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Improving market-based forecasts of short-term interest rates: Time-varying stationarity and the predictive content of switching regime-expectations

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Improving market-based forecasts of short-term interest rates: Time-varying stationarity and the predictive content of switching regime-expectations[*]

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Abstract: Modeling short-term interest rates as following regime-switching processes has become increasingly popular. Theoretically, regime-switching models are able to capture rational expectations of infrequently occurring discrete events. Technically, they allow for potential time-varying stationarity. After discussing both aspects with reference to the recent literature, this paper provides estimations of various univariate regime-switching specifications for the German three-month money market rate and bivariate specifications additionally including the term spread. However, the main contribution is a multi-step out-of-sample forecasting competition. It turns out that forecasts are improved substantially when allowing for state-dependence. Particularly, the informational content of the term spread for future short rate changes can be exploited optimally within a multivariate regime-switching framework.

Keywords: interest rates, term structure, peso problem, regime-switching, forecasting

JEL classification: C32, C53; E43, E44; G12

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1. Introduction

Although the importance of interest rate forecasts for economists, practitioners and policymakers is obvious, there are relatively few out-of-sample studies on this issue found in international journals. This is surprising, because in contrast to stock market forecasts, where success would in general not be consistent with the hypothesis of efficient capital markets, the prediction of interest rates is virtually a consequence of rational individuals maximizing their utility in informational efficient debt markets.

The oldest and most prominent theoretical explanation of this predictability is the expectations theory of the term structure. The expectations hypothesis postulates that, for any choice of holding period, investors do not expect to realize different returns from holding bonds or bills of different maturities. Thus, a downward sloping yield curve implies an expected fall in interest rates which equalizes the ex ante returns of different investment opportunities. As a result, the current long-term rate is an average of expected future short-term rates, and the current term spread - defined as the difference between the long-term interest rate and the rate on a short-term instrument - contains information about future movements of long- and short-term rates as expected by the market. If, in addition, market participants form their expectations rationally, the expectations hypothesis becomes the rational expectations hypothesis, and the market forecasts do on average predict future interest rate movements successfully.

The experience with term structure-based forecasts documented in the literature so far is threefold: First, market based forecasts perform better out-of-sample than predictions of professional forecasters. Second, the yield curve performs better than simple univariate or multivariate time series techniques which do not account for contemporary market information. Third, though the yield curve contains expectations about future interest rate movements, the rational expectations hypothesis is generally rejected by the data across all countries.

As regards the last point, some authors argue that the rejection is due to time-varying risk premia. In this case, the term spread would also include information about future movements of the premium an investor will receive when he buys a long-term instrument. Although the rational expectations hypothesis is consistent with time-varying premia, it is difficult to find a stochastic representation which is both theoretically adequate and empirically successful. Another explanation for the failure of the expectations hypothesis is irrational market behaviour, for instance an overreaction of the term spread to expected short-rate changes. An economically more promising and also empirically appealing third explanation for the term structure puzzle is offered by the so called "peso-problem". In general, peso-problems are caused by important discrete economic events that occur less frequently in the sample under consideration than was rationally anticipated by economic agents. In such cases, the standard rational expectations hypothesis is a sensible description of expectations formation and its rejection is caused by a failure of the asymptotic distribution theory used in empirical tests. From a theoretical point of view, peso-problem behaviour in interest rates is a sound hypothesis which forecasters should take into account.

The econometric strategy of this paper builds upon recent research by Bekaert, Hodrick and Marshall (1997a) and Ang and Bekaert (1998) who propose univariate regime-switching models of short-term interest rates as well as bivariate regime-switching specifications additionally including the term spread. Bekaert, Hodrick and Marshall (1997a) explicitely choose the regime-switching approach to formalize and test the idea of a generalized peso-problem in the term structure. Indeed, they obtain estimation results in favour of regime-switching behaviour in the short rate. Moreover, building econometric inference upon small sample distributions generated by their regime-switching model substantially weakens the evidence against the expectations hypothesis. Having all these considerations in mind, the central argument motivating this study is as follows: when the expectations hypothesis is rejected in small samples because of peso problems, and, additionally, infrequently occcuring discrete events causing peso-problems can be captured by a regime-switching probability process, then we expect interest rate forecasts based on the yield curve slope to be even more successful as they are already reported to be, provided that the forecasting regression is modelled as a nonlinear regime-switching process.

Ang and Bekaert (1998) offer several methodological contributions and show that vector-autoregressive (VAR) models allowing for endogenous regime-shifts produce better one-step forecasts than univariate models and single regime VAR specifications. One special feature of their approach is to enable a time varying degree of mean reversion in the short rate, with the result that one of the estimated states describes the series as a slow mean reverting process whereas the other one characterizes the process as being integrated of order one. While using the same basic time series models and the same estimation technique as Ang and Bekaert, our study is essentially about out-of-sample forecasting. Over the reasonably large period from 1991 to 1998 we generate multi-step predictions of the German three-month interest rate. Furthermore, this study experiments on different stochastic representations, especially in relation to the question of stationarity. In contrast to Ang and Bekaert, we treat the short rate in some models as being integrated of order one in both states, which is according to recent empirical and methodological results an obvious misperception. Thus, together with the econometric modeling of peso problems, the adequate stochastic specification of the short rate is in the center of this study.

In the emprical part of our paper we will demonstrate that, from a conventional statistical point of view, two-state regime-switching models do describe the data of the short-term interest rate and the term spread well. However, the main empirical finding is the possibility to generate indeed better forecasts with regime-switching term structure models than with single regime specifications. Moreover, in many cases, the "no change"-forecast of the random walk model - a classical benchmark when predicting financial prices - turns out to be inferior over all forecasting horizons. To our surprise, modeling the short rate as an I(1) process across both regimes is clearly the superior forecasting strategy than allowing for mean reversion. Because this finding is not justified on a priori considerations, it deserves further attention.

In the next section the basic ideas and problems of the standard rational expectations hypothesis are characterized. It follows a discussion of peso-problem behaviour in the term structure and the potential role of regime-switching models to capture rational market expectations of discrete events. In section 3 we will show that regime-switching behaviour in interest rates is also empirically motivated by observed stochastic properties of the short-term rate. This section concludes with some principal results of recent research studies dealing with out-of-sample forecasting issues. Section 4 contains specifications and estimations of various models followed by the results of the forecasting competition. Section 5 concludes with a short summary of this study.

2. Regime-switching, rational expectations and the term structure

2.1 The expectations hypothesis

The expectations theory requires that the interest rate $R_{t,n}$ on a long term-bond with maturity n is an average of expected future one-period short-term interest rates r_t. In the special case of pure discount instruments and continuously compounded long and short rates we have:[1]

$$R_{t,n} = \frac{1}{n}\sum_{i=0}^{n-1} E_t r_{t+i} + \theta_n \, , \tag{1}$$

where θ_n denotes a constant term premium.

