took place.

L 4 (Figure 4.24) used a sketch by drawing a whole cake. He then divided the cake in half, using an incorrect circular representation and promptly wrote the following:

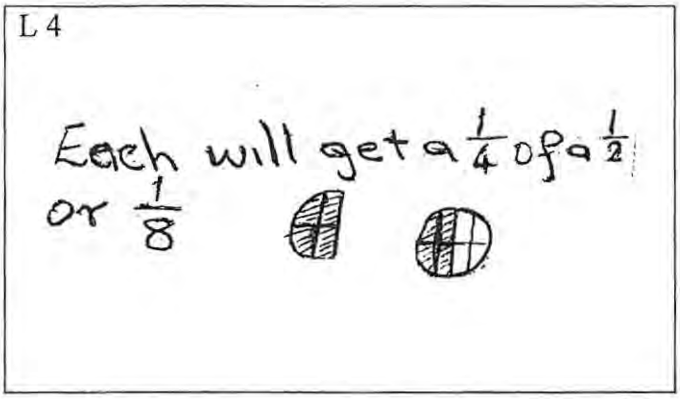


Figure 4.24 a drawing showing the representation

L 5 said:

> L: I was going to make a quarter but I didn't go well and I made how many eighths in a quarter and they were two. And how many $\frac{4}{8}$ in a quarter and they made a $\frac{1}{2}$. That is why I made $\frac{1}{8}$ for 4 children.
>
> I: Four children can get how many eighths altogether?
>
> L: Four children get one eighth each.

L 8 (Figure 4.25) also used an incorrect representation, a round cake (although he drew a $\frac{1}{2}$ cake) divided into four to get a $\frac{1}{4}$.

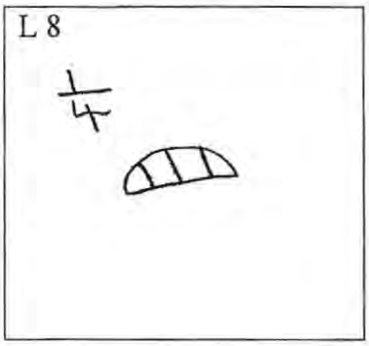


Figure 4.25 a drawing showing an incorrect representation

L 9 attempted explaining what he had done. It was very confusing, although he still got one eighth.

L 9: I should give each child one eighth because in one half we've got two quarters and in a quarter we have one eighth and in a half we have one eighth.

T 1 and T 2 were clear in their simple explanation of sharing the half cake into four equal parts with each child then getting an eighth.

T 3 (Figure 4.26) used the multiplicative strategy of solving the problem. She said:

T 3: It is said $\frac{1}{2}$ a cake is shared between four children. So if you divide a half you get a quarter, then that $\frac{2}{4}$ is $\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{1}{4}$ is an eighth so each will get an eighth.

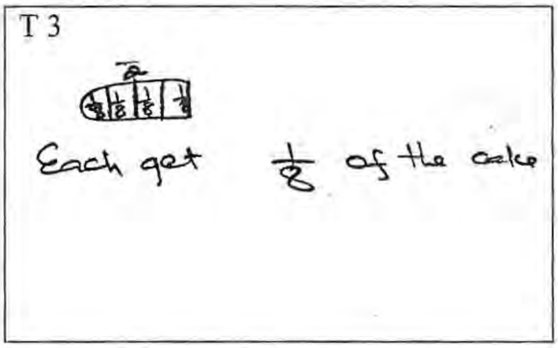


Figure 4.26 uses the multiplicative strategy

T 4 was asked to explain 'what does it mean a half divided by 4'. Teacher replied:

T 4: It's a half a cake shared amongst 4 children.

I: What fraction does each child get?

T 4: I changed the division sign to multiplication sign because it's easier to work with, then I did the inverse of four, that is a quarter and then I multiplied by a half and got an eighth.

T 7 also explained the problem using the invert and multiply strategy which showed her procedural knowledge, while teacher 8 explained it like this:

T 8: I started with the whole. I divided the whole into two equal parts. Then I divided, I discovered that each part is a half. Then I've divided each $\frac{1}{2}$ into two equal parts. So I discovered that a half of a half is a quarter. So each half is a quarter, then I went straight to the question, then I divided the half into 4 children so I ended up having an eighth for each child.

This strategy shows that the teacher has some conceptual understanding of what a fraction is.

4.9.5 Comparison between learner and teacher solution strategies

There appeared to be only two instances where learners matched the teachers' strategies. It was interesting to note the number of learners who used a drawing strategy to try and solve the problem with four of the seven succeeding. This seems to indicate that learners were using their own prior knowledge in order to understand the problem. It shows that in order to understand something, learners need to be given opportunities to use a strategy that they have thought about and are then able to execute correctly. Although T 5 and T 6 (Figure 4.27) were unable to solve the problem correctly, their learners on the other hand capably handled the division problem with ease. Both T 9 and L 9 got the correct answer, but their strategy and explanation was somewhat confusing. It appears that they were looking for some pattern like $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$ but they failed to explain it in a coherent manner.

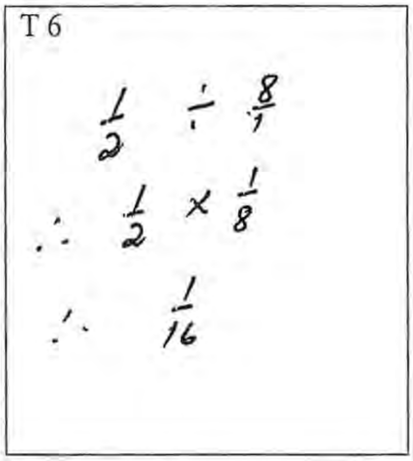


Figure 4. 27 where a half is divided by eight instead of four

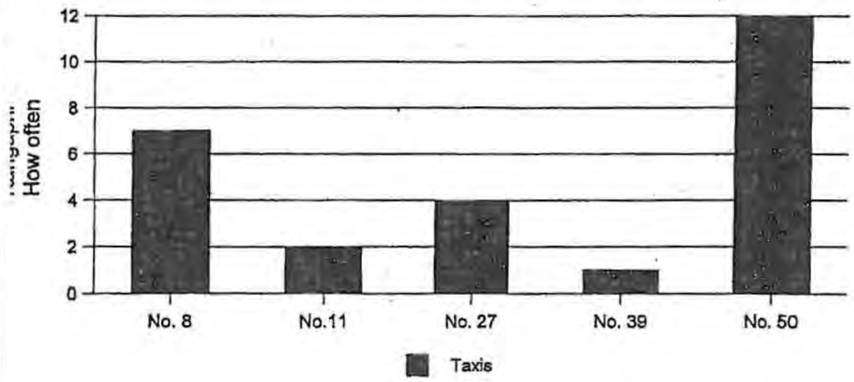
4.10 TASK 8

4.10.1 Description of the task

8. Babalwa makes a graph of taxis passing her school in 1 hour.

UBabalwa ubonakalisa igrafu yeeteksi ezigqitha esikolweni sakhe nge-yure enye.

Iiteksi ezidlulayo ngeyure enye/ Taxis passing in one hour



1. How many times does taxi number 27 pass in 1 hour?
   Idlule kangaphi iTeksi engunombolo 27 ngeyure enye ?

   ____

2. How many more number 8 taxis pass in the hour than number 11 taxis?
   Iteksi engunombolo 8 iyodlula kangakanani engu -11 ngeyure ?

   ____

3. Babalwa says: Taxi number 39 passes least often in the hour. Explain how the graph shows this.
   UBabalwa uthi : iTeksi engu -39 yeyona idlula amathuba ambalwa ngeyure.
   Chaza ukuba igrafu iyibonakalisa njani le nto.

   ____

   ____

4.10.2 Categorisation of the task

1.Content Area Category	Data Representation, probability & statistics
2.Subcategory	Data Representation and analysis
3.Subordinate Subcategory	Not applicable
4.Performance Expectation Category	Communication
5.Subcategory	Relating representations

Figure 4.28 TIMSS Curriculum Framework categorisation of task 8

The reason for putting in a task on data handling was to see whether learners would be able to understand and interpret the questions. Figure 4.28 shows the content category being data representation, with its subcategory being the ability able to analyse the graph.

4.10.3 Open coding

Table 4.8 Summary of strategies used by both learners and teachers in solving task 8

PC = Partially correct TC = Totally correct

Learner	PC	TC	Strategies	Teacher	PC	TC	Strategies
1	✓		Reading and applying information to the graph	1	✓		Reading and applying information to the graph
2	✓			2	✓		
3	✓			3		✓✓	
4		✓✓		4	✓		
5	✓			5	✓		
6	✓			6	✓		
7		✓✓		7		✓✓	
8	✓			8		✓✓	
9		✓✓		9		✓✓	
33 % correctly answered by learners				56 % correctly answered by teachers			

Open coding was not used in this task. The strategy required was to be able to read the graph accurately and to interpret the questions (Table 4.8). Properties were not applicable to this particular task.

4.10.4 Discussion of solution strategies

Dunkels (as cited in Nickson 2000: 87), mentions that:

> data handling is an important topic in the learning of mathematics in the primary school. It is a valuable medium for the development of number sense. Being able to read information from a two-dimensional representation, children learn to interpret data, which may relate to them and be useful to them

According to Orton & Frobisher (1996: 148) "graphs are an important means of communication, and we should obviously be aiming to enable children to communicate clearly, accurately and attractively, through graphs."

L's 4, 7 and 9 all gave clear explanations and interpreted the graph correctly.

L's 1, 2, 3, 5, 6, and 8 were able to partially interpret the graph correctly, but the second question requiring them to subtract seemed to be a problem.

Many of the teachers were at first unsure and remarked that the questions required a lot of thinking. T 3 said: "I had a problem with this one." T 7 said: "I've taken a long time with this one", but both eventually succeeded in answering all the questions.

T's 4, 5 and 6 were hesitant and had difficulty in answering all the questions.

T 1 and T 9 were very clear about their explanations although they took a long time in completing this question.

4.10.5. Comparison between learner and teacher solution strategies

In the data-handling question, similarities were found with two teachers and learners (7 and 9). It was obvious that this topic had been dealt with in class. The rest of the teachers and learners were able to answer only some of the questions that required careful reading and interpretation.

This is how L 7 and T 7 answered the task.

> L 7: Here it is asked, how many times does taxi number 27 pass in one hour. I wrote 4 times. How many more number 8 taxis pass in the hour than number 11 taxi, I counted and I said 7-2.
> I: Where did you get this 7?
>
> L 7: This 7 of this and I minus 2 of this and I got this 5 and Babalwa says, Taxi number 39 passes least often in the hour.
> I: Explain how the graph shows this.
> L 7: I said number 39 is in the half or number 1. The graph shows that number 39 is the least often in hour.

> T 7: I've taken a long time with this one. I couldn't understand where the hours are indicated here but at the same time, I could see taxi number 8 and so on. Then, the first question. How many times does taxi number 27 pass in one hour. So I thought taxi number 27 passes four times an hour. And then taxi number 8 passes 7 times in an hour and taxi 11 passes 2 times. So the question goes how many more number 8 taxis pass in the hour than taxi number 11. I subtracted the number of times of taxi number 8 passes which is seven times and the number of times of number 11 so I've got that taxi number 8 has past 5 times more than taxi number 7. Taxi number 39 passes the least number. Uhhh, I said, the graph has just moved one time from 0 up to 1 between 0 and 2, there is. So that is how it has been shown here

4.11 TASK 9

4.11.1 Description of the task

Thobeka obtains 45 marks out of a total of 75 marks for a science test. What was Thobeka's percentage?

4.11.2 Categorisation of the task

1. Content Category	Number
2.Subcategory	Fractions & Decimals
3.Subordinate Subcategory	Percentages
4.Performance Expectation Category	Using routine procedures
5.Subcategory	Performing routine procedures

Figure 4.29 TIMSS Curriculum Framework categorisation of task 9

The reason for inserting a question on percentages was to see whether learners could connect the fraction-percentage relationship. Working with percentages is used daily in most classrooms.

Figure 4.29 shows the content category used which was number and the subcategory, fractions and decimals. The subordinate subcategory is percentages.

4.11.3 Open coding

Table 4.9 Summary of strategies used by both learners and teachers in solving task 9

CS = Correct solution IS = Incorrect solution

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1		×	Subtraction	1	✓		Formal calculation
2		×	Counting	2	✓		Formal calculation
3		×	Multiples of 2	3	✓		Formal calculation
4		×	Addition	4	✓		Formal calculation
5		×	Simplification	5	✓		Formal calculation
6		×	Formal calculation	6	✓		Formal calculation
7		×	No idea	7	✓		Formal calculation
8		×	'guess'	8	✓		Formal calculation
9	✓		Formal calculation	9	✓		Formal calculation
11 % correctly answered by learners				100 % correctly answered by teachers			

The following was the strategy used in this task:

Formal percentage calculation

I have called this strategy the 'formal calculation' strategy because it involves writing down the fraction and then multiplying by 100 in order to obtain the percentage.

