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Realized Beta: Persistence and Predictability*

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Abstract:
A large literature over several decades reveals both extensive concern with the question of time-varying betas and an emerging consensus that betas are in fact time-varying, leading to the prominence of the conditional CAPM. Set against that background, we assess the dynamics in realized betas, vis-à-vis the dynamics in the underlying realized market variance and individual equity covariances with the market. Working in the recently-popularized framework of realized volatility, we are led to a framework of nonlinear fractional cointegration: although realized variances and covariances are very highly persistent and well approximated as fractionally-integrated, realized betas, which are simple nonlinear functions of those realized variances and covariances, are less persistent and arguably best modeled as stationary I(0) processes. We conclude by drawing implications for asset pricing and portfolio management.

JEL Classification: C1, G1

Keywords: quadratic variation and covariation, realized volatility, asset pricing, CAPM, equity betas, long memory, nonlinear fractional cointegration, continuous-time methods.

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1. Introduction

One of the key insights of asset pricing theory is also one of the simplest: only systematic risk should be priced. Perhaps not surprisingly, however, there is disagreement as to the sources of systematic risk. In the one-factor capital asset pricing model (CAPM), for example, systematic risk is determined by covariance with the market (Sharpe, 1963; Lintner, 1965a, b), whereas, in more elaborate pricing models, additional empirical characteristics such as firm size and book-to-market are seen as proxies for another set of systematic risk factors (Fama and French, 1993).[1]

As with most important scientific models, the CAPM has been subject to substantial criticism (e.g., Fama and French, 1992). Nevertheless, to paraphrase Mark Twain, the reports of its death are greatly exaggerated. In fact, the one-factor CAPM remains alive and well at the frontier of both academic research and industry applications, for at least two reasons. First, recent work reveals that it often works well – despite its wrinkles and warts – whether in traditional incarnations (e.g., Ang and Chen, 2003) or more novel variants (e.g., Cohen, Polk and Voulteenaho, 2002; Campbell and Vuolteenaho, 2002). Second, competing multi-factor pricing models, although providing improved statistical fit, involve factors whose economic interpretations in terms of systematic risks remain unclear, and moreover, the stability of empirically-motivated multi-factor asset pricing relationships often appears tenuous when explored with true out-of-sample data, suggesting an element of data mining.[2]

In this paper, then, we study the one-factor CAPM, which remains central to financial economics nearly a half century after its introduction. A key question within this setting is whether stocks' systematic risks, as assessed by their correlations with the market, are constant over time – i.e., whether stocks' market betas are constant. And if betas are not constant, a central issue becomes how to understand and formally characterize their persistence and predictability vis-à-vis their underlying components.

The evolution of a large literature over several decades reveals both extensive concern with this question and, we contend, an eventual implicit consensus that betas *are* likely time-varying.[3] Several pieces of evidence support our contention. First, leading texts echo it. For example, Huang and Litzenberger (1988) assert that "It is unlikely that risk premiums and betas on individual assets are stationary over time" (p. 303). Second, explicitly dynamic betas are often modeled nonstructurally via time-varying parameter regression, in a literature tracing at least to the early "return to normality" model of Rosenberg (1973), as

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[1] The Roll (1977) critique is also relevant. That is, even if we somehow knew what factor(s) should be priced, it is not clear that the factor proxies measured in practice would correspond to the factor required by the theory.

[2] See Keim and Hawawini (1999) for a good discussion of the difficulty of interpreting additional empirically-motivated factors in terms of systematic risk.

[3] There are of course qualifications, notably Ghysels (1998), which we discuss subsequently.

implemented in the CAPM by Schaefer, Brealey, Hodges and Thomas (1975). Third, even in the absence of explicit allowance for time-varying betas, the CAPM is typically estimated using moving estimation windows, usually of five to ten years, presumably to guard against beta variation (e.g., Fama, 1976; Campbell, Lo and MacKinlay, 1997). Fourth, theoretical and empirical inquiries in asset pricing are often undertaken in conditional, as opposed to unconditional, frameworks, the essence of which is to allow for time-varying betas, presumably because doing so is viewed as necessary for realism.

The motivation for the conditional CAPM comes from at least two sources. First, from a theoretical perspective, financial economic considerations suggest that betas may vary with conditioning variables, an idea developed theoretically and empirically in a large literature that includes, among many others, Dybvig and Ross (1985), Hansen and Richard (1987), Ferson, Kandel and Stambaugh (1987), Ferson and Harvey (1991), Jagannathan and Wang (1996), and Wang (2003).[4] Second, from a different and empirical perspective, the financial econometric volatility literature (see Andersen, Bollerslev and Diebold, 2004, for a recent survey) has provided extensive evidence of wide fluctuations and high persistence in asset market conditional variances, and in individual equity conditional covariances with the market. Thus, even from a purely statistical viewpoint, market betas, which are ratios of time-varying conditional covariances and variances, might be expected to display persistent fluctuations, as in Bollerslev, Engle and Wooldridge (1988). In fact, unless some special cancellation occurs – in a way that we formalize – betas would inherit the persistence features that are so vividly present in their constituent components.

Set against this background, we assess the dynamics in betas vis-à-vis the widely documented persistent dynamics in the underlying variance and covariances. We proceed as follows. In section 2 we sketch the framework, both economic and econometric, in which our analysis is couched. In section 3 we present the empirical results with an emphasis on analysis of persistence and predictability. In section 4 we formally assess the uncertainty in our beta estimates. In section 5 we offer summary, conclusions and directions for future research.

2. Theoretical Framework

Our approach has two key components. First, in keeping with the recent move toward nonparametric volatility measurement, we cast our analysis within the framework of realized variances and covariances, or equivalently, empirical quadratic variation and covariation. That is, we do not entertain a null hypothesis of period-by-period constant betas, but instead explicitly allow for continuous evolution in betas. Our "realized betas" are (continuous-record) consistent for realizations of the underlying ratio between the integrated stock

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[4] The idea of conditioning in the CAPM is of course not unrelated to the idea of multi-factor pricing mentioned earlier.

and market return covariance and the integrated market variance.[5] Second, we work in a flexible econometric framework that allows for – without imposing – fractional integration and/or cointegration between the market variance and individual equity covariances with the market.

Realized Quarterly Variances, Covariances, and Betas

We provide estimates of quarterly betas, based on nonparametric realized quarterly market variances and individual equity covariances with the market. The quarterly frequency is appealing from a substantive financial economic perspective, and it also provides a reasonable balance between efficiency and robustness to microstructure noise. Specifically, we produce our quarterly estimates using underlying daily returns, as in Schwert (1989), so that the sampling frequency is quite high relative to the quarterly horizon of interest, yet low enough so that contamination by microstructure noise is not a serious concern for the highly liquid stocks that we study. The daily frequency further allows us to utilize a long sample of data, which is not available for higher return sampling frequencies.

Suppose that the logarithmic $N\times 1$ vector price process, p_t, follows a multivariate continuous-time stochastic volatility diffusion,

$$dp_t \quad = \quad \mu_t \, dt \;\; + \;\; \Omega_t \, dW_t \,, \tag{1}$$

where W_t denotes a standard N-dimensional Brownian motion, and both the process for the $N\times N$ positive definite diffusion matrix, Ω_t, and the N-dimensional instantaneous drift, μ_t, are strictly stationary and jointly independent of the W_t process. For our purposes it is helpful to think of the N'th element of p_t as containing the log price of the market and the i'th element of p_t as containing the log price of the i'th individual stock included in the analysis, so that the corresponding covariance matrix contains both the market variance, say $\sigma^2_{M,t} = \Omega_{(NN),t}$, and the individual equity covariance with the market, $\sigma_{iM,t} = \Omega_{(iN),t}$. Then, conditional on the sample path realization of μ_t and Ω_t, the distribution of the continuously compounded h-period return, $r_{t+h,h} \equiv p_{t+h} - p_t$, is

$$r_{t+h,h} \;\; / \; \sigma\{ \mu_{t+\tau}, \Omega_{t+\tau}\}_{\tau=0}^{h} \quad \sim \quad N\left(\int_0^h \mu_{t+\tau} \, d\tau, \;\; \int_0^h \Omega_{t+\tau} \, d\tau \right) , \tag{2}$$

where $\sigma\{ \mu_{t+\tau}, \Omega_{t+\tau}\}_{\tau=0}^{h}$ denotes the σ-field generated by the sample paths of $\mu_{t+\tau}$ and $\Omega_{t+\tau}$ for $0 \le \tau \le h$. The integrated diffusion matrix $\int_0^h \Omega_{t+\tau} \, d\tau$ therefore provides a natural measure of the true latent h-period

[5] The underlying theory and related empirical strategies are developed in Andersen, Bollerslev, Diebold and Labys (2001, 2003), Andersen, Bollerslev, Diebold and Ebens (2001), and Barndorff-Nielsen and Shephard (2003). Here we sketch only the basics; for a more rigorous treatment in the framework of special semimartingales, see the survey and unification by Andersen, Bollerslev and Diebold (2004).

volatility.[6] The requirement that the innovation process, W_t, is independent of the drift and diffusion processes is rather strict and precludes, for example, the asymmetric relations between return innovations and volatility captured by the so-called leverage or volatility feedback effects. However, from the results in Meddahi (2002), Barndorff-Nielsen and Shephard (2003) and Andersen, Bollerslev and Meddahi (2004), we know that the continuous-record asymptotic distribution theory for the realized covariation continues to provide an excellent approximation for empirical high-frequency realized volatility measures.[7] As such, even if the conditional return distribution result (2) does not apply in full generality, the evidence presented below, based exclusively on the realized volatility measures, remains trustworthy in the presence of asymmetries in the return innovation-volatility relations.

By the theory of quadratic variation, we have that under weak regularity conditions, and regardless of the presence of leverage or volatility feedback effects, that

$$\Sigma_{j=1,\ldots,[h/\Delta]} r_{t+j\cdot\Delta,\Delta} \cdot r'_{t+j\cdot\Delta,\Delta} - \int_0^h \Omega_{t+\tau} d\tau \rightarrow 0, \tag{3}$$

almost surely for all t as the sampling frequency of the returns increases, or $\Delta \rightarrow 0$. Thus, by summing sufficiently finely-sampled high-frequency returns, it is possible to construct ex-post *realized* volatility measures for the integrated latent volatilities that are asymptotically free of measurement error. This contrasts sharply with the common use of the cross-product of the h-period returns, $r_{t+h,h} \cdot r'_{t+h,h}$, as a simple ex-post (co-)variability measure. Although the squared return (innovation) over the forecast horizon provides an unbiased estimate for the integrated volatility, it is an extremely noisy estimator, and predictable variation in the true latent volatility process is typically dwarfed by measurement error. Moreover, for longer horizons any conditional mean dependence will tend to contaminate this variance measure. In contrast, as the sampling frequency is lowered, the impact of the drift term vanishes, thus effectively annihilating the mean.

These assertions remain valid if the underlying continuous time process in equation (1) contains jumps, so long as the price process is a special semimartingale, which will hold if it is arbitrage-free. Of course, in this case the limit of the summation of the high-frequency returns will involve an additional jump component, but the interpretation of the sum as the realized h-period return volatility remains intact.

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[6] This notion of integrated volatility already plays a central role in the stochastic volatility option pricing literature, in which the price of an option typically depends on the distribution of the integrated volatility process for the underlying asset over the life of the option. See, for example, the well-known contribution of Hull and White (1987).

[7] Formal theoretical asymptotic justification for this finding has very recently been provided by Barndorff-Nielsen and Shephard (2004).

Finally, with the realized market variance and realized covariance between the market and the individual stocks in hand, we can readily define and empirically construct the individual equity "realized betas." Toward that end, we introduce some formal notation. Using an initial subscript to indicate the corresponding element of a vector, we denote the realized market volatility by

$$\hat{v}^2_{M,t,t+h} \quad = \quad \Sigma_{j=1,\ldots,[h/\Delta]} \; r^2_{(N),t+j\cdot\Delta,\Delta} \,, \tag{4}$$

and we denote the realized covariance between the market and the *i*th individual stock return by

$$\hat{v}_{iM,t,t+h} \quad = \quad \Sigma_{j=1,\ldots,[h/\Delta]} \; r_{(i),t+j\cdot\Delta,\Delta} \cdot r_{(N),t+j\cdot\Delta,\Delta} \,. \tag{5}$$

We then define the associated realized beta as

$$\hat{\beta}_{i,t,t+h} \quad = \quad \frac{\hat{v}_{iM,t,t+h}}{\hat{v}^2_{M,t,t+h}} \,. \tag{6}$$

Under the assumptions invoked for equation (1), this realized beta measure is consistent for the true underlying integrated beta in the following sense:

$$\hat{\beta}_{i,t,t+h} \quad \rightarrow \quad \beta_{i,t,t+h} \quad = \quad \frac{\int_0^h \Omega_{(iN),t+\tau}\, d\tau}{\int_0^h \Omega_{(NN),t+\tau}\, d\tau} \,, \tag{7}$$

almost surely for all *t* as the sampling frequency increases, or $\Delta \rightarrow 0$.

A number of comments are in order. First, the integrated return covariance matrix, $\int_0^h \Omega_{t+\tau} d\tau$, is treated as stochastic, so both the integrated market variance and the integrated covariances of individual equity returns with the market over $[t, t+h]$ are ex-ante, as of time *t*, unobserved and governed by a non-degenerate (and potentially unknown) distribution. Moreover, the covariance matrix will generally vary continuously and randomly over the entire interval, so the integrated covariance matrix should be interpreted as the average realized covariation among the return series. Second, equation (3) makes it clear that the realized market volatility in (4) and the realized covariance in (5) are continuous-record consistent estimators of the (random) realizations of the underlying integrated market volatility and covariance. Thus, as a corollary, the realized beta will be consistent for the integrated beta, as stated in (7). Third, the general representation here encompasses the standard assumption of a constant beta over the measurement or estimation horizon, which is attained for the degenerate case of the Ω_t process being constant throughout each successive h-period measurement interval, or $\Omega_t = \Omega$. Fourth, the realized beta estimation procedure in equations (4)-(6) is implemented through a simple regression (without a constant term) of individual high-frequency stock returns on the corresponding market return. Nonetheless, the interpretation is very different from a standard regression, as the OLS point estimate now represents a consistent estimator of the ex-post realized regression coefficient obtained as the ratio of unbiased estimators of the average realized covariance and the realized market variance. The associated continuous-record asymptotic theory developed by Barndorff-Nielsen and Shephard (2003) explicitly recognizes the diffusion setting underlying this regression interpretation and hence facilitates the construction of standard errors for our beta estimators.

Nonlinear Fractional Cointegration: A Common Long-Memory Feature in Variances and Covariances

The possibility of common persistent components is widely recognized in modern multivariate time-series econometrics. It is also important for our analysis, because there may be common persistence features in the underlying variances and covariances from which betas are produced.

The idea of a common feature is a simple generalization of the well-known cointegration concept. If two variables are integrated but there exists a function f of them that is not, we say that they are cointegrated, and we call f the conintegrating function. More generally, if two variables have property X but there exists a function of them that does not, we say that they have common feature X. A key situation is when X corresponds to *persistence*, in which case we call the function of the two variables that eliminates the persistence the *copersistence function*. It will prove useful to consider linear and nonlinear copersistence functions in turn.

Most literature focuses on linear copersistence functions. The huge cointegration literature pioneered by Granger (1981) and Engle and Granger (1987) deals primarily with linear common long-memory $I(1)$ persistence features. The smaller copersistence literature started by Engle and Kozicki (1993) deals mostly with linear common short-memory $I(0)$ persistence features. The idea of fractional cointegration, suggested by Engle and Granger (1987) and developed by Cheung and Lai (1993) and Robinson and Marinucci, (2001), among others, deals with linear common long-memory $I(d)$ persistence features, $0<d<1/2$.

Our interest is closely related but different. First, it centers on *nonlinear* copersistence functions, because betas are ratios. There is little literature on nonlinear common persistence features, although they are implicitly treated in Granger (1995). We will be interested in nonlinear common long-memory $I(d)$

persistence features, $0<d<1/2$, effectively corresponding to nonlinear fractional cointegration.[8]

Second, we are interested primarily in the case of *known* cointegrating relationships. That is, we may not know whether a given stock's covariance with the market is fractionally cointegrated with the market variance, but if it is, then there is a good financial economic reason (i.e., the CAPM) to suspect that the cointegrating function is the *ratio* of the covariance to the variance. This provides great simplification. In the integer-cointegration framework with known cointegrating vector under the alternative, for example, one could simply test the cointegrating combination for a unit root, or test the significance of the error-correction term in a complete error-correction model, as in Horvath and Watson (1995). We proceed in analogous fashion, examining the integration status (generalized to allow for fractional integration) of the realized market variance, realized individual equity covariances with the market, and realized market betas.

Our realized beta series are unfortunately relatively short compared to the length required for formal testing and inference procedures regarding (fractional) cointegration, as the fractional integration and cointegration estimators proposed by Geweke-Porter Hudak (1983), Robinson and Marinucci (2001) and Andrews and Guggenberger (2003) tend to behave quite erratically in small samples. In addition, there is considerable measurement noise in the individual beta series so that influential outliers may have a detrimental impact on our ability to discern the underlying dynamics. Hence we study the nature of the long range dependence and short-run dynamics in the realized volatility measures and realized betas through intentionally less formal but arguably more informative graphical means, and via some robust procedures that utilize the joint information across many series, to which we now turn.

3. Empirical Analysis

We examine primarily the realized quarterly betas constructed from daily returns. We focus on the dynamic properties of market betas vis-à-vis the dynamic properties of their underlying covariance and variance components. We quantify the dynamics in a number of ways, including explicit measurement of the degree of predictability in the tradition of Granger and Newbold (1986).

Dynamics of Quarterly Realized Variance, Covariances and Betas

This section investigates the realized quarterly betas constructed from daily returns obtained from the Center for Research in Security Prices from July 1962 to September 1999. We take the market return $r_{m,t}$ to be the thirty Dow Jones Industrial Average (DJIA), and we study the subset of twenty-five DJIA stocks as of

[8] One could of course attempt a linear cointegration approach by taking logs of the realized volatilities and covariances, but there is no theoretical reason to expect all covariances to be positive, and our realized covariance measures are indeed sometimes negative, making logarithmic transformations problematic.

March 1997 with complete data from July 2, 1962 to September 17, 1999, as detailed in Table 1. We then construct quarterly realized DJIA variances, individual equity covariances with the market, and betas, 1962:3 - 1999:3 (149 observations).

In Figure 1 we provide a time series plot of the quarterly realized market variance, with fall 1987 included (top panel) and excluded (bottom panel). It is clear that the realized variance is quite persistent and, moreover, that the fall 1987 volatility shock is unlike any other ever recorded, in that volatility reverts to its mean almost instantaneously. In addition, our subsequent computation of asymptotic standard errors reveals that the uncertainty associated with the fall 1987 beta estimate is enormous, to the point of rendering it entirely uninformative. In sum, it is an exceptional outlier with potentially large influence on the analysis, and it is measured with huge imprecision. Hence, following many other authors, we drop the fall 1987 observation from this point onward.

In Figures 2 and 3 we display time series plots of the twenty-five quarterly realized covariances and realized betas.[9] Like the realized variance, the realized covariances appear highly persistent. The realized betas, in contrast, appear noticeably less persistent. This impression is confirmed by the statistics presented in Table 2: the mean Ljung-Box Q-statistic (through displacement 12) is 84 for the realized covariance, but only 47 for the realized beta.[10]

The impression of reduced persistence in realized betas relative to realized covariances is also confirmed by the sample autocorrelation functions for the realized market variance, the realized covariances with the market, and the realized betas shown in Figure 4.[11] Most remarkable is the close correspondence between the shape of the realized market variance correlogram and the realized covariance correlograms. This reflects an extraordinary high degree of dependence in the correlograms across the individual realized covariances with the market, as shown in Figure 5. In Figure 4, it makes the median covariance correlogram appear as a very slightly dampened version of that for the market variance. This contrasts sharply with the lower and gently declining pattern for the realized beta autocorrelations. Intuitively, movements of the realized market variance are largely reflected in movements of the realized covariances; as such, they largely "cancel" when we form ratios (realized betas). Consequently, the correlation structure across the individual realized beta series in Figure 6 is much more dispersed than is the case for the realized covariances in Figure 5. This results in an effective averaging of the noise and the point estimates of the median correlation values

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[9] We compute the quarterly realized variance, covariances and betas from slightly different numbers of observations due to the different numbers of trading days across the quarters.