Equation (1) has the well-known implication that the weighted term spread $\left(R_{t,n} - r_t\right)$ has predictive content with respect to future changes of the long-term interest rate over the maturity of the short-term instrument. For testing this assumption Campbell and Shiller (1991) propose the following regression:

$$R_{t+1,n-1} - R_{t,n} = \alpha_0 + \alpha_1 \left[\frac{1}{n-1}\left(R_{t,n} - r_t\right)\right] + u_{t+1} \, . \tag{2}$$

From the expectations theory (1) it also follows that the term spread has predictive content with respect to cumulative future changes of the short rate over the maturity of the long term-

[1] An excellent introduction to the expectations hypothesis and its implications for predicting interest rates is given by Campbell, Lo and MacKinlay (1997), pp. 413-424.

bond. This assumption is of central importance for the forecasting exercises reported below. Campbell and Shiller (1991) propose to test it by:

$$\sum_{i=1}^{n-1}\left[1-\frac{i}{n}\right]\left(r_{t+i}-r_{t+i-1}\right)=d_0+d_1\left(R_{t,n}-r_t\right)+u_{t+n-1}\,. \tag{3}$$

Under the expectations hypothesis the slope coefficients α_1 in (2) and δ_1 in (3) should both equal unity. However, numerous empirical studies have lead to its overall empirical rejection. Some of the most cited contributions in this field are Campbell and Shiller (1991) for the U.S. and Hardouvelis (1994) and Gerlach and Smets (1997) for some more countries including Germany. Two aspects of this overall failure are particularly puzzling: while the term spread forecasts the wrong direction for the short-term changes in the long-term yield, it gives a forecast in the right direction for long-term changes in short rates. Nevertheless, the coefficient δ_1 is significantly different from unity. Another puzzle is the so called 'U-shaped' pattern of the term spread's predictive content with respect to future short rate changes: predictive ability is found to be quite good for forecast horizons that are no longer than about one month, while at horizons from three months to one year predictive power disappears. However, at horizons longer than one year, the forecasting power appears to improve.[2]

One intuitive interpretation of the discouraging evidence related to regression (3) is the existence of time-varying risk premia. However, a convincing theoretical and empirical solution has not been offered until now: while general equilibrium macroeconomic models of the term structure generally fail to explain the observations (see Bekaert, Hodrick and Marshall (1997b)), empirical specifications as GARCH models (see Hurn, McDonald Moody (1995)) are not motivated by sound theoretical hypotheses.

The overreaction of the spread to expected short rate changes as a special case of irrational market behaviour was originally suggested by Mankiw and Summers (1984) as a reason for the rejection of the expectations hypothesis. While this argument is also used by Campbell and Shiller (1991) and Hardouvelis (1994) as a possible explanation for the negative

2 See the results of twelve studies collected in Rudebusch (1995), p. 249.

coefficient α_1 in regression (1), Froot (1989) finds no evidence in favour of irrationality at the short end of the term structure using U.S.-survey data.

2.2 Peso-problem behaviour in the term structure

Empirical evidence supporting peso-problem behaviour in interest rates was given first by Lewis (1991) and Evans and Lewis (1994). In the most recent study on this issue, Bekaert, Hodrick and Marshall (1997a, p. 2) define peso problems in the following way:

> "(...) as arising whenever the *ex post* frequencies of states within the data sample differ substantially from their *ex ante* probabilities, and where these deviations distort econometric inference. When a peso problem is present, the sample moments calculated from the available data do not coincide with the population moments that agents actually use when making their decisions".

As this definition suggests, peso problems may exist when a state-dependent economy is subject to discrete events. The definition further implies that peso-problems are only relevant in small samples. However, again by definition, small samples are characterized by an unrepresentative number of states or regime shifts and not by the number of observations. Thus, even a sample spanning many decades can still be too small for relying on asymptotic distribution theory when regime shifts occur infrequently.

The presence of peso-problems may have severe consequences for the estimation of econometric models and the evaluation of forecasts. It is well-known that in circumstances where the number of discrete shifts observed in the sample is unrepresentative of the underlying distribution, forecast errors viewed ex post may appear biased and correlated with ex ante information though market participants form their expectations rationally. From this implication it follows further that coefficient estimates found in conventional regressions are affected by peso-problems, too (see Evans (1996)). Particularly, in the case of empirical testing the expectations hypothesis, a correlation between rational forecast errors and the term spread causes biased estimates of α_1 and δ_1 in the regressions (2) and (3).

In the literature, different economic explanations for peso-problem behaviour in the term structure are offered. While possible consequences of an erroneous anticipation of infrequent changes in interest rate targets may explain peso-problems in the medium run, conceptually similar considerations apply to rational errors in long run forecasts. It seems to be a useful general assumption that all possible regimes characterizing debt markets are associated with macroeconomic phenomena like monetary policy objectives, inflation or the business cycle. Then, a peso problem may arise when, for example, an expected shift to a high inflation regime is captured by the term spread, but fails to occur ex post.

2.3 Formalizing peso-problems

A useful characterization of economic models capturing the situations described in section 2.2 is suggested by Evans (1996, p. 613):

> "(...) 'peso problem' models focus on how the potential for discrete shifts in the distribution of future shocks to the economy can affect the rational expectations held by market participants, and hence the behavior of asset prices".

The econometric identification and exploitation of market forecasts in the presence of peso problems is difficult and requires that the two following conditions are met. First, to be distinguished from irrational expectations, market expectations have to be linked to discrete shifts estimated in the data. Obviously, such a distinction would be impossible when the expected discrete events are never observed in the past. Second, there have to be infrequent but repeated discrete shifts in the distribution of the data and not just one single event. Of course, models which are designed exclusively to explain economic variables around a particular past event, for instance a structural break like the German unification, are not expected to have predictive content.

A convenient method to formalize peso problems in the sense described above is the application of regime-switching models as suggested by Hamilton (1988). More precisely, regime-switching models can capture the phenomena of "generalized peso-problems", which occur when economic agents are not only affected by uncertainty about future regimes, but additionally cannot directly observe current or past regimes. In this setting, market participants have to make probabilistic inferences about the actual state of the economy. They further assign probabilities to the transition from one state to another. Combining both concepts, the regime probability of the actual state and the transition probability, leads to regime probabilities associated with future states. In the presence of a peso problem, and provided that the regime-switching model can capture it, market forecasts of discrete events are supposed to have an impact on the characteristics of the states as well as on the transition probabilities.

Actually, an increasing number of studies shows that there is regime-switching behaviour in interest rates and term spreads. Note, however, that regime-switching models can be constructed to explain term structure anomalies, but by definition, a formal test for peso-problems using the small sample of data is impossible. This problem can be solved when inference is based on Monte Carlo empirical distributions generated by the respective regime-switching model. Contributions which use such Monte Carlo experiments find evidence for rejecting economic hypotheses too often when inferences are based on asymptotic distributions (Evans and Lewis (1995), Bekaert, Hodrick and Marshall (1997a)). Hence, regime-switching models seem to be useful for describing the behaviour of financial time series in the presence of peso-problems. In section 4 it is shown that the various probabilities described above are part of the results obtained by estimations of regime-switching models.

3. Time-varying stationarity: stochastic properties of the short rate

3.1 Monetary policy and the stochastic representation of the short-term rate

In combining Modigliani and Shiller's (1973) preferred-habitat model with the rational expectations hypothesis, Sargent (1976) demonstrates that in an efficient market long-term interest rates will approximately exhibit random walk characteristics. This is due to the fact that the one-period variation of the equilibrium return on a long-term bond is assumed to be small relative to all other sources of one-period variations in returns which are caused by the arrival of new information. By contrast, market efficiency does not imply random walk behaviour of short-term rates. With a holding period being equal to the maturity of a short-term instrument, the one period return equals the interest rate at the beginning of the holding period which is known with certainty. Thus, variation in one-period returns is due solely to changes in the expected returns, and an expectations solution to the term structure is consistent with any stochastic representation of the short-term rate (Pesando (1981)).