Only one property was identified and I have labelled it a 'simple percentage problem'.

4.11.4. Discussion of solution strategies

Only L 9 was able to solve this problem and it was done using the formal percentage computation.

L 5 (Figure 4.30) used the percentage computation but lacked the skill of simplification.

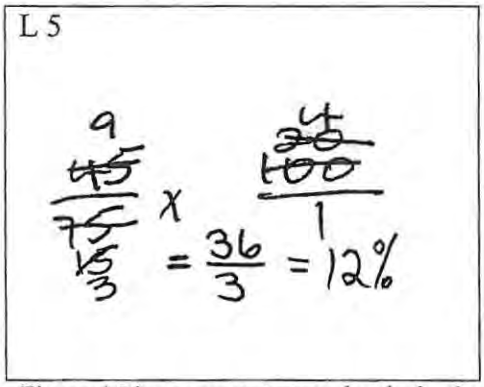


Figure 4.30 a correct strategy but lacks the skill of simplification

All the other learners did not know how to calculate a percentage from a total mark. There seemed to be no conceptual understanding of percentages.

> L 1: Here is the total 75. This is 45 of Thobeka. I subtract 45 from 75, I get 30%
>
> I: If you get 9 out of 10 for a test, what percentage would you get?
>
> L 1: 1%
>
> I: Would you only get 1%? If you got 6 out of 10, what would that be?
>
> L: 4%

All of the teachers used the formal percentage strategy, which is simplifying the numerators and denominators first by dividing by 5 and then by 3 before simplifying by 100 and dividing by 5 (Figure 4.31).

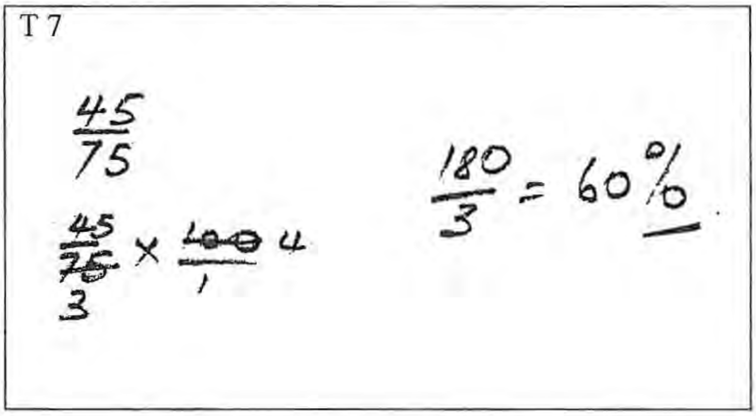


Figure 4.31 shows a formal percentage strategy

T 8 multiplied the numerators together and the denominators together and then simplified the fraction to get to 60%.

4.11.5. Comparison between learner and teacher solution strategies

Although the teachers all managed to complete this problem, only one learner was able to solve it correctly. L 9 matched his teacher and both used the same learnt strategy (see Figure 4.32).

L 9	T 9
9. $\frac{\cancel{45}^{\,3}}{\cancel{75}_{\,\cancel{15}}} \times \frac{\cancel{100}^{\,20}}{1} = 60\%$	$\frac{\cancel{45}^{\,3}}{\cancel{75}_{\,\cancel{15}\;1}} \times \frac{\cancel{100}^{\,20}}{1}$ $= 60\%$

Figure 4.32 shows the identical strategy used by L 9 and T 9.

4.12 TASK 10

4.12.1 Description of the task

If four packets of sugar together weigh 48 kg, how much does a $\frac{1}{3}$ of a packet of sugar weigh?

4.12.2 Categorisation of the task

1.Content Area Category	Number
2.Subcategory	Proportionality problem
3.Subordinate Subcategory	Not applicable
4.Performance Expectation Category	Investigating and problem solving
5.Subcategory	Solving and verifying

Figure 4.33 TIMSS Curriculum Framework categorisation of task 10

Figure 4.33 shows the categories used to classify the task. The task is part of number and is a proportional problem because it requires learners to first calculate how much one-kilogram of sugar weighs, before working out $\frac{1}{3}$ kilogram.

My reason for including this question in the task was that proportion is an important aspect of the problem solving process and learners and teachers had to solve more than one operation when investigating this problem.

4.12.3 Open coding

Table 10 Summary of strategies used by both learners and teachers in solving task 10

CS = Correct solution IS = Incorrect solution

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1		×	Division	1	✓		Proportion calculation
2		×	Counting	2	✓		Proportion calculation
3		×	Counting	3		×	Proportion calculation
4	✓		Proportion calculation	4	✓		Proportion calculation
5		×	No idea	5	✓		Proportion calculation
6		×	Multiplication 'of'	6	✓		Calculation
7		×	Multiplication 'of'	7	✓		Proportion calculation
8		×	Multiplication 'of'	8	✓		Algebraic calculation
9		×	Division	9	✓		Proportion calculation
11 % correctly answered by learners				89 % correctly answered by teachers			

Table 10 shows the following strategies used in solving the problem:

Proportion calculation

In this strategy simple proportion was used to find 1 kg and then a third was calculated.

Algebraic calculation

This strategy made use of algebra and then used 'cross multiplication'.

Division

In this strategy the fraction was converted to a decimal and then a simple computation took place.

Counting

Counting in threes was the strategy used here in order to get 48. Another counting strategy was to count first in fours to 48 and then threes to 12.

Multiplication

Computing a third of 48 was the strategy used here to solve the problem.

The properties found in this question are simplification and making use of multiplication and division (simple proportion).

4.12.4 Discussion of solution strategies

L 4 gave a clear explanation of how she arrived at 4 kg. L 1 worked out one packet to be 12 kg and then said $\frac{1}{3}$ would be 12 kg. After much prompting by the interviewer, she was able to say a third would be 4 kg.

The same can also be said for learner 3 who also eventually got 4 kg. L 2 and 5 had no idea of what to do while L 8, after being prompted by the interviewer to make a drawing, was able to get 4 kg.

L 9: I am not sure with this. And wrote down the following:

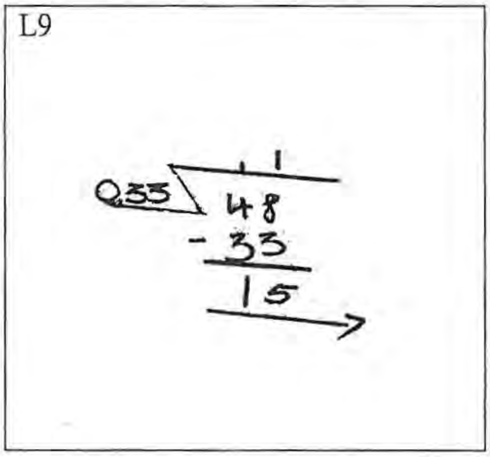


Figure 4.34 shows a somewhat confused strategy

When L 9 (Figure 4.34) was prompted by the interviewer to explain how much $\frac{1}{3}$ of a packet of sugar weighed?

L: 15 kg

The interviewer then asked:

I: Draw a packet of sugar.

L: I am not good at drawing.

L: 12 kg

I: How much is a $\frac{1}{3}$ of a packet of sugar? [No response.]

I: How much does 1 packet of sugar weigh?

L: 12 kg

I: How much does $\frac{1}{2}$ a packet of sugar weigh?

L: 6 kg

I: How much does a $\frac{1}{3}$ of a packet of sugar weigh?

L: 3 kg

I: Are you sure? How did you get that 6 ?

L: Half of 12 is 6 and add 6 plus 6 equals 12.

I: What does that 2 tell you?

L: Divide 12 by 3 is equal to 4 kg.

I: Great, okay.

Seven of the nine teachers solved this proportion sum by using a proportional strategy. That meant that they worked out what one packet of sugar weighs and then found a $\frac{1}{3}$ of the one packet. Only T 8 used an algebraic way to solve it by making use of cross multiplication. T 5 did not fully read the question and said:

> T 5: Now it is said if 4 packets of sugar together weigh 48 kg, what will a third of a packet of sugar weigh. Now I calculated one third of 48 and I got 16 kg.

4.12.5 Comparison between learner and teacher solution strategies

This problem was challenging to teachers and learners with only L 4 (Figure 4.35) being able to solve the problem.

L 4: The packets of sugar were 4. I divided 48 by 4 to get how much ....

I: How much does 1 packet of sugar weigh?

L 4 : I got 12 kg. Then I divided 12 by 3.

I: Why 3 ?

L 4: Because the question said, "How much does $\frac{1}{3}$ of a packet of sugar weigh?"

I: How did you get 4?

L 4: I divided by 3.

L 4

$48kg \div 4 = 12\ kg$

$12\ kg \div 3 = 4\ kg$

The $\frac{1}{3}$ of a packet of sugar weigh 4 kg

Figure 4.35 shows a correct solution

The above description compared favourably with the teacher's description (Figure 4.36)

> T 4: It says 4 packets of sugar together weigh 48 kg. I looked for a packet and then I said 48 divided by 4 = 12 kg so I looked for a 1/3 of a packet, which is 4 kg.

T 4

$48 \div 4 = 12$ Kg a packet

$\frac{1}{3}$ of 12 kg

$\frac{1}{\cancel{3}_{1}} \times \frac{\cancel{12}^{4}}{1}$ Kg

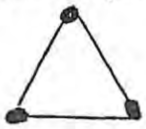
$= 4$ Kg $= \frac{1}{3}$ of a packet

Figure 4.36 shows a near identical solution to L 4

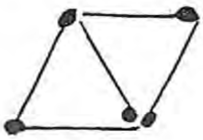
4.13 TASK 11

4.13.1 Description of the task

11. A pattern of triangles is made from matches.
I-patheni yoonxantathu (triangles) yenziwe ngėmicinga yematshisi.



3 matches make 1 triangle
Emi - 3 yenza unxantathu o -1



5 matches make 2 triangles
Emi - 5 yenza oonxantathu a - 2

7 matches make 3 triangles
Esi - 7 yenza oonxantathu a - 3

How many matches do I need to make 5 triangles?
Show how you worked this out.
Kwakufuneka ndibenemicinga emingaphi ukuze ndenze oonxantathu a- 5. Bonisa indlela ofumene ngayo impendulo yakho ?

Working out/Bala apha.

If I have 19 matches, how many triangles can I make?
Show how you worked this out.
Ukuba ndinemicinga e -19 , ndakukwenza onxantathu abangaphi ? Bonisa indlela ofumene ngayo impendulo yakho ?
Working out

4.13.2 Categorisation of the task

1.Content Area Category	Functions, patterns & relations
2.Subcategory	Patterns, relations & functions
3.Subordinate Subcategory	Not applicable
4.Performance Expectation Category	Mathematical reasoning
5.Subcategory	Generalising

Figure 4.37 TIMSS Curriculum Framework categorisation of task 11

Figure 4.37 shows the category in which this task was placed. The content category is functions, patterns and relations. This task requires careful reasoning and understanding to work out the pattern in order to come to some generalisation.

The reason for putting in such a problem was to see whether learners could get to the 'rule' of generalising.

4.13.3 Open coding

Table 4.11 Summary of strategies used by both learners and teachers in solving task 11

CS = Correct solution IS = Incorrect solution

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1	✓		Drawing	1	✓		Counting/pattern
2	✓		Addition	2	✓		Table/drawing
3	✓		Counting	3	✓		Table/pattern
4	✓		Differences between triangles	4	✓		Table
5	✓		Counting/drawing	5	✓		Drawing
6	✓		Counting	6	✓		Drawing/pattern
7	✓		Drawing/counting	7	✓		Table/drawing
8	✓		Counting/addition	8	✓		Table
9	✓		Drawing	9	✓		Addition
100 % correctly answered by learners				100 % correctly answered by teachers			

Table 4.11 shows the following strategies used by teachers and learners:

Making use of a drawing and then counting

In this strategy each triangle of match sticks was drawn and then counted.

Constructing a table

Using the pattern of triangles given, a table was constructed.

Differences between triangles

By looking at the differences between the patterns provided, a common difference was found and then applied to the rest of the table.

The following properties were identified: counting; tabular format and relating the number of triangles to match sticks.

4.13.4. Discussion of solution strategies

Describing number patterns, the relationship between variables, and forming generalisations have been perceived to be important in the Department of Education (2002). The focus of this learning outcome was to formalise the rules generating patterns.