[10] Note also that the Dickey-Fuller statistics indicate that unit roots are not present in the market variance, individual equity covariances with the market, or market betas, despite their persistent dynamics.

[11] For the realized covariances and realized betas, we show the median autocorrelations functions.

are effectively zero beyond ten quarters for the beta series.[12]

The work of Andersen, Bollerslev, Diebold and Ebens (2001) and Andersen, Bollerslev, Diebold and Labys (2003), as well as that of many other authors, indicates that asset return volatilities are well-described by a pure fractional noise process, typically with the degree of integration around $d \approx .4$.[13] That style of analysis is mostly conducted on high-frequency data. Very little work has been done on long memory in equity variances, market covariances, and market betas at the quarterly frequency, and it is hard to squeeze accurate information about d directly from the fairly limited quarterly sample. It is well-known, however, that if a flow variable is *I(d),* then it remains *I(d)* under temporal aggregation. Hence, we can use the results of analyses of high-frequency data, such as Andersen, Bollerslev, Diebold and Labys (2003), to help us analyze the quarterly data. Inspired by their results we settle on the point estimate of $d = .42$.

In Figure 7 we graph the sample autocorrelations of the quarterly realized market variance, the median realized covariances with the market, and the median realized betas, all prefiltered by $(1-L)^{.42}$. It is evident that the dynamics in the realized variance and covariances are effectively annihilated by filtering with $(1-L)^{.42}$, indicating that the pure fractional noise process with $d = .42$ is indeed a good approximation to their dynamics. Interestingly, however, filtering the realized *betas* with $(1-L)^{.42}$ appears to produce *overdifferencing*, as evidenced by the fact that the first autocorrelation of the fractionally differenced betas is often negative. Compare, in particular, the median sample autocorrelation function for the prefiltered realized covariances to the median sample autocorrelation function for the prefiltered realized betas. The difference is striking in the sense that the first autocorrelation coefficient for the betas is negative and much larger than those for all of the subsequent lags. Recall that the standard error band for the median realized beta (not shown in the lower panels, as it depends on the unknown cross-sectional dependence structure) should be considerably narrower than for the other series in Figure 7, thus likely rendering the first-order correlation coefficient for the beta series significantly negative. This finding can be seen to be reasonably consistent across the individual prefiltered covariance and beta correlation functions displayed in Figures 8 and 9.

If fractional differencing of the realized betas by $(1-L)^{.42}$ may be "too much," then the question

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[12] The standard error band (under the null of an i.i.d. series) indicated in Figure 4 is only valid for the realized market variance. It should be lower for the two other series, reflecting the effective averaging in constructing the median values. In fact, it should be considerably lower for the beta series due to the near uncorrelated nature of the underlying beta dynamics, while the appropriate reduction for the covariance series would be less because of the strong correlation across the series. We cannot be more precise on this point without imposing some direct assumptions on the correlation structure across the individual series.

[13] A partial list of references not written by the present authors includes Breidt and de Lima (1998), Comte and Renault (1998), Harvey (1998), and Robinson (2001), as well as many of the earlier papers cited in Baillie (1996).

naturally arises as to how much differencing is "just right." Some experimentation revealed that differencing the betas by $(1-L)^{.20}$ was often adequate for eliminating the dynamics. However, for short samples it is almost impossible to distinguish low-order fractional integration from persistent but strictly stationary dynamics. We are particularly interested in the latter alternative where the realized betas are $I(0)$. To explore this possibility, we fit simple $AR(p)$ processes to realized betas, with p selected by the AIC. We show the estimated roots in Table 3, all of which are indicative of covariance stationarity. In Figure 10 we show the sample autocorrelation functions of quarterly realized betas prefiltered by the estimated $AR(p)$ lag-operator polynomials. The autocorrelation functions are indistinguishable from those of white noise.

Taken as a whole, the results suggest that realized betas are integrated of noticeably lower order than are the market variance and the individual equity covariances with the market, corresponding to a situation of nonlinear fractional cointegration. $I(d)$ behavior, with $d \in [0,\ .25]$, appears accurate for betas, whereas the market variance and the individual equity covariances with the market are better approximated as $I(d)$ with $d \in [.35,\ .45]$. Indeed, there is little evidence against an assertion that betas are $I(0)$, whereas there is strong evidence against such an assertion for the variance and covariance components.

Predictability

Examination of the *predictability* of realized beta and its components provides a complementary perspective and additional insight. Granger and Newbold (1986) propose a measure of the predictability of covariance stationary series under squared-error loss, patterned after the familiar regression R^2,

$$G(j) = \frac{var(\hat{x}_{t+j,t})}{var(x_t)} = 1 - \frac{var(e_{t+j,t})}{var(x_t)}, \tag{8}$$

where j is the forecast horizon of interest, $\hat{x}_{t+j,t}$ is the optimal (i.e., conditional mean) forecast, and $e_{t+j,t} = x_{t+j} - \hat{x}_{t+j,t}$. Diebold and Kilian (2001) define a generalized measure of predictability, building on the Granger-Newbold measure, as

$$P(L,\ \Omega,\ j,\ k) = 1 - \frac{E(L(e_{t+j,t}))}{E(L(e_{t+k,t}))}, \tag{9}$$

where L denotes the relevant loss function, Ω is the available univariate or multivariate information set, j is the forecast horizon of interest, and k is a long but not necessarily infinite reference horizon.

Regardless of the details, the basic idea of predictability measurement is simply to compare the expected loss of a short-horizon forecast to the expected loss of a very long-horizon forecast. The former will be much smaller than the latter if the series is highly predictable, as the available conditioning information will then be very valuable. The Granger-Newbold measure, which is the canonical case of the Diebold-Kilian measure (corresponding to $L(e) = e^2$, univariate Ω, and $k = \infty$) compares the 1-step-ahead forecast error variance to that of the ∞-step-ahead forecast error variance, i.e., the unconditional variance of the series being forecast (assuming that it is finite).

In what follows, we use predictability measures to provide additional insight into the comparative dynamics of the realized variances and covariances versus the realized betas. Given the strong evidence of fractional integration in the realized market variance and covariances, we maintain the pure fractional noise process for the quarterly realized market variance and the realized covariances, namely *ARFIMA*(0, .42, 0). We then calculate the Granger-Newbold predictability $G(j)$ analytically, conditional upon the *ARFIMA*(0, .42, 0) dynamics, and we graph it in Figure 11 for j = 1, ... , 7 quarters.[14] The graph starts out as high as .4 and decays only slowly over the first seven quarters. If the realized beta likewise follows a pure fractional noise process but with a smaller degree of integration, say *ARFIMA*(0, .20, 0), which we argued was plausible, then the implied predictability is much lower, as also shown in Figure 11. As we also argued, however, the integration status of the realized betas is difficult to determine. Hence, for the realized betas we also compute Granger-Newbold predictability using an estimated *AR*(p) sieve approximation to produce estimates of $var(e_{t+j,t})$ and $var(x_t)$; this approach is valid regardless of whether the true dynamics are short-memory or long-memory. In Figure 12 we plot the beta predictabilities, which remain noticeably smaller and more quickly-decaying than the covariance predictabilities, as is further clarified by comparing the median beta predictability, also included in Figure 11, to the market variance and equity covariances predictability. It is noteworthy that the shorter-run beta predictability – up to about four quarters – implied by the *AR(p)* dynamics is considerably higher than for the *I(.20)* dynamics. Due to the long-memory feature of the *I(.20)* process this eventually reverses beyond five quarters.

4. Assessing Precision: Interval Estimates of Betas

Thus far we have largely abstracted from the presence of estimation error in the realized betas. It is possible to assess the (time-varying) estimation error directly using formal continuous-record asymptotics.

Continuous-Record Asymptotic Standard Errors

We first use the multivariate asymptotic theory recently developed by Barndorff-Nielsen and Shephard (2003) to assess the precision of our realized betas which are, of course, estimates of the underlying integrated betas. This helps us in thinking about separating "news from noise" when examining temporal

[14] Note that only one figure is needed, despite the many different realized covariances, because all $G(j)$ are identical, as all processes are assumed to be *ARFIMA*(0, .42, 0).

movements in the series.

From the discussion above, realized beta for stock i in quarter t is simply

$$\hat{\beta}_{it} = \frac{\sum_{j=1}^{N_t} r_{ijt} r_{mjt}}{\sum_{j=1}^{N_t} r_{mjt}^2}, \tag{10}$$

where r_{ijt} is the return of stock i on day j of quarter t, r_{mjt} is the return of the DJIA on day j of quarter t, and N_t is the number of units (e.g., days) into which quarter t is partitioned.[15] Under appropriate regularity conditions that allow for non-stationarity in the series, Barndorff-Nielsen and Shephard (2003) derive the limiting distribution of realized beta. In particular, as $N \rightarrow \infty$,

$$\frac{\hat{\beta}_{it} - \beta_{it}}{\sqrt{\left(\sum_{j=1}^{N_t} r_{mjt}^2\right)^{-2} \hat{g}_{it}}} \Rightarrow N(0, 1) , \tag{11}$$

where

$$\hat{g}_{it} = \sum_{j=1}^{N_t} a_{ij}^2 - \sum_{j=1}^{N_t - 1} a_{ij} a_{i\,j+1} \tag{12}$$

and

$$a_{ij} = r_{ijt} r_{mjt} - \hat{\beta}_{it} r_{mjt}^2 . \tag{13}$$

Thus a feasible and asymptotically valid α-percent confidence interval for the underlying integrated beta is

$$\beta_{it} \in \hat{\beta}_{it} \pm z_{\alpha/2} \sqrt{\left(\sum_{j=1}^{N_t} r_{mjt}^2\right)^{-2} \hat{g}_{it}} , \tag{14}$$

where $z_{\alpha/2}$ denotes the appropriate critical value of the standard normal distribution.

In Figure 13 we plot the pointwise ninety-five percent confidence intervals for the quarterly betas. They are quite wide, indicating that daily sampling is not adequate to drive out all measurement error. They

[15] N has a time subscript because the number of trading days varies slightly across quarters.

are, given the width of the bands, moreover, consistent with the conjecture that there is only limited (short range) dependence in the realized beta series. In addition, notice the extremely wide bands associated with the outliers. Hence, potentially influential outliers are generally quite imprecisely estimated and the impact of such incidents on the analysis should be monitored closely.

The continuous record asymptotics discussed above directly points to the advantage of using finer sampled data for improved beta measurements. However, the advent of reliable high-frequency intraday data is, unfortunately, a relatively recent phenomenon and we do not have access to such data for the full 1962:3 - 1999:3 sample period used in the empirical analysis so far. Nonetheless, to see how the reduction in measurement error afforded by the use of finer sample intradaily data manifests itself empirically in more reliable inference, we reproduce in Figure 14 the pointwise ninety-five percent confidence bands for the quarterly betas over the shorter 1993:1 - 1999:3 sample. These bands may be compared directly to the corresponding quarterly realized beta standard error bands over the identical time span based on a fifteen-minute sampling scheme reported in Figure 15.[16] The improvement is readily visible in the narrowing of the bands. It is also evident from Figure 15 that there is quite pronounced positive dependence in the realized quarterly beta measures. In other words, the high-frequency beta measures importantly complement the results for the betas obtained from the lower frequency daily data, by more clearly highlighting the dynamic evolution of individual security betas. In the appendix to this paper we perform a preliminary analysis of realized betas computed from high-frequency data over the shorter seven-year sample period. The results are generally supportive of the findings reported here, but the relatively short sample available for the analysis invariably limits the power of our tests for fractional integration and non-linear cointegration. In the concluding remarks to this paper we also sketch a new and powerful econometric framework that we plan to pursue in future, much more extensive, work using underlying high-frequency data.

<u>HAC Asymptotic Standard Errors</u>

As noted previously, the quarterly realized betas are just regression coefficients computed quarter-by-quarter from CAPM regressions using intra-quarter daily data. One could obtain consistent estimates of the standard errors of those quarterly regression-based betas using HAC approaches, such as Newey-West, under the *very stringent* auxiliary assumption that the period-by-period betas are constant. For comparison to the continuous-record asymptotic bands discussed above, we also compute these HAC standard error bands.

In Figure 16 we provide the Newey-West ninety-five percent confidence intervals for the quarterly realized betas. Comparing the figure to Figure 13, there is not much difference in the assessment of the

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[16] The high-frequency tick-by-tick data underlying the fifteen-minute returns was obtain from the TAQ (Trade And Quotation) database. We refer to the appendix to this paper, available at www.ssc.upenn.edu/~fdiebold, for a more detailed description of the data capture, return construction, and high-frequency beta measurements.

estimation uncertainly inherent in the quarterly beta measures obtained from the two alternative procedures based on daily data. However, as noted above, there are likely important gains to be had from moving to high-frequency intraday data.

5. Summary, Concluding Remarks, and Directions for Future Research

We have assessed the dynamics and predictability in realized betas, relative to the dynamics in the underlying market variance and covariances with the market. Key virtues of the approach include the fact that it does not require an assumed volatility model, and that it does not require an assumed model of time variation in beta. We find that, although the realized variances and covariances fluctuate widely and are highly persistent and predictable (as is well-known), the realized betas, which are simple nonlinear functions of the realized variances and covariances, display much less persistence and predictability.

The empirical literature on systematic risk measures, as captured by beta, is much too large to be discussed in a sensible fashion here. Before closing, however, we do want to relate our approach and results to the literature on latent factor models and two key earlier papers that have important implications for the potential time variation of betas and the further use of the techniques developed here.

First, our results are closely linked to the literature on the latent factor volatility model, as studied by a number of authors, including Diebold and Nerlove (1989), Harvey, Ruiz and Shephard (1994), King, Sentana and Wadhwani (1994), Fiorentini, Sentana and Shephard (1998), and Jacquier and Marcus (2000). Specifically, consider the model,

$$r_{it} = \beta_i f_t + v_{it}, \quad f_t \,|\, I_t \sim (0, h_t) \tag{15a}$$

$$v_{it} \overset{iid}{\sim} (0, \omega_i^2), \qquad cov(v_{it} v_{jt'}) = 0, \ \forall\, i \neq j,\ t \neq t', \tag{15b}$$

where $i, j = 1, ..., N$, and $t = 1, ..., T$. The i^{th} and j^{th} time-t conditional (on h_t) variances, and the ij-th conditional covariance, for arbitrary i and j, are then given by

$$h_{it} = \beta_i^2\, h_t + \omega_i^2, \qquad h_{jt} = \beta_j^2\, h_t + \omega_j^2, \qquad cov_{ijt} = \beta_i \beta_j\, h_t. \tag{16}$$

Assume, as is realistic in financial contexts, that all betas are nonnegative, and consider what happens as h_t increases, say: all conditional variances increase, and all pairwise conditional covariances increase. Hence the market variance increases, and the covariances of individual equities with the market increase. Two observations are immediate: (1) both the market variance and the covariances of individual equities with the market are time-varying, and (2) because the market variance moves together with the covariances of individual equities with the market, the market betas may not vary as much – indeed in the simple one-factor case sketched here, the betas are constant, by construction! The upshot is that wide fluctuations in the market variance and individual equity covariances with the market, yet no variation in betas, is precisely what one expects to see in a latent (single) factor volatility model. It is also, of course, quite similar to what we found in the data: wide variation and persistence in market variance and individual equity covariances with the market, yet less variation and persistence in betas. Notice also the remarkable similarity in the correlograms for the individual realized covariances in Figure 5. This is another indication of a strong coherence in the dynamic evolution of the individual covariances, consistent with the presence of one dominant underlying factor.

Second, our results also complement and expand upon those of Braun, Nelson and Sunier (1995), who study the discrepancy in the time series behavior of betas relative to the underlying variances and covariances for twelve industry portfolios using bivariate EGARCH models. They also find variation and persistence in the conditional variances and covariances, and less variation and persistence in betas. Moreover, they find the strong asymmetric relationship between return innovations and future return volatility to be entirely absent in the conditional betas.[17] Hence, at the portfolio level they document similar qualitative behavior between the variances and covariances relative to the betas as do we. However, their analysis is linked directly to a specific parametric representation, it studies industry portfolios, and it never contemplates the hypothesis that the constituent components of beta – variances and covariances – may be of a long memory form. This latter point has, of course, been forcefully argued by numerous subsequent studies. Consequently, our investigation can be seen as a substantive extension of their findings performed in a fully nonparametric fashion.

Third, our results nicely complement and expand upon those of Ghysels (1998), who argues that the constant beta CAPM, as bad as it may be, is nevertheless not as bad as some popular conditional CAPMs. We provide some insight into why allowing for time-varying betas may do more harm than good when estimated from daily data, even if the true underlying betas display significant short memory dynamics: it may not be possible to estimate reliably the persistence or predictability in individual realized betas, so good in-sample fits may be spurious artifacts of data mining.[18] We also establish that there should be a real potential for the use of high-frequency intraday data to resolve this dilemma.

In closing, therefore, let us sketch an interesting framework for future research using high-frequency

[17] In contrast, on estimating a similar EGARCH model for individual daily stock returns, Cho and Engle (2000) find that daily company specific betas *do* respond asymmetrically to good and bad news.

[18] Chang and Weiss (1991) argue that a strictly stationary ARMA(1,1) model provides an adequate representation for most individual quarterly beta series. Their sample size is smaller than ours, however, and their estimated betas are assumed to be constant over each quarter. Also, they do not provide any separate consideration of the persistence of the market variance or the individual covariances with the market.

intraday data, which will hopefully deliver superior estimates of integrated volatilities by directly exploiting insights from the continuous-record asymptotics of Barndorff-Nielsen and Shephard (2003). Consider the simple state-space representation:

$$\hat{\beta}_{i,t} = \beta_{i,t} + u_{i,t} \tag{17a}$$

$$\beta_{i,t} = a_0 + a_1 \beta_{i,t-1} + v_{i,t} \tag{17b}$$

$$u_{i,t} \sim N\left(0, \left(\sum_{j=1}^{N_t} r_{mjt}^2\right)^{-2} \hat{g}_{it}\right), \qquad v_{i,t} \sim N(0, \sigma^2_{v,i,t}). \tag{17c}$$

The measurement equation (17a) links the observed realized beta to the unobserved true underlying integrated beta by explicitly introducing a normally-distributed error with the asymptotically valid variance obtained from the continuous-record distribution of Barndorff-Nielsen and Shephard (2003). The transition equation (17b) is a standard first-order autoregression with potentially time-varying error variance.[19] The simplest approach would be to let $v_{i,t}$ have a constant variance, but it is also straightforward to let the variance change with the underlying variability in the realized beta measure, so that the beta innovations become more volatile as the constituent parts, the market variance and the covariance of the stock return with the market, increase. This approach directly utilizes the advantages of high-frequency intraday beta measurements by incorporating estimates of the measurement errors to alleviate the errors-in-variables problem while explicitly recognizing the heteroskedasticity in the realized beta series. We look forward to future research along these lines.

[19] Generalization to an arbitrary ARMA process or other stationary structures for the evolution in the true betas is, of course, straightforward.