According to Mankiw and Miron (1986) the rejection of the expectations hypothesis discussed in section 2.1 is attributable to the high variance of predicted changes in the short rate. Furthermore, the short rate has been approximately following a random walk since the founding of the Federal Reserve System. Mankiw and Miron explain this observation and thus the empirical failure of the expectations theory by the Fed's commitment to stabilizing interest rates. However, Rudebusch (1995) points out that under these circumstances the documented empirical failure does not imply a rejection of the rational expectations hypothesis, because there is no predictable variation in future short rates at all. In his paper, Rudebusch develops and estimates a daily model of Federal Reserve interest rate targeting behaviour and thereby explains the varying and 'U-shaped' predictive ability of the term structure to forecast future changes in short rates: while the Fed allows for predictable interest rate movements in the very short and the very long run, it eliminates predictable movements in the medium run by interest rate smoothing and by setting the Fed funds target rate at a level the Fed expects to maintain.

In a related study Balduzzi, Bertola and Foresi (1997) demonstrate that erroneous anticipation of future changes in monetary policy, especially discrete and infrequent changes in interest rate targets, mainly influence the term spread and thus can explain the disappointing outcome of empirical tests of the expectations hypothesis. Note, that this result directly points to the peso-problem which is characterized in section 2 of this study. The empirical analysis of Roberds, Runkle and Whiteman (1996) supports the assumption of time-varying predictive content of the yield curve caused by actual *and* expected central bank behaviour. In particular, they find information in the short end of the term structure to be present primarily within periods surrounding the reserve account settlements.

3.2 Implications of short rate persistence for empirical tests of the expectations hypothesis

Results of standard unit root tests generally do not reject the null hypothesis of a unit root in short-term interest rates (see, for example, Pagan, Hall and Martin (1996)). Motivated by the well-known low power of standard tests against stationary alternatives in small samples, Wu and Zhang (1996) apply a multivariate test procedure which pools the three-months interest rate series of twelve OECD countries. The results of the estimation strongly reject the null hypothesis of a unit root. Nevertheless, interest rates are found to follow slow mean-reverting processes. According to the AR(1)-coefficients estimated by Wu and Zhang, one-time innovations to the interest rates have a half-life of approximately three years. Recent evidence obtained by nonparametric estimations of short rate dynamics suggests that mean reversion depends on the level of interest rates. Particularly, the short rate exhibits random walk behaviour when it is in the middle of its historical range, but outside this range it is mean reverting (Aït-Sahalia (1996)).

The high persistence of the short rate has central implications for empirical tests of the expectations hypothesis. Bekaert, Hodrick and Marshall (1997c) document extreme small sample biases and deviations from asymptotic distribution theory in standard tests of the regressions (2) and (3), even under large sample sizes of 524 monthly observations. Unfortunately, the small sample distributions of the test statistics strengthen the evidence against the expectations hypothesis.[3]

3.3 Out-of-sample performance of market-based interest rate forecasts

As already stated in the introduction, evaluations of out-of-sample interest rate forecasts are hard to find in the economic and finance literature. Nevertheless, the basic findings on this issue, reported in some recent studies, can be summarized as follows. First, compared to the random walk model, the performance of recorded professional or survey predictions is worse in the case of long-term rates, while to a lesser extent this is also the case for recorded forecasts of short rates (Pesando (1981), Pesando and Plourde (1988), Hafer, Hein and MacDonald (1992), Deaves (1996)). Second, time series based short rate forecasts using term structure information perform better than no-change predictions (Deaves (1996)). In particular, the exploitation of market expectations using cointegration relationships within an vector error-correction framework turns out to be very successful (Hall, Anderson and Granger (1992), Bradley and Lumpkin (1992), Arshanapalli and Doukas (1994)). Third, there is evidence that the stochastic nature of short-term interest rates changes over time. In particular, series which are reasonably modeled as I(1)-processes over the whole sample exhibit mean-reverting behaviour in sub-sample estimations (Deaves (1996)). Consequently, choosing an appropriate stochastic representation seems to be crucial for the success of forecasting exercises (Lin and Tsay (1996)). Fourth, the forecasting ability of professionals as well as various market-based techniques is time-varying, too. It seems to depend on economic regimes characterized primarily by central bank behaviour (Pesando and Plourde (1988), Deaves (1996)).

Notwithstanding their different origins, all the considerations presented in the three subsections above suggest a time-varying or state dependent behaviour of the short rate. Consequently, the case for regime-switching models, which are already motivated in section

[3] Ball and Torous (1996) show that near unit-root behaviour could also imply severe drawbacks to the estimation of single-factor continuous-time models explaining movements of the short-term rate.

2 by peso-problem behaviour, is strengthened. Therefore, a strategy for market-based forecasts has to take into account regime-switching in interest rates.

4. Model specifications, estimations and forecasting results

4.1 The basic regime-switching model

In order to describe the stochastic process of the short rate and the term spread we estimate univariate and bivariate regime-switching models with two states as suggested originally by Hamilton (1988) and develloped further by, among others, Sola and Driffill (1994), Kugler (1996) and Ang and Bekaert (1998). In the univariate case, the conditional mean μ and the conditional variance h of a stationary series y are allowed to follow two different processes. The behaviour of the series depends on the value of an unobserved state variable S_t. Thus, under conditional normality, the observed realization y_t is presumed to be drawn from a $N(m_{1t}, h_{1t})$ distribution when $S_t = 1$, whereas y_t is distributed $N(m_{2t}, h_{2t})$ when $S_t = 2$.

The regime indicator S_t is parameterized as a first-order Markov process and the switching or transition probabilities P and Q have the typical Markov structure:

$$
\begin{aligned}
&\Pr[S_t = 1 | S_{t-1} = 1] = P \\
&\Pr[S_t = 2 | S_{t-1} = 1] = (1 - P) \\
&\Pr[S_t = 2 | S_{t-1} = 2] = Q \\
&\Pr[S_t = 1 | S_{t-1} = 2] = (1 - Q) \quad .
\end{aligned}
\tag{4}
$$

Under the assumption of conditional normality for each regime, the conditional distribution of y_t is a mixture of normal distributions,

$$
y_t | \Phi_{t-1} \sim \begin{cases} N(m_{1t}, h_{1t}) & \text{w. p. } p_{1t} \\ N(m_{2t}, h_{2t}) & \text{w. p. } p_{2t} = (1 - p_{1t}) \end{cases} , \tag{5}
$$

Intuitively, these models assume the short rate to be mean reverting, but estimations of discrete-time versions generally find it to be highly persistent (Campbell, Lo and MacKinlay (1997), pp. 449-451).

where $p_{1t} = \Pr(S_t = 1 | \Phi_{t-1})$ is the probability that the analyzed process is in regime 1 at time t conditional on information available at time t-1. The probability p_{1t} is called 'ex ante regime probability', because it is based solely on information already available and because it forecasts the prevailing regime in the next period.