The NCTM of the Principles and Standards (2000) state that:

> students should investigate both numerical and geometric patterns and express them mathematically in words or symbols. They need to analyse the structure of a pattern and watch it grow or change, organise the information systematically and develop generalisations about the mathematical relationship.

In the example of the matchstick pattern, learners were encouraged to use the idea of a variable as they thought about how to describe a rule.

Garcia-Cruz & Martinon (1997) in their research found that learners checked their rules by either counting or drawing or extending the numerical sequence. This was also my finding in this research. Four learners made use of a drawing with three more making use of a counting strategy in order to reach the pattern that was emerging.

L 4 immediately when asked how she solved the pattern said:

> L 4: I first looked for a rule.
>
> I: How did you work out the rule?
>
> L 4: (Pause) ....I first saw that one triangle was made up of three match sticks, two triangles was made up of 5 match sticks and three triangles was made up of 7 match sticks. Now I looked for the

difference here. It was 2. Then I added here 7 plus 2, I got 9 and said any number multiplied by 2 and add 1.
I: So how many matchsticks for five triangles?

L 4: 11

I: In the second part of the question?

L 4: I made the opposite of this to get to the number of triangles for 19 matchsticks. I said 19 divided by 2 minus 1 is 9.

I: Just one moment. Are you sure what you have said is correct?

L 4: I said 19 minus 1 = 18 divided by 2 , a half of 18 is 9 triangles.

Sasman, Linchevski, Olivier & Libenberg (1998) found in their research that when problems were formed in terms of pictures, learners instinctively constructed a 'table' and then only used the table of values to solve the solution.

Five of the nine teachers in this question constructed a table and made a drawing or looked for the pattern to get to the generalisation.

T 1 said: Here I,I,I, constructed a pattern and counted and discovered that you always multiply the triangle number by 2 and add 1. So for 19 sticks, you construct 9 triangles because I divided this 19 by 2 and there was one odd stick left, which was the one I always add.

T 9: I've done this, they like it. 1 2 3 and then they'll say 1 2 3 4 5, ja and even I've asked them. How do you say and they just say plus 2 ne, like this one, 3 5 7 9 11. They would say 11. also 19 match sticks, how many triangles can I make? So they've got a battle there. So they'll say + 2 + 6 would be 13 + 7; 16 + 2: 17 + 2 would be 19. So 9 triangles form 19 matches.

4.13.5 Comparison between learner and teacher solution strategies

In this problem all learners and teachers managed to complete the problem with the majority either using a drawing and then counting on or drawing up a table and looking for a pattern. There were a number of similarities between teachers and learners.

Figure 4.38 shows how L 4 and T 4 approached the task.

<table>
<tr><th>L 4</th><th>T 4</th></tr>
<tr>
<td>The rule of this is $N \times 2+1$ any nur 2
$5 \times 2+1 = 11$
9 will need 11 match sticks

$19 \div 2 - 1 = 9$
$19 - 1 = 18 \div 9$</td>
<td><table>
<tr><td>triangle no.</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>n</td></tr>
<tr><td>no of matches</td><td>3</td><td>5</td><td>7</td><td>9</td><td>11</td><td>13</td><td>$n \times 2+1$</td></tr>
<tr><td></td><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td></td><td></td></tr>
</table>
19 matches = n triangles
$(19 - 1) \div 2$ from the Rule $n \times 2+1$
$18 \div 2 = 9$</td>
</tr>
</table>

Figure 4.38 shows the different ways L 4 and T 4 solved the problem

4.14 TASK 12

4.14.1. Description of the task

Mrs Khumalo has a bag of sweets to give to her Grade 7 classes. She gives the first class 157 sweets and the second class 248 sweets. She then has 35 sweets left in her bag. How many sweets were in her bag at the start?

4.14.2 Categorisation of the task

1.Content Area Category	Number
2.Subcategory	Whole numbers
3.Subordinate Subcategory	Properties of operations
4.Performance Expectation Category	Using routine procedures
5.Subcategory	Performing routine procedures

Figure 4.39 TIMSS Curriculum Framework categorisation of task 12

The task in Figure 4.39 is part of number. It is an addition word problem whose subcategory is whole numbers. The reason for setting such a task was to explore whether or not learners and teachers would use the same traditional vertical algorithm.

4.14.3 Open coding

Table 4.12 Summary of strategies used by both learners and teachers in solving task 12

CS = Correct solution IS = Incorrect solution

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1	✓		Vertical algorithm	1	✓		Vertical algorithm
2	✓		Decomposition of numbers	2	✓		Vertical algorithm
3	✓		Vertical algorithm	3	✓		Vertical algorithm
4	✓		Vertical algorithm	4	✓		Vertical algorithm
5	✓		Vertical algorithm	5	✓		Vertical algorithm
6	✓		Vertical algorithm	6	✓		Horizontal & Vertical algorithm
7	✓		Vertical algorithm	7	✓		Horizontal & Vertical algorithm
8	✓		Vertical algorithm	8	✓		Vertical algorithm
9	✓		Vertical algorithm	9	✓		Vertical algorithm
100 % correctly answered by learners				100 % correctly answered by teachers			

The strategies used in Figure 4.12 include the following:

Vertical algorithm

When using this strategy, numbers are placed underneath each other according to their place value and added.

Decomposition of numbers or expanded notation

In this strategy, each number is decomposed into hundreds, tens, and ones and then added.

Horizontal algorithm

This strategy uses decomposition of numbers or expanded notation and the numbers are then added.

The properties that have been identified were partitioning of numbers and vertical addition.

4.14.4. Discussion of solution strategies

The addition word problem was straightforward and clear. All learners and teachers succeeded in completing it. Except for L 2 who decomposed the numbers, all the other learners used the vertical addition algorithm. This suggests to me that 'rule' like behaviour does have its advantages. However, Hiebert *et al.* (as cited in Nickson 2000) relates that children should not only have a variety of models at their disposal to help them carry out number operations, but that they should be encouraged to use their own strategies. In task 12 it seemed that most learners and teachers felt comfortable with the vertical algorithm strategy. They were flexible in applying it and it was the most efficient strategy to use. Fuson (as cited in Nickson 2000), contends that children who understand the structures of numbers and can see the relationships among the numbers, are able to work with them flexibly. For example 364 = (300 + 60 + 4).

Figure 4.40 shows how L 2 solved the problem correctly. She combined and partitioned numbers as her strategy.

L2

100+200=300
57+48=105
5+35=40
400+40=440

Figure 4.40 shows decomposing and combining numbers

4.14.5. Comparison between learner and teacher solution strategies

I started by looking at similarities among the strategies of the nine teachers. All of them added in columns, with T 7 using the decomposition strategy as well. T 6 (Figure 4.39) also used expanded notation in doing the first part of the sum.

T 6

157
+248
405
35
440

or

100 + 50 + 7
200 + 40 + 8
300 + 90 + 1
405
+ 35
440

Figure 4.41 shows the traditional vertical algorithm and a decomposition strategy

Besides L 2 (as mentioned above) and L 7, the rest of the learners used the traditional algorithm by adding in columns. All learners had no difficulty in the computation of the sum or the understanding of the word problem. Figure 4.42 shows teachers and learners using the same solution strategy.

L 8	T 8
157 248 + 35 440 - Sweets	157 +248 405 + 35 =440

Figure 4.42 shows the identical strategy used by L 8 and T 8

4.15 TEACHERS' PERCEPTIONS OF THE TEST

In the previous section of this chapter, aspects of grounded theory were used to analyse and classify learners and teachers' solution strategies.

In order to ascertain whether the standard of questions asked in the test was consistent with learners' ability at a Grade 7 level, a short structured interview was conducted with the nine teachers from the schools who took part in the study. My initial expectations were that the teachers would comment that the questions were of a high standard and too difficult for their learners to answer.

4.15.1 The general standard of the test

All nine teachers stated that the standard of the tasks were appropriate. T 7 commented by saying that, "the questions were challenging. Children had to think." T1 stated, "I think that we have covered all the stated outcomes that is number sense, shape and space, geometry." T4 stated that, "they [the tasks] are the correct standard for Grade 7. They allow for different types of questions like choosing the correct answer and calculations."

After assessing the test and looking at the results, I felt that maybe the tasks were of a standard which was above that expected of learners at the Grade 7 level. However, as seen from the above comments, the teachers felt differently.

4.15.2 The translation of questions into isiXhosa

There was unanimous agreement by all nine teachers that it was a good idea to translate the questions into mother tongue. Both T 1 and T 4 said the learners had a language problem and that they lacked understanding of English. T 1 further commented that the task was "easy for them". This seems to possibly suggest that the task instructions were easy for the learners to understand.

T 8 commented, "Yes, ja because our learners have a problem of the medium of instruction which is English so in order to make them understand, we used to, to, to, to use the, the mother tongue so that they could be able to understand what you want."

T 9 said, "Yes, because in fact it is not their mother tongue the medium of instruction and then some of the learners may become confused. And then now if she comes to her language, then now it becomes clear."

4.15.3 The accuracy of the isiXhosa translation

Only two teachers commented on the accuracy of the translation. T 3 said, "It is accurately translated. I think the translator is a very good person." T 1 said that he could not comment, as he had not read the isiXhosa translation. He concentrated on the English version only. T 4 said, "To be honest, I don't normally look at the isiXhosa translation. When I am answering myself, I look just at the English." The priority for the teachers seems to suggest that they wanted to complete the test and were not so concerned about the standard of the translation.

4.15.4 The asking of multiple-choice questions

Multiple-choice questions were useful in evaluating learners' thinking processes. According to Charles, Lester & O'Daffer (1987: 42):

> a multiple choice test is made up of items that consist of a problem or question and a list of possible solutions or answers... are versatile and can measure the ability to get a correct answer as well as the ability to use problem solving thinking skills

My intention in setting multiple-choice questions (section 1 of the test) was to explore the different solution strategies that learners would use. Although space had been provided for learners to show their working out, many of them did not utilize it. I then decided to concentrate only on section 2 of the test, which would help me understand how learners were thinking about aspects of problem solving and how they were employing alternative solution strategies.

Comments made by the teachers about using multiple-choice questions were varied and interesting. Three teachers had mixed feelings about using this type of a question. T 4 commented and said, "I don't think it is very good because some [learners] are so lazy. When there is multiple-choice, they tend to choose without calculating." T 6 had a similar comment. She said, "Well, children become lazy when doing multiple-choice questions. They don't think them through." T 3 thought it was a good idea. He remarked, "it is a good idea if he is aware of what is happening here, it's not just guessing. You have to calculate before you can choose a correct answer. They have got to calculate all the time so they are exposing their strategies and skills."

T 8 said, "I think it's a good idea because it encourages the learners to think critically about what is being asked."

4.15.5 The level of difficulty of the test

The teachers were positive when asked what they thought about the level of difficulty of the test. All the teachers agreed that the test was not too difficult. T 9 said, "Not so difficult. They have to think because mathematics is about thinking."

T 5 commented that, "No, I would not say they are not difficult, because now it's the use of language, that is the understanding."

4.15.6 Content knowledge covered in the test

I tried, when drawing up the test, to cover all five learning outcomes, namely number sense, patterns, geometry, measurement, and data handling.

When I asked teachers to comment on whether they had covered all the work that was included in the test, some said 'yes', with T 9 saying, "I did all but not too much on geometry."

T 3 said, "No because of time. I have not done decimal fractions and percentages."

T 5 said: "Ja, in fact I would say three quarters of it. I have not done graphs."

T 7 commented, "Yes, I am worried. I did not touch percentages, otherwise I've covered most of the work."

4.15.7 Reflection and summary

The range of opinions and comments made by the teachers pleasantly surprised me. As mentioned earlier, I personally expected that the teachers would find the test too difficult and the translation not sufficiently accurate. Yet despite what was expected, learners and teachers appeared to enjoy answering the test. Teachers appeared open to not having covered all the content knowledge and the comments on using multiple-choice questions was also interesting.

It confirmed then that the pilot study, which I had carried out the previous year, had been well worth the effort and that the tasks were reliable and valid for this study.

4.16 CONCLUSION

This chapter describes the 12 tasks of the test. It categorised each task into categories, subcategories, subordinate categories and performance expectations. The discussion of solution strategies was framed by the TIMSS Curriculum Frameworks. Using aspects of grounded theory, namely open coding, strategies for each task were identified, and properties of the tasks were formulated. A discussion of various solution strategies used by learners and teachers followed. This was followed by a comparison of solution strategies using a substantial amount of material taken from the interview responses to verify and strengthen the analysis. Finally, the teachers' perceptions of their overall impression of the test were investigated.

The next chapter uses the analysis of chapter four to discuss the findings, draw implications and conclusions from this study, and to formulate a number of recommendations.