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Table1
The Dow Jones Thirty

Company Name	Ticker	Data Range
Alcoa Inc.	AA	07/02/1962-09/17/1999
Allied Capital Corporation	ALD	07/02/1962-09/17/1999
American Express Co.	AXP*	05/31/1977-09/17/1999
Boeing Co.	BA	07/02/1962-09/17/1999
Caterpillar Inc.	CAT	07/02/1962-09/17/1999
Chevron Corp.	CHV	07/02/1962-09/17/1999
DuPont Co.	DD	07/02/1962-09/17/1999
Walt Disney Co.	DIS	07/02/1962-09/17/1999
Eastman Kodak Co.	EK	07/02/1962-09/17/1999
General Electric Co.	GE	07/02/1962-09/17/1999
General Motors Corp.	GM	07/02/1962-09/17/1999
Goodyear Tire & Rubber Co.	GT	07/02/1962-09/17/1999
Hewlett- Packard Co.	HWP	07/02/1962-09/17/1999
International Business Machines Corp.	IBM	07/02/1962-09/17/1999
International Paper Co.	IP	07/02/1962-09/17/1999
Johnson & Johnson	JNJ	07/02/1962-09/17/1999
JP Morgan Chase & Co.	JPM*	03/05/1969-09/17/1999
Coca-Cola Co.	KO	07/02/1962-09/17/1999
McDonald's Corp.	MCD*	07/05/1966-09/17/1999
Minnesota Mining & Manufacturing Co.	MMM	07/02/1962-09/17/1999
Philip Morris Co.	MO	07/02/1962-09/17/1999
Merck & Co.	MRK	07/02/1962-09/17/1999
Procter & Gamble Co.	PG	07/02/1962-09/17/1999
Sears, Roebuck and Co.	S	07/02/1962-09/17/1999
AT&T Corp.	T	07/02/1962-09/17/1999
Travelers Group Inc.	TRV*	10/29/1986-09/17/1999
Union Carbide Corp.	UK	07/02/1962-09/17/1999
United Technologies Corp.	UTX	07/02/1962-09/17/1999
Wal-Mart Stores Inc.	WMT*	11/20/1972-09/17/1999
Exxon Corp.	XON	07/02/1962-09/17/1999

Notes: The table summarizes company names and tickers, and the range of the data examined. We use the Dow Jones Thirty as of March 1997. Tickers with asterisks denote stocks with incomplete data, which we exclude from the analysis.

Table 2
The Dynamics of Quarterly Realized Market Variance, Covariances and Betas

	Q	ADF^1	ADF^2	ADF^3	ADF^4
v_{mt}^2	109.50	-5.159	-3.792	-4.014	-3.428

	$cov(r_{mt}, r_{it})$					β_{it}				
	Q	ADF^1	ADF^2	ADF^3	ADF^4	Q	ADF^1	ADF^2	ADF^3	ADF^4
Min.	47.765	-6.188	-4.651	-4.621	-4.023	6.6340	-8.658	-6.750	-5.482	-5.252
0.10	58.095	-5.880	-4.383	-4.469	-3.834	15.026	-7.445	-6.419	-5.426	-4.877
0.25	69.948	-5.692	-4.239	-4.352	-3.742	26.267	-6.425	-5.576	-5.047	-4.294
0.50	84.190	-5.478	-4.078	-4.179	-3.631	46.593	-6.124	-5.026	-3.896	-3.728
0.75	100.19	-5.235	-3.979	-4.003	-3.438	66.842	-5.431	-4.188	-3.724	-3.313
0.90	119.28	-4.915	-3.777	-3.738	-3.253	106.67	-4.701	-3.404	-3.225	-2.980
Max.	150.96	-4.499	-3.356	-3.690	-2.986	134.71	-4.600	-3.315	-2.808	-2.493
Mean	87.044	-5.435	-4.085	-4.159	-3.580	53.771	-6.090	-4.925	-4.245	-3.838
St.Dev	24.507	0.386	0.272	0.250	0.239	35.780	1.026	0.999	0.802	0.729

Notes: The table summarizes aspects of the time-series dependence structure of quarterly realized market variance, covariances and realized betas. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation, and ADF^i denotes the augmented Dickey-Fuller unit root test with i augmentation lags. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations. We calculate the quarterly realized variance, covariances and betas from daily returns.

Table 3
Inverted Roots of AR(p) Models for Quarterly Realized Betas

Stock	Inverted Roots	(and Modulus of Dominant Inverted Root)				
AA	0.49 -.25i	0.49+0.25i	-0.10 -0.41i	-0.10+0.41i	-0.57	(0.57)
ALD	0.50	-0.30				(0.50)
BA	0.80	-0.30+0.49i	-0.30 -0.49i			(0.80)
CAT	0.39					(0.39)
CHV	0.80	-0.29 -0.44i	-0.29+0.44i			(0.80)
DD	0.20					(0.20)
DIS	0.86	0.20 -0.48i	0.20+0.48i	-0.50 -0.59i	-0.50+0.59i	(0.86)
EK	0.73	-0.25+0.38i	-0.25 -0.38i			(0.73)
GE	0.50	-0.28				(0.50)
GM	0.84	-0.29+0.44i	-0.29 -0.44i			(0.84)
GT	0.83	-0.33+0.41i	-0.33 -0.41i			(0.83)
HWP	0.36	-0.13+0.27i	-0.13 -0.27i			(0.36)
IBM	0.66	0.09+0.68i	0.09 -0.68i	-0.76		(0.76)
IP	0.10					(0.10)
JNJ	0.33					(0.33)
KO	0.79	0.04+0.50i	0.04 -0.50i	-0.63		(0.79)
MMM	0.47	-0.13+0.31i	-0.13 -0.31i			(0.47)
MO	0.83	0.16+0.61i	0.16 -0.61i	-0.48 -0.35i	-0.48+0.35i	(0.83)
MRK	0.60 -0.11i	0.60+0.11i	-0.07 -0.73i	-0.07+0.73i	-0.81	(0.81)
PG	0.72	-0.45				(0.72)
S	0.59	0.25	-0.57			(0.59)
T	0.87	0.17 -.64i	0.17+0.64i	-0.56+0.34i	-0.56 -0.34i	(0.87)
UTX	0.77	0.08+.63i	0.08 -0.63i	-0.68		(0.77)
UK	0.80	-0.10+.58i	-0.10 -0.58i	-0.42		(0.80)
XON	0.58	-0.29				(0.58)

Notes: The table shows the inverted roots and modulus of the dominant root of the autoregressive lag operator polynomials $(1-\hat{\phi}_1 L - \hat{\phi}_2 L^2 - \ldots - \hat{\phi}_p L^p)$, where $\hat{\phi}_1$, $\hat{\phi}_2$, ..., $\hat{\phi}_p$ are the least squares estimates of the parameters of $AR(p)$ models fit to the realized betas, with p selected by the AIC. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations. We calculate the quarterly realized variance, covariances and betas from daily returns.

Figure 1a
Time Series Plot of Quarterly Realized Market Variance, Fall 1987 Included

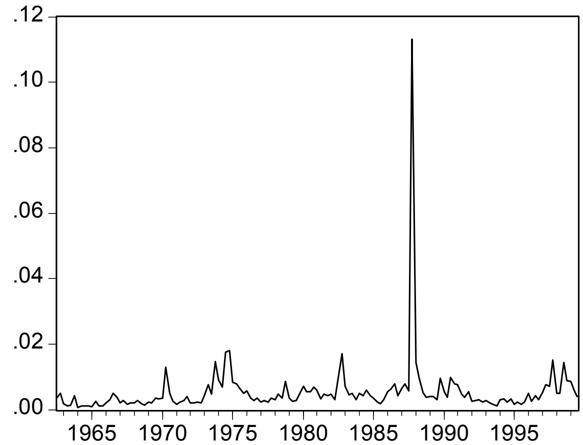
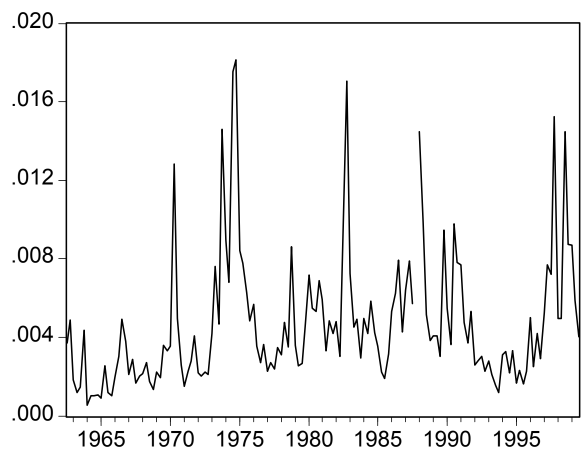
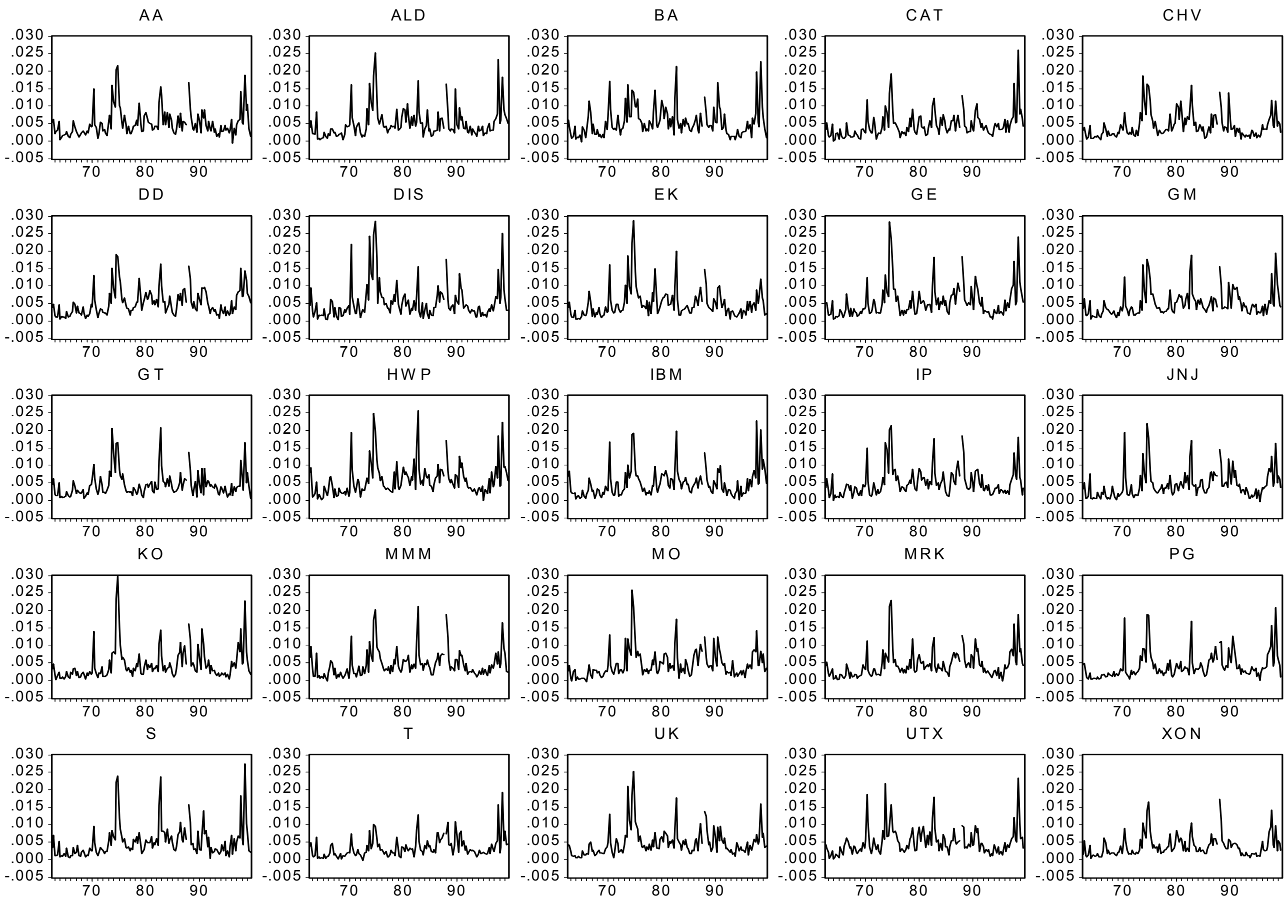


Figure 1b
Time Series Plot of Quarterly Realized Market Variance, Fall 1987 Excluded



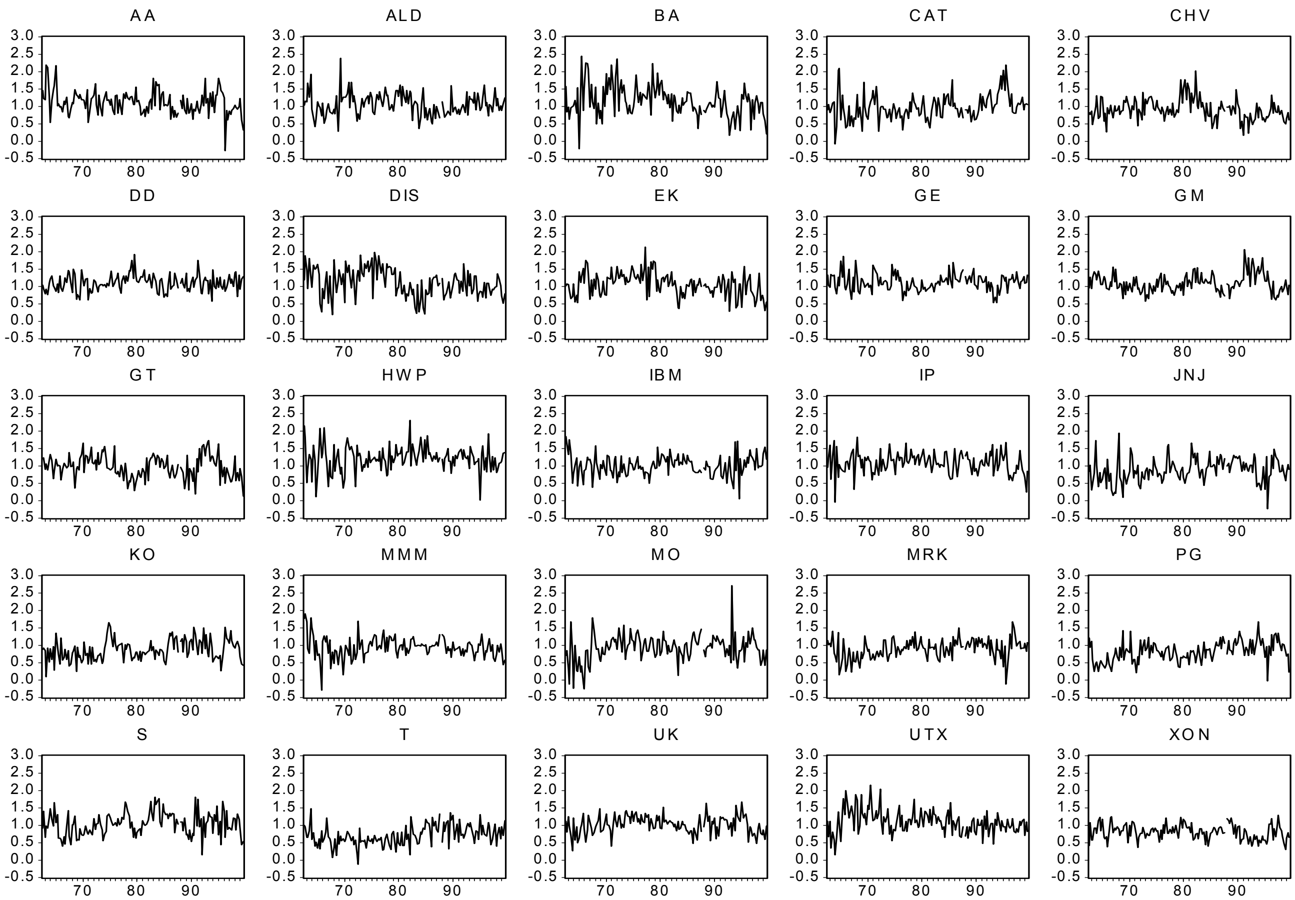
Notes: The two subfigures show the time series of quarterly realized market variance, with the 1987:4 outlier included (Figure 1a) and excluded (Figure 1b). The sample covers the period from 1962:3 through 1999:3, for a total of 149 observations. We calculate the realized quarterly market variances from daily returns.

Figure 2
Time Series Plots of Quarterly Realized Covariances



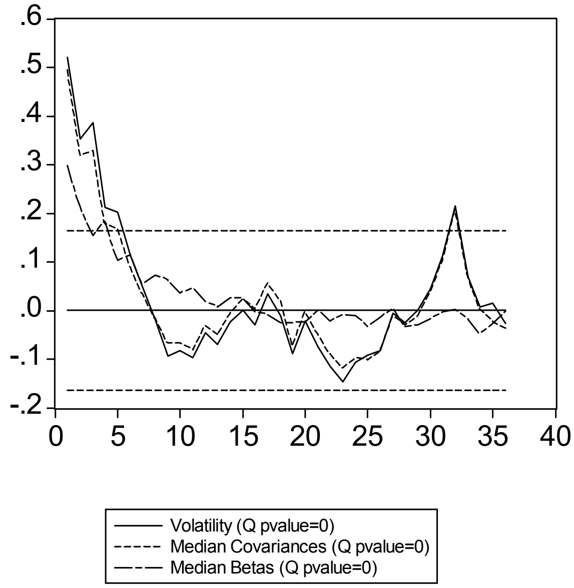
Notes: The Figure shows the time series of quarterly realized covariances, with the 1987:4 outlier excluded. The sample covers the period from 1962:3 through 1999:3, for a total of 148 observations. We calculate the realized quarterly covariances from daily returns.

Figure 3
Time Series Plots of Quarterly Realized Betas



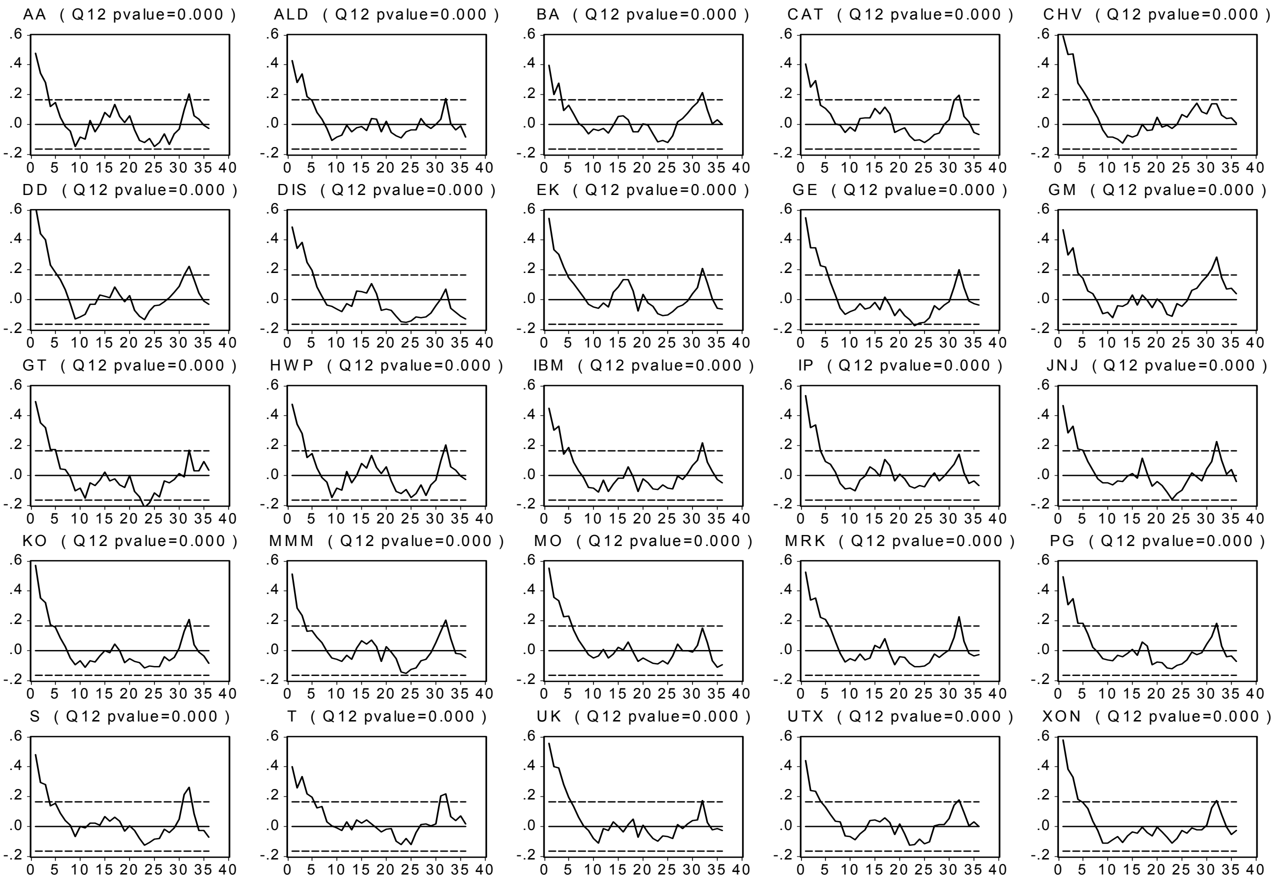
Notes: The Figure shows the time series of quarterly realized betas, with the 1987:4 outlier excluded. The sample covers the period from 1962:3 through 1999:3, for a total of 148 observations. We calculate the realized quarterly betas from daily returns.