Following Hamilton (1994) and Gray (1996) we formulate the unobserved regime probabilities as a recursive process,

$$p_{1t} = P\left[\frac{f_{1t-1}\,p_{1t-1}}{f_{1t-1}\,p_{1t-1} + f_{2t-1}(1-p_{1t-1})}\right] + (1-Q)\left[\frac{f_{2t-1}(1-p_{1t-1})}{f_{1t-1}\,p_{1t-1} + f_{2t-1}(1-p_{1t-1})}\right], \tag{6}$$

with the regime-dependent conditional distributions $f_{1t} = f(y_t | S_t = 1)$ and $f_{2t} = f(y_t | S_t = 2)$. This specification is very similar to a GARCH model where unobserved conditional variances follow a recursive structure with unknown parameters. The recursive representation of the regime-switching model allows us to construct the log-likelihood function conveniently as

$$\begin{aligned} L = \sum_{t=1}^{T} \log\Bigg[& p_{1t}\frac{1}{\sqrt{2p\,h_{1t}}}\exp\left\{\frac{-(y_t - m_{1t})^2}{2\,h_{1t}}\right\} \\ & + (1-p_{1t})\frac{1}{\sqrt{2p\,h_{2t}}}\exp\left\{\frac{-(y_t - m_{2t})^2}{2\,h_{2t}}\right\}\Bigg]. \end{aligned} \tag{7}$$

All models used in the following subsections were estimated by maximum likelihood. Parameter estimates were obtained using the BFGS algorithm, and the reported t-statistics are based on heteroskedastic-consistent standard errors (White (1982)). The estimates are derived from a monthly data set of German interest rates which are taken from the monthly report of the Bundesbank. The short rate is the three-month money market rate. To calculate the term spread for bivariate estimations, the "yield on bonds outstanding issued by residents" is selected as the long-term rate. The sample extends from January 1970 to December 1990. The out-of-sample period starts in January 1991 and ends in December 1998. Note, that the forecasting period begins before the short rate has achieved its maximum in 1992. Forecasts are generated without updating the parameters on the out-of-sample period. The series of the short rate, its first differences and the term spread are contained in Figure 1, 2 and 3, respectively.

[Figure 1, Figure 2, Figure 3]

4.2 Quality of regime classification and forecast error measures

In order to check the statistical fit of regime-switching models, we calculate the regime classification measure, RCM, statistic proposed by Ang and Bekaert (1998):

$$\mathrm{RCM} = 400\,\frac{1}{T}\sum_{t=1}^{T} p_{1t}\left(1 - p_{1t}\right). \tag{8}$$

An exact regime classification obtained by an ideal model would be reflected in an ex-ante probability p_{1t} being close to one or zero. According to (8) this would imply a low RCM statistic value. In contrast, models which cannot successful distinguish between the two regimes are associated with ex ante probabilities close to a half resulting in a high RCM statistic value. Because the constant normalizes the statistic to be between zero and 100, a value of zero indicates perfect regime classification while a value of 100 implies that no information about the regime is obtained. Out-of-sample forecasting performance is measured conventionally by the mean absolute error (MAE) and the root mean squared error (RMSE). Performance relative to the random walk model is measured by Theil's coefficient of inequality (TU) which is defined as the ratio of the RMSE of the technique being evaluated to the RMSE of the no-change forecast.

4.3 Modelling the short rate as a mean reverting process

Following Hamilton (1988), Gray (1996) and Ang and Bekaert (1998), in this subsection, the short rate r_t is assumed to be stationary. For the conditional mean, we adopt an AR(1) specification with a state-dependent autoregressive coefficient:

$$r_t = g(S_t) + a_1(S_t) r_{t-1} + s(S_t) e_t . \tag{9}$$

In contrast to the general model introduced in 4.1 the conditional variance is restricted to be constant within both regimes: $h_{1t} = s_1^2$ and $h_{2t} = s_2^2$. Thus, the only source of heteroskedasticity is due to regime changes. Because this study is not about forecasting second moments, such a simplification seems to be reasonable. Moreover, estimation of complex models, for instance regime-switching GARCH models, is avoided, and the number of possible specifications is reduced considerably.

As a first step, we compare the statistical fit of three models: a traditional single regime AR(1) model, denoted AR(1), the regime-switching model described by (9), denoted RSH-AR(1),[4] and a restricted regime-switching model with constant variances ($s_1^2 = s_2^2$) across regimes, denoted RS-AR(1). Maximum likelihood estimates are reported in Table 1. According to the results, the short rate is highly persistent. While the near-unit-root behaviour becomes obvious in the AR(1) model parameters, some evidence in favour of a slow mean reverting process in regime two can be found when looking at the RSH-AR(1) estimates. Although there is strong evidence in favour of a high and a low volatility regime, restricting the variance to be independent of states leads to a better regime classification. Note however, that this has the consequence of a lower log-likelihood value. Moreover, the autoregressive parameter in regime one becomes explosive.

[Table 1]

[4] The "H" denotes (state-dependent) heteroskedasticity; see Krolzig (1997, p.14), who suggests a general classification scheme for regime-switching models.

Comparing the forecasting ability documented in Table 2 leads to the following insights. The AR(1) as well as the RSH-AR(1) model perform worse than the random walk over horizons within one year. Over long horizons their predictive ability improves. In contrast, the RS-AR(1) model is superior over all horizons when looking at the RMSE criterion. Most importantly, forecast errors decrease dramatically in the case of 12, 24 and 36-step forecasts. One can conclude that the increasing forecasting accuracy as horizons become longer reflects slow mean reversion in the short rate. As far as the RCM statistic is concerned, it seems to indicate predictive ability, too.

[Table 2]

4.4 Modeling the short rate as an I(1)-process

In contrast to the approach followed in section 4.3, the short rate can also be assumed to be nonstationary in both regimes (Sola and Driffill (1994), Kugler (1996)). Consequently, the regime switching models in this subsection describe the behaviour of the first differences of the series r_t. Again, an AR(1) process with a state-dependent autoregressive coefficient characterizes the conditional mean:

$$\Delta r_t = g(S_t) + a_1(S_t)\,\Delta r_{t-1} + s(S_t)\,e_t, \tag{10}$$

whereas the conditional variances are: $h_{1t} = s_1^2$ and $h_{2t} = s_2^2$.

As Table 3 reveals, there is substantial improvement in the log-likelihood function, compared to the estimates shown in Table 1, where the short rate is assumed to be I(1). Again, the RSH-AR(1) model seems to capture primarily state-dependent heteroskedasticity, while in the RS-AR(1) model regimes are characterized by their conditional mean dynamics exclusively, which results in a more exact regime classification.

[Tables 3, 4]

According to Table 4 all autoregressive models perform better out-of-sample than the random walk. While the RSH-AR(1) model shows no improvement over the single regime specification, RS-AR(1) is the superior model for all forecasting horizons. In the case of 1, 3, 6, and 9-step forecasts, its predictive ability is even higher than that of the corresponding mean reverting specification documented in Table 2. However, out of all six univariate specifications considered so far for the 12, 24 and 36-step horizons the mean reverting RS-AR(1) model yields the most exact predictions.

4.5 Modeling the short rate and the term spread as bivariate processes

Because univariate estimations in sections 4.4 and 4.5 provided mixed results with regard to (non-) stationarity of the short rate, two basic bivariate models including the term spread are considered. The first one treats the short rate as a stationary variable,

$$\begin{aligned} r_t &= g(S_t) + a_1 r_{t-1} + a_2 (R_{t-1} - r_{t-1}) + s_1(S_t) e_t \\ (R_t - r_t) &= n(S_t) + b_2 r_{t-1} + b_1 (R_{t-1} - r_{t-1}) + s_2(S_t) u_t , \end{aligned} \tag{11}$$

while the second assumes unit root behaviour in both states:

$$\begin{aligned} \Delta r_t &= g(S_t) + a_1 \Delta r_{t-1} + a_2 (R_{t-1} - r_{t-1}) + s_1(S_t) e_t \\ (R_t - r_t) &= n(S_t) + b_2 \Delta r_{t-1} + b_1 (R_{t-1} - r_{t-1}) + s_2(S_t) u_t . \end{aligned} \tag{12}$$

Following Ang and Bekaert (1998), a lag order of one is selected for all bivariate systems. As in the univariate cases, we estimate models with regime dependent variances (RSH-VAR(1)) as well as restricted specifications with constant variances across regimes (RS-VAR(1)). To obtain parsimony in modeling, the coefficients of the regime switching VAR models are restricted to be independent of regimes. The single-regime benchmark is a linear first-order VAR (VAR(1)). Table 5 contains the estimation results of the basic specification (11).