CHAPTER FIVE

DISCUSSION OF FINDINGS, IMPLICATIONS, RECOMMENDATIONS AND AVENUES FOR FURTHER RESEARCH

5.1 INTRODUCTION

In this chapter I discuss the findings of this study under the headings of

a) number sense
b) patterns and functions
c) geometry
d) data handling

as itemised in the TIMSS Curriculum Frameworks (see chapter 4, page 73 - 74).

I use these headings as a framework for the formulation of a number of implications and recommendations. The chapter also identifies avenues for possible further research with a final section reflecting on the entire research process.

The main goal of the research was to investigate and analyse the solution strategies adopted by nine Grade 7 learners and their teachers of mathematics in solving a given set of tasks. In order to achieve this goal, each task was deconstructed into categories using the TIMSS Curriculum Frameworks (McNeeley 1997). Further, I employed aspects of grounded theory as a methodology to analyse the different solution strategies used by the participants.

5.2 DISCUSSION OF THE FINDINGS

5.2.1 Number or number sense

Traditionally, number activities in classrooms have been characterised by children copying the teachers' ideas or simply supplying the correct answer. Howden (1989: 11) however suggests that:

> Number sense can be described as good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualising them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.

The shift in instruction implicit in this quotation is from an individual learner memorising facts, rules and procedures to developing into a mathematical thinker.

The following tasks formed part of number sense: task 1, task 3, task 4, task 6, task 7, task 9, task 10 and task 12. They constituted 67% of the test. 17% were simple computation problems, while 50% of the tasks were made up of problem solving type, word problems.

Task 1 involved the magic triangle that required the simple manipulation of adding or subtracting multiples of 10 from 20 to 60 to make a total of 90 along each side of the triangle. The investigation was interesting because a wide range of problem solving strategies or heuristics, which can be both general and specific, were used by the participants (Suydam, as cited in Krulik & Reys 1980). These included trial and error, using consecutive multiples and deduction. All nine teachers and six of the learners answered the task correctly.

Task 3, which was a multiplication word problem, was only solved correctly by 44% of the learners. Those learners who forged ahead and successfully solved the multiplication algorithm, almost exclusively used the traditional vertical algorithm strategy, but very few were able to explain in a logical manner the processes involved. Evidence from the interviews I conducted shows that learners often started off well but got lost along the way or lacked the mastery of the multiplication tables. The solution very often did not make sense and was seen as just another 'rule' to follow. Bryant (1995) and Prior (2000) have shown that mastery of basic arithmetic underpins the learning of more complex mathematical capacities that depend on an accurate memory for basic facts. By extending the repertoire of strategies we teach, learners will come to the realisation that substituting addition for a multiplication computation is not always feasible. This of course depends on the size of the numbers used. Learners when solving such problems need to 'mathematize', i.e. to reason mathematically by looking carefully at the numbers to be used before deciding which strategy to use The strategy chosen should be efficient, clear and fit the problem (Fosnot & Dolk 2001). As many of the basic mathematics concepts are learned in primary schools, it is important that concepts and skills from all strands of mathematics need to be strengthened and expanded (Perso 1992). Drill and practise still needs to be reinforced, while teaching computations should not be seen as detached activities, but need to be connected to a conceptual base or located within a significant setting.

Task 4 was a fraction subtraction computation. It was poorly answered with only 44% of the selected sample of learners capable of doing it. Two learners who made use of a drawing were unable to explain how they arrived at their solution. One learner was unable to explain his model of a drawing. He just said $\frac{2}{4}$. When asked what $\frac{1}{2}$ - $\frac{1}{4}$ was the learner replied, $\frac{3}{4}$. His next sentence was inaudible and he just said, "I don't understand". Four of the learners simply subtracted the numerators from each other and wrote down the first denominator given. McIntosh *et al.* (1997) found the same error in his sample across Grades 3-10. It was clear from the interviews that the concept of a fraction had not been clearly understood. In this computation, interviews revealed that procedural understanding had been emphasised at the expense of conceptual understanding (see chapter 4, page 90), Some learners had approached the computation from a whole number point of view. The most common strategy was to find the lowest common multiple, while two teachers and one learner took a more flexible and efficient approach and used equivalent fractions. This example showed that learners experienced conceptual understanding that required reasoning for justifying procedures.

Task 6, the whole number division computation task was successfully completed by only 11% of the selected sample of learners. Some of the learners not only lacked procedural knowledge, but they also lacked a clear understanding of how to solve the computation (see chapter 4, page 107-108). Evidence from the interviews showed that some learners relied heavily on memorisation, which hindered them in the execution of the task. There was evidence from the interviews that some teachers teach division as a 'goes into' model. According to Fosnot & Dolk (2001) it is an ineffective model as digits are treated separately. This confuses learners and makes little sense to them.

Teachers tend not to emphasise the reciprocal relationship between multiplication and division or what the meaning of a remainder is. It appeared that the meaning of the remainder when applied to a paper and pencil algorithm had not been properly investigated. It seemed that the connection when doing division using a calculator had also not been properly explored. From the interviews there was evidence to suggest that teachers are perpetuating the methodology they learnt in their own school careers and this had been transferred to the learners whom they now teach.

Task 7, the fraction division word problem, although fairly complex, was well done, considering learners in Grade 7 only encounter division of fractions towards the end of the year. 67% of the learners answered the problem correctly as against 78% of the teachers. The 'invert and multiply' strategy was the most common strategy used by four of the teachers, while only one learner used this strategy. An interesting aspect of this task was that five of the learners used a drawing as the mode of representation to solve the problem. It was used as a tool by the learners to display and support their mathematical learning and understanding. In comparison, only one teacher applied this mode.

Task 9 was a percentage task and was successfully completed by 11% of the learners. Teachers traditionally teach percentages as a discrete section of work. Many appear not to connect them to the fraction-decimal relationship that exists. For example the relationship that 50% can also be written as 0,5 or $\frac{1}{2}$ (see chapter 4, page 125). Only one strategy was evident, which was used by both learner and teachers. It appeared that these learners lacked the necessary understanding that the structure of practical situations like percentages can be modelled by a particular operation.

Task 10 was a proportional problem. Only 11% of learners and 89% of teachers correctly completed the task. This problem required a deeper understanding than the other word problems, as it required a number of different operations. For example, division of whole numbers and multiplication of a fractional part of a whole. The learners attempted a number of different strategies but were mostly unsuccessful. They were unable to make sense of the problem, focussing more on the answer instead of looking at the investigative processes involved (Southwood & Spanneberg 1996).

Task 12 was an addition problem, which was successfully completed by the entire sample. It has been my experience in working with teachers that many of them spend a considerable amount of time doing addition computations and word problems involving addition at the expense of the other operations. Some of the strategies used were based on the place-value principle of adding numbers in columns, the so-called vertical algorithm and decomposing numbers into hundreds, tens and ones.

The number sense word problems were generally better completed than the computations. Computation "is a particular form of mathematical problem solving ... involving the performance of basic numerical operations" (Bass 2003:322). According to Bass (2003) 'computing' and 'calculating' are words often used interchangeably. Word problems on the other hand require careful reading and understanding. A possible reason for the learners doing better at word problems was that the test was written in English and translated into their mother tongue, isiXhosa. Because the learners were able to read each task not only in English, but also in their own language, they could in my opinion comprehend what they had to do and transfer it to symbols. Instead of just focussing on the symbols themselves (i.e. the language itself), they were now focussing on the meaning of the symbols. This is in contrast to research done on word problems. Research done by De Corte & Verschaffel (1989) found that a large number of learners and teachers find word problems difficult to learn and to teach. They also found that current educational practice does not consider the variety and flexibility in learners' informal solution strategies. They further state that learners who had not had formal instruction in a particular operation, for example addition or subtraction, could solve simple word problems using a wide variety of informal strategies. The findings of my interviews and the written work presented by the learners and teachers in the word problems, showed that many of the solution strategies depended on the learner's procedural understanding (see chapter 4, page 90). The types of strategies used were indicative of their own level of understanding. However, there were instances, for example in the multiplication word problem in task 3, where a variety of informal strategies were identified. They ranged from simple counting strategies to decomposing the multiplier. Some learners concentrated on the strategies of using place value and the 'short method' of multiplying by 25 (see chapter 4, page 89). This confirms the findings of De Corte & Verschaffel (1989), as discussed above.

5.2.2 Patterns and functions

From the time that learners enter the Foundation Phase and throughout their primary school years, they should ideally have regular, daily opportunities for exploring number patterns. For example, teachers might help children notice that red-blue-blue-red-blue-blue can be extended with another red-blue-blue sequence or help them predict that the twelfth term is blue, assuming that the red-blue-blue pattern repeats indefinitely. Initially, learners may describe the regularity in patterns verbally rather than with mathematical symbols. The study of patterns in the Intermediate and Senior Phases should gradually shift to the study of functions as the learners become increasingly confident. In these two phases, learners should be given experiences in conjunction with appropriate teaching resources. Mathematical problems should lead learners to build connections between the concrete context and the numerical pattern. From using such patterns, learners begin to use variables and describe algebraic relationships.

The study of patterns is the core of learning outcome 2 in the RNCS (2002:35). It states, "the study of numeric and geometric patterns develops the concepts of variable relationships and functions"; concepts which are central to an understanding of algebra. It further states, "investigating pattern and relationships allows the learner to … develop thinking skills such as generalising, explaining, describing, observing, inferring, specialising, creating, justifying, representing, refuting and predicting."

Task 2 and **task 11** addressed this learning outcome. Task 2 was the function machine. In the input-output flow diagram, 56% of the learners and 78% of the teachers identified the pattern. The task required learners to use the idea of a variable to enable them to explore pattern work and to look for relationships.

The interviewing process revealed that strategies used by the participants included looking for a rule and finding a relationship between the input and output numbers.

The ability to link new facts to known facts and to structure a variety of facts into the formation of mental patterns contributes to learners mastering number facts. It was helpful in this task for learners to discover the relationship amongst facts, instead of just simply requiring learners to know the facts.

Task 11 was the matchstick number pattern. Both learners and teachers scored 100% for this task. Learners and teachers solved this problem using a number of alternative representations by visualising the pattern found. The solution strategies identified showed that 67% of the learners used a drawing. Gardner (as cited in Thornton 2001:389) states, "there is no more effective aid in understanding algebraic identities than a good diagram." I fully support this statement, as the next step would be to get learners to understand how the visualisation can be described symbolically. In the task, some teachers on the other hand drew up a table and also made a drawing, while others just drew up a table. Most teachers then wrote down the number pattern in the form of an algebraic expression. The constructive approach used by some learners was to draw and visualise each pattern of matchsticks and then physically count the number of ensuing matchsticks. The strategies used ranged from the simple to more complex ones. This shows aspects of teachers and learners being able to think flexibly in acquiring the knowledge of number patterns and executing more advanced strategies.

5.2.3 Geometry

Shape and space are the words used to describe geometry in the RNCS document (2002). Too often this topic is taught using formulae, rather than the practical experience which will enable learners to understand the properties of the most common shapes and their classification (Merttens 1987).

The only task covering geometry was **task 5**. The van Hiele model, which focusses on levels of thinking in geometry, provided a useful framework in which to design the one geometric task (van Hiele 1986). According to the van Hiele theory, learners move through five (later revised to three) hierarchical levels of geometric understanding. The first three levels are appropriate to the teaching and learning of geometry at the Intermediate and Senior Phase. For this study only the first level, the recognition level, was used. At this level, learners are able to identify, name and compare geometric figures on the basis of the appearance of the shape as a whole.

The geometric **task 5**, required participants to identify two shapes and find one similarity between them. Only 22% of the learners and 78% of the teachers correctly and completely solved the task. In the identification of two-dimensional shapes, some learners confused a square with a rectangle and a rhombus with a general parallelogram. Finding a similarity between the two shapes was fairly well done. Only some of the learners and teachers thus appear to be on the first level of the van Hiele levels of geometric understanding. The van Hieles also argue that a learner cannot perform with understanding on one level if he/she has not passed through preceding levels. This study was not able to confirm or deny this comment.

5.2.4. Data Handling

Data handling is a fairly new mathematical strand for most teachers in South Africa to teach. Although it was prescribed in previous curriculum documents, data handling– which includes being able to collect information in statements, draw graphs and tables in my view appears to be marginalized by some teachers. The focus of data handling is the gaining of skills to gather and summarise data so that it can be interpreted and analysed. Learning Outcome 5 of the RNCS document states, "... collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, and to interpret and determine chance variation" (Department of Education 2002:38).

The bar graph in **task 8** required learners and teachers to read the questions related to the graph and to correctly interpret it. 33% of the learners and 56% of the teachers correctly interpreted **all** the questions. To be able to read and interpret information is an important element for the development of number sense.