Figure 4
Sample Autocorrelations of Quarterly Realized Market Variance, Median Sample Autocorrelations of Quarterly Realized Covariances and Median Sample Autocorrelations of Quarterly Realized Betas



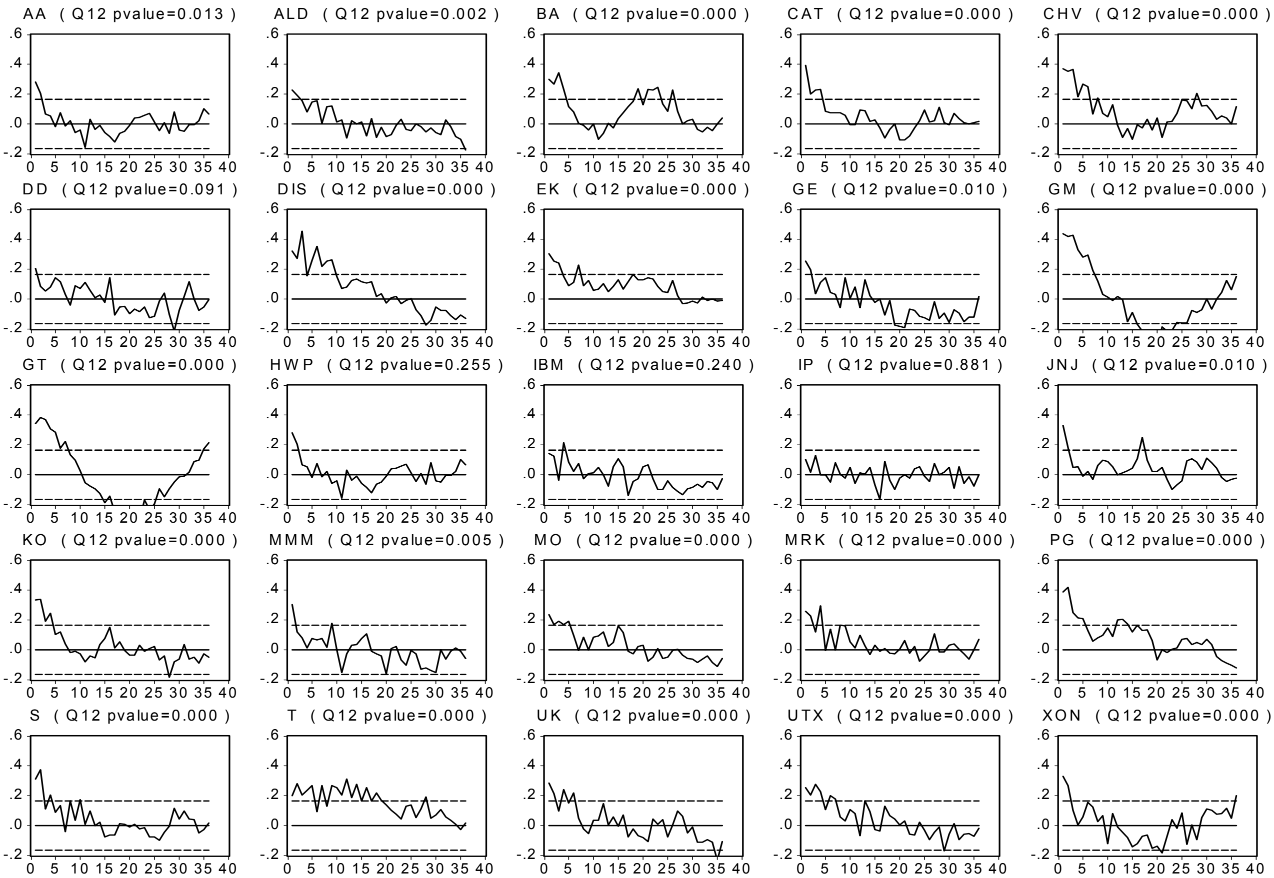
Notes: The figure shows the first 36 sample autocorrelations of quarterly realized market variance, the medians across individual stocks of the first 36 sample autocorrelations of quarterly realized covariances and the medians across individual stocks of the first 36 sample autocorrelations of quarterly realized betas. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations. We calculate the quarterly realized variance, covariances and betas from daily returns.

Figure 5
Sample Autocorrelations of Quarterly Realized Covariances



Notes: The figure shows the first 36 sample autocorrelations of quarterly realized covariances. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. ***Q*** denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations. We calculate the quarterly realized covariances from daily returns.

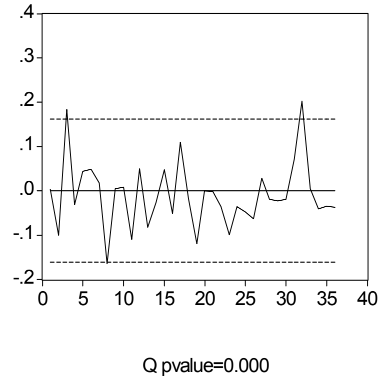
Figure 6
Sample Autocorrelations of Quarterly Realized Betas



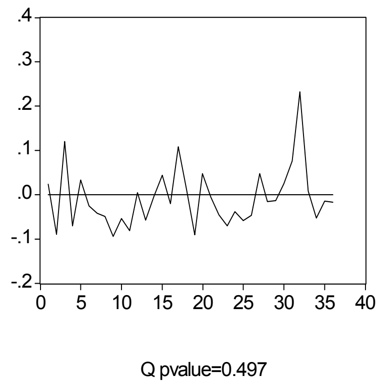
Notes: The figure shows the first 36 sample autocorrelations of quarterly realized betas. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations. We calculate the quarterly realized betas from daily returns.

Figure 7

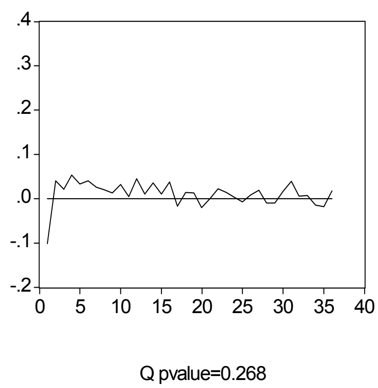
Sample Autocorrelations of Quarterly Realized Market Variance Prefiltered by $(1-L)^{.42}$



Median Sample Autocorrelations of Quarterly Realized Covariances Prefiltered by $(1-L)^{.42}$

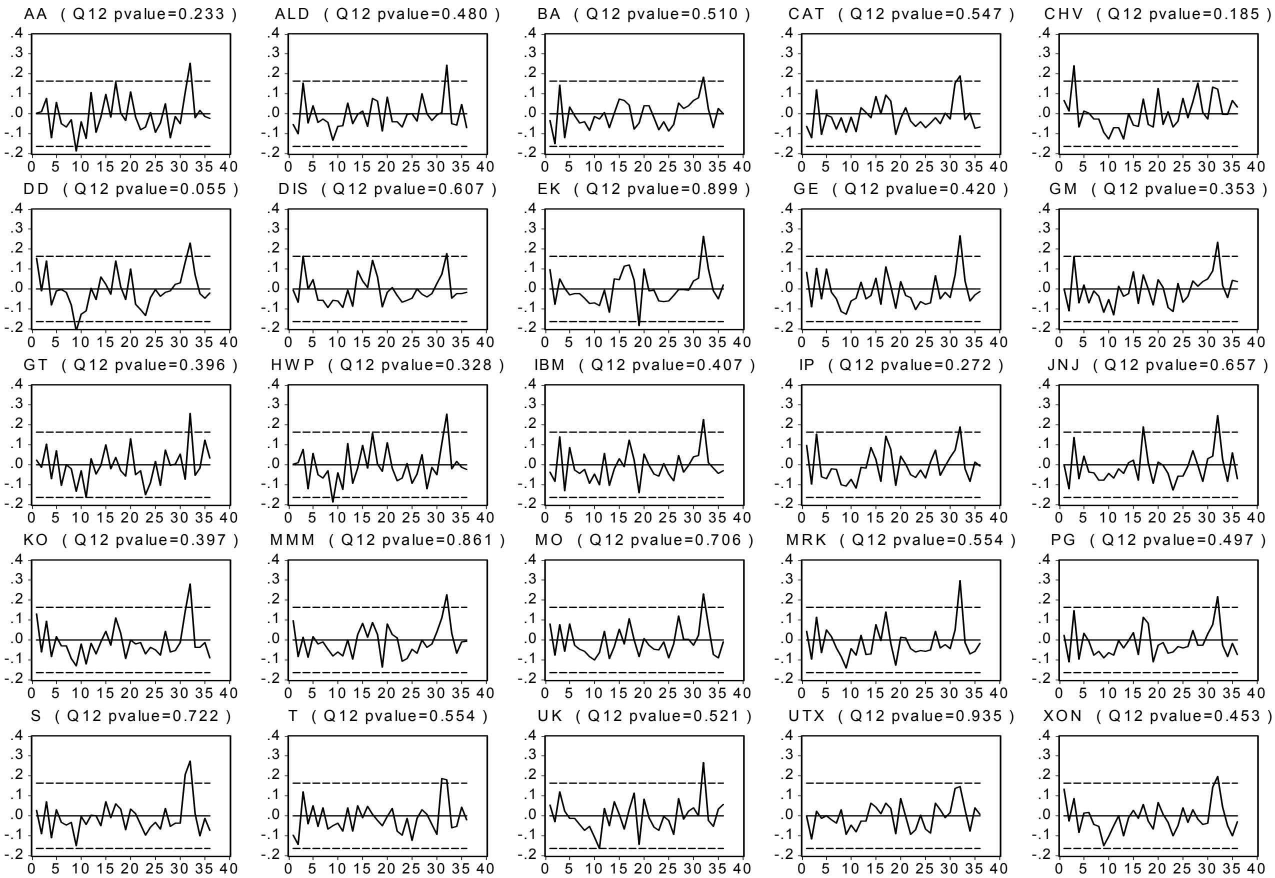


Median Sample Autocorrelations of Quarterly Realized Betas Prefiltered by $(1-L)^{.42}$



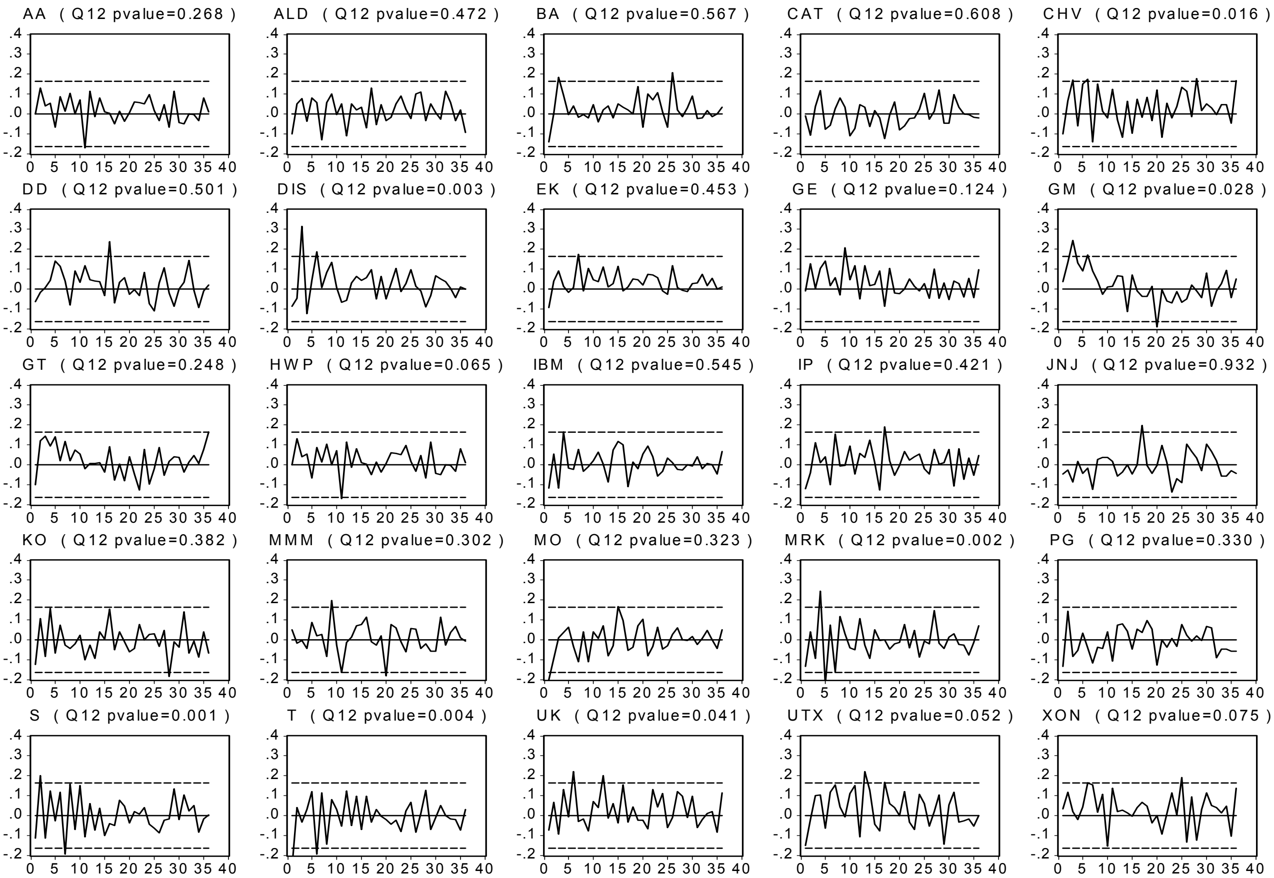
Notes: The three subfigures show the first 36 sample autocorrelations of quarterly realized market variance, the medians across individual stocks of first 36 sample autocorrelations of quarterly realized covariances and the medians across individual stocks of first 36 sample autocorrelations of quarterly realized betas all prefiltered by $(1-L)^{.42}$. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the median of Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations. We calculate the quarterly realized variance from daily returns.

Figure 8
Sample Autocorrelations of Quarterly Realized Covariances Prefiltered by $(1-L)^{.42}$



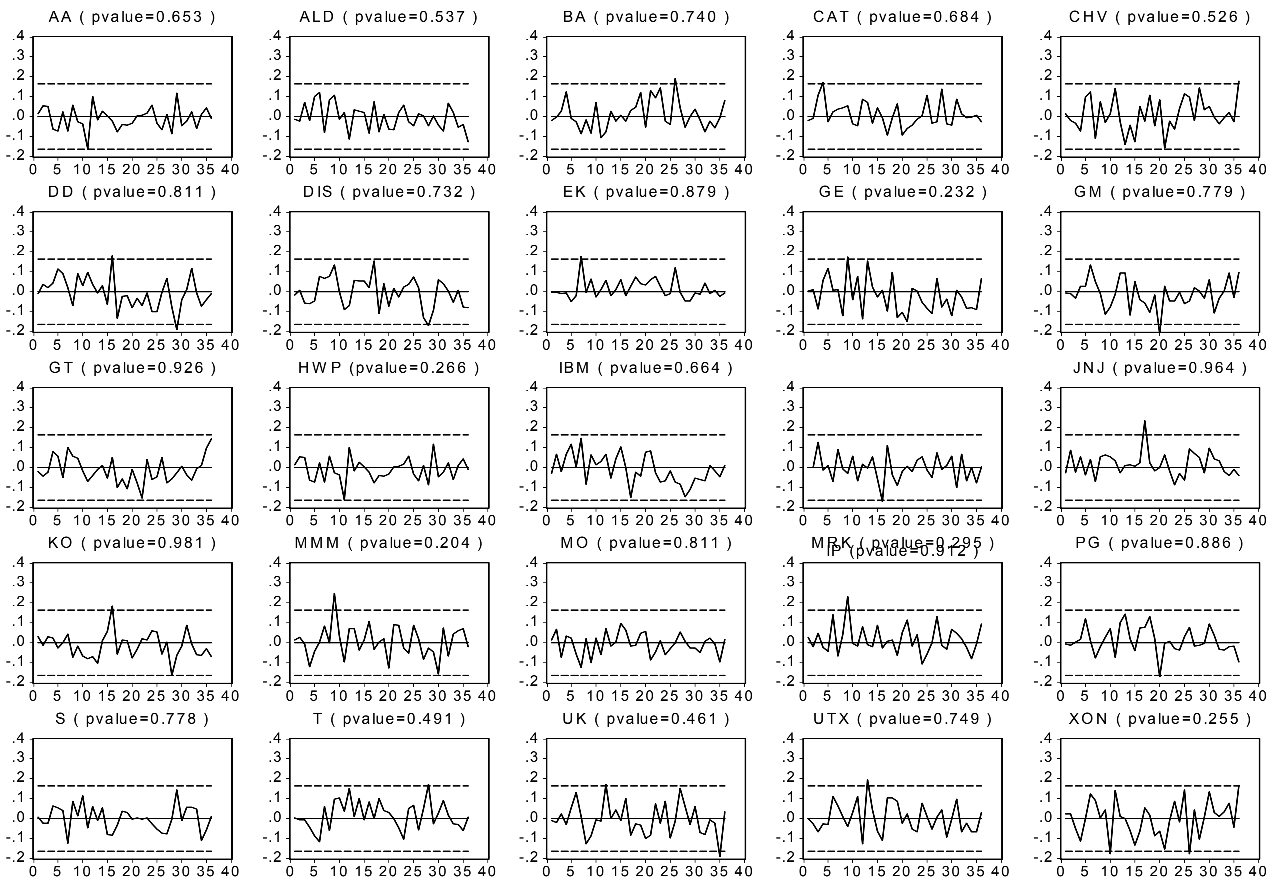
Notes: The figure shows the first 36 sample autocorrelations of quarterly realized covariances prefiltered by $(1-L)^{.42}$. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations. We calculate the quarterly realized covariances from daily returns.

Figure 9
Sample Autocorrelations of Quarterly Realized Betas Prefiltered by $(1-L)^{.42}$



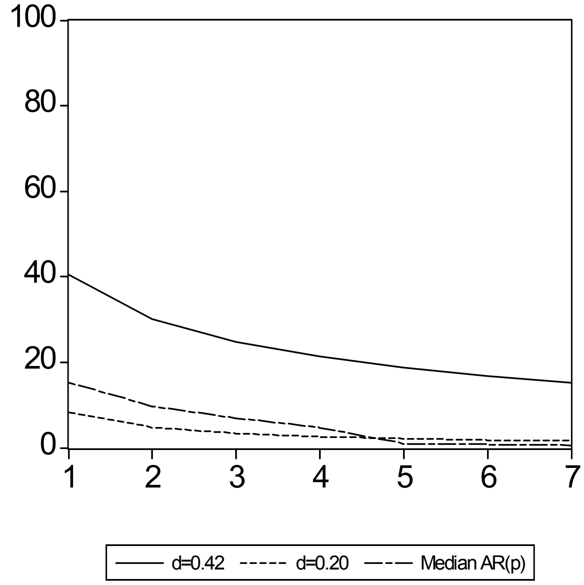
Notes: The figure shows the first 36 sample autocorrelations of quarterly realized betas prefiltered by $(1-L)^{.42}$. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations. We calculate the quarterly realized betas from daily returns.