[Table 5]

All three models are characterized by highly significant autoregressive coefficients a_1 and b_1, indicating strong persistence in both series. In contrast, the estimates of the parameters a_2 and b_2 are insignificantly different from zero. As expected, allowing for regime switching substantially improves the log-likelihood value. A further improvement can be achieved when estimating the RSH-VAR(1) model which captures regime-dependent heteroskedasticity. For both the short rate and the spread, volatility in regime 2 is more than ten times as high as in regime 1. The second regime's covariance cov_2 is more than forty times as high as the one associated with regime one. However, like in the univariate cases, restricting the variance to be constant leads to a better regime classification. Furthermore, three intercept parameters become significant when regime characteristics are exclusively driven by conditional mean dynamics.

[Table 6]

Compared to the random walk, superior forecasts over all horizons are only generated by the RSH-VAR(1) model. This result may imply that, unlike the forecasting performance of univariate models documented in Tables 2 and 4 suggests, state-dependent conditional volatility could be valuable information for predicting the mean. Notwithstanding its impressing predictive ability over long horizons, the RSH-VAR(1) does not better out-of-sample than the univariate RS-AR(1) for the short rate in levels.

[Table 7]

As Table 7 reveals, including first differences of the short rate in the bivariate models has the consequence that all VAR coefficients are significant at the 1% level. The volatility characteristics are the same as reported in Table 5. However, the improvement in out-of-sample forecasting is impressing, particularly when applying the RSH-VAR(1) model. With the exception of the one and the three month horizons, it clearly dominates all other specifications in this study. Over long horizons, its TU decreases to almost 0.25. Note, that the single-regime VAR model is strongly dominated over all horizons, in most cases also by the RS-VAR(1) model.

[Table 8]

Investigating the RSH-VAR(1) model further by looking at the estimated ex-ante probabilities in Figure 4 reveals that there was substantial regime uncertainty at the end of the sample. One can suppose that this uncertainty was associated with the expected course of monetary policy. Because restrictive monetary policy ended not before 1992, after estimation, tentative evidence in favour of a peso-problem is obtained. Figures 5 and 6 show the contribution of the Markov chain to the conditional mean of short rate changes and the spread respectively. The bold lines can be interpreted as conditional one-step in-sample forecasts of the respective series. It becomes obvious, that the model does explain the spread better than the short rate changes.

[Figures 4, 5, 6]

The impressing predictive ability of the RSH-VAR(1) model relative to the linear VAR(1) specification is shown in Figure 7 which compares the series of out-of-sample forecast errors.

[Figure 7]

5. Summary

Despite of being far away from a comprehensive approach of explaining rational expectations formation under the presence of state-dependent stationarity and predicted regime-changes, recent theoretical and empirical research strengthens the case for market-based forecasts using term structure information. Relying on this work, our study attempts to bring back out-of-sample interest rate forecasting to economic science. The regime-switching approach applied in this paper has two interrelated advantages over traditional forecasting techniques. Theoretically, it is able to capture rational expectations of infrequently occurring discrete events. Technically, it allows for time-varying dynamics, particularly time-varying stationarity, in the series under consideration. After discussing both aspects with reference to recent literature, the paper contains estimation results and documents our try to forecast the German money market rate over different horizons.

The main results can be summarized as follows. First, there is strong evidence in favour of regime-switching behaviour in the short-term interest rate. Because the estimated regimes are not found to be highly persistent, one can conclude that expected discrete events do affect the behaviour of market participants in the sample. Consequently, peso-problems should be taken into account when generating interest rate forecasts. Second, empirical results suggest that (non-) stationarity of the short rate is state-dependent. However, the short rate reverts to its mean only slowly in the stationary regime. Third, after a careful examination of different models, some specifications turn out to have superior forecasting power. Especially regime-switching AR(1) models with constant variances outperform the random walk as well as their single regime counterparts. Thus, in order to predict the first moment of the short rate with univariate regime-switching models, one should restrict the variance to be constant across regimes. Fourth, while allowing for mean reversion produces the better long horizon forecasts, an I(1)-specification of the short rate across both regimes performs better for horizons within a year. Finally and most importantly, our forecasting results show that bivariate regime-switching models including the term spread exploit market expectations in the term structure better than single regime term structure models. Within a bivariate framework, however, the short rate should be included in first differences as suggested by Sola and Driffill (1994), and the variance should not be restricted to be independent of states. Because forecast errors can be substantially reduced over all horizons, one can conclude that identification, econometric modeling and extraction of switching regime-expectations should be an indispensable component of all marked-based forecasting strategies.

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Figure 1: Short-term interest rate
1970-1998