5.2.5 General comments

The findings in the interviews and the written work presented by the learners and teachers, show that some learners struggled to communicate their thought processes in a coherent way. Without their being able to give clear explanations, there is little evidence of learners' real understanding and application of mathematical processes. It highlights the importance of social discourse and understanding. If learning takes place with understanding, these processes will become more long lasting and valuable to both learners and teachers. The linking of fractions, decimal fractions and percentages would be such an example.

The findings further suggest that many of the teachers in this sample do not allow enough time and opportunity for learners to develop alternative solution strategies, which results in a lack of capacity to produce new understandings.

Of the twelve tasks analysed, half of them could be classified as 'typical' word problems. There is evidence in the interviews that suggests procedural understanding had been emphasised at the expense of conceptual understanding. In the fraction subtraction computation, learners and teachers relied heavily on procedural understanding (see chapter 4, page 94-101). Most learners saw the fraction computation as four separate whole numbers to be subtracted. Because they had learnt the operations and algorithms of whole numbers, they thought that the same processes could be applied when calculating fractions. This suggests that the recall of memorized data does not necessarily help with understanding. This also suggests that giving problems in context teaches learners to think about and understand the problems first, rather than just applying some step-by-step sequence of a rule that they have learnt.

The solution strategies of whole numbers used by learners were similar to those used by the teachers. This was particularly evident in the whole number multiplication, addition and division calculations. In these cases it appears that learners had learnt the algorithm from their teachers in isolation from concepts, with no effort made to relate them to understanding. Learners were often unable to 'mathematize' (Fosnot & Dolk

2001), to look at the numbers carefully before deciding on what solution strategy to use. The same was evident with the fraction subtraction computation.

The learners and teachers who did use alternative solution strategies showed that if given a chance to construct their own solutions, and make reasoned choices, they were thinking for themselves. For them sense making with understanding was seen as a priority.

5.3. SUMMARY OF FINDINGS

- Most of the solution strategies that the participating learners used were straight forward procedures that they had learnt. They relied mostly on procedural understanding at the expense of conceptual understanding.
- The solution strategies of whole numbers adopted by the learners in this study were similar to the whole number solution strategies used by their teachers.
- Some teachers and learners in the sample did employ their own constructed solution strategies. They were able to make sense of the problems and to 'mathematize'.
- Teachers in the sample placed more emphasis on addition computations and word problems than on the other three basic operations.
- Language played a role in that learners sometimes struggled to communicate their thought processes in a coherent manner. Nevertheless, number sense word problems were better completed than computations as the tasks had been translated into mother tongue. This is in contrast to research done on word problems.
- Both learners and teachers used the heuristic or problem-solving strategies to solve some of the problems. For example they used strategies like trial-and-error, constructing a table, looking for patterns and making drawings.

5.4 IMPLICATIONS AND RECOMMENDATIONS

This study suggests that there are several important points that teachers should consider in teaching computation and word problems involving number sense.

Teachers should ensure that learners be given sufficient opportunities to solve problem-solving type problems. Besides addition, the other whole number operations (subtraction, multiplication and division) must also be given the recognition they deserve. In each lesson planned time should be taken to allow learners to master the basic concepts. For example, understanding the concept of a fraction as compared to whole number operations. More emphasis needs to be placed on developing conceptual understanding, allowing learners to explore and use their own constructed solution strategies (see chapter 4, page 90).

This study has shown that teachers need to take cognisance of language problems when teaching word problems. They should note that if word problems are written in English and mother tongue, reading, comprehension and encoding errors would be lessened. Olivares (as cited in Lee & Jung 2004:271) recommends that limited-English-proficient students, " be allowed to rely on their native language to make sense of mathematics communication, since background knowledge is the basis of any learning process in mathematics." The findings in this study with its bilingual approach show that the errors made by learners were mainly defective algorithms, careless or process errors, but did not relate to not understanding the task. The implication is that teachers need to spend more time allowing learners to look at the processes involved in arriving at a solution, rather than focussing on the solution.

This study has shown that learners in Grade 7 can and are able to invent their own strategies to solve complex word problems. The fraction division and whole number multiplication tasks are such examples. Kamii & Dominick (as cited in Morrow & Kenney 1998) found that learners using their own calculating procedures were able to solve more problems correctly than children who had been taught the standard algorithm. Carpenter, Fennema & Franke (1996) found in CGI classrooms that teachers do not prescribe procedures for learners to solve or expect them to use a particular algorithm. Learners solve problems on their own by working out their solutions, which are composed of fundamental number concepts and alternative solution strategies, which they then communicate to their peers and teachers.

By allowing learners to develop their own solution strategies, teachers will be sensitized to the thinking and reasoning of learners as they strive to make sense of the mathematics. Carroll & Porter, (as cited in Kenney & Morrow 1998) assert that "although it is advantageous for all students to know at least one written procedure for each of the operations, the standard algorithms taught in schools are often not the most appropriate or understandable. However, teaching algorithms as 'fixed' procedures restricts the thinking ability of learners to reason, communicate, and consequently, their ability to do mathematics. Teaching for understanding should be emphasised at the expense of 'teacher taught procedures'. Hiebert (1996) asserts that there is increasing evidence suggesting that learners, who have memorized and practised procedures without understanding, have difficulty in making sense of their work.

The above is also documented by Kamii & Dominick (as cited in Morrow & Kenney 1998) who believe, and which I fully support, that conventional algorithms are harmful because they encourage children to give up their own thinking, they un-teach place value, thereby preventing children from developing number sense.

This study has also shown the 'negative' effects of teaching one written standard algorithm with no understanding. Kamii & Plunkett (as quoted in Zarszycki 2001) report that doing this discourages a learner's logical mathematical thinking and contributes to the poor success rate of large numbers of learners. This study fully supports this statement because by giving learners the freedom to develop their own strategies, they are likely to use different 'methods' to those a teacher might expect from them.

According to Scharton (2004) being computationally fluent is an essential goal of mathematics. She argues that if learners are given opportunities to explain and analyse their computational methods, this will result in their developing efficient, accurate and flexible strategies. As their knowledge of different strategies grows, so does their computational fluency. Solution strategies that learners use should be grounded in understanding. This means that an understanding of the meaning of operations, for example subtraction and division, and their relationship, needs to be emphasised (see chapter 4, page 112). In this study the problems learners were given and asked to solve possibly yielded greater problem solving competence and possibly equal or better computational competence. Through exposing learners to problems the 'threads' of problem solving and computational competence become interwoven and together lead to a development of understanding. Learners need to be able to compute using more than one method. The various strategies they use need to be efficient and carried out by using ideas gained through discussion with their peers in the class. An aspect of the problem solving process is that learners need to record carefully and flexibly when choosing a strategy. It may require the knowledge of more than one approach to solve a particular problem. Orehovec (1984) found that learners could succeed when problem solving is taught, but for it to be a success there must be an understanding on the part of teachers and others to fully put the problem solving process into effect. I agree with Naidoo (1991) that more in-service professional development courses using the problem solving approach for teachers are necessary to allow them to change from the transmission mode of teaching to adopting the role of a facilitator in the learning process.

5.5 SIGNIFICANCE OF THE STUDY

The results of this study have contributed to the mathematical understanding of learners' solution strategies in the following way:

- Firstly, the test was not only seen as a means of assessing learners, but as a tool that created opportunities for learning. The significance of this study lies in the methodology used and the analysis of solution strategies.
- Secondly, the importance of seeing non-routine solution strategies being constructed by learners and teachers contributed to the study.
- Thirdly, the significance of the study has informed teacher practice in that it has promoted the idea of allowing learners to use their own solution strategies in the execution of any problem solving task. In this study, only one of the nine teachers demonstrated complete insight into all tasks. She understood and was able to articulate her thinking and provide correct solutions to all the tasks in the test.
- Fourthly, this study has shown that through the efforts of outcomes-based teaching, learners are starting to work towards the outcome of 'constructing'

their own knowledge by interacting with the problems, together with their peers in the social environment.

5.6 LIMITATIONS OF THE STUDY

On reflection, there were numerous limitations and shortcomings in this study.

- The limited use of grounded theory as a tool for analysis could be viewed as a limitation of this study. In hindsight, it could be argued that grounded theory only enabled me to analyse the various solution strategies to the level of open coding. I initially set out to use grounded theory to analyse solution strategies in more depth, but my data did not suit this strategy.
- Another limitation I found was in the clinical interviews that I conducted. The limited information about the social construction of meaning and knowledge in the classroom environment was evident. I was not able to witness the learners working in groups, negotiating meaning with their peers and with the teacher. Nevertheless the clinical interviews continued to be a powerful tool to gain insight into the learners and teachers' construction of knowledge, their understanding and sense making.
- A further shortcoming of this research was the small sample size. The solution strategies of only nine learners and nine teachers were analysed, although 341 learners took part in the overall study.
- Another limitation I saw was in the semi-structured teacher interview schedule. I should have asked teachers questions pertaining to the teaching and learning of problem solving. For example: questions such as whether they used the problem solving approach in their daily teaching, whether they knew anything about the problem-centred approach, and if there were any benefits to using such an approach should have been explored.
- In hindsight, I also felt that some of the tasks in the test were not cognitively demanding enough.

5.7 AVENUES FOR FUTURE RESEARCH

The findings of this study suggest that a more in-depth longitudinal study into the development of solution strategies needs to be carried out in a range of schools from Grades 4 to grade 7. The RNCS document now being implemented has been made more explicit, which will hopefully allow teachers to set more demanding cognitive activities for their learners. As this study was located in a rural and semi-rural context, similar research needs to be conducted in urban areas.

A further area where further research is needed is in the professional development of teachers, where they need to be trained in looking at strengthening the multiplicity of solution strategies that learners can use. Teachers need to become more experienced at being able to gain insight into what a learner has done and to give guidance as to how to refine the solution strategy used.

Another avenue for research is an exploration of how teachers assist learners with misconceptions. Teachers need to acquire the skills to be able to help all learners in acquiring alternative solution strategies and to identify misconceptions and be able to deal with them. The approach with the learner should involve discussion, communication, reflection and negotiation.

5.8 REFLECTION OF THE RESEARCH PROCESS

The entire research process has been a fascinating and enriching experience. From the outset, when I formulated my research goals to the methodology I intended using, I was naïve, as it all seemed so simple. However, I encountered many challenges with the methodology and reflecting back, I would now not choose grounded theory as a tool for analysing solution strategies.

Administering the test to the learners made me realise the poor conditions under which some teachers have to work and learners have to learn. Some schools lacked basic equipment such as chairs and tables, while others had broken windows, no doors and few resource materials. My dismay at seeing teachers struggle with some of the tasks in the test is testimony to the amount of professional development and training that is needed in our schools. I am of the opinion that this research will help teachers see the need to allow their learners to construct and use a variety of solution strategies when solving problems. Teachers will need to develop the insight and skills to help their learners refine inefficient strategies they use, and equip them and offer guidance where necessary.

The analysis of the solution strategies I found difficult and frustrating. I was disappointed in the low level of alternative strategies, which both teachers and learners used. The TIMSS Curriculum Frameworks, however, helped me categorise and deconstruct the solution strategies.

I found the conducting of the clinical interviews to be a real challenge. Interviewing learners and teachers in their second language was difficult. Not being able to communicate exactly what they wanted to say often stifled some participants. Although I had an isiXhosa-speaking co-worker to assist me, many of the learners did not want to speak in their mother tongue and hence struggled to give precise descriptions.

Notwithstanding the above, I have come to think more deeply about my own teaching and learning. My experience when working with teachers and learners has been to allow them to construct their own solution strategies, which may emerge as they solve tasks presented to them. I have also realised the importance of teaching learners to become computationally fluent and that both conceptual knowledge and conceptual understanding are integral aspects that need to be reinforced.

5.9 CONCLUSION

This qualitative study was situated in the interpretive paradigm and was underpinned by constructivism. As an empirical study, it was framed by the problem-centred approach. The TIMSS Curriculum Frameworks (McNeeley 1997) and aspects of grounded theory were used to analyse the solution strategies.

The data used to answer the research goal in this study was collected using three instruments. The first instrument, a test, was used to gather the necessary information concerning the different types of strategies exhibited by the learners and the teachers at a Grade 7 level. This instrument was also used to assess the learners' cognitive performance. This information was of importance to schools that took part, although it was not used for this study. Also included in the study was a semi-structured interview schedule on teachers' perceptions of the tasks. The third instrument was the clinical interviews conducted with nine learners and their mathematics teachers to gather information about the different strategies used and their understanding of the problems. Evidence obtained from the interviews and the test was analysed using aspects of grounded theory as a methodology and the TIMSS Curriculum Frameworks Document. Strategies were then analysed and a comparison was made between the strategies used by learners and their teachers.