Figure 10
Sample Autocorrelations of Quarterly Realized Betas Prefiltered by $(1-\hat{\phi}_1 L-\hat{\phi}_2 L^2-\ldots-\hat{\phi}_p L^p)$



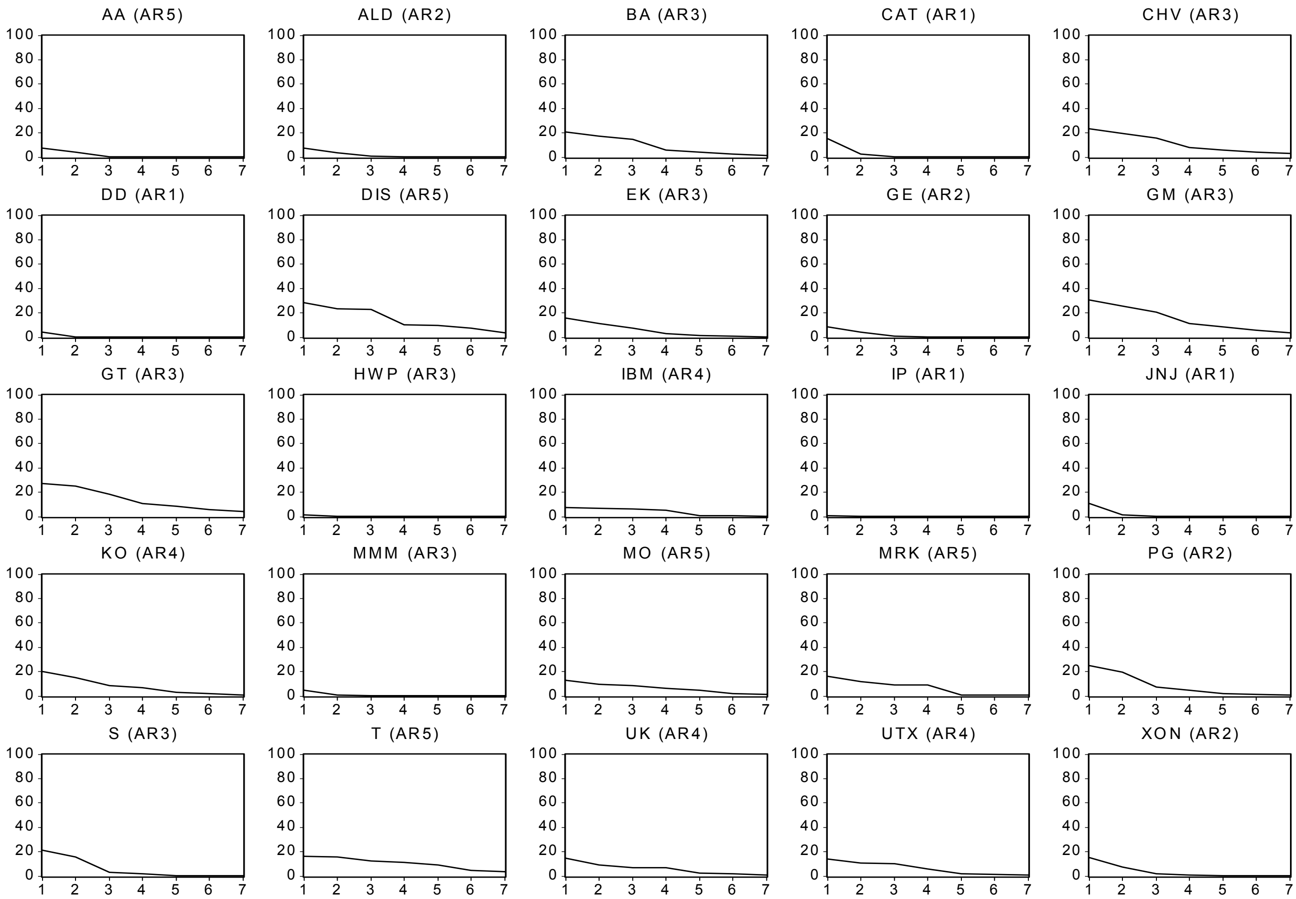
Notes: The figure shows the first 36 sample autocorrelations of quarterly realized betas prefiltered by $(1-\hat{\phi}_1 L-\hat{\phi}_2 L^2-\ldots-\hat{\phi}_p L^p)$, where $\hat{\phi}_1$, $\hat{\phi}_2$, ..., $\hat{\phi}_p$ are the least squares estimates of the parameters of $AR(p)$ models fit to the realized betas, with p selected by the AIC. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations. We calculate the quarterly realized variance, covariances and betas from daily returns.

Figure 11
Predictability of Market Volatility, Individual Equity Covariances with the Market, and Betas



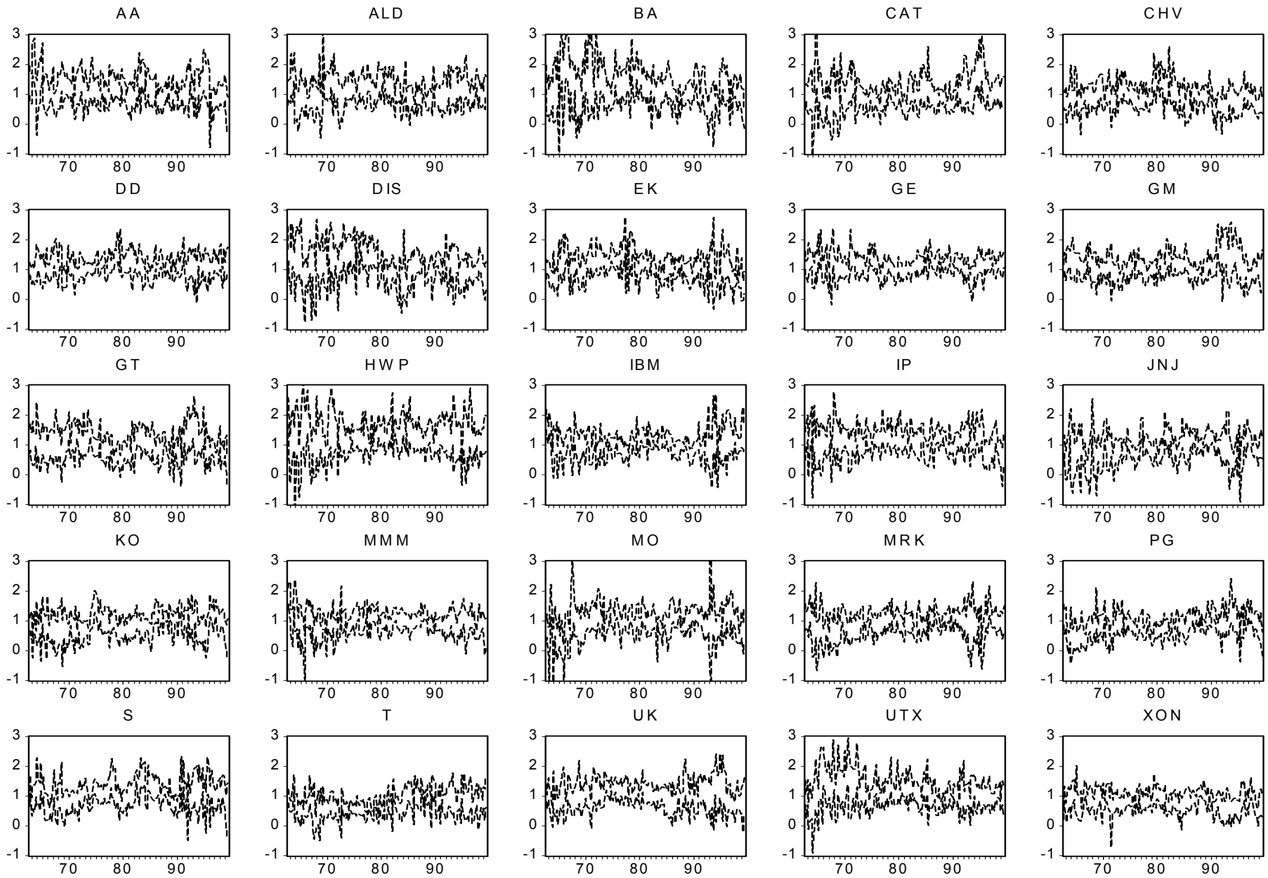
Notes: We define predictability as $P_j = 1 - var(e_{t+j,t})/var(e_{t+40,t})$, where $var(e_{t+j,t}) = \sigma^2 \sum_{i=0}^{j-1} b_i^2$, σ_t^2 is the variance of the innovation ε_t, and the b_i's are moving average coefficients; i.e., the Wold representation is $y_t = (1 + b_1L + b_2L^2 + b_2L^3 + ...)\varepsilon_t$. We approximate the dynamics using a pure long-memory model, $(1 - L)^{.42} y_t = \varepsilon_t$, in which case $b_0 = 1$ and $b_i = (-1)b_{i-1}(d - i + 2)/(i-1)$ and plot P_j for $j = 1,...,7$ in the solid line. Moreover, because we take d=0.42 for market volatility and for all covariances with the market, all of their predictabilities are the same at all horizons. As one approximation for the dynamics of the betas we use a pure long-memory model, $(1 - L)^{.20} y_t = \varepsilon_t$, in which case $b_0 = 1$ and $b_i = (-1)b_{i-1}(d - i + 2)/(i-1)$ and plot P_j for $j = 1,...,7$ in the dotted line. We also approximate the beta dynamics using an $AR(p)$ model, with the autoregressive lag order p determined by the AIC and plot the median of P_j for $j = 1,...,7$ among all 25 stocks in the mixed dotted line. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations.

Figure 12
Predictability of Betas based on AR(p) Sieve Approximation of Dynamics



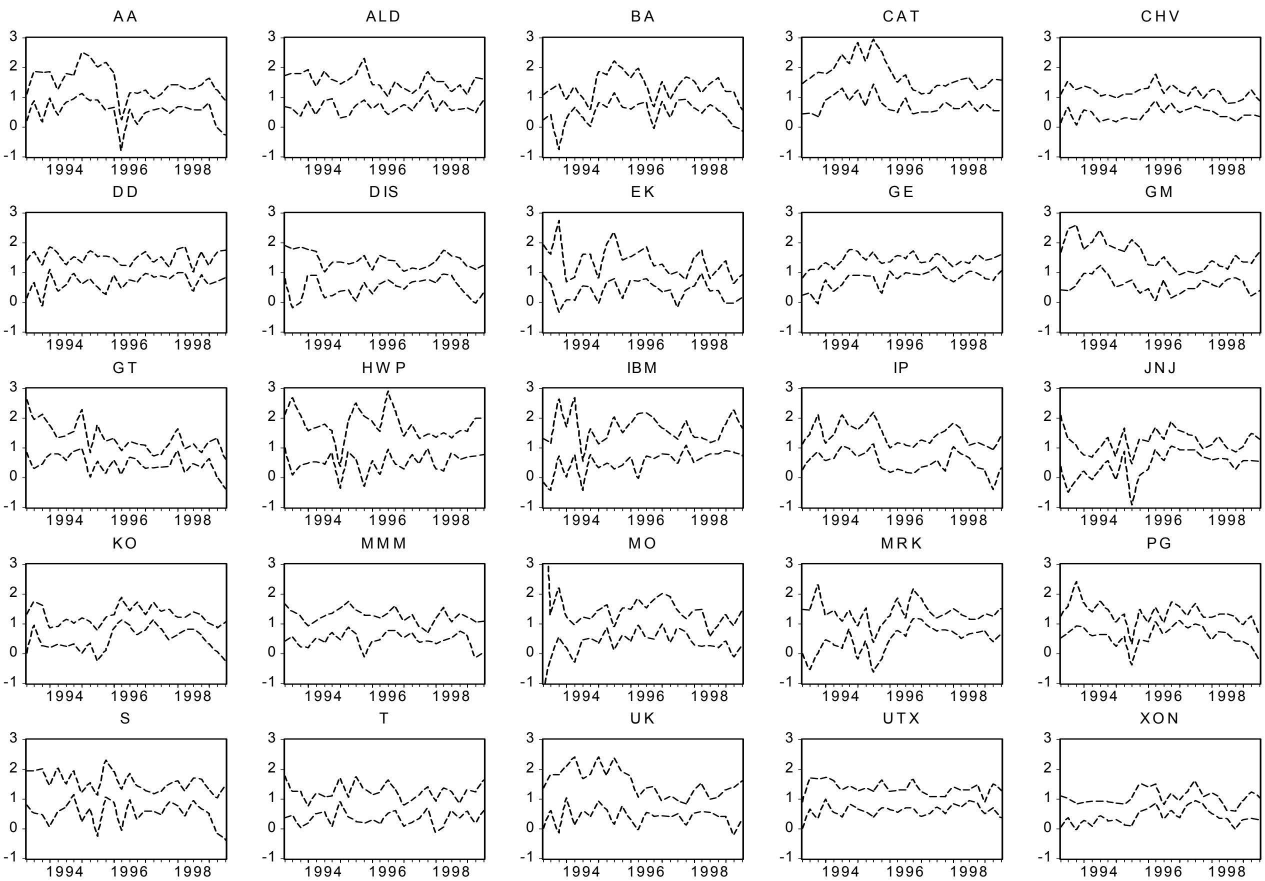
Notes: We define predictability as $P_j = 1 - var(e_{t+j,t})/var(y_t)$, where $var(e_{t+j,t}) = \sigma^2 \sum_{i=0}^{j-1} b_i^2$, $var(y_t) = \sigma^2 \sum_{i=0}^{\infty} b_i^2$, σ_t^2 is the variance of the innovation ε_t, so that the b_i's correspond to the moving average coefficients in the Wold representation for y_t. We approximate the dynamics using an *AR*(*p*) model, with the autoregressive lag order *p* determined by the AIC, and plot P_j for $j = 1, \ldots, 7$. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded, for a total of 148 observations. We calculate the quarterly realized betas from daily returns.

Figure 13
Ninety-Five Percent Confidence Intervals for Quarterly Beta, Long Sample, Daily Sampling



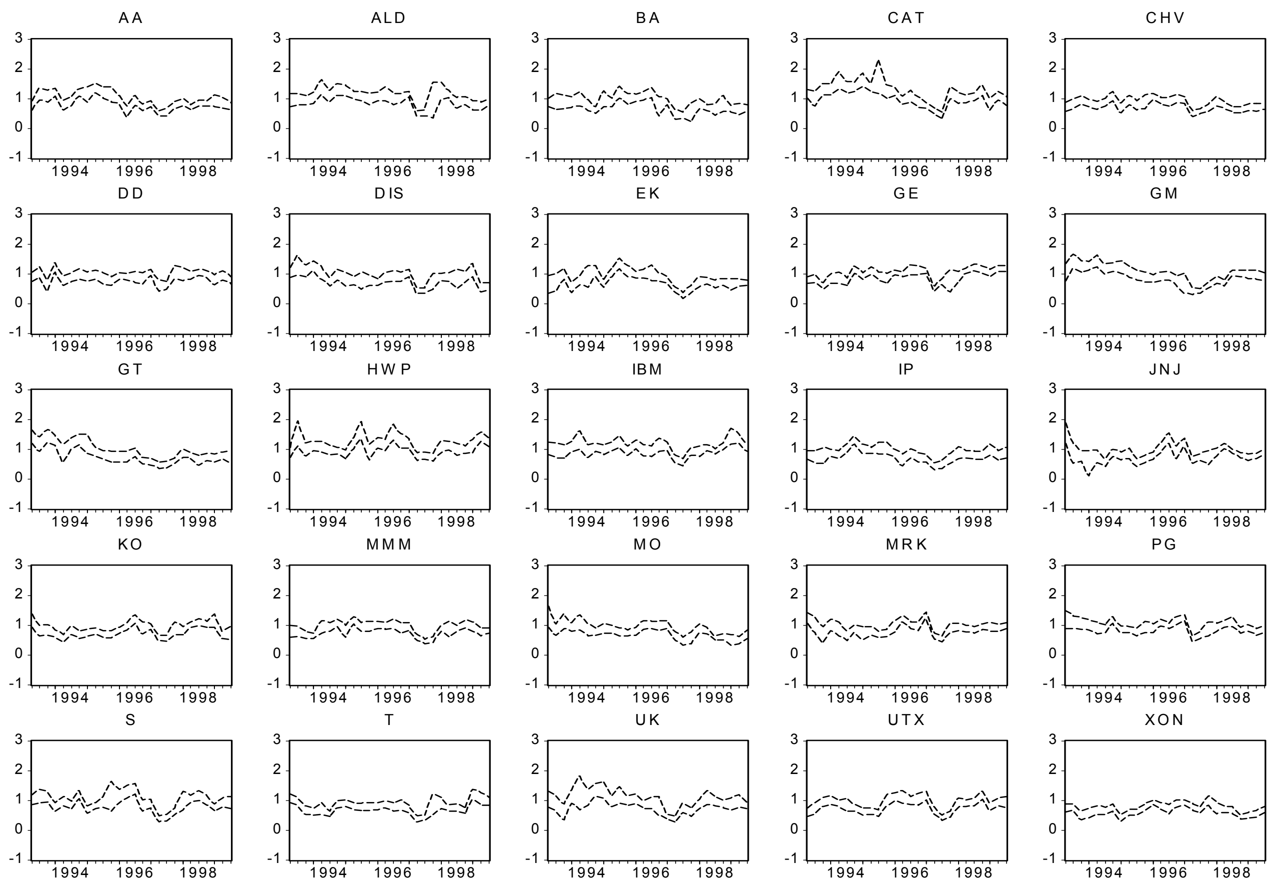
Notes: The figure shows the time series of ninety-five percent confidence intervals for the underlying quarterly integrated beta, calculated using the results of Barndorff-Nielsen and Shephard (2003). The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded. We calculate the realized quarterly betas from daily returns.

Figure 14
Ninety-Five Percent Confidence Intervals for Quarterly Beta, Short Sample, Daily Sampling



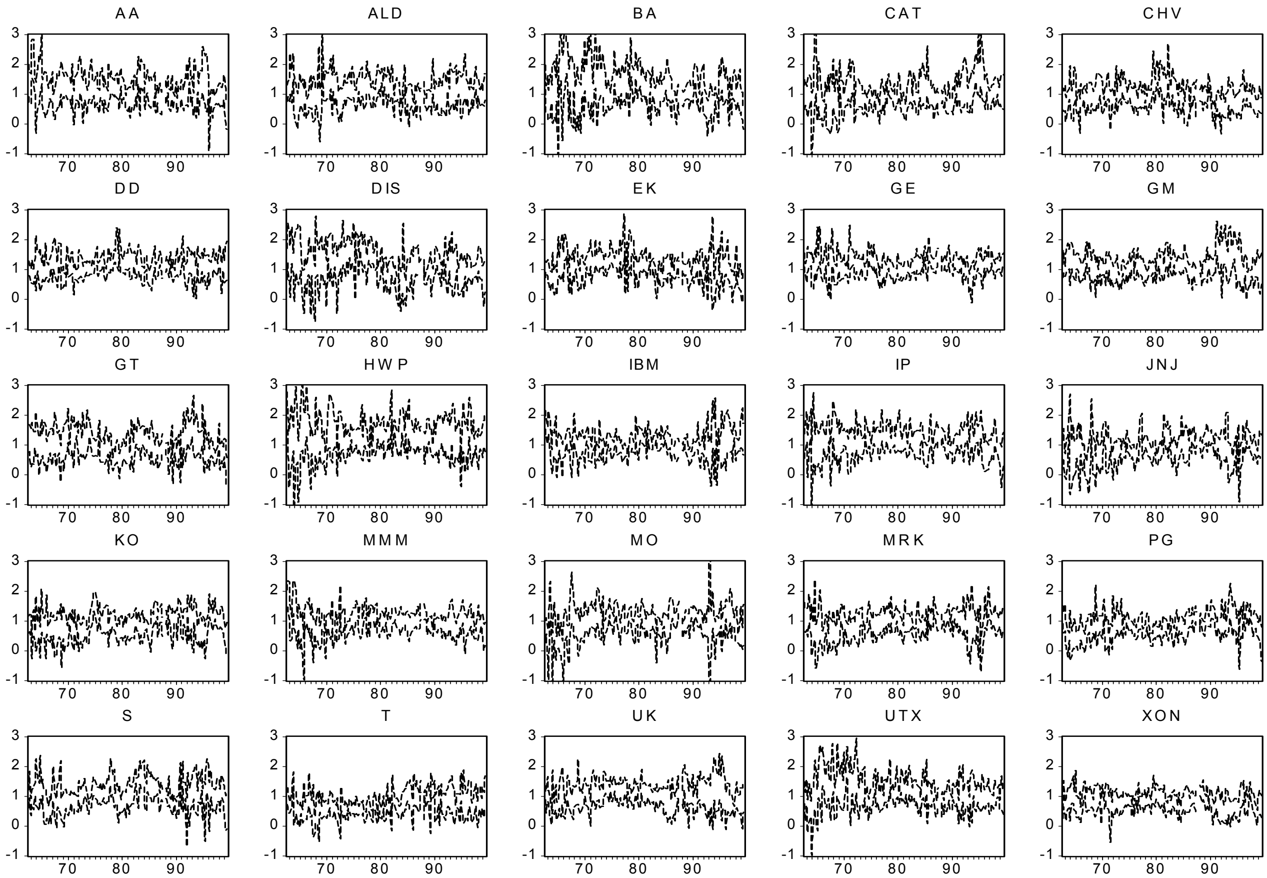
Notes: The figure shows the time series of ninety-five percent confidence intervals for the underlying quarterly integrated beta, calculated using the results of Barndorff-Nielsen and Shephard (2003). The sample covers the period from 1993:2 through 1999:3. We calculate the realized quarterly betas from daily returns.