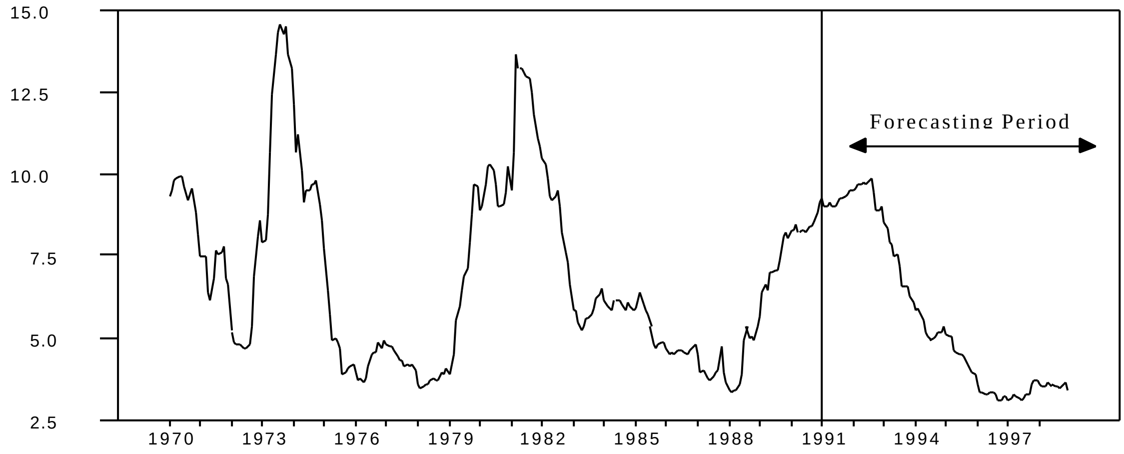

Figure 2: First differences of short-term interest rate
1970-1998

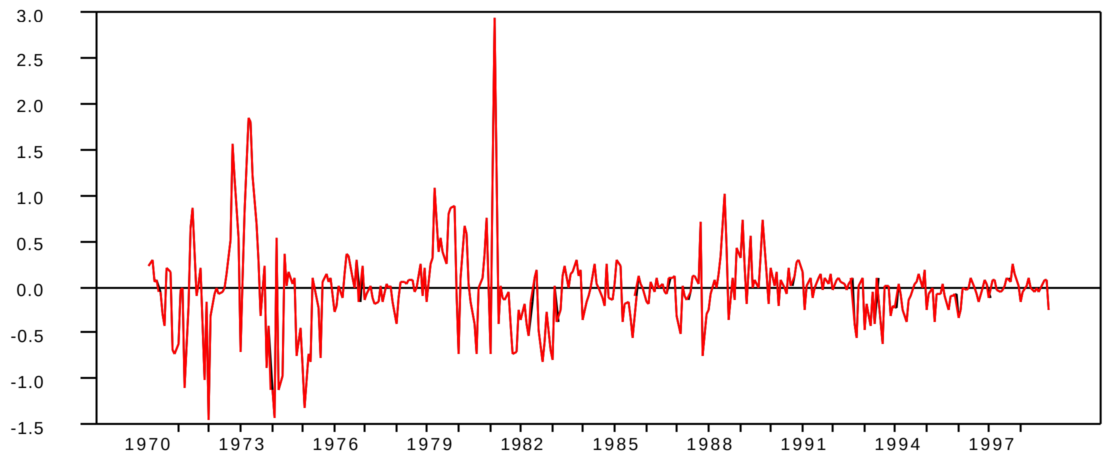

Figure 3: Term spread
1970-1998

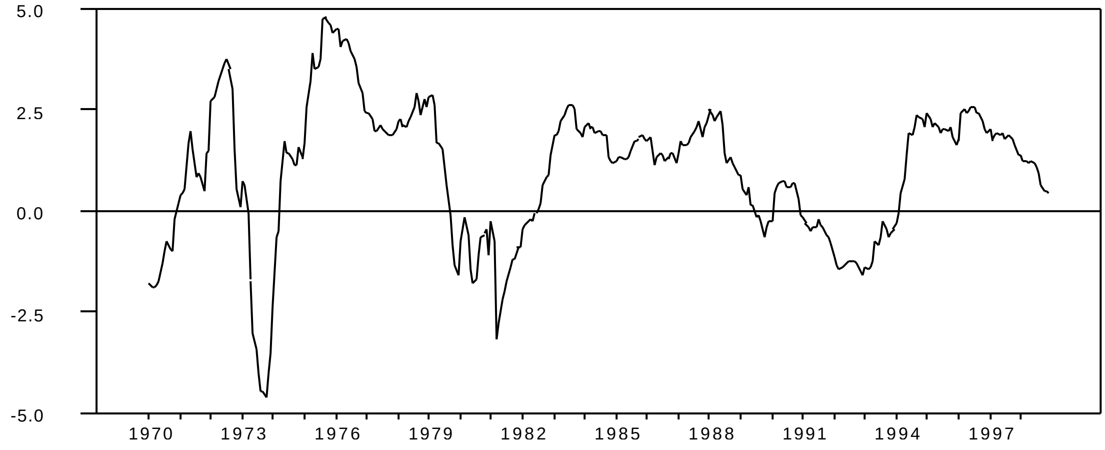

TABLE 1

Univariate AR(1) models for the short rate:

RSH-AR(1): $r_t = g(S_t) + a_1(S_t)\, r_{t-1} + s(S_t)\, e_t$

	AR(1)	*RS-AR(1)*	*RSH-AR(1)*
γ_1	0.12 (1.62)	- 0.53 (1.01)	0.01 (0.08)
a_{11}	0.98*** (75.83)	1.20*** (15.22)	1.00*** (50.73)
γ_2		0.14** (2.23)	0.32 (1.63)
a_{12}		0.97*** (92.54)	0.96** (45.60)
s_1^2	0.26*** (5.54)	0.15*** (7.83)	0.03*** (4.31)
s_2^2			0.52*** (5.36)
P		0.74*** (6.05)	0.91*** (19.48)
Q		0.97*** (83.05)	0.89*** (14.85)
Log-Likelihood	- 182.71	- 144.70	- 118.90
RCM		24.89	60.42

Notes: The sample contains monthly observations from January 1970 to December 1990. The short rate is the three-month money market rate. t-statistics in parentheses are based on heteroskedastic-consistent standard errors. * (**) (***) denotes significance at the 10% (5%) (1%) level.

TABLE 2

Univariate AR(1) models for the short rate:

MAE's and RMSE's for h-step predictions of the short rate

	h=1 (N=96)	*h=3 (N=94)*	*h=6 (N=91)*	*h=9 (N=88)*	*h=12 (N=85)*	*h=24 (N=73)*	*h=36 (N=61)*
Random Walk							
MAE	0.1255	0.2919	0.5354	0.7733	1.0300	2.0293	3.1567
RMSE	0.1806	0.3916	0.7040	1.0116	1.3195	2.4299	3.5025
AR(1)							
MAE	0.1408	0.3456	0.6365	0.8996	1.1533	2.0245	2.9117
RMSE	0.1878	0.4140	0.7386	1.0400	1.3237	2.2169	2.9979
TU	1.0394	1.0573	1.0491	1.0281	1.0032	0.9123	0.8559
RS-AR(1)							
MAE	0.1425	0.3154	0.5534	0.7413	0.9054	1.2686	1.5281
RMSE	0.1796*	0.3789*	0.6513*	0.8846*	1.0788*	1.3803*	1.6349*
TU	0.9943	0.9675	0.9251	0.8744	0.8176	0.5680	0.4668
RSH-AR(1)							
MAE	0.1325	0.3103	0.5718	0.8213	1.0651	2.0430	3.1610
RMSE	0.1848	0.4042	0.7191	1.0258	1.3264	2.3762	3.3627
TU	1.0230	1.0277	1.0214	1.0140	1.0052	0.9779	0.9601

Notes: Forecasts are generated out-of-sample over the period from January 1991 to December 1998. h denotes the forecasting horizon, N is the number of observations. * denotes the lowest RMSE.

TABLE 3

Univariate AR(1) models for the first differences of the short rate:

RSH-AR(1): $\Delta r_t = g(S_t) + a_1(S_t)\Delta r_{t-1} + s(S_t)e_t$

	AR(1)	*RS- AR(1)*	*RSH- AR(1)*
γ_1	- 0.00 (0.03)	- 0.04 (1.53)	0.00 (0.48)
a_{11}	0.44*** (3.89)	0.35*** (3.36)	0.23** (2.52)
γ_2		1.37*** (5.59)	- 0.01 (0.18)
a_{12}		0.49 (1.51)	0.47*** (3.48)
s_1^2	0.21*** (6.16)	0.16*** (10.60)	0.03*** (4.21)
s_2^2			0.40*** (4.46)
P		0.98*** (113.01)	0.93*** (20.36)
Q		0.45** (2.05)	0.92*** (15.87)
Log-Likelihood	- 156.90	- 140.12	- 102.49
RCM		7.84	54.74

Notes: The sample contains monthly observations from January 1970 to December 1990. The short rate is the three-month money market rate. t-statistics in parentheses are based on heteroskedastic-consistent standard errors. * (**) (***) denotes significance at the 10% (5%) (1%) level.