This study has shown that teaching one standard algorithm may not be very useful to learners, as it does not inspire logical mathematical thinking. It 'un-teaches' place value thereby preventing learners from developing number sense. One of the main findings of the study was that teachers appeared to place more emphasis on procedural understanding at the expense of solid conceptual understanding. The study has also shown that learners are capable of inventing and constructing their own solution strategies, provided teachers allow enough time and opportunities for learners to communicate their understanding and thinking to each other. Three avenues for further research have also been suggested. They are that a more in-depth longitudinal study be undertaken into the development of solution strategies in a range of schools, both rural and urban. Two other areas where research is needed is in the professional development of teachers, and exploring learners' misconceptions.

This research study evolved like the unravelling of a tight ball of string. As the results of the TIMSS study were recognised and the challenges of designing problem solving tasks unfolded, so too has the meaningful exploration of solution strategies constructed by learners and teachers been disentangled. As a teacher using the problem-centred approach, I have made it a daily routine of mine to allow learners to struggle towards finding solutions to problems and I avoid providing rule-like procedures as much as possible. However, I do provide my learners with scaffolding to keep them interested in the problem and encourage them to look meaningfully at all tasks.

This study has shown that learners are quite capable and ingenuous enough to develop their own constructed solution strategies once they have understood and made sense of the mathematics involved and have learnt to 'mathematize.'

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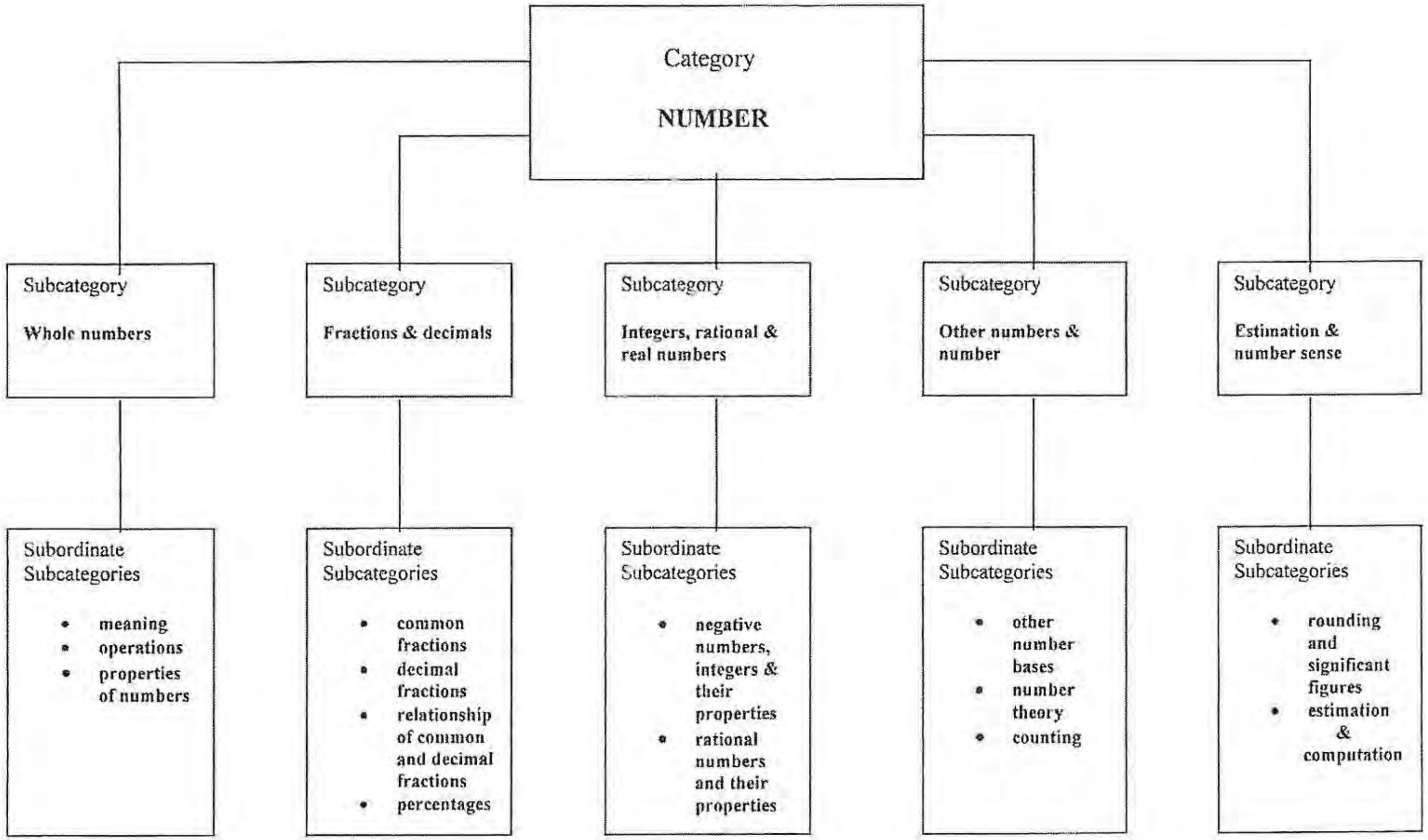
Appendix A

TIMSS Curriculum Frameworks (McNeeley 1997)

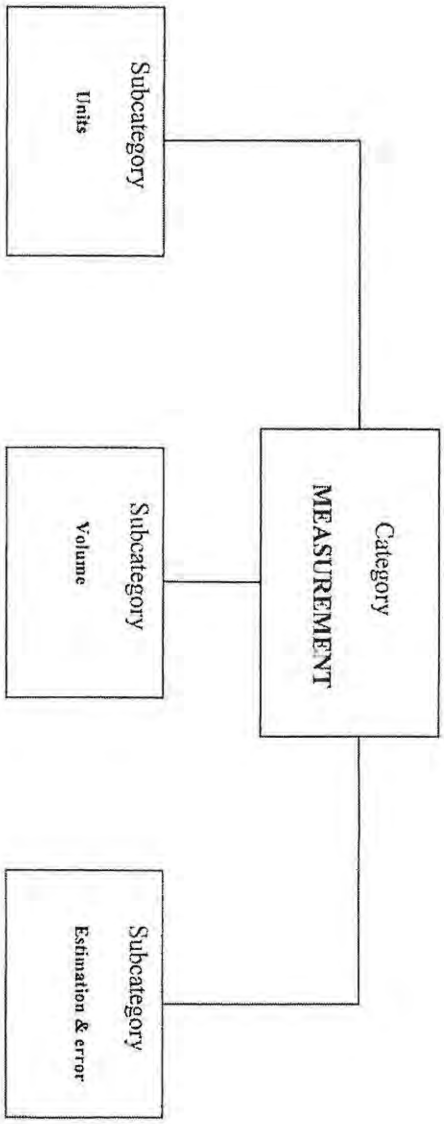
Content Category: Number



Appendix A

TIMSS Curriculum Frameworks (McNeeley 1997)

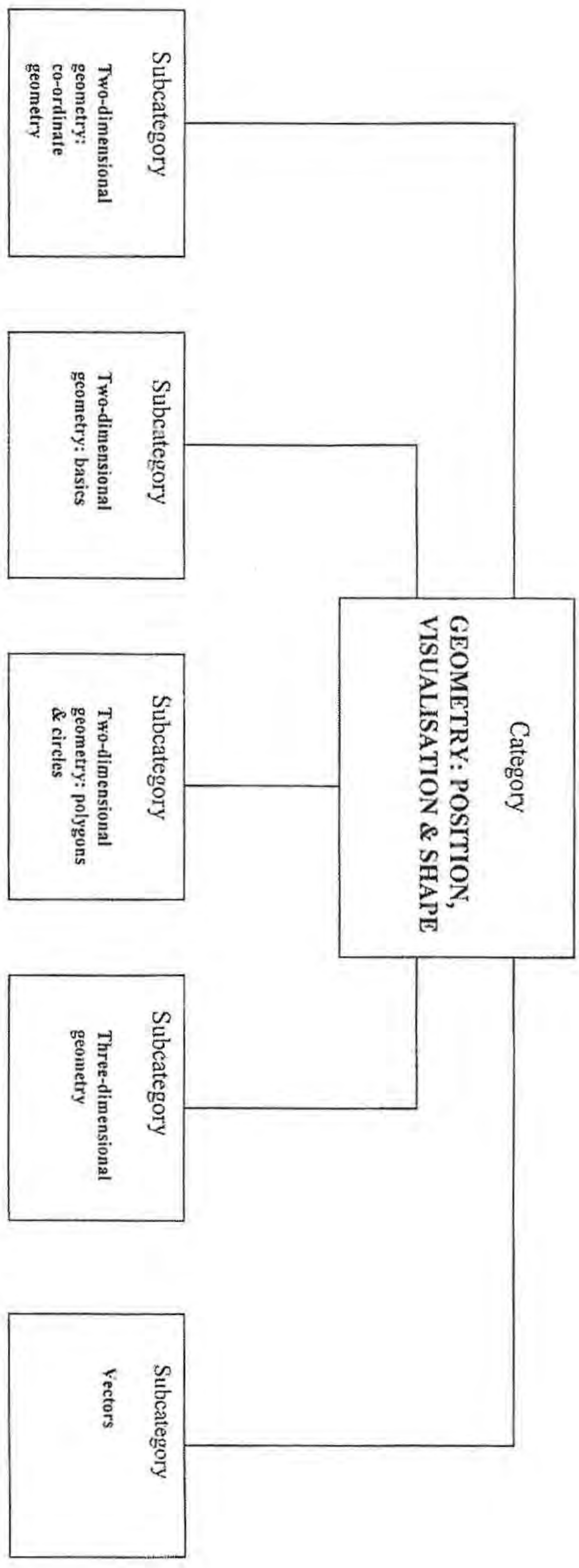
Content Category: Measurement



Appendix A

TIMSS Curriculum Frameworks (McNeeley 1997)

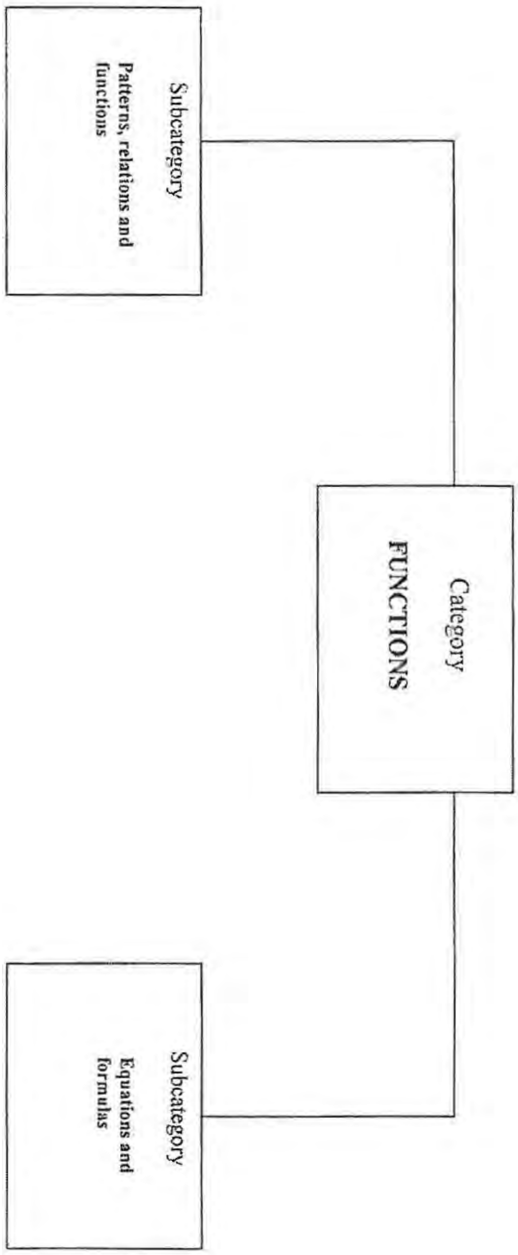
Content Category: Geometry: Position, Visualisation and Shape



Appendix A

TIMSS Curriculum Frameworks (McNeeley 1997)