Figure 15
Ninety-Five Percent Confidence Intervals for Quarterly Beta, Short Sample, Fifteen-Minute Sampling



Notes: The figure shows the time series of ninety-five percent confidence intervals for the underlying quarterly integrated beta, calculated using the results of Barndorff-Nielsen and Shephard (2003). The sample covers the period from 1993:2 through 1999:3. We calculate the realized quarterly betas from fifteen-minute returns.

Figure 16
Ninety-Five Percent Confidence Intervals for Quarterly Beta, Long Sample, Daily Sampling (Newey-West)



Notes: The figure shows the time series of Newey-West ninety-five percent confidence intervals for the underlying quarterly integrated beta. The sample covers the period from 1962:3 through 1999:3, with the 1987:4 outlier excluded. We calculate the realized quarterly betas from daily returns.

(Appendix to Realized Beta: Persistence and Predictability, by Andersen, Bollerslev, Diebold and Wu, 2004)

Analysis of Monthly Realized Beta Based on High-Frequency Intraday Data*

by

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Here we further explore the dynamic nature of realized betas and their components by studying monthly realized betas constructed from high-frequency intraday returns. The time span covered is shorter, because availability of intraday data is limited, but the improvement in the quality of the measurement still allows a precise characterization of the dynamics.

Data

Our analysis is based on data from the TAQ (Trade And Quotation) database. The TAQ data files contain continuously recorded information on the trades and quotations for the securities listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and the National Association of Security Dealers Automated Quotation system (NASDAQ). The database is published monthly, and has been available on CD-ROM from the NYSE since January 1993; we refer the reader to the corresponding data manual for a more complete description of the actual data and the method of data-capture. Our sample extends from January 4, 1993 until September 30, 1999, for a total of 1,366 trading days. A complete analysis based on all trades for all stocks, although straightforward conceptually, is infeasible in practice. We therefore restrict our analysis to the same set of DJIA firms that we analyzed using daily data above. This also helps ensure a reasonable degree of liquidity. We display the specific stocks in Table 1H ("H" indicates "high-frequency," in reference to the underlying 15-minute sampling). Finally, the corresponding market return is the DJIA cash index.

Although the DJIA stocks are among the most actively traded U.S. equities, the median inter-trade duration for all stocks across the full sample is 23.1 seconds, ranging from a low of 7 seconds for Merck & Co. Inc. (MRK) to a high of 54 seconds for United Technologies Corp. (UTX). As such, it is not practically feasible to push beyond this level the continuous record asymptotics and the length of the observation interval Δ in equation (3) of the main text. Moreover, because of the organizational structure of the market, the available quotes and transaction prices are subject to discrete clustering and bid-ask bounce effects. Such market microstructure features are generally not important when analyzing longer horizon interdaily returns but can seriously distort the distributional properties of high-frequency intraday returns; see, e.g., the textbook treatment by Campbell, Lo and MacKinlay (1997). For simplicity, we rely on artificially constructed equally-spaced intraday returns, as discussed for example by Andersen and Bollerslev (1997).[1] It turns out that a 15-minute sampling frequency is sufficient to alleviate the non-synchronous trading problem so that the market return is nearly uncorrelated and the individual stock return series are approximately serially uncorrelated and orthogonal to all but the first lagged and leading 15-minute market

[1] An alternative but much more complicated approach would be to utilize all of the observations by explicitly modeling the high-frequency frictions; see, also Andersen, Bollerslev, Diebold and Labys (2000), Andreou and Ghysels (2002), Hansen and Lunde (2003), Bandi and Russell (2003), and Zhang, Mykland and Ait-Sahalia (2003), among others, for further discussion or alternative procedures in this context.

returns. Following the logic of Scholes and Williams (1977),[2] our high-frequency based estimate of beta then consists of the contemporaneously measured beta plus the first-order lag and lead beta obtained from the covariance of the individual 15-minute stock return with the lagged (respectively lead) 15-minute market return divided by the market return variance estimate adjusted for the (almost negligible) first-order serial correlation in the market index. With the daily transaction record extending from 9:30 EST until 16:00 EST, there are a total of 26 fifteen-minute returns for each day, corresponding to $\Delta = 1/26 \approx 0.0385$ in the notation above, and thus about 570 underlying intraday observations for the monthly beta calculations. This frequency should be sufficiently high to reduce the measurement errors substantially while also avoiding the confounding influences from market microstructure frictions. The main advantage of using the intraday returns is that we obtain less noisy measures of the relevant variance and covariances – conditional on the chosen 15-minute frequency controlling appropriately for market microstructure features – because we are pushing closer to continuous sampling, while the drawback is the much shorter time span available for analysis. Because the analysis in the main text points toward the potential presence of short-run dependence in betas, we focus on the monthly interval to explore this possibility in some depth.

Point Estimates of Monthly Betas

Figures 1H and 2H reveal a strong shift upward in both market volatility and the stocks covariances with the market in the latter half of the sample, where the values are both higher and much more volatile. In contrast, there is no visual evidence of any shift in the realized beta series in Figure 3H. The latter figures occasionally display distinct outliers but these are not concentrated toward the second half of the sample, and the enhanced volatility in the market variance and the individual covariances in the second part of the 1990s is certainly not visible, implying that the dominant dynamic components in the variances and covariances tend to annihilate each other. As with the quarterly realized betas based on daily data, there is no sign of integer integration in any of the realized variance, covariance or beta measures and the median Ljung-Box statistic for the beta series of 48 in Table 2H is below that of the covariances, 59 and the market variance, 79, supporting the visual impression of relatively less serial dependence in the realized monthly betas.

Further confirmation of the relative strength of the serial dependence across the series is evident in Figures 4H-6H. The autocorrelations generally appear significant, or nearly so, for the realized market variance and the individual covariances out to an order of fifteen months. This is consistent with the significance of the autocorrelations out to lags four to six in the quarterly measures in Figures 4-6. Moreover, the median correlogram for the realized covariances again appears to be a dampened version of that for the market variance. In contrast, the individual realized betas rarely are significant beyond a lag

[2] See also the more recent discussion in Bollerslev and Zhang (2003).

length of five or six months, consistent with only the first few autocorrelation coefficients being significant in Figure 6. From Figure 6H it is evident that there is a large degree of heterogeneity in the results across the individual stocks, but the dominant pattern is clearly one with significantly positive coefficients at lower lags but also one with a sharply declining correlogram. This is again consistent with a cancellation occurring between the variance and covariance measures in the construction of the betas. Because none of the individual realized beta series display significant negative correlation at lower lags, the results point strongly toward the presence of a relatively short-lived but highly significant degree of positive serial dependence in the betas, as also evidenced by the median beta autocorrelation pattern in Figure 4H.

Continuing along the lines above, Figures 7H-9H present sample autocorrelations of the monthly realized measures, each prefiltered by $(1-L)^{.42}$. Because the degree of fractional integration is invariant to the sampling frequency and temporal aggregation, it is natural to compare these figures to those from the quarterly realized volatility measures. The message is in important respects similar to that for quarterly realized betas computed from daily data: the market variance and the covariance measures seem consistent with a pure fractionally integrated process with $d = .42$. In contrast, however, the monthly realized beta series behave differently from their quarterly counterparts, as there is no direct indication of over-differencing this time. Of course, at the monthly frequency the serial dependence, as manifest in the correlogram, should appear stronger than for the quarterly series and thus generally blur our ability to distinguish the short run dependent case from the fractionally integrated one, in particular because the calendar period is now shorter. The discrepancy in behavior for the monthly and quarterly series for the realized betas is nevertheless an indication that the invariance property with respect to sampling frequency and time aggregation, as should occur if the series is truly $I(d)$, for $d > 0$, may be violated. This lends additional credibility to our earlier assertion in the main text that, all told, $I(0)$ dynamics for realized betas seemed reasonable. Consequently, we again also model the realized betas using a standard autoregression. Table 3H presents results for low-order $AR(p)$ models for each individual realized monthly beta series. The modulus of the dominant inverted root of the *AR* polynomial is now generally higher compared to Table 3, as is to be expected given the shorter monthly horizon for the betas. Figures 10H and 11H document that prefiltering using the estimated $AR(p)$ polynomial produces residuals with white noise characteristics and that the resulting predictability is lower than for the constituent components, the market variance and the covariances with the market. Overall, there are no signs that a null hypothesis of short memory dependence for the realized betas can be rejected.

All told, the results across the two sampling frequencies and time horizons are generally consistent. There is strong evidence of a much lower degree of dependence in the realized betas compared to the realized market variance and the realized covariances with the market return. Although the realized betas can be approximated by a pure fractionally integrated process with a *d* of around .20 in the quarterly data – and perhaps an even higher degree of fractional integration in the shorter monthly sample – this may well be an artifact of the short sample span. There is clearly some heterogeneity across the stock betas but a standard short memory autoregressive process with significantly positive serial correlation for each of the individual realized betas appears robust across both estimation horizons and sample lengths.

Interval Estimates of Monthly Betas

Once we pass to the monthly beta estimation frequency, even the intraday sampling provides somewhat imprecise inference, as seen in Figure 12H. Nonetheless, the standard error bands remain tighter than for the quarterly beta estimates based on daily data, and it is feasible to establish significant time-variation in the monthly betas from Figure 12H.

Recall from the main text that there was not much difference in the assessment of the estimation uncertainly inherent in the quarterly beta measures obtained from the alternative Barndorff-Nielsen-Shephard and Newey-West procedures. This conclusion is largely confirmed from a comparison of the Newey-West standard error bands for the monthly betas based on the intraday data in Figure 13H with Figure 12H discussed above. The two sets of figures again appear qualitatively similar, although there is now evidence of significant differences in the width of the bands associated with some of the outliers in the series.

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Table1H
The Dow Jones Thirty

Company Name	Ticker	Data Range
Alcoa Inc.	AA	04/01/1993-09/30/1999
Allied Capital Corporation	ALD	04/01/1993-09/30/1999
American Express Co.	AXP*	04/01/1993-09/30/1999
Boeing Co.	BA	04/01/1993-09/30/1999
Caterpillar Inc.	CAT	04/01/1993-09/30/1999
Chevron Corp.	CHV	04/01/1993-09/30/1999
DuPont Co.	DD	04/01/1993-09/30/1999
Walt Disney Co.	DIS	04/01/1993-09/30/1999
Eastman Kodak Co.	EK	04/01/1993-09/30/1999
General Electric Co.	GE	04/01/1993-09/30/1999
General Motors Corp.	GM	04/01/1993-09/30/1999
Goodyear Tire & Rubber Co.	GT	04/01/1993-09/30/1999
Hewlett- Packard Co.	HWP	04/01/1993-09/30/1999
International Business Machines Corp.	IBM	04/01/1993-09/30/1999
International Paper Co.	IP	04/01/1993-09/30/1999
Johnson & Johnson	JNJ	04/01/1993-09/30/1999
JP Morgan Chase & Co.	JPM*	04/01/1993-09/30/1999
Coca-Cola Co.	KO	04/01/1993-09/30/1999
McDonald's Corp.	MCD*	04/01/1993-09/30/1999
Minnesota Mining & Manufacturing Co.	MMM	04/01/1993-09/30/1999
Philip Morris Co.	MO	04/01/1993-09/30/1999
Merck & Co.	MRK	04/01/1993-09/30/1999
Procter & Gamble Co.	PG	04/01/1993-09/30/1999
Sears, Roebuck and Co.	S	04/01/1993-09/30/1999
AT&T Corp.	T	04/01/1993-09/30/1999
Travelers Group Inc.	TRV*	04/01/1993-09/30/1999
Union Carbide Corp.	UK	04/01/1993-09/30/1999
United Technologies Corp.	UTX	04/01/1993-09/30/1999
Wal-Mart Stores Inc.	WMT*	04/01/1993-09/30/1999
Exxon Corp.	XON	04/01/1993-09/30/1999

Notes: The table summarizes company names and tickers, and the range of the data examined. We use the Dow Jones Thirty as of March 1997. Tickers with asterisks denote stocks with incomplete data, which we exclude from the analysis.

Table 2H
The Dynamics of Monthly Realized Market Variance, Covariances and Betas

	Q	ADF^1	ADF^2	ADF^3	ADF^4
v_{mt}^2	78.99	-3.989	-3.317	-2.814	-2.619

	$cov(r_{mt}, r_{it})$					β_{it}				
	Q	ADF^1	ADF^2	ADF^3	ADF^4	Q	ADF^1	ADF^2	ADF^3	ADF^4
Min.	35.841	-4.658	-3.941	-3.531	-3.357	10.818	-5.474	-4.763	-5.732	-5.217
0.10	38.289	-4.543	-3.860	-3.451	-3.029	17.559	-4.753	-4.419	-4.158	-3.776
0.25	40.752	-4.358	-3.644	-3.253	-2.908	23.250	-4.475	-3.746	-3.332	-3.144
0.50	58.915	-4.099	-3.460	-3.094	-2.769	47.954	-3.822	-2.994	-2.778	-2.385
0.75	69.659	-3.848	-3.313	-2.815	-2.537	70.443	-3.392	-2.737	-2.576	-2.204
0.90	84.754	-3.685	-3.023	-2.486	-2.335	134.90	-3.173	-2.486	-2.194	-1.908
Max.	89.932	-3.599	-2.878	-2.374	-2.132	209.29	-2.445	-2.164	-1.802	-1.718
Mean.	57.340	-4.117	-3.464	-3.049	-2.735	58.182	-3.918	-3.224	-3.068	-2.736
St.Dev.	16.966	0.319	0.292	0.332	0.275	46.307	0.691	0.697	0.939	0.824

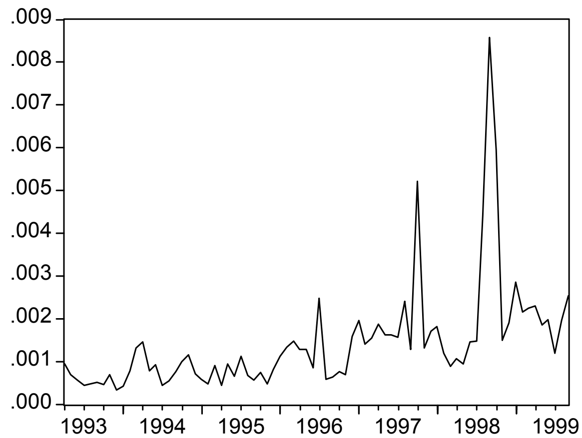
Notes: The table summarizes aspects of the time-series dependence structure of monthly realized market variance, covariances and realized betas. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation, and ADF^i denotes the augmented Dickey-Fuller unit root test with i augmentation lags. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized variance, covariances and betas from fifteen-minutes returns.

Table 3H
Inverted Roots of *AR*(*p*) Models for Monthly Realized Betas

Stock	Inverted Roots (and Modulus of Dominant Inverted Root)					
AA	0.71	-0.28				(0.71)
ALD	0.78	-0.26 -0.34i	-0.26+0.34i			(0.78)
BA	0.68	-0.53				(0.68)
CAT	0.90	0.05+0.65i	0.05 -0.65i	-0.68		(0.90)
CHV	0.28					(0.28)
DD	0.54 -0.51i	0.54+.51i	-0.50 -0.49i	-0.50+0.49i		(0.74)
DIS	0.49					(0.49)
EK	0.82	0.14 -0.20i	0.14+0.20i	-0.42+0.41i	-0.42 -0.41i	(0.82)
GE	0.54					(0.54)
GM	0.85	-0.49				(0.85)
GT	0.85	-0.29 -0.43i	-0.29+0.43i			(0.85)
HWP	0.55+0.34i	0.55 -0.34i	-0.15	-0.35+0.31i	-0.35 -0.31i	(0.65)
IBM	0.25					(0.25)
IP	0.83	-0.29 -0.47i	-0.29+0.47i			(0.83)
JNJ	0.54					(0.54)
KO	0.77	-0.25 -0.46i	-0.25+0.46i			(0.77)
MMM	0.81	-0.20 -0.59i	-0.20+0.59i			(0.81)
MO	0.22					(0.22)
MRK	0.70+0.46i	0.70 -0.46i	-0.33+0.59i	-0.33 -0.59i	-0.39	(0.84)
PG	0.29					(0.29)
S	0.71	-0.43				(0.71)
T	0.62 -0.19i	0.62+0.19i	-0.46 -0.51i	-0.46+0.51i		(0.69)
UTX	0.69	-0.59				(0.69)
UK	0.77	-0.15+0.53i	-0.15 -0.53i			(0.77)
XON	0.70+0.27i	0.70 -0.27i	-0.23+0.74i	-0.23 -0.74i	-0.79	(0.79)

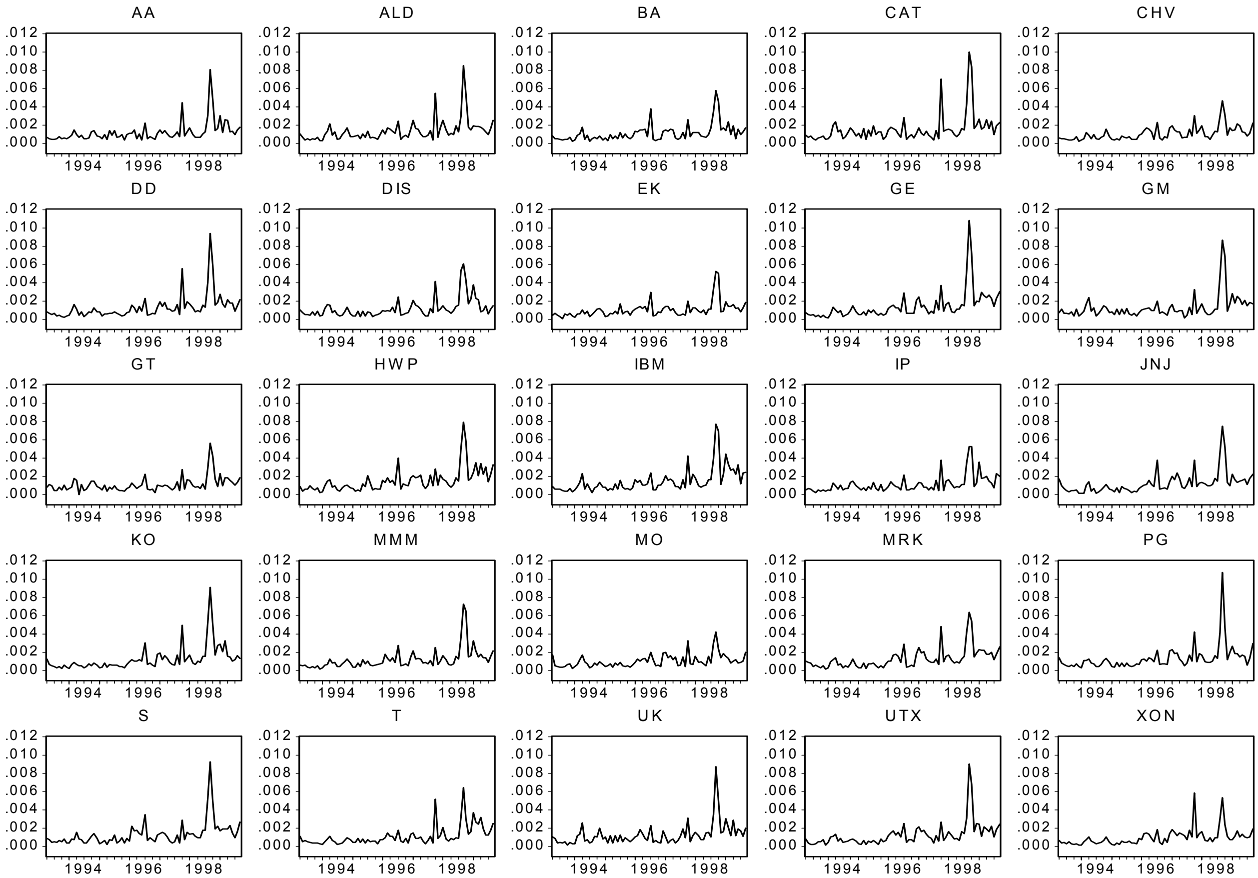
Notes: The table shows the inverted roots and modulus of the dominant root of the autoregressive lag operator polynomials $(1-\hat{\phi}_1 L-\hat{\phi}_2 L^2-\ldots-\hat{\phi}_p L^p)$, where $\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_p$ are the least squares estimates of the parameters of *AR*(*p*) models fit to the realized betas, with *p* selected by the AIC. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized variance, covariances and betas from fifteen-minute returns.