TABLE 4

Univariate AR(1) models for the first differences of the short rate:

MAE's and RMSE's for h-step predictions of the short rate

	h=1 (N=96)	*h=3 (N=94)*	*h=6 (N=91)*	*h=9 (N=88)*	*h=12 (N=85)*	*h=24 (N=73)*	*H=36 (N=61)*
Random Walk							
MAE	0.1255	0.2919	0.5354	0.7733	1.0300	2.0293	3.1567
RMSE	0.1806	0.3916	0.7040	1.0116	1.3195	2.4299	3.5025
AR(1)							
MAE	0.1228	0.2666	0.5004	0.7262	0.9676	1.9520	3.0224
RMSE	0.1712	0.3625	0.6478	0.9465	1.2484	2.3458	3.3883
TU	0.9476	0.9258	0.9202	0.9357	0.9461	0.9654	0.9674
RS-AR(1)							
MAE	0.1216	0.2598	0.4823	0.6864	0.8885	1.5935	2.1891
RMSE	0.1668*	0.3445*	0.6019*	0.8608*	1.1139*	1.9058*	2.5917*
TU	0.9235	0.8798	0.8549	0.8510	0.8442	0.7843	0.7399
RSH-AR(1)							
MAE	0.1223	0.2669	0.5050	0.7347	0.9856	1.9741	3.0669
RMSE	0.1712	0.3640	0.6585	0.9603	1.2649	2.3680	3.4201
TU	0.9478	0.9295	0.9354	0.9493	0.9586	0.9745	0.9765

Notes: Forecasts are generated out-of-sample over the period from January 1991 to December 1998. h denotes the forecasting horizon, N is the number of observations. * denotes the lowest RMSE.

TABLE 5

Bivariate VAR(1) models for the short rate and the spread:

RSH-VAR(1):

$$r_t = g\left(S_t\right) + a_1\, r_{t-1} + a_2\left(R_{t-1} - r_{t-1}\right) + s_1\left(S_t\right) e_t$$

$$\left(R_t - r_t\right) = n\left(S_t\right) + b_2\, r_{t-1} + b_1\left(R_{t-1} - r_{t-1}\right) + s_2\left(S_t\right) u_t$$

	VAR(1)	*RS-VAR(1)*	*RSH-VAR(1)*
γ_1	0.29	0.32*	0.00
	(1.77)	(1.89)	(0.00)
ν_1	- 0.02	- 0.04	0.21**
	(1.74)	(0.32)	(2.33)
γ_2		1.79**	0.16
		(2.55)	(1.01)
ν_2		- 1.20**	0.09
		(2.47)	(0.89)
a_1	0.96***	0.95***	0.99***
	(45.40)	(56.82)	(63.33)
a_2	- 0.03	- 0.02	0.03
	(1.07)	(0.63)	(1.22)
b_1	0.98***	0.97***	0.93***
	(36.64)	(31.01)	(49.72)
b_2	0.01	0.02	- 0.01
	(0.42)	(1.52)	(1.09)
s_{11}^2	0.25***	0.16***	0.04***
	(5.65)	(6.15)	(5.54)
s_{21}^2	0.18***	0.13***	0.03***
	(5.53)	(4.80)	(5.30)
s_{12}^2			0.56***
			(4.22)
s_{22}^2			0.40***
			(4.17)
cv_1	- 0.19***	- 0.12***	- 0.01*
	(4.91)	(4.36)	(1.95)
cv_2			- 0.44***
			(3.97)
P		0.98***	0.93***
		(70.35)	(38.09)
Q		0.65***	0.90***
		(4.64)	(15.02)
Log-Likelihood	76.69	111.10	157.71
RCM		12.57	53.05

Notes: The sample contains monthly observations from January 1970 to December 1990. The short rate is the three-month money market rate, while the term spread variable is calculated as the percentage difference between the interest rate on long-term government bonds ("yield on bonds outstanding issued by residents") and the short rate. t-statistics in parentheses are based on heteroskedastic-consistent standard errors. * (**) (***) denotes significance at the 10% (5%) (1%) level.

TABLE 6

Bivariate VAR(1) models for the short rate and the spread:

MAE's and RMSE's for h-step predictions of the short rate

	h=1 (N=96)	*h=3 (N=94)*	*h=6 (N=91)*	*h=9 (N=88)*	*h=12 (N=85)*	*h=24 (N=73)*	*H=36 (N=61)*
Random Walk							
MAE	0.1255	0.2919	0.5354	0.7733	1.0300	2.0293	3.1567
RMSE	0.1806	0.3916	0.7040	1.0116	1.3195	2.4299	3.5025
VAR(1)							
MAE	0.1587	0.4130	0.7674	1.1008	1.3932	2.3104	3.1115
RMSE	0.2086	0.4862	0.8770	1.2270	1.5423	2.4940	3.2211
TU	1.1548	1.2417	1.2458	1.2129	1.1688	1.0264	0.9196
RS-VAR(1)							
MAE	0.1734	0.4307	0.7565	1.0305	1.2476	1.6925	2.0693
RMSE	0.2087	0.4748	0.8228	1.1039	1.3280	1.7784	2.1074
TU	1.1551	1.2125	1.1687	1.0912	1.0065	0.7319	0.6017
RSH-VAR(1)							
MAE	0.1374	0.2912	0.5058	0.7200	0.9041	1.3092	1.6596
RMSE	0.1723*	0.3586*	0.6181*	0.8584*	1.0727*	1.5017*	1.8739*
TU	0.9538	0.9159	0.8780	0.8486	0.8129	0.6180	0.5350

Notes: Forecasts are generated out-of-sample over the period from January 1991 to December 1998. h denotes the forecasting horizon, N is the number of observations. * denotes the lowest RMSE.

TABLE 7

Bivariate VAR(1) models for the first differences of the short rate and the spread:

RSH-VAR(1):

$$\Delta r_t = g\left(S_t\right) + a_1\, \Delta r_{t-1} + a_2\left(R_{t-1} - r_{t-1}\right) + s_1\left(S_t\right) e_t$$
$$\left(R_t - r_t\right) = n\left(S_t\right) + b_2\, \Delta r_{t-1} + b_1\left(R_{t-1} - r_{t-1}\right) + s_2\left(S_t\right) u_t$$

	VAR(1)	*RS-VAR(1)*	*RSH-VAR(1)*
γ_1	- 0.04	0.06	- 0.10**
	(0.28)	(1.35)	(2.19)
ν_1	0.06*	0.14	0.08***
	(1.74)	(1.10)	(3.30)
γ_2		- 0.10	0.03
		(1.28)	(0.30)
ν_2		0.01	0.02
		(0.19)	(0.40)
a_1	0.46***	0.45***	0.35***
	(4.16)	(4.38)	(3.86)
a_2	0.04**	0.04**	0.04***
	(2.13)	(2.20)	(2.78)
b_1	0.95***	0.95***	0.95***
	(61.72)	(57.54)	(101.50)
b_2	- 0.36***	- 0.37***	- 0.24***
	(4.13)	(4.88)	(3.42)
s_{11}^2	0.20***	0.20***	0.04***
	(5.90)	(5.92)	(2.64)
s_{21}^2	0.15***	0.15***	0.03***
	(5.84)	(5.13)	(4.62)
s_{12}^2			0.44***
			(5.07)
s_{22}^2			0.33***
			(4.77)
cv_1	- 0.15***	- 0.15***	- 0.02
	(5.08)	(5.18)	(1.48)
cv_2			- 0.34***
			(4.62)
P		0.72***	0.94***
		(7.04)	(27.35)
Q		0.86***	0.91***
		(12.43)	(15.28)
Log-Likelihood	102.90	111.59	171.22
RCM		76.82	49.90

Notes: The sample contains monthly observations from January 1970 to December 1990. The short rate is the three-month money market rate, while the term spread variable is calculated as the percentage difference between the interest rate on long-term government bonds ("yield on bonds outstanding issued by residents") and the short rate. t-statistics in parentheses are based on heteroskedastic-consistent standard errors. * (**) (***) denotes significance at the 10% (5%) (1%) level.