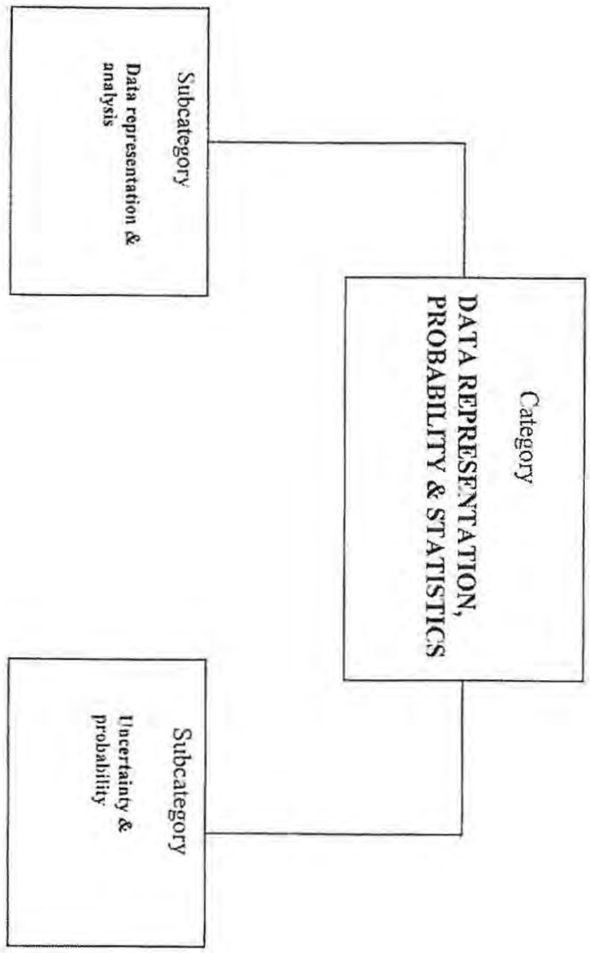
Content Category: Functions



Appendix A

TIMSS Curriculum Frameworks (McNeeley 1997)

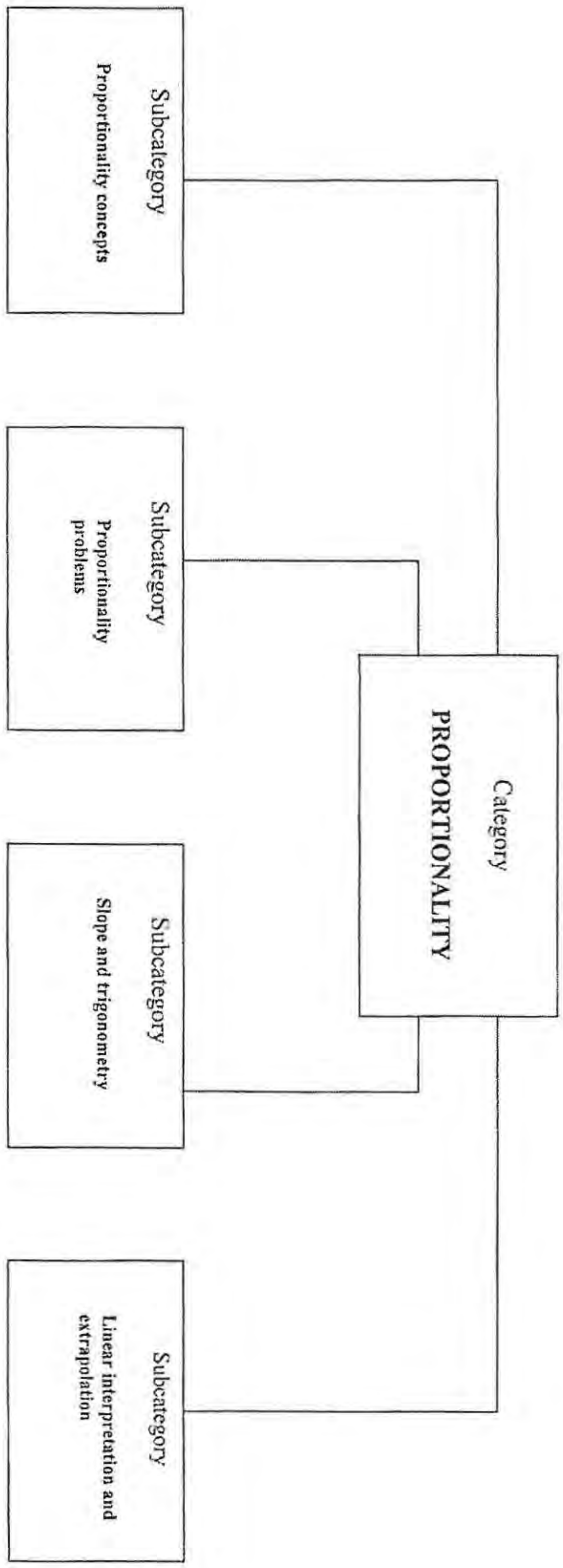
Content Category: Data Representation, Probability & Statistics



Appendix A

TIMSS Curriculum Frameworks (McNeeley 1997)

Content Category: Proportionality



Igama: Name: .... Iminyaka: Age: ....
Igama likatitshalakazi: Name of teacher: .... Umhla: Date: ....
Igama lesikolo: Name of school: .... Intombi / Inkwenkwe Girl / Boy

Let us do these 2 examples together.
Masenze le mizekelo mibini kunye.

Show your working in the space provided.
Wubonise umsebenzi wakho kulendawo kungabhalwanga kuyo.

Example 1./ Umzekelo 1	Working out/ Bala apha
How many minutes in 2 ½ hours? Mingaphi imizuzu kwiyure eziyi - 2 ½ ? A. 60 minutes / imizuzu B. 120 minutes / imizuzu C. 150 minutes / imizuzu D. 180 minutes / imizuzu	

Example 2./ Umzekelo 2	Working out / Bala apha
Calculate / Bala: 4 + (3 x 6) + 1 A. 23 B. 43 C. 32 D. 90	

Benchmarks for Grade 7

Section/ Icandelo 1.

1. You multiply 3 by 5 and add 7 to the product. You get 22. Which number sentence represents this statement? Xa uphinda-phinde u 3 ngo 5 uze udibanise u 7 kwisiphumo eso ufumana u 22. Sesiphi isivakalisi samanani esibonisa oko kwezi zilandelayo. A. $5 \times 3 + 7 = 22$ B. $3 \times 5 - 7 = 22$ C. $5 \times 3 \times 7 = 22$ D. $5\,(3+7) = 22$	Working out / Bala apha

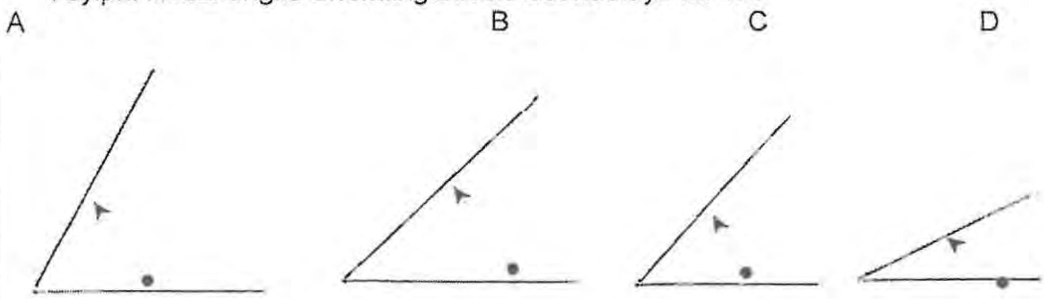
2. These shapes are arranged in a pattern. Le mizobo ibekwe ngokohlobo eyahluke ngayo. ○ ▲ ○ ○ ▲ ▲ ○ ○ ○ ▲ ▲ ▲ Which line of shapes is arranged in the same pattern as the one shown above? Loluphi uludwe lwemizobo kule ilandelayo olubekwe ngohlobo olufana nolu luboniswe ngasentla ? A. ▬ □ ▬ □ ▬ ▬ □ □ ▬ ▬ □ □ B. □ ▬ □ □ ▬ □ □ □ ▬ □ □ □ □ C. ▬ □ ▬ ▬ □ □ ▬ ▬ ▬ □ □ □ D. □□ ▬ ▬ □ ▬ □ □ ▬ ▬ □ ▬	Working out / Bala apha

3. A piece of wire 28 cm long is made into a rectangle. The width of this rectangle is 4 cm. What is its length ? Ucingo olubude buyi 28 cm lwenziwe uxande (rectangle). Ububanzi balo buyi 4 cm. Bungakanani ubude balo ? A. 6 cm B. 8 cm C. 9 cm D. 10 cm	Working out / Bala apha

4. Which of the following is the biggest decimal fraction? Leliphi elona qhezu likhulu kula alandelayo ? A. 0,19 B. 0,9 C. 0,091 D. 0,109	Working out/Bala apha

5. Which of these angles has a size closest to 45°?

Yeyiphi kwezi engile enomlinganiselo osondeleyo ku 45?



A.

B.

C.

D.

6. In which list of fractions are all the fractions equivalent?

Loluphi uluhlu lwamaqhezu olunamaqhezu alinganayo ?

A. $\frac{1}{2}, \frac{4}{8}, \frac{5}{12}$

B. $\frac{2}{5}, \frac{4}{7}, \frac{8}{15}$

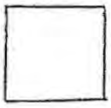
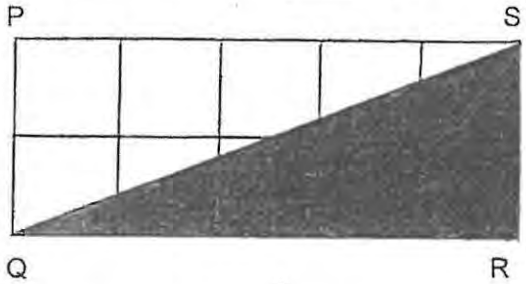
C. $\frac{2}{8}, \frac{8}{32}, \frac{12}{48}$

D. $\frac{1}{2}, \frac{5}{12}, \frac{10}{50}$

Working out / Bala apha

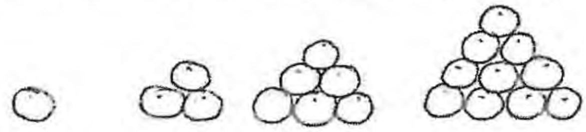
7. A newspaper reported that about 14 600 trees had been planted in a New township in Peddie. The number was rounded off to the nearest hundred. Which of these could have been the actual number of trees planted? Iphephandaba lithi imithi eqikelelwa kwi 14 600 iye yalinywa kwindawo yokuhlala entsha edolophini yase Ngqushwa. Eli nani libhalwe ngokwekhulu elisondele kulo (Rounded off to the nearest hundred). Ingaba ibileliphi elona nani lemithi eye yatyalwa ? A. 14 348 B. 14 672 C. 14 463 D. 14 567	Working out / Bala apha

8. What is the next number? Leliphi inani elilandelayo? 1, 4, 8, 13, ... A. 16 B. 17 C. 18 D. 19	Working out / Bala apha

9. Rectangle PQRS is divided into equal squares. Uxande (rectangle) u- PQRS lahlulwe lazizikwere ezilinganayo. This is one square unit Nasi esinye isikwere.   The area of the triangle SQR is .... square units. I-area ka nxantathu (triangle) u - SQR ingu.....(square units) A. 2 B. 5 C. 8 D. 10	Working out / Bala apha
10. Monde is 11 years old. His age is a quarter of his father's. How old is Monde's father? UMonde uneminyaka eyi 11. Iminyaka yakhe iyikota yeminyaka katata wakhe. Mdala kangakanani utata KaMonde? A. 22 yrs B. 33 yrs C. 44 yrs D. 50 yrs	Working out / Bala apha

11. Here are piles of tomatoes. How many tomatoes will be in row 5?

Nalu isicuku seetumato. Zakubangaphi itumato eziya kuba kwindawo yesi - 5 ?



Row 1 Row 2 Row 3 Row 4

A. 14 B. 15 C. 16 D. 20	Working out / Bala apha

12. On one day the minimum temperature for Queenstown is -7° C and the maximum temperature is 17°C. By how much does the temperature rise? Kusuku olunye iqondo lobushushu baseKomani eliphantsi liba ngu -7 C , lize eliphezulu libe ngu 17 C. Bunyuke kangakanani ubushushu ? A. 20° B. 27° C. 24° D. 10°	Working out / Bala apha

13. The sum of 3 and □ is equal to 12. What is the value of □?

    Xa kudityaniswe u 3 nenani elisebhokisini isiphumo ngu 12. Ingaba ngubani inani elimelwe yibhokisi ?

A. 4

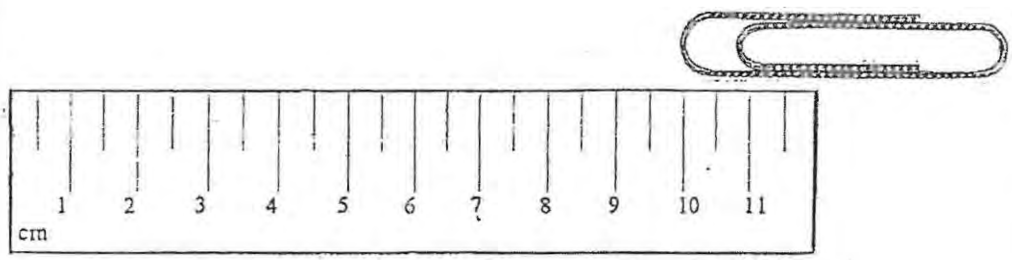
B. 9

C. 15

D. 36

Working out / Bala apha

14.



What is the length of the paper clip in the figure?
Bungakanani ubude besiqhoboshi-maphepha esikulo mzobo?

A. 4 cm

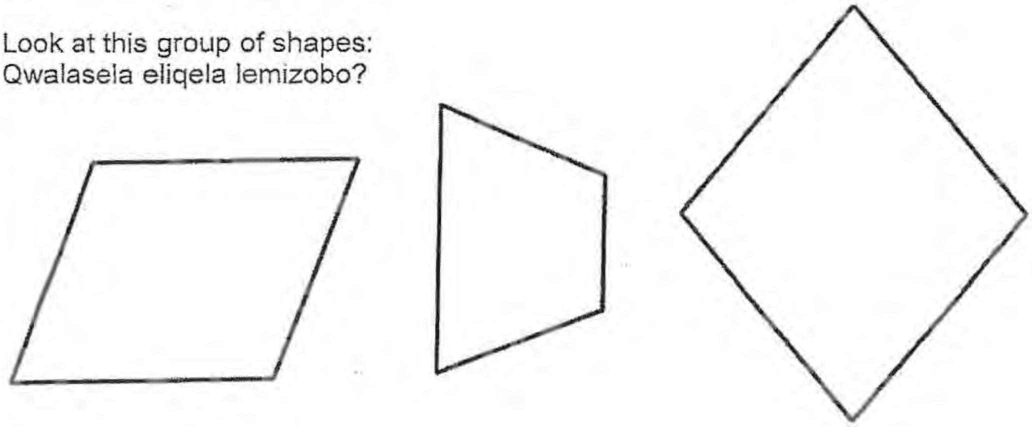
B. 4,5 cm

C. 5 cm

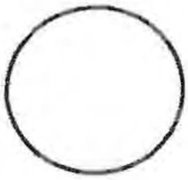
D. 12,5 cm

15.

Look at this group of shapes:
Qwalasela eliqela lemizobo?



Which shape below is included in this group? Why ?
Kule mizobo ingezantsi ngowuphi ofakelwe kule ingentla ? Kutheni ?

A. 

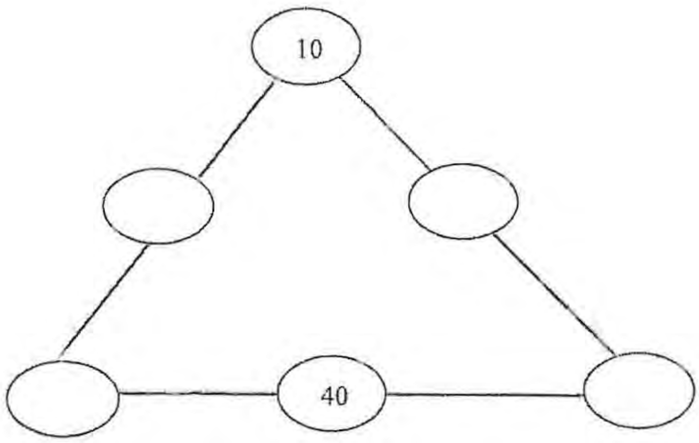
B. 

C. 

D. 

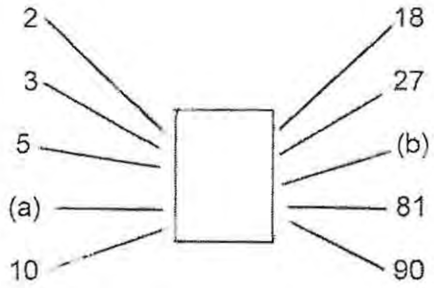
Section / Icandelo 2

1. In the magic triangle all the numbers along each edge must add up to 90.
   Put all the numbers 20, 30, 50 and 60 in the circles to make totals correct.
   Kulo nxantathu umangalisayo onke amanani asemacaleni xa edityanisiwe kufuneka enze u-90.
   Fakela la manani u 20, 30, 50 kwakunye no 60 kwizangqa ezingenanto ukuze kuphume isiphumo esichanekileyo, ukutsho u-90.



Working out

2.	Working out (a) (b)



2.1 Fill in the missing numbers in (a) and (b)

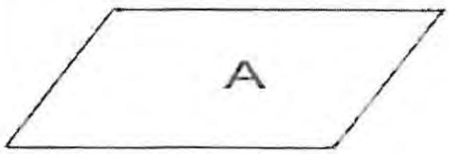
Fakela amanani ashiyiweyo kwindawo eno (a) naku (b)

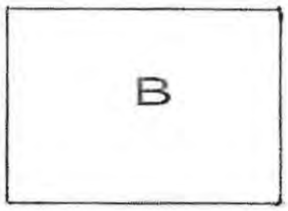
3. 25 learners go on an outing to the beach. They each buy an ice-cream which costs R3,50. How much must they pay altogether? Abafundi abayi-25 banohambo oluya elwandle. Emnye kubo uthenga iayisiskhrim exabisa R3,50. Ingaba kufuneka bebhatele imalini xa imali yabo idityanisiwe ?	Working out

4. Calculate / Bala : $\frac{3}{4} - \frac{1}{3}$	Working out

5. Look at the following two shapes

Khangela lemizobo mibini ilandelayo





What do we call shape A?
Yintoni igama lomzobo ongu A ....

What do we call shape B ?
Yintoni igama lomzobo ongu B ? ....

Write one thing which is the same about both shapes.
Bhala into ibenye lemizobo efana ngayo.

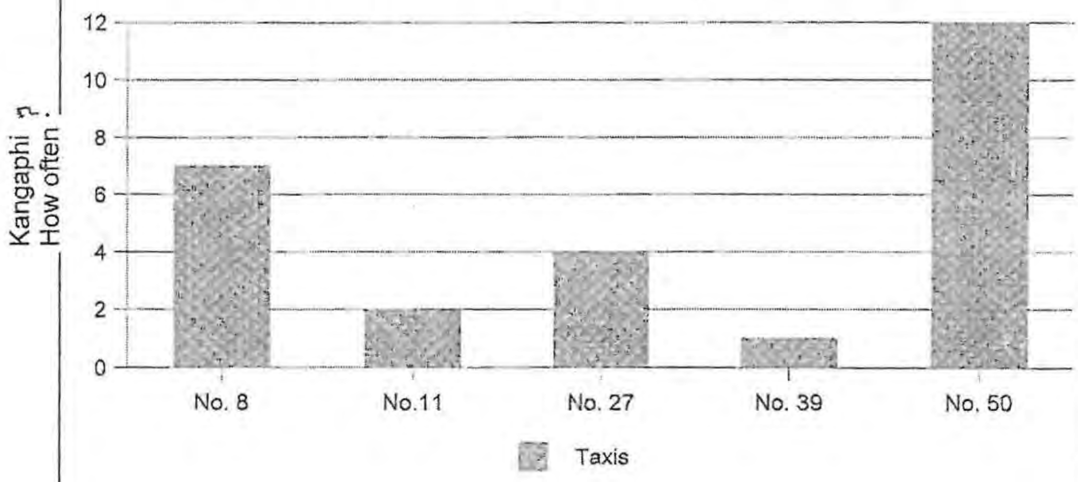
6. Calculate/ Bala : 2806 ÷ 8	Working out

7. Half a cake is shared fairly between four children. What fraction of cake would each child get? Isiqingatha (ihafu) yekeyiki yahlulwe ngokulinganayo isahlulelwa abantwana a - 4. Ingaba leliphi iqhezu umntwana ngamnye ayakulifumana ?	Working out

8. Babalwa makes a graph of taxis passing her school in 1 hour.

UBabalwa ubonakalisa igrafu yeeteksi ezigqitha esikolweni sakhe nge-yure enye.

Iiteksi ezidlulayo ngeyure enye/ Taxis passing in one hour



1. How many times does taxi number 27 pass in 1 hour?
   Idlule kangaphi iTeksi engunombolo 27 ngeyure enye ?

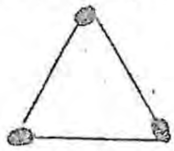
2. How many more number 8 taxis pass in the hour than number 11 taxis?
   Iteksi engunombolo 8 iyodlula kangakanani engu -11 ngeyure ?

3. Babalwa says: Taxi number 39 passes least often in the hour. Explain how the graph shows this.
   UBabalwa uthi : iTeksi engu -39 yeyona idlula amathuba ambalwa ngeyure.
   Chaza ukuba igrafu iyibonakalisa njani le nto.

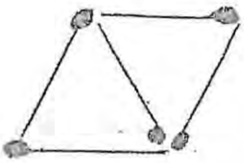
9. Thobeka obtains 45 marks out of a Total of 75 marks for a science test. Kumanqaku ayi -75 kwi-testi yeScience (nzululwazi), Uthobeka ufumana amanqaku ay -45. What was Thobeka's percentage? Ingaba ayintoni amanqaku kaThobeka ekhulwini (%) ?	Working out

10. If 4 packets of sugar together weigh 48 kg, how much does ⅓ of a packet of sugar weigh? Ukuba iipakethi zeswekile ezi 4 zizonke zinobunzima obunggu 48 kg, ingaba i -1/3 yepakethi enye ibunzima bungakanani na ?	Working out

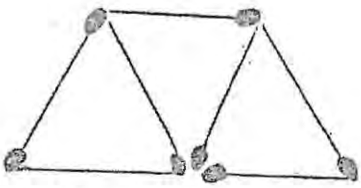
11. A pattern of triangles is made from matches.
I-patheni yoonxantathu (triangles) yenziwe ngemicinga yematshisi.



3 matches make 1 triangle
Emi - 3 yenza unxantathu o -1



5 matches make 2 triangles
Emi - 5 yenza oonxantathu a - 2



7 matches make 3 triangles
Esi - 7 yenza oonxantathu a - 3

How many matches do I need to make 5 triangles?
Show how you worked this out.
Kwakufuneka ndibenemicinga emingaphi ukuze ndenze oonxantathu a- 5. Bonisa indlela ofumene ngayo impendulo yakho ?

Working out/Bala apha.

If I have 19 matches, how many triangles can I make?
Show how you worked this out.
Ukuba ndinemicinga e -19 , ndakukwenza onxantathu abangaphi ? Bonisa indlela ofumene ngayo impendulo yakho ?
Working out

12. Mrs Khumalo has a bag of sweets to give to her grade 7 classes. She gives the first class 157 sweets, and the second class 248 sweets. She then has 35 sweets left in her bag. How many sweets were in her bag at the start? UNkosikazi Khumalo unengxowa yeelekese afuna ukuzinika abantwana bakhe bakwa grade 7. Unike iklasi yokuqala zayi 157, wanika eyesibini zayi 248. Ushiyekelwe ngoku zilekese eziyi35 engxoweni yakhe. Bezingaphi iilekese zakhe engxoweni ekuqaleni ? Bonisa ukuba ubale njani .	<u>Working out</u>

Appendix C

Notes to the administrators/ field workers were as follows:

- The benchmark activities are the property of RUMEP and no activity books will be left for the school or the class teacher. The administrator is responsible for handing out material, administering the benchmark test and collecting the booklets at the end.
- The introduction, especially if the learners have not had any prior experience with multiple-choice questions, should take between 5-10 minutes.
- The introductory section needs to be done thoroughly.
- Check that each learner has circled the correct letter and not the answer.
- Explain to the learners that if they make a mistake they are to cross out the incorrect answer and ring the correct one. Stress that no erasers are allowed.
- There is only one correct answer; therefore indicate that two answers cannot be circled at the same time.
- The multiple-choice section should take between 25-30 minutes and the long questions should take 40 - 50 minutes.
- No calculators may be used.
- Read all the tasks clearly to the learners.
- All tasks are to be read twice. In the pilot study this was done in English only. During the actual study, tasks were translated and read once in English and once in the mother tongue. This enabled learners to return to the task (should they so wish) and read it again in either English or isiXhosa before attempting to answer it.
- Allow enough time for individual learners to answer the test. Always be guided by the speed set by the learners themselves. Only move to the next task once each person has completed the work.
- Learners are required to sit in rows and should cover their work while they are waiting for the next task to be read.
- Stress that you want to see how the learners worked things out. Tell them to show all their working out in the space provided. You will have to continually tell them this and prevent them from writing on hands, desks and scraps of paper.