Figure 1H
Time Series Plot of Monthly Realized Market Variance



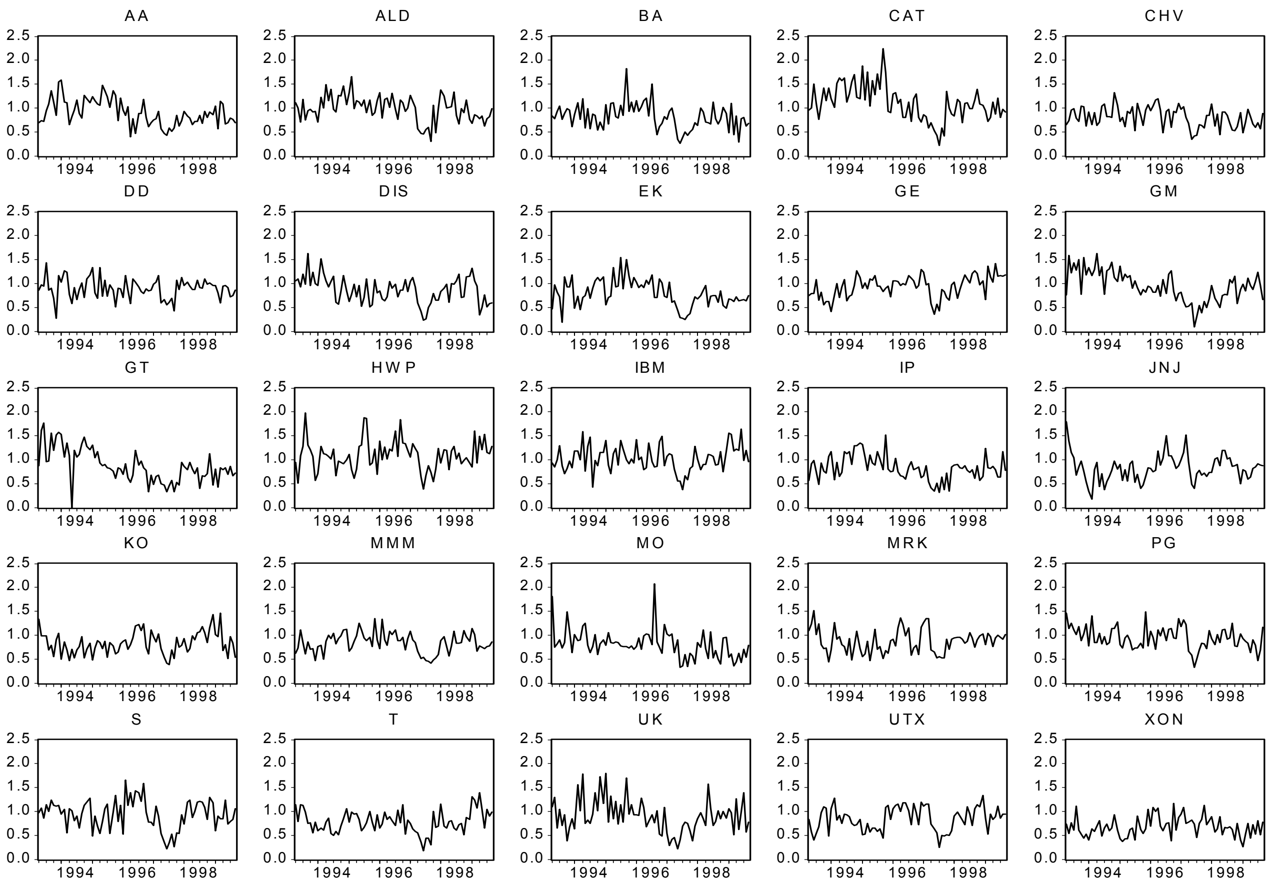
Notes: The figure shows the time series of monthly realized market variance. The sample covers the period from 1993.4 through 1999.9, for a total of 78 observations. We calculate the realized monthly market variances from fifteen-minute returns.

Figure 2H
Time Series Plots of Monthly Realized Covariances



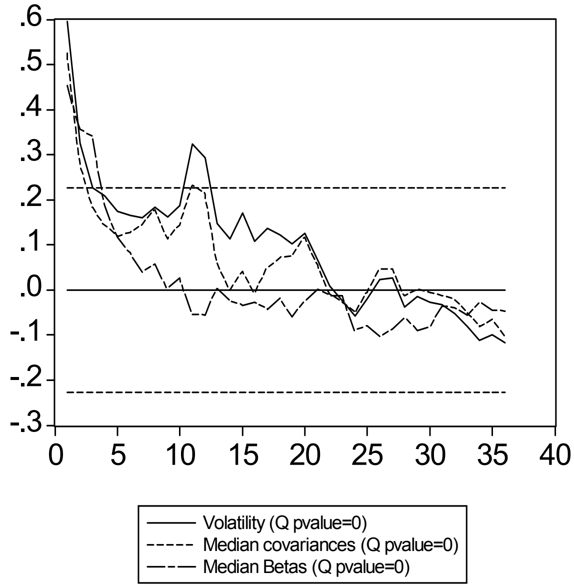
Notes: The Figure shows the time series of monthly realized covariances. The sample covers the period from 1993.4 through 1999.9, for a total of 78 observations. We calculate the realized monthly covariances from fifteen-minute returns.

Figure 3H
Time Series Plots of Monthly Realized Betas



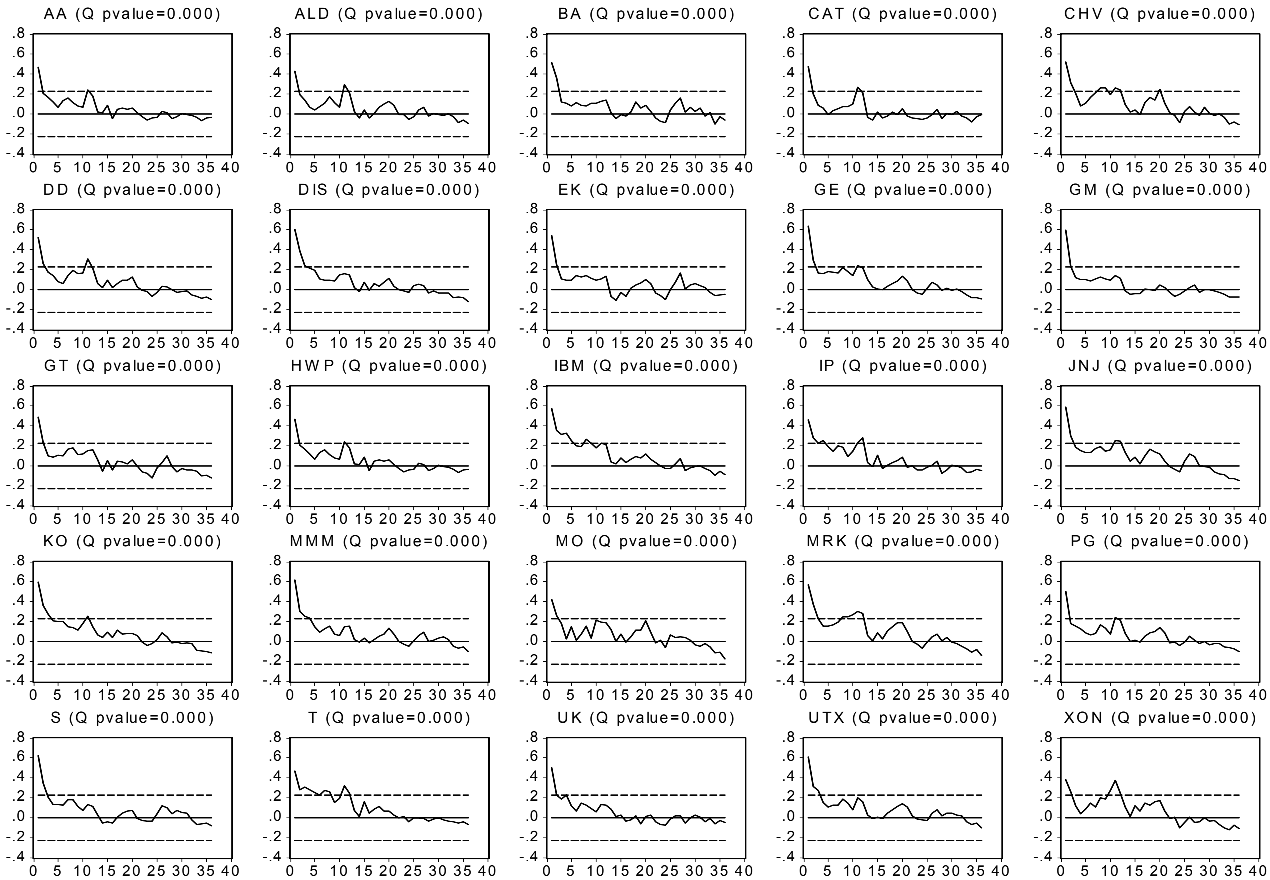
Notes: The Figure shows the time series of monthly realized betas. The sample covers the period from 1993.4 through 1999.9, for a total of 78 observations. We calculate the realized monthly betas from fifteen-minute returns.

Figure 4H
Sample Autocorrelations of Monthly Realized Market Variance, Median Sample Autocorrelations of Monthly Realized Covariances and Median Sample Autocorrelations of Monthly Realized Betas



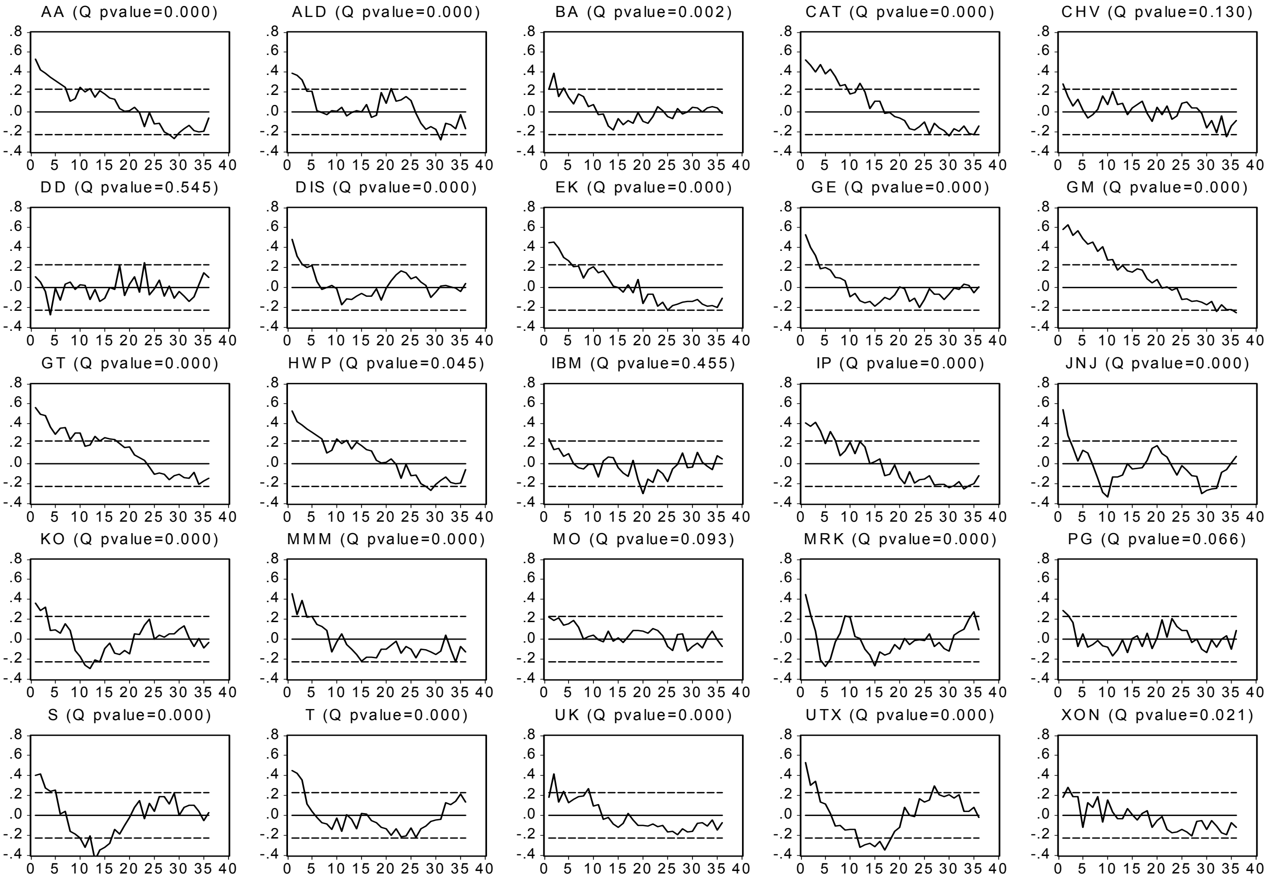
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized market variance, the medians across individual stocks of the first 36 sample autocorrelations of monthly realized covariances and the medians across individual stocks of the first 36 sample autocorrelations of monthly realized betas. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized variance, covariances and betas from fifteen-minute returns.

Figure 5H
Sample Autocorrelations of Monthly Realized Covariances



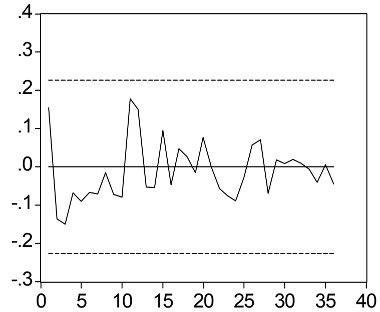
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized covariances. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized covariances from fifteen-minute returns.

Figure 6H
Sample Autocorrelations of Monthly Realized Betas



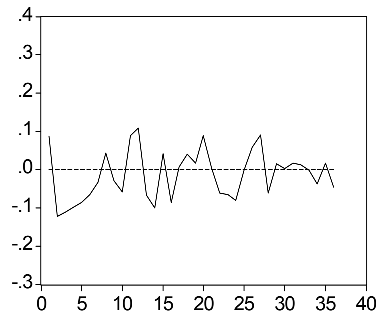
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized betas. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. *Q* denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized betas from fifteen-minute returns.

Figure 7H
Sample Autocorrelations of Monthly Realized Market Variance Prefiltered by $(1 - L)^{.42}$



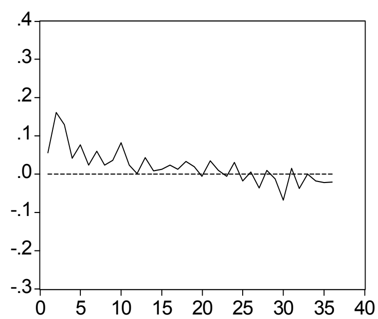
Q pvalue=0.337

Median Sample Autocorrelations of Monthly Realized Covariances Prefiltered by $(1 - L)^{.42}$



Q pvalue=0.478

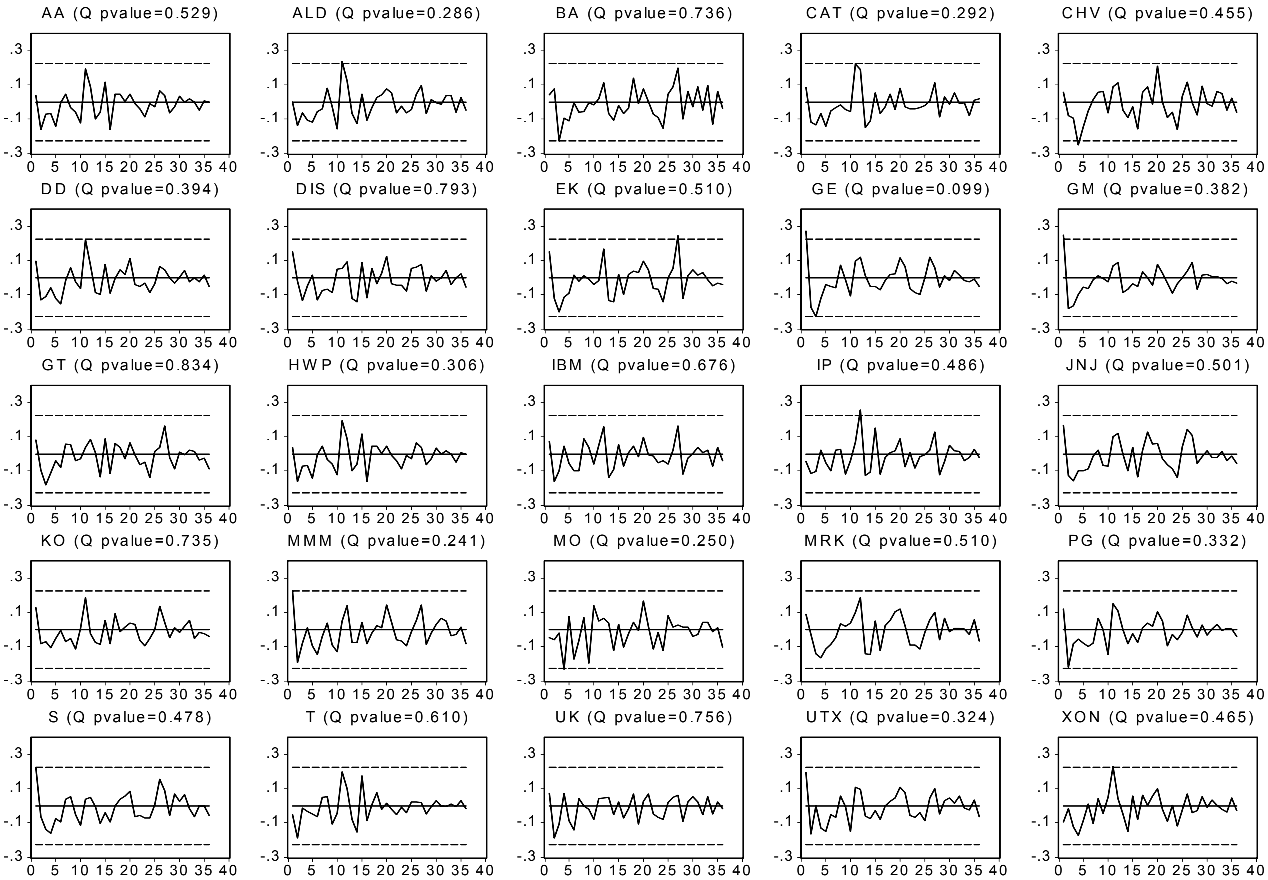
Median Sample Autocorrelations of Monthly Realized Betas Prefiltered by $(1 - L)^{.42}$