TABLE 8

Bivariate VAR(1) models for the first differences of the short rate and the spread:

MAE's and RMSE's for h-step predictions of the short rate

	h=1 (N=96)	*h=3 (N=94)*	*h=6 (N=91)*	*h=9 (N=88)*	*h=12 (N=85)*	*h=24 (N=73)*	*h=36 (N=61)*
Random Walk							
MAE	0.1255	0.2919	0.5354	0.7733	1.0300	2.0293	3.1567
RMSE	0.1806	0.3916	0.7040	1.0116	1.3195	2.4299	3.5025
VAR(1)							
MAE	0,1310	0,2907	0,4978	0,7073	0,8943	1,5198	2,3741
RMSE	0,1700	0,3528	0,6049	0,8510	1,0822	1,7353	2,4378
TU	0,9410	0,9011	0,8592	0,8412	0,8202	0,7141	0,6960
RS-VAR(1)							
MAE	0.1370	0.3131	0.4831	0.6667	0.8178	0.8955	0.9547
RMSE	0.1729	0.3726	0.6134	0.8260	1.0103	1.1347	1.0923
TU	0.9570	0.9514	0.8713	0.8165	0.7656	0.4670	0.3119
RSH-VAR(1)							
MAE	0.1324	0.2892	0.4541	0.6243	0.7539	0.7694	0.8216
RMSE	0.1675*	0.3523*	0.5898*	0.7973*	0.9599*	0.9676*	0.9344*
TU	0.9269	0.8997	0.8378	0.7881	0.7274	0.3982	0.2668

Notes: Forecasts are generated out-of-sample over the period from January 1991 to December 1998. h denotes the forecasting horizon, N is the number of observations. * denotes the lowest RMSE.

Figure 4: Ex-ante probability (Regime 1)
RSH-VAR(1) model for differenced short rate and spread

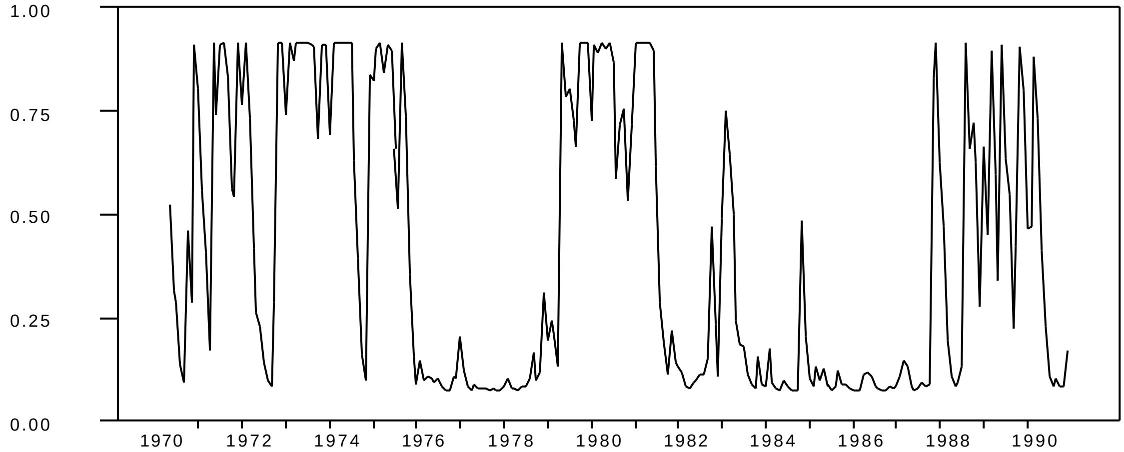

Figure 5: Contribution of the Markov chain to short rate changes
RSH-VAR(1) model for differenced short rate and spread

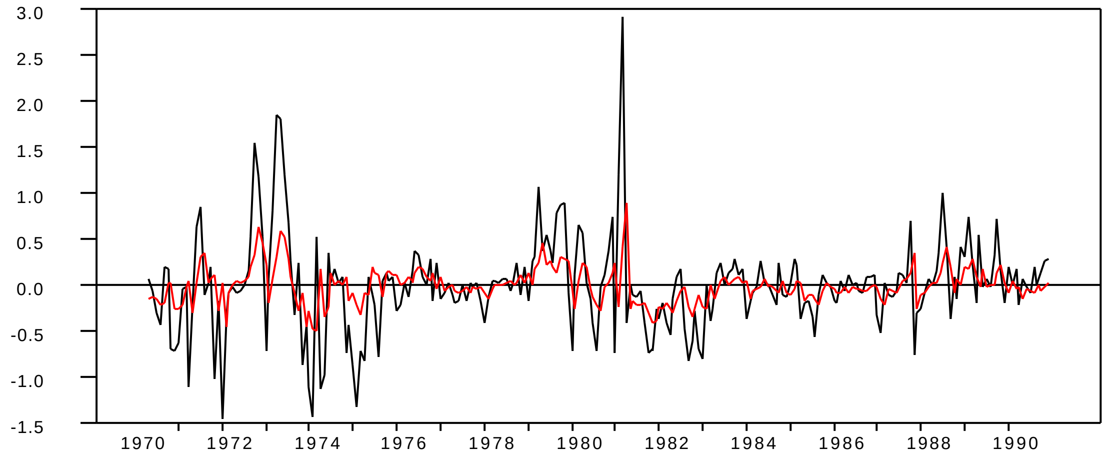

Figure 6: Contribution of the Markov chain to the spread
RSH-VAR(1) model for differenced short rate and spread

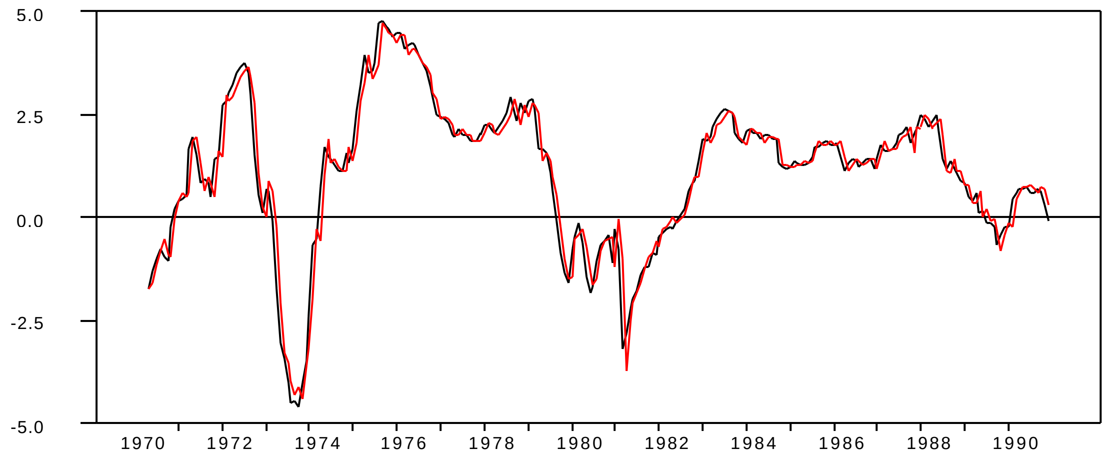

Figure 7: Series of h-step errors of short rate predictions

VAR(1) and RSH-VAR(1) model for differenced short rate and spread