Q pvalue=0.252

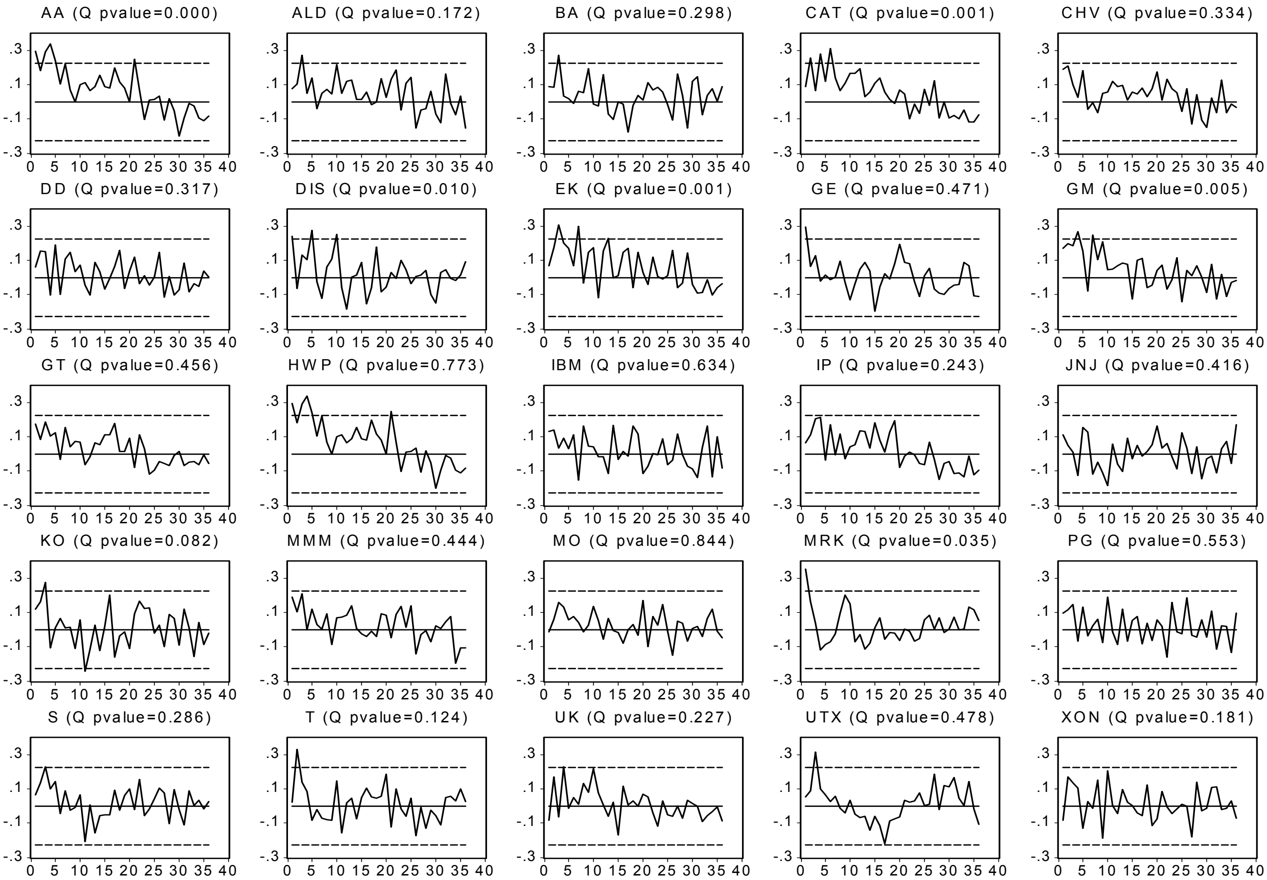
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized market variance, the medians across individual stocks of the first 36 sample autocorrelations of monthly realized covariances and the medians across individual stocks of the first 36 sample autocorrelations of monthly realized betas all prefiltered by $(1-L)^{.42}$. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the median of Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized variance from fifteen-minute returns.

Figure 8H
Sample Autocorrelations of Monthly Realized Covariances Prefiltered by (1 - L)$^{.42}$



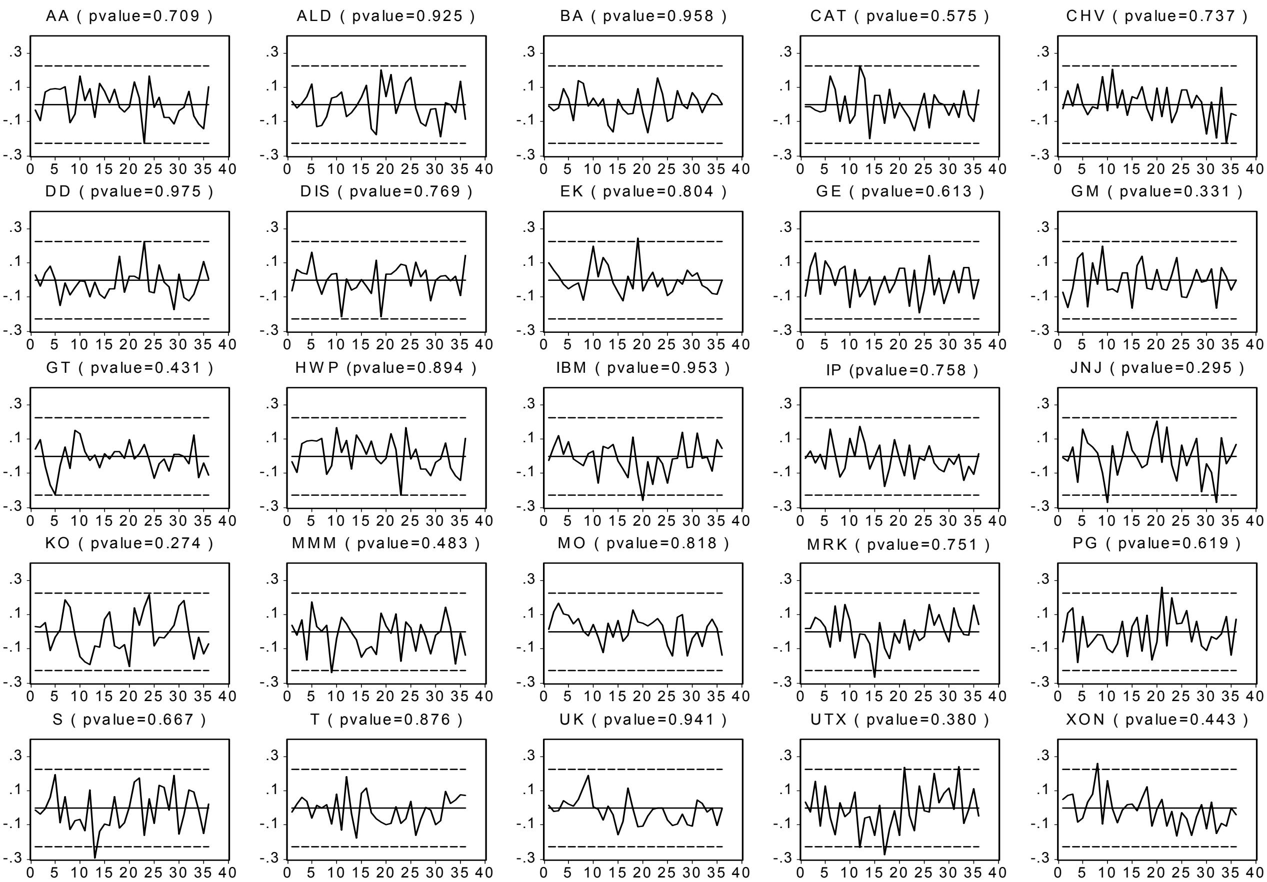
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized covariances prefiltered by $(1-L)^{.42}$. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized covariances from fifteen-minute returns.

Figure 9H
Sample Autocorrelations of Monthly Realized Betas Prefiltered by (1 - L)$^{.42}$



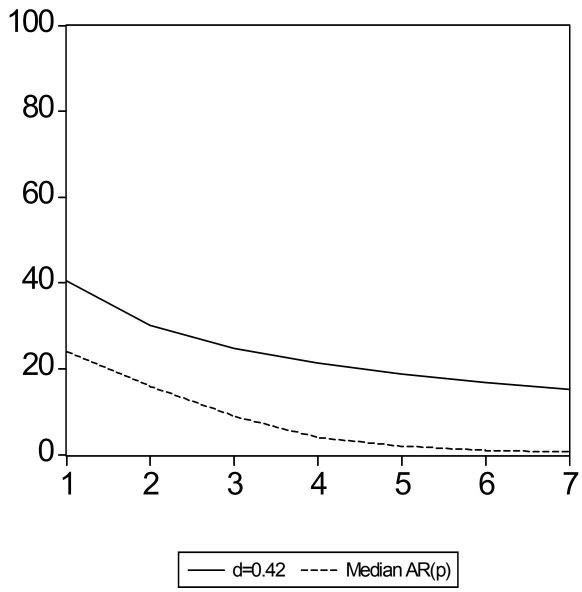
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized betas prefiltered by $(1-L)^{.42}$. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. Q denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized betas from fifteen-minute returns.

Figure 10H
Sample Autocorrelations of Monthly Realized Betas Prefiltered by Estimated *AR*(*p*) Lag Operator Polynomial



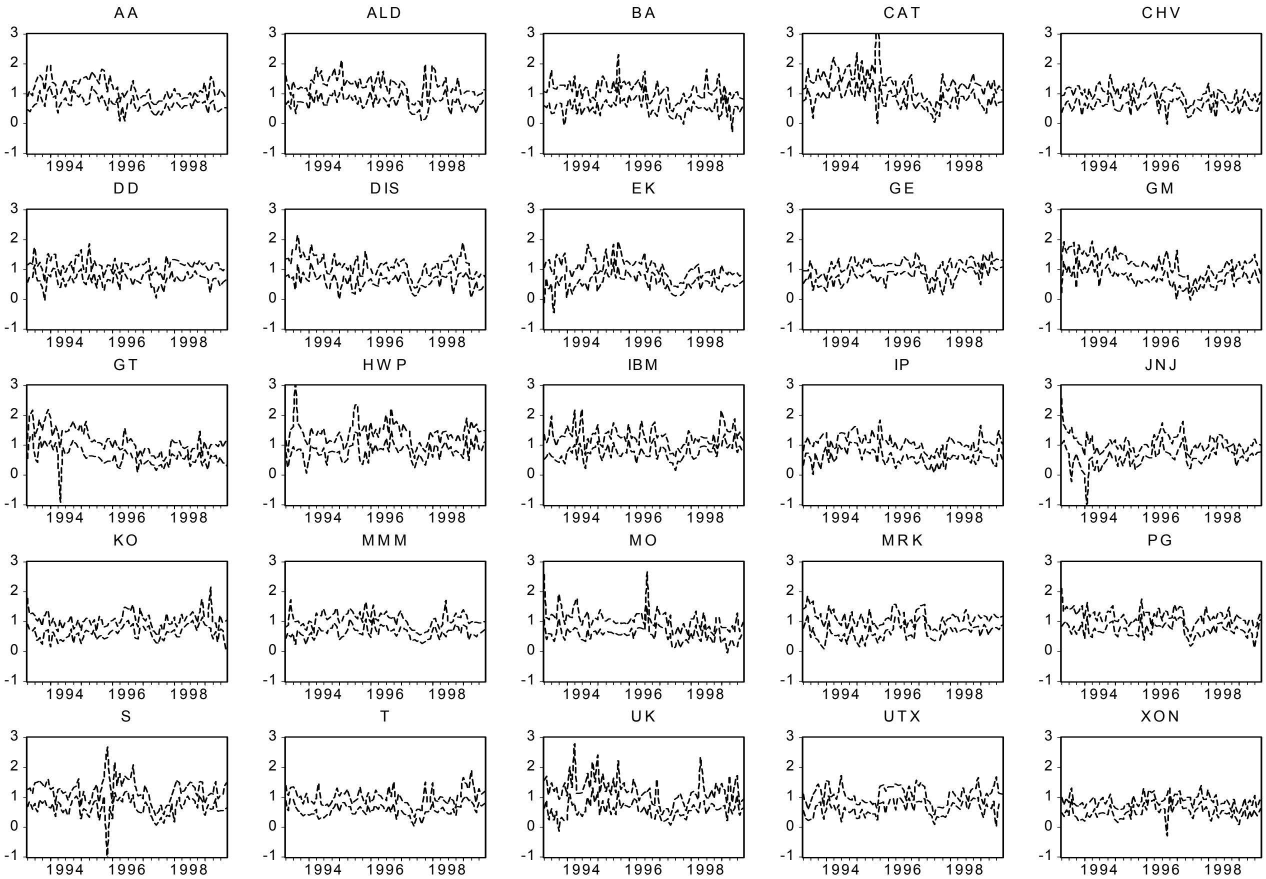
Notes: The figure shows the first 36 sample autocorrelations of the monthly realized betas prefiltered by $(1 - \hat{\phi}_1 L - \hat{\phi}_2 L^2 - ... - \hat{\phi}_p L^p)$, where $\hat{\phi}_1$, $\hat{\phi}_2$, ..., $\hat{\phi}_p$ are the least squares estimates of the parameters of *AR*(*p*) models fit to the realized betas, with *p* selected by the AIC. The dashed lines denote Bartlett's approximate 95 percent confidence band in the white noise case. *Q* denotes the Ljung-Box portmanteau statistic for up to twelfth-order autocorrelation. The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations. We calculate the monthly realized variance, covariances and betas from fifteen-minute returns.

Figure 11H
Predictability of Market Volatility, Individual Equity Covariances with the Market, and Betas



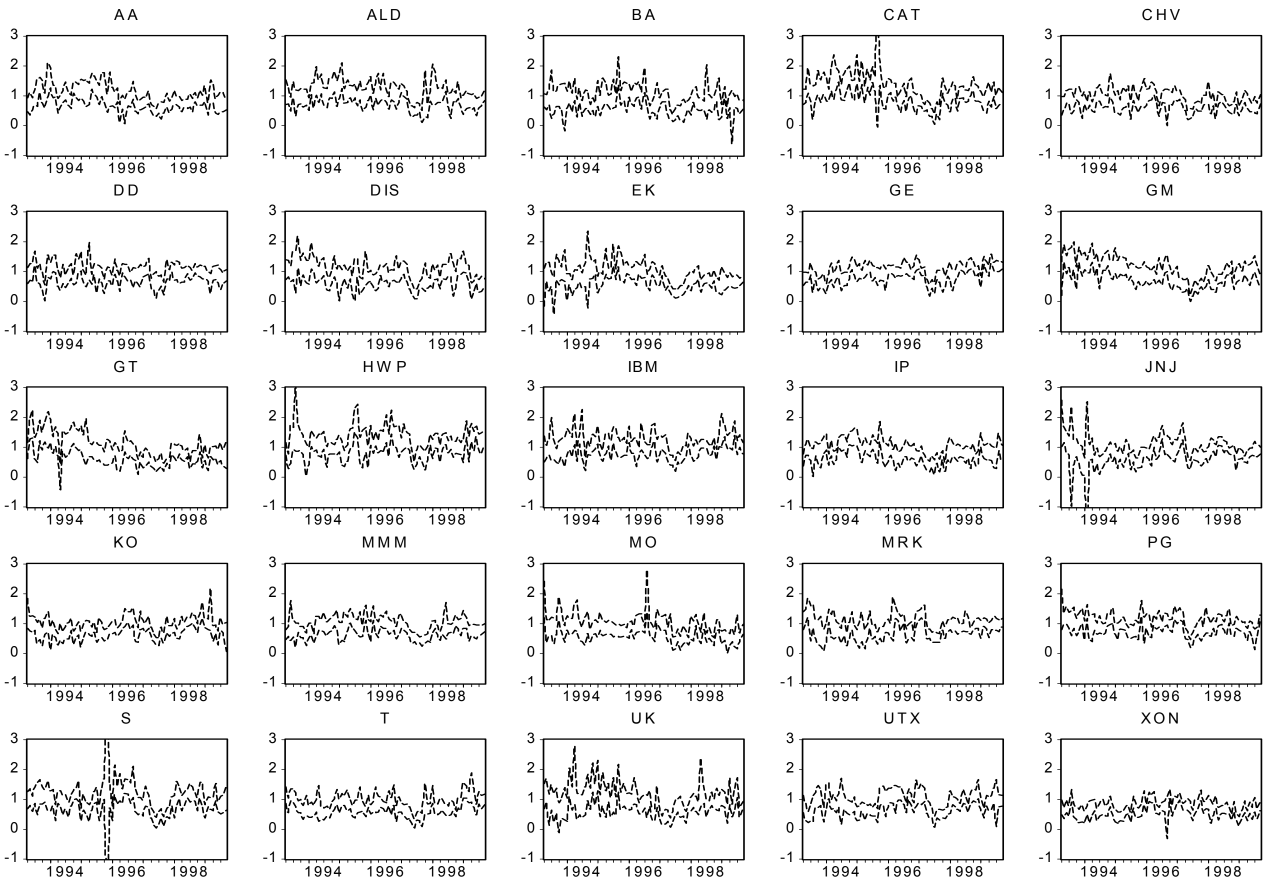
Notes: We define predictability as $P_j \ = \ 1 - var(e_{t+j,t})/var(e_{t+40,t})$, where $var(e_{t+j,t}) \ = \ \sigma^2 \sum_{i=0}^{j-1} b_i^2$, σ_t^2 is the variance of the innovation ε_t, and the b_i's are moving average coefficients; i.e., the Wold representation is $y_t \ = \ (1 + b_1 L + b_2 L^2 + b_2 L^3 + ...)\varepsilon_t$. We approximate the dynamics using a pure long-memory model, $(1 - L)^{.42} y_t \ = \ \varepsilon_t$, in which case $b_0 = 1$ and $b_i = (-1) b_{i-1} (d - i + 2)/(i - 1)$ and plot P_j for $j = 1,...,7$ (solid line). Moreover, because we take d=0.42 for market volatility and for all covariances with the market, all of their predictabilities are the same at all horizons. We approximate the dynamics using an *AR*(*p*) model, with the autoregressive lag order *p* determined by the AIC and plot the median of P_j for $j = 1,...,7$ among all 25 stocks (dashed line). The sample covers the period from 1993.4 through 1999.9 for a total of 78 observations.

Figure 12H
Ninety-Five Percent Confidence Intervals for Monthly Beta, Fifteen-Minute Sampling



Notes: The figure shows the time series of ninety-five percent confidence intervals for the underlying monthly integrated beta, calculated using the results of Barndorff-Nielsen and Shephard (2003). The sample covers the period from 1993.4 through 1999.9. We calculate the realized monthly betas from fifteen-minute returns.

Figure 13H
Ninety-Five Percent Confidence Intervals for Monthly Beta, Fifteen-Minute Sampling (Newey-West)



Notes: The figure shows the time series of Newey-West ninety-five percent confidence intervals for the underlying monthly integrated beta. The sample covers the period from 1993.4 through 1999.9. We calculate the realized monthly betas from fifteen-minute returns.